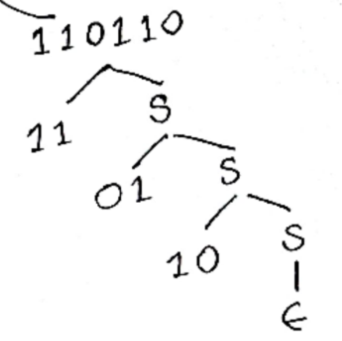
CSE331: Automata and Computability
Quiz 03
Total Marks: 30

No need to show the parse tree, included for better understanding

Obtained Marks

Name: **ID:**

Problem 1: CFG



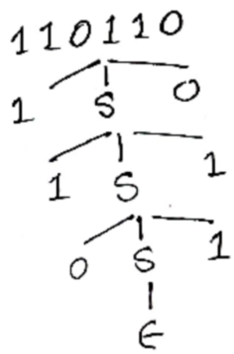
Consider the following two languages.

L1 = {w ∈ {0, 1}*: The length of w is even.}
L2 = {w ∈ {0, 1,#}*: $w = x\#1^n\#0^{2n+1}$ where x ∈ L1 and n ≥ 0 }

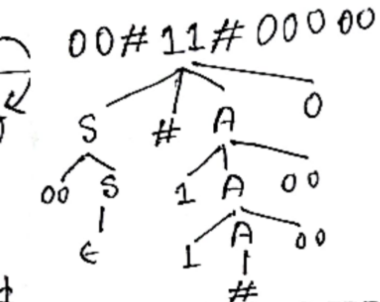
a) Design a context-free grammar whose language is L1. **[Points 6]**

Solution 1: $S \to 00S \mid 01S \mid 10S \mid 11S \mid \epsilon$

Solution 2: ~~On~~, $S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \epsilon$



