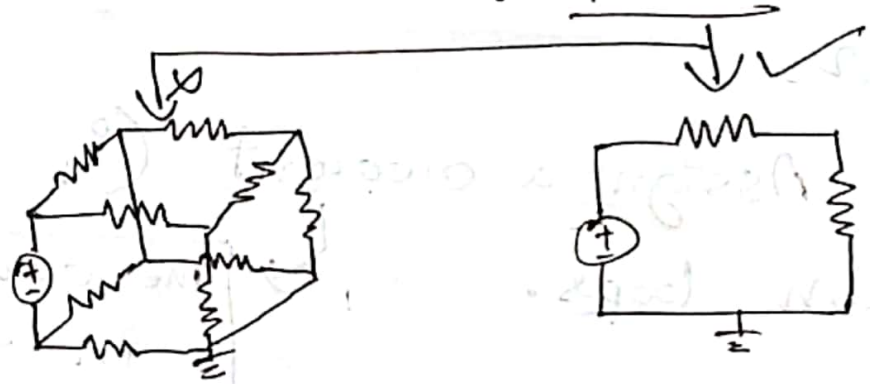
Week 4 Part 2

Mesh Analysis

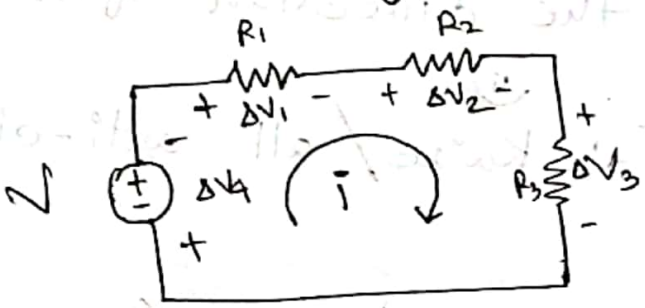
- Works using ~~KCL~~ → KVL ✓
- Component Equation for Resistance ⇒ $\Delta V = IR$
- Applicable in case of Planar circuits.



Non - Planar

Planar

KVL



$$\Rightarrow \sum \Delta V = 0$$

$$\Rightarrow \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

$$\Rightarrow iR_1 + iR_2 + iR_3 - V = 0$$

⊡ Current (i) direction is clockwise or anti-clockwise in all loops

Steps → Of Mesh Analysis

Step-1

Find out the number of loops → n in the Circuit → Assign a number ~~(0 to n-1)~~ → (1 to n).

Step-2

Assign a current (i_1 to i_n) to each loops.

→ The current circulating in that loop.

Step-3

Select the direction for each current.

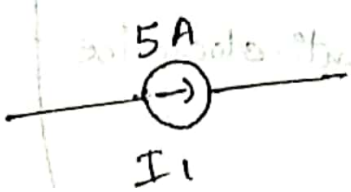
Take all clockwise / all anti-clockwise.

Step-4

Write component equations for all the

Current Sources → [Current = Labeled value]

example



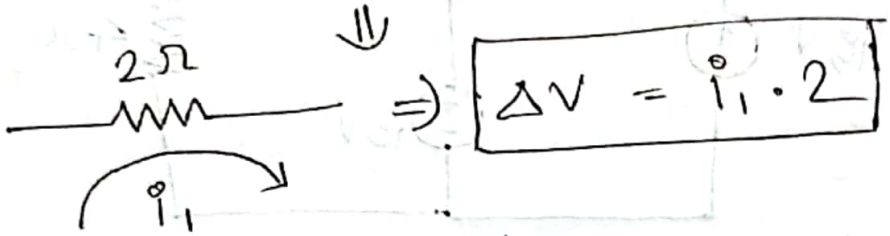
⇒ $I_1 = 5$

Step-5

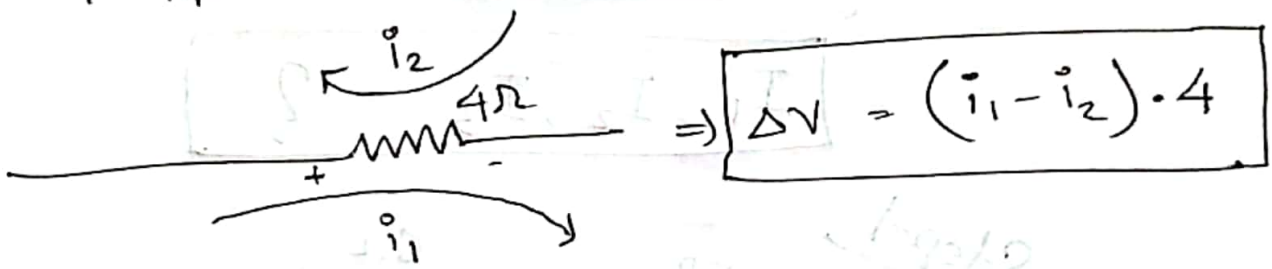
Determine the voltage drop across each resiston for each loops using Ohm's Law.

