PS4_Sharon

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November 6, 2019

Factor Analysis

CFA v. EFA

Confirmatory factor analysis is a statistical method used to test where the data fit into researcher's understanding of the model based a hypothetical construct, which is supported by some prior theories; In the case of EFA, on the other hand, there are no apriori hypothesis on what the patterns of the measured variables would be. EFA is used to try and find those latent variables that actually contribute to the variations. The factor loading assumes all items are related to all factors.

Load and prepare data:

```
library(lattice)
library(psych)
```

```
## Warning: package 'psych' was built under R version 3.5.3
```

```
library(GPArotation)
library(ggplot2)
```

```
##
## Attaching package: 'ggplot2'
```

```
## The following objects are masked from 'package:psych':
##
##     %+%, alpha
```

```
# Load the dataset

countries <- read.csv("C:/Users/sharo/Box Sync/UChicago Courses/Unsupervised Machine L
earning/PCA-master/countries.csv", header = TRUE)
summary(countries)
```

```
##           X         idealpoint          polity           polity2
##  Albania   :  1   Min.   :-1.6797   Min.   :-10.000   Min.   :-10.000
##  Algeria   :  1   1st Qu.:-0.7060   1st Qu.: -3.000   1st Qu.: -3.000
##  Angola    :  1   Median :-0.3624   Median :  6.000   Median :  6.000
##  Armenia   :  1   Mean   :-0.0879   Mean   :  3.065   Mean   :  3.065
##  Australia :  1   3rd Qu.: 0.7294   3rd Qu.:  8.500   3rd Qu.:  8.500
##  Azerbaijan:  1   Max.   : 1.7447   Max.   : 10.000   Max.   : 10.000
##  (Other)   :101
##      democ            autoc            unreg          physint
##  Min.   : 0.000   Min.   : 0.000   Min.   :  2.0   Min.   :0.000
##  1st Qu.: 1.000   1st Qu.: 0.000   1st Qu.:  2.0   1st Qu.:2.500
##  Median : 6.000   Median : 0.000   Median :142.0   Median :4.000
##  Mean   : 5.159   Mean   : 2.093   Mean   :147.6   Mean   :4.318
##  3rd Qu.: 8.500   3rd Qu.: 4.000   3rd Qu.:150.0   3rd Qu.:6.000
##  Max.   :10.000   Max.   :10.000   Max.   :419.0   Max.   :8.000
##
##      speech        new_empinx         wecon           wopol
##  Min.   :0.000   Min.   : 0.000   Min.   :0.000   Min.   :0.00
##  1st Qu.:1.000   1st Qu.: 5.000   1st Qu.:1.000   1st Qu.:2.00
##  Median :1.000   Median : 9.000   Median :1.000   Median :2.00
##  Mean   :1.065   Mean   : 8.421   Mean   :1.327   Mean   :1.85
##  3rd Qu.:2.000   3rd Qu.:12.000   3rd Qu.:2.000   3rd Qu.:2.00
##  Max.   :2.000   Max.   :14.000   Max.   :3.000   Max.   :3.00
##
##      wosoc           elecsd        gdp.pc.wdi        gdp.pc.un
##  Min.   :0.000   Min.   :0.000   Min.   :  128.6   Min.   :  103.8
##  1st Qu.:1.000   1st Qu.:0.000   1st Qu.:  546.7   1st Qu.:  568.6
##  Median :1.000   Median :1.000   Median : 1461.0   Median : 1461.6
##  Mean   :1.206   Mean   :1.112   Mean   : 5183.3   Mean   : 5110.2
##  3rd Qu.:2.000   3rd Qu.:2.000   3rd Qu.: 5074.4   3rd Qu.: 4803.9
##  Max.   :3.000   Max.   :2.000   Max.   :37299.6   Max.   :37634.4
##
##     pop.wdi             amnesty        statedept         milper
##  Min.   :5.642e+05   Min.   :1.000   Min.   :1.000   Min.   :   1.0
##  1st Qu.:4.963e+06   1st Qu.:2.000   1st Qu.:2.000   1st Qu.:  13.0
##  Median :1.092e+07   Median :3.000   Median :2.000   Median :  51.0
##  Mean   :4.744e+07   Mean   :2.664   Mean   :2.477   Mean   : 143.6
##  3rd Qu.:2.986e+07   3rd Qu.:3.000   3rd Qu.:3.000   3rd Qu.: 138.5
##  Max.   :1.263e+09   Max.   :5.000   Max.   :5.000   Max.   :2810.0
##
##       cinc             domestic9
##  Min.   :0.0000456   Min.   :   0.0
##  1st Qu.:0.0005530   1st Qu.:   0.0
##  Median :0.0015271   Median :   0.0
##  Mean   :0.0068052   Mean   : 651.8
##  3rd Qu.:0.0053466   3rd Qu.: 406.0
##  Max.   :0.1557135   Max.   :8687.0
##
```

```
scale(countries[,-1])
```

```
##          idealpoint      polity     polity2       democ       autoc
##   [1,] -0.420801504 -0.93085372 -0.93085372 -1.08888601  0.65103113
##   [2,]  1.617151774  0.29689787  0.29689787  0.22022414 -0.37338550
##   [3,] -0.654583552 -1.69819847 -1.69819847 -1.35070804  2.01691996
##   [4,]  0.433563359  0.29689787  0.29689787  0.22022414 -0.37338550
##   [5,]  1.253733987  1.06424262  1.06424262  1.26751226 -0.71485771
##   [6,]  0.183650087 -1.54472952 -1.54472952 -1.35070804  1.67544775
##   [7,] -0.639876876 -0.62391582 -0.62391582 -1.08888601 -0.03191329
##   [8,]  1.645344019  1.06424262  1.06424262  1.26751226 -0.71485771
##   [9,] -0.873852782 -0.93085372 -0.93085372 -1.35070804  0.30955892
##  [10,] -0.888042946  0.45036682  0.45036682  0.22022414 -0.71485771
##  [11,]  1.409047722  0.75730472  0.75730472  0.74386820 -0.71485771
##  [12,] -0.894749292 -1.85166742 -1.85166742 -1.35070804  2.35839217
##  [13,] -0.349617620 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [14,] -0.085992777  0.91077367  0.91077367  1.00569023 -0.71485771
##  [15,]  0.135361678  0.75730472  0.75730472  0.74386820 -0.71485771
##  [16,] -0.903051832 -2.00513637 -2.00513637 -1.35070804  2.69986438
##  [17,]  1.512576022  1.06424262  1.06424262  1.26751226 -0.71485771
##  [18,]  0.063349465  0.91077367  0.91077367  1.00569023 -0.71485771
##  [19,] -0.725547482 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [20,] -0.624645151  0.14342892  0.14342892 -0.04159789 -0.37338550
##  [21,] -0.695156669 -1.08432267 -1.08432267 -1.08888601  0.99250334
##  [22,] -0.816320905 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [23,] -0.289919163  0.60383577  0.60383577  0.48204617 -0.71485771
##  [24,] -1.593353538 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [25,]  1.535289280  1.06424262  1.06424262  1.26751226 -0.71485771
##  [26,] -0.258892301  0.75730472  0.75730472  0.74386820 -0.71485771
##  [27,] -1.122201644 -0.93085372 -0.93085372 -1.08888601  0.65103113
##  [28,] -0.306245001  0.45036682  0.45036682  0.22022414 -0.71485771
##  [29,] -1.205201786 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [30,] -0.547428611 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [31,]  1.438595282  1.06424262  1.06424262  1.26751226 -0.71485771
##  [32,]  1.546535873  1.06424262  1.06424262  1.26751226 -0.71485771
##  [33,]  1.931920473  0.91077367  0.91077367  1.00569023 -0.71485771
##  [34,]  2.203887902  1.06424262  1.06424262  1.26751226 -0.71485771
##  [35,]  1.154526702  0.29689787  0.29689787 -0.04159789 -0.71485771
##  [36,] -0.846102488 -0.16350897 -0.16350897 -0.56524195 -0.37338550
##  [37,] -0.625304770 -0.62391582 -0.62391582 -1.08888601 -0.03191329
##  [38,] -0.736861179 -1.23779162 -1.23779162 -1.35070804  0.99250334
##  [39,]  1.470947274  1.06424262  1.06424262  1.26751226 -0.71485771
##  [40,]  0.156401838  0.75730472  0.75730472  0.74386820 -0.71485771
##  [41,] -0.447911660  0.45036682  0.45036682  0.22022414 -0.71485771
##  [42,] -0.234551822  0.60383577  0.60383577  0.48204617 -0.71485771
##  [43,]  1.383691178  0.75730472  0.75730472  0.74386820 -0.71485771
##  [44,]  1.681888229  1.06424262  1.06424262  1.26751226 -0.71485771
##  [45,] -1.072472512  0.45036682  0.45036682  0.48204617 -0.37338550
##  [46,] -0.755841127  0.91077367  0.91077367  1.00569023 -0.71485771
##  [47,]  1.170512930  1.06424262  1.06424262  1.26751226 -0.71485771
```

```
##  [48,] -1.714261624 -0.01004002 -0.01004002 -0.30341992 -0.37338550
##  [49,]  1.486025308  1.06424262  1.06424262  1.26751226 -0.71485771
##  [50,] -0.282203125  0.91077367  0.91077367  1.00569023 -0.71485771
##  [51,] -1.071250323 -0.77738477 -0.77738477 -0.82706398  0.65103113
##  [52,]  1.028448335  1.06424262  1.06424262  1.26751226 -0.71485771
##  [53,]  0.600443455 -1.08432267 -1.08432267 -1.08888601  0.99250334
##  [54,] -0.502341464 -0.77738477 -0.77738477 -0.82706398  0.65103113
##  [55,]  0.264374177 -0.93085372 -0.93085372 -1.08888601  0.65103113
##  [56,] -0.723820808 -0.16350897 -0.16350897 -0.56524195 -0.37338550
##  [57,]  0.869094149  0.75730472  0.75730472  0.74386820 -0.71485771
##  [58,] -0.590276301 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [59,] -1.325122547 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [60,] -1.914298723 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [61,] -0.901341513  0.29689787  0.29689787  0.22022414 -0.37338550
##  [62,]  1.589677475  0.75730472  0.75730472  0.74386820 -0.71485771
##  [63,] -0.584539360 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [64,]  1.234816213  0.60383577  0.60383577  0.48204617 -0.71485771
##  [65,] -0.477902614  0.75730472  0.75730472  0.74386820 -0.71485771
##  [66,]  1.308577601  0.45036682  0.45036682  0.22022414 -0.71485771
##  [67,] -0.781918983  0.45036682  0.45036682  0.22022414 -0.71485771
##  [68,] -0.719873076  0.29689787  0.29689787 -0.04159789 -0.71485771
##  [69,] -0.930500156 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [70,] -0.448585229  0.45036682  0.45036682  0.22022414 -0.71485771
##  [71,] -0.735801219 -0.01004002 -0.01004002 -0.30341992 -0.37338550
##  [72,] -0.749738720  0.45036682  0.45036682  0.22022414 -0.71485771
##  [73,] -0.869166300  0.29689787  0.29689787  0.22022414 -0.37338550
##  [74,] -0.781832276  0.14342892  0.14342892 -0.30341992 -0.71485771
##  [75,]  0.001381293  0.75730472  0.75730472  0.74386820 -0.71485771
##  [76,] -0.724823765  0.45036682  0.45036682  0.48204617 -0.37338550
##  [77,]  1.075517345  1.06424262  1.06424262  1.26751226 -0.71485771
##  [78,] -0.862722840 -1.39126057 -1.39126057 -1.35070804  1.33397554
##  [79,] -0.603902452  0.75730472  0.75730472  0.74386820 -0.71485771
##  [80,]  1.550290349  0.91077367  0.91077367  1.00569023 -0.71485771
##  [81,]  1.410495637  1.06424262  1.06424262  1.26751226 -0.71485771
##  [82,]  0.234899734  0.60383577  0.60383577  0.48204617 -0.71485771
##  [83,]  1.488222435  0.75730472  0.75730472  0.74386820 -0.71485771
##  [84,]  0.393882682  0.45036682  0.45036682  0.22022414 -0.71485771
##  [85,] -0.403569636 -1.08432267 -1.08432267 -1.35070804  0.65103113
##  [86,] -0.929186570 -2.00513637 -2.00513637 -1.35070804  2.69986438
##  [87,] -0.377798921  0.75730472  0.75730472  0.74386820 -0.71485771
##  [88,] -0.211134810  0.60383577  0.60383577  0.48204617 -0.71485771
##  [89,]  1.484970640  0.91077367  0.91077367  1.00569023 -0.71485771
##  [90,]  1.471346533  1.06424262  1.06424262  1.26751226 -0.71485771
##  [91,]  1.359946100  1.06424262  1.06424262  1.26751226 -0.71485771
##  [92,] -1.894131341 -1.54472952 -1.54472952 -1.35070804  1.67544775
##  [93,] -1.095548711 -0.77738477 -0.77738477 -1.08888601  0.30955892
##  [94,] -0.733247766 -0.77738477 -0.77738477 -1.08888601  0.30955892
##  [95,] -0.330127295  0.91077367  0.91077367  1.00569023 -0.71485771
##  [96,]  0.263258777 -0.62391582 -0.62391582 -0.82706398  0.30955892
```

```
##  [97,]  0.191674360 -1.85166742 -1.85166742 -1.35070804  2.35839217
##  [98,] -0.213943331  1.06424262  1.06424262  1.26751226 -0.71485771
##  [99,] -1.033199753 -0.93085372 -0.93085372 -1.08888601  0.65103113
## [100,]  0.937253647  0.60383577  0.60383577  0.74386820 -0.37338550
## [101,]  0.619049398  0.45036682  0.45036682  0.22022414 -0.71485771
## [102,]  0.156133781  1.06424262  1.06424262  1.26751226 -0.71485771
## [103,]  0.802615266 -1.85166742 -1.85166742 -1.35070804  2.35839217
## [104,] -0.505213362  0.60383577  0.60383577  0.48204617 -0.71485771
## [105,]  0.158595308  0.91077367  0.91077367  1.00569023 -0.71485771
## [106,] -0.471600938 -0.31697792 -0.31697792 -0.56524195 -0.03191329
## [107,] -0.799512228 -0.93085372 -0.93085372 -1.08888601  0.65103113
##              unreg    physint      speech new_empinx      wecon      wopol
##   [1,] -1.05539260 -1.5381108 -1.48527973 -0.5930132 -0.5814092  0.2738383
##   [2,]  0.01741035 -0.6109116 -0.09120139  0.1419568 -0.5814092  0.2738383
##   [3,] -0.04057900  1.2434868 -1.48527973 -2.0629533  1.1960417 -3.3887492
##   [4,] -0.04057900 -0.1473120 -0.09120139 -0.5930132 -0.5814092 -1.5574555
##   [5,] -1.00465192  1.2434868  1.30287695  1.1219169  1.1960417  0.2738383
##   [6,] -0.04057900 -0.1473120 -1.48527973 -1.0829933 -0.5814092  0.2738383
##   [7,] -1.05539260 -1.5381108 -1.48527973 -0.8380032 -0.5814092  0.2738383
##   [8,]  0.01741035  1.7070863 -0.09120139  0.8769269  1.1960417  0.2738383
##   [9,] -1.05539260 -0.1473120 -0.09120139  0.3869469 -2.3588600  0.2738383
##  [10,] -0.04057900 -1.0745112 -1.48527973 -0.3480232 -0.5814092  0.2738383
##  [11,]  0.01741035 -0.1473120 -0.09120139 -0.3480232 -0.5814092  0.2738383
##  [12,] -0.04057900 -0.6109116 -1.48527973 -1.0829933 -0.5814092 -3.3887492
##  [13,]  0.01741035 -0.1473120 -1.48527973 -1.0829933 -0.5814092  0.2738383
##  [14,]  1.96730221 -0.1473120 -0.09120139  0.6319369 -0.5814092  0.2738383
##  [15,]  1.96730221 -1.0745112  1.30287695  1.1219169 -0.5814092  0.2738383
##  [16,] -0.04057900 -0.1473120 -1.48527973 -1.0829933  1.1960417 -1.5574555
##  [17,] -0.91766790  1.2434868 -0.09120139  1.1219169  1.1960417  2.1051321
##  [18,]  1.96730221 -0.1473120  1.30287695  1.1219169 -0.5814092  0.2738383
##  [19,] -0.04057900 -0.6109116 -1.48527973 -1.8179633 -0.5814092  0.2738383
##  [20,] -1.05539260 -1.5381108 -0.09120139 -0.5930132  1.1960417  0.2738383
##  [21,] -1.05539260 -1.5381108 -0.09120139 -0.8380032 -0.5814092  0.2738383
##  [22,] -1.05539260 -0.6109116 -0.09120139  0.3869469 -0.5814092  0.2738383
##  [23,]  1.96730221 -2.0017104  1.30287695  0.8769269 -0.5814092  0.2738383
##  [24,]  1.96730221 -0.1473120 -1.48527973 -1.5729733  1.1960417  0.2738383
##  [25,]  0.01741035  1.2434868 -0.09120139  0.6319369  1.1960417  0.2738383
##  [26,]  1.96730221 -0.1473120  1.30287695  0.6319369 -0.5814092  0.2738383
##  [27,] -1.05539260 -0.1473120 -0.09120139 -0.3480232 -0.5814092  0.2738383
##  [28,]  1.96730221  0.7798872 -0.09120139 -0.1030332 -0.5814092  0.2738383
##  [29,] -1.05539260 -0.1473120 -0.09120139 -1.0829933 -0.5814092  0.2738383
##  [30,] -1.05539260  0.7798872 -0.09120139 -0.3480232 -0.5814092  0.2738383
##  [31,]  0.01741035 -0.1473120  1.30287695  1.3669069  1.1960417  0.2738383
##  [32,]  0.01741035  1.7070863  1.30287695  1.3669069  2.9734925  2.1051321
##  [33,]  0.01741035  0.7798872  1.30287695  1.1219169  1.1960417  0.2738383
##  [34,]  0.01741035  0.7798872  1.30287695  0.8769269  1.1960417  0.2738383
##  [35,] -0.04057900  0.3162876  1.30287695  0.1419568 -0.5814092  0.2738383
##  [36,] -1.05539260  0.3162876 -0.09120139  0.1419568 -0.5814092  0.2738383
##  [37,] -1.05539260 -1.0745112 -0.09120139 -1.0829933 -0.5814092  0.2738383
```

```
##  [38,] -1.05539260  0.7798872 -0.09120139 -0.5930132 -0.5814092 -1.5574555
##  [39,]  0.01741035  1.2434868  1.30287695  0.6319369  1.1960417  0.2738383
##  [40,]  1.96730221 -1.0745112  1.30287695  1.1219169 -0.5814092  0.2738383
##  [41,]  1.96730221  0.7798872  1.30287695  1.1219169 -0.5814092  0.2738383
##  [42,]  1.96730221  0.7798872  1.30287695  1.3669069 -0.5814092  0.2738383
##  [43,]  0.01741035  0.7798872 -0.09120139 -0.1030332 -0.5814092  0.2738383
##  [44,]  0.01741035  1.7070863  1.30287695  0.8769269  1.1960417  0.2738383
##  [45,] -0.04057900 -1.5381108 -0.09120139 -0.5930132 -0.5814092  0.2738383
##  [46,] -0.04057900 -1.5381108 -0.09120139 -0.5930132 -0.5814092  0.2738383
##  [47,]  0.01741035  1.7070863  1.30287695  1.3669069  1.1960417  0.2738383
##  [48,] -0.04057900 -1.5381108 -1.48527973 -1.8179633 -2.3588600 -1.5574555
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##  [28,]  0.3366267  0.4787219 -0.26176350 -0.2946907590  3.06110935
##  [29,]  0.3366267  0.4787219  0.93139769  0.1323153526 -0.46560518
##  [30,]  1.3371562 -0.4359789  0.17266955 -0.2832842019 -0.46560518
##  [31,] -0.6639027 -1.3506796  0.06865037  0.2332100628  2.03459755
##  [32,] -1.6644321 -1.3506796 -0.34130757 -0.2510465805 -0.46560518
##  [33,] -0.6639027 -1.3506796  0.46025199  0.7777321869 -0.37631222
##  [34,] -0.6639027 -1.3506796  0.20938220  0.8516877695 -0.19772631
##  [35,]  0.3366267 -0.4359789 -0.35660451 -0.3483959341 -0.46560518
##  [36,] -0.6639027 -0.4359789 -0.41779227 -0.3244215062 -0.46560518
##  [37,]  0.3366267  1.3934227 -0.40861410 -0.3578483575  1.05237505
##  [38,]  0.3366267  0.4787219 -0.43614859 -0.3774064434 -0.46560518
##  [39,] -0.6639027 -1.3506796  0.04723466 -0.1647632064 -0.46560518
##  [40,]  0.3366267  0.4787219 -0.34436696 -0.3346388334  0.07015255
##  [41,]  0.3366267 -0.4359789 -0.43308920 -0.3768369531 -0.46560518
##  [42,]  0.3366267  0.4787219 -0.41473288 -0.3564413813 -0.46560518
##  [43,] -1.6644321 -1.3506796 -0.25258533 -0.3292565911 -0.46560518
##  [44,] -0.6639027 -0.4359789 -0.30459492 -0.2768299778 -0.46560518
##  [45,]  1.3371562  1.3934227  0.46943016  0.3923044555  1.98959390
##  [46,]  1.3371562  1.3934227  3.54717414  3.3565074067 -0.37631222
##  [47,] -1.6644321 -1.3506796 -0.40249533 -0.3481335218 -0.46560518
##  [48,]  0.3366267  1.3934227  1.13025789  0.3130057133 -0.24272996
##  [49,] -0.6639027 -1.3506796  0.32869832  0.6729794586 -0.46560518
##  [50,]  0.3366267  0.4787219 -0.43002982 -0.3692046655 -0.46560518
##  [51,]  0.3366267 -0.4359789 -0.12103166 -0.3032386978 -0.46560518
##  [52,] -1.6644321 -1.3506796  0.28586690  2.6632477982 -0.46560518
##  [53,]  0.3366267  0.4787219 -0.24340717 -0.2002000238  1.76671869
##  [54,]  0.3366267  0.4787219 -0.37190145 -0.3005308269 -0.46560518
##  [55,] -0.6639027  0.4787219 -0.41167349 -0.3593558321 -0.46560518
##  [56,]  0.3366267 -0.4359789 -0.01089371 -0.2868630581 -0.46560518
##  [57,] -0.6639027 -0.4359789  1.65035379  1.0385029433 -0.46560518
##  [58,] -0.6639027 -1.3506796 -0.39331717 -0.2906317443 -0.46560518
##  [59,]  0.3366267  0.4787219 -0.35048574 -0.3552633179 -0.46560518
##  [60,]  0.3366267  0.4787219 -0.20669452 -0.2760539076 -0.46560518
##  [61,]  1.3371562  2.3081234 -0.08737840 -0.2802915859  1.76671869
##  [62,] -1.6644321 -1.3506796 -0.42391104 -0.3617287086 -0.28701927
##  [63,] -0.6639027  0.4787219  0.16655078 -0.1673761622 -0.28701927
##  [64,] -0.6639027 -0.4359789 -0.40861410 -0.3526447789 -0.46560518
##  [65,]  0.3366267  0.4787219  0.15125384  0.3631320322  2.43605867
##  [66,] -0.6639027 -0.4359789 -0.39025778 -0.3628788558 -0.46560518
```

