# ESTIMATE WIND & FIND ORIENTATION - II

Assume Wind also induces moments:

$$\left[\begin{array}{l} {}^{B}\vec{\mathbb{F}} = m\left({}^{B}\vec{\omega} \times {}^{B}\vec{v} + {}^{B}\dot{\vec{v}}\right) \quad \text{— ①} \\ {}^{B}\vec{\tau} = {}^{B}\vec{\omega} \times J\,{}^{B}\vec{\omega} + J\,{}^{B}\dot{\vec{\omega}} \quad \text{— ②} \end{array}\right]$$

① $${}^{B}\vec{\mathbb{F}} = {}^{B}\vec{F}_T + {}^{B}\vec{F}_g + {}^{B}\underbrace{\vec{F}_w}_{\text{wind}}$$

(Analysis mostly remains same)

② $${}^{B}\vec{\tau} = \underbrace{\begin{bmatrix} M_{p_x} \\ M_{p_y} \\ M_{p_z} \end{bmatrix}}_{\text{Moments due to Propellors}} + \underbrace{\begin{bmatrix} M_{w_x} \\ M_{w_y} \\ M_{w_z} \end{bmatrix}}_{\text{Moments due to wind}}$$

"${}^{B}\tau_p$"

"${}^{B}\vec{\tau}_{w_{est}}$"

$$\boxed{{}^{B}\vec{F}_{w_{est}} = m\left({}^{B}\vec{\omega}_{IMU} \times {}^{B}\vec{v}_{est} + {}^{B}\dot{\vec{v}}_{IMU}\right) - \begin{bmatrix} 0 \\ 0 \\ -u_4 \end{bmatrix} - Q_{est}^{T}\begin{bmatrix} 0 \\ 0 \\ -(9.8)m \end{bmatrix}}$$

Estimated Wind vector in Body frame

Sensors
EKF 2
Control Data
Truth

$${}^{B}\tau_p + {}^{B}\tau_w = J\,{}^{B}\dot{\vec{\omega}} + {}^{B}\vec{\omega} \times J\,{}^{B}\vec{\omega}$$

☐ sensor
• integratⁿ ???

☑ IMU sensor
(gyro)

$$\underbrace{\left[\text{refer...}\right]}_{4\times4}\underbrace{\begin{bmatrix} F_{T-1} \\ F_{T-2} \\ F_{T-3} \\ F_{T-4} \end{bmatrix}} + {}^{B}\tau_{w_{est}} = J\,{}^{B}\dot{\vec{\omega}} + {}^{B}\vec{\omega} \times J\,{}^{B}\vec{\omega}$$

${}^{B}\tau_{w_{est}}$: need to find

$J\,{}^{B}\dot{\vec{\omega}}$: ☐ sensor • integratⁿ ???

${}^{B}\vec{\omega}$, ${}^{B}\vec{\omega}$: ☑ IMU sensor (gyro)

$F_{T-1}, \dots, F_{T-4}$:
☑ individual propellor thrusts
☑ maybe read from ESC/PWM data?

---

$$\boxed{{}^{B}\tau_{w_{est}} = J\,{}^{B}\dot{\vec{\omega}} + {}^{B}\vec{\omega} \times J\,{}^{B}\vec{\omega} - \underbrace{\left[\text{refer...}\right]}_{4\times4}\underbrace{\begin{bmatrix} F_{T-1} \\ F_{T-2} \\ F_{T-3} \\ F_{T-4} \end{bmatrix}}}$$

${}^{B}\tau_{w_{est}}$: need to find

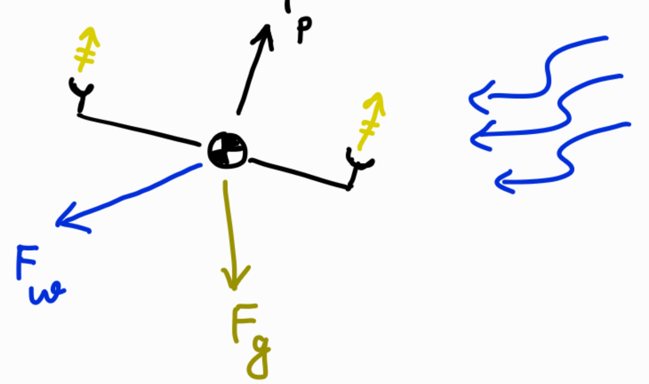
$J\,{}^{B}\dot{\vec{\omega}}$: ☐ sensor • integratⁿ ???

