Solution

Name: ID #

Quiz # 5 (April 12, 2023) CSE330 (01) Marks: /10

MCQ: Choose Only One Answer.

1. A linear system is defined by the matrix equation $Ax = b$. Which of the following matrix representation of A will yield a unique solution?

   A. $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$. **B.** $\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$. **C.** $\begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$. **D.** $\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$.

   1. A

2. How many operations are required to solve a linear system where $Lx = b$? Here, L is a lower triangular 4×4 matrix.

   A. 2. **B.** 16. **C.** 9. **D.** 14.

   2. B

3. We obtain the lower triangular matrix $\begin{pmatrix} 1 & 0 \\ L_{21} & 1 \end{pmatrix}$ from the matrix $\begin{pmatrix} 2 & 3 \\ 4 & 9 \end{pmatrix}$ by using the LU decomposition method. What will be the value of L_{21}?

   A. 4. **B.** 3. **C.** 2. **D.** 1.

   3. C

4. A 6×6 square matrix, A, is changed to an upper triangular form by row operations in Gaussian elimination method. After the completion of the 3rd row operation, how many matrix elements of A have been changed to zero by the row operations?

   A. 3. **B.** 18. **C.** 6. **D.** 12.

   4. D

5. Which of the following statement is NOT true about the Gaussian elimination method?

   A. The row operation changes all matrix elements of the matrix A.
   B. $\det(A)$ does not change.
   C. Both of the above. **D.** None of the above.

   5. A

6. Suppose, you are constructing the upper triangular matrix from $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{pmatrix}$. What will be the value of row multiplier, m_{32} in the next step of this calculation ?

   A. 1. **B.** -2. **C.** -4. **D.** 0.5.

   6. B

Problems: Marks are as indicated

7. (2+2 marks) A linear system described by $Ax = b$ has the following expressions,

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Evaluate the Frobenius matrices $F^{(1)}$ and $F^{(2)}$. Here the symbols have their usual meanings.

For 1st row operation: $m_{21} = \frac{a_{21}}{a_{11}} = \frac{-1}{1} = -1$

and $m_{31} = \frac{a_{31}}{a_{11}} = \frac{2}{1} = 2$

Therefore, the Frobenius matrix, $F^{(1)}$, is

$$F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix} \Rightarrow \boxed{F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}}$$

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Now, $A^{(2)} = F^{(1)} A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & -1 & -1 \end{pmatrix}$$

So, For 2nd row operation, $m_{32} = \dfrac{a_{32}}{a_{22}} = \dfrac{-1}{1} = -1.$

Hence, the Frobenious matrix, $F^{(2)}$, is:

$$F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix} \Rightarrow \boxed{F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}}$$ ✓