



普通物理I PHYS1181.03

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研究方向: 超快光谱、X射线阿秒脉冲产生、阿秒瞬态吸收光谱、
强场激光物理、飞秒激光成丝。

<https://spst.shanghaitech.edu.cn/2021/0115/c2349a59066/page.htm>



例 如图,一半径为 R 、质量为 m 的均质圆盘平放在粗糙的水平面上,设圆盘与水平面的摩擦系数为 μ_k ,摩擦力均匀地分布于圆盘的底面。若圆盘绕垂直于圆盘中心的 OO' 轴转动,初速度为 ω_0 。

求 经过多长时间圆盘会停止?

解 受力分析如图,以 ω_0 的转动方向为正。

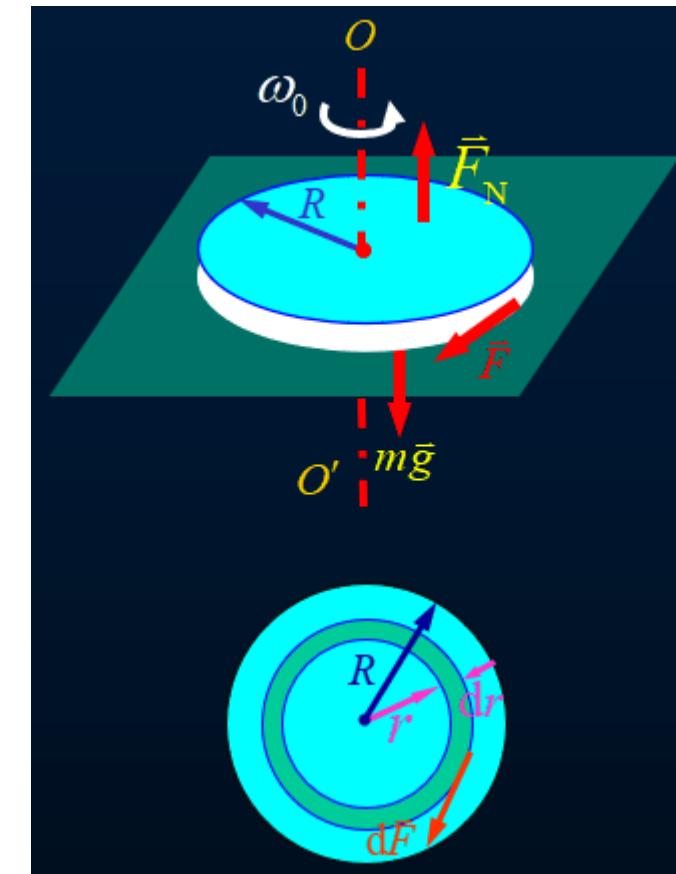
任取一半径为 r ,宽度 dr 的圆环。

所取圆环的质量为

$$dm = \sigma \cdot 2\pi r dr \quad \leftarrow \quad \sigma = \frac{m}{\pi R^2}$$

则这一圆环受到的摩擦力为

$$dF = \mu_k dm g = \frac{2m\mu_k gr dr}{R^2}$$



圆环受到的摩擦力矩为

$$dM = -rdF = -\frac{2m\mu_k gr^2 dr}{R^2}$$

(式中力矩取负号是因为摩擦力矩的方向与选取的正方向相反)

整个薄圆盘受到的摩擦力矩为

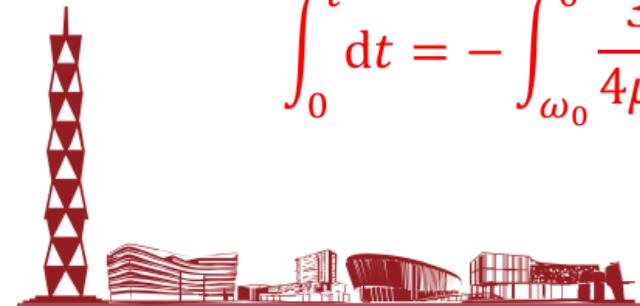
$$M = \int dM = - \int_0^R \frac{2m\mu_k gr^2}{R^2} dr = -\frac{2}{3}\mu_k mgR$$

由刚体定轴转动定律有

$$-\frac{2}{3}\mu_k mgR = J \frac{d\omega}{dt} \quad (J = \frac{1}{2}mR^2) \quad \xrightarrow{\hspace{1cm}} \quad dt = -\frac{3R}{4\mu_k g} d\omega$$

积分并代入初始条件，有

$$\int_0^t dt = - \int_{\omega_0}^0 \frac{3R}{4\mu_k g} d\omega \quad \xrightarrow{\hspace{1cm}} \quad t = \frac{3R\omega_0}{4\mu_k g}$$



例 如图，在光滑水平面上放一质量为 m 、长为 l 的均质细棒，细棒可绕中心固定的光滑竖直轴转动，细棒开始静止。若有一质量为 m_0 的小球，以垂直于细棒的水平速度 v_0 冲击细棒的一个顶端，设冲击是完全弹性碰撞。

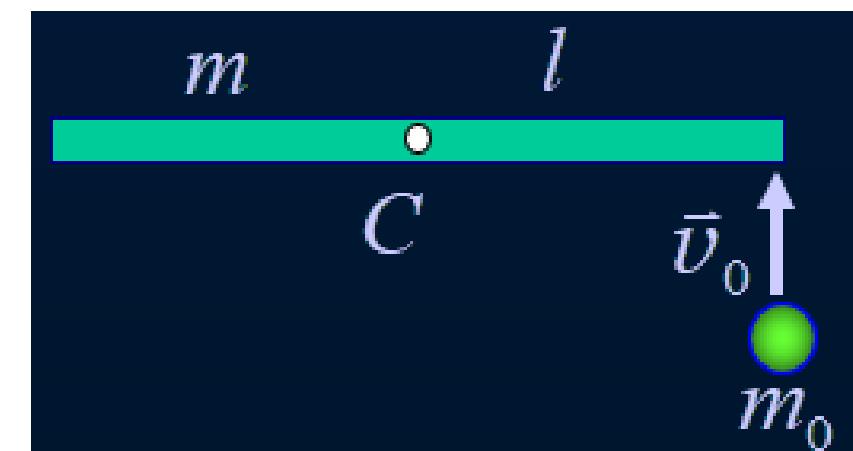
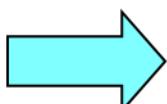
求 碰撞后小球的反弹速度 v 和细棒的角速度 ω 。

解 外力对转轴C的合外力矩为零，碰撞时系统角动量守恒，有

$$\frac{1}{2}m_0v_0l = J_C\omega - \frac{1}{2}m_0vl \quad (J_C = \frac{1}{12}ml^2)$$

由于碰撞是完全弹性碰撞，系统机械能守恒，则

$$\frac{1}{2}m_0v_0^2 = \frac{1}{2}m_0v^2 + \frac{1}{2}J_C\omega^2$$



$$\left. \begin{array}{l} v = \frac{3m_0 - m}{3m_0 + m} v_0 \\ \omega = \frac{12m_0 v_0}{(3m_0 + m)l} \end{array} \right\}$$



质点的运动规律与刚体的定轴转动规律的比较

质点的运动		刚体的定轴转动	
速度	$\bar{v} = \frac{d\bar{r}}{dt}$	角速度	$\omega = \frac{d\theta}{dt}$
加速度	$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2}$	角加速度	$\beta = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
质量	m	转动惯量	$J = \int r^2 dm$
力	\bar{F}	力矩	M
运动规律	$\bar{F} = m\bar{a}$	转动定律	$M = J\beta$
动量	$\bar{p} = m\bar{v}$	动量	$\bar{p} = \sum \Delta m_i \bar{v}_i$
角动量	$\bar{L} = \bar{r} \times \bar{p}$	角动量	$L = J\omega$
动量定理	$\bar{F} = \frac{d(m\bar{v})}{dt}$	角动量定理	$M = \frac{d(J\omega)}{dt}$



