CSE331
Assignment - 02
CFG Solution
Faculty: KKP
(IF you spot any error, please notify me)

3) Design a Context Free Grammar for the Language:

a) L = {w ∈ {a,b,c,p,q,r,#}*: $a^i\#^n c^k p^{2x} q^y r^z b^j$ where i=j+k, y=3x+z, n is odd and i,j,k,n,x,y,z ≥ 0}

b) L = {w ∈ {0,1,2}*: w = $0^i 2^j 1^k$, [where ......conditions......] }

where...

~~i) i = k, i,k ≥ 1 and j ≥ 2~~

ii) i = 3k, j is odd and i,j,k ≥ 0

iii) i is a multiple of two, k is two more than a multiple of 3, j = k+i, and i,j,k ≥ 0

iv) i+j > k and i,j,k ≥ 0

v) i+k is even, j = i+k and j>=1

c) L = {w ∈ {0,1}*: the parity of 0s and 1s is different in w}

d) L = {w ∈ {0,1}*: the number of 0s and 1s are different in w} [Hint: First, try to solve for an equal number of 0s and 1s in w]

e) L = {$1^i 0 2^j 1^k$| i, j, k ≥ 0, 3i ≥ 4k + 2, j is not divisible by three}

f) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}

L1 = {w ∈ Σ^*: w contains exactly two 1s}

L2 = {x#y : x ∈ Σ^*, y ∈ L1, |x| = |y|}

Construct a CFG for L2.

g) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}

L1 = {w ∈ Σ^*: w contains at least three 1s}

L2 = {x#y : x ∈ $(\Sigma\Sigma)^*$, y ∈ L1, |x| = |y|}

Construct a CFG for L2.

$$L = \{ w \in \{a, b, c, p, q, r, \#\}^* : a^i \#^n c^k p^{2x} q^y r^z b^j,$$
$$\text{where } i = j + k,\ y = 3x + z,\ n \text{ is odd and}$$
$$i, j, k, n, x, y, z \geq 0 \}$$

$$a^i \#^n c^k p^{2x} q^y r^z b^j$$

$$\Rightarrow a^{j+k} \#^n c^k p^{2x} q^{3x+z} r^z b^j$$

$$\Rightarrow a^j a^k \#^n c^k p^{2x} q^{3x} q^z r^z b^j$$

$S \rightarrow aSb \mid T$

$T \rightarrow ABC$

$A \rightarrow aAc \mid X$

$X \rightarrow \#\#X \mid \#$

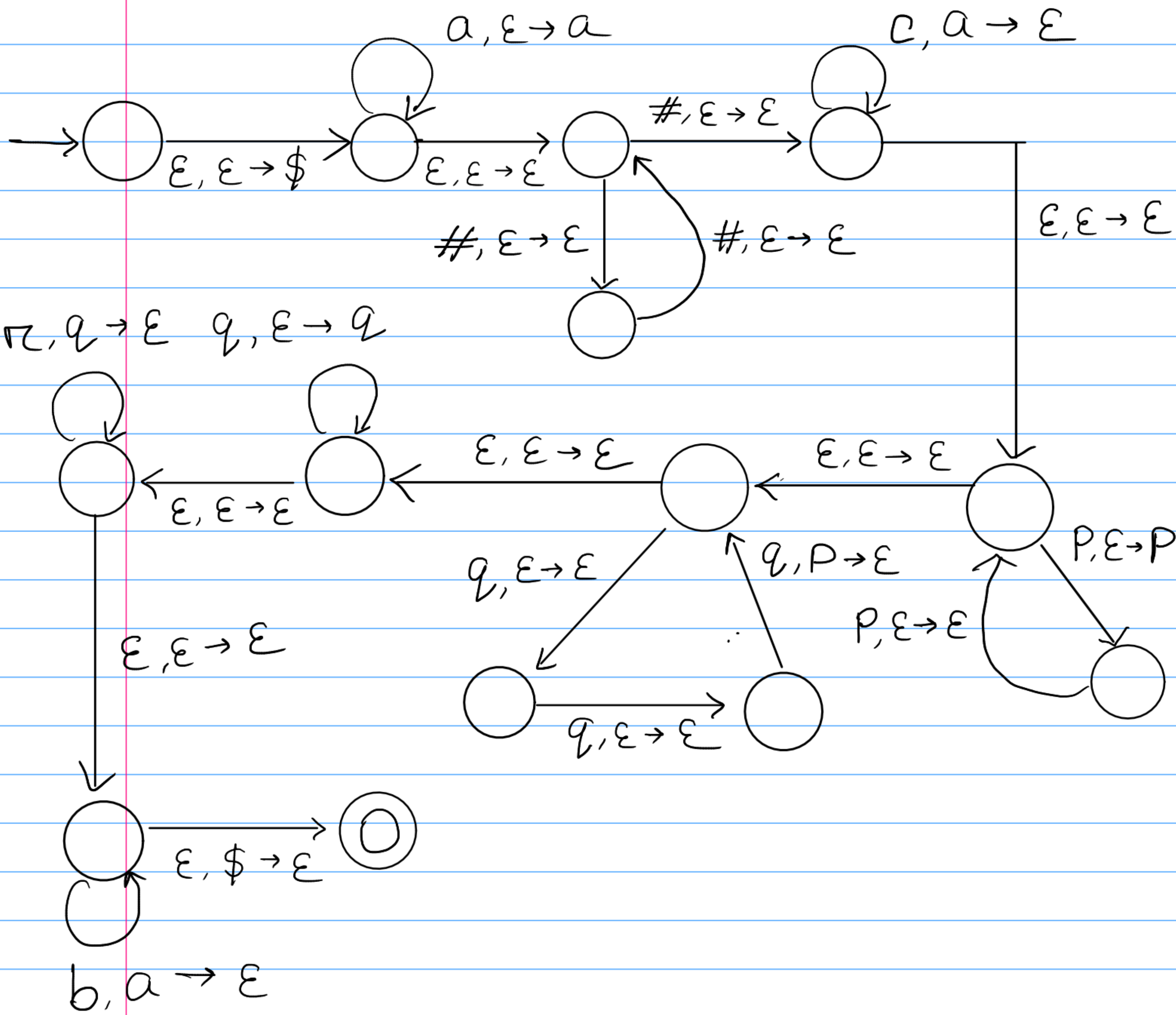
$B \rightarrow ppBqqq \mid \varepsilon$

$C \rightarrow qCr \mid \varepsilon$

$L = \{\omega \in \{a, b, c, p, q, r, \#\}^* : a^i \#^n c^k p^{2x} q^y r^z b^j,$
where $i = j + k$, $y = 3x + z$, n is odd and
$i, j, k, n, x, y, z \geq 0\}$

$$a^i \#^n c^k p^{2x} q^y r^z b^j$$

$$\Rightarrow \quad a^j a^k \#^n c^k p^{2x} q^{3x} q^z r^z b^j$$



$$L = \{ \omega \in \{0,1,2\}^* : w = 0^i 2^j 1^k, \text{ where } i = k, \; i,k \geq 1, \; j \geq 2 \}$$

$0^i 2^j 1^k$

$\Rightarrow 0^i 2^j 1^i$

0221, 00022111, $022221 \in L$

22, 01, 021, $022 \notin L$

Solution:

$i,k \geq 1$

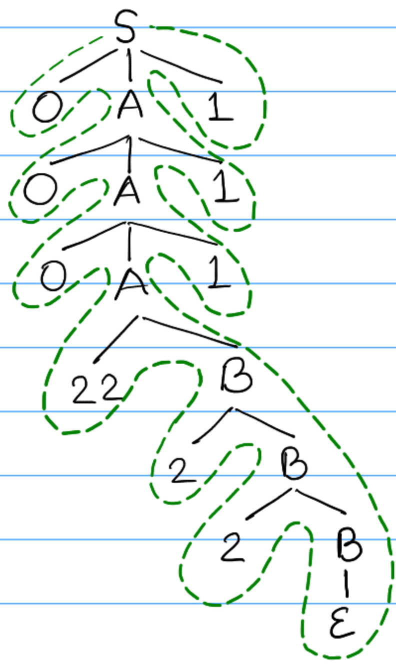
$S \rightarrow 0A1$

$A \rightarrow 0A1 \mid 22B$

$B \rightarrow 2B \mid \varepsilon$

$j \geq 2$

0002222111



Another solution:

$S \rightarrow 0S1 \mid 0\underbrace{22}_{j \geq 2}A1$ ($i,k \geq 1$)

