" Answer to the question no : 1 "

Given,

When the values of function repeats at regular intervals.

It is called as periodic function.

Sket for example, $y = \sin x$

Here, Amplitude $= 1 = b$

Period $= \frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi$

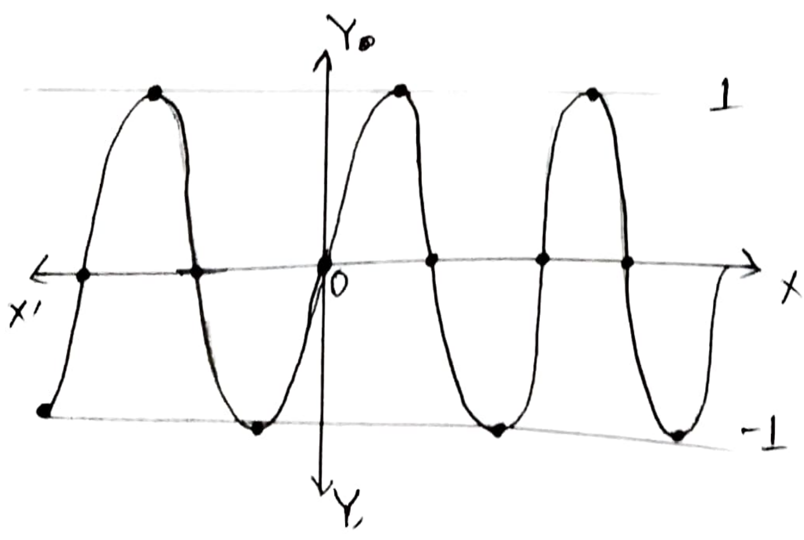
Increment $= \frac{\text{Period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

Now,

| $x$ | 0 | $\pi/2$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 1 | 0 | -1 | 0 | 1 | 0 |

So, here the value of $y$ is repeasts. So, it is a periodic function.

Sketch:

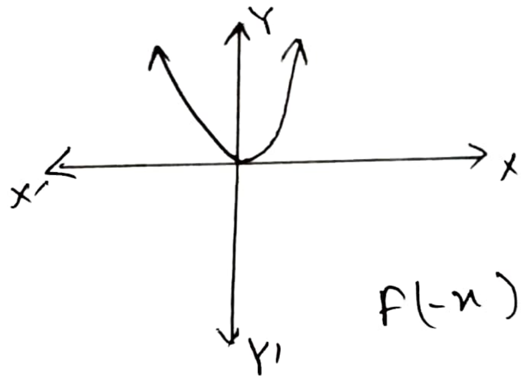



→ This is the neat sketch of the periodic function. $y = \sin x$.

"Answer ~~tho~~ the question no: 2"

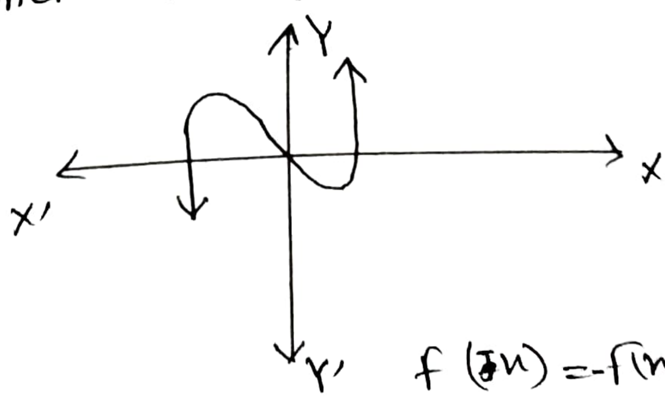
We know. For even function, it is symmetric with respect to the y-axis.

