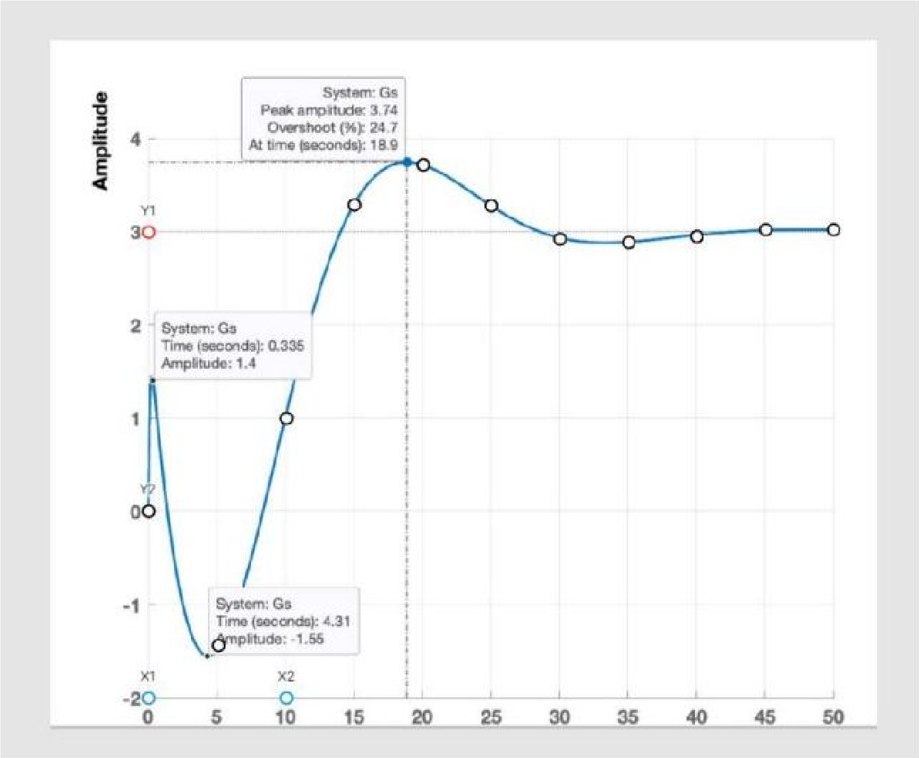
Q1 – Part a)

The general form of the required transfer function will be as given above. Data points are extracted from all the three given plots using an online Plot Digitizer as shown below for the step response

