

CS101A DATA STRUCTURE(H)
QUIZ [2] DS1-5. BASIC DATA STRUCTURES

NAME: _____ STUDENT ID: _____

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Start Time: 20:40 PM

End Time: 21:00 PM

Q. 1 TRUE OR FALSE (14 pts)

1. Starting empty and doubling capacity only when full, there exists a sequence of m `push_backs` where one push costs $\Theta(m)$ while the average cost over all pushes is $O(1)$. ☐ T ☐ F
2. In a singly linked list with both head and tail pointers, `push_front`, `pop_front`, and `push_back` each run in worst-case $O(1)$ time. ☐ T ☐ F
3. Given a pointer to a non-head and non-tail node in a singly linked list, deleting that node is a $\Theta(1)$ operation. ☐ T ☐ F
4. Evaluating the RPN expression `5 1 2 + 4 * + 3 -` yields 14. ☐ T ☐ F
5. With a hash table of size $m = 7$, hash $h(k) = k \bmod 7$, and *quadratic probing* $h_i(k) = (h(k) + i^2) \bmod 7$, inserting 10, 17, 24, 31 occupies indices 3, 4, 0, 6, and inserting 38 then succeeds at index 1. ☐ T ☐ F
6. $n + \lfloor \log_2 n \rfloor = \Theta(n)$ and $(\log n)^2 = o(n^{0.01})$. ☐ T ☐ F
7. For fixed constants $A > B > 0$, $(\log n)^A = \omega((\log n)^B)$; moreover, for any $C > 0$, $(\log n)^C = o(n^\varepsilon)$ for every $\varepsilon > 0$. ☐ T ☐ F

Q. 2 ARRAY CAPACITY (8 pts)

Suppose there are two initially empty arrays of capacity 4. You continuously push elements into these arrays. When you want to push an element into a full array, you must increase the array's capacity and copy all the old elements to the new array. The first array's capacity increases by +2 each time. The second array's capacity increases by a factor of 2 each time. Answer the following questions; the subparts are independent.

- (a) Suppose we insert 7 elements into the **first** array, the unused memory is ____, the total number of copies is ____.
- (b) Suppose we insert 7 elements into the **second** array, the unused memory is ____, the total number of copies is ____.
- (c) Suppose we insert 17 elements into the **first** array, the unused memory is ____, the total number of copies is ____.
- (d) Suppose we insert 17 elements into the **second** array, the unused memory is ____, the total number of copies is ____.

Q. 3 STACK VIA TWO QUEUES (18 pts)

In the symmetry of data structures lies a quiet elegance: FIFO and LIFO are twin paradigms, they built from the same parts, but with opposite choreography. Building on this duality, revisit the classic trick in reverse: use two FIFO queues `q1` and `q2` to emulate a LIFO stack. Implement `push(x)`, `pop()`, and `top()` by completing the placeholders `/*(1)*/`/`/*(6)*/` in the provided skeleton.

```

class MyStack {
    queue<int> q1, q2;
    void push(int x) {
        /*(1)*/;
        while (/*(2)*/) {
            int v = /*(3)*/;
            /*(4)*/;
        }
        std::swap(q1, q2);
    }
    int top() {
        /*(5)*/;
    }
    int pop() {
        int v = /*(6)*/;
        return v;
    }
    bool empty() const { return q1.empty(); }
};

```

Fill in the blanks here

- (1) _____
- (2) _____
- (3) _____
- (4) _____
- (5) _____
- (6) _____

Q. 4 ASYMPTOTIC ANALYSIS (10 pts)

```

inline void tiny_mix(int& x, int& y, int& z){
    int t = x ^ (y + 0x9e3779b9);
    x = y ^ (z + 0x7f4a7c15);
    y = z ^ (t + 0x85ebca6b);
}

void SolveB(vector<int>& a, int l, int r){
    int n = r - l + 1;
    if(n <= 1) return;
    int c1 = l + (3*n)/5;
    int c2 = l + (4*n)/5;
    int i1 = l, i2 = c1-1, i3 = c2-1;
    if(i1 >= l && i1 <= r && i2 >= l && i2 <= r && i3 >= l && i3 <= r){
        tiny_mix(a[i1], a[i2], a[i3]);
    }
    SolveB(a, l, c1-1);
    SolveB(a, l, c2-1);
}

```

First **write the correct recurrence** for the *number of calls* $T(n)$ made by `SolveB` on an input of size n , then **find a function** g such that $T(n) = \Theta(g(n))$. Show your reasoning; only the final $\Theta(\cdot)$ answer is worth 2 points.