CONTROL SYSTEMS FINAL PROJECT

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TOPIC CHOSEN:

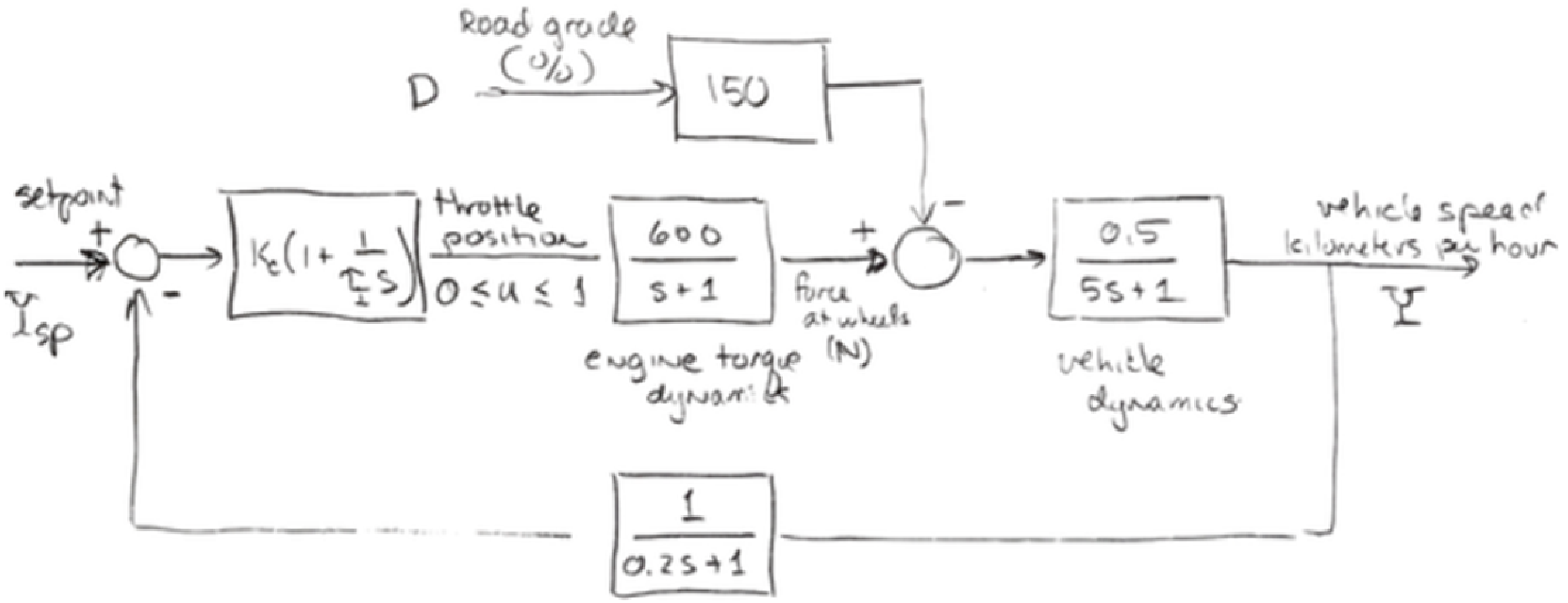
CAR CRUISE CONTROL

1) Let's imagine you own a car with a mass of 1,500 kg (including passengers) that is traveling down a level stretch of highway at a desired speed of 100 kilometers per hour. You wish to design a cruise controller that will adjust the throttle position 'u' within the limits 0 to 1 in order to maintain constant speed.

A block diagram for this system is shown in the following sketch:

Source: https://jckantor.github.io/CBE30338/05.02-Closed--Loop-Transfer-Functions-for-Car-Cruise-Control.html

Block Diagram
of the System



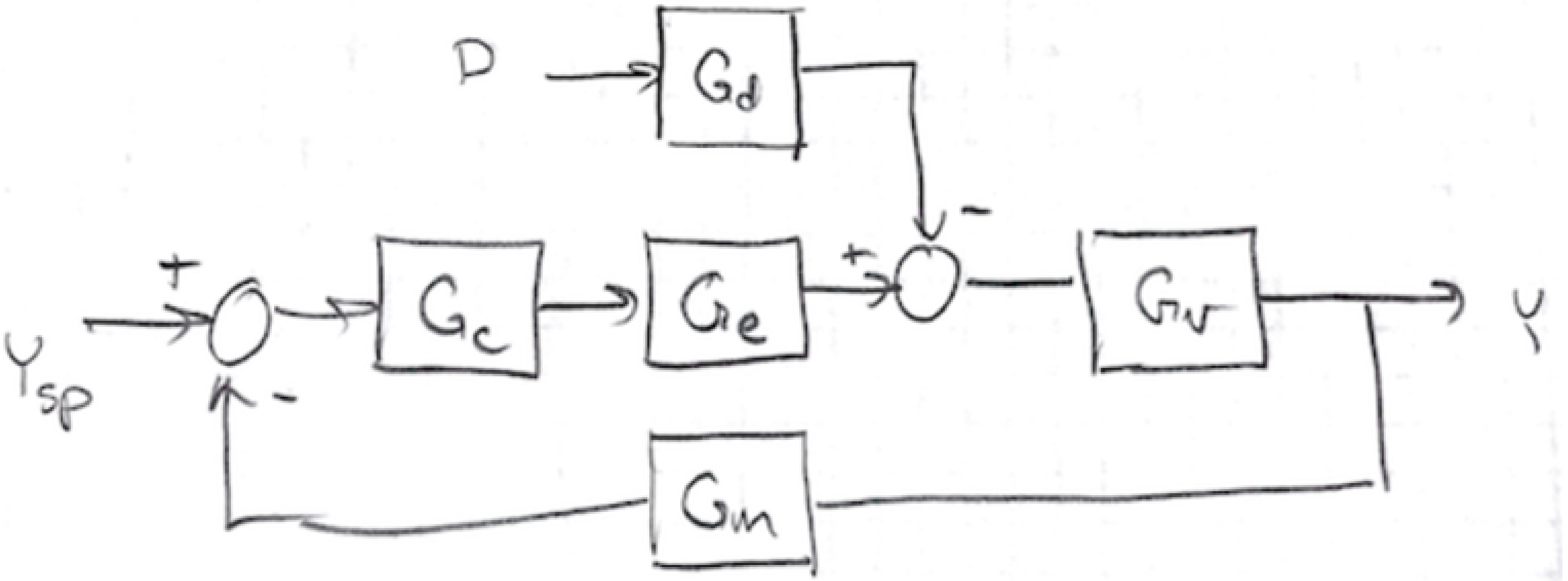
$$G_e = \frac{600}{s+1}$$

$$G_v = \frac{0.5}{5s+1}$$

$$G_m = \frac{1}{0.2s+1}$$

$$G_d = 150$$

$$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right)$$



Transfer Function:

This is the transfer function for :
Vehicle response to a change in setpoint

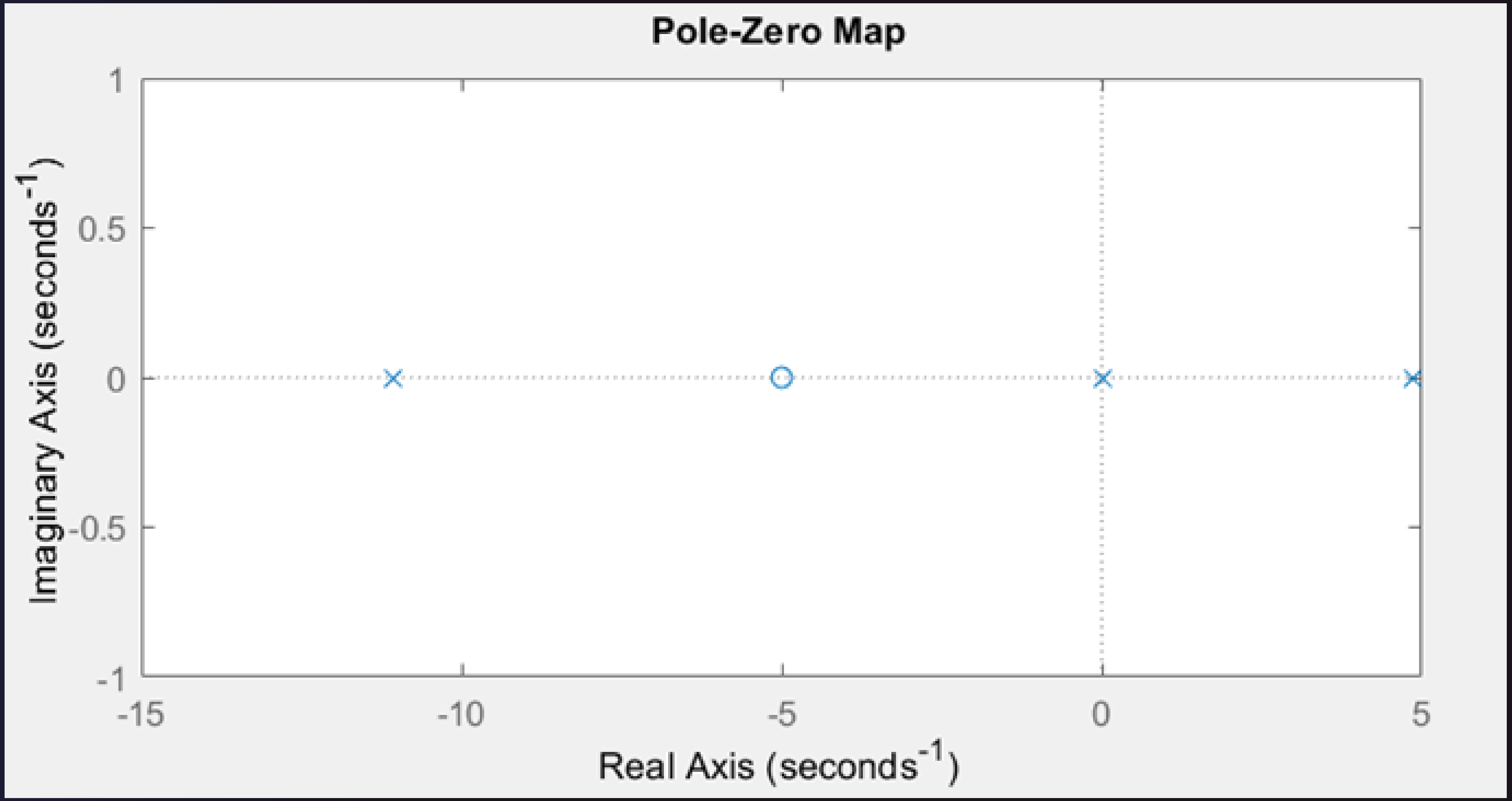
```
open_loop =

          60 s + 300
  --------------------------
  s^3 + 6.2 s^2 - 53.8 s + 1
```

DELIVERABLES

2. LOCATION OF POLE AND ZEROES OF OPEN LOOP SYSTEM
3. 'POLE-ZERO' MAP FOR THE OPEN LOOP SYETEM

Open Loop Poles and Zeroes With Pole-Zero Map

```
Open-loop poles:
  -11.0687
    4.8501
    0.0186

Open-loop zeroes:
    -5
```

