

# Transverse field Ising model spectrum

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## Abstract

We calculate  $\chi''(\omega)/\omega$  spectrum by interpolating the Masubara frequency correlation function  $\chi(i\omega_n)$  data obtained by cmpo method using Nevalinna analytical continuation algorithm.

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## I. WHAT'NEW IN OCTOBER 27, 2021'S UPDATE

1. fix typo of eq. (6)
2. add  $\chi''(\omega)$  definition eq. (4)
3. add  $\chi''(\omega)/\omega$  result with different  $D$ , see fig. 5.

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## II. THE ISING CHAIN IN A TRANSVERSE FIELD

Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - Jg \sum_i \sigma_i^x \quad (1)$$

where  $\sigma_i^\alpha$ ,  $\alpha = x, y, z$  are Pauli matrices,  $\langle .. \rangle$  stands for nearest neighbor,  $g = \Gamma/J$  and we set  $J = 1.0$ .

At finite temperature, the local two time correlation  $\chi(\tau)$  is defined as:

$$\chi(\tau) = \langle \sigma_i^z(\tau) \sigma_i^z(0) \rangle \quad (2)$$

We can limit ourselves to  $\tau \in [0, \beta]$  by the boundary conditions in  $\tau$ .

Its Fourier transform is:

$$\chi(i\omega_n) = \int_0^\beta d\tau \chi(\tau) e^{i\omega_n \tau} \quad (3)$$

Let  $\chi(\omega) = \chi(i\omega_n \rightarrow \omega + i0^+)$ , define:

$$\chi(i\omega_n) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega - i\omega_n} \quad (4)$$

one has  $\chi''(\omega) = 2\text{Im}\chi(\omega)$ , it related to the structure factor  $S(\omega)$  via:

$$S(\omega) = \frac{\chi''(\omega)}{1 - e^{-\beta\omega}} \quad (5)$$

From eq. (4),  $\chi''(\omega)$  has the following sum rule:

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega} = \chi(T) = \chi(i\omega_n = 0) \quad (6)$$

### III. QUANTUM CRITICAL POINT

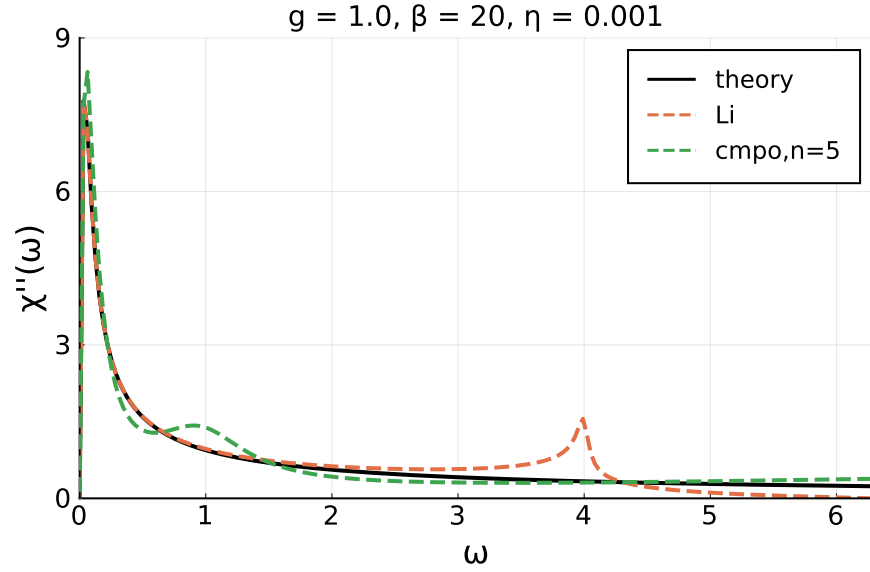


FIG. 1. The solid line is the semi-classical theoretical results. The orange dash line is the numerical results obtain by Zi-Long Li; the green dash line is the Nevalinna analytical continuation results.  $n = 5$  means we use the first 5 Matsubara frequencies data, start from  $i\omega_1$ , which is the best fit.

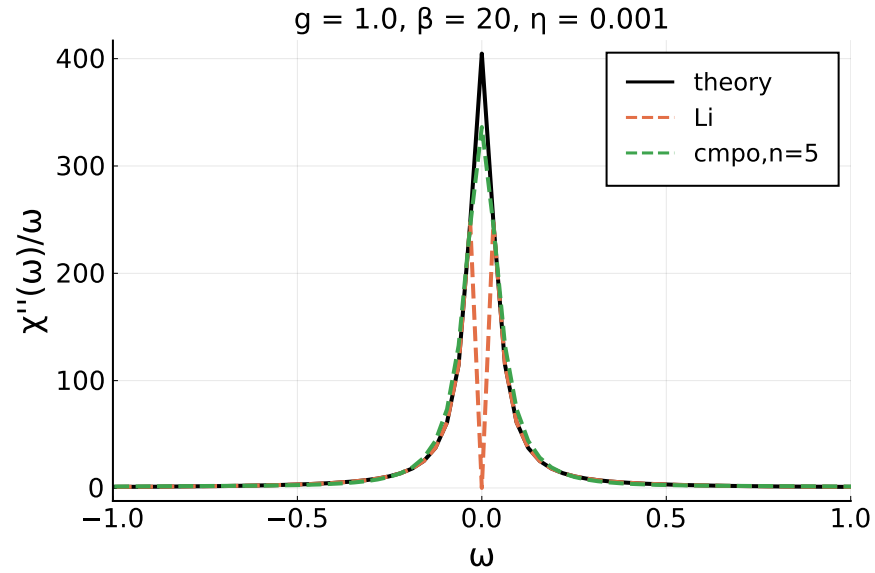


FIG. 2.

