# Transverse field Ising model spectrum

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## Abstract

We calculate $\chi''(\omega)/\omega$ spectrum by interpolating the $G(i\omega_n)$ data obtained by cmpo method using Nevalinna analytical continuation algorithm.

**CONTENTS**



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## I. THE ISING CHAIN IN A TRANSVERSE FIELD

Hamiltonian:

$$H = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x = -J \sum_{<i,j>} \sigma_i^z \sigma_j^z - Jg \sum_i \sigma_i^x \tag{1}$$

where $\sigma_i^\alpha$, $\alpha = x, y, z$ are Pauli matrices, $< .. >$ stands for nearest neighbor, $g = \Gamma/J$ and we set $J = 1.0$.

At finite temperature, the local two time correlation $\chi(\tau)$ is defined as:

$$\chi(\tau) = \langle \sigma_i^z(\tau) \sigma_i^z(0) \rangle \tag{2}$$

We ca limit ourselves to $\tau \in [0, \beta]$ by the boundary conditions in $\tau$.

Its Fourier transform is:

$$\chi(i\omega_n) = \int_o^\beta d\tau \chi(\tau) e^{i\omega_n \tau} \tag{3}$$

Let $\chi(\omega) = \chi(i\omega_n \to \omega + i0^+)$, define $\chi''(\omega) = 2\mathrm{Im}\chi(\omega)$, it related to the structure factor $S(\omega)$ via:

$$S(\omega) = \frac{\chi''(\omega)}{1 - e^{-\beta\omega}} \tag{4}$$

then $\chi''(\omega)$ has the following sum rule:

$$\int_{-\infty}^{\infty} d\omega \frac{\chi''(\omega)}{\omega} = \chi(T) = -G(i\omega_n = 0) \tag{5}$$

## II. QUANTUM CRITICAL POINT

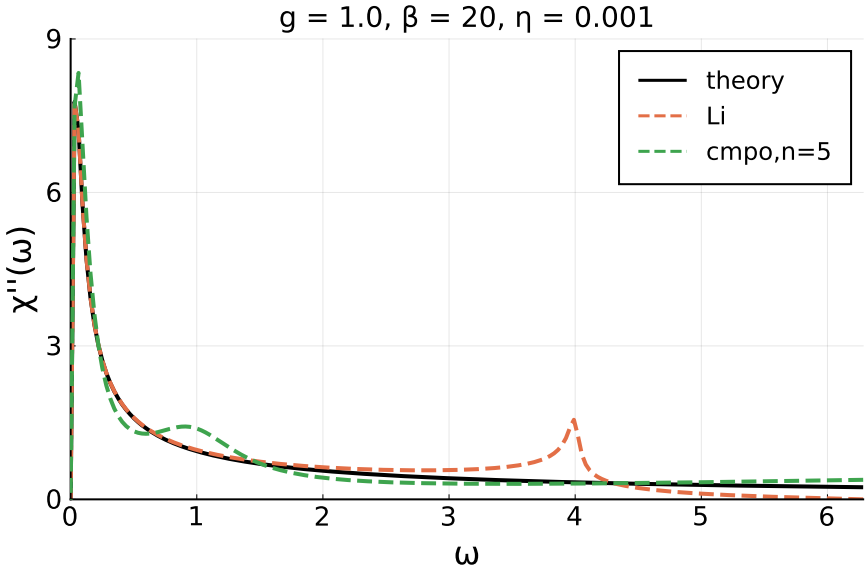


FIG. 1. The solid line is the semi-classical theoretical results. The orange dash line is the numerical results obtain by Zi-Long Li; the green dash line is the Nevalinna analytical continuation results. $n = 5$ means we use the first 5 Mausubara freqencies data, start from $i\omega_1$,, which is the best fit.

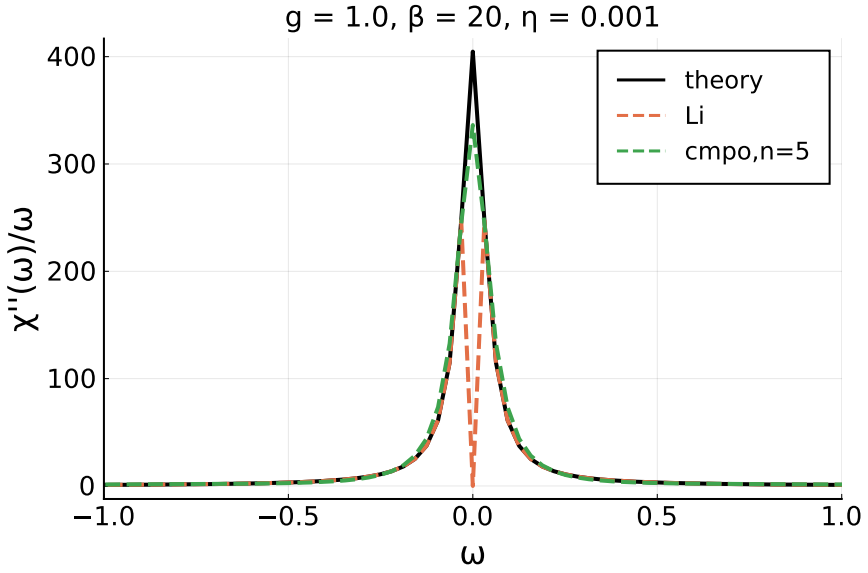


FIG. 2.

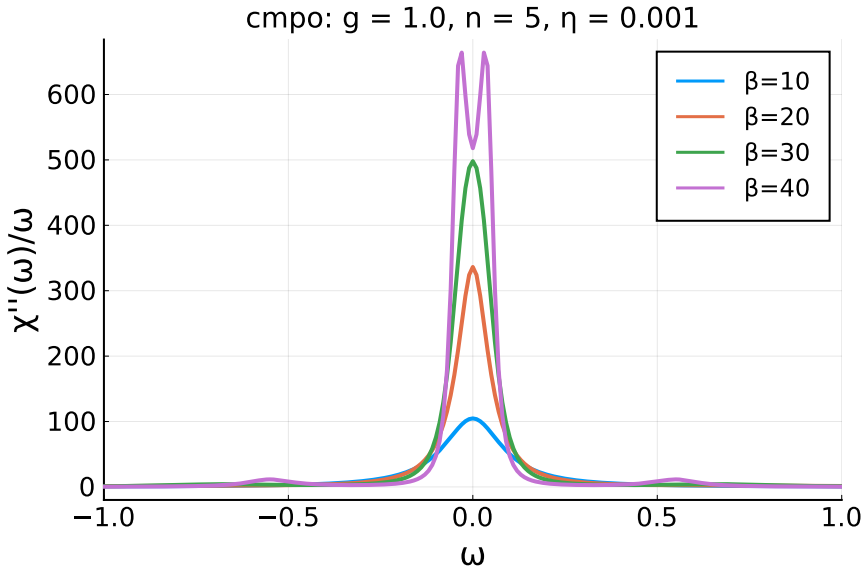


FIG. 3. $\chi''(\omega)/\omega$ at different temperatures. $\chi''(\omega)/\omega$ should diverge at $T = 0$.

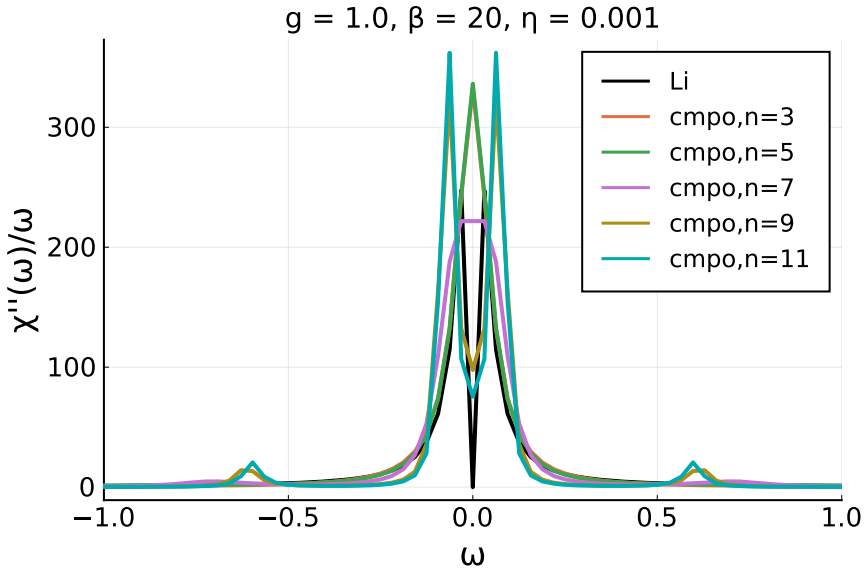


FIG. 4. $\chi''(\omega)/\omega$ obtained from fitting different number of data points.

### A. Sum rule check

| sum rule check: $g = 1.0, n = 5$ | | | | |
|---|---|---|---|---|
| | $\beta = 10$ | $\beta = 20$ | $\beta = 30$ | $\beta = 40$ |
| $-2\pi G(i\omega_0)$ | 31.00523 | 52.35123 | 71.05310 | 88.18143 |
| cmpo | 30.98179 | 52.51833 | 71.00934 | 87.75747 |

## III. PARAMAGNETIC PHASE

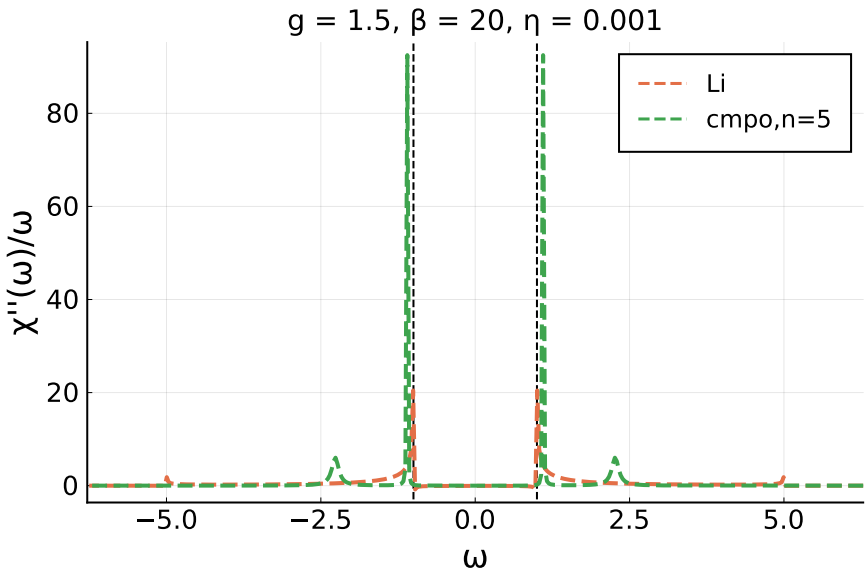


FIG. 5.

| sum rule check: $g = 1.5, n = 5$ | | | | |
|---|---|---|---|---|
| | $\beta = 10$ | $\beta = 20$ | $\beta = 30$ | $\beta = 40$ |
| $-2\pi G(i\omega_0)$ | 6.51806 | 6.51806 | 6.51806 | 6.51806 |
| cmpo | 6.41328 | 6.41328 | 7.84681 | 4.75610 |

| sum rule check:$g = 2.0, n = 5$ | | | | |
|---|---|---|---|---|
| | $\beta = 10$ | $\beta = 20$ | $\beta = 30$ | $\beta = 40$ |
| $-2\pi G(i\omega_0)$ | 3.89954 | 3.89954 | 3.89954 | 3.89954 |
| cmpo | 3.81870 | 3.76048 | 2.99155 | 3.29656 |

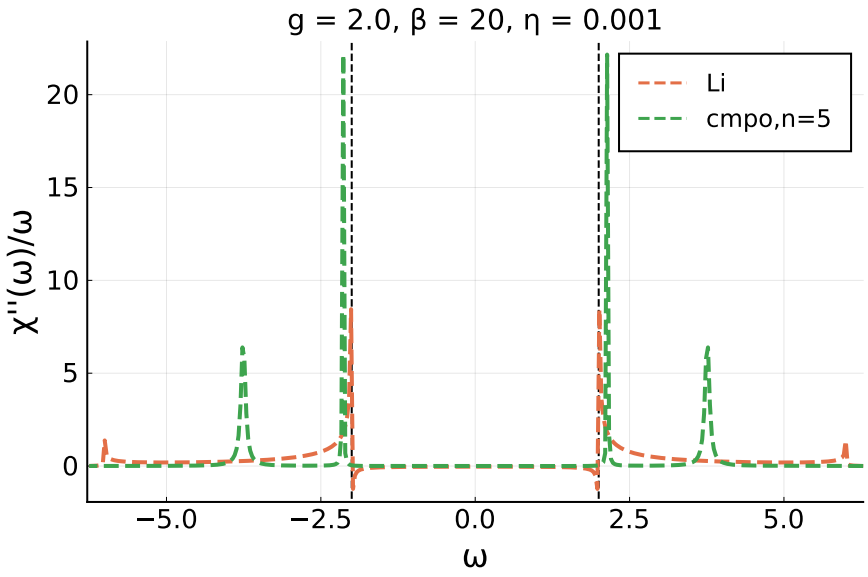


FIG. 6.