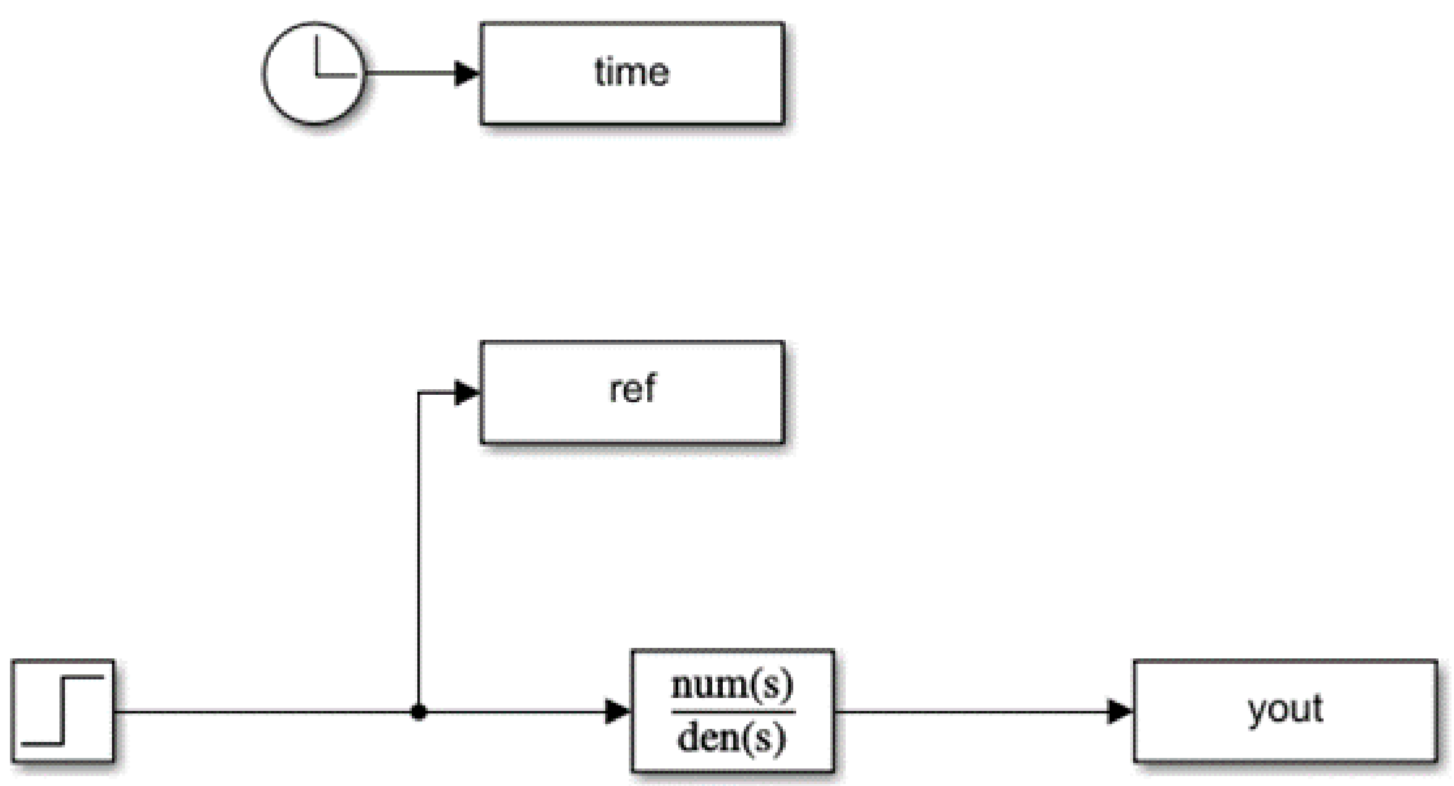


DELIVERABLES

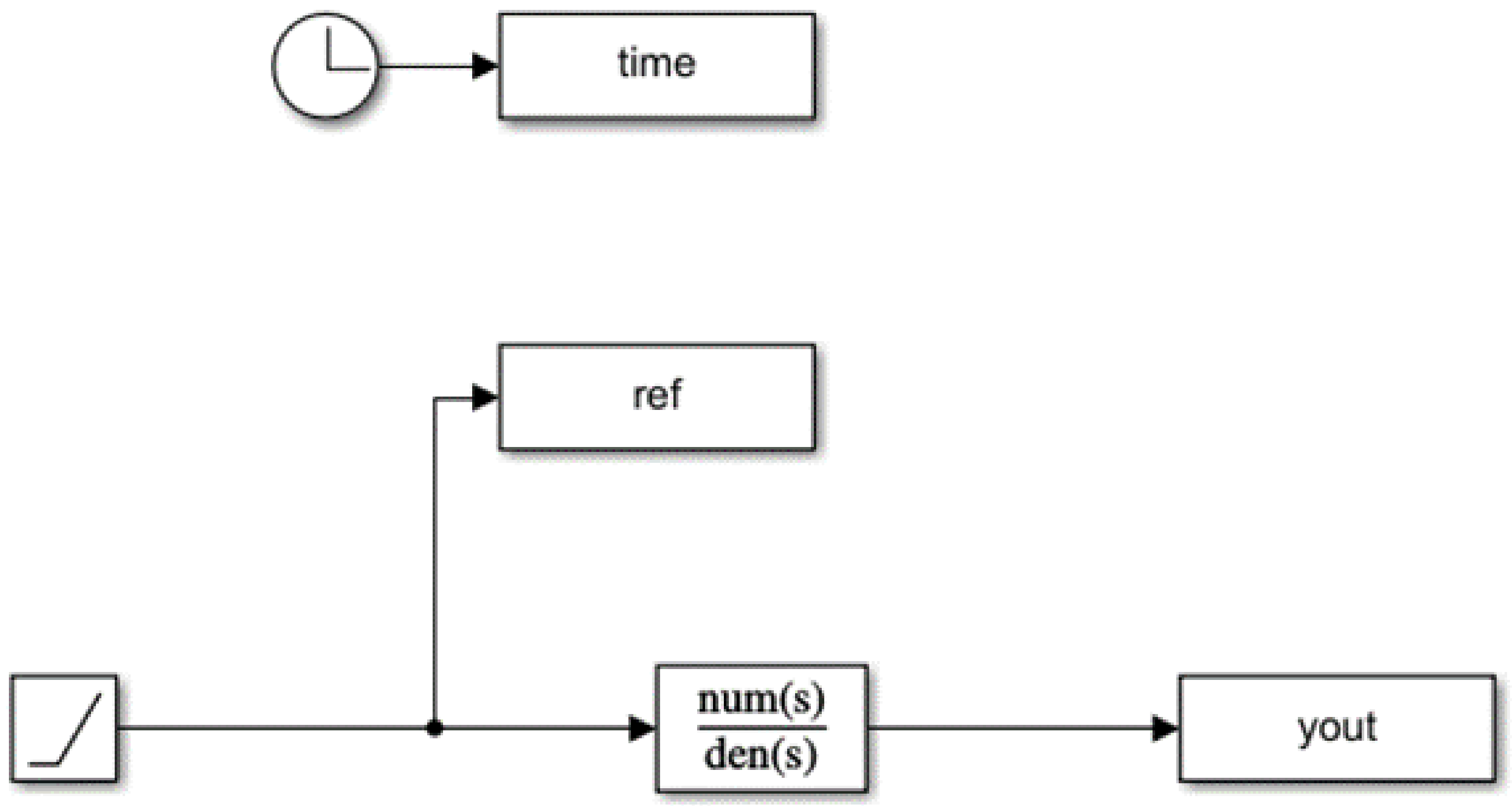
4. OBTAIN THE OPEN LOOP RESPONSE FOR (A) UNIT STEP INPUT, (B) UNIT RAMP INPUT. DISCUSS THE RESULTS WHILE REFFERRING TO THE POLE- ZERO MAP.COMPUTE THE RISE TIME, MAXIMUM PEAK,PEAL OVERSHOOT,SETTLING TIME AND STEADY STATE ERROR FROM THE STEP RESPONSE GRAPHS

