CSE331

Spring 2025

Faculty: Kabbya Kantam Patwary

Lecture Updates:

1. Lecture 01 - DFA
2. Lecture 02 - DFA
3. Lecture 03 - DFA
4. Lecture 04 - Regular Operations
5. Lecture 05 - NFA
6. Regular Expressions Intro - Resources and How to Practice
7. Lecture 06 - Regular Expressions
8. Lecture 07 - Regular Expressions
9. Lecture 08 - Quiz01 and Regular Expressions
10. Lecture 09 - Converting NFA to DFA
11. Lecture 10 - Regular Expressions to NFA
12. Lecture 11 - DFA to Regular Expressions
13. Lecture 12 - Quiz02 and Lecture 9 Review
14. Midterm Syllabus
15. Lecture 13 - Context Free Grammar (CFG-1)
16. Lecture 14 - Context Free Grammar (CFG-2)
17. Lecture 15.1 - Context Free Grammar (CFG-3)
18. Lecture 15.2 - Derivation, Parse Tree, and Ambiguity
19. Lecture 16 - Pumping Lemma-1
20. Lecture 17 - Pushdown Automata (PDA-1)
21. Lecture 18 - Pushdown Automata (PDA-2)
22. Lecture 19 - Turing Machine-1
23. Lecture 20 - Turing Machine-2

Important Links:

- CSE331 All Sections Drive Folder Link Spring 2025: Here
- CSE331 Course Materials Link: Here
- Previous Semester's Midterm Questions: Here

Lecture 01

Date: Feb 9, 2025 Feb 10, 2025

1. We tried to understand what we would learn in this course from a bird's-eye view.

2. Learned the definition of
    a. Symbol
    b. Alphabet
    c. String, Length of String, Empty String, and
    d. Language (Finite, Non-Finite)
        i. Finite Language: the set of all strings of length two over the $\Sigma = \{0,1\}$
        ii. Infinite Language: the set of all strings that contain at least two 0s where $\Sigma = \{0,1\}$, the set of all strings that don't contain 1 where $\Sigma = \{0,1\}$
        iii. Is there any difference between {} and {ε}?

3. Discussed two types of computational problems.
    a. Decision-making problem (Yes/No Problem)
    b. Optimization problem

4. We tried to get an idea of how DFA works.

5. Solved a problem on DFA:

    a. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains **at least** two 1s.}
        i. Test Strings: 11, 01, 0100, 01001, 0000, 010110011, ε *[These test strings won't be provided in the exams]*

6. DFA (Deterministic Finite Automata):
    a. What do you understand by Deterministic?
    b. If a DFA has M states and Σ has N elements, then how many transitions can the DFA have?
    c. Why is DFA called Finite?

7. Practice problem:

    a. Construct a DFA of the language, L = { **w ∈ {0,1,2}***: w contains at least two 1s.}
        i. Test Strings: Make your own

    b. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains **exactly** two 1s.}

i. Test Strings: 11, 111, 01100, 0111010 *[These test strings won't be provided in the exams]*

c. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains **at most** two 1s.}

i. Test Strings: ε, 0, 00, 01, 11, 01100, 00011101 *[These test strings won't be provided in the exams]*

d. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is at least two.}

i. Test Strings: Make your own

e. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is exactly two.}

i. Test Strings: Make your own

f. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is at most two.}

i. Test Strings: Make your own

Note:

I. Please try to have some idea on (2), (5) and (6) before attending the next class, otherwise, you may face difficulty in the next class.

II. The practice problems are for your Brainstorming, there is no need to submit them. It is highly recommended that you solve the given practice problems on your own.

Lecture 02

Date: Feb 11, 2025

1. Review of previous class:

   a. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains at least two 1s.}

b. Construct a DFA of the language, L = { w ∈ {0,1,#}*: w contains at least two 1s.}

2.

a. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains exactly two 1s.}

b. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains at most two 1s.}

3.

a. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is at least three.}

b. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is exactly three.}

c. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is at most three.}

4.

