Name: Solution ID #

Quiz # 6 (April 29, 2023) CSE330 (01) Marks: /10

MCQ: Choose Only One Answer.

1. We will fit a least-squares straight-line to the data: $f(1) = 7$, $f(0) = 5$, $f(-2) = 1$. The matrix expression of $Ax = b$ is

A. $\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix}$. **B.** $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix}$.

C. $\begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix}$. **D.** $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix}$.

1. A

2. Which of the following statement(s) about the QR-decomposition method is(are) true? Note that A is a $m \times n$ order matrix.

(a) Q has same order as A.

(b) R has same order as A.

(c) Q is a set of orthonormal vectors.

(d) R is a lower triangular matrix.

A. (a) and (d) only. **B.** (a) and (c) only. **C.** (a) and (b) only. **D.** (c) and (d) only.

2. B

3. Compute upper bound of error of numerical integration of the function e^x for the interval $[0, 1]$ using the Trapezium rule. Consider only up to two significant figures.

A. 0.90. **B.** 0.84. **C.** 0.63. **D.** 0.23.

3. D

4. In composite Newton-Cotes formula, we

A. for each sub-interval, apply the trapezoidal rule, and multiply them.
B. for one sub-interval, apply closed Newton-Cotes formula and then for the next we apply open Newton-Cotes formula. Thus we get result for all the sub-intervals and we multiply all of them.
C. for each sub-interval, apply the trapezoidal rule, and add them up.
D. for one sub-interval, apply closed Newton-Cotes formula and then for the next we apply open Newton-Cotes formula. Thus we get result for all the sub-intervals and we add all of them.

4. C

5. In Simpson's rule, interpolating polynomial has degree

A. 1. **B.** 2. **C.** 3. **D.** 4.

5. B

Problems: Marks are as indicated

6. Consider the function $f(x) = e^{0.5x} + \frac{1}{30}x^2$ which is continuous on the interval $[0, 2]$. Answer the following questions:

(a) (2 marks) Compute the exact value of integration $I(f)$.

(b) (2+1 marks) Evaluate the approximate value of the integration using Composite Newton Cotes formula with 4 segments $C_{1,4}$. Then calculate the relative percent error using the result of Part-(a).

(a) $I(f) = \int_0^2 \left[e^{0.5x} + \frac{1}{30}x^2\right] dx = \left[\frac{e^{0.5x}}{0.5} + \frac{x^3}{3 \cdot 30}\right]_{x=0}^{x=2} \Rightarrow \boxed{I(f) = 3.52545..}$ ✓

(b) $C_{1,4} = \frac{b-a}{2m}\left[f(a) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(b)\right]$

$= \frac{2-0}{2 \cdot 4}\left[f(0) + 2f(0.5) + 2f(1.0) + 2f(1.5) + f(2)\right]$

$= \frac{1}{4}\left[1 + 2(1.29236) + 2(1.68705) + 2(2.19200) + 2.85162\right] \Rightarrow \boxed{C_{1,4} = 3.54611}$ ✓

$\therefore \%\,\text{Error} = \left|\frac{3.52545 - 3.54611}{3.52545}\right| \times 100\% \Rightarrow \boxed{\%\,\text{Error} = 0.58602}$ ✓

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