UNIT STEP AND RAMP INPUT FOR OL SYSTEM

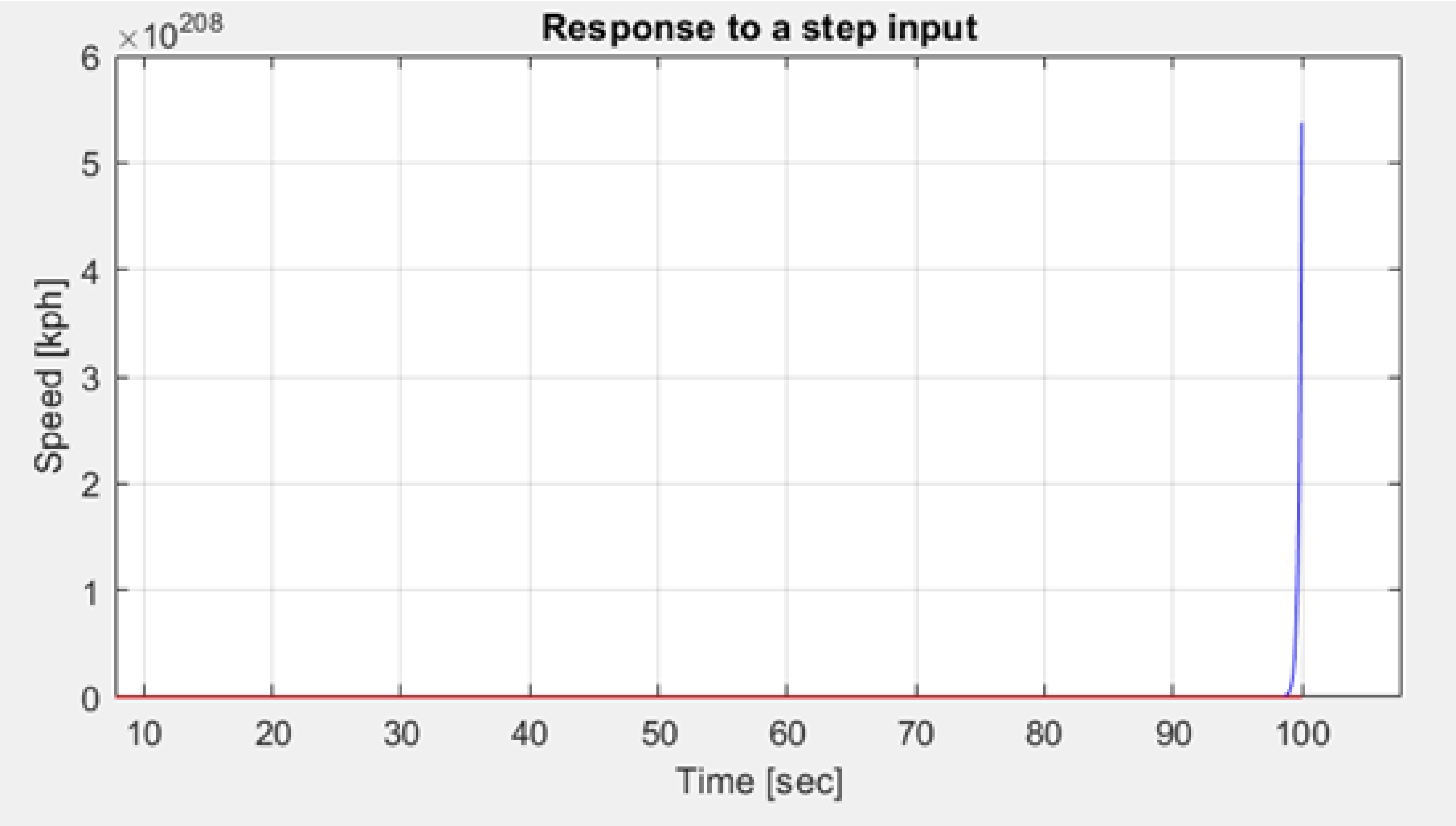
Unit Step

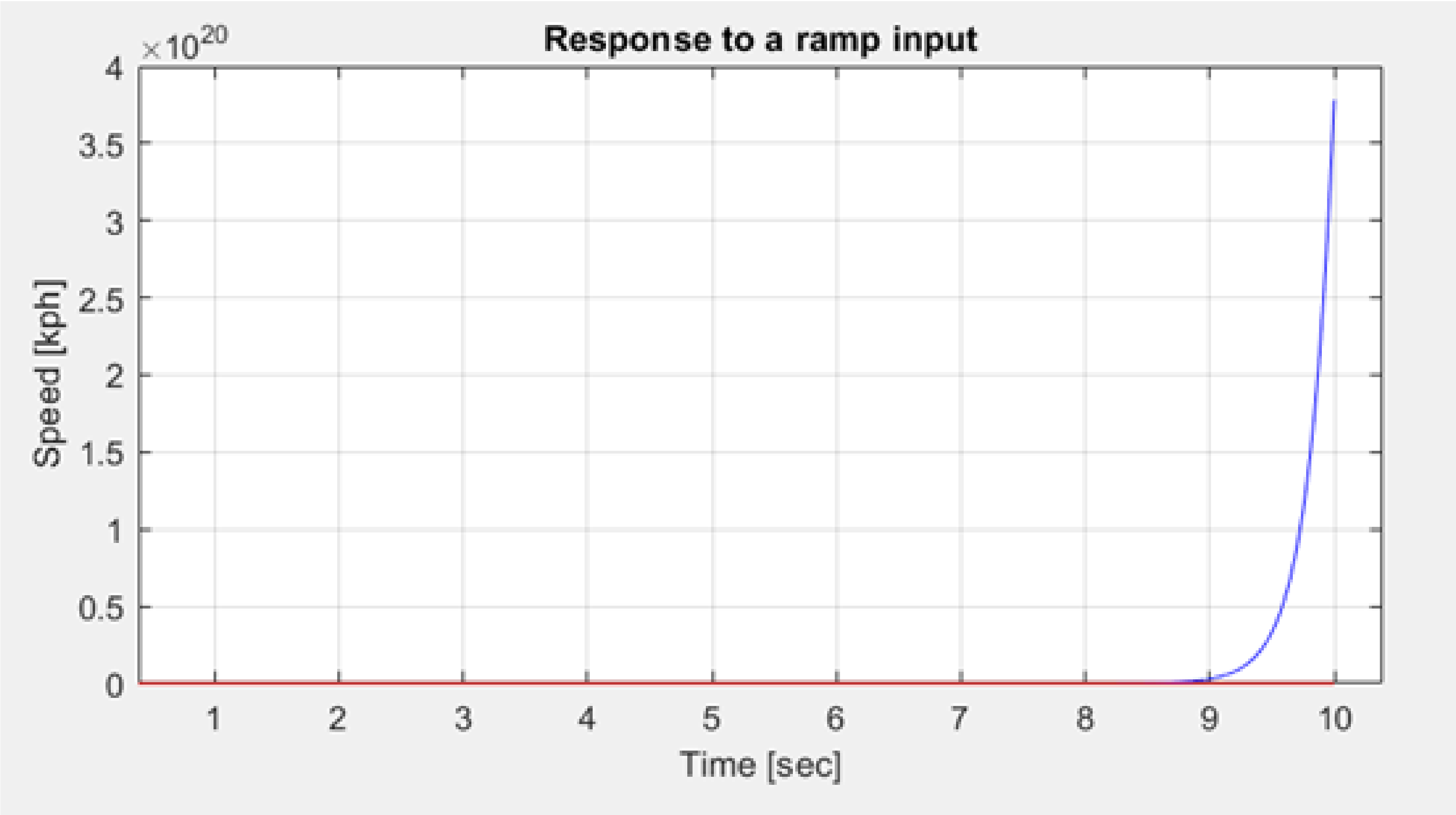


Unit Ramp



UNIT STEP AND RAMP INPUT FOR OL SYSTEM





ANALYSIS :

```
        RiseTime: 4.5272
   TransientTime: 999.9480
    SettlingTime: 999.9480
     SettlingMin: 6.8593e+210
     SettlingMax: 6.8593e+210
       Overshoot: 0
      Undershoot: 0
            Peak: 6.8593e+210
        PeakTime: 1000
```

RESPONSE CHARACTERISTICS

We observe that the open loop step response does not settle to a value

ANALYSIS :

RESPONSE CHARACTERISTICS

The open loop system is not stable as the output does not settle to a value and it keeps increasing for both step and ramp inputs

DELIVERABLES

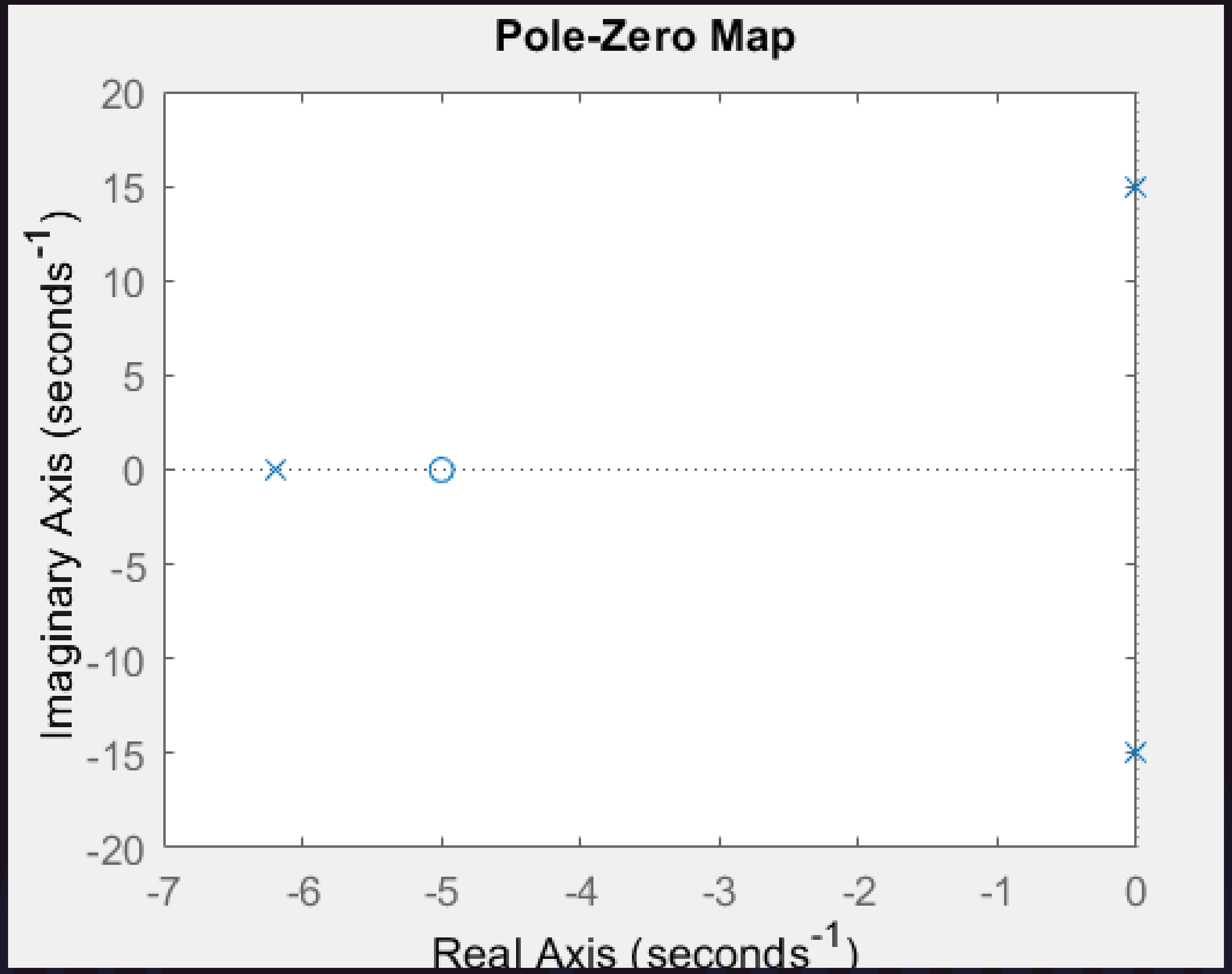
5.ASSUME NOW THE SYSTEM TO BE PART OF UNITY NEGATIVE FEEDBACK SYSTEM. ALSO CONSIDER GAIN K KEPT IN CASCADE TO THE GIVEN SYSTEM. EVALUTE THE RANGE OF K FOR WHICH THE CLOSED LOOP SYATEM IS STABLE FOR A UNIT STEP INPUT. COMPUTE PERFORMACE SPECIFICATIONS. WHAT IS THE VALUE OF K WHEN THE OUTPUT STARTS OSCILLATING? WHAT IS THE FREQUENCY OF OSCILLATIONS?

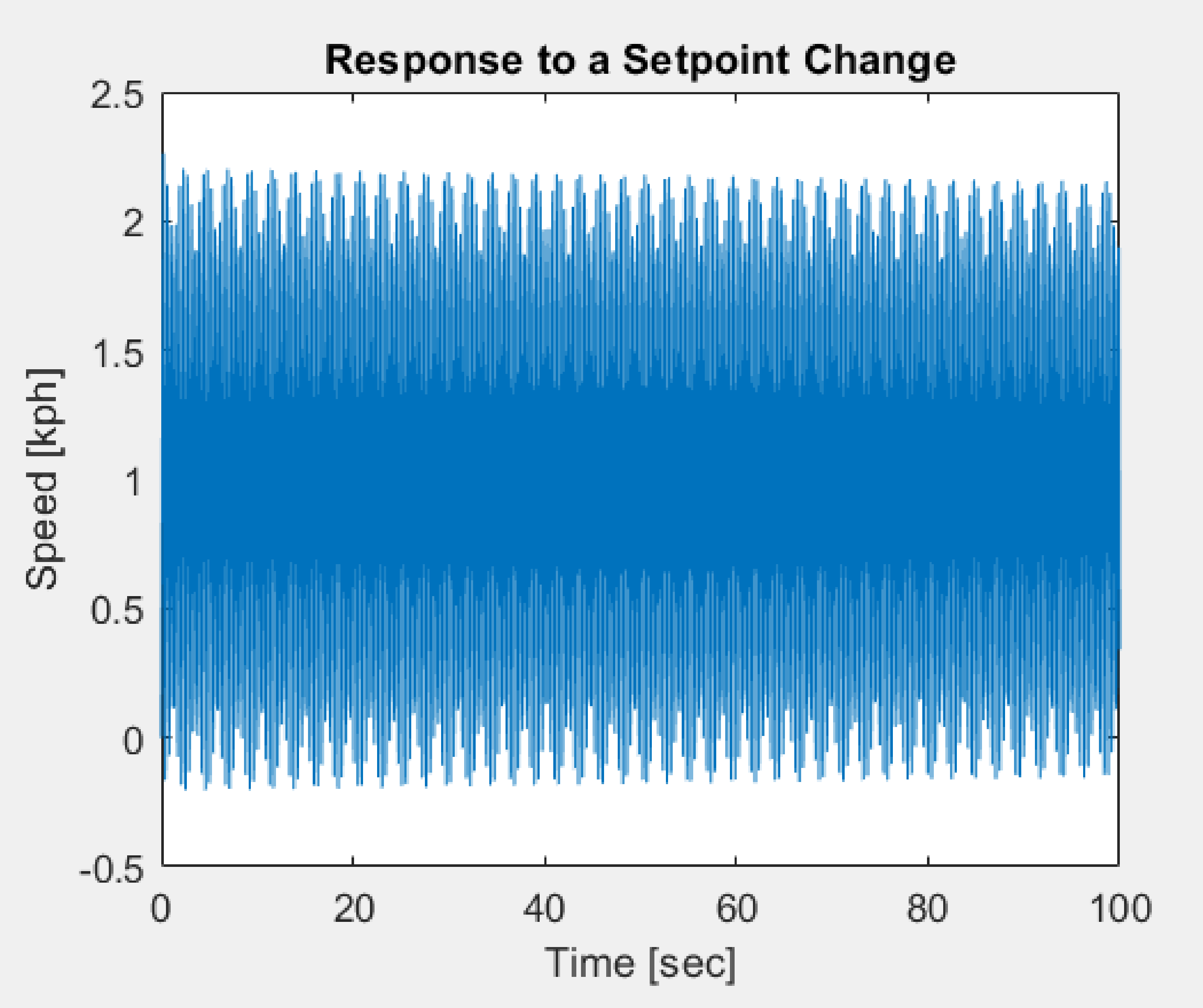
The closed loop transfer function without any gain is given by:

```
closed_loop =

          60 s + 300
  ----------------------------
  s^3 + 6.2 s^2 + 6.2 s + 301
```

Range of K for stability:

By Routh-Hurwitz criterion,
K > 4.65
for stability





K VALUE FOR OSCILLATIONS:

To find the value of K at which the output starts oscillating, we need to find the value of K at which the closed-loop system becomes marginally stable. Therefore, the value of K for which the output starts oscillating is K = 4.65. For K < 4.65, the closed loop system becomes unstable, and for K > 4.65, the system remains stable but without oscillations. The frequency of oscillations is 15 rad/sec

Routh-Hurwitz criterion:-

The characteristic equation, with gain k is given by:

$$s^3 + 6.2s^2 + (60k - 53.8)s + (300k+1) = 0.$$

s^3	1	$60k-53.8$
s^2	6.2	$300k+1$
s^1	b_1	
s^0	$300k+1$	

$$b_1 = \frac{-1}{6.2}\begin{vmatrix} 1 & 60k-53.8 \\ 6.2 & 300k+1 \end{vmatrix}$$

$$0 = \frac{-1}{6.2}\left[300k+1 - [6.2(60k-53.8)]\right]$$

$$0 = \frac{-1}{6.2}\left[300k - 372k + 1 + 333.56\right]$$

$$0 = +11.6k - \cancel{334.56}\ 53.96$$

$k = 4.65$ for marginally stable.

