Problem 7.5.1

Graph one trace of the sample mean of a Poisson $(\alpha = 1)$ random variable. Calculate (using a central limit theorem approximation) and graph the corresponding 0.9 confidence interval estimate.

Problem 8.2.6

Some telephone lines are used only for voice calls. Others are connected to modems and used only for data calls. The duration of a voice telephone call is an exponential random variable V with expected value $E[V] = 3$ minutes. The duration of a data call is an exponential random variable D with expected value $E[D] = \mu_D = 6$ minutes. The null hypothesis of a binary hypothesis test is H_0 : a line is used for voice calls. The alternative hypothesis is H_1: a line is a data line. The probability of a voice line is $P[V] = 0.8$. The probability of a data line is $P[D] = 0.2$.

A binary hypothesis test observes n calls from one telephone line and calculates $M_n(T)$, the sample mean of the duration of a call. The decision is H_0 if $M_n(T) \leq t_0$ minutes. Otherwise, the decision is H_1.

(a) Use the central limit theorem to write a formula for the false alarm probability as a function of t_0 and n.

(b) Use the central limit theorem to write a formula for the miss probability as a function of t_0 and n.

(c) Calculate the maximum likelihood decision time, $t_0 = t_{\text{ML}}$, for $n = 9$ calls monitored.

(d) Calculate the maximum a posteriori probability decision time, $t_0 = t_{\text{MAP}}$ for $n = 9$ calls monitored.

(e) Draw the receiver operating curves for $n = 9$ calls monitored and $n = 16$ calls monitored.

SKK: Do this in tutorial. Did the theory in class. Revise theory quickly and then get to the matlab part (e)

Problem 8.2.7

In this problem, we repeat the voice/data line detection test of Problem 8.2.6, except now we observe n calls from one line and records whether each call lasts longer than t_0 minutes. The random variable K is the number of calls that last longer than t_0 minutes. The decision is H_0 if $K \leq k_0$. Otherwise, the decision is H_1.

(a) Write a formula for the false alarm probability as a function of t_0, k_0, and n.

(b) Find the maximum likelihood decision number $k_0 = k_{ML}$ for $t_0 = 4.5$ minutes and $n = 16$ calls monitored.

(c) Find the maximum a posteriori probability decision number $k_0 = k_{MAP}$ for $t_0 = 4.5$ minutes and $n = 16$ calls monitored.

(d) Draw the receiver operating curves for $t_0 = 4.5$ minutes and $t_0 = 3$ minutes. In both cases let $n = 16$ calls monitored.

SKK: Do this in tutorial. Did the theory in class. Revise theory quickly and then get to the matlab part (d).