January 8, 2022

Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I

Final Assessment, Fall 2021

Time: 1 Hour (6:00 pm to 7:00 pm)

Total Marks: 20

Answer any two questions.

1. As shown in Fig. 1, a block of mass *m* is falling from some height h along an inclined plane. It then slides across a frictionless surface and enters a circular loop of radius R = 5 meters. The magnitude of gravitational acceleration is given by 9.81 m/s^2.

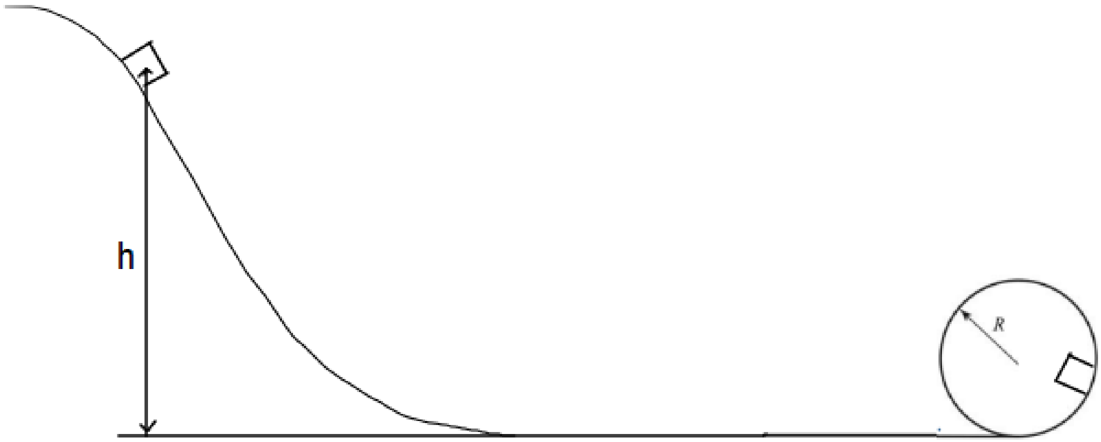


Fig. 1

(a) (*3 marks*) If h = 20 meters, what is the magnitude of velocity of the block when it is travelling along the horizontal surface?

(b) (*3 marks*) Calculate the magnitude of minimum tangential velocity with which the block must travel when it is at the top position of the circular loop such that it does not fall off. (For the minimum speed, the block will almost lost contact with the surface of the loop, making the normal reaction force zero)

(c) (*4 marks*) Find the minimum height h from where the block needs to be released on the inclined plane such that it can make a complete turn along the circular loop.

2. Two identical stars with radius R and mass M, orbits around their center of mass. The orbit is circular, and the stars are always on the opposite side of the circle.

(a) (*2 marks*) Find the gravitational force of one star on the another.

(b) (*4 marks*) Find the orbital speed of each star and their periods.

(c) (*4 marks*) How much energy would be needed to separate the two stars to the infinity?

3. A particle of mass 20 g is connected to one end of a spring on a frictionless horizontal surface and moving in a simple harmonic motion about the origin. The displacement given by $x = 2\ \sin 3t$, where *x* is in meters and time, *t* is in seconds. The motion starts when *t* = 0.

(a) (*3 marks*) What is the total distance travelled by the particle at time *t* = 3T/4, where T is the time period?

(b) (*3 marks*) Find the kinetic and potential energy of the particle at time *t* = 3T/4.

(c) (*4 marks*) What is the acceleration of the particle when it first comes to a rest?

Answer to the question no: 1

(a) Given,

$h = 20m$

$R = 5m$

$g = 9.8ms^{-2}$

We know,

$P = mgh$ [Consider mass of the block is m]

$= m(20 \times 9.8)$

$= 196m$

Now,

$196\cancel{m} = \frac{1}{2}\cancel{m}v^2$

$\Rightarrow v^2 = 196 \times 2$

$\Rightarrow v^2 = 392$

$\Rightarrow v = \sqrt{392}$

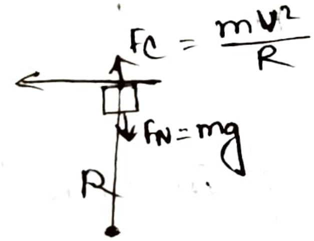
$v = 19.799 ms^{-1}$

(Ans)

