Мамонов Антон ЗИСиП-19-1

Практическая работа №1.
"Построение интерполяционного многочлена в форме Лагранжа".
Выполнил студент ЗИСиП-19-1.
Мамонов Антон
Вариант 9.

X	-3	-2	-1	0	1	2	5
y	-93	-37	13	17	5	127	24757

$w_1(x) = (x+2)(x+1)(x-0)(x-1)(x-2)(x-5) = (x^2-4)(x^2-1)(x)(x-5) =$

$= (x^3-4x)(x^2-1)(x-5) = (x^5 - x^3 - 4x^3 + 4x)(x-5) =$

$= x^6 - 5x^5 - x^4 + 5x^3 - 4x^4 + 20x^3 + 4x^2 - 20x = \boxed{x^6 - 5x^5 - 5x^4 + 25x^3 + 4x^2 - 20x}$

$w_2(x) = (x+3)(x+1)(x-0)(x-1)(x-2)(x-5) = (x^2-1)(x+3)(x-5)x =$

$x^6 = (x+3)(x^2-1)x(x-2)(x-5) = (x^2+3x)(x^2-1)(x-2)(x-5) =$

$= (x^4 - x^2 + 3x^3 - 3x)(x-2)(x-5) = (x^5 - 2x^4 - x^3 + 2x^2 + 3x^4 - 6x^3 - 3x^2 + 6x)$

$(x-5) = (x^6 - 5x^5 - 2x^5 + 10x^4 + x^4 + 5x^3 + 2x^3 - 10x^2 + 3x^5 - 15x^4 - 6x^4 +$

$+ 30x^3 - 3x^3 + 15x^2 + 6x^2 - 30x = \boxed{x^6 - 4x^5 - 12x^4 + 34x^3 + 11x^2 - 30x}$

$w_3(x) = (x+3)(x+2)(x-0)(x-1)(x-2)(x-5) = (x+3)(x^2-4)x(x-1)(x-5) =$

$= (x^2+3x)(x^2-4)(x-1)(x-5) = (x^4 - 4x^2 + 3x^3 - 12x)(x-1)(x-5) =$

$= (x^5 - x^4 - 4x^3 + 4x^2 + 3x^4 - 3x^3 - 12x^2 + 12x)(x-5) = x^6 - 5x^5 - x^5 + 5x^4 -$

$- 4x^4 + 20x^3 + 4x^3 - 20x^2 + 3x^5 - 15x^4 - 3x^4 + 15x^3 - 12x^3 + 60x^2 + 12x^2 - 60x =$

$= \boxed{x^6 - 3x^5 - 17x^4 + 27x^3 + 52x^2 - 60x}$

$w_4(x) = (x+3)(x+2)(x+1)(x-1)(x-2)(x-5) = (x+3)(x^2-4)(x^2-1)(x-5) =$

$= (x^3 - 4x + 3x^2 - 12)(x^2-1)(x-5) = (x^5 - x^3 - 4x^3 + 4x + 3x^4 - 3x^2 - 12x + 12)$

$(x-5) = x^6 - 5x^5 - x^4 + 5x^3 - 4x^4 + 20x^3 + 4x^2 - 20x + 3x^5 - 15x^4 - 3x^3 +$

$+ 15x^2 - 12x^3 + 60x^3 + 12x - 60 = \boxed{x^6 - 2x^5 - 20x^4 + 101x^3 + 79x^2 - 8x - 60}$

$w_5(x) = (x+3)(x+2)(x+1)(x-0)(x-2)(x-5) = (x^2+3x)(x^2-4)(x+1)(x-5) =$

$= (x^4 - 4x^2 + 3x^3 - 12x)(x+1)(x-5) = x^6 - 5x^5 + x^5 - x^4 - 4x^4 + 20x^3 - 4x^3$

$+ 20x^2 + 3x^5 - 15x^4 + 3x^4 - 15x^3 - 12x^3 + 60x^2 + 12x^2 + 60x = \boxed{x^6 - x^5 - 21x^4 + 111x^3 + 68x^2 + 60x}$

$w_6(x) = (x+3)(x+2)(x+1)(x-0)(x-2)(x-5) = (x+3)(x^2-4)(x+1)x(x-5) =$

$= (x^2+3x)(x^2-4)(x+1)(x-5) = (x^4-4x^2+3x^3-12x)(x+1)(x-5) =$

$= (x^5+x^4-4x^3-4x^2+3x^4+3x^3-12x^2-12x)(x-5) = (x^6-5x^5+x^5-x^4-4x^4+$

$+20x^3-4x^3+20x^2+3x^5-15x^4+3x^4-15x^3-12x^3+60x^2-12x^2+60x =$

$= x^6-20x^4-30x^3+19x^2+30x$

$w_7(x) = (x+3)(x+2)(x+1)(x-0)(x-1)(x-2) = (x+3)(x^2-4)(x^2-1)x =$

$= (x^2+3x)(x^2-4)(x^2-1) = (x^4-4x^2+3x^3-12x)(x^2-1) = (x^6-x^4-4x^4+4x^2+$

$3x^5-3x^3-12x^3+12x = x^6+3x^5-5x^4-15x^3+4x^2+12x$

$w_1(x_1) = -3^6-5\cdot(-3)^5-5\cdot(-3)^4+25\cdot(-3)^3+4\cdot(-3)^2+20\cdot(-3) =$

