$$\tau_{P_2} = \underset{\text{Second largest } \tau}{\underbrace{12}} + \underset{\text{Half of largest neglected } \tau}{\underbrace{\frac{4}{2}}} = 14$$

$$D = \underset{\text{Due to the zero}}{\underbrace{2}} + \underset{\text{Existing delay}}{\underbrace{2}} + \underset{\text{Half of largest neglected } \tau}{\underbrace{\frac{4}{2}}} + \underset{\text{Remaining time constants}}{\underbrace{1}} = 7$$

$$\therefore G_1(s) = \frac{-e^{-7s}}{(20s+1)(14s+1)}$$

c) We need to find an SOPTD approximation using least squares for the frequency response. As the delay D does not affect the magnitude, we will find values of $K_p$, $\tau_{P_1}$, $\tau_{P_2}$ from the magnitude vs frequency Bode plot. Using these values for least squares fitting for Phase vs frequency we can find the delay.

From the MATLAB code, we get $K_p = 0.99977$. We know that $K_p$ is -ve but it does not affect the magnitude.

$\therefore$ We will take $K_p = -0.99977$ as we know that gain has a -ve sign which is not reflected in the magnitude plot, but can be observed in the phase plot.

MATLAB code and output is as follows-

From the code,

$K_P = -0.99977$

$\tau_1 = 16.508$

$\tau_2 = 16.508$

$D = 2.1078$

The transfer function from the least squares approximation is given as

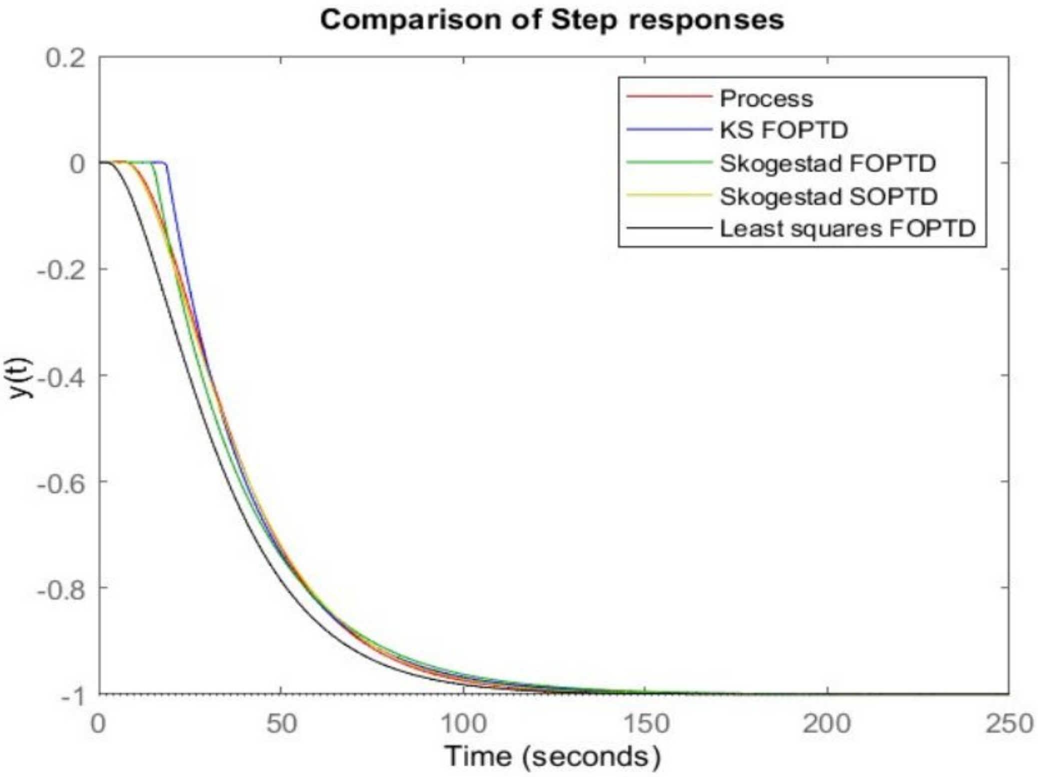
$$G(s) = \frac{-0.99977\, e^{-2.1078s}}{(16.508s+1)(16.508s+1)}$$

d) The step and frequency responses for the actual process and the various approximations are generated using MATLAB.

