$$\therefore G(s) = -\frac{e^{-7s}}{(20s+1)(14s+1)}$$

We need to find an SOPTD approximation using least squares for the frequency response. As the delay D does not affect the magnitude, we will find values of $k_p, \tau_{p_1}.\ \tau_{p_2}$ from the magnitude vs frequency Bode plot. Using these values for least squares fitting for phase vs frequency we can find the delay.

Using MATLAB, we get $Kp = 0.99977$ We know that K_p is negative but it does not affect the magnitude $\therefore$ We will take $Kp = -0.99977$ as we know that gain has a negative sign which is not reflected in the magnitude plot but can be observed in the phase plot.

Using MATLAB we obtain the following values,

$$K_p = -0.99977$$
$$\tau_1 = 16.508$$
$$\tau_2 = 16.508$$
$$D = 2.1078$$

The transfer function from the least squares approximation is given as

$$G(S) = \frac{-0.99977e^{-2.1078s}}{(16.508s+1)(16.508s+1)}$$

The step and frequency responses for the actual process and the various approximations are generated using MATLAB. The plots are attached below, and the observations for the step response are also tabulated.

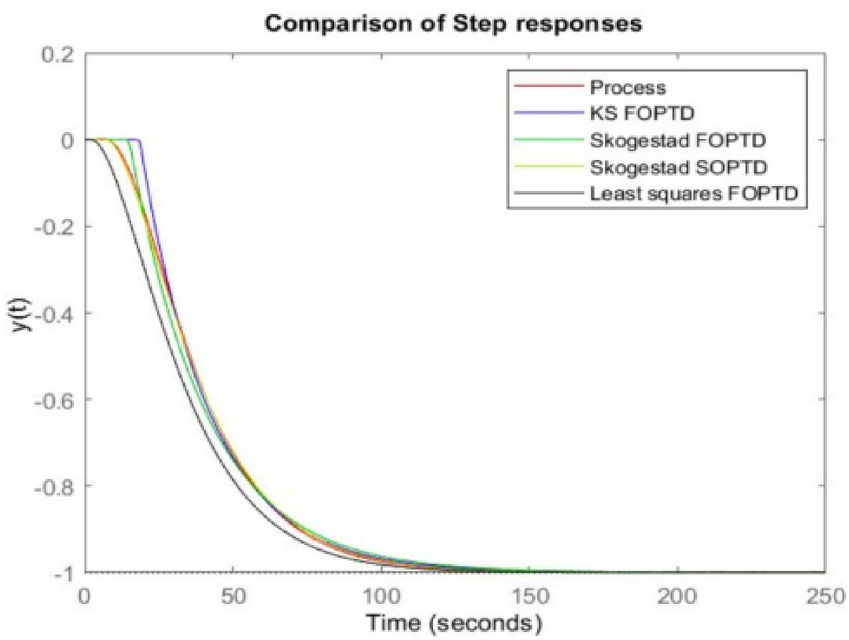

Figure 3: Step Response of the process

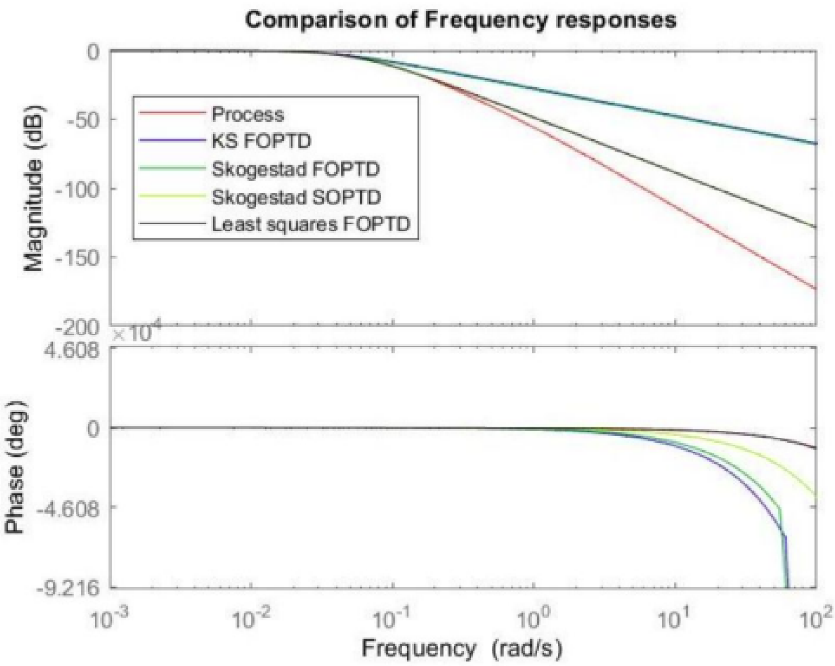

Figure 4: Frequency Response of the process

Approximation	**Observation**
KS FOPTD	Delay is higher than the true value, thus, the initial response does not match. The remaining process is seen to match well
Skogestad's FOPTD	Delay is higher than the true value, thus, the initial response does not match. The remaining process is seen to match well, but not as well as KS FOPTD
Skogestad's SOPTD	Matches best among all approximations and fits the actual step response. SOPTD models do better in capturing the process characteristics, when compared to FOPTD models, due to more parameters
Least Squares SOPTD	Matches least among all the approximations. Due to lower delay, initial response does not match. The step responses match only at steady state, but good approximation for frequency response.

