"Problem Set 1.1"

26

From the given matrixes we can write these two equations,

$2c+d=8$ & $c+3d=14$ — (ii)

$\Rightarrow 2c = 8-d$

$\Rightarrow c = \frac{8-d}{2}$ — (i)

adding c value in (ii), $\frac{8-d}{2}+3d=14$

$\Rightarrow \frac{8-d+6d}{2}=14$

$\Rightarrow 8+5d=28$

$\Rightarrow 5d=20$

$\Rightarrow \boxed{d=4}$

Now, putting the value d in (i), $c = \frac{8-4}{2}$

$= \frac{4}{2}$

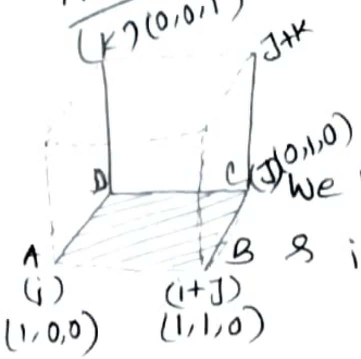
$\therefore \boxed{c=2}$

Lastly, we can write,

$2(1,2)+4(3,1)=(4,8)$, this is the equation for linear combination.

Ans

27



Given, $i=(1,0,0)$ &

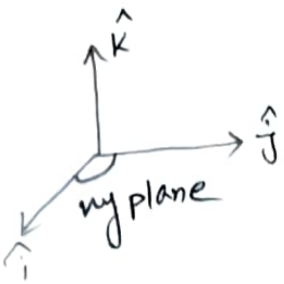
$i+J=(1,1,0)$

We have to find the plane where $i=(1,0,0)$ & $i+J=(1,1,0)$

Let, ABCD be the plane in xy

$\therefore (0,0,0)$ will be the bottom corner.

Ans



28

Given, $v+w = (4,5,6)$ ... ① & $(v-w) = (2,5,8)$ ... ⑪ If we

$u - v +$

adding, ①+⑪, $v+\cancel{w}+v-\cancel{w} = (4+2,\ 5+5,\ 8+6)$

$\Rightarrow 2v = (6, 10, 14)$

$\Rightarrow v = \left(\frac{6}{2}, \frac{10}{2}, \frac{14}{2}\right)$

$= (3,5,7)$

Subtracting ①−⑪, $v+w-v+w = (4-2,\ 5-5,\ 6-8)$

$\Rightarrow 2w = (2, 0, -2)$

$\Rightarrow w = \left(\frac{2}{2}, \frac{0}{2}, -\frac{2}{2}\right)$

$= (1, 0, -1)$

$\therefore v = (3,5,7)$ & $w = (1,0,-1)$ Ans

29

If we take any three vectors u, v & w in the plane, always there will not be two different combinations that produces $b=(0,1)$. Three vectors u, v & w in the x, y plane could fail to produce b if all three lie on a line that does not contain b. But yes, if 1 combination produces b then ~~to~~ 2 and many combinations will produce b. This is true even if $u=0$ and the combinations can have different cu. We can write combination,

$-2u+v+0w = b \Rightarrow -2u+v+0w = -2\,(1,3)+(2,7)+0(1,5)$

$= (-2,-6)+(2,7)$

$= (0,1)$

$= b$

if we check another combination,

$u - v + w = b \Rightarrow u - v + w = (1,3) - (2,7) + (1,5)$

$= (-1,-4) + (1,5)$

$= (0,1)$

$= b$

Therefore, No, three vectors in x, y plane could fail to produce b if all 3 lie on a line that does not contain b. Ans

30

The combination of v & w will fill the plane unless v & w lie on the same line through $(0,0)$.

∴ Four vectors whose combinations fill 4-dimensional space, for example "Standard basis", we can write, $v = (0,1,0,0)$, $w = (0,0,1,0)$ &

$z = (0,0,0,1)$

& the linear combination of $v = (a,b)$ & $w = (c,d)$ fill the plane unless v & w are dependent.

Ans

31

Given,

$$u = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \;\&\; b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Condition, $cu + dv + ew = b$

$$\Rightarrow c\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + e\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2c \\ -c \\ 0 \end{bmatrix} + \begin{bmatrix} -d \\ 2d \\ -d \end{bmatrix} + \begin{bmatrix} 0 \\ -e \\ 2e \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2c - d + 0 \\ -c + 2d - e \\ 0 - d + 2e \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Here,

$2c - d = 1$ — (i), $\quad -c + 2d - e = 0$ — (ii), $\quad -d + 2e = 0$ — (iii)

from (ii) & (iii), $2 \times$ (ii) $\rightarrow -2c + 4d - \cancel{2e} = 0$

(iii) $\rightarrow -d + \cancel{2e} = 0$

$$-2c + 3d = 0 \quad \text{— (iv)}$$

from (i) & (iv),

(i) $\rightarrow \cancel{2c} - d = 1$

(iv) $\rightarrow \cancel{-2c} + 3d = 0$

$$2d = 1$$

$$\Rightarrow d = 1/2$$

Putting value of d in (i),

$2c - d = 1 \Rightarrow 2c = 1 + d \Rightarrow c = \dfrac{1 + \frac{1}{2}}{2} = \dfrac{\frac{3}{2}}{2} = 3/4$