$$\Delta V = IR$$

$$\Downarrow$$



$$\Rightarrow \boxed{\Delta V = i_1 \cdot 2}$$



$$\Rightarrow \boxed{\Delta V = (i_1 - i_2) \cdot 4}$$

Step-6

Write KVL equations for all the loops/ meshes and supermeshes with unknown current.

$$\Downarrow$$

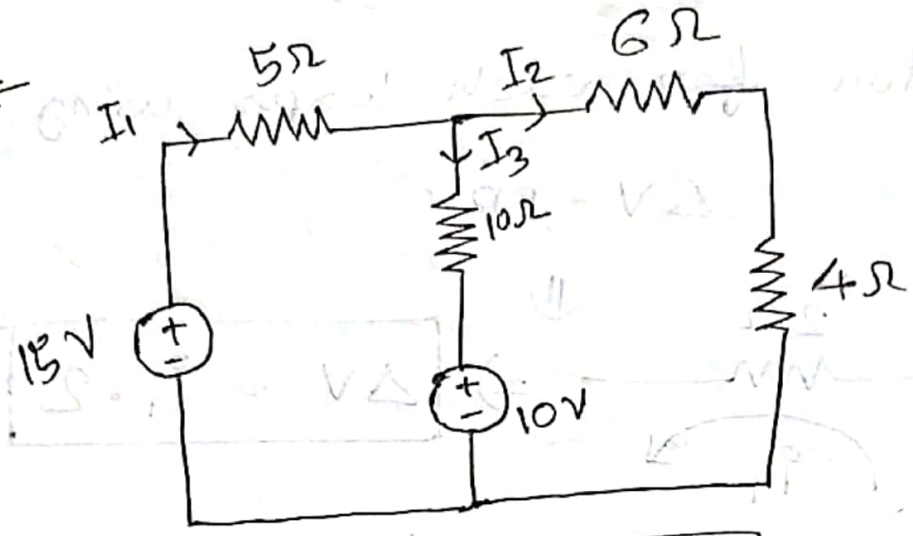
[Not needed in loops with known value of i]

Step-7

Solve all the equations to find all the mesh currents. [Calculaton / By substitution]

