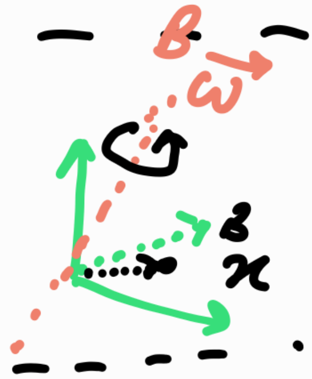
$${}^{W}x = Q\,{}^{B}x$$

$${}^{W}\dot{x} = \frac{d}{dt}\left(Q\,{}^{B}x\right) = \dot{Q}\,{}^{B}x + Q\left(\cancelto{0}{{}^{B}\dot{x}}\right)$$

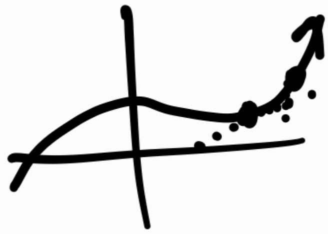
$$\boxed{{}^{W}\dot{x} = \dot{Q}\,{}^{B}x} \quad \text{— (1)}$$



$${}^{W}v = Q\,{}^{B}v$$

$${}^{W}v = Q\left({}^{B}\vec{\omega} \times {}^{B}x\right)$$

$${}^{W}\dot{x} = Q\left({}^{B}\vec{\omega} \times {}^{B}x\right) = \dot{Q}\,{}^{B}x$$



$$\Rightarrow \quad \dot{Q}\,\cancel{{}^{B}x} = Q\,{}^{B}\hat{\omega}\,\cancel{{}^{B}x}$$

$$\boxed{\boxed{\dot{Q} = Q\,{}^{B}\hat{\omega}}}$$

$$\Rightarrow \quad Q_{t+\Delta t} \approx Q_t + \underbrace{\dot{Q}(\Delta t)}_{@\, t=0}$$

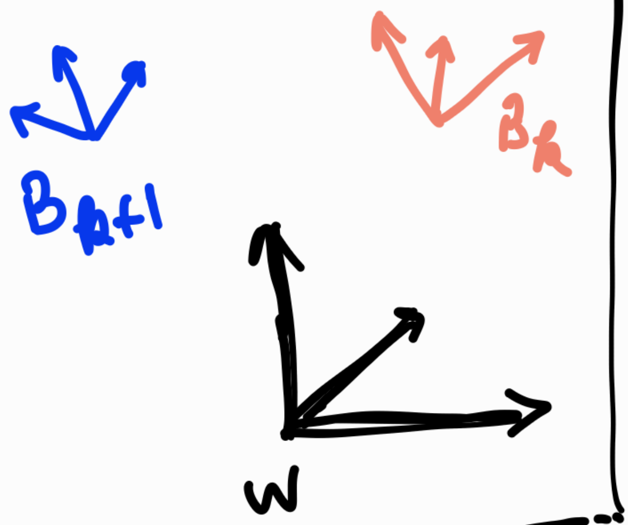
$$\Rightarrow \quad Q_{0+\Delta t} = Q_{\Delta t} = Q_0 + \dot{Q}(\Delta t)$$

$$Q_{\Delta t} = Q_0 + \left(Q_0\,{}^{B}\hat{\omega}_0\right)\Delta t \quad ✓$$

✓ ✓ initial orientation

✓ IMU data for every time

known step!



$$w_k\, x = Q_k^{B_k} x$$

$$w_{k+1}\, x = Q_{k+1}^{B_{k+1}} x$$

$$^{B_k}_{B_{k+1}} x = Q_\Delta^{B_k} x$$

$$^{B_k}_{w} x = Q_{k+1} \underbrace{{}^{B_k}_{B_{k+1}} x}$$

$$\underbrace{^{B_k}_{w} x} = Q_{k+1} Q_\Delta^{B_k} x$$

$$Q_k^{B_k} \cancel{x} = Q_{k+1} Q_\Delta^{B_k} \cancel{x}$$

$$Q_k = Q_{k+1} Q_\Delta$$

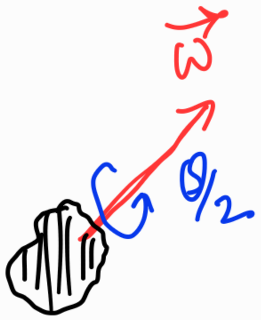
$$\boxed{Q_\Delta = Q_{k+1} \, Q_k}$$

"Error Rotatn"