$\therefore c = 3/4$

Putting value of c in (iii),

~~$-d + 2c = 0 \Rightarrow 2c = 1 + d = \frac{1 + 1/2}{2} = \frac{\frac{3}{2}}{2}$~~

$-d + 2e = 0 \Rightarrow 2e = d \Rightarrow 2e = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{2 \times 2} = 1/4$

Therefore, $c = \dfrac{3}{4}$, $d = \dfrac{1}{2}$ & $e = \dfrac{1}{4}$ Ans

"Worked Example"

1.2A

The dot product is $v.w = 3\times4 + 4\times3 = 24$.
The length of v is $\|v\| = \sqrt{9+16} = 5$ & also $\|w\| = 5$. The sum $= (7,7)$ has length $7\sqrt{2} < 10$.

Schwarz inequality, $|v.w| \le \|v\|\|w\|$ is $24 < 25$.

Triangle inequality, $\|v+w\| \le \|v\| + \|w\|$ is, $7\sqrt{2} < 5+5$

Cosine of angle, $\cos\theta = \frac{24}{25}$, Thin angle from $v = (3,4)$ to $w = (4,3)$

Ans

1.2B

For a unit vector, u, dividing v by length $\|v\| = 5$.
For a perpendicular vector V we can choose $(-4, 3)$
since the dot product $v.V$ is $3\times(-4) + 4\times3 = 0$.
for a unit vector U, dividing V by it's length $\|V\|$,

$$u = \frac{v}{\|v\|} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$U = \frac{V}{\|V\|} = \left(-\frac{4}{5}, \frac{3}{5}\right)$$

$$u.U = 0$$

Therefore,
the only other perpendicular unit vector would be $-U = (4/5, -3/5)$. Ans

"Problem set 1.2"

We know.

1

Given, $u = \begin{bmatrix} -.6 \\ .8 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ & $w = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

Firstly, $u.v = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (-.6) \times 3 + .8 \times 4$

$= -\frac{9}{5} + \frac{12}{5}$

$= 0$

$u.w = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

1

Given, $u = \begin{bmatrix} -.6 \\ .8 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

Firstly,

For, $u.v$ if $u = \begin{pmatrix} a \\ b \end{pmatrix}$ & $v = \begin{pmatrix} c \\ d \end{pmatrix}$ then.

$u.v = ac + bd$

$u.v = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (-.6) \times 3 + (.8) \times 4 = -9/5 + 16/5 = \boxed{\frac{7}{5}}$ Ans

Secondly, $u.w = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 6 \end{bmatrix} = (-.6) \times 8 + (.8) \times 6 = -\frac{24}{5} + \frac{24}{5} = \boxed{0}$ Ans

Thirdly,

$u.(v+w) = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 8 \\ 6 \end{bmatrix} \right\}$

$= \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 10 \end{bmatrix}$

$= (-.6) \times 11 + (.8) \times 10$

$= -\frac{33}{5} + 8$

$= \boxed{\frac{7}{5}}$ Ans

Lastly, $w.v = \begin{bmatrix} 8 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 8 \times 3 + 6 \times 4 = 24 + 24 = \boxed{48}$ Ans

7(a)

We know, $\cos\theta = \dfrac{\vec{a}\,\vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$ —① here θ is angle b/w vector $\vec{a}$ + $\vec{b}$

Given, $V = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ & $W = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$V \cdot W = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 + 0 = 1$

$\|V\| = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

$\|W\| = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$

From (1),

$\cos\theta = \dfrac{V \cdot W}{\|V\| \cdot \|W\|} = \dfrac{1}{2 \times 1} = \dfrac{1}{2}$

$~~\cos 1/2~~ \Rightarrow \cos\theta = \dfrac{1}{2}$

$\theta = \cos^{-1}(1/2)$

$= \dfrac{\pi}{3}$ Ans

7(b)

Given, $V = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ & $W = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$V W = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = 2\times2 + 2\times(-1) + (-1)\times 2 = 4 - 2 - 2 = 0$

$\|V\| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{4+4+1} = 3$

$\|W\| = \sqrt{2^2 + (-1)^2 + (2)^2} = \sqrt{4+1+4} = 3$

From(1) formula,

$\cos\theta = \dfrac{V \cdot W}{\|V\| \cdot \|W\|} = \dfrac{0}{3\times3} = \dfrac{0}{9} = 0$

$\therefore \cos\theta = 0 \Rightarrow \theta = \cos^{-1}(0) = \pi/2$ Ans

1.3A Worked Problem

Firstly, $x_1 = b_1$ $[\because Ax = b]$

Secondly, $x_2 = b_2 + b_1$ thirdly, $x_3 = b_1 + b_2 + b_3$

Meaning, $x = A^{-1}b = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

The first column of A^{-1} is the ~~column x of A^{-1}~~ solution for $b = (1, 0, 0)$, the second column is the solution $b = (0, 1, 0)$ & third column is x of A^{-1} the solution for $Ax = b = (0, 0, 1)$

The three columns of A are independent. They don't lie in a plane. The combinations of those three columns, using the right weights x_1, x_2, x_3 can produce any three dimentional vector $b = (b_1, b_2, b_3)$. Those weights come from $x = A^{-1}b$.

