Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Fall 2021)

Assignment-5

Total Marks: 20

Answer all questions.

1. A pair of stars revolves about their common center of mass as in Fig. 1. One of the stars has a mass M that is twice the mass m of the other star. Their centers are a distance d apart (d being large compared to the size of either star). All units are in SI.

(a) *(4 marks)* Derive an expression for the period of revolution of the stars about their common center of mass in terms of d, m, and G.

(b) *(3 marks)* Compare the angular momenta of the two stars about their common center of mass by calculating the ratio L_m / L_M.

(c) *(3 marks)* Compare the kinetic energies of the two stars by calculating the ratio K_m / K_M.

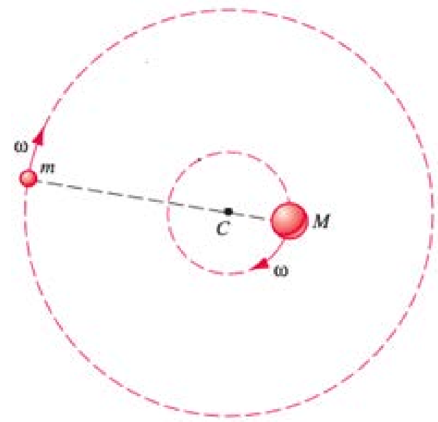

Fig. 1

2. A metal block of mass M kg is attached to a spring of negligible mass and spring constant k as shown in Fig. 2, and is free to slide on a frictionless, horizontal surface. A clay ball of mass M/4 kg is fired at the block with velocity V m/s and sticks to it. The block is initially at rest and the spring is initially uncompressed.

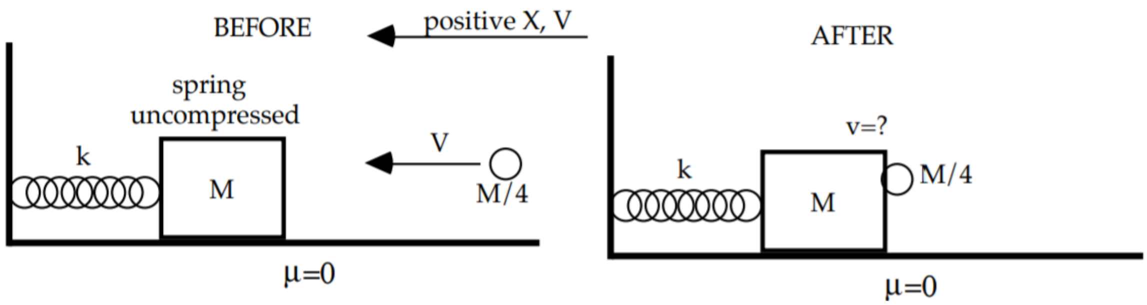

Fig. 2

(a) *(3 marks)* What is the speed of the block and ball system immediately after the impact?

(b) *(4 marks)* Write an equation for the position of the block as a function of time after the collision, assume that at $t = 0$ s, the instant of the impact, it is at $X = 0$ m, which is the unstretched point of the spring. Determine values for the amplitude, angular frequency in terms of the given quantities. Assume that X and V are positive to the left as shown.

(c) *(3 marks)* Calculate the time period, maximum velocity and maximum acceleration of the motion of the spring block and ball system.

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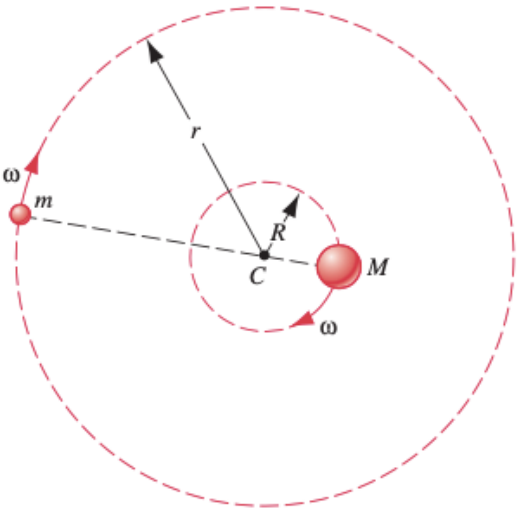
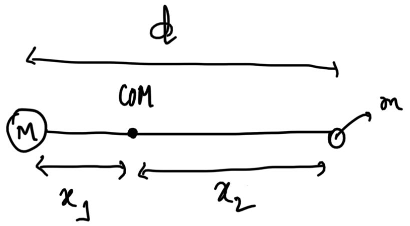


