

## EE 746-Neuromorphic Computing, Homework 1

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The codes for each of the parts has been named and numbered accordingly and stored in a single folder titled "Code". Within this folder the code for say question is contained in the file "1\_lif.py". To see the results of any of the codes, simply open a terminal and type "python filename.py", replacing filename with the name of the file to be run.

All the code has been written in Python 2.7. Furthermore, for all four parts, the code has been written so that an arbitrary number of neurons can be simulated together without using a for loop to iterate across said neurons. The initial conditions and parameters for individual neurons can also be changed with ease. The numpy package has been used to do this, and hence the code is fully vectorised.

1) A) The required expression is:

$$V_{steady-state} = E_L + \frac{I_0}{g_L}$$

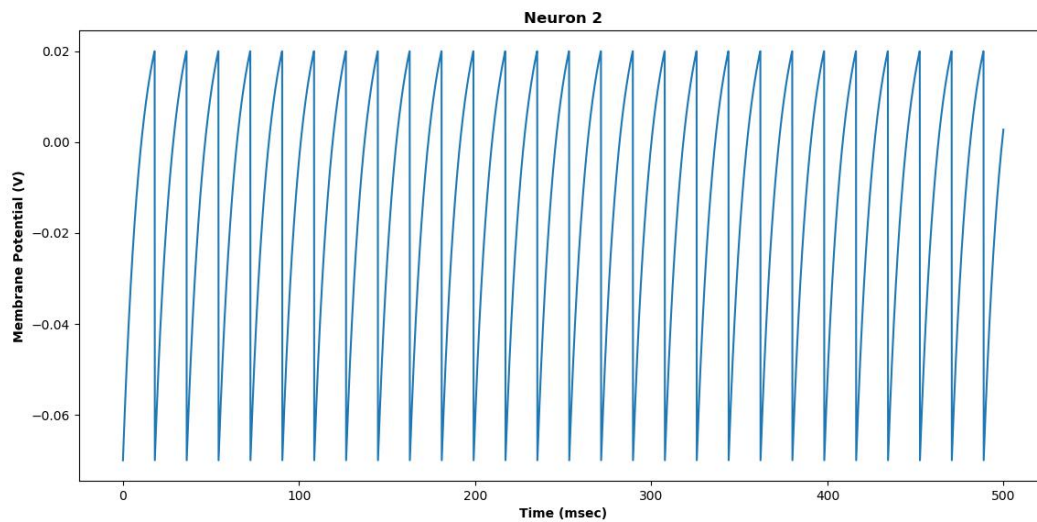
The critical value of current is that current which leads to the steady-state voltage being equal to the firing threshold. Hence:

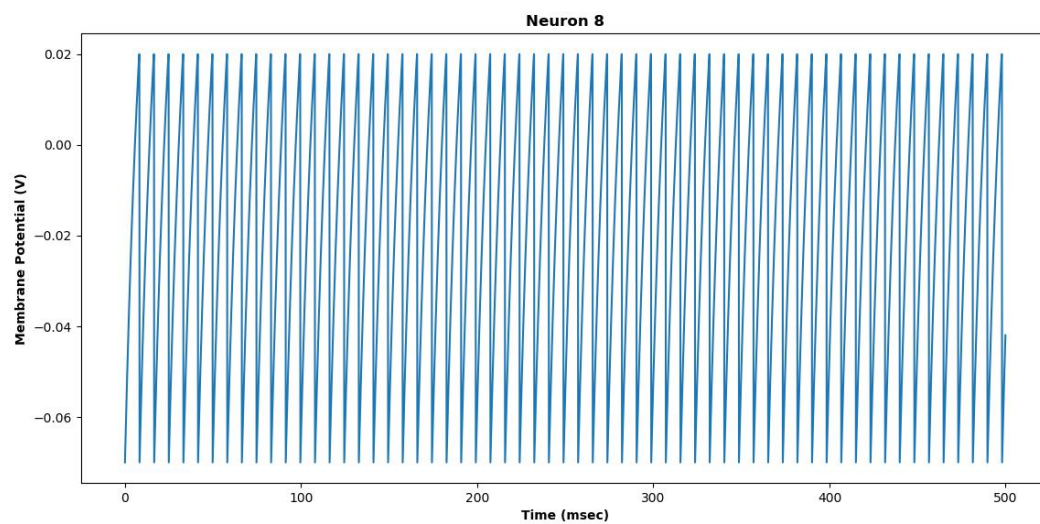
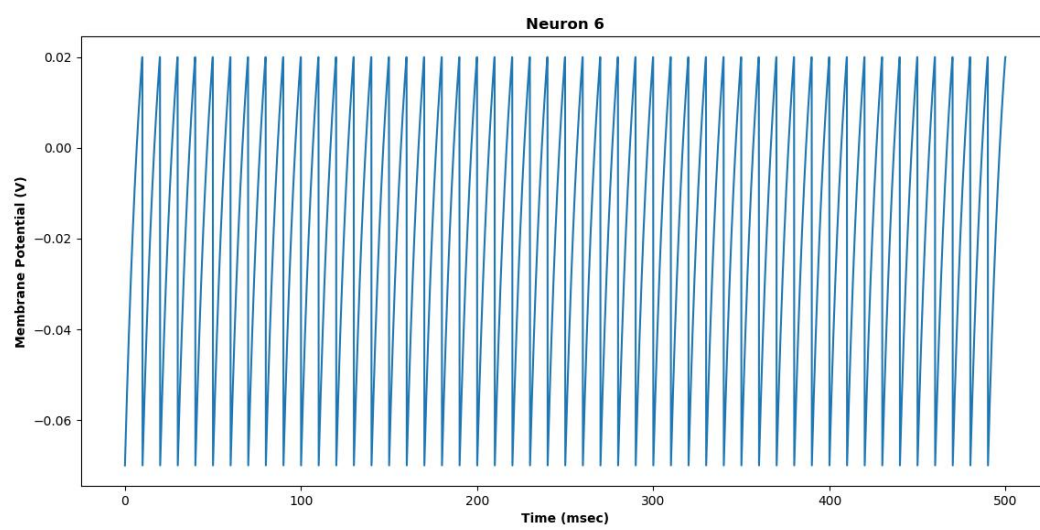
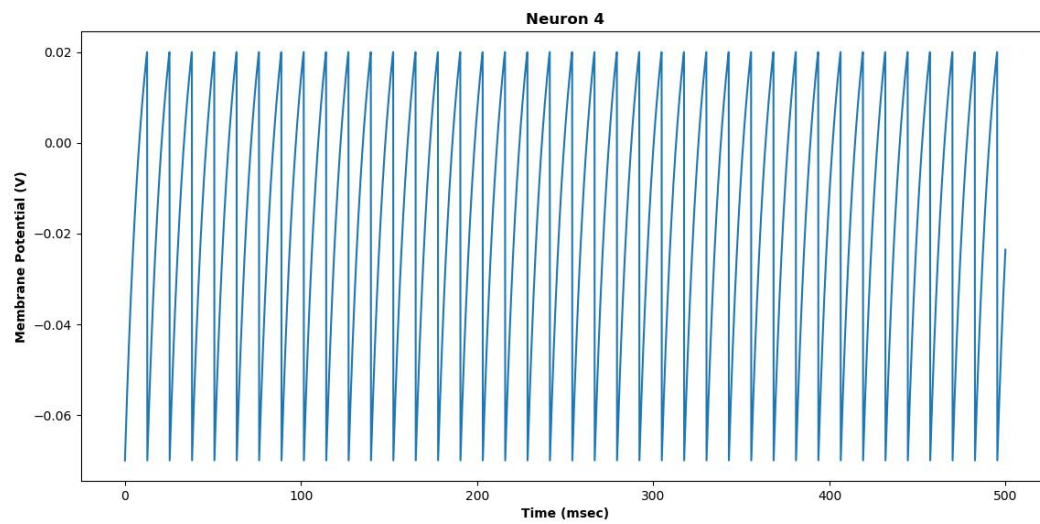
$$I_c = g_L \times (V_T - E_L)$$

