Transverse field Ising model spectrum

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Abstract

We calculate $\chi''(\omega)/\omega$ spectrum by interpolating the Masubara frequency correlation function $\chi(i\omega_n)$ data obtained by cmpo method using Nevalinna analytical continuation algorithm.

CONTENTS

I. WHAT'S NEW IN NOVEMBER 4, 2021'S UPDATE

1. update all figures, fix $\omega = 0$ data and add damping factor $\eta = 0.001$

2. add sum rule check of $D = 8$ case and Li's data

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II. THE ISING CHAIN IN A TRANSVERSE FIELD

Hamiltonian:

$$H = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - Jg \sum_i \sigma_i^x \tag{1}$$

where σ_i^α, $\alpha = x, y, z$ are Pauli matrices, $< .. >$ stands for nearest neighbor, $g = \Gamma/J$ and we set $J = 1.0$.

At finite temperature, the local two time correlation $\chi(\tau)$ is defined as:

$$\chi(\tau) = \langle \sigma_i^z(\tau) \sigma_i^z(0) \rangle \tag{2}$$

We ca limit ourselves to $\tau \in [0, \beta]$ by the boundary conditions in τ.

Its Fourier transform is:

$$\chi(i\omega_n) = \int_o^\beta d\tau \chi(\tau) e^{i\omega_n \tau} \tag{3}$$

Define:

$$\chi(i\omega_n) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega - i\omega_n} \tag{4}$$

one has $\chi''(\omega) = 2\mathrm{Im}\chi(i\omega_n \to \omega + i0^+)$, it related to the structure factor $S(\omega)$ via:

$$S(\omega) = \frac{\chi''(\omega)}{1 - e^{-\beta\omega}} \tag{5}$$

From eq. (4), $\chi''(\omega)$ has the following sum rule:

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega} = \chi(T) = \chi(i\omega_n = 0) \tag{6}$$

III. QUANTUM CRITICAL POINT

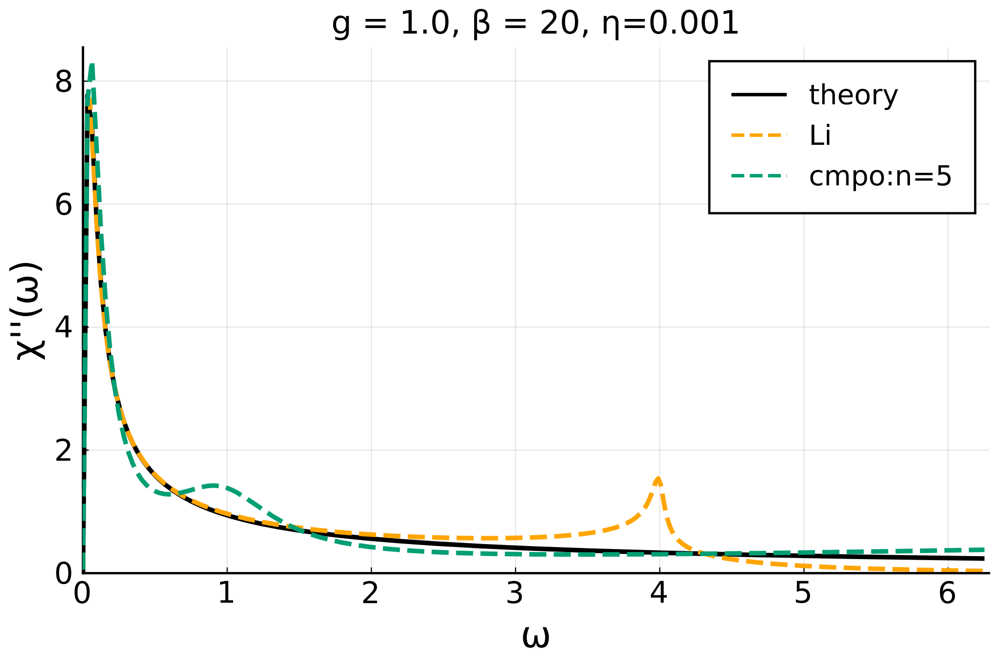

FIG. 1. The solid line is the semi-classical theoretical results. The orange dash line is the numerical results obtained by Zi-Long Li; the green dash line is the Nevalinna analytical continuation results. $n = 5$ means we use the first 5 Mausubara freqencies data, start from $i\omega_1$,, which is the best fit. cmpo bond dimension $D = 2 \times 8$