$= 729+1215-405-675+36+60 = 960$

$w_2(x_2) = -2^6-4\cdot(-2)^5-12\cdot(-2)^4+34\cdot(-2)^3+11\cdot(-2)^2-30\cdot(-2) =$

$= 64+128-192-272+16+60 = -168$

$w_3(x_3) = 1^6-5\cdot(-1)^5-21\cdot(-1)^4-11\cdot(-1)^3+68\cdot(-1)^2+60\cdot(-1) =$

$= 1+5-21+11+68-60 = 0$

$w_4(x_4) = 0^6-2\cdot0^5-20\cdot0^4+10\cdot0^3+19\cdot0^2-3\cdot0-60 = -60$

$w_5(x_5) = 1^6-1^5-21\cdot1^4-11\cdot1^3+68\cdot1^2+60\cdot1 = 1-1-21-11+68+60 = 96$

$w_6(x_6) = 2^6-20\cdot2^4-30\cdot2^3+19\cdot2^2+30\cdot2 = 64-320-240+76+60 = -360$

$w_7(x_7) = 5^6+3\cdot5^5-5\cdot5^4-15\cdot5^3+4\cdot5^2+12\cdot5 = 20160$

$f(x) = \frac{-43}{960}(x^6-5x^5-5x^4+25x^3+4x^2-20x) + \frac{-37}{168}(x^6-4x^5-12x^4+34x^3+11x^2-30x) +$

$+\frac{13}{72}(x^6-3x^5-17x^4+27x^3+52x^2-60x) + \frac{17}{60}(x^6-2x^5-20x^4+10x^3+19x^2-3x-60) +$

$+\frac{5}{96}(x^6-x^5-21x^4-11x^3+68x^2+60x) + \frac{127}{-360}(x^6-20x^4-30x^3+19x^2+30x) +$

$+\frac{24757}{20160}(x^6+3x^5-5x^4-15x^3+4x^2+12x) = x^6\left(\frac{-43}{960}+\frac{37}{168}+\frac{13}{72}-\frac{17}{60}+\frac{5}{96}-\frac{127}{360}+\frac{24757}{20160}\right.$

$+x^5\left(\frac{43\cdot5}{960}-\frac{37\cdot4}{168}-\frac{13\cdot3}{72}+\frac{17\cdot2}{60}-\frac{5\cdot1}{96}+\frac{127\cdot30}{360}+\frac{24757\cdot3}{20160}\right)+x^4\left(\frac{43\cdot5}{960}-\frac{37\cdot12}{168}-\right.$

$-\frac{13\cdot17}{72}+\frac{17\cdot20}{60}-\frac{5\cdot21}{96}+\frac{127\cdot20}{360}-\frac{24757\cdot5}{20160}\Big)+$

$$+x^3\left(\frac{-43\cdot25}{960}+\frac{37\cdot34}{168}+\frac{13\cdot27}{72}-\frac{17\cdot10}{60}-\frac{5\cdot11}{96}+\frac{127\cdot30}{360}-\frac{24757\cdot1}{20160}\right)$$

$$+x^2\left(\frac{-43\cdot4}{960}+\frac{37\cdot11}{168}+\frac{13\cdot52}{72}-\frac{17\cdot79}{60}+\frac{5\cdot68}{96}-\frac{127\cdot19}{360}+\frac{24757\cdot7}{20160}\right)$$

$$+x\left(\frac{43\cdot20}{960}-\frac{37\cdot30}{168}-\frac{13\cdot60}{72}+\frac{17\cdot8}{60}+\frac{5\cdot60}{96}-\frac{127\cdot30}{360}+\frac{24757\cdot13}{20160}\right)$$

$$+\frac{17\cdot60}{60}=\left(\frac{-43\cdot21+37\cdot120+13\cdot280-17\cdot336+5\cdot210-127\cdot56+24757}{20160}\right)x^6+\left(\frac{215\cdot21-148\cdot120-39\cdot280+34\cdot336-5\cdot210+24757}{20160}\right)x^5+\left(\frac{215\cdot21-444\cdot120-221\cdot280+340\cdot336-105\cdot210+2540\cdot56-123785}{20160}\right)x^4+\left(\frac{-1075\cdot21+1258\cdot120+351\cdot280-170\cdot336-55\cdot210+3810\cdot56+371355}{20160}\right)x^3+$$

$$+\left(\frac{-172\cdot21+407\cdot120+676\cdot280-1343\cdot336+340\cdot210-2413\cdot56+297084}{20160}\right)x^2+\left(\frac{860\cdot21-1110\cdot120-780\cdot280+300\cdot210-3810\cdot56+297084}{20160}\right)x+17=$$

$$=\boxed{x^6+3x^5-9x^2-7x+17}$$

Проверка:

1) $-3^6+3(-3)^5-9\cdot(-3)^2-7(-3)+17=729-729+81+21+17=(-43)$ Верно

2) $-2^6+3(-2)^5-9(-2)^2-7(-2)+17=64-96-36+14+17=(-37)$ Верно

3) $-1^6+3(-1)^5-9(-1)^2+7(-1)+17=1-3-9+7+17=(13)$ Верно

4) $0^6+3(0)^5-9(0)^2-7(0)+17=(17)$ Верно

5) $1^6+3(1)^5-9(1)-7(1)+17=1+3-9-7+17=(5)$ Верно

6) $2^6+3(2)^5-9(2)-7(2)+17=64+96-18-14+17=(127)$ Верно

7) $5^6+3(5)^5-9(5)^2-7(5)+17=15625+9375-225-35+17=(24757)$ Верно