CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-7

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TOPICS TO BE COVERED

- Hamming code

1. If the generator has more than one term and coefficient of x° is 1, all single bit error can be detected.
2. If a generator cannot divide $\boxed{x^t + 1}$ (t between 0 and n – 1) then all isolated Double error can be detected
3. A generator that contains a Factor of $\boxed{x + 1}$ can detect all odd numbered errors.

① data = 1011
CRC generator = 1 (generator is not Valid)

② data = 1011

CRC generator = $x = 1\cdot x^1 + 0\cdot x^0 = 10$

Sender

$$
\begin{array}{r|l}
10 & 10110 \\
& \underline{10} \\
& 00110 \\
& \quad\underline{10} \\
& \quad 010 \\
& \quad\ \underline{10} \\
& \quad\ 00
\end{array}
$$

CRC or Remainder

Transmitted data

$\boxed{10110}$

Received data

$\boxed{10010}$

1 bit error

Receiver

$$
\begin{array}{r|l}
10 & 10010 \\
& \underline{10} \\
& 00010 \\
& \quad\ \underline{10} \\
& \quad\ 00
\end{array}
$$

syndrom = 0 means No error.

dataword Accepted

"Receiver Fails to Detect one bit error"

Note

generator should not contain only 'x'

(2) data = 1011

CRC generator = $x+1 = 1\cdot x^1 + 1\cdot x^0 = 11$

Sender

```
11 ) 10110 (
     11
     01110
      11
      0010
        11
        01   CRC or Remainder
```

Transmitted data | Received data

(i) $\boxed{10111}$ $\boxed{10011}$

1 bit error

Receiver

```
11 ) 10011 (
     11
     01011
      11
      0111
       11
       001   syndrom ≠ 0 [Error]
```

dataword Rejected

"Receiver can detect one bit error"

(ii) Transmitted data

10111

Received data

10100

2 bit error

Receiver

```
11 ) 10100
     11
     -----
     01100
      11
     -----
      0000  ⇒ syndrom = 0 (No error)
```

⇓

"dataword Accepted"

"Receiver Fails to detect 2 bit error"

(iii) Transmitted data

$\boxed{10111}$

3 bit error

Received data

$\boxed{10000}$

Receiver

"Receiver can detect 3 bit error"

```
11 ) 10000
     11
     -----
     01000
      11
      ----
      0100
       11
       ---
       010
        11
        --
        01
```

$01 \Rightarrow$ syndrom $\neq 0$ (error) dataword Rejected

A good polynomial generator needs to have the following characteristics:

1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $\boxed{x^t + 1,}$ for t between 2 and n - 1.
4. It should have the factor $\boxed{x + 1.}$

Q

Let G(x) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(x) to detect odd number of bits in error?

(A) G(x) contains more than two terms

(B) G(x) does not divide 1+x^k, for any k not exceeding the frame length

(C) 1+x is a factor of G(x)

(D) G(x) has an odd number of terms.

$\boxed{\text{GATE-2008}}$

Hamming Code

- Hamming code can correct 1 bit error only.
- Hamming code can detect upto 2 bit error.
- Hamming code is used for error correction.

Transmitted data

Message = m bits — $n = m + r$

or data

r = redundant bits or Parity bits or check bits or extra bits

According to the hamming code, number of redundant bits

$\boxed{m + r + 1 \le 2^r}$ where r = lower limitation

(1) data = 1010111 , $m = 7$ bit , $r = 4$, $n = m + r$, $n = 7 + 4 = 11$
OR
msg

$$\boxed{m + r + 1 \le 2^r}$$

$r = 1 \rightarrow 7 + 1 + 1 \le 2^1$, $9 \le 2$ (No)

$r = 2 \rightarrow 7 + 2 + 1 \le 2^2$, $10 \le 4$ (No)

$r = 3 \rightarrow 7 + 3 + 1 \le 2^3$, $11 \le 8$ (No)

✓ $r = 4 \rightarrow 7 + 4 + 1 \le 2^4$, $12 \le 16$ (yes)

$r = 5 \rightarrow 7 + 5 + 1 \le 2^5$, $13 \le 32$ (yes)

$r = 6 \rightarrow 7 + 6 + 1 \le 2^6$, $14 \le 64$ (yes)