(b) Given, $R = 5m$

$g = 9.81 \ ms^{-2}$

We ~~know~~, from the equation,



$$\frac{m v_{min}^2}{R} = mg$$

$$\Rightarrow v_{min} = \sqrt{Rg} \quad \text{—— (1)}$$

$$\Rightarrow V_{min} = \sqrt{5 \times 9.81}$$

$$\therefore V_{min} = 7.0036 \ ms^{-1}$$

(c) ~~From~~ ~~b,~~ ~~we~~ Given, $R = 5$

We know, $E_I = mgh$

$$E_F = mg(2R) + \tfrac{1}{2} m(Rg) \quad \boxed{\text{from equation (1)}}$$

$$\therefore E_F = \tfrac{5}{2} mgR$$

Now, $E_I = E_F$

$$\Rightarrow \cancel{m}gh = \tfrac{5}{2} \cancel{m}gR$$

$\Rightarrow h = 5/2\, R$

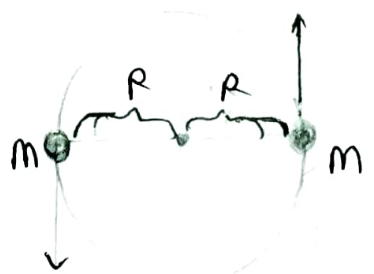
$\Rightarrow h = 5/2 \times 5$

$\therefore h = 12.5$ m

(Ans)

Answer to the question no 12

(a)



We know,

$$F_G = \frac{G(M \times M)}{(2R)^2}$$

$$\Rightarrow F_G = \frac{GM^2}{4R^2}$$

(b) We know,

$$F_C = \frac{mv^2}{R}$$

From a, $F_G = \frac{GM^2}{4R^2}$

Now, $F_C = F_G$

$$\Rightarrow \frac{Mv^2}{R} = \frac{GM^2}{4R^2}$$

$$\Rightarrow v^2 = \frac{G\cancel{M^2}^{M}R}{4\cancel{R^2}_{R}\cancel{M}}$$

$$\Rightarrow v = \sqrt{\frac{GM}{4R}}$$

And $T = \frac{2\pi R}{v}$

$$\Rightarrow T = \frac{2\pi R}{\sqrt{\frac{GM}{4R}}}$$

$$= 2\pi R \times 2 \cdot \sqrt{\frac{R}{GM}}$$

$$= 4\pi R \cdot \sqrt{\frac{R}{GM}}$$

(c) From the equation,

$E =$ Final energy − Initial energy

$$= 0 - \left(\cancel{2}\times \frac{1}{2} Mv^2 - \frac{GM^2}{2R}\right)$$

$$= -M\frac{GM}{4R} + \frac{GM^2}{2R} \quad \left[\text{From b}\right]$$

$$= -\frac{GM^2}{4R} + \frac{GM^2}{2R}$$

$$= \frac{GM^2}{4R}$$

(Ans)

Answer to the question no: 3

(a) Given, $m = 20g = .02\,kg$

$$x = 2\sin 3t \quad \text{——} \ (1)$$

For $t = \frac{3\pi}{4}$,

$$x = 2\sin 3\left(\frac{3\pi}{4}\right)$$

$$= 2\sin\left(\frac{9\pi}{4}\right)$$

$$= 2\sin\left(\frac{9\pi}{4}\right) \quad \text{——} \ (11)$$

Now, Comparing (1) with $x = A\sin \omega t$

$\omega = 3$ $\qquad\qquad$ $\omega = 3$

$\Rightarrow \cancel{f = \frac{3}{2\pi}}$ $\qquad\qquad$ $\Rightarrow \frac{2\pi}{T} = 3$

$$\Rightarrow T = \frac{2\pi}{3}$$

Using T in equation (11),

$$\cancel{x = \sin \frac{3(2\pi/3)}{2}}$$

$$\cancel{\Rightarrow \sin}$$

$$= \sin 3\pi$$

$$\cancel{= 0}$$

~~(Ans)~~

$$x = 2 \sin \frac{\cancel{3}^{3}(2\pi)}{\cancel{4}_{2} \times \cancel{3}}$$

$$= 2 \sin \frac{3\pi}{2}$$

$$= 2 \times (-1)$$

$$= -2$$

(Ans)

(b) We know,

$$\cancel{k = \sqrt{\omega/m}}$$

$$\omega = \sqrt{k/m}$$

$$\Rightarrow \omega^2 = k/m$$

$$\Rightarrow k = \omega^2 m$$

$$P = \frac{1}{2} k x^2$$

$$= \frac{1}{2} \times \omega^2 m \, x^2$$

$$= \frac{1}{2} \times 3^2 \times .02 \times (-2)^2$$

$$= 0.36 \text{ J}$$

$$K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\cancel{= \frac{1}{2} \times 0.02 \times 3^2 \times (-2)^2}$$

Here, $A = 2$

$$\cancel{= 0.36 \text{ J}}$$

$$K = \frac{1}{2} \times 0.02 \times 3^2 \times (2^2 - (-2)^2)$$

$$= 0 \text{ J}$$

(c) We know,

$$a = \frac{KA}{m}$$

$$\Rightarrow a = \omega^2 A$$

$$a = 3^2 \times 2$$

$$= 18 \ ms^{-2}$$

(Ans)