

Solution

Name: _____

ID # _____

Quiz # 2

CSE 330 (01)

Marks: _____ /10

MCQ: Choose Only One Answer.

1. A data set has three values. The Vandermonde Matrix corresponding to these data values is of order
A. 3×3 . B. 2×2 . C. 2×3 . D. 3×2 .
1. A
2. Which of the following methods is useful for a dynamic system?
A. Lagrange method. B. Newton's divided/difference method. C. Vandermonde matrix method.
D. All of the above methods are useful.
2. B
3. A system is solved by Vandermonde matrix method where the Vandermonde matrix, V , is of order 3×3 . If the same system is solved by the Lagrange method, how many Lagrange basis elements will be needed?
A. 1. B. 2. C. 3. D. 4.
3. C
4. The Lagrange basis element, $l_3(x)$, of a system with two nodes is a
A. linear function of x . B. quadratic function of x . C. cubic function of x . D. It does not exist.
4. D
5. The Lagrange basis element, $l_1(x)$, of a system with four nodes is a
A. cubic function of x . B. quadratic function of x . C. linear function of x . D. None of the above.
5. A
6. Consider a system with four nodes: x_0, x_1, x_2 and x_3 . The Newton basis element $n_2(x)$ is expressed as
A. $(x - x_0)$. B. $(x - x_0)(x - x_1)$. C. $(x - x_0)(x - x_1)(x - x_2)$. D. $(x - x_0)(x - x_1)(x - x_2)(x - x_3)$.
6. B

Problems: Marks are as indicated

7. (4 marks) Consider the function $f(x) = e^{-x}$ with nodes at 0, 1 and 2 in the interval $[-0.25, 2.25]$. Working to 3 significant figures, compute the upper bound of the estimated error if $f(x)$ is interpolated by a degree two polynomial.

Solution: here: $n=2$ & $I = [-0.25, 2.25]$. & $f(x) = e^{-x}$.

$$\text{Upper bound} = \left| \frac{1}{(n+1)!} f^{(n+1)}(\xi) \cdot \omega_3(x) \right|_{\max} \leq \frac{1}{3!} \left| f'''(\xi) \right|_{\max} \left| \omega_3(x) \right|_{\max} \text{ with } \xi, x \in I.$$

$$\text{Now, } \left| f'''(\xi) \right|_{\max} = \left| \frac{d^3}{dx^3} (e^{-x}) \right|_{\max} = \left| (-1)^3 e^{-x} \right|_{\max} = e^{-(-0.25)} = 1.284.$$

$$\text{and for } \omega_3: \omega_3'(x) = 0 \Rightarrow \left| \frac{d}{dx} [(x-0)(x-1)(x-2)] \right|_{\max} = 0$$
$$\Rightarrow \frac{d}{dx} (x^3 - 3x^2 + 2x) = 0 \Rightarrow 3x^2 + 6x + 2 = 0$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{36 - 24}}{2 \cdot 3}$$
$$= -1 \pm \frac{\sqrt{2}}{3} = \frac{-3 \pm \sqrt{2}}{3}$$
$$= -0.528, -1.471$$

$\notin I$.

But ~~$x = -0.528, -1.471 \notin I$~~

$$\text{So, } \left| \omega_3(x) \right|_{\max} = \left| \omega(0.25) \right| = 0.703$$

$$\therefore \text{Upper Bound} \leq \frac{1}{6} (1.284) (0.703) = 0.150$$

So, the critical points are outside the interval