Solution : Quiz # 3

Name: ~~ID #~~

Quiz # 3 CSE 330 (01) Marks: /10

MCQ: Choose Only One Answer.

1. A set of data values are given as $x = 3.0, 3.1, 3.2, 3.3$ and $f(x) = 2.5, 2.8, 3.2, 3.4$ respectively. The first derivative of $f(x)$ at $x = 3.1$ according to the central difference formula is

   A. 3.5. **B.** 3.0. **C.** 4.0. **D.** 2.0.

   1. A

2. A set of data values are given as $x = 3.0, 3.1, 3.2, 3.3$ and $f(x) = 2.5, 2.8, 3.2, 3.4$ respectively. The first derivative of $f(x)$ at $x = 3.2$ according to the backward difference formula is

   A. 1.0. **B.** - 2.0. **C.** 3.0. **D.** 4.0.

   2. D

3. The truncation error of the forward difference method varies according to which of the following rules ($h > 0$)?

   A. Truncation error is inversely proportional to $-h$ **B.** Truncation error is directly proportional to $-h$.
   C. Truncation error is directly proportional to h^2. **D.** None of the above.

   3. B

4. The rounding error increase as

   A. $h \to 0$. **B.** $h \to 1$. **C.** $h \to \infty$. **D.** None of these.

   4. A

5. If we compute the 2nd order Richardson extrapolated numerical derivative $D_h^{(2)}$, the leading error term in the expansion is of order

   A. h^2. **B.** h^4. **C.** h^6. **D.** h^8.

   5. C

Problems: Marks are as indicated

6. (5 marks) As we derived in the class, the total error in computing the derivative using the central difference method is given by

$$\left|\text{Total Error}\right| \le \frac{h^2}{6}\left|f'''(\xi)\right|_{\max} + \frac{\epsilon_M}{h}\left|f(\xi)\right|_{\max}, \quad \text{with} \quad \xi \in [x_0 - h, x_0 + h] \, ,$$

where the first term on the right hand side is the truncation error and the 2nd term is the rounding error. Show that the error is minimum when

$$h = \left(3\epsilon_M \times \frac{|f(\xi)|_{\max}}{|f'''(\xi)|_{\max}}\right)^{1/3},$$

where the symbols have their usual meanings.

Here $Err(h) = \frac{h^2}{6}|f'''(\xi)|_{max} + \frac{\epsilon_M}{h}|f(\xi)|_{max}$.

Note that $|f'''(\xi)|_{max}$ and $|f(\xi)|_{max}$ are constant values.

$$\therefore \left|\frac{d\,Err(h)}{dh}\right| = \left|\frac{2h}{6}|f'''(\xi)|_{max} + \epsilon_M\left(-\frac{1}{h^2}\right)|f(\xi)|_{max}\right|$$

And for minimum: $\left|\frac{d\,Err(h)}{dh}\right| = 0$

$$\Rightarrow \frac{2h}{6}|f'''(\xi)|_{max} = \frac{\epsilon_M}{h^2}|f(\xi)|_{max}$$

$$\Rightarrow h^3 = \frac{3\epsilon_M}{|f'''(\xi)|_{max}} \cdot |f(\xi)|_{max}$$

$$\Rightarrow \boxed{h = \left(3\epsilon_M \cdot \frac{|f(\xi)|_{max}}{|f'''(\xi)|_{max}}\right)^{1/3}}$$