CSE331
Automata and Computability

MIDTERM EXAM
TOTAL MARKS: 50
DURATION: 90 MINUTES

B

There are a total of five problems. You have to solve the first four. Problem 5 is optional.

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ. Note that we define 0^m to be the string $\overbrace{000\ldots000}^{m \text{ times}}$. 1^n is defined analogously.

$$L_1 = \{w : w \text{ does not contain } 01 \text{ as a substring}\}$$

$$L_2 = \{0^m : m \geq 0\}$$

$$L_3 = \{1^n : n \text{ is even}\}$$

$$L_4 = L_2 \circ L_3$$

Now solve the following problems.

(a) **Give** the state diagram for a DFA that recognizes L_1. (4 points)

(b) **Give** the state diagram for a DFA that recognizes L_3. (4 points)

(c) **Find** all the four and five-letter strings in L_4. (1 point)

(d) **Give** the state diagram for a DFA that recognizes L_4. (2 points)

(e) If you were to use the "cross product" construction shown in class to obtain a DFA for the language $L_1 \cap L_4$, how many states would it have? (1 point)

(f) **Find** all five-letter strings in $L_1 \cap L_4$. (1 point)

(g) **Give** the state diagram for a DFA that recognizes $L_1 \cap L_4$ using only five states. (2 points)

Problem 2 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{a, b\}$. **Give** regular expressions generating each of the following languages over Σ.

(a) $\{w$: the first and last letters of w are a and b respectively$\}$ (3 points)

(b) $\{w$: the length of w is odd$\}$ (3 points)

(c) $\{w$: every a in w is followed by an even number of bs$\}$ (3 points)

(d) $\{w$: w does not contain ab$\}$ (3 points)

(e) $\{w$: ab appears in w exactly once$\}$ (3 points)
(Hint: If $w = x\mathtt{ab}y$, what can you say about x and y?)

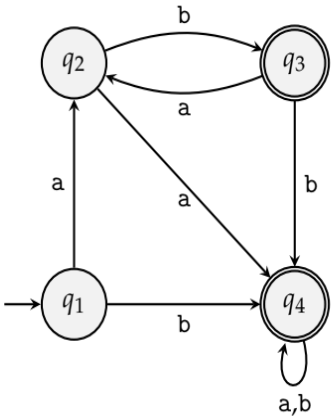
Problem 3 (CO3): Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression over $\Sigma = \{a, b, c\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$(bc)^*ab + (bc + a^*)^*a$$

Problem 4 (CO3): Converting Finite Automata to Regular Expressions (10 points)

Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_2, then q_3, and finally q_4. You must show work.



Problem 5 (Bonus): Sum of Squares Modulo Three (5 points)

Disclaimer: This is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!

Let $\Sigma = \{0, 1\}$. For $w \in \Sigma^*$, we denote by $h(w)$ the number of 1s in w. For example, $h(100101) = 3$. Consider the following language over Σ. As always, $|w|$ is the length of the string w.

$$L = \{w : |w|^2 + h(w)^2 \text{ is one more than a multiple of three}\}$$

Give the state diagram for a DFA that recognizes L.