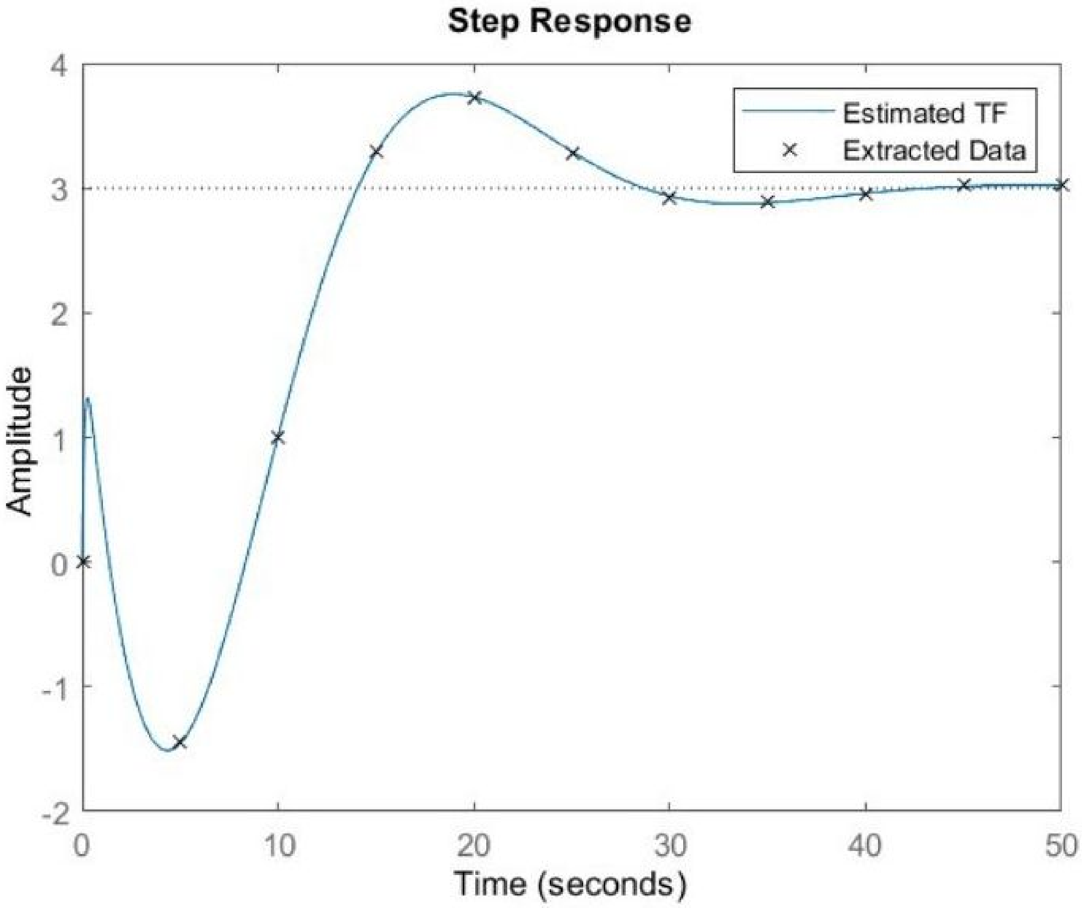


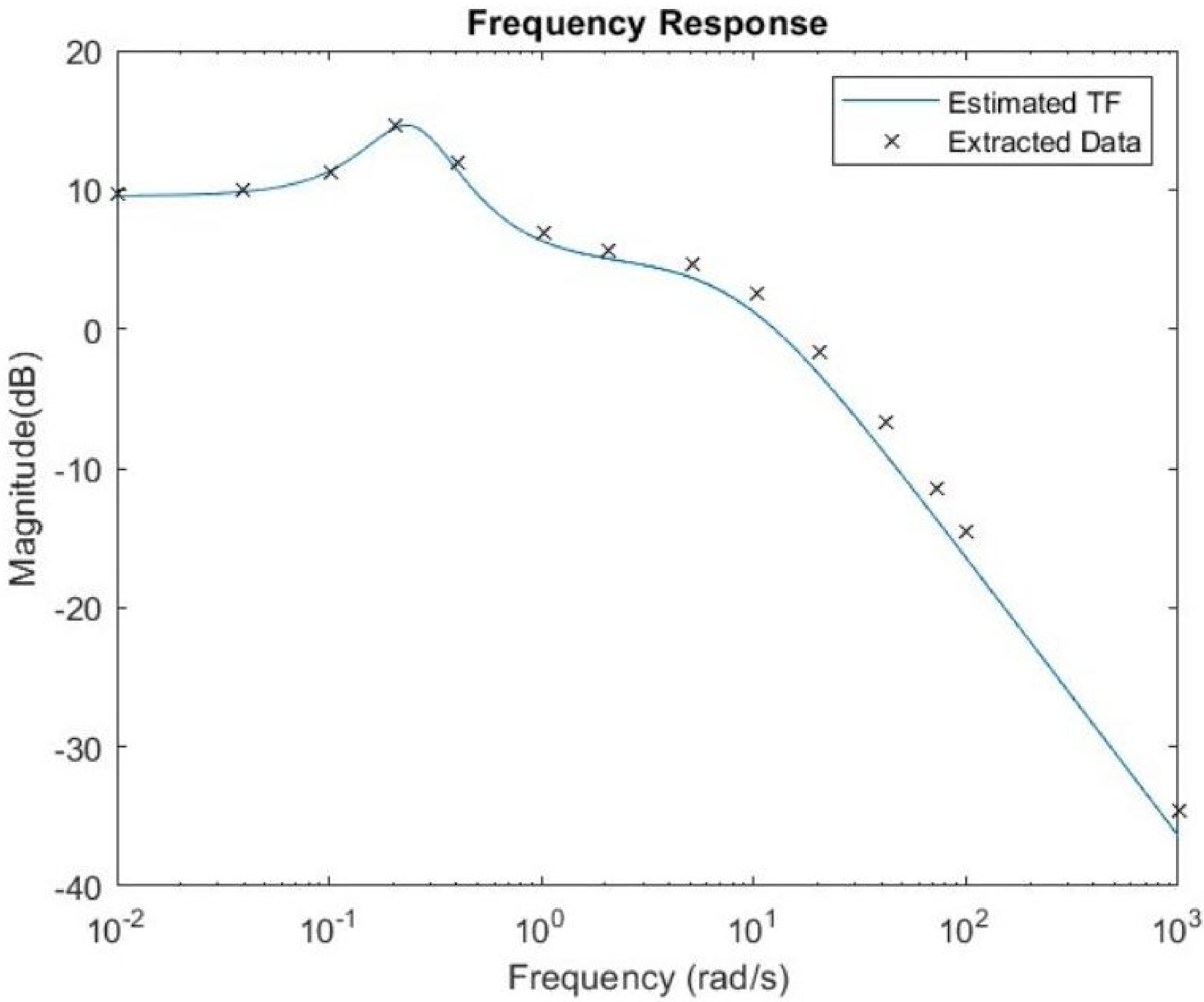
To find the parameters - $\tau_1, \tau_2, \tau_3, \tau_4$ and ξ we will do a least squares fit with the extracted data points of the step response using lsqcurvefit in MATLAB.