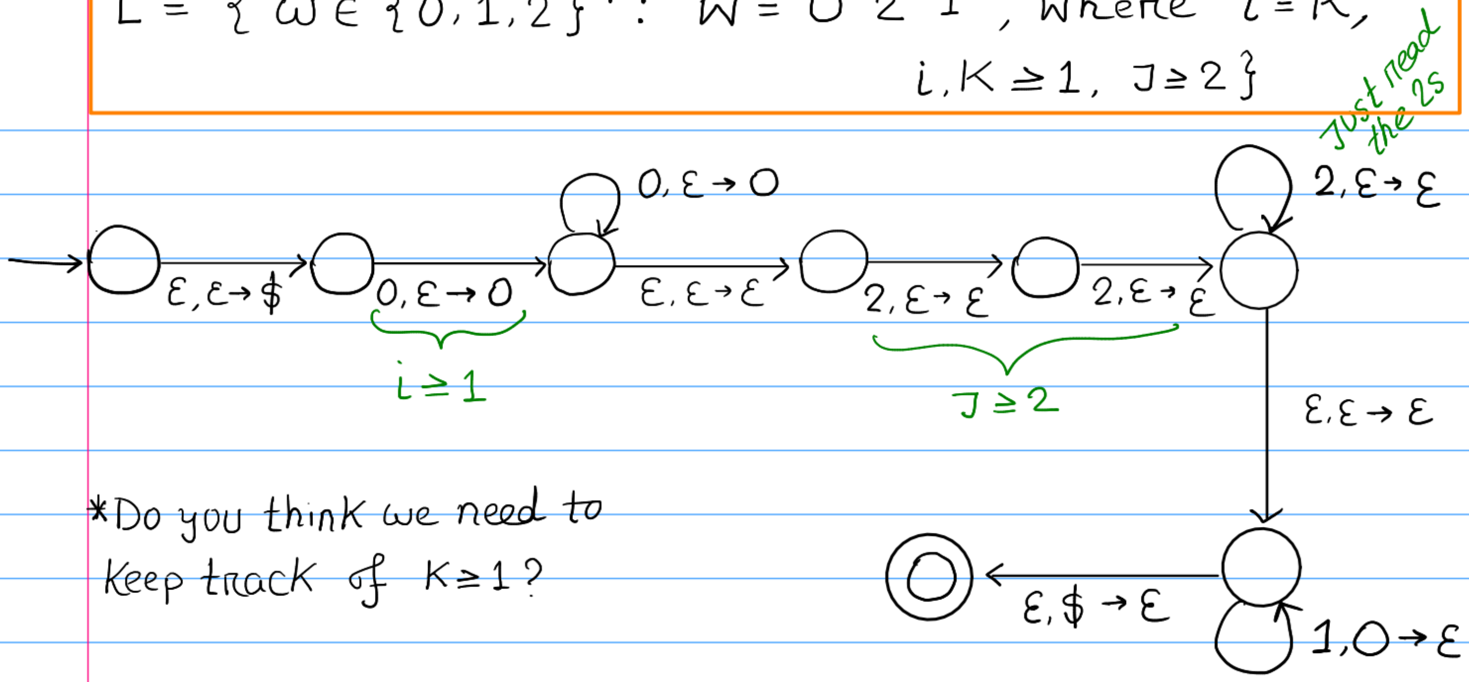
$A \rightarrow 2A \mid \varepsilon$

Another Solution:

$S \rightarrow 0S1 \mid 0A1$ $\rightarrow i,k \geq 1$

$A \rightarrow 2A \mid 22$ $\rightarrow j \geq 2$

$$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^J 1^K, \text{ where } i = K, \; i, K \geq 1, \; J \geq 2 \}$$



*Do you think we need to keep track of K ≥ 1?

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, where $i = 3k$, j is odd and $i, j, k \geq 0 \}$

$0^i 2^j 1^k$

$\Rightarrow 0^{3k} 2^j 1^k$

$k = 0$, 'odd 2s'

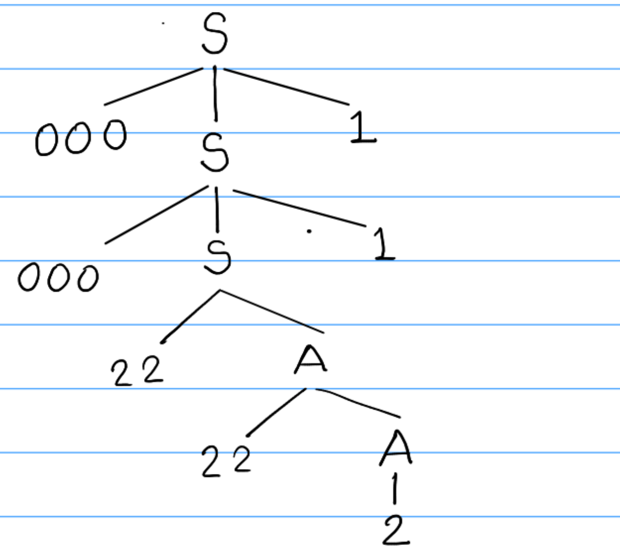
$k = 1$, 000 'odd 2s' 1

$k = 2$, 000000 'odd 2s' 11

Solution:

$S \rightarrow 000S1 \mid A$

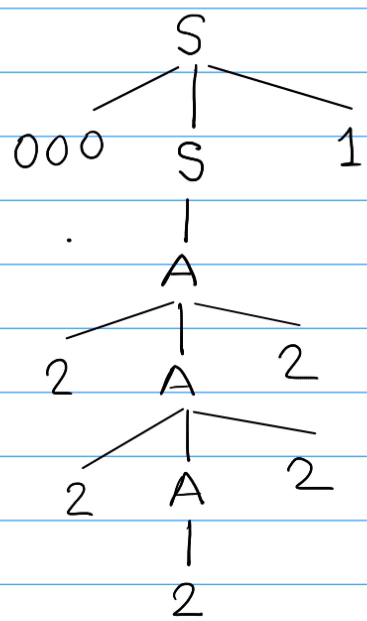
$A \rightarrow 22A \mid 2$



Another Solution:

$S \rightarrow 000S1 \mid A$

$A \rightarrow 2A2 \mid 2$



* It is not mandatory to draw the parse tree in the qs of CFG. I have drawn for the understanding purpose.

$$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k, \text{ where } i = 3k, \; j \text{ is odd and } i,j,k \geq 0 \}$$

