CSE331: Automata and Computability
Quiz on: Context-Free Grammar

Construct CFG for the following grammars.

a) L = {w ∈ {(,)}*: w is a valid parenthesis.}

$$S \rightarrow (S) \mid SS \mid \varepsilon$$

d) L = {w ∈ {a, b, 1}*: $a^n 1^{2n+m+3} b^m$, where n, m ≥ 0}

$$a^n \, 1^{2n+m+3} \, b^m$$

$$\Rightarrow \underbrace{a^n 1^{2n}} \, \overbrace{1^m 1^3 b^m}$$

$$S \rightarrow PQ$$
$$P \rightarrow aP11 \mid \varepsilon$$
$$Q \rightarrow 1Qb \mid 111$$

or $\underbrace{a^n 1^{2n}} \, 1^3 \, \overbrace{1^m b^m}$

$$S \rightarrow P111Q$$
$$P \rightarrow aP11 \mid \varepsilon$$
$$Q \rightarrow 1Qb \mid \varepsilon$$

b) Convert the following Regular Expressions into a CFG: ((aa+ bc)* a)* + cb

$$\underbrace{(\underbrace{\underbrace{(\overbrace{aa+bc}^{F})^*}_{M}\, a}_{P})^*}_{X} + \underbrace{cb}_{Y}$$

$$S \rightarrow X \mid Y$$
$$X \rightarrow PX \mid \varepsilon$$
$$P \rightarrow Ma$$
$$M \rightarrow FM \mid \varepsilon$$
$$F \rightarrow aa \mid bc$$
$$Y \rightarrow cb$$

c) L = {w ∈ {0, 1}*: length of w is odd and the mid is 1.}

$$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 1$$

or, $S \rightarrow XSX \mid 1$

$$X \rightarrow 0 \mid 1$$

Wrong Solve:

$$\cancel{S \rightarrow A1A}$$
$$\cancel{A \rightarrow 0A \mid 1A \mid \varepsilon}$$

d) L = {w ∈ {0,1}*: w is an odd-length palindrome.}

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1$$

d) L = {w ∈ {0,1}*: w is an even-length palindrome.}

$$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$$

e) L = {w ∈ {0, 1}*: the length of w is three more than the multiple of four.}

$A \rightarrow MMMMA \mid MMM$

$M \rightarrow 0 \mid 1$

f) L = {w ∈ {0, 1}*: the length of w is two more than the multiple of three.}

$A \rightarrow MMMA \mid MM$

$M \rightarrow 0 \mid 1$

g) L1 = {w ∈ {0, 1}*, length of w is odd.}
L2 = { w1#w2, where w1, w2 ∈ L1 and |w1| = |w2|}.
Construct a CFG for L2.

$S \rightarrow XXSXX \mid X\#X$

$X \rightarrow 0 \mid 1$

h) L1 = {w ∈ {0, 1}*: count of 0 is a multiple of three.}
L2 = { w1#w2, where w1 ∈ {0,1}*, w2 ∈ L1 and |w1| = |w2|}.
Construct a CFG for L2.

$A \rightarrow XA1 \mid XB0 \mid \#$

$B \rightarrow XB1 \mid XC0$

$C \rightarrow XC1 \mid XA0$

i) L = {w ∈ {a, b, c, d}*: $a^n c^m d^m b^{3n}$, where $n \geq 0$ and $m \geq 1$ }