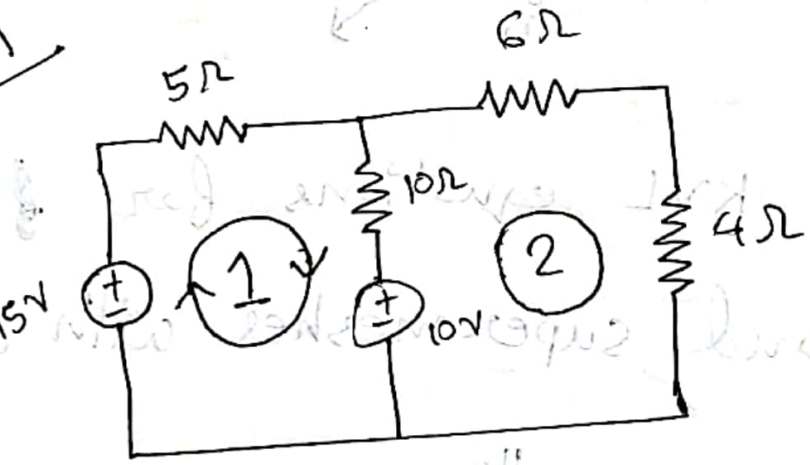
Mesh Analysis

1



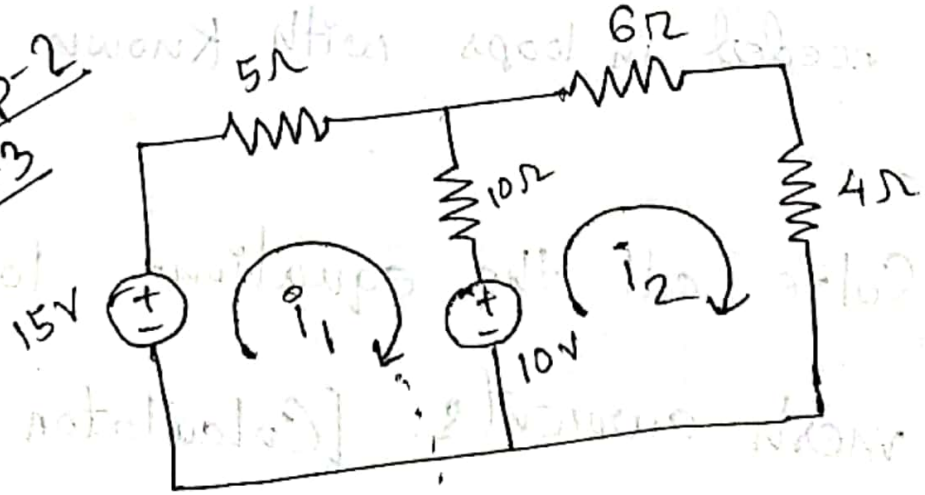
$I_1, I_2, I_3 = ?$

Step-1



Step-2

+3



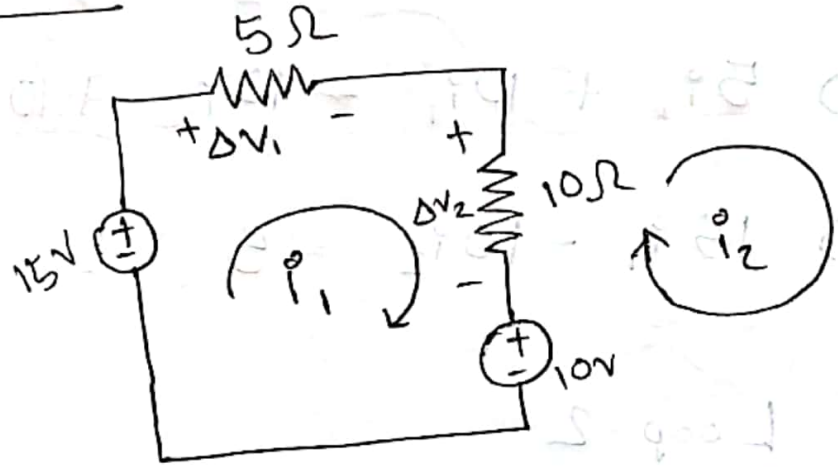
∴ Clockwise

Step-4

No current sources

Step-5

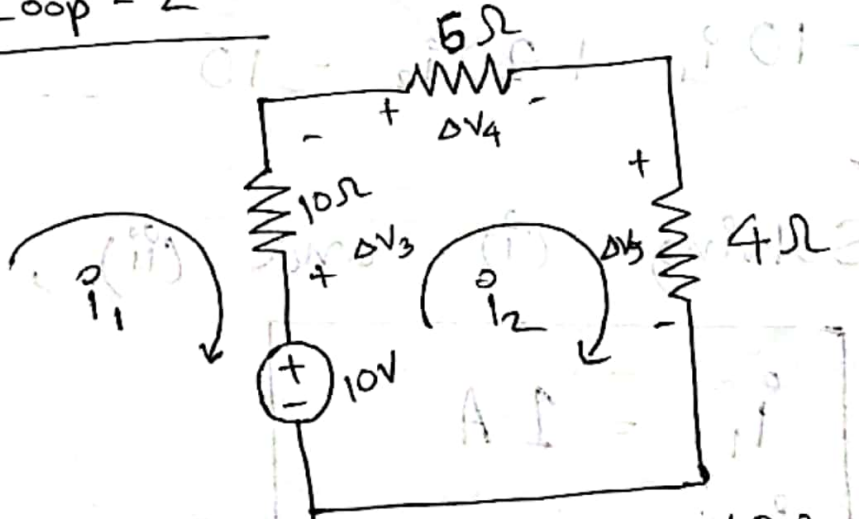
Loop-1



$$\Delta V_1 = i_1 \times 5 = 5i_1$$

$$\Delta V_2 = (i_1 - i_2) \times 10 = 10i_1 - 10i_2$$

Loop-2



$$\Delta V_3 = (i_2 - i_1) \times 10 = 10i_2 - 10i_1$$

$$\Delta V_4 = i_2 \times 6 = 6i_2$$

$$\Delta V_5 = i_2 \times 4 = 4i_2$$

Step-6

Loop-1

→ Current i_1 entering from + side.

