BASIC STATISTICS

PROBABILITY IS THE NUMERICAL VALUE GIVEN TO THE LIKELIHOOD OF OCCURRENCE OF AN EVENT.

$$P(X) = \frac{f}{n}$$

Eg: coin tossing experiment

S{H,T}

{H} {T}

RANDOM EXPERIMENT

SAMPLE POINT- Every indecomposable outcome of a random experiment

<u>PROPERTIES(axioms)</u>

1. $0 \le P(X) \le 1$
2. P(S) = 1

   P{H} =1/2 P{T}=1/2

   P(S) = p{H} + P{T} = 1
3. If A AND B ARE MUTUALLY EXCLUSIVE EVENTS

   P(A U B) = P(A) + P(B)

UNION AN INTERSECTION

<u>A ={ 1,2,3,4} B={3,4,5,6}</u>

A U B={1,2,3,4,5,6}

$$\boldsymbol{A \cap B = \{3,4\}}$$

Conditional probability

Probability of an event A given that B has happened is called conditional probability.

2 CHILDREN

S={GG,BG,GB,BB}

ATLEST 1 GIRL

A = { GG,BG,GB}

ATLEAST ONE BOY

B ={BG,GB,BB}

P{A} = $\frac{3}{4}$ P(B) =$\frac{3}{4}$

P(A|B) = $\frac{2}{3}$

Inverse Probability

Suppose an event A has happened as a result of several causes. Then we are interested to find out the probability of a particular cause which really affect the event to happen. Problems of this type are called inverse probability.

BAYES THEOREM

If an event A happen only if one or the other of a set of mutually exclusive events B1,B2,B3…Bn happens.Then

P(Bk|A)=$\frac{P(Bk)P(A|Bk)}{P(B1)P(A|B1)+P(B2)P(A|B2)+\cdots.P(Bk)P(A|Bk)}$

A is a commonly action elements for all B's

<u>EVENT</u>

<u>S={HH,HT,TT,TH}</u>

<u>S=(1,2,3,4,5,6)</u>

<u>S={2,4,6}   S={1,3,5}</u>

<u>SURE EVENT - CERTAINITY</u>

EG:

GETTING A WHITE BALL FROM A BAG CONTAINING WHITE BALL

IMPOSSIBLE EVENT

GETTING A BLACK BALL FROM A BAG CONTAINING WHITE BALL

UNCERTAIN EVENT

GETTING A WHITE BALL FROM A BAG CONTAINING WHITE BALL AND BLACK BALL

6*6=36

S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

PROBABILITY DISTRIBUTIONS

P(X)= P[X=x]   x 0,1,2,3,4,

1. Find the probability distribution of total no: of tails obtained in 3 tosses of a coin.
   X = NO:OF TAILS
   S = {HHH,HHT,HTH,THH,TTT,TTH,THT,HTT}
   0 1 1 1 3 2 2 2

   X={0,1,2,3} x1=0 x2 = 1 x3=2 x4= 3

P{X=0} = $\frac{1}{8}$ P{X=1} =$\frac{3}{8}$

BINOMIAL DISTRIBUTION -DISCRETE

f(x) =$nCxp^xq^{n-x}$; $x = 0, 1, \dots\dots. n$
p , q=1-p

ncx =$\frac{n!}{x!(n-x)!}$

1. Four coins a tossed .What is the probability of getting 2 heads.
   n = 4
   p=P(getting head in a single trial) =$\frac{1}{2}$
   q = (1/2)
   P(X=2) =

POISSON DISTRIBUTION

f(x) =$\frac{e^{-m}m^{x}}{x!}$ **; x=0,1,2,…∞**

1. If 3% of bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs exactly five bulbs are defective.

n=100 m = np

p=0.03

correlation –relation b/w two variables

CAUSATION - CAUSE AND EFFECT RELATIONSHIP

RICE TEA RAINFALL

BASIC MATHEMATICS

LINEAR ALGEBRA

Linear algebra is the study of linear combinations. It is the study of vector spaces, lines and planes, and some mappings that are required to perform the linear transformations. It includes vectors, matrices and linear functions. .