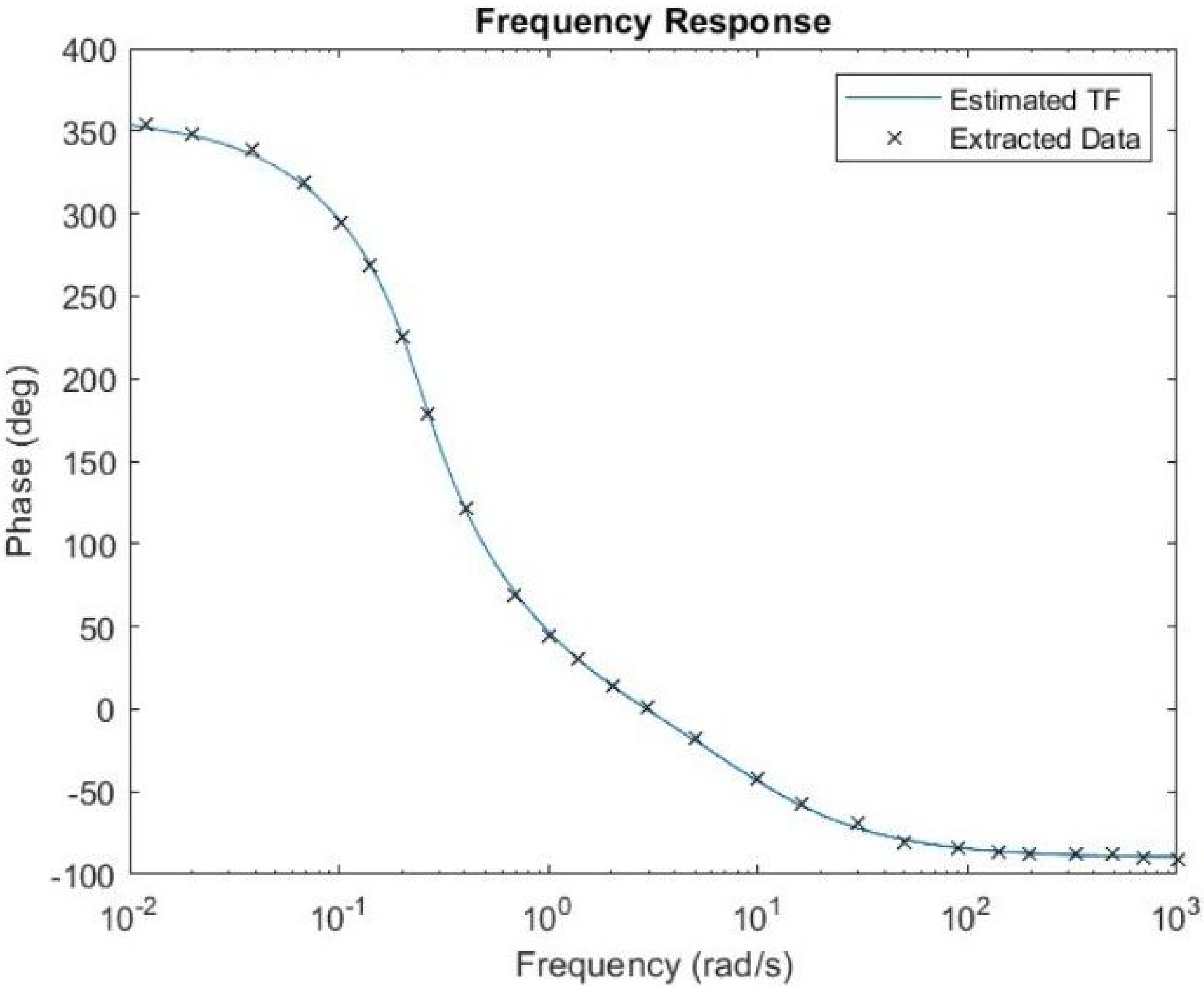
MATLAB Code:

```
clc; clear all; close all; format compact; format shortg;

%Data points extracted from the phase in Bode Plot
xdata_phase = [0.101081767
0.019909715
0.038795807
0.068619658
0.199097231
0.400685499
0.693620204
2.012511656
5.077009349
10.00000821
29.6457906
1
0.266215719
0.141101818
1.380978623
2.96457906
16.22859007
49.68923478
89.79912072
199.0977216
491.5746531
326.6028031
1000.002463
```


Step Response
Estimated TF
Extracted Data
Amplitude
Time (seconds)
4
3
2
1
0
-1
-2
0
10
20
30
40
50

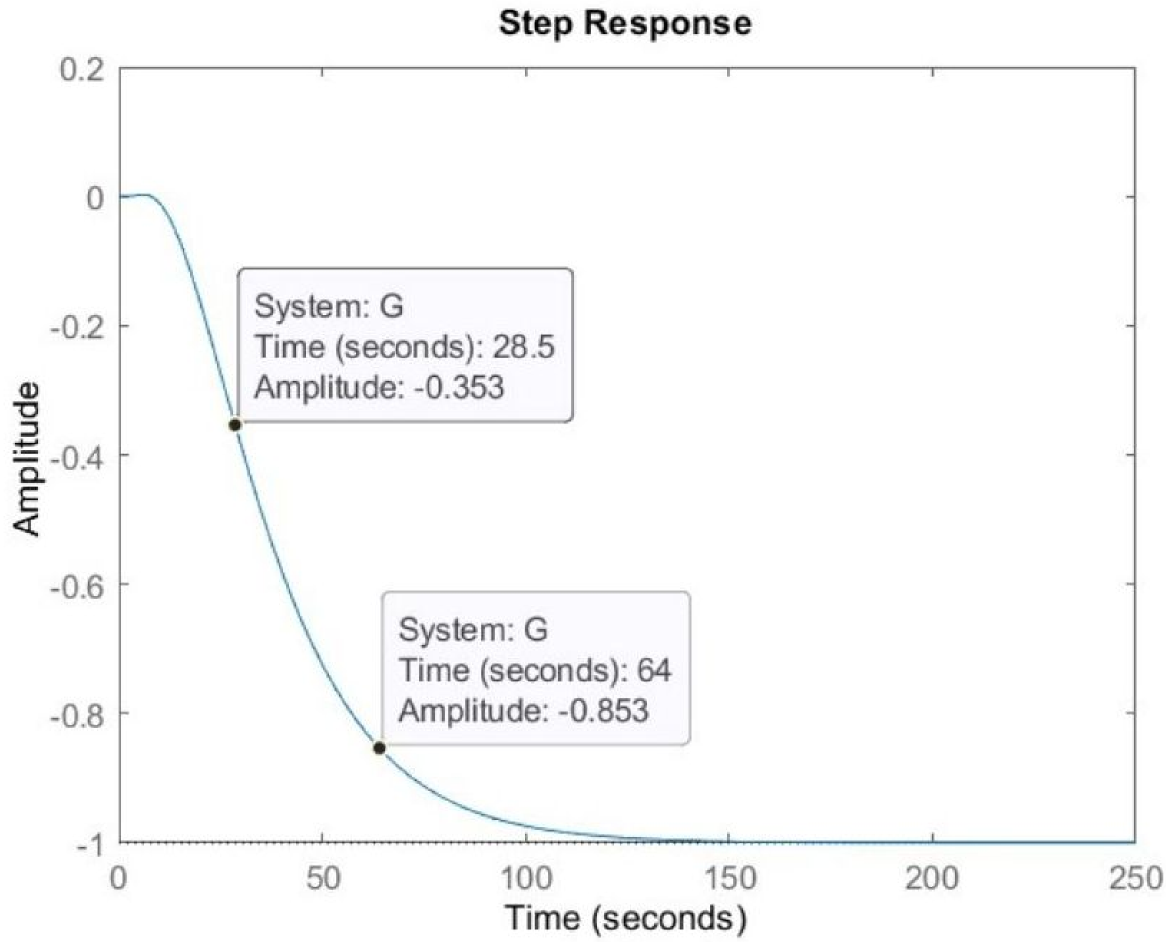

Frequency Response
Estimated TF
Extracted Data
Magnitude(dB)
Frequency (rad/s)
20
10
0
-10
-20
-30
-40
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