```
##  [67,] -1.6644321 -0.4359789 -0.41779227 -0.3530132727 -0.46560518
##  [68,]  0.3366267  0.4787219 -0.42085165 -0.3262751415 -0.06414405
##  [69,]  0.3366267 -0.4359789 -0.39025778 -0.3657598071 -0.46560518
##  [70,] -0.6639027 -0.4359789 -0.42391104 -0.3544816645 -0.46560518
##  [71,] -0.6639027 -0.4359789 -0.14550676 -0.1795029571 -0.46560518
##  [72,]  0.3366267 -0.4359789 -0.41167349 -0.3711252997 -0.46560518
##  [73,] -0.6639027 -0.4359789 -0.42391104 -0.3551684029 -0.46560518
##  [74,]  0.3366267  0.4787219 -0.20669452 -0.0005545669  0.69520323
##  [75,] -0.6639027 -0.4359789 -0.39025778 -0.3578706905 -0.46560518
##  [76,]  1.3371562  1.3934227 -0.28623860 -0.3125347906 -0.46560518
##  [77,] -1.6644321 -1.3506796 -0.41167349 -0.3346667496 -0.46560518
##  [78,]  0.3366267  1.3934227  1.43313727  0.3493470153 -0.24272996
##  [79,]  1.3371562  1.3934227 -0.11491289 -0.0494804895  2.03459755
##  [80,] -0.6639027 -1.3506796  0.22467914  0.0777391922 -0.46560518
##  [81,] -1.6644321 -1.3506796 -0.30153554 -0.2698006800 -0.46560518
##  [82,]  0.3366267  0.4787219 -0.37802023 -0.3507129782  1.32025391
##  [83,]  0.3366267 -0.4359789  0.19408527 -0.1346918817 -0.46560518
##  [84,]  2.3376856  1.3934227  2.63241723  2.3851020023  1.98959390
##  [85,]  1.3371562  1.3934227 -0.22505084 -0.3343596715 -0.28701927
##  [86,]  0.3366267  0.4787219  0.17878833  0.1893593045 -0.46560518
##  [87,]  0.3366267 -0.4359789 -0.41167349 -0.3469722082  0.87378914
##  [88,]  0.3366267  0.4787219 -0.38719839 -0.3477538616 -0.46560518
##  [89,] -0.6639027 -1.3506796 -0.31989186 -0.2846967614 -0.46560518
##  [90,] -0.6639027 -1.3506796 -0.41167349 -0.3603831480 -0.46560518
##  [91,] -1.6644321 -1.3506796 -0.27706043 -0.1416262644 -0.46560518
##  [92,]  0.3366267  0.4787219  0.52755852 -0.1441443052 -0.46560518
##  [93,]  0.3366267  1.3934227 -0.34742635 -0.3481446883 -0.46560518
##  [94,]  0.3366267 -0.4359789 -0.41779227 -0.3655364775 -0.46560518
##  [95,]  0.3366267 -0.4359789  0.48166771  0.0243801780 -0.46560518
##  [96,] -0.6639027  0.4787219 -0.42085165 -0.3601877346 -0.46560518
##  [97,] -0.6639027  0.4787219 -0.39637655 -0.3486192636 -0.46560518
##  [98,] -0.6639027 -1.3506796 -0.43002982 -0.3572286180 -0.24272996
##  [99,]  0.3366267  0.4787219 -0.33212941 -0.3329973612 -0.46560518
## [100,]  1.3371562  0.4787219  1.42701849  0.4577400155  1.76671869
## [101,] -0.6639027  0.4787219  0.49084587  0.4824960968 -0.46560518
## [102,] -0.6639027 -0.4359789 -0.36578268 -0.3512433859 -0.46560518
## [103,] -0.6639027  0.4787219 -0.25870411 -0.2309134210  1.76671869
## [104,] -0.6639027  0.4787219 -0.26788227 -0.1279864118  0.11444186
## [105,]  1.3371562 -0.4359789 -0.24646656  0.0004504161 -0.46560518
## [106,]  0.3366267 -0.4359789 -0.37190145 -0.3407636465 -0.46560518
## [107,]  1.3371562  0.4787219 -0.31683247 -0.3219257984 -0.15343701
## attr(,"scaled:center")
##    idealpoint        polity       polity2         democ         autoc
## -8.790160e-02  3.065421e+00  3.065421e+00  5.158879e+00  2.093458e+00
##         unreg       physint        speech    new_empinx         wecon
##  1.475981e+02  4.317757e+00  1.065421e+00  8.420561e+00  1.327103e+00
##         wopol         wosoc        elecsd    gdp.pc.wdi     gdp.pc.un
##  1.850467e+00  1.205607e+00  1.112150e+00  5.183262e+03  5.110174e+03
##       pop.wdi       amnesty     statedept        milper          cinc
```

```
##  4.744495e+07  2.663551e+00  2.476636e+00  1.435607e+02  6.805233e-03
##     domestic9
##  6.517944e+02
## attr(,"scaled:scale")
##   idealpoint       polity      polity2        democ        autoc
## 8.315408e-01 6.515976e+00 6.515976e+00 3.819388e+00 2.928496e+00
##        unreg      physint       speech   new_empinx        wecon
## 1.379564e+02 2.157034e+00 7.173198e-01 4.081799e+00 5.626035e-01
##        wopol        wosoc       elecsd   gdp.pc.wdi    gdp.pc.un
## 5.460620e-01 7.615565e-01 8.614063e-01 8.196742e+03 8.076768e+03
##      pop.wdi      amnesty    statedept       milper         cinc
## 1.594928e+08 9.994709e-01 1.093254e+00 3.268628e+02 1.791075e-02
##    domestic9
## 1.399886e+03
```

```
ct_scaled <- scale(countries[,-1])

# Store the correlation matrix and round the number

cor(ct_scaled)
```

```
##             idealpoint      polity     polity2        democ       autoc
## idealpoint  1.00000000  0.60847112  0.60847112  0.666743593 -0.48428632
## polity      0.60847112  1.00000000  1.00000000  0.973791515 -0.95499127
## polity2     0.60847112  1.00000000  1.00000000  0.973791515 -0.95499127
## democ       0.66674359  0.97379151  0.97379151  1.000000000 -0.86249525
## autoc      -0.48428632 -0.95499127 -0.95499127 -0.862495245  1.00000000
## unreg       0.11861430  0.35124748  0.35124748  0.389559842 -0.27346455
## physint     0.51989079  0.32135851  0.32135851  0.391163975 -0.20486872
## speech      0.45867264  0.64899241  0.64899241  0.636642862 -0.61370503
## new_empinx  0.57145463  0.83427786  0.83427786  0.828332534 -0.77596518
## wecon       0.30815345  0.26174422  0.26174422  0.335594124 -0.14470049
## wopol       0.35915906  0.50918968  0.50918968  0.468354961 -0.52212408
## wosoc       0.56576374  0.42882102  0.42882102  0.510847945 -0.28788185
## elecsd      0.50177851  0.84746610  0.84746610  0.843291344 -0.78579985
## gdp.pc.wdi  0.48863673  0.29298378  0.29298378  0.400660785 -0.12934836
## gdp.pc.un   0.47670143  0.28273276  0.28273276  0.390873867 -0.11930384
## pop.wdi    -0.11225091 -0.01742067 -0.01742067 -0.001144892  0.03726823
## amnesty    -0.51074942 -0.30079160 -0.30079160 -0.366449269  0.19134014
## statedept  -0.60459383 -0.39112065 -0.39112065 -0.472431904  0.25410036
## milper     -0.06805411 -0.06649447 -0.06649447 -0.045782794  0.08824125
## cinc        0.02621179  0.01765334  0.01765334  0.046666228  0.02158366
## domestic9  -0.07238929  0.10385585  0.10385585  0.074056289 -0.13449652
##                   unreg     physint      speech  new_empinx       wecon
## idealpoint  0.118614302  0.51989079  0.45867264  0.57145463  0.30815345
## polity      0.351247476  0.32135851  0.64899241  0.83427786  0.26174422
## polity2     0.351247476  0.32135851  0.64899241  0.83427786  0.26174422
## democ       0.389559842  0.39116398  0.63664286  0.82833253  0.33559412
## autoc      -0.273464548 -0.20486872 -0.61370503 -0.77596518 -0.14470049
## unreg       1.000000000  0.08409644  0.29751394  0.34156801 -0.02272164
## physint     0.084096437  1.00000000  0.40104199  0.48291871  0.41106826
## speech      0.297513940  0.40104199  1.00000000  0.78313419  0.15686266
## new_empinx  0.341568008  0.48291871  0.78313419  1.00000000  0.27228585
## wecon      -0.022721637  0.41106826  0.15686266  0.27228585  1.00000000
## wopol       0.068071599  0.14484129  0.29014058  0.51098958  0.31425348
## wosoc       0.157754863  0.51117626  0.33780389  0.49391836  0.65623647
## elecsd      0.348887710  0.33604797  0.69032779  0.85041267  0.23505117
## gdp.pc.wdi  0.025376495  0.51162824  0.30816831  0.33095195  0.47194565
## gdp.pc.un   0.024661898  0.50933189  0.30206755  0.32160353  0.46539043
## pop.wdi     0.006114645 -0.22753618 -0.09692815 -0.17017220 -0.12453396
## amnesty    -0.042110192 -0.65446236 -0.28481703 -0.35348071 -0.33930876
## statedept  -0.019609903 -0.79692560 -0.37697290 -0.48718694 -0.43993031
## milper     -0.002045234 -0.22176895 -0.13591452 -0.23304563 -0.17322446
## cinc        0.017010333 -0.12219545 -0.04601143 -0.11014391 -0.09421149
## domestic9   0.219153041 -0.43572526 -0.02174488 -0.01799559 -0.11082183
##                   wopol        wosoc       elecsd  gdp.pc.wdi    gdp.pc.un
## idealpoint  0.359159055  0.565763741  0.501778515  0.48863673  0.476701432
## polity      0.509189676  0.428821022  0.847466099  0.29298378  0.282732760
## polity2     0.509189676  0.428821022  0.847466099  0.29298378  0.282732760
```

```
## democ       0.468354961  0.510847945  0.843291344  0.40066079  0.390873867
## autoc      -0.522124076 -0.287881848 -0.785799846 -0.12934836 -0.119303840
## unreg       0.068071599  0.157754863  0.348887710  0.02537649  0.024661898
## physint     0.144841292  0.511176259  0.336047975  0.51162824  0.509331888
## speech      0.290140579  0.337803894  0.690327794  0.30816831  0.302067550
## new_empinx  0.510989583  0.493918364  0.850412668  0.33095195  0.321603528
## wecon       0.314253478  0.656236467  0.235051175  0.47194565  0.465390426
## wopol       1.000000000  0.414913045  0.437108127  0.01590115  0.004707645
## wosoc       0.414913045  1.000000000  0.395943251  0.50373308  0.492531051
## elecsd      0.437108127  0.395943251  1.000000000  0.29706393  0.290374836
## gdp.pc.wdi  0.015901152  0.503733082  0.297063929  1.00000000  0.999416976
## gdp.pc.un   0.004707645  0.492531051  0.290374836  0.99941698  1.000000000
## pop.wdi     0.038115702 -0.067098401 -0.065845664 -0.05790695 -0.057696071
## amnesty    -0.058479926 -0.428819499 -0.317361148 -0.53599197 -0.533676283
## statedept  -0.100723728 -0.504073931 -0.387879020 -0.57948640 -0.574701507
## milper     -0.035784365 -0.091538579 -0.101815189 -0.03302240 -0.033637844
## cinc        0.019731283 -0.007570257  0.002816845  0.13143354  0.132573190
## domestic9   0.080893353 -0.104759887  0.039511620 -0.13767772 -0.137363398
##                 pop.wdi     amnesty  statedept       milper         cinc
## idealpoint -0.112250911 -0.51074942 -0.6045938 -0.068054109  0.026211791
## polity     -0.017420669 -0.30079160 -0.3911206 -0.066494475  0.017653344
## polity2    -0.017420669 -0.30079160 -0.3911206 -0.066494475  0.017653344
## democ      -0.001144892 -0.36644927 -0.4724319 -0.045782794  0.046666228
## autoc       0.037268235  0.19134014  0.2541004  0.088241246  0.021583665
## unreg       0.006114645 -0.04211019 -0.0196099 -0.002045234  0.017010333
## physint    -0.227536176 -0.65446236 -0.7969256 -0.221768949 -0.122195447
## speech     -0.096928149 -0.28481703 -0.3769729 -0.135914521 -0.046011431
## new_empinx -0.170172200 -0.35348071 -0.4871869 -0.233045630 -0.110143909
## wecon      -0.124533957 -0.33930876 -0.4399303 -0.173224463 -0.094211492
## wopol       0.038115702 -0.05847993 -0.1007237 -0.035784365  0.019731283
## wosoc      -0.067098401 -0.42881950 -0.5040739 -0.091538579 -0.007570257
## elecsd     -0.065845664 -0.31736115 -0.3878790 -0.101815189  0.002816845
## gdp.pc.wdi -0.057906952 -0.53599197 -0.5794864 -0.033022399  0.131433543
## gdp.pc.un  -0.057696071 -0.53367628 -0.5747015 -0.033637844  0.132573190
## pop.wdi     1.000000000  0.31462571  0.2420838  0.889757944  0.896113320
## amnesty     0.314625711  1.00000000  0.7438803  0.351068292  0.251583758
## statedept   0.242083756  0.74388034  1.0000000  0.245479935  0.140453690
## milper      0.889757944  0.35106829  0.2454799  1.000000000  0.939914363
## cinc        0.896113320  0.25158376  0.1404537  0.939914363  1.000000000
## domestic9   0.063476297  0.40178558  0.4361176  0.094880177  0.078178773
##              domestic9
## idealpoint -0.07238929
## polity      0.10385585
## polity2     0.10385585
## democ       0.07405629
## autoc      -0.13449652
## unreg       0.21915304
## physint    -0.43572526
## speech     -0.02174488
```

```
## new_empinx -0.01799559
## wecon      -0.11082183
## wopol       0.08089335
## wosoc      -0.10475989
## elecsd      0.03951162
## gdp.pc.wdi -0.13767772
## gdp.pc.un  -0.13736340
## pop.wdi     0.06347630
## amnesty     0.40178558
## statedept   0.43611763
## milper      0.09488018
## cinc        0.07817877
## domestic9   1.00000000
```

```
round(cor(ct_scaled), 4)
```

```
##            idealpoint  polity polity2   democ   autoc   unreg physint
## idealpoint     1.0000  0.6085  0.6085  0.6667 -0.4843  0.1186  0.5199
## polity         0.6085  1.0000  1.0000  0.9738 -0.9550  0.3512  0.3214
## polity2        0.6085  1.0000  1.0000  0.9738 -0.9550  0.3512  0.3214
## democ          0.6667  0.9738  0.9738  1.0000 -0.8625  0.3896  0.3912
## autoc         -0.4843 -0.9550 -0.9550 -0.8625  1.0000 -0.2735 -0.2049
## unreg          0.1186  0.3512  0.3512  0.3896 -0.2735  1.0000  0.0841
## physint        0.5199  0.3214  0.3214  0.3912 -0.2049  0.0841  1.0000
## speech         0.4587  0.6490  0.6490  0.6366 -0.6137  0.2975  0.4010
## new_empinx     0.5715  0.8343  0.8343  0.8283 -0.7760  0.3416  0.4829
## wecon          0.3082  0.2617  0.2617  0.3356 -0.1447 -0.0227  0.4111
## wopol          0.3592  0.5092  0.5092  0.4684 -0.5221  0.0681  0.1448
## wosoc          0.5658  0.4288  0.4288  0.5108 -0.2879  0.1578  0.5112
## elecsd         0.5018  0.8475  0.8475  0.8433 -0.7858  0.3489  0.3360
## gdp.pc.wdi     0.4886  0.2930  0.2930  0.4007 -0.1293  0.0254  0.5116
## gdp.pc.un      0.4767  0.2827  0.2827  0.3909 -0.1193  0.0247  0.5093
## pop.wdi       -0.1123 -0.0174 -0.0174 -0.0011  0.0373  0.0061 -0.2275
## amnesty       -0.5107 -0.3008 -0.3008 -0.3664  0.1913 -0.0421 -0.6545
## statedept     -0.6046 -0.3911 -0.3911 -0.4724  0.2541 -0.0196 -0.7969
## milper        -0.0681 -0.0665 -0.0665 -0.0458  0.0882 -0.0020 -0.2218
## cinc           0.0262  0.0177  0.0177  0.0467  0.0216  0.0170 -0.1222
## domestic9     -0.0724  0.1039  0.1039  0.0741 -0.1345  0.2192 -0.4357
##              speech new_empinx   wecon   wopol   wosoc  elecsd gdp.pc.wdi
## idealpoint   0.4587     0.5715  0.3082  0.3592  0.5658  0.5018     0.4886
## polity       0.6490     0.8343  0.2617  0.5092  0.4288  0.8475     0.2930
## polity2      0.6490     0.8343  0.2617  0.5092  0.4288  0.8475     0.2930
## democ        0.6366     0.8283  0.3356  0.4684  0.5108  0.8433     0.4007
## autoc       -0.6137    -0.7760 -0.1447 -0.5221 -0.2879 -0.7858    -0.1293
## unreg        0.2975     0.3416 -0.0227  0.0681  0.1578  0.3489     0.0254
## physint      0.4010     0.4829  0.4111  0.1448  0.5112  0.3360     0.5116
## speech       1.0000     0.7831  0.1569  0.2901  0.3378  0.6903     0.3082
## new_empinx   0.7831     1.0000  0.2723  0.5110  0.4939  0.8504     0.3310
## wecon        0.1569     0.2723  1.0000  0.3143  0.6562  0.2351     0.4719
## wopol        0.2901     0.5110  0.3143  1.0000  0.4149  0.4371     0.0159
## wosoc        0.3378     0.4939  0.6562  0.4149  1.0000  0.3959     0.5037
## elecsd       0.6903     0.8504  0.2351  0.4371  0.3959  1.0000     0.2971
## gdp.pc.wdi   0.3082     0.3310  0.4719  0.0159  0.5037  0.2971     1.0000
## gdp.pc.un    0.3021     0.3216  0.4654  0.0047  0.4925  0.2904     0.9994
## pop.wdi     -0.0969    -0.1702 -0.1245  0.0381 -0.0671 -0.0658    -0.0579
## amnesty     -0.2848    -0.3535 -0.3393 -0.0585 -0.4288 -0.3174    -0.5360
## statedept   -0.3770    -0.4872 -0.4399 -0.1007 -0.5041 -0.3879    -0.5795
## milper      -0.1359    -0.2330 -0.1732 -0.0358 -0.0915 -0.1018    -0.0330
## cinc        -0.0460    -0.1101 -0.0942  0.0197 -0.0076  0.0028     0.1314
## domestic9   -0.0217    -0.0180 -0.1108  0.0809 -0.1048  0.0395    -0.1377
##             gdp.pc.un pop.wdi amnesty statedept  milper    cinc domestic9
## idealpoint     0.4767 -0.1123 -0.5107   -0.6046 -0.0681  0.0262   -0.0724
## polity         0.2827 -0.0174 -0.3008   -0.3911 -0.0665  0.0177    0.1039
## polity2        0.2827 -0.0174 -0.3008   -0.3911 -0.0665  0.0177    0.1039
```

```
## democ         0.3909 -0.0011 -0.3664   -0.4724 -0.0458  0.0467    0.0741
## autoc        -0.1193  0.0373  0.1913    0.2541  0.0882  0.0216   -0.1345
## unreg         0.0247  0.0061 -0.0421   -0.0196 -0.0020  0.0170    0.2192
## physint       0.5093 -0.2275 -0.6545   -0.7969 -0.2218 -0.1222   -0.4357
## speech        0.3021 -0.0969 -0.2848   -0.3770 -0.1359 -0.0460   -0.0217
## new_empinx    0.3216 -0.1702 -0.3535   -0.4872 -0.2330 -0.1101   -0.0180
## wecon         0.4654 -0.1245 -0.3393   -0.4399 -0.1732 -0.0942   -0.1108
## wopol         0.0047  0.0381 -0.0585   -0.1007 -0.0358  0.0197    0.0809
## wosoc         0.4925 -0.0671 -0.4288   -0.5041 -0.0915 -0.0076   -0.1048
## elecsd        0.2904 -0.0658 -0.3174   -0.3879 -0.1018  0.0028    0.0395
## gdp.pc.wdi    0.9994 -0.0579 -0.5360   -0.5795 -0.0330  0.1314   -0.1377
## gdp.pc.un     1.0000 -0.0577 -0.5337   -0.5747 -0.0336  0.1326   -0.1374
## pop.wdi      -0.0577  1.0000  0.3146    0.2421  0.8898  0.8961    0.0635
## amnesty      -0.5337  0.3146  1.0000    0.7439  0.3511  0.2516    0.4018
## statedept    -0.5747  0.2421  0.7439    1.0000  0.2455  0.1405    0.4361
## milper       -0.0336  0.8898  0.3511    0.2455  1.0000  0.9399    0.0949
## cinc          0.1326  0.8961  0.2516    0.1405  0.9399  1.0000    0.0782
## domestic9    -0.1374  0.0635  0.4018    0.4361  0.0949  0.0782    1.0000
```

