Car Cruise Control System

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April 27, 2023

Introduction

A car cruise control system is a system that accurately maintains the speed set by the driver without any external intervention. It automatically controls the speed of a car and only allows the vehicle to hit a speed set by the driver. It does not allow the car to cross that speed limit.

The cruise control system takes different parameters(responses) as its inputs and adjusts the speed of the vehicle accordingly to ensure it lies in the specified limits.

The cruise control system

Problem parameters

The following transfer functions have been taken from a github repository by [Kan21a] , and his work [Kan21b]

Consider a car with a mass of 1,500 kg (including passengers) that is traveling down a level stretch of highway at a desired speed of 100 kilometers per hour. You wish to design a cruise controller that will adjust the throttle position $\boldsymbol{u}$ within the limits 0 to 1 in order to maintain constant speed.

A block diagram for this system is shown in the following sketch:

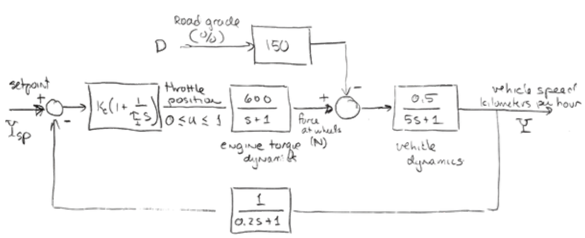


Figure 1: Block diagram showing the different parameters

The disturbance to the system is a change in road grade measured in percent. (A one percent grade corresponds to 1 foot of rise in 100 feet of horizontal travel, or a tangent of 0.01). The dynamics are represented by three transfer functions denoting:

the response of the engine torque to throttle position,

$$G_e = \frac{600}{s+1}$$

the response of the vehicle to engine and external forces,

$$G_v = \frac{0.5}{5s+1}$$

and the dynamics of the speed measurement,

$$G_m = \frac{1}{0.2s+1}$$

However, we exclude the controller given in the block diagram as we will proceed to design a controller ourselves.

Transfer function

We consider G_c and G_v in cascade, and find our G_{eq} by transferring G_m onto the main block.

The open loop transfer function is given by:

$$G_{eq}(s) = GH(s) = \frac{60s + 300}{s^3 + 6.2s^2 - 53.8s + 1}$$

Pole-zero Map and location of poles of open loop system

The poles and zeroes of the open loop system are found to be:

Open-loop poles:
-11.0687
4.8501
0.0186

Open-loop zeroes:
-5

Pole-zero map

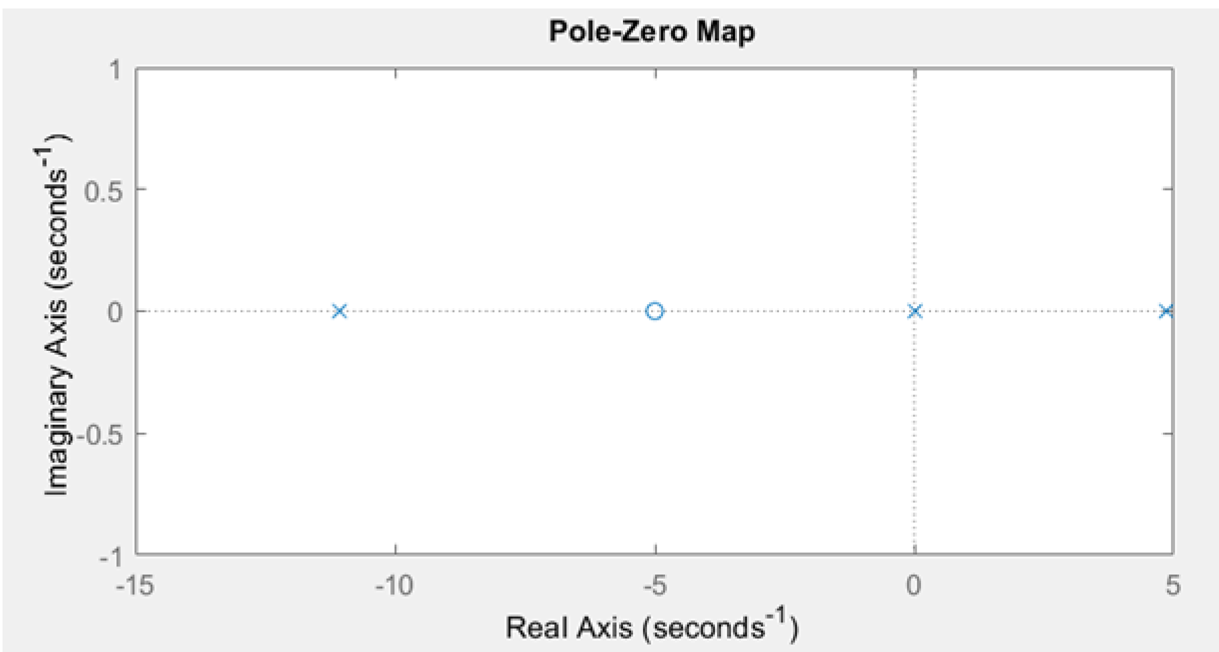


Figure 2: Pole-zero map of the open loop TF

From this, we can infer that the open loop system is unstable as it has 2 poles on the RHP of the s-plane.

Step and ramp response of the open loop system

The step and ramp responses of the open loop TF are:

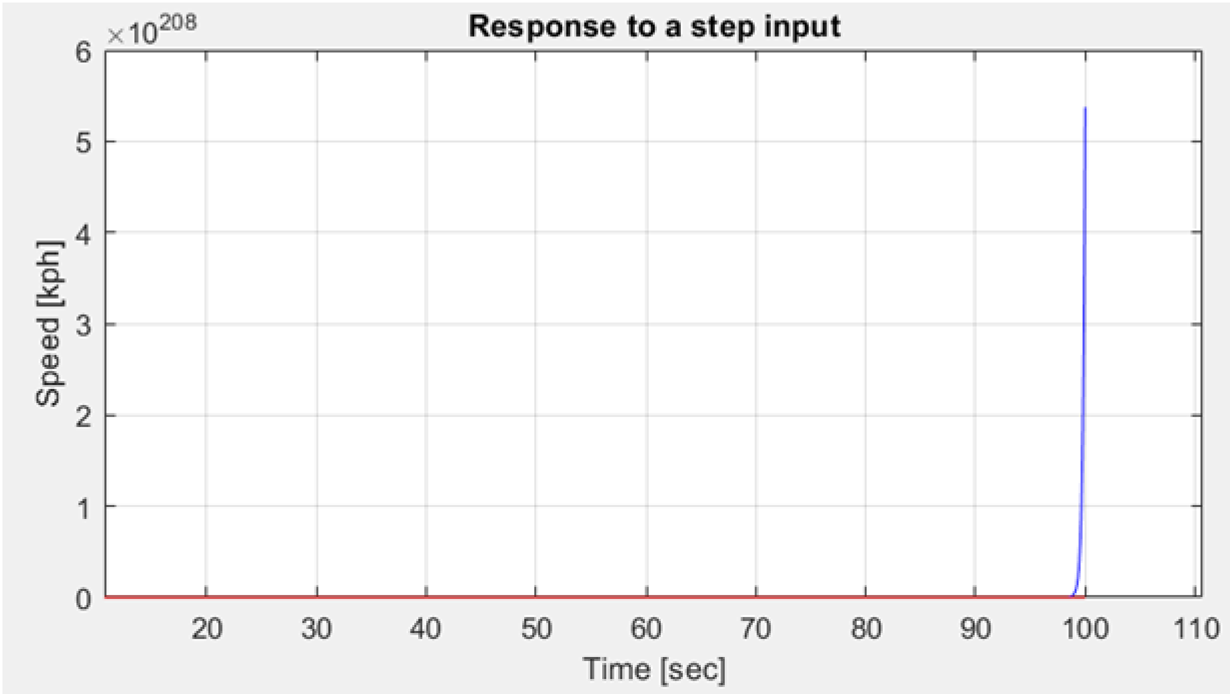


Figure 3: Step response

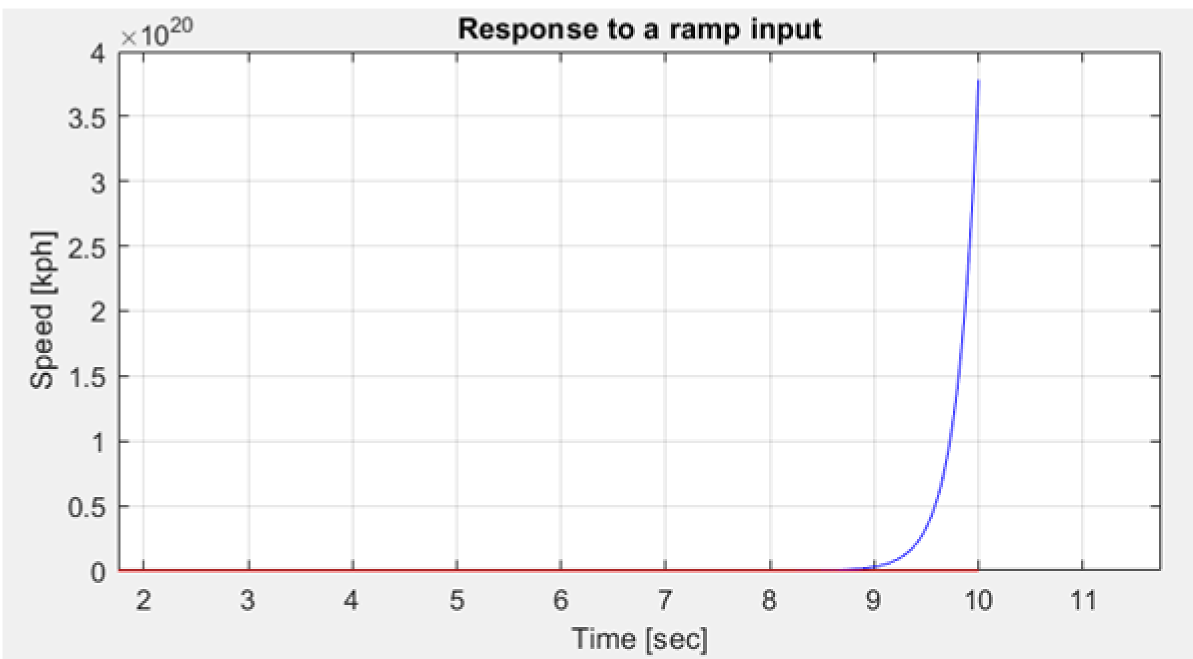


Figure 4: Ramp response

From the plots, we can infer that the open loop system is clearly unstable, as the step response does not settle and shoots up to a very large value.

Closed loop characteristics

Considering the system to be part of a unity negative feedback system, we get the closed loop transfer function as:

$$G_{CL}(s) = \frac{60s + 300}{s^3 + 6.2s^2 + 6.2s + 301}$$

Cascading a gain K, we now check for the range of K for which the closed loop system is stable.

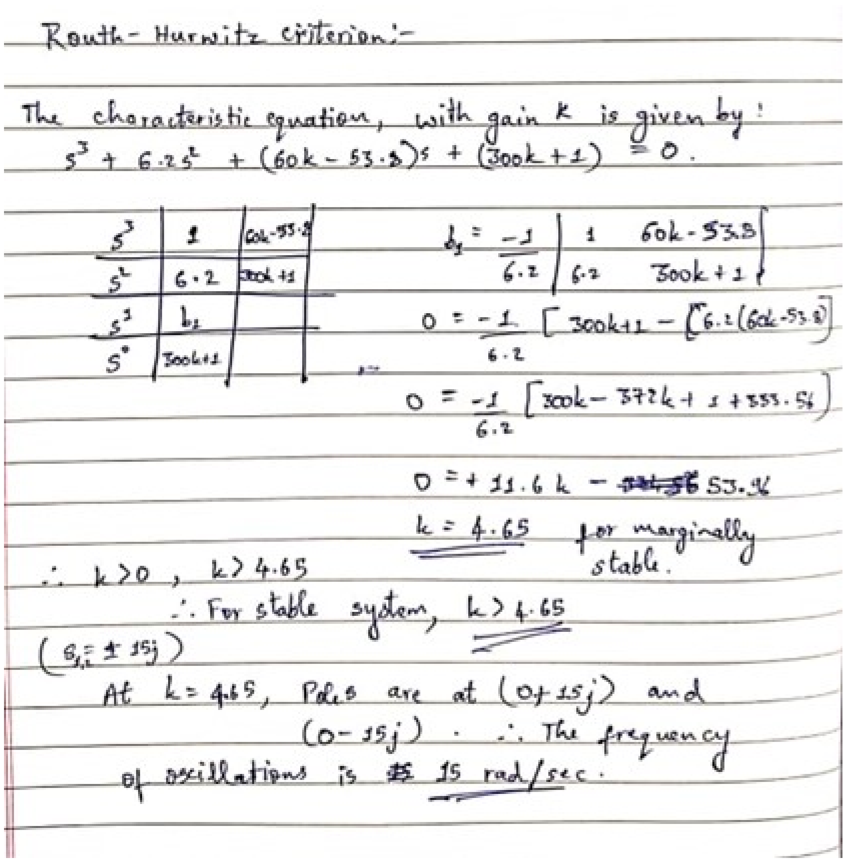


Figure 5: Calculations for R-H criterion

From the Routh-Hurwitz criterion, we find that the system is marginally stable for K = 4.65, and the frequency of oscillations is found to be 15 rads/sec.