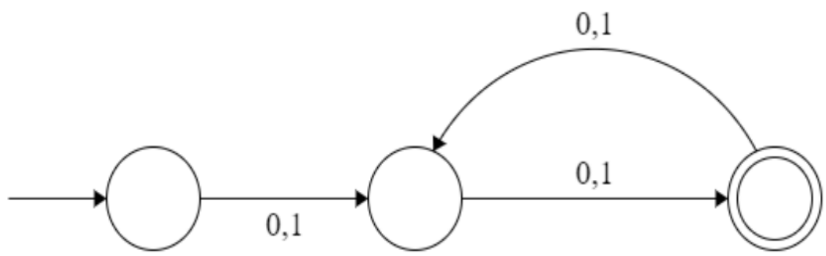
a. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is Even.}

b. Or(a), Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is multiple of two.}

c. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is multiple of three.}

d. Construct a DFA of the language, L = { w ∈ {0,1}*: the length of w is one more than multiple of three.}

6. Your friend has constructed a DFA for **4(b)**. Is the solution correct?



L = { w ∈ {0,1}*: the length of w is multiple of two.}

Note, for a regular language L, the DFA will be correct only if it accepts all the strings, w ∈ L, and doesn't accept any strings, w ∉ L.

7. Practice problem:

a. Construct a DFA of the language, L = { w ∈ {0,1}*: the count of 1s in w is one more than multiple of three.}

b. Construct a DFA of the language, L = { w ∈ {a,b,#}*: the count of "a" in w is Odd.}
c. Construct a DFA of the language, L = { w ∈ {0,1}*: w starts with "0".}
d. Construct a DFA of the language, L = { w ∈ {0,1}*: w starts with "001".}
e. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains "00".}
f. Construct a DFA of the language, L = { w ∈ {0,1}*: w ends with "0".}

Note:

I. If you are having trouble understanding the topics / have missed the classes, please go through Mursalin Sir's first two lecture videos on DFA from Here

   A. 1. DFA Part A Feb 09

   B. 2. DFA Part B Feb 14

II. Don't worry if you find it difficult to solve the practice problems. Spending some time thinking about the solution would be very appreciable.

Lecture 03

Date: Feb 16, 2025 Feb 17, 2025

1.

a. Construct a DFA of the language, L = { w ∈ {a,b}*: the count of 'a' in w is multiple of three.}

2.

a. Construct a DFA of the language, L = { w ∈ {0,1}*: w starts with "0".}
b. Construct a DFA of the language, L = { w ∈ {0,1}*: w starts with "001".}

3.

a. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains "00".}
b. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains "01".}
c. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains "1011".}
d. Construct a DFA of the language, L = { w ∈ {0,1}*: w doesn't contain "1011".}

4.

a. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains even numbers of 0s and odd numbers of 1s}

   i. Solving Directly

   ii. Solving Using Cross Product Method

In Cross Product Method:

- What will be the starting state?
- How to choose accepted states in the Cross Product Method if the question mentions 'And'?
- How to choose accepted states in the Cross Product Method if the question mentions 'Or'?
- If language L1 has m states and language L2 has n states then constructing L using Cross Product Method will have how many states?

4. Practice Problem:

a. Construct a DFA of the language, L = { w ∈ {0,1}*: w ends with 01.}

b. Construct a DFA of the language, L = {w ∈ {0,1}*: w ends with 11 and the length of w is odd.}

[Medium Difficulty Level Problem]

c. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains at least two "00" as a substring.}

d. Construct a DFA of the language, L = { w ∈ {0,1}*: w contains at most two "00" as a substring.}

[Try to answer: How many "00" can you find in the string "000"?]

Lecture 04

Date: Feb 18, 2025 Feb 19, 2025

a. Construct a DFA of the language, L = { $w \in \{0,1\}^*$: w ends with 01.}
   i. Practice:
      i. Construct a DFA of the language, L = { $w \in \{a,b\}^*$: w ends with aab.}
      ii. Construct a DFA of the language, L = { $w \in \{a,b\}^*$: w ends with aba.}

b. What is Regular Language?

c. Regular Operations on Regular Languages: Union, Intersection, Concatenation, Kleene Closure/ Star

d. If A = {Good, Bad}, B = {Pen, Car}, then find $A \cup B$, $A \cap B$, $A \circ B$, A^*.