⋮

Redundant bits Position $= 2^i \; (i \ge 0)$
or
Parity bits $= 2^0, 2^1, 2^2, 2^3, 2^4 \ldots$
or $= 1, 2, 4, 8, 16$
check bits
or
extra bits

(By using even Parity)

P_1	P_2	3	P_4	5	6	7	P_8	9	10	11
1	0	1	1	0	1	0	1	1	1	1
2^0	2^1		2^2				2^3			

P_1

1	3	5	7	9	11
$\boxed{1}$	1	0	0	1	1

P_4

4	5	6	7
$\boxed{1}$	0	1	0

P_2

2	3	6	7	10	11
$\boxed{0}$	1	1	0	1	1

P_8

8	9	10	11
$\boxed{1}$	1	1	1

$1 = 2^0$
$2 = 2^1$
$3 = 2^1 + 2^0$
$4 = 2^2$
$5 = 2^2 + 2^0$
$6 = 2^2 + 2^1$
$7 = 2^2 + 2^1 + 2^0$
$8 = 2^3$
$9 = 2^3 + 2^0$
$10 = 2^3 + 2^1$
$11 = 2^3 + 2^1 + 2^0$

Transmitted data = $\overset{1}{1}\overset{2}{0}\overset{3}{1}\overset{4}{1}\overset{5}{0}\overset{6}{1}\overset{7}{0}\overset{8}{1}\overset{9}{1}\overset{10}{1}\overset{11}{1}$

(i) IF Receiver Received uncorrupted data

Received data = $\overset{1}{1}\overset{2}{0}\overset{3}{1}\overset{4}{1}\overset{5}{0}\overset{6}{1}\overset{7}{0}\overset{8}{1}\overset{9}{1}\overset{10}{1}\overset{11}{1}$

P_1

1 3 5 7 9 11

1 1 0 0 1 1 → even ($P_1 = 0$)

P_2

2 3 6 7 10 11

0 1 1 0 1 1 → even ($P_2 = 0$)

P_4

4 5 6 7

1 0 1 0 → even ($P_4 = 0$)

P_8

8 9 10 11

1 1 1 1 → even ($P_8 = 0$)

P_8	P_4	P_2	P_1
0	0	0	0

→ (No error)

(ii) If Receiver Received corrupted data (1 bit error)

	1	2	3	4	5	6	7	8	9	10	11
Received data =	1	0	1	1	0	1	0	1	0	1	1

P_1

1	3	5	7	9	11	
1	1	0	0	0	1	→ odd ($P_1 = 1$)

P_2

2	3	6	7	10	11	
0	1	1	0	1	1	→ even ($P_2 = 0$)

P_4

4	5	6	7	
1	0	1	0	→ even ($P_4 = 0$)

P_8

8	9	10	11	
1	0	1	1	→ odd ($P_8 = 1$)

P_8	P_4	P_2	P_1	
1	0	0	1	→ Non zero means Error

⇓

decimal value = 9^{th} bit got corrupted

(ii) If Receiver Received corrupted data (2 bit error)

1	2	3	4	5	6	7	8	9	10	11

Received data = 1 0 1 1 1 1 0 1 0 1 1

P_1

1 3 5 7 9 11

1 1 1 0 0 1 → even ($P_1 = 0$)

P_2

2 3 6 7 10 11

0 1 1 0 1 1 → even ($P_2 = 0$)

P_4

4 5 6 7

1 1 1 0 → odd ($P_4 = 1$)

P_8

8 9 10 11

1 0 1 1 → odd ($P_8 = 1$)

P_8	P_4	P_2	P_1
1	1	0	0

→ Non zero means Error

⇓

decimal Value = ~~12th bit got corrupted~~

"It can't correct 2 bit error"

Problem Solving on Hamming Code

Q.1 If a 7 bit hamming code word received by receiver is 1011011. assume even parity state whether the received code word is correct or not ?if it is incorrect then locate the bit having error.

Q.2 Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is $d_8d_7d_6d_5c_8d_4d_4d_3d_2c_4d_1c_2c_1$, where the data bits and check bits are given in

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

A. x is 1 and y is 0

B. x is 1 and y is 1

C. x is 0 and y is 0

D. x is 0 and y is 1

Q.3 Consider hamming code (Signal bit error detection and correction technique), the minimum parity bits needed for 60 data bits is ____.

