Transverse field Ising model spectrum

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Abstract

We calculate $\chi''(\omega)/\omega$ spectrum by interpolating the Masubara frequency correlation function $\chi(i\omega_n)$ data obtained by cmpo method using Nevalinna analytical continuation algorithm.

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I. WHAT'S NEW IN OCTOBER 27, 2021'S UPDATE

1. fix typo of eq. (6)
2. add $\chi''(\omega)$ definition eq. (4)
3. add $\chi''(\omega)/\omega$ result with different D, see fig. 5.

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II. THE ISING CHAIN IN A TRANSVERSE FIELD

Hamiltonian:

$$H = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - Jg \sum_i \sigma_i^x \tag{1}$$

where σ_i^α, $\alpha = x, y, z$ are Pauli matrices, $< .. >$ stands for nearest neighbor, $g = \Gamma/J$ and we set $J = 1.0$.

At finite temperature, the local two time correlation $\chi(\tau)$ is defined as:

$$\chi(\tau) = \langle \sigma_i^z(\tau) \sigma_i^z(0) \rangle \tag{2}$$

We ca limit ourselves to $\tau \in [0, \beta]$ by the boundary conditions in τ.

Its Fourier transform is:

$$\chi(i\omega_n) = \int_o^\beta d\tau \chi(\tau) e^{i\omega_n \tau} \tag{3}$$

Let $\chi(\omega) = \chi(i\omega_n \rightarrow \omega + i0^+)$, define:

$$\chi(i\omega_n) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega - i\omega_n} \tag{4}$$

one has $\chi''(\omega) = 2\text{Im}\chi(\omega)$, it related to the structure factor $S(\omega)$ via:

$$S(\omega) = \frac{\chi''(\omega)}{1 - e^{-\beta\omega}} \tag{5}$$

From eq. (4), $\chi''(\omega)$ has the following sum rule:

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\chi''(\omega)}{\omega} = \chi(T) = \chi(i\omega_n = 0) \tag{6}$$

III. QUANTUM CRITICAL POINT

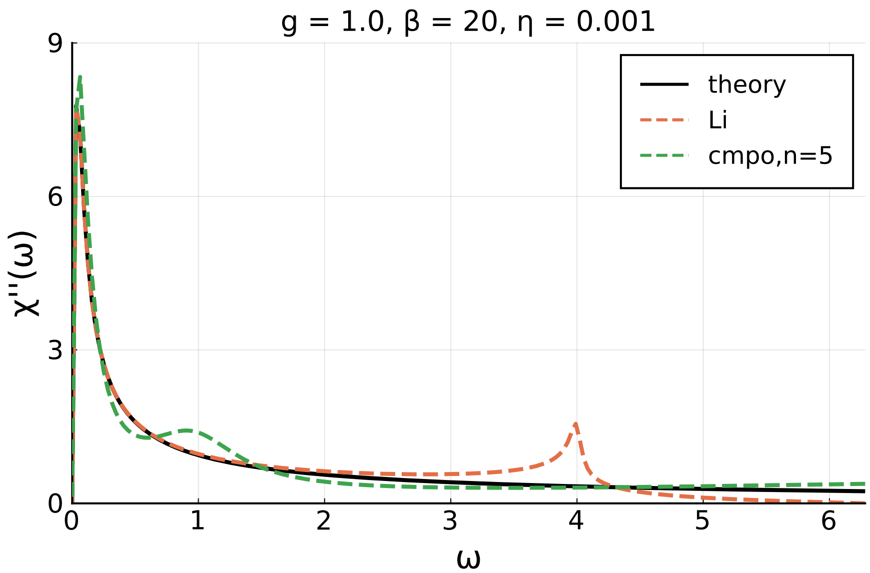

FIG. 1. The solid line is the semi-classical theoretical results. The orange dash line is the numerical results obtain by Zi-Long Li; the green dash line is the Nevalinna analytical continuation results. $n = 5$ means we use the first 5 Mausubara freqencies data, start from $i\omega_1$,, which is the best fit.

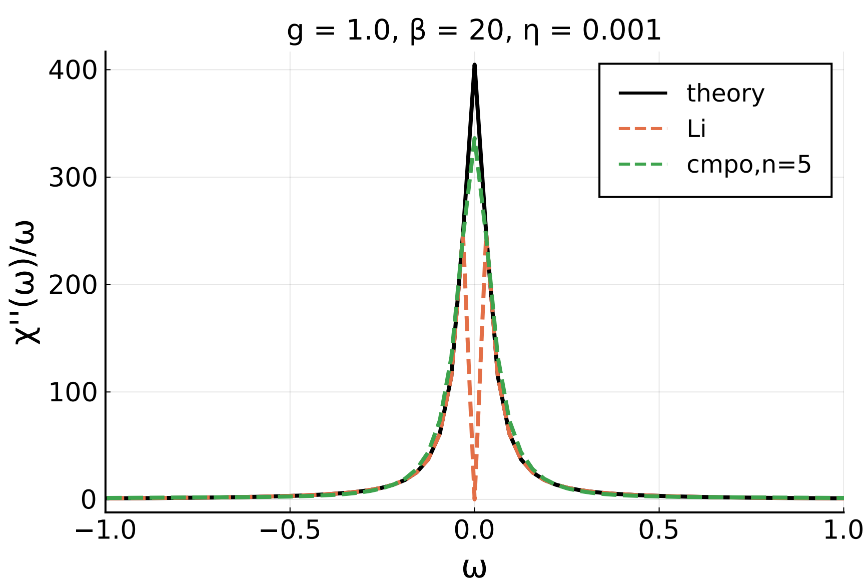

FIG. 2.

