1. Using the usual process, $N = \frac{n\langle c \rangle}{4}$  $P = \frac{1}{3} n m \langle c^2 \rangle$

$$\boxed{\langle c^2 \rangle = \frac{3RT}{M} = \frac{3k_BT}{m}} \quad \& \quad \boxed{\langle c \rangle = \sqrt{\frac{8}{\pi}\frac{k_BT}{m}}}$$

$$\frac{N}{P} = \frac{3}{4} \cdot \frac{1}{m} \cdot \frac{\langle c \rangle}{\langle c^2 \rangle} = \frac{3}{4m} \cdot \frac{\sqrt{\frac{8}{\pi}\frac{k_BT}{m}}}{\frac{3k_BT}{m}}$$

$$= \frac{3}{4m} \cdot \frac{\sqrt{\frac{8}{\pi}\frac{k_BT}{m}}}{\frac{3k_BT}{m}} = \frac{\cancel{3}}{4} \cdot \frac{\sqrt{\frac{8}{\pi}}}{\cancel{3}} \cdot \frac{\sqrt{m}}{m} \cdot \frac{1}{\sqrt{k_BT}}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{mk_BT}}$$

$$\boxed{\frac{N}{P} = \frac{1}{\sqrt{2m\pi k_BT}}}$$

2) The ~~number~~ momentum change for ~~a~~ collisions is given by, (per unit time),

$$\left(2mc\cos\theta\right)\left(dn_c\,dV\right)\cdot\left(\frac{c}{\lambda_c}\right)\cdot\left(\frac{\delta S\cos\theta}{4\pi r^2}\right)\left(e^{-r/\lambda_c}\right)$$

momentum change for a single collision

number of molecules from dV with velocity $\in (c, c+dc)$

no. of collisions.

probability of reaching $\delta S$ assuming no collision.

probability of travelling a distance r without collision.

$$\frac{P}{\delta S} = \frac{\cancel{2mc\cos\theta}\ 2m}{4\pi\lambda_c}\int_0^\infty c^2 dn_c \int_0^{\pi/2}\cos^2\theta\sin\theta\, d\theta \int_0^{2\pi} d\phi \int_0^\infty dr\, e^{-r/\lambda_c}$$

$$= \frac{\cancel{2}m}{\cancel{4\pi}\cancel{\lambda_c}}\cdot\frac{1}{3}\cdot\cancel{2\pi}\cdot\cancel{\lambda_c}\int_0^\infty c^2 dn_c$$

$$= \boxed{\frac{m}{3}\int_0^\infty c^2 dn_c}$$

$$\int_0^\infty e^{-r/\lambda_c}dr = \lambda_c e^{-r/\lambda_c}\Big|_\infty^0 = \frac{1-0}{\cancel{\lambda_c}} = \frac{1}{\cancel{\lambda_c}} = \lambda_c$$

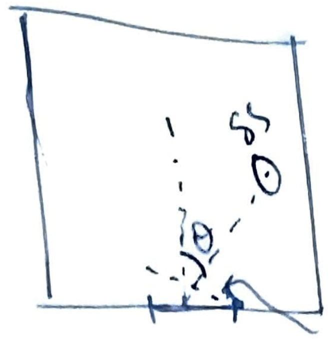
but $\int_0^\infty c^2 dn_c = \langle c^2\rangle \times n$

$$= \frac{3k_BT}{m}\times n.$$

$$\frac{P}{\delta S} = \frac{m}{3} \cdot n \times \frac{3k_BT}{\cancel{m}}$$

$$= \boxed{n \cdot k_BT}$$

3)




$\Delta l$ $\quad$ $\Delta l \cos\theta$.

Let $f(u)du$ be the probability that $c_x$ lies in $(u, u+du)$.

Then, number of molecules in the range $(u, u+du), (v, v+dv)$

$$dN = N f(u) f(v)\, du\, dv$$

(they must be equal because the directions are equivalent)

now, due to isotropy, we can also write,

$$dN = N F(\vec{c})\, d\tau \qquad (d\tau \to \text{volume element in velocity space})$$

$$\Rightarrow F(c) = f(u) f(v)$$

This suggests an exponential nature of $f$.

we write $\quad f(u) = a e^{-u^2/\alpha^2}$

> Normalisation: since, $\displaystyle\int_{-\infty}^{\infty} a e^{-u^2/\alpha^2} = 1$,

$$\Rightarrow a \times \alpha\sqrt{\pi} = 1$$

$$\Rightarrow a = \frac{1}{\alpha\sqrt{\pi}}$$

This gives us,

$$dn = n F(c)\, d\tau$$

$$= n \cdot a^2 e^{-c^2/\alpha^2} \cdot d\tau.$$

$$dn = \frac{n}{\alpha^2 \pi} e^{-c^2/\alpha^2} c\, d\theta\, dc.$$

but velocity does not have θ dependence. hence,
we integrate dθ between 0, 2π.

$$\Rightarrow \cancel{dn = \frac{2\pi}{\alpha^2}} \quad \boxed{dn = \frac{2n}{\alpha^2} \cdot e^{-c^2/\alpha^2} \cdot c\, dc.}$$

The RMS speed is defined as,

$$\langle c^2 \rangle = \frac{\int_0^\infty c^2 dn_c}{n} = \frac{1}{n} \int \frac{2n}{\alpha^2} \cdot e^{-c^2/\alpha^2} c^3 dc$$

$$= \frac{2}{\alpha^2} \int_0^\infty c^3 e^{-c^2/\alpha^2} dc \qquad (\Gamma(4))$$

$$= \frac{2}{\alpha^2} \cdot \frac{\alpha^4}{2} = \boxed{\alpha^2.}$$

Sharon.k
19MS015.

4). $PV_0 = N_0 k_B T$

Standard

$P = 1 atm$

$V = 22.4 L$

$T = 273.15 K$

$k_B = 1.38 \times 10^{-23} JK^{-1}$

$$N_0 = \frac{PV_0}{k_B T} = \frac{101325 Nm^{-2} \cdot 22.4 \times 10^{-3} m^3}{1.38 \times 10^{-23} JK^{-1} \cdot 273.15 K}$$

$$= \boxed{6.0212 \times 10^{23}}$$

6) ~~$PV^\gamma$~~ $V^{\gamma-1} T = \text{const}$.

number of collisions in $\Delta t = \frac{\langle c \rangle}{\lambda} = \sqrt{2} n \pi \sigma^2 \times \sqrt{\frac{8}{\pi} \frac{k_B T}{m}}$

ie, no. of collisions $\propto$ ~~$\Delta t$~~ $\sqrt{T}$

here $(V_1, T_1) \rightarrow (V_2, T_2)$ with $V_2 = 2V_1$.

$$V_2^{\gamma-1} \cdot T_2 = T_1 \cdot V_1^{\gamma-1}$$

$$\Rightarrow (2V_1)^{\gamma-1} \cdot T_2 = T_1 \cdot V_1^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{2V_1}\right)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 \left(\frac{1}{2}\right)^{\gamma-1}$$

but $N \propto \sqrt{T}$
$N^2 \propto T$ $\Rightarrow N_2^2 = N_1^2 \left(\frac{1}{2}\right)^{\gamma-1}$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \left(\left(\frac{1}{2}\right)^{\gamma-1}\right)^{1/2} = 0.870}$$