The general linear equation is represented as

$$a_1x_1 + a_2x_2 ....+a_nx_n = b$$

Here,

a's – represents the coefficients

x's – represents the unknowns variables

b – represents the constant

can be written as AX = B

1. Find the value of x, y and z for the given system of linear equations.

$$2x + y - z = 8$$

$$-3x - y + 2z = -11$$

$$-2x + y + 2z = -3$$

Solution:given

$2x + y - z = 8$

$-3x - y + 2z = -11$

$-2x + y + 2z = -3$

The matrix is of the form, AX = B

$$A = \begin{matrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{matrix} \qquad X = \begin{matrix} x \\ y \\ z \end{matrix} \qquad B = \begin{matrix} 8 \\ -11 \\ -3 \end{matrix}$$

After performing elementary row operation and augmented matrix, it is reduced to the form

$$A : B = \begin{matrix} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & 11 \\ -2 & 1 & 2 & -3 \end{matrix}$$

Now the reduced echelon form of the above matrix is,

$$A : B = \begin{matrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{matrix}$$

Therefore, the unique solution for this is,

x = 2

y = 3

z = -1

VECTOR SPACE

A vector space is defined as the collection of objects called vectors, which may be added together and multiplied (i.e. scaled) by numbers, called scalars.

A Non empty set V with 2 operations + and * is a vector space over a field F if it satisfies the following axioms:

1. Closure property x + y $\in V$
   V ={-1,1,0,1,2,3,4,5}

   X=2 Y=3 x+y =3+2 =5

2. Associative property
   (X +y)+ z =x+(y+z)
3. Existence of identity
   X+0 = x
4. Existence of inverse
   X +(-x) =0
5. Commutative property x+y =y+x
6. Distributive property
   1. $\propto.(x+y) = \propto x + \propto y$
   2. $(\propto \beta).x = \propto.(\beta x)$
   3. $(\propto + \beta).x = \propto.x + \beta.x$
7. For every x$\in V$ $1 \in F, 1.x = x$

LINEAR COMBINATION

Consider V over a field F .Let x1,x2,…,xn$\in V$ and c1,c2,…,cn$\in F$ .Then combination of these vectors and constants also belongs to V is called linear combination.

i.e., c1x1+c2x2+…cnxn$\in V$

1. Express (3,-1,3) as a linear combination of (1,-3,1),(1,1,-1), and (2,0,3)

Solution: Consider (1,-3,1) as V1,(1,1,-1) as V2 and (2,0,3) as V3.

Let a,b,c be the constants

Then $aV1 + bV2 + cV3 = (3, -1, 3)$

i.e., $a(1, -3, 1) + b(1, 1, -1) + c(2, 0, 3) = (3, -1, 3) - - - - - - -(1)$

$$(a, -3a, a) + (b, b, -b) + (2c, 0, 3c) = (3, -1, 3)$$

$$(a + b + 2c, -3a + b, a - b + 3c) = (3, -1, 3)$$

$$a + b + 2c = 3$$

$$-3a + b = -1$$

$$a - b + 3c = 3$$

The matrix is of the form, AX = B

A = $\begin{matrix} 1 & 1 & 2 \\ -3 & 1 & 0 \\ 1 & -1 & 3 \end{matrix}$ X = $\begin{matrix} a \\ b \\ c \end{matrix}$ B = $\begin{matrix} 3 \\ -1 \\ 3 \end{matrix}$

After performing elementary row operation and augmented matrix, it is reduced to the form

A = $\begin{matrix} 1 & 1 & 2 & 3 \\ -3 & 1 & 0 & -1 \\ 1 & -1 & 3 & 3 \end{matrix}$

Using Gauss Elimination Method

A = $\begin{matrix} 1 & 1 & 2 & 3 \\ 0 & 4 & 6 & 8 \\ 0 & 0 & 8 & 8 \end{matrix}$

$$a + b + 2c = 3$$

$$4b + 6c = 8$$

$$8c = 8$$

Solving these equations

a = $\frac{1}{2}$,

b =$\frac{1}{2}$,

c=1

Equation (1) can be rewritten as

$$\frac{1}{2}(1,-3,1)+\frac{1}{2}(1,1,-1)+1(2,0,3)=(3,-1,3)$$

LINEAR SPAN

The span of a set of vectors, also called linear span, is the linear space formed by all the vectors that can be written as linear combinations of the vectors belonging to the given set.

Let V be a linear space. Let $x1, x2 \dots, xn \in V$ be n vectors. The linear span of $x1, x2, \dots, xn$, denoted by $span(x1, x2, \dots, xn)$ is the set of all the linear combinations y $= \propto 1x1 + \propto 2x2 + \cdots + \propto nxn$

that can be obtained by arbitrarily choosing n scalars $\propto 1, \propto 2 \dots, \propto n$

BASIS OF A VECTOR SPACE

Let V denote a vector space and S = {u1,u2, …, un} a subset of V . S is called a basis for V if the following is true:

1. S spans V .

2. S is linearly independent.

DIMENSION OF VECTOR

Let V denote a vector space. Suppose a basis of V has n vectors (therefore all bases will have n vectors). n is called the dimension of V . We write dim (V) = n.

