

**Lecture 1** 

Alt. Representation, CSE250 Review, IV Characteristics

Prepared By:

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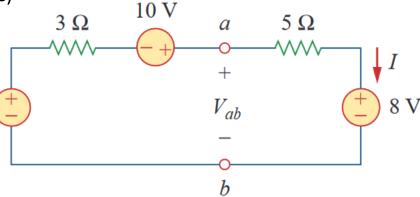
### Alternative Circuit Representation: Line diagrams

#### Steps to decompose circuits to line diagram

- 1. Set a ground so that number of **floating voltage** sources are minimized.
- 2. Detach the ground
- 3. Convert the non-floating voltage sources (current sources) into:
  - Arrow : (→) Fixed/Constant voltage source
  - Open circle dot: (-0) Input/Output node voltage (may or may not be a source)
  - Filled circle dot: (- Known node voltage (may or may not be a source)
- 4. Keep passive elements as they are.

#### Floating voltage sources:

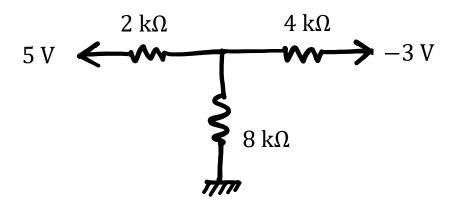
Voltage sources which are **not connected the ground** terminal. In the diagram, the **10 V** voltage source is floating



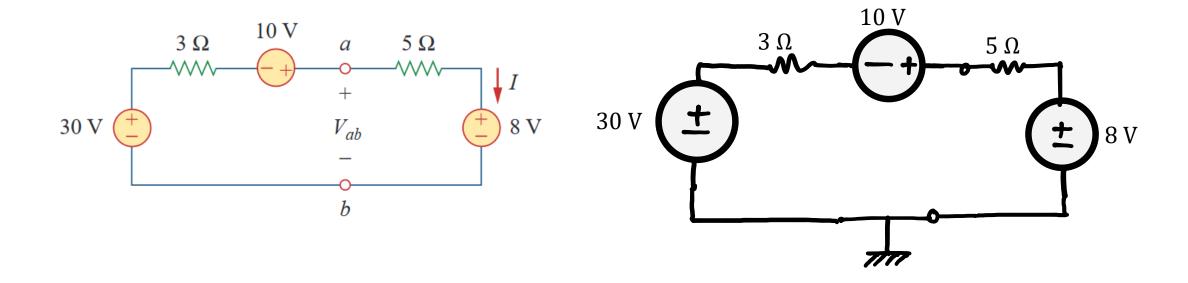
30 V

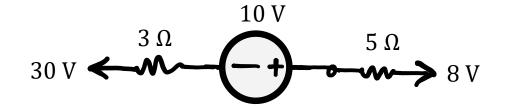
# Line diagrams: Example 1





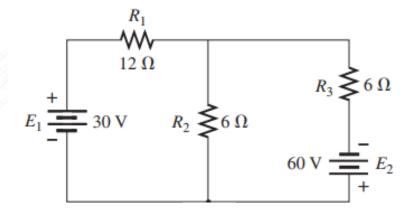
# Line diagrams: Example 2



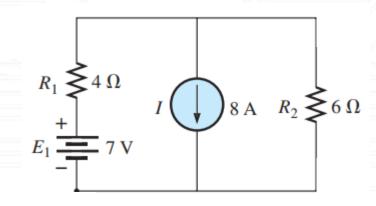


# More Examples

### Difficulty: 2/5



### Difficulty: 3/5

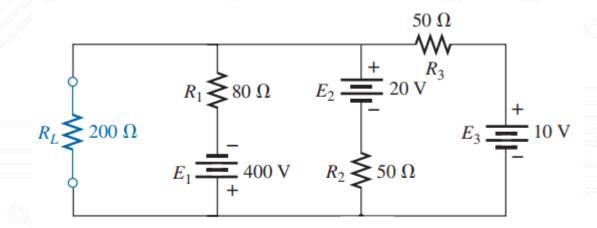


Example: 2

Example: 3

### More Examples

### Difficulty: 4/5



Example: 4

Step – (4) Make all the active elements (dc/ac type, voltage/current sources) into single terminals (arrows/circles) using the voltages you wrote as much as you can [THERE MIGHT BE CASES WHERE YOU CAN'T DO THAT]

