CSE331
Automata and Computability

FINAL EXAM
TOTAL MARKS: 50
DURATION: 105 MINUTES

**B**

There are a total of six problems. You have to solve all of them.

## Problem 1 (CO5): Pumping Lemma (5 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 1^a 0^b 0^c 1^d, \text{ where } a + d = b + c \text{ and } a, b, c, d \geq 0\}$$

Use the pumping lemma to **demonstrate** that $L$ is not regular language.

## Problem 2 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages. Recall that for a string $w$, $|w|$ denotes the length of $w$.

$$L_1 = \{w \in \Sigma^* : w \text{ is a odd length palindrome}\}$$
$$L_2 = \{w \in \Sigma^* : \text{length of } w \text{ is multiple of two}\}$$
$$L_3 = \{x000y : x, y \in L_2, |x| = |y|\}$$
$$L_4 = L_1 \cap L_3$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language $L_1$. (3 points)

(b) **Give** a context-free grammar for the language $L_3$. (4 points)

(c) **Give** a context-free grammar for the language $L_4$.(3 point)

## Problem 3 (CO4): The CYK Algorithm (5 points)

**Apply** the CYK algorithm to fill up the table for the string aabaa using the following grammar. Here a and b are terminals, and the rest are variables.

$S \to BA \mid BC$
$A \to AB \mid AC \mid a$
$B \to CB \mid CC \mid b$
$C \to CA \mid a$

| | | | | |
|---|---|---|---|---|
| 1,5 {?} | | | | |
| 1,4 {?} | 2,5 $\{S, A\}$ | | | |
| 1,3 {?} | 2,4 $\{S, A\}$ | 3,5 {?} | | |
| 1,2 $\{A, B, C\}$ | 2,3 {?} | 3,4 $\{S\}$ | 4,5 $\{A, B, C\}$ | |
| 1,1 $\{A, C\}$ | 2,2 $\{A, C\}$ | 3,3 $\{B\}$ | 4,4 $\{A, C\}$ | 5,5 $\{A, C\}$ |
| $a$ | $a$ | $b$ | $a$ | $a$ |

## Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

$$L_1 = \{w \mid w \text{ starts and ends with the different character}\}$$

$$L_2 = \{w \mid \text{the number of } 0\text{s in } w \text{ is not same as the number of } 1\text{s}\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes $L_1$. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes $L_2$. (6 points)

## Problem 5 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to 1A \mid 0B \mid 00A$$
$$B \to 1B \mid 1C \mid 0A \mid 00B$$
$$C \to 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid 0 \mid 1$$

(a) **Give** a leftmost derivation for the string 10110010. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

(d) **Find** a string $w$ of length six such that $w$ has exactly one parse tree in the grammar above. (1 point)

(e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

## Problem 6 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (5 points)

$$P \to \mathtt{a} \mid PR \mid \varepsilon$$
$$Q \to \mathtt{b}P \mid R$$
$$R \to QQ \mid \mathtt{a}T$$
$$T \to QT \mid \varepsilon$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $A \to \varepsilon$ is removed. (3 points)

$$S \to AA \mid \mathtt{b}B$$
$$A \to CB \mid \mathtt{a}A \mid \varepsilon$$
$$B \to ABA \mid \mathtt{b} \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$A \to 0A1B1 \mid C$$
$$B \to 1B \mid B$$
$$C \to BC \mid A \mid B$$