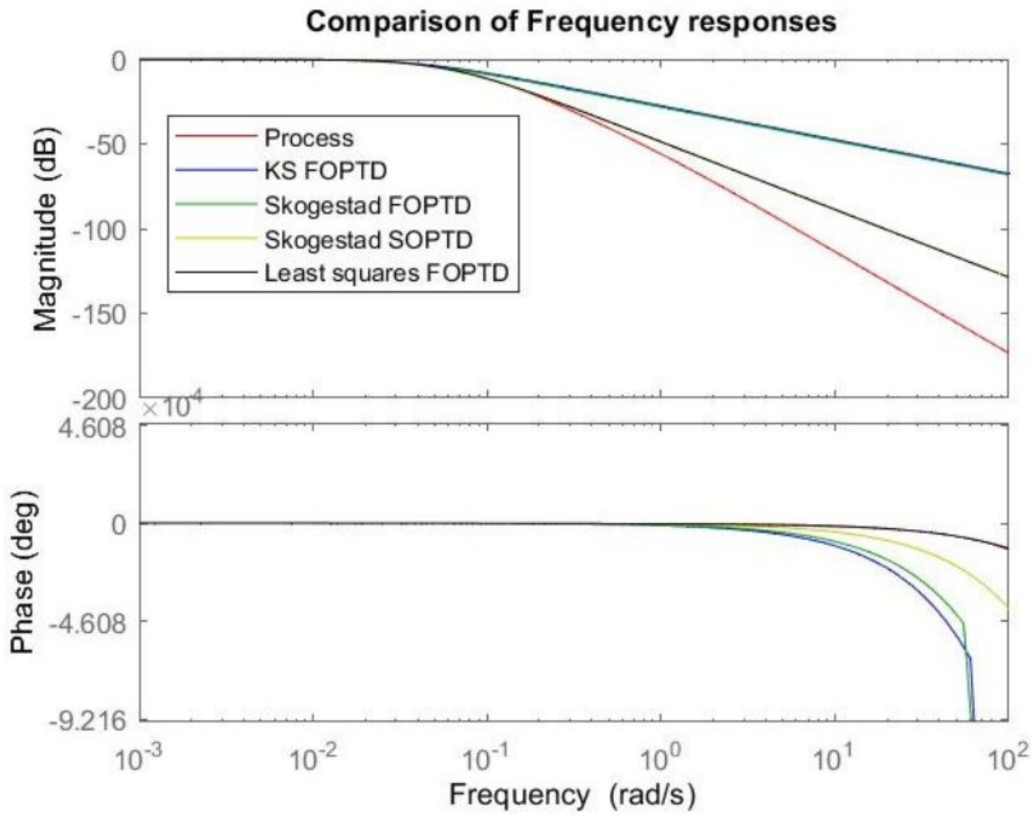
The plots are attached below and the observations for the step response are also tabulated.

Q2 Part d)
Comparison of the step and frequency responses of the various approximations:

Step Response:




Frequency Response:



Tabulating the observations of Step Response for all the approximations:

All the models give the same gain as the given transfer function.

| Approximation | Observations | Parameters | | | |
|---|---|---|---|---|---|
| | | $K_p$ | $\tau_1$ | $\tau_1$ | D |
| KS FOPTD | Highest delay compared to the process, so it does not match well in the initial response. But it is a good approximation for the rest of the response | -1 | 23.785 | - | 18.49 |
| Skogestad's FOPTD | Similar to the above for this FOPTD approximation also the delay is higher. The KS FOPTD is a better fit than this model for the rest of the step response | -1 | 26 | - | 15 |
| Skogestad's SOPTD | Skogestad's SOPTD is the best approximation among the four, it matches with the actual step response. Compared to FOPTD models, the SOPTD models capture the characteristics of the response better. | -1 | 20 | 14 | 7 |
| Least squares SOPTD | This is the worst approximation of the 4.<br>The delay found from least squares is lower than the actual process which is why it does not match the initial response and is faster than the actual process. It gives a good approximation for the step response only near the steady state. But this approximation gives better for the frequency response | -0.99977 | 16.508 | 16.508 | 2.1078 |

We can approximately say that Skogestad's SOPTD > Skogestad's FOPTD > KS FOPTD > Least squares SOPTD in terms of the quality of fit to the step response.