b) Design a context-free grammar whose language is L2. **[Points 4]**



$W = X\#1^n\#0^{2n+1}$

$= X\#1^n\#0^{2n}0$

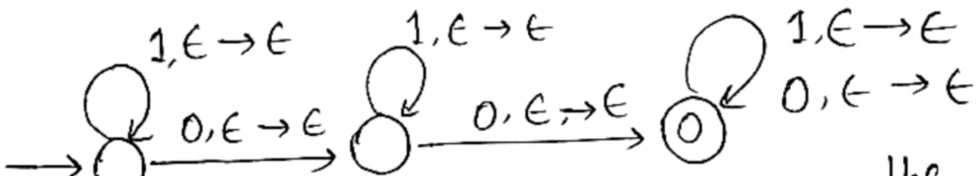
$S' \to S\#A0$

$S \to 00S \mid 01S \mid 10S \mid 11S \mid \epsilon$

$A \to 1A00 \mid \#$

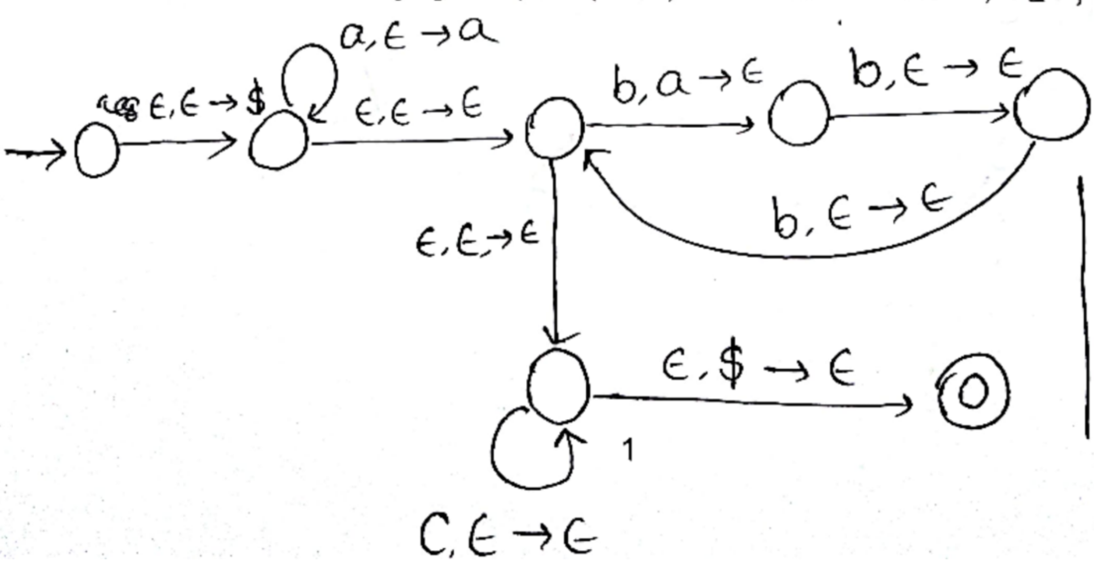
Problem 2: PDF

a) Give a PDA for the language L = { w ∈ {0, 1}*: w contains at least two 0s.} **[4 Points]**



You may push in the stack. But, since the language is regular, no need to use the stack.

b) Give a PDA for the language L = { w ∈ {a, b, c}* : $w = a^nb^{3n}c^m$ where n,m ≥ 0 } **[6 Points]**



Solution 2: You may also push three 'a' by reading one 'a' in the stack.

Problem 3 : Derivations, Parse Trees, and Ambiguity

a) Consider the following context free grammar:

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid 01 \mid 10$$

i) Give a left derivation for the string 011010. **[2 Points]**

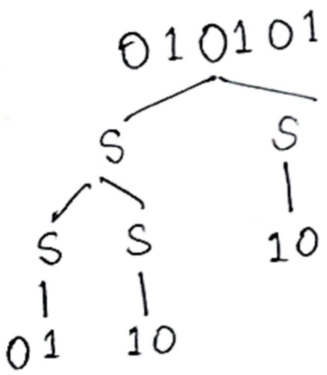
$S \rightarrow SS$

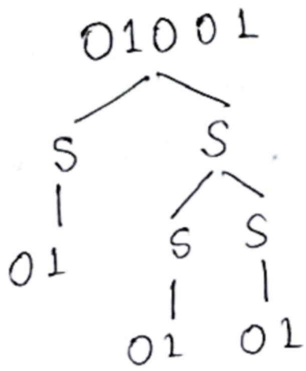
$\rightarrow 01S$

$\rightarrow 01SS$

$\rightarrow 0110S \rightarrow 011010$

ii) Show that the grammar above is ambiguous by demonstrating two different parse trees for 010101. **[3 Points]**

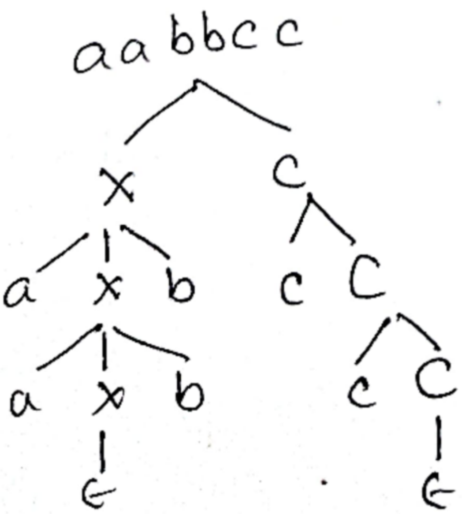


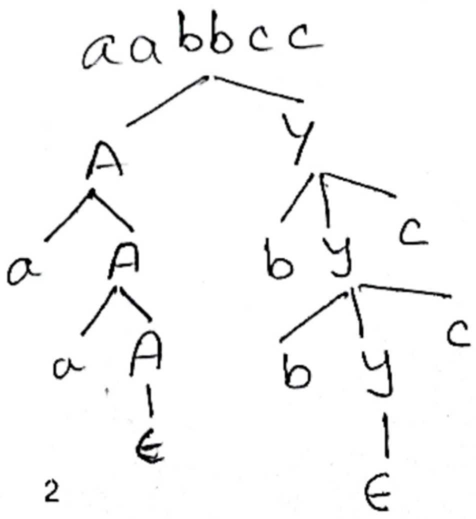


b) Consider the following context free grammar:

$$S \rightarrow XC \mid AY$$
$$X \rightarrow aXb \mid \varepsilon$$
$$Y \rightarrow bYc \mid \varepsilon$$
$$A \rightarrow aA \mid \varepsilon$$
$$C \rightarrow cC \mid \varepsilon$$

Show that the grammar above is ambiguous by finding a length 6 string with two parse trees. **[5 Points]**





Question 03

a) ω starts with '0' and the length of ω is even.

$S \rightarrow 0AB$ $\qquad 0(\Sigma\Sigma)^*(0+1)$

$A \rightarrow 00A \mid 01A \mid 10A \mid 11A \mid \epsilon$

$B \rightarrow 0 \mid 1$

b) Every second letter in ω is b

$((a+b)\, b)^* \cancel{(\epsilon \mid a \mid b)}\, (\epsilon + a + b)$

$S \rightarrow AB$

$A \rightarrow XA \mid \epsilon$

$X \rightarrow PQ$

$P \rightarrow a \mid b$

$Q \rightarrow b$

$B \rightarrow a \mid b \mid \epsilon$

Solution 2 :

$S \rightarrow abS \mid bbS \mid A$ $\qquad$ _b_b_b_

$A \rightarrow a \mid b \mid \epsilon$

c) the length of ω is divisible by ~~3~~ three.

$(\Sigma\Sigma\Sigma)^*$

$\rightarrow ((a+b)(a+b)(a+b))^*$

} No need to write ~~it~~ ~~st~~ in the answer script (Optional)

$S \rightarrow AS \mid \epsilon$

$A \rightarrow BBB$

$B \rightarrow a \mid b$

d) ω starts and ends with different latters.

$a(a+b)^*b + b(a+b)^*a$

$S \rightarrow aAb \mid bAa$

$A \rightarrow aA \mid bA \mid \epsilon$

e) the number of a is at least the number of b in ω

more 'a's

$S \rightarrow \underbrace{aSbS \mid bSaS}_{} \mid aS \mid \epsilon$

ensuring #'b's equal to #'a's, means the amount of b's won't exceed the amont of 'a's

Similar Qs:

ω contain more ~~as~~ 'a's than ~~1s~~ 'b's

$S \rightarrow XaX$

$X \rightarrow aXbX \mid bXaX \mid \cancel{\ } aX \mid \epsilon$

f) $w = 0^n 1^n$, where n is odd.

wrong solution, since this doesn't ensure equal numbers of 0s & 1s.

$S \to AB$ ✗
$A \to 0X$
$X \to 00X \mid \epsilon$
$B \to 1Y$
$Y \to 11Y \mid \epsilon$

$S \to 00S11 \mid A$
$A \to 01$

g) $w = 0^i 1^j 2^k$, where $j \geq 2i + 3k$

$S \to ABC$
$A \to 0A11 \mid \epsilon$
$B \to 1B \mid \epsilon$
$C \to 111C2 \mid \epsilon$

* first try to solve $j = 2i + 3k$

$0^i 1^j 2^k$

$\to 0^i 1^{2i+3k} 2^k$

$\to \underbrace{0^i 1^{2i}} \, \underbrace{1^{3k} 2^k}$

the additional 1s ($j>$) will occur in this place

J) $A = w$ contains at least two 0s

$L = w : w = 0^{3i} v \, 1^{2i}, \; v \in A, \; i \geq 0$

atleast two 0s

$\_\, 0 \,\_\, 0 \,\_$

any combinations of 0s and 1s

$S \rightarrow 000S11 \mid A$

$A \rightarrow X0X0X$

$X \rightarrow 0X \mid 1X \mid \epsilon$