

**Problem 1: Cascading Behavior in Networks**

1.  $\{6, 7, 11\}$  are capable of causing a complete cascade of A for  $q = \frac{1}{2}$ .
2. Followings are different set of three initial adopters who are capable of complete cascade of A:
  - I.  $\{4, 9, 15\}$
  - II.  $\{10, 12, 16\}$
3. We know that,

$$\text{Cluster Density} = \frac{\text{Actual Connection}}{\text{Potential Connection}}$$

Followings are the clusters where cluster density is greater than  $\frac{1}{2}$

- I.  $\{1, 4, 5, 6, 10\} \rightarrow \text{Density} = \frac{3}{5} > \frac{1}{2}$
  - II.  $\{3, 7, 8, 9, 12\} \rightarrow \text{Density} = \frac{3}{5} > \frac{1}{2}$
  - III.  $\{11, 13, 14, 15, 16\} \rightarrow \text{Density} = \frac{3}{5} > \frac{1}{2}$
4. In this case,  $q = \frac{1}{2}$  that deduce  $(1 - q) = \frac{1}{2}$ . It implies that if there is any network that has cluster of density more than  $(1 - q)$ , then there will be no complete cascade.

From (3), we know that there are three clusters in the given network and each of them have cluster density more than  $q$  i.e.,  $\frac{1}{2}$ . Now if we choose a set of two initial adopter (for example:  $\{5, 14\}$ ), there will be one cluster remaining (in this case, cluster  $\{3, 7, 8, 9, 12\}$ ) that will have density  $> \frac{1}{2}$  and this cluster will stall the cascading. As a result, the network will fail to be complete cascade of A.

**Problem 2: Centrality and Clustering**

We know that a graph is known as complete graph if every node in the connected with each other i.e., for a graph with  $n$  nodes, each node relates to  $(n-1)$  nodes.

$$1. \text{ Degree Centrality} = \frac{n-1}{n-1} = 1$$

So normalized degree centrality of a complete graph is 1.

$$2. \text{ Closeness Centrality} = \frac{n-1}{n-1} = 1$$

So normalized closeness centrality of a complete graph is 1.

3. There is no via node because all nodes are connected to each other.

$$\textit{Betweenness} = \frac{0}{n - 1}$$

So normalized betweenness of a complete graph is 0.

### Problem 3: Following the Crowd

1. Individual 1 does not have any prior information that's why he will follow his own instinct and for individual 2, he has only one neighbor so he will follow his own judgement rather than being influenced. That's why, y the decision problems faced by individuals 1 and 2 are unchanged by this modification to the information network.
2. Individual 3 could infer that action of individual 2 is same as what 2 observes.
3. According to the given decision model, individual 2 will make decision based on his observation regardless of the action of 1<sup>st</sup> individual and any person can only observe the action of his immediate previous person. That's why. Individual 3 can not infer anything about 1's signal from 2's action.
4. If 3 observes a high signal and he knows that 2 Accepted, then he will also Accept.  
If 3's signal was low and 2 Accepted, then he will Reject.
5. Based on the given network, information cascade is not possible because here every individual is making decision based on their own observation, and they can only see their immediate previous person's action. Since information is not passing throughout the network, information cascading will fail in this case.