There are a total of four problems. You have to solve all the problems.

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ.

$$L_1 = \{w : \text{length of } w \text{ is exactly three}\}$$

$$L_2 = \{w : \text{every even position in } w \text{ is } 1\}$$

$$L_3 = \{w : 10 \text{ appears even number of times in } w \text{ as a substring}\}$$

$$L_4 = L_1 \cap L_2 \cap L_3$$

$$L_5 = \{w : 1^m 0^n, \text{where } m, n \geq 0\}$$

Now solve the following problems.

(a) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

(b) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(c) **Give** the state diagram for a DFA that recognizes L_3. (3 points)

(d) If you were to use the "cross product" construction shown in class to obtain a DFA for the language L_4, how many states would it have? (1 point)

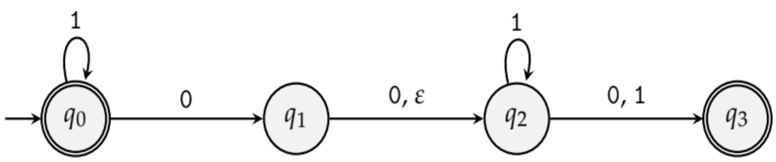
(e) **Find** all the strings in L_4. (1 point)

(f) **Give** the state diagram for a DFA that recognizes L_4 using only five states. (2 points)

(g) Is L_4 is a subset of L_5? **Give** justification for your answer. (2 points)

Problem 2 (CO2): Converting NFA to DFA (5 points)

Consider the following NFA:



Now solve the following problems. Note that you do not need to convert the given NFA into its equivalent DFA in order to answer the following questions.

(a) If you convert the given NFA into an equivalent DFA using the subset construction method, what is the maximum number of states the DFA can have? (1 Point)

(b) **Write** the subsets of states of the given NFA that will be the rejecting states in its equivalent DFA (1 Point)

(c) **Write** the ε-closure of state q_1 in the given NFA. (1 Point)

(d) What is $\delta(\{q_0, q_2\}, 1)$ in the given NFA? **Write** all the states. [Recall: $\delta(\{q\}, a)$ is the set of states the NFA transitions to when it is in state q and receives input a.] (2 Points)

Problem 3 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$. Consider the following languages over Σ.

$$L_1 = \{w \text{ does not contain consecutive } \texttt{0}\}$$

$$L_2 = \{w \text{ starts with } \texttt{1}\}$$

$$L_3 = \{w \text{ starts and ends with the different character}\}$$

$$L_4 = L_2 \setminus L_3$$

Now solve the following problems.

(a) **Give** a regular expression for the language L_1. (3 points)

(b) **Give** a regular expression for the language $\overline{L_2}$. [Recall: $\overline{L}_2$ denotes the complement of the language L_2 i.e., $\overline{L}_2 = \Sigma^* - L_2$] (3 points)

(c) **Give** a regular expression for the language L_3. (3 points)

(d) **Write** four four-letter strings in L_4. (2 point)

(e) **Give** a regular expression for the language L_4. [Recall: $L_2 \setminus L_3$ contains all strings that are in L_2 but not in L_3] (2 points)

(f) **Give** a regular expression for the language $\overline{L_4}$. (2 points)

Problem 4 (CO2): Converting Regular Expressions to NFA (10 points)

Convert the following regular expression over $\Sigma = \{\texttt{a}, \texttt{b}\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$(\texttt{bb} + \texttt{a}^*\texttt{b})^*\texttt{a} + \texttt{b}(\texttt{aa})^*$$

After you are done with the test, please indicate where you stand on the smiley face spectrum.