QUATERNIONS

$$q = \begin{bmatrix} \cos(\theta/2) \\ \vec{\omega}\sin(\theta/2) \end{bmatrix}$$

could be expressed in any frame ...



$$q = \begin{bmatrix} \text{scalar} \\ \overrightarrow{\text{vector}} \end{bmatrix}_{4\times 1} = \begin{bmatrix} S \\ V \end{bmatrix}$$

$$\Rightarrow q_1 \otimes q_2 = \begin{bmatrix} S_1 S_2 - \vec{V}_1 \cdot \vec{V}_2 \\ S_1 \vec{V}_2 + S_2 \vec{V}_1 + \vec{V}_1 \times \vec{V}_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} S_1 & -V_1^T \\ V_1 & S_1 I + \hat{V}_1 \end{bmatrix}}_{L(q_1)} \begin{bmatrix} S_2 \\ V_2 \end{bmatrix} = L(q_1)\, q_2$$

$$= \begin{bmatrix} S_2 & -V_2^T \\ V_2 & S_2 I - \hat{V}_2 \end{bmatrix} \begin{bmatrix} S_1 \\ V_1 \end{bmatrix} = R(q_2)\, q_1$$

$R(q_2)$

$\Rightarrow$ Rotate vector ${}^B\vec{a}$:

$${}^W\vec{\ } = Q\ {}^B\vec{\ } \quad \text{(Rotatⁿ Mat Q)}$$

$$\begin{bmatrix} 0 \\ {}^W\vec{a} \end{bmatrix} = q \otimes \begin{bmatrix} 0 \\ {}^B\vec{a} \end{bmatrix} \otimes q^* \quad \rightarrow \quad q \rightleftharpoons$$

$$\left\{ \text{Note: } \quad q^* = \begin{bmatrix} s \\ -\vec{v} \end{bmatrix} \right\}$$

$$= q \otimes \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{H} {}^B\vec{a} \otimes q^*$$

$$= \left\{ L(q)\, H\, {}^B a \right\} \otimes q^*$$

$$= R(q^*)\, L(q)\, H\, {}^B\vec{a}$$

$$H\, {}^W\vec{a} = \underline{\underline{R^T(q)\, L(q)\, H\, {}^B\vec{a}}}$$

(or)

$$= q \otimes \left\{ R(q^*)\, H\, {}^B\vec{a} \right\}$$

$$\begin{bmatrix} 0 \\ \cdots \\ {}^{W}\vec{a} \end{bmatrix} = M\,{}^{W}\vec{a} = \underline{L(q)\,R^T(q)\,M\,{}^{B}\vec{a}}$$

$$\Rightarrow \boxed{\begin{aligned} {}^{W}\vec{a} &= \{M^T L(q) R^T(q) M\}\,{}^{B}\vec{a} \\ {}^{W}\vec{a} &= \{M^T R^T(q) L(q) M\}\,{}^{B}\vec{a} \end{aligned}}$$

$\Rightarrow$ q_1 then q_2 :

$$q_{final} = q_2 \otimes q_1$$

$${}^{W}\vec{a} = (q_2 \otimes q_1) \otimes \begin{bmatrix} 0 \\ \cdots \\ {}^{B}\vec{a} \end{bmatrix} \otimes \underbrace{(q_2 \otimes q_1)^*}_{q_1^* \otimes q_2^*}$$

$${}^{W}\vec{a} = q_2 \otimes q_1 \otimes \begin{bmatrix} 0 \\ \cdots \\ {}^{B}\vec{a} \end{bmatrix} \otimes q_1^* \otimes q_2^*$$