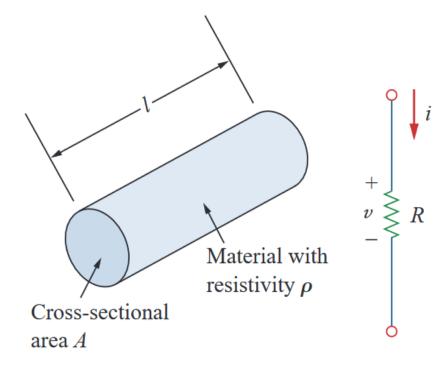
### The fundamentals ...

### Ohm's Law -

• the voltage v across a resistor is directly proportional to the current i flowing

through the resistor (R)

$$v \propto i$$
 $v = iR$ 



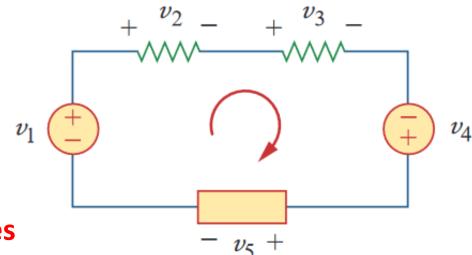
## KVL: Kirchhoff's voltage law

The <u>algebraic sum</u> of all <u>voltages</u> around a closed path (or loop) is zero.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$

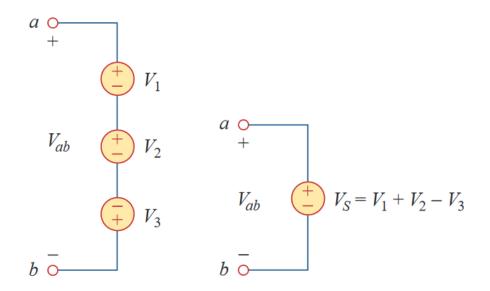
Sum of voltage drops = Sum of voltage rises



## KVL: Kirchhoff's voltage law

$$-V_{ab} + V_1 + V_2 + V_3 = 0$$

$$V_{ab} = V_1 + V_2 + V_3$$



**Equivalent Circuits** 

### KVL – Example 1

Find I and  $V_{ab}$  in the circuit

#### **Solution:**

**KVL** 

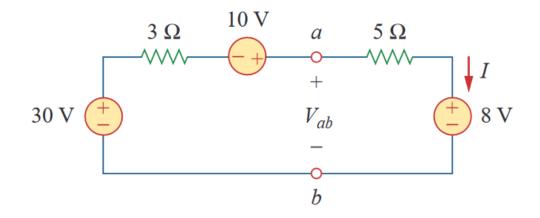
$$-30 + 3I - 10 + 5I + 8 = 0$$

$$I = \frac{32}{8}A = 4A$$

**KVL** 

$$-V_{ab} + 5I + 8 = 0$$

$$V_{ab} = 28 \text{ V}$$



Tip: If you find resistance values in  $k\Omega$  instead of  $\Omega$ , don't convert the  $k\Omega$  values to  $\Omega$ . Just find currents in mA instead of A.

### KVL – Example 2

Find  $v_1, v_2, v_3, i_1, i_2$  and  $i_3$  in the circuit

#### **Solution:**

KVL in first loop

$$-5 + 2\mathbf{i_1} + 8(\mathbf{i_1} - \mathbf{i_3}) = 0$$

$$10i_1 - 8i_3 = 5$$

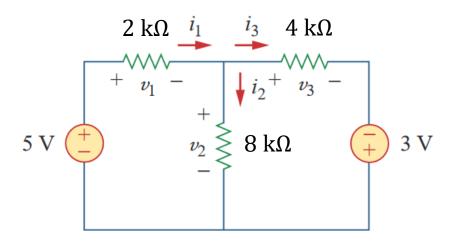
KVL in second loop

$$-8(\mathbf{i_1} - \mathbf{i_3}) + 4\mathbf{i_3} - 3 = 0$$

$$-8i_1 + 12i_3 = 3$$

Solving:

$$i_1 = 1.5 \text{ mA}$$
  $v_1 = 3 \text{ V}$   
 $i_3 = 1.25 \text{ mA}$   $v_2 = 2 \text{ V}$   
 $i_2 = i_1 - i_3 = 0.25 \text{ mA}$   $v_3 = 5 \text{ V}$ 



