1. (a)

$$i = \int_{R/3}^{R} \vec{J} \cdot d\vec{A}$$

$$= \int_{R/3}^{R} J \, dA$$

$$= \int_{R/3}^{R} J \cdot 2\pi r \, dr$$

$$= J 2\pi \int_{R/3}^{R} r \, dr$$

$$= 2\pi J \left. \frac{r^2}{2} \right|_{R/3}^{R}$$

$$= \pi J \left(R^2 - \frac{R^2}{9} \right) = \frac{\pi J \, 8R^2}{9}$$

$$= 3.77 \text{ A}$$

2nd part of (c)

$$\pi J \left. r^2 \right|_0^{R/2}$$

$$\Rightarrow \frac{\pi J R^2}{4}$$

$$= 1.06 \text{ A}$$

(b)

$$I = \int_{R/3}^{R} J \, dA = 2\pi \int_{R/3}^{R} e^{-r^2} r \, dr$$

$$= \cancel{2\pi} \left. -\pi e^{-r^2} \right|_{R/3}^{R}$$

$$= -\pi e^{-R^2} + \pi e^{-R^2/9}$$

$$r^2 = u$$

$$\Rightarrow 2r \, dr = du$$

$$\pi \int e^{-u} du$$

$$= -\pi e^{-u}$$

$$= -\pi e^{-r^2}$$

$= 2.51 \times 10^{-5}$ A

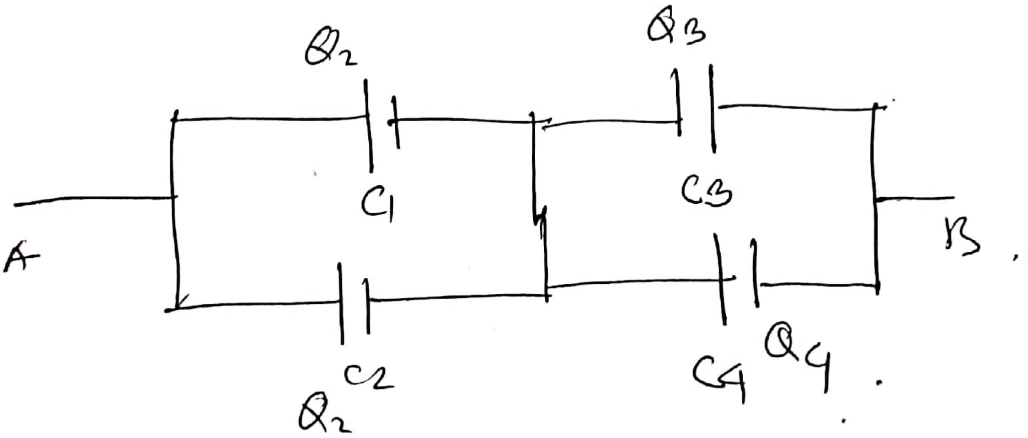
for 0 to $R/2$

$$-\pi e^{-r^2}\Big|_0^{R/2}$$

$$= -\pi e^{-R^2/4} + \pi$$

$= 7.069 \times 10^{-6}$ A.

(2)



$Q_1 = C_1 V_1 \Rightarrow V_1 = \dfrac{Q_1}{C_1}$

$= \dfrac{20 \times 10^{-3}\,C}{8 \times 10^{-3}\,F}$

$= 2.5\,V.$

C_1 and C_2 parallel so $V_1 = V_2$

$C_2 V_2 = Q_2 \Rightarrow Q_2 = (15 \times 10^{-3}) \times (2.5)$

$= 0.0375\ C$

$= 37.5\ mC$

total charge in C_1 and C_2

$= (20 + 37.5)\ mC = 57.5\ mC.$

C_3 and C_4 has to have same amount of charge in total as C_3 and C_4 are together in series with C_1 and C_2.

so total charge in (C_3 and C_4) together is

57.5

and $Q_3 = 28.75\ mC$

$Q_4 = 28.75\ mC.$

as $C_3 = C_4$,

so $V_3 = \frac{Q_3}{C_3} = 1.92\ V.$

so $V_A - V_B = V_1 + V_3 = 2.5 + 1.92$

$= 4.42$ volts.

(b)

$$C_{p_1} = C_1 + C_2$$

$$C_{p_2} = C_3 + C_4$$

$$C_s = \frac{1}{\frac{1}{C_1+C_2} + \frac{1}{C_3+C_4}} = \frac{1}{\frac{1}{23\times10^{-3}} + \frac{1}{30\times10^{-3}}}$$

$$= 13.02\times10^{-3}\,F$$

$$= 13.02\,mF$$

$$C_{new} = KC_s \quad = 6\times13.02\times10^{-3}\,F$$

$$= 0.078\,F$$

$$= 78\,mF.$$

q remains the same.

so $q = 57.5\,mC$

or $0.0575\,C$.

(c) without dielectric.

$$EA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 A} = \frac{57.5 \times 10^{-3}}{8.854 \times 10^{-12} \times 120 \times 10^{-4}}$$

$$= 5.41 \times 10^{11} \ NC^{-1}$$

with dielectric

$$KEA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{K \epsilon_0 A} = \frac{57.5 \times 10^{-3}}{6 \times 8.854 \times 10^{-12} \times 120 \times 10^{-4}}$$

$$= 9 \times 10^{10} \ NC^{-1}$$