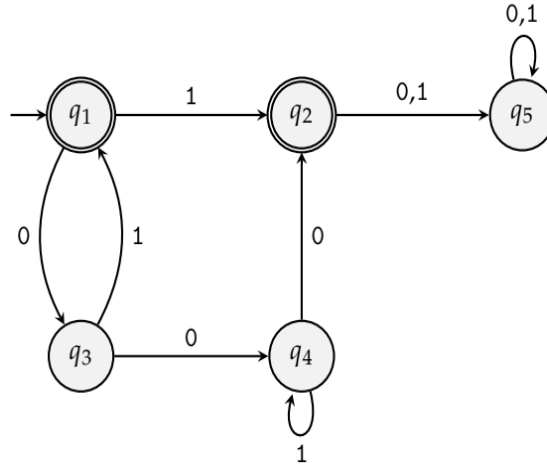


There are six problems in total. You must solve the first five, and problem 6 is optional.

Problem 1 (CO2): Converting Finite Automata to Regular Expressions (10 points)

**Convert** the following finite automata into an equivalent regular expression using the state elimination method. You must eliminate  $q_5$  first, then  $q_4$ , then  $q_3$ , then  $q_1$ , and finally  $q_2$ . Show each step of the process.



Problem 2 (CO5): Pumping Lemma (5 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following language over  $\Sigma$ .

$$L = \{w = 0^n 1^{n+1} \text{ where } n \text{ is 1 more than multiple of 4}\}$$

Use the pumping lemma to **demonstrate** that  $L$  is not a regular language.

Problem 3 (CO3): Designing Context-Free Grammars (15 points)

$$L_1 = \{w \in \{a, b\}^* : w \text{ is a odd length palindrome}\}$$

$$L_2 = \{w \in \{a, b\}^* : \text{every second letter of } w \text{ is } b\}$$

$$L_3 = \{w \in \{0, 1\}^* : w \text{ contains exactly two 0s}\}$$

$$L_4 = \{w_1 \# w_2 : w_1 \in L_2, w_2 \in \{0, 1\}^* \text{ and } |w_1| = |w_2|\}$$

$$L_5 = \{w_1 \# w_2 : w_1 \in L_2, w_2 \in L_3 \text{ and } |w_1| = |w_2|\}$$

Now solve the following problems.

- Give** a context-free grammar for the language  $L_1$ . (3 points)
- Give** a context-free grammar for the language  $L_2$ . (3 points)
- Give** a context-free grammar for the language  $L_3$ . (3 points)
- Find** all strings  $w \in L_4$  such that  $w$  ends with  $\#1011$  and has a length of 9. (1 point)
- Give** a context-free grammar for the language  $L_4$ . [Recall: For a string  $w$ ,  $|w|$  denotes the length of  $w$ .] (3 points)
- Give** a context-free grammar for the language  $L_5$ . (2 points)

Problem 4 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following grammar over  $\Sigma$ .

$$\begin{aligned} S &\rightarrow 0S1 \mid 1S0 \mid A \\ A &\rightarrow 00A \mid 01A \mid 10A \mid 11A \mid \varepsilon \end{aligned}$$

- (a) **Give** a leftmost derivation for the string 01011001. (3 points)
- (b) **Draw** the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the given string in (a). (4 points)
- (d) How many four-letter strings will have exactly one parse tree in the given grammar? (1 point)

Problem 5 (CO3): Designing Pushdown Automata (10 points)

$$\begin{aligned} L_1 &= \{w \in \{0, 1\}^* : w \text{ contains at most two } 0\text{s}\} \\ L_2 &= \{w \in \{0, 1\}^* : w = 0^{2n}1^{3n} \text{ where } n \geq 0\} \\ L_3 &= \{w\#x : w, x \in \{0, 1\}^* \text{ and } w^R \text{ is a substring of } x\} \end{aligned}$$

- (a) **Give** the state diagram of a pushdown automaton that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram of a pushdown automaton that recognizes  $L_2$ . (3 points)
- (c) **Give** the state diagram of a pushdown automaton that recognizes  $L_3$ . [Recall: For a string  $w$ ,  $w^R$  denotes  $w$  in reverse order.] (4 points)

Problem 6 (Bonus): Pumping Lemma (4 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following language over  $\Sigma$ .

$$L = \{w \text{ is not a palindrome.}\}$$

Use the pumping lemma to **demonstrate** that  $L$  is not a regular language.

After you are done with the test, please indicate where you stand on the smiley face spectrum.