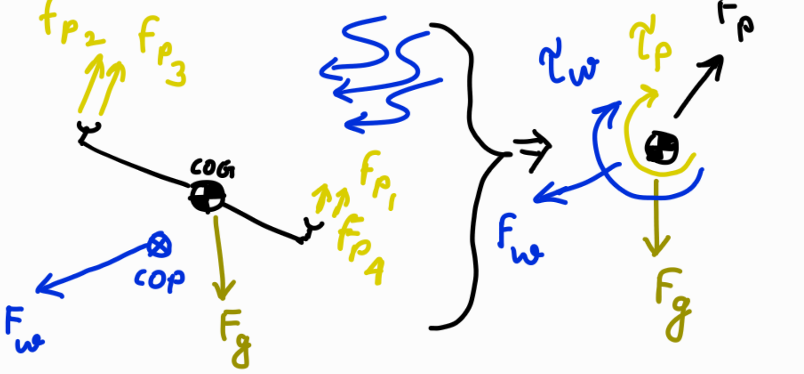
${}^{B}\vec{\omega}$, ${}^{B}\vec{\omega}$: ☑ IMU sensor (gyro)

$F_{T-1}, \dots, F_{T-4}$:
☑ individual propellor thrusts
☑ maybe read from ESC/PWM data?

★ THIS IS THE WIND-TORQUE ESTIMATE IN NON-EQUILIBRIUM CONDITION.

↳ . . . ★




**Forces**

Prop-Force $(f_p) \neq 0$

Wind Force $(F_w) \neq 0$

Gravity $(F_g) \neq 0$

**Moments**

Roll $\rightsquigarrow$ ${}^{B}M_x = 0$

Pitch $\rightsquigarrow$ ${}^{B}M_y = 0$

Yaw $\rightsquigarrow$ ${}^{B}M_z = 0$

Control Moments

**Forces**

Prop-Force $(f_p) \neq 0$

Wind Force $(F_w) \neq 0$

Gravity $(F_g) \neq 0$

**Moments**

Roll $\rightsquigarrow$ ${}^{B}M_x \neq 0$

Pitch $\rightsquigarrow$ ${}^{B}M_y \neq 0$

Yaw $\rightsquigarrow$ ${}^{B}M_z \neq 0$

Control Moments $(\tau_p)$

---

$\hookrightarrow \ldots$ ★ ★ WE FOUND THE WIND-TORQUE ESTIMATE IN NON-EQUILIBRIUM CONDITION. (IN BODY FRAME)

★ THE DRONE HAS NON-ZERO $\dot{\vec{\omega}}$ & $\dot{\vec{v}}$ IMPLYING THAT "HOVER-CONDITION" IS NOT MET YET... THE REQUIRED ORIENTATION TO ACHIEVE THAT WILL BE OBTAINED BY ANALYSIS DISCUSSED IN "HOVER-FIND1" (Case 1)

★ THE REQUIRED CONTROL-MOMENTS WILL BE OBTAINED LIKE THIS:

OPTION-2

$${}^{W}\vec{z}_{body} = -\frac{{}^{W}\vec{F}_{T_{sp}}}{\|{}^{W}\vec{F}_{T_{sp}}\|}$$

mc_pos_control
↳ /PositionControl
↳ /ControlMath.cpp
(Line 70-120

→ normalize$({}^{W}\vec{z}_{body})$

→ $$\vec{y}_{dummy} = \begin{bmatrix} -\sin(\psi_{sp}) \\ \cos\psi_{sp} \\ 0.0 \end{bmatrix}$$