→ Current i_1 entering from − side.

$$\Delta V_1 + \Delta V_2 + 10 - 15 = 0$$

$$\Rightarrow 5i_1 + 10i_1 - 10i_2 + 10 - 15 = 0$$

$$\Rightarrow 15i_1 - 10i_2 = 5 \quad \text{---} \quad (i)$$

Loop-2

→ Current i_2 entering from − side

$$\Delta V_3 + \Delta V_4 + \Delta V_5 - 10 = 0$$

$$\Rightarrow 10i_2 - 10i_1 + 6i_2 + 4i_2 - 10 = 0$$

$$\Rightarrow -10i_1 + 20i_2 = 10 \quad \text{---} \quad (ii)$$

Step-7

Solving (i) and (ii),

$$i_1 = 1\ A$$

$$i_2 = 1\ A$$

I_1, I_2, I_3 ?

$I_1 = i_1$ | From circuit diagram |

$\therefore \boxed{I_1 = 1\ A}$

$I_2 = i_2$

$\therefore \boxed{I_2 = 1\ A}$

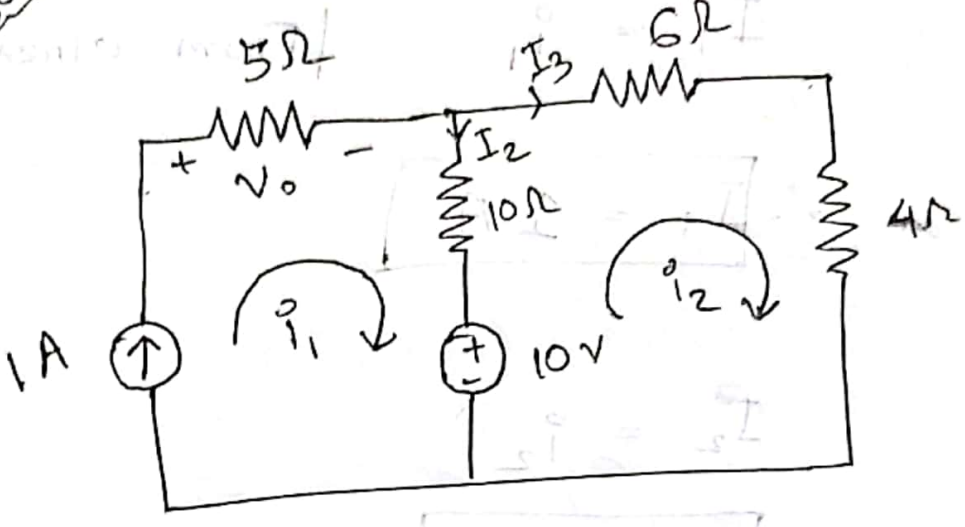
$I_3 = i_1 - i_2$

$\therefore I_3 = (1 - 1)\ A$

$\therefore \boxed{I_3 = 0\ A}$

(Ans.)

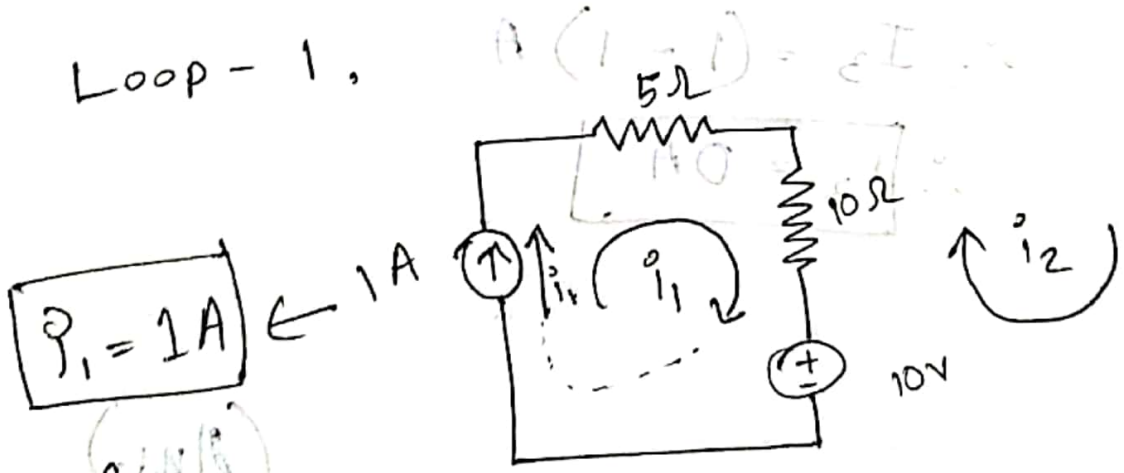
Mesh Analysis with Current Source



Step 1-3

Step-4

There is a current source in Loop - 1.