**Tip:** If you find resistance values in  $\mathbf{k}\Omega$  instead of  $\Omega$ , don't convert the  $\mathbf{k}\Omega$  values to  $\Omega$ . Just find currents in  $\mathbf{m}\mathbf{A}$  instead of  $\mathbf{A}$ .

### KCL: Kirchoff's Current Law

The <u>algebraic sum</u> of the <u>currents</u> entering a node (closed boundary)

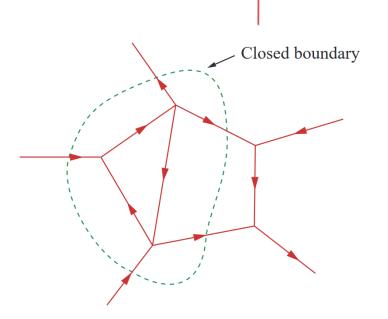
is equal to the sum of the currents leaving the node.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Current Entering node: Positive

Current Exiting node: Negative

Or vice versa...



### KCL- Example 1

Find  $v_1, v_2, v_3, i_1, i_2$  and  $i_3$  in the circuit

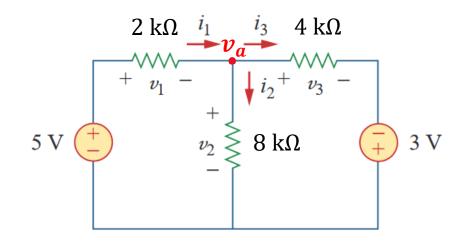
#### **Solution:**

KCL in node  $v_a$ . (PS:  $v_a = v_2$ )

$$\frac{5 - \mathbf{v_2}}{2} - \frac{\mathbf{v_2} - (-3)}{4} - \frac{\mathbf{v_2} - 0}{8} = 0$$

$$v_2\left(-\frac{1}{2} - \frac{1}{4} - \frac{1}{8}\right) = -\left(\frac{5}{2} - \frac{3}{4}\right)$$

$$v_2 = \frac{7}{4} \cdot \frac{8}{7} \text{ V} = 2 \text{ V}$$
 $v_1 = 5 - v_2 = 3 \text{ V}$ 
 $v_3 = v_2 - (-3) = 5 \text{ V}$ 

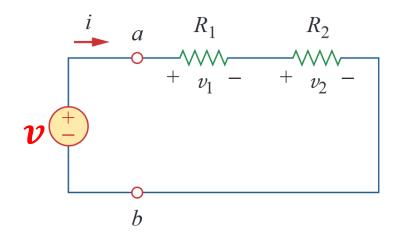


### Series Resistors and Voltage Division

The **equivalent resistance** of any number of resistors **connected in series** is the <u>sum of the individual resistances</u>.

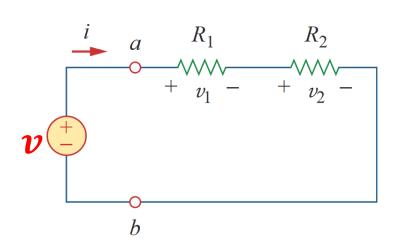
### Principle of voltage division

**Source voltage** v - is divided among the resistors in <u>direct proportion to their resistances</u>; the larger the resistance, the larger the voltage drop.



