Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

---

1. Answer the following questions:
   (a) (3 marks) Show that the set,
   $$S = \left\{ \frac{1}{\sqrt{5}}\Big(2, -1, 0\Big)^{\mathrm{T}},\ \frac{1}{\sqrt{30}}\Big(1, 2, -5\Big)^{\mathrm{T}},\ \frac{1}{\sqrt{24}}\Big(2, 4, 2\Big)^{\mathrm{T}} \right\}$$
   is orthonormal.

   (b) (6 marks) Consider the values of $f(x) = \sin x$ at the points $x_0 = 4$, $x_1 = 9$ and $x_2 = -6$. Consider only up to 3 decimal places after rounding. Now, evaluate the best fit straight line using the Discrete Square Approximation for the given function.

2. Consider a set of four data points given below:
   $$f(0) = 3, \quad f(4) = -2, \quad f(-1) = 2, \quad \text{and} \quad f(1) = 1\ .$$
   Use the above data values to find the best fit polynomial of degree 2 by using the QR-decomposition method by answering the questions below step by step.

   (a) (2.5+1.5 marks) Identify the matrix A and b. Also identify the linearly independent column vectors u_1, u_2 and u_3 from the matrix A. Explain why there are only three linearly independent vectors u_1, u_2 and u_3.

   (b) (1+2+3 marks) Using Gram-Schmidt process construct the orthonormal column matrices (or vectors) q_1, q_2 and q_3 from the linearly independent column vectors obtained in the previous part, and then write down the Q matrix.

   (c) (3 marks) Now calculate the matrix elements of R , and write down the matrix R.

   (d) (4 marks) Compute Rx and $Q^{\mathrm{T}}b$.

   (e) (4 marks) Let $x = (a_0\ a_1\ a_2)^{\mathrm{T}}$ being the coefficients of the polynomial $p_2(x)$. Evaluate these coefficients and write down the polynomial $p_2(x)$.

Motto: Mathematics is NOT difficult, but what is difficult is to believe that mathematics is NOT difficult.