Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Fall 2021)

Assignment-4

Total Marks: 20

Answer all questions.

1. A pinball machine launch ramp consisting of a spring and a 30° ramp of length L as shown in Fig. 1.

(a) (*3 marks*) If the spring is compressed a distance x from its equilibrium position and is then released at $t = 0$, the pinball (a sphere of mass m and radius r) reaches the top of the ramp at $t = T$. Derive the expression for the spring constant k in terms of $m, g, x,$ and L.

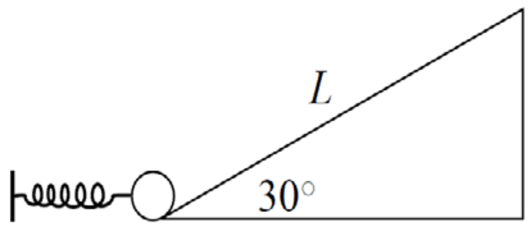


Fig. 1

[Assume that the friction is sufficient, and the ball begins rolling without slipping immediately after launch.]

(b) (*2 marks*) What is the potential energy of the ball when it is at the midpoint of the ramp?

(c) (*3 marks*) Derive the expression of the speed of the ball immediately after being launched in terms of g and L.

2. Fig. 2 shows a solid sphere of mass 2 kg and a massless string of total length 4 meters which is wrapped around the equator of the sphere (total length includes the length of wrapped string and the length of the string hanging from the ceiling). The sphere then falls vertically while rolling down (without slipping) the whole wrapped string and falls on top of an inclined plane. Due to the collision with the inclined plane, it loses all its kinetic energy within a very short time and

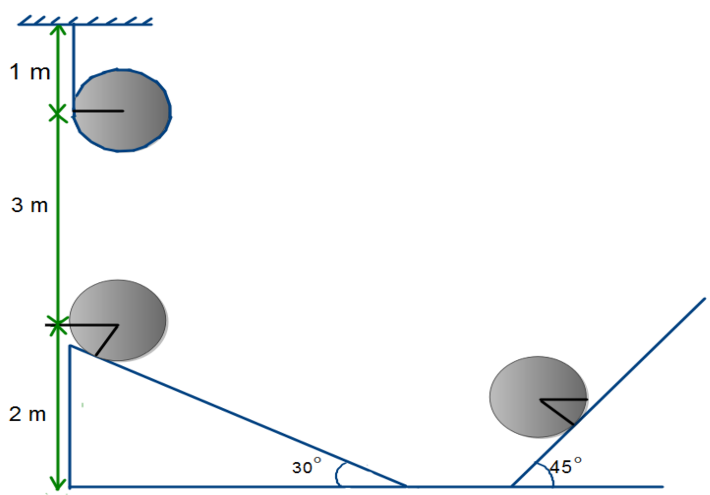


Fig. 2

starts rolling along the inclined plane with zero initial velocity. Consider that a total of 10 Joules of energy is lost during the motion of the sphere along the planes. The magnitude of gravitational acceleration g = 9.8 m/s^2.

(a) (*2 marks*) Find the value of the moment of inertia of the sphere about its axis of rotation.

(b) (*2 marks*) What is the magnitude of angular velocity of the sphere when it touches the inclined plane?

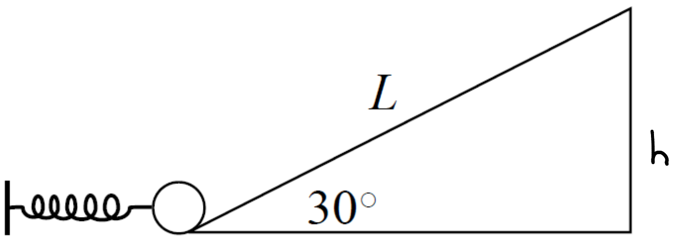
(c) (*2 marks*) How far along the second inclined plane (45º) on the right will the center of the sphere travel before it comes to a stop?

3. A target in a shooting board consists of a vertical square wooden board, 0.250 m on a side and with a mass 750 g and pivots around a horizontal axis along its top edge. The board is stuck face-on at its center by a bullet of mass 1.90 g, travelling at 360 m/s that remains embedded in the board.

(a) (*2 marks*) What is the angular speed of the board just after the bullet's impact?

(b) (*2 marks*) What maximum height does the center of the board reach from the equilibrium before it starts swinging down again?

(c) (*2 marks*) What minimum bullet speed is needed for the board to swing all the way over after the impact?



(a) Let, the height of the ramp is h.

$$\frac{h}{L} = \sin 30^\circ$$

$$\Rightarrow h = \frac{L}{2}$$

We can set the zero of gravitational potential at the bottom of the ramp. At the top of the ramp, the pinball's potential energy is $\frac{mgL}{2}$ and kinetic energy is zero. Because the ball barely reaches the top of the ramp. So total energy is $\frac{mgL}{2}$.

When the spring is compressed a distance x from its equilibrium position and then released at $t=0$, it comes back to its original position with an energy $\frac{1}{2}Kx^2$ and the pinball takes this energy and reaches the top of the ramp barely.