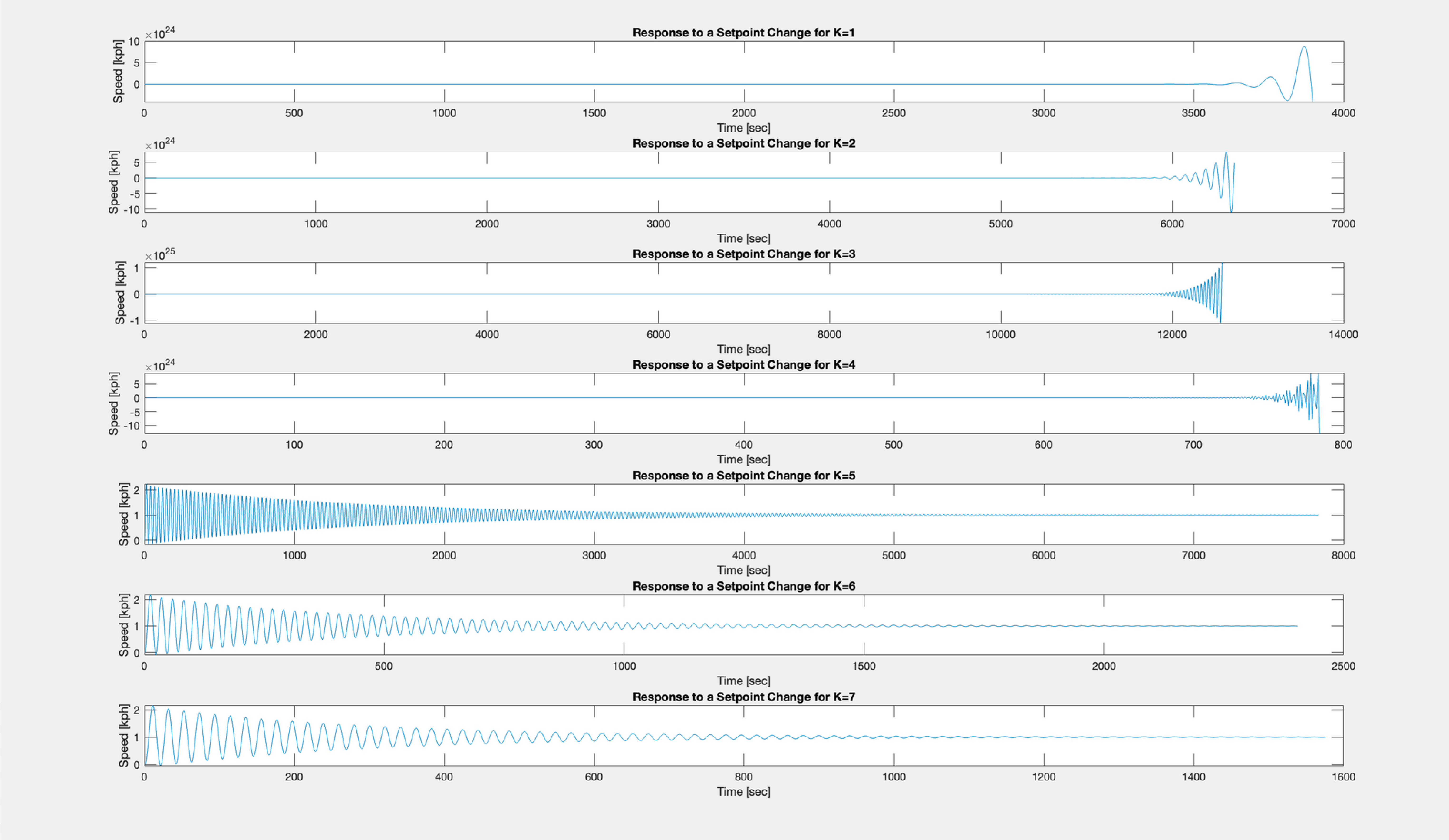
$\therefore$ $k > 0$, $k > 4.65$

$\therefore$ For stable system, $k > 4.65$

$(s_{1,2} = \pm 15j)$

At $k = 4.65$, Poles are at $(0 + 15j)$ and $(0 - 15j)$. $\therefore$ The frequency of oscillations is ~~15~~ 15 rad/sec.

STEP RESPONSE FOR DIFFERENT VALUES OF K



K=1
RiseTime: 12.0789
TransientTime: 999.9363
SettlingTime: 999.9590
SettlingMin: 5.3666e+63
SettlingMax: 5.3666e+63
Overshoot: 0
Undershoot: 55.1862
Peak: 5.3666e+63
PeakTime: 1000

K=2
RiseTime: 28.7734
TransientTime: 999.9236
SettlingTime: 999.9556
SettlingMin: -1.0832e+33
SettlingMax: -1.0832e+33
Overshoot: 0
Undershoot: 72.0340
Peak: 1.0832e+33
PeakTime: 1000

K=3
RiseTime: 65.8735
TransientTime: 999.9028
SettlingTime: 999.9495
SettlingMin: -8.4941e+14
SettlingMax: -8.4941e+14
Overshoot: 0
Undershoot: 92.4649
Peak: 8.4941e+14
PeakTime: 1000

K=4
RiseTime: 219.9399
TransientTime: 999.9641
SettlingTime: 999.9823
SettlingMin: -3.2897e+04
SettlingMax: 3.3937e+04
Overshoot: 0
Undershoot: 103.1611
Peak: 3.3937e+04
PeakTime: 998

K=5
RiseTime: 0.6902
TransientTime: 998.1613
SettlingTime: 998.6122
SettlingMin: -0.1556
SettlingMax: 2.2381
Overshoot: 121.0895
Undershoot: 15.3680
Peak: 2.2381
PeakTime: 3

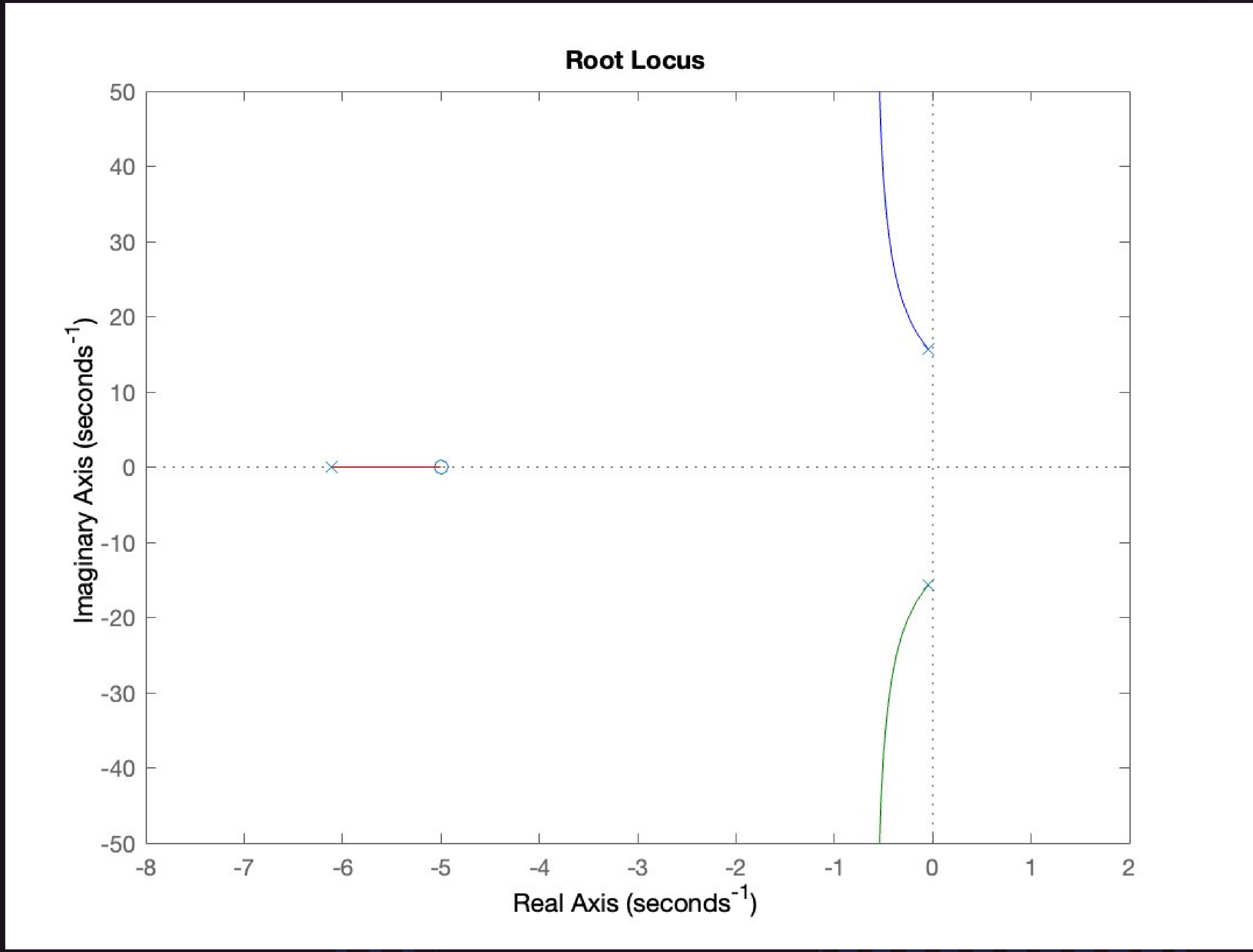
K=6
RiseTime: 0.5983
TransientTime: 278.0081
SettlingTime: 278.0788
SettlingMin: -0.0052
SettlingMax: 2.0964
Overshoot: 109.7613
Undershoot: 0.5172
Peak: 2.0964
PeakTime: 3

K=7
RiseTime: 0.5406
TransientTime: 187.0463
SettlingTime: 187.0624
SettlingMin: 0.0991
SettlingMax: 2.0242
Overshoot: 102.5154
Undershoot: 0
Peak: 2.0242
PeakTime: 6

DELIVERABLES

6. OBTAIN THE ROOT LOCUS FOR THE GIVEN SYSTEM WITH GAIN K IN CASCADE. (verifying the obatined analytical results for the range of K for which the given system is stable through root locus.)

Root Locus



ANALYSIS

For K= 5, the closed loop poles lie on the LHP of the s-plane and hence the system is stable for K>4.65

DELIVERABLES

7.DIFFERENCES IN THE BEHAVIOUR OF SYSTEM WHEN IT IS CHNAGED FROM OPEN LOOP TO CLOSED LOOP

DIFFERENCES BETWEEN CLOSED LOOP AND OPEN LOOP SYSTEM

OPEN LOOP

Open-loop systems tend to be simple and inexpensive as they do not provide feedback from the machine movement to the controller. In other words, open-loop systems act solely on the basis of the input and do not use feedback from the output to self-correct while the test is running. Thus, the test procedure entered into an open-loop controller may vary due to external disturbances, like noise.

CLOSED LOOP

closed-loop control systems have a feedback loop that continuously sends information from the closed-loop controller to the plant and also from the plant to the closed-loop controller. This constant feedback allows certain variables to remain as specified throughout the tests. Closed-loop systems provide higher accuracy due to the ability to react immediately to possible changes.

DIFFERENCES BETWEEN CLOSED LOOP AND OPEN LOOP SYSTEM IN THIS CASE

OPEN LOOP

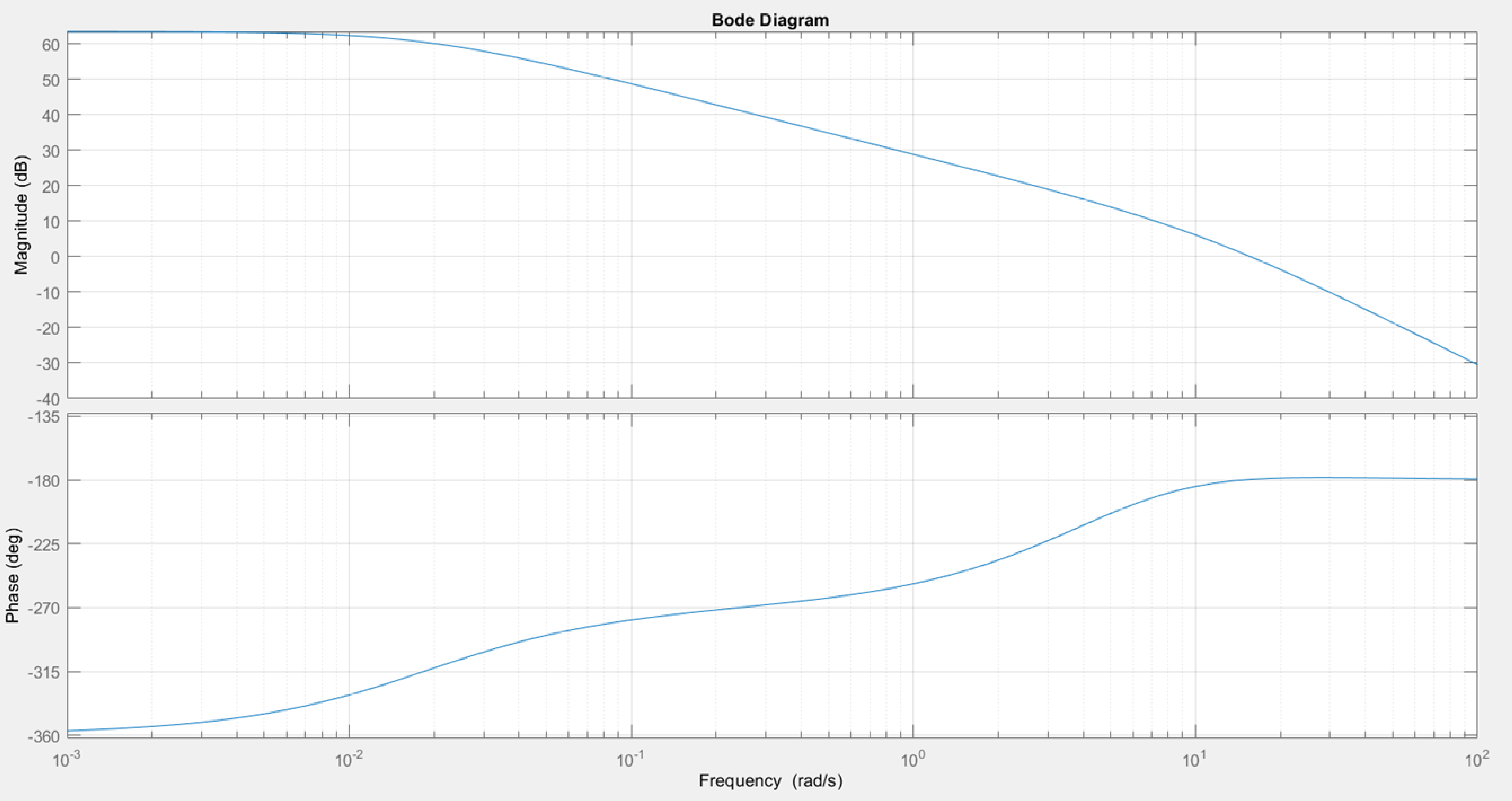
The system is unstable
The step response does not settle
Two of the poles lie on the RHP of the s plane