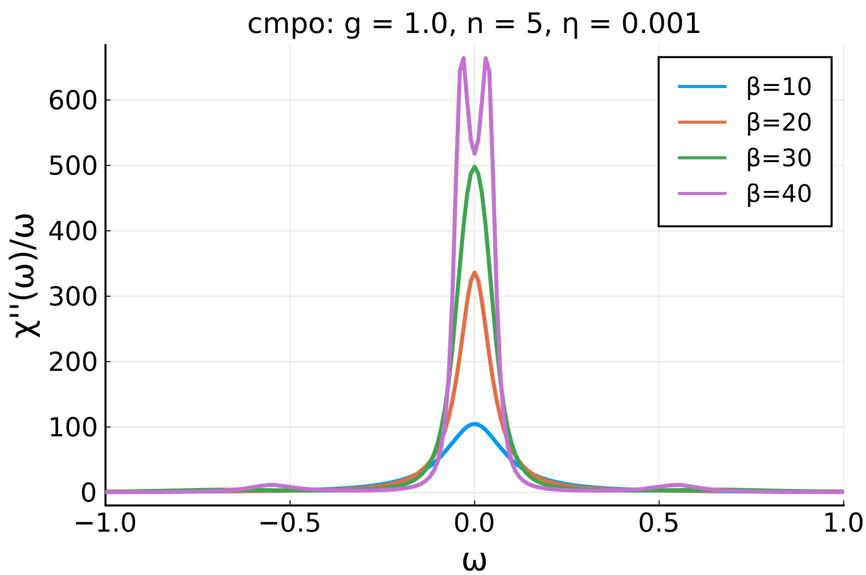

FIG. 3. $\chi''(\omega)/\omega$ at different temperatures. $\chi''(\omega)/\omega$ should diverge at $T=0$.

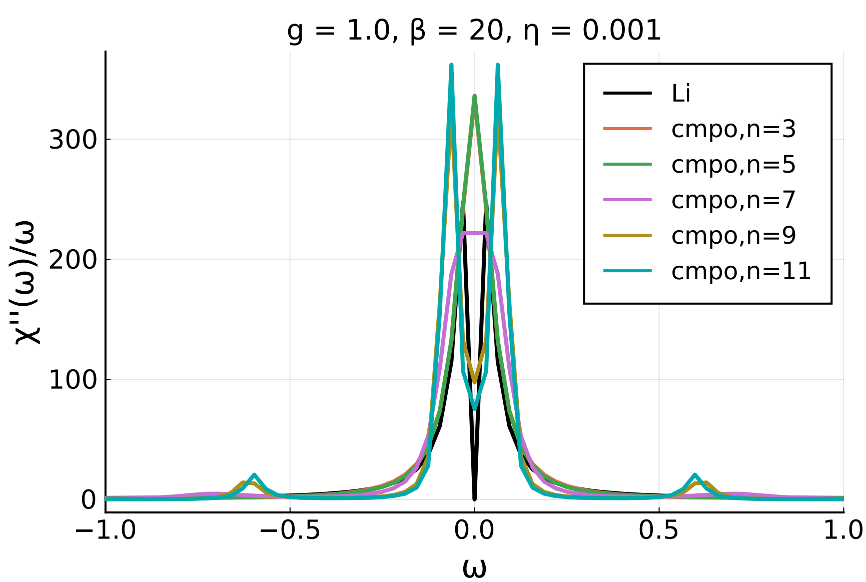

FIG. 4. $\chi''(\omega)/\omega$ obtained from fitting different number of data points.cmpo bond dimension $D = 2\times 8$.

sum rule check: $g=1.0, n=5$				
	$\beta=10$	$\beta=20$	$\beta=30$	$\beta=40$
$2\pi\chi(i\omega_0)$	31.00523	52.35123	71.05310	88.18143
cmpo	30.98179	52.51833	71.00934	87.75747