质点的运动规律与刚体的定轴转动规律的比较（续）

质点的运动	刚体的定轴转动
动量守恒 $\sum F_i = 0$ 时 $\sum m_i v_i = \text{恒量}$	角动量守恒 $M = 0$ 时 $\sum J\omega = \text{恒量}$
力的功 $A_{ab} = \int_a^b \bar{F} \cdot d\bar{r}$	力矩的功 $A_{ab} = \int_{\theta_1}^{\theta_2} M d\theta$
动能 $E_k = \frac{1}{2}mv^2$	转动动能 $E_k = \frac{1}{2}J\omega^2$
动能定理 $A = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$	动能定理 $A = \frac{1}{2}J\omega_2^2 - \frac{1}{2}J\omega_1^2$
重力势能 $E_p = mgh$	重力势能 $E_p = mgh_c$
机械能守恒 $A_{\text{外}} + A_{\text{非保内}} = 0$ 时 $E_k + E_p = \text{恒量}$	机械能守恒 $A_{\text{外}} + A_{\text{非保内}} = 0$ 时 $E_k + E_p = \text{恒量}$



1. 平衡条件

2. 应变

3. 应力

4. 胡克定律

5. 杨氏模量





平衡

质点 -- {x, y, z} 三个运动自由度

质点系 - {x, y, z} *n 个运动自由度

刚体 - {x, y, z} + { α , β , γ } 六个运动自由度 -- 刚体的运动可以分解为质心的平动 + 绕质心的转动

质点的平衡 -- 牛顿第一定律：合力为零

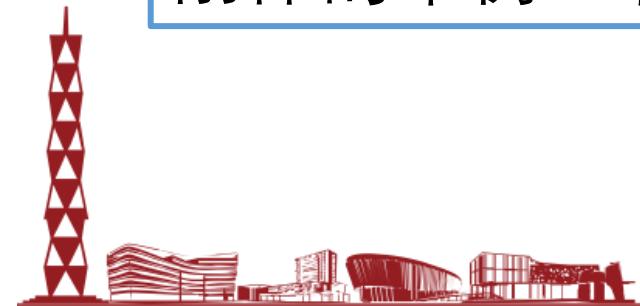
刚体的平衡 - 合力为零 + 合力矩为零

First condition for equilibrium:
For the center of mass of a body
at rest to remain at rest ...

$$\sum \vec{F} = 0 \quad \dots \text{net external force on the body must be zero.}$$

Second condition for equilibrium:
For a nonrotating body to remain
nonrotating ...

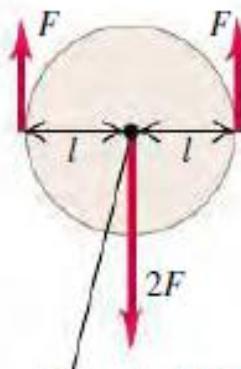
$$\sum \vec{\tau} = 0 \quad \dots \text{net external torque around any point on the body must be zero.}$$



合力为0+合力距为0

(a) This body is in static equilibrium.

Equilibrium conditions:



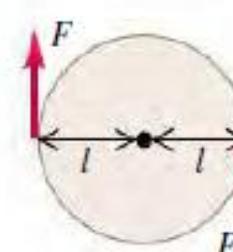
First condition satisfied:
Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied:
Net torque about the axis = 0, so body at rest has no tendency to start rotating.

Axis of rotation (perpendicular to figure)

合力为0+合力距不为0

(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.

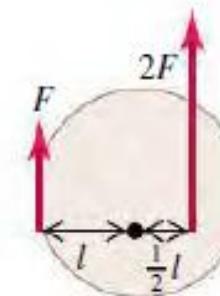


First condition satisfied:
Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

合力不为0+合力距为0

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.



First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied:
Net torque about the axis = 0, so body at rest has no tendency to start rotating.



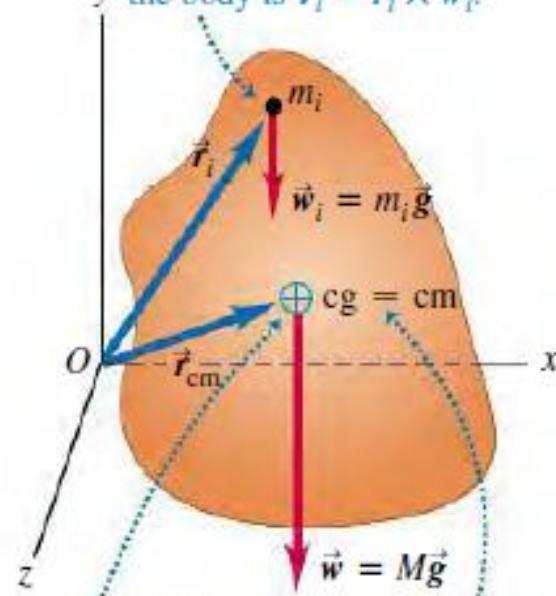
$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i y_i}{\sum_i m_i} \quad (\text{center of mass})$$

$$z_{\text{cm}} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

Position vector of center of mass of a system of particles $\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$
 Position vectors of individual particles
 Masses of individual particles

The gravitational torque about O on a particle of mass m_i within the body is $\vec{\tau}_i = \vec{r}_i \times \vec{w}_i$.



If \vec{g} has the same value at all points on the body, the cg is identical to the cm.

The net gravitational torque about O on the entire body is the same as if all the weight acted at the cg: $\vec{\tau} = \vec{r}_{\text{cm}} \times \vec{w}$.

重力的力矩

$$\vec{\tau}_i = \vec{r}_i \times \vec{w}_i = \vec{r}_i \times m_i \vec{g}$$

The *total* torque due to the gravitational forces on all the particles is

$$\begin{aligned}\vec{\tau} &= \sum_i \vec{\tau}_i = \vec{r}_1 \times m_1 \vec{g} + \vec{r}_2 \times m_2 \vec{g} + \dots \\ &= (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots) \times \vec{g} \\ &= \left(\sum_i m_i \vec{r}_i \right) \times \vec{g}\end{aligned}$$

When we multiply and divide this by the total mass of the body,

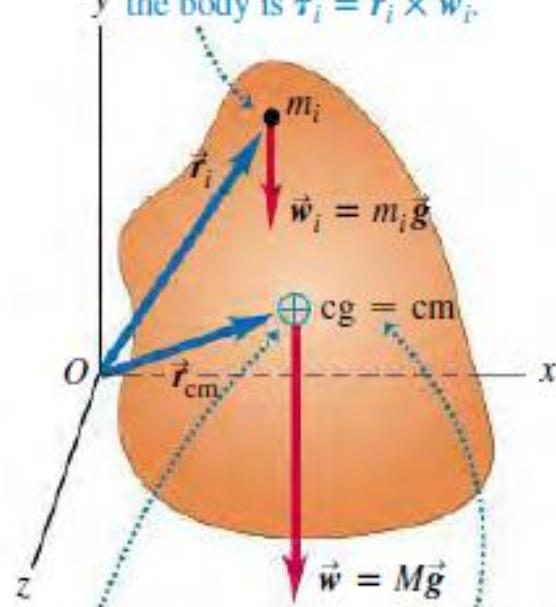
$$M = m_1 + m_2 + \dots = \sum_i m_i$$

we get

$$\vec{\tau} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots} \times M \vec{g} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \times M \vec{g}$$

$$\vec{\tau} = \vec{r}_{cm} \times M \vec{g} = \vec{r}_{cm} \times \vec{w}$$

The gravitational torque about O on a particle of mass m_i within the body is $\vec{\tau}_i = \vec{r}_i \times \vec{w}_i$.



If \vec{g} has the same value at all points on the body, the cg is identical to the cm.

The net gravitational torque about O on the entire body is the same as if all the weight acted at the cg: $\vec{\tau} = \vec{r}_{cm} \times \vec{w}$.

平衡条件判断重心位置

$$\vec{\tau} = \vec{r}_{\text{cm}} \times M\vec{g} = \vec{r}_{\text{cm}} \times \vec{w}$$



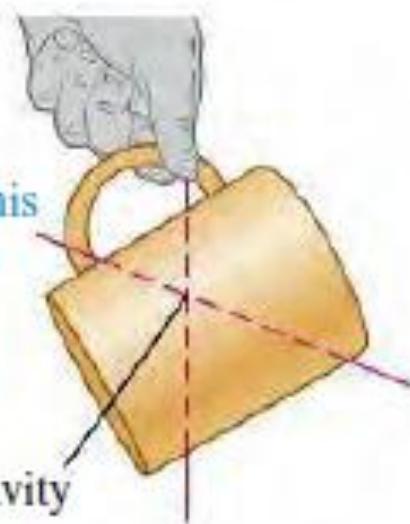
Where is the center of gravity of this mug?