The closed loop system is stable for all K>4.65

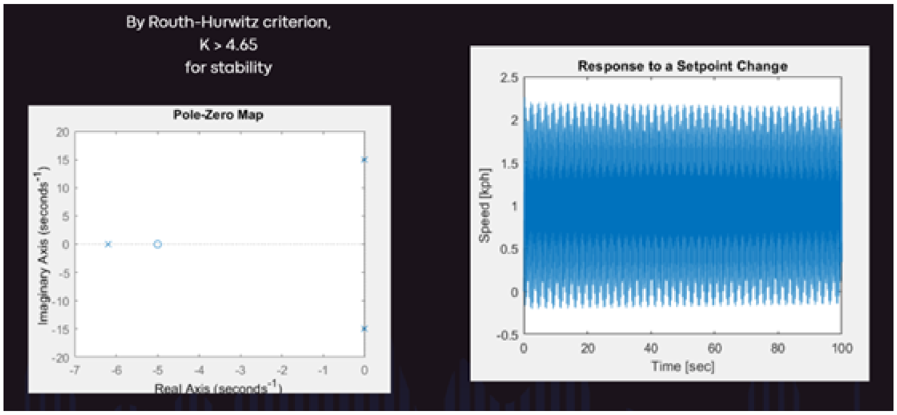


Figure 6: Step response and PZ map for K=4.65

Plots for different values of K

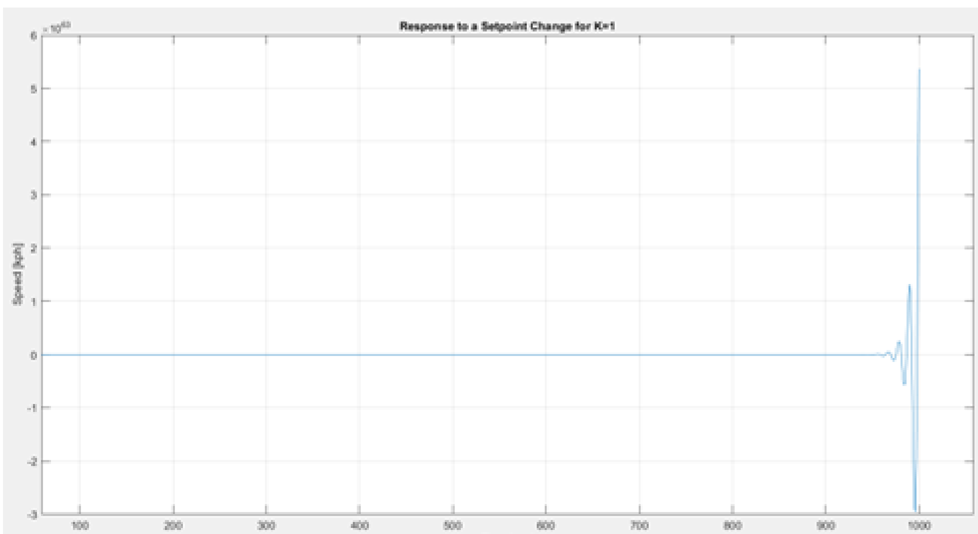


Figure 7: Step response for K=1

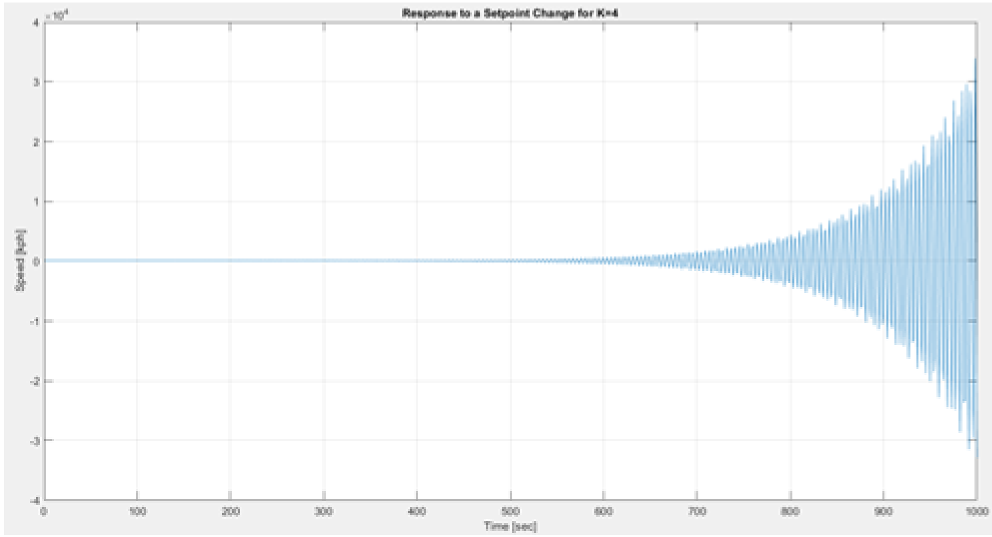


Figure 8: Step response for K=4

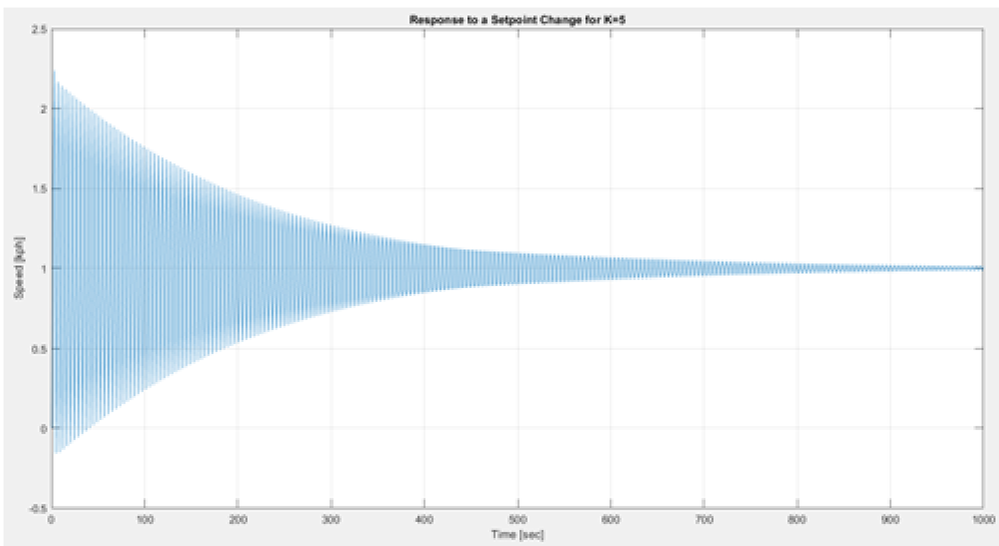


Figure 9: Step response for K=5

The step response is observed to be settling to 1 after a finite period of time for K=5, and therefore, we can conclude that the closed loop system is stable for all values of K>4.65

Root Locus of the closed loop system with gain K

Let us consider the closed loop system with a gain K=5 in cascade, and plot the root locus of this system.

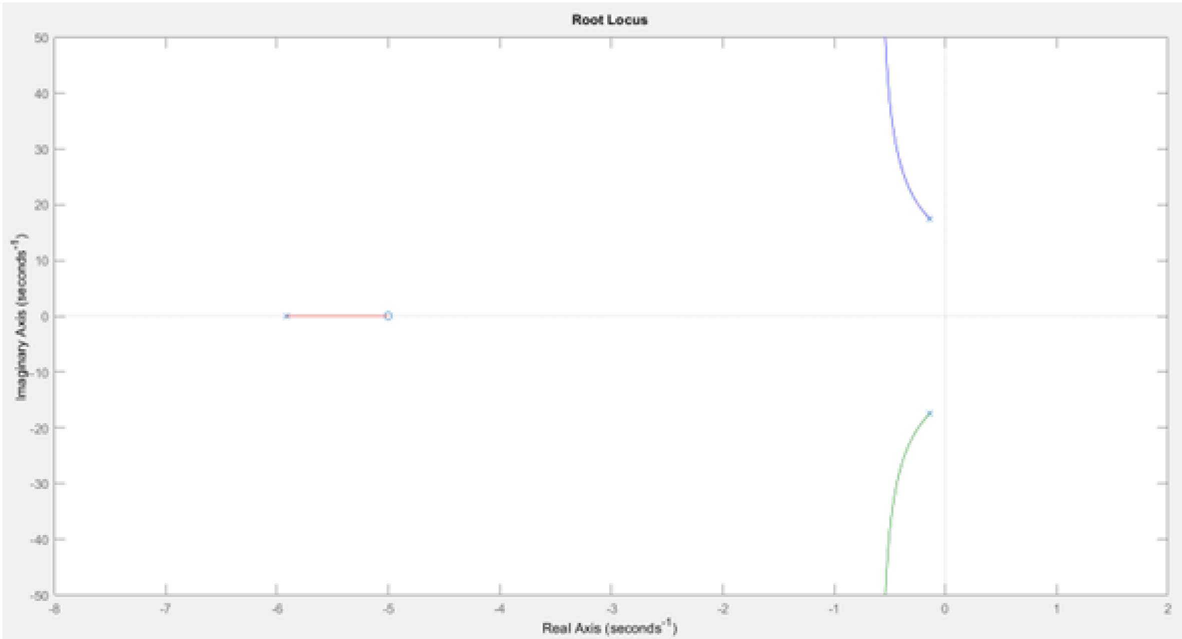


Figure 10: Root locus for K=5

We observe that the root locus does not intersect with the jω axis, and that it does not branch off from a point on the real axis.

