CSE331
Automata and Computability

FINAL EXAM
TOTAL MARKS: 50
DURATION: 100 MINUTES

B

There are a total of five problems. You have to solve all of them.

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following pair of languages. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{w \in \Sigma^* : w \text{ contains exactly two } 0\text{s}\}$$
$$L_2 = \{x\#y : x \in L_1, y \in \Sigma^*, |x| = |y|\}$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language L_1. (4 points)

(b) **Write** down four seven-letter strings in L_2. (1 point)

(c) **Give** a context-free grammar for the language L_2. (5 points)

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \to BA \mid Abb \mid SS$$
$$A \to \mathtt{a} \mid \varepsilon$$
$$B \to \mathtt{b}$$

(a) **Give** a leftmost derivation for the string `babbba`. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse tree (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** a string w of length five such that w has exactly one parse tree in the grammar above. (1 point)

Problem 3 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string `yxxzy` can be derived in the following grammar. You must show the entire CYK table. Here `x`, `y` and `z` are terminals, and the rest are variables.

$$S \to AB$$
$$A \to AC \mid \mathtt{y}$$
$$B \to BD \mid EB \mid \mathtt{z}$$
$$C \to AE \mid \mathtt{x}$$
$$D \to \mathtt{y}$$
$$E \to BD \mid \mathtt{x}$$

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$S \to \mathtt{a} \mid AB$$
$$A \to SB \mid \mathtt{ba} \mid B$$
$$B \to b \mid \varepsilon$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $X \to \varepsilon$ is removed. (4 points)

$$S \to \mathtt{a}X\mathtt{b}X\mathtt{c}X \mid X$$
$$X \to \mathtt{a} \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$S \to A\mathtt{a}$$
$$A \to BC \mid C$$
$$B \to \mathtt{b}$$
$$C \to A \mid AA \mid A\mathtt{a}B$$

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Note that we define $c(w, x)$ to be the count of x in the string w.

$$L_1 = \{w \mid \ w \text{ is a palindrome and the length of } w \text{ is even.}\}$$
$$L_2 = \{w_1\#w_2\#w_3 \mid \ c(w_1, \mathtt{b}) = c(w_3, \mathtt{b}) \text{ or } c(w_2, \mathtt{a}) = c(w_3, \mathtt{a})\}$$

Now solve the following problems.

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Find** all strings $w \in L_2$ such that w starts with `babbab#abba#` and has a length of 15. (1 point)

(c) **Give** the state diagram of a pushdown automaton that recognizes L_2. (5 points)