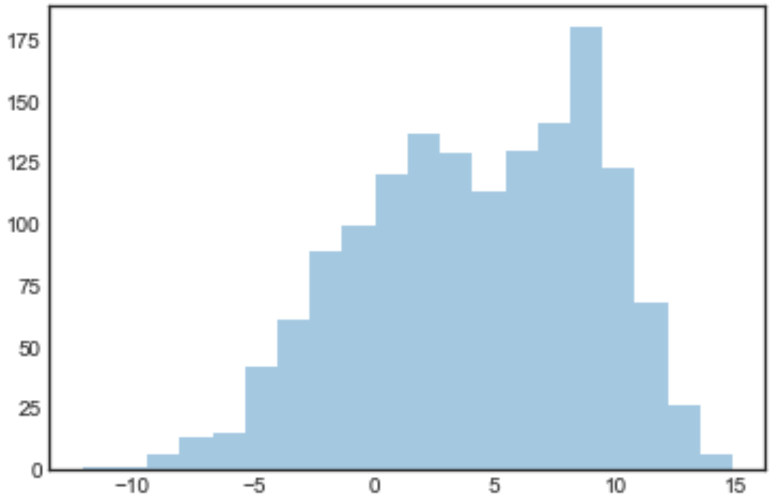
In [2]:
```
# import libraries

# For plotting
"seaborn: statistical data visualization"
import matplotlib.pyplot as plt
import seaborn as sns
sns.set_style("white")
%matplotlib inline
#for matrix math
import numpy as np
#for normalization + probability density function computation
from scipy import stats
#for data preprocessing
import pandas as pd
from math import sqrt, log, exp, pi
from random import uniform
print("import done")
```

```
import done
```

In [3]:
```
Mean1 = 2  # Input parameter, mean of first normal probability dist
ribution
Standard_dev1 = 4 #@param {type:"number"}
Mean2 = 9 # Input parameter, mean of second normal  probability dis
tribution
Standard_dev2 = 2 #@param {type:"number"}

# generate data
y1 = np.random.normal(Mean1, Standard_dev1, 1000)
y2 = np.random.normal(Mean2, Standard_dev2, 500)
data=np.append(y1,y2)
print(data)

# For data visiualisation calculate left and right of the graph
Min_graph = min(data)
Max_graph = max(data)
x = np.linspace(Min_graph, Max_graph, 2000) # to plot the data

sns.distplot(data, bins=20, kde=False)
```

```
[-0.87012123  8.72030264  3.37633571 ...  9.06582318  6.66174451
  7.73636841]
```

Out[3]: <matplotlib.axes._subplots.AxesSubplot at 0x1146070b8>

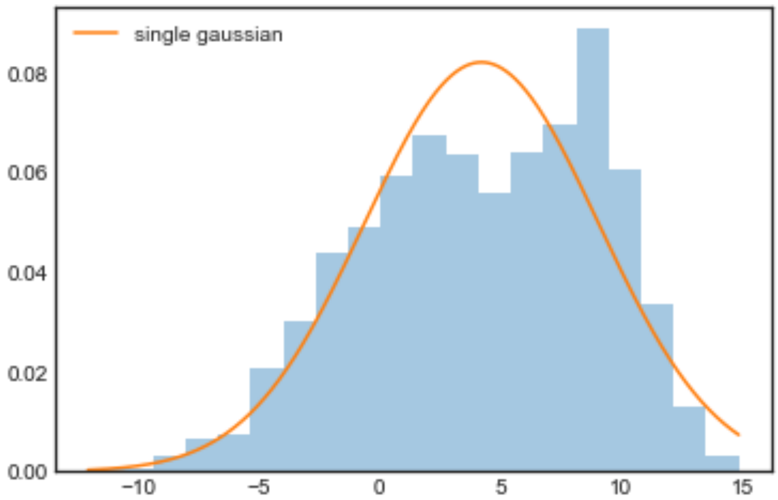


In [4]:
```
class Gaussian:
    "Model univariate Gaussian"
    def __init__(self, mu, sigma):
        #mean and standard deviation
        self.mu = mu
        self.sigma = sigma

    #probability density function
    def pdf(self, datum):
        "PDF Of gaussian distribution"
        "Probability of a data point given the current parameters"
        u = (datum - self.mu) / abs(self.sigma)
        y = (1 / (sqrt(2 * pi) * abs(self.sigma))) * exp(-u * u / 2
)
        return y

    def __repr__(self):
        return 'Gaussian({0:4.6}, {1:4.6})'.format(self.mu, self.si
gma)
print("done")
```

```
done
```

In [9]:
```
"A single Gaussion will not fit the data well"

#gaussian of best fit
best_single = Gaussian(np.mean(data), np.std(data))
print('Best single Gaussian: μ = {:.2}, σ = {:.2}'.format(best_sing
le.mu, best_single.sigma))


#fit a single gaussian curve to the data
print('Mean of fitting gaussian is '+ str(best_single.mu))
print('Standard deviation of fitting gaussian is '+ str(best_single
.sigma))
g_single = stats.norm(best_single.mu, best_single.sigma).pdf(x)
sns.distplot(data, bins=20, kde=False, norm_hist=True)
plt.plot(x, g_single, label='single gaussian')
plt.legend();
```

```
Best single Gaussian: μ = 4.2, σ = 4.9
Mean of fitting gaussian is 4.2146301403131705
Standard deviation of fitting gaussian is 4.850670553053463
```



