There are a total of five problems. You have to solve all of them.

Set A CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

(a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$\mathtt{a}^*(\mathtt{ba} + \mathtt{bb})^* + \mathtt{\#}$$

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{\#}\}$. Consider the following languages over Σ. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{\mathtt{a}^{3i} \text{ , where } i \geq 0\}$$

$$L_2 = \{\mathtt{b}^{2j} \text{ , where } j \geq 0\}$$

$$L_3 = \{x\mathtt{\#}y \text{ , where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

(b) Your Friend writes the following context-free grammars for the language L_1. **Choose** the correct grammar for L_1. There could be more than one correct grammar. (3 points)

(i) $S \to ABS \mid \varepsilon$
$A \to a \mid \varepsilon$
$B \to aa \mid \varepsilon$

(ii) $S \to aaaS \mid \varepsilon$

(iii) $S \to AAAS \mid \varepsilon$
$A \to aA \mid \varepsilon$

(iv) $S \to aP \mid aQ \mid \varepsilon$
$P \to aR$
$R \to aS$
$Q \to aR$

(c) **Write** down a string of any length in L_3. (1 point)

(d) **Give** a context-free grammar for the language L_3. (3 points)

Set B CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

(a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$\mathtt{a}(\mathtt{ba})^* + \mathtt{\#a}^*\mathtt{b}$$

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{\#}\}$. Consider the following languages over Σ. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{\mathtt{a}^{4i} \text{ , where } i \geq 0\}$$

$$L_2 = \{\mathtt{b}^{2j} \text{ , where } j \geq 0\}$$

$$L_3 = \{x\#y \text{ , where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

(b) Your Friend writes the following context-free grammars for the language L_1. **Choose** the correct grammar for L_1. There could be more than one correct grammar. (3 points)

(i) $S \to ASA \mid \varepsilon$
$A \to aa$

(ii) $S \to AAS \mid \varepsilon$
$A \to aa \mid \varepsilon$

(iii) $S \to AAAAS \mid \varepsilon$
$A \to a$

(iv) $S \to aP \mid aQ \mid \varepsilon$
$P \to aR$
$R \to aQ$
$Q \to aS$

(c) **Write** down a string of any length in L_3. (1 point)

(d) **Give** a context-free grammar for the language L_3. (3 points)

Set C CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

(a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$(\mathtt{ab}^* + \mathtt{\#a})^* + b$$

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{\#}\}$. Consider the following languages over Σ. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{\mathtt{b}^{5i} \text{ , where } i \geq 0\}$$

$$L_2 = \{\mathtt{a}^{j} \text{ , where } j \geq 0\}$$

$$L_3 = \{x\#y \text{ , where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

(b) Your Friend writes the following context-free grammars for the language L_1. **Choose** the correct grammar for L_1. There could be more than one correct grammar. (3 points)

(i) $S \to ASA \mid \mathtt{b}$
$A \to bb$

(ii) $S \to AAAAAS \mid \varepsilon$
$A \to bA \mid b$

(iii) $S \to AASB \mid \varepsilon$
$A \to bb$
$B \to b$

(iv) $S \to bP \mid bbbbQ \mid \varepsilon$
$P \to bbR$
$R \to bQ$
$Q \to bS$

(c) **Write** down a string of any length in L_3. (1 point)

(d) **Give** a context-free grammar for the language L_3. (3 points)

Set A

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \to SS \mid 0S1 \mid 1S0$$
$$S \to \varepsilon$$

(a) **Give** a leftmost derivation for the string 011010. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing three more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

(e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

Set B

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to 0A1 \mid AA \mid 1A0$$
$$A \to \varepsilon$$

(a) **Give** a leftmost derivation for the string `100101`. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing three more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

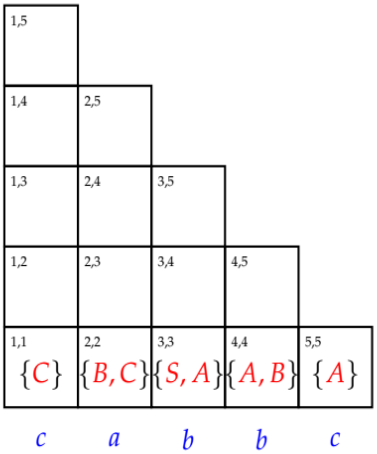
(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

(e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

set A for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string `cabbc`.



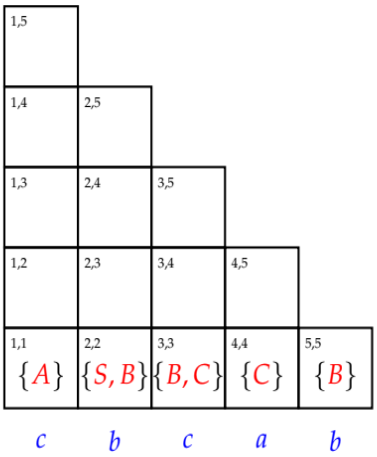
Apply the CYK algorithm to determine whether the string `cabbc` can be derived in the following grammar. You must show the entire CYK table. Here `a`, `b` and `c` are terminals, and the rest are variables.

$$S \to AB \mid BC \mid \texttt{a}$$
$$A \to BC \mid \texttt{c} \mid \texttt{a}$$
$$B \to AB \mid CB \mid \texttt{b}$$
$$C \to BA \mid \texttt{c}$$

set B for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string `cbcab`.