Sol^n:

a)



$M = 2m$

$d = x_1 + x_2$

Common C.O.M $\Rightarrow$ $M x_1 = m x_2$

$\Rightarrow 2\cancel{m} x_1 = \cancel{m} x_2$

$\Rightarrow x_2 = 2x_1$

$\therefore d = 3x_1 \Rightarrow x_1 = \frac{d}{3}$, $x_2 = \frac{2d}{3}$

Since the gravitional force provides the necessary centripetal acceleration for circular motion.

$$\therefore \cancel{m} x_2 \omega^2 = \frac{G(2m)\cancel{m}}{d^2}$$

$$\Rightarrow \frac{\cancel{2}d}{3}\omega^2 = \frac{\cancel{2}Gm}{d^2}$$

$$\Rightarrow \omega^2 = \frac{3Gm}{d^3} \quad —(1)$$

We know, $\omega = \frac{2\pi}{T}$

$\therefore$ ① $\Rightarrow$ $\frac{4\pi^2}{T^2} = \frac{3Gm}{d^3}$

$\therefore$ $T^2 = \frac{4\pi^2 d^3}{3Gm}$ $\Rightarrow T = \sqrt{\frac{4\pi^2 d^3}{3Gm}}$

b) $L_m = m\omega^2 x_2^2 = m\omega^2\left(\frac{2d}{3}\right)^2 = \frac{4m\omega^2 d^2}{9}$ (angular momentum)

$L_M = M\omega^2 x_1^2 = 2m\omega^2\left(\frac{d}{3}\right)^2 = \frac{2m\omega^2 d^2}{9}$

$\therefore$ $\frac{L_m}{L_M} = \frac{4m\omega^2 d^2}{\cancel{9}} \times \frac{\cancel{9}}{2m\omega^2 d^2} = 2$

c)

Since the gravitional force provides the necessary centripetal acceleration for circular motion, we can write, for M, with velocity V_M.

$$\frac{GMm}{d^2} = \frac{MV_M^2}{x_1} \quad ①$$

and for m, with velocity V_m we can write,

$$\frac{GMm}{d^2} = \frac{mV_m^2}{x_2} \quad ②$$

$$\therefore \quad \frac{M v_M^2}{x_1} = \frac{m v_m^2}{x_2}$$

$$\Rightarrow \quad \frac{\frac{1}{2} M v_M^2}{\frac{d}{3}} = \frac{\frac{1}{2} m v_m^2}{\frac{2d}{3}}$$

$$\Rightarrow \quad \frac{3 K_M}{d} = \frac{3 K_m}{2d}$$

$$\Rightarrow \quad \frac{K_M}{K_m} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{K_m}{K_M} = 2$$

2. A metal block of mass M kg is attached to a spring of negligible mass and spring constant k as shown in Fig. 2, and is free to slide on a frictionless, horizontal surface. A clay ball of mass M/4 kg is fired at the block with velocity V m/s and sticks to it. The block is initially at rest and the spring is initially uncompressed.

