Option Pricer Neural Net

FINN paper

- accurate pricing of models are needed
- based on dynamic hedging from Black Scholes Model
- loss function defined by BS model
- learns pricing patterns while respecting no-arb conditions

Contributions
- FINN
- Theory embedded Neural Net training
  - loss defined by no-arb / dynamic hedging strats
- Generalization / Robustness across stochastic processes
- Generalization of FINN towards Delta Gamma Hedging

Delta Risk – risk of loss due to small changes in underlying asset prices
- long Δ + stock ↓ = loss   Δ ∝ stock

Gamma Risk – risk due to changes in delta
- Δ – , move in the underlying can cause delta to shift

Delta Gamma Hedging – aims to make portfolio insensitive to small movements in the underlying
= neutralizing delta + neutralizing gamma
(protect against small price movements) (delta doesn't shift with large price moves)

2.1 Dynamic Hedging for Option Pricing

European option w/ PDE

Call (Buy later at price)

if strike price (K) > asset price (S):
    option is worthless
elif $K < S$:
    option $= S - K$

Put (sell) is just reverse

Delta $= \frac{\partial V}{\partial S}$ ← option price

determines optimal offsetting position in the underlying asset

2.1.1 Specifications of Asset Price Dynamics

Underlying asset price follow Geometric Brownian motion

$$\frac{dS_t}{S_t} = \mu^x dt + \sigma^x dW_t$$

S_t = asset price at time t

μ^x = expected return (reflects actual returns inc. risk premiums

σ^x = assets volatility

W_t = standard brownian motion

Model requires risk bank account ; Equation denotes growth of one unit of currency invested at risk free rate r

$$\frac{dB_t}{B_t} = r dt$$

No random component (dW_t term)

Itô's Lemma

$$dg_t = \left(\partial_t g + \mu_t^x \partial_x + \frac{1}{2}(\sigma_t^x S_t)^2 \partial_{xx} g\right) dt + \left(\sigma_t^x S_t \partial_x g\right) dW_t$$

Wiener process inc

$\partial_x g$ = denote first derivative to asset price

zero cost = costs no money to enter position for the portfolio

2.1.3 Construction of Zero Cost, Self financing Portfolio

S ← underlying asset, B ← Bank Account, g ← derivative of what

Properties:
i. zero cost: no cost to establish portfolio positions
ii. Self financing: Any changes in portfolio value come from asset prices changes

2.1.4 Enforcing No-Arb on Portfolio

No arb: In efficient market, it is impossible to generate risk-free profits w/ initial investment/taking on any risk

- zero cost
- self financing
- riskless
- strictly positive returns

Limitations on Black Scholes

i. Volatility is not constant
- volatility depends on the option not the stock

ii. BS assumes stocks follow lognormal distribution, returns are normally distributed
- actual market returns have big price jumps more than the model accepts (Fat tails)
- there is negative skew meaning crashes are more common than booms

2.2 Machine Learning Approaches to Option Pricing

direct price prediction

ANN → RNNs and LSTMs because they model temporal dependencies in option prices with market sentiment

Gaussian Process models provide probabilistic price predictions

RL
- Variants of Q-learning have been employed to develop optimal hedging strats

NN-based approximations of PDE solutions

PDE describes how option prices evolve over time
SDE (Stochastic Differential Equation) describes how underlying asset behaves inc. randomness

You can go from PDE → SDE using Itô Lemma

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...to develop optimal

...how underlying

establishes Forward - Backward Stochastic Differential Equation (FBSDE)!
with known terminal conditions which can be solved by NNs

Limitations of ML-Based Approach
- black box systems that lack transparency

3 Finance Informed Neural Network

$$f(I_t) \to g_t^\theta(I_t)$$

I_t = Feature Set $[S_t, K, t]$
stock price, strike, TTM

$$dV_t = \alpha_t dS_t + \beta_t dB_t - dg_t^\theta(I_t)$$: Portfolio Value Process
$= 0$ ☆

Portfolio positions :

$$\alpha_t = \partial_x g_t, \quad \beta_t = \frac{g_t - \partial_x g S_t}{B_t}, \quad \frac{dB_t}{B_t} = r_t dt$$

$$\text{loss} = \Big(\underbrace{\partial_x g_t^\theta(I_t)}_{\text{delta}(\Delta)} dS_t + r_t\big(g_t^\theta(I_t) - \underbrace{\partial_x g_t^\theta(I_t)}_{\text{delta}(\Delta)} S_t\big)dt - dg_t^\theta(I_t)\Big)^2$$

Portfolio : $(\alpha_t, \beta_t, \eta_t, -1)$

α_t : position in underlying asset

β_t : position in risk free bank account

η_t : position in hedging instrument

-1 : short position in option priced

Ref to Theorem 1

4 Experimental Setup

Testing is done on predefined SDEs on GBM and Heston

FINN
- forward batch processing
- generates option price predictions that are evaluated using dynamic hedging
- early stopping on held-out validation set

Ref example

Structure
- 2 hidden layers each containing 50 neurons
- Output layer producing normalized ops (C/K)
- softmax activation