In PX4, this $\psi_{sp}$ comes from user input

This means PX4 assumes world frame to be N-E-Down

→ $${}^{W}\vec{x}_{body} = \vec{y}_{dummy} \times {}^{W}\vec{z}_{body}$$

→ normalize$({}^{W}\vec{x}_{body})$

→ sanity check for upside-down case

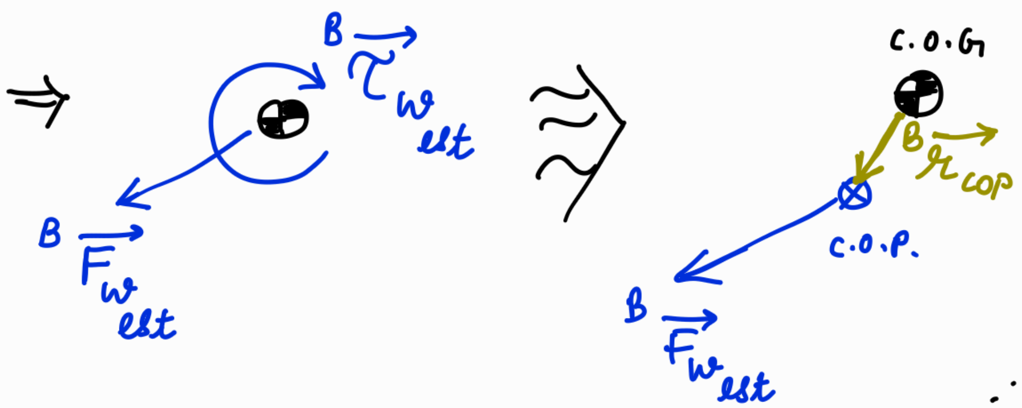
→ $${}^{W}\vec{y}_{body} = {}^{W}\vec{z}_{body} \times {}^{W}\vec{x}_{body}$$

→ normalize$({}^{W}\vec{y}_{body})$

→ $$\boxed{Q_{sp} = \begin{bmatrix} {}^{W}\vec{x}_{body} & {}^{W}\vec{y}_{body} & {}^{W}\vec{z}_{body} \end{bmatrix}}$$

∵ We have estimated wind-torque (in current Body-Frame) and wind-force (in both → current Body-Frame as well as World-Frame).

⇒




$$\boxed{{}^{B}\vec{\tau}_{W_{est}} = {}^{B}\vec{r}_{COP} \times {}^{B}\vec{F}_{W_{est}}}$$

all in Body Frame

(**Check if true?**)

$$\left[{}^B\vec{\tau}_{w_{est}}\right]_{3\times1} = \left[{}^B\hat{r}_{cop}\right]_{3\times3}\left[{}^B\vec{F}_{w_{est}}\right]_{3\times1}$$

$$ {}^B\vec{\tau}_{w_{est}} = -\left[\hat{F}_{w_{est}}\right]_{3\times3}\left[{}^B\vec{r}_{cop}\right]_{3\times1}$$

$$\left({}^B\hat{F}_{w_{est}}\right)^T\left({}^B\vec{\tau}_{w_{est}}\right) = -\left({}^B\hat{F}_{w_{est}}\right)^T\left({}^B\hat{F}_{w_{est}}\right)\left[{}^B\vec{r}_{cop}\right]_{3\times1}$$

$$\boxed{{}^B\vec{r}_{cop_{est}} = -\left[\left\{\left({}^B\hat{F}_{w_{est}}\right)^T\left({}^B\hat{F}_{w_{est}}\right)\right\}^{-1}\left({}^B\hat{F}_{w_{est}}\right)^T\right]\left({}^B\vec{\tau}_{w_{est}}\right)}$$

Estimated Centre Of Pressure offset from Centre of Gravity in Drone-Body-Frame

$$ {}^B\vec{\tau} = {}^B\vec{\tau}_p + {}^B\vec{\tau}_{w_{est}} = \cancelto{0}{{}^B\vec{\omega}\times J\,{}^B\vec{\omega}} + \cancelto{0}{J\,{}^B\dot{\vec{\omega}}}$$