Frequency Response
Estimated TF
Extracted Data
Phase (deg)
400
350
300
250
200
150
100
50
0
-50
-100
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

Q2 Part a)

Following is the step response of the system, required to find t_1 and t_2 for the Krishnaswamy and Sundaresan's two point method:



From the plot, we can see that

t_1 = 28.5 sec

t_2 = 64 sec

$\tau_{P2} = \underset{\text{Second largest } \tau}{\underbrace{12}} + \underset{\text{Half of largest neglected } \tau}{\underbrace{\frac{4}{2}}} = 14$

$D = \underset{\text{Due to the zero}}{\underbrace{2}} + \underset{\text{Existing delay}}{\underbrace{2}} + \underset{\text{Half of largest neglected } \tau}{\underbrace{\frac{4}{2}}} + \underset{\text{Remaining time constants}}{\underbrace{1}} = 7$

$\therefore G(s) = \frac{-e^{-7s}}{(20s+1)(14s+1)}$

c) We need to find an SOPTD approximation using least squares for the frequency response. As the delay D does not affect the magnitude, we will find values of K_p, τ_{P1}, τ_{P2} from the magnitude vs frequency Bode plot. Using these values for least squares fitting for Phase vs frequency we can find the delay.

From the MATLAB code, we get $K_p = 0.99977$
We know that K_p is -ve but it does not affect the magnitude
$\therefore$ We will take $K_p = -0.99977$ as we know that gain has a -ve sign which is not reflected in the magnitude plot, but can be observed in the phase plot.

MATLAB code and output is as follows-

Q2 Part c)

The MATLAB code for the least squares method is given below. The gain and time constants are found using the magnitude vs frequency response after which the delay is found using the phase vs frequency response

MATLAB Code:

```
clc; clear all; close all; format compact; format shortg;
G = tf([2,-1],[960,1328,404,37,1],'IOdelay',2); %Given Process TF

%Krishnaswami and Sundaresan's two point method
figure;
step(G)

%From the step response,
%t1 = 28.5 sec
%t2 = 64 sec
G_KS = tf(-1,[23.785,1],'IOdelay',18.49); %Krishnaswami and Sundaresan
Approximation - FOPTD

%Skogestad's Half Rule method
%FOPTD
G_foptd = tf(-1,[26,1],'IOdelay',15);
%SOPTD
G_soptd = tf(-1,[280,34,1],'IOdelay',7);

%SOPTD using least squares for frequency response
[mag,phase,w] = bode(G);
mag = squeeze(mag); phase = squeeze(phase);
magdb = 20*log10(mag);

%Least squares fitting for the magnitude to estimate tau1, tau2, Kp
pars = lsqcurvefit(@magfun,[1,1,1],w,mag);
%Least sqaures fitting for the phase to find D
D = lsqcurvefit(@(D,w) phasefun(D,w,pars),2,w,phase);

tau1 = pars(1);
tau2 = pars(2);
Kp = -pars(3); %Taking gain as negative as magnitude does not consider sign of Kp

%Approximate TF from least squares
Ghat = tf(Kp, [tau1*tau2,tau1+tau2,1], 'IOdelay',D);

%Plotting the step and frequency responses for all the approximations
figure; %Step Response
step(G,'-r',G_KS,'-b',G_foptd,'-g',G_soptd,'-y',Ghat,'-k')
ylabel('y(t)')
title('Comparison of Step responses')
legend('Process','KS FOPTD','Skogestad FOPTD','Skogestad SOPTD','Least squares
FOPTD')

figure; %Frequency Response
bode(G,'-r',G_KS,'-b',G_foptd,'-g',G_soptd,'-y',Ghat,'-k')
title('Comparison of Frequency responses')
legend('Process','KS FOPTD','Skogestad FOPTD','Skogestad SOPTD','Least squares
FOPTD')
```