$0^i 2^j 1^k$

$\Rightarrow 0^{3k} 2^j 1^k$

$k = 1$,

000 'odd 2s' 1

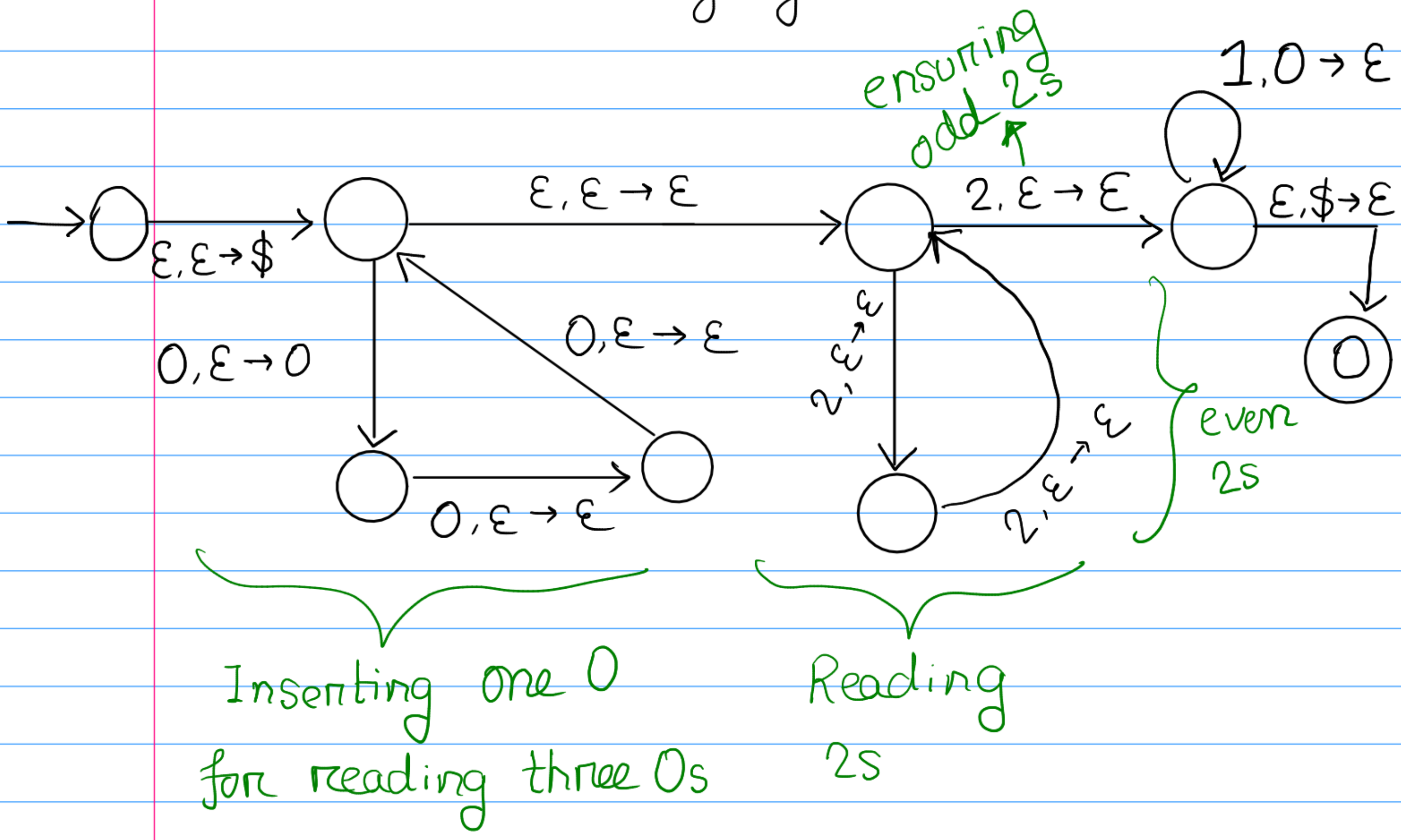
0 'odd 2s' 1

$k = 2$,

$\underbrace{000}_{0}\underbrace{000}_{0}$ 'odd 2s' 1 1

0 0 'odd 2s' 1 1

So, if we insert one 0 for reading three 0s, the language becomes 0^k odd 2 1^k



$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, i is multiple of two, k is two more than multiple of three, $j = k + i$, and $i, j, k \geq 0 \}$

Be careful with maintaining the same order

$$0^i \, 2^j \, 1^k$$

$$\Rightarrow 0^{2i} \, 2^{3k+2+2i} \, 1^{3k+2}$$

$$\Rightarrow \underbrace{0^{2i} \, 2^2 \, 2^{2i}}_{A} \; \underbrace{2^{3k} \, 1^2 \, 1^{3k}}_{B}$$

$S \rightarrow AB$

$A \rightarrow 00A22 \mid 22$

$B \rightarrow 222B111 \mid 11$

Some 0s.... Some 2s.... Some 1s

Another Solution:

$$0^i \, 2^j \, 1^k$$

$$\Rightarrow 0^{2i} \, 2^{3k+2+2i} \, 1^{3k+2}$$

$$\Rightarrow \underbrace{0^{2i} \, 2^{2i}} \; \underbrace{2^2} \; \underbrace{2^{3k} \, 1^{3k}} \; \underbrace{1^2}$$

$S \rightarrow ABCD$

$A \rightarrow 00A22 \mid \varepsilon$

$B \rightarrow 22$

$C \rightarrow 222C111 \mid \varepsilon$

$D \rightarrow 11$

Another Solution:

some 0s.... some 2s.... some 1s

Be careful with maintaining the same order

$$0^i\, 2^j\, 1^k$$

$$\Rightarrow 0^{2i}\, 2^{3k+2+2i}\, 1^{3k+2}$$

$$\Rightarrow 0^{2i}\, 2^{3k}\, 2^2\, 2^{2i}\, 1^{3k}\, 1^2$$

$$\Rightarrow 0^{2i}\, 2^{2i}\, 2^{3k}\, \underbrace{2^2}\, \underbrace{1^2}\, 1^{3k}$$

$$S \rightarrow AB$$
$$A \rightarrow 00A22 \mid \varepsilon$$
$$B \rightarrow 222B111 \mid 2211$$

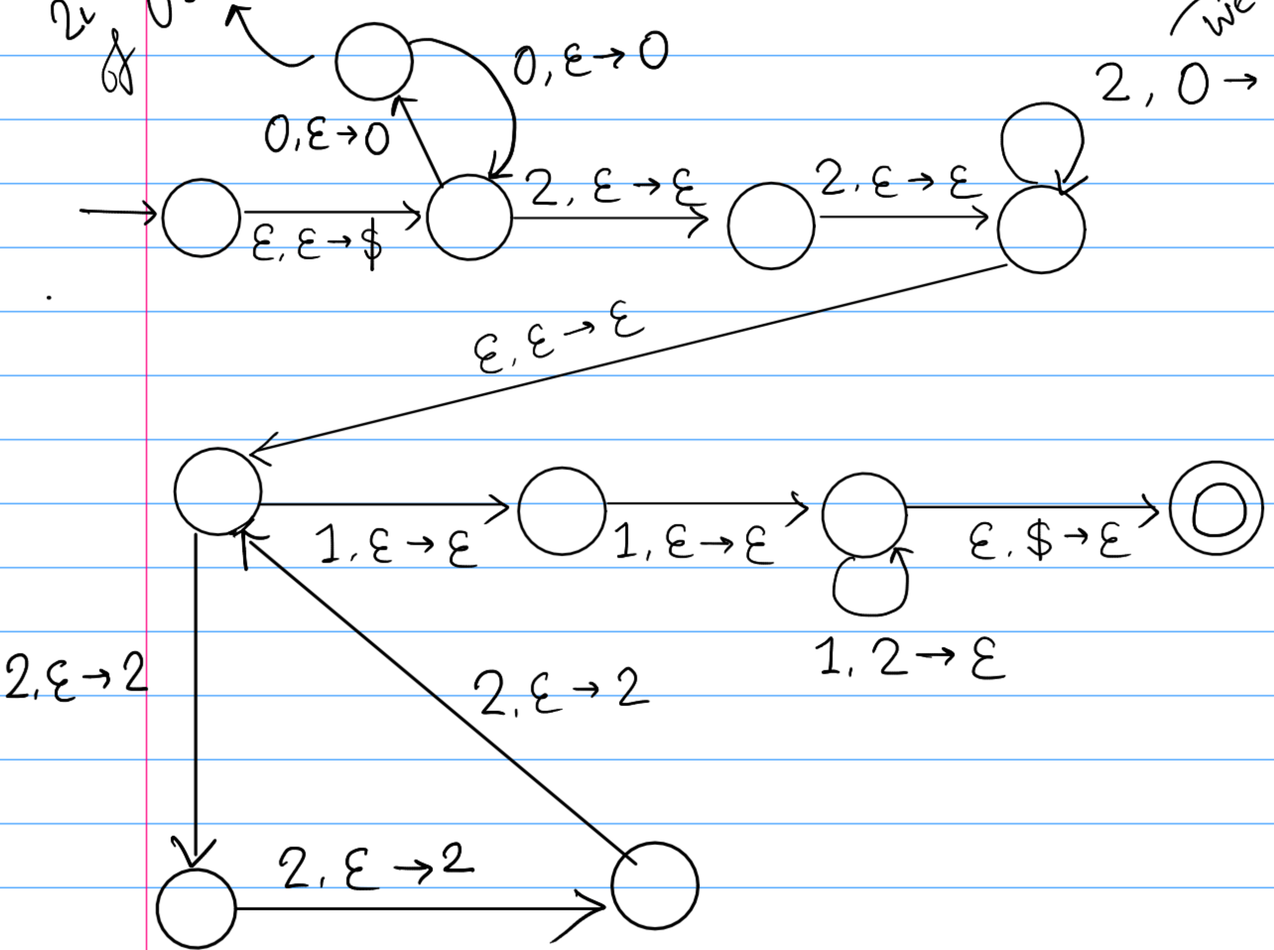
$L = \{ \omega \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, i is multiple of two, K is two more than multiple of three, $J = K + i$, and $i, J, K \geq 0 \}$

$$0^i 2^j 1^K$$

$$\Rightarrow 0^{2i} \, 2^{3K+2+2i} \, 1^{3K+2}$$

$$\Rightarrow \underbrace{0^{2i} \, 2^2 \, 2^{2i}}_{A} \, \underbrace{2^{3K} \, 1^2 \, 1^{3K}}_{B}$$