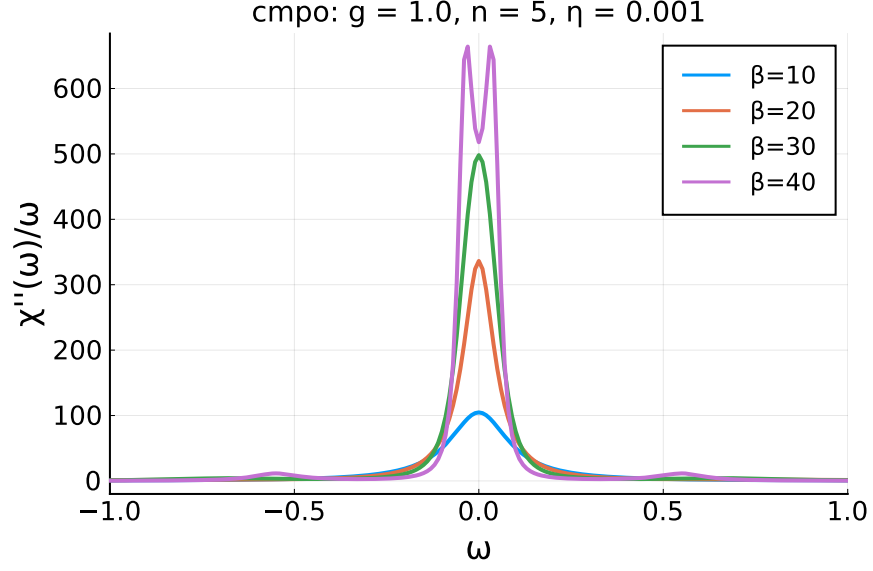


FIG. 3.  $\chi''(\omega)/\omega$  at different temperatures.  $\chi''(\omega)/\omega$  should diverge at  $T = 0$ .

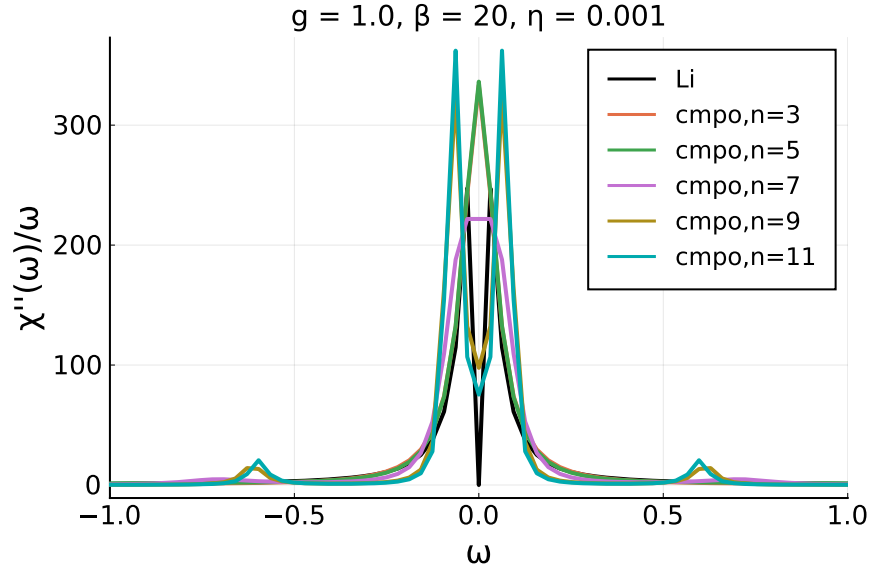


FIG. 4.  $\chi''(\omega)/\omega$  obtained from fitting different number of data points. cmpto bond dimension  $D = 2 \times 8$ .

sum rule check: $g = 1.0, n = 5$				
	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	31.00523	52.35123	71.05310	88.18143
cmpto	30.98179	52.51833	71.00934	87.75747