e. CLOSURE UNDER THE REGULAR OPERATIONS:

   i. Regular languages are closed under intersection, i.e., if L1 and L2 are regular then $L1 \cap L2$ is also regular.
   ii. Regular Languages are closed under union, i.e., if L1 and L2 are regular then $L1 \cup L2$ is also regular.
   iii. The class of regular languages is closed under concatenation, i.e., if L1 and L2 are regular then $L1 \circ L2$ is also regular.
   iv. If L is regular, then L^* is regular.

f. Cross-product method (**Related to D(i) and D(ii)**)

   i. L1 = {$w \in \{0,1\}^*$: w ends with 01.}
      L2 = {$w \in \{0,1\}^*$: length of w is odd.}
      $L = L1 \cup L2$
      Proof L is regular using DFA.

   ii. L1 = {$w \in \{0,1\}^*$: w ends with 01.}
      L2 = {$w \in \{0,1\}^*$: length of w is odd.}
      $L = L1 \cap L2$
      Proof L is regular using DFA.

g. Introduction to NFA. (**Related to D(iii)**)

   i. L1 = {$w \in \{0,1\}^*$: w ends with 01.}
      L2 = {$w \in \{0,1\}^*$: length of w is odd.}
      $L = L1 \circ L2$

Proof L is regular using NFA.

Lecture 05

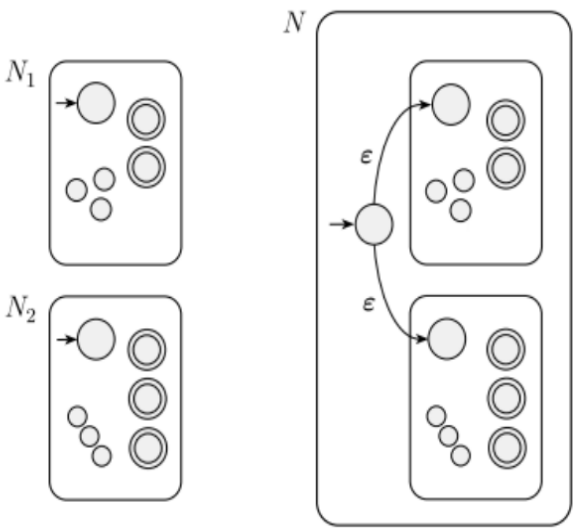
Date: Feb 24, 2025 Mar 2, 2025

a. Introduction to NFA.
    i. How NFA works.
    ii. We tried to understand the non-determinism aspect of NFA.

b. Regular Languages are closed under union, i.e., if L1 and L2 are regular then L = L1 ∪ L2 is also regular. Prove this by constructing the following regular language L using NFA.
    i. L1 = {w ∈ {0,1}*: w ends with 11.}
       L2 = {w ∈ {0,1}*: length of w is odd.}

       L = L1 ∪ L2

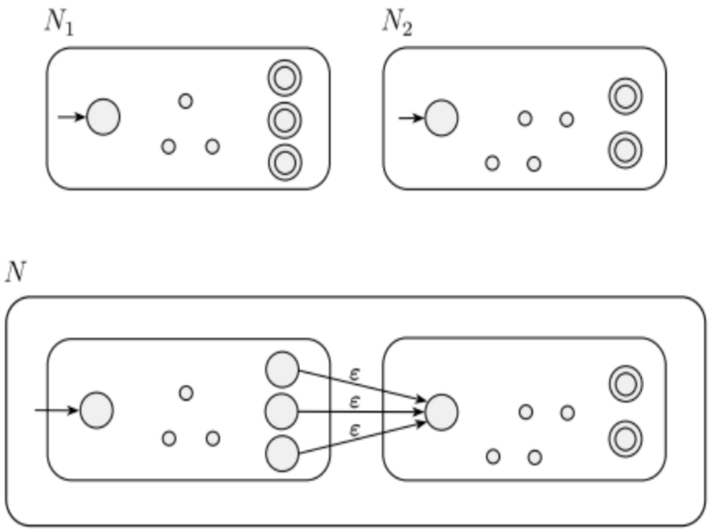
       Construct L by NFA.



