Set A

Problem 1 (CO1): DFA and Regular Languages (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ.

$$L_1 = \{w : w = 0^m 1^n, \text{ where } m, n \geq 0\}$$

$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$

Now solve the following problems.

(a) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

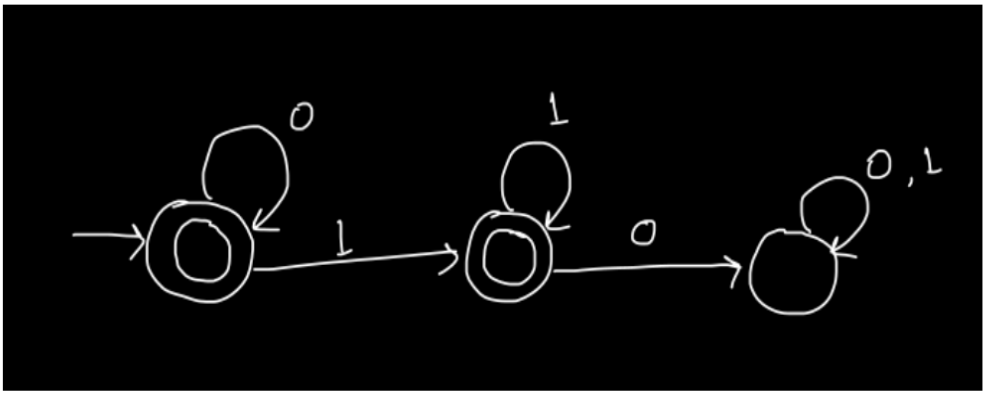
(b) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(c) If you were to use the "cross product" construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have? (1 point)

(d) **Find** all five-letter strings in $L_1 \cap L_2$. (1 point)

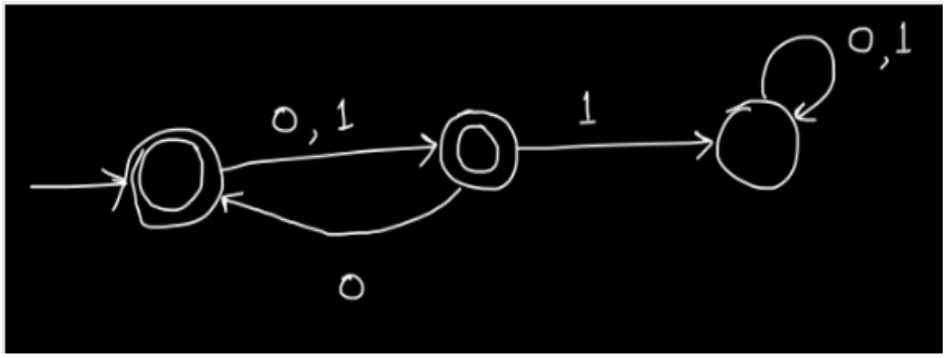
(e) **Give** the state diagram for a DFA that recognizes $L_1 \cap L_2$ using only four states. (2 points)

(a)



A correct DFA is worth the full 3 points. If the student misses one of the accepting states (with everything else being correct), award up to 2 points. Anything unsalvageably wrong is worth 0 points.

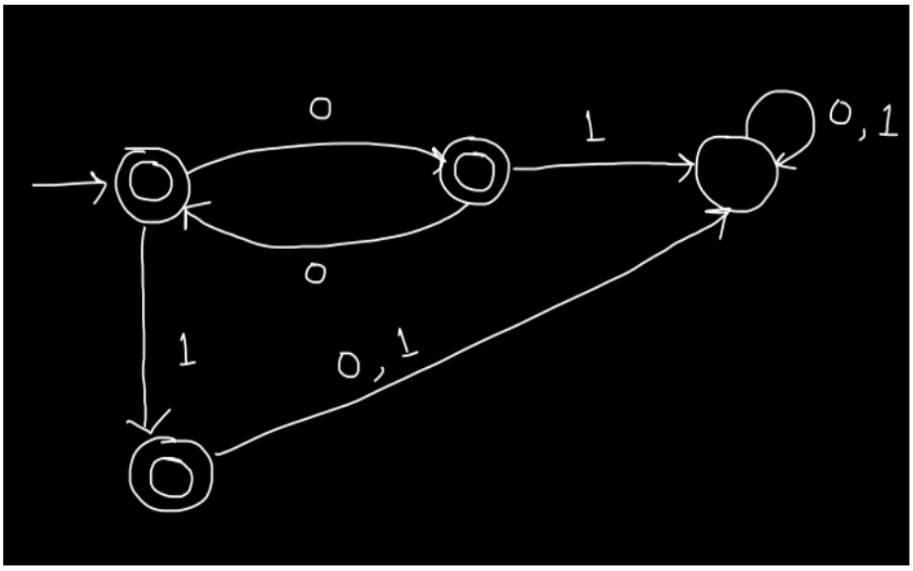
(b)



- A correct DFA is worth 3 points
- If the student misses one of the accepting states, award up to 2.5 points (with everything else being correct)
- If the student tries something involving two states to keep track of whether or not they are at an even position, award up to 1.5 points.

