CSE331
Automata and Computability

FINAL EXAM
TOTAL MARKS: 50
DURATION: 105 MINUTES

A

There are a total of six problems. You have to solve all of them.

Problem 1 (CO5): Pumping Lemma (5 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 0^a 1^b 1^c 0^d, \text{ where } a + b = c + d \text{ and } a, b, c, d \geq 0\}$$

Use the pumping lemma to **demonstrate** that L is not regular language.

Problem 2 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{w \in \Sigma^* : w \text{ is an even length palindrome}\}$$
$$L_2 = \{w \in \Sigma^* : \text{length of } w \text{ is even}\}$$
$$L_3 = \{x11y : x, y \in L_2, |x| = |y|\}$$
$$L_4 = L_1 \cap L_3$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language L_1. (3 points)

(b) **Give** a context-free grammar for the language L_3. (4 points)

(c) **Give** a context-free grammar for the language L_4.(3 point)

Problem 3 (CO4): The CYK Algorithm (5 points)

Apply the CYK algorithm to fill up the table for the string aaaba using the following grammar. Here a and b are terminals, and the rest are variables.

$S \to BA \mid BC$
$A \to AB \mid AC \mid a$
$B \to CB \mid CC \mid b$
$C \to CA \mid a$

1,5 {?}				
1,4 $\{S, A, B, C\}$	2,5 {?}			
1,3 {?}	2,4 $\{A, B, C\}$	3,5 {?}		
1,2 $\{A, B, C\}$	2,3 $\{A, B, C\}$	3,4 {?}	4,5 $\{S\}$	
1,1 $\{A, C\}$	2,2 $\{A, C\}$	3,3 $\{A, C\}$	4,4 $\{B\}$	5,5 $\{A, C\}$
a	*a*	*a*	*b*	*a*

Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

$$L_1 = \{w \mid w \text{ starts and ends with the same character}\}$$

$$L_2 = \{w \mid \text{the number of 0s in } w \text{ is not same as the number of 1s}\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)

Problem 5 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to 1A \mid 1C \mid 0B \mid 00A$$
$$B \to 0A \mid 1B \mid 00B$$
$$C \to 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string 01011001. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

(e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

Problem 6 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (5 points)

$$M \to \varepsilon \mid \mathtt{x} \mid MN$$
$$N \to NC \mid \mathtt{yy} \mid P$$
$$P \to QQ \mid \mathtt{z}$$
$$Q \to \mathtt{y}Q \mid \varepsilon$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $B \to \varepsilon$ is removed. (3 points)

$$S \to A\mathtt{a} \mid BB$$
$$A \to CA \mid B\mathtt{a}B \mid \varepsilon$$
$$B \to AB \mid \mathtt{b} \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$X \to 0X1Y1 \mid Y$$
$$Y \to XY \mid YY \mid Z$$
$$Z \to 1Z \mid Z$$