Since, $0^{2i} = 2^{2i}$ we are not checking $2i$, you can check though



$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, where $i+j > k$ and $i,j,k \geq 0 \}$

let's first solve for $i+j=k$

$$0^i 2^j 1^k$$

$$\Rightarrow 0^i 2^j 1^{i+j}$$

$$\Rightarrow 0^i 2^j 1^j 1^i$$

Now since $i+j>k$,

$0^i 2^j 1^j 1^i$ or $0^i \underbrace{2^j} 1^j 1^i$

there could be more 0s and equal 2s & 1s

means, in $i+j>k$, $j=k$ and $i>k$

00 00 2 2 2 1 1 1 1 1

more 0s

there could be more 2s and equal 0s & 1s

means, in $i+j>k$, $i=k$ and $j>k$

00 2 2 2 2 2 1 1 1

you can also think in this way

00 00 2 2 2 1 1 1 1 1

more 0s

or, there could be more 0s and 2s more than 1s both,

means in $i+j>k$

$i>k$ and $j>k$

Too summarize,

$0^i \_\, 2^j \; 1^j \; 1^i$

→ equal 0s & 1s

equal 2s & 1s

$000\ldots2222$

↳ either more 0s

↳ either more 2s

↳ or both

→ equal 0s & 1s

$P \to 0P1 \mid X$

$X \to ABQ \mid CDQ$

where $i>k$, $j=k$

$i+j>k$

$A \to 0A \mid 0$ → more 0s

$B \to 2B \mid \varepsilon$ → there could be more 2s (handling the both case) → $i>k$ & $j>k$

where $j>k$, $i=k$

$i+j>k$

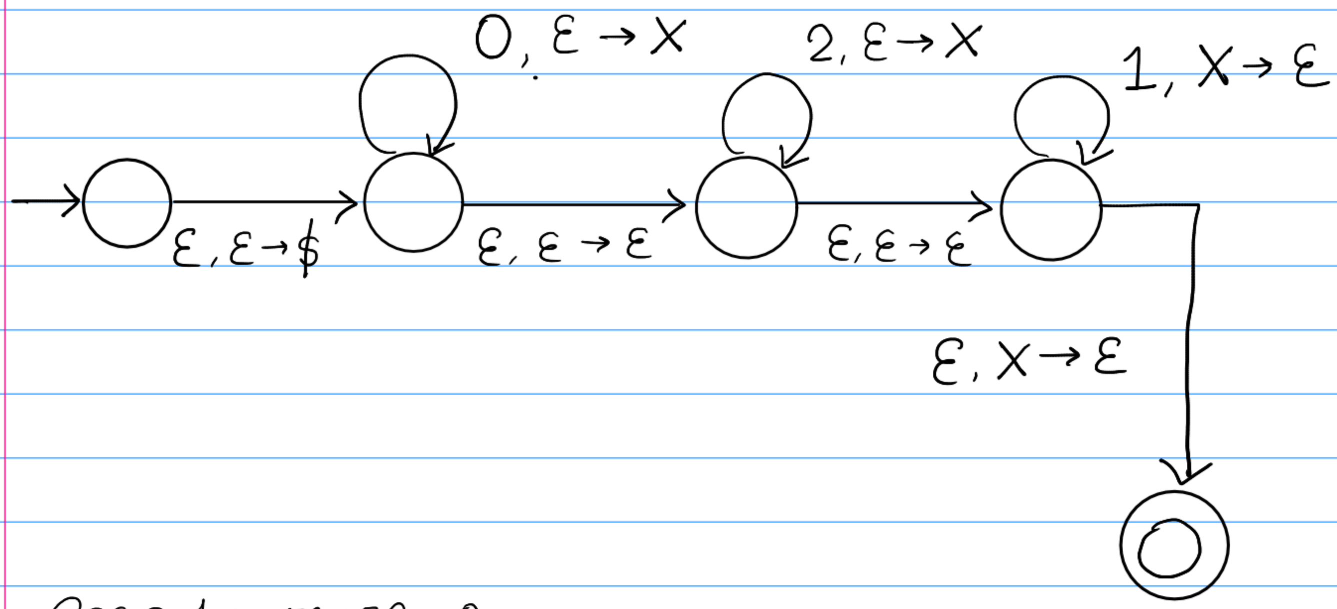
$C \to 0C \mid \varepsilon$

$D \to 2D \mid 2$ → more 2s

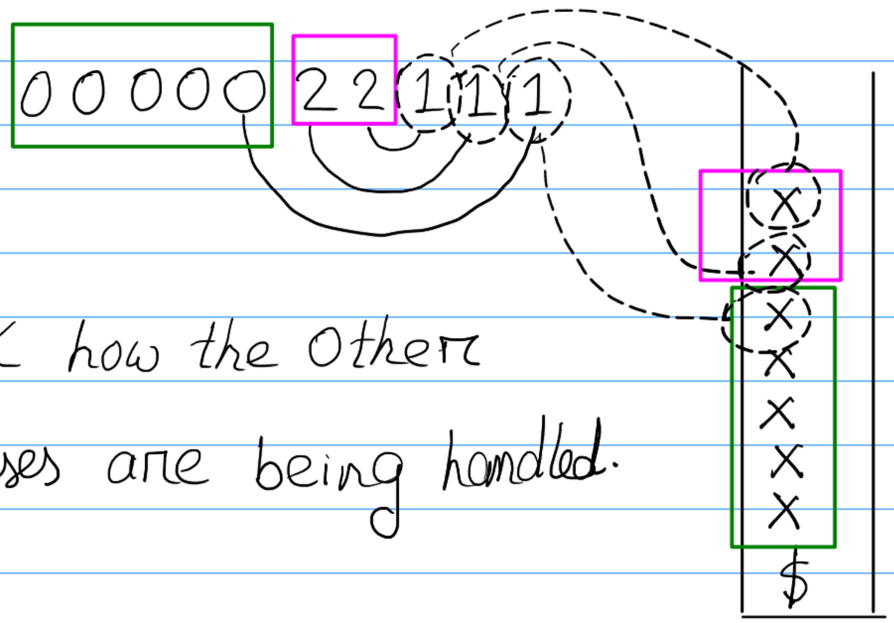
$Q \to 2Q1 \mid \varepsilon$ → equal 2s & 1s

this also handle the both case, can be skipped, since we already have handled the case previously.

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k, \text{ where } i + j > k \text{ and } i, j, k \geq 0 \}$



Case 1: more 0s



Think how the other cases are being handled.

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, where $i+k$ is even, $j = i+k$ and $j \geq 1 \}$

let's first solve,

$L1 = \{ w \in \{0,1\}^* : 0^i 1^k$, where $i+k$ is even$\}$

Now, $i+k$ can be even in two ways
- i & k both even
- i & k both odd

$S \to AC \mid BD$

even 0s ← $A \to 00A \mid \varepsilon$

odd 0s ← $B \to 00B \mid 0$

$C \to 11C \mid \varepsilon$

$D \to 11D \mid 1$

Another solution:

$S \to AB \mid 0A1B$ → making odd

$A \to 00A \mid \varepsilon$

$B \to 11B \mid \varepsilon$

Now, Solve the question given with $j \geq 0$

$L2 = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k$, where $i+k$ is even $j = i+k$ and $\cancel{j \geq 0} \}$

$0^i 2^j 1^k$

$\Rightarrow 0^i 2^{i+k} 1^k$

$\Rightarrow \underbrace{0^i 2^i} \underbrace{2^k 1^k}$

- if i is even then k is even
- if i is odd, then k is odd

$S \to AC \mid BD$

$A \to 00A22 \mid \varepsilon$

$B \to 00B22 \mid 02$

$C \to 22C11 \mid \varepsilon$

$D \to 22D11 \mid 21$

Now let's solve the initial question

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^J 1^K$, where $i+K$ is even, $J = i+K$ and $J \geq 1 \}$

Since $J \geq 1$
i) only $i \geq 1$ and $K = 0$
ii) only $K \geq 1$ and $i = 0$
iii) both $i, K \geq 1$

⇒ when i even & K even, then we have to handle case i, ii and iii

⇒ when i odd & K odd then only the case iii needs to consider

When we have ensured both i & K can't be 0 at the same time i even, K even & $i, K \geq 1$ are also get handled