- ① Suspend the mug from any point. A vertical line extending down from the point of suspension passes through the center of gravity.



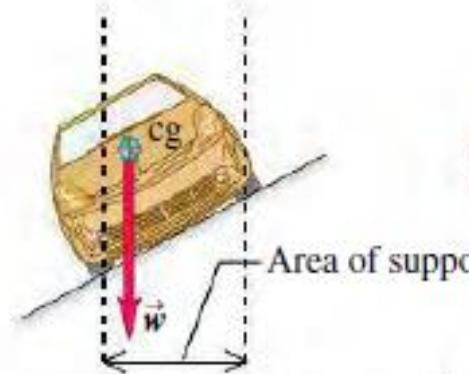
- ② Now suspend the mug from a different point. A vertical line extending down from this point intersects the first line at the center of gravity (which is inside the mug).

Center of gravity



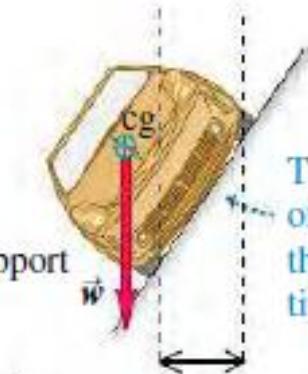
11.5 In (a) the center of gravity is within the area bounded by the supports, and the car is in equilibrium. The car in (b) and the truck in (c) will tip over because their centers of gravity lie outside the area of support.

(a)



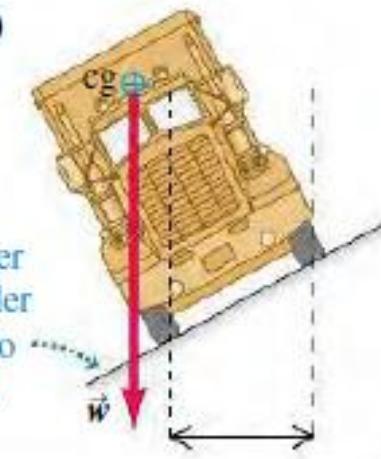
Center of gravity is over the area of support: car is in equilibrium.

(b)



The higher the center of gravity, the smaller the incline needed to tip the vehicle over.

(c)



Center of gravity is outside the area of support: vehicle tips over.

平衡：重心在支撑范围内

不平衡：重心超出支撑范围

降低重心+增加支持区域面积





四腿的动物脚比较小



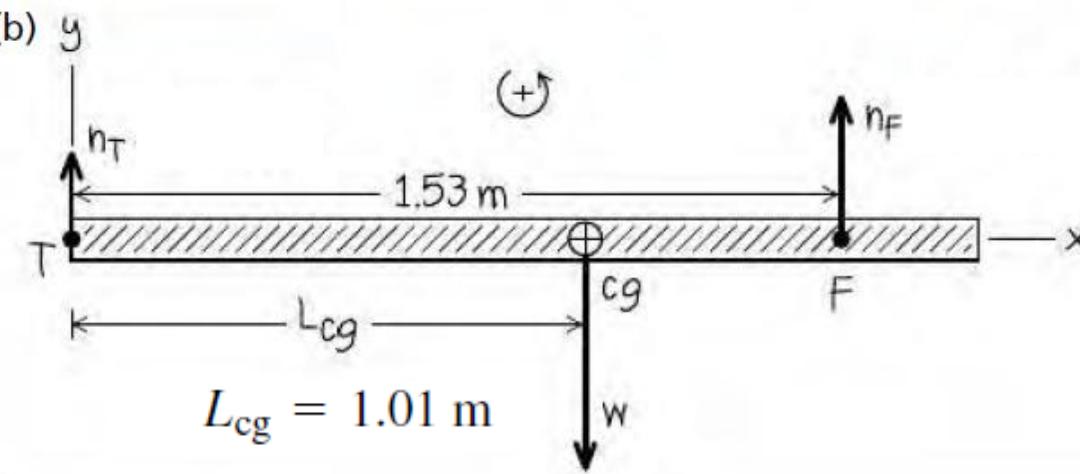
两腿的动物脚比较大



(a)



(b)



$$\sum \tau_R = 0.340w(0) - wL_{cg} + 0.660w(1.53 \text{ m}) = 0$$

平板支撑
肘部的支撑力大于脚部的支撑力

应力、应变、胡克定律



$$\text{Hooke's law: } \frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus}$$

Measure of forces applied to deform a body
应力

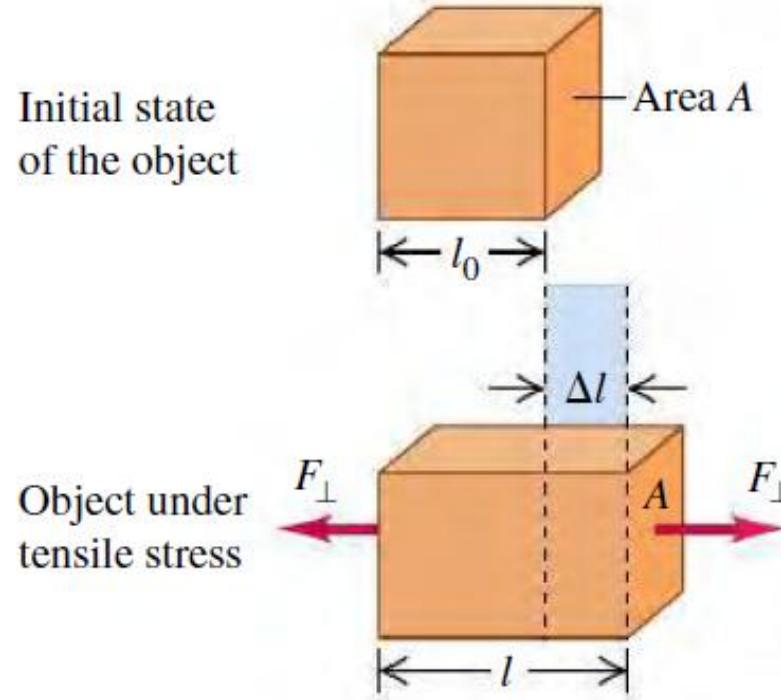
Property of material
of which body is made
弹性模量

Measure of how much deformation results from stress
应变



Tensile stress and strain

拉伸应力和应变



$$\text{Tensile stress} = \frac{F_{\perp}}{A} \quad \text{Tensile strain} = \frac{\Delta l}{l_0}$$

$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$

$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

人站在地面上: $65\text{Kg} \times 9.8\text{N/Kg} / (8 \times 24 \times 2 \text{ cm}^2) = 1.7\text{e}4 \text{ N/m}^2$

$$\text{Tensile strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

1m的刚性材料 – 一般“常规受力”弹性形变在十微米 ($\text{e-}5\text{m}$)量级

杨氏模量 - Young's modulus

Young's modulus for tension

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{F_{\perp}}{A} \frac{l_0}{\Delta l}$$

Force applied perpendicular to cross section (see Fig. 11.14)

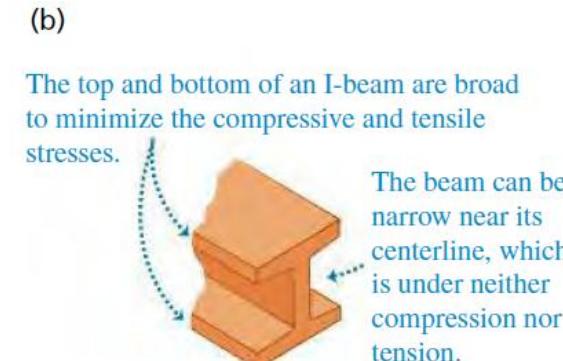
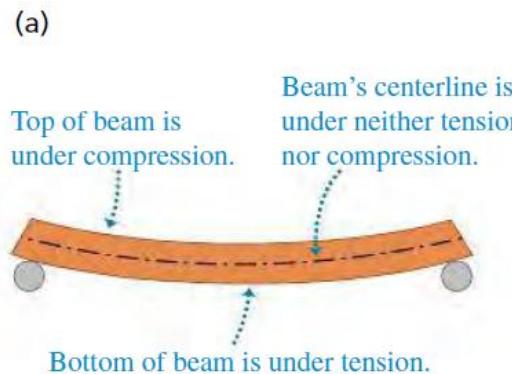
Cross-sectional area of object (see Fig. 11.14)