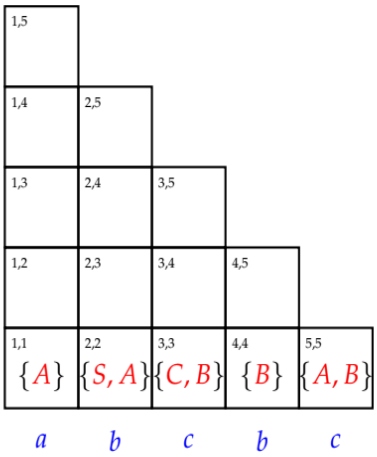
Apply the CYK algorithm to determine whether the string `cbcab` can be derived in the following grammar. You must show the entire CYK table. Here `a`, `b` and `c` are terminals, and the rest are variables.

$$S \to AB \mid BC \mid \texttt{a}$$
$$A \to BC \mid \texttt{c} \mid \texttt{a}$$
$$B \to AB \mid CB \mid \texttt{b}$$
$$C \to BA \mid \texttt{c}$$

set C for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string `abcbc`.



Apply the CYK algorithm to determine whether the string `abcbc` can be derived in the following grammar. You must show the entire CYK table. Here `a`, `b` and `c` are terminals, and the rest are variables.

$$
\begin{aligned}
S &\to AB \mid BC \mid \texttt{a} \\
A &\to BC \mid \texttt{c} \mid \texttt{a} \\
B &\to AB \mid CB \mid \texttt{b} \\
C &\to BA \mid \texttt{c}
\end{aligned}
$$

Set A for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$S \to AB \mid \varepsilon$$
$$A \to AS \mid \mathtt{a} \mid \varepsilon$$
$$B \to Bb \mid A\mathtt{a}B \mid \mathtt{b}$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $X \to \varepsilon$ is removed. (4 points)

$$S \to X\mathtt{a}X\mathtt{b}X \mid X$$
$$X \to \mathtt{a}AX \mid \varepsilon$$
$$A \to AA \mid a$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$S \to AB \mid CA$$
$$A \to C \mid BA \mid \mathtt{a}$$
$$B \to A\mathtt{ba} \mid B\mathtt{bb} \mid C\mathtt{a}$$
$$C \to AB \mid AC \mid A$$

Set B for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$
\begin{aligned}
S_0 &\to S \mid \varepsilon \\
S &\to AB \\
A &\to AS \mid \mathtt{a}Ab \\
B &\to BS_0 \mid \varepsilon
\end{aligned}
$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $A \to \varepsilon$ is removed. (4 points)

$$
\begin{aligned}
S &\to \mathtt{a}ABA \mid A \\
A &\to ABAB \mid \varepsilon \\
B &\to BAb \mid bBABb
\end{aligned}
$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$
\begin{aligned}
S &\to X\mathtt{a} \mid Y\mathtt{b} \\
X &\to Y \mid ZZ \mid \mathtt{a}X \\
Y &\to XY \mid Z\mathtt{b} \mid X \\
Z &\to Z\mathtt{a}Z \mid ZX \mid ZXX
\end{aligned}
$$

Set C for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$
\begin{aligned}
S &\to AA \mid AS \mid \varepsilon \\
A &\to B\mathtt{c}D \mid BD \\
B &\to CD \mid \varepsilon \\
C &\to AD \mid B \\
D &\to AB
\end{aligned}
$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $Y \to \varepsilon$ is removed. (4 points)

$$
\begin{aligned}
S &\to XY\mathtt{a}Y\mathtt{c}YX \mid Y \\
Y &\to X \mid \varepsilon \\
X &\to XX \mid \mathtt{d}
\end{aligned}
$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$
\begin{aligned}
S &\to PQ \mid \mathtt{aa}QQ \\
P &\to RS \mid \mathtt{a}Q \mid R \\
Q &\to Q\mathtt{b} \mid SR \mid \mathtt{a}P \\
R &\to QS \mid \mathtt{a}P \mid P
\end{aligned}
$$

Set A for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. Consider the following language.

$$L_1 = \{\text{Length of } w \text{ is even}\}$$

$$L_2 = \{\mathtt{0}^{2n}(\mathtt{10})^n \text{ , where } n \geq \mathtt{0}\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)

Set B for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. Consider the following language.

$$L_1 = \{w \text{ contains at least two } \mathtt{1}\}$$

$$L_2 = \{(\mathtt{10})^n\mathtt{1}^{2n} \text{ , where } n \geq \mathtt{0}\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)

Set C for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L_1 = \{w \text{ contains } 01 \text{ as a substring}\}$$

$$L_2 = \{w = (01)^n 0^{3n} \text{ , where } n \geq 0\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)