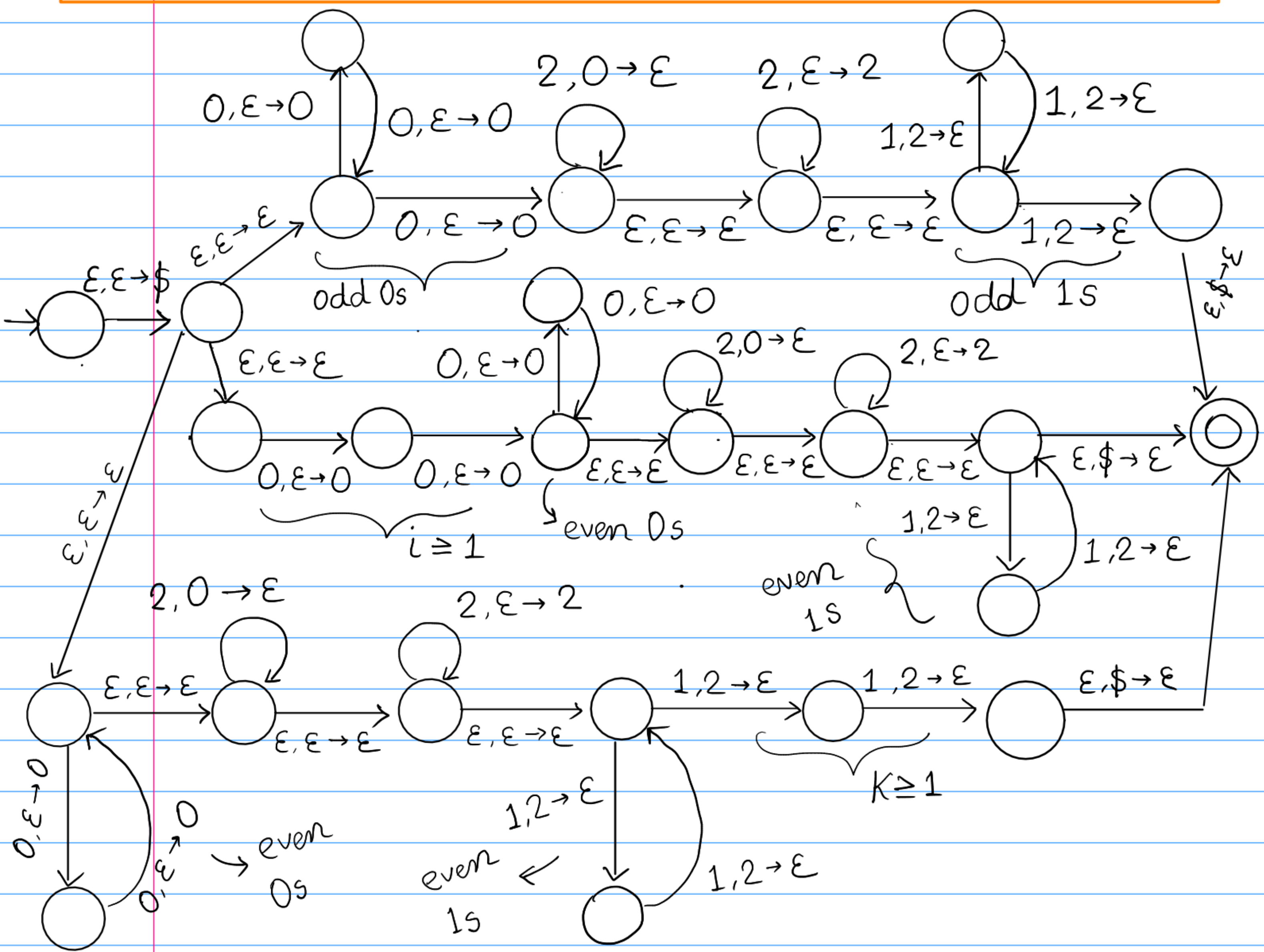
$S \rightarrow AP \mid BQ \mid CR$

$A \rightarrow 00A22 \mid 0022$
$P \rightarrow 22P11 \mid \varepsilon$
} i even, K even and $i \geq 1$

$B \rightarrow 00B22 \mid \varepsilon$
$Q \rightarrow 22Q11 \mid 2211$
} i even, K even and $K \geq 1$

$C \rightarrow 00C22 \mid 0022$
$R \rightarrow 22R11 \mid 2211$
} i odd, K odd and $i, K \geq 1$

$L = \{ \omega \in \{0,1,2\}^* : w = 0^i 2^j 1^K$, where $i+K$ is even, $J = i+K$ and $J \geq 1 \}$



$$\boxed{L = \{w \in \{0,1\}^* : \text{parity of number of 0s and 1s is different}\}}$$

Case 1: even 0s and odd 1s

Case 2: odd 0s and even 1s

So, the problem can be boiled down into $L = \{\text{length of } w \text{ is odd}\}$

$$S \longrightarrow 00S \mid 01S \mid 10S \mid 11S \mid 0 \mid 1$$

This can also be written as

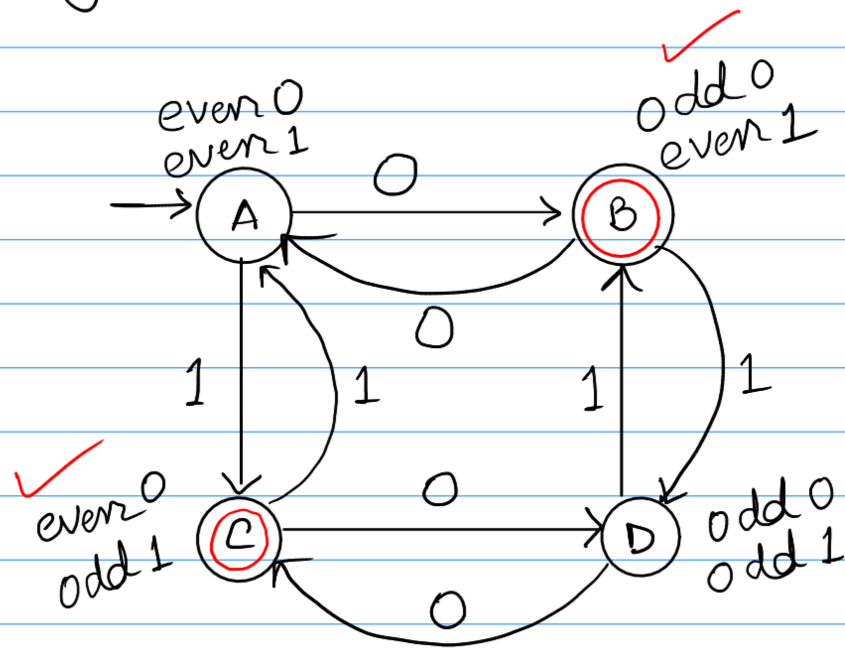
$$S \longrightarrow XXS \mid X$$
$$X \longrightarrow 0 \mid 1$$

Another Solution:

$$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0 \mid 1$$

Another solution:

if you couldn't figure out the previous idea, then no worry. You may also recall the following DFA we had done in the class:



So another solution can be

$A \to 0B \mid 1C$

$B \to 0A \mid 1D \mid \varepsilon$

$C \to 0D \mid 1A \mid \varepsilon$

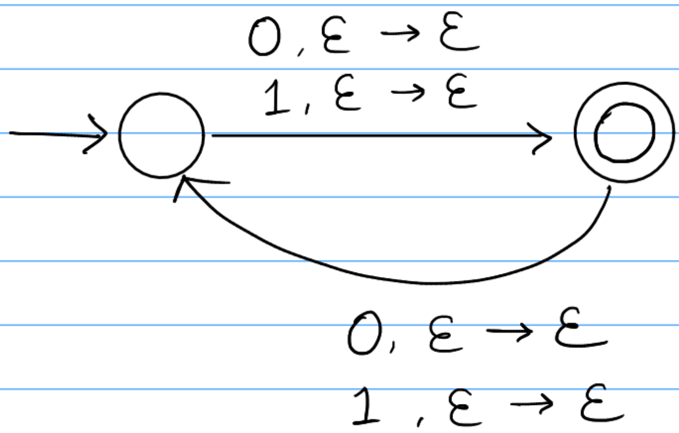
$D \to 0C \mid 1B$

$L = \{w \in \{0,1\}^*$: parity of number 0s and 1s is different$\}$

Case 1: even 0s and odd 1s

Case 2: odd 0s and even 1s

So, the problem can be boiled down into $L = \{$ length of w is odd $\}$



Updated previously accepting state was marked incorrectly.

$L1 = \{ w \in \{0,1\}^* :$ the number of 0s and 1s are different in w.

Before solving L1, first we try to solve

$L2 = \{ w \in \{0,1\}^* :$ w contains equal numbers of 0s and 1s$\}$ → 0s & 1s are parsed in pairs, so both the count of 0s & 1s will be same.

$$S \rightarrow 0S1 \mid 1S0 \mid \varepsilon$$

However, this solution is partially correct. For example, 0110 can't be parsed.

If we take a string, $w \in L2$, and if it has equal numbers of 0s and 1s, then it means, in w, there are one or more substring in w, having equal 0s & 1s.

$\boxed{001011}\ \boxed{10}$

↓

We can devide the string into two substrings, having equal 0s and 1s.

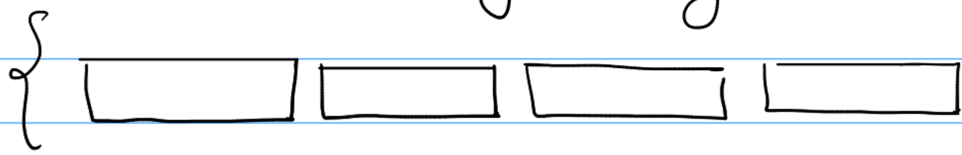
Now, recall the solution for valid parentheses. and let's fix the grammar.

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \varepsilon$$

☐ Draw parse tree for 11010101000011

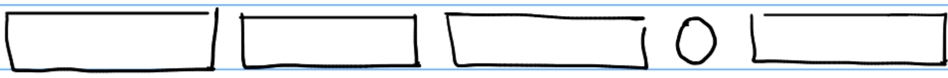
Now, let's come back to our original question. let's consider a string having equal 0s & 1s

each block having equal 0s & 1s



Now, if there is more 0s then having at least one additional 0 will be enough.

assuming the additional 0 is here.