Original length (see Fig. 11.14)

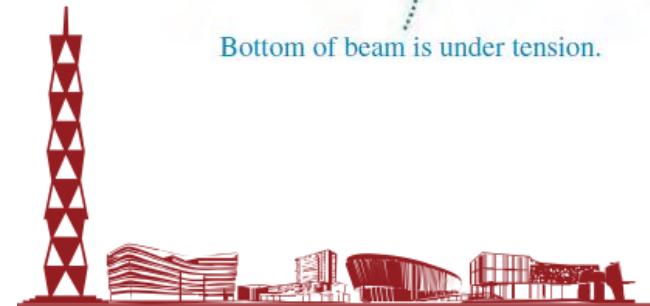
Elongation (see Fig. 11.14)

Y 越小，材料越容易变形。

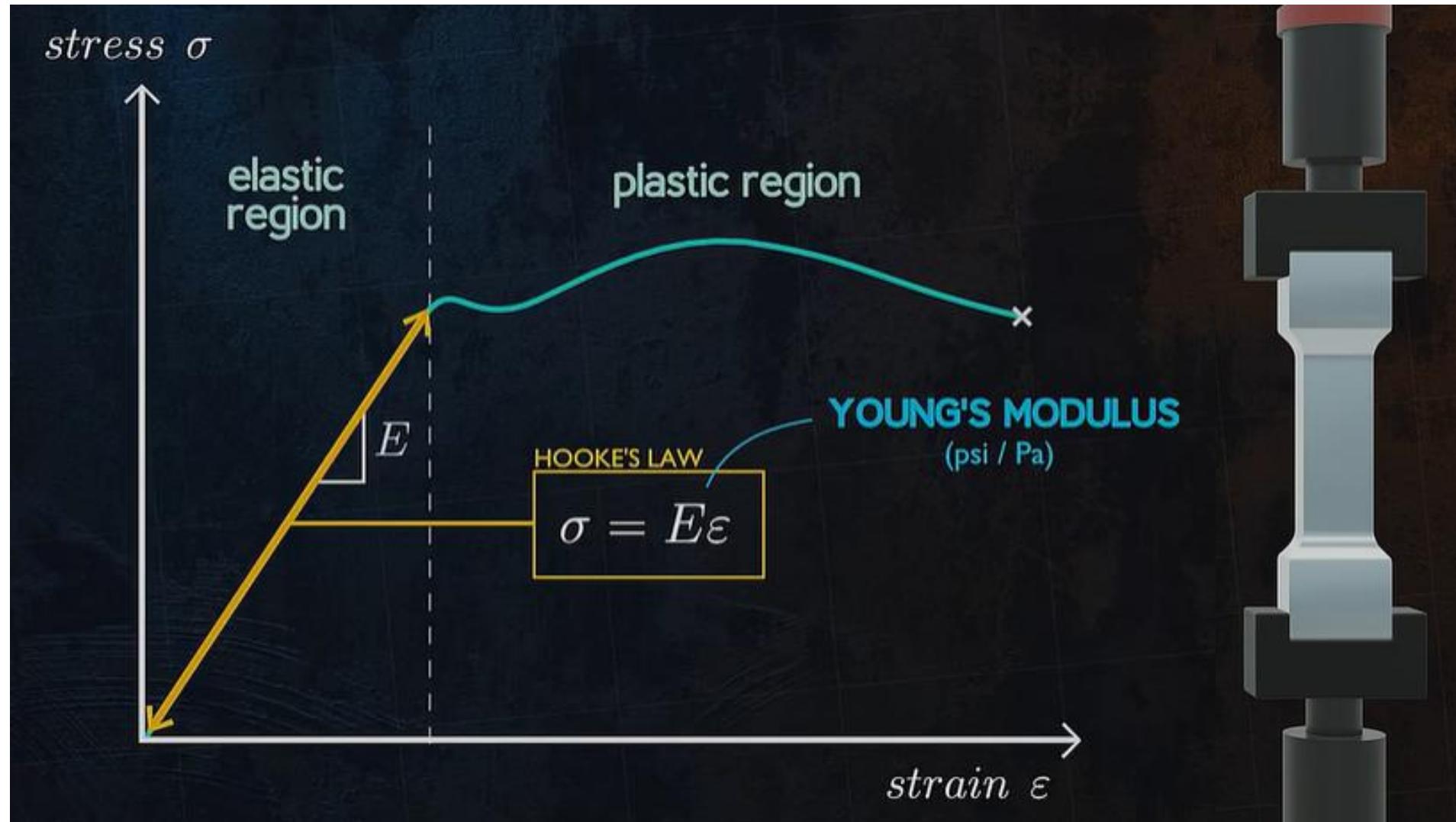
Y 一样，同样受力，面积越小越容易变形。



工形钢，上端压缩，下端拉伸，中间受力小



杨氏模量 - Young's modulus



杨氏模量 - Young's modulus

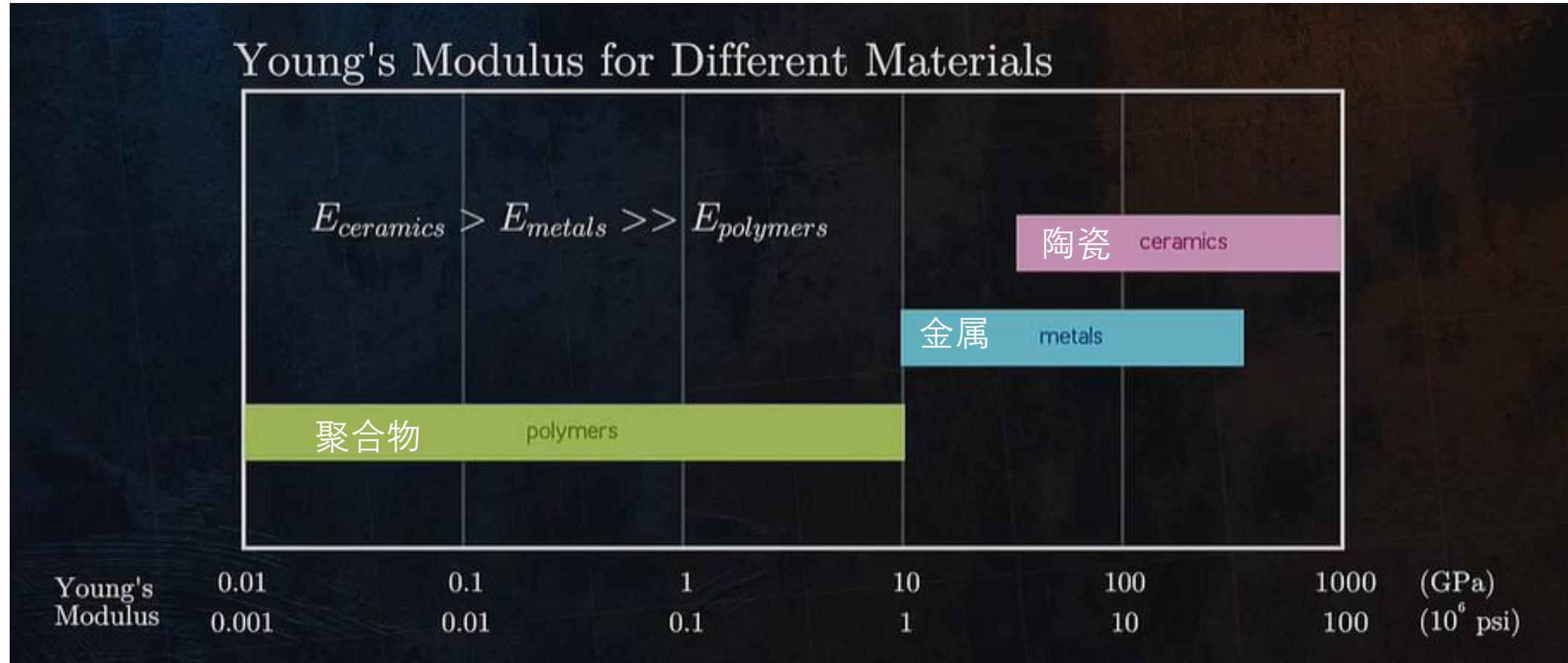
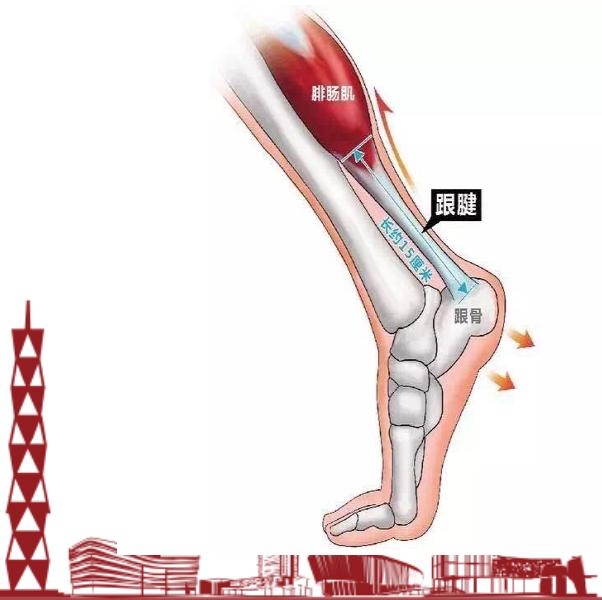
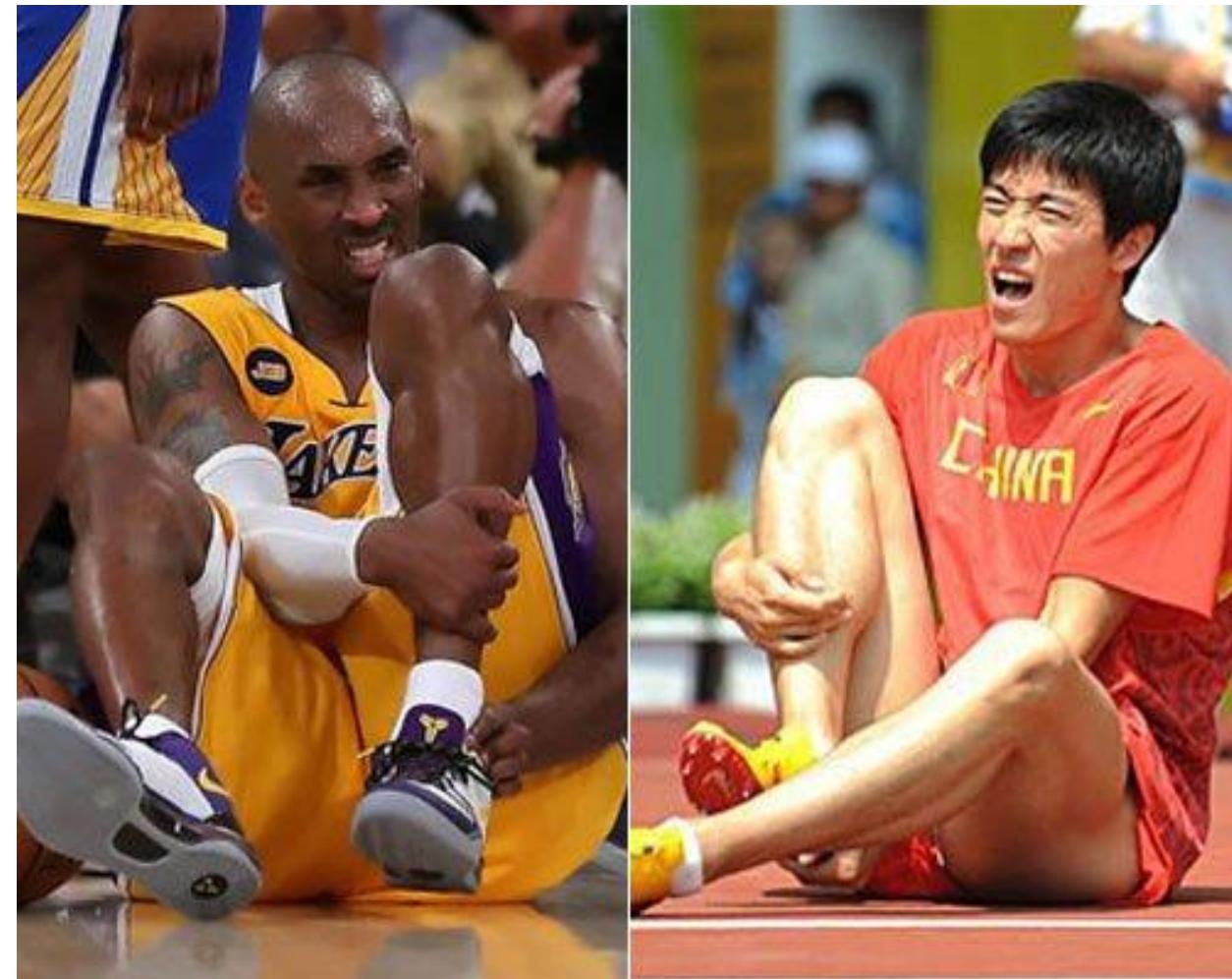


TABLE 11.1 Approximate Elastic Modulus

Material	Young's Modulus, Y (Pa)
Aluminum	7.0×10^{10}
Brass	9.0×10^{10}
Copper	11×10^{10}
Iron	21×10^{10}
Lead 铅	1.6×10^{10}
Nickel	21×10^{10}
