Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Fall 2021)

Assignment-1

Total Marks: 20

Answer all questions.

1. Three vectors are given by,

$$\vec{R}_1 = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k},$$

$$\vec{R}_2 = 2\hat{\imath} - m\hat{\jmath} - 8\hat{k} \text{ and}$$

$$\vec{R}_3 = n\hat{\imath} + 4.5\hat{\jmath} - 9\hat{k}$$

(a) *(Marks: 2.5)* Explain why $\vec{R}_1$ & $\vec{R}_2$ are not parallel to each other.

(b) *(Marks: 5)* If $\vec{R}_1 \parallel \vec{R}_3$ & $2|\vec{R}_2 \times \vec{R}_3| = [\vec{R}_1 . (\vec{R}_2 \times \vec{R}_3)] + \sqrt{58.8n}$, calculate the value of *m*.

(c) *(Marks: 2.5)* Calculate the angles that the vector $(\vec{R}_3 - \vec{R}_2)$ makes with $\hat{\imath}$, $\hat{\jmath}$ & $\hat{k}$ axis.

2. A projectile fired at *O* follows a parabolic trajectory (Fig. 1), given in parametric form by

$$x = 66t \;\; and \;\; y = 86t - 4.91t^2$$

where *x* and *y* are measured in meters and *t* in seconds. Determine:

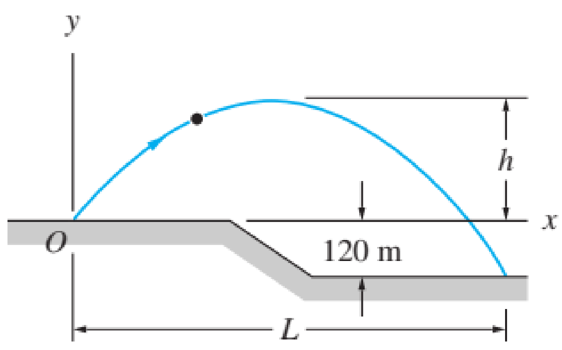


Fig. 1

(a) *(Marks: 1)* the acceleration vector throughout the flight,

(b) *(Marks: 1)* the velocity vector at *O*,

(c) *(Marks: 3)* the maximum height *h*; and the range *L*.

3. A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown in Fig. 2. The ball bounces horizontally off the plane at point P_1 with the same speed with which it struck the plane and strikes the plane again at point P_2. In terms of g and h determine each of the following quantities:

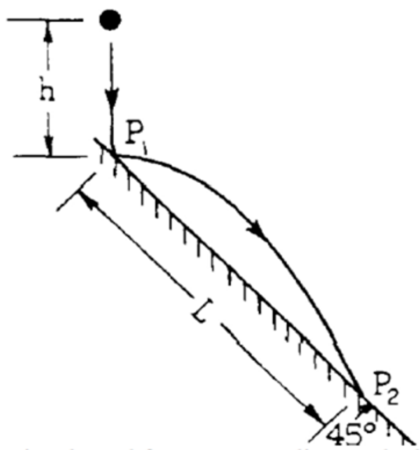


Fig. 2

(a) *(Marks: 1)* The speed of the ball just after it first bounces off the plane at P_1.

(b) *(Marks: 3)* The distance L along the plane from P_1 to P_2.

(c) *(Marks: 1)* The speed of the ball just before it strikes the plane at P_2.

(1) Given,

$\bar{R}_1 = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$\bar{R}_2 = 2\hat{i} - m\hat{j} - 8\hat{k}$

$\bar{R}_3 = n\hat{i} + 4.5\hat{j} - 9\hat{k}$

(a) $\bar{R}_1$ and $\bar{R}_2$ will be parallel if the ratio of their x components, y components and z components are equal. Since,

$$\frac{2}{2} \neq \frac{-6}{-8}$$

or, $1 \neq \frac{3}{4}$

So, $\bar{R}_1$ and $\bar{R}_2$ are not parallel to each other.

