Three Inverter Ring Oscillator Simulation

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November 17, 2016

The ring oscillator is composed of three CMOS inverters as shown in Figure 1. We use Modified Nodal Analysis (MNA) to simulate the circuit.

1 MNA Equations

Using Gnd node as reference, the remaining nodes are numbered as v_1, v_2, v_3 and v_4 as shown in Figure 1. The current through the voltage source is denoted by i_s. The source currents of p-channel MOSFETs (p_1, p_2 and p_3) are denoted by i_{p1}, i_{p2} and i_{p3} respectively. Similarly, the drain currents of n-channel MOSFETs are denoted by i_{n1}, i_{n2} and i_{n3}.

The expression for i_{pj}, $j = 1, 2, 3$ in different modes of operations of p-channel MOSFETs are

1. *Cut-off mode:* $v_{sg} < v_{th}$

$$i_{pj} = 0$$

2. *Linear mode:* $v_{sg} - v_{th} < v_{sd}$

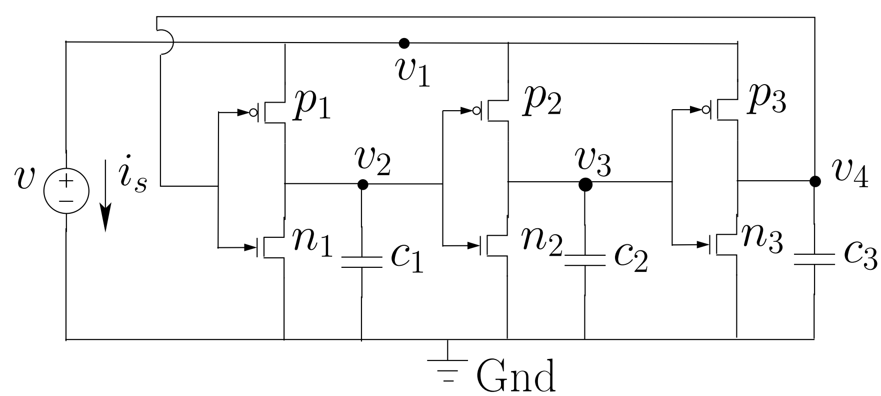

Figure 1: Three inverter ring oscillator

$$i_{pj} = K_p \left((v_{sg} - v_{th}) v_{sd} - \frac{v_{sd}^2}{2} \right)$$

3. *Saturation mode:*

$$i_{pj} = K_p \left(v_{sg} - v_{th}\right)^2 \left(1 + \lambda v_{sd}\right)$$

Since the transistor currents are functions of v_{sg} and v_{sd}, we can also write the approximate current expressions as

$$\dot{i}_p = \frac{\partial i_p}{\partial v_{sg}} \dot{v}_{sg} + \frac{\partial i_p}{\partial v_{sd}} \dot{v}_{sd} \tag{1}$$

We denote $\frac{\partial i_p}{\partial v_{sg}}$ by g_m^p and $\frac{\partial i_p}{\partial v_{sd}}$ by g_o^p. Linearizing (1), we get

$$i_p = \hat{i}_p + g_m^p \left(v_{sg} - \hat{v}_{sg}\right) + g_o^p \left(v_{sd} - \hat{v}_{sd}\right)$$

where $\hat{\cdot}$ denoted current guess value.

Similar expressions can be given for nMOS currents.

The capacitor current is given by

$$i_c = \frac{c}{\Delta t} \left(v_c - v_{c,prev}\right)$$

where $v_{c,prev}$ is the value of capacitor voltage at previous time instant.

The current balance equations at the nodes are:

1. Node 1: $i_s + i_{p1} + i_{p2} + i_{p3} = 0$
2. Node 2: $i_{p1} = i_{n1} + i_{c1}$
3. Node 3: $i_{p2} = i_{n2} + i_{c2}$
4. Node 4: $i_{p3} = i_{n3} + i_{c3}$