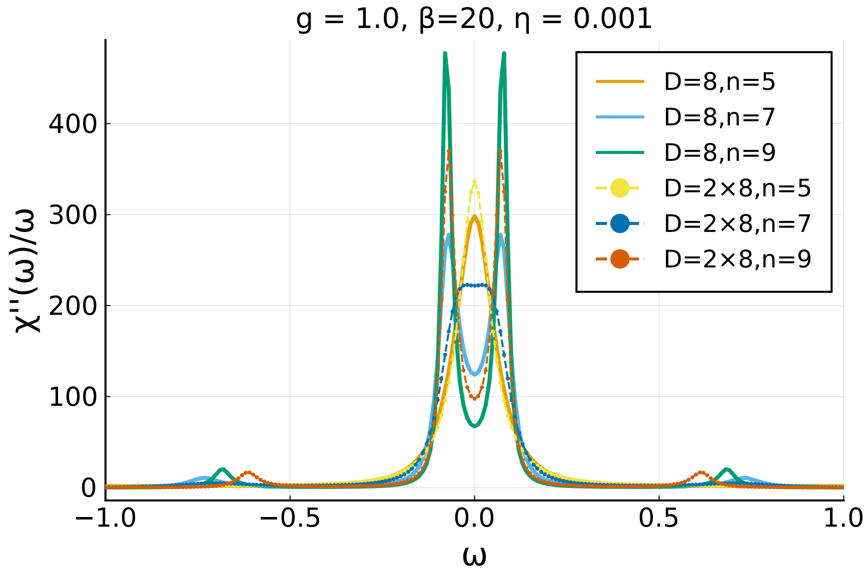

FIG. 5. $\chi''(\omega)/\omega$ obtained from fitting different number of data points and different bond dimension.

IV. PARAMAGNETIC PHASE

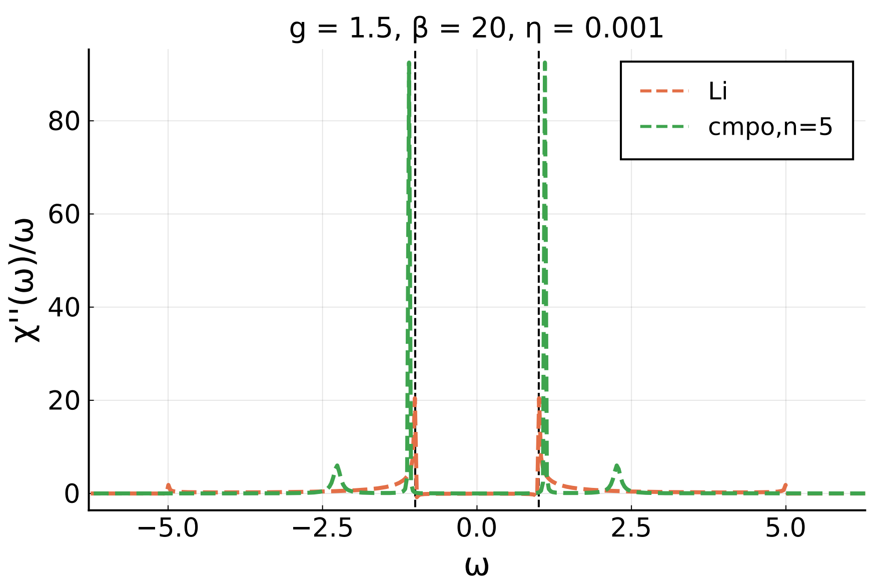

FIG. 6.

sum rule check: $g = 1.5, n = 5$				
	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	6.51806	6.51806	6.51806	6.51806
cmpo	6.41328	6.41328	7.84681	4.75610

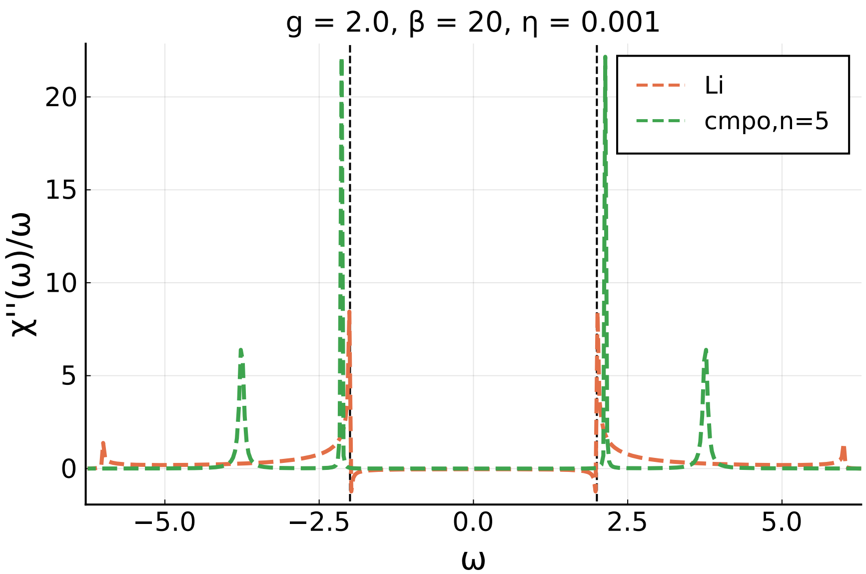

FIG. 7.

sum rule check:$g = 2.0, n = 5$				
	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 40$
$2\pi\chi(i\omega_0)$	3.89954	3.89954	3.89954	3.89954
cmpo	3.81870	3.76048	2.99155	3.29656