Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file. The **deadline is March 01, 2023** in class.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. (6 marks) One of the Hermite basis element that we discussed during a class is
$$h_k(x) = \Big(1 - 2\big(x - x_k\big)l'_k(x_k)\Big)l_k^2(x) \ .$$
Very that $h'_k(x_j) = 0 \ \forall \ j, k$.

2. A function is given by $f(x) = xe^{-3x} + x^2$. Now answer the following up to five significant figures.
   (a) (4 marks) Approximate the derivative of $f(x)$ at $x_0 = 2$ with step size $h = 0.1$ using the central difference method.
   (b) (4 marks) Calculate the truncation error of $f(x)$ at $x_0 = 2$ using $h = 0.1$ using the central difference method.
   (c) (6 marks) Compute $D_{0.1}^{(1)}$ at $x_0 = 2$ using Richardson extrapolation method and calculate the truncation error.

3. During the class, we derived in detail the first order Richardson extrapolated derivative, by using $h \to h/2$,
$$D_h^{(1)} \equiv f'(x_0) - \frac{h^4}{480} f^{(5)}(x_0) + \mathcal{O}(h^6) \ .$$
   (a) (6 marks) Using $h \to h/2$, derive the expression for $D_h^{(2)}$ which is the second order Richardson extrapolation.
   (b) (4 marks) If $f(x) = x^2 \ln x$, $x_0 = 1$, $h = 0.1$, find the upper bound of error for $D_h^{(1)}$.

Motto: Mathematics is NOT difficult, but what is difficult is to believe that mathematics is NOT difficult.