Additionally, we have expression $V_1 = V$. Expanding and simplifying these equations, we get the following MNA matrix equation

$$\begin{bmatrix} -\hat{i}_{p1} - \hat{i}_{p2} - \hat{i}_{p3} + \left(g_m^{p1} + g_o^{p1} + g_m^{p2} + g_o^{p2} + g_m^{p3} + g_o^{p3}\right)\hat{v}_1 - \left(g_m^{p2} + g_o^{p1}\right)\hat{v}_2 - \left(g_m^{p3} + g_o^{p2}\right)\hat{v}_3 - \left(g_m^{p1} + g_o^{p3}\right)\hat{v}_4 \\ -\hat{i}_{p1} - \hat{i}_{n1} + \left(g_m^{p1} + g_o^{p1}\right)\hat{v}_1 - \left(g_m^{p1} + g_m^{n1}\right)\hat{v}_4 - \left(g_o^{p1} + g_o^{n1} + \frac{c_1}{\Delta t}\right)\hat{v}_2 \\ -\hat{i}_{p2} + \hat{i}_{n2} + \left(g_m^{p2} + g_o^{p2}\right)\hat{v}_1 - \left(g_m^{p2} + g_m^{n2}\right)\hat{v}_2 - \left(g_o^{p2} - g_o^{n2} + \frac{c_2}{\Delta t}\right)\hat{v}_3 \\ -\hat{i}_{p3} + \hat{i}_{n3} + \left(g_m^{p3} + g_o^{p3}\right)\hat{v}_1 - \left(g_m^{p3} + g_m^{n3}\right)\hat{v}_3 - \left(g_o^{p3} - g_o^{n3} + \frac{c_3}{\Delta t}\right)\hat{v}_4 \\ v \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_s \end{bmatrix}$$

$$= \begin{bmatrix} -\hat{i}_{p1} - \hat{i}_{p2} - \hat{i}_{p3} + \left(g_m^{p1} + g_o^{p1} + g_m^{p2} + g_o^{p2} + g_m^{p3} + g_o^{p3}\right)\hat{v}_1 - \left(g_m^{p2} + g_o^{p1}\right)\hat{v}_2 - \left(g_m^{p3} + g_o^{p2}\right)\hat{v}_3 - \left(g_m^{p1} + g_o^{p3}\right)\hat{v}_4 \\ -\hat{i}_{p1} - \hat{i}_{n1} + \left(g_m^{p1} + g_o^{p1}\right)\hat{v}_1 - \left(g_m^{p1} + g_m^{n1}\right)\hat{v}_4 - \left(g_o^{p1} + g_o^{n1}\right)\hat{v}_2 - \frac{c_1}{\Delta t} v_{2,prev} \\ -\hat{i}_{p2} + \hat{i}_{n2} + \left(g_m^{p2} + g_o^{p2}\right)\hat{v}_1 - \left(g_m^{p2} + g_m^{n2}\right)\hat{v}_2 - \left(g_o^{p2} - g_o^{n2}\right)\hat{v}_3 - \frac{c_2}{\Delta t} v_{3,prev} \\ -\hat{i}_{p3} + \hat{i}_{n3} + \left(g_m^{p3} + g_o^{p3}\right)\hat{v}_1 - \left(g_m^{p3} + g_m^{n3}\right)\hat{v}_3 - \left(g_o^{p3} - g_o^{n3}\right)\hat{v}_4 - \frac{c_3}{\Delta t} v_{4,prev} \\ v \end{bmatrix}$$

We solve this equation iteratively to obtain the simulation results as shown in Figure 2. The parameter values are given in m-files of the simulation.

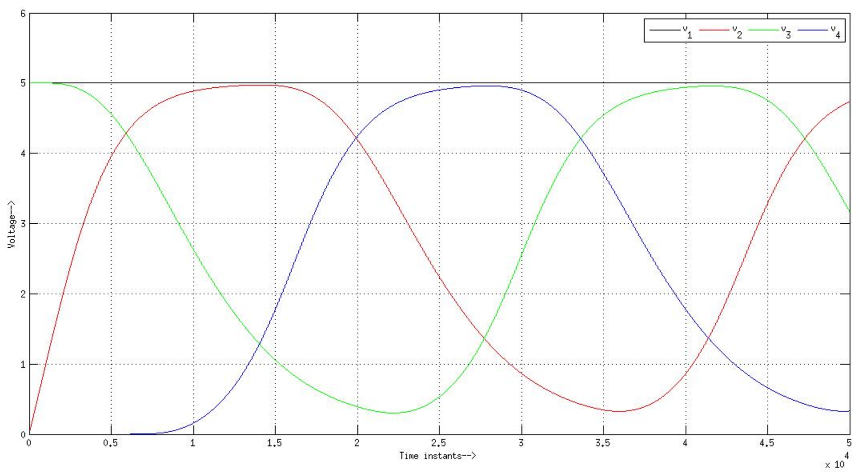

Figure 2: Simulation waveform of ring oscillator