Assignment on Fourier Analysis

1. What is a periodic Function? Provide an example with neat sketch. Determine the period of the function $y = 2023sin^{2022}11x + 12$

2. Define Odd and Even Function with figures. Provide example.

3. From the definition of Fourier Series $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n cos\left(\frac{n\pi x}{L}\right) + b_n sin\left(\frac{n\pi x}{L}\right)$, which is period over the interval $[-L, +L]$, derive the formula for the coefficients a_0, a_n, b_n.

4. Using Euler's Identities, prove that the Fourier series can be expressed as $f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{in\pi x}{L}}$

5. Define Orthogonal Functions. Using $\int_{-\pi}^{+\pi} sin\ nx\ dx = 0, and\ \int_{-\pi}^{+\pi} cos\ nx\ dx = 0, where\ m, n \in Z$, prove the following identities –
   a. $\int_{-\pi}^{\pi} \cos nx \cos mx\ \ dx = \begin{cases} 2\pi & \text{if } n = m = 0 \\ \pi & \text{if } n = m \neq 0 \\ 0 & \text{if } n \neq m \end{cases}$
   b. $\int_{0}^{\pi} \cos nx \cos mx\ dx = \begin{cases} \pi & \text{if } n = m = 0 \\ \frac{\pi}{2} & \text{if } n = m \neq 0 \\ 0 & \text{if } n \neq m \end{cases}$
   c. $\int_{-\pi}^{\pi} \sin nx \sin mx\ dx = \begin{cases} \pi & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$
   d. $\int_{0}^{\pi} \sin nx \sin mx\ dx = \begin{cases} \frac{\pi}{2} & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$
   e. $\int_{-\pi}^{\pi} \sin nx \cos mx\ dx = 0$

6. Draw sketches and determine the Fourier Series for the following functions.
   a. $s(x) = \frac{x}{\pi}$, for $-\pi < x < +\pi$
   b. $s(x) = 3|\sin x|$ for $0 \leq x < 2\pi$
   c. $s(x) = \begin{cases} 2\sin x & \text{for } 0 \leq x < \pi \\ 0 & \text{for } \pi \leq x < 2\pi \end{cases}$
   d. $s(x) = \begin{cases} 1 & \text{for } 0 \leq x < \pi \\ 0 & \text{for } \pi \leq x < \pi \end{cases}$
   e. $s(x) = A - \frac{Ax}{P}$ for $0 \leq x < P$