CLOSED LOOP

The system is stable for a gain of K>4.65
The step response is stable and it settles to a finite value
All the poles lie on the LHP for K>4.65

DELIVERABLES

8. OBTAIN THE BODE PLOT FOR OPEN LOOP TRANSFER FUNCTION OF YOUR SYSTEM CHOOSING K FROM THE OBATINED RANGE.
WHAT ARE GM AND PM? IS THE CLOSED LOOP SYSTEM STABLE? DOES IT MATCH THE INFERENCE ON RANGE OF K FOR STABILITY




```
Phase Margin: 0.28 degrees
Gain Margin: -0.64 dB
```

ANALYSIS

From the Bode Plot:

Phase Margin: 0.28 degrees

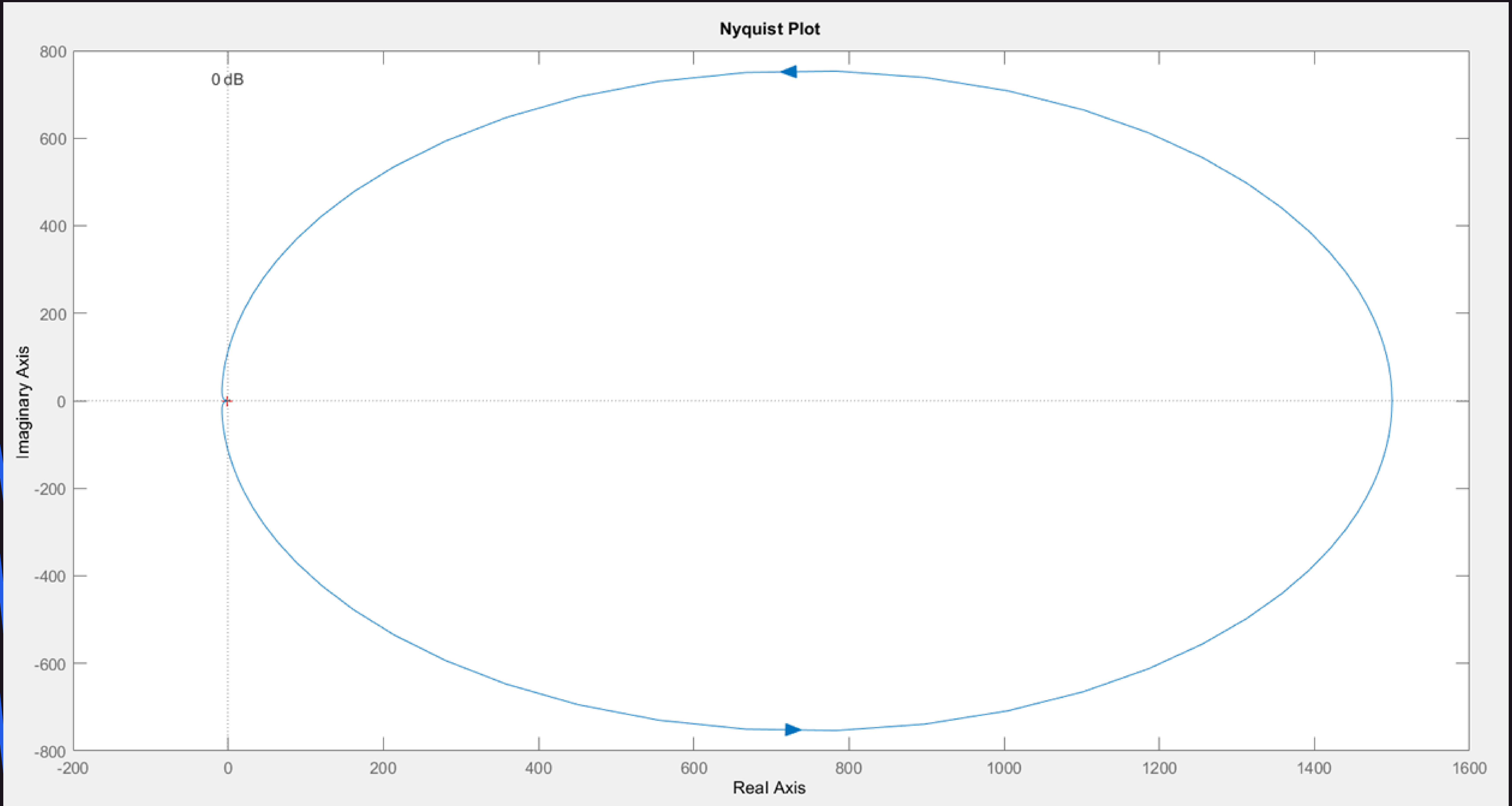
Gain Margin: -0.64 dB

Hence, PM is positive, and it is inferred that the system is stable

This matches with the inference we have drawn previously that the closed loop system is stable

DELIVERABLES

9. OBTAIN NYQUIST PLOT AND COMMENT ON SAME QUESTIONS AS FOR BODE PLOT



Nyquist Plot
0 dB
Imaginary Axis
Real Axis
800
600
400
200
0
-200
-400
-600
-800
-200
0
200
400
600
800
1000
1200
1400
1600

ANALYSIS

From the Nyquist Plot :

The number of encircements of the point -1+j0 is 0 times(N=0), number of poles on the RHP is 0 (P=0), and hence Z= N+P = 0

Therefore, there are no poles on the RHP for the closed loop TF.

This matches with the inference we have drawn previously that the closed loop system is stable, and matches with the stability inference from the Bode plot

DELIVERABLES

DESIGN OF CONTROLLERS

Design Specifications:

Damping co-efficient = 0.8, settling time = 2 seconds

P CONTROLLER

Design of P controller with the given design specifications is not possible for the chosen transfer function

P Controller:

$$1 + K_1\left(\frac{60s+300}{s^3+6.2s^2 \cancel{+6.2s} - 53.8s+1}\right)=0$$

$$s^3+6.2s^2-53.8s+1+K_1(60s+300)=0 \rightarrow (1).$$

$$\frac{4}{\zeta\omega_n}=2 \; ; \; \zeta=0.8$$

$$\Rightarrow \omega_n = 2.5.$$

from this,

$$(s^2+4s+6.25)(s+a)$$

$$a>20$$

$$\therefore (s^2+4s+6.25)(s+20)$$

$$= s^3+24s^2+86.25s+125 \rightarrow (2)$$

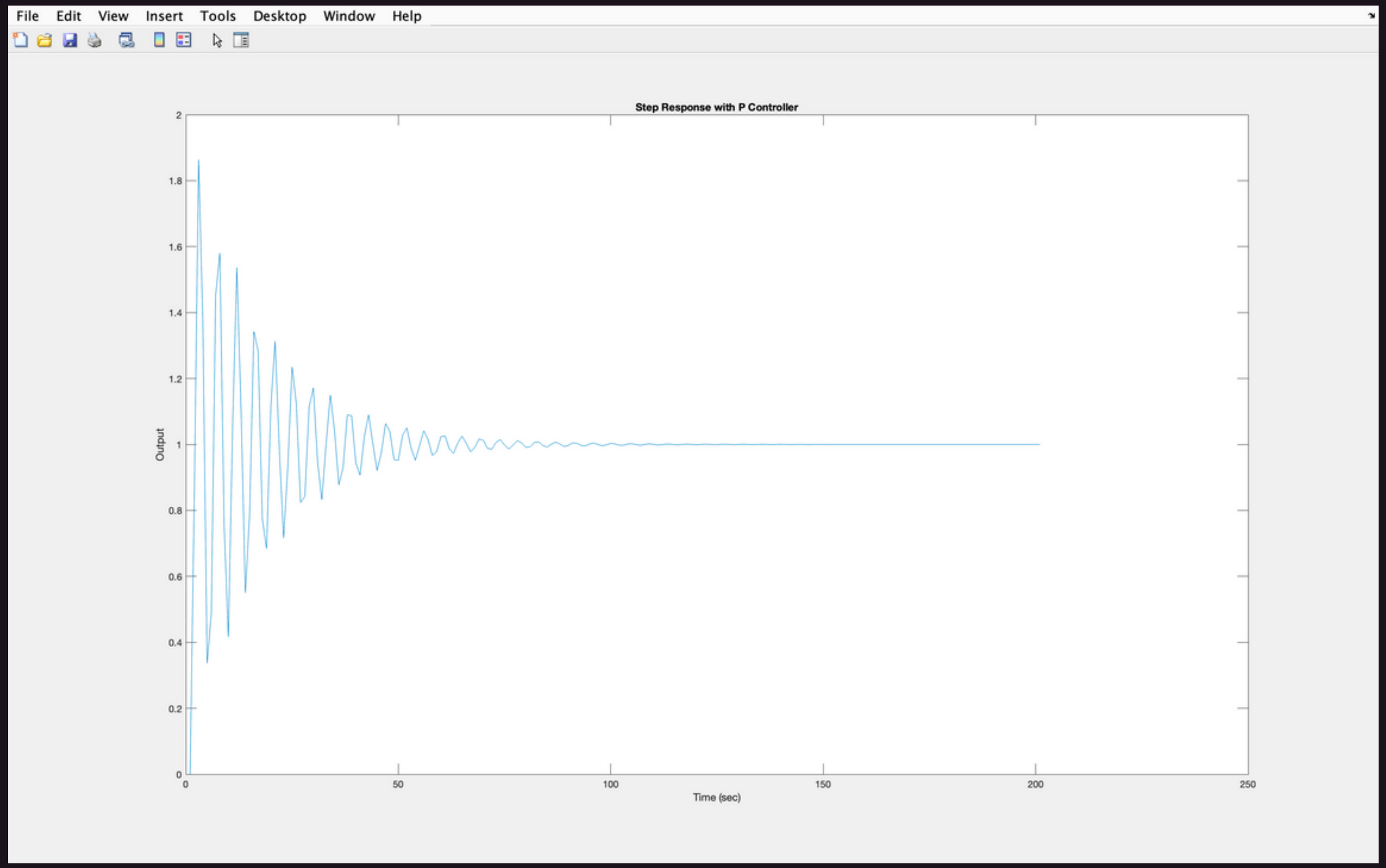
from (1), $s^3+6.25s^2-(53.8+60K_1)s+(1+300K_1)=0 \rightarrow (3)$

Comparing (2) & (3) we find that no value of K_1 satisfies the necessary conditions. $T_s \neq 2s$.

$\therefore$ No P-Controller is possible for this plant.

