Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Fall 2021)

Assignment-3

Total Marks: 20

Answer all questions.

1. A person is jumping from a height h_0 = 10 m with zero initial velocity and a cricket ball is thrown directly upward with an initial velocity v_0 = 9.8 m/s at the same instant when time t = 0 s as shown in Fig. 1. Consider h = 11.5 m and $\hat{\imath}$ and $\hat{\jmath}$ to be the unit vectors along the X and Y axis. Gravity is acting along the -$\hat{\jmath}$ direction and the magnitude of gravitational acceleration is given by g = 9.81 m/s^2. The red dot on the person represents his center of mass.

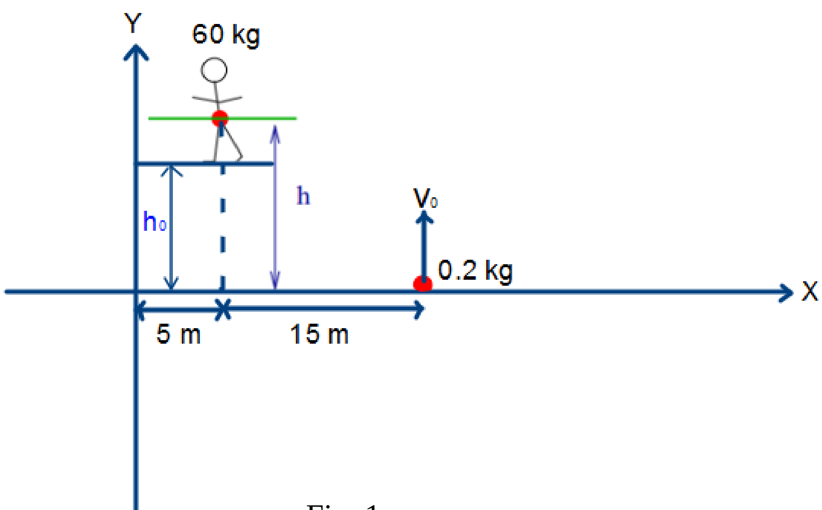


Fig. 1

(a) (*4 marks*) If the person and the cricket ball consist of a system, find the position and velocity of the center of mass of the system at time t = 0.5 s.

(b) (*2 marks*) If it takes 0.01 seconds for the person to come to complete rest when he hits the ground, calculate the average force acting on the person.

(c) (*4 marks*) Now consider when the person hits the ground, he bends his knees such that the center of mass of his body is lowered by 15 cm. If the maximum force that the bone of his legs can withstand is 50000 N, will he break his legs?

[Hint: Use the change in center of mass as the distance travelled during the collision process to calculate the deceleration. Remember that the reaction force by the ground to his legs is responsible for breaking them.]

2. At a sharp horizontal circular turn a train slows down going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Assume the train continues to slow down at the same rate. At the moment the train speed reaches 50.0 km/h calculate:

(a) (*3 marks*) the magnitude and direction of the tangential acceleration.
(b) (*3 marks*) the radial acceleration (magnitude and direction).
(c) (*4 marks*) the magnitude and direction of the total acceleration.

Solution 1

(a) Position of the com and the cricket ball does not change with time in the X-axis.

$$\therefore X_{com} = \frac{60\times5 + 0.2\times20}{60+0.2} \text{ m} = 5.05 \text{ m}$$

After 0.5 s, in the Y-axis,

position of com of the person is,

$$\vec{y}_p = \vec{y}_{op} + \vec{v}_{y_{op}}t + \frac{1}{2}\vec{a}_{y_p}t^2$$

$$= \left\{11.5\hat{j} + 0\hat{j} - \frac{1}{2}\times9.81\times(0.5)^2\hat{j}\right\} \text{ m}$$

$$= 10.274 \text{ m } \hat{j}$$

position of the cricket ball is,

$$\vec{y}_b = \vec{y}_{ob} + \vec{v}_{y_{ob}}t + \frac{1}{2}\vec{a}_{y_b}t^2$$

$$= 0\hat{j} + \left\{(9.81\times0.5)\hat{j} - (0.5\times9.81\times(0.5)^2\hat{j}\right\} \text{ m}$$

$$= 3.679 \text{ m } \hat{j}$$

$$\therefore Y_{com} = \frac{60\times10.274 + 0.2\times3.679}{60+0.2} \text{ m} = 10.252 \text{ m}$$

So, the position of com of the system after 0.5 s is,

$$\vec{r}_{com} = 5.05 \text{ m } \hat{i} + 10.252 \text{ m } \hat{j}$$

Now, the velocity of the com of the person and the cricket ball are both zero along the X-axis.