Silicone rubber 硅胶	0.001×10^{10}
Steel	20×10^{10}
Tendon (typical) 跟腱	0.12×10^{10}



合理运动，远离伤病



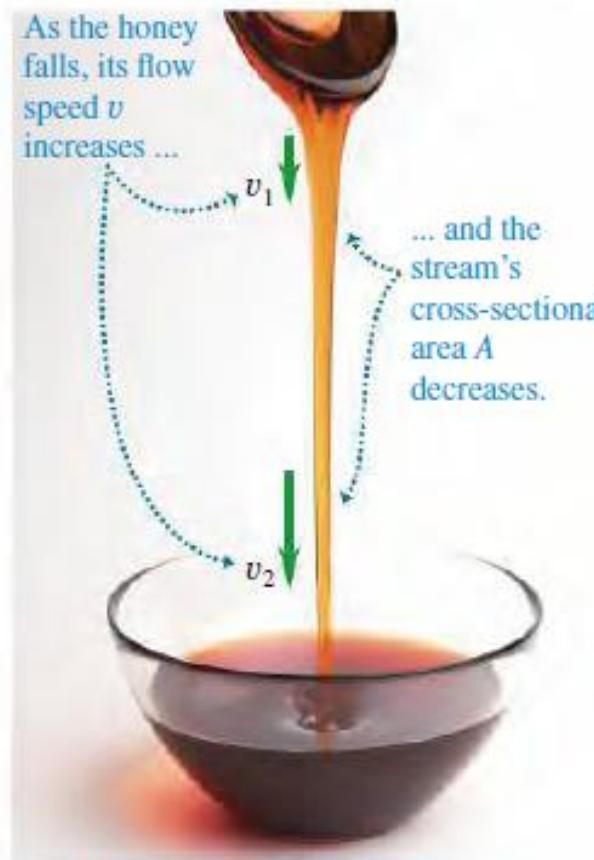


1. 流体静力学
2. 帕斯卡定律
3. 流体动力学
4. 连续性方程
5. 伯努利方程



蜂蜜形状

12.22 The continuity equation, Eq. (12.10), helps explain the shape of a stream of honey poured from a spoon.



静水流深 Still waters run deep



喷泉、水枪。。。





一. 基本概念：

1. 流体：

具有流动性的液体和气体；

2. 流体动力学：

研究流体的运动规律以及流体与其他物体之间相互作用的力学；

二. 流体动力学的应用：

- 生物体液和氧分的输送，动物体内血液的循环，土壤中水分的运动，农田排灌、昆虫迁飞；





一. 基本概念

- 1. 流体的粘滞性:

实际流体在流动时. 其内部有相对运动的相邻两部分之间存在类似两固体相对运动时存在的摩擦阻力(内摩擦力), 流体的这种性质称为粘滞性。

- 2. 流体的可压缩性:

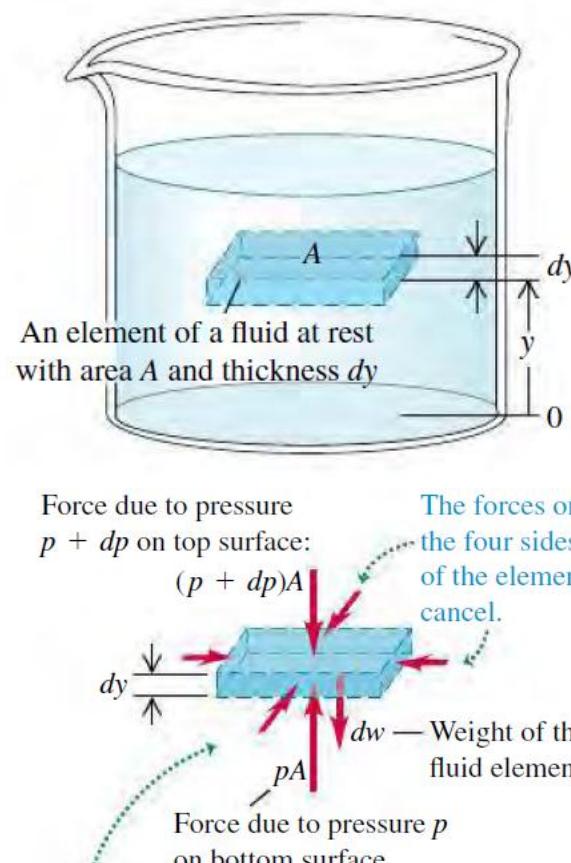
实际流体在外界压力作用下、其体积会发生变化，即具有可压缩性；

- 3. 理想流体模型:

- ◆ 绝对不可压缩、没有粘滞性的流体叫做**理想流体**；
- ◆ 一般情况下，密度不发生明显变化的气体或者液体、粘滞性小的流体均可看成理想流体.

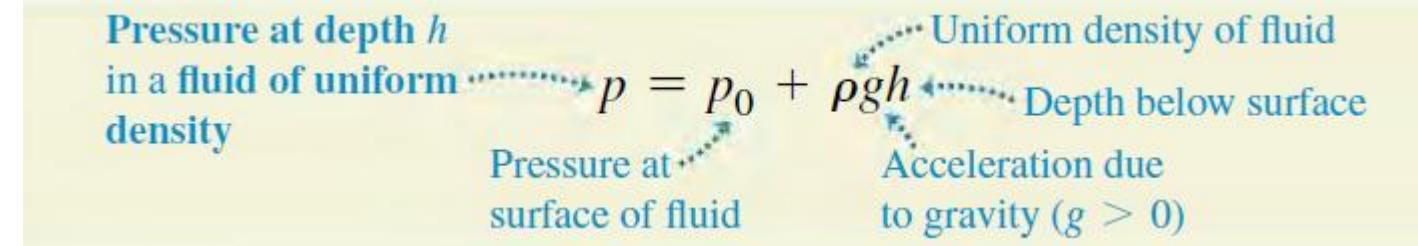


压强



$$\sum F_y = 0 \quad \text{so} \quad pA - (p + dp)A - \rho g A dy = 0$$

$$\frac{dp}{dy} = -\rho g$$



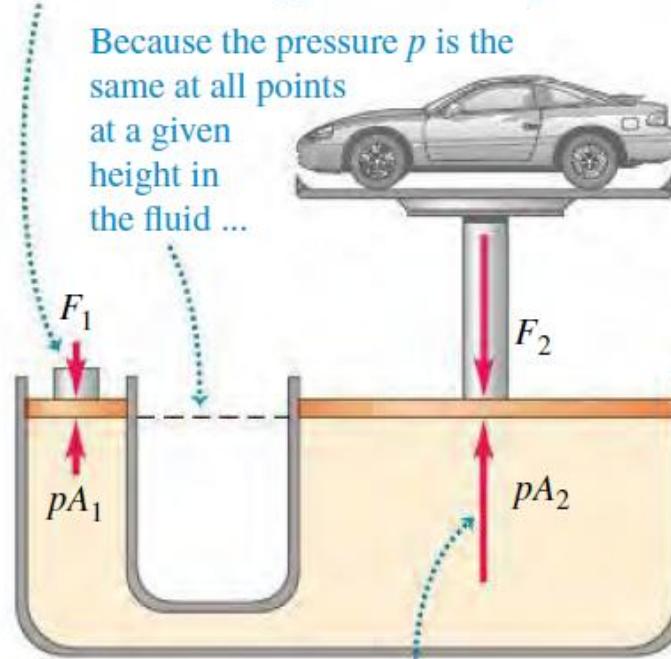
帕斯卡定律：不可压缩静止流体中任一点受外力产生压强增值后，此压强增值瞬时间传至静止流体各点。

压强

液压升降机

12.7 The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.

A small force is applied to a small piston.



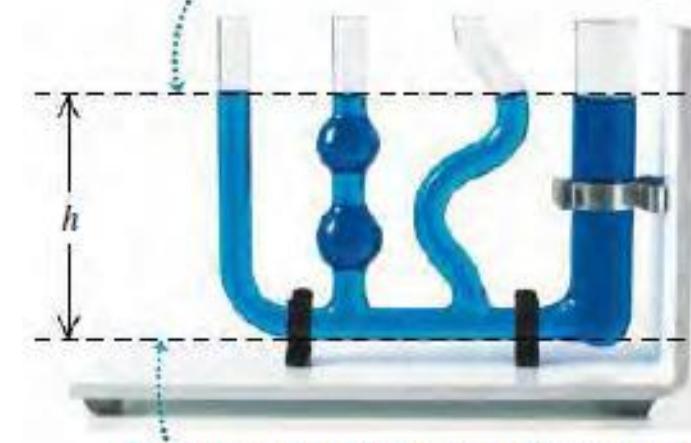
$$F_1 = pA_1$$

$$F_2 = pA_2$$

连通器



The pressure at the top of each liquid column is atmospheric pressure, p_0 .

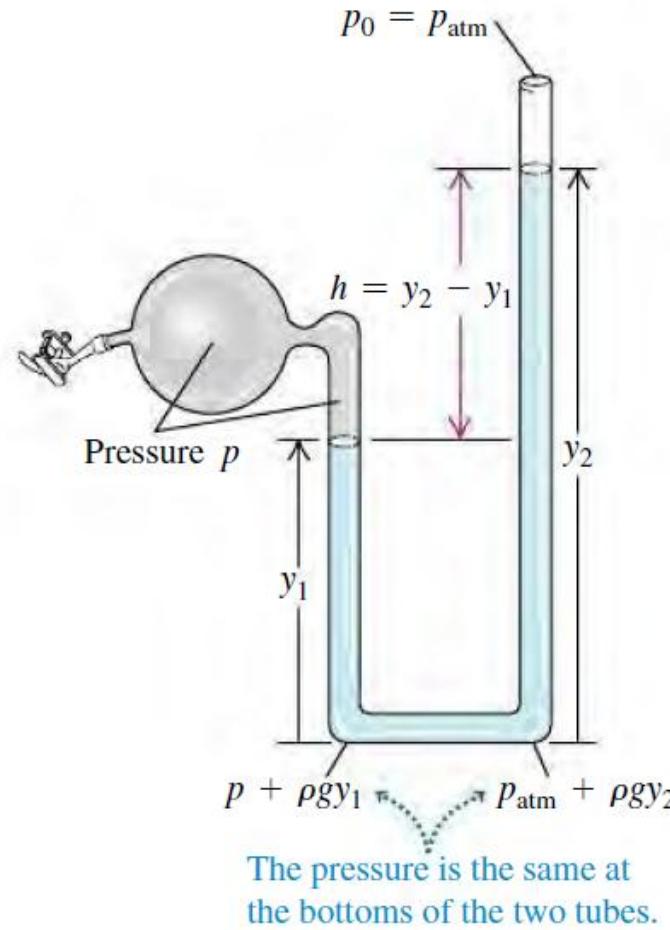


The pressure at the bottom of each liquid column has the same value p .