So, we can write

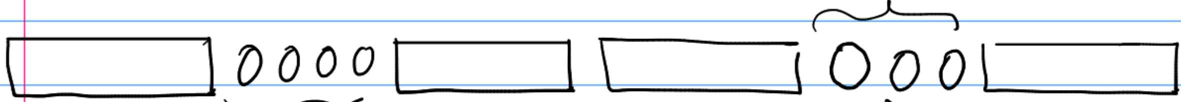
where S produce equal 0s and 1s

$$T \rightarrow S0S$$

$$S \rightarrow 0S1 \mid 1S1 \mid SS \mid \varepsilon$$

However, there could be more than one additional 0 than 1s. So, those 0s should be parsed as well.

additional 0s



additional 0s

$$T \rightarrow S0S$$

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid 0S \mid \varepsilon$$

However, we have handled only one case - more 0s than 1s. There could be more 1s than 0s also.

So, the final solution :

$$S \longrightarrow A \mid B$$

$$A \longrightarrow X0X$$

$$X \rightarrow 0X1 \mid 1X0 \mid XX \mid 0X \mid \varepsilon$$

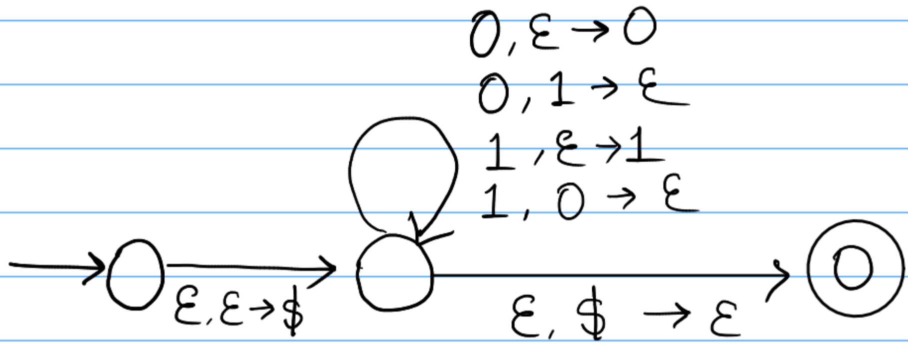
$$B \longrightarrow Y1Y$$

$$Y \rightarrow 0Y1 \mid 1Y0 \mid YY \mid 1Y \mid \varepsilon$$

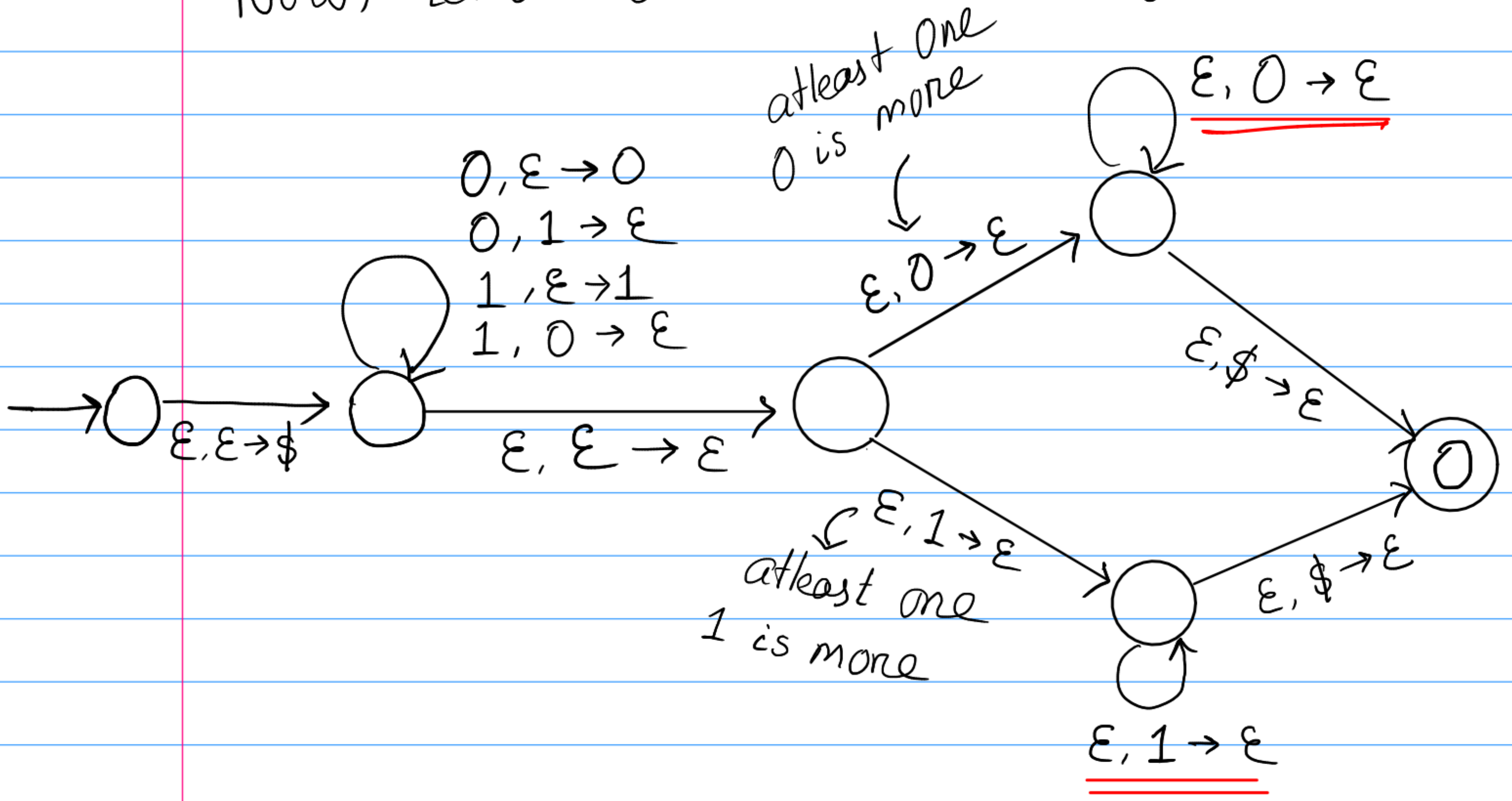
$$L1 = \{ \omega \in \{0,1\}^* : \text{the number of 0s and 1s are different in } \omega. $$

Again before solving L1, first we try to solve

$$L2 = \{ w \in \{0,1\}^* : w \text{ contains equal numbers - 0s and 1s} \}$$



Now, let's construct the PDA for L1



verifying there is more 1s. We have to ensure that there are no 0s left in the stack. If we don't Check if the stack has only 1s or not, then $w \notin L1$ will get accepted.

Recall that for a string w, |w| denotes the length of w. $\Sigma = \{0,1\}$

$L1 = \{w \in \Sigma^*: w \text{ contains exactly two 1s}\}$

$L2 = \{x\#y : x \in \Sigma^*, y \in L1, |x| = |y|\}$

Construct a CFG for L2.

Before solving this problem, let's try to solve a few similar problems.

$$L = \{ W_1 \# W_2 \mid W_1, W_2 \in \{0,1\}^* \text{ and } |W_1| = |W_2| \}$$

$$1001\#0010$$

$$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \#$$

We can also write it as

$$S \rightarrow XSX \mid \#$$
$$X \rightarrow 0 \mid 1$$

Now let's say,

$$L1 = \{ W_1 \# W_2 \mid W_1 \in \{0,1\}^*, W_2 \in L2 \text{ and } |W_1| = |W_2| \}$$

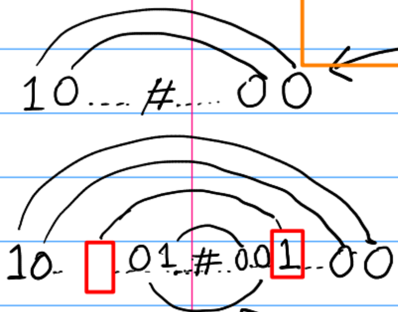
$$L2 = \{ W \in \{0,1\}^* : W \text{ is even} \}$$

$$S \rightarrow XXSXX \mid \#$$
$$X \rightarrow 0 \mid 1$$

If you understood the previous two solutions. then we try to solve our original question.

$$L1 = \{ w \in \{0,1\}^* : w \text{ contains exactly two 1s} \}$$

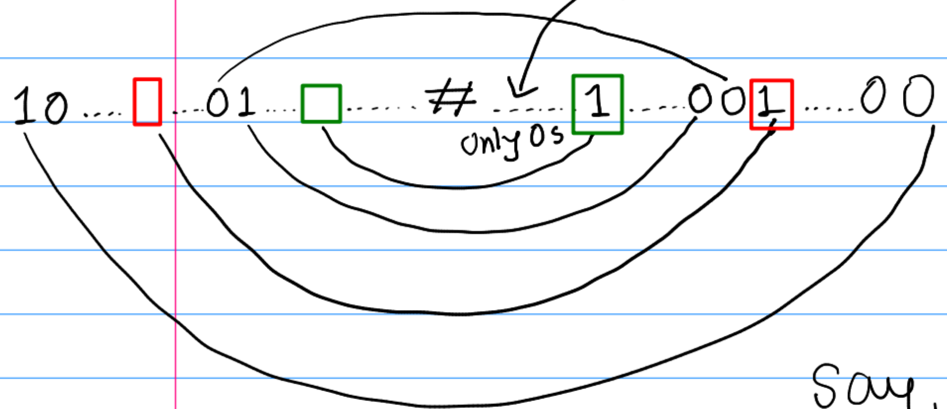
$$L2 = \{ x\#y,\ x \in \{0,1\}^*,\ y \in L1 \text{ and } |x| = |y| \}$$



$A \rightarrow QA0 \mid \boxed{QB1}$ we have found the first 1

$B \rightarrow QB0 \mid \boxed{QC1}$ found the second 1

$C \rightarrow QC0 \mid \#$



Here, we can say, if we are at production rule A, then have seen no 1 in y, if we are at the production rule B, then we have found exactly one 1 in y. Next, if we are at rule C, then we have seen exactly two 1s in the y.