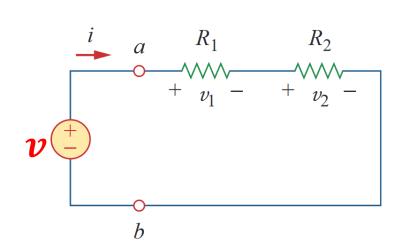
$$v_1 = \frac{R_1}{R_1 + R_2} v$$
  $v_2 = \frac{R_2}{R_1 + R_2} v$ 

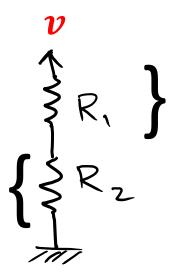
# Line diagram: Example 3



$$v_{2} = \frac{R_{2}}{R_{1} + R_{2}} v \left\{ \begin{cases} R_{1} \\ R_{2} \end{cases} \right\} \frac{R_{1}}{R_{1} + R_{2}} v$$

## Line diagram: Example 3





KVL (acts along a line instead of a loop)

$$\mathbf{v} - iR_1 - iR_2 = 0$$

### Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors **connected** in **parallel** is the <u>inverse</u> of the <u>sum</u> of the individual **conductances**.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$\rightarrow R_{eq}$$

$$R_1 \geqslant R_2 \geqslant R_3 \geqslant R_3 \geqslant R_N$$

Simplification for the case when  $R_1 = R_2 = R_3 \cdots = R_N$ 

$$R_{eq} = \frac{R_1}{N}$$

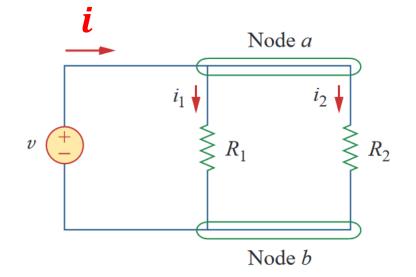
### Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors **connected** in **parallel** is the <u>inverse</u> of the <u>sum</u> of the individual **conductances**.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_{eq} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

Simplification for the case when  $R_1 = R_2$ 

$$R_{eq} = \frac{R_1}{2}$$



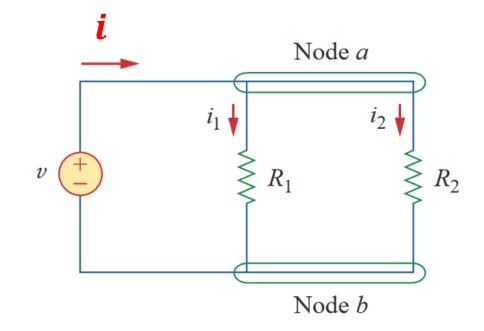
### Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors **connected** in **parallel** is the <u>inverse</u> of the <u>sum</u> of the individual **conductances**.

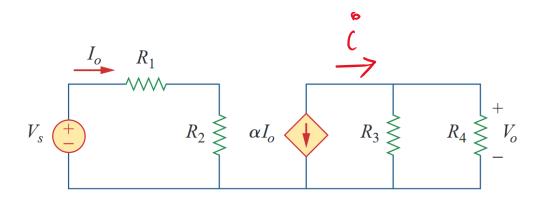
### **Principle of current division**

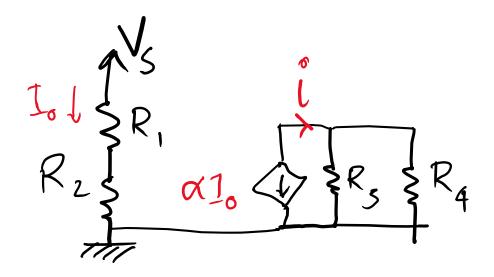
**Source current** *i* - is divided among the resistors in <u>direct inverse</u> proportion to their resistances; the larger the resistance, the larger the voltage drop.

$$i_1 = \frac{1/R_1}{1/R_1 + 1/R_2}i$$
  $i_2 = \frac{1/R_2}{1/R_1 + 1/R_2}i$ 



# Line diagrams: Example 4





### Practice Problem 1

For the circuit, find  $\left| \frac{V_o}{V_s} \right|$  in terms of  $\alpha$ ,  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$ .

If  $R_1 = R_2 = R_3 = R_4$  what value of  $\alpha$  will produce  $\left| \frac{V_o}{V_S} \right| = 10$ ?