Differences found between the open loop and closed loop system

Open Loop	Closed Loop
The system is unstable	The system is stable for a gain of K>4.65
The step response does not settle	The step response is stable and it settles to a finite value
Two of the poles lie on the RHP of the s plane	All the poles lie on the LHP for K>4.65

Bode plot for the open loop transfer function

Taking the gain K as 5, we proceed to obtain the Bode plot for the open loop TF, to infer the stability of the closed loop system based on gain and phase margins.

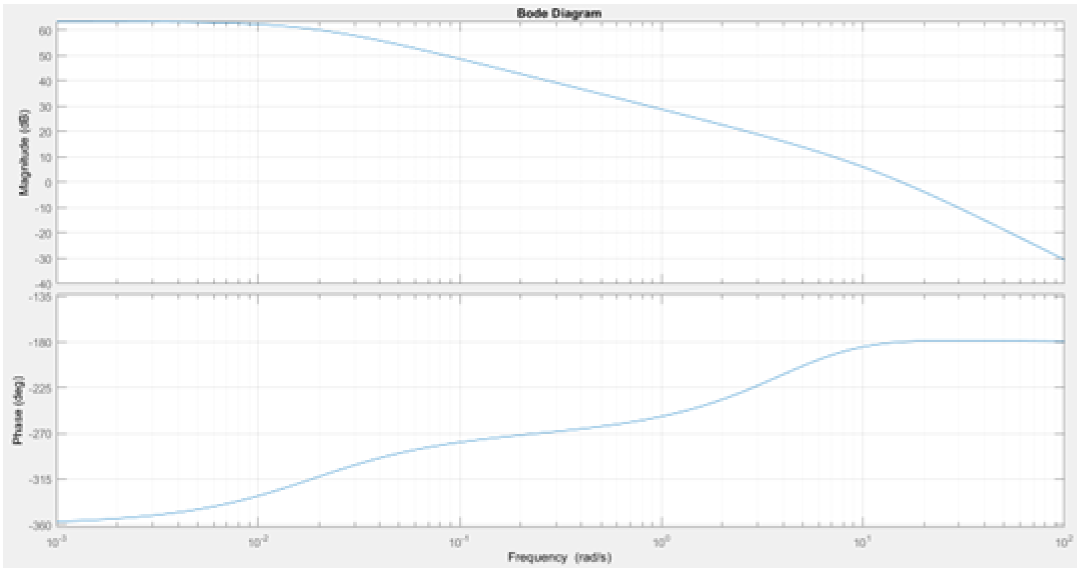


Figure 11: Bode plot for the open loop TF

The phase and gain margins are found to be:

Phase margin:
0.28°

Gain margin:
-0.64 dB

The phase margin is positive, and hence the closed loop system is stable.

Nyquist plot

Obtaining the Nyquist plot for the open loop TF, we can infer the stability of the closed loop system.

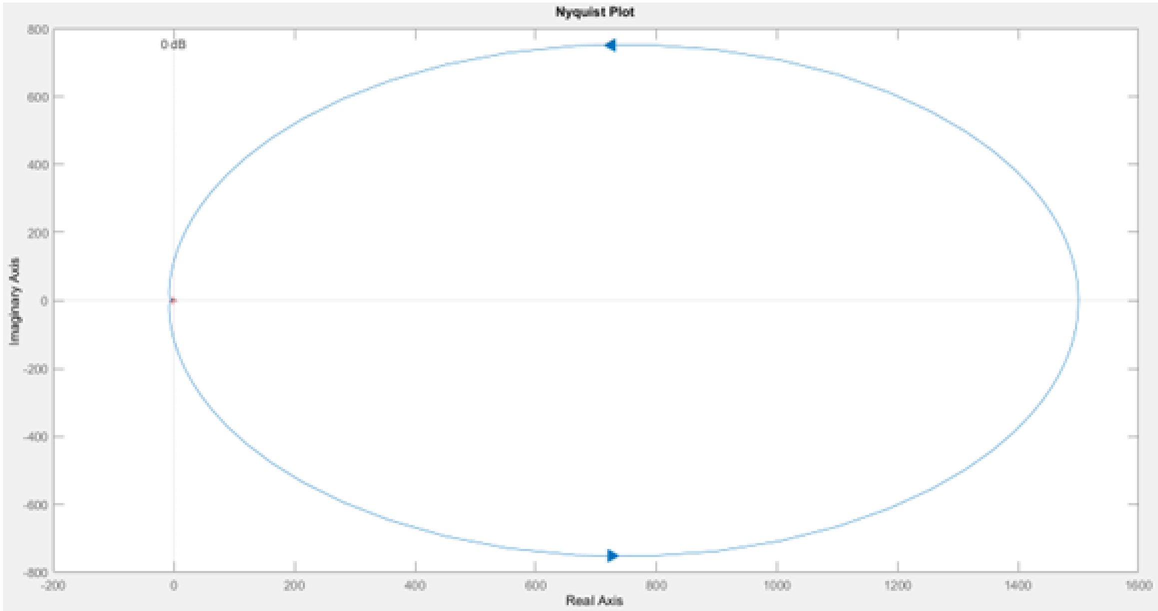


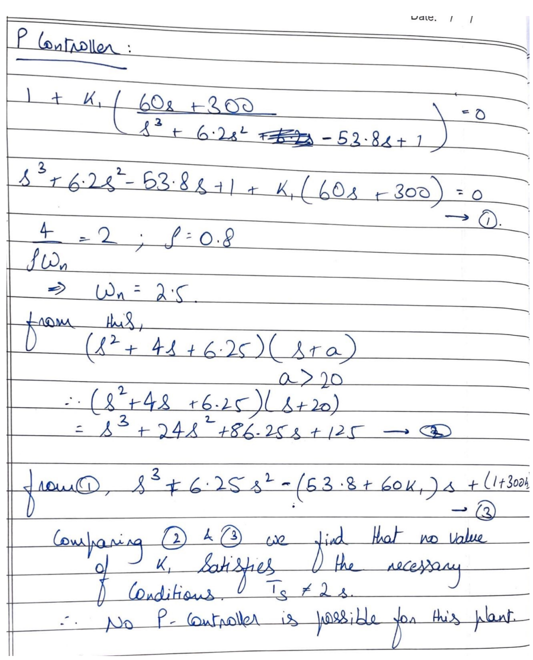
Figure 12: Nyquist plot for the open loop TF

The number of clockwise encircements of the point (-1+j0) is 0 times(N=0), number of poles on the RHP is 0 (P=0), and hence Z= N+P = 0 Therefore, there are no poles on the RHP for the closed loop TF. This matches with the inference we have drawn previously that the closed loop system is stable, and matches with the stability inference from the Bode plot

Controllers

P controller

A proportional controller could not be designed to meet the desired specifications of t_s=2s and a damping coeffecient of 0.8



P Controller :

$$1 + K_1\left(\frac{60s + 300}{s^3 + 6.2s^2 - 53.8s + 1}\right) = 0$$

$$s^3 + 6.2s^2 - 53.8s + 1 + K_1(60s + 300) = 0 \rightarrow (1)$$

$$\frac{4}{\zeta\omega_n} = 2 \ ; \ \zeta = 0.8$$

$$\Rightarrow \omega_n = 2.5$$

from this,

$$(s^2 + 4s + 6.25)(s + a)$$

$$a > 20$$

$$\therefore (s^2 + 4s + 6.25)(s + 20)$$

$$= s^3 + 24s^2 + 86.25s + 125 \rightarrow (2)$$

from (1), $s^3 + 6.25s^2 - (53.8 + 60K_1)s + (1 + 300K_1)$ $\rightarrow (3)$

Comparing (2) & (3) we find that no value of K_1 satisfies the necessary conditions. $T_s \neq 2$ s.