Ans

The colu

"Worked Examples"

2.1 A

The column picture asks for a linear combination that produces b from the three columns of A. In this example b is minus(-) the second column So, the solution is, $x = 0, y = -1, z = 0$. To show that $(0, -1, 0)$ is the only solution we have to know that "A is invertible" & "the columns are independent" and "the determinant isn't zero" ~~for Those words~~ this we need a full set of three nonzero pivots.

if, the right side changes to $b = (4, 4, 8)$ = sum of the first two columns. Then the good combination has the solution becomes $x = (1, 1, 0)$
$x = 1, y = 1, z = 0$

Ans

2.1 B

① Multiplying the equations by 1, 1, -1 & adding gives $0 = 1$

$$
\begin{aligned}
x + 3y + 5z &= 4 \\
x + 2y - 3z &= 5 \\
-[2x + 5y + 2z &= 8] \\
\hline
0 + 0 + 0 &= 1
\end{aligned}
$$

$\boxed{\text{No solution}}$

The planes don't meet at a point, even though no two planes are parallel. For a plane parallel to $x + 3y + 5z = 4$, change the "4". The parallel plane $x + 3y + 5z = 0$ goes through the origin $(0, 0, 0)$. And the equation multiplied by any nonzero constant still gives the same plane, as in $2x + 6y + 10z = 8$.

Ans

Problem Set 1.2

27

we know, $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$

Since, base of a right angled triangle, can never be greater than it's hypotenuse with respect to any angle of it.

$\therefore$ The value of $\cos\theta$ can never be greater than 1

The cosine of θ, $\dfrac{0}{\sqrt{x^2+y^2}}$, near side over hypotenuse.

then $|\cos\theta|^2 = \dfrac{x^2}{(x^2+y^2)} \leq 1$

Ans

Problem Set 2.1

16

ⓐ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, this is 2×2 identity matix.

Ans

ⓑ $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, this is 2×2 exchange matrix.

26

Given, $A = \text{ones}(4,4) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$V = \text{ones}(4,1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Firstly,

$$A * V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \text{ Ans}$$

Secondly,

$B = \text{eye}(4) + \text{ones}(4,4)$ times W

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot W$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} W & W & W & W \\ W & W & W & W \\ W & W & W & W \\ W & W & W & W \end{bmatrix}$$

$$= \begin{bmatrix} 1+W & W & W & W \\ W & 1+W & W & W \\ W & W & 1+W & W \\ W & W & W & 1+W \end{bmatrix}$$

Thirdly,

$W = \text{zeros}(4,1) + 2*\text{ones}(4,1)$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

Lastly,

$$B * W = \begin{bmatrix} 1+W & W & W & W \\ W & 1+W & W & W \\ W & W & 1+W & W \\ W & W & W & 1+W \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2W & 2W & 2W & 2W \\ 2W & 2+2W & 2W & 2W \\ 2W & 2W & 2+2W & 2W \\ 2W & 2W & 2W & 2+2W \end{bmatrix} \text{ Ans}$$

Problem set 2.2

12

From the question we can write,

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R'2 \to R2 - 2R1$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R'3 \to R3 + 2R2$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{array}\right]$$

Upper triangular Matrix, the pivots are $(2, 1, 8)$.

From the upper triangular matrix,

$8z = 8 \Rightarrow z = 1$

from 2nd row : $y + 3z = 4$

$y = 4 - 3 = 1$

so, $2x + 3y + z = 8$

$\Rightarrow 2x + 3 + 1 = 8$

$\Rightarrow 2x = 4$

$\therefore x = 2$ Ans

Answer: $(2, 1, 1)$

From the question we can write,

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 4 & -5 & 1 & 7 \\ 2 & -1 & -3 & 5 \end{array}\right]$$

The three row operations,

$R3 = R3 - R1$

$R2 = R2 - 2R1$

$R3 = R3 - 2R2$

Upper triangular matrix,

$$\begin{bmatrix} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5 & 0 \end{bmatrix}$$

The pivotes are $(2, 1, -5)$

From the above matrix,

$-5z = 0$ ~~$[z = 0]$~~

$\Rightarrow z = 0$

so, $y + z = 1$

$\Rightarrow y = 1$

and $2x - 3y = 3$

$2x = 6$

$\therefore x = 3$

Answer $(3, 1, 0)$

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(a) If (x,y,z) & (X,Y,Z) are two solutions, then any linear combination of both solutions is also a solution. For Example $(x+X, y+Y, z+Z)$ is a solution. Ans