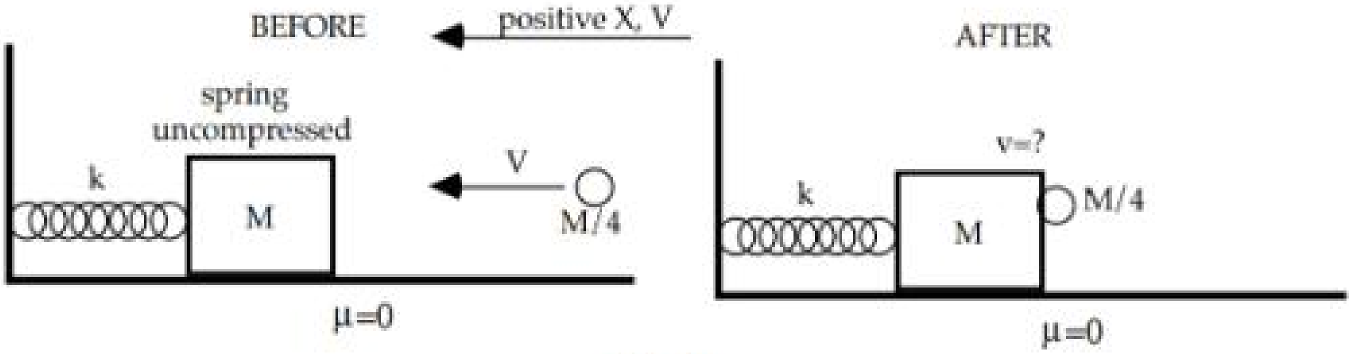


Fig. 2

(a) *(3 marks)* What is the speed of the block and ball system immediately after the impact?

(b) *(4 marks)* Write an equation for the position of the block as a function of time after the collision, assume that at t = 0 s, the instant of the impact, it is at X = 0 m, which is the unstretched point of the spring. Determine values for the amplitude, angular frequency in terms of the given quantities. Assume that X and V are positive to the left as shown.

(c) *(3 marks)* Calculate the time period, maximum velocity and maximum acceleration of the motion of the spring block and ball system.

2) (a) According to the momentum conservation law,

Initial momentum of the system = Final momentum of the system

$$\Rightarrow M\times 0+\left(\frac{M}{4}\times V\right)=\left(M+\frac{M}{4}\right)v$$

$$\Rightarrow \frac{M}{4}V=\frac{5M}{4}v$$

$$\Rightarrow \boxed{v=\frac{V}{5}}$$

(b) According to energy conservation law,

Energy at maximum compressed position = Energy at uncompressed position

$$\Rightarrow \frac{1}{2}KA^2=\frac{1}{2}\left(M+\frac{M}{4}\right)v^2$$

Here, A = amplitude of the motion

$$\Rightarrow \frac{1}{2}KA^2=\frac{5M}{8}\left(\frac{V}{5}\right)^2$$

$$\Rightarrow KA^2=\frac{5M}{4}\left(\frac{V}{5}\right)^2$$

$$\therefore \boxed{A=\frac{V}{5}\sqrt{\frac{5M}{4K}}}$$

Angular frequency, $\omega=\sqrt{\frac{K}{m}}$

$$\Rightarrow \omega=\sqrt{\frac{K}{(M+M/4)}}$$

$$\therefore \omega=\sqrt{\frac{4K}{5M}}$$

Now, the equation for the position of the block,

$$X=A\sin(\omega t+\phi) \qquad \phi=0$$

$$=\frac{V}{5}\sqrt{\frac{5M}{4K}}\sin\left(\sqrt{\frac{4K}{5M}}\cdot t\right)$$

(c) Time period $= \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\sqrt{\frac{4K}{5M}}}$$

$$= 2\pi\sqrt{\frac{5M}{4K}}$$

Maximum velocity $= \omega A$

$$= \sqrt{\frac{4K}{5M}} \times \frac{V}{5}\sqrt{\frac{5M}{4K}}$$

$$= \frac{V}{5} = v$$

Maximum acceleration $= \omega^2 A$

$$= \left(\sqrt{\frac{4k}{5m}}\right)^2 \times \frac{V}{5}\sqrt{\frac{5M}{4K}}$$

$$= \frac{V}{5}\sqrt{\frac{4K}{5M}}$$