$\therefore$ No P-controller is possible for this plant.

Figure 13: Calculations for P controller

The corresponding output from MATLAB is as follows: For K_p=100

```
RiseTime: 0.9235
TransientTime: 67.1447
 SettlingTime: 67.1447
  SettlingMin: 0.3362
  SettlingMax: 1.8636
    Overshoot: 86.3685
   Undershoot: 0
         Peak: 1.8636
     PeakTime: 3
```

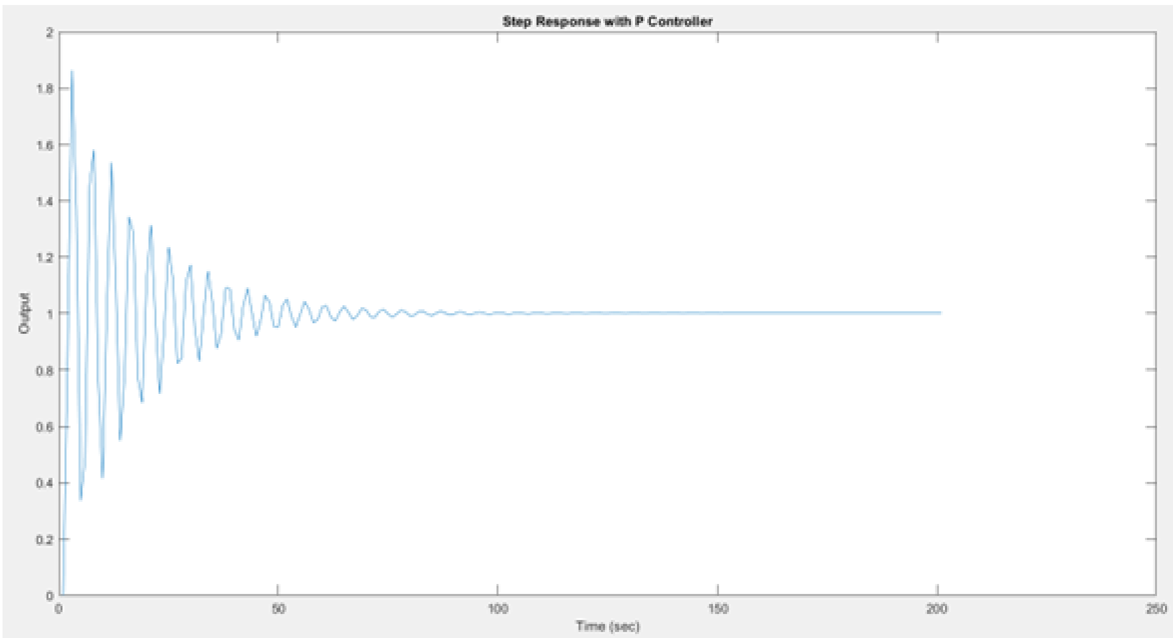


Figure 14: Step response of P controller with K_p=100

For K_p=1000

```
RiseTime: 1.8489
TransientTime: 64.4142
 SettlingTime: 64.4142
  SettlingMin: 0.4626
  SettlingMax: 1.7380
    Overshoot: 73.7962
   Undershoot: 0
         Peak: 1.7380
     PeakTime: 6
```

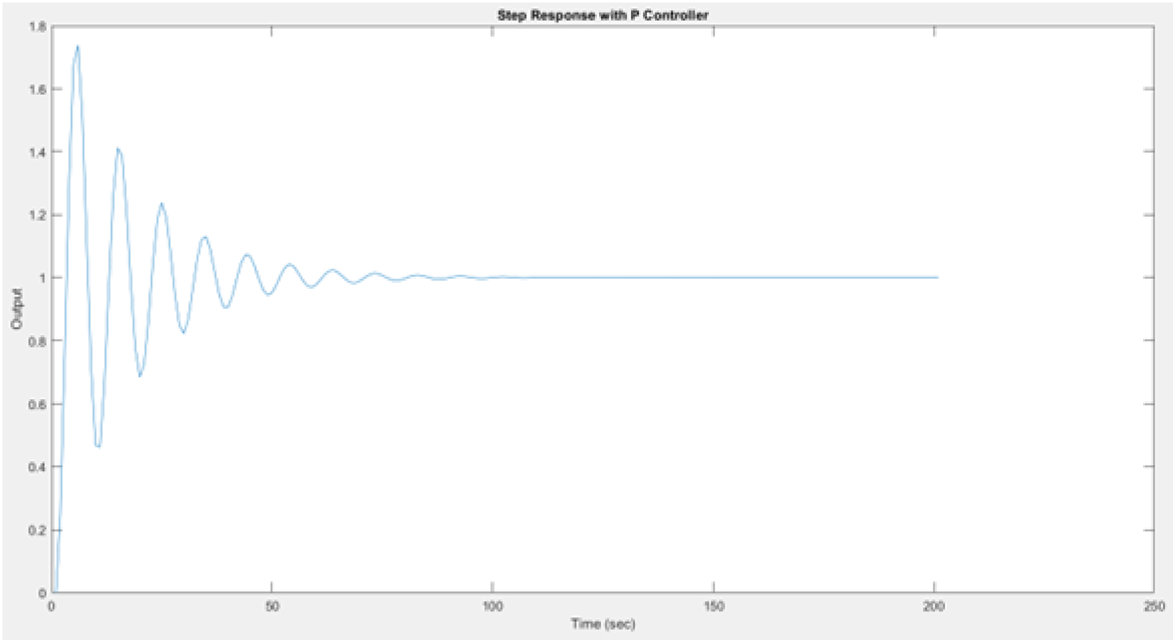
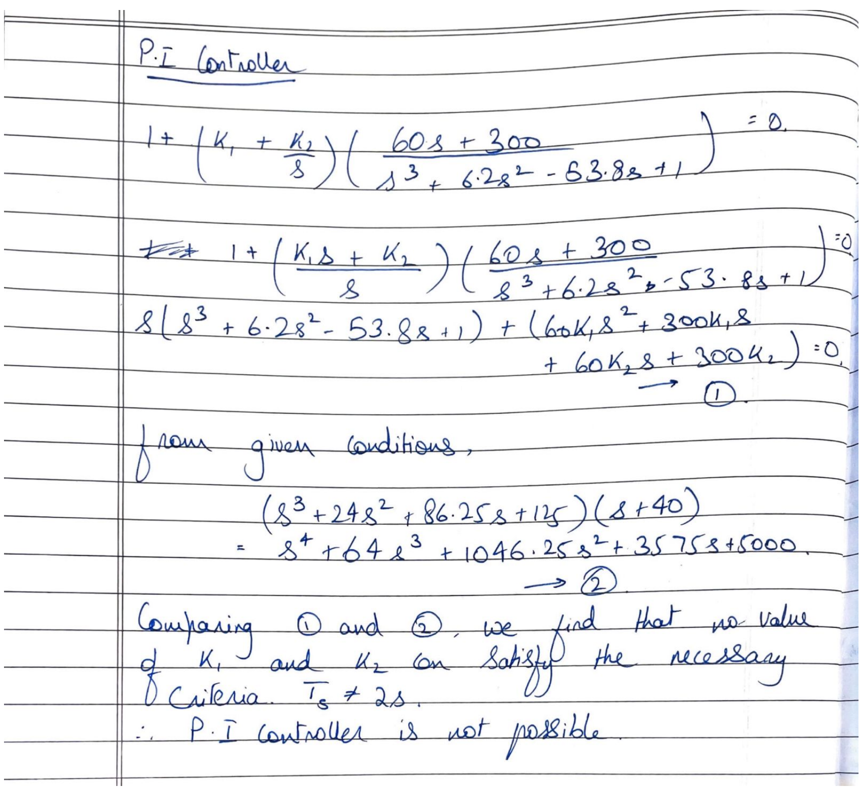