Recall that for a string w, |w| denotes the length of w. $\Sigma = \{0,1\}$

$L1 = \{w \in \Sigma^*: w \text{ contains exactly two 1s}\}$

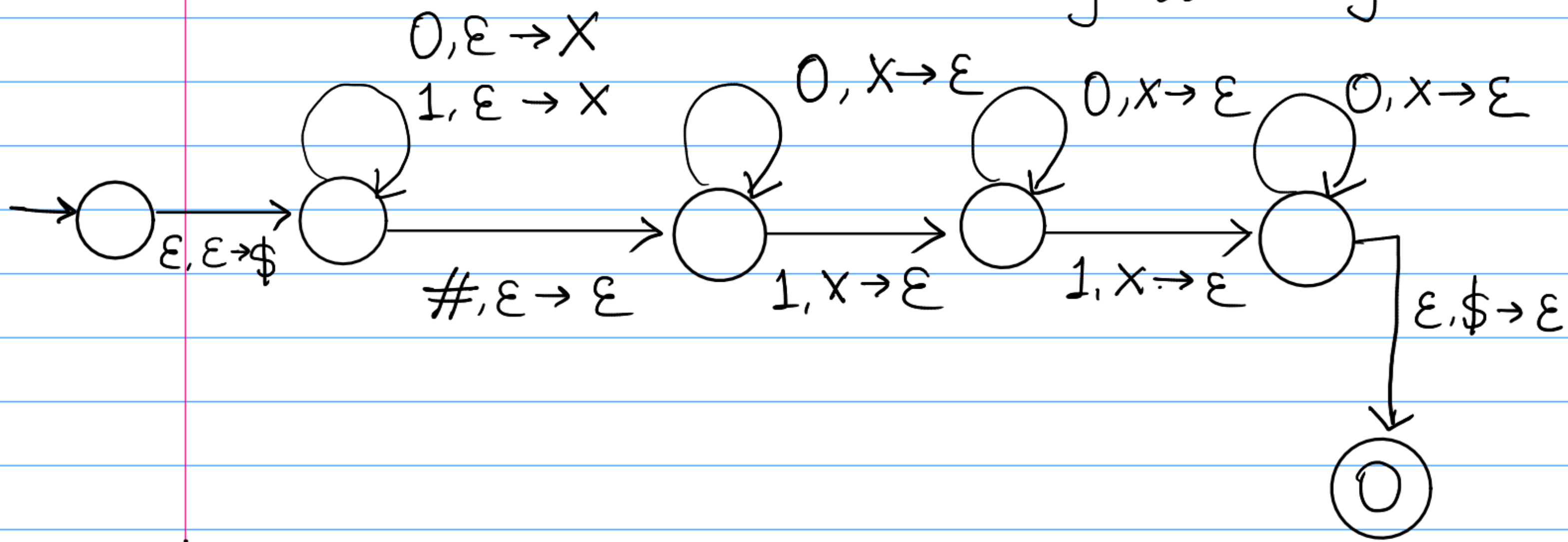
$L2 = \{x\#y : x \in \Sigma^*, y \in L1, |x| = |y|\}$

Construct a CFG for L2.

has exactly two 1s

$\underbrace{0010110}_{x}\#\underbrace{0001010}_{y}$

⇒ we have to check |x| = |y| and y has exactly two 1s



Recall that for a string w, |w| denotes the length of w. Σ = {0,1}

L1 = {w $\in \Sigma^*$: w contains at least three 1s}

L2 = {x#y : x $\in (\Sigma\Sigma)^*$, y $\in$ L1, |x| = |y|}

Construct a CFG for L2.

$\rightarrow$ even length string

since $x \in \{\Sigma\Sigma\}^*$ and $|x| = |y|$ hence $y \in \{\Sigma\Sigma\}^*$

also, $y \in L1 \rightarrow y$ contains at least three 1s

$\boxed{00}\boxed{01}\boxed{01}\boxed{00}\boxed{00}\#\boxed{10}\boxed{11}\boxed{01}\boxed{00}\boxed{10}$

Count of 1 = 0

$\hookrightarrow S \rightarrow XXS00 \mid XXA01 \mid XXA10 \mid XXB11$

Count of 1 = 1

$\hookrightarrow A \rightarrow XXA00 \mid XXB01 \mid XXB10 \mid XXC11$

Count of 1 = 2

$\hookrightarrow B \rightarrow XXB00 \mid XXC01 \mid XXC10 \mid XXC11$

Count of $1 \geq 3$

$\hookrightarrow C \rightarrow XXCXX \mid \#$

Try solving

Recall that for a string w, |w| denotes the length of w. Σ = {0,1}

L1 = {w $\in \Sigma^*$: w contains ~~at least~~ exactly three 1s}

L2 = {x#y : x $\in (\Sigma\Sigma)^*$, y $\in$ L1, |x| = |y|}

Construct a CFG for L2.

Recall that for a string w, |w| denotes the length of w. Σ = {0,1}

L1 = {w ∈ Σ*: w contains at least three 1s}

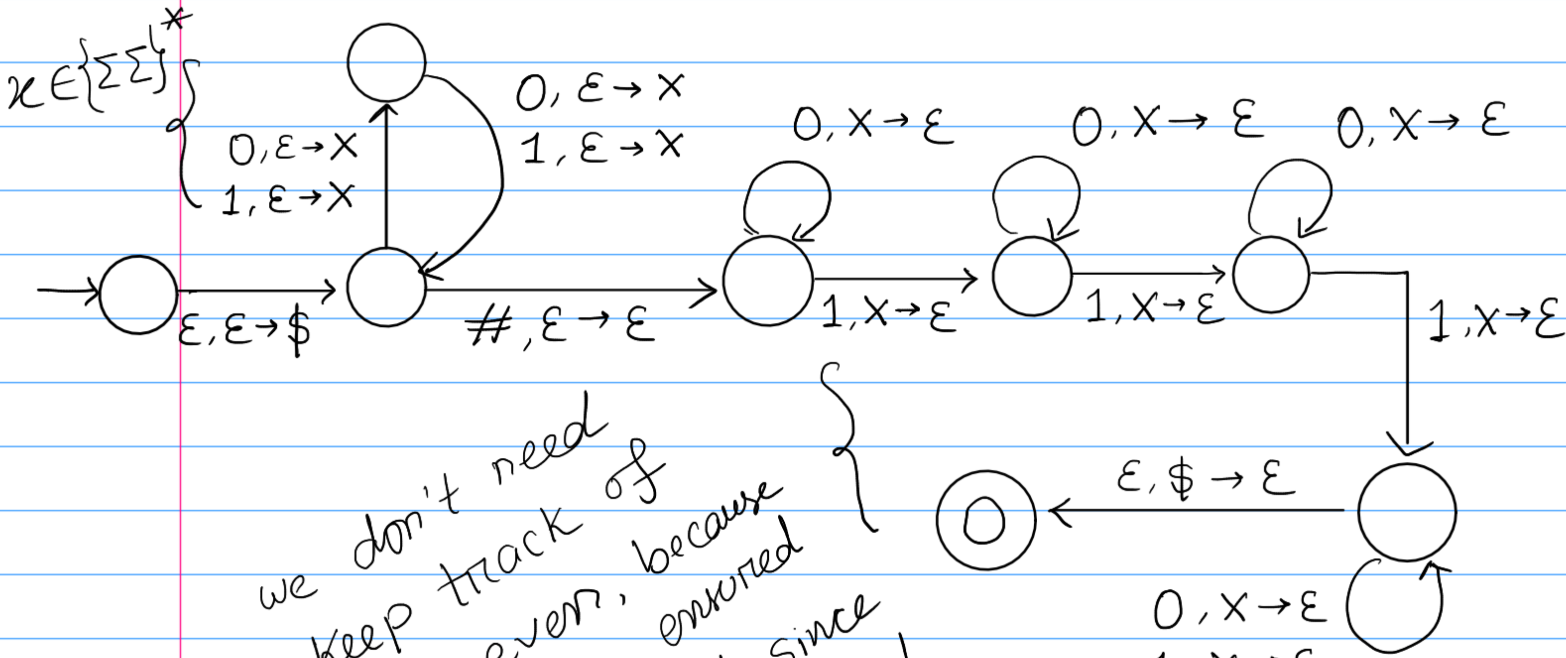
L2 = {x#y : x ∈ (ΣΣ)*, y ∈ L1, |x| = |y|}

Construct a CFG for L2.