#### **Solution:**

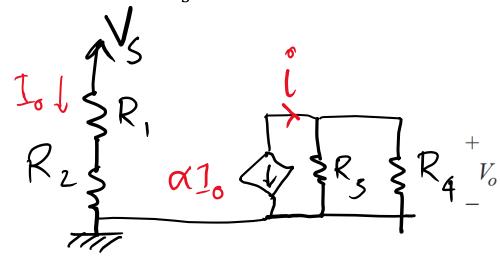
Ohm's Law across  $R_1 + R_2$ .

$$I_O = \frac{V_S}{R_1 + R_2}$$

$$i = -\alpha I_{o}$$

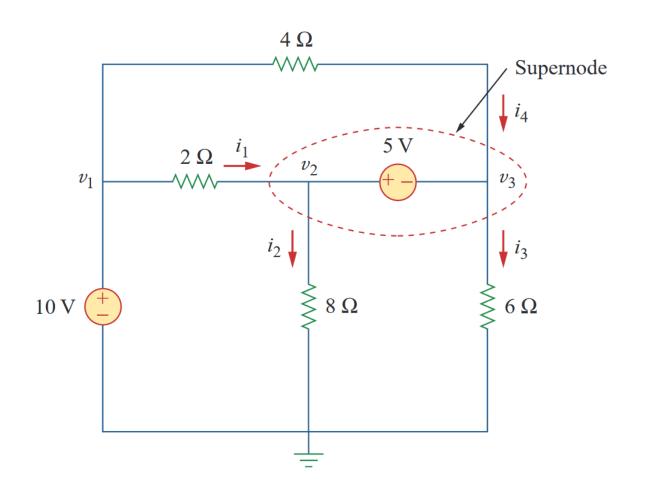
Voltage across Parallel Resistors  $R_3$ ,  $R_4$ 

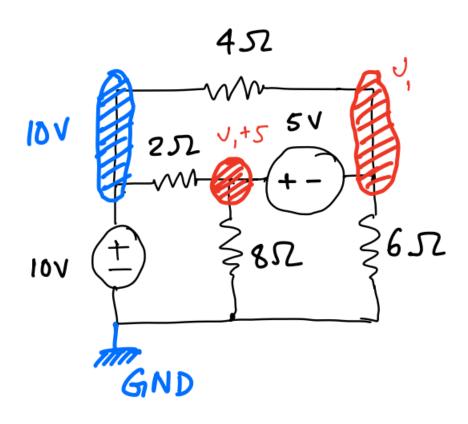
$$V_O = i(\mathbf{R_3}||\mathbf{R_4}) = -\frac{\alpha V_S}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$



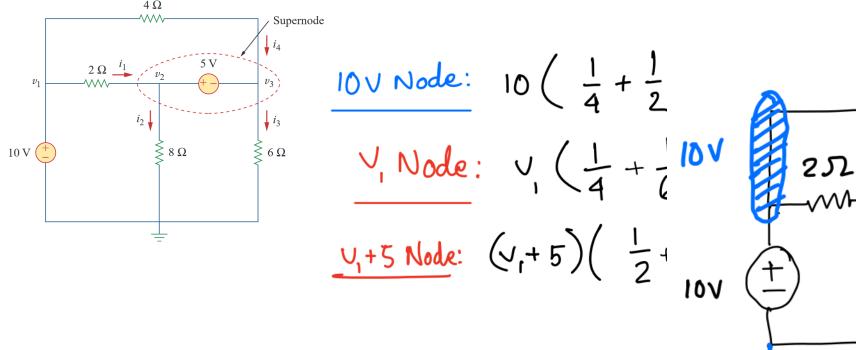
$$\left|\frac{V_o}{V_s}\right| = \frac{\alpha}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

