Construct CFG for the following grammars.

a) L = {w ∈ {(,)}*: w is a valid parenthesis.}

$$S \rightarrow (S) | SS | E$$

d) L = {w \in {a, b, 1}*: $a^n 1^{2n+m+3}b^m$, where n, m \ge 0}

$$a^{n} 1^{2n+m+3} b^{m}$$

$$\Rightarrow a^{n} 1^{2n} 1^{m} 1^{3} b^{m}$$

$$S \rightarrow PQ$$

$$P \rightarrow a P11 | E$$

$$Q \rightarrow 1Qb | 111$$

$$Q \rightarrow 1Qb | E$$

b) Convert the following Regular Expressions into a CFG: ((aa+ bc)* a)* + cb

$$((aa+bc), a) + cb$$

$$S \rightarrow X \mid Y$$

$$X \rightarrow PX \mid E$$

$$P \rightarrow Ma$$

$$M \rightarrow FM \mid E$$

$$F \rightarrow aa \mid bc$$

$$Y \rightarrow cb$$

c) L = $\{w \in \{0, 1\}*: length of w is odd and the mid is 1.\}$

$$S o 050 | 051 | 150 | 151 | 1$$

Or, $S o XSX | 1$
 $X o 0 | 1$

Wrong Solve:

 $S o A 1A$
 $A o 0A | 1A | E$

d) L = $\{w \in \{0,1\}*: w \text{ is an odd-length palindrome.}\}$

$$S \rightarrow OSO | 1S1 | O | 1$$

d) L = {w ∈ {0,1}*: w is an even-length palindrome.}

e) $L = \{w \in \{0, 1\} *: the length of w is \}$ three more than the multiple of four.}

$$A \rightarrow MMMMA | MMM$$

 $M \rightarrow 0|1$

- $M \rightarrow 011$
 - g) $L1 = \{w \in \{0, 1\}^*, \text{ length of } w \text{ is odd.} \}$ $L2 = \{ w1\#w2, where w1, w2 \in L1 \text{ and } |w1| = 1 \}$ |w2|. Construct a CFG for L2.

$$S \rightarrow XXSXX \mid X\#X$$

 $X \rightarrow 0 \mid 1$

- S→ XXSXX X#X
- i) $L = \{w \in \{a, b, c, d\}^*: a^n c^m d^m b^{3n}, d^n b^{3n}\}$ where $n \ge 0$ and $m \ge 1$ }

$$a^{n}c^{m}d^{m}b^{3n}$$

$$S \to aSbbb \mid X$$

$$X \to cXd \mid cd$$

$$m \ge 1$$
Alternate solution
$$S \to aSbbb \mid cXd$$

$$X \to cXd \mid \varepsilon$$

f) $L = \{w \in \{0, 1\} *: the length of w is two \}$ more than the multiple of three.

$$A \rightarrow MMMA \mid MM$$

 $M \rightarrow 011$

h) $L1 = \{w \in \{0, 1\}^* : \text{ count of } 0 \text{ is a multiple of three.} \}$ $L2 = \{ w1 \# w2, where w1 \in \{0,1\} *, w2 \in L1 \text{ and } |w1| \}$ = |w2|.

Construct a CFG for L2.

$$A \rightarrow XA1 \mid XB0 \mid \#$$

 $B \rightarrow XB1 \mid XC0$
 $C \rightarrow XC1 \mid XA0$

j) $L = \{w \in \{0, 1\}^*: 0^n 1^m, \text{ where } m \ge 3n\}$ [Hint: First, try to solve for m=3n]

on
$$3n$$

since $m \ge 3n$

on $3n$

on

if m = 3n

k)
$$L = \{w \in \{a, b, c, d\}^*: a^n c^{m+3} d^m b^{3n},$$

where $n \ge 0$ and $m \ge 1$

$$a^{n} c^{m+3} d^{m} b^{3n}$$

$$\Rightarrow a^{n} c^{m} c^{3} d^{m} b^{3n}$$

$$S \rightarrow aSbbb/cAd \rightarrow m \ge 1$$

 $A \rightarrow cAd/c$

Alternate solution:

$$S \rightarrow aSbbb \mid A_{m \ge 1}$$

 $A \rightarrow cAd \mid c ccc d$

1) L3 = { $w \in \{a, b, 1\}^*: a^n 1^{k+3} b^m$, where k=2n+m and $n, m, k \ge 0$

$$a^{n} 1^{K+3} m$$
 $a^{n} 1^{K+3} m$
 $\Rightarrow a^{n} 1^{2n+m+3} m$
 $\Rightarrow a^{n} 1^{m+3} m$
Previously solved,
Same as $a^{n} 3^{m+3} m$

m)
$$L = \{w \in \{0, 1\}^*: length of w is a multiple of six.\}$$

n) $L = \{w \in \{0, 1\}^*: w1\#w2, where w1, w2 \in \{0, 1\}^*\}$ 1}* and |w1| = |w2|.}

$$S \rightarrow XSX \mid \#\#$$

$$X \rightarrow 0 \mid 1$$

o) L1 = { w
$$\in$$
 {0, 1}*: length of w is multiple of six}
L2 = {w \in {0, 1}*: w1##w2, where w1,w2 \in

 $\{0, 1\}$ * and |w1| = |w2|.

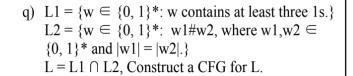
 $L = L1 \cap L2$, Construct a CFG for L.

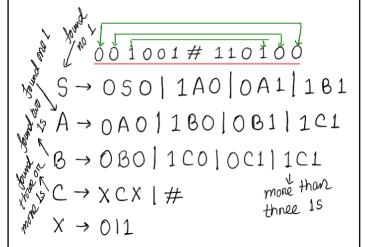
p) $L = \{w \in \{0, 1\}^*: w \text{ contains at least three 1s.} \}$

$$(0+1)^*1(0+1)^*1(0+1)^*1(0+1)^*$$

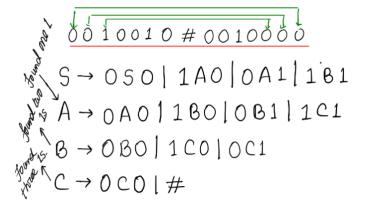
 $S \to X 1 X 1 X 1 X$

$$X \rightarrow 0 \times |1 \times | \epsilon$$





r) $L1 = \{w \in \{0, 1\}^*: w \text{ contains exactly three 1s.} \}$ $L2 = \{w \in \{0, 1\}^*: w1\#w2, where w1, w2 \in \{0, 1\}^*\}$ 1}* and |w1| = |w2|.} $L = L1 \cap L2$, Construct a CFG for L.



s) L1 = $\{w \in \{0, 1\}^*: w \text{ contains exactly three 1s.} \}$ $L2 = \{w \in \{0, 1\}^*: w1\#w2, where w1 \in \{0, 1\}^*\}$ 1}*, $w2 \in L1$, and |w1| = |w2|. Construct a CFG for L2.

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$$S \rightarrow XSO \mid XA1$$

 $A \rightarrow XAO \mid XB1$
 $B \rightarrow XBO \mid XC1$
 $C \rightarrow XCO \mid \#$
 $C \rightarrow O11$