CSE331
Automata and Computability

FINAL EXAM
TOTAL MARKS: 50
DURATION: 90 MINUTES

A

There are a total of six problems. You have to solve the first five. Problem 6 is optional.

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{\#}\}$. Consider the following languages over Σ.

$$L_1 = \{\texttt{a}^i\texttt{\#}\texttt{b}^j \mid i \text{ is a multiple of three and } i, j \geq 0\}$$
$$L_2 = \{\texttt{a}^i\texttt{\#}\texttt{b}^j \mid j = 2 + 3i \text{ and } i, j \geq 0\}$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language L_1. (5 points)

(b) **Give** a context-free grammar for the language L_2. (5 points)

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below.

$$\begin{aligned} S &\to PE \\ P &\to \texttt{a}P\texttt{a} \mid \texttt{b}P\texttt{b} \mid \texttt{a} \mid \texttt{b} \\ E &\to \texttt{aa}E \mid \texttt{ab}E \mid \texttt{ba}E \mid \texttt{bb}E \mid \varepsilon \end{aligned}$$

Now solve the following problems.

(a) **Give** a leftmost derivation for the string ababab. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** how many parse trees the string abababa has in the grammar above. (1 point)

Problem 3 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string ababc can be derived in the following grammar. You must show the entire CYK table. Here a, b and c are terminals and the rest are variables.

$$\begin{aligned} S &\to AB \mid BC \mid \texttt{a} \\ A &\to BC \mid \texttt{a} \mid \texttt{c} \\ B &\to AB \mid \texttt{b} \\ C &\to AC \mid \texttt{b} \mid \texttt{c} \end{aligned}$$

Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$. Consider the following language.

$$L = \{x\texttt{\#}y : x, y \in \Sigma^*, \text{ and the number of occurrences of } \texttt{01} \text{ in } x \text{ is equal to the number of occurrences of } \texttt{10} \text{ in } y\}$$

Solve the following problems.

(a) Find all strings $w \in L$ such that w starts with 110011010# and has a length of 15. (3 points)

(b) Give the state diagram of a pushdown automaton that recognizes L. (7 points)

Problem 5 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (5 points)

$$\begin{aligned} S &\to \mathtt{a}X\mathtt{b}Y \mid X \\ X &\to \mathtt{a} \mid \mathtt{ba} \mid YY \\ Y &\to \mathtt{c} \mid \varepsilon \end{aligned}$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $Y \to \varepsilon$ is removed. (3 points)

$$\begin{aligned} S &\to \mathtt{a}X\mathtt{b}Y \mid X \\ X &\to \mathtt{a} \mid \mathtt{ba} \mid YY \\ Y &\to \mathtt{c} \mid \varepsilon \end{aligned}$$

(c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned} S &\to \mathtt{a}A\mathtt{b}B \mid A \\ A &\to \mathtt{a} \mid \mathtt{b} \mid B \\ B &\to \mathtt{c} \mid \mathtt{d} \mid A \end{aligned}$$

Problem 6 (Bonus): Even Distances in Reverse (5 points)

Disclaimer: This is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!

Given a string w over some alphabet Σ, let $w^{\mathcal{R}}$ be its reverse. For example, if $w = \mathtt{1011}$, then $w^{\mathcal{R}} = \mathtt{1101}$. The *Hamming distance* between two strings of equal length is the number of positions at which the corresponding symbols differ. For example, the Hamming distance between the strings $\mathtt{11011}$ and $\mathtt{10001}$ is two since these strings differ at two positions: position two and position four.

Now consider the following language over $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

$$L = \{xy : \text{the Hamming distance between } x \text{ and } y^{\mathcal{R}} \text{ is even}\}$$

(a) **Show** that L is context-free by giving a pushdown automaton for it. (2 points)

(b) **Give** a context-free grammar for L as well. (3 points)