Solution: Assignment-4

#1 Here $f(x) = x^3 - 7x^2 + 4x + 12$, and $x_* = -1$

(a) So $f(x) = x^2(x+1) - 8x(x+1) + 12(x+1)$

$= (x+1)(x^2 - 8x + 12)$

$= (x+1)(x-6)(x-2)$

The remaining roots are: $\boxed{x_* = 2, 6}$.

(b) (i) $f(x) = 0 \Rightarrow x^3 - 7x^2 + 4x + 12 = 0$

$\Rightarrow x(x^2 - 7x + 4) = -12$

$\Rightarrow x = -\frac{12}{x^2 - 7x + 4} \Rightarrow \boxed{g_1(x) = -\frac{12}{x^2 - 7x + 4}}$

(ii) $f(x) = 0 \Rightarrow x^3 - 7x^2 + 4x + 12 = 0$

$\Rightarrow 4x = -x^3 + 7x^2 - 12$

$\Rightarrow x = \frac{1}{4}(-x^3 + 7x^2 - 12) \Rightarrow \boxed{g_2(x) = \frac{1}{4}(-x^3 + 7x^2 - 12)}$

(c) $\lambda = \left|\frac{dg_1}{dx}\right|_{x_*} = \left|\frac{(-12)(-1)(2x-7)}{(x^2 - 7x + 4)^2}\right|_{x_*} = \left|\frac{12(2x-7)}{(x^2 - 7x + 4)^2}\right|_{x_*}$

$$= \begin{cases} \frac{3}{4}, \text{ if } x_* = -1 \Rightarrow \text{converging} \\ 1 \text{ if } x_* = 2 \Rightarrow \text{diverging} \\ 15 \text{ if } x_* = 6 \Rightarrow \text{diverging.} \end{cases}$$

$g_1(x)$ is converging to -1.

$\lambda = \left|\frac{dg_2}{dx}\right|_{x_*} = \left|\frac{1}{4}(-3x^2 + 14x)\right|_{x_*} = \begin{cases} 4.25 \text{ if } x_* = -1 \Rightarrow \text{diverging} \\ 4 \text{ if } x_* = 2 \Rightarrow \text{diverging} \\ 6 \text{ if } x_* = 6 \Rightarrow \text{diverging} \end{cases}$

For $g_2(x)$ there is no convergence.

(d) $x_* = 6 \in I = [4.25, 8.95]$. $\Rightarrow a_0 = 4.25$ & $b_0 = 8.95$.

k	a_k	m_k	b_k	$f(a_k)$	$f(m_k)$	$f(b_k)$	$x_* \in I_k$
0	4.25	6.6	8.95	<0	>0	>0	$[a_0, m_0]$
1	4.25	5.425	6.6	<0	<0	>0	$[m_1, b_1]$
2	5.425	6.0125	6.6	<0	>0	>0	$[a_2, m_2]$
3	5.425	5.71875	6.0125	<0	<0	>0	$[m_3, b_3]$
4	5.71875	5.8656	6.0125	<0	<0	>0	$[m_4, b_4]$.

After 4 iterations, x_* is in the interval $[5.8656, 6.0125]$ //

(e) Here $\delta = \epsilon_M = 1.4 \times 10^{-18}$ and $a_0 = 4.25$ & $b_0 = 8.95$.

$$\therefore\ n \geq \frac{\log|b_0 - a_0| - \log(\delta)}{\log(2)} - 1$$

$$= \frac{\log|8.95 - 4.25| - \log(1.4 \times 10^{-18})}{\log 2} - 1$$

$$= \frac{0.672098 - (-17.853872)}{0.301030} - 1$$

$$= 60.54193$$

$$\Rightarrow \boxed{n \geq 61} \ //$$

(f) Here $f_k(x) = x_k^3 - 7x_k^2 + 4x_k + 12$

so, $f_k'(x) = 3x_k^2 - 14x_k + 4$

$$\therefore\ x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^3 - 7x_k^2 + 4x_k + 12}{3x_k^2 - 14x_k + 4}$$

We have: $x_0 = 9.26$ $(k=0)$

$$\therefore x_1 = x_0 - \frac{x_0^3 - 7x_0^2 + 4x_0 + 12}{3x_0^2 - 14x_0 + 4} = 7.41483 \quad \cancel{2.00263}.$$

$$x_2 = x_1 - \frac{x_1^3 - 7x_1^2 + 4x_1 + 12}{3x_1^2 - 14x_1 + 4} = 6.42504$$

$$\therefore \hat{x}_2 = x_0 - \frac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0} = 5.27972.$$

$$x_3 = \hat{x}_2 - \frac{\hat{x}_2^3 - 7\hat{x}_2^2 + 4\hat{x}_2 + 12}{3\hat{x}_2^2 - 14\hat{x}_2 + 4} = 6.36173$$

$$x_4 = x_3 - \frac{x_3^3 - 7x_3^2 + 4x_3 + 12}{3x_3^2 - 14x_3 + 4} = 6.04220$$

$$\hat{x}_4 = x_2 - \frac{(x_3 - \hat{x}_2)^2}{x_4 - 2x_3 + \hat{x}_2} = \underline{\underline{6.11505}}$$