(b) There were the 25 planes meet the solution of a system of linear equations. So, also here any linear combination of the two points is a solution. Any linear combination of the two points makes a line going through the 2 points. There-fore, the 25 Planes meet everywhere on the line going through the 2 points. Ans

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The argumented matrix of the given system is,

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right] \quad \cancel{[R_2' = R_2 - 2R_1]} \quad [R_1 \leftrightarrow R_2]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -2 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right] \quad [R_2' \to R_2 - 2R_1]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right] \quad [R_2 \leftrightarrow R_3]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right] \quad [R_3' = R_3 + 3R_2]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 5/4 & 5 \end{array}\right] \quad [R_4' = R_4 - \tfrac{1}{4} R_3]$$

$\therefore$ The pivots are $(1, 1, 4, 5/4)$.

From the last matrix the given system can be written as,

$$x + 2y + z = 0$$
$$y + 2z + t = 0$$
$$4z + 3t = 0$$
$$5/4\, t = 5$$

Using the back substitution, we get $t = 4$, $z = -3$ & $x = -1$

Ans

Worked Example

2.3B

$$[A \quad b] = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 4 & 8 & 9 & 3 \\ 0 & 3 & 2 & 1 \end{bmatrix} \;\&\; E_{21}[A \quad b] = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

P_{32} exchanges equation 2 & 3. Back substitution produces (x, y, z),

$$P_{32}E_{21}[A \quad b] = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

For the matrix $P_{32}E_{21}$ that does both steps at once applying P_{32} to E_{21}

$$P_{32}E_{21} = \text{exchange the rows of } E_{21}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -4 & 1 & 0 \end{bmatrix} \qquad \underline{\text{Ans}}$$

2.3C

Rows of A times columns of B are dot products of vectors,

row 1 · col 1 $= [3 \quad 4] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 10$, is the (1,1) entry of AB

row 2 · col 1 $= [1 \quad 5] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 7$, is the (2,1) entry of AB.

The first columns of AB are (10, 7, 4) & (16, 9, 8). We need 6 dot products, 2 multiplications each 12 in all (3·2·2). The same AB comes from columns of A times rows of B.

$$AB = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} [2 \quad 4] + \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} [1 \quad 1] = \begin{bmatrix} 6 & 12 \\ 2 & 4 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 5 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 16 \\ 7 & 9 \\ 4 & 8 \end{bmatrix} \quad \underline{\text{Ans}}$$

"Worked Examples"

2.2 C

Success / then / failure

$$x+y+z=7 \qquad x+y+z=7$$
$$x+y-z=5 \qquad x+y-z=5$$
$$x-y+z=3 \qquad -x-y+z=3$$

For the first system, substract equation 1 from equation 2 & 3 (the multipliers are $l_{21}=1$ & $l_{32}=1$). The 2,2 entry becomes 0, so ~~ex~~ exchange equations.

success,
$$x+y+z=7 \qquad\qquad x+y+z=7$$
$$0y-2z=-2 \quad \text{exchang to,} \quad -2y+0z=-4$$
$$-2y+0z=-4 \qquad\qquad -2z=-2$$

Then back substitution gives $z=1$ & $y=2$ & $x=4$ The pivots are 1, -2, -2. For the second system, substracting equation 1 from 2 as before. Adding equation 1 to 3. This leaves zero in 2,2 entry & also,

Failure
$$x+y+z=7$$
$$0y-2z=-2$$
$$0y+2z=10$$

There is no pivot in column 2 (it was column 1). A further elimanation step gives $0z=8$. The three planes don't meet.

Plane 1 meets plane 2 in a line, Plane 1 meets plane 3 in parallel line. If we change the (3) in original third equation to -5. then the elimination would lead to $0 = 0$.

" Assignment/Worksheet : 2 "

" Probem set 2.3 "

$$E = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix}$$

$$E \cdot F = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \quad E^2 = E \cdot E = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix}$$

$$F \cdot E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ ac+b & c & 1 \end{pmatrix}$$

$$F^3 = F \cdot F \cdot F = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2c & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{pmatrix}$$ Ans

$$F^{100} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 100c & 1 \end{pmatrix}$$ Ans

$$E \cdot F = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \quad \underline{\text{Ans}}$$

$$F \cdot E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ ac+b & c & 1 \end{pmatrix} \quad \underline{\text{Ans}}$$

$$E^2 = E \cdot E = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{pmatrix} \quad \underline{\text{Ans}}$$

24

$$An = \left[\begin{array}{cc:c} 2 & 3 & 1 \\ 4 & 1 & 17 \end{array}\right] \text{ (argumented matrix)}$$

$$= \left[\begin{array}{cc:c} 2 & 3 & 1 \\ 0 & -5 & 15 \end{array}\right] \text{ by elimination}$$