B)

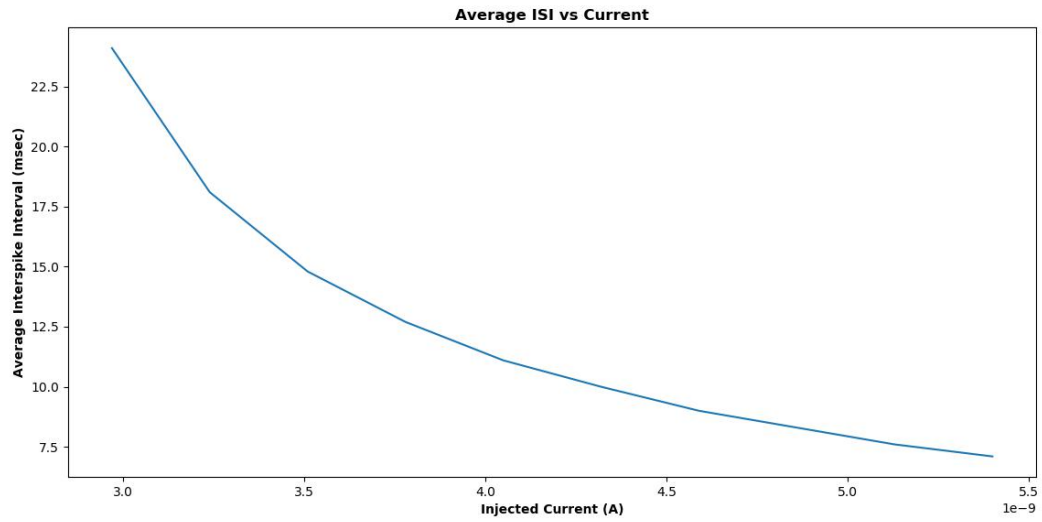
Note that in my code, I have used MxN matrices, instead of NxM matrices as was specified in the question. This has been done for better readability of the code. If desired, this difference can be removed by simply performing an additional transpose operation.

C) Plots:





D) Average ISI versus injected current:



2) A) We get:

$$U = b \times (V - E_r)$$

Substituting this into the quadratic equation, we get:

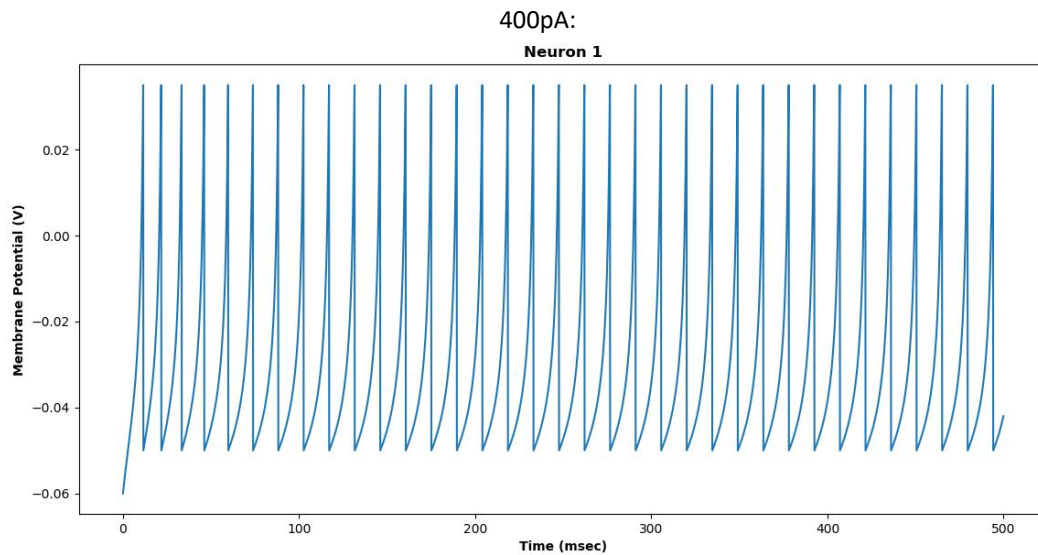
$$V = E_r \text{ or } V = E_t + \frac{b}{k_z}$$

B) The difference equations are:

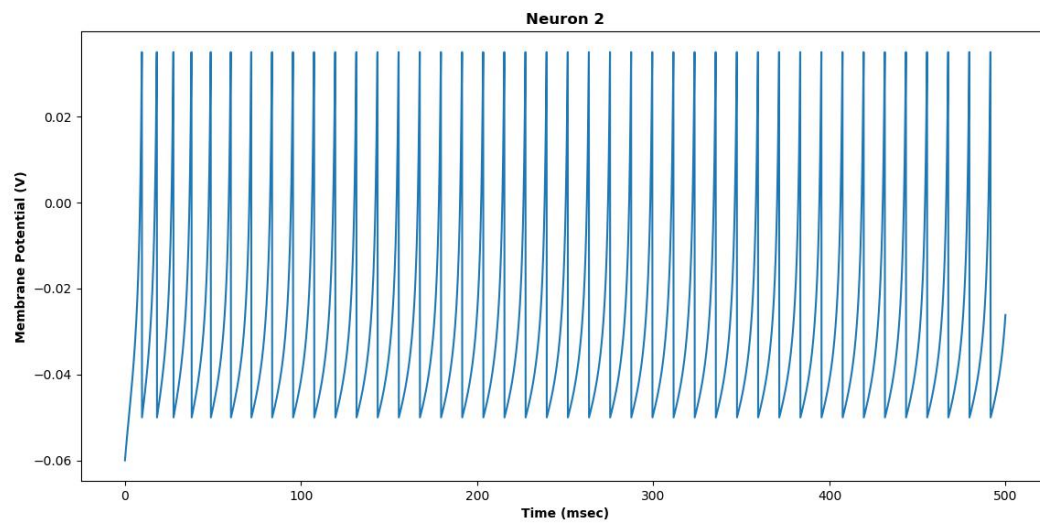
$$C \frac{V[n+1] - V[n]}{\Delta n} = k_z (V[n] - E_r)(V[n] - E_t) - U[n] + I_{app}[n]$$