Step-6

Loop - 1

$$i_1 = 1 \quad \text{---- (i)}$$

Loop-2

Like before,

$$-10 i_1 + 20 i_2 = 10 \quad \text{---(ii)}$$

Step-7

$$\Rightarrow -10 \cdot 1 + 20 i_2 = 10$$

$$\Rightarrow 20 i_2 = 10 + 10 = 20$$

$$\Rightarrow i_2 = \frac{20}{20}$$

$$\therefore i_2 = 1$$

$$\therefore i_1 = 1 \text{ A}, \quad i_2 = 1 \text{ A}.$$

V_o

$$V_o = i_1 \cdot 5 \text{ V} \quad \text{[From Circuit]}$$

$$= 1 \cdot 5 \text{ V}$$

$$= \boxed{5 \text{ V}}$$

I_2

$$I_2 = i_1 - i_2 \quad \text{[From Circuit]}$$

$$= \boxed{0 \text{ A}}$$

I_3

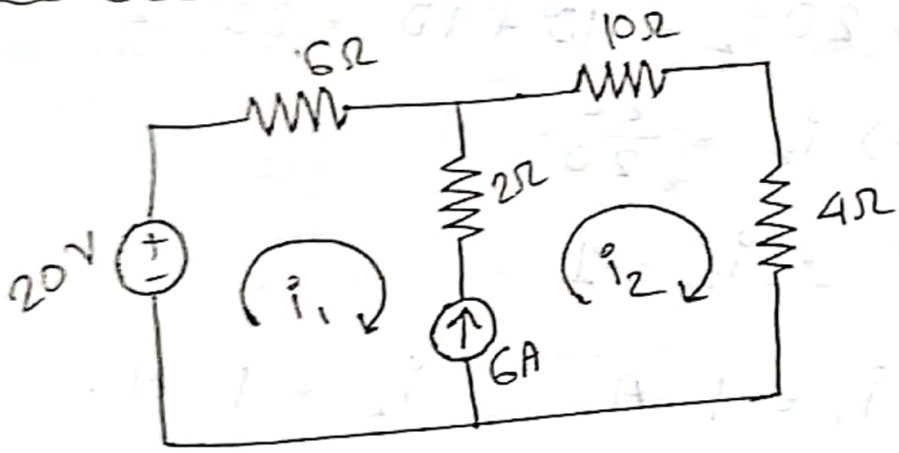
$$I_3 = i_2 = \boxed{1 \text{ A}}$$

(Ans.)

Supermesh

↳ A Current Source in between two loops → Those two loops (Meshes) together become Supermesh.

Practise Example



Step-4

6A Current source in between loop 1 and loop 2 → Supermesh.

$$i_2 - i_1 = 6 \quad ----(i)$$

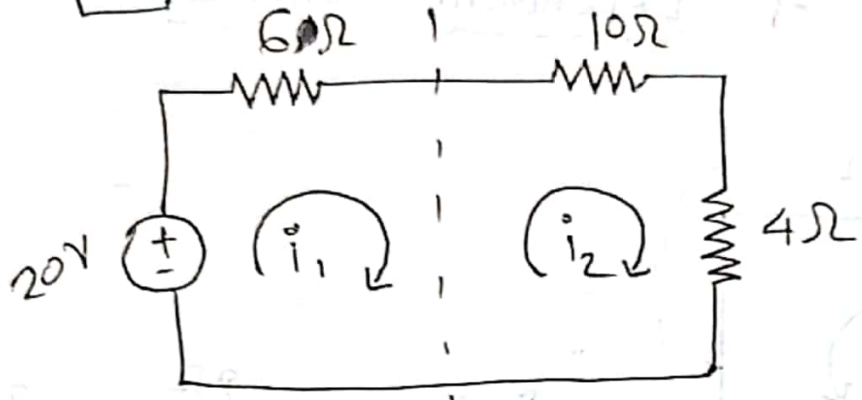
Step-5

$$\Delta V_{6\Omega} = 6 i_1$$

$$\Delta V_{10\Omega} = 10 i_2$$

$$\Delta V_{4\Omega} = 4 i_2$$

Step-6 → Ignore all common components bet^{n} those two loops.