The triangular system,

$$2x_1 + 3x_2 = 1 \quad \Rightarrow 5x_2 = 15$$

Back substitution, $x_1 = 5$ & $x_2 = -3$ Ans

25

$$An = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 5 & 7 & 6 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right] \quad \left[\text{Row operating on } [A:6]\ [A:b] = \frac{R_2 - 2R_1}{R_3 - (R_2 + R_1)}\right]$$

$$\text{So, } An = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

or, comparing both sides,

$$x + 2y + 3z = 1 \quad \dots \text{(i)}$$
$$-y - 2z = 0 \quad \dots \text{(ii)}$$
$$0 = 3 \quad \dots \text{(iii)}$$

Now, $0 = 3$ can not happen. It will give no solution.

$$[A:b] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 5 & 7 & 3 \end{array}\right] \quad \text{[replaced by 3]}$$

By row operation,

$$A_{n} = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[[A:b] = \frac{R_2 - 2R_1}{R_3 - (R_1 + R_2)} \right]$$

Now,

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Comparing, $x + 2y + 3z = 1$ — (iv)

$y - 2z = 0$ — (v)

$0 = 0$ — (vi)

$y = 2z$

Using (v) in (iv),

$x + 2(2z) + 3z = 1$

$\Rightarrow x + 4z + 3z = 1$

$\therefore x = 1 - 7z$

Therefore, $x = 1 - 7z$, $y = 2z$ & z is free.

Ans

Thus,

for any value of z we get different value of x, y. There are many different solution for different values of z.

$\therefore$ The system has infinitely many solution.

Ans

adding two columns,

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] = \left[\begin{array}{cccc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \rightarrow x = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

$$\& \; x^* = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Ans

27

$$[A \; b] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & d & c \end{array}\right]$$

(a) For no solution : Range of A & Rank of $[A\,b]$ must not be equal, if $d=0$ then Range of A will be 2. & if $a=5$, $b=2$ & $c=1$,

$$[A \; b] = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 4 & 5 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

so, Rank of $[A\,b]$ will be 3

& Rank of A $\neq$ Range of $[A \; b]$

$\therefore$ No solution for $a=5$, $b=2$, $c=1$ & $d=0$.

Ans

(b) Rank of A = Rank of $[A \cdot b] = 2$ $\quad [A \cdot b] = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & d \end{bmatrix}}_{A}\begin{matrix} a \\ b \\ c \end{matrix}$

for, $d = 0$, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

similarly, if $c = 0$, $[A \; b] = \left[\begin{array}{ccc:c} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & 0 & 0 \end{array}\right]$

So, Rank $[A \; b] = 2$ for $a = b \in Z - \{0\}$

$\therefore a = b \in Z - \{0\}$, $c = d = 0$

For infinitely many solution. Ans

28

Given, $AB = I$ & $BC = I$

Now multiplying both sides of $AB = I$ by C,

$(AB)C = I.C$

$\Rightarrow AB.C = C$ $[IC = CI = C]$

$\Rightarrow A.BC = C$ $[AB.C = A.BC]$

$\Rightarrow AI = C$ $[BC = I]$

$\Rightarrow A = C$

[Proved]

Problem set 2.4

12

ⓐ $AB = BA$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

$\therefore b = c = 0$

ⓑ $AC = CA$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$$

Hence, $a = d$ [Proved]

13

For, $A^2 - B^2 = (A-B)^2 = (A-B)(A-B)$

$= (A-B)A - (A-B) \cdot B$

$= A^2 - BA - AB + B^2$

$(A-B)^2 = A^2 - AB - BA + B^2$

Ans

In a typical case (when $AB \neq BA$) the matix $A^2 - 2AB + B^2$ is different from $(A-B)^2$.

~~For $(B-A)^2$,~~

14

(a) True, A^2 is defined only if A is square matrix,

(b) False, if AB & BA are defined then A & B are need to be square matrices.

(c) True, $A = 5\times3$, $B = 3\times5$, $AB = 3\times3$, $BA = 5\times5$, so they are square.

(d) True, $AB = B \Rightarrow ABB^{-1} = BB^{-1} \Rightarrow AI = I \Rightarrow A = I$.

15

Given, A is m by n, we have found the number of separate multiplications are involved in each part.

(a) A multiplies a vector x with n components.

In the product of Ax every entry involves mn separate multiplications.

(b) A multiplies an n by p matrix B & then AB is m by p.

Since, A is m by n & B is n by p & AB is of order m by p. So in the product AB every entry involves mnp separate multiplications.

(c) A multiplies itself to produce A^2 & here $m = n$.

Since, $m = n$, the product A^2 involves n^3 separate products & these are n^2 dot products.