MATRIX

An m × n matrix is usually written as:

$$A = \begin{matrix} a11 & a12 \ldots & a1n \\ a21 & a22 \ldots & a2n \\ am1 & am2 \ldots & amn \end{matrix}$$

In brief, the above matrix is represented by A = $[a_{ij}]_{mxn}$. The number a_{11}, a_{12}, ….. etc., are known as the elements of the matrix A, where a_{ij} belongs to the i^{th} row and j^{th} column and is called the $(i, j)^{th}$ element of the matrix A = $[a_{ij}]$.

Types of Matrices

- **(i) Symmetric Matrix:** A square matrix A =$[aij]$ is called a symmetric matrix if aij=aji for all i, j.
- **(ii) Skew-Symmetric Matrix:** when aij = -aji
- **(iii) Hermitian and skew – Hermitian Matrix:** A =A^{θ}(Hermitian matrix) (A^{θ} represents conjugate transpose) $A^{\theta} = -A$ (skew-Hermitian matrix)
- **(iv) Orthogonal matrix:** if A=$AA^{T} = I_{n} = A^{T}A$
- **(v) Idempotent matrix:** if $A^{2} = A$

Transpose of Matrix

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called the transpose of matrix A and is denoted by A^{T} or A’. From the definition it is obvious that if the order of A is m x n, then the order of A^T becomes n x m; E.g. transpose of matrix

$$A = \begin{matrix} a1 & a2 & a3 \\ b1 & b2 & b3 \end{matrix} \qquad A^T = \begin{matrix} a1 & b1 \\ a2 & b2 \\ a3 & b3 \end{matrix}$$

1. If $\begin{matrix} 1 & -2 & 3 \\ -2 & 4 & 5 \end{matrix} \quad and \quad B = \begin{matrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{matrix}$

. then prove that $(AB)^T = B^TA^T$

$$\begin{matrix} 1 & -2 & 3 \\ -2 & 4 & 5 \end{matrix} * \begin{matrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{matrix}$$

AB =

$$\begin{matrix} (1*1)+(-2*-1)+(2*3) & (1*3)+(-2*0)+(3*4) \\ (-2*1)+(4*-1)+(5*2) & (-2*3)+(4*0)+(5*4) \end{matrix}$$

$$= \begin{matrix} 9 & 15 \\ 4 & 8 \end{matrix}$$

$$((AB)^T = \begin{matrix} 9 & 4 \\ 15 & 8 \end{matrix}$$

$$B^TA^T = \begin{matrix} 1 & -1 & 2 \\ 3 & 0 & 4 \end{matrix} * \begin{matrix} 1 & -2 \\ -2 & 4 \\ 3 & 5 \end{matrix}$$

$$= \begin{matrix} 9 & 4 \\ 15 & 8 \end{matrix}$$

|AB| = Rank =2

RANK OF A MATRIX

Rank of A is defined as the order of the largest square sub matrix A whose determinant is non zero. It is denoted by$\rho(A)$.

Rank of 2$\times$ 2 matrix –Possible ranks are 2,1,0.

1. Find determinant of matrix
2. If it is non zero then rank is 2
3. If it is zero rank is 1
4. If the matrix is a null matrix then rank is 0

1. Find rank of A =$\begin{matrix} 4 & 6 \\ 5 & 6 \end{matrix}$

   |A|=(24-30) = -6 >0

   RANK = 2

2. Find rank of A=$\begin{matrix} 4 & 1 & 5 \\ -1 & -2 & 2 \\ 1 & -1 & 1 \end{matrix}$

   |A| = 4$\begin{matrix} -2 & 2 \\ -1 & 1 \end{matrix}$ -1$\begin{matrix} -1 & 2 \\ 1 & 1 \end{matrix}$ +5$\begin{matrix} -1 & -2 \\ 1 & -1 \end{matrix}$

   = 4((-2*1)-(-1*2))-1((-1*1)-(1*2))+5((-1*-1)-((-2*1))

   =(4*0)-(1*-3)+(5+3)

   =18 > 0

   RANK = 3

EIGEN VALUE(SPECTRUM) AND EIGEN VECTORS

Direction of movement is the eigen vector and how much it is directed is the eigen value. The basic equation is $AX = \lambda X$. The number λ is an eigen value of A

Eg: $A = \begin{matrix} a11 & a12 \\ a21 & a22 \end{matrix}$

Characteristic equation = |A-λI|=0 i.e., $\lambda^2 - S1\lambda + S2 = 0$

S1 =Sum of main diagonal element

=a11+a22

S2=|A|

Substitute S1 and S2 in characteristic equation and solve we get eigen values of λ