The difference between p and p_0 is ρgh , where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

压强计

(a) Open-tube manometer



$$p + \rho gy_1 = p_{\text{atm}} + \rho gy_2$$

$$p - p_{\text{atm}} = \rho g(y_2 - y_1) = \rho gh$$

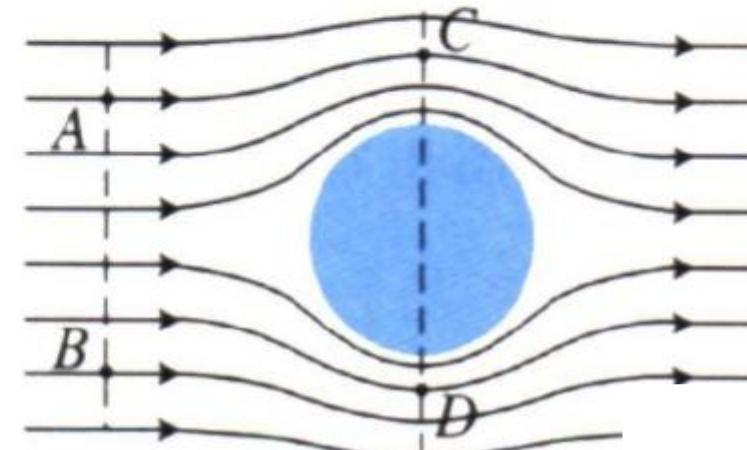
二. 流体的运动形式:

1. 一般流动形式:

- 通常流体看做是由大量**流体质点**所组成的连续介质。
- 一般情况流体运动时，由于流体各部分可以有相对运动，各部分质点的流动速度是空间位置的函数，又是时间t的函数

2. 定常流动:

- 流体质点经过空间各点的流速虽然可以不同，但如果空间每一点的流速不随时间而改变，这样的流动方式称为**定常流动**，也称为**稳定流动**
- 是一种理想化的流动方式。



三. 流线、流管

1. **流线:** 为了形象地描述定常流动的流体而引入的假想的直线或曲线

- 流线上任意点的**切线方向**就是流体质点流经该点的速度方向
- 稳定流动时，流线的形状和分布不随时间变化，且流线与流体质点的运动轨迹重合；
- 流线的疏密程度可定性地表示流体流速的大小；
- 流线不相交；

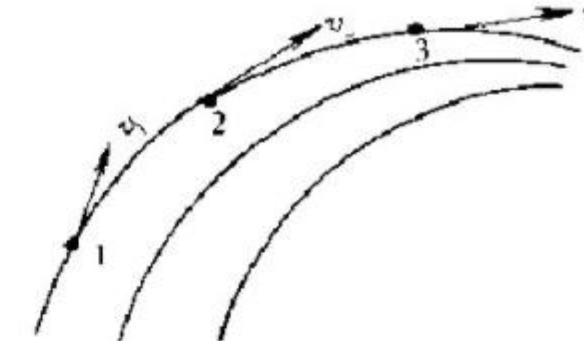


图 1-1 稳定流动、流线

2. **流管:** 流体内部，通过某一个截面的流线围成的管状空间；

- 流体质点不会任意穿出或进入流管；（与实际管道相似）
- 流体可视为由无数个稳定的流管组成，分析每个流管中流体的运动规律，是掌握流体整体运动规律的基础；



四. 连续性原理

1. 推导过程:

假设：

- ①. 取一个截面积很小的**细流管**，垂直于流管的同一截面上的各点流速相同；
- ②. 流体由左向右流动；
- ③. 流体具有不可压缩性；
- ④. 流体质点不可能穿入或者穿出流管；
- ⑤. 在一个较短的时间 Δt 内，流进流管的流体质量等于流出流管的流体质量（**质量守恒**），即：

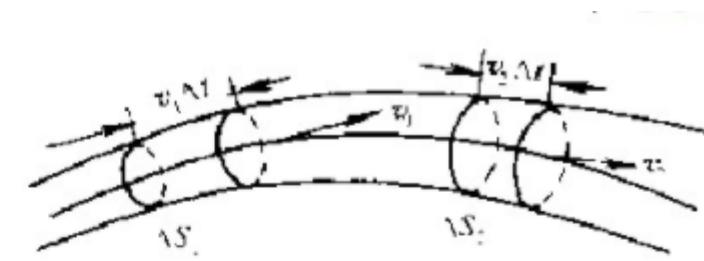


图 1-2 连续性方程推导

$$\rho S_1 v_1 \Delta t = \rho S_2 v_2 \Delta t$$

$$S_1 v_1 = S_2 v_2$$





2. 理想流体的连续性方程(连续性原理、流量方程):

$$Sv = \text{恒量}$$

体积流量：表示单位时间内流过任意截面S的流体体积，称为**体积流量**，简称**流量**，用 Q_v 表示，单位为 m^3/s .

- ◆ 流体在同一**细流管**中作稳定流动时，通过任一截面S的**体积流量**保持不变。
- ◆ 推广，对于不可压缩的实际流体，**任意流管、真实导流管、流体管道**都满足连续性原理。
- ◆ 如果同一截面上流速相同，不可压缩的流体在流管中做稳定流动时流体的流速 v 与流管的截面积 S 成反比，即**截面大处流速小，狭窄处流速大**。



补充例题



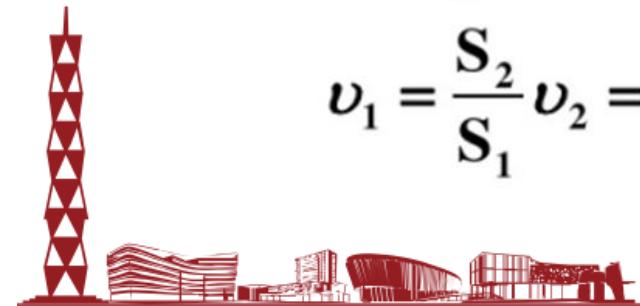
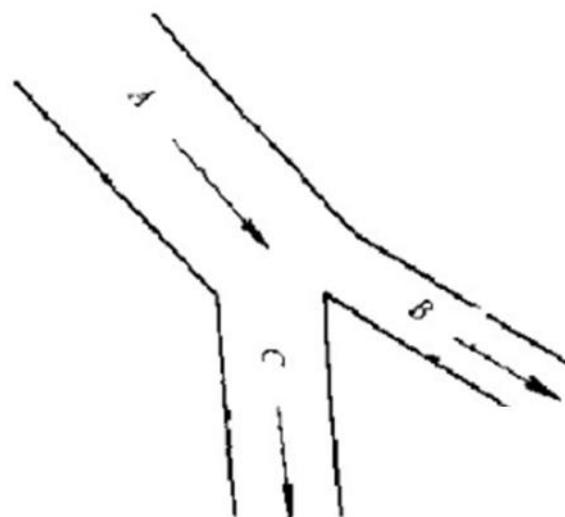
有一条灌溉渠道，横截面是梯形，底宽2m，水面宽4m，水深1m，这条渠道再通过两条分渠道把水引到田间，分渠道的横截面也是梯形，底宽1m，水面宽2m，水深0.5m，如果水在两条渠道内的流速均为0.2m/s，求水在总渠道中的流速？

$$S_1 v_1 = S_2 v_2$$

$$S_1 = \frac{1}{2}(4+2) \times 1 = 3 m^2$$

$$S_2 = \frac{1}{2}(2+1) \times 0.5 \times 2 = 1.5 m^2$$

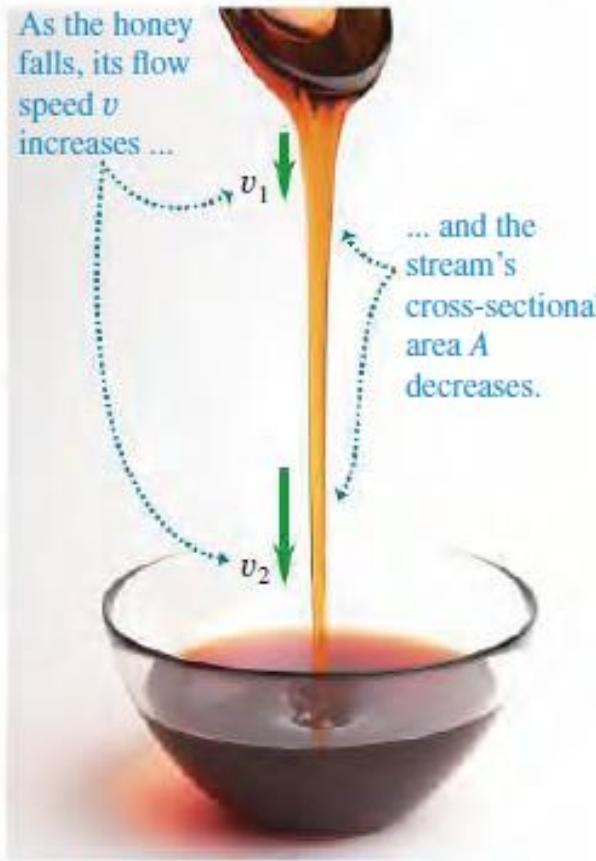
$$v_1 = \frac{S_2}{S_1} v_2 = 0.1 m/s$$



连续性方程的应用 $Sv = \text{恒量}$

蜂蜜形状

12.22 The continuity equation, Eq. (12.10), helps explain the shape of a stream of honey poured from a spoon.



静水流深 Still waters run deep



喷泉、水枪。。。



伯努利方程及其应用



伯努利方程是瑞士物理学家伯努利提出来的，是理想流体作稳定流动时的基本方程，对于确定流体内部各处的压力和流速有很大的实际意义、在水利、造船、航空等部门有着广泛的应用。