17

$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

(a) $AB = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$, column 2 of $AB = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Ans

(b) Row 2 of $AB = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ Ans

(c) $A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$\therefore$ Row 2 of $A^2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ Ans

(d) $A3 = A \cdot A \cdot A = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$\therefore$ Row 2 of $A3 = \begin{bmatrix} 3 & -2 \end{bmatrix}$ Ans

20

Given,

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \& \quad v = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Ans}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Ans}$$

$$A^4 = A^3 \cdot A = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Ans}$$

$$A \cdot v = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2y \\ 2z \\ 2t \\ 0 \end{bmatrix} \quad \text{Ans}$$

$$A^2 \cdot v = \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 4z \\ 4t \\ 0 \\ 0 \end{bmatrix} \quad \text{Ans}$$

$$A^3 \cdot v = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 8t \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Ans}$$

$$A4.\ AV = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Ans

21

Given,

$$A = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} \ \& B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} \cdot \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} = \begin{bmatrix} .25+.25 & .25+.25 \\ .25+.25 & .25+.25 \end{bmatrix} = \begin{bmatrix} 0.5 & .5 \\ .5 & .5 \end{bmatrix}$$

$A^2 = A \Rightarrow A^2 - A = 0$

$\therefore A^2 = A$

$\Rightarrow A(A^2) = A(A)$

$\Rightarrow A^3 = A^2 = A$

$\therefore A^3 = A$

Now, $A^3 = A \Rightarrow (A)^3 \cdot A = A \cdot A \Rightarrow A^4 = A^2 = A$

$\therefore A^4 = A$

Similarly, $A4 = A$

$$\therefore A = A^2 = A^3 = \cdots = A^n = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} .5 & -.5 \\ .5 & -.5 \end{bmatrix}$$

$$(AB)^2 = (AB) \cdot (AB) = \begin{bmatrix} .5 & -.5 \\ .5 & -.5 \end{bmatrix} \cdot \begin{bmatrix} .5 & -.5 \\ .5 & -.5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(AB)^3 = (AB)^2 \cdot (AB) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} .5 & -.5 \\ .5 & -.5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ Except (AB) all are null matrices.

Ans

32

The matrix AX has columns Ax_1, Ax_2 & Ax_3.

$$AX = [Ax_1 \quad Ax_2 \quad Ax_3]$$

Therefore, $AX = I$

Ans

33

From the question,

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Ans

" Problem set 2.5 "

If a 2×2 matrix, inverse of A,

$$A^{-1} = \frac{1}{|A|} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

where, $|A| = ad - bc$

= det. of matrix A

$A = \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$, $|A| = 0\times0 - 3\times4 = -12$

$A^{-1} = \frac{1}{-12} \begin{bmatrix} 0 & -3 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/4 \\ 1/3 & 0 \end{bmatrix}$ Ans

For B, $|B| = \begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} = 2\times2 - 4\times0 = 4$

$B^{-1} = \frac{1}{4} \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1 & 1/2 \end{bmatrix}$ Ans

For C,

$|C| = \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 3\times7 - 4\times5 = 21-20 = 1$

$\therefore C^{-1} = \frac{1}{1} \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$ Ans

2

Given,

let $P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ & $P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

check if inverse matix will be the transpose.

$$P_1^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ Ans

in this case, it the matrix itself.

Now, check for another P,

$$P_2^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$ Ans

3

Given,

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{— (1)}$$

Now, $$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}^{-1} = \frac{1}{500-400} \begin{bmatrix} 50 & -20 \\ -20 & 10 \end{bmatrix}$$

$$= \frac{1}{100} \times \begin{bmatrix} 50 & -20 \\ -20 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 50/100 & -20/100 \\ -20/100 & 10/100 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -1/5 \\ -1/5 & 1/10 \end{bmatrix}$$

From (1),

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 & -1/5 \\ -1/5 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/5 \end{bmatrix} \quad \underline{\text{Ans}}$$

similarly,

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/5 \\ -1/5 & 1/10 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -1/5 \\ 1/10 \end{bmatrix} \quad \underline{\text{Ans}}$$

4

Given,

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1} B$$

And $A^{-1} = \dfrac{1}{|A|} \operatorname{Adj} A$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 2 \times 3 = 0$$

$\therefore$ There is no inverse as magnitude is 0.

Ans

7

Given, A has (1+2) row = 3 row & A is not invertible.

ⓐ If, row 1 + row 2 = row 3

as A^{-1} does not exist then magnitude 0.

Let, $u_1, u_2, u_1 + u_2$ be the rows of A,

$$A = \begin{matrix} u_1 \\ u_2 \\ u_2 + u_1 \end{matrix}$$

Then, $Ax = \left[\cancel{u_1 \cdot x \; u_2 \cdot x \; u_1 \cdot x} \right. \begin{bmatrix} u_1 x \\ u_2 x \\ u_1 x + u_2 x \end{bmatrix}$

Therefore, row 3 of vector Ax is always the sum of rows 1 & 2 of Ax. Since, this isn't true for vector (1,0,0) it's not possible for Ax to even equal (1,0,0).

ⓑ We at least require $b_3 = b_1 + b_2$.

But this maybe not enough, depending on what those first two rows of A look like.

