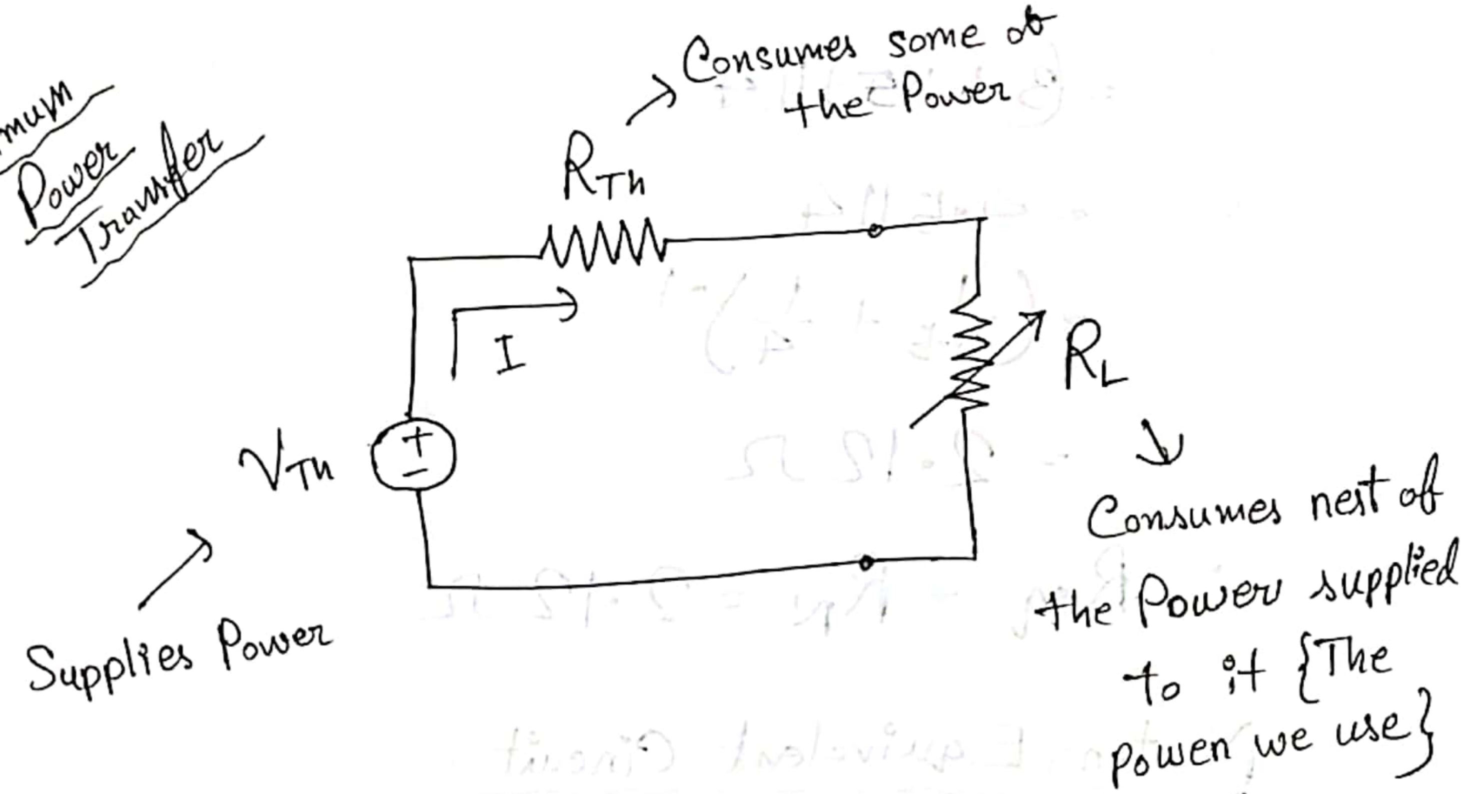
Lecture - 11: Superposition Principle & Maximum Power Transfer

Maximum Power Transfer



• Sometimes it's desirable to maximize the power delivered to the Load [R_L here].