伯努利个人简介：（Daniel Bernoulli, 1700~1782）瑞士物理学家、数学家、医学家。他是伯努利这个数学家族（4代10人）中最杰出的代表，16岁时就在巴塞尔大学攻读哲学与逻辑，后获得哲学硕士学位，17~20岁又学习医学，并于1721年获医学硕士学位，成为外科名医并担任过解剖学教授。但在父兄熏陶下最后仍转到数理科学。伯努利成功的领域很广，除**流体力学**这一主要领域外，还有天文测量、引力、行星的不规则轨道、磁学、海洋、潮汐等等。

伯努利方程：理想流体在重力场**中作稳定流动时，**能量守衡定律**在流动液体中的表现形式。**

一. 伯努利方程的推导:

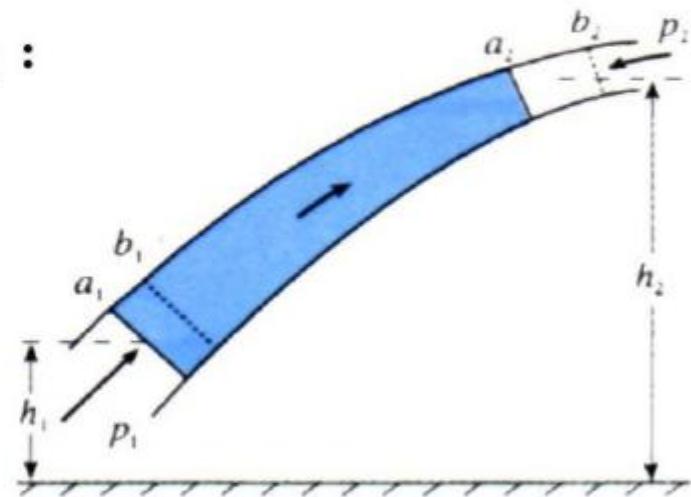
稳定流动的理想流体中，忽略流体的粘滞性，任意**细流管**中的液体满足**能量守恒和功能原理**！

设：流体密度 ρ ，细流管中分析一段流体 $a_1 a_2$ ：

a_1 处： S_1, v_1, h_1, p_1

a_2 处： S_2, v_2, h_2, p_2

经过微小时间 Δt 后，流体 $a_1 a_2$ 移到了 $b_1 b_2$ ，从整体效果看，相当于将流体 $a_1 b_1$ 移到了 $a_2 b_2$ ，设 $a_1 b_1$ 段流体的质量为 Δm ，则：



$$\Delta E_1 = \frac{1}{2} \Delta m v_1^2 + \Delta m g h_1$$

$$\Delta E_2 = \frac{1}{2} \Delta m v_2^2 + \Delta m g h_2$$

机械能的增量： $\Delta E = \Delta E_2 - \Delta E_1$



功能原理：系统受到非保守力做功，系统机械能的增量等于非保守力对系统作的功；

外界对系统作的功？

受力分析=端面压力+侧壁压力

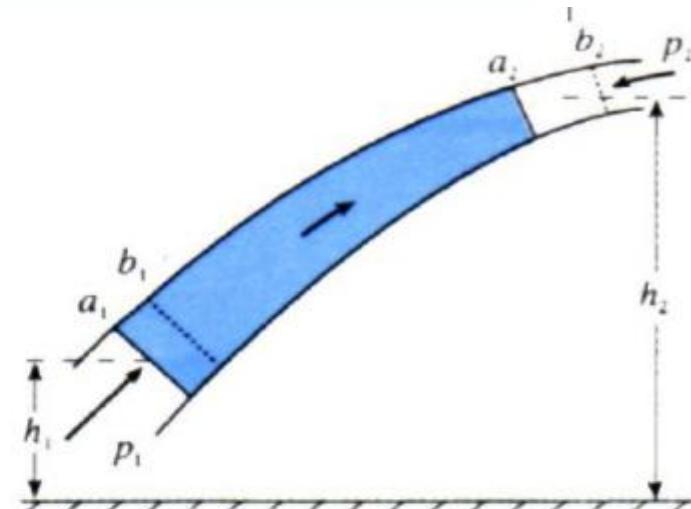
$$W = p_1 S_1 v_1 \Delta t - p_2 S_2 v_2 \Delta t$$

$$V = S_1 v_1 \Delta t = S_2 v_2 \Delta t$$

$$\frac{1}{2} \Delta m v_2^2 + \Delta m g h_2 - (\frac{1}{2} \Delta m v_1^2 + \Delta m g h_1) = p_1 S_1 v_1 \Delta t - p_2 S_2 v_2 \Delta t$$

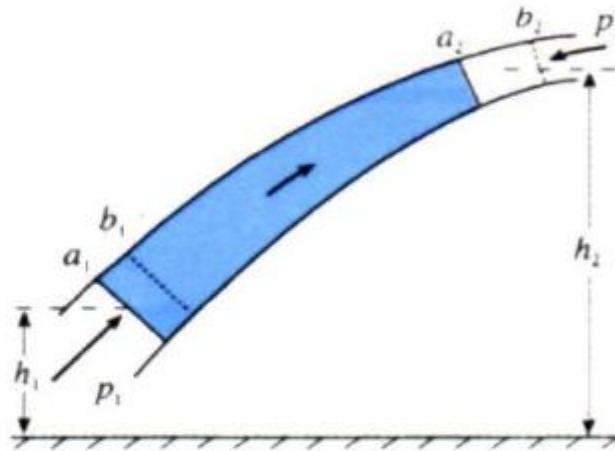
$$\frac{1}{2} \rho V v_2^2 + \rho V g h_2 - (\frac{1}{2} \rho V v_1^2 + \rho V g h_1) = p_1 V - p_2 V$$

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$



二. 对于同一流管的任意截面，伯努利方程：

$$p + \frac{1}{2} \rho v^2 + \rho g h = \text{恒量}$$



•**含义：**对于理想流体作稳定流动，在同一流管中任一处，每单位体积流体的**动能、势能和该处压强之和**是一个恒量。

- 伯努利方程**，是理想流体作稳定流动时的基本方程；
- 对于实际流体，如果粘滞性很小，如：水、空气、酒精等，可应用伯努利方程解决实际问题；
- 对于确定流体内部各处的压力和流速有很大的实际意义、在水利、造船、航空等部门有着广泛的应用。



1. 在海洋中平行逆向航行的两艘大轮船，相互不能靠得太近，否则就会有相撞的危险，为什么？
2. 逆流航行的船只行到水流很急的岸边时，会自动地向岸靠拢；
3. 汽车驶过时，路旁的纸屑常被吸向汽车；
4. **简单的实验：**用两张窄长的纸条，相互靠近，用嘴从两纸条中间吹气，会发现二纸条不是被吹开而是相互靠拢，就是“速大压小”的道理。



补充例题,

水管里的水在压强为 $p=4\times 10^5 \text{ Pa}$ 的作用下流入房间，水管的内直径为2.0 cm，管内水的流速为4 m/s。引入到5 m高处二楼浴室的水管，内直径为1.0 cm，

试求浴室水管内水的流速和压强？

(已知水的密度为 $\rho=10^3 \text{ kg/m}^3$)。

$$Sv = \text{恒量} \quad p + \frac{1}{2}\rho v^2 + \rho gh = \text{恒量}$$

$$v_2 = 16 \text{ m/s}$$

$$p_2 = 2.25 \times 10^5 (\text{Pa})$$