	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	31.00523	52.35123	71.05310	88.18143
$D = 8$	31.06081	52.54714	70.87341	87.69007
$D = 2 \times 8$	31.06504	52.79551	71.30617	88.02769
Li	30.85385	51.47069	69.25598	85.43683

TABLE I. sum rule check: $g = 1.0, n = 5$, sum $\omega \in (-4\pi, 4\pi)$, step= $\pi/400$.

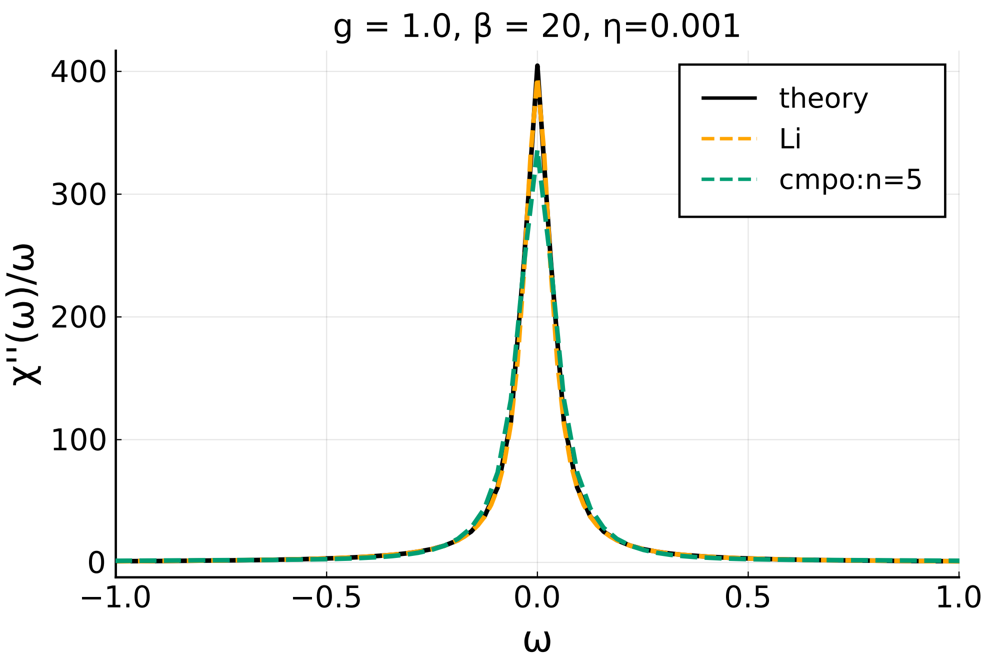

FIG. 2. $\chi''(\omega)/\omega$. cmpo bond dimension $D = 2 \times 8$