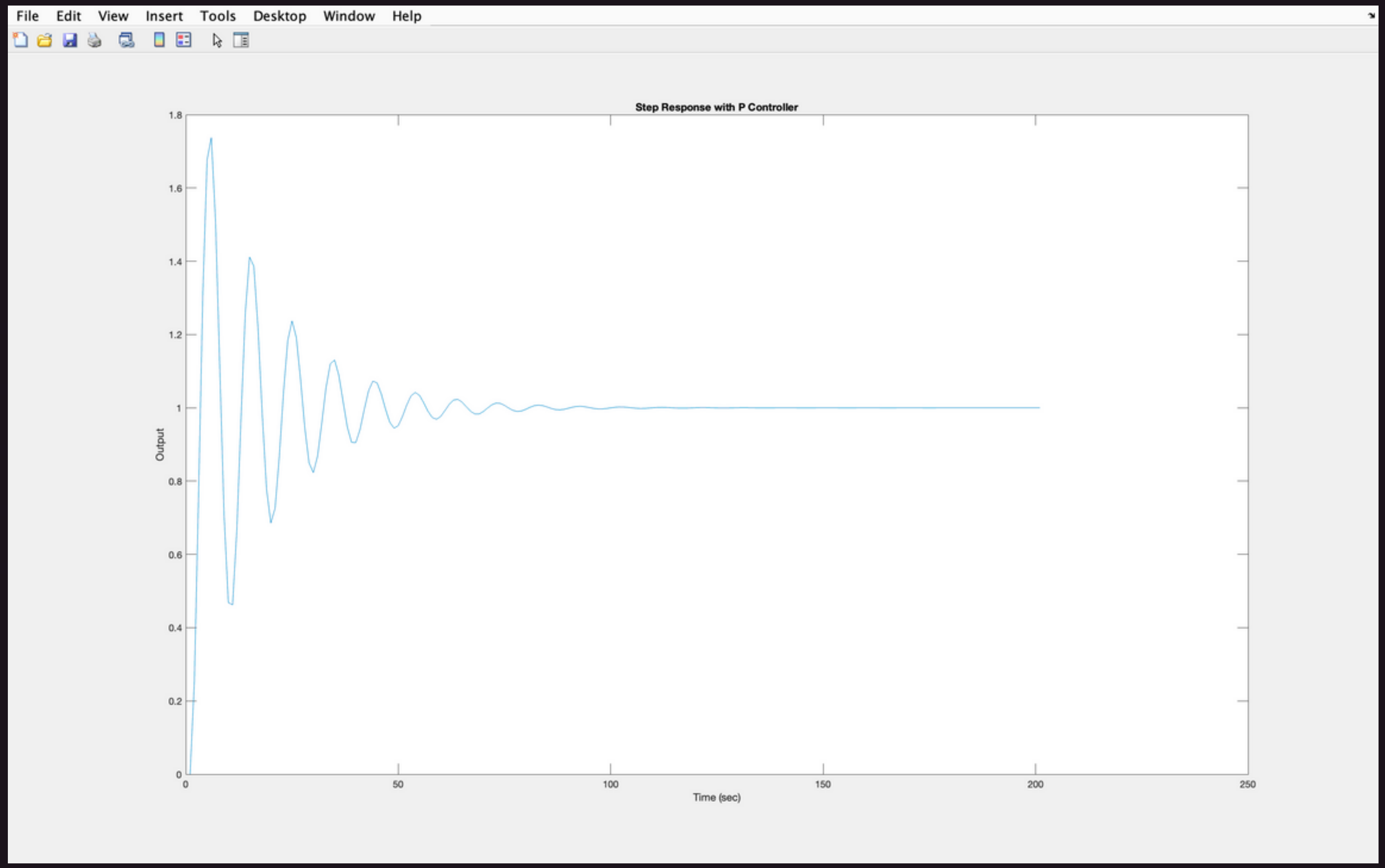
K = 100

```
ans =

  struct with fields:

         RiseTime: 0.9235
    TransientTime: 67.1447
     SettlingTime: 67.1447
      SettlingMin: 0.3362
      SettlingMax: 1.8636
        Overshoot: 86.3685
       Undershoot: 0
             Peak: 1.8636
         PeakTime: 3


         Pole              Damping       Frequency      Time Constant
                                       (rad/seconds)      (seconds)

 -5.05e+00                1.00e+00       5.05e+00         1.98e-01
 -5.75e-01 + 7.71e+01i    7.46e-03       7.71e+01         1.74e+00
 -5.75e-01 - 7.71e+01i    7.46e-03       7.71e+01         1.74e+00
fx >>
```



K = 1000

```
ans =

  struct with fields:

         RiseTime: 1.8489
    TransientTime: 64.4142
     SettlingTime: 64.4142
      SettlingMin: 0.4626
      SettlingMax: 1.7380
        Overshoot: 73.7962
       Undershoot: 0
             Peak: 1.7380
         PeakTime: 6


         Pole              Damping       Frequency      Time Constant
                                       (rad/seconds)      (seconds)

 -5.01e+00                1.00e+00       5.01e+00         2.00e-01
 -5.97e-01 + 2.45e+02i    2.44e-03       2.45e+02         1.67e+00
 -5.97e-01 - 2.45e+02i    2.44e-03       2.45e+02         1.67e+00
fx >>
```



PI CONTROLLER

Design of PI controller with the given design specifications is not possible for the chosen transfer function

P.I Controller

$$1+\left(K_1+\frac{K_2}{s}\right)\left(\frac{60s+300}{s^3+6.2s^2-63.8s+1}\right)=0$$

$$1+\left(\frac{K_1 s+K_2}{s}\right)\left(\frac{60s+300}{s^3+6.2s^2-53.8s+1}\right)=0$$

$$s(s^3+6.2s^2-53.8s+1)+(60K_1s^2+300K_1s+60K_2s+300K_2)=0 \rightarrow (1)$$

from given conditions,

$$(s^3+24s^2+86.25s+125)(s+40)$$

$$= s^4+64s^3+1046.25s^2+3575s+5000 \rightarrow (2)$$

Comparing (1) and (2), we find that no value of K_1 and K_2 can satisfy the necessary criteria. $T_s \neq 2s$.

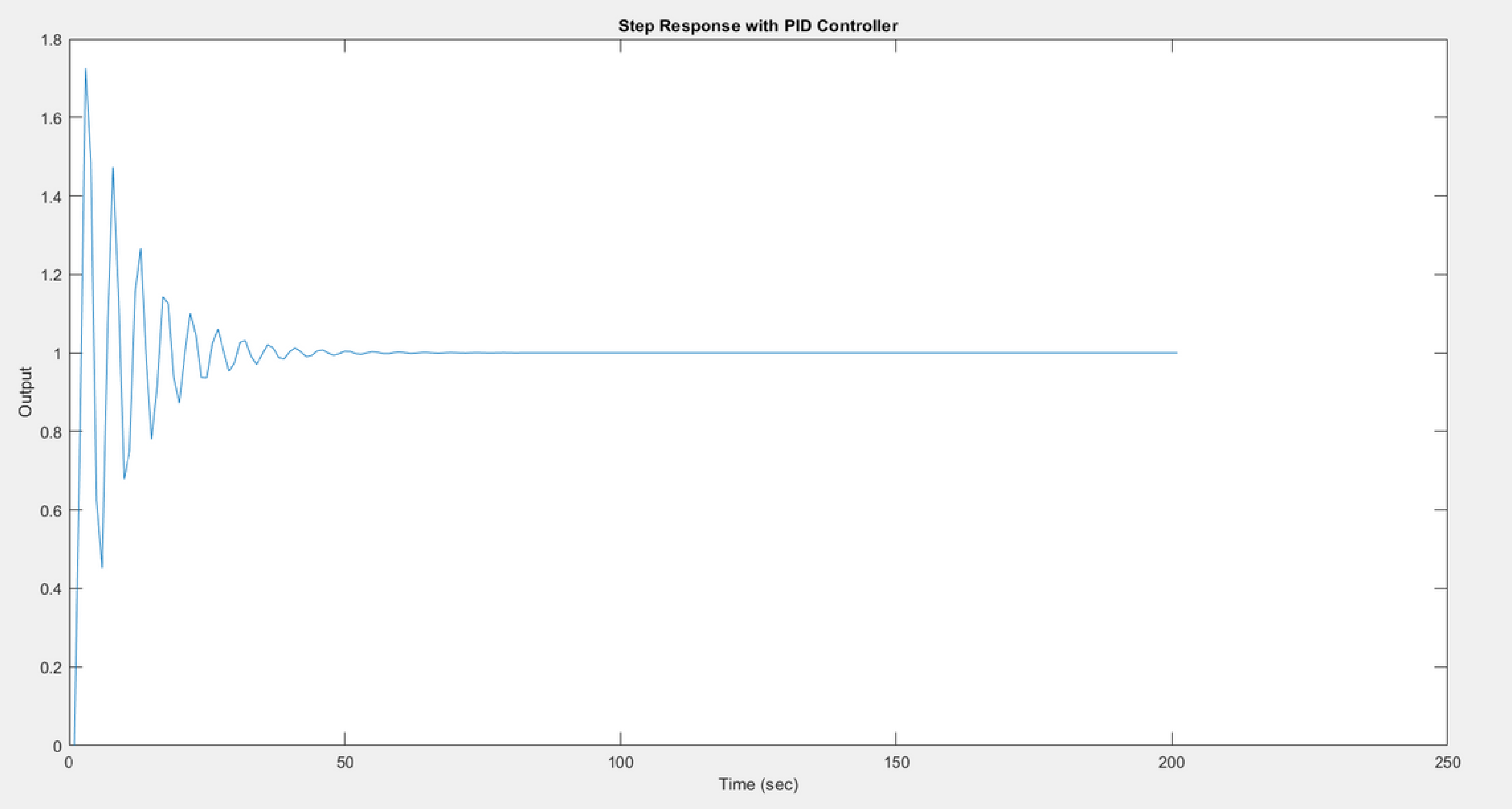
$\therefore$ P.I controller is not possible.

Kp = 100

Ki=2

RiseTime: 0.9969
TransientTime: 36.1363
SettlingTime: 36.1363
SettlingMin: 0.4523
SettlingMax: 1.7250
Overshoot: 72.5047
Undershoot: 0
Peak: 1.7250
PeakTime: 3

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-2.00e-02	1.00e+00	2.00e-02	5.00e+01
-3.89e+00	1.00e+00	3.89e+00	2.57e-01
-1.08e+00 + 7.62e+01i	1.41e-02	7.62e+01	9.29e-01
-1.08e+00 - 7.62e+01i	1.41e-02	7.62e+01	9.29e-01

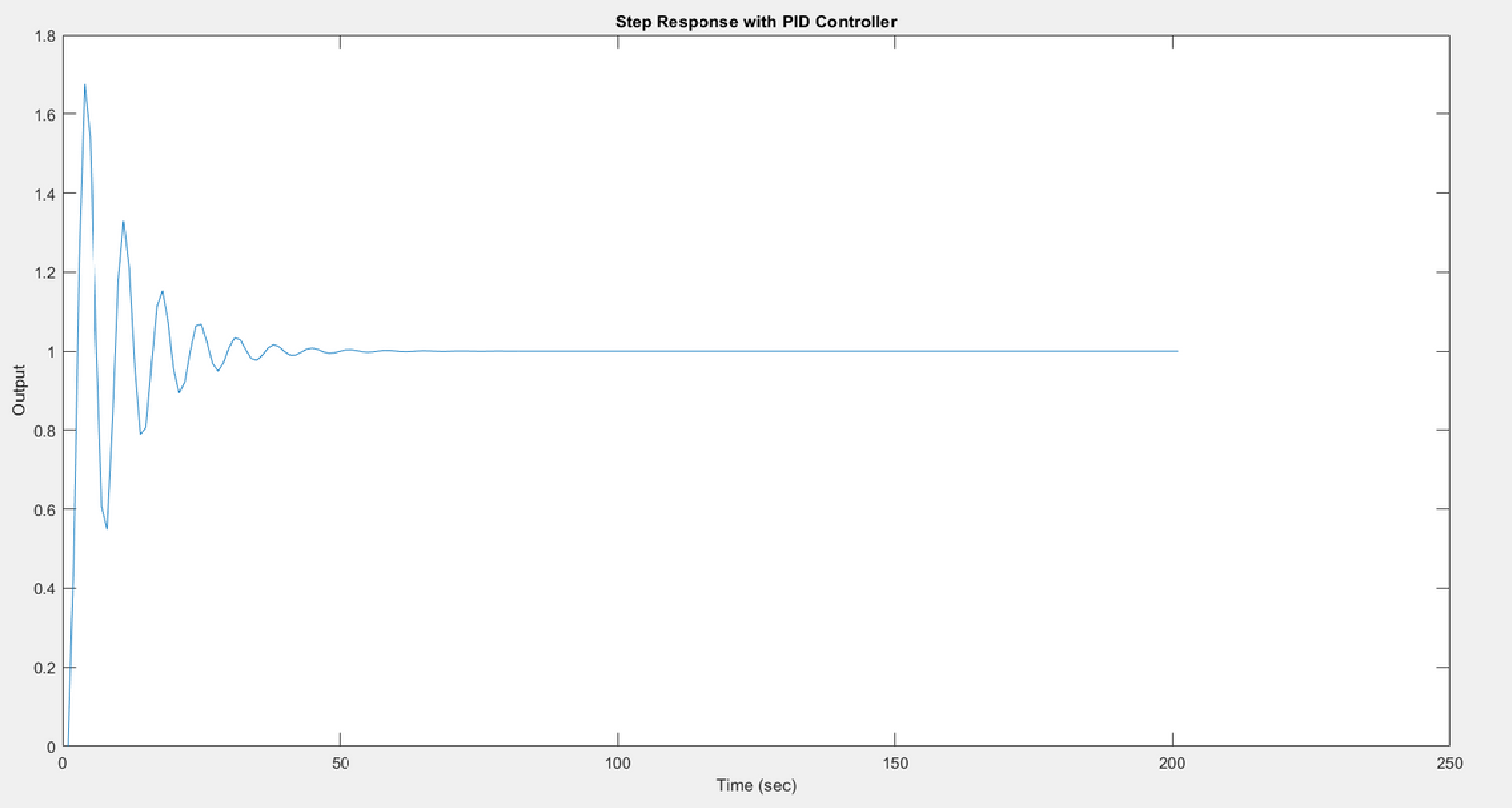


Kp = 1000

Ki=5

RiseTime: 1.3528
TransientTime: 35.2435
SettlingTime: 35.2435
SettlingMin: 0.5486
SettlingMax: 1.6755
Overshoot: 67.5529
Undershoot: 0
Peak: 1.6755
PeakTime: 4

