Solution to Assignment #1

#1 (a) For Eq(1): $\text{Max} = (0.1111)_2 \cdot 2^6$

For Eq(2): $\text{Max} = (1.1111)_2 \cdot 2^6$

& For Eq(3): $\text{Max} = (0.11111)_2 \cdot 2^6$

(b) For eq(1): $\text{Min} = (0.1000)_2 \cdot 2^{-3}$

For eq(2): $\text{Min} = (1.0000)_2 \cdot 2^{-3}$

& For eq(3): $\text{Min} = (0.10000)_2 \cdot 2^{-3}$

#2 (a) Min of $|x| \equiv |x|_{min} = (1.0000)_2 \cdot 2^{e_{min}}$

$$= 1\times 2^0 \cdot 2^{-1} = \frac{1}{2} = (0.5)_{10}.$$

(b) $\epsilon_M = \frac{1}{2}\beta^{-m} = \frac{1}{2}\cdot 2^{-4} = \frac{1}{2}\cdot\frac{1}{2^4} = \frac{1}{32} = 0.03125.$

(c) $|\delta|_{max} \equiv \epsilon_M = \frac{1}{2}\beta^{1-m} = \frac{1}{2}\cdot 2^{1-4} = \frac{1}{2^4} = \frac{1}{16} = 0.0625.$

#3 Here $x_0 = 0$. The Taylor expansion of $f(x)$ is

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2!}f''(x_0)(x-x_0)^2 + \cdots$$

$$= a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \cdots$$

So, $a_0 = f(x_0) = f(0) = e^0 - \sin(0) + 0 - 1 = 0.$

$a_1 = f'(x_0) = f'(0) = e^0 - \cos 0 + 1 = 1.$

$a_2 = \frac{1}{2}f''(x_0) = f''(0) = \frac{e^0 + \sin 0}{2} = \frac{1}{2}$

Hence, $f(x) \simeq P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)^2$

$$= 0 + 1(x-0) + \frac{1}{2}(x-0)^2$$

$$\Rightarrow \boxed{P_2(x) = \frac{1}{2}x^2 + x}$$