Figure 15: Step response of P controller with K_p=1000

PI controller

A proportional-integral controller could not be designed to meet the desired specifications of t_s=2s and a damping coeffecient of 0.8


P.I Controller

$$1 + \left(K_1 + \frac{K_2}{s}\right)\left(\frac{60s + 300}{s^3 + 6.2s^2 - 53.8s + 1}\right) = 0$$

$$1 + \left(\frac{K_1 s + K_2}{s}\right)\left(\frac{60s + 300}{s^3 + 6.2s^2 - 53.8s + 1}\right) = 0$$

$$s(s^3 + 6.2s^2 - 53.8s + 1) + (60K_1 s^2 + 300K_1 s + 60K_2 s + 300K_2) = 0 \rightarrow (1)$$

from given conditions,

$$(s^3 + 24s^2 + 86.25s + 125)(s + 40) = s^4 + 64s^3 + 1046.25s^2 + 3575s + 5000 \rightarrow (2)$$

Comparing (1) and (2), we find that no value of K_1 and K_2 can satisfy the necessary criteria. $T_s \neq 2s$.

∴ P.I controller is not possible.

Figure 16: Calculations for PI controller

The corresponding output from MATLAB is as follows:

For K_p=100, K_i=2

```
 RiseTime: 0.9234
 TransientTime: 67.2437
  SettlingTime: 67.2437
   SettlingMin: 0.3334
   SettlingMax: 1.8654
     Overshoot: 86.5460
    Undershoot: 0
          Peak: 1.8654
      PeakTime: 3
```

For K_p=1000, K_i=5

```
 RiseTime: 1.8487
 TransientTime: 64.4570
  SettlingTime: 64.4570
   SettlingMin: 0.4612
   SettlingMax: 1.7389
     Overshoot: 73.8886
    Undershoot: 0
          Peak: 1.7389
      PeakTime: 6
```

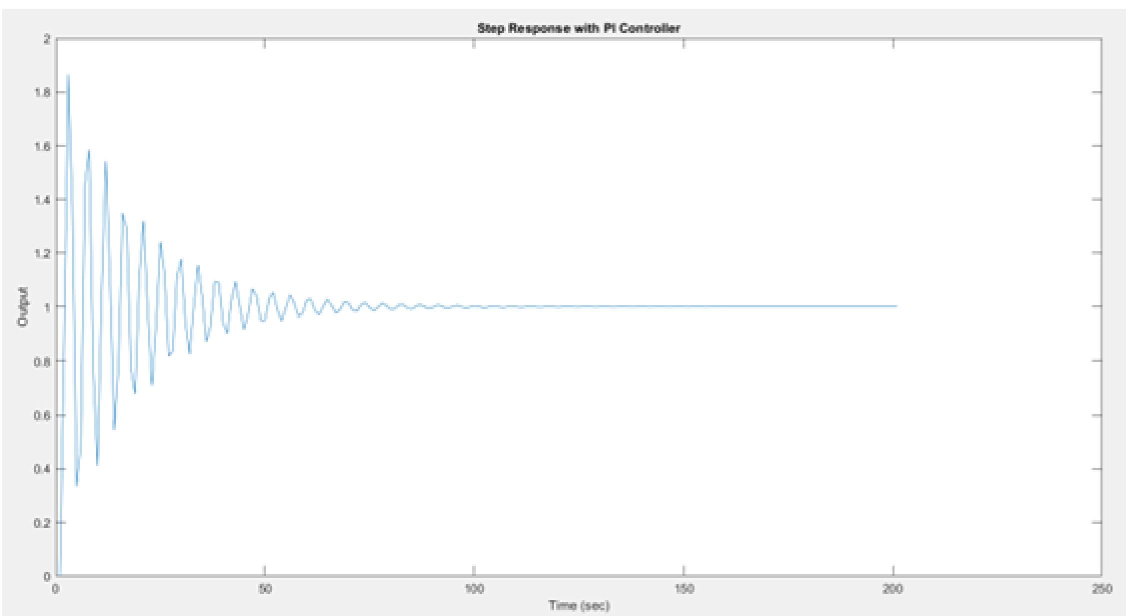


Figure 17: Step response of PI controller with K_p=100, K_i=2

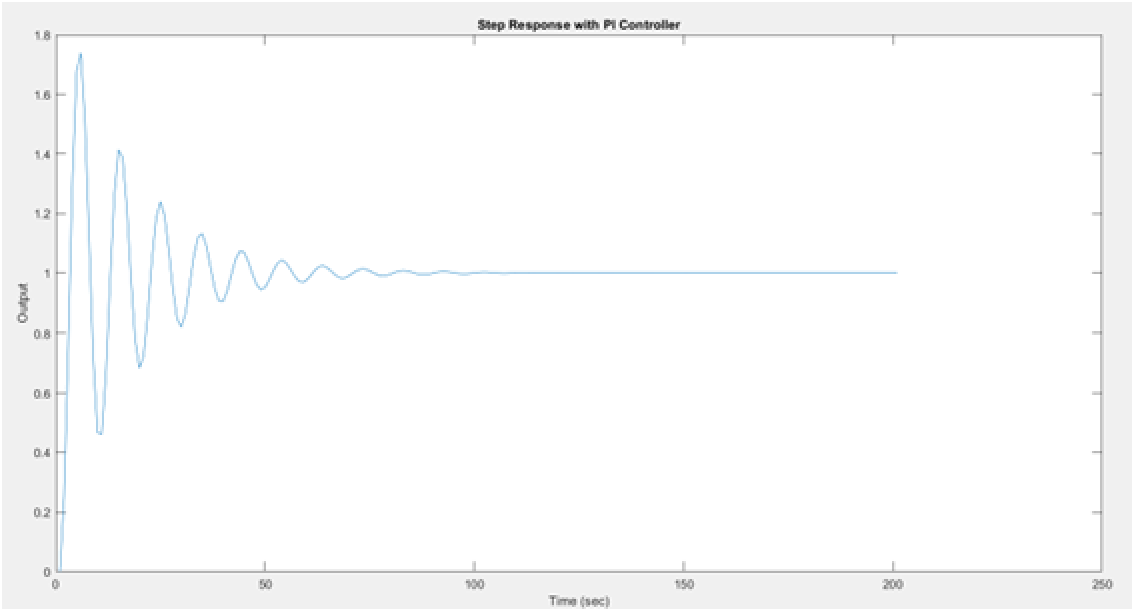


Figure 18: Step response of PI controller with K_p=1000, K_i=5

PD Controller

The design of a PD controller with the given design specifications of t_s=2s and a damping coeffecient of 0.8, is possible for the chosen transfer function for: K_p=10 and K_d=10