Table 1: Observations for different approximations

Question 3

(a) Given $G(s) = \dfrac{2(-s+2)e^{-2s}}{(8s+1)(5s+1)}$

Chosen a sample interval of $T_s = 0.4s$.

(b)
```
%MATLAB script for generating the data
% Created by Rajesh, March 2023
N = 2555;
Ts = 0.4;
```

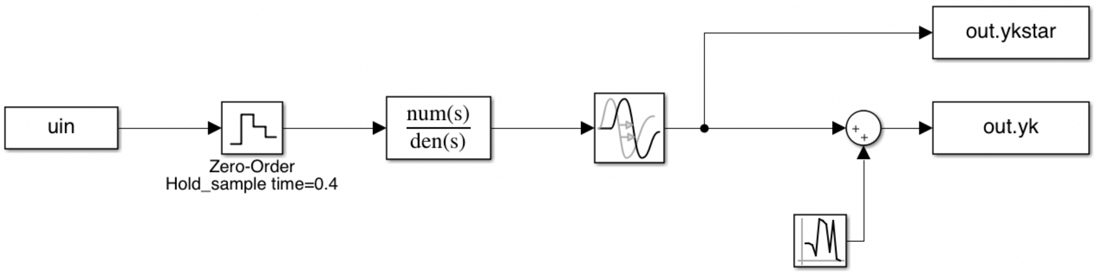

Figure 5: Simulink model for the given process

```
uk = idinput(N,[0,0.2],[-2,2]);
uin = [(0:1:length(uk)-1)'*Ts, (uk)]; %uk=ukstar and run the Simulink model

% Creating iddata object
size(uk)
size(out.yk.Data)
data = iddata(out.yk.Data(1:2500),uk(1:2500),0.4);

% Creating test and train data sets
data_train = data(1:1300);
data_test = data(1300:end)
;
% Removing the trend
[ztrain,Tr] = detrend(data_train,0);
ztest = detrend(data_test,Tr);
impulse_est= impulseest(ztrain);
figure
impulse(impulse_est,'sd',3);
figure
step(impulse_est)
```

(c) From (7a) and (7b), we observe that the delay comes out as eight samples. The gain of the system is 3.4. It seems sluggish, which made us observe a large delay. So guessing that it is a second-order system.

(d) With the model order of 2, and delay of 8, we fit an output error model.

```
% MATLAB script for creating OE model
 M = oe(data_train,[1 2 8]);
 resid(M,data_train)
```

The model we get is

$$y[k] = \frac{0.01557z^{-8}}{1 - 1.869z^{-1} + 0.8727z^{-2}}u[k] + e[k] \tag{11}$$

The noise variance($\hat{\sigma}_e^2$) is 1.2978. The standard errors of the coefficients of the model are reported in the script below.

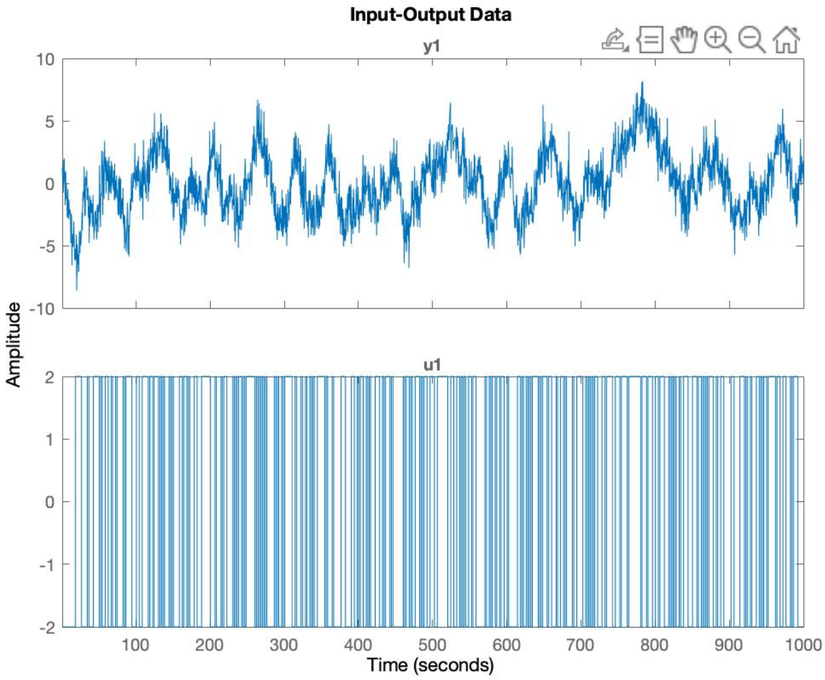

Figure 6: Snapshot of 2500 samples of output and input

(e)

(f)

```
present(M)
dcgain(impulse_est) %Gain from the data
dcgain(M) % Gain from the OE model estimate


    M =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.01557 (+/- 0.0006322) z^-8
  F(z) = 1 - 1.869 (+/- 0.005623) z^-1 + 0.8727 (+/-
     0.005526) z^-2
Sample time: 0.4 seconds
Parameterization:
   Polynomial orders:   nb=1   nf=2   nk=8
   Number of free coefficients: 3

ans=
3.4004
ans=
4.0187
```

As we can observe that the gains are different. The one from the non-parametric is 3.4004, and the OE model gives 4.0187.