* Even function~~tion~~ graph: For example, $f(x) = x^2 + 6$ is an even function, the graph of this will be,




$f(-x) = f(x)$

* Odd function graph: For example, $f(x) = x^3 - 8x$ is an odd function, the graph of this function will be,




$f(-x) = -f(x)$

" Answer to the question no: 3 "

Given,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

interval $= [-L, +L]$

Let, $f(x)$ be the periodic function with period $2\pi$.

So, for interval $[L, +L]$,

$$a_0 = \frac{1}{P}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{P}\int_{-L}^{L} f(x)\cos(nx)\,dx$$

$$\& \ b_n = \frac{1}{P}\int_{-L}^{L} f(x)\sin(nx)\,dx$$

(Answer)

- Answer to the question no: 4 -

We know,

fourier series, $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right.$

using euler's identities,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\& \ e^{-i\theta} = \cos\theta - i\sin\theta$$

where, $i = \sqrt{-1}$, the fourier series can be written a

$$f(x) = e^{i\theta} = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}}$$

where, $$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{-\frac{in\pi x}{L}}\,dx$$

(Proved)

"Answer to the question no: 5"

(a)

Given,

$$\int_{-\pi}^{\pi} \cos nx \cos mx \, dx = \begin{cases} 2\pi & \text{if } n = m = 0 \\ \pi & \text{"} \quad n = m \neq 0 \\ 0 & \text{"} \quad n \neq m \end{cases}$$

$$\frac{1}{2}\int_{-\pi}^{\pi} 2\cos mx \cos nx = \int_{-\pi}^{\pi}\left[\cos(m+n)x + \cos(m-n)x\right] dx$$

$2\pi$ if $n=m=0$

$$= \left[\frac{\sin(m+n)x}{(m+n)} + \frac{\sin(m-n)x}{(m-n)}\right]_{-\pi}^{\pi}$$

1st case,

$$\int_{-\pi}^{\pi} dx = x \Big|_{-\pi}^{\pi}$$

$$= \pi + \pi = 2\pi \quad \text{Ans}$$

2nd case, $\pi$ if, $n = m \neq 0$

$$\int_{-\pi}^{\pi} \cos^2 mx$$

$$= \int_{-\pi}^{\pi} \frac{1+\cos 2mx}{2} \, dx$$

$$= \frac{x}{2} + \frac{\sin 2mx}{4m} \Big|_{-\pi}^{\pi}$$

$$= \frac{\pi}{2} + 0 - \frac{-\pi}{2} - 0$$

$$= \pi \quad \text{Ans}$$

3rd case, 0 if, $m \neq n$ then $m, n \in I$

so, $0 - 0 = 0$

Ans

[Proved]

(b)

Given,

$$\int_0^{\pi} \cos(nx) \cdot \cos(mx)\, dx$$

$$= \frac{1}{2}\int_0^{\pi} \cos(nx+mx) + \cos(nx-mx)\, dx$$

$$= \frac{1}{2}\left( \frac{\sin(\pi n+\pi m)}{n+m} + \frac{\sin(\pi n-\pi m)}{n-m} \right)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$(nx+mx) \qquad (nx-mx)$$

1st case, $\pi$ if $n=m=0$

$$\int_0^{\pi} dx = x \Big|_0^{\pi} = (\pi + 0) = \pi \text{ Ans}$$

2nd case, $\frac{\pi}{2}$ if, $n=m \neq 0$

$$= \int_0^{\pi} \cos^2 mx$$

$$= \int_0^{\pi} \frac{1+\cos 2mx}{2}\, dx$$

$$= \frac{x}{2} + \frac{\sin 2mx}{4m} \Big|_0^{\pi}$$

$$= \frac{\pi}{2} + 0 = \pi/2 \text{ Ans}$$

3rd case, $0$ if, $n \neq m$

So, $\int_0^{\pi} 0 \cdot dx = 0 - 0 = 0$ Ans

[Proved]

(c)