$a^n c^m d^m b^{3n}$

$S \rightarrow aSbbb \mid X$

$X \rightarrow cXd \mid cd$

$m \geq 1$

Alternate solution

$S \rightarrow aSbbb \mid cXd$

$X \rightarrow cXd \mid \varepsilon$

j) L = {w ∈ {0, 1}*: $0^n 1^m$, where $m \geq 3n$}
[Hint: First, try to solve for m=3n]

if $m = 3n$

$0^n 1^{3n}$

since $m \geq 3n$

$0^n 1^{3n}$ can have more 1s

$m = 3n$

$S \rightarrow 0S111 \mid A$

$A \rightarrow A1 \mid \varepsilon$

k) $L = \{w \in \{a, b, c, d\}^*: a^n c^{m+3} d^m b^{3n}$, where $n \geq 0$ and $m \geq 1\}$ $a^n c^{m+3} d^m b^{3n}$ $\Rightarrow a^n c^m \underbrace{c^3} d^m b^{3n}$ $S \to aSbbb \mid cAd \to m \geq 1$ $A \to cAd \mid c$ Alternate solution: $S \to aSbbb \mid A_{m \geq 1}$ $A \to cAd \mid c\underbrace{ccc}_{c^3}d$	l) $L3 = \{w \in \{a, b, 1\}^*: a^n 1^{k+3} b^m$, where $k=2n+m$ and $n, m, k \geq 0\}$ $a^n 1^{K+3} b^m$ $\Rightarrow a^n 1^{2n+m+3} b^m$ Previously solved, same as Q5:
m) $L = \{w \in \{0, 1\}^*$: length of w is a multiple of six.} $S \to XXXXXXS \mid \varepsilon$ $X \to 0 \mid 1$	n) $L = \{w \in \{0, 1\}^*$: w1##w2, where w1,w2 $\in \{0, 1\}^*$ and $\lvert w1 \rvert = \lvert w2 \rvert$.} $S \to XSX \mid \#\#$ $X \to 0 \mid 1$
o) $L1 = \{w \in \{0, 1\}^*$: length of w is multiple of six} $L2 = \{w \in \{0, 1\}^*$: w1##w2, where w1,w2 $\in \{0, 1\}^*$ and $\lvert w1 \rvert = \lvert w2 \rvert$.} $L = L1 \cap L2$, Construct a CFG for L. $S \to XXXSXXX \mid XX\#\#XX$ $X \to 0 \mid 1$ multiple of six Since ## in fixed, need to adjust the terminate length	p) $L = \{w \in \{0, 1\}^*$: w contains at least three 1s.} $(0+1)^* 1 (0+1)^* 1 (0+1)^* 1 (0+1)^*$ $S \to X1X1X1X$ $X \to 0X \mid 1X \mid \varepsilon$

q) L1 = {w ∈ {0, 1}*: w contains at least three 1s.}
L2 = {w ∈ {0, 1}*: w1#w2, where w1,w2 ∈ {0, 1}* and |w1| = |w2|.}
L = L1 ∩ L2, Construct a CFG for L.

found no 1

0 0 1 0 0 1 # 1 1 0 1 0 0

found one 1

S → 0S0 | 1A0 | 0A1 | 1B1

found two 1s

A → 0A0 | 1B0 | 0B1 | 1C1

found three or more 1s

B → 0B0 | 1C0 | 0C1 | 1C1

C → XCX | #

more than three 1s

X → 0|1

r) L1 = {w ∈ {0, 1}*: w contains exactly three 1s.}
L2 = {w ∈ {0, 1}*: w1#w2, where w1,w2 ∈ {0, 1}* and |w1| = |w2|.}
L = L1 ∩ L2, Construct a CFG for L.

0 0 1 0 0 1 0 # 0 0 1 0 0 0 0 0

found one 1

S → 0S0 | 1A0 | 0A1 | 1B1

found two 1s

A → 0A0 | 1B0 | 0B1 | 1C1

found three 1s.

B → 0B0 | 1C0 | 0C1

C → 0C0 | #

s) L1 = {w ∈ {0, 1}*: w contains exactly three 1s.}
L2 = {w ∈ {0, 1}*: w1#w2, where w1 ∈ {0, 1}*,
w2 ∈ L1, and |w1| = |w2|.}
Construct a CFG for L2.

0 0 0 1 1 0 1 1 # 0 0 1 0 1 1 0 0

S → XS0 | XA1

A → XA0 | XB1

B → XB0 | XC1

C → XC0 | #

C → 0|1