

Step 1: Training Phase

Calculate the Model 1 and Model 2 for Linear Regression and Non-Linear Regression with data in training phase

Linear Regression

$$y = a + bx$$

Data	x	y	x ²	xy
1	1	1.5	1	1.5
2	2	2	4	4
3	3	2	9	6
4	4	3.5	16	14
5	5	5	25	25
6	1.1	1.2	1.21	1.32
7	2.2	1.9	4.84	4.18
8	2.5	3.5	6.25	8.75
9	3.8	3.7	14.44	14.06
10	4.8	5.1	23.04	24.48
SUM	29.4	29.4	104.78	103.29

$$\begin{aligned}\text{Slope}(b) &= (N\sum XY - (\sum X)(\sum Y)) / (N\sum X^2 - (\sum X)^2) \\ &= (10 * 103.29 - 29.4 * 29.4) / (10 * 104.78 - 29.4^2) \\ &= 0.918775\end{aligned}$$

$$\begin{aligned}\text{Intercept}(a) &= (\sum Y - b(\sum X)) / N \\ &= (29.4 - 0.918775 * 29.4) / 10 \\ &= 0.238803\end{aligned}$$

$$y = 0.238803 + 0.918775x$$

Non-Linear Regression

$$y = a + bx^2$$

Data	x	y	P=x*x	P ²	Py
1	1	1.5	1	1	1.5
2	2	2	4	16	8
3	3	2	9	81	18
4	4	3.5	16	256	56
5	5	5	25	625	125
6	1.1	1.2	1.21	1.4641	1.452
7	2.2	1.9	4.84	23.4256	9.196

8	2.5	3.5	6.25	39.0625	21.875
9	3.8	3.7	14.44	208.5136	53.428
10	4.8	5.1	23.04	530.8416	117.504
SUM	29.4	29.4	104.78	1782.307	411.955

$$\begin{aligned}\text{Slope}(b) &= (N\sum PY - (\sum P)(\sum Y)) / (N\sum P^2 - (\sum P)^2) \\ &= (10*411.955 - 104.78*29.4) / (10*1782.307 - 104.78^2) \\ &= 0.151809\end{aligned}$$

$$\begin{aligned}\text{Intercept}(a) &= (\sum Y - b(\sum P)) / N \\ &= (29.4 - 0.151809*104.78) / 10 \\ &= 1.349341\end{aligned}$$

$$y = 1.349341 + 0.151809 x^2$$

Step 2: Validation Phase

➔ Calculate the Real y values based on the equation calculated in training phase for x values in validation

Model 1: $y = 0.238803 + 0.918775x$

Model 2: $y = 1.349341 + 0.151809 x^2$

Validation Phase			
Real Data Set 2 25% of the collected data		Model 1	Model 2
x	y	$y=a1+b1*x$	$y=a2+b2*x$
1.5	1.7	1.6169655	1.6909113
2.9	2.7	2.9032505	2.6260547
3.7	2.5	3.6382705	3.4276062
4.7	2.8	4.5570455	4.7028018
5.1	5.5	4.9245555	5.2978931

➔ Calculate MSE for Training data and Validation data

Training Set

	Model 1	Model 2		
y	$y1=a1+b1*x$	$y2=a2+b2*x$	$(y1-y)^2$	$(y2-y)^2$
1.5	1.157578	1.50115	0.117253	1.3225E-06
2	2.076353	1.956577	0.00583	0.001885557
2	2.995128	2.715622	0.99028	0.512114847
3.5	3.913903	3.778285	0.171316	0.077442541
5	4.832678	5.144566	0.027997	0.020899328

1.2	1.2494555	1.53302989	0.002446	0.110908908
1.9	2.260108	2.08409656	0.129678	0.033891543
3.5	2.5357405	2.29814725	0.929796	1.444450033
3.7	3.730148	3.54146296	0.000909	0.025133993
5.1	4.648923	4.84702036	0.20347	0.063998698
		SUM	2.578974	2.290726771

Model 1

$$\begin{aligned} \text{MSE} &= 2.578974/10 \\ &= 0.25789741 \end{aligned}$$

Model 2

$$\begin{aligned} \text{MSE} &= 2.290726771/10 \\ &= 0.22907268 \end{aligned}$$

Validation Set

Model 1		Model 2		
y	y1=a1+b1*x	y2=a2+b2*x	(y1-y)^2	(y2-y)^2
1.7	1.6169655	1.69091125	0.006895	8.26054E-05
2.7	2.9032505	2.62605469	0.041311	0.005467909
2.5	3.6382705	3.42760621	1.29566	0.860453281
2.8	4.5570455	4.70280181	3.087209	3.620654728
5.5	4.9245555	5.29789309	0.331136	0.040847203
		SUM	4.76221	4.527505726

Model 1

$$\begin{aligned} \text{MSE} &= 4.76221/5 \\ &= 0.9524421 \end{aligned}$$

Model 2

$$\begin{aligned} \text{MSE} &= 4.527505726/5 \\ &= 0.90550115 \end{aligned}$$

➔ Compare Model 1 and Model 2 and select the better model

Model 1:

$$0.9524421 / 0.25789741 = 3.693105$$

Model 2:

$$0.90550115 / 0.22907268 = 3.952899$$

Model 1 is slightly better than Model 2

Step 3: Test Phase

Calculate the y value of x in Test Phase based on the Model selected

x	$y = 0.238803 + 0.918775x$
1.4	1.525088
2.5	2.5357405
3.6	3.546393
4.5	4.3732905
5.4	5.200188