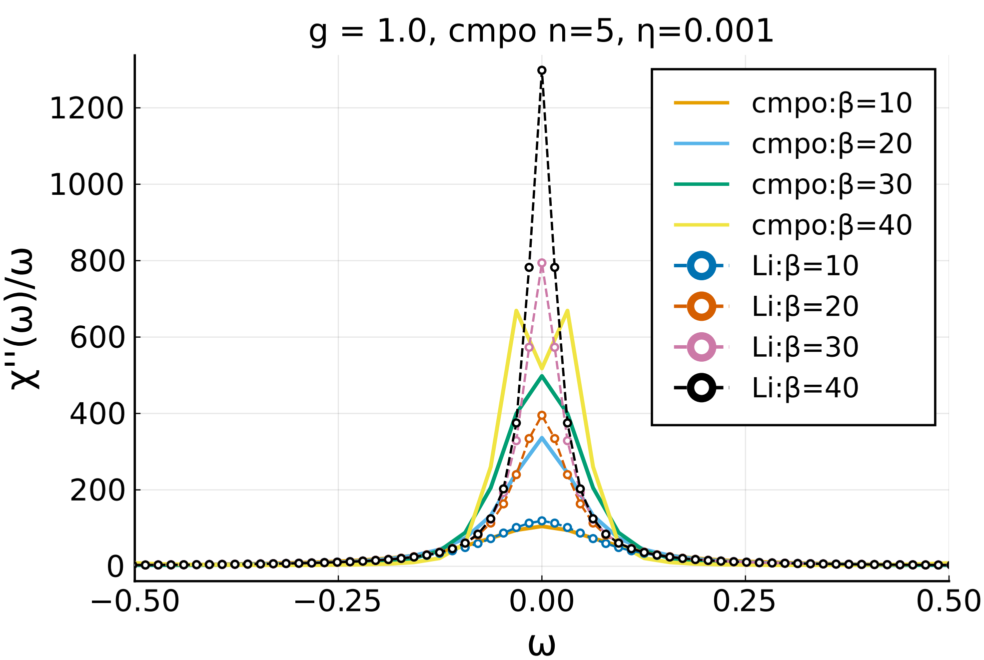

FIG. 3. $\chi''(\omega)/\omega$ at different temperatures. The solid lines are cmpo results and the dash line with circles are numerical results obtained by Zi-Long Li cmpo bond dimension $D = 2 \times 8$. $\chi''(\omega)/\omega$ should diverge at $T = 0$.

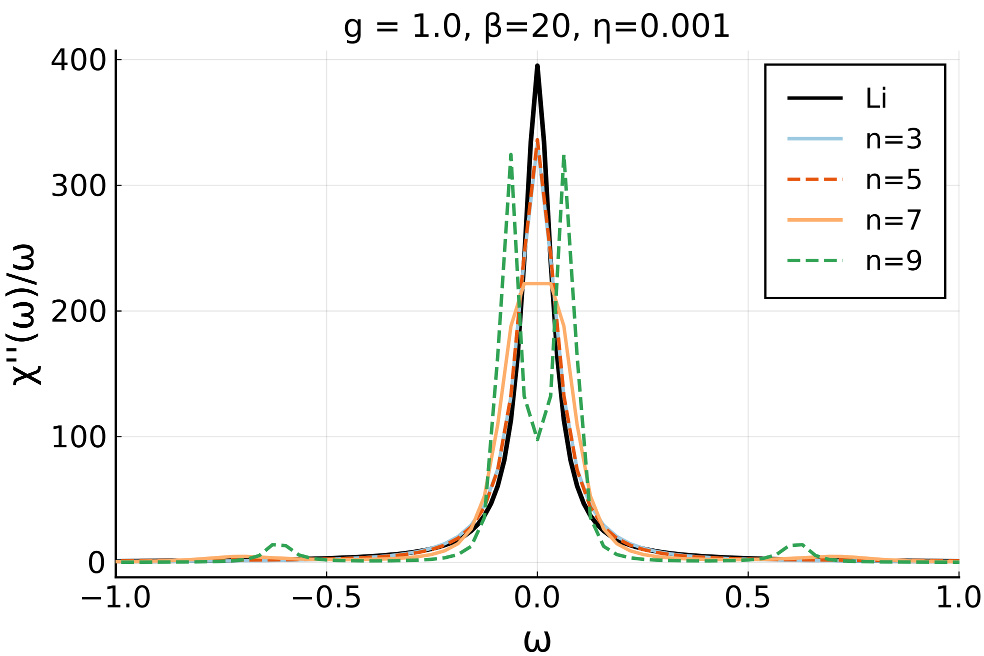

FIG. 4. $\chi''(\omega)/\omega$ obtained from fitting different number of data points.cmpo bond dimension $D = 2 \times 8$.