Q3)

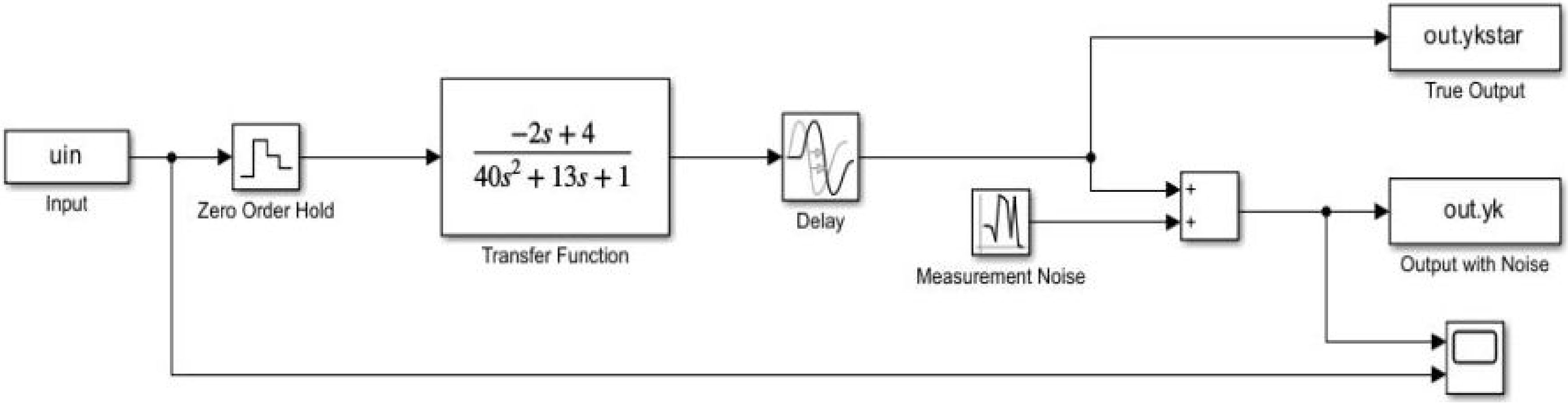
a) $G(s) = \dfrac{2(-s+2)e^{-2s}}{(8s+1)(5s+1)}$

The simulink model is attached below.

To select an appropriate sampling time $T_s$,
Dominating time const. $\tau = 8$
$T_s$ should be between $\frac{\tau}{5}$ and $\frac{\tau}{10}$ ie 1.6 and 0.8

$\therefore$ We will choose $T_s = 1$ sec. as the sampling time interval.

Q3-Part a)
Simulink Model:



Q3-Part b)
MATLAB code:

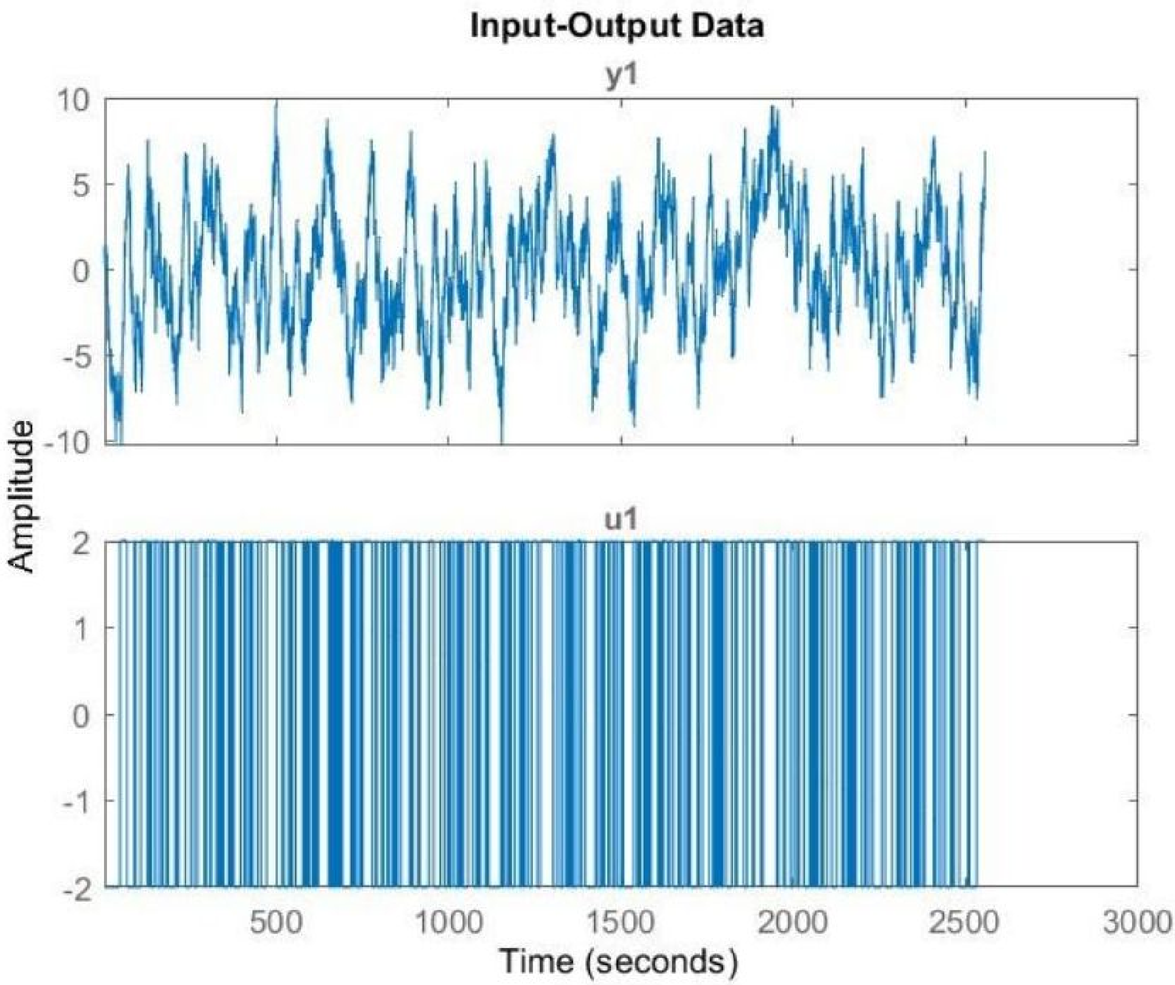
```
%Generating the PRBS input to the model
b = 1/5;
Ts= 1; % Sampling Time
usig = idinput(2555,'prbs',[0 b],[-2 2]);
uin = [(0:1:length(usig)-1)'*Ts (usig)];

%Simulating the response to the above input
out = sim("Q3model.slx");
yk = out.yk;

%Creating a data object to store the input output data
data=iddata(yk(1:length(usig)),uin(:, 2),1);
plot(data) %Plotting the input output data

%Splitting the data into test and train sets
data_train=data(1:1300);
data_test=data(1300:end);
```

Plot of PRBS input and the response of the system:

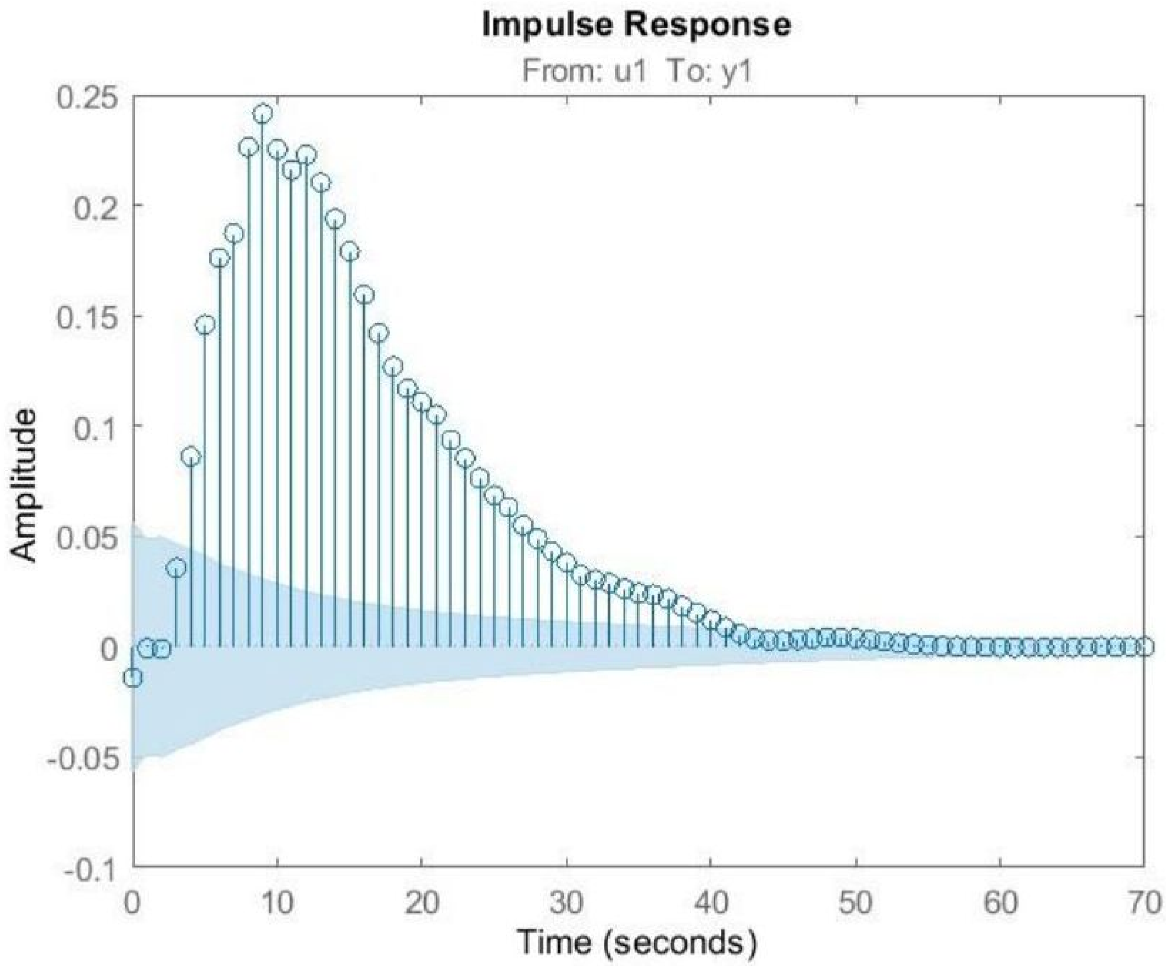


Q3-Part c)

```
%Impulse response model using the training data
[ztrain,Tr]=detrend(data_train,0);
ztest=detrend(data_test,Tr);
impulse_est= impulseest(ztrain);
%PLotting the impusle response
figure
impulse(impulse_est,'sd',3);

%Plotting the step response based on the estimated impulse response
figure
step(impulse_est)
```

Plot of estimated impulse response



Q3-Part d)

MATLAB code:

```
%Guesses for the parameters of the OE model
m=2;
n=2;
d=4;

%Estimating the OE model
model_oe=oe(ztrain, [m, n, d]);

%Plotting the cross and auto residual corelations
figure
resid(model_oe, ztrain);

%Getting the estimated parameters for the OE model
present(model_oe);

%Comparison between the repsonse of OE model and actual system response
figure
compare(model_oe, ztest);
```

```
Model Properties
>> present(model_oe)

model_oe =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.08617 (+/- 0.009128) z^-4 - 0.0006899 (+/- 0.01269) z^-5

  F(z) = 1 - 1.7 (+/- 0.01483) z^-1 + 0.7215 (+/- 0.01383) z^-2

Sample time: 1 seconds
```

From this the parameters for the OE model are

a1 = 1.7

a2 = 0.7215

b1, b2, b3 = 0

b4 = 0.08617

b5 = -0.0006899

Q3-Part e)

```
Model Properties
>> present(model_oe)

model_oe =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.08617 (+/- 0.009128) z^-4 - 0.0006899 (+/- 0.01269) z^-5

  F(z) = 1 - 1.7 (+/- 0.01483) z^-1 + 0.7215 (+/- 0.01383) z^-2

Sample time: 1 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=4
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 4, Number of function evaluations: 9

Estimated using OE on time domain data "ztrain".
Fit to estimation data: 68.86%
FPE: 1.297, MSE: 1.289
More information in model's "Report" property.
```