Here, Power delivered to R_L is, [The Load]

$$P = I^2 R_L$$

$$= \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 R_L$$

☐ For maximum power, $\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L = 0$

Maximum Power Theorem

Maximum power is transfered to the load when the load resistance = Thevenin Resistance seen from the load $\Rightarrow$ $\boxed{R_L = R_{Th}}$

$\therefore$ In the circuit we saw, $P = I^2 R_L$ is maximum when $\boxed{R_L = R_{Th}}$

Proof

$$\frac{dp}{dR_L} = \frac{d}{dR_L}\left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L = 0$$

$$= V_{th}^2 \frac{d}{dR_L}\left\{\frac{R_L}{(R_{th}+R_L)^2}\right\}$$

$\left[\frac{d}{dx}\left(\frac{u}{v}\right) \text{ formula}\right] \leftarrow$

$$= V_{th}^2 \frac{(R_{th}+R_L)^2 \frac{d}{dR_L} R_L - R_L \frac{d}{dR_L}(R_{th}+R_L)^2}{(R_{th}+R_L)^4}$$

$$= V_{th}^2 \frac{(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^4}$$

$$= V_{th}^2 \frac{(R_{th}+R_L)(R_{th}+R_L-2R_L)}{(R_{th}+R_L)^4}$$

$$= V_{th}^2 \frac{R_{th} + R_L - 2R_L}{(R_{th} + R_L)^3}$$

$$= V_{th}^2 \frac{R_{th} - R_L}{(R_{th} + R_L)^3}$$

$$\therefore V_{th}^2 \frac{R_{th} - R_L}{(R_{th} + R_L)^2} = 0$$

$$\Rightarrow R_{th} - R_L = 0$$

$$\therefore \boxed{R_{th} = R_L}$$

| Proved |

Maximum Power

$$P_{max} = I^2 R_L \quad \text{when, } R_L = R_{th}$$

$$\therefore P_{max} = I^2 R_{th}$$

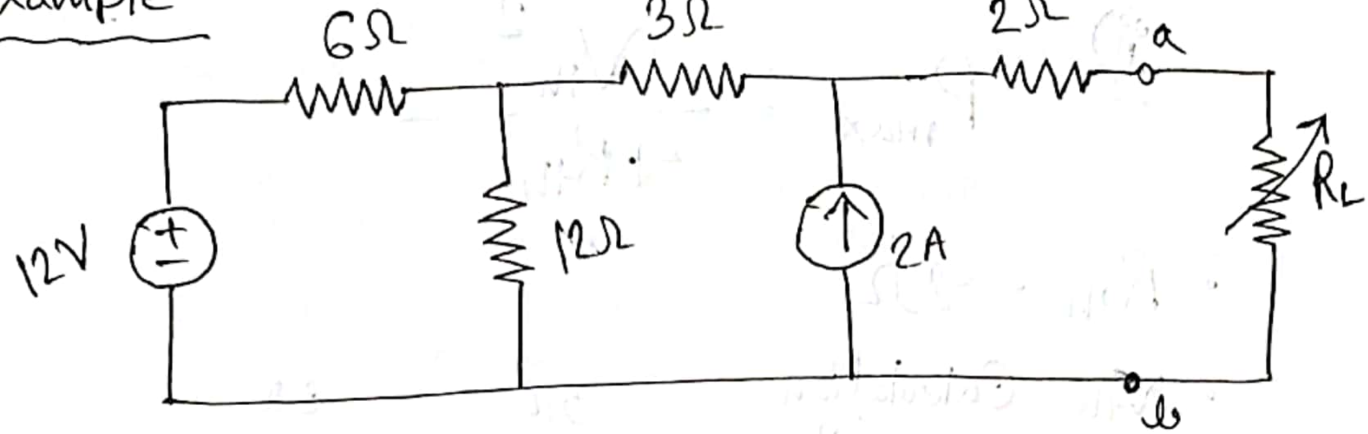
$$= \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 R_{th} = \left(\frac{V_{th}}{R_{th} + R_{th}}\right)^2 R_{th}$$

$$= \frac{V_{th}^2}{4R_{th}^2} R_{th}$$

$$\therefore \boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

(Ans.)