c. The class of regular languages is closed under concatenation. For arbitrary regular languages, L1 and L2 then L = L1.L2 is a regular language. Prove this by constructing the following regular language L using NFA.
    i. L1 = {w ∈ {0,1}*: w ends with 11.}
       L2 = {w ∈ {0,1}*: the length of w is odd.}

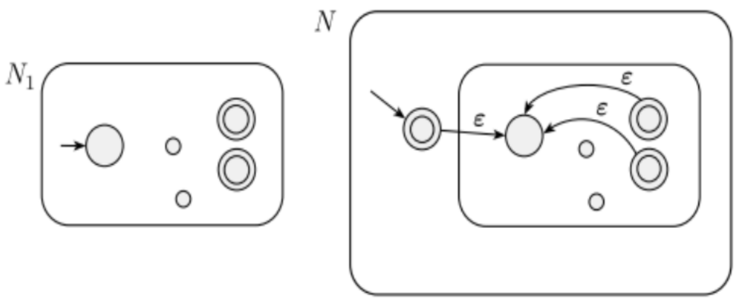
       L = L1.L2

       Construct L by NFA.



d. The class of regular languages is closed under the star operation. If L is regular then L* is also a regular language.
   a. L1 = {01}
      Construct L1* by NFA.

   b. L1 = {01, 001}
      Construct L1* by NFA.



e. Construct NFA
   a. Construct an **NFA** of the language, L = { w ∈ {0,1}*: w contains "101".}
   b. Construct an NFA of the language, L = { w ∈ {0,1}*: w ends with "0".}
   c. Construct an NFA of the language, L = { w ∈ {0,1}*: w contains even numbers of 0s.}

Practice on NFA construction:

a. Construct an **NFA** of the language, L = { w ∈ {0,1}*: w ends with "001".}
b. Construct an NFA of the language, L = { w ∈ {0,1}*: w starts with "001".}
c. Construct an NFA of the language, L = { w ∈ {a,b}*: the third last symbol in w is "a".}

f. Regular Languages are closed under intersection, i.e., if L1 and L2 are regular then L1 ∩ L2 is also regular. Proving this using NFA could be very complex. However, since we know DFA is also an NFA, we can use the DFA way (cross-product) to prove this situation as well.

Regular Expressions Intro:

Practice Materials on Regular Expressions:

- Practice Sheet- Link
- Previous semester assignment on R.E. - Link
- Class Lectures: Part_A, Part_B, Part C

How to practice Regular Expressions:

a) Don't see the solution at the beginning
b) Write down some strings that should be accepted. Be careful about the edge/corner cases
c) Write down the regular expression
d) Check if your regular expression generates the strings you have listed
e) Check your solutions from your class notebook
    i) If correct - Move to the next question
    ii) If wrong
        1) Find out the strings that your strings don't generate and rewrite your regular expression
        2) If your regular expression generates all the strings then your solution also might be the correct one, since there could be multiple solutions for a language

Lecture 06

Date: Feb 23, 2025 Feb 26, 2025

1. Extended definition of Regular Language.

2. We tried to understand that DFAs and NFAs are recognizers, while Regular Expressions are generators.

3. Write the strings that the following Regular Expressions will generate:

1. (0+1)*
2. 0+1
3. (00)*
4. 0*1*
5. (0*1*)*
6. (01*)*
7. 0*1* + (ab)*
8. (0*1* + (ab)*)*
9. a (0+1) b
10. a (0+1)* b
11. (a (0+1)* b)*
12. ab + 1*01*

4. Is (0+1)* and (0*1*)* the same? Justify your answer.

5. Write the shortest string that will be generated by this regular expression –
   a. aa*b (0+1) + 1*0 (baa*)
   b. (0+1) 1 (00 + 0(11)* + 100) + 1* (00 (010 + 01*0)) 01

6. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w contains "101" as a substring.}

7. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w starts with "101".}

8. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w ends with "101".}

9. L = { w ∈ $\{0,1\}^*$: w starts with "0".}
   Construct a Regular Expression for $\overline{L}$.

10.
   a. Construct a Regular Expression that generates the language L1 = { w ∈ $\{0,1\}^*$: w contains "00" or "11".}

   b. Construct a Regular Expression that generates the language L2 = { w ∈ $\{0,1\}^*$: w contains "00" and "11".} [Practice]

   c. Construct a regular expression for $\overline{L1}$ [Practice]

11.
   a. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w contains at least two 1s.}

   b. For the previous question, your three friends write three different solutions.
      (0+1)* 1 (0+1)* 1 (0+1)*, (0+1)* 1 0* 1 0* and 0* 1 0* 1 (0+1)*
      Find the correct solution.

12. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: w contains exactly two 1s.}

13. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: w contains at most two 1s.}

Lecture 07

Date: Feb 28, 2025 Feb 3, 2025

1.
   a. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: w contains at most two 1s.}
   b. Your friend claims that one of the correct regular expressions for the previous question is 0* (0+1) 0* (0+1) 0* + ε. Do you agree or disagree?

2.
   a. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The length of w is even or multiple of 2.}
   b. Your friend claims that one of the correct regular expressions for the previous question is (00)* + (01)* + (10)* + (11)*. Do you agree or disagree?

3. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The length of w is odd.}

4. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The length of w is multiple of 3.}

5. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The length of w is two more than multiple of four..}

6. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The length of w is **not** multiple of 3.}

7. Construct a Regular Expression that generates the Language L = { w ∈ $\{0,1\}^*$: The number of 1s in w is a multiple of 3.} [Practice]

8. Construct a Regular Expression that generates the language L = { w ∈ $\{a,b\}^*$: w starts and ends with different symbols.}

9. Construct a Regular Expression that generates the language L = { w ∈ $\{a,b\}^*$: w starts and ends with the same symbols.}

10. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w doesn't end with 01.}

Lecture 08

Date: Mar 4, 2025 Feb 5, 2025

1. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: 0's and 1's alternates in w.}

2.
   a. L = { w ∈ $\{0,1\}^*$: w contains "00" or "11".}
      Construct a regular expression for $\overline{L}$

   b. L = { w ∈ $\{0,1\}^*$: w contains "01" and "10".}
      Construct a regular expression for $\overline{L}$

3. L = { w ∈ $\{0,1\}^*$: w contains 0 in every third position.}
   a. Construct a regular expression for L
   b. Construct a regular expression for $\overline{L}$

4. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w doesn't contain 00.}

5. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w doesn't contain 000.} [Practice]

6. Construct a Regular Expression that generates the language L = { w ∈ $\{0,1\}^*$: w doesn't contain 10.}

Lecture 09

Mar 11, 2025 Mar 12, 2025

NFA to DFA using Subset Construction

- Lecture video:

    - Part A: Link (Why the method is called Subset Construction)
    - Part B: Link (Example)
- Sample Questions on NFA - Fall 2024 Midterm Paper

Lecture 10
Date: Mar 9, 2025 Feb 7, 2025

Regular Expressions to NFA

- Practice from Previous Midterm Questions
- Lecture Video: Link

Lecture 11
Date: Mar 10, 2025 Feb 16, 2025

DFA to Regular Expression

- Lecture Video on DFA to RE: Link
- Practice Sheet: Link
- Sample Solutions: Link
- Practice from Previous Midterm Questions

Midterm Syllabus:

- Alphabets, Strings, and Languages
- DFAs and Regular Languages
- The Regular Operations (Union, Concatenation, and Kleene star)
- Nondeterminism and NFAs
- Equivalence between NFAs and DFAs (Subset Construction)
- Closure under the Regular Operations
- Regular Expressions
- Equivalence between Regular Expressions and Finite Automata
    - Converting Regular Expressions to NFAs
    - Converting DFAs to Regular Expressions

Best of luck with your midterm exam.