$$\frac{U[n+1] - U[n]}{\Delta n} = a [b(V[n] - E_r) - U[n]]$$

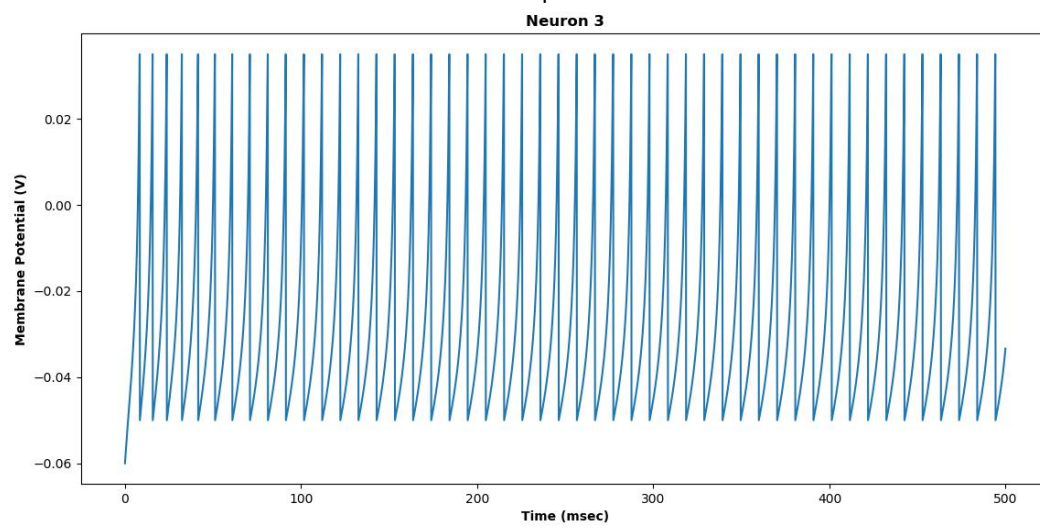
C) RS:



500pA:

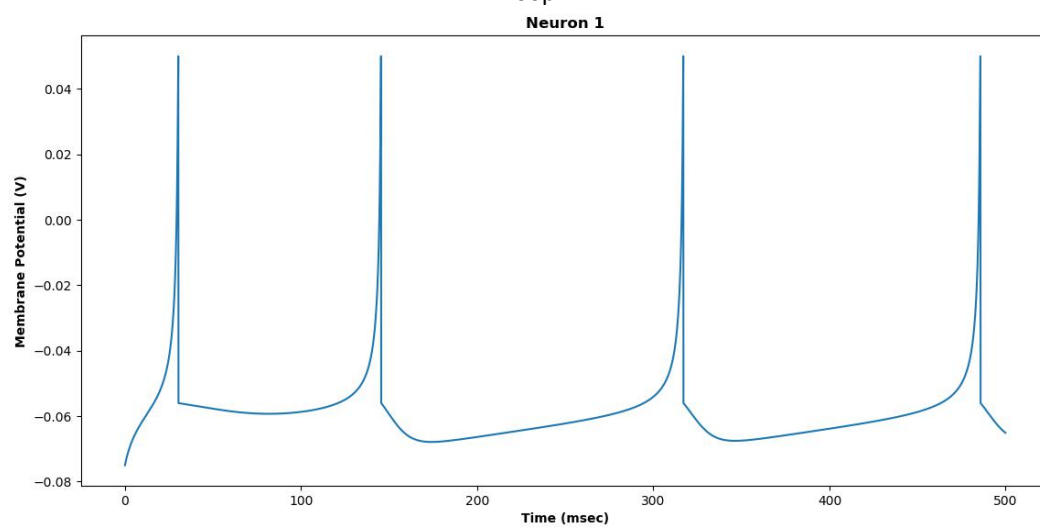


600pA:

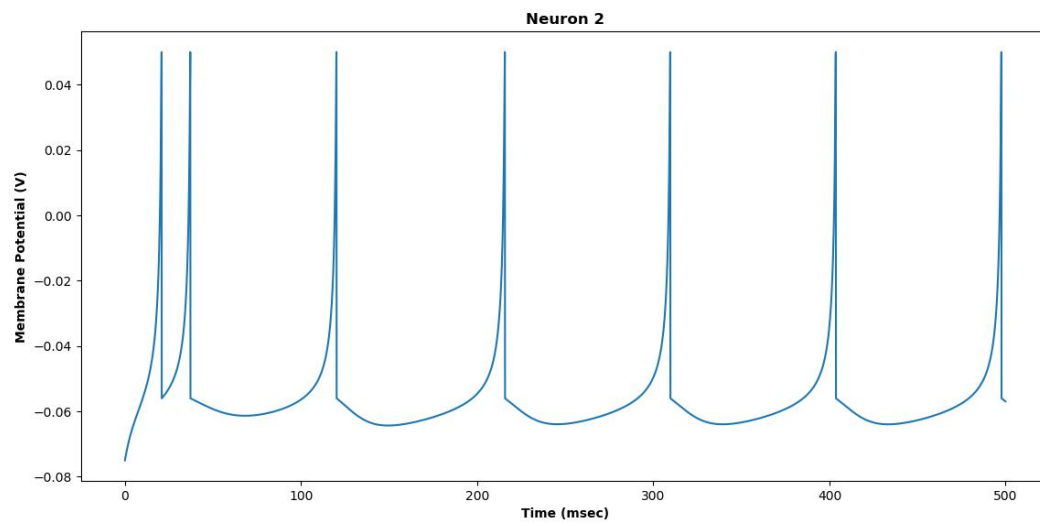


**IB:**

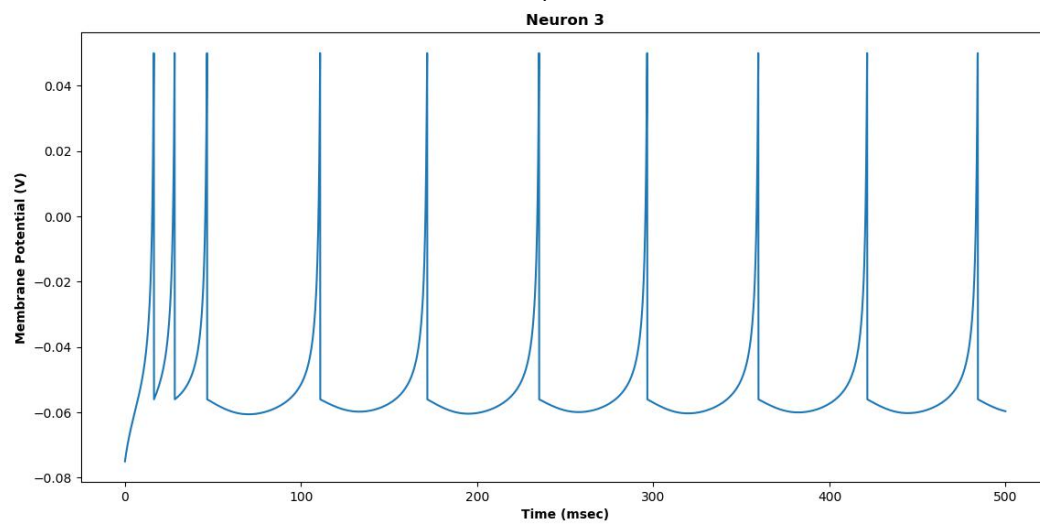
400pA:



500pA:

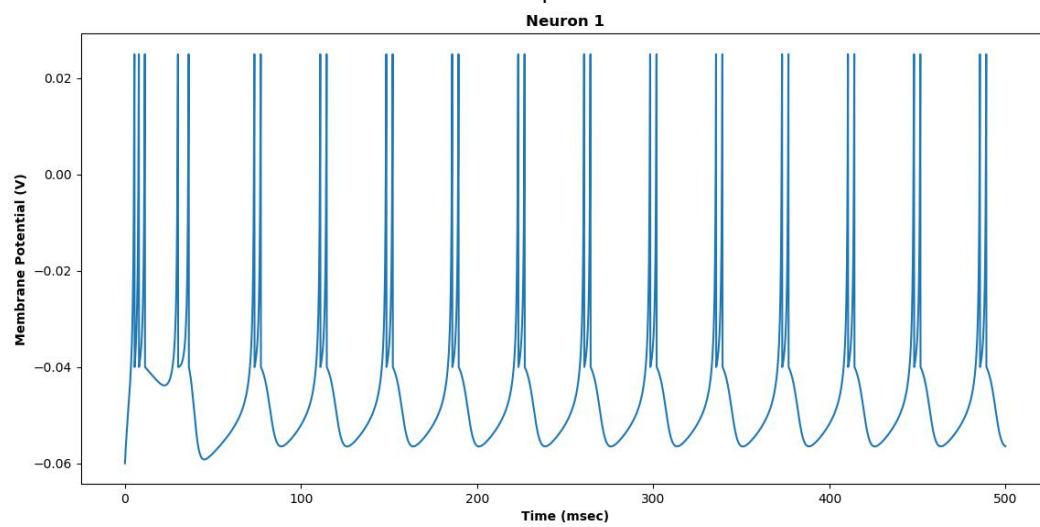


600pA:

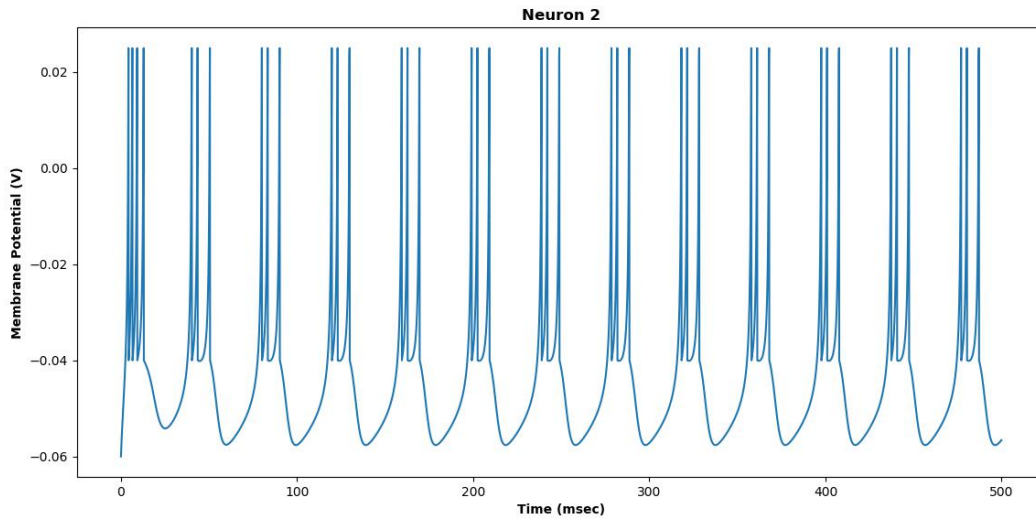


CH:

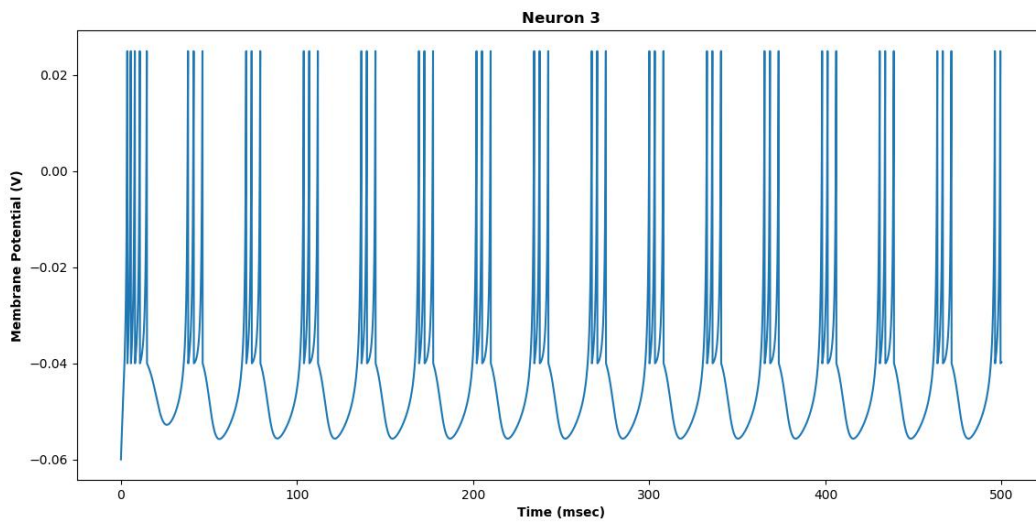
400pA:



500pA:



600pA:



3) A) The equivalent difference equations are:

$$C \frac{V[n+1] - V[n]}{\Delta n} = -g_L(V[n] - E_L) + g_L \Delta_T \exp\left(\frac{V[n] - V_T}{\Delta_T}\right) - U[n] + I_{app}[n]$$

$$\tau_w \frac{U[n+1] - U[n]}{\Delta n} = a[V[n] - E_L] - U[n]$$

B) To compute the steady state values of V and U, we simply simulate a neuron for a long enough time period, with zero injected current. The value that the variables settle to is the desired steady state values, that we have computed numerically. The neuron was simulated for 10000ms and convergence to steady state was observed both visually and numerically. The steady-state values obtained were:

**RS:** V = -0.0699999246265, U = 1.54226308265e-16

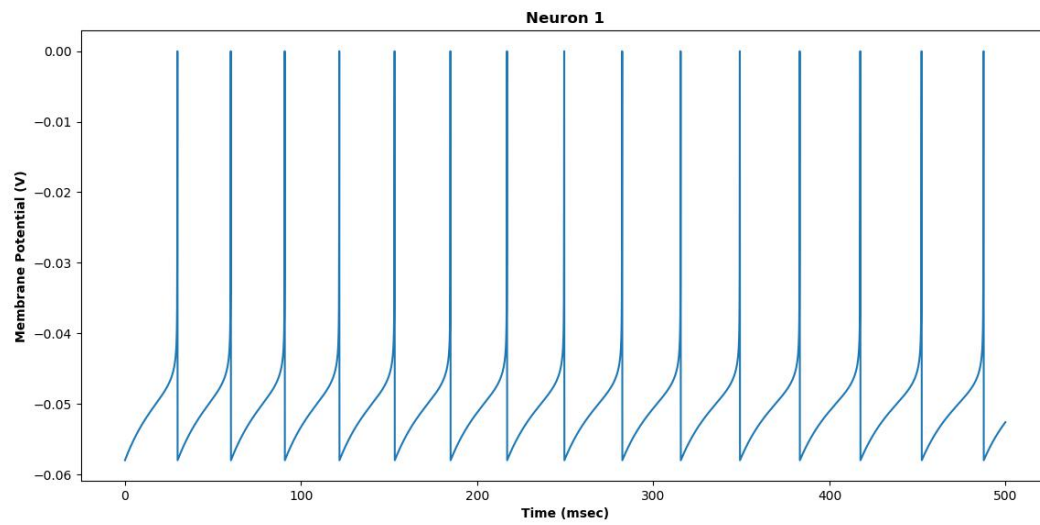
**IB:** V = -0.0579695694851, U = 1.21722814456e-13

**CH:** V = -0.0579690571373, U = 6.25810736469e-14

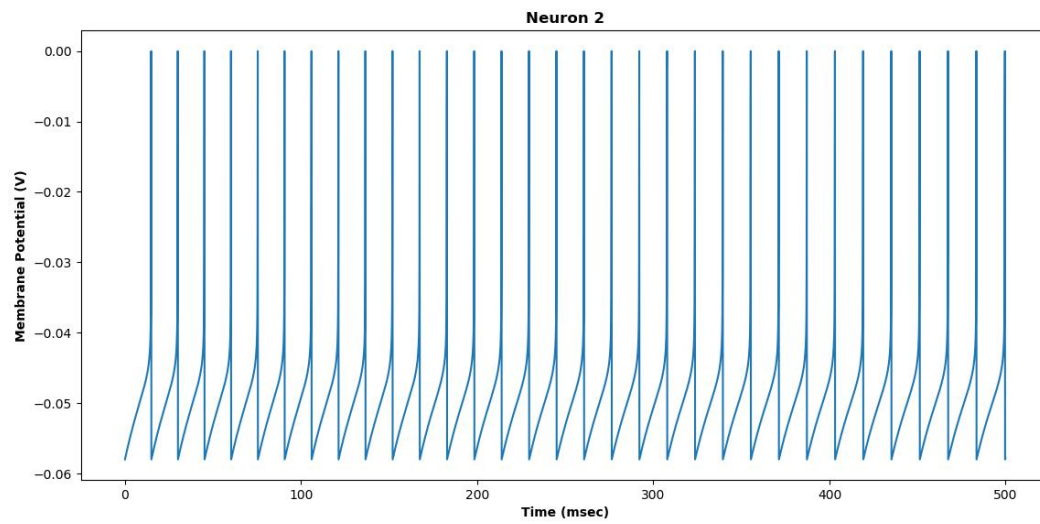


C)  
RS:

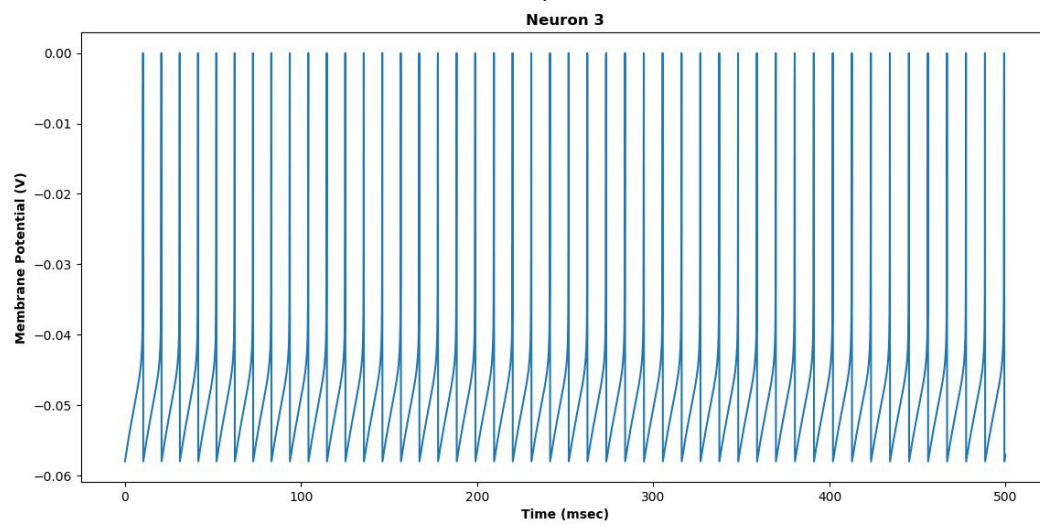
250pA:



350pA:



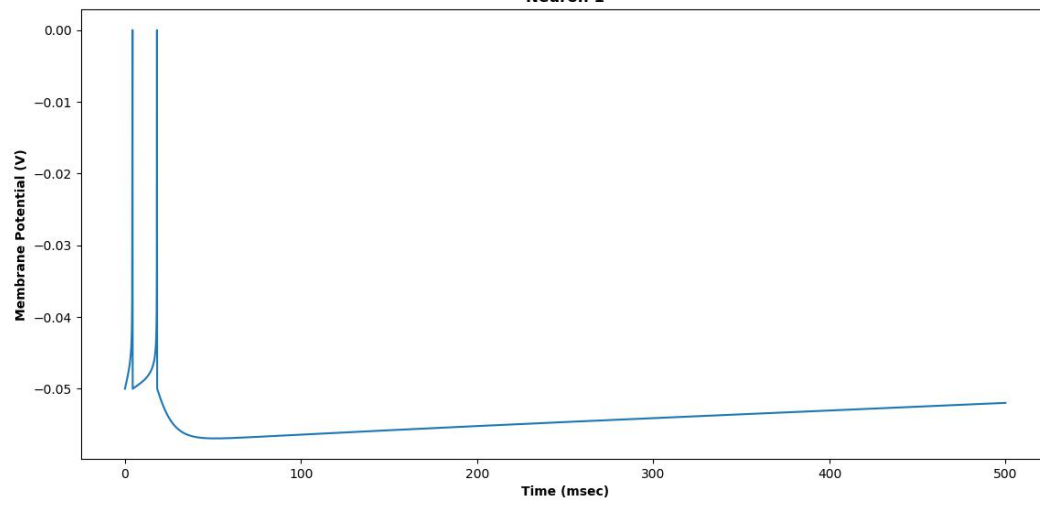
450pA:



**IB:**

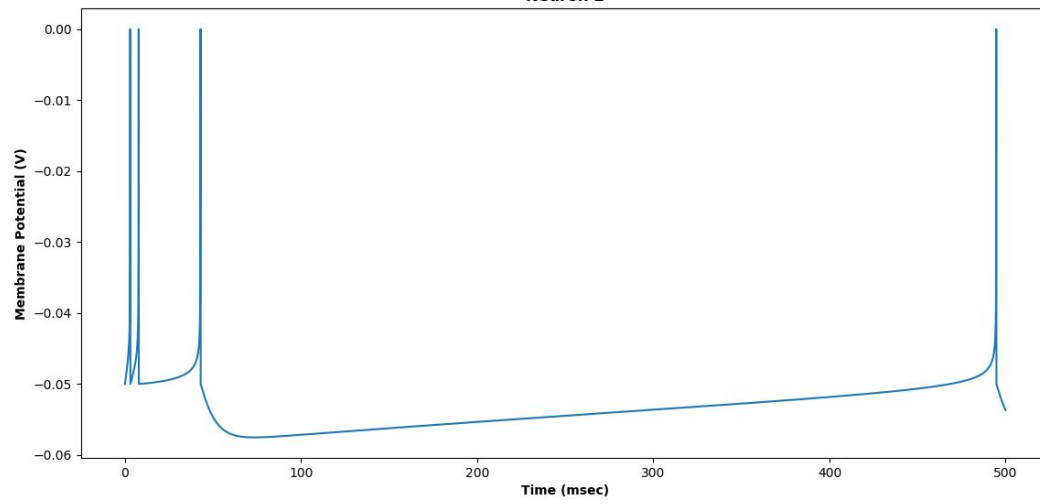
**250pA:**

**Neuron 1**



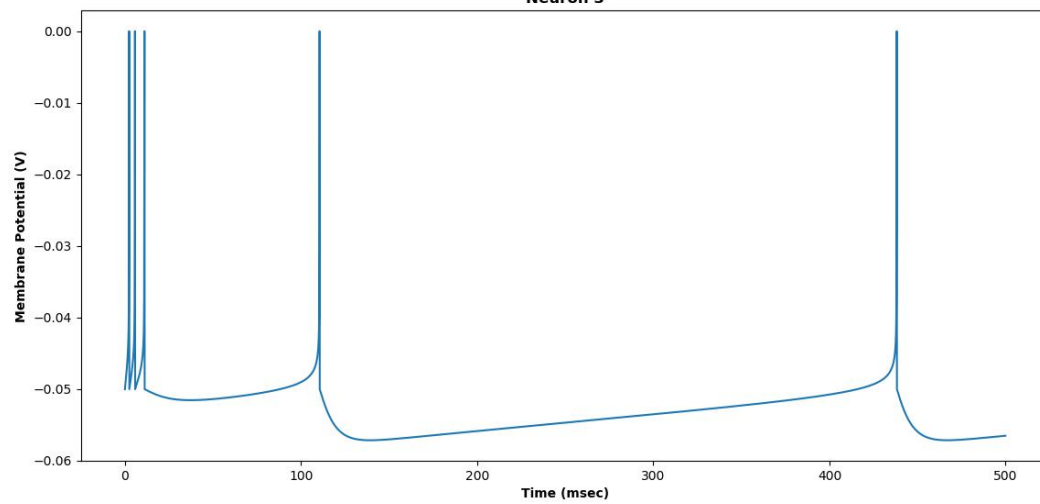
**350pA:**

**Neuron 2**



**450pA:**

**Neuron 3**

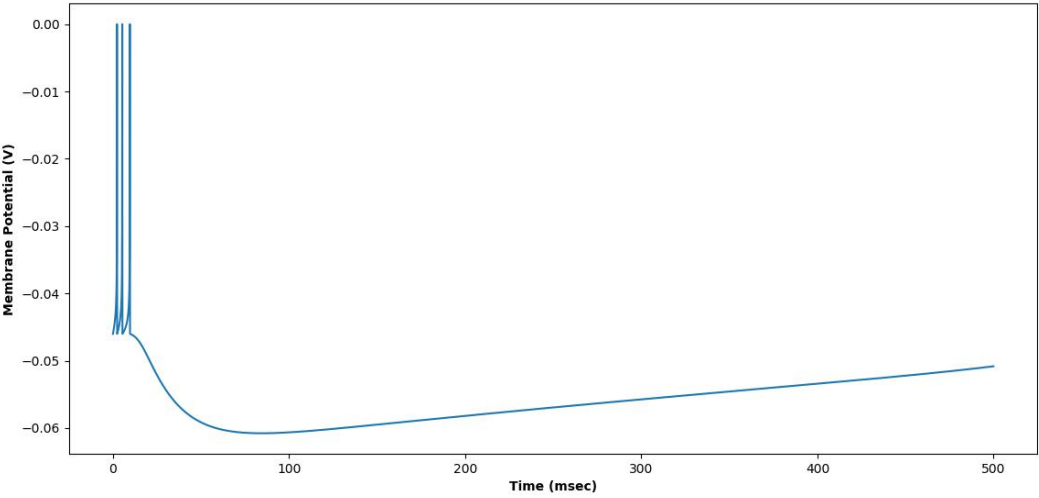




CH:

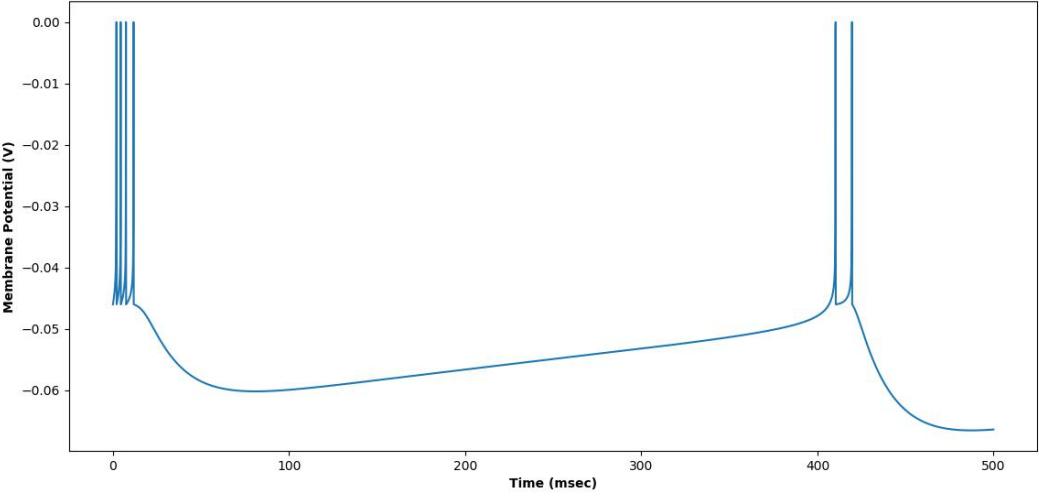
250pA:

Neuron 1



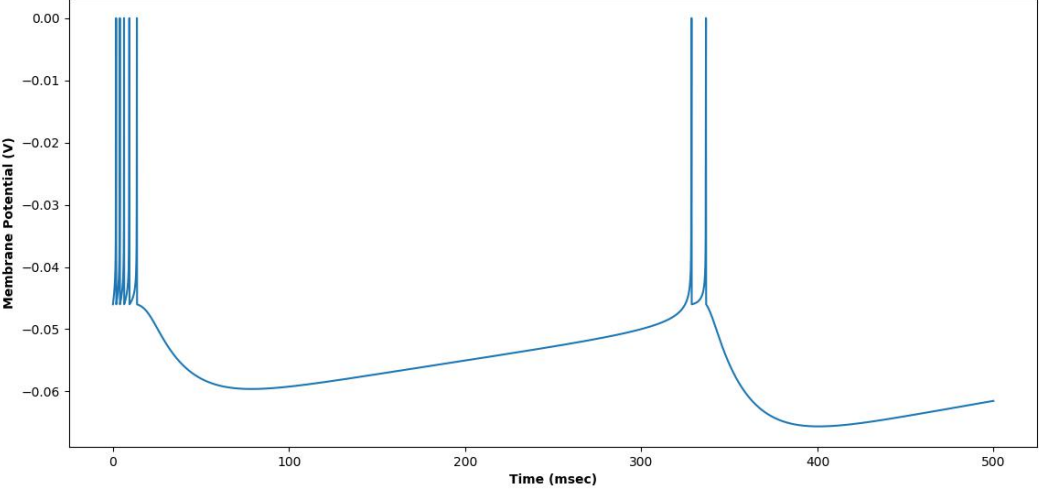
350pA:

Neuron 2



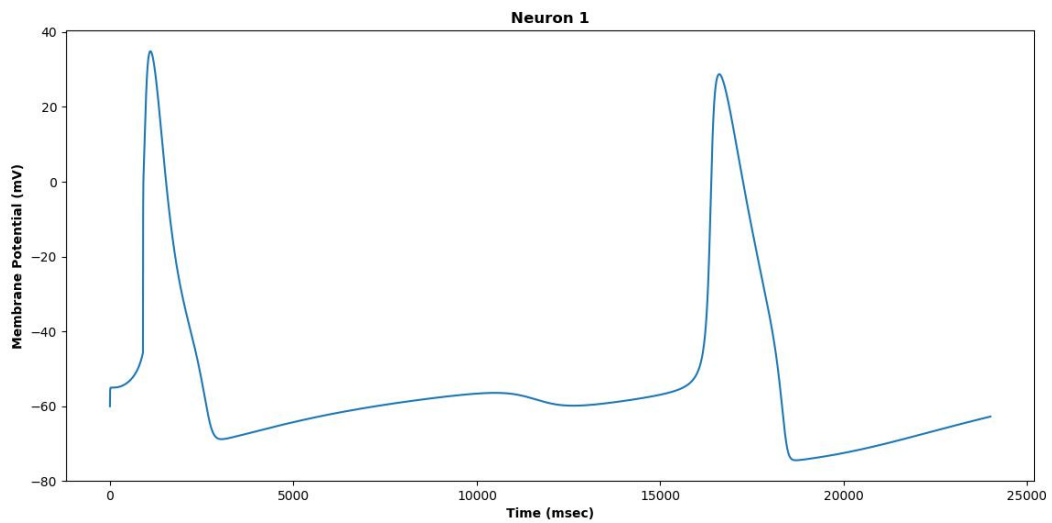
450pA:

Neuron 3



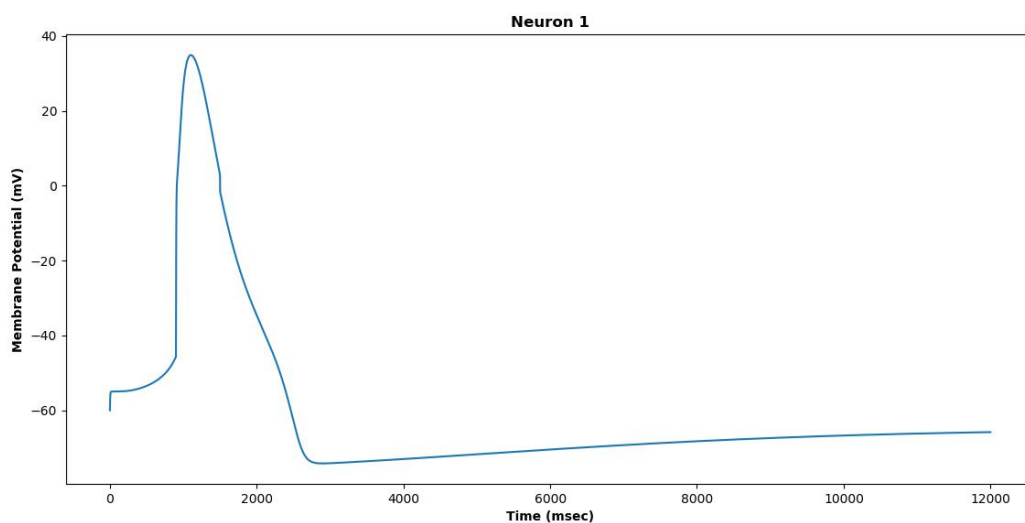
4) Plots:

A) The following plot is obtained when a constant current is started at 900ms and held at this constant value until the end of the simulation, with a total simulation time of 18000ms.



