1 Cascading Behavior in Networks (32%)

Consider the model we studied for diffusion of a new behavior in social networks. Recall that we have a network, a behavior B that everyone starts with, and a threshold q for switching to a new behavior A; any node will switch to A if at least a q fraction of its neighbors have adopted A. Suppose that $q = 1/2$, i.e. any node will switch to A if at least half of its neighbors have adopted A. In the network of Figure 1:

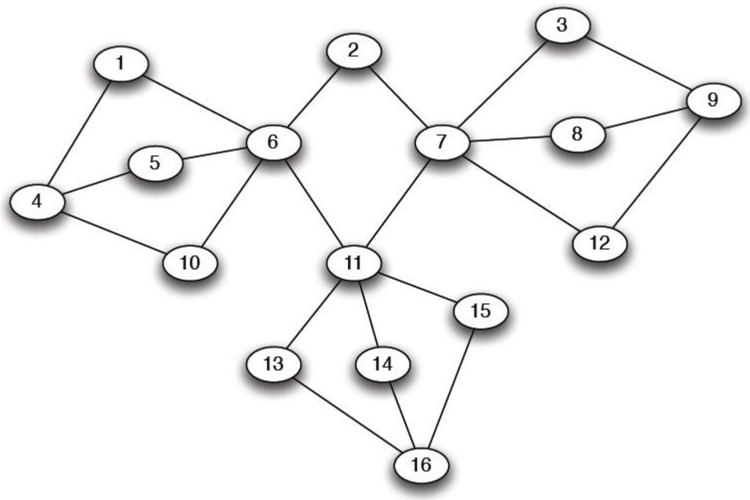

Figure 1: Network for cascading behavior analysis.

1. Find a set of three nodes in the network with the property that if they act as the three initial adopters of A, then A will spread to all nodes. (In other words, find three nodes who are capable of causing a complete cascade of A.)
2. Is the set of three nodes you found in (1) the only set of three initial adopters capable of causing a cascade of A, or can you find a different set of three initial adopters who could also cause a cascade of A?
3. Find three clusters in the network, each of density greater than 1/2, with the property that no node belongs to more than one of these clusters.
4. How does your answer to (3) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a complete cascade of A?

2 Centrality & Clustering (28%)

Consider a complete graph with n nodes.

1. Compute normalized degree centrality for each node as a function of n.
2. Compute normalized closeness centrality for each node as a function of n.
3. Compute normalized betweenness centrality for each node as a function of n.

3 Following the Crowd (40%)

In this question, we will ask whether an information cascade can occur if each individual sees only the action of his immediate neighbor rather than the actions of all those who have chosen previously. Let's keep the same setup as in the Information Cascades model discussed in the class, except than when individual *i* chooses he observes only his own signal and the action of individual *i - 1*.

1. Briefly explain why the decision problems faced by individuals 1 and 2 are unchanged by this modification to the information network.
2. Individual 3 observes the action of individual 2, but not the action of individual 1. What can 3 infer about 2's signal from 2's action?
3. Can 3 infer anything about 1's signal from 2's action? Briefly explain.
4. What should 3 do if he observes a high signal and he knows that 2 Accepted? What if 3's signal was low and 2 Accepted?
5. Do you think that a cascade can form in this world? Briefly explain why or why not (a formal proof is not necessary).