(c) The answer is 3*3=9. The student does not have to arrive at the correct answer. She only has to demonstrate the knowledge that you have to multiply the state counts. In other words, award her the full 1 point if the student has m states in her DFA for Problem (a), n states in her DFA for Problem (b), and has m*n as her answer in (c).

(d) The strings are 00000 and 00001. 0.5 points for each correct string listed.

(e) 

- A correct DFA is worth 2 points.
- With everything else being correct, deduct 0.5 points if the student does not mark the start state as an accepting state.
- Any attempt at determining the parity of which index you're on using two states should be worth at least 0.5 points.

Problem 2 (CO1): Regular Expressions (10 points)

Consider the following languages over $\Sigma = \{0, 1\}$.

$$L_1 = \{w : w \text{ does not contain } 11\}$$
$$L_2 = \{w : \text{every } 1 \text{ in } w \text{ is followed by at least one } 0\}$$
$$L_3 = \{w : \text{the number of times } 1 \text{ appears in } w \text{ is even}\}$$

Now solve the following problems.

(a) **Give** a regular expression for the language L_1. (2 points)

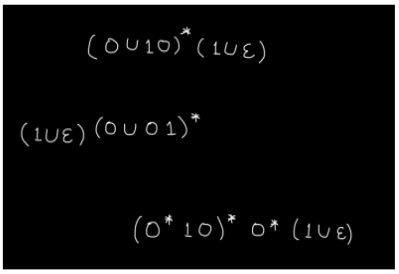
(b) Your friend claims that $L_1 = L_2$. **Prove** her wrong by writing down a five-letter string in $L_1 \setminus L_2$. Recall that $L_1 \setminus L_2$ contains all strings that are in L_1 but not in L_2. (2 points)

(c) **Give** a regular expression for the language $L_1 \setminus L_2$. (2 points)

(d) **Give** a regular expression for the language L_3. (2 points)

(e) **Give** a regular expression for the language $L_2 \setminus L_3$. (2 points)

(a) Possible correct answers include the following:



- Award the full 2 points if the student has anything equivalent to the answers above.
- Deduct 0.5 points if the student misses the (1 \cup \varepsilon) from the first two answers. These are small corner cases.
- Award at least 0.5 points if the student's answer does not match any string in L_1'. It is possible that his answer misses some subset in L_1. Depending on the nature of these subsets, this 0.5 can become a 1.5 (see, for example, the last point). Grade at your discretion.
- Award 0 points, if the student's answer matches any infinite subset of L_1'.

(b) Any five-letter string of the form (0|10)*1 is correct and worth the full 2 points. An incorrect answer is worth nothing.

(c) The answer is anything equivalent to (0|10)*1. No points for partial progress Anything equivalent to (0*10*1)*0* is worth the full 2 points.

- If the student's answer does not match the strings of the form 00…0000, then deduct 0.5 points.
- If the student's answer does not match any string in L_3', award at least 0.5 points. It is possible that his answer misses some subset in L_3. Depending on the nature of these subsets, this 0.5 can become a 1.5 (see, for example, the last point). Grade at your discretion.
- Award 0 points, if the student's answer matches any infinite subset of L_3'.

(d) Anything equivalent to (0*100*10)*0*100* is worth the full 2 points. Instructions similar to (d) also apply here. Grade at your discretion.

