ALL BRANCHES

ME,CE,EC,EE,CS

Probability and Statistics

Lecture No- 01

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Topics to be Covered

01 Basic probability

02 Basic Theorems on probability

03 Conditional probability

04 Bayes' Theorem

Revision

1. Single Variable calculus
2. Multi Variable calculus

$\}\rightarrow$ All branches.

3. Ordinary Differential Equations
4. Partial Differential Equations
5. Vector calculus

$\}\rightarrow$ ME, CE, EC, EE

6. Complex Calculus $\rightarrow$ ME, EC, EE

Basic Probability

Probability → chance (or) Possibility.

Experiments → Determinstic Experiments → outcome is certain
→ Undeterminstic Experiments → outcome is uncertain.
(Random Experiments)
→ Probability exist since outcome is unpredictable.

→ Sample Space (S): The set containing all the Possible outcomes of a Random experiment is called Sample space. It is denoted with 'S'.

① If Random Experiment is flipping a coin, S = {Head, Tail}

② If Random Experiment India playing an ODI with Pakistan, S = {Win, Lose, Draw}

③ If an aspirant write GATE exam, S = {Qualified, Disqualified}

④ If Relationship is a Random Experiment, S = {Marriage, Break-up}

Event (E): Any subset of Sample Space is called an Event.

Ex: $\underbrace{\text{'Getting a Head'}}_{\text{Event}}$ when a $\underbrace{\text{coin is flipped}}_{\text{Random Experiment}}$

S = {Head, Tail}

E = {Head}

Probability of an Event:

If $n(S)$ is the total number of elements in the Sample space of a Random Experiment and if $n(E)$ is the number of