FINAL EXAM
TOTAL MARKS: 50
DURATION: 100 MINUTES

**A**

There are a total of five problems. You have to solve all of them.

## Problem 1 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$. Consider the following pair of languages. Recall that for a string $w$, $|w|$ denotes the length of $w$.

$$L_1 = \{w \in \Sigma^* : w \text{ contains exactly two } \texttt{1}\text{s}\}$$
$$L_2 = \{x\#y : x \in \Sigma^*, y \in L_1, |x| = |y|\}$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language $L_1$. (4 points)

(b) **Write** down four seven-letter strings in $L_2$. (1 point)

(c) **Give** a context-free grammar for the language $L_2$. (5 points)

## Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \to AB \mid \texttt{b}AA \mid SS$$
$$A \to \texttt{a} \mid \texttt{a}A$$
$$B \to \texttt{b} \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string abaaab. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse tree (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** a string $w$ of length five such that $w$ has exactly one parse tree in the grammar above. (1 point)

## Problem 3 (CO4): The CYK Algorithm (10 points)

**Apply** the CYK algorithm to determine whether the string yxzyz can be derived in the following grammar. You must show the entire CYK table. Here x, y and z are terminals, and the rest are variables.

$$S \to AB$$
$$A \to AC \mid \texttt{y}$$
$$B \to BD \mid EB \mid \texttt{z}$$
$$C \to AE \mid \texttt{x}$$
$$D \to \texttt{y}$$
$$E \to BD \mid \texttt{x}$$

## Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$\begin{aligned} S &\to \texttt{a} \mid AB \\ A &\to SB \mid \texttt{ba} \mid B \\ B &\to b \mid \varepsilon \end{aligned}$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $X \to \varepsilon$ is removed. (4 points)

$$\begin{aligned} S &\to \texttt{a}X\texttt{b}X\texttt{c}X \mid X \\ X &\to \texttt{a} \mid \varepsilon \end{aligned}$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned} S &\to A\texttt{a} \\ A &\to BC \mid C \\ B &\to \texttt{b} \\ C &\to A \mid AA \mid A\texttt{a}B \end{aligned}$$

## Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$. Note that we define $c(w, x)$ to be the count of $x$ in the string $w$.

$$\begin{aligned} L_1 &= \{w \mid w \text{ is a palindrome and the length of } w \text{ is odd.}\} \\ L_2 &= \{w_1\#w_2\#w_3 \mid c(w_1, \texttt{0}) = c(w_2, \texttt{0}) \text{ or } c(w_2, \texttt{1}) = c(w_3, \texttt{1})\} \end{aligned}$$

Now solve the following problems.

(a) **Give** the state diagram of a pushdown automaton that recognizes $L_1$. (4 points)

(b) **Find** all strings $w \in L_2$ such that $w$ starts with `00010#1001#` and has a length of 14. (1 point)

(c) **Give** the state diagram of a pushdown automaton that recognizes $L_2$. (5 points)