(b) If $\bar{R}_1 \parallel \bar{R}_3$, then,

$$\frac{2}{n} = \frac{3}{4.5}$$

$\Rightarrow n = 3$

So, $\bar{R}_3 = 3\hat{i} + 4.5\hat{j} - 9\hat{k}$

$$\bar{R}_2 \times \bar{R}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -m & -8 \\ 3 & 4.5 & -9 \end{vmatrix}$$

$= \hat{i}(9m + 36) + \hat{j}(-24 + 18) + \hat{k}(9 + 3m)$

$= \hat{i}(9m + 36) - 6\hat{j} + \hat{k}(3m + 9)$

$|\bar{R}_2 \times \bar{R}_3|$

$= \left[(9m + 36)^2 + 36 + (3m + 9)^2\right]^{1/2}$

$= \left(81m^2 + 648m + 1296 + 36 + 9m^2 + 54m + 81\right)^{1/2}$

$= \left(90m^2 + 702m + 1413\right)^{1/2}$

$$\bar{R}_1 \cdot (\bar{R}_2 \times \bar{R}_3) = \bar{R}_2 \cdot (\bar{R}_3 \times \bar{R}_1)$$

$$|\bar{R}_3 \times \bar{R}_1| = R_3 R_1 \sin\theta$$

$$= 0 \quad [\because \bar{R}_1 \parallel \bar{R}_3, \theta = 0^\circ \text{ and } \sin 0^\circ = 0]$$

so, $\bar{R}_2 \cdot (\bar{R}_3 \times \bar{R}_1) = 0$ [since, $|\bar{R}_3 \times \bar{R}_1| = 0$]

Given, $2(90m^2 + 702m + 1413)^{1/2} = (58.8\,n)^{1/2}$

$\Rightarrow 90m^2 + 702m + 1413 = 44.1 \quad [\because n = 3]$

$\Rightarrow 90m^2 + 702m + 1368.9 = 0$

So, $\boxed{m = -3.9}$

(c) $\bar{R}_2 = 2\hat{i} + 3.9\hat{j} - 8\hat{k}$

$\bar{R}_3 = 3\hat{i} + 4.5\hat{j} - 9\hat{k}$

$\bar{R}_3 - \bar{R}_2 = \hat{i} + 0.6\hat{j} - \hat{k}$

Let, $(\bar{R}_3 - \bar{R}_2)$ makes angles α, β, γ with $\hat{i}, \hat{j}, \hat{k}$ axis respectively.

$$(\bar{R}_3 - \bar{R}_2) \cdot \hat{i} = |\bar{R}_3 - \bar{R}_2| \cos\alpha$$

$$\Rightarrow \cos\alpha = \frac{1}{1.536}$$

$$\Rightarrow \boxed{\alpha = 49.38^\circ}$$

$$(\bar{R}_3 - \bar{R}_2) \cdot \hat{j} = |\bar{R}_3 - \bar{R}_2| \cos\beta$$

$$\Rightarrow \cos\beta = \frac{0.6}{1.536}$$

$$\Rightarrow \boxed{\beta = 67.007^\circ}$$

$$(\bar{R}_3 - \bar{R}_2) \cdot \hat{k} = |\bar{R}_3 - \bar{R}_2| \cos \gamma$$

$$\Rightarrow \cos \gamma = -\frac{1}{1.536}$$

$$\Rightarrow \boxed{\gamma = 130.62^\circ}$$

2. A projectile fired at O follows a parabolic trajectory (Fig. 1), given in parametric form by

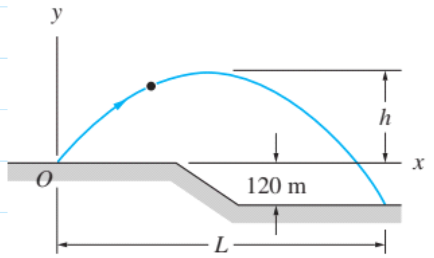
$$x = 66t \text{ and } y = 86t - 4.91t^2$$

where x and y are measured in meters and t in seconds. Determine:

(a) (Marks: 1) the acceleration vector throughout the flight,

(b) (Marks: 1) the velocity vector at O,

(c) (Marks: 3) the maximum height h; and the range L.