Problem 3 (CO3): Converting Regular Expressions to NFAs (10 points)

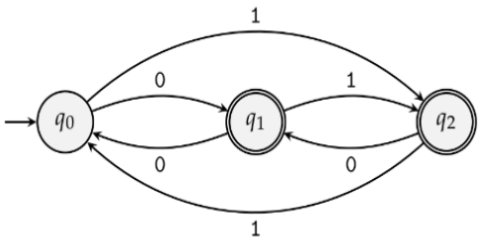
Convert the following regular expression over $\Sigma = \{a, b, c, d\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$a(b^*c)^* + ac(b^*c)^* + (c^* + d)^*$$

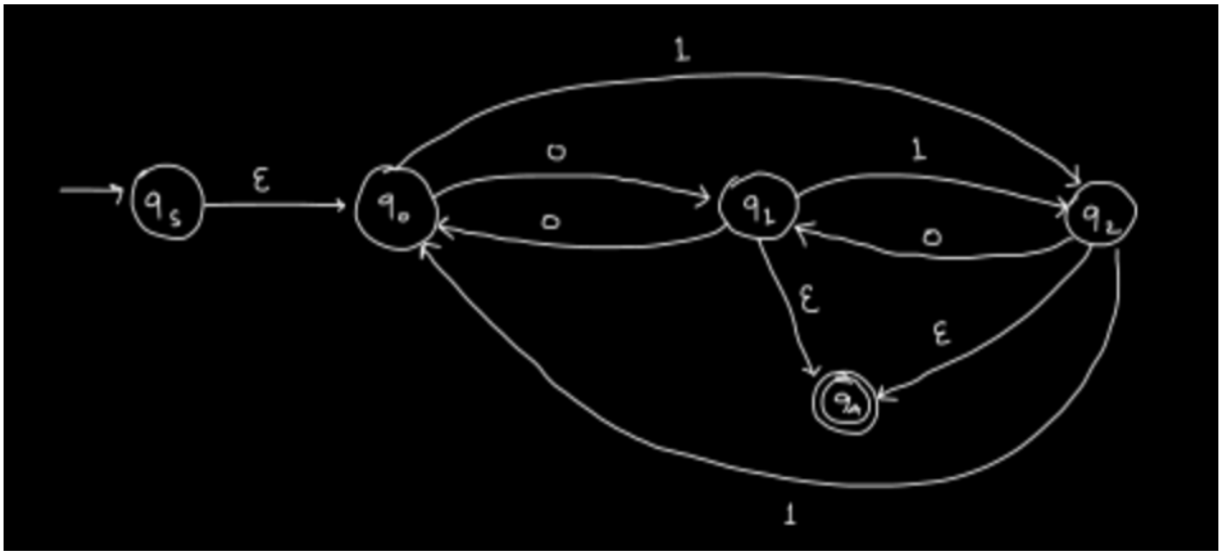
Grade at your own discretion.

Problem 4 (CO3): Converting Finite Automata to Regular Expressions (10 points)

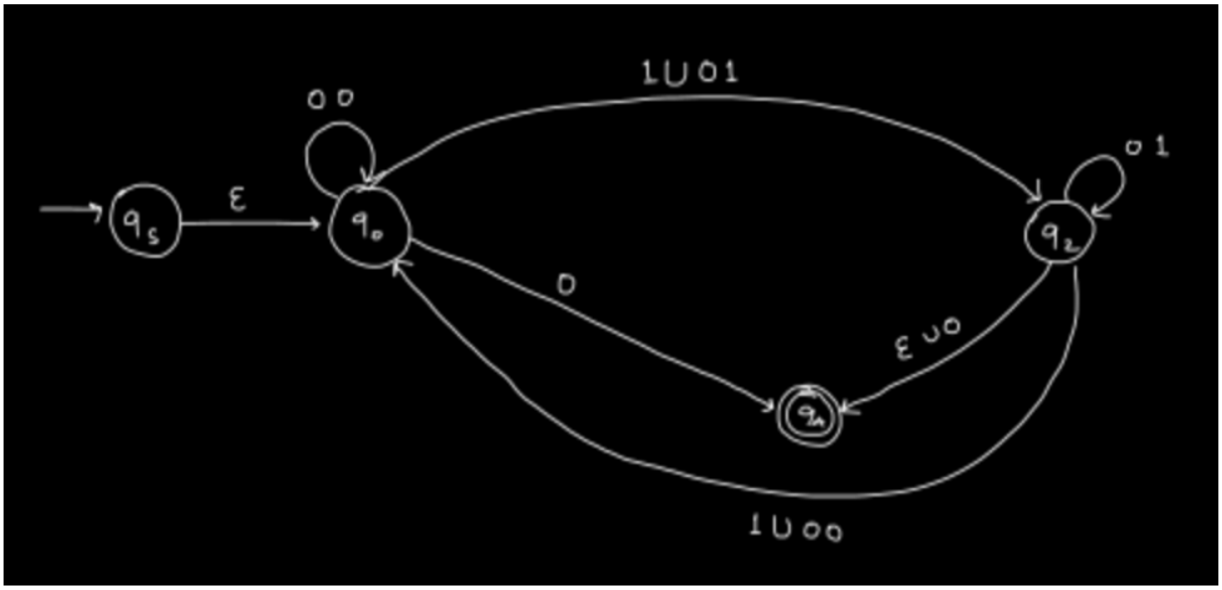
Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_1, then q_2, and finally q_0. You must show work.



