Solution

MCQ: Choose Only One Answer.

1. $\left(\dfrac{41}{4}\right)_{10}$ equals to

   A. $(1010.01)_2$.   B. $(100.101)_2$.   C. $(101.111)_2$.   D. $(1001.11)_2$.

   1. _____ **A** _____

2. Up to 5 significant figure, $\left(\dfrac{41}{32}\right)_{10}$ equals to

   A. $(1.0001)_2$.   B. $(1.0101)_2$.   C. $(1.0100)_2$.   D. $(1.1010)_2$.

   2. _____ **B** _____

3. In 2-digit decimal arithmetic, $\text{fl}[5.9 + (5.5 + 0.4)]$ equals

   A. 10.0.   B. 11.0.   C. 12.0.   D. None of these.

   3. _____ **C** _____

4. If we work to 3 decimal digits, the average of 5.01 and 5.02 will be

   A. 5.015.   B. 5.02.   C. 5.01.   D. 5.0.

   4. _____ **D** _____

5. Using the Taylor expansion of $\cosh(x) = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \cdots$, how many terms on the right-hand side will give the value of $\cosh(0.1)$ up to 9 significant figures?

   A. 1.   B. 2.   C. 3.   D. 4.

   5. _____ **C** _____

Problems: Marks are as indicated

6. (5 marks) Consider the quadratic equation, $x^2 - 60x + 1 = 0$. Working to 6 significant figures, compute the roots of the quadratic equation and check that there is a loss of significance. Find the correct roots such that loss of significance does not occur.

The roots are: $x = \dfrac{-(-60) \pm \sqrt{(-60)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = 30 \pm \sqrt{899}$

$\Rightarrow x_1 = 30 + \sqrt{899} = 30 + 29.9833 = 59.9833.$ ✓

& $x_2 = 30 - \sqrt{899} = 30 - 29.9833 = 0.0167.$

Now Check that

$x_1 + x_2 = 60$, but $x_1 x_2 = 1.00172 \neq 1$. This is due to loss of significance when $\sqrt{899}$ is subtracted from 30 to get x_2. According to algebraic properties: $x_1 + x_2 = 60$ & $x_1 x_2 = 1$.

Since $x_1 + x_2 = 60$ is alright, to avoid loss of significance we calculate x_2 by using x_1 from $x_1 x_2 = 1$. That is

$x_2 = \dfrac{1}{x_1} = \dfrac{1}{59.9833} = 0.0166713.$ ✓

- Now, check that: $x_1 + x_2 = 59.9833 + 0.0166713 = 59.9999713 \approx 60.0000$ ✓

and of course: $x_1 x_2 = 1$.

So, the algebraic properties are satisfied.