# Example 1- Nodal Analysis

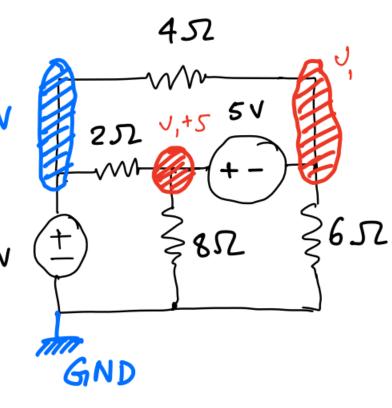




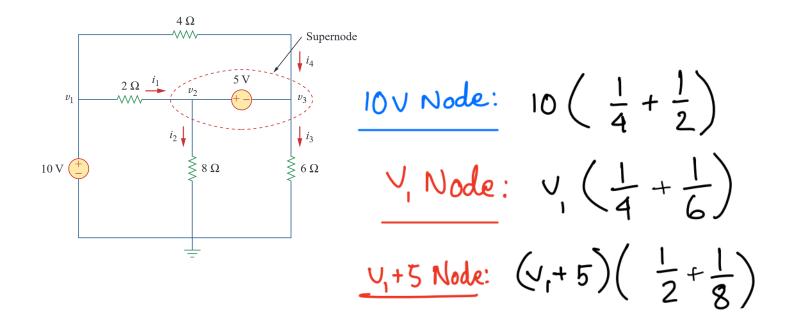
## Example 1- Nodal Analysis



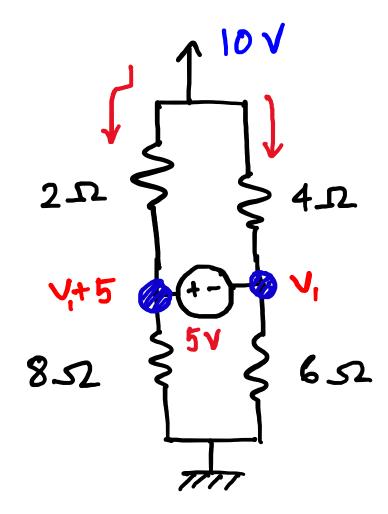
$$v_1\left(\frac{1}{4} + \frac{1}{6}\right) + (v_1 + 5)\left(\frac{1}{2} + \frac{1}{8}\right) - 10\left(\frac{1}{2} + \frac{1}{4}\right)$$



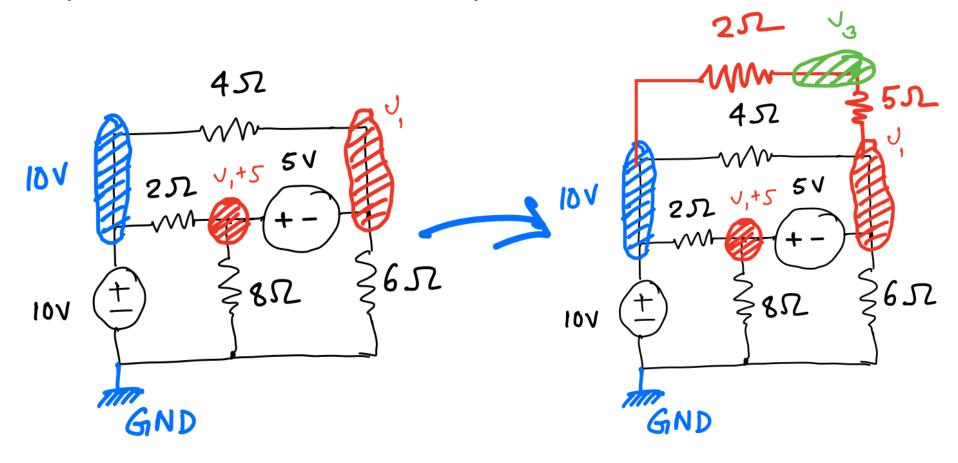
### Example 1- Nodal Analysis



$$v_1\left(\frac{1}{4} + \frac{1}{6}\right) + (v_1 + 5)\left(\frac{1}{2} + \frac{1}{8}\right) - 10\left(\frac{1}{2} + \frac{1}{4}\right) = 0$$



## Example 2- Nodal Analysis – Home Task 1

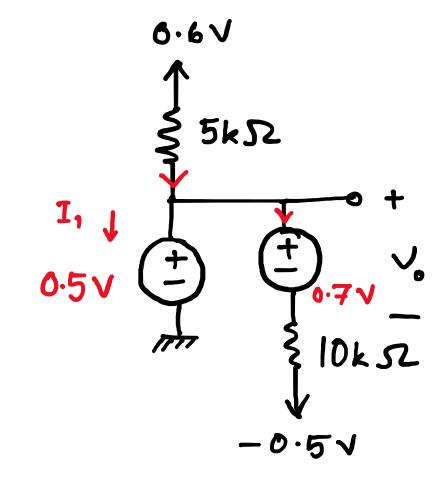


Find the two node  $v_1$  and  $v_3$  equations!