Given, $\int_{-\pi}^{\pi} \sin nx \sin mx \, dx$

$= \frac{1}{2} \int_{-\pi}^{\pi} 2\sin(mx)\sin(nx)\, dx$

$= \frac{1}{2} \int_{-\pi}^{\pi} [\cos(m-n)x - \cos(m+n)x]\, dx$

$= \frac{1}{2} \left[ \frac{\sin(m-n)x}{(m-n)} - \frac{\sin(m+n)x}{(m+n)} \right]_{-\pi}^{\pi}$

First case, $\pi$ if $n=m$

$= \int_{-\pi}^{\pi} \sin^2(nx)\, dx = \int_{-\pi}^{\pi} \frac{1-\cos(2n\pi)}{2}\, dx$

$= \frac{1}{2}\left[ x - \frac{\sin(2nx)}{2n} \right]_{-\pi}^{\pi}$

$= \frac{1}{2}\left[ (\pi - 0) - (-\pi - 0) \right]$

$= \frac{1}{2}(\pi - 0 + \pi - 0)$

$= \frac{2\pi}{2} = \pi$

Ans

2nd case,

$= \frac{1}{2}\left[ (0-0) - (0-0) \right] = 0$

Ans

[Proved]

(d)

From "c", we can write,

$\int_{0}^{\pi} \sin nx \sin mx \, dx$

$= \frac{1}{2} \left[ \frac{\sin(m-n)x}{(m-n)} - \frac{\sin(m+n)x}{(m+n)} \right]$

First Case, $\pi/2$ if $n=m$,

$\int_{0}^{\pi} \sin^2(nx)\, dx = \int_{0}^{\pi} \frac{1-\cos(2\pi x)}{2}\, dx$

$= \frac{1}{2}\left[ x - \frac{\sin(2nx)}{2n} \right]_{0}^{\pi}$

$= \frac{1}{2}\left[ (\pi - 0) - (0 - 0) \right]$

$= \frac{1}{2}(\pi)$

$= \frac{\pi}{2}$ (Answer)

2nd Case, 0 if $n \neq m$,

$\frac{1}{2}[(0-0)-(0-0)]$

$= \frac{1}{2} \times 0 = 0$ Ans [Proved]

(e)

Given, L.S $= \int_{-\pi}^{\pi} \sin nx \cdot \cos mx \cdot dx$

$= \int_{-\pi}^{\pi} \frac{\sin(n+m)x + \sin(n-m)x}{2} dx$ [Product formula]

$= \frac{1}{2} \left( \int_{-\pi}^{\pi} \sin(n+m)x \cdot dx + \int_{-\pi}^{\pi} \sin(n-m)x \cdot dx \right)$

$= \int_{-\pi}^{\pi} \sin v \cdot dv + \int_{-\pi}^{\pi} \sin(v)\, dv$

$= -\cos(v) - \cos(v)$

$= -\frac{\cos(n+m)x}{2(n+m)} - \frac{\cos(n-m)x}{2(n-m)}$

$= 0$ [Proved]

" Answer to the question no: 6 "

We know, Fourier series of function $f(x)$ on interval $-L \le x \le L$ is defined as,

$$f(x) = A_0 + \sum_{n=1}^{\infty} \left( A_n \cos\left(\frac{n\pi x}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

Where,

$$A_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx$$

$$A_n = \frac{1}{L} \cdot \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad , n > 0$$

$$B_n = \frac{1}{L} \cdot \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad , n > 0$$

(a)

Finding $A_0$, $A_n$ & $B_n$,

Given, $s(x) = \frac{x}{\pi}$ for, $-\pi < x < +\pi$

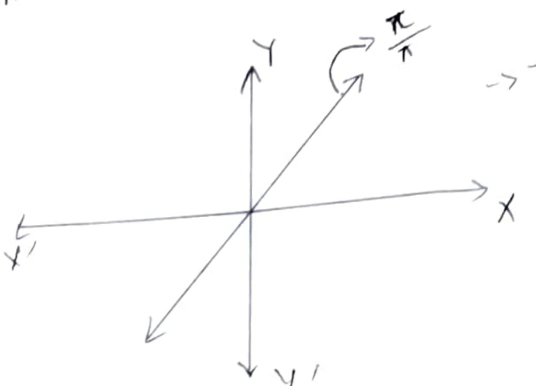
$$A_0 = \frac{1}{2\cdot\infty}\int_0^{\infty}\left(\frac{x}{\pi}\right)dx = \text{diverges}$$