From the code,

$K_p = -0.99977$

$\tau_1 = 16.508$

$\tau_2 = 16.508$

$D = 2.1078$

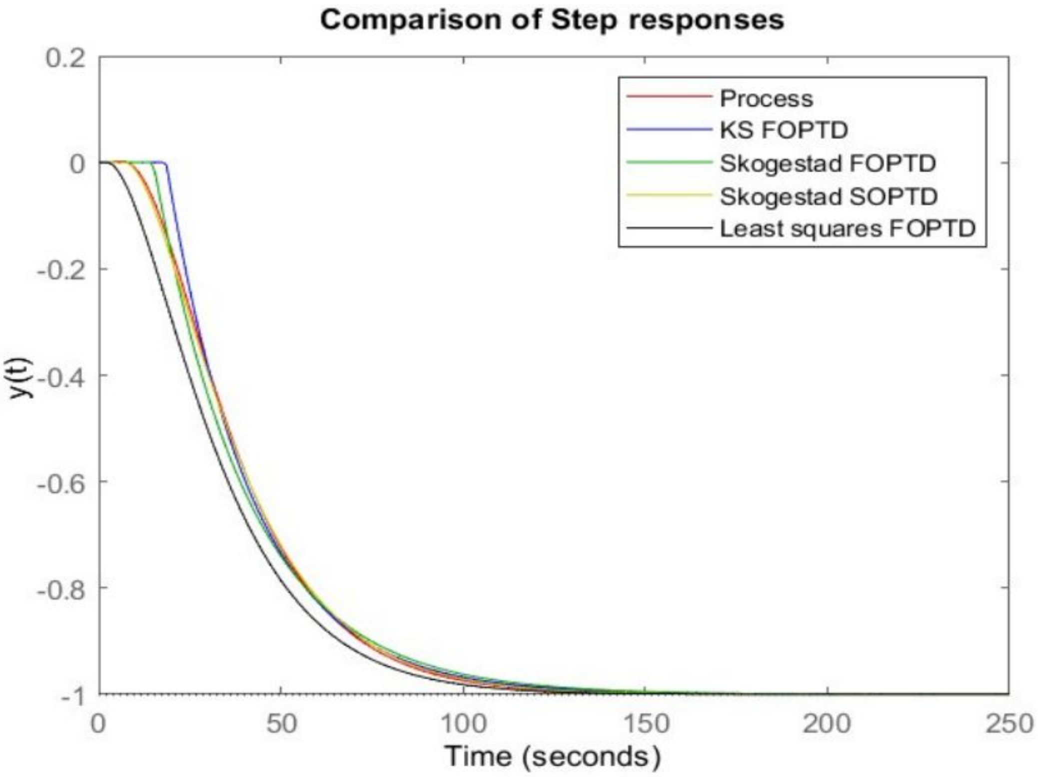
The transfer function from the least squares approximation is given as

$$G(s) = \frac{-0.99977\, e^{-2.1078s}}{(16.508s+1)(16.508s+1)}$$

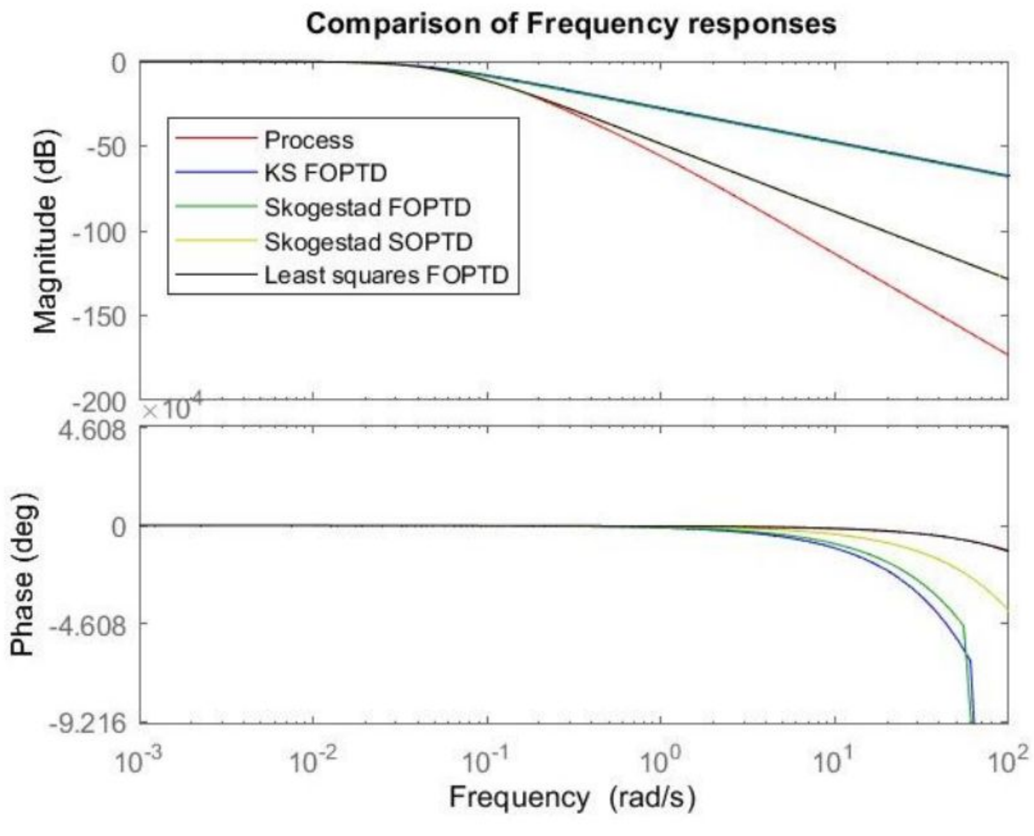
d) The step and frequency responses for the actual process and the various approximations are generated using MATLAB.
The plots are attached below and the observations for the step response are also tabulated.

Q2 Part d)
Comparison of the step and frequency responses of the various approximations:

Step Response:



Frequency Response:



Tabulating the observations of Step Response for all the approximations:

All the models give the same gain as the given transfer function.

Approximation	Observations	Parameters			
		K_p	τ_1	τ_1	D
KS FOPTD	Highest delay compared to the process, so it does not match well in the initial response. But it is a good approximation for the rest of the response	-1	23.785	-	18.49
Skogestad's FOPTD	Similar to the above for this FOPTD approximation also the delay is higher. The KS FOPTD is a better fit than this model for the rest of the step response	-1	26	-	15
Skogestad's SOPTD	Skogestad's SOPTD is the best approximation among the four, it matches with the actual step response. Compared to FOPTD models, the SOPTD models capture the characteristics of the response better.	-1	20	14	7
Least squares SOPTD	This is the worst approximation of the 4. The delay found from least squares is lower than the actual process which is why it does not match the initial response and is faster than the actual process. It gives a good approximation for the step response only near the steady state. But this approximation gives better for the frequency response	-0.99977	16.508	16.508	2.1078

We can approximately say that Skogestad's SOPTD > Skogestad's FOPTD > KS FOPTD > Least squares SOPTD in terms of the quality of fit to the step response.