Mesh Eqn.

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

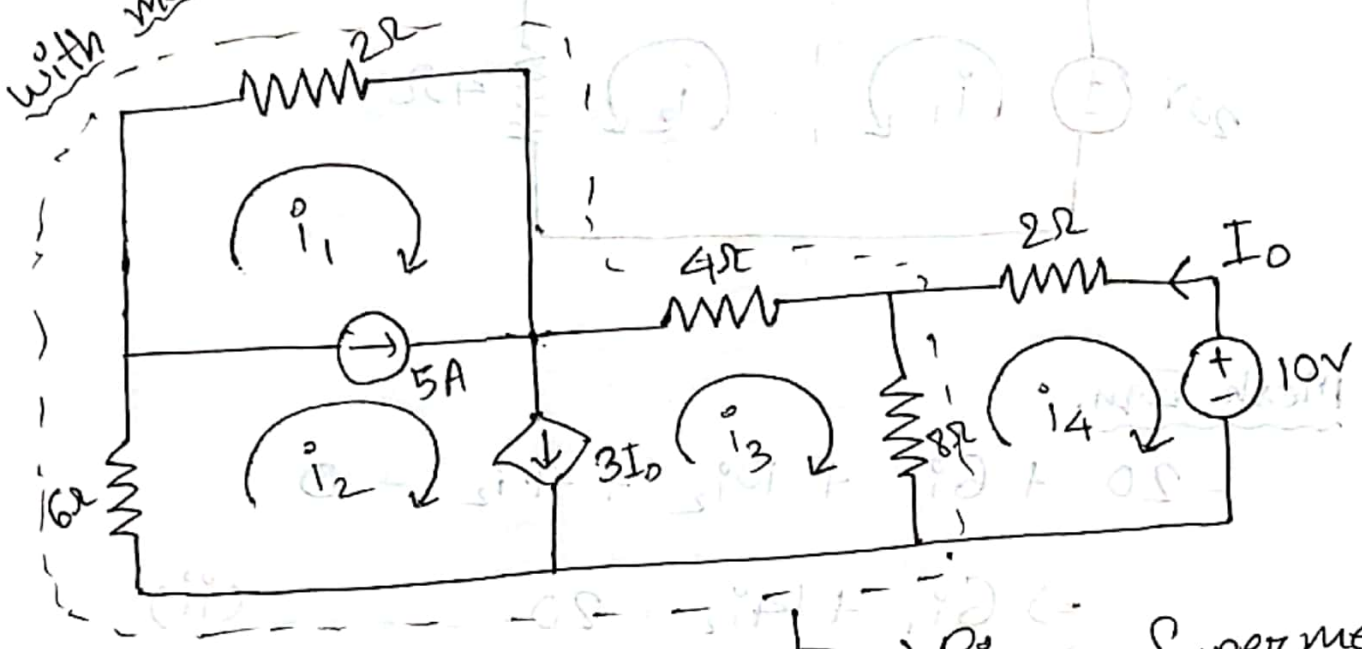
$$\Rightarrow 6i_1 + 14i_2 = 20 \quad ---- \quad (ii)$$

Step-7

Solving (i) & (ii),

$$i_1 = -3.2\,A \quad , \quad i_2 = 2.8\,A$$

Supermesh with more than two loops + Dependent Source.