Lecture 13

Date: Apr 8, 2025 Apr 9, 2025

Lecture 14

Date: Apr 13, 2025 Apr 16, 2025

Lecture 15.1

Date: Apr 15, 2025 Apr 21, 2025

Context Free Grammar

Study Materials:

- Handnotes: Context Free Grammar (CFG)
    - First, go through the handnotes on CFG-1,2,3; the lecture videos follow the same notes.
    - After you complete the first three handnotes, check the handnote - CFG_updated [will be updated]
- Lecturer Videos: Link-1, Link-2, Link-3 (Till 41:20)
- Previous Semester Assignment: Context Free Grammar_Central Assignment_Summer24

Lecture 15.2

Derivation, Parse Tree, and Ambiguity

Study Materials:

- Handnotes: Derivation, Parse Tree, Ambiguity
- Lecture Vides: Link-3 (From 41:20)

Lecture 16

Date: Apr 20, 2025 Apr 23, 2025

Pumping Lemma

1. Study Materials:
    a. Mursalin Sir Class Lecture: Link
    b. A few more Lecture videos: Link-1, Link-2
    c. **Previous Semester Assignment with Solution: Link**

2. Identify regular and non-regular languages from the following languages.

a. L = { w ∈ {0,1}*: 0^n1^n, n ≥ 0}

b. L = { w ∈ {0,1}*: 0^n1^n, 0 ≤ n ≤ 3}

c. L = { w ∈ {0,1}*: 0^n1^m, n,m ≥ 0}

d. L = { w ∈ {0,1}*: w contains equal numbers of 0s and 1s}

e. L = { w ∈ {0,1}*: w contains equal numbers of 0s and 1s}

3. *From the book Introduction to the Theory of Computation: Sipser*

THE PUMPING LEMMA FOR REGULAR LANGUAGES

Our technique for proving nonregularity stems from a theorem about regular languages, traditionally called the ***pumping lemma***. This theorem states that all regular languages have a special property. If we can show that a language does not have this property, we are guaranteed that it is not regular. The property states that all strings in the language can be "pumped" if they are at least as long as a certain special value, called the ***pumping length***. That means each such string contains a section that can be repeated any number of times with the resulting string remaining in the language.

4. *From the book Introduction to the Theory of Computation: Sipser*

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,

2. $|y| > 0$, and

3. $|xy| \leq p$.

Recall the notation where $|s|$ represents the length of string s, y^i means that i copies of y are concatenated together, and y^0 equals ε.

When s is divided into xyz, either x or z may be ε, but condition 2 says that $y \neq \varepsilon$. Observe that without condition 2 the theorem would be trivially true. Condition 3 states that the pieces x and y together have length at most p. It is an extra technical condition that we occasionally find useful when proving certain languages to be nonregular. See Example 1.74 for an application of condition 3.

5. Summarization

a. Property of a regular language: ∀ long strings w, there exists ∃ xyz with |xy|≤p and y≠ϵ such that ∀ i≥0, xy^iz∈L

b. To prove a language is nonregular, we have to show: There exists ∃ a long string w, ∀ xyz with |xy|≤p and y≠ϵ such that ∃ i≥0, xy^iz∉L.

6. Proof L is a nonregular language
   a. L = { w ∈ {0,1}*: 0^n1^n, n ≥ 0}
   b. L = { w ∈ {0,1}*: ww}
   c. L = { w ∈ {0,1}*: $0^n1^n2^n$, n ≥ 0}

7. Practice Problems:
   a. L = { w ∈ {0,1}*: w contains equal numbers of 0s and 1s}
   b. L = {w ∈ {1}*: 1^{n^2}, $n \geq 0$}
   c. L = {w ∈ {0, 1}*: 0^i1^j , where $i > j$}

In the example section of the pumping lemma chapter, you may solve the problems and check the solution from Sipser's textbook.

Lecture 17
Date: Apr 22, 2025
Lecture 18
Date:

Pushdown Automata (PDA)

Study Materials:
- Handnotes: Pushdown Automata (PDA)
- Lecturer Videos: [Link-1](), [Link-2]()

Lecture 19
Date:
Lecture 20
Date:

Turing Machine

Study Materials:

- Sipser's book. (Page:165, section: 3.1, 3.7, 3.8, 3.9, definition: 3.5, 3.6)
- Handnotes: Turing Machine
    - Theoretical Concepts
    - Designing Turing Machine [Doesn't include the basic problems, mostly practice sheet problems]
- Lecture Video: Link [Discussed a few problems, this lecture is on the second class on TM]