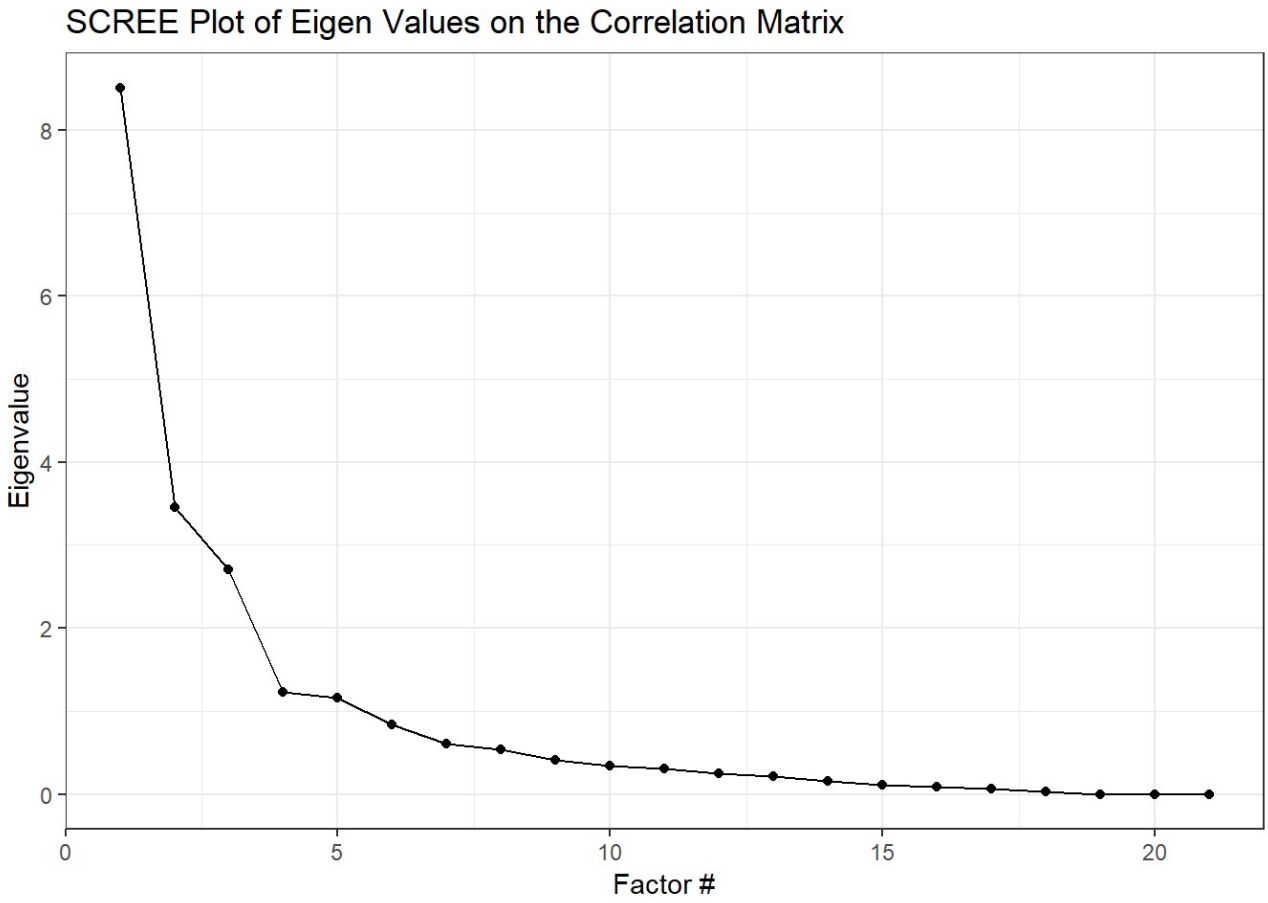
```
countrycor <- cor(ct_scaled)


# Generate the eigenvalues and store

ev <- eigen(countrycor)
ev$values
```

```
##  [1]  8.510913e+00  3.459670e+00  2.702458e+00  1.225737e+00  1.158450e+00
##  [6]  8.333739e-01  6.112287e-01  5.321391e-01  4.150076e-01  3.446022e-01
## [11]  3.043095e-01  2.434553e-01  2.147082e-01  1.595778e-01  1.073515e-01
## [16]  8.416652e-02  5.927696e-02  3.317877e-02  3.962048e-04  1.230485e-15
## [21] -9.239694e-18
```

Fit three EFA models:

I. 2-Factor Model

```
# Generate Scree plot
qplot(y = ev$values,
      main = 'SCREE Plot of Eigen Values on the Correlation Matrix',
      xlab = 'Factor #',
      ylab = 'Eigenvalue') +
    geom_line() +
    theme_bw()
```




```
# Fit the factor analysis model with 2 factors
fa2 <- fa(ct_scaled,
               nfactors = 2)
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs
## = np.obs, : The estimated weights for the factor scores are probably
## incorrect. Try a different factor extraction method.
```

```
## In factor.scores, the correlation matrix is singular, an approximation is used
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
# Inspect output
summary(fa2)
```

```
##
## Factor analysis with Call: fa(r = ct_scaled, nfactors = 2)
##
## Test of the hypothesis that 2 factors are sufficient.
## The degrees of freedom for the model is 169  and the objective function was  51.41
## The number of observations was  107  with Chi Square =  4978.17  with prob <  0
##
## The root mean square of the residuals (RMSA) is  0.12
## The df corrected root mean square of the residuals is  0.14
##
## Tucker Lewis Index of factoring reliability =  0.081
## RMSEA index =  0.543  and the 10 % confidence intervals are  0.506 NA
## BIC =  4188.46
##  With factor correlations of
##      MR1  MR2
## MR1 1.00 0.41
## MR2 0.41 1.00
```

```
# Inspect individual values, e.g., loadings, scores
fa2$loadings
```

```
##
## Loadings:
##             MR1    MR2
## idealpoint  0.449  0.429
## polity      0.995
## polity2     0.995
## democ       0.931
## autoc      -0.969  0.159
## unreg       0.412 -0.131
## physint             0.782
## speech      0.631  0.154
## new_empinx  0.802  0.197
## wecon               0.509
## wopol       0.551
## wosoc       0.286  0.497
## elecsd      0.852
## gdp.pc.wdi          0.673
## gdp.pc.un           0.671
## pop.wdi     0.204 -0.476
## amnesty            -0.821
## statedept          -0.849
## milper      0.158 -0.468
## cinc        0.211 -0.366
## domestic9   0.288 -0.479
##
##                  MR1   MR2
## SS loadings    6.523 4.527
## Proportion Var 0.311 0.216
## Cumulative Var 0.311 0.526
```

```
fa2$scores
```

```
##                MR1          MR2
##   [1,] -0.846907574 -1.398257402
##   [2,]  0.142462169 -0.139677550
##   [3,] -1.928228455  1.373115046
##   [4,]  0.001438971  0.029198535
##   [5,]  1.229504870  1.822660481
##   [6,] -1.461627415 -0.123783026
##   [7,] -0.840725088 -1.583068475
##   [8,]  1.091725388  1.886073307
##   [9,] -0.937870057 -0.026980591
##  [10,]  0.162933143 -0.745609646
##  [11,]  0.479380112 -0.136402880
##  [12,] -1.623113220  0.177381965
##  [13,] -1.449616166 -0.290176139
##  [14,]  0.826406208 -0.091934797
##  [15,]  0.880915622 -0.813179254
##  [16,] -1.721407236  0.751569301
##  [17,]  1.148602881  1.695953734
##  [18,]  0.951255841  0.305469841
##  [19,] -1.056534477 -2.776170672
##  [20,]  0.007250929 -0.825443345
##  [21,] -1.022495227 -1.128770081
##  [22,] -1.020114280 -0.652826674
##  [23,]  0.733753704 -1.630772024
##  [24,] -1.573064355 -0.248083418
##  [25,]  1.039883169  1.033770191
##  [26,]  0.745999837 -0.106828608
##  [27,] -0.820681731 -1.117690121
##  [28,]  0.220901864 -0.347762918
##  [29,] -1.241827023 -0.441133215
##  [30,] -1.158253086 -0.087422331
##  [31,]  1.200980506  1.098133470
##  [32,]  1.208931404  2.146246668
##  [33,]  0.992091729  1.360058242
##  [34,]  1.144736584  1.621560324
##  [35,]  0.090414194 -0.054704248
##  [36,] -0.263336068  0.109592338
##  [37,] -0.865636295 -0.933899654
##  [38,] -1.142320570 -0.194057058
##  [39,]  1.073185174  1.218670924
##  [40,]  0.872376390 -0.356657044
##  [41,]  0.510652791  0.031176334
##  [42,]  0.796623168 -0.092308655
##  [43,]  0.458685996  0.847279312
##  [44,]  1.209280956  0.821273440
##  [45,]  0.390077562 -1.315282897
##  [46,]  0.808197035 -2.085684196
##  [47,]  1.159205680  1.990450372
```

```
##  [48,] -0.457216898 -1.452260586
##  [49,]  1.291058785  1.520977003
##  [50,]  1.098650635  0.144626487
##  [51,] -0.993539207 -0.244545616
##  [52,]  1.262743220  1.575189779
##  [53,] -1.207870856 -0.357373369
##  [54,] -0.906141379 -0.606588850
##  [55,] -1.065045768  0.062110347
##  [56,] -0.319608603 -0.202533506
##  [57,]  0.776999502  0.310847603
##  [58,] -1.659337503  0.800888519
##  [59,] -1.445025599 -0.190450830
##  [60,] -1.492954228 -0.279597346
##  [61,]  0.372034168 -1.483713773
##  [62,]  0.774562540  1.031690393
##  [63,] -1.400042672 -0.285560178
##  [64,]  0.550959552  0.324368076
##  [65,]  0.717553612 -0.501869103
##  [66,]  0.548951840  0.489110783
##  [67,]  0.574177874  0.571063887
##  [68,]  0.120228643 -0.562773688
##  [69,] -1.313125028  0.188905258
##  [70,]  0.372236886  0.325840728
##  [71,] -0.390037175 -0.070567188
##  [72,]  0.554046063  0.264675508
##  [73,]  0.328212939  0.228938194
##  [74,] -0.281624389 -0.915157023
##  [75,]  0.746916882  0.218680616
##  [76,]  0.413160960 -1.200376273
##  [77,]  1.316043280  1.663664482
##  [78,] -1.144743244 -1.203239922
##  [79,]  0.858377331 -0.983709649
##  [80,]  0.997768240  0.757650043
##  [81,]  1.246418946  1.462577215
##  [82,]  0.663661737 -0.282794032
##  [83,]  0.658040466 -0.027728707
##  [84,]  0.470992293 -1.756261197
##  [85,] -1.100525284 -1.057404551
##  [86,] -1.736735013 -0.263168388
##  [87,]  0.773224444 -0.010793770
##  [88,]  0.691806443 -0.160822163
##  [89,]  0.814899530  1.114318193
##  [90,]  1.042327872  1.176492769
##  [91,]  1.181903121  2.309847524
##  [92,] -1.514502923 -0.511018158
##  [93,] -0.826618943 -0.806435735
##  [94,] -1.029387028 -0.021955890
##  [95,]  0.821978192  0.015275552
##  [96,] -0.810208135 -0.499958664
```

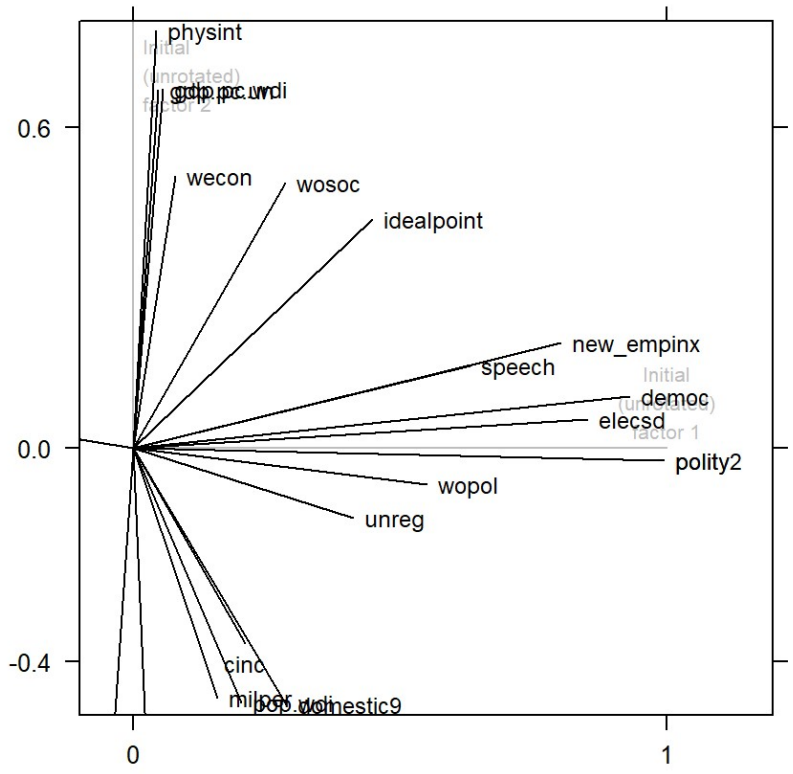
```
##  [97,] -1.672861868 -0.135304728
##  [98,]  1.134238805  1.033153664
##  [99,] -1.034202293 -0.356948254
## [100,]  0.438187138 -0.897239120
## [101,]  0.266613050 -0.379102515
## [102,]  1.059709035  0.525112501
## [103,] -1.753520293 -0.233424190
## [104,]  0.403333694 -0.102426585
## [105,]  1.040176736  0.001515734
## [106,] -0.467656476 -0.039366044
## [107,] -0.963759317 -0.610118136
```

Summary of this 2-factor model shows that the root mean square of the residuals is 0.12, which signals it may not be the best model fit given the magnitude of the number. A minimum residual method is used in this case for the model.

Inspection of the loadings generates results with a fairly large proportion of cross-loading dimensions to both factors, such as “idealpoint” and “wosoc”. It is unclear how in terms of the pattern as both political and non-political indicators seems to be mixed together in the loading.

Plotting 2-factor loading:

```
# Plot unrotated factor pattern
fa2.pattern <- as.data.frame(fa2$loadings[1:21,])


xyplot(MR2 ~ MR1, data = fa2.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa2.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa2.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "2 Factor Pattern",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```



From the xyplot above, we see some policy-related indicators/features are indeed loaded to the same factor. For example, democracy and freedom of speech, represented by input variable “democ” and “speech” are both close to the x-axis, meaning they are fitted into factor 1. One feature “idealpoint” seems to be sit right in the diagonal of this panel, suggesting there could be some collinearity.

II. 3-Factor Model

```
# Fit the factor analysis model with 3 factors
fa3 <- fa(ct_scaled,
               nfactors = 3)
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs
## = np.obs, : The estimated weights for the factor scores are probably
## incorrect. Try a different factor extraction method.
```

```
## In factor.scores, the correlation matrix is singular, an approximation is used
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
# Inspect output
summary(fa3)
```

```
##
## Factor analysis with Call: fa(r = ct_scaled, nfactors = 3)
##
## Test of the hypothesis that 3 factors are sufficient.
## The degrees of freedom for the model is 150  and the objective function was  46.65
## The number of observations was  107  with Chi Square =  4486.65  with prob <  0
##
## The root mean square of the residuals (RMSA) is  0.06
## The df corrected root mean square of the residuals is  0.07
##
## Tucker Lewis Index of factoring reliability =  0.06
## RMSEA index =  0.549  and the 10 % confidence intervals are  0.509 NA
## BIC =  3785.72
##  With factor correlations of
##       MR1   MR2   MR3
## MR1  1.00  0.38 -0.05
## MR2  0.38  1.00 -0.12
## MR3 -0.05 -0.12  1.00
```

```
# Inspect factor loading
fa3$loadings
```

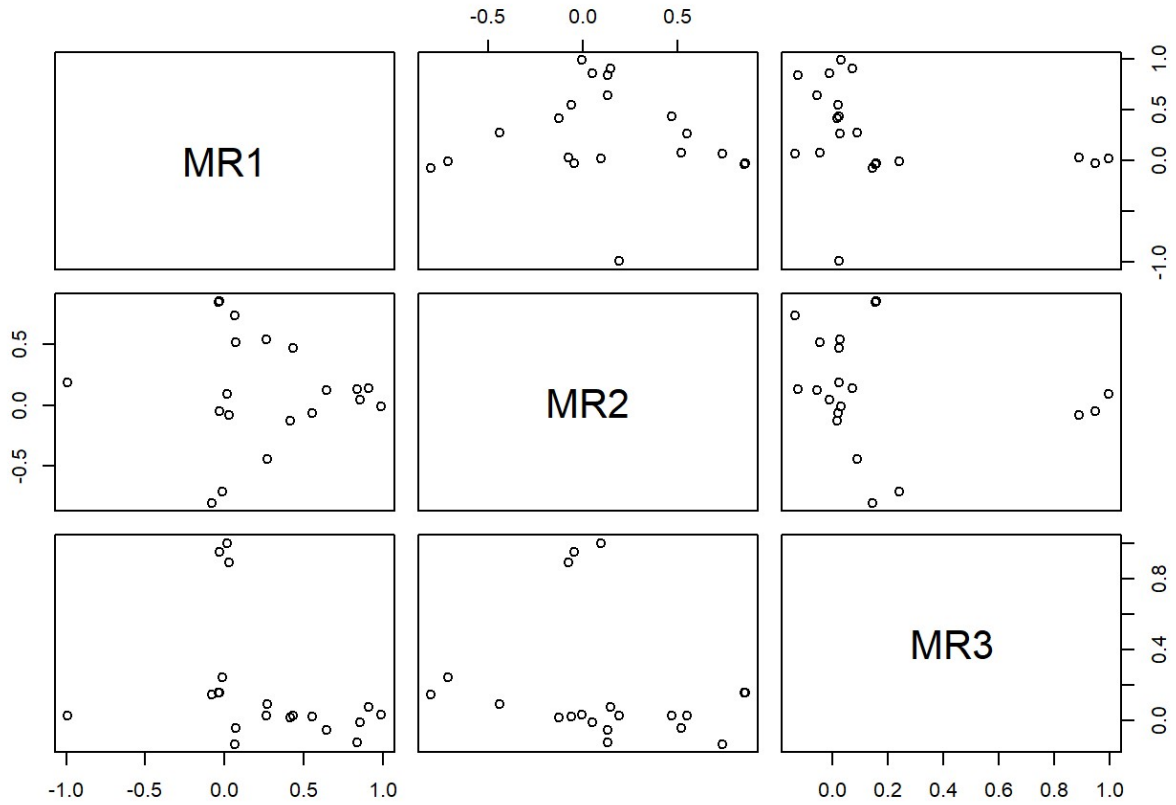
```
##
## Loadings:
##            MR1    MR2    MR3
## idealpoint  0.432  0.468
## polity      0.992
## polity2     0.992
## democ       0.910  0.144
## autoc      -0.994  0.191
## unreg       0.413 -0.129
## physint             0.737 -0.136
## speech      0.646  0.128
## new_empinx  0.840  0.131 -0.125
## wecon               0.518
## wopol       0.552
## wosoc       0.263  0.547
## elecsd      0.858
## gdp.pc.wdi          0.856  0.158
## gdp.pc.un           0.853  0.157
## pop.wdi                    0.892
## amnesty            -0.715  0.243
## statedept          -0.803  0.144
## milper                     0.949
## cinc                       0.999
## domestic9   0.269 -0.443
##
##                  MR1   MR2   MR3
## SS loadings    6.466 4.275 2.881
## Proportion Var 0.308 0.204 0.137
## Cumulative Var 0.308 0.512 0.649
```

Summary of 3-factor model shows a root mean sqaure of residuals value of 0.06, which indicates it is a better fit than the 2-factor model;

Inspection of the loadings still displays cross-loading for some variables, and it overall demonstrates greater proportion of variations explained by such three factors, evident by the cumulative variance of 0.649. However, there is still unexplained variance under this 3-factor model, suggesting maybe we could consider four factors.

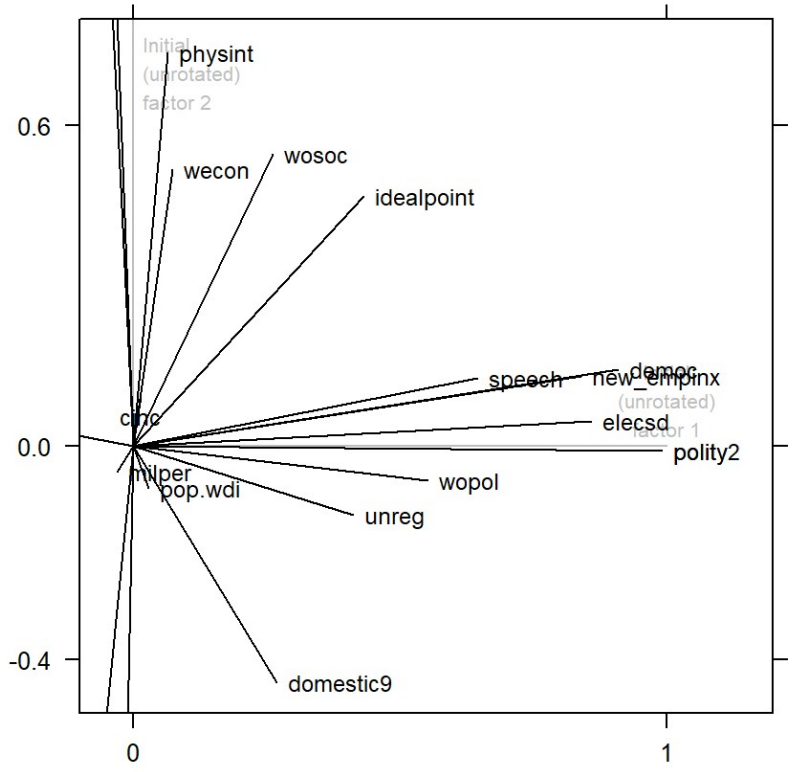
Plotting 3-factor loading:

```
# Plot unrotated factor pattern
pairs(fa3$loadings)
```


MR1
MR2
MR3
-0.5
0.0
0.5
1.0
-1.0
0.2
0.4
0.6
0.8

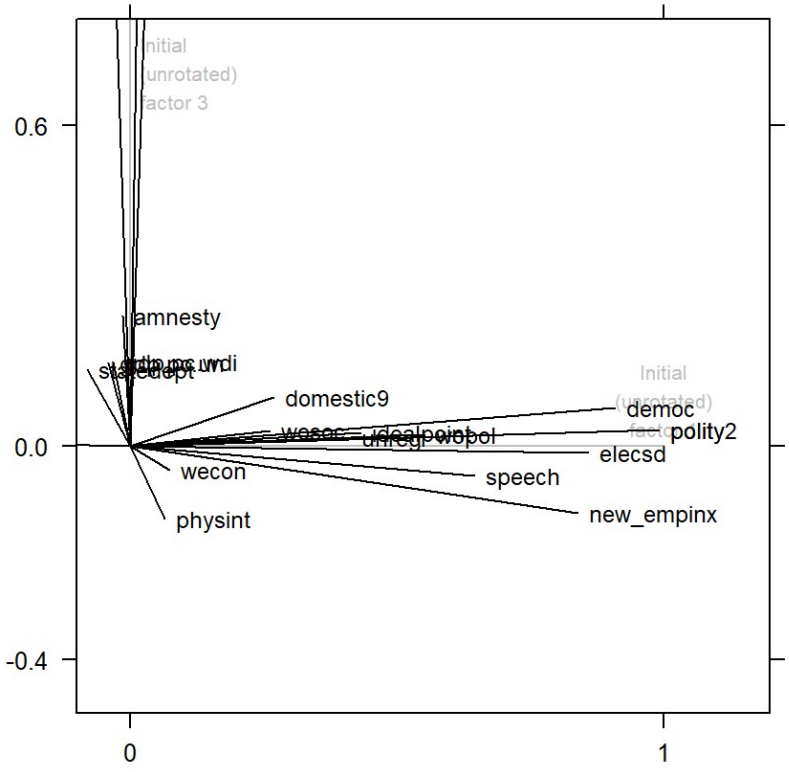
```
fa3.pattern <- as.data.frame(fa3$loadings[1:21,])


par(mfrow = c(2,2))

xyplot(MR2 ~ MR1, data = fa3.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa3.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa3.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Three Factor 1-2 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Three Factor 1-2 Loading

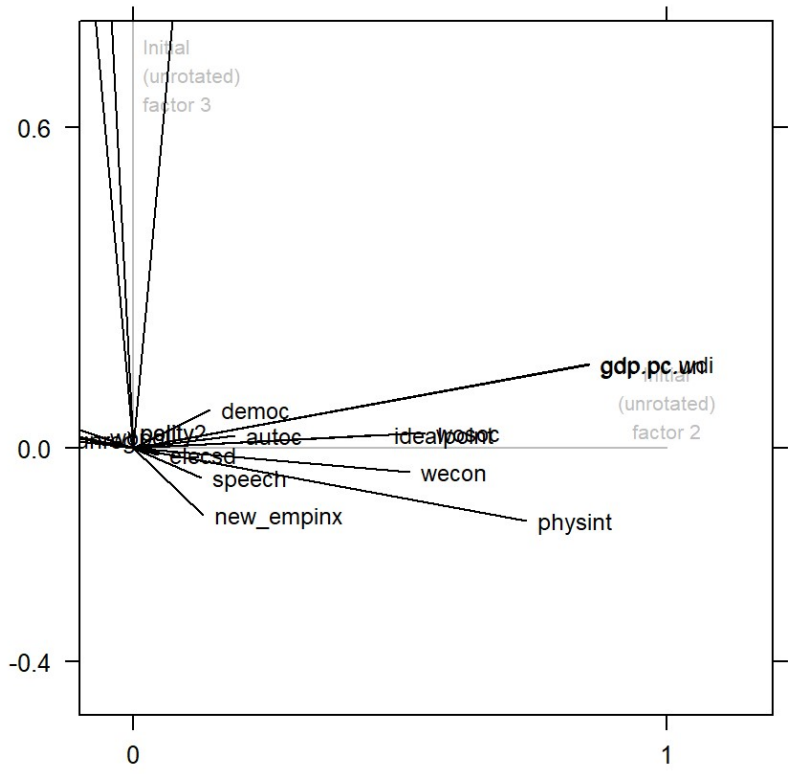


```
xyplot(MR3 ~ MR1, data = fa3.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 3",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa3.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa3.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Three Factor 1-3 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Three Factor 1-3 Loading



```
xyplot(MR3 ~ MR2, data = fa3.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 3",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa3.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa3.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Three Factor 1-3 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```



An initial attempt to present the factor loading in scatterplot is not able to visualize the pattern clearly; therefore, xyplots are used to show the loading by pairing two factors at each time. It is evident that some features are heavily loaded to one factor space, for example, polity2 is almost parallel to the x-axis for factor 1, so does elecsd. Additionally, with the introduction of the 3rd factor, features like “milper”" and “cinc” are able to fit in better in the model with high factor loading scores.

III. 4-Factor Model

```
# Fit the factor analysis model with 2 factors
fa4 <- fa(ct_scaled,
               nfactors = 4)
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs
## = np.obs, : The estimated weights for the factor scores are probably
## incorrect. Try a different factor extraction method.
```

```
## In factor.scores, the correlation matrix is singular, an approximation is used
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
# Inspect output
summary(fa4)
```

```
##
## Factor analysis with Call: fa(r = ct_scaled, nfactors = 4)
##
## Test of the hypothesis that 4 factors are sufficient.
## The degrees of freedom for the model is 132  and the objective function was  43.56
## The number of observations was  107  with Chi Square =  4160.02  with prob <  0
##
## The root mean square of the residuals (RMSA) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Tucker Lewis Index of factoring reliability =  0
## RMSEA index =  0.566  and the 10 % confidence intervals are  0.523 NA
## BIC =  3543.21
##  With factor correlations of
##       MR1   MR3   MR4   MR2
## MR1  1.00 -0.06  0.32 -0.29
## MR3 -0.06  1.00  0.00  0.23
## MR4  0.32  0.00  1.00 -0.52
## MR2 -0.29  0.23 -0.52  1.00
```

```
# Inspect loadings
fa4$loadings
```

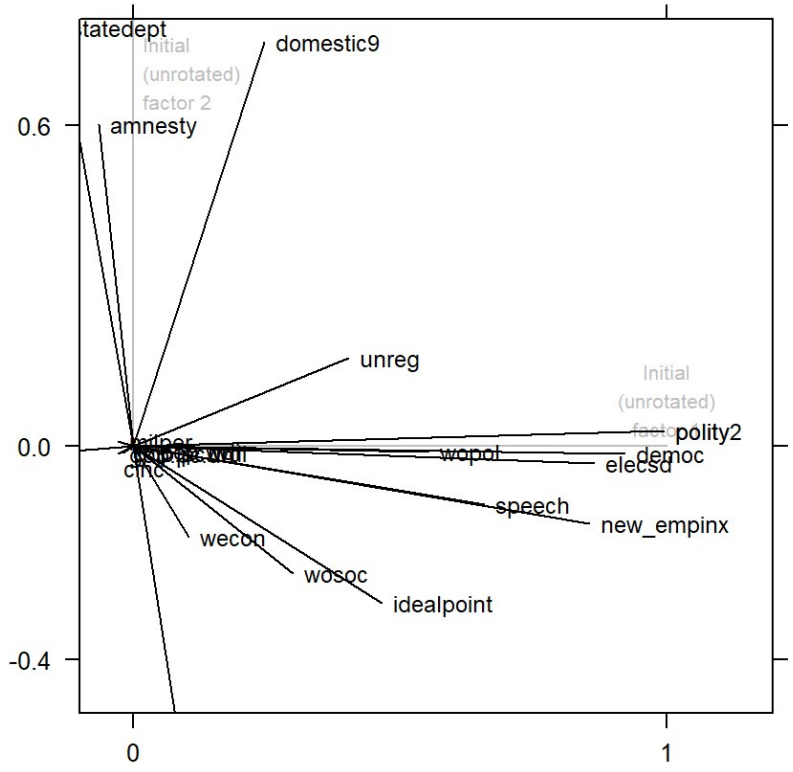
```
##
## Loadings:
##             MR1    MR3    MR4    MR2
## idealpoint  0.467          0.214 -0.294
## polity      0.995
## polity2     0.995
## democ       0.922          0.127
## autoc      -0.986          0.146
## unreg       0.405                 0.165
## physint     0.119                -0.761
## speech      0.658                -0.109
## new_empinx  0.855                -0.145
## wecon       0.105          0.390 -0.170
## wopol       0.555
## wosoc       0.300          0.350 -0.239
## elecsd      0.865
## gdp.pc.wdi                 0.986
## gdp.pc.un                  0.979
## pop.wdi             0.923
## amnesty             0.177 -0.197  0.602
## statedept  -0.137         -0.139  0.783
## milper              0.965
## cinc                0.981  0.111
## domestic9   0.247          0.204  0.757
##
##                   MR1   MR3   MR4   MR2
## SS loadings     6.605 2.811 2.426 2.370
## Proportion Var 0.315 0.134 0.116 0.113
## Cumulative Var 0.315 0.448 0.564 0.677
```

The root mean square of the residuals is 0.04, indicating that this is a fairly good model fit. A closer look at the factor loading shows that majority of the features tend to be heavily loaded in factor 1, and the overall variation explained by the factors improved just marginally from 3-factor to 4-factor model, with a cumulative variance of 0.677.

Plotting 4-factor model:

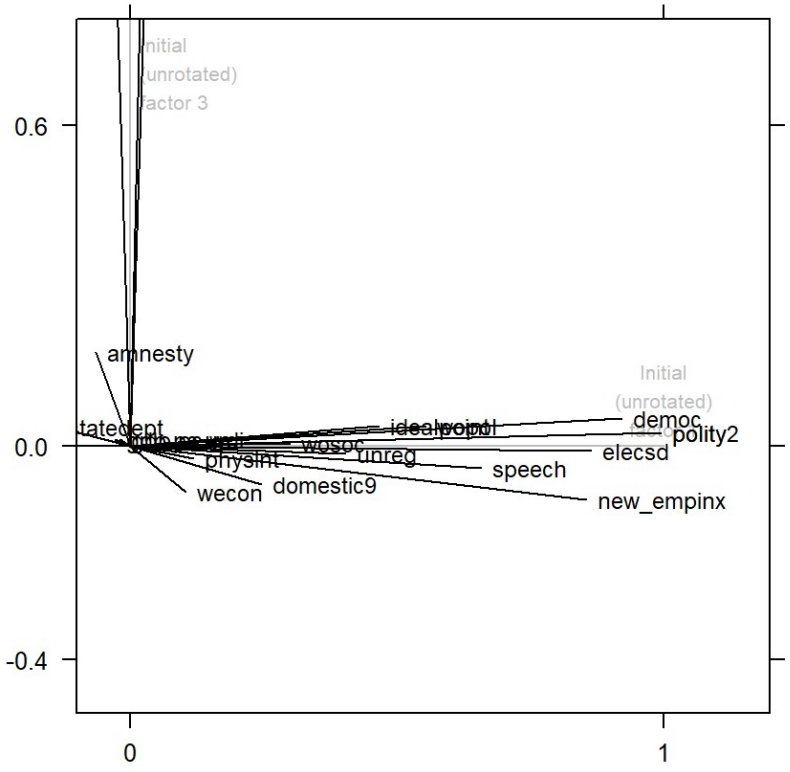
```
# Plot  4factor pattern
fa4.pattern <- as.data.frame(fa4$loadings[1:21,])

par(mfrow = c(3,3))

xyplot(MR2 ~ MR1, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 1-2 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Four Factor 1-2 Loading



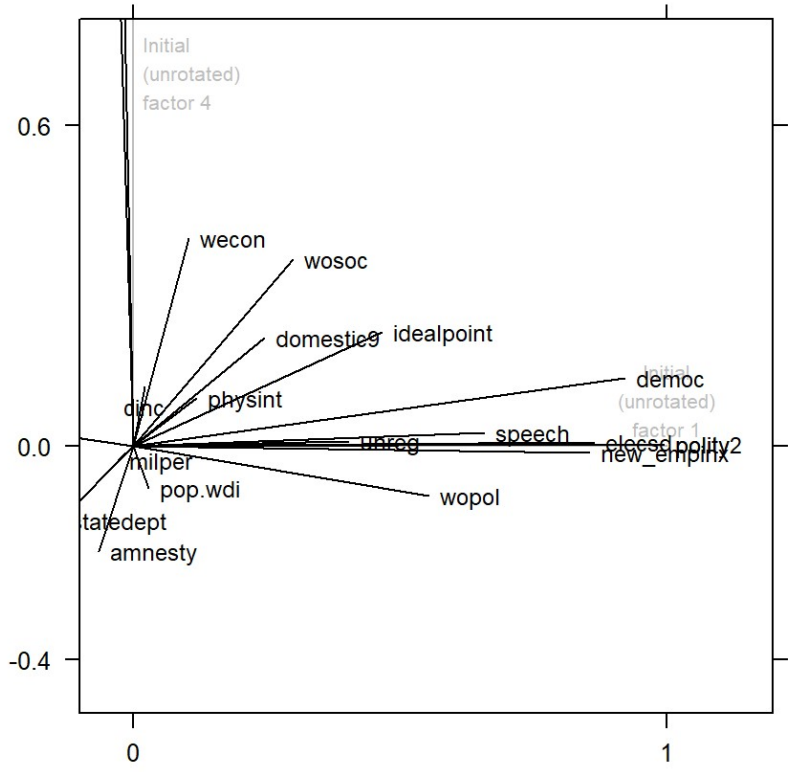
```
xyplot(MR3 ~ MR1, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 3",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 1-3 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Four Factor 1-3 Loading



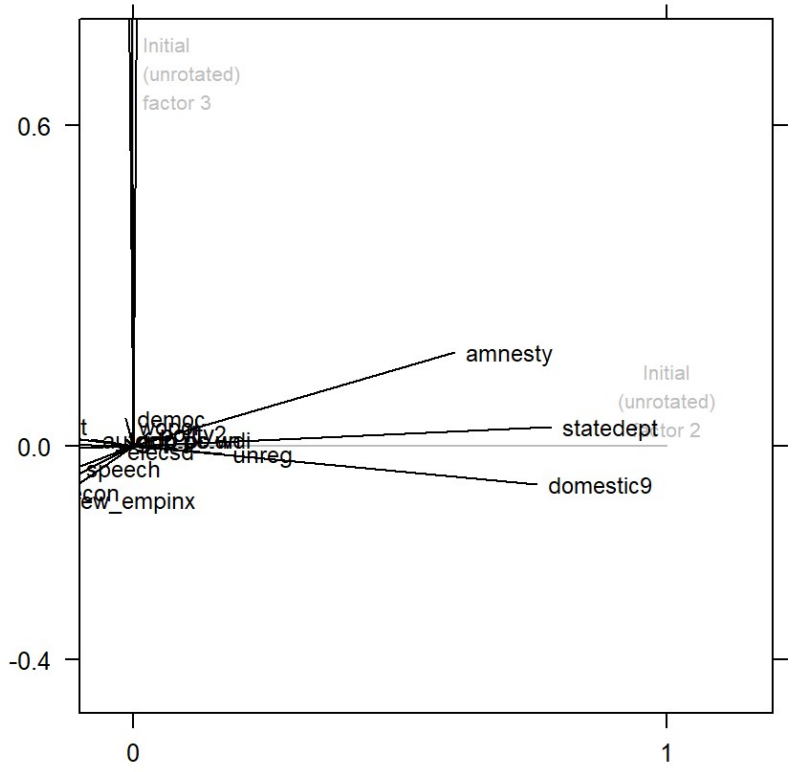
```
xyplot(MR4 ~ MR1, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 1",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 4",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 1-4 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Four Factor 1-4 Loading



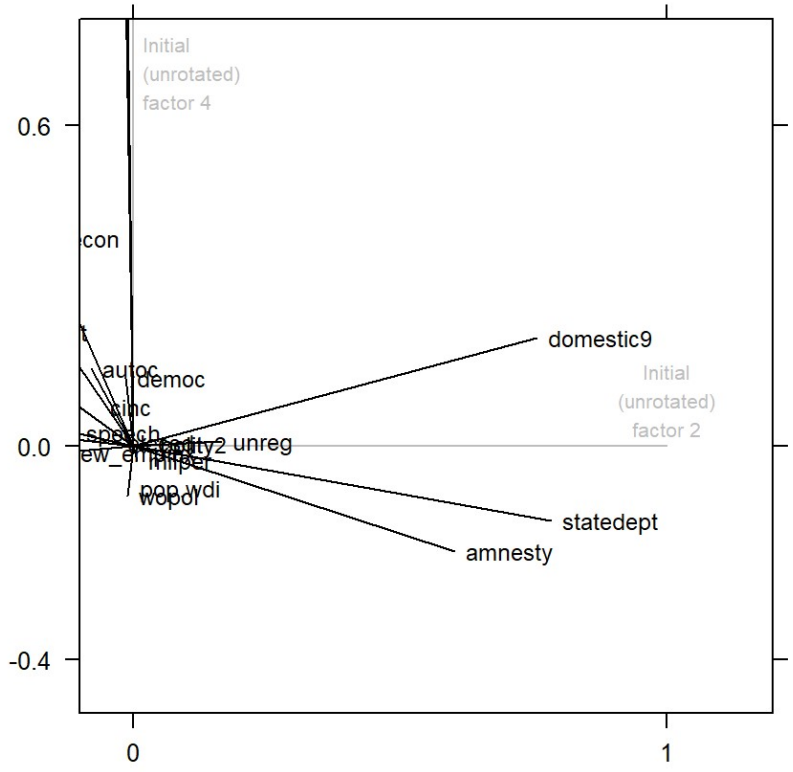
```
xyplot(MR3 ~ MR2, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 3",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 2-3 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Four Factor 2-3 Loading

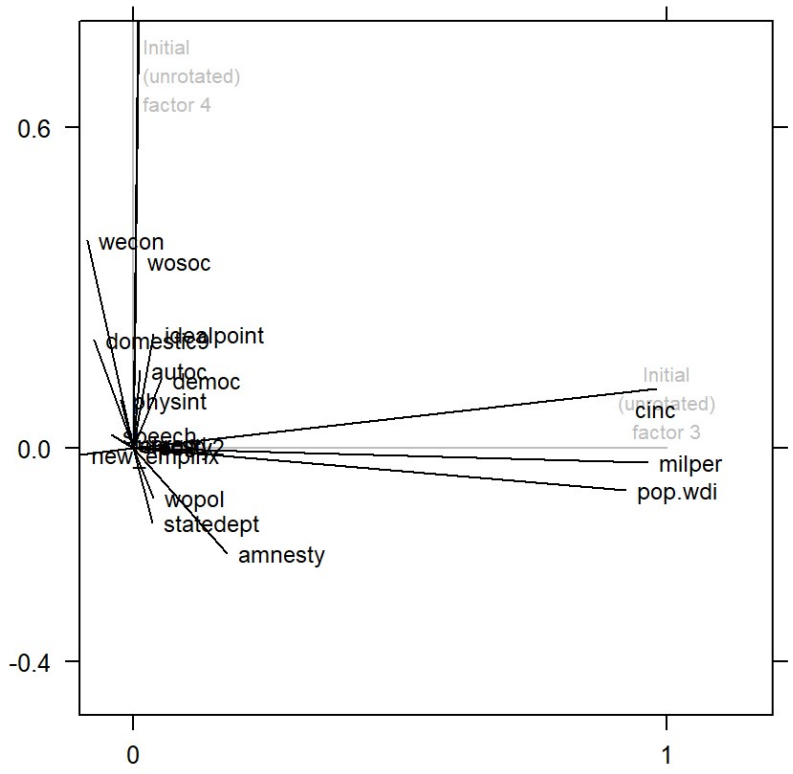


```
xyplot(MR4 ~ MR2, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 2",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 4",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 2-4 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```

Four Factor 2-4 Loading



```
xyplot(MR4 ~ MR3, data = fa4.pattern,
       aspect = 1,
       xlim = c(-.1, 1.2),
       ylim = c(-.5, .8),
       panel = function (x, y) {
         panel.segments(c(0, 0), c(0, 0),
             c(1, 0), c(0, 1), col = "gray")
         panel.text(1, 0, labels = "Initial\n(unrotated)\nfactor 3",
                    cex = .65, pos = 3, col = "gray")
         panel.text(0, .7, labels = "Initial\n(unrotated)\nfactor 4",
                    cex = .65, pos = 4, col = "gray")
         panel.segments(rep(0, 21), rep(0, 21), x, y,
            col = "black")
         panel.text(x[-20], y[-20], labels = rownames(fa4.pattern)[-20],
            pos = 4, cex = .75)
         panel.text(x[20], y[20], labels = rownames(fa4.pattern)[20],
                    pos = 1, cex = .75)
       },
       main = "Four Factor 3-4 Loading",
       xlab = "",
       ylab = "",
       scales = list(x = list(at = c(0, 1)),
                     y = list(at = c(-.4, 0, .6)))
)
```



Rotated the 3-factor model:

```
## Oblique Rotation, i.e. assuming factors are correlated
fa3rot <- fa(ct_scaled,
                 nfactors = 3,
                 rotate = "Promax")
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate =
## rotate, : A loading greater than abs(1) was detected. Examine the loadings
## carefully.
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs
## = np.obs, : The estimated weights for the factor scores are probably
## incorrect. Try a different factor extraction method.
```