Q3)

a) $$G(s) = \frac{2(-s+2)e^{-2s}}{(8s+1)(5s+1)}$$

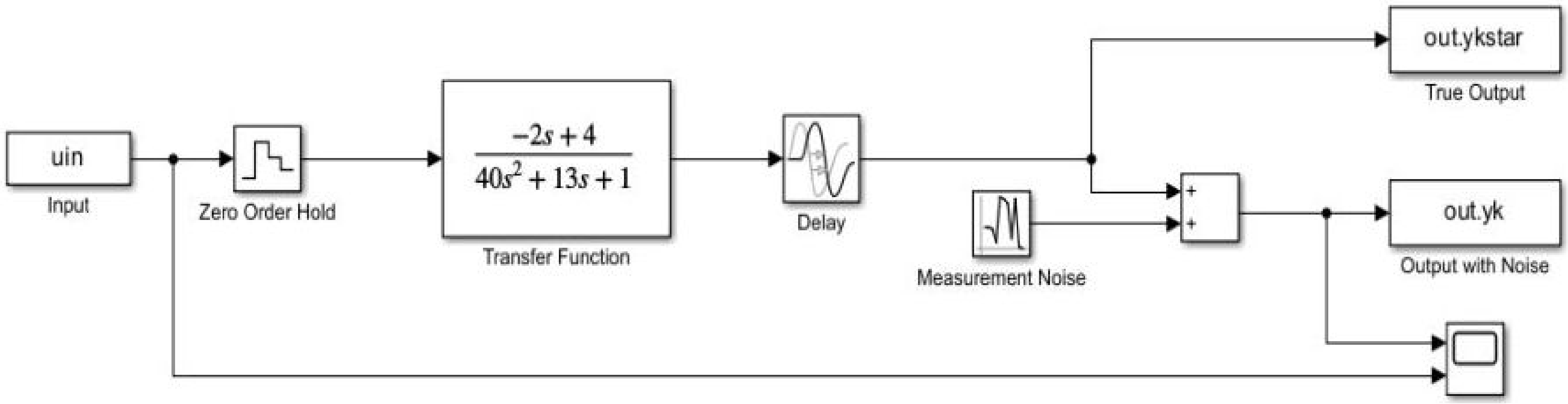
The simulink model is attached below.

To select an appropriate sampling time T_s,
Dominating time const. $\tau = 8$
T_s should be between $\frac{\tau}{5}$ and $\frac{\tau}{10}$ ie 1.6 and

0.8

$\therefore$ We will choose T_s = 1 sec. as the sampling time interval.

Q3-Part a)
Simulink Model:



Q3-Part b)
MATLAB code:

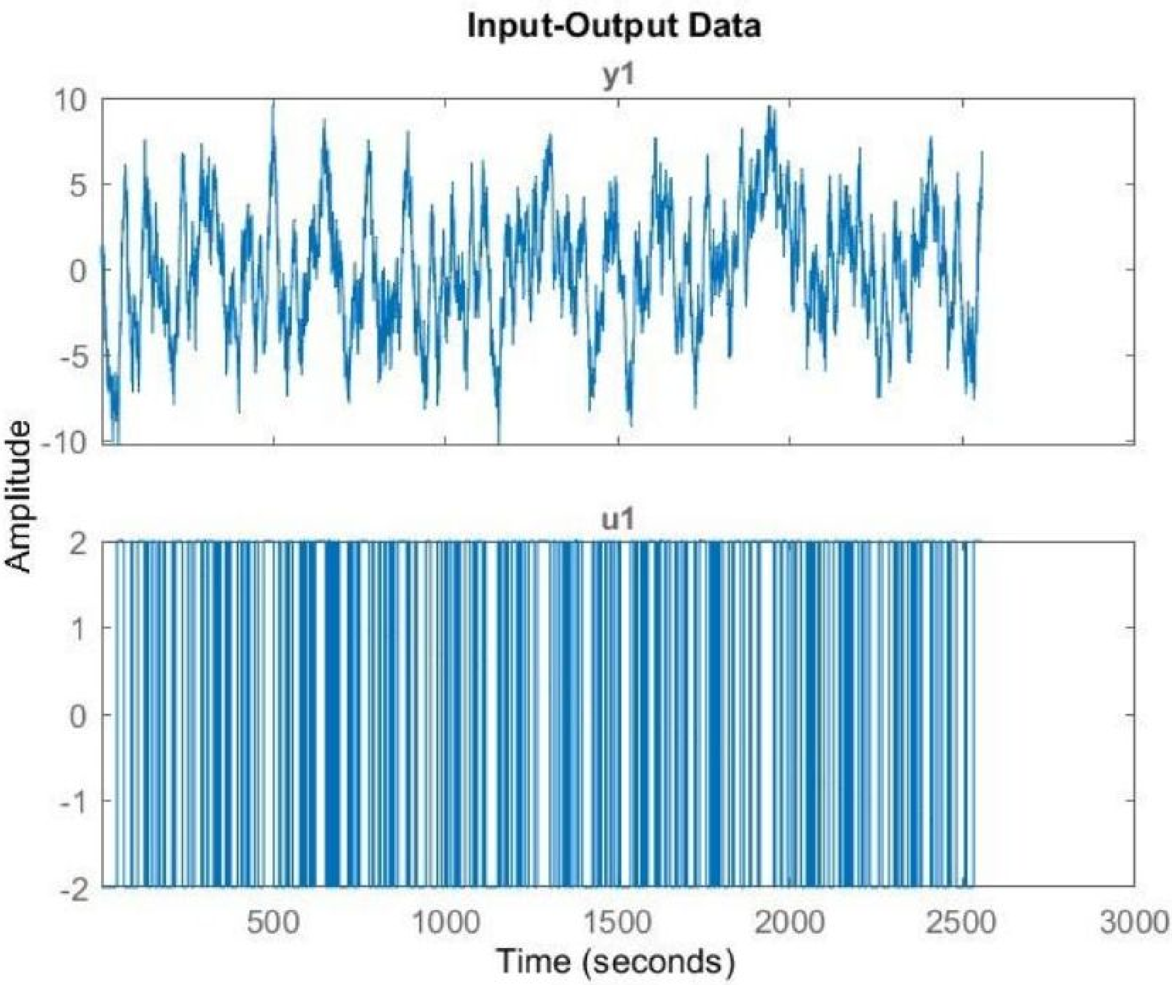
```
%Generating the PRBS input to the model
b = 1/5;
Ts= 1; % Sampling Time
usig = idinput(2555,'prbs',[0 b],[-2 2]);
uin = [(0:1:length(usig)-1)'*Ts (usig)];

%Simulating the response to the above input
out = sim("Q3model.slx");
yk = out.yk;

%Creating a data object to store the input output data
data=iddata(yk(1:length(usig)),uin(:, 2),1);
plot(data) %Plotting the input output data

%Splitting the data into test and train sets
data_train=data(1:1300);
data_test=data(1300:end);
```