B)

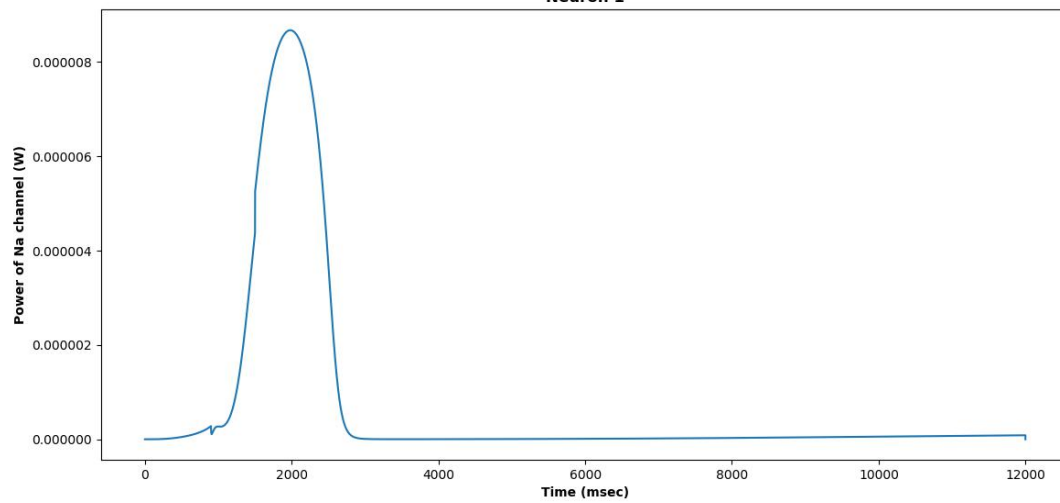
We will compute power and total energy for one spike by looking at the interval in which the potential looks like:



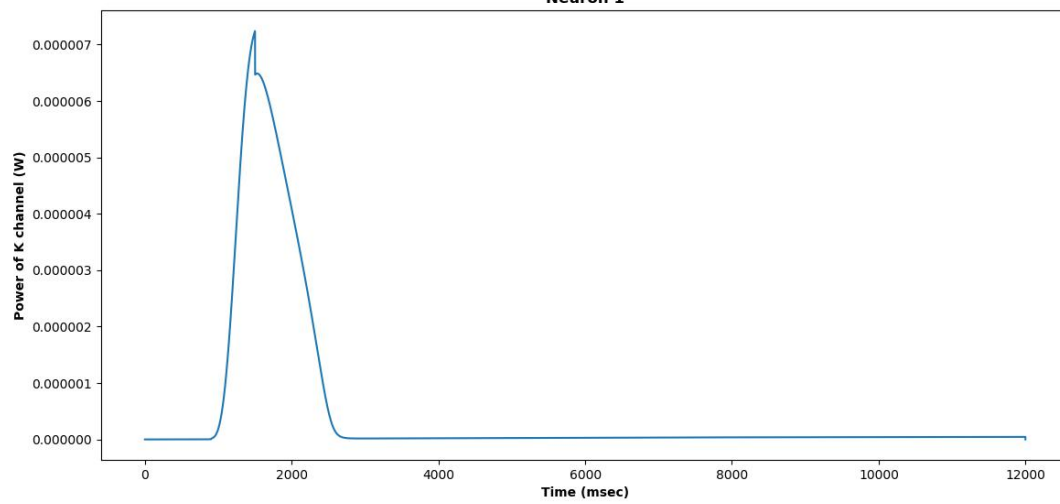
This is clearly a good approximate of one complete spike. This was simulated by injecting constant current between 900ms and 1500ms, with a total simulation time of 12000ms.

The corresponding plots of instantaneous power are:

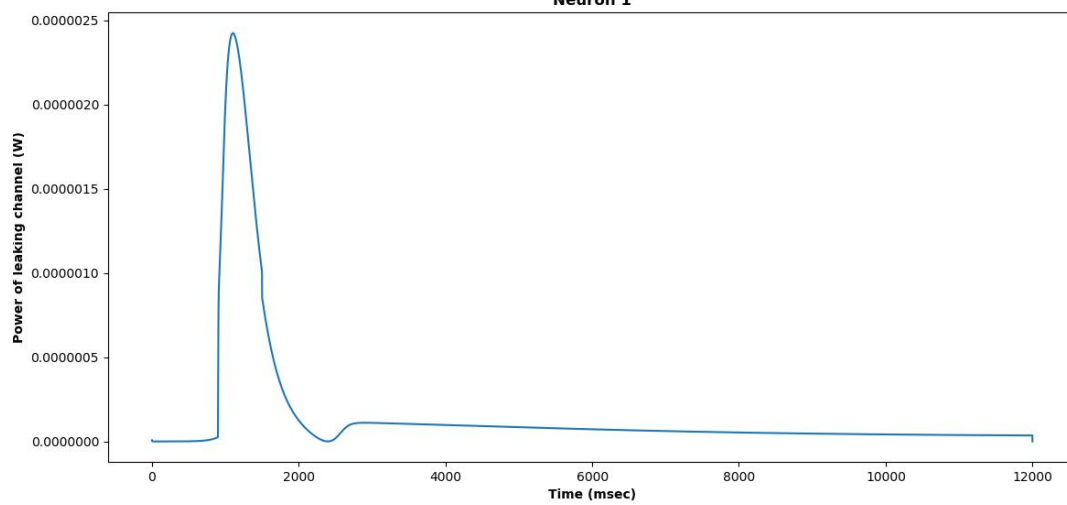
Na channel:  
Neuron 1

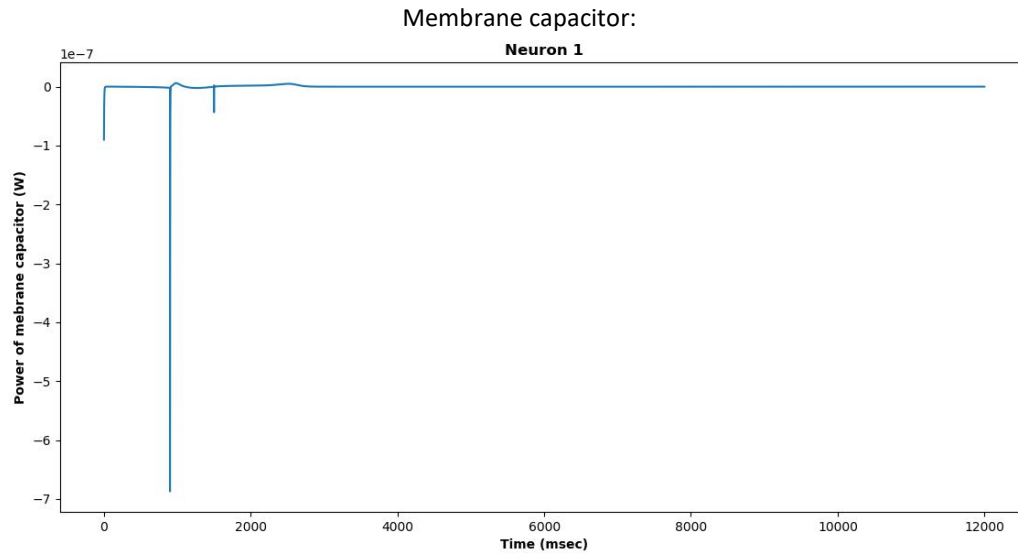


K channel:  
Neuron 1



Leaking channel:  
Neuron 1





D)

For the same time period as part C, we compute the total energy by simply summing up the instantaneous power for all time-steps and multiplying by the size of one time-step.

Total energy of Na channel:  $8.86938473 \times 10^{-6}$  W

Total energy of K channel:  $6.18275243 \times 10^{-6}$  W

Total energy of leaking channel:  $1.91458049 \times 10^{-6}$  W

Total energy of membrane capacitor:  $3.25920288 \times 10^{-10}$  W