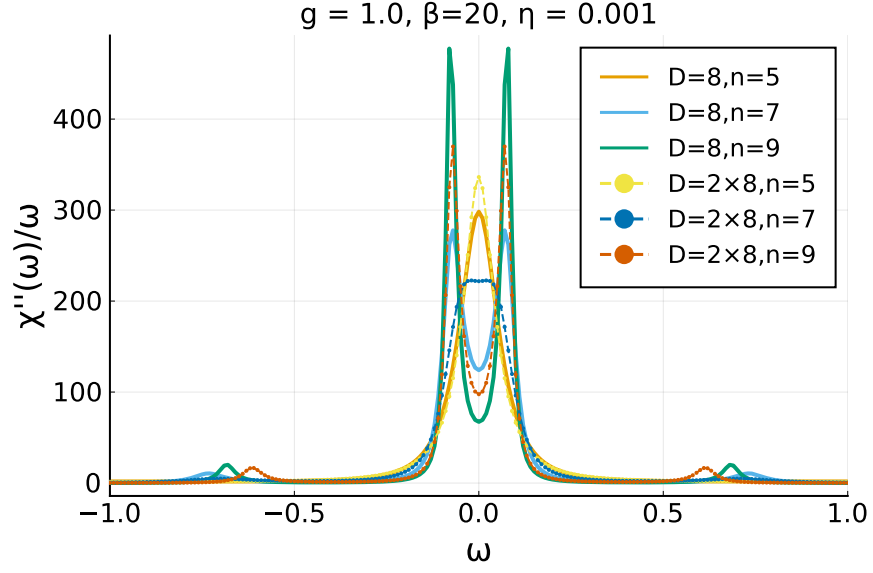


FIG. 5.  $\chi''(\omega)/\omega$  obtained from fitting different number of data points and different bond dimension.

#### IV. PARAMAGNETIC PHASE

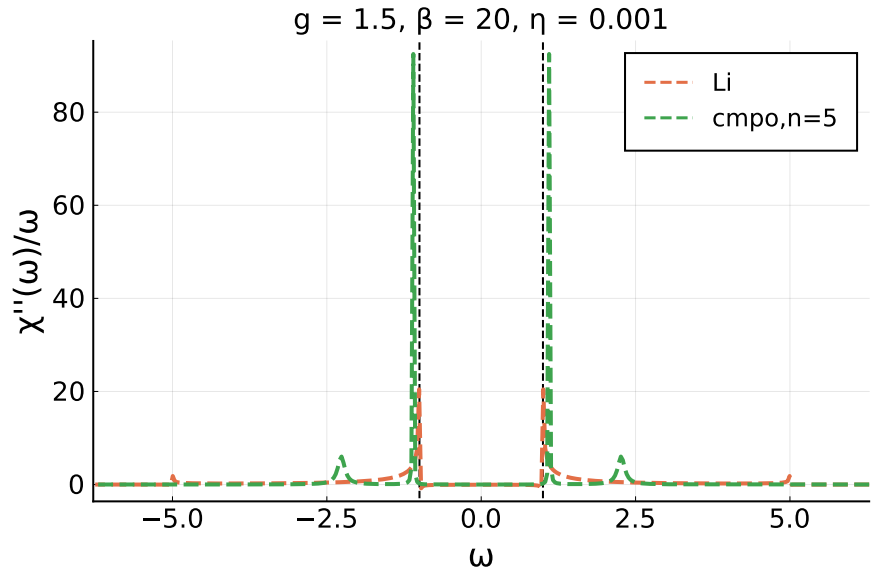


FIG. 6.

sum rule check: $g = 1.5, n = 5$				
	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	6.51806	6.51806	6.51806	6.51806
cmpo	6.41328	6.41328	7.84681	4.75610

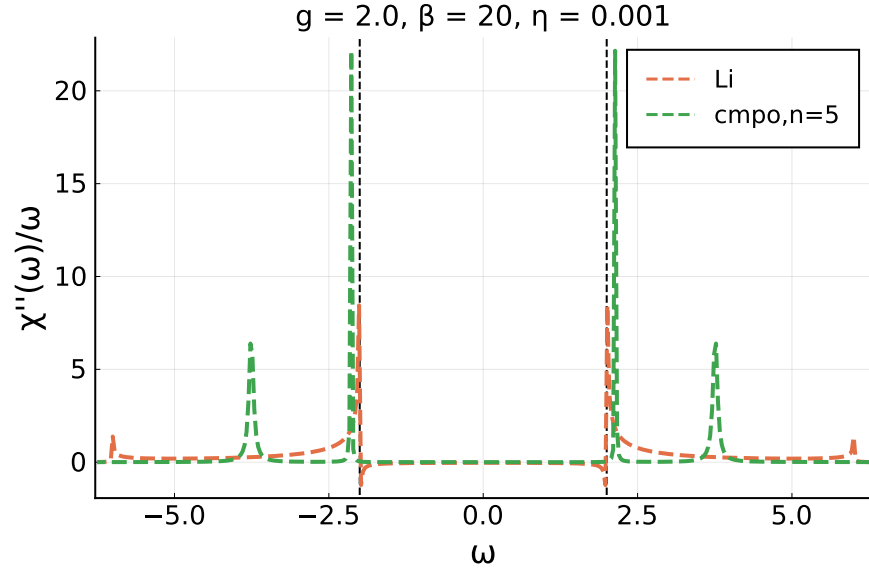


FIG. 7.

sum rule check: $g = 2.0, n = 5$				
	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	3.89954	3.89954	3.89954	3.89954
cmpo	3.81870	3.76048	2.99155	3.29656