Plot of PRBS input and the response of the system:

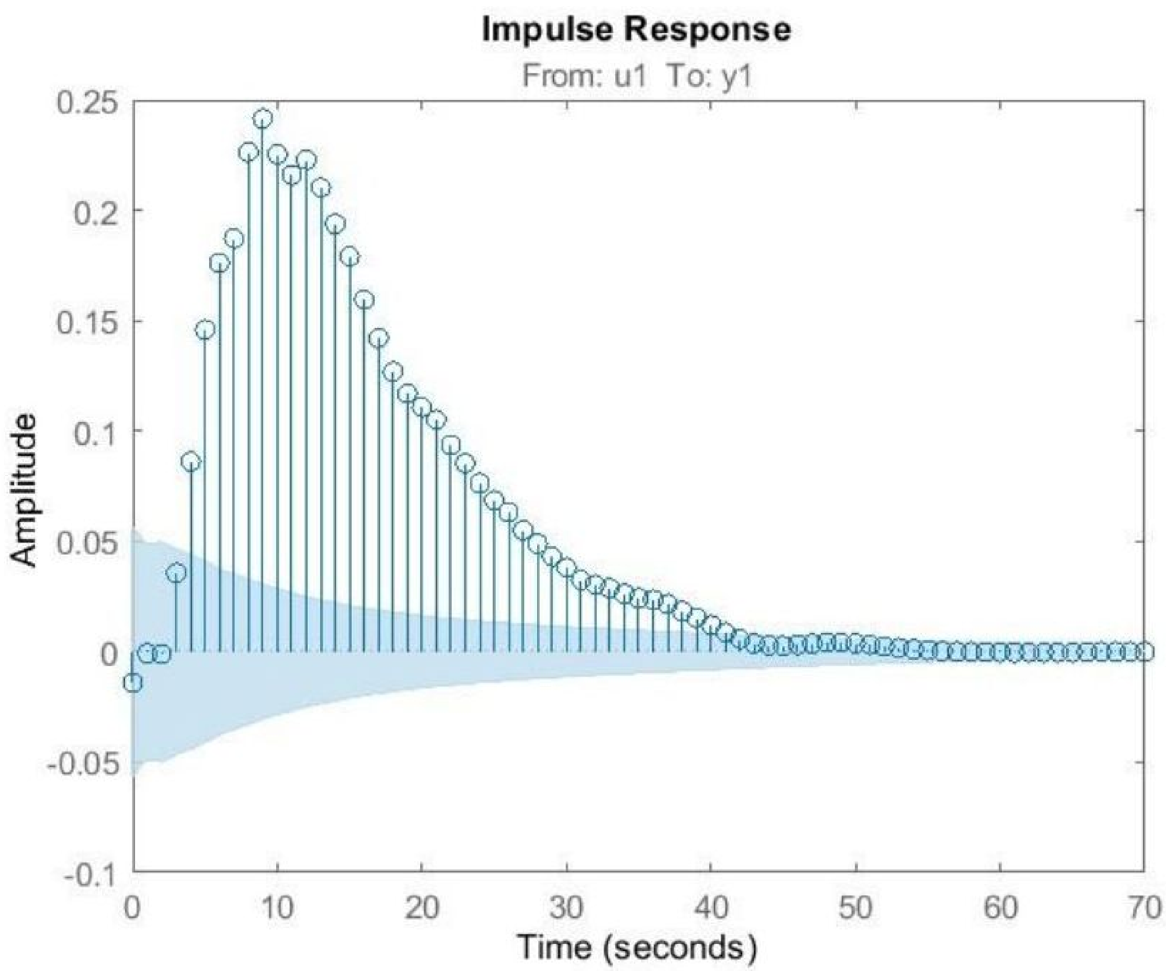


Q3-Part c)