In [19]:
```
class GaussianMixture_self:
    "Model mixture of two univariate Gaussians and their EM estimat
ion"

    def __init__(self, data, mu_min=min(data), mu_max=max(data), si
gma_min=1, sigma_max=1, mix=.5):
        self.data = data
        #todo the Algorithm would be numerical enhanced by normaliz
ing the data first, next do all the EM steps and do the de-normalis
ing at the end
        #init with multiple gaussians

        #'Uniform' draws samples from a uniform distribution.


        self.one = Gaussian(uniform(mu_min, mu_max),
                            uniform(sigma_min, sigma_max))
```

```
        self.two = Gaussian(uniform(mu_min, mu_max),
                            uniform(sigma_min, sigma_max))

        #how well to mix the gaussians
        self.mix = mix

    def Estep(self):
        "E(stimation)-step, assign each point to gaussian 1 or 2 wi
th a percentage"

        # compute weights
        self.loglike = 0. # = log(p = 1)
        for datum in self.data:
            # unnormalized weights
            wp1 = self.one.pdf(datum) * self.mix
            wp2 = self.two.pdf(datum) * (1. - self.mix)
            # compute denominator
            den = wp1 + wp2
            # normalize
            wp1 /= den
            wp2 /= den     # wp1+wp2= 1, it either belongs to gauss
ian 1 or gaussion 2
            # add into loglike
            self.loglike += log(wp1 + wp2) #freshening up self.logl
ike in the process
            # yield weight tuple
            yield (wp1, wp2)

    def Mstep(self, weights):
        "Perform an M(aximization)-step"
        # compute denominators
        (left, rigt) = zip(*weights)
        one_den = sum(left)
        two_den = sum(rigt)

        # compute new means
        self.one.mu = sum(w * d  for (w, d) in zip(left, data)) / o
ne_den
        self.two.mu = sum(w * d  for (w, d) in zip(rigt, data)) / t
wo_den

        # compute new sigmas
        self.one.sigma = sqrt(sum(w * ((d - self.one.mu) ** 2)
                                  for (w, d) in zip(left, data)) /
one_den)
        self.two.sigma = sqrt(sum(w * ((d - self.two.mu) ** 2)
                                  for (w, d) in zip(rigt, data)) /
two_den)
        # compute new mix
        self.mix = one_den / len(data)


    "Verbose is a general programming term for produce lots of logg
```

```
ing output. You can think of it as asking the program to tell me ev
erything about what you are doing all the time."

    def iterate(self, N=1, verbose=False):
        "Perform N iterations, then compute log-likelihood"
        for i in range(1, N+1):
            self.Mstep(self.Estep()) #The heart of the algorith, pe
rform E-stepand next M-step
            if verbose:
                print('{0:2} {1}'.format(i, self))
        self.Estep() # to freshen up self.loglike

    def pdf(self, x):
        return (self.mix)*self.one.pdf(x) + (1-self.mix)*self.two.p
df(x)

    def __repr__(self):
        return 'GaussianMixture({0}, {1}, mix={2.03})'.format(self.
one,
                                                              self.
two,
                                                              self.
mix)

    def __str__(self):
        return 'Mixture: {0}, {1}, mix={2:.03})'.format(self.one,
                                                        self.two,
                                                        self.mix)
print("done")
```

```
done
```

In [20]:
```
# See the algorithem in action
n_iterations = 5
best_mix = None
best_loglike = float('-inf')
mix = GaussianMixture_self(data)
for _ in range(n_iterations):
    try:
        #train!
        mix.iterate(verbose=True)
        if mix.loglike > best_loglike:
            best_loglike = mix.loglike
            best_mix = mix

    except (ZeroDivisionError, ValueError, RuntimeWarning): # Catch
division errors from bad starts, and just throw them out...
        print("one less")
        pass
```

```
 1 Mixture: Gaussian(-8.99228, 1.37431), Gaussian(4.28471, 4.76586
), mix=0.00528)
 1 Mixture: Gaussian(-8.33561, 1.46212), Gaussian(4.25674, 4.80318
), mix=0.00334)
 1 Mixture: Gaussian(-7.82462, 1.38221), Gaussian(4.24806, 4.81513
), mix=0.00277)
 1 Mixture: Gaussian(-7.47141, 1.24351), Gaussian(4.24516, 4.81962
), mix=0.00261)
 1 Mixture: Gaussian(-7.26184, 1.12581), Gaussian(4.24498, 4.82065
), mix=0.00264)
```

In [21]:
```
# Find best Mixture Gaussian model
n_iterations = 300
n_random_restarts = 4
best_mix = None
best_loglike = float('-inf')
print('Computing best model with random restarts...\n')
for _ in range(n_random_restarts):
    mix = GaussianMixture_self(data)
    for _ in range(n_iterations):
        try:
            mix.iterate()
            if mix.loglike > best_loglike:
                best_loglike = mix.loglike
                best_mix = mix
        except (ZeroDivisionError, ValueError, RuntimeWarning): # C
atch division errors from bad starts, and just throw them out...
            pass
print('Best Gaussian Mixture : μ = {:.2}, σ = {:.2} with μ = {:.2},
σ = {:.2}'.format(best_mix.one.mu, best_mix.one.sigma, best_mix.two
.mu, best_mix.two.sigma))

#Show mixture
sns.distplot(data, bins=20, kde=False, norm_hist=True)
g_both = [best_mix.pdf(e) for e in x]
plt.plot(x, g_both, label='gaussian mixture');
g_left = [best_mix.one.pdf(e) * best_mix.mix for e in x]
plt.plot(x, g_left, label='gaussian one');
g_right = [best_mix.two.pdf(e) * (1-best_mix.mix) for e in x]
plt.plot(x, g_right, label='gaussian two');
plt.legend();
```

```
Computing best model with random restarts...

Best Gaussian Mixture : μ = 1.8, σ = 4.1 with μ = 8.9, σ = 2.0
```