Example



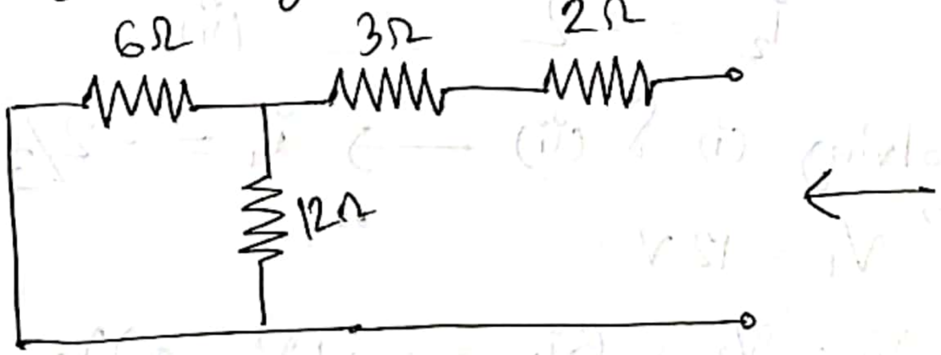
i) R_L for maximum Power transfer = ?

ii) P_{max} = ?

i) $\boxed{R_L = R_{th}}$

Measuring R_{th}

Switching off all independent sources:



$$\therefore R_{th} = R_{eq} = 2\Omega + 3\Omega + (6\Omega \parallel 12\Omega)$$

$$= \left\{2 + 3 + \left(\frac{1}{6} + \frac{1}{12}\right)^{-1}\right\} \Omega$$

$$= (2 + 3 + 4)\,\Omega$$

$$= \boxed{9\Omega} = R_L$$

(Ans.)

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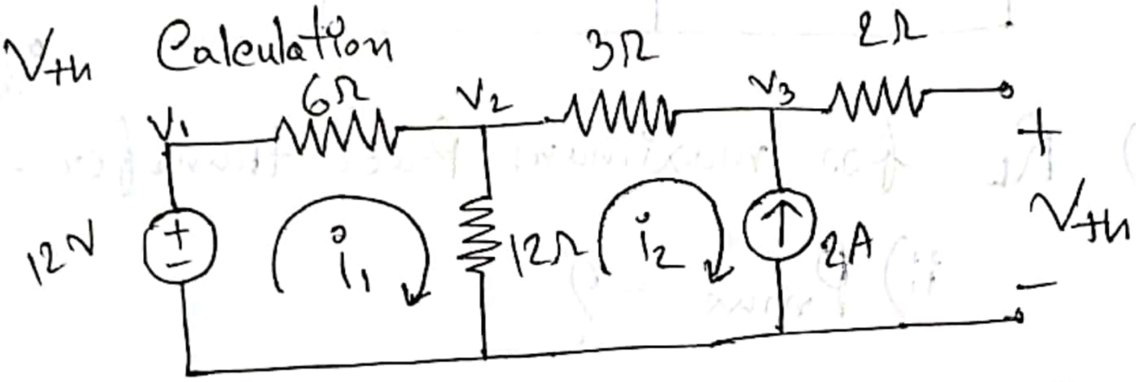
ii) $P_{max} = \frac{V_{th}^2}{4R_{th}}$

- $R_{th} = 9\Omega$
- V_{th} Calculation



Loop-1

$6i_1 + 12(i_1 - i_2) - 12 = 0$

$\Rightarrow 18i_1 - 12i_2 = 12$ —— (i)

Loop-2

$i_2 = -2$ —— (ii)

Solving (i) & (ii) $\longrightarrow i_1 = -2/3\ A$

Now,
$V_1 = 12V$

$V_1 - V_2 = 6i_1 = -4V \Rightarrow V_2 = V_1 + 4 = 16V$

$V_2 - V_3 = 3i_2 = -6V \Rightarrow V_3 = V_2 + 6 = 22V$

$\therefore V_{th} = V_3 = \boxed{22V}$

$\therefore P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{22^2}{4\times 9} W = \boxed{13.44W}$

(Ans.)

Superposition Principle

- **Linearity**
  - → Linear Relationship between cause & Effect.
  - → Homogeneity
  - → Additivity

i) Homogeneity →
- Input $\xrightarrow[\text{Element}]{\text{Linear}}$ Output
- Input X Constant $\xrightarrow[\text{Element}]{\text{Linear}}$ Output X Constant

Example: A Resistor.

$$V = iR$$

$$\boxed{KV = KiR}$$

ii) Additivity →
- Input 1 $\xrightarrow[\text{Element}]{\text{Linear}}$ Output 1
- Input 2 $\xrightarrow[\text{Element}]{\text{Linear}}$ Output 2
- Input 1 + Input 2 $\xrightarrow[\text{Element}]{\text{Linear}}$ Output 1 + Output 2