$$A_n = \frac{1}{\infty}\int_0^{\infty}\left(\frac{\pi}{x}\right)\cos\left(\frac{n\pi x}{\infty}\right)dx = \text{undifined}$$

$$B_n = \frac{1}{\infty}\int_0^{\infty}\left(\frac{\pi}{x}\right)\sin\left(\frac{n\pi x}{\infty}\right)dx = \text{Undifined}$$

As we can not calculate $A_n$ & $B_n$ as we can't do any operations with $\infty$ Therefore, there is no fourier series for this function.

And sketch for $s(x) = \frac{x}{\pi}$ for $\pi < x < +\pi$,




→ This function is not peridic.

(b)

Findin $A_0$, $A_n$ & $B_n$,

Given, $s(x) = 3|\sin x|$ for, $[0, 2\pi]$

$$A_0 = \frac{1}{2\cdot 2\pi}\int_0^{2\pi} 3|\sin x|\,dx = \cancel{\frac{3}{\pi}}$$

$$\cancel{A_n = \frac{1}{2\pi}\int_0^{2\pi}(3|\sin x|).\cos\left(\frac{n\pi x}{2\pi}\right)dx = \frac{3nx^2}{2\pi}}$$

$$= \frac{1}{2\cdot 2\pi}\int_0^{2\pi} 3|\sin x|$$

$$= \frac{3}{4\pi}\int_0^{2\pi}\frac{\sin x}{|\sin x|} - \int \sin x\,dx$$

$$= \frac{3}{4\pi}\cdot\int_0^{2\pi} - \frac{\cos x\sin x}{\sin x}$$

$$= \frac{1}{4\pi} \left[ \frac{3\cos u \sin}{\sin u} \right]_0^{2\pi}$$

$$= 12 \times \frac{1}{4\pi} = 3/\pi$$

$$\therefore A_0 = 3/\pi$$

$$A_n = \frac{1}{2\pi} \int_0^{2\pi} 3|\sin n| \cdot \cos\left(\frac{n\pi u}{2\pi}\right) du$$

$$\int_0^{2\pi} \cos\left(\frac{n\pi u}{2\pi}\right) du$$

$$= \int_0^{2\pi} \cos(u)$$

$$= \frac{2}{n} \int_0^{\pi n} \cos u \, du$$

$$= \frac{2}{n} [\sin u]_0^{\pi n}$$

$$= \frac{2}{n} \quad 0$$

$$= 0$$

$$A_n = \frac{1}{2\pi} \times 12 \times 0 = 0$$

$$B_n = \frac{1}{2\pi} \int_0^{2\pi} 3|\sin n| \cdot \sin\left(\frac{n\pi u}{2\pi}\right) du$$

$$\int_0^{2\pi} \sin\left(\frac{n\pi u}{2\pi}\right) du$$

$$= \int_0^{\pi n} \frac{2\sin u}{n} du$$

$$= \frac{2}{n} \int_0^{\pi n} \sin u \, du$$

$$= \frac{2}{n} [-\cos u]_0^{\pi n}$$

$$= \frac{2(+1^n)+1}{n} = \frac{2}{n}\left(-\cos\left(\frac{n\pi n}{2}\right) + \cos\left(\frac{n0}{2}\right)\right)$$

$$= -\frac{2\cos\left(\frac{n2\pi}{2}\right)+1}{n}$$

$$\text{So, } B_n = \frac{1}{2\pi} \times \overset{6}{\cancel{12}} \times -\frac{2\cos\left(\frac{n2\pi}{2}\right)+1}{n}$$

