CFG

$L = \{ \omega \in \{0,1\}^* :$ ω starts and ends with different symbol.$\}$

$$S \rightarrow 0A1 \mid 1A0$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

Another Approach:

Since the language is regular, we can first write the Regular Expression and then convert it to CFG.

$$\underbrace{0\overbrace{(0+1)^*}^{X}1}_{A} + \underbrace{1(0+1)^*1}_{B}$$

$$S \rightarrow A \mid B$$

$$A \rightarrow 0X1$$

$X \rightarrow 0X \mid 1X \mid \epsilon$ $\longrightarrow$ Another way of writing this

$$X \rightarrow MX \mid \epsilon$$

$$M \rightarrow 0 \mid 1$$

$$B \rightarrow 1X0$$

convert the regular expression into CFG

$$\underbrace{\Big(\overbrace{\underbrace{\overbrace{(ab+bc)}^{P}\,\overbrace{bb}^{Q}\,\overbrace{(\overbrace{ca+cc}^{W})^*}^{R}}_{X}+\overbrace{aa}^{y}}^{T}\Big)^*}_{S}$$

$S \rightarrow TS \mid \epsilon$

$T \rightarrow X \mid y$

$X \rightarrow PQR$

$P \rightarrow ab \mid bc$

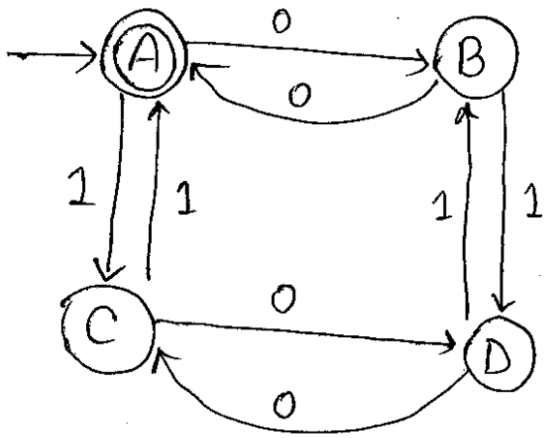
$Q \rightarrow bb$

$R \rightarrow WR \mid \epsilon$

$W \rightarrow ca \mid cc$

$y \rightarrow aa$

$L = \{w \in \{0,1\}^* : w \text{ contains even number of 0s and 1s}\}$



Since A is accepting state

$A \rightarrow 0B \mid 1C \mid \epsilon$

$B \rightarrow 0A \mid 1D$

$C \rightarrow 1A \mid 0D$

$D \rightarrow 0C \mid 1B$

So as we can see, it is also possible to make the CFG from its DFA design if the language is Regular.

Practice:

$L = \{w \in \{0,1\}^* :$ w contains equal numbers of 0s and 1s$\}$