Example: A Resistor

$V_1 = i_1 R$, $V_2 = i_2 R$

$$\boxed{V = (i_1 + i_2)R = i_1 R + i_2 R = V_1 + V_2}$$

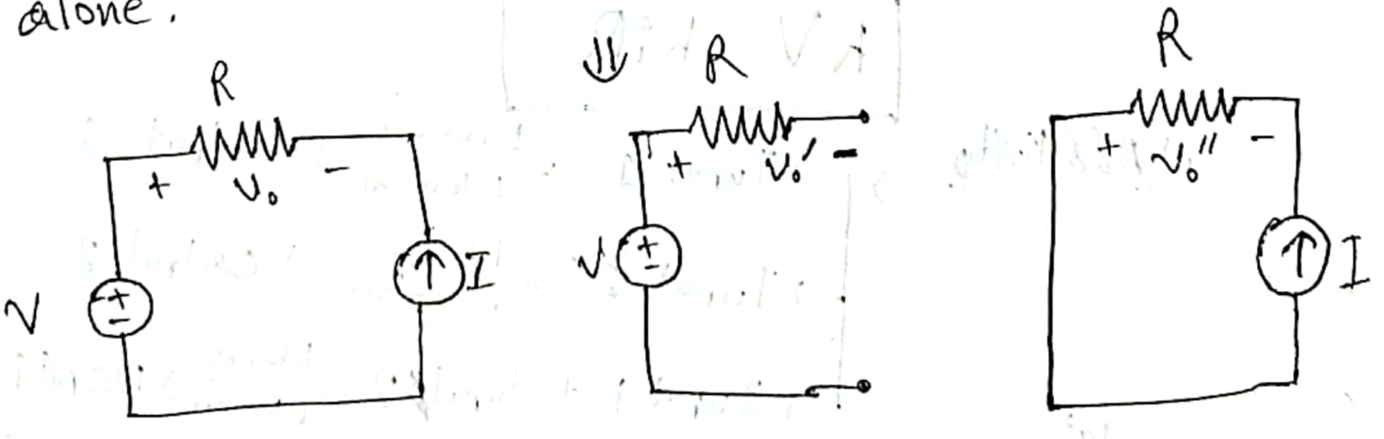
What about Power? :)

A Resistor is a linear element because voltage-current relationship satisfies homogeneity & additivity property.

- A linear Circuit ⟶ A circuit whose output is linearly related (directly proportional) to its input.

Super Position Principle

" The Voltage across / Current through an element in a linear circuit is the sum (algebraic) of the voltag across / current through that element due to each independent source acting alone.



$$\boxed{V_o = V_o' + V_o''}$$

Things to keep in mind:

- Voltage source off → 0 V ⟶ Short Circuit
- Current source off → 0 A ⟶ Open Circuit
- Only deal with Independent sources, not dependent sources.

Steps

Step-1

Turn off all the independent sources (see above) except one.

Step-2

Find the output (voltage V_o / current i) due to that one active source. (Using other techniques)

Step-3

Repeat step-1-2 for each of the independent sources.

Step-4

Add up all the outputs calculated for each of the sources → The overall output.

Disadvantage

→ Involved more work.

Advantage

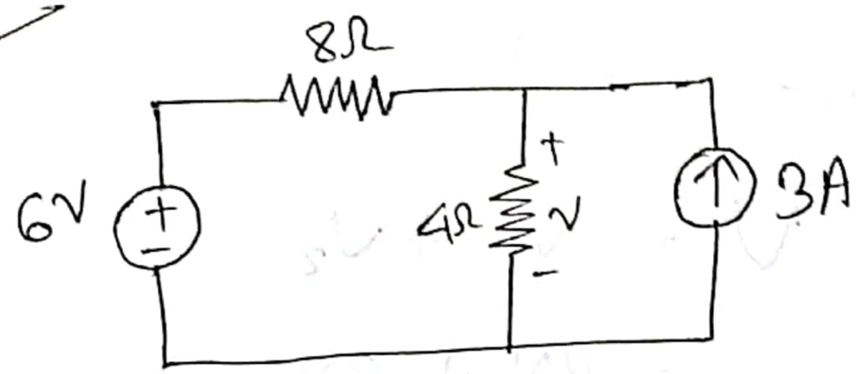
→ Simpler concept.

