Coding Theory:

* Binary symmetric channel: 0's and 1's are transmitted. Symmetric: prob of 0 and 1 is same

* $\boxed{r = c + e}$ $e \to$ error pattern
  $c \to$ word transmitted
  $r \to$ received word

$\boxed{\text{word } c \in Z_2^n \text{ transmitted}} \xrightarrow{\text{Transmission channel}} \boxed{\text{word } T(c) \in Z_2^n \text{ received}}$

* $\boxed{Z_2^n}$ $\Rightarrow$ length of data

* If 'p' is prop of '0' transmitted & '1' received and vice versa. (incorrect transmission $\Rightarrow p$)
  Then, '1-p' will be the prob of correct transmission.

Note:
when you know the no. of errors & positions:
* $p(1-p)^{n-1}$ is the prob that r differs from c in exactly one place.
* $p^k(1-p)^{n-k}$ is ~ ~ ~ ~ k places.

when we do not know:
* if k errors are made in the transmission of c and if received word r differs from c in k places since c has n places and k of these n places can be chosen in ^nC_k no. of ways.
* The prob that k errors are made in transmission: $\left({}^nC_k\right)p^k(1-p)^{n-k}$

Q) word $c = 1010110$, $e = 0101101$, $r = ?$, $p = 0.05$ (incorrectly received). Find prop with with r is received

$\Rightarrow$ $c = 1010110$
$e = 0101101$
$r = c + e$
$= 1111011$

$\Rightarrow$ $c = 1010110$
1111011
$0101101 \Rightarrow 4$ errors

$\Rightarrow$ $p^4 \times (1-p)^{7-4}$
$(0.05)^4 \times (0.95)^3 = 5.359 \times 10^{-6}$

Q) Consider string c=10110. c is an element of Z_2^5, obtained from the direct product of five copies of $(Z_2,+)$. When sending each bit of c through channel we assume that p=0.05 is the probability of incorrect transmission. Find the probability of sending c and receiving r= 00110 and find the error pattern? What inference you can draw by looking at the error pattern?

$\Rightarrow$ $c = 10110$
$r = 00110$
$e = 10000$

$\Rightarrow$ no. of 1s determine the no. of errors in bits.
$\Rightarrow$ There is one error.
$\Rightarrow p^1 \cdot (1-p)^4 = 0.05 \times (0.95)^4$ //

Q) Consider string c=10110. c is an element of Z_2^5 obtained from the direct product of five copies of $(Z_2,+)$. When sending each bit of c through channel we assume that p=0.05 is the probability of incorrect transmission.
i) Find the probability of sending c and receiving r which differs in exactly two places?
ii) Find the probability of sending c and receiving r which differs in atmost two places?(≤2)
iii) Find the probability of sending c and receiving r which differs in atleast three places?(≥3)

$\Rightarrow$ $c = 10110$

$p = 0.05 \Rightarrow$ incorrect transmission.

(i) $\Rightarrow$ $\underbrace{{}^5C_2\, p^2 (1-p)^{5-2}}_{\text{2 errors}}$

(ii) $\Rightarrow \underbrace{{}^5C_0(1-p)^4}_{\text{no errors}} + \underbrace{{}^5C_1 p(1-p)^4}_{\text{one error}} + \underbrace{{}^5C_2 p^2(1-p)^3}_{\text{two errors}}$

(iii) $\underbrace{{}^5C_3 p^3(1-p)^2}_{\text{3 errors}} + \underbrace{{}^5C_4 \times p^4 \times (1-p)^1}_{\text{4 errors}} + \underbrace{{}^5C_5 \times p^5 \times (1-p)^{5-5}}_{\text{5 errors}}$

Q) $C = 1010110$ is sent through a binary symmetric channel. If $p = 0.02$ is the prob of incorrect receipt of a signal. Find the prob that c is received as $r = 1011111$, $e = ?$

$\Rightarrow$ $C = 1010110$
$r = 1011111$
$e = 0001001 \Rightarrow$ 2 errors

$\Rightarrow (0.02)^2 \times (1-0.02)^{7-2} \Rightarrow$ ans //

Q) $p = 0.05$, $c = 011011101$; find the prob that (i) single error occurs. $k=1$
(ii) double error occurs $k=2$
(iii) Triple error occurs. $k=3$
(iv) 3 errors occur, no two of them are consecutive.

$\Rightarrow p = 0.05$, $c = 011011101$, $\boxed{n=9}$, ${}^{n}C_k \times p^k \times (1-p)^{n-k}$

(i) $k=1 \Rightarrow {}^9C_1 \times (0.05)^1 \times (1-0.05)^{9-1} \Rightarrow$ ans //

(ii) $k=2 \Rightarrow {}^9C_2 \times (0.05)^2 \times (1-0.05)^{9-2} \Rightarrow 0.06285$ //

(iii) $k=3 \Rightarrow {}^9C_3 \times (0.05)^3 \times (1-0.05)^{9-3} \Rightarrow 0.0077185$ //

(iv) no two are consecutive $\Rightarrow$ let p_1 be prob of error occurs in 1, 3, 5, 7, 9 place $(n=5)$
let p_2 be prob of error occurs in 2, 4, 6, 8 places. $(n=4)$

3 errors occurs ($k=3$) $\Rightarrow p_1 = {}^5C_3 \times (0.05)^3 \times (1-p)^{5-3} = 0.001128$ //
$p_2 = {}^4C_3 \times (0.05)^3 \times (1-p)^{4-3} = 0.000475$ //

$p_1 + p_2 =$ ans //

Q) $c = 10110$, $e = 01100$, $r = ?$

$\Rightarrow c = 10110$
$e = 01100$
$r = c + e$
$= 11010$ //

Q) $r = 10110$, $e = 00100$, $c = ?$

$\Rightarrow r = 10110$
$e = 00100$
$c = r + e \Rightarrow 10010$