Q.4 For single bit error correcting hamming code ,the code length for 12 data bit is ______

Q.5 After encoding using Hamming method, the pattern for 1010111 is (consider Even Parity)

A. 10110101111

B. 10111101111

C. 11110101111

D. 10011101111

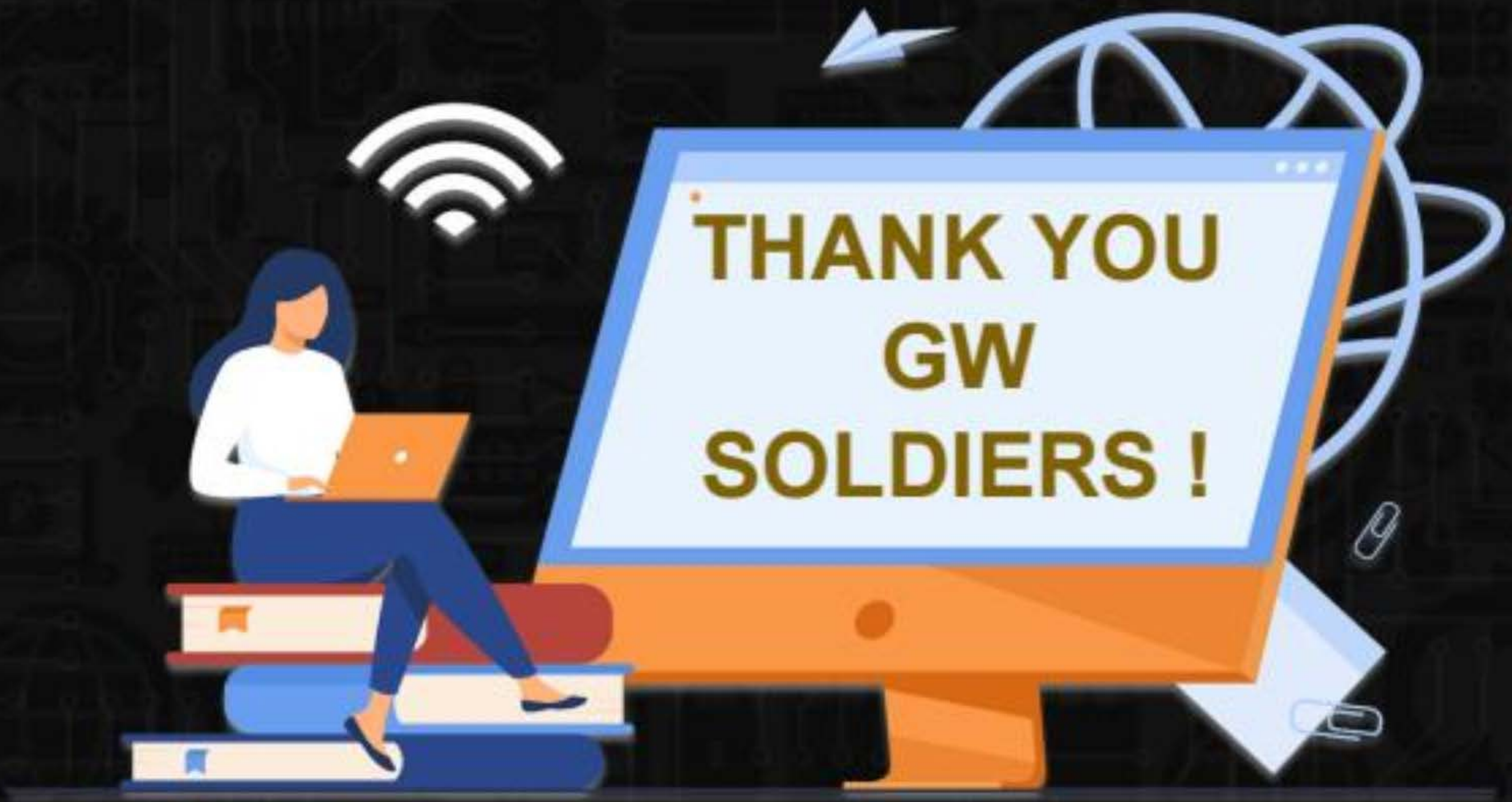
Q.6 Identify valid 7 bit hamming code.

A. 0110011

B. 1011011

C. Both A & B

D. None of these


THANK YOU
GW
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