ⓒ Two steps are sure needed for eliminatio,

1. Substract row 1 from row 3
2. Substract row 2 from row 3. The result is that row 3 becomes a row of zeros

Given,

10

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix}$$

$A = AI$

$A^{-1} \cdot A = I$

$$A \cdot I = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad [R_1 \leftrightarrow R_4]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \left[\frac{R_1}{5}\right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad [R_2 \leftrightarrow R_3]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \left[\frac{R_2}{4}\right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \left[\frac{R_3}{3}\right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{bmatrix} \quad \left[\frac{R_4}{2}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{bmatrix} \quad \text{Ans}$$

Now,

$$BI = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2/3 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\frac{R_1}{3}\right]$$

$$= \begin{bmatrix} 4 & 8/3 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 4/3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_1 \times 4]$$

$$= \begin{bmatrix} 1 & 8/3 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 4/3 & 0 & 0 & 0 \\ -4/3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_2' = R_2 - R_1]$$

$$= \begin{bmatrix} 1 & 2/3 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1/3 & 0 & 0 & 0 \\ -4/3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_1 \times 1/4]$$

$$= \begin{bmatrix} 1 & 2/3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 0 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [C_2 \div 1/3]$$

$$= \begin{bmatrix} 1 & 2/3 & 0 & 0 \\ 0 & 2/3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 0 & 0 & 0 \\ -8/3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_2 \times 2/3]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -8/3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_1' = R_1 - R_2]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [R_2 \times 3/2]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/6 \\ 0 & 0 & 7 & 6 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\frac{R_3}{6}\right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & \frac{35}{6} \\ 0 & 0 & 7 & \frac{6}{6} \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 7/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [R_3 \times 7]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 35/6 \\ 0 & 0 & 0 & 1/6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 7/6 & 0 \\ 0 & 0 & -7/6 & 1 \end{bmatrix} \quad [R_4' = R_3 - R_4]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/6 \\ 0 & 0 & 0 & 1/6 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & -7/6 & 1 \end{bmatrix} \quad \left[\frac{R_3}{7}\right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & -7 & 6 \end{bmatrix} \quad [R_4 \times 6]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/6 \\ 0 & 0 & 0 & 5/6 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & -\frac{35}{6} & 5 \end{bmatrix} \quad [R_4 \times 5/6]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5/6 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -\frac{35}{6} & 5 \end{bmatrix} \quad [R_3' = R_3 - R_4]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix} \quad \left[R_4 \times \frac{6}{5}\right]$$

$$\therefore B^{-1} \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix}$$

Ans

11

(a) Let $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

then $|A| = -1$ & $|B| = -1$

so, A & b are invertible.

But, $A+B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|A+B| = 0$$

hence, A+B is not invertible. Ans

(b) Let, $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

then, $|A| = 0$ & $|B| = 0$

$\therefore$ A & B are singular.

But $A+B = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

& $|A+B| = -2 \neq 0$

$\therefore$ $(A+B)$ is invertible. Ans

15

Let, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & 0 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & 0 \\ \vdots & & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & 0 \end{bmatrix}$

A be a matrix with 0 column.

If a matrix has inverse then A has n pivot position. Since, $n \times n$ matrix A has no pivot position in the last column so the matrix A is not invertible.

Ans

21

Let, the matrixes with one 1's, with 2×2 form,

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Det. of these matrixes are 0. So, they arce not invertible.

Let, matrixes be with two 1's,

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

Determinant ~~non~~zero & ~~invertible~~ not inventible.

matrixes with three 1's,

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Det. arce nonzero & invertible.

consider, 1's positioned on the diagonal & anti-diagonal

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

consider, the matrix with all 1's & 0's

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Det. are zero & not invertible.

Therefore, from sixteen (2×2) matrixes with 0's & 1's only 6 are invertible.

Ans

22

$[A\ I]$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Ans

$$[A\ I] = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -3 & 4/3 \\ 0 & 1 & 1 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Ans

23

$$[A \;\; I] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 1 & -1/2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 1 & -1/2 & 1 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & -2/3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 0 & -3/4 & 3/2 & -3/4 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 3/2 & -1 & 1/2 \\ 0 & 3/2 & 0 & -3/4 & 3/2 & -3/4 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{-2}{3} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & -1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 3/4 \end{bmatrix}$$

$$= [I \;\; A^{-1}]$$

Ans

24

$$\left[\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_1' = R_1 - aR_2 \\ R_2' = R_2 - cR_2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \left[R_1' = R_1 - (b-ac)R_3\right]$$