↳ Bigger Supermesh created by two Supermeshes

Step-4

$$i_2 - i_1 = 5 \quad \text{---(i)}$$

$$i_2 - i_3 = 3I_o \quad \text{---(ii)}$$

Now, $i_4 = -I_o \Rightarrow I_o = -i_4$ |From Loop 4|

$\therefore i_2 - i_3 = -3i_4$

$\Rightarrow i_2 - i_3 + 3i_4 = 0 \quad \text{---(iii)}$

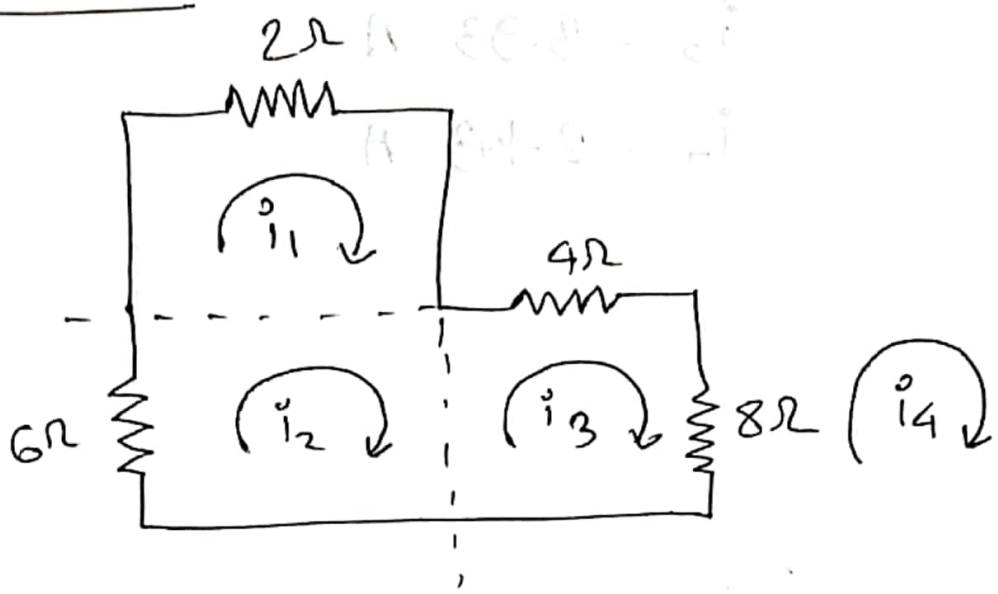
Step-5

You can skip this part now with practise

:)

Step-6

Supermesh



Eqn.

$$2i_1 + 6i_2 + 4i_3 + 8(i_3 - i_4) = 0$$

$$\Rightarrow 2i_1 + 6i_2 + 12i_3 - 8i_4 = 0 \quad ---(iv)$$

Loop-4

$$8(i_4 - i_3) + 2i_4 + 10 = 0$$

$$\Rightarrow 10i_4 - 8i_3 = -10 \quad ---(v)$$

Step-7

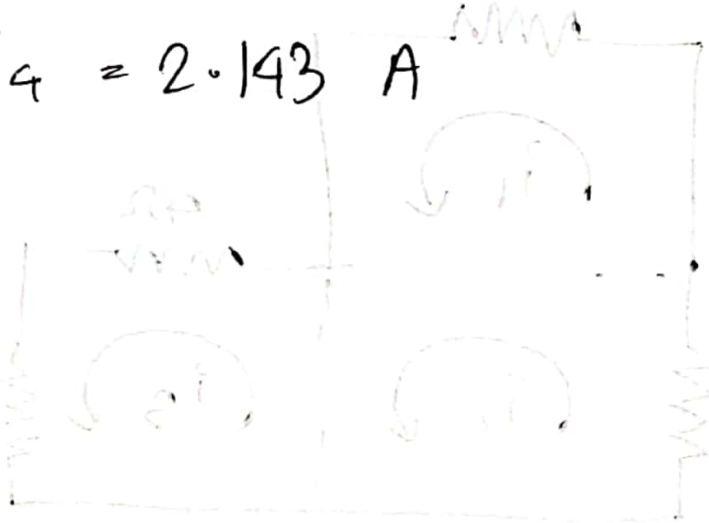
Solving $\overset{\circ}{i}, \overset{\circ\circ}{iii}, \overset{\circ}{iv}, V$

$$\overset{\circ}{i}_1 = -7.5 \text{ A}$$

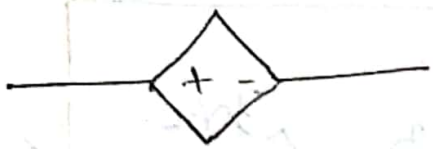
$$\overset{\circ}{i}_2 = -2.5 \text{ A}$$

$$\overset{\circ}{i}_3 = 3.93 \text{ A}$$

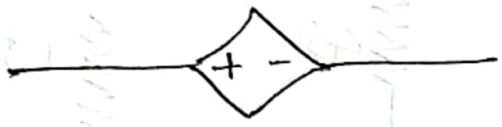
$$\overset{\circ}{i}_4 = 2.143 \text{ A}$$