```
## In factor.scores, the correlation matrix is singular, an approximation is used
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
summary(fa3rot)
```

```
##
## Factor analysis with Call: fa(r = ct_scaled, nfactors = 3, rotate = "Promax")
##
## Test of the hypothesis that 3 factors are sufficient.
## The degrees of freedom for the model is 150  and the objective function was  46.65
## The number of observations was  107  with Chi Square =  4486.65  with prob <  0
##
## The root mean square of the residuals (RMSA) is  0.06
## The df corrected root mean square of the residuals is  0.07
##
## Tucker Lewis Index of factoring reliability =  0.06
## RMSEA index =  0.549  and the 10 % confidence intervals are  0.509 NA
## BIC =  3785.72
##  With factor correlations of
##       MR1   MR2   MR3
## MR1  1.00  0.44 -0.08
## MR2  0.44  1.00 -0.20
## MR3 -0.08 -0.20  1.00
```

```
# view new loadings
fa3rot$loadings
```

```
##
## Loadings:
##             MR1    MR2    MR3
## idealpoint  0.406  0.479
## polity      0.998
## polity2     0.998
## democ       0.907  0.139
## autoc      -1.012  0.212
## unreg       0.423 -0.138
## physint             0.756 -0.104
## speech      0.642  0.121
## new_empinx  0.836  0.119 -0.120
## wecon               0.533
## wopol       0.559
## wosoc       0.232  0.563
## elecsd      0.861
## gdp.pc.wdi          0.891  0.197
## gdp.pc.un           0.888  0.196
## pop.wdi                    0.895
## amnesty            -0.729  0.213
## statedept          -0.824  0.109
## milper                     0.954
## cinc                0.134  1.010
## domestic9   0.298 -0.457
##
##                  MR1   MR2   MR3
## SS loadings    6.513 4.544 2.914
## Proportion Var 0.310 0.216 0.139
## Cumulative Var 0.310 0.527 0.665
```

```
scores <- data.frame(fa3rot$scores)
scores
```

```
##             MR1         MR2          MR3
## 1   -0.81933818 -1.42142449 -0.051926799
## 2    0.18610714 -0.26380699 -0.460657963
## 3   -2.05881675  1.61196944 -0.622049649
## 4    0.05981509 -0.11394440 -0.174328435
## 5    1.17007277  1.99409592 -0.187018325
## 6   -1.44219258 -0.28122283 -0.286311854
## 7   -0.75985544 -1.64050345 -0.146318615
## 8    1.05002480  2.09370183 -0.364280381
## 9   -0.81016497 -0.33121544 -0.634888404
## 10   0.19270879 -0.71331063  0.096628884
## 11   0.46820609 -0.13075472 -0.261588364
## 12  -1.63640611  0.18674587 -0.270066011
## 13  -1.44425610 -0.28510958 -0.131347084
## 14   0.85112632 -0.14252564 -0.327416466
## 15   0.88040093 -0.52207021  0.965957292
## 16  -1.70315792  0.52262178 -0.271789619
## 17   1.06268554  1.88501149 -0.115585675
## 18   0.96373980  0.27152680 -0.180523094
## 19  -1.56483891 -1.23977689  8.211930746
## 20   0.04877651 -0.92993573  0.001363672
## 21  -1.02158692 -1.08631966 -0.188566645
## 22  -0.98519047 -0.71063558 -0.355837095
## 23   0.73937282 -1.62057066  0.207720397
## 24  -1.56894962 -0.24570827 -0.121046993
## 25   1.04270796  0.98744326 -0.342515356
## 26   0.77708947 -0.16531402 -0.246353916
## 27  -0.82288833 -1.03731860  0.072823493
## 28   0.27521715 -0.43978260 -0.286980003
## 29  -1.34273064 -0.47894381  0.347350769
## 30  -1.18235606 -0.20603106 -0.435538576
## 31   1.16848176  1.15192678  0.112988906
## 32   1.14494735  2.29301856 -0.449913044
## 33   0.92770727  1.71788674  0.505842252
## 34   1.06580277  2.22901726  0.665937663
## 35   0.16190387 -0.19339133 -0.512041036
## 36  -0.16022734 -0.15346072 -0.577251736
## 37  -0.79130179 -1.02562960 -0.179902268
## 38  -1.06274781 -0.34962479 -0.356743188
## 39   1.00829029  1.30165840 -0.225930833
## 40   0.92060147 -0.41592882 -0.278591733
## 41   0.58899568 -0.22719843 -0.701089136
## 42   0.87213361 -0.21113369 -0.461710041
## 43   0.47422875  0.71316855 -0.426926653
## 44   1.20229643  0.72864126 -0.350125883
## 45   0.29745803 -1.06138166  0.642223052
## 46   0.49595255 -1.30159693  3.425273313
## 47   1.09740725  2.15726661 -0.582837787
```

```
## 48  -0.53622599 -1.27196223  0.814181706
## 49   1.22761013  1.87449301  0.495654523
## 50   1.12274069  0.08873844 -0.320768811
## 51  -0.97442648 -0.32613962 -0.159696620
## 52   1.16038681  2.50055267  2.068743414
## 53  -1.18401798 -0.42466136 -0.254898896
## 54  -0.87669494 -0.61537237 -0.173918278
## 55  -0.97110497 -0.15839092 -0.386578265
## 56  -0.24500456 -0.41227066 -0.397644615
## 57   0.64926127  0.52520918  0.992124500
## 58  -1.67741360  0.77787223 -0.566410949
## 59  -1.43092243 -0.25647293 -0.147555314
## 60  -1.53763493 -0.23307292 -0.118218746
## 61   0.38070910 -1.47733476 -0.042811139
## 62   0.83712516  0.75319992 -0.656241713
## 63  -1.40221030 -0.45611775 -0.002314370
## 64   0.63103833  0.05245579 -0.480826951
## 65   0.71766229 -0.37899607  0.372125770
## 66   0.65345604  0.20460573 -0.702780076
## 67   0.66494309  0.25220401 -0.420245962
## 68   0.21024447 -0.74033812 -0.281569344
## 69  -1.22023495 -0.07444081 -0.388847365
## 70   0.44139829  0.12238075 -0.397546080
## 71  -0.34958488 -0.21679485 -0.131235308
## 72   0.61810039  0.09043629 -0.408367397
## 73   0.41677565 -0.01438097 -0.409688950
## 74  -0.23255977 -0.90317784  0.089117290
## 75   0.82788521  0.01500322 -0.483130402
## 76   0.38909207 -1.09194195 -0.056372711
## 77   1.24999134  1.54323350 -0.599467838
## 78  -1.25069010 -1.28345641  0.624638221
## 79   0.87729110 -0.98985778  0.319920552
## 80   0.99649434  0.64035841 -0.196196625
## 81   1.24386669  1.32915138 -0.434400698
## 82   0.73967571 -0.40027983 -0.393635938
## 83   0.64536573 -0.11415015 -0.189620142
## 84   0.34840814 -1.27439418  2.524115976
## 85  -1.06499201 -1.09649399 -0.203170511
## 86  -1.85738393 -0.15550432  0.317985523
## 87   0.79722728 -0.09782410 -0.318249921
## 88   0.75172514 -0.23859110 -0.433410106
## 89   0.86827226  1.33190730 -0.096214739
## 90   1.04086349  1.11983097 -0.452716091
## 91   1.10990989  2.68201971 -0.210030836
## 92  -1.58503016 -0.59727153  0.213797382
## 93  -0.77778156 -0.85690495 -0.234900452
## 94  -0.93052842 -0.22965416 -0.400090535
## 95   0.75677935 -0.01404971  0.263421163
## 96  -0.67615475 -0.75854816 -0.426729645
```

```
## 97  -1.71417861 -0.27586864 -0.326475435
## 98   1.15079862  0.88855459 -0.448709266
## 99  -0.97607118 -0.49327990 -0.200978396
## 100  0.26302945 -0.72910926  0.684224555
## 101  0.28050149 -0.36863539  0.558427524
## 102  1.05165898  0.51252923 -0.434102441
## 103 -1.78925079 -0.31675074 -0.115233914
## 104  0.46248496 -0.18850372  0.001583829
## 105  1.02064052  0.09048757  0.007994400
## 106 -0.41140781 -0.14894258 -0.306783275
## 107 -0.94916033 -0.61541656 -0.229313039
```

```
## VISUALIZATION
# Manual plotting non-rotation vs. rotation
## Initial (unrotated) factor solution
fa3nonrot <- fa(ct_scaled,
        nfactors = 3,
        rotate = "none",
        residuals = TRUE)
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs
## = np.obs, : The estimated weights for the factor scores are probably
## incorrect. Try a different factor extraction method.
```

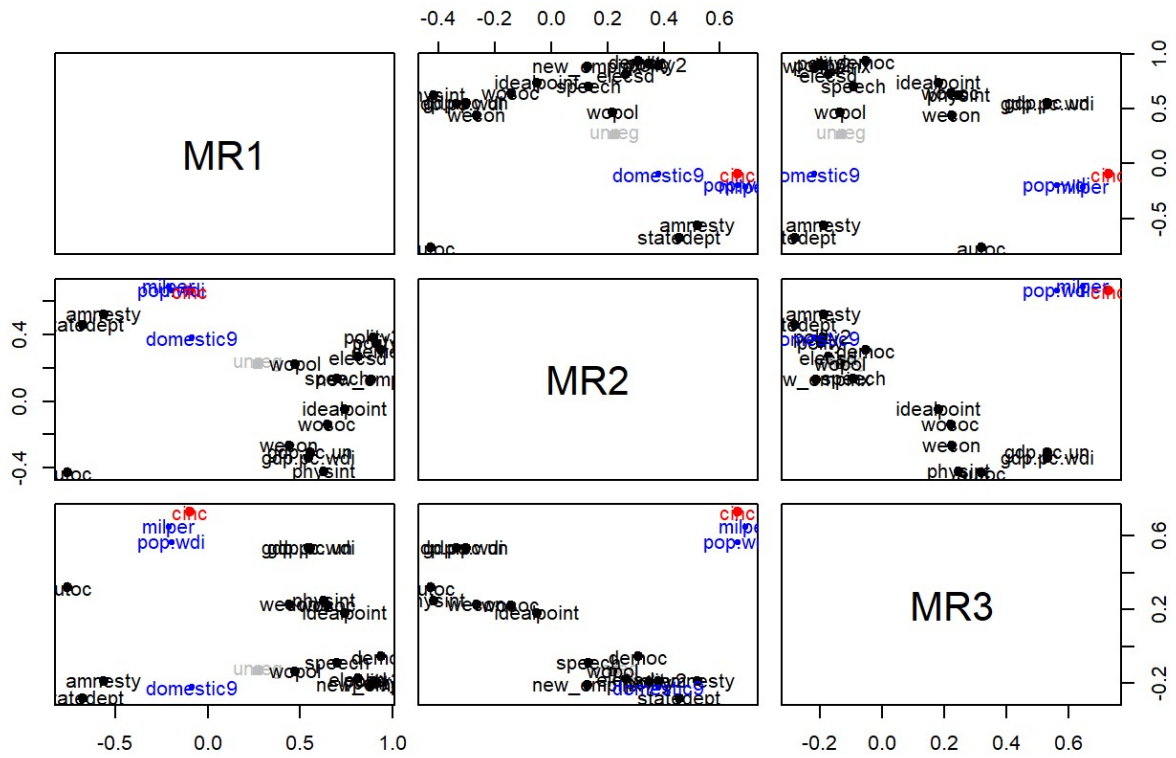
```
## In factor.scores, the correlation matrix is singular, an approximation is used
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was
## done
```

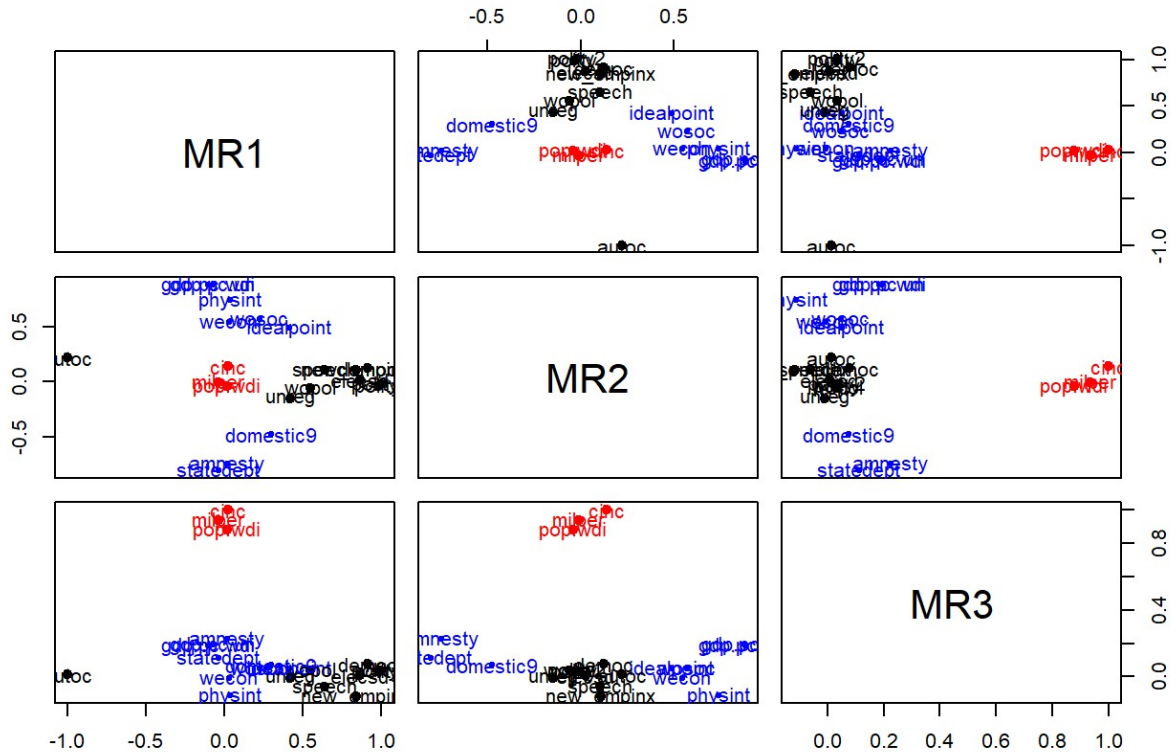
```
# loadings / structure
fa3nonrot$loadings
```

```
##
## Loadings:
##             MR1    MR2    MR3
## idealpoint  0.726         0.162
## polity      0.898  0.366 -0.189
## polity2     0.898  0.366 -0.189
## democ       0.925  0.292
## autoc      -0.778 -0.417  0.319
## unreg       0.283  0.216 -0.139
## physint     0.610 -0.434  0.260
## speech      0.693  0.120 -0.108
## new_empinx  0.884  0.135 -0.196
## wecon       0.445 -0.260  0.213
## wopol       0.456  0.236 -0.132
## wosoc       0.627 -0.158  0.238
## elecsd      0.822  0.263 -0.163
## gdp.pc.wdi  0.558 -0.319  0.543
## gdp.pc.un   0.547 -0.322  0.543
## pop.wdi    -0.176  0.675  0.572
## amnesty    -0.563  0.517 -0.186
## statedept  -0.671  0.468 -0.285
## milper     -0.217  0.680  0.639
## cinc                0.662  0.734
## domestic9           0.373 -0.214
##
##                  MR1   MR2   MR3
## SS loadings    8.258 3.203 2.512
## Proportion Var 0.393 0.153 0.120
## Cumulative Var 0.393 0.546 0.665
```

```
# Plot unrotated and rotated factor pattern

par(mfrow=c(1,2))

factor.plot(fa3nonrot, cut = 0.3, jiggle = TRUE, amount = 0.02,
            labels = rownames(fa3nonrot$loadings),
            title = "Unrotated 3Factor Loading ")
```




```
factor.plot(fa3rot, cut = 0.3, jiggle = TRUE, amount = 0.02,
            labels = rownames(fa3rot$loadings),
            title = "Rotated 3Factor Loading ")
```



In this exercise, Promax is used for oblique rotation to account for inter factor correlations. As we see from the summary, cross-loading problem is slightly ameliorated as opposed to the previous 3-factor version (which uses default oblimin transformation); and the overall proportion of variance increases from 0.649 to 0.665. As compared to a non-rotated model, the cross-loading is much better, while the cumulative variance appears similar.

Principal Components Analysis

PCA v. FA

The difference between PCA and FA is that the former is to derive component(s) by constructing a linear combination from a set of measured variables without any assumptions about these variables, whereas the latter assumes there is a latent component or components that is the cause for the set of measured variables.

As expressed through equations, for PCA, for example, for the first principal component, Comp1 = L1X1 + L2X2 +⋯+ LkXk, meaning each variable/feature (Xi) has an assigned weight (Li) and combining the optimal choice of measured variables they create comp1. For FA, on the other hand, each variable included in the feature space is assumed to be associated with the latent factor, F. More precisely, measured variables X1…Xn are caused by F, therefore the equation X1 = b1F + d1U1, where the relationship between F and each X is weighted according to b, its coefficient, and d1U1 is error term, i.e. the variance in each X that is unexplained by that factor.

Fit a PCA model:

```
library(tidyverse)
```

```
## -- Attaching packages ---------------------------------------------------------
--------------- tidyverse 1.2.1 --
```

```
## v tibble  1.4.2     v purrr   0.2.5
## v tidyr   0.8.1     v dplyr   0.7.6
## v readr   1.1.1     v stringr 1.3.1
## v tibble  1.4.2     v forcats 0.3.0
```

```
## -- Conflicts ------------------------------------------------------------------
--------- tidyverse_conflicts() --
## x ggplot2::%+%()   masks psych::%+%()
## x ggplot2::alpha() masks psych::alpha()
## x dplyr::filter()  masks stats::filter()
## x dplyr::lag()     masks stats::lag()
```

```
library(ggfortify)
```

```
## Warning: package 'ggfortify' was built under R version 3.5.3
```

```
pca <- prcomp(ct_scaled,
              center = TRUE); summary(pca)
```

```
## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6
## Standard deviation     2.9173 1.8600 1.6439 1.10713 1.07631 0.91289
## Proportion of Variance 0.4053 0.1648 0.1287 0.05837 0.05516 0.03968
## Cumulative Proportion  0.4053 0.5700 0.6987 0.75708 0.81225 0.85193
##                           PC7     PC8     PC9    PC10    PC11    PC12
## Standard deviation     0.78181 0.72948 0.64421 0.58703 0.55164 0.49341
## Proportion of Variance 0.02911 0.02534 0.01976 0.01641 0.01449 0.01159
## Cumulative Proportion  0.88104 0.90638 0.92614 0.94255 0.95704 0.96864
##                           PC13   PC14    PC15    PC16    PC17    PC18
## Standard deviation     0.46337 0.3995 0.32765 0.29011 0.24347 0.18215
## Proportion of Variance 0.01022 0.0076 0.00511 0.00401 0.00282 0.00158
## Cumulative Proportion  0.97886 0.9865 0.99157 0.99558 0.99840 0.99998
##                           PC19      PC20      PC21
## Standard deviation     0.01990 7.605e-16 2.858e-16
## Proportion of Variance 0.00002 0.000e+00 0.000e+00
## Cumulative Proportion  1.00000 1.000e+00 1.000e+00
```

```
names(pca) # rotation = loadings; x = scores
```

```
## [1] "sdev"     "rotation" "center"   "scale"    "x"
```

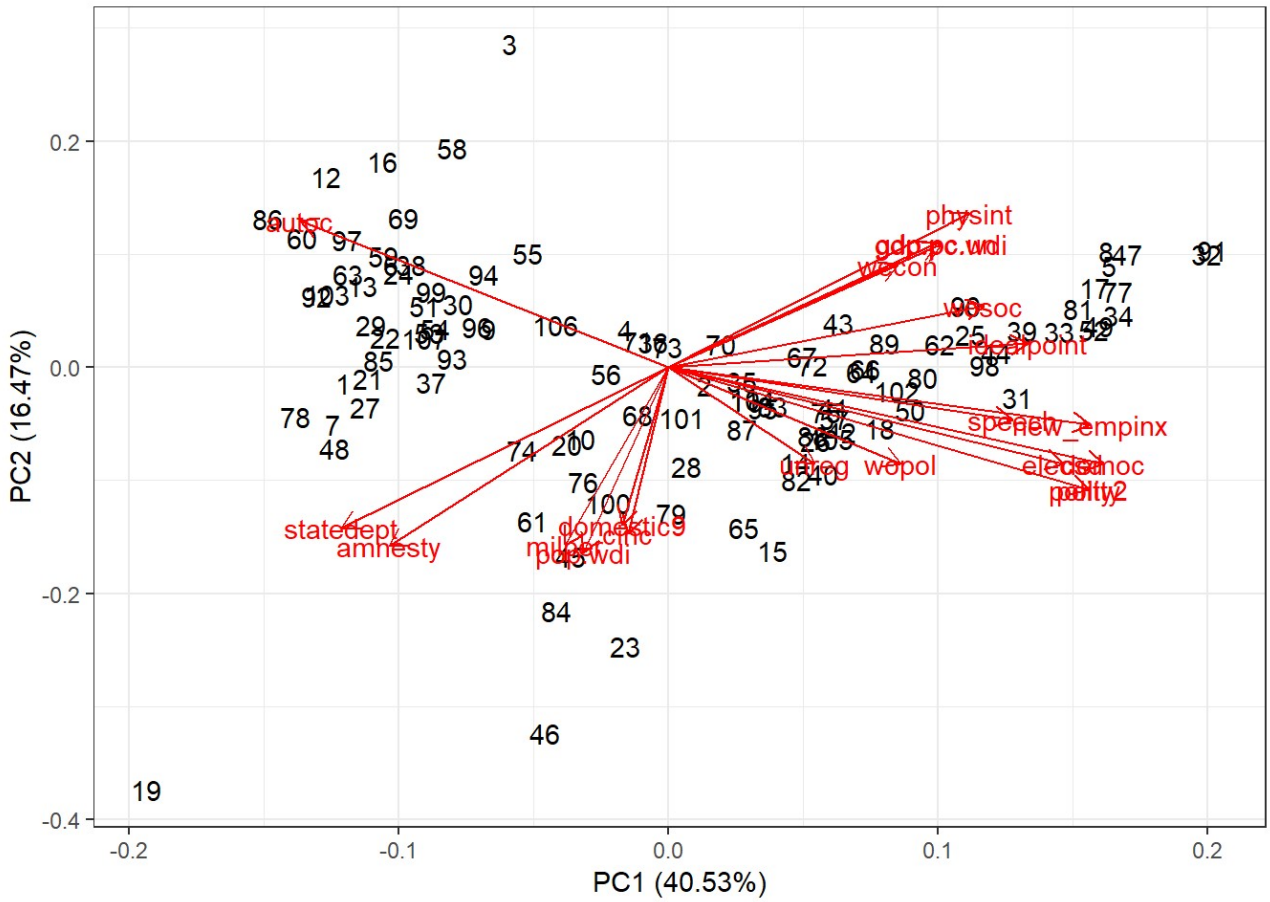
```
pca$rotation
```