"p" for propellors... = Torque due to control moments...

$$ {}^B\vec{\tau}_p = -{}^B\vec{\tau}_{w_{est}}$$

$${}^B\vec{\tau} \quad \begin{bmatrix} {}^BM_x \end{bmatrix} \quad {}^B\vec{\tau}$$

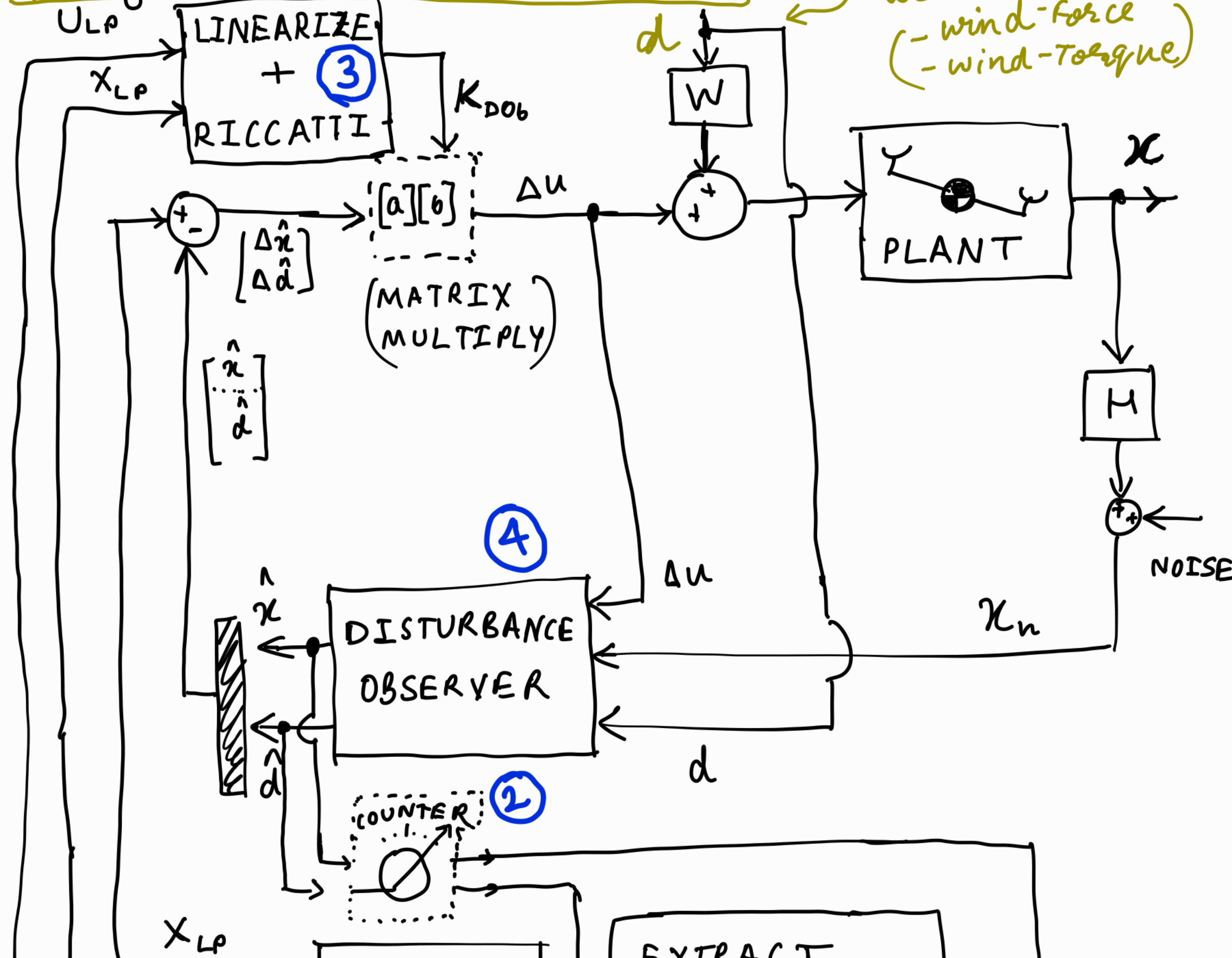
$$\tau_p = \begin{bmatrix} {}^{B}M_x \\ {}^{B}M_y \\ {}^{B}M_z \end{bmatrix} = \tau_{w_{est}}$$

→ This gives us the Hover-Condition for Controls vector

→ We will linearize about this point for linear controls...