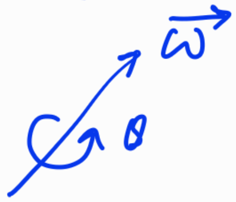
$\Rightarrow$ Quaternion derivatives:

Let $\delta q = \begin{bmatrix} \cos(d\theta/2) \\ \vec{\omega} \sin(d\theta/2) \end{bmatrix}$ $\quad (d\theta \to 0)$

$$\approx \begin{bmatrix} 1 \\ \vec{\omega} \dfrac{d\theta}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vec{\omega} \dfrac{d\theta}{2} \end{bmatrix}$$

Side: Axis Angle:

$\Downarrow$



$$\boxed{\vec{\phi} = \vec{\omega}\theta}$$

$$\vec{\omega} = \frac{\vec{\phi}}{\|\vec{\phi}\|}$$

$$\|\vec{\phi}\| = \theta$$

$$\delta q = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vec{\phi} \end{bmatrix}$$

or

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix} (d\theta/2)$$

$\hookrightarrow$ Method-1: $\to$ quat 'q' exists.

→ disturb it by δq

→ just like $x + \Delta x$

$$\Rightarrow \quad q_{\text{disturbed}} = \delta q \otimes q$$

$$= \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix} \left(\frac{d\theta}{2}\right) \right\}^{\otimes} q$$

$$= I_q \otimes q + \left(\frac{d\theta}{2}\right) \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix}^{\otimes} q$$

$$q_{\text{disturbed}} = q + \left(\frac{d\theta}{2}\right) \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix}^{\otimes} q$$

$$\Rightarrow \quad \frac{q_{\text{dist}} - q}{d\theta} = \frac{1}{2} \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix}^{\otimes} q$$

$$\lim_{d\theta \to 0} \left(\frac{q_{\text{dist}} - q}{d\theta} \right) = \boxed{\dot{q} = \frac{1}{2} \begin{bmatrix} 0 \\ \vec{\omega} \end{bmatrix}^{\otimes} q}$$

① if $\vec{\omega}$ representing

② ~~if $\vec{\omega}$ here is~~

disturbance quat δq is expressed in world frame:

$$ {}^{W}\vec{\omega} = q \otimes \begin{bmatrix} 0 \\ {}^{B}\vec{\omega} \end{bmatrix} \otimes q^{*} $$

$$ \Rightarrow \dot{q} = \frac{1}{2} \begin{bmatrix} 0 \\ {}^{W}\vec{\omega} \end{bmatrix} \otimes q $$

$$ = \frac{1}{2} \left\{ q \otimes \begin{bmatrix} 0 \\ {}^{B}\vec{\omega} \end{bmatrix} \otimes \underbrace{q^{*} \right\} \otimes q}_{I_q} $$

$$ \boxed{\dot{q} = \frac{1}{2} \left\{ q \otimes \begin{bmatrix} 0 \\ {}^{B}\vec{\omega} \end{bmatrix} \right\}} $$

in body

$$ \begin{bmatrix} 0 \\ {}^{B}\vec{\omega} \end{bmatrix} = (q^{*}) \otimes \begin{bmatrix} 0 \\ {}^{W}\vec{\omega} \end{bmatrix} \otimes (q^{*})^{*} $$

$$ \Rightarrow \dot{q} = \frac{1}{2} \begin{bmatrix} 0 \\ {}^{B}\vec{\omega} \end{bmatrix} \otimes q $$

$$ = \frac{1}{2} \left\{ q^{*} \otimes \begin{bmatrix} 0 \\ {}^{W}\vec{\omega} \end{bmatrix} \otimes q \otimes q \right. $$

(?)

Prolly doesn't work if expressed in body frame...

...would work only if

$$ q_{dist} = \underbrace{q}_{(body)} \otimes \underbrace{\delta q}_{(body)} $$