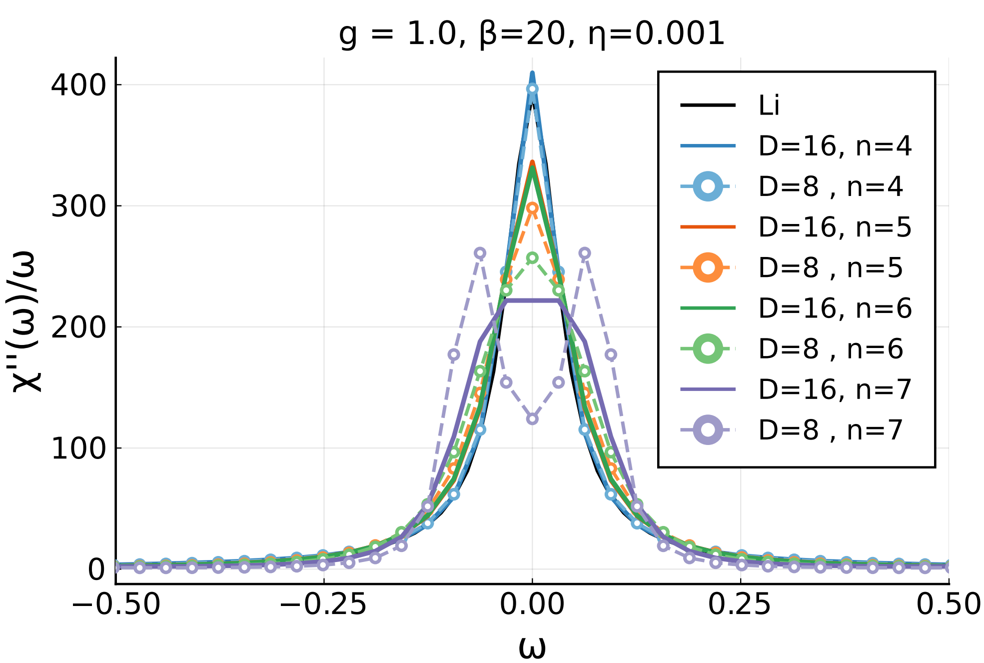

FIG. 5. $\chi''(\omega)/\omega$ obtained from fitting different number of data points and different bond dimension.