```
##                    PC1         PC2         PC3         PC4         PC5
## idealpoint  0.25905085  0.04118340 -0.09502442 -0.00940566  0.04815128
## polity      0.30244275 -0.20995526  0.07988403  0.03264443  0.01088367
## polity2     0.30244275 -0.20995526  0.07988403  0.03264443  0.01088367
## democ       0.31326036 -0.16611579  0.01740663  0.01322431 -0.04334737
## autoc      -0.26438375  0.25050495 -0.15504195 -0.05538732 -0.08075072
## unreg       0.10527297 -0.16509946  0.09087275  0.11062289 -0.43869691
## physint     0.21701243  0.26401130 -0.11452036  0.19296917  0.14062832
## speech      0.24755349 -0.08928371  0.07139126  0.21451614 -0.08821487
## new_empinx  0.30361647 -0.09756571  0.11637792  0.07758241  0.02950739
## wecon       0.16500969  0.17645917 -0.12593011 -0.53724726  0.12436941
## wopol       0.16752647 -0.16580778  0.07083120 -0.38639554  0.45220464
## wosoc       0.22723513  0.10555860 -0.14452531 -0.38852986  0.10840565
## elecsd      0.28422120 -0.16492615  0.08304143  0.10093184 -0.02604124
## gdp.pc.wdi  0.19663527  0.21094312 -0.30912955 -0.10404755 -0.36221893
## gdp.pc.un   0.19323489  0.21303244 -0.30990323 -0.09895129 -0.36959847
## pop.wdi    -0.06107865 -0.31750032 -0.43241407  0.07230271  0.13653136
## amnesty    -0.20055319 -0.30654595  0.05621632 -0.19589140 -0.02169226
## statedept  -0.23491693 -0.27527869  0.12112108 -0.19757118 -0.10646581
## milper     -0.07430216 -0.30574069 -0.45614767  0.08839068  0.07036212
## cinc       -0.02868693 -0.28575076 -0.49677936  0.07797889  0.02633609
## domestic9  -0.03283672 -0.26982336  0.11798395 -0.42047789 -0.48153708
##                     PC6          PC7          PC8          PC9         PC10
## idealpoint -0.112626834 -0.567213678 -0.378198952 -0.114044591 -0.38382877
## polity     -0.111473142 -0.080416697  0.224812161 -0.037182477 -0.02517233
## polity2    -0.111473142 -0.080416697  0.224812161 -0.037182477 -0.02517233
## democ      -0.038665140 -0.111285826  0.216610085 -0.050678785 -0.06410504
## autoc       0.197602845  0.033788500 -0.217706504  0.016635901 -0.02759769
## unreg       0.773932392 -0.109431378  0.089520638  0.252565999 -0.02569712
## physint     0.231966243 -0.002177720 -0.204172096 -0.191920508  0.45925585
## speech      0.004382801  0.497834184 -0.481957919 -0.203910334 -0.09749271
## new_empinx  0.034012455  0.235200939 -0.199554485  0.013020189  0.03759950
## wecon       0.175535297  0.257412617  0.365165321 -0.332612491  0.09898381
## wopol       0.048754403 -0.017382051 -0.285786760  0.604554816  0.27500900
## wosoc       0.316032630 -0.033344990 -0.128451881 -0.141951889 -0.43845539
## elecsd     -0.036649348  0.196981852  0.085132062  0.024492799  0.03142022
## gdp.pc.wdi -0.229645775  0.139052803  0.004675724  0.275470819 -0.03106233
## gdp.pc.un  -0.230505464  0.145056575  0.008293369  0.279342414 -0.01907956
## pop.wdi     0.083138178  0.041601241  0.077751256 -0.022945755  0.07891144
## amnesty    -0.020916035  0.302400604 -0.142241350 -0.200853843 -0.21176257
## statedept  -0.040999290  0.155267715 -0.024292945  0.227893322 -0.26375488
## milper      0.050221520 -0.060162478 -0.022669641 -0.095328162  0.01030265
## cinc        0.009519597  0.001959336 -0.054167445  0.007885001  0.07833898
## domestic9  -0.126658876 -0.262503325 -0.265686014 -0.293723836  0.46602048
##                    PC11          PC12        PC13        PC14         PC15
## idealpoint  0.131442430 -0.3466154179 -0.23423114  0.18661785 -0.168626286
## polity      0.077228746 -0.0040790064  0.16322781 -0.01033419  0.018983812
## polity2     0.077228746 -0.0040790064  0.16322781 -0.01033419  0.018983812
```

```
## democ       0.080447948  0.0021718576  0.02313964 -0.04302857 -0.126924642
## autoc      -0.066914453  0.0119084594 -0.33300685 -0.03312467 -0.207776469
## unreg       0.111223960 -0.1861581486 -0.02696118 -0.04521685  0.043427356
## physint     0.364080242  0.1972011243  0.24949064  0.45876770 -0.075840027
## speech     -0.281441569 -0.3831435237  0.21190632 -0.06316281  0.173016872
## new_empinx  0.038024578  0.2132648338 -0.16625913 -0.13542936 -0.440663894
## wecon      -0.023302598 -0.4460275649 -0.15960732  0.14704707 -0.045175033
## wopol       0.044475586 -0.1288898159 -0.02025270 -0.10130925  0.117872642
## wosoc      -0.207818857  0.5397412219  0.23246386 -0.10103721  0.126273003
## elecsd     -0.188245040  0.2971967918 -0.67933879  0.29297034  0.110904335
## gdp.pc.wdi  0.122290210  0.0118431952  0.05110046 -0.02171313 -0.004180094
## gdp.pc.un   0.122924075  0.0154840432  0.04936519 -0.01463368 -0.008426744
## pop.wdi    -0.190211535 -0.0365009759  0.12637889 -0.17542301 -0.612001463
## amnesty     0.739372192  0.0699038266 -0.11338754 -0.15725924  0.005478594
## statedept  -0.103454415  0.0002247493  0.24053928  0.72506478 -0.212683785
## milper      0.004216548 -0.0031497038 -0.03887817  0.09693246  0.444576830
## cinc       -0.003891716  0.0154836568 -0.09998195  0.07583191  0.133776619
## domestic9  -0.171656353  0.1086538890  0.01524335 -0.04676860 -0.013546950
##                     PC16         PC17         PC18          PC19
## idealpoint  0.004754042 -0.140540467  0.081155618 -0.0085752206
## polity     -0.039162666  0.085258263 -0.066585235  0.0005169327
## polity2    -0.039162666  0.085258263 -0.066585235  0.0005169327
## democ      -0.214158877  0.569241009 -0.284469288  0.0061724836
## autoc      -0.192171311  0.552710896 -0.222855306  0.0069000575
## unreg       0.021359798 -0.106599731  0.042382077  0.0002288703
## physint    -0.140968548 -0.002085242  0.001194248  0.0063916498
## speech     -0.168578130  0.041034984 -0.075391868  0.0008896513
## new_empinx  0.669211646  0.149873764  0.150055726 -0.0080086116
## wecon       0.143517503 -0.027742765  0.028800340 -0.0025715545
## wopol      -0.116131152  0.058074156 -0.012064203 -0.0015550894
## wosoc      -0.053875637 -0.069984200 -0.034853565 -0.0052601070
## elecsd     -0.319656065 -0.182520937  0.079817613  0.0041631182
## gdp.pc.wdi -0.030217208 -0.022490855  0.100174887  0.7075697904
## gdp.pc.un  -0.039639351 -0.028371099  0.059204439 -0.7058685151
## pop.wdi    -0.345980249 -0.197058898  0.214076952 -0.0020600664
## amnesty    -0.174649403 -0.082528341  0.003599354 -0.0007567902
## statedept   0.074909557  0.104275335 -0.010473978  0.0022458343
## milper      0.166210093  0.393223986  0.522600646 -0.0192909419
## cinc        0.303486819 -0.214098601 -0.694623074  0.0197528372
## domestic9  -0.018112287 -0.005234861  0.017202997  0.0016763621
##                     PC20          PC21
## idealpoint -5.802069e-16  7.867543e-16
## polity     -1.062799e-01  8.531169e-01
## polity2     8.357150e-01 -2.017033e-01
## democ      -4.275639e-01 -3.818310e-01
## autoc       3.278324e-01  2.927669e-01
## unreg      -6.852158e-17  6.245005e-17
## physint     3.955170e-16 -3.330669e-16
## speech      5.100087e-16  8.326673e-17
```

```
## new_empinx -5.377643e-16 -4.440892e-16
## wecon       1.578598e-16  2.775558e-17
## wopol       9.714451e-17 -1.387779e-16
## wosoc       1.630640e-16  1.110223e-16
## elecsd     -4.857226e-17  1.387779e-16
## gdp.pc.wdi  2.827599e-15 -1.026956e-15
## gdp.pc.un  -3.070461e-15  8.604228e-16
## pop.wdi    -4.423545e-17 -2.012279e-16
## amnesty    -1.630640e-16  1.387779e-16
## statedept  -4.163336e-17 -2.498002e-16
## milper     -1.435484e-16 -1.595946e-16
## cinc        2.044805e-16  2.584738e-16
## domestic9   2.836273e-16  2.428613e-17
```

```
# visualize
autoplot(pca,
         shape = F,
         loadings.label = T) +
theme_bw()
```



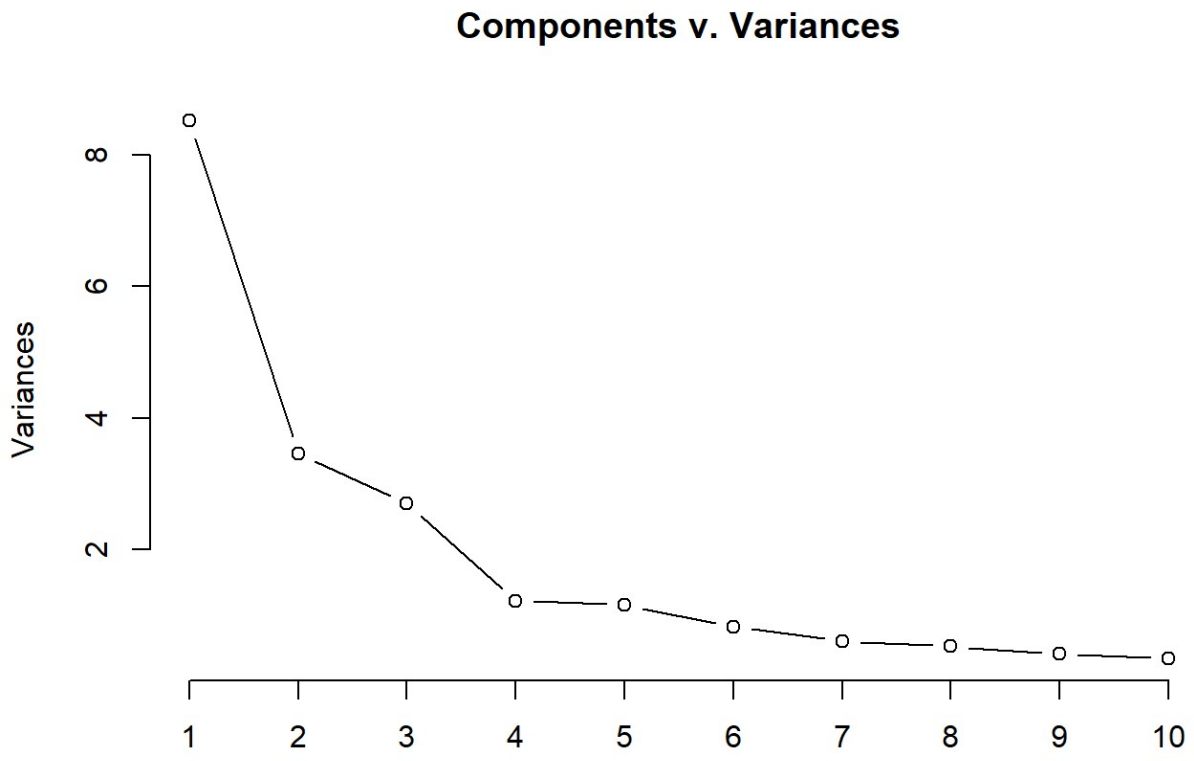
```
# loadings and scores
loadings <- pca$rotation
loadings[1:10, 1:10]
```

```
##                   PC1         PC2         PC3         PC4         PC5
## idealpoint  0.2590509  0.04118340 -0.09502442 -0.00940566  0.04815128
## polity      0.3024428 -0.20995526  0.07988403  0.03264443  0.01088367
## polity2     0.3024428 -0.20995526  0.07988403  0.03264443  0.01088367
## democ       0.3132604 -0.16611579  0.01740663  0.01322431 -0.04334737
## autoc      -0.2643837  0.25050495 -0.15504195 -0.05538732 -0.08075072
## unreg       0.1052730 -0.16509946  0.09087275  0.11062289 -0.43869691
## physint     0.2170124  0.26401130 -0.11452036  0.19296917  0.14062832
## speech      0.2475535 -0.08928371  0.07139126  0.21451614 -0.08821487
## new_empinx  0.3036165 -0.09756571  0.11637792  0.07758241  0.02950739
## wecon       0.1650097  0.17645917 -0.12593011 -0.53724726  0.12436941
##                    PC6         PC7         PC8         PC9        PC10
## idealpoint -0.112626834 -0.56721368 -0.37819895 -0.11404459 -0.38382877
## polity     -0.111473142 -0.08041670  0.22481216 -0.03718248 -0.02517233
## polity2    -0.111473142 -0.08041670  0.22481216 -0.03718248 -0.02517233
## democ      -0.038665140 -0.11128583  0.21661009 -0.05067878 -0.06410504
## autoc       0.197602845  0.03378850 -0.21770650  0.01663590 -0.02759769
## unreg       0.773932392 -0.10943138  0.08952064  0.25256600 -0.02569712
## physint     0.231966243 -0.00217772 -0.20417210 -0.19192051  0.45925585
## speech      0.004382801  0.49783418 -0.48195792 -0.20391033 -0.09749271
## new_empinx  0.034012455  0.23520094 -0.19955449  0.01302019  0.03759950
## wecon       0.175535297  0.25741262  0.36516532 -0.33261249  0.09898381
```

```
scores <- pca$x
scores[1:10, 1:10]
```

```
##              PC1        PC2         PC3         PC4         PC5        PC6
##  [1,] -3.6399214 -0.2672129  0.61794318 -1.15914878  0.68409947 -0.7339635
##  [2,]  0.4130631 -0.3079309  0.79222392  0.44603842  0.60670923 -0.3673925
##  [3,] -1.7735942  5.4981470 -3.61876060 -0.03256528 -3.59482373 -0.4704274
##  [4,] -0.4803014  0.6320023  0.53232376  1.27958406 -0.11368204 -0.1949473
##  [5,]  4.9340546  1.7301635 -1.55885993 -0.60720269  0.01285816 -0.8868693
##  [6,] -3.1032132  1.7390704 -0.07516694 -0.18543181  0.58515420  0.7443185
##  [7,] -3.7505535 -0.9765606  1.30698392 -1.18679404 -0.03745869 -1.4925618
##  [8,]  4.9025193  1.9749611 -1.71693340 -0.77608010 -0.37918750 -0.1366512
##  [9,] -2.0126927  0.6475680  1.04828020  1.99526353  0.75338411 -0.9891571
## [10,] -0.9812800 -1.2197379  0.53792692  0.00982798  0.69138492 -0.2749670
##               PC7         PC8         PC9        PC10
##  [1,]  0.09629664 -0.07737017  0.97115943 -1.32048628
##  [2,] -1.16522549 -0.39071765  0.03849354 -0.97487385
##  [3,]  0.49535850  1.24739632 -0.12905567  0.32765276
##  [4,] -1.17642955  0.67027541 -0.90568080 -0.65864797
##  [5,]  0.38862856 -0.20542156 -0.27575523 -0.51741578
##  [6,] -1.19676898 -0.62700497  0.95954374 -0.04008785
##  [7,] -0.22477474  0.15789231  0.73770796 -0.05149526
##  [8,] -0.64731974  0.36697805  0.23354675 -0.37419400
##  [9,]  0.07982751 -1.06111988  1.07699633  0.99441769
## [10,] -0.44741170  1.57457891  0.77719781 -0.25015210
```

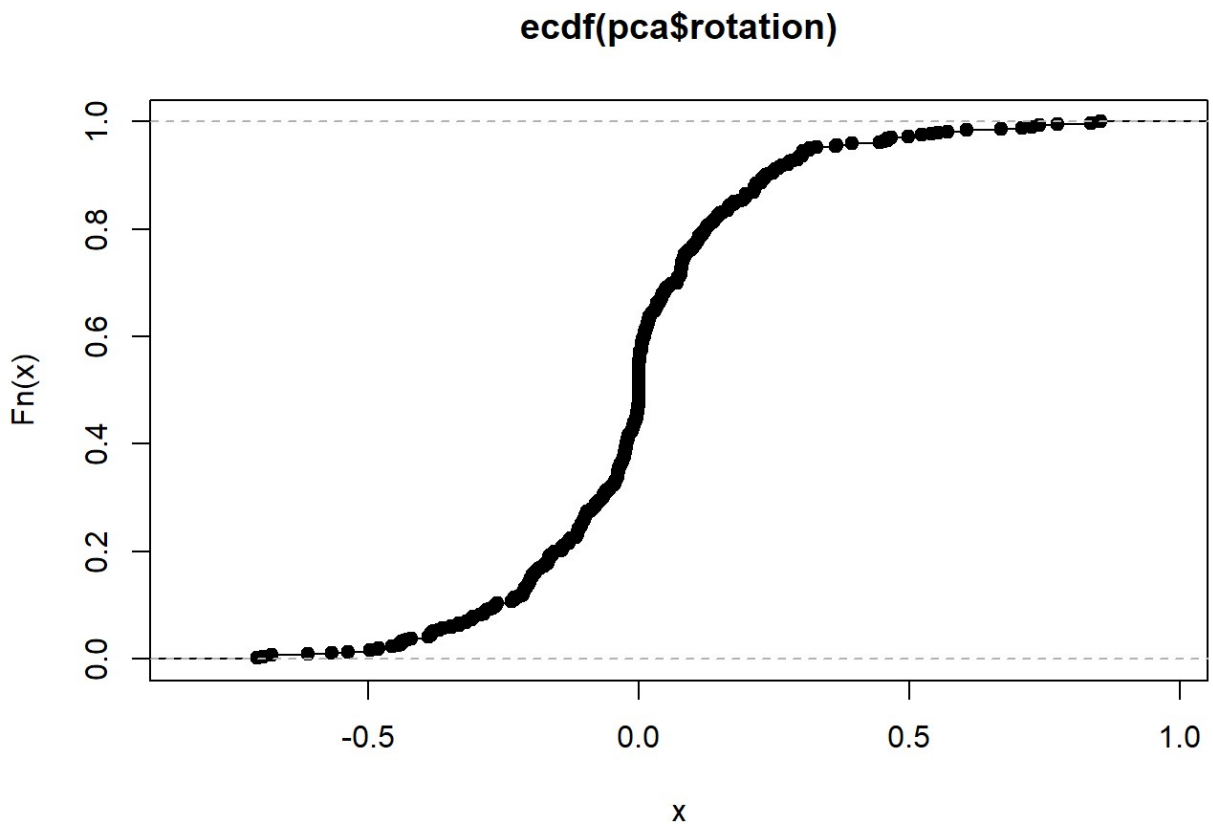
```
# Plot PCA object, which returns a scree plot of the variances (y-axis) associated wit
h the PCs (x-axis)
plot(pca,
     type="l",
     main = "Components v. Variances")
```




