MCQ: Choose Only One Answer.

1. A function $f(x)$ has a ZERO in the interval $[a, b]$ if

   A. $f(a)f(b) < 0$. **B.** $f(a)f(b) = 0$. **C.** $f(a)f(b) > 0)$. **D.** None of these.

   1. **A**

2. A function $f(x)$ has two ZEROs in the interval $[a, b]$ at $x = x_1$ and at $x = x_2$. How do you find the roots of $f(x)$ by using the Interval Bisection method?

   A. By diving the interval as $[a, b] = [a, x_1] \cup [x_1, b]$.
   B. By diving the interval as $[a, b] = [a, x_1] \cup [x_1, x_2] \cup [x_2, b]$.
   C. By diving the interval as $[a, b] = [a, x_2] \cup [x_2, b]$.
   D. The Interval Bisection method can not be applied in this case.

   2. **B**

3. A function $f(x)$ has roots at -1.5 and 3.0. Let $g(x)$ is the converging fixed point function derived from $f(x)$. The iteration starts from $x_0 = 0.5$, the iteration will converge to

   A. 3.0. **B.** 0.5. **C.** -1.5. **D.** It can not be determined..

   3. **C**

4. Which of the following is/are propertie(s) of Newton's method to find the roots of $f(x)$?

   A. It is a superlinear method. **B.** The first derivative of $f(x)$ must not be zero.
   C. There should not be any turning points in between two successive iterations. **D.** All of these.

   4. **D**

5. The Aitken acceleration can only be applied when

   A. the convergence is linear. **B.** the convergence is superlinear
   C. Both of the above. **D.** None of the above.

   5. **C**

Problems: Marks are as indicated

6. (5 marks) Consider a fixed-point function $g(x) = (9x - 1)^{1/3}$. The corresponding nonlinear function $f(x)$ has a solution $x_\star \in \mathbb{R}$. Show that $g(x)$ would lead to linear convergence if $x_\star > \frac{1}{9}\left(1 + \sqrt{27}\right)$. Hint: Use the condition for linear convergence.

Solution: Here, we have,

$$g'(x) = \frac{1}{3}\frac{9}{(9x-1)^{2/3}} = \frac{3}{(9x-1)^{2/3}} \,.$$

Therefore, for linear convergence, we must have,

$$\begin{aligned} g'(x_\star) &< 1 \\ \implies \frac{3}{(9x_\star - 1)^{2/3}} &< 1 \\ \implies \sqrt{27} &< 9x_\star - 1 \\ \implies x_\star &> \frac{1}{9}\left(1 + \sqrt{27}\right) . \checkmark \end{aligned}$$