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Limit and Continuity

1.1 Introduction

Limit is the fundamental concept of calculus. In real sense, calculus is the study of limits. It was developed independently by Sir Isaac Newton (1642 – 1727) and Gottfried Wilhelm Leibnitz (1646–1716). Intuitively, limit is a value to which a function approaches as the variable approaches a given value. The limit has many applications in calculus as well as in other branches of Mathematics.

We have discussed about a function and its graph in first semester. We now give the definition of function, domain and range of a function and value of the function at a point as a review.

Function

Let X and Y be two non-empty sets. A function f from a set X to set Y is a rule which associates each element of X with a unique element of Y . The set X is called the **domain of f** and the set of values of $f(x)$ for every $x \in X$ is called the **range of f** and the set Y is called the **co-domain of f** . In symbol, we write $f: X \rightarrow Y$ in order to mean f is a function from X to Y .

Value of a Function

Let $f: X \rightarrow Y$ be a function and $x = a$ be an element in X . The value of $f(a)$ is called the value of the function f at $x = a$. If $f(a)$ is a finite number then we say that $f(x)$ exists or is defined at $x = a$. If $f(a)$ is not finite, we say $f(x)$ does not exist or is not defined at $x = a$.

Examples:

- (i) The function $f(x) = 2x + 1$ is defined at $x = 2$ since

$$f(2) = 2 \times 2 + 1$$

= 5 which is a finite number.