*[This doc file contains both the **Assignment** and **[Practice Sheet](#) [Click Here]** on the final syllabus.]*

**CSE331: Automata and Computability**
**Assignment 02**

**Group Formation:** You may form a group of **at most 3 people**.
**Deadlines and Submission Links:**

- CFG (Question 3):
  - Deadline: Apr 30, 2025 [Extended for one day]
  - Submission Link: https://forms.gle/1YubwdbKkGP5RnndA

- Pumping Lemma and Ambiguity (Questions 2 and 5):
  - Deadline: May 5, 2025
  - Submission Link: https://forms.gle/ijS3SKqYzQhFMMEb6

- Pushdown Automata (Questions 1, 4, ~~6~~):
  - Deadline: May 15, 2025 [Extended for one day]
  - Submission Link: https://forms.gle/uerhKmbbY2xfVsUv6 [sorry for the delay]

**Assignment & Practice Sheet Solution Folder:** Solutions

Solve all the questions given below:

1) List five use cases where you can apply the knowledge from our CSE331 course.

2) Proof L is a non regular language using the Pumping lemma:
   a) $L = \{w \in \{0,1\}^*: \ w = 0^{n!}, \ n \geq 0\}$
   b) $L = \{w \in \{0,1\}^*: w = 0^a1^b1^c0^d$, where $a + b = c + d$ and $a, b, c, d \geq 0\}$ [Be careful for (b): check if xyyz works]
   c) $L = \{w \in \Sigma^*: w = a^ib^j$, where $i > j$, and $j \geq 0\}$

**3) Design a Context Free Grammar for the Language:**

   a) $L = \{w \in \{a,b,c,p,q,r,\#\}^*: a^i\#^nc^kp^{2x}q^yr^zb^j$ where $i=j+k$, $y=3x+z$, $n$ is odd and $i,j,k,n,x,y,z \geq 0\}$
   b) $L = \{w \in \{0,1,2\}^*: \ w = 0^i2^j1^k$, [where ……conditions…… ] $\}$

where…

~~i) $i = k$, $i,k \geq 1$ and $j \geq 2$~~

ii) $i = 3k$, $j$ is odd and $i,j,k \geq 0$

iii) $i$ is a multiple of two, $k$ is two more than a multiple of 3, $j = k+i$, and $i,j,k \geq 0$

iv) $i+j > k$ and $i,j,k \geq 0$

v) $i+k$ is even, $j = i+k$ and $j \geq 1$

c) $L = \{w \in \{0,1\}^*:$ the parity of 0s and 1s is different in w$\}$

d) $L = \{w \in \{0,1\}^*:$ the number of 0s and 1s are different in w$\}$

[Hint: First, try to solve for an equal number of 0s and 1s in w]

e) $L = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, 3i \geq 4k + 2,$ j is not divisible by three$\}$

f) Recall that for a string w, $|w|$ denotes the length of w. $\Sigma = \{0,1\}$

$L1 = \{w \in \Sigma^*:$ w contains exactly two 1s$\}$

$L2 = \{x\#y : x \in \Sigma^*, y \in L1, |x| = |y|\}$

Construct a CFG for L2.

g) Recall that for a string w, $|w|$ denotes the length of w. $\Sigma = \{0,1\}$

$L1 = \{w \in \Sigma^*:$ w contains at least three 1s$\}$

$L2 = \{x\#y : x \in (\Sigma\Sigma)^*, y \in L1, |x| = |y|\}$

Construct a CFG for L2.

4) For all the problems in Question(3), now construct the Pushdown Automata (PDA).

**5) Derivation, Parse Tree, Ambiguity**

i) Given the Context-Free Grammar, answer the following questions

$$A \rightarrow A1 \mid 0A1 \mid 01$$

(a) Give a leftmost derivation for the string 001111.

(b) Sketch the parse tree corresponding to the derivation you gave in (a).
(c) Demonstrate that there are two more parse trees (apart from the one you already found in (b)) for the same string.
(d) Find a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

ii) Given the Context-Free Grammar, answer the following questions:

$$A \rightarrow 1A \mid 1C \mid 0B \mid 00A$$
$$B \rightarrow 0A \mid 1B \mid 00B$$
$$C \rightarrow 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \varepsilon$$

(a) Give a leftmost derivation for the string 01011001. (3 points)
(b) Sketch the parse tree corresponding to the derivation you gave in (a). (2 points)
(c) Demonstrate that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
(d) Find a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)
(e) Design an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