KCL at node  $v_o$ 

$$\frac{0.6 - 0.5}{5} = \frac{(0.5 - 0.7) - (-0.5)}{10} + I_1$$

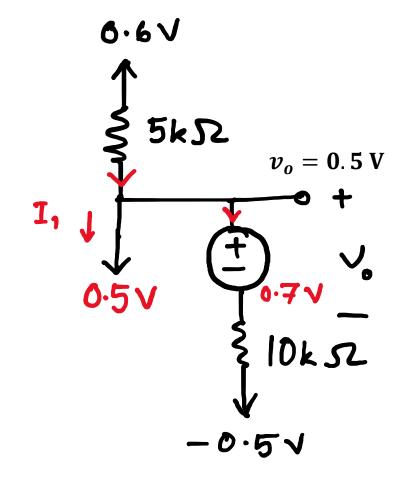
$$I_1 = -0.01 \text{ mA}$$



KCL at node  $v_o$ 

$$\frac{0.6 - 0.5}{5} = \frac{(0.5 - 0.7) - (-0.5)}{10} + I_1$$

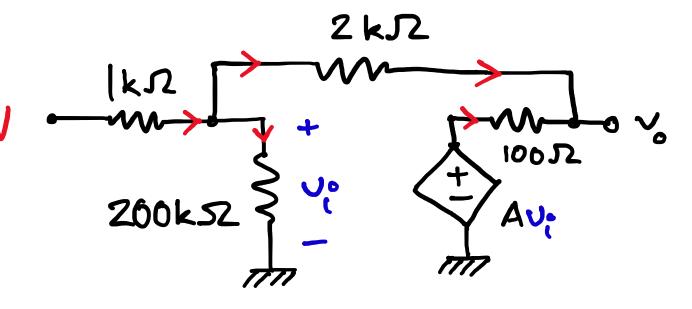
$$I_1 = -0.01 \text{ mA}$$



KCL at node 
$$\frac{v_i}{2 - v_i} = \frac{v_i - v_o}{2} + \frac{v_i}{200}$$
 
$$\frac{301}{200}v_i - \frac{1}{2}v_o = 2$$

KCL at node 
$$\frac{v_o}{2}$$
  $\frac{v_i - v_o}{2} + \frac{Av_i - v_o}{0.1} = 0$ 

$$(2 \times 10^6 + 0.5)v_i - 10.5v_o = 0$$



$$A = 2 \times 10^5$$

KCL at node 
$$v_i$$

$$\frac{0 - v_i}{5} = \frac{v_i - v_o}{40} + \frac{v_i - 1}{200}$$

$$\frac{23}{100}v_i - \frac{1}{40}v_o = \frac{1}{200}$$

KCL at node  $v_o$ 

$$\frac{v_i - v_o}{40} + \frac{Av_i - v_o}{0.1} = \frac{v_o}{20}$$

 $\frac{40 \text{ kJ2}}{777}$   $\frac{5 \text{ kJ2}}{200 \text{ kJ2}}$   $\frac{1000 \text{ J2}}{400 \text{ kJ2}}$ 

$$(2 \times 10^6 + 0.025)v_i - 10.075v_o = 0$$

## Example 6 – Home Task 2

For  $\emph{\textbf{R}}=100~\Omega$ ,  $\emph{\textbf{R}}_{\emph{\textbf{L}}}=10~\mathrm{k}\Omega$ ,  $\emph{\textbf{r}}_{\emph{\textbf{Z}}}=20~\Omega$ ,  $\emph{\textbf{V}}_{\emph{\textbf{Z}}\emph{\textbf{0}}}=3~\mathrm{V}$ , and  $\emph{\textbf{I}}_{\emph{\textbf{Z}}}=1~\mathrm{mA}$ .

- a. Find  $V_{o}$
- b. Find  $I_L$
- c. Find I
- d. Find  $V^+$