So, using energy conservation laws,

$$\frac{1}{2}Kx^2 = \frac{mgL}{2}$$

$$\boxed{\Rightarrow K = \frac{mgL}{x^2}}$$

(b) At the midpoint of the ramp, height of the pinball

$$h' = \sin 30^\circ \times \frac{L}{2}$$

$$\Rightarrow h' = \frac{L}{4}$$

Potential energy $= \frac{mgL}{4}$

(c) There is no change in potential energy of the ball, immed-
-iately after being launched, so the ball's total energy is
kinetic.

The pinball's translational kinetic energy, $T_{tr} = \frac{1}{2} mv^2$

and its rotational kinetic energy, $T_r = \frac{1}{2} I\omega^2$

Total kinetic energy, $T = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$$= \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{2}{5} mr^2\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{5} mv^2$$

$$= \frac{7}{10} mv^2$$

Using energy conservation laws,

$$\frac{7}{10} mv^2 = \frac{mgL}{2}$$

$$\Rightarrow v^2 = \frac{5}{7} gL$$

$$\boxed{\Rightarrow v = \sqrt{\frac{5}{7} gL}}$$

(a) Moment of inertia of a solid sphere about an axis going through the origin is given by,

$$I = \frac{2}{5}MR^2$$

The length of the string wrapped around the equator of the solid sphere is $= (4-1)\,m = 3\,m$

$$\therefore 2\pi R = 3$$

$$\therefore R = \frac{3}{2\pi}$$

$$\therefore I = \frac{2}{5} \times 2 \times \left(\frac{3}{2\pi}\right)^2 = 0.18238 \text{ kg-m}^2$$

(b) We will use the conservation of energy concept here.

We have,

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$$

$h = 3m$

If the sphere rolls without slipping, then-

$$V_{cm} = \omega R$$

$$\therefore Mgh = \frac{1}{2}M\omega^2R^2 + \frac{1}{2}I\omega^2$$

$$\therefore \omega^2 = \frac{Mgh}{\frac{1}{2}(MR^2+I)} = 184.234 \text{ rad}^2\text{s}^{-2}$$

$$\therefore \omega = 13.573 \text{ rads}^{-1}$$

(a)

When the sphere starts rolling along the inclined plane on the left, it has only gravitational potential energy. When it comes to a stop after travelling some distance along the inclined plane on the right, it also has only gravitational potential energy.

If the ~~sp~~ center of the sphere reaches a height of h', then,

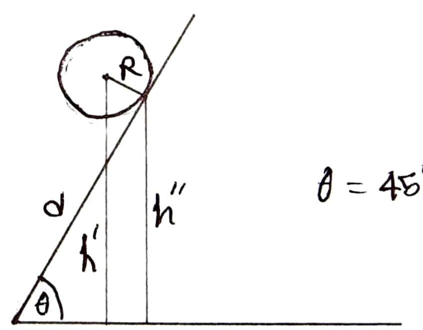
$$Mgh = Mgh' + 10$$

$$\therefore h' = \frac{Mgh - 10}{Mg}$$

$$= 1.4898 \text{ m}$$

$h = 2m$

If the center of the sphere travels a distance of d along the inclined plane, then –

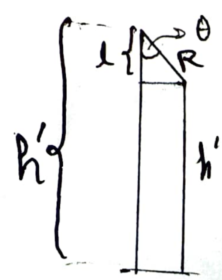


$\theta = 45°$

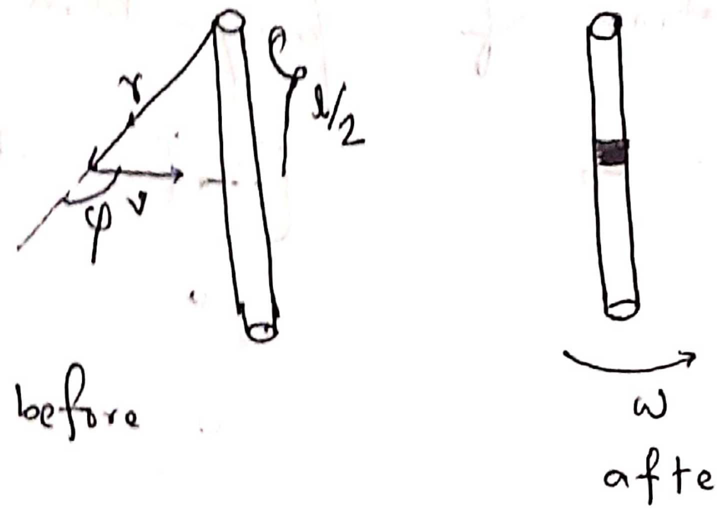
$$d = \frac{h''}{\sin\theta} = \frac{h' - l}{\sin\theta}$$

$$= \frac{h' - R\cos\theta}{\sin\theta}$$

$$= 1.6295\text{m}$$



(a)



$l = 0.250\ m$, $m_{board} = 0.75\ kg$.