```
summary(pca)
```

```
## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6
## Standard deviation     2.9173 1.8600 1.6439 1.10713 1.07631 0.91289
## Proportion of Variance 0.4053 0.1648 0.1287 0.05837 0.05516 0.03968
## Cumulative Proportion  0.4053 0.5700 0.6987 0.75708 0.81225 0.85193
##                           PC7     PC8     PC9    PC10    PC11    PC12
## Standard deviation     0.78181 0.72948 0.64421 0.58703 0.55164 0.49341
## Proportion of Variance 0.02911 0.02534 0.01976 0.01641 0.01449 0.01159
## Cumulative Proportion  0.88104 0.90638 0.92614 0.94255 0.95704 0.96864
##                           PC13   PC14    PC15    PC16    PC17    PC18
## Standard deviation     0.46337 0.3995 0.32765 0.29011 0.24347 0.18215
## Proportion of Variance 0.01022 0.0076 0.00511 0.00401 0.00282 0.00158
## Cumulative Proportion  0.97886 0.9865 0.99157 0.99558 0.99840 0.99998
##                           PC19      PC20      PC21
## Standard deviation     0.01990 7.605e-16 2.858e-16
## Proportion of Variance 0.00002 0.000e+00 0.000e+00
## Cumulative Proportion  1.00000 1.000e+00 1.000e+00
```

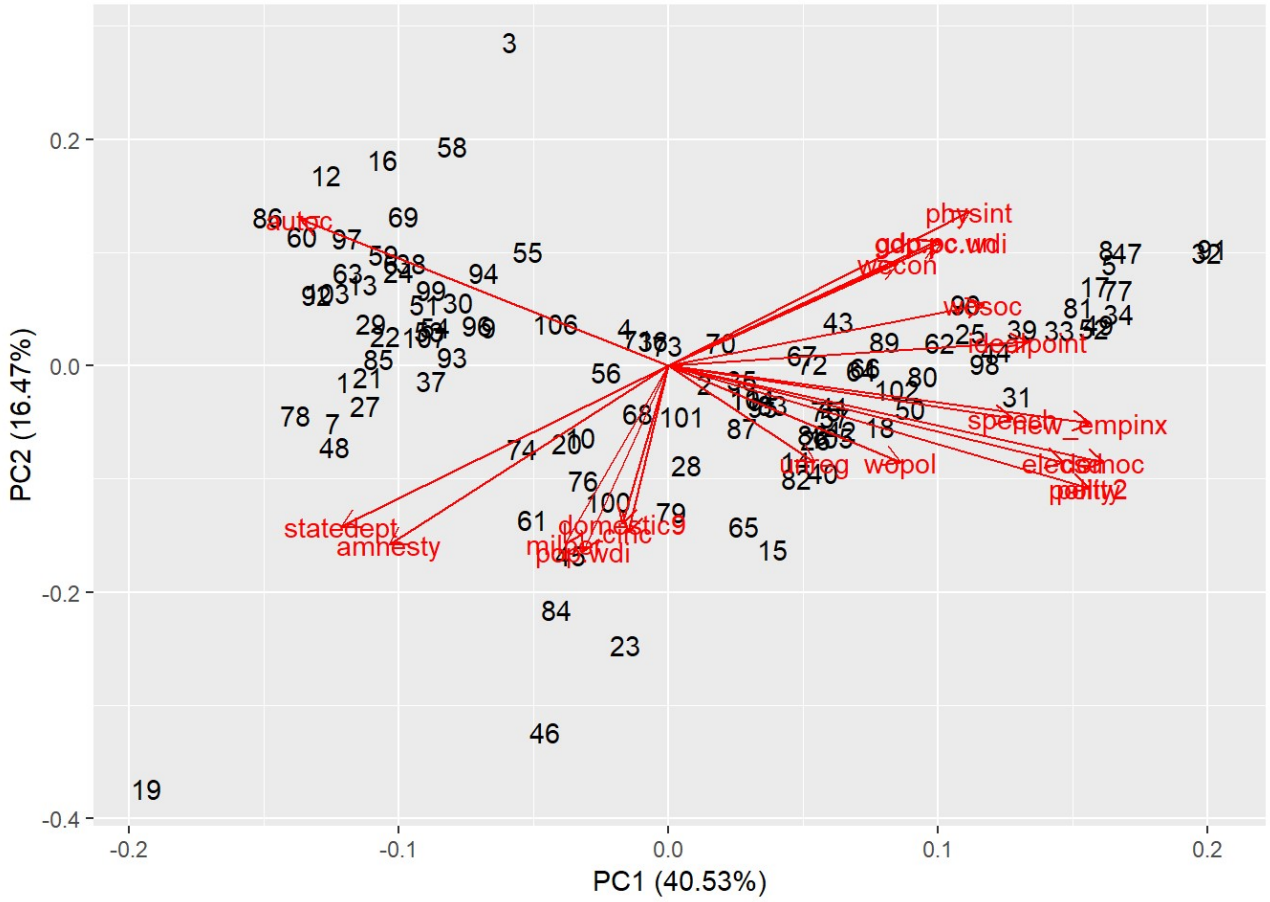
```
# Plot cumulative density for overall variance

plot(ecdf(pca$rotation))
```



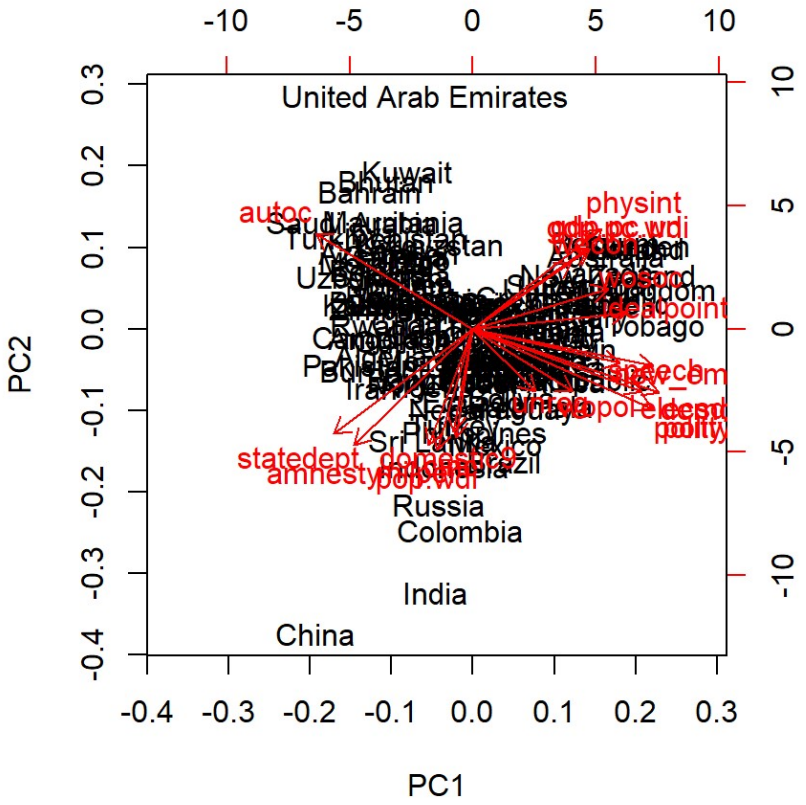
According to the plot, great proportion of the overall variances are explained once we reach components number of 4, after which the marginal cumulative variance for each additional component diminishes. A summary of the PCA data frame also shows that cumulative proportion of variance gradually achieves 0.75708 from PC1 to PC4, and such increase in proportion for later introduced components is not as significant as the previous four.

Presenting the biplot of the PCA:

```
# prety version from above (i.e., ggplot2)
autoplot(pca,
         shape = F, # labaels instead of points
         loadings.label = T)
```




```
par(mfrow=c(1,1))
# simple biplot version in base R
biplot(pca,
       xlabs = countries$X)
```



According to the biplot, it seems like asian countries are clustered down towards first principal component, mostly based on the political indicators such as democracy and polity scores; and african countries are clustered towards top left hand corner, which is primarily determined by “autoc”, i.e. institutionalized autocracy.

PCA Extension

Fit a sparse PCA model:

```
library(sparsepca)
```

```
## Warning: package 'sparsepca' was built under R version 3.5.3
```

```
spca <- spca(ct_scaled, center = TRUE); summary(spca)
```

```
## [1] "Iteration:    1, Objective: 6.79879e+00, Relative improvement Inf"
## [1] "Iteration:   11, Objective: 6.76709e+00, Relative improvement 3.84129e-04"
## [1] "Iteration:   21, Objective: 6.74437e+00, Relative improvement 3.04899e-04"
## [1] "Iteration:   31, Objective: 6.72565e+00, Relative improvement 2.61948e-04"
## [1] "Iteration:   41, Objective: 6.70896e+00, Relative improvement 2.38216e-04"
## [1] "Iteration:   51, Objective: 6.69373e+00, Relative improvement 2.21149e-04"
## [1] "Iteration:   61, Objective: 6.67954e+00, Relative improvement 2.06385e-04"
## [1] "Iteration:   71, Objective: 6.66627e+00, Relative improvement 1.94172e-04"
## [1] "Iteration:   81, Objective: 6.65358e+00, Relative improvement 1.85640e-04"
## [1] "Iteration:   91, Objective: 6.64148e+00, Relative improvement 1.79731e-04"
## [1] "Iteration:  101, Objective: 6.62968e+00, Relative improvement 1.76529e-04"
## [1] "Iteration:  111, Objective: 6.61809e+00, Relative improvement 1.72226e-04"
## [1] "Iteration:  121, Objective: 6.60686e+00, Relative improvement 1.68873e-04"
## [1] "Iteration:  131, Objective: 6.59577e+00, Relative improvement 1.67404e-04"
## [1] "Iteration:  141, Objective: 6.58482e+00, Relative improvement 1.64935e-04"
## [1] "Iteration:  151, Objective: 6.57406e+00, Relative improvement 1.62183e-04"
## [1] "Iteration:  161, Objective: 6.56350e+00, Relative improvement 1.60268e-04"
## [1] "Iteration:  171, Objective: 6.55301e+00, Relative improvement 1.59683e-04"
## [1] "Iteration:  181, Objective: 6.54257e+00, Relative improvement 1.59348e-04"
## [1] "Iteration:  191, Objective: 6.53221e+00, Relative improvement 1.56416e-04"
## [1] "Iteration:  201, Objective: 6.52207e+00, Relative improvement 1.54850e-04"
## [1] "Iteration:  211, Objective: 6.51197e+00, Relative improvement 1.54887e-04"
## [1] "Iteration:  221, Objective: 6.50194e+00, Relative improvement 1.53818e-04"
## [1] "Iteration:  231, Objective: 6.49199e+00, Relative improvement 1.52803e-04"
## [1] "Iteration:  241, Objective: 6.48212e+00, Relative improvement 1.50950e-04"
## [1] "Iteration:  251, Objective: 6.47234e+00, Relative improvement 1.51031e-04"
## [1] "Iteration:  261, Objective: 6.46257e+00, Relative improvement 1.51257e-04"
## [1] "Iteration:  271, Objective: 6.45282e+00, Relative improvement 1.50951e-04"
## [1] "Iteration:  281, Objective: 6.44317e+00, Relative improvement 1.49101e-04"
## [1] "Iteration:  291, Objective: 6.43359e+00, Relative improvement 1.47819e-04"
## [1] "Iteration:  301, Objective: 6.42413e+00, Relative improvement 1.46984e-04"
## [1] "Iteration:  311, Objective: 6.41470e+00, Relative improvement 1.47035e-04"
## [1] "Iteration:  321, Objective: 6.40526e+00, Relative improvement 1.47356e-04"
## [1] "Iteration:  331, Objective: 6.39588e+00, Relative improvement 1.46193e-04"
## [1] "Iteration:  341, Objective: 6.38661e+00, Relative improvement 1.44870e-04"
## [1] "Iteration:  351, Objective: 6.37736e+00, Relative improvement 1.44685e-04"
## [1] "Iteration:  361, Objective: 6.36813e+00, Relative improvement 1.45037e-04"
## [1] "Iteration:  371, Objective: 6.35888e+00, Relative improvement 1.45426e-04"
## [1] "Iteration:  381, Objective: 6.34969e+00, Relative improvement 1.44675e-04"
## [1] "Iteration:  391, Objective: 6.34050e+00, Relative improvement 1.45151e-04"
## [1] "Iteration:  401, Objective: 6.33128e+00, Relative improvement 1.45723e-04"
## [1] "Iteration:  411, Objective: 6.32208e+00, Relative improvement 1.45622e-04"
## [1] "Iteration:  421, Objective: 6.31286e+00, Relative improvement 1.46180e-04"
## [1] "Iteration:  431, Objective: 6.30362e+00, Relative improvement 1.46821e-04"
## [1] "Iteration:  441, Objective: 6.29435e+00, Relative improvement 1.47521e-04"
## [1] "Iteration:  451, Objective: 6.28509e+00, Relative improvement 1.45208e-04"
## [1] "Iteration:  461, Objective: 6.27604e+00, Relative improvement 1.44065e-04"
## [1] "Iteration:  471, Objective: 6.26700e+00, Relative improvement 1.44245e-04"
```

```
## [1] "Iteration:  481, Objective: 6.25796e+00, Relative improvement 1.44146e-04"
## [1] "Iteration:  491, Objective: 6.24893e+00, Relative improvement 1.44629e-04"
## [1] "Iteration:  501, Objective: 6.23989e+00, Relative improvement 1.44529e-04"
## [1] "Iteration:  511, Objective: 6.23088e+00, Relative improvement 1.44555e-04"
## [1] "Iteration:  521, Objective: 6.22187e+00, Relative improvement 1.45020e-04"
## [1] "Iteration:  531, Objective: 6.21283e+00, Relative improvement 1.45592e-04"
## [1] "Iteration:  541, Objective: 6.20379e+00, Relative improvement 1.45993e-04"
## [1] "Iteration:  551, Objective: 6.19472e+00, Relative improvement 1.46638e-04"
## [1] "Iteration:  561, Objective: 6.18562e+00, Relative improvement 1.47329e-04"
## [1] "Iteration:  571, Objective: 6.17652e+00, Relative improvement 1.46062e-04"
## [1] "Iteration:  581, Objective: 6.16752e+00, Relative improvement 1.45925e-04"
## [1] "Iteration:  591, Objective: 6.15851e+00, Relative improvement 1.46383e-04"
## [1] "Iteration:  601, Objective: 6.14948e+00, Relative improvement 1.46939e-04"
## [1] "Iteration:  611, Objective: 6.14047e+00, Relative improvement 1.45330e-04"
## [1] "Iteration:  621, Objective: 6.13161e+00, Relative improvement 1.43862e-04"
## [1] "Iteration:  631, Objective: 6.12279e+00, Relative improvement 1.44137e-04"
## [1] "Iteration:  641, Objective: 6.11395e+00, Relative improvement 1.44569e-04"
## [1] "Iteration:  651, Objective: 6.10510e+00, Relative improvement 1.45006e-04"
## [1] "Iteration:  661, Objective: 6.09624e+00, Relative improvement 1.45519e-04"
## [1] "Iteration:  671, Objective: 6.08736e+00, Relative improvement 1.46135e-04"
## [1] "Iteration:  681, Objective: 6.07845e+00, Relative improvement 1.46807e-04"
## [1] "Iteration:  691, Objective: 6.06951e+00, Relative improvement 1.47483e-04"
## [1] "Iteration:  701, Objective: 6.06054e+00, Relative improvement 1.48136e-04"
## [1] "Iteration:  711, Objective: 6.05155e+00, Relative improvement 1.48904e-04"
## [1] "Iteration:  721, Objective: 6.04252e+00, Relative improvement 1.49703e-04"
## [1] "Iteration:  731, Objective: 6.03345e+00, Relative improvement 1.50527e-04"
## [1] "Iteration:  741, Objective: 6.02439e+00, Relative improvement 1.50070e-04"
## [1] "Iteration:  751, Objective: 6.01534e+00, Relative improvement 1.50748e-04"
## [1] "Iteration:  761, Objective: 6.00625e+00, Relative improvement 1.51527e-04"
## [1] "Iteration:  771, Objective: 5.99713e+00, Relative improvement 1.52355e-04"
## [1] "Iteration:  781, Objective: 5.98800e+00, Relative improvement 1.52553e-04"
## [1] "Iteration:  791, Objective: 5.97886e+00, Relative improvement 1.52746e-04"
## [1] "Iteration:  801, Objective: 5.96973e+00, Relative improvement 1.52961e-04"
## [1] "Iteration:  811, Objective: 5.96059e+00, Relative improvement 1.53610e-04"
## [1] "Iteration:  821, Objective: 5.95145e+00, Relative improvement 1.53511e-04"
## [1] "Iteration:  831, Objective: 5.94230e+00, Relative improvement 1.54167e-04"
## [1] "Iteration:  841, Objective: 5.93312e+00, Relative improvement 1.54594e-04"
## [1] "Iteration:  851, Objective: 5.92394e+00, Relative improvement 1.54874e-04"
## [1] "Iteration:  861, Objective: 5.91481e+00, Relative improvement 1.53921e-04"
## [1] "Iteration:  871, Objective: 5.90569e+00, Relative improvement 1.54542e-04"
## [1] "Iteration:  881, Objective: 5.89655e+00, Relative improvement 1.55268e-04"
## [1] "Iteration:  891, Objective: 5.88754e+00, Relative improvement 1.52737e-04"
## [1] "Iteration:  901, Objective: 5.87853e+00, Relative improvement 1.53470e-04"
## [1] "Iteration:  911, Objective: 5.86949e+00, Relative improvement 1.54230e-04"
## [1] "Iteration:  921, Objective: 5.86043e+00, Relative improvement 1.54769e-04"
## [1] "Iteration:  931, Objective: 5.85138e+00, Relative improvement 1.54203e-04"
## [1] "Iteration:  941, Objective: 5.84235e+00, Relative improvement 1.54816e-04"
## [1] "Iteration:  951, Objective: 5.83330e+00, Relative improvement 1.54691e-04"
## [1] "Iteration:  961, Objective: 5.82426e+00, Relative improvement 1.55355e-04"
```

```
## [1] "Iteration:  971, Objective: 5.81526e+00, Relative improvement 1.54329e-04"
## [1] "Iteration:  981, Objective: 5.80628e+00, Relative improvement 1.54831e-04"
## [1] "Iteration:  991, Objective: 5.79732e+00, Relative improvement 1.54421e-04"
```

```
##                           PC1   PC2   PC3   PC4   PC5   PC6   PC7   PC8
## Explained variance     8.507 3.456 2.699 1.222 1.155 0.830 0.608 0.529
## Standard deviations    2.917 1.859 1.643 1.106 1.075 0.911 0.780 0.727
## Proportion of variance 0.405 0.165 0.129 0.058 0.055 0.040 0.029 0.025
## Cumulative proportion  0.405 0.570 0.698 0.756 0.811 0.851 0.880 0.905
##                           PC9  PC10  PC11  PC12  PC13  PC14  PC15  PC16
## Explained variance     0.412 0.341 0.301 0.240 0.211 0.156 0.104 0.081
## Standard deviations    0.642 0.584 0.549 0.490 0.460 0.395 0.323 0.284
## Proportion of variance 0.020 0.016 0.014 0.011 0.010 0.007 0.005 0.004
## Cumulative proportion  0.925 0.941 0.955 0.967 0.977 0.984 0.989 0.993
##                          PC17  PC18  PC19  PC20  PC21
## Explained variance     0.056 0.030 0.000 0.000 0.000
## Standard deviations    0.237 0.174 0.018 0.000 0.000
## Proportion of variance 0.003 0.001 0.000 0.000 0.000
## Cumulative proportion  0.996 0.997 0.997 0.997 0.997
```

```
names(spca)
```

```
## [1] "loadings"    "transform"   "scores"      "eigenvalues" "center"
## [6] "scale"       "objective"   "sdev"        "var"
```