Ans

25

$$[A\ I] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad [R_1/2]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3/2 & 1/2 & -1/2 & 1 & 0 \\ 0 & 1/2 & 3/2 & -1/2 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 1/2 & 3/2 & -1/2 & 0 & 1 \end{array}\right] \quad [R_2 \times 2/3]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 2/3 & -2/3 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & -1/3 & -1/3 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_3' = R_3 - \frac{1}{2}R_2 \\ R_1' = R_1 - \frac{1}{2}R_2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 2/3 & -1/3 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right] \quad [R_3 \times 3/4]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & -1/4 & -1/4 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right] \quad \left[\begin{array}{l} R_1' = R_1 - \frac{1}{3}R_3 \\ R_2' = R_2 - \frac{1}{3}R_3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0.75 & -0.25 & -0.25 \\ -0.25 & 0.75 & -0.25 \\ -0.25 & -0.25 & 0.75 \end{bmatrix} \quad \underline{\text{Ans}}$$

$$[B|I] = \left[\begin{array}{ccc|ccc} 2 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1/2 & -1/2 & 1/2 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \quad [R_1' = R_1/2]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 3/2 & -3/2 & 1/2 & 1 & 0 \\ 0 & -3/2 & 3/2 & 1/2 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 + R_1 \\ R_3' = R_3 + R_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 1/3 & 2/3 & 0 \\ 0 & -3/2 & 3/2 & 1/2 & 0 & 1 \end{array}\right] \quad [R_2 \times 2/3]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2/3 & 1/3 & 0 \\ 0 & 1 & -1 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_1' = R_1 + \frac{1}{2}R_2 \\ R_3' = R_3 + \frac{3}{2}R_2 \end{array}\right]$$

$\therefore$ No inverse for B.

$\therefore B^{-1}$ is not exists.

<u>Ans</u>

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A : I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad [R_2' = R_2 - 2R_1]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad [R_2' = R_2 - 3R_3]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Ans}$$

$$[A : I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_1' = R_1 - R_2 \\ R_3' = R_3 - R_2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \quad [R_2' = R_2 - R_3]$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{Ans}$$

$$[A\ I] = \left[\begin{array}{cccc|cc} 0 & 2 & 1 & 0 & 1 & 0 \\ 2 & 2 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} -2 & 0 & 1 & -1 \\ 2 & 2 & 0 & 1 \end{array}\right] \quad [R_1' = R_1 - R_2]$$

$$= \left[\begin{array}{cc|cc} -2 & 0 & 1 & -1 \\ 0 & 2 & 1 & 0 \end{array}\right] \quad [R_2' = R_1 + R_2]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 1/2 & 0 \end{array}\right] \quad \left[R_1 \times \frac{1}{-2} \ \& \ R_2 \times \frac{1}{2}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 0 \end{bmatrix} \quad \underline{\text{Ans}}$$

" Problem set 2.7 "

1

$$A = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 9 \\ 0 & 3 \end{bmatrix} \text{ Ans}$$

$$[A\ I] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 9 & 3 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 3 & -9 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1/3 \end{array}\right] \quad [R_2 - 9R_1]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1/3 \end{array}\right] \quad \left[\frac{R_2}{3}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1/3 \end{bmatrix} \text{ Ans}$$

$$(A^{-1})^T = \begin{bmatrix} 1 & -3 \\ 0 & 1/3 \end{bmatrix} \text{ Ans}$$

$$(A^T)^{-1} = \frac{1}{\det\begin{pmatrix} 1 & 9 \\ 0 & 3 \end{pmatrix}} \times \begin{pmatrix} 3 & -9 \\ -0 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \times \begin{pmatrix} 3 & -9 \\ -0 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 3 & -9 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 \\ 0 & 1/3 \end{pmatrix}$$

$$\therefore (A^T)^{-1} = \begin{pmatrix} 1 & -3 \\ 0 & 1/3 \end{pmatrix} \text{ Ans}$$

$(AB)^T = B^T A^T$

if, $(AB)^T = (BA)^T$

$\Rightarrow B^T A^T = A^T B^T$

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ & $AB = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ — (1)

$\therefore (AB)^T = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ — (11)

From (1) & (11), $\boxed{(AB)^T = B^T A^T}$

If, $AB \neq BA$ & if $AB = BA$

$(AB)^T = B^T A^T$

$\Rightarrow (BA)^T = A^T B^T$

$\therefore AB = BA \Rightarrow A^T B^T = B^T A^T$

Intgreal, $AB = BA = I$ (If there is inverse to each other)

$B = A^{-1} \Rightarrow A^{-1} = B$

$\therefore (AB)^T = A^T B^T$

$\Rightarrow (BA)^T = B^T A^T$

$\Rightarrow (A \cdot A^{-1})^T = (A^{-1})^T A^T$

$\Rightarrow (A^{-1} \cdot A)^T = (A^{-1})^T \cdot A^T$

$\Rightarrow A^T (A^{-1})^T = I$ Ans

$$\overbrace{}^{5}$$

(a) $x^T A y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$= \underset{1\times 2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}$

$\therefore x^T A y = \begin{bmatrix} 5 \end{bmatrix}$ Ans

(b) $x^T A = \underset{1\times 2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$

$\therefore x^T A = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

$\therefore$ row $x^T A = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$ times the column,

$y = (0, 1, 0)$ Ans

(c) $x^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$

$Ay = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$\therefore x^T = \begin{bmatrix} 0 \cdot 1 \end{bmatrix}$ times the column $Ay = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Ans