$m_{bullet} = 1.9 \times 10^{-3}\ kg$, $v = 360\ m/s$.

$L_1 = m_{bul} v r \sin\varphi = m_{bull} v\, l/2$

$= (1.9 \times 10^{-3}\ kg)(360\ ms^{-1})(0.125\ m)$

$= 0.086\ kg\, m^2 s^{-1}$

$L_2 = I_2 \omega_2$

$I_2 = I_{board} + I_{bullet}$

$= \frac{1}{3} M l^2 + m r^2$

$= \frac{1}{3}(0.750\ kg)(0.250)^2$

$+ (1.9 \times 10^{-3}\ kg)(0.125\ m)^2$

$= 0.01565\ kg\, m^2$

$= 0.01557\ kg\, m^2$

From conservation . angular momentum .

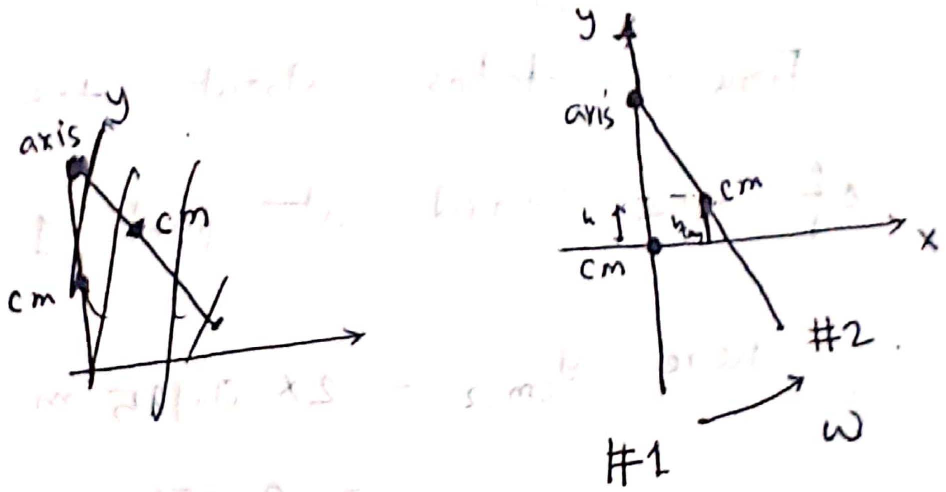
$$L_1 = L_2 .$$

$$\Rightarrow 0.086 = I_2 \omega_2$$

$$\Rightarrow 0.086 = 0.0156\, \omega_2$$

$$\Rightarrow \omega_2 = 5.46 \text{ rad/s}$$

(b)



$$y_{cm_1} = 0$$

$$y_{cm,2} = h .$$

$$m = m_{bulet} + m_{board} = (1.9\times10^{-3} + 0.75) \text{ kg.}$$

$$K_1 + U_1 = K_2 + U_2 .$$

$$\Rightarrow \frac{1}{2} I\omega^2 = mgh$$

$$\Rightarrow h = \frac{1}{2} I\omega^2 \cdot \frac{1}{mg} = 0.0317\,m = 3.17\,cm .$$

(c)

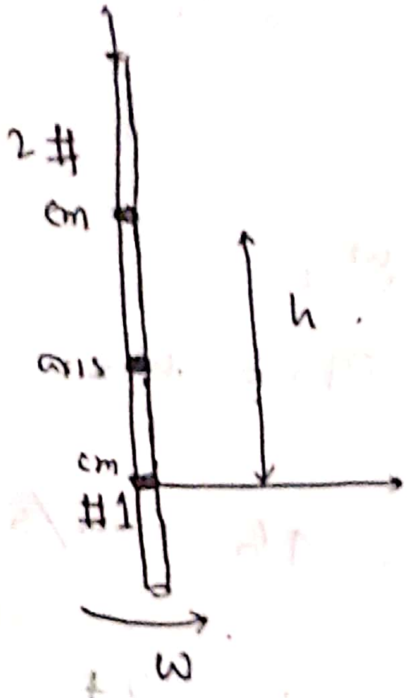


Figure states about the position of the board at point 1 and 2.

here $y_{cm.2} = 2 \times 0.125\ m$

$= 0.250\ m$

$m = m_{board} + m_{bullet}$

$U_1 + \cancel{K_1} = U_2 + K_2$

$\Rightarrow \frac{1}{2} I \omega^2 = mgh.$

$\Rightarrow \omega^2 = \frac{2mgh}{I}$

$\Rightarrow \omega = \sqrt{\frac{2mgh}{I}}$

$\Rightarrow \omega = 15.34\ rad/s$

Now as

$$L_1 = L_2$$

$$\Rightarrow m_{bullet} v \, l/_2 = I_2 \omega_2 .$$

$$\Rightarrow v = \frac{I_2 \omega_2}{m_{bullet} \times l/_2} = \frac{(0.0156)(15.39)}{(1.90 \times 10^{-3})(0.125)}$$

$$= 1010 \ ms^{-1}$$