→ even length string

since $x \in \{\Sigma\Sigma\}^*$ and $|x| = |y|$ hence $y \in \{\Sigma\Sigma\}^*$

also, $y \in L1 \longrightarrow$ y contains at least three 1s



we don't need to keep track of |y| is even, because |x| has been ensured to be even and since we pop $ at the end |y| has to of equal to |x|

$L = \{ w \in \{0,1\}^* : 0^i 1^K \text{ where } i, K \geq 0 \text{ and } 3i \geq 4K+2 \}$

This means, if we have i amount of 0s and K amount of 1s, then (3 * total 0s) should be greater than or equal to (4 * total 1s + 2)

So, let's first figure out, what is the minimum amount of 0s we need to have for $K = 0, 1, 2 \ldots$ satisfying the condition.

if $K=0$, $i \geq 1$

$K=1$, $i \geq 2$

$K=2$, $i \geq 4$

$K=3$, $i \geq 5$

$K=4$, $i \geq 6$

$K=5$, $i \geq 8$

$K=6$, $i \geq 9$

$3i \geq 4K+2$

$i \geq \left\lceil \frac{4K+2}{3} \right\rceil$

Now, can you find any pattern? Think in respect of $K \% 3$.

[since, $3i$]

$K \% 3$

$K = 0 \Rightarrow K \% 3 = 0, \; i \geq 1$
$K = 1 \Rightarrow K \% 3 = 1, \; i \geq 2$
$K = 2 \Rightarrow K \% 3 = 2, \; i \geq 4$
$K = 3 \Rightarrow K \% 3 = 0, \; i \geq 5$
$K = 4 \Rightarrow K \% 3 = 1, \; i \geq 6$
$K = 5 \Rightarrow K \% 3 = 2, \; i \geq 8$
$K = 6 \Rightarrow K \% 3 = 0, \; i \geq 9$
$K = 7 \Rightarrow K \% 3 = 1, \; i \geq 10$
$K = 8 \Rightarrow K \% 3 = 2, \; i \geq 12$

if we do $K \% 3$, then we have three patterns. The minimum length of strings for each pattern

$$0^i 1^K$$

0	001	000011
$K=0, K\%3 = 0$	$K=1, K\%3 = 1$	$K=2, K\%3 = 2$

Now, see what amount of 0s and 1s you need to go the next pattern in same $K\%3$

0 $\boxed{0000111}$	00 $\boxed{00001111}$ 1	0000 $\boxed{0000111}$ 11
$K=3, K\%3 = 0$	$K=4, K\%3 = 1$	$K=5, K\%3 = 2$

So, in each pattern we see, we can jump to the next string by adding 0000111

$$L = \{ w \in \{0,1\}^* : 0^i 1^k, \text{ where } i, k \geq 0 \}$$

So, now, if the condition was $\boxed{3i = 4k+2}$ then,

$$S \to 0A \mid 00A1 \mid 0000A11$$
$$A \to 0000A111 \mid \varepsilon$$

Now, since, the condition is $\boxed{3i \geq 4k+2}$, hence, we will have some additional 0s as well. So,

$$S \to 0S \mid \overset{k\%3=0}{0A} \mid \overset{k\%3=1}{00A1} \mid \overset{k\%3=2}{0000A11}$$
$$A \to 0000A111 \mid \varepsilon$$

Another Approach: Based on the increment of 0s $\to i = 0, 1, 2, 4, 5, 6, 8$

$$\underbrace{0,1}_{1},\underbrace{1,2}_{1},\underbrace{2,4}_{2},\underbrace{4,5}_{1},\underbrace{5,6}_{1},\underbrace{6,8}_{2}$$

$k = 0, 1, 2, 3, 4, 5, 6$

if $3i = 4k+1$

$$S \to 0A \mid \varepsilon$$
$$A \to 0B1 \mid \varepsilon$$
$$B \to 00C1 \mid \varepsilon$$
$$C \to 0A1 \mid \varepsilon$$

Now, for $3i \geq 4k+1$

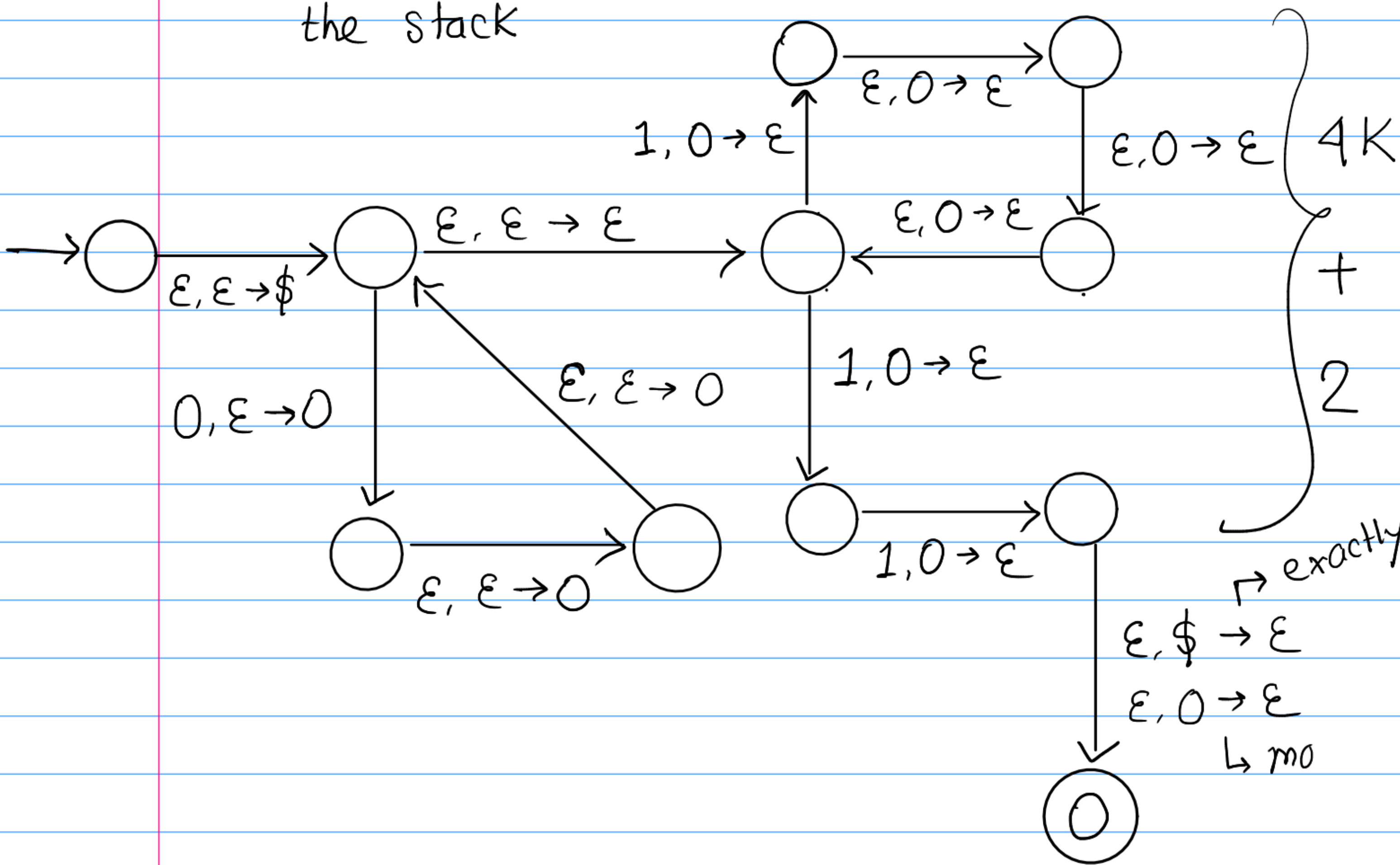
$$S \to 0S \mid 0A \mid \varepsilon$$
$$A \to 0B1 \mid \varepsilon$$
$$B \to 00C1 \mid \varepsilon$$
$$C \to 0A1 \mid \varepsilon$$

$L = \{ w \in \{0,1\}^* : 0^i 1^K \text{ where } i, K \geq 0$
$\text{and } 3i \geq 4K + 2 \}$

000 .... 0011 1

we push (3 * total 0s) amount of 0s in the stack

for each 1 we pop four 0s.



Now Solve:

$L = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, 3i \geq 4k + 2, j$ is not divisible by three$\}$