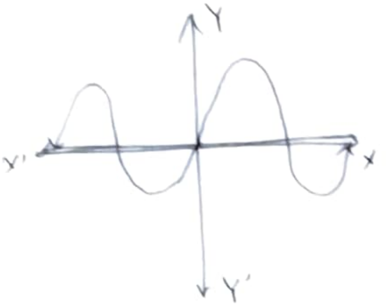
$$= \frac{6}{\pi} \cdot \frac{(\cos\left(\frac{n2\pi}{2}\right)+1)}{n}$$

∴ Fourier Series

$$S(x) = \frac{3}{\pi} + \sum_{n=1}^{\infty} \left( 0 + \frac{6}{\pi} - \frac{\cos\left(\frac{n2\pi}{2}\right)+1}{n} \right)$$

Ans

Sketch: $\sum_{n=0}^{2\pi} 3\sin(x)$



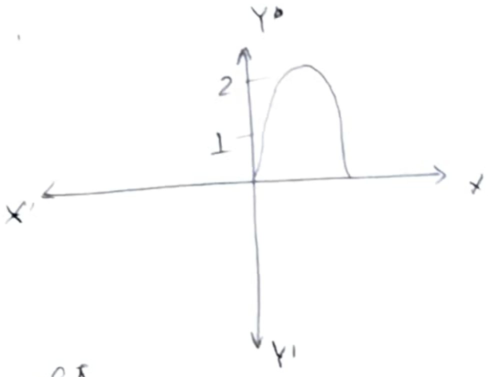

~~The sketch can not be defined~~

(c)

Given,

$$S(x) = \begin{cases} 2\sin x & \text{for } 0 \le x < \pi \\ 0 & \pi \le x < 2\pi \end{cases}$$

Sketch:




$$a_0 = \frac{1}{2\pi} \int_0^{\pi} 2\sin(x)\,dx$$

$$= \frac{2}{2\pi} \int_0^{\pi} \sin(x)\,dx$$

$$= \frac{1}{\pi} \left[ -\cos(x) \right]_0^{\pi}$$

$$= \frac{1}{\pi} \times 2$$

$$\therefore a_0 = \frac{2}{\pi} \text{ Ans}$$

$a_n = \frac{1}{\cancel{2}\pi} \int_0^{\pi} 2\sin x \cdot \cos \frac{\pi n x}{\pi} \cdot dx$

for

$= \cancel{\frac{2}{\pi}\int_0^{\pi} \sin x} = \int_0^{\pi} \cos \frac{\pi n x}{\pi} \cdot dx$

$= \frac{1}{n}\int_0^{n\pi} \cos v \cdot dv$

$= \frac{1}{n}\left[\sin(v)\right]_0^{n\pi}$

$= \frac{1}{n}\, 0$

$= 0$

$= \frac{1}{\pi} \cdot 4 \times 0$

$= 0$

$b_n = \frac{1}{\pi}\int_0^{\pi} 2\sin x \cdot \sin \frac{\pi n x}{\pi} \cdot dx$

for, $\int_0^{\pi} \sin\left(\frac{\pi n x}{\pi}\right) \cdot dx$

$= \frac{1}{n}\left[-\cos v\right]_0^{n\pi}$

$= \frac{1}{n}\left(-(-1)^n + 1\right)$

$= \frac{-(-1^n)+1}{n}$

if, $n = 1$

$= \frac{1+1}{1} = 2$

$n = 2 = \frac{1+1}{2} = 0$

$\therefore$ Fourier Series, $f(x) = \frac{2}{\pi} + 0 + \frac{+1^n+1}{n}$

(Ans)

$$A_0 = \frac{1}{2.2\pi} \int_{\pi}^{2\pi} 0 \,.\, dx$$

$$= [0]_{\pi}^{2\pi} \times \frac{1}{4\pi}$$

$$\therefore A_0 = 0$$

$$A_n = \frac{1}{2\pi} \int_{\pi}^{2\pi} 0 \,.\, \cos\left(\frac{\pi n x}{2\pi}\right)$$