Dependent Sources

- 

→ Voltage Controlled Voltage Source

- 

→ Current Controlled Voltage Source

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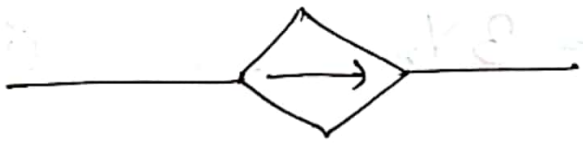
→ Voltage Controlled Current Source

- 

→ Current Controlled Current Source

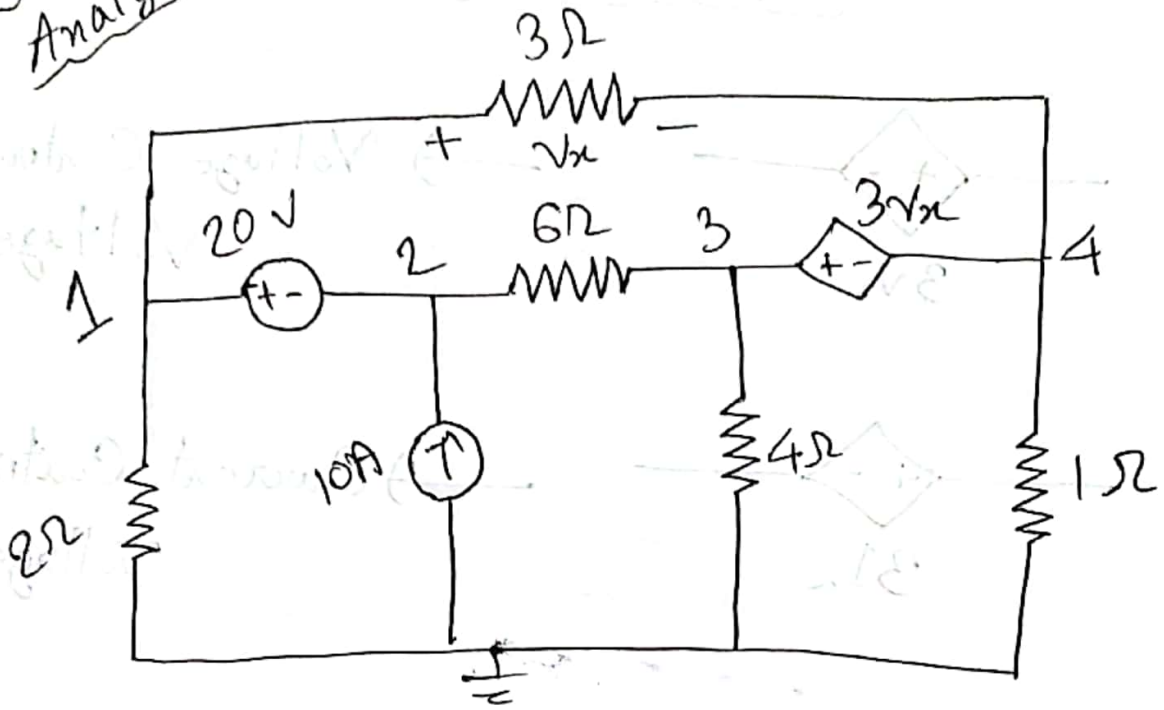
How to Deal with them?

Simply replace v_x / i_x with the corresponding component equation from the circuit.

Example : Previous One.

They also form Supernodes and Supermeshes.

Example for Nodal Analysis



Step-2

$$V_1 - V_2 = 20\,V \quad ---(i)$$

$$V_3 - V_4 = 3V_x \quad ----(ii)$$

Now, $V_x = V_1 - V_4$

$\therefore V_3 - V_4 = 3(V_1 - V_4)$

$\Rightarrow 3V_1 - V_3 - 2V_4 = 0 \quad ---(iii)$

Step-3

No need anymore I guess?

Step-4

$$+ - \frac{V_3}{6}$$

Super Node 1-2

$$V_1\left(\frac{1}{2}+\frac{1}{3}\right) - \frac{V_4}{3} + V_2\left(\frac{1}{6}\right)_1 - 10 = 0$$

$$- - - \quad (iv)$$

Super Node 3-4

$$V_3\left(\frac{1}{6}+\frac{1}{4}\right) - \frac{V_2}{6} + V_4\left(1+\frac{1}{3}\right) - \frac{V_1}{3} = 0$$

$$- - - \quad (v)$$

Step-5

Solve (i), (iii), (iv), (v). to get

V_1, V_2, V_3, V_4.