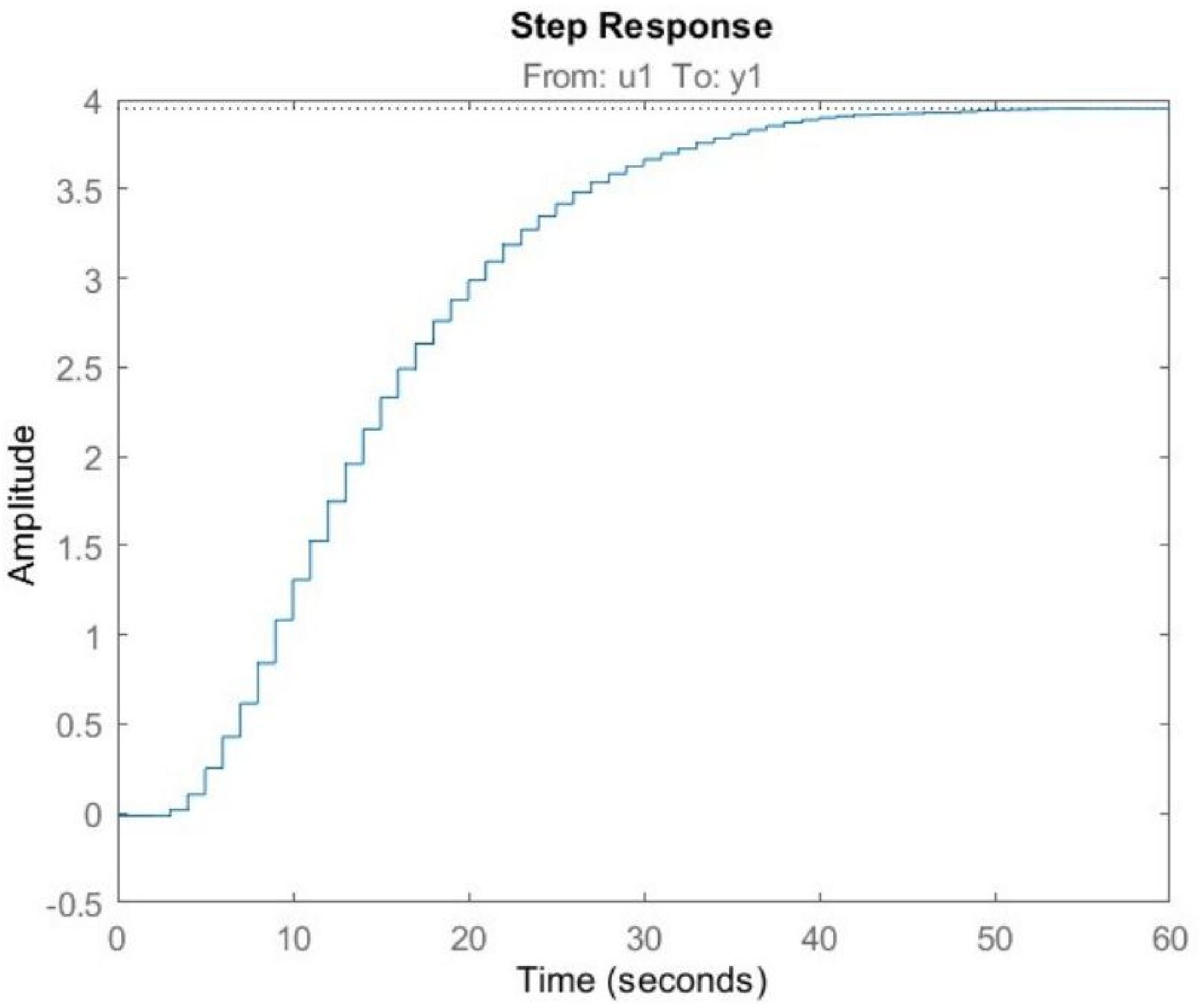
```
%Impulse response model using the training data
[ztrain,Tr]=detrend(data_train,0);
ztest=detrend(data_test,Tr);
impulse_est= impulseest(ztrain);
%PLotting the impusle response
figure
impulse(impulse_est,'sd',3);

%Plotting the step response based on the estimated impulse response
figure
step(impulse_est)
```

Plot of estimated impulse response



Plot of estimated step response



c) From the step response we can see that the system shows a small inverse response initially. So, it has a RHP zero.

From the impulse response we see a delay of 4 samples ie 4 sec as $T_s = 1$ sec.

The gain from the step response is around 3.95 which is close to the actual gain of 4.

The time constant assuming a first order system is around 16 seconds. But based on the step response, the system can also be a overdamped second order system.

Q3-Part d)
MATLAB code:

```
%Guesses for the parameters of the OE model
m=2;
n=2;
d=4;

%Estimating the OE model
model_oe=oe(ztrain, [m, n, d]);

%Plotting the cross and auto residual corelations
figure
resid(model_oe, ztrain);

%Getting the estimated parameters for the OE model
present(model_oe);

%Comparison between the repsonse of OE model and actual system response
figure
compare(model_oe, ztest);
```

```
Model Properties
>> present(model_oe)

model_oe =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.08617 (+/- 0.009128) z^-4 - 0.0006899 (+/- 0.01269) z^-5

  F(z) = 1 - 1.7 (+/- 0.01483) z^-1 + 0.7215 (+/- 0.01383) z^-2

Sample time: 1 seconds
```

From this the parameters for the OE model are

a1 = 1.7

a2 = 0.7215

b1, b2, b3 = 0

b4 = 0.08617

b5 = -0.0006899

Q3-Part e)

```
Model Properties
>> present(model_oe)

model_oe =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.08617 (+/- 0.009128) z^-4 - 0.0006899 (+/- 0.01269) z^-5

  F(z) = 1 - 1.7 (+/- 0.01483) z^-1 + 0.7215 (+/- 0.01383) z^-2

Sample time: 1 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=4
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 4, Number of function evaluations: 9

Estimated using OE on time domain data "ztrain".
Fit to estimation data: 68.86%
FPE: 1.297, MSE: 1.289
More information in model's "Report" property.
```