IV. PARAMAGNETIC PHASE

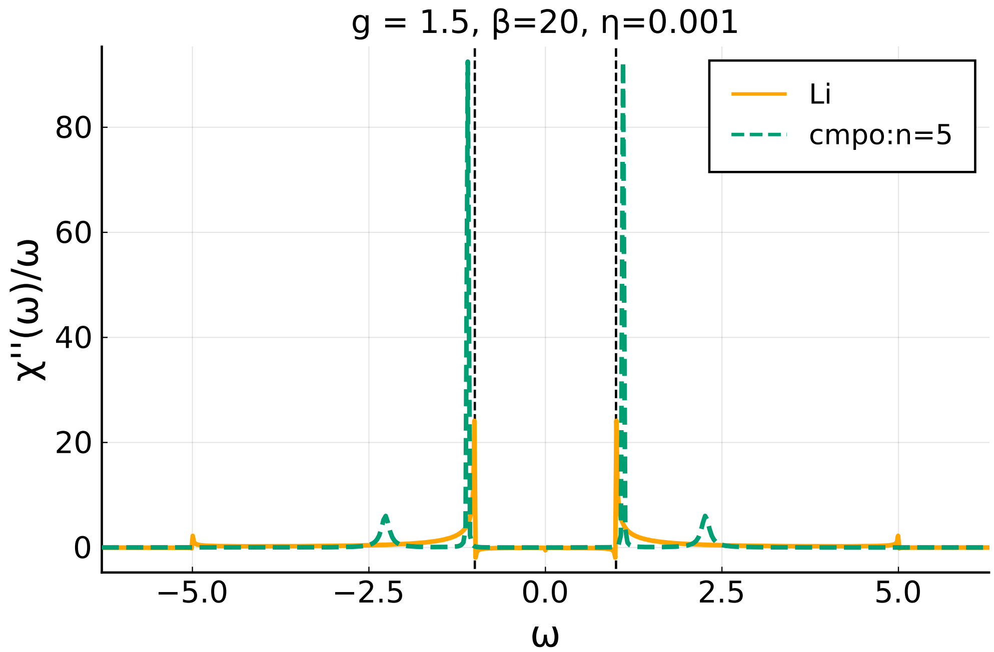

FIG. 6. cmpo bond dimension $D = 2 \times 8$

	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	6.51806	6.51806	6.51806	6.51806
$D = 8$	6.43377	5.79316	5.22786	4.27285
$D = 2 \times 8$	6.44330	6.17900	6.18920	4.83817
Li	6.75017	6.74622	6.74212	6.73801

TABLE II. sum rule check: $g = 1.5, n = 5$, sum $\omega \in (-4\pi, 4\pi)$, step$= \pi/400$.

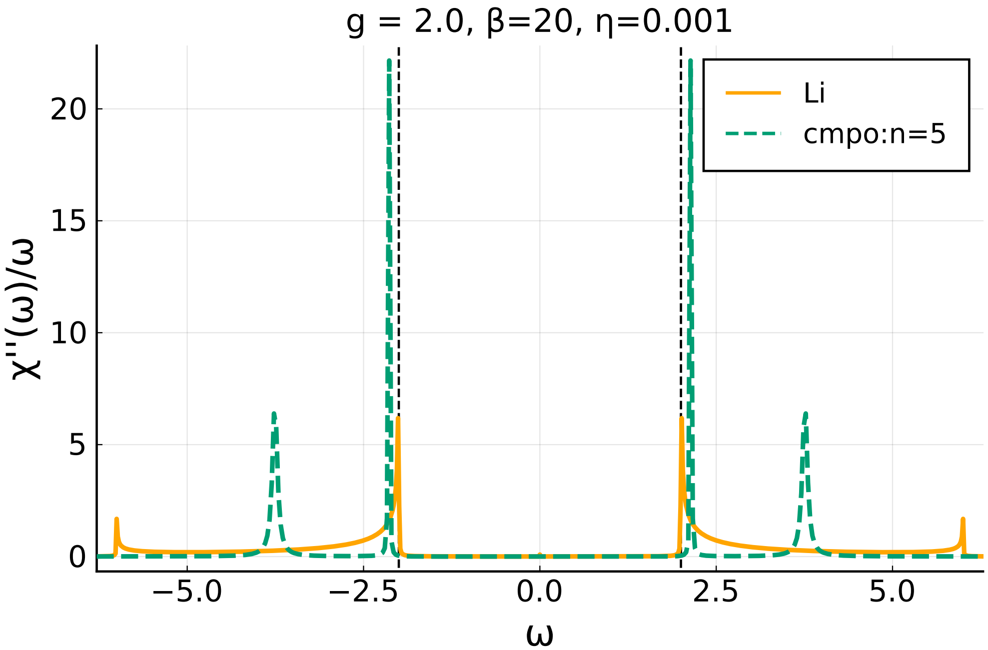

FIG. 7. cmpo bond dimension $D = 2 \times 8$

	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	3.89954	3.89954	3.89954	3.89954
$D = 8$	3.83354	3.55442	4.05804	3.07066
$D = 2 \times 8$	3.82526	3.72291	3.97656	3.02695
Li	4.02838	4.02908	4.02977	4.03047

TABLE III. sum rule check: $g = 2.0, n = 5$, sum $\omega \in (-4\pi, 4\pi)$, step= $\pi/400$.