# DISTURBANCE REJECTION

Q) Why did we do all this?




**Ans)** To find out $X_{LP}$ & $U_{LP}$ meaning the "Linearization-Point" about which we will be linearizing the non-linear drone dynamics (under wind).

This LP is not straightforward under the presence of wind-force & wind-induced torque. The corresponding equations have been discussed in this pdf (cop ≠ cog) & "Hover-Finding" pdf (cop coincides with cog)

BLOCK 1

| | |
|---|---|
| Centre of Pressure (COP) coincides with Centre of Gravity (COG) | Centre of Pressure (COP) offset from Centre of Gravity (COG) |

Intuition :

Augmented State-Disturbance Vector:

$$\begin{bmatrix} \hat{x} \\ \hdashline \hat{d} \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hdashline \vec{F}_{w_{est}} \end{bmatrix}$$

Augmented State-Disturbance Vector:

$$\begin{bmatrix} \hat{x} \\ \hdashline \hat{d}_1 \\ \hdashline \hat{d}_2 \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hdashline \vec{F}_{w_{est}} \\ \hdashline \vec{\tau}_{w_{est}} \end{bmatrix}$$

* $\hat{d}_1$ affects drone-orientation-setpoint $Q_{sp}$ or $q_{sp}$ and Thrust-magnitude setpoint $F_T$

* $\hat{d}_2$ affects only the Control-Moment setpoints $\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$ Roll Pitch Yaw

$$\Rightarrow \quad X_{LP} = \begin{bmatrix} x_{desired} \end{bmatrix}$$

$$\begin{bmatrix} y_{desired} \\ z_{desired} \\ \hdashline q_{sp} \\ \hdashline 0 \\ \hdashline 0 \\ \hdashline {}^{W}\vec{F}_{w_{est}} \\ \hdashline {}^{B}\vec{\tau}_{w_{est}} \end{bmatrix}$$

discussed before

${}^{B}\vec{v} = 0$

${}^{B}\vec{\omega} = 0$

most recent estimates

$$\Rightarrow U_{LP} = \begin{bmatrix} {}^{B}M_x \\ {}^{B}M_y \\ {}^{B}M_z \\ \hdashline F_T \end{bmatrix} = \begin{bmatrix} -{}^{B}\vec{\tau}_{w_{est}} \\ \hdashline F_T \end{bmatrix}$$

most recent estimate

discussed before :

$$\left\| Q_{sp}^{T} \left( {}^{W}\vec{F}_{w_{est}} - {}^{W}\vec{F}_{g} \right) \right\|_2$$

Ignore

Note :

↳ ${}^{W}\vec{F}_{w}$ (affects) $X_{LP}$

$\omega_{est}$ $\Rightarrow$ $U_{LP}$

$\hookrightarrow$ $\overrightarrow{B}$ $\tau_{w_{est}}$ (affects) $\rightsquigarrow$ $U_{LP}$ only

**★ USING $x_{LP}$ & $U_{LP}$**

**BLOCK ③ : LINEARIZE**

Nonlinear Dynamics:

$$\dot{x} = f(x, u)$$

Linearize about $x_{LP}$ & $U_{LP}$:

$$\dot{\left(x_{LP} + \Delta x\right)} \approx f(x_{LP}, u_{LP}) + \left.\frac{\partial f}{\partial x}\right|_{@(x_{LP}, u_{LP})} \Delta x + \left.\frac{\partial f}{\partial u}\right|_{@(x_{LP}, u_{LP})} \Delta u$$