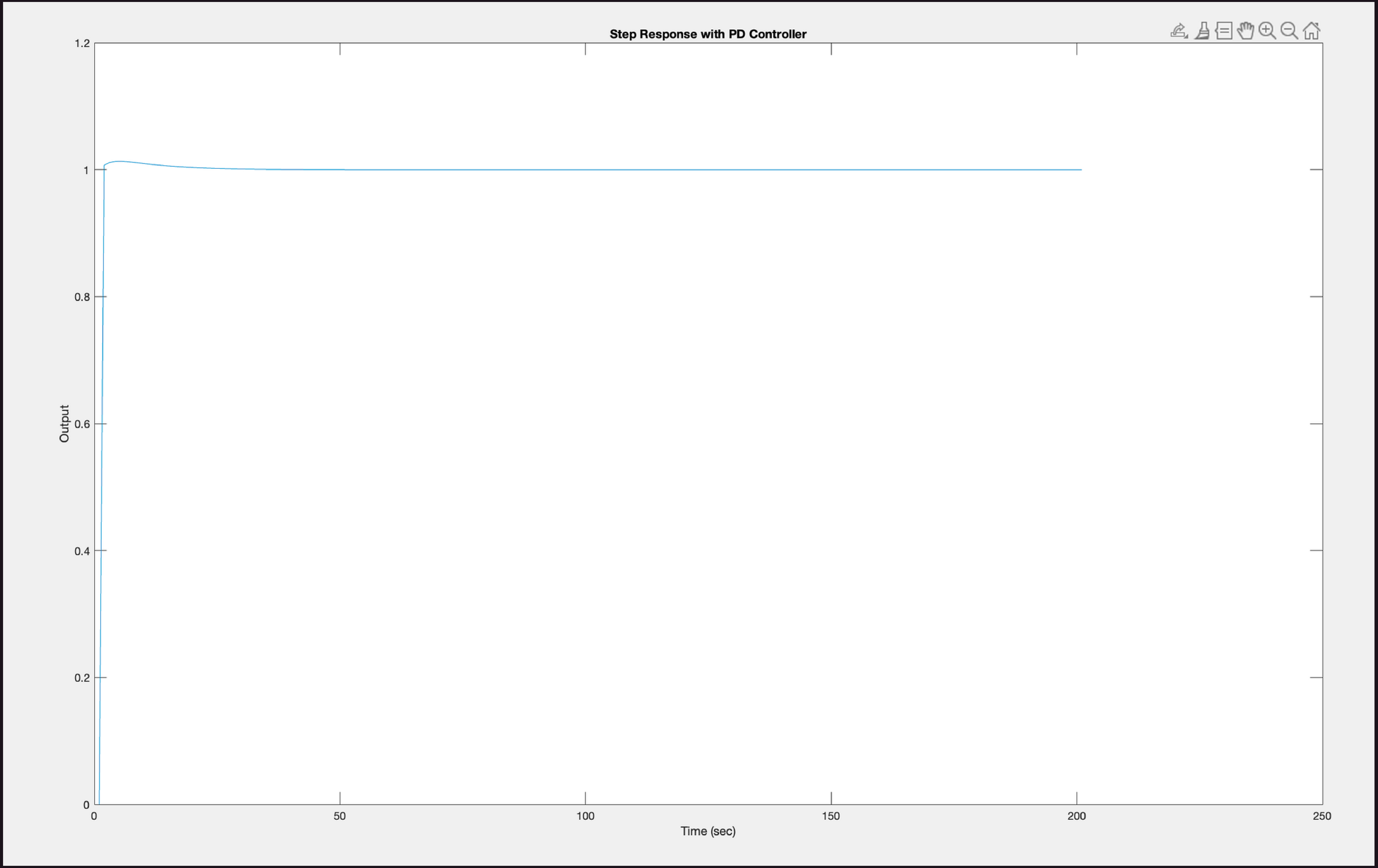
Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-5.00e-03	1.00e+00	5.00e-03	2.00e+02
-3.85e+00	1.00e+00	3.85e+00	2.60e-01
-1.10e+00 + 2.42e+02i	4.55e-03	2.42e+02	9.07e-01
-1.10e+00 - 2.42e+02i	4.55e-03	2.42e+02	9.07e-01



PD CONTROLLER

Design of PD controller with the given design specifications is possible for the chosen transfer function for values of K1 = 10, K2 = 10

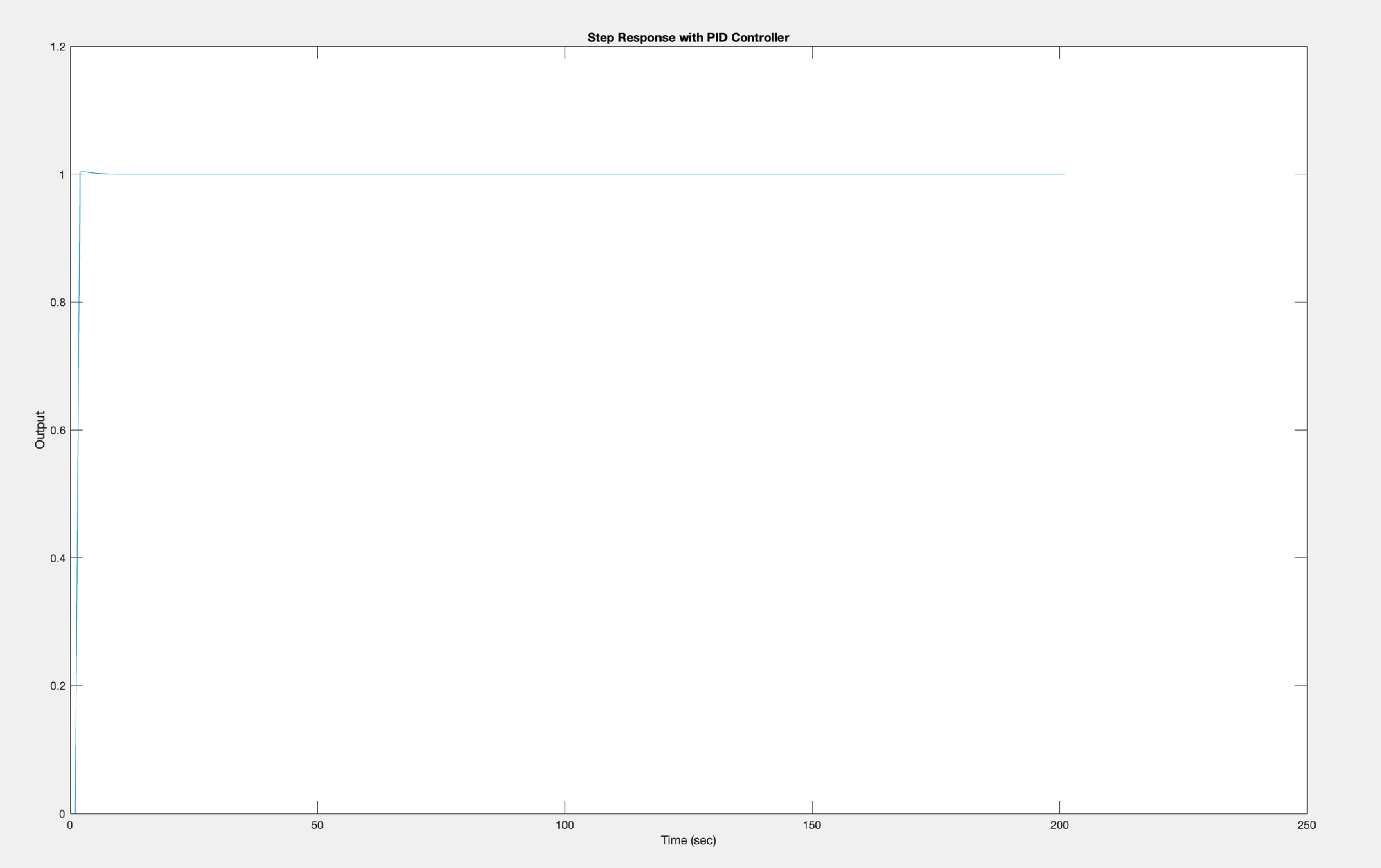
```
ans =

  struct with fields:

        RiseTime: 0.7942
   TransientTime: 1.9729
    SettlingTime: 1.9729
     SettlingMin: 0.9997
     SettlingMax: 1.0132
       Overshoot: 1.3535
      Undershoot: 0
            Peak: 1.0132
        PeakTime: 5


    Pole          Damping       Frequency      Time Constant
                               (rad/seconds)     (seconds)

 -1.03e+00       1.00e+00       1.03e+00        9.75e-01
 -4.87e+00       1.00e+00       4.87e+00        2.05e-01
 -6.00e+02       1.00e+00       6.00e+02        1.67e-03
>>
```


Step Response with PD Controller
Output
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250

PID CONTROLLER

Design of PID controller with the given design specifications is possible for the chosen transfer function for values of K1 = 100, K2 = 2, K3 = 5

```
ans =

  struct with fields:

         RiseTime: 0.7970
    TransientTime: 1.9764
     SettlingTime: 1.9764
      SettlingMin: 0.9999
      SettlingMax: 1.0040
        Overshoot: 0.4039
       Undershoot: 0
             Peak: 1.0040
         PeakTime: 3


         Pole                Damping       Frequency      Time Constant
                                         (rad/seconds)      (seconds)

 -5.00e-02                  1.00e+00       5.00e-02        2.00e+01
 -5.06e+00                  1.00e+00       5.06e+00        1.98e-01
 -6.05e+01 + 4.76e+01i      7.86e-01       7.70e+01        1.65e-02
 -6.05e+01 - 4.76e+01i      7.86e-01       7.70e+01        1.65e-02
>>
```


Step Response with PID Controller
Output
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250