Adding new start and accepting states (1 point)



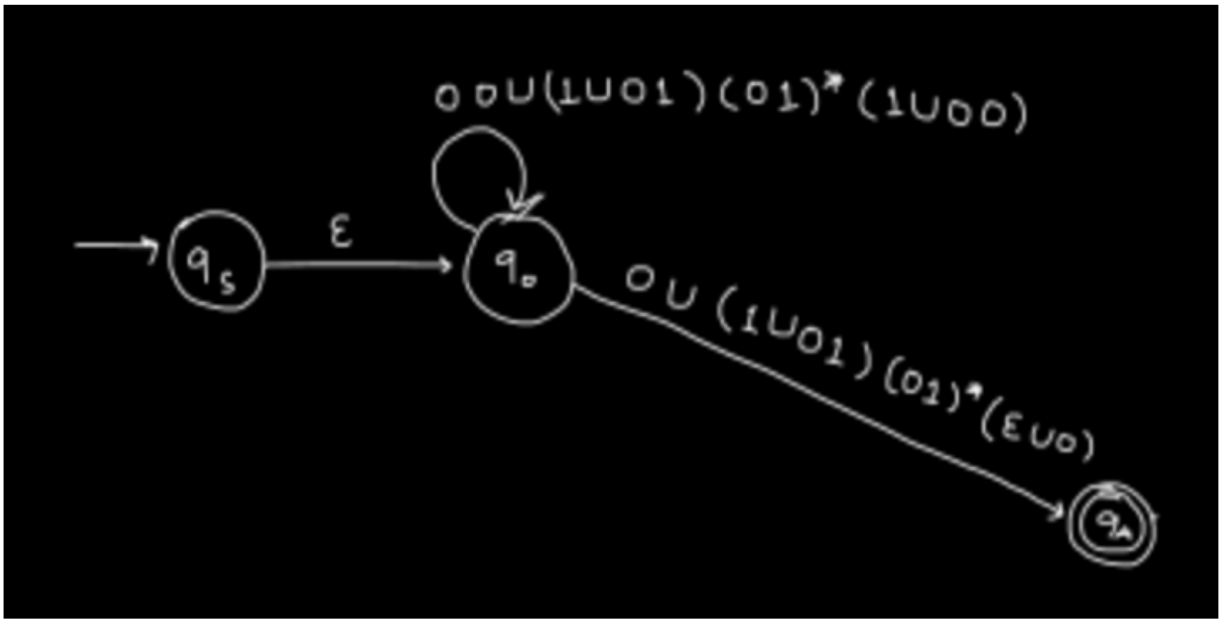
Eliminate q_1 (3 points)



- Transition q0 → q0
- Transition q0 → q2
- Transition q2 → q0
- Transition q2 → q2
- Transition q0 → qA
- Transition q2 → qA

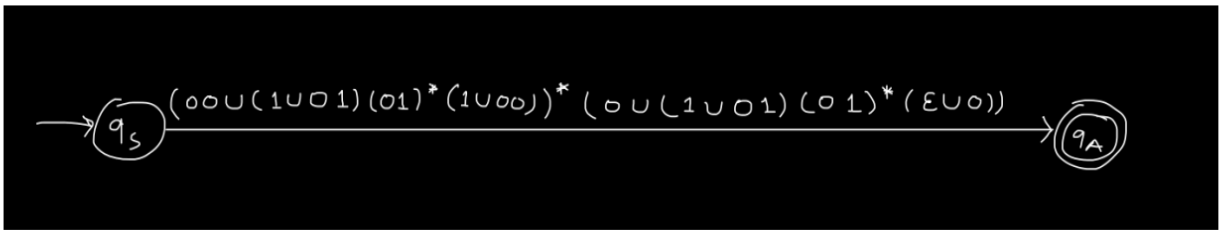
Each of these transitions is worth 0.5 points

Eliminate q2 (4 points)



The transitions q0→q0 and q0→qA are both worth 2 points

Eliminate q0 (2 points)



Watch the parentheses!