6) **Turing Machine [no need to submit, for practice only]**

a) L = $\{1^{3^n} \mid n \geq 0\}$
b) L = $\{w \in \{0,1,2\}^*: 0^i 1^j 2^k$ where $i=j=k$ and $i,j,k \geq 0\}$
c) L = $\{w \in \{0,1,2\}^*: 0^i \# 1^j \# 2^k$ where $i*j=k$ and $i,j,k \geq 0\}$ [Idea described in Sipser's Book]
d) L = {w ∈ {a,b}*: w is an odd length palindrome}
e) L = $\{w\#w^R\#w \mid w \in \{a,b\}^*\}$ [$w^R$ means reverse of w]

Study Materials on Turing Machine: Class Updates Spring 25

**CSE331: Automata and Computability**
**Practice Sheet**
**Prepared By: KKP**

[There are also some practice problems in the handnotes]

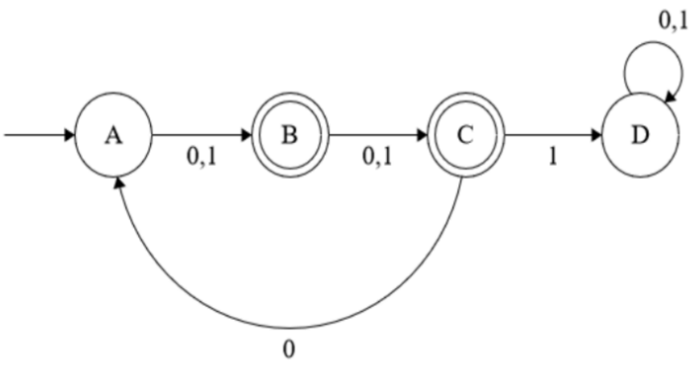
**Question 1: Designing Context-Free Grammar**

Give Context-free Grammar that generates the language

a) L = {w ∈ {0,1}*: w starts with '**0**' and the length of w is even.}
b) L = {w ∈ {a,b}*: every second letter in w is a '**b**'.}
c) L = {w ∈ {a,b}*: the length of w is divisible by three}
d) L = {w ∈ {a,b}*: the length of w is two more than multiple of six}
e) L = {w ∈ {0,1}*: w = $0^n 1^{2n+3}$, n ≥ 0 }
f) L = {w ∈ {0,1}*: w = $0^n 1^n$, where n is odd.}
g) L = {w ∈ {a,b}*: the number of '**a**' is at least the number of '**b**' in w.}
   [**Hint**: Link]
h) L = {w ∈ {0,1,2}*: w=$0^i 1^j 2^k$ where j ≥ 2i + 3k, and i, k ≥ 0}
i) L = {w ∈ {0,1,2,3}*: w=$0^i 1^j 2^k 3^m$ where i=m, j ≥ 3k+2, and m, k ≥ 0}
j) L = {w ∈ {0,1,2}*: w=$0^i 1^j 2^k$ where i > 2j + 3k, and j, k ≥ 0}
k) If A = {w ∈ {0, 1}* : w contains at least two 0s}, then construct L = {w ∈ {0, 1}*: w = $0^{3i} v 1^{2i}$ where v ∈ A and i ≥ 0}
l) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}
   L1 = {w ∈ $\Sigma^*$: w contains 11}
   L2 = {x#y : y ∈ L1, x ∈ $\Sigma^*$, |x| = |y|}
   Construct a CFG for L2.
m) L = {w ∈ {0,1}* : $w_1$#$w_2$: number of 0s $w_1$ is equal to number of 1s in $w_2$.}
n) L = {w ∈ {0,1}* : $w_1$#$w_2$: length of $w_2$ is double of length of $w_1$.}
o) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}
   L1 = {w ∈ $\Sigma^*$: w contains exactly two 1s}
   L2 = {x#y : x ∈ $\Sigma^*$, y ∈ L1, |x| = |y|}
   Construct a CFG for L2.
p) L = {$1^i 0 2^j 1^k$| i, j, k ≥ 0, 3i ≥ 4k + 2, j is not divisible by three}

Construct a Context-free Grammar for the language expressed by the following Finite Automata:

a) Convert the Regular Expression into a CFG: ( (ab)* + (a+b$^3$) cb)*
b) Convert the Regular Expression into a CFG: ( a*b* + (ac+b*c) b)*
c) Convert the following DFA into a CFG:




**Question 2: Derivation, Parse tree, Ambiguity**

I.