```
spca$loadings
```

```
##              [,1]        [,2]        [,3]        [,4]          [,5]
##  [1,]  0.29381532  0.02288480 -0.04281433  0.00000000  0.0183421168
##  [2,]  0.35573582 -0.19596343  0.08268949  0.00000000  0.0000000000
##  [3,]  0.35573582 -0.19596343  0.08268949  0.00000000  0.0000000000
##  [4,]  0.35987224 -0.15515946  0.05256899  0.00000000  0.0000000000
##  [5,] -0.31463465  0.22612662 -0.10788930 -0.01401594  0.0000000000
##  [6,]  0.06021595 -0.12960713  0.04390352  0.09294084 -0.3815498820
##  [7,]  0.24561933  0.31716581 -0.15206401  0.17971556  0.1822159236
##  [8,]  0.22919059 -0.04645770  0.00888112  0.15346120 -0.0237555743
##  [9,]  0.22912100 -0.04311076  0.09833092  0.03494560  0.0000000000
## [10,]  0.06826121  0.23949682 -0.13921052 -0.58157817  0.1147455323
## [11,]  0.10538005 -0.09267341  0.03459803 -0.39971571  0.4602363014
## [12,]  0.17446019  0.16853101 -0.12183116 -0.41986384  0.0473881263
## [13,]  0.25622892 -0.13713380  0.01877174  0.03783766  0.0007258234
## [14,]  0.15602030  0.27617981 -0.23840357 -0.06281784 -0.4109372128
## [15,]  0.15095336  0.27739639 -0.23771007 -0.05664808 -0.4192265067
## [16,] -0.01043035 -0.23214786 -0.49127375  0.00000000  0.1018324305
## [17,] -0.10978972 -0.29050876  0.05666113 -0.15206300 -0.0133860249
## [18,] -0.26261844 -0.24886228  0.09434001 -0.12862413 -0.0890366129
## [19,] -0.03933835 -0.29977336 -0.49728666  0.07223905  0.0417780980
## [20,] -0.02641255 -0.25372033 -0.51632567  0.05061153  0.0280566015
## [21,] -0.04331759 -0.30995559  0.10895016 -0.44456778 -0.4715286975
##               [,6]         [,7]         [,8]         [,9]        [,10]
##  [1,] -0.037106136 -0.638516637 -0.378163581 -0.066501771 -0.374243739
##  [2,] -0.071048461 -0.008529955  0.214988707  0.000000000  0.000000000
##  [3,] -0.071048461 -0.008529955  0.214988707  0.000000000  0.000000000
##  [4,] -0.021890036 -0.037450267  0.185685665  0.000000000 -0.006355993
##  [5,]  0.121999681  0.000000000 -0.228645579  0.000000000  0.000000000
##  [6,]  0.842164125 -0.011759327  0.058118748  0.275421773  0.000000000
##  [7,]  0.208746143 -0.004924839 -0.145776160 -0.201652911  0.486352275
##  [8,]  0.000000000  0.553066069 -0.536002587 -0.174002562 -0.082397456
##  [9,]  0.002421908  0.200184329 -0.150405759  0.000000000  0.000000000
## [10,]  0.137916757  0.199424897  0.361701664 -0.285005084  0.004901130
## [11,]  0.015530911  0.000000000 -0.270604161  0.662077873  0.262955242
## [12,]  0.253805707  0.000000000 -0.146516953 -0.134473086 -0.441916156
## [13,]  0.000000000  0.182709925  0.020405133  0.000000000  0.000000000
## [14,] -0.234785653  0.095492392  0.000000000  0.285364293 -0.019202253
## [15,] -0.237130040  0.103007956  0.000000000  0.289084030 -0.006531289
## [16,]  0.029734309  0.000000000  0.028470195 -0.016257367  0.048198859
## [17,] -0.005284484  0.283672684 -0.135885886 -0.102138010 -0.185757802
## [18,] -0.085140844  0.123441753  0.008350163  0.189581574 -0.254390112
## [19,]  0.015141947 -0.032609328  0.000000000 -0.040117844  0.000000000
## [20,]  0.000000000  0.000000000  0.000000000 -0.002431232  0.004998845
## [21,] -0.085784397 -0.210781971 -0.278637817 -0.282512915  0.481484873
##              [,11]       [,12]        [,13]         [,14]         [,15]
##  [1,]  0.141127095 -0.35375995 -0.146634857  0.1429988046 -0.0583864991
##  [2,]  0.016482814  0.00000000  0.143698190  0.0000000000  0.0000000000
##  [3,]  0.016482814  0.00000000  0.143698190  0.0000000000  0.0000000000
```

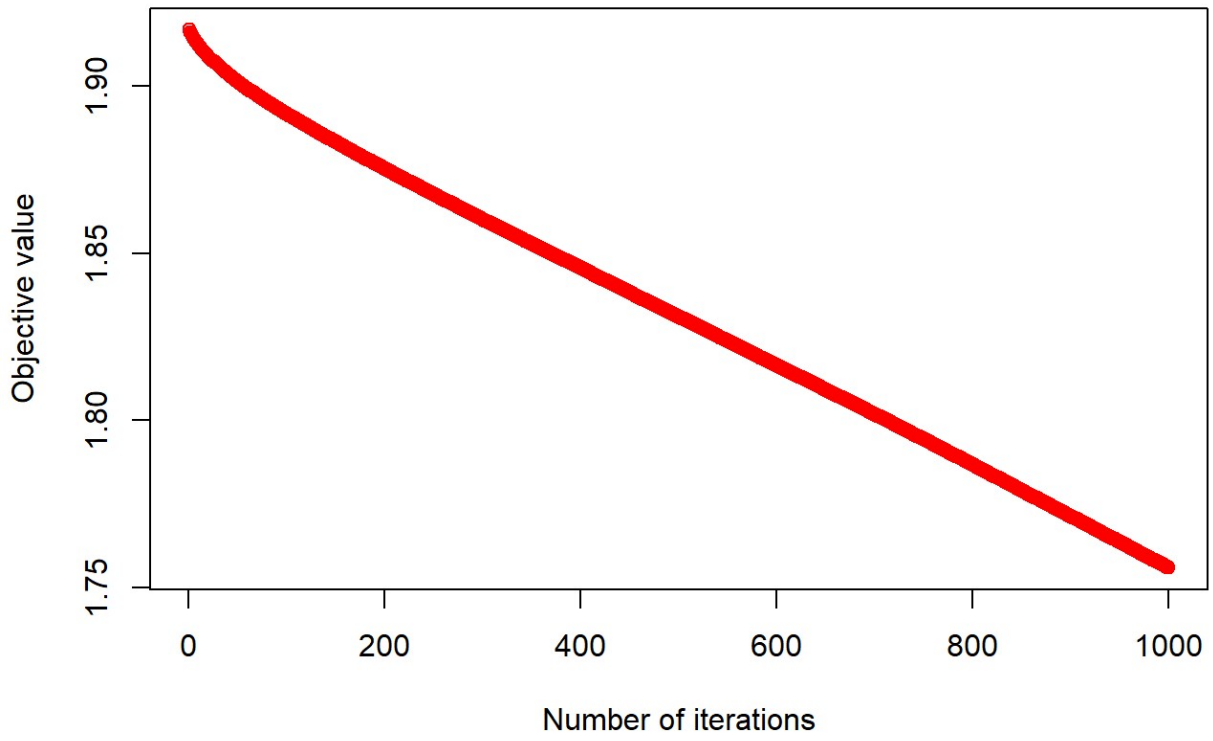
```
##  [4,]  0.004327098  0.00000000  0.024654721  0.0000000000 -0.0688403387
##  [5,] -0.023495616  0.00000000 -0.280041383  0.0000000000 -0.1568253811
##  [6,]  0.023972007 -0.14832679  0.000000000 -0.0166652112  0.0000000000
##  [7,]  0.343456053  0.17415315  0.189674062  0.4491395918 -0.0133325134
##  [8,] -0.284898536 -0.34696679  0.182479195 -0.0175172092  0.1103317514
##  [9,]  0.010739033  0.11259418 -0.165877919 -0.0457693346 -0.4421399059
## [10,]  0.000000000 -0.48853898 -0.140486184  0.1100199646 -0.0011880866
## [11,]  0.000000000 -0.05975610  0.000000000 -0.0574336053  0.0780545704
## [12,] -0.148632089  0.59062997  0.215654625 -0.0602385669  0.0780894847
## [13,] -0.111025159  0.25583158 -0.782882891  0.2446999079  0.0904598296
## [14,]  0.106374054  0.00000000  0.005868844  0.0000000000  0.0000000000
## [15,]  0.106047766  0.00000000  0.006409134  0.0000000000  0.0000000000
## [16,] -0.113196200  0.00000000  0.069983992 -0.1149777044 -0.6716831693
## [17,]  0.819458493  0.02188939 -0.045833724 -0.1598454451 -0.0007409286
## [18,] -0.052553201  0.00000000  0.196244007  0.7867188907 -0.1484677124
## [19,] -0.004205042  0.00000000 -0.012981873  0.0456661297  0.4413244125
## [20,]  0.000000000  0.00000000 -0.025266850  0.0180418793  0.1271022607
## [21,] -0.113636984  0.07565260  0.000000000 -0.0002130629  0.0000000000
##              [,16]        [,17]        [,18]      [,19]      [,20]
##  [1,]  0.000000000 -0.064990578  0.010493276  0.0000000  0.0000000
##  [2,]  0.000000000  0.026684497  0.000000000  0.0000000  0.0000000
##  [3,]  0.000000000  0.026684497  0.000000000  0.0000000  0.6774843
##  [4,] -0.205399081  0.625008652 -0.244205069  0.0000000 -0.2748413
##  [5,] -0.088851653  0.572991441 -0.160902710  0.0000000  0.1868845
##  [6,]  0.000000000 -0.039785063  0.000000000  0.0000000  0.0000000
##  [7,] -0.034007211 -0.001321243  0.000000000  0.0000000  0.0000000
##  [8,] -0.149067779  0.000000000 -0.016002915  0.0000000  0.0000000
##  [9,]  0.745496646  0.098845881  0.110584304  0.0000000  0.0000000
## [10,]  0.090194210 -0.004710228  0.003098939  0.0000000  0.0000000
## [11,] -0.008643091  0.000000000  0.000000000  0.0000000  0.0000000
## [12,] -0.018896241  0.000000000  0.000000000  0.0000000  0.0000000
## [13,] -0.241914965 -0.171503951  0.014787944  0.0000000  0.0000000
## [14,] -0.012768619  0.000000000  0.036796902  0.5494879  0.0000000
## [15,] -0.025513034  0.000000000  0.000000000 -0.5467614  0.0000000
## [16,] -0.337467153 -0.132057488  0.173441322  0.0000000  0.0000000
## [17,] -0.121657261  0.000000000  0.000000000  0.0000000  0.0000000
## [18,]  0.016524548  0.040627082  0.000000000  0.0000000  0.0000000
## [19,]  0.174453157  0.310985730  0.505487574  0.0000000  0.0000000
## [20,]  0.276084425 -0.166086917 -0.678968729  0.0000000  0.0000000
## [21,]  0.000000000  0.000000000  0.000000000  0.0000000  0.0000000
##             [,21]
##  [1,]  0.00000000
##  [2,]  0.69336372
##  [3,] -0.07076568
##  [4,] -0.23330649
##  [5,]  0.15505939
##  [6,]  0.00000000
##  [7,]  0.00000000
##  [8,]  0.00000000
```

```
##  [9,]  0.00000000
## [10,]  0.00000000
## [11,]  0.00000000
## [12,]  0.00000000
## [13,]  0.00000000
## [14,]  0.00000000
## [15,]  0.00000000
## [16,]  0.00000000
## [17,]  0.00000000
## [18,]  0.00000000
## [19,]  0.00000000
## [20,]  0.00000000
## [21,]  0.00000000
```

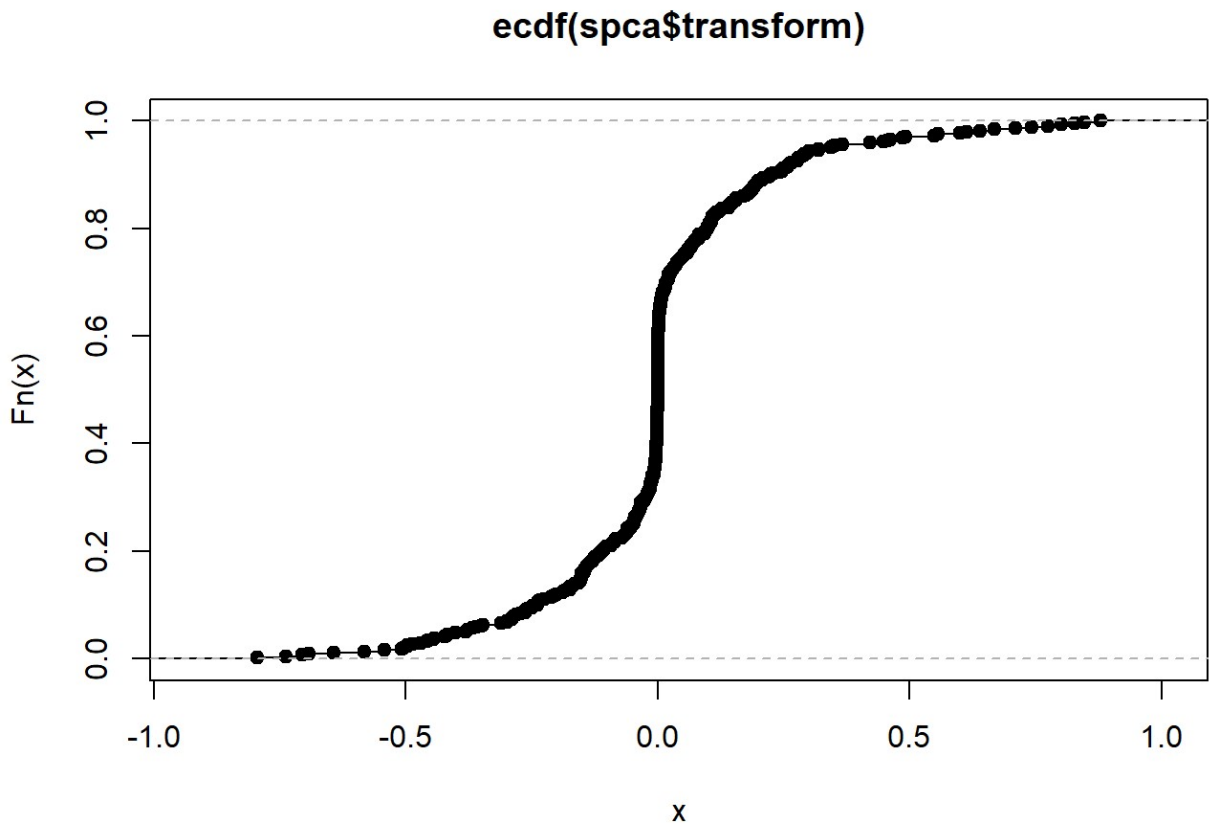
```
summary(spca)
```

```
##                           PC1   PC2   PC3   PC4   PC5   PC6   PC7   PC8
## Explained variance     8.507 3.456 2.699 1.222 1.155 0.830 0.608 0.529
## Standard deviations    2.917 1.859 1.643 1.106 1.075 0.911 0.780 0.727
## Proportion of variance 0.405 0.165 0.129 0.058 0.055 0.040 0.029 0.025
## Cumulative proportion  0.405 0.570 0.698 0.756 0.811 0.851 0.880 0.905
##                           PC9  PC10  PC11  PC12  PC13  PC14  PC15  PC16
## Explained variance     0.412 0.341 0.301 0.240 0.211 0.156 0.104 0.081
## Standard deviations    0.642 0.584 0.549 0.490 0.460 0.395 0.323 0.284
## Proportion of variance 0.020 0.016 0.014 0.011 0.010 0.007 0.005 0.004
## Cumulative proportion  0.925 0.941 0.955 0.967 0.977 0.984 0.989 0.993
##                          PC17  PC18  PC19  PC20  PC21
## Explained variance     0.056 0.030 0.000 0.000 0.000
## Standard deviations    0.237 0.174 0.018 0.000 0.000
## Proportion of variance 0.003 0.001 0.000 0.000 0.000
## Cumulative proportion  0.996 0.997 0.997 0.997 0.997
```

```
# Plot PCA object, which returns a scree plot of the variances (y-axis) associated wit
h the PCs (x-axis)
plot(log(spca$objective), col='red', xlab='Number of iterations', ylab='Objective valu
e')
```




```
# Plot cumulative density for overall variance

plot(ecdf(spca$transform))
```



As shown by the summasry statistics, the first principal component of a sparse PCA model explains 40.5% of variance, the second component 16.5%, third 12.9%, fourth 5.8%, with the rest of components at low single digit values.

Fit a probabilistic PCA model:

```
library(pcaMethods)
```

```
## Loading required package: Biobase
```

```
## Loading required package: BiocGenerics
```

```
## Loading required package: parallel
```

```
##
## Attaching package: 'BiocGenerics'
```

```
## The following objects are masked from 'package:parallel':
##
##     clusterApply, clusterApplyLB, clusterCall, clusterEvalQ,
##     clusterExport, clusterMap, parApply, parCapply, parLapply,
##     parLapplyLB, parRapply, parSapply, parSapplyLB
```

```
## The following objects are masked from 'package:dplyr':
##
##     combine, intersect, setdiff, union
```

```
## The following objects are masked from 'package:stats':
##
##     IQR, mad, sd, var, xtabs
```

```
## The following objects are masked from 'package:base':
##
##     anyDuplicated, append, as.data.frame, basename, cbind,
##     colMeans, colnames, colSums, dirname, do.call, duplicated,
##     eval, evalq, Filter, Find, get, grep, grepl, intersect,
##     is.unsorted, lapply, lengths, Map, mapply, match, mget, order,
##     paste, pmax, pmax.int, pmin, pmin.int, Position, rank, rbind,
##     Reduce, rowMeans, rownames, rowSums, sapply, setdiff, sort,
##     table, tapply, union, unique, unsplit, which, which.max,
##     which.min
```

```
## Welcome to Bioconductor
##
##     Vignettes contain introductory material; view with
##     'browseVignettes()'. To cite Bioconductor, see
##     'citation("Biobase")', and for packages 'citation("pkgname")'.
```

```
##
## Attaching package: 'pcaMethods'
```

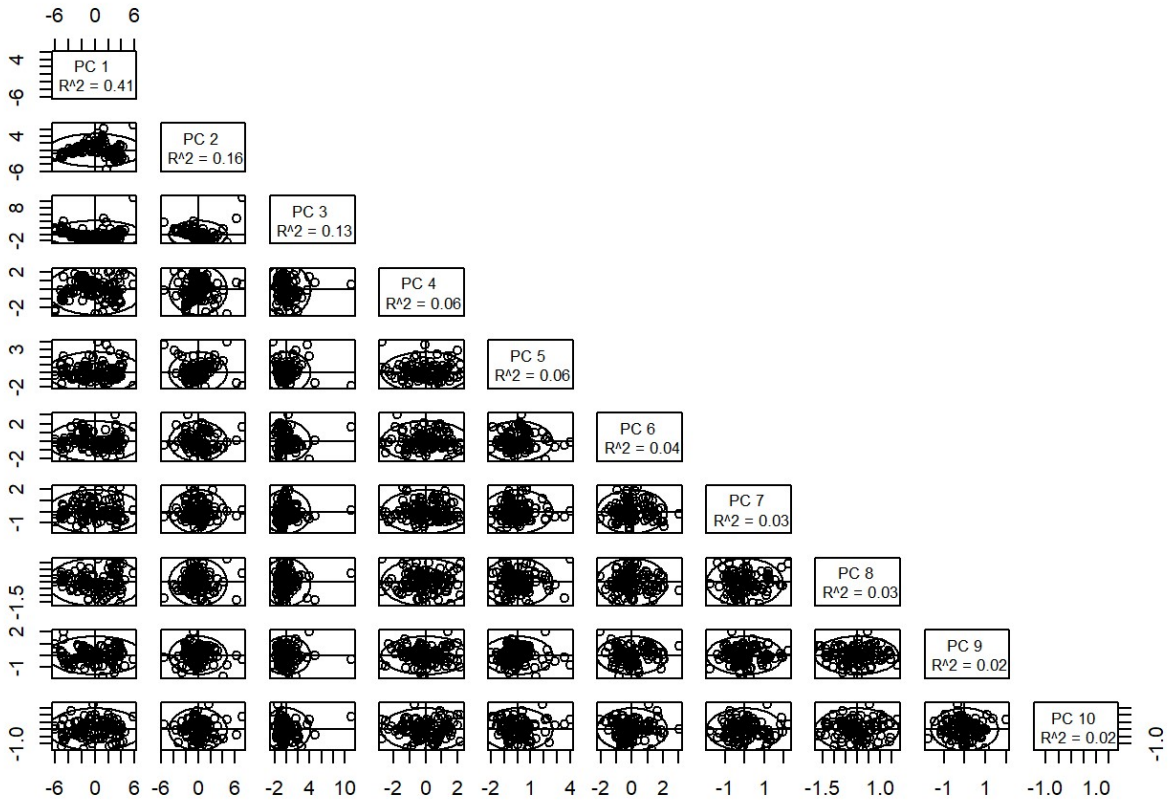
```
## The following object is masked from 'package:psych':
##
##     pca
```

```
## The following object is masked from 'package:stats':
##
##     loadings
```

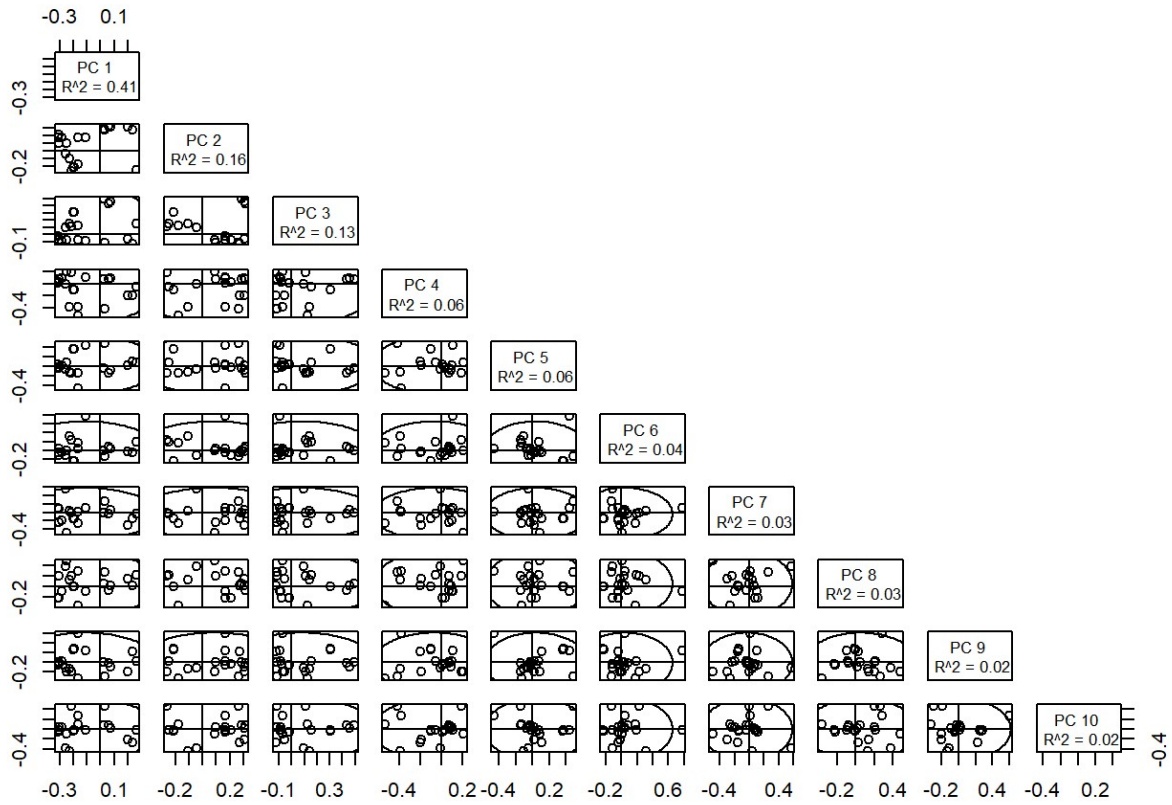
```
## Perform probabilistic PCA using the 10 largest components
ppca <- pca(ct_scaled, method="ppca", nPcs=10, seed=123)
summary(ppca)
```

```
## ppca calculated PCA
## Importance of component(s):
##                   PC1    PC2    PC3     PC4     PC5     PC6     PC7     PC8
## R2            0.4053 0.1647 0.1287 0.05837 0.05516 0.03968 0.02911 0.02534
## Cumulative R2 0.4053 0.5700 0.6987 0.75708 0.81225 0.85193 0.88104 0.90638
##                    PC9    PC10
## R2             0.01976 0.01641
## Cumulative R2 0.92614 0.94255
```

```
## Get the estimated complete observations
cObs <- completeObs(ppca)

## Plot the scores
plotPcs(ppca, type = "scores")
```




```
plotPcs(ppca, type = "loadings")
```



The probabilistic PCA model is showing a simialr loading pattern with the first three or four components explaining cumulatively 75% of total variance.

The difference between a sparse PCA and a probabilistic PCA is that while the former tries to introduce sparsity structure to the input variables (in other words, finding linear combinations that contain just a few variables instead of all) using lasso regression, the latter marginalizes out the latent component(s) by fitting a expectation-maximization algorithm, which allows for a bayesian inference.