Solⁿ:- (a) Given that, $x = 66t$ and

$y = 86t - 4.91t^2$

Now, acceleration, $\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$

$\Rightarrow \vec{a} = 0 + (-9.82)\hat{j}$

$\Rightarrow \boxed{\vec{a} = -9.82\hat{j} \ m/s^2}$ Ans

(b) velocity, $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$\Rightarrow \vec{v} = 66\hat{i} + (86 - 9.82 \times t)\hat{j}$

But at point O, $t=0$. So,

$\boxed{\vec{v}_0 = 66\hat{i} + 86\hat{j}}$ Ans

(c) For maximum height, $v_y = 0$. So,

$v_y^2 = v_{0y}^2 - 2gy = 0$

$\Rightarrow y = \frac{v_{0y}^2}{2g} = \frac{(86)^2}{2\times 9.8} = 377.34$

$$\Rightarrow \quad y = \frac{v_{0y}^2}{2g} = \frac{\quad}{2\times 9.8} = 377.34$$

$$\text{or} \quad \boxed{y = h = 377.34\ m}$$

Ans

Now, for the range L, the time of flight t

$$y = \cancel{y_0}^{\,0} + v_{0y}t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad -120 = 86t - 4.91t^2$$

$$\Rightarrow \quad 4.91t^2 - 86t - 120 = 0$$

$$\therefore \quad \boxed{t = 18.81\ \text{sec}} \quad \text{or} \quad t = -1.29\ \text{sec}\ [\text{Not acceptable}]$$

Now, the range,

$$L = x = 66t$$

$$\Rightarrow \quad L = 66 \times 18.81$$

$$\Rightarrow \quad \boxed{L = 1241.46\ m}$$

Ans

3. (a) Initial speed of the ball, $V_i = 0\ ms^{-1}$

Let, V_f is the speed after falling a height, h

Now, we know,

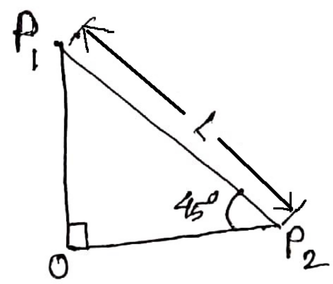
$$V_f^2 = V_i^2 + 2gh$$

$$\Rightarrow V_f = \sqrt{0^2 + 2gh}$$

$$\therefore V_f = \sqrt{2gh}\ (ms^{-1})$$

(b) According to the question at P_1 point, the velocity along the horizontal is equal to V_f and vertical velocity is zero.

$$\therefore V_{1x} = \sqrt{2gh} \text{ and } V_{1y} = 0$$



Again, during the flight from P_1 to P_1 the ball travels a horizontal distance of $\frac{L}{\sqrt{2}}$ and vertical distance of $\frac{L}{\sqrt{2}}$.

$$\cos 45^\circ = \frac{OP_2}{P_1P_2}$$

$$\Rightarrow OP_2 = P_1P_2 \cos 45^\circ = \frac{L}{\sqrt{2}}$$

Similarly, $\sin 45^\circ = \frac{OP_1}{P_1P_2}$

$$\therefore OP_1 = P_1P_2 \sin 45^\circ = \frac{L}{\sqrt{2}}$$

Now, we know,

$$x - x_0 = V_{0x} t$$

$$\Rightarrow \frac{L}{\sqrt{2}} = V_{1x} t$$

$$\Rightarrow \frac{L}{\sqrt{2}} = (\sqrt{2gh})t \quad \text{——— (I)}$$

Again, $y - y_0 = V_{0y} t + \frac{1}{2} g t^2$

$$\Rightarrow \frac{L}{\sqrt{2}} = V_{1y} t + \frac{1}{2} g t^2$$

$$\Rightarrow \frac{L}{\sqrt{2}} = \frac{1}{2} g t^2 \quad \text{——— (II)}$$

From equation (I) and (II), $(\sqrt{2gh})t = \frac{1}{2} g t^2$

$$\therefore t = \sqrt{\frac{8h}{g}}$$

Now, from equation (I), $\frac{L}{\sqrt{2}} = \sqrt{2gh} \times \sqrt{\frac{8h}{g}}$

$$\therefore L = 4\sqrt{2}h = 5.66h \text{ (Ans)}$$

(c) Let, $\vec{V}$ is the velocity at point P_2

We Know, $\vec{V} = V_x \hat{i} + V_y \hat{j}$

$$= V_{1x} \hat{i} + (V_{1y} + gt) \hat{j}$$

$$= \sqrt{2gh}\, \hat{i} + \left(0 + g\sqrt{\frac{8h}{g}}\right) \hat{j}$$

$$= \sqrt{2gh}\, \hat{i} + \sqrt{8gh}\, \hat{j}$$

$\therefore$ The speed at point P_2,

$$|\vec{V}| = \sqrt{(\sqrt{2gh})^2 + (\sqrt{8gh})^2}$$

$$= \sqrt{10gh}\ (ms^{-1})$$