S→ASB|SS|SAS|A
A→ASS|BS|B
B→00|11|01|1

Given the above Context-Free Grammar, answer the following questions:

a) Show the Leftmost Derivation of 00010111
b) Sketch the parse tree corresponding to the derivation you gave in (a).
c) Show the Rightmost Derivation of 00010111
d) Sketch the parse tree corresponding to the derivation you gave in (c).

II. S → 0S1 | 1S0 | SS | 01 | 10

Given the above Context-Free Grammar, answer the following questions:

a) Show that the grammar above is ambiguous by demonstrating two different parse trees for 011010.
b) Find a string w of length six such that w has exactly one parse tree in the grammar above.

## Question 3: Pushdown Automata [Each Question Carries 5 marks]

a) L = { w ∈ {0,1}* : w = $0^n1^n$, where n is odd.}

b) L = { w ∈ {0,1}* : w = $0^m1^n$, where m,n ≥ 1 and m≥n.}

c) L = { w ∈ {0,1}*: w | the length of w is divisible by four}

d) L1 = { w ∈ {0,1}*: the number of 1s in w is multiple of 3.}
L2 = { w ∈ {0,1}* : w contains even numbers of 0.}
Construct a PDA for L = { w ∈ {0,1}*: w = uv, where u ∈ L1,v ∈ L2 and |u| = |v| }

e) L = { w ∈ {0,1}* : w = $0^i1^j0^k$, where j = i+k and i, k ≥ 0.}

f) L = { w ∈ {0,1}* : w = $0^i1^j0^k$, where i+j = k and i, k ≥ 0.}

g) L = { w ∈ {0,1}* : $w_1\#w_2$: number of 00 substrings in $w_1$ is equal to number of 11 in $w_2$.}

h) L = { $w\overline{w}^R$ : w ∈ {0,1}* }

$$L = \{w\overline{w}^{\mathcal{R}} : w \in \{0,1\}^*\}$$

Here, $\overline{w}^R$ denotes the reverse complement of the string w.

For example, $01001101 \in L$, because the second half of the string, $1101$, is the reverse complement of the first half, $0100$, i.e. $1101 = \overline{0100}^R$

## Question 4: Chomsky Normal Form - CNF [Each Question Carries 10 marks]

**[Not included in the Spring 2025 Final Syllabus]**

a) Convert the following grammar into Chomsky Normal Form. You must show work.

S → bBS | ε
B → bYb | bY
C → Ccb | c | A
Y → bcY | ε | YBF

Here b, and c are terminals and the rest are variables.

b) Convert the following grammar into Chomsky Normal Form. You must show work.

S→YaSb
X→aXY | bX | Y
Y→X | b | ϵ

Here a, and b are terminals and the rest are variables.

c) Convert the following grammar into Chomsky Normal Form. You must show work.

$$S \rightarrow bXaY \mid ZXb$$
$$X \rightarrow aY \mid bY \mid Y$$
$$Y \rightarrow X \mid c \mid \varepsilon$$
$$Z \rightarrow ZaX$$

Here a, b, c are terminals and the rest are variables.

**d)**

**Problem 3 (CO2): Chomsky Normal Form (10 points)**

(a) **List** the rules that violate the conditions of Chomsky Normal form in the following grammar. Here a, b, and c are terminals and the rest are variables.

$$A \rightarrow BC \mid bB \mid a$$
$$B \rightarrow bb \mid Cb \mid b \mid C$$
$$C \rightarrow c$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production, $B \rightarrow \varepsilon$ is removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow AB \mid 1$$
$$A \rightarrow BAB \mid ABA \mid B \mid 11$$
$$B \rightarrow 00 \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if the unit productions are removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow XYX \mid YX \mid X \mid Y$$
$$Y \rightarrow XY \mid X0 \mid 0$$
$$X \rightarrow 1 \mid Y$$

**Question 5: CYK Algorithm [Each Question Carries 10 marks]**
**[Not included in the Spring 2025 Final Syllabus]**

a) Consider the following grammar in CNF form.

$$S \rightarrow BC \mid CD$$
$$B \rightarrow CB \mid c$$
$$C \rightarrow DD \mid c$$
$$D \rightarrow BC \mid b$$

Show if the string w=cccbb can be derived from the grammar above.

b) Apply the CYK algorithm to determine whether the string "abcaa" can be derived in the following grammar. You must show the entire CYK table. Here a, b, c are terminals and the rest are variables.

$$
\begin{aligned}
S &\rightarrow CA \\
A &\rightarrow AA \mid AD \mid a \\
B &\rightarrow AB \mid BC \mid b \\
C &\rightarrow CA \mid BC \mid c \\
D &\rightarrow a
\end{aligned}
$$