$$\cancel{\dot{x}_{LP}} + \dot{\Delta x} \approx \cancel{f(x_{LP}, u_{LP})} + A(\Delta x) + B(\Delta u)$$

$$\star \quad \boxed{\dot{\Delta x} = \underset{\sim}{A}\,\underset{\sim}{(\Delta x)} + \underset{\sim}{B}\,\underset{\sim}{(\Delta u)}} \quad //$$

using Derivative of quaternion for LQR discussed later

$(X_k - X_{LP})$ $(U_k - U_{LP})$

Evaluated @ $X_{LP}$ and $U_{LP}$

$(X_{LP}, U_{LP})$ & $(X_k, U_k)$ ARE DIFFERENT

| | time steps | Actual Wind | Estimate | wind chosen for Linearizat^n |
|---|---|---|---|---|
| COUNTER TRIGGER | $k = 1$ | 5 N | $\boxed{4.8\text{ N}}$ | 4.8 N |
| (BLOCK ②) | $k = 2$ | 7 N | 6 N | — " — |
| | $k = 3$ | 10 N | 10.5 N | — " — |
| | $k = 4$ | 9 N | 8.7 N | — " — |
| COUNTER TRIGGER | $k = 5$ | 11 N | $\boxed{9.8\text{ N}}$ | 9.8 N |
| | ⋮ | ⋮ | ⋮ | — " — |
| | | | | — " — |
| | $k = 9$ | 10 N | 11 N | — " — |
| COUNTER TRIGGER | $k = 10$ | 6 N | $\boxed{6.3\text{ N}}$ | 6.3 N |
| | ⋮ | ⋮ | ⋮ | ⋮ |

$\underbrace{\quad}_{X_k,\, U_k}$ changes every time step.

$\underbrace{\quad}_{X_{LP},\, U_{LP}}$ for next 5 steps

# BLOCK 3 : RICCATTI

→ use A & B matrices for further processing for Riccatti Recursion to find optimal gain matrix K

☆ LQR with quaternions requires some special pre-processing of state-error vector $\begin{bmatrix} \Delta x \\ \hdashline \Delta d \end{bmatrix}$ (not discussed here)

↳ Quaternion ⟷ Axis-Angle vector

↳ Error Quaternion

↳ Jacobian of Quaternion that maps axis-angle to quaternion, then quaternion to changed axis-angle vector

# CONTROL LAW

$$\Delta U = -K_{LQR}(\Delta X)$$

$$U_k - U_{LP} = -\begin{bmatrix} K_{\hat{x}} & K_{\hat{d}_1} & K_{\hat{d}_2} & \cdots \end{bmatrix} \begin{bmatrix} \Delta \hat{x} \\ \hline \Delta d_1 \\ \Delta d_2 \\ \vdots \end{bmatrix}$$

$$U_k = U_{LP} - \left[\, K_{\hat{x}} \;\vdots\; K_{\hat{d}_1} \;\vdots\; K_{\hat{d}_2} \cdots \right] \begin{bmatrix} x_k - x_{LP} \\ \hline \vec{F}_{w_{est}} - \vec{F}_{w_{LP}} \\ \hline \vec{\tau}_{w_{est}} - \vec{\tau}_{w_{LP}} \\ \hline \vdots \end{bmatrix}$$

(extra derivative terms)

* Everything boils down to this!

* Intuition: $\Delta U = -K_{\hat{x}}(\Delta x) - K_{\hat{d}}(\Delta d)$

Normal LQR
- stabilizes Attitude
- Does not stick to setpoint because wind blew it away

Extra Disturbance Compensation
- Puts extra effort to compensate for lost-setpoint

Overall:
- stabilizes Attitude
- Puts extra effort to compensate for lost setpoint
  - $\odot(K_{\hat{d}}, \Delta \vec{F}_{wind})$

compensates for translation errors

$\hookrightarrow$ 2 $\left(K_{\hat{d}_2} \, \overrightarrow{\Delta \tilde{\tau}_{wind}}\right)$

compensates for Roll/pitch/Yaw disturbances.