Step Response of the Controller Function

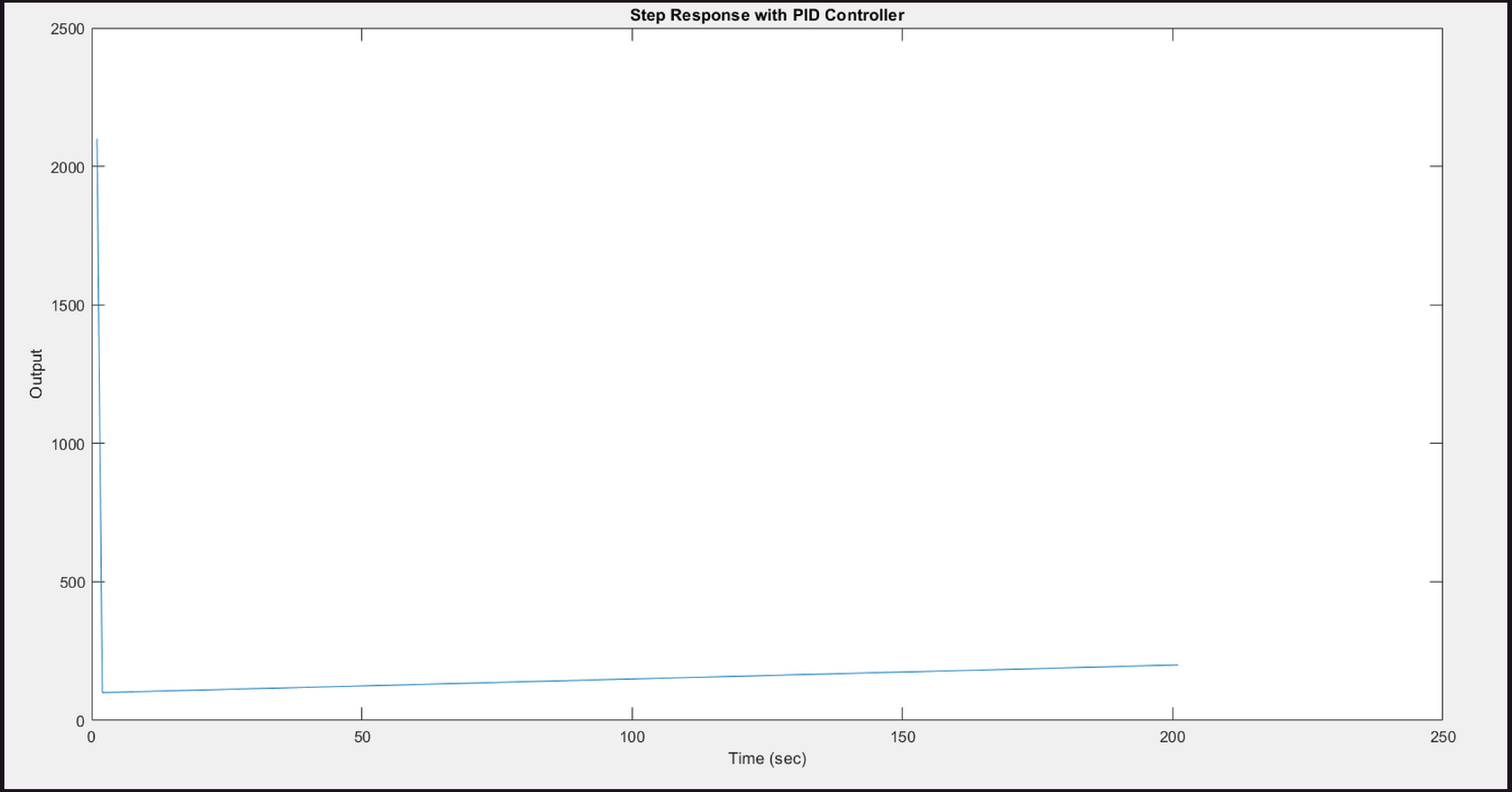
```
controller =

  2.1 s^2 + 100 s + 5
  -------------------
     0.001 s^2 + s
```

For :

Kp = 100

Ki = 5

Kd = 2



DELIVERABLES

ROBUSTNESS CHECK:

Having frozen the controller gains, change the co-efficients in the plant transfer function by + 30 % first and then by – 30 %. Simulate for a unit step input for both cases and verify as to whether the design specifications are met, while there is an uncertainty in the plant parameters.

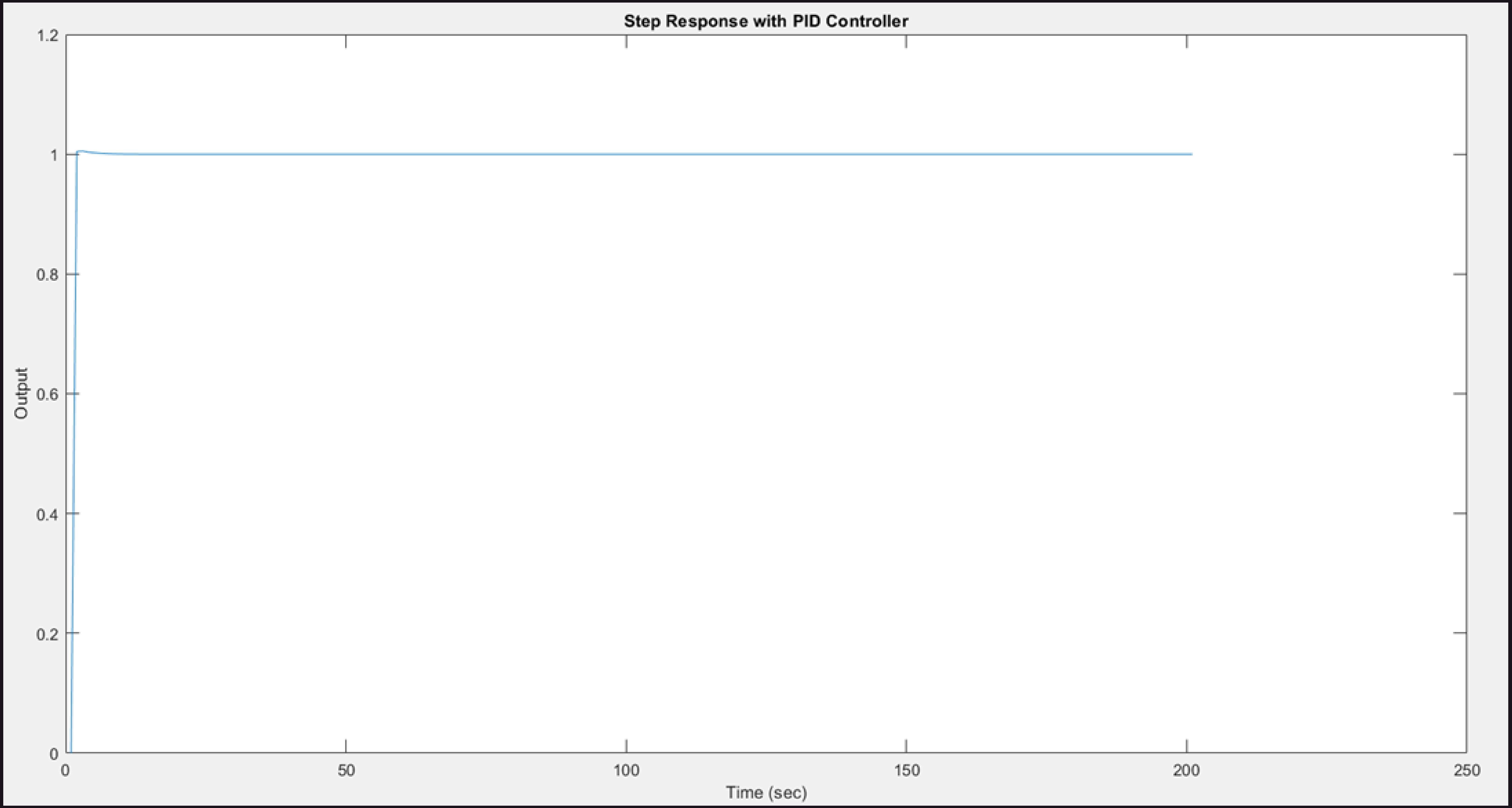
Increasing the coefficients by 30%
with K1 = 100, K2 = 2, K3 = 5,
we get the plant function as :

```
plant =

              78 s + 300
  ---------------------------------
  1.33 s^3 + 8.06 s^2 - 69.94 s + 1
```

```
        RiseTime: 0.7963
   TransientTime: 1.9754
    SettlingTime: 1.9754
     SettlingMin: 0.9999
     SettlingMax: 1.0052
       Overshoot: 0.5242
      Undershoot: 0
            Peak: 1.0052
        PeakTime: 3
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-5.00e-02	1.00e+00	5.00e-02	2.00e+01
-3.89e+00	1.00e+00	3.89e+00	2.57e-01
-5.97e+01 + 4.72e+01i	7.84e-01	7.61e+01	1.67e-02
-5.97e+01 - 4.72e+01i	7.84e-01	7.61e+01	1.67e-02

The Step response is:



Decreasing the coefficients by 30%
with K1 = 100, K2 = 2, K3 = 5,
we get the plant function as :

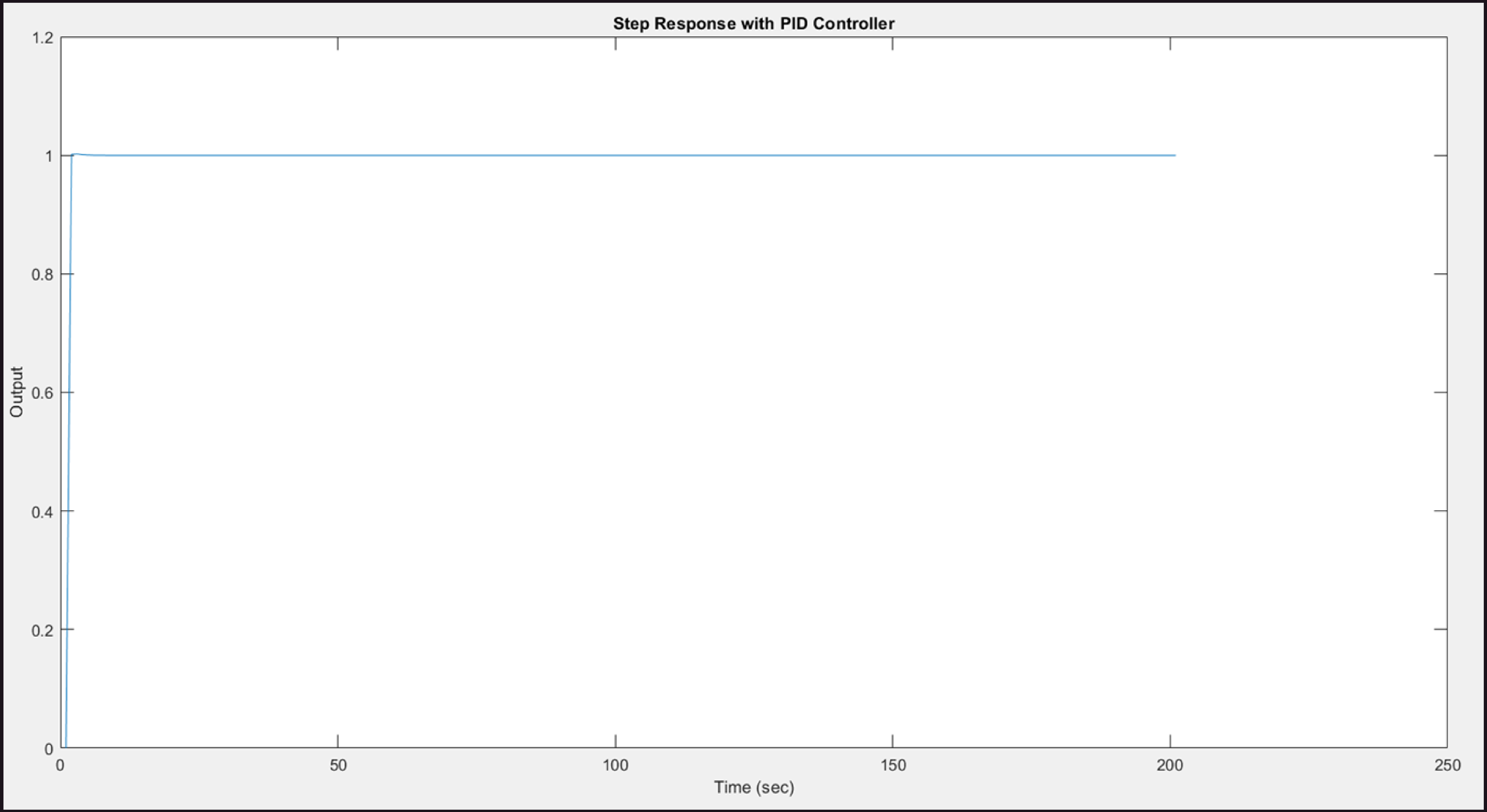
plant =

$$\frac{42\ s + 300}{0.67\ s^3 + 4.34\ s^2 - 37.66\ s + 1}$$

```
        RiseTime: 0.7985
   TransientTime: 1.9782
    SettlingTime: 1.9782
     SettlingMin: 0.9999
     SettlingMax: 1.0022
       Overshoot: 0.2214
      Undershoot: 0
            Peak: 1.0022
        PeakTime: 3
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-5.00e-02	1.00e+00	5.00e-02	2.00e+01
-7.21e+00	1.00e+00	7.21e+00	1.39e-01
-6.23e+01 + 4.82e+01i	7.91e-01	7.88e+01	1.61e-02
-6.23e+01 - 4.82e+01i	7.91e-01	7.88e+01	1.61e-02

The Step response is:



ANALYSIS

From the plots and data obtained :
We infer that the settling time is below 2 seconds in both cases and damping remains around 0.784 - 1, so the plant meets the desired specs of 2s even when there is uncertainty in the plant parameters.

Thank You