CSE331
Automata and Computability

MIDTERM EXAM
TOTAL MARKS: 50
DURATION: 80 MINUTES

B

There are a total of five problems. You have to solve the first four. Problem 5 is optional.

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ.

$$L_1 = \{w \text{ starts with } 10\}$$
$$L_2 = \{w \text{ doesn't contain } 11\}$$
$$L_3 = \{w \text{ doesn't contain } 00\}$$
$$L_4 = \{w = 10\}$$
$$L_5 = L_2 \cap L_3$$

Now solve the following problems.

(a) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

(b) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(c) If you were to use the "cross product" construction shown in class to obtain a DFA for the language L_5, how many states would it have? (1 point)

(d) **Find** all four-letter strings in L_5. (1 point)

(e) **Give** the state diagram for a DFA that recognizes L_5 using only four states. (2 points)

(f) **Find** one six-letter string in $L_4{}^*$. (1 point)

(g) **Give** the state diagram for a DFA that recognizes $L_4{}^*$. (2 points)

(h) Is $L_4{}^*$ and $L_1 \cap L_5$ same? **Give** justification for your answer. (2 points)

Problem 2 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{a, b\}$. Consider the following languages over Σ.

$$L_1 = \{\text{length of } w \text{ is odd}\}$$
$$L_2 = \{w \text{ doesn't end with } a\}$$
$$L_3 = \{\text{every third position in } w \text{ is } b\}$$
$$L_4 = \{\text{every } b \text{ in } w \text{ is followed by at least two } a\}$$

Now solve the following problems.

(a) **Give** a regular expression for the language L_1. (3 points)

(b) **Give** a regular expression for the language L_2. (3 points)

(c) **Give** a regular expression for the language L_3. (3 points)

(d) **Write** a five-letter string that belongs to $L_3 \cap L_4$. (1 point)

(e) **Give** a regular expression for the language $L_3 \cap L_4$. (2 points)

(f) **Give** a regular expression for the language $\overline{L}_4$. Here $\overline{L}$ denotes the complement of the language L i.e., $\overline{L} = \Sigma^* - L$. (3 points)

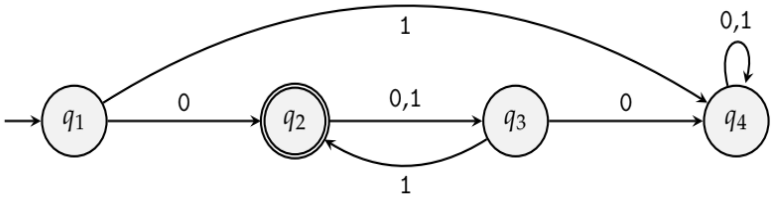
Problem 3 (CO2): Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$\mathtt{b}(\mathtt{b} + \mathtt{ba})^*(\mathtt{ab} + \mathtt{ba}^*\mathtt{b})$$

Problem 4 (CO2): Converting Finite Automata to Regular Expressions (10 points)

Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_2, then q_3 and finally q_4. You must show work.



Problem 5 (Bonus): Ternary Number System (5 points)

Disclaimer: This is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!

Let $\Sigma = \{0, 1, 2\}$.

$$L = \{w, \text{when interpreted in three base number system, is divisible by nine}\}$$

Give a state diagram for a DFA that recognizes L.