for, $\int_{\pi}^{2\pi} \cos\left(\frac{nx}{2}\right)$

$$= \frac{2}{n} \int_{\frac{\pi n}{2}}^{\pi n} \cos u \, du$$

$$= \frac{2}{n} \left[\sin(u)\right]_{\frac{\pi n}{2}}^{\pi n}$$

$$A_n = -\frac{2\sin\left(\frac{\pi n}{2}\right)}{n} \times 0 \times \frac{1}{2\pi} = 0$$

$B_n = 0$

There is no ~~skeath~~ skeatch
as $a_0 = b_n = a_n = 0$

So, The Fourier Siries,

$$a_0 + \sum_{n=1}^{\infty} \left(0 \,\cancel{\cos\frac{\pi n x}{}} + 0\right)$$

(Ans)

d

$$S(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{"} \ \pi \le x < \pi \end{cases}$$

$$A_0 = \frac{1}{2.\pi} \int_0^{\pi} 1 \, dx$$

$$= \frac{1}{2.\pi} \,.\, (-\pi)$$

$$= \frac{1}{2}$$

$$B_n = \frac{1}{\pi} \int_0^{\pi} 1 \,.\, \sin\left(\frac{\pi n x}{\pi}\right) dx$$

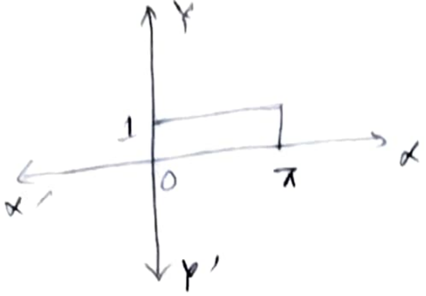
$$= \frac{1}{\pi} \times \frac{1}{2} \times 0$$

$$= 0$$

$$A_n = \frac{1}{\pi} \int_0^{\pi} 1 \,.\, \cos\left(\frac{\pi n x}{\pi}\right) dx$$

$$= \frac{1}{\pi} \times \frac{1}{2} \times \frac{\sin \cancel{2}\pi n}{\cancel{\pi} n} 0 = \frac{\sin \cancel{2}\pi n}{2\cancel{\pi} n} \ 0$$

sketch




~~$a_0 = \frac{Y}{\pi}$~~

Fourier Series

$$\frac{1}{2} + \sum_{n=1}^{\infty} (0 + 0)$$

Ans

$$a_0 = \frac{1}{2\pi} \int_{\pi}^{\pi} 0$$

$= 0$

$$A_n = \frac{4}{\pi} \cdot \int_{\pi}^{\pi} 0 \cdot \cos\left(\frac{\pi n x}{\pi}\right)$$

$= 0$

$b_n = 0$

Fourier serier,

$S(x) = 0 + 0$ , there is no Fourier series

And no sketch.

(e)

Given,

$$f(x) = A - \frac{Ax}{P} \quad 0 \leq x \leq P$$

$$a_0 = \frac{1}{2P} \cdot \int_0^P A - \frac{Ax}{P}$$

$$= -\frac{1}{2P} \int_0^P \frac{Ax}{P} dx + \int_0^P A\,dx$$

$$= \frac{1}{2P} \div \frac{AP}{2} + AP \quad = \frac{4P - \cancel{4AP} + \cancel{4AP}}{\cancel{4P}}$$

$= 0$

$$a_n = \frac{1}{p} \int_0^p A - \frac{Au}{p} \cos\left(\frac{\pi n u}{p}\right)$$

$$= \frac{1}{p} \times 0 \times \cos \frac{\pi n u}{p}$$

$$= 0$$

& $b_n = 0$

So, ~~fourier series~~, there will be no fourier

series and no sketch.