The corresponding output from MATLAB is as follows:

```
    RiseTime: 0.7942
    TransientTime: 1.9729
     SettlingTime: 1.9729
      SettlingMin: 0.9997
      SettlingMax: 1.0132
        Overshoot: 1.3535
       Undershoot: 0
             Peak: 1.0132
         PeakTime: 5
```

```
   Pole          Damping        Frequency       Time Constant
                               (rad/seconds)      (seconds)

 -1.03e+00      1.00e+00         1.03e+00          9.75e-01
 -4.87e+00      1.00e+00         4.87e+00          2.05e-01
 -6.00e+02      1.00e+00         6.00e+02          1.67e-03
```

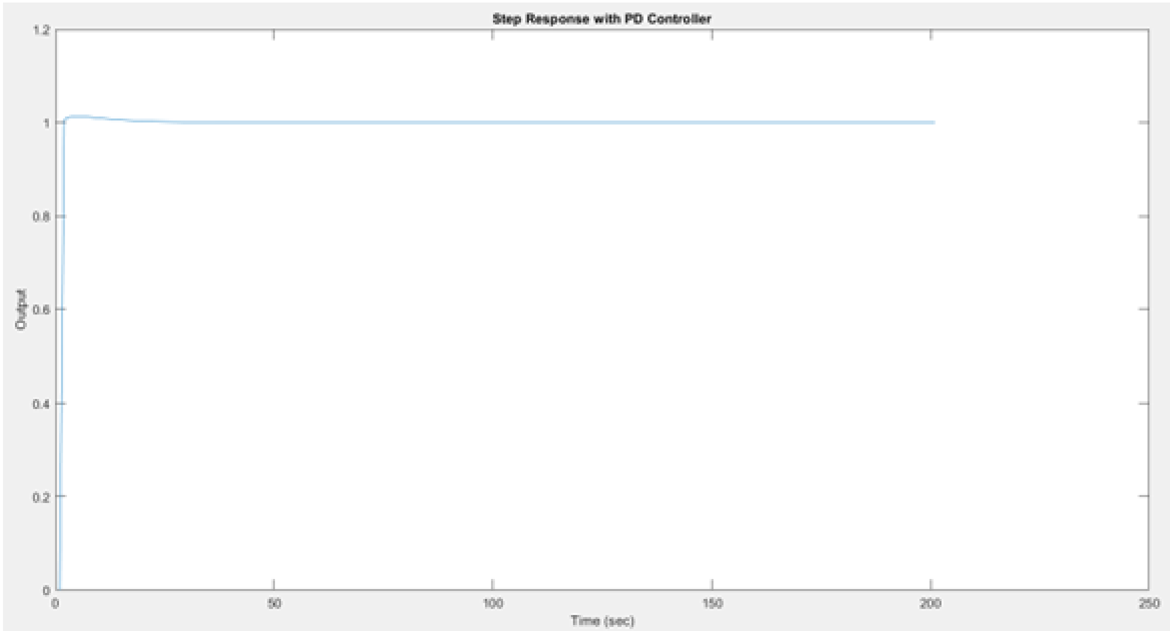


Figure 19: Step response of PD controller with K_p=10, K_d=10

PID Controller

The design of a PD controller with the given design specifications of t_s=2s and a damping coeffecient of 0.8, is possible for the chosen transfer function for: K_p=10, K_i=2, and K_d=5

The corresponding output from MATLAB is as follows:

For K_p=10, K_d=5, K_i=2 :

```
     RiseTime: 0.7970
TransientTime: 1.9764
 SettlingTime: 1.9764
  SettlingMin: 0.9999
  SettlingMax: 1.0040
    Overshoot: 0.4039
   Undershoot: 0
         Peak: 1.0040
     PeakTime: 3
```

```
     Pole                 Damping       Frequency      Time Constant
                                      (rad/seconds)      (seconds)

-5.00e-02                 1.00e+00       5.00e-02         2.00e+01
-5.06e+00                 1.00e+00       5.06e+00         1.98e-01
-6.05e+01 + 4.76e+01i     7.86e-01       7.70e+01         1.65e-02
-6.05e+01 - 4.76e+01i     7.86e-01       7.70e+01         1.65e-02
```

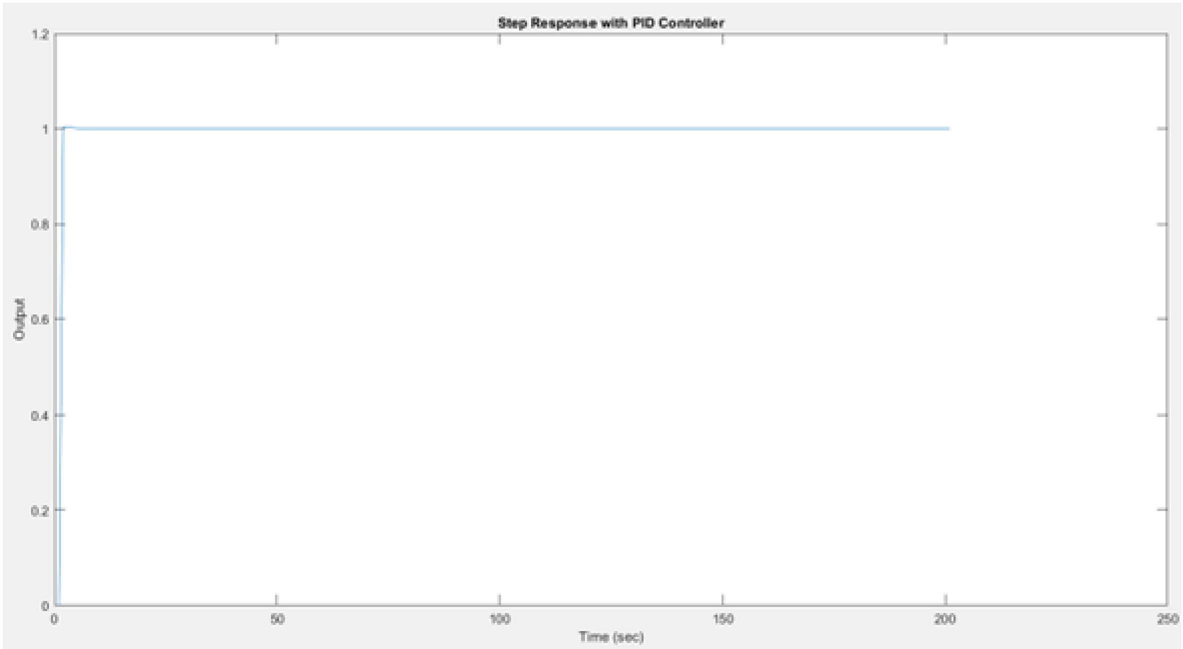


Figure 20: Step response of PID controller with K_p=10, K_d=5, K_i=2

Robustness check

The aim of this robustness check is to make sure that the system with the designed PID controller can meet the design specifications even with slight variations in the plant TF coefficients.