$\therefore V_{x,com} = 0 \; ms^{-1}$

After 0.5 s, in the Y-axis,

velocity of com of the person is,

$$\vec{V_P} = V_{op}\,\hat{j} - gt\,\hat{j}$$

$$= -4.905 \; ms^{-1}\,\hat{j}$$

velocity of the ball is,

$$\vec{V_b} = V_{ob}\,\hat{j} - gt\,\hat{j}$$

$$= 4.905 \; ms^{-1}\,\hat{j}$$

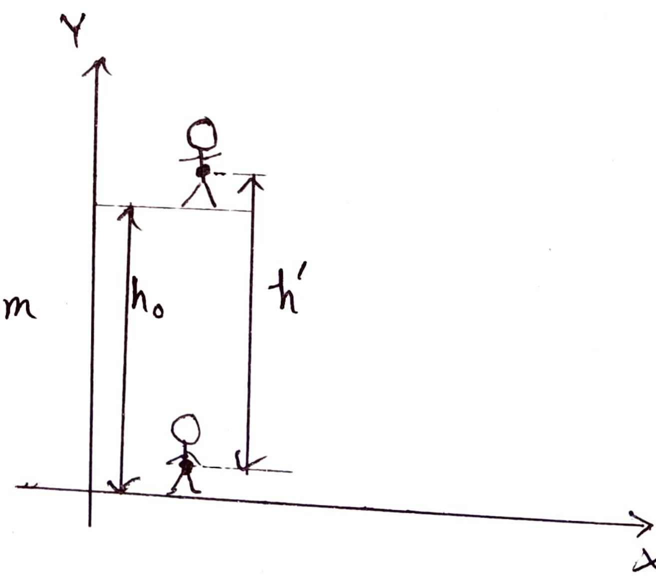
$$\therefore V_{y,com} = \frac{60\times(-4.905) + 0.2\times 4.905}{60+0.2} \; ms^{-1}$$

$$= -4.872 \; ms^{-1}$$

$$\therefore \vec{V}_{com} = 0 \; ms^{-1}\,\hat{i} - 4.872 \; ms^{-1}\,\hat{j}$$

(b) Impulse, $\vec{J} = \vec{P_f} - \vec{P_i}$

$$= m\vec{V_f} - m\vec{V_i}$$



Now, distance travelled by the com of the person when his feet touches the ground is,

$$h' = h_o = 10\,m$$

Now, $\vec{V_i} = -\sqrt{2gh'}\,\hat{j} = -14.007 \; ms^{-1}\,\hat{j}$

$$\vec{V_f} = 0\,\hat{j}$$

$\therefore\ \vec{J} = \{0\hat{j} - (-60\times 14.007)\hat{j}\}$ Ns

$= 840.42$ Ns $\hat{j}$

$\therefore\ \vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{840.42 \text{ Ns } \hat{j}}{0.01 \text{ s}} = 84042 \text{ N } \hat{j}$

(c) Distance travelled during the collision process with the ground by the com of the person is = 15 cm

If we consider the magnitude, then,

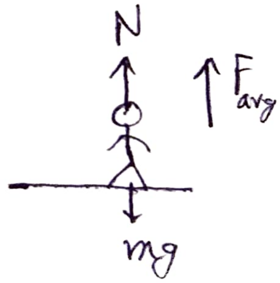
$$h - h_0 = 0.15 \text{ m}$$

Now, $V_f^2 = V_i^2 + 2a(h - h_0)$

$\therefore\ a = \frac{V_f^2 - V_i^2}{2(h-h_0)} = 654 \text{ ms}^{-2}$

This is the deceleration of the person during the collision period with the ground.

So, the average net force on the person is,



$$F_{avg} = (60\times 654) \text{ N} = 39240 \text{ N}$$

In vector notation, $\vec{F}_{avg} = 39240 \text{ N } \hat{j}$

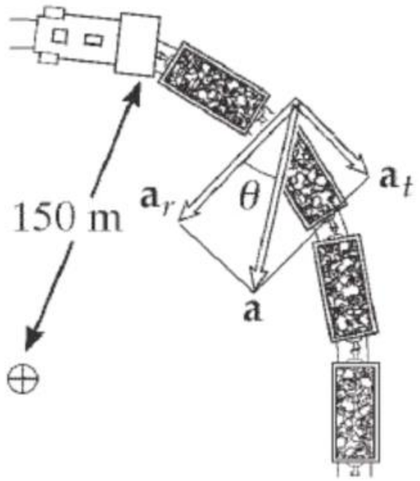
Now, as shown in the figure,

$$F_{avg}\hat{j} = N_{avg}\hat{j} - mg\hat{j}$$

$$\therefore\ N_{avg} = F_{avg} + mg$$

$\therefore N_{avg} = 39828.6 \text{ N}$

Since the normal force provided by the ground is less than the maximum force which the person's legs can withstand, he won't break his legs.



v_i = 90.0 km/h = (90.0 km/h)(103 m/km)(1 h/3600 s)
= 25.0 m/s
v_f = 50.0 km/h = (50.0 km/h)(103 m/km)(1 h/3600 s)
= 13.9 m/s

(a) The tangential acceleration

$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.9-25.0}{15.0}\ \text{m/s}^2$$

= - 0.741 m/s^2 (backward to the train motion)

(b) And radial acceleration

$$a_r = \frac{v^2}{r} = \frac{(13.9)^2}{150}\ \text{m/s}^2$$

= 1.29 m/s^2 (toward the center)

(c) The total acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.29)^2+(-0.741)^2}\ \text{m/s}^2 = 1.48\ \text{m/s}^2$$

and the direction

$$\theta = \tan^{-1}\left(\frac{|a_t|}{a_r}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right) = 29.9^\circ,\ \text{clockwise with radial direction}$$