N.B: Power → Not linear for R

⇓

No Superposition.

Example 1

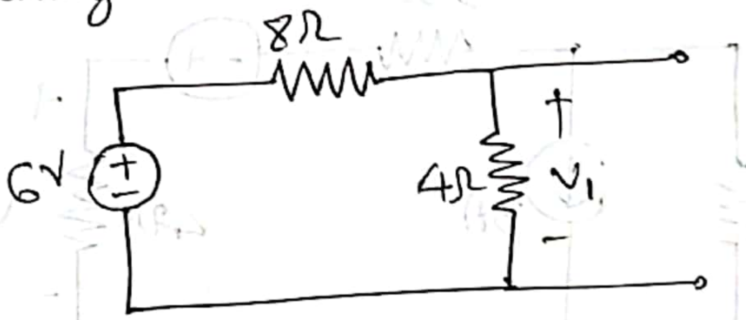


2 Independent sources

$\therefore$ Let, $V = V_1 + V_2$

Step-1

Turning Current source off,



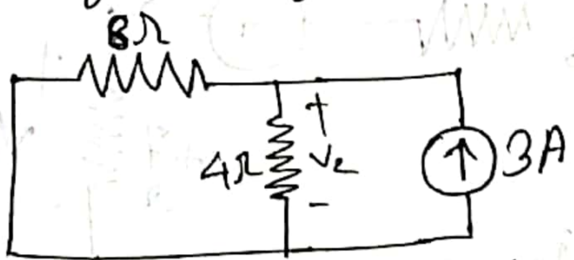
Step-2

$\therefore V_1 = 6 \times \frac{4}{8+4} V$ |Voltage divider|

$= \boxed{2V}$

Step-3

Turning Voltage Source off,



$\therefore V_2 = I R_{eq} = 3 \times \left(\frac{1}{8} + \frac{1}{4}\right)^{-1} V = \boxed{8V}$

|Equivalent Resistance|

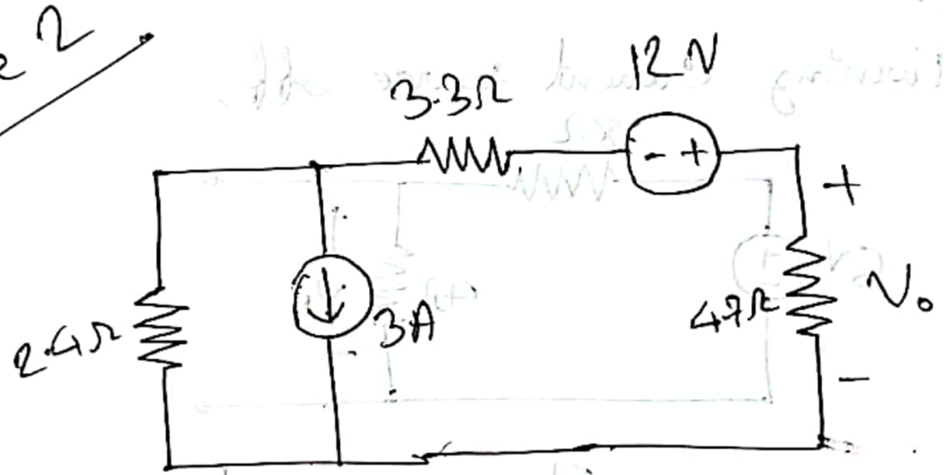
Step-4

$\therefore V = V_1 + V_2$

$= 2V + 8V$

$\therefore \boxed{V = 10V}$ (Ans.)

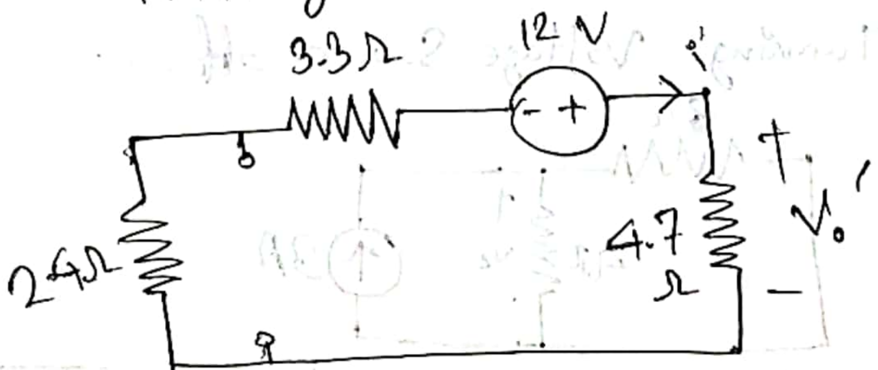
Example 2



Let, $V_o = V_o' + V_o''$

Step-1

Turning Current Source off,



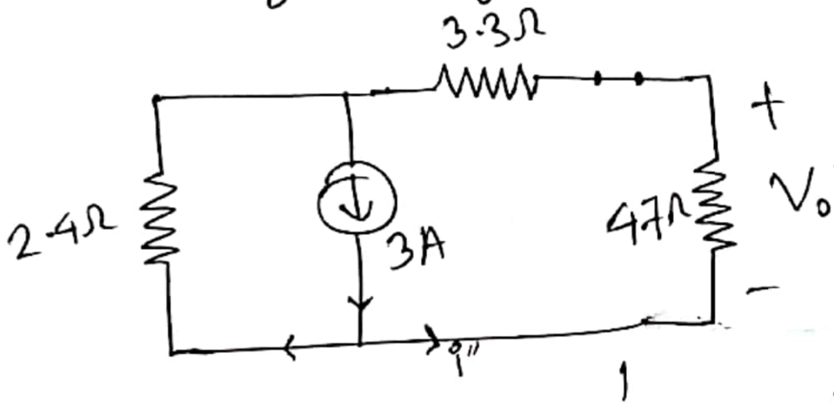
Step-2

$$\therefore V_0' = 12 \times \frac{4.7}{2.4+3.3+4.7} \text{ V}$$ | Voltage Divider |

$$= 5.423 \text{ V}$$

Step-3

Turning Voltage source off,



$$\therefore i'' = 3 \times \left(\frac{\frac{1}{4.7+3.3}}{\frac{1}{2.4} + \frac{1}{4.7+3.3}} \right) \text{ A}$$

$$= 0.692 \text{A}$$

Current entering from the negative terminal.

$$\therefore V_0'' = -(0.692 \times 4.7) \text{ V}$$

$$= -3.254 \text{ V}$$

Step-4

$$V_0 = V_0' + V_0'' = 5.423 \text{ V} - 3.254 \text{ V}$$

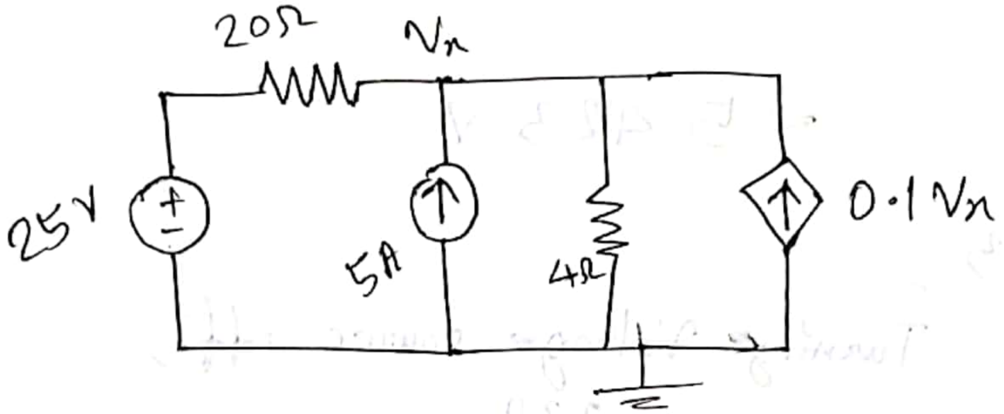
$$= \boxed{2.169 \text{ V}}$$ (Ans.)

Example-3

→ What if there are dependent sources?

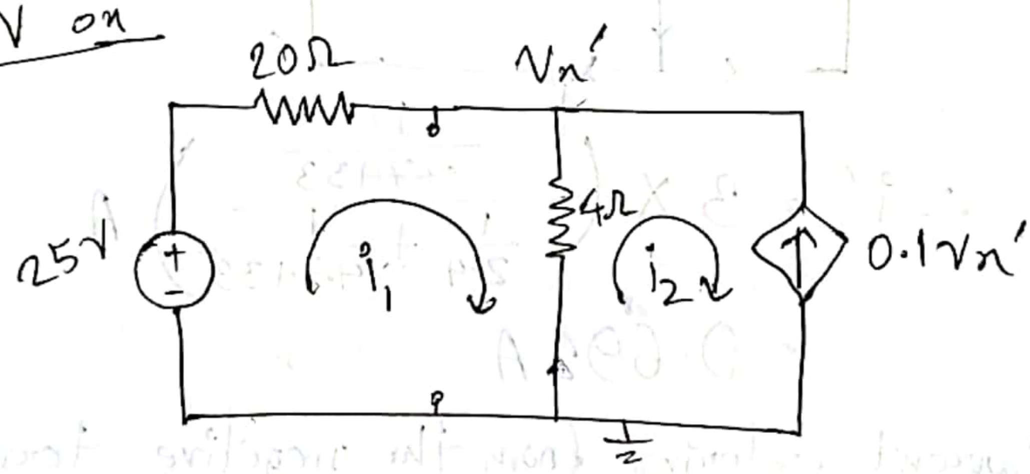
⇓

Just leave them alone!



$$V_x = V_x' + V_x''$$

25 V on



$$V_x' = (i_1 - i_2)4$$

Loop-1

$$-25 + 20i_1 + 4(i_1 - i_2) = 0$$

$$\Rightarrow 24i_1 - 4i_2 = 25 \quad \text{—— (i)}$$

Loop-2

$$i_2 = -0.1(i_1 - i_2)4 \quad = -0.1 V_x'$$

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$$\Rightarrow i_2 = -0.4 i_1 + 0.4 i_2$$

$$\Rightarrow 0.4 i_1 + 0.6 i_2 = 0 \quad \text{———} \quad (ii)$$

Solving (i) & (ii),

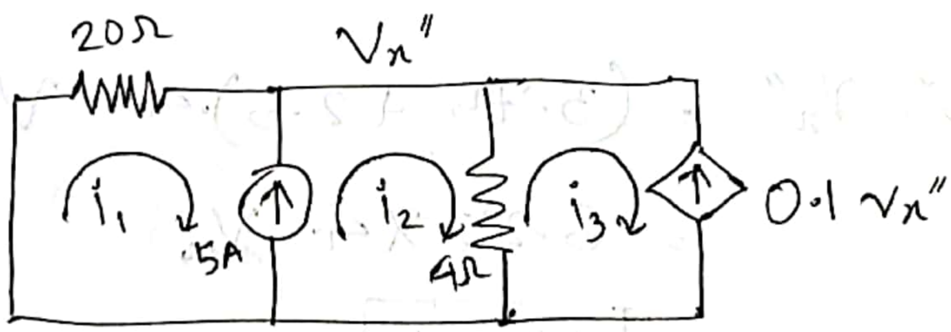
$$i_1 = 0.9375 \text{ A}$$

$$i_2 = -0.625 \text{ A}$$

$$\therefore V_x' = (0.9375 + 0.625)4 \text{ V}$$

$$= \boxed{6.25 \text{ V}}$$

5A on



$$V_x'' = (i_2 - i_3)4$$

Super mesh 1 & 2

$$i_2 - i_1 = 5 \quad \text{———} \quad (i)$$

$$20 i_1 + 4(i_2 - i_3) = 0$$

$$\Rightarrow 20 i_1 + 4 i_2 - 4 i_3 = 0 \quad \text{———} \quad (ii)$$

Loop-3

$$i_3 = -0.1 V_x''$$

$$\Rightarrow i_3 = -0.1(i_2 - i_3)4$$

$$\Rightarrow 0.4 i_2 + 0.6 i_3 = 0 \quad \text{——— (iii)}$$

Solving (i), (ii), (iii),

$$i_1 = -1.25 \text{ A}$$

$$i_2 = 3.75 \text{ A}$$

$$i_3 = -2.5 \text{ A}$$

$$\therefore V_x'' = (3.75 + 2.5) \cdot 4 \text{ V}$$

$$= 6.25 \times 4 \text{ V}$$

$$= \boxed{25 \text{ V}}$$

$$\therefore V_x = V_x' + V_x''$$

$$= (6.25 + 25) \text{ V}$$

$$= \boxed{31.25 \text{ V}}$$

(Ans)