Increasing the coefficients by 30%

Increasing the coefficients by 30% with K_p=10, K_i=2, and K_d=5, we get the plant function as :

$$G_{+30\%}(s) = \frac{78s + 300}{1.33s^3 + 8.06s^2 - 69.94s + 1}$$

The corresponding output from MATLAB is as follows:
For K_p=10, K_d=5, K_i=2 :

```
RiseTime: 0.7963
TransientTime: 1.9754
 SettlingTime: 1.9754
  SettlingMin: 0.9999
  SettlingMax: 1.0052
    Overshoot: 0.5242
   Undershoot: 0
         Peak: 1.0052
     PeakTime: 3
```

```
        Pole               Damping       Frequency      Time Constant
                                       (rad/seconds)      (seconds)

-5.00e-02                  1.00e+00       5.00e-02         2.00e+01
-3.89e+00                  1.00e+00       3.89e+00         2.57e-01
-5.97e+01 + 4.72e+01i      7.84e-01       7.61e+01         1.67e-02
-5.97e+01 - 4.72e+01i      7.84e-01       7.61e+01         1.67e-02
```

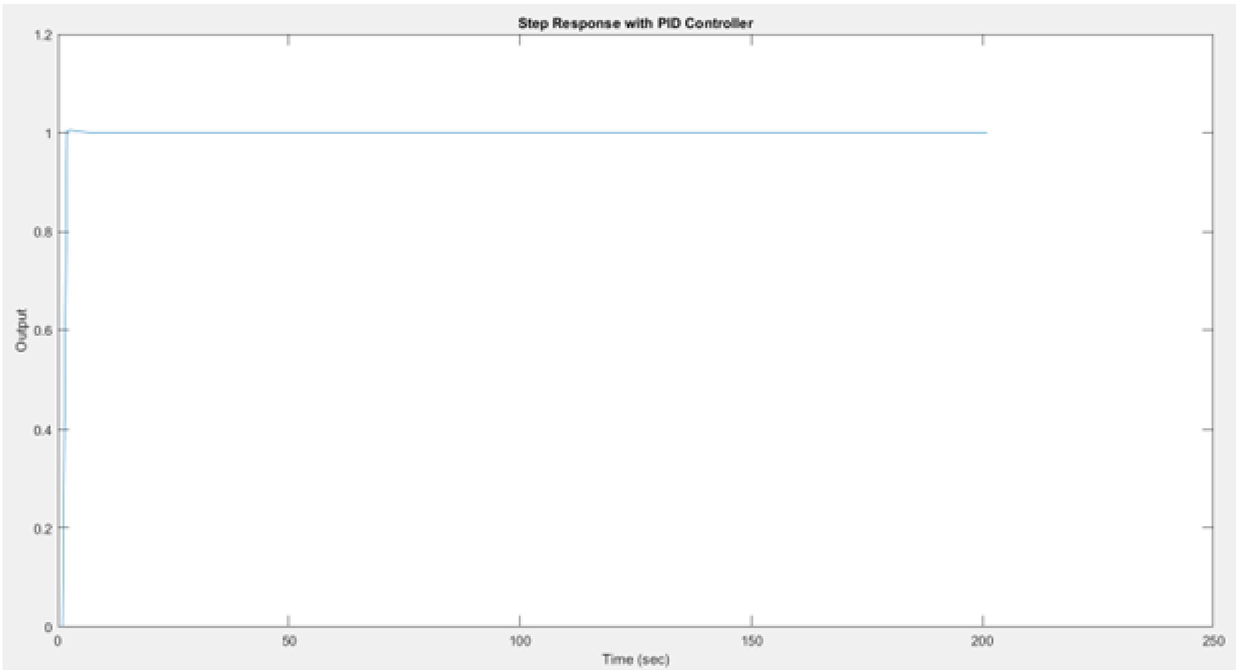


Figure 21: Step response after increasing coefficients by 30%

Decreasing the coefficients by 30%

Decreasing the coefficients by 30% with K_p=10, K_i=2, and K_d=5, we get the plant function as :

$$G_{-30\%}(s) = \frac{42s + 300}{0.67s^3 + 4.34s^2 - 37.66s + 1}$$

The corresponding output from MATLAB is as follows:
For K_p=10, K_d=5, K_i=2 :

```
     RiseTime: 0.7985
TransientTime: 1.9782
 SettlingTime: 1.9782
  SettlingMin: 0.9999
  SettlingMax: 1.0022
    Overshoot: 0.2214
   Undershoot: 0
         Peak: 1.0022
     PeakTime: 3
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-5.00e-02	1.00e+00	5.00e-02	2.00e+01
-7.21e+00	1.00e+00	7.21e+00	1.39e-01
-6.23e+01 + 4.82e+01i	7.91e-01	7.88e+01	1.61e-02
-6.23e+01 - 4.82e+01i	7.91e-01	7.88e+01	1.61e-02

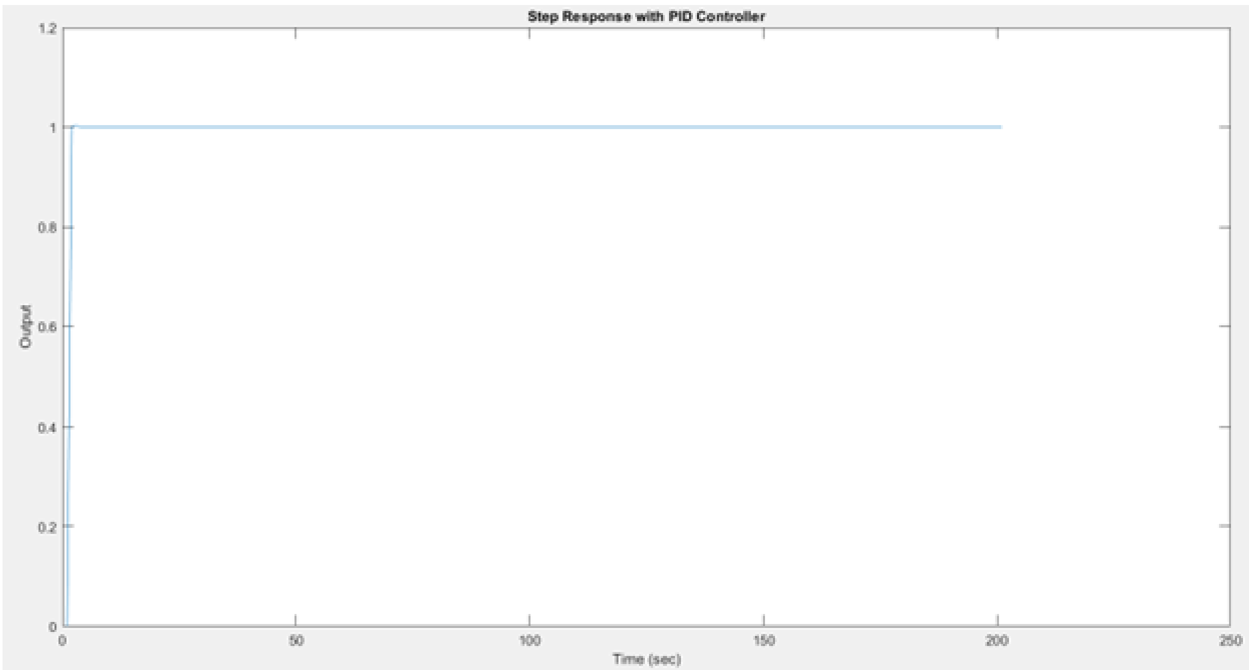


Figure 22: Step response after decreasing coefficients by 30%

Hence, we infer that the settling time is below 2 seconds in both cases and damping remains around 0.784 - 1, so the plant meets the desired specifications of t_s=2s and a damping coeffecient of 0.8, even when there is uncertainty in the plant parameters.

References

[Kan21a] Jeffery Kantor. Github repository. https://https://github.com/jckantor/CBE30338, 2021.

[Kan21b] Jeffery Kantor. Textbook. https://jckantor.github.io/CBE30338/05.02-Closed--Loop-Transfer-Functions-for-Car-Cruise-Control.html, 2021.