Tutorial 1, Solution - PH 2202, Spring 21, AK

Q 1:

$$pV_0 = RT$$ [$p, V_0, T \rightarrow$ pressure, volume temperature of 1 mole of a gas; R is unknown]

choose freezing point of water (1 atmos) $\rightarrow T_1$

choose boiling point of water (1 atomos) $\rightarrow T_2$

(both are fixed temperatures, but unknown till now)

so, $p_1 V_{01} = RT_1$ (at T_1)

$p_2 V_{02} = RT_2$ (at T_2)

p_1, p_2, V_{01}, V_{02} can be measured. Gives.

$\frac{T_2}{T_1} \rightarrow$ known number (found out to be 1.3661)

choose, $T_2 - T_1 = 100$ (to match with centigrade scale)

these two give: $T_1 = 273.16$, $T_2 = 373.16$;

Putting the value in any one of the equations: $R = \frac{p_1 V_{01}}{T_1}$ or $\frac{p_2 V_{02}}{T_2}$, turns out to be 8.317 Joule/Kelvin (see Q3).

Q 2:

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3k_B N_A T}{m N_A}} = \sqrt{\frac{3RT}{M}}$$

~~For~~ At, $T = 300K$,

(m = mass of one molecule

M = molar mass)

for N_2:

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3 \times 8.317 \times 300}{28 \times 10^{-3}}} = 517 \text{ m/s}$$

Next $\rightarrow$

Tutorial 1, Sol., PH2202, Spr 21 - AK

for electron: $m = 9.11 \times 10^{-31}$ Kg.

So, $\sqrt{\overline{c^2}} = \sqrt{\dfrac{3K_BT}{m_e}} = \sqrt{\dfrac{3 \times 1.38 \times 10^{-23} \times 300}{9.11 \times 10^{-31}}}$ m/s

$= 116762$ m/s $\cong 116$ Km/s !

or, accelerated by a potential V:

$eV = \frac{1}{2} m_e v^2 \Rightarrow v = \sqrt{\dfrac{2eV}{m_e}}$;

Take, $e = 1.6 \times 10^{-19}$ Coulomb ;

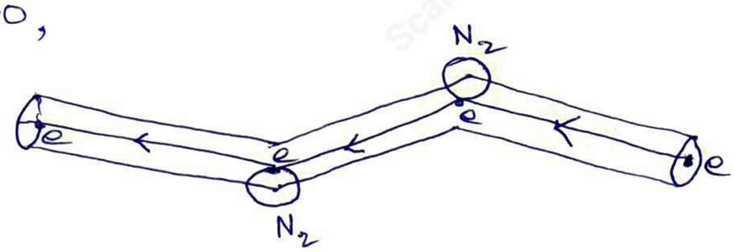
with $V = 1$; $v = \sqrt{\dfrac{2 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}}$ m/s

$= 592674$ m/s $\cong 593$ Km/s !

This is around 1200 times faster than N_2.

Further, electron radius is neglible compared to N_2 molecule. [10^{-13} cm $<<$ 10^{-8} cm]

So,



e is like a point-mass compared to N_2;

$u = v$ (as N_2 is virtually stationary to e)

Then, radius of 'cylinder of influence' $= r$

$=$ radius of N_2;

So, $\lambda = \dfrac{u\,\Delta t}{n \pi r^2 u\,\Delta t} = \dfrac{1}{n \pi r^2}$; [Pg. 22 notes]

Next →

Tutorial 1, Sol., PH 2202, Spr 21, AK

Q3:

$$R = \frac{pV_0}{T} = \frac{76 \times 13.6 \times 981 \times 22.4 \times 10^3}{273} \text{ erg/mole/K}$$

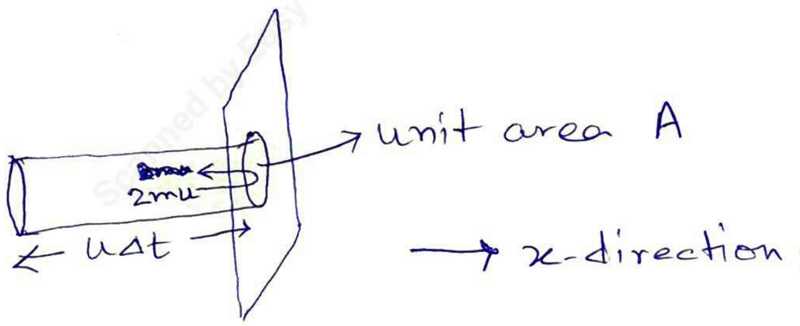
$$= 8.31 \times 10^7 \text{ erg/mole/K} = 8.31 \text{ J/mole/K};$$

[p is 76 cm of column of mercury;
V_0 is volume of 1 mole of gas at STP → 22.4 litre]

[Note:

$$pV_0 = RT \Rightarrow pV_0 = N_A K_B T$$
$$\Rightarrow p = n K_B T \Rightarrow pV = N K_B T = \frac{N}{N_A} RT]$$

Q4:



~~Vele~~ Velocity → $\vec{c}\,(u, v, w)$;

Momentum change 1 molecule $= 2mu$

In Δt time half molecules in cylinder of length $u\Delta t$ will hit area A (other half → opposite direction)

So, total change in momentum in Δt (unit area)

$$\frac{n u \Delta t . 1}{2} \times 2mu \rightarrow m \sum n_i u_i^2 \quad \text{(sets with different velocities)}$$
$$= m n \overline{u^2}\, \Delta t$$

From symmetry, $\overline{u^2} = \overline{v^2} = \overline{w^2}$

Again, $c^2 = u^2 + v^2 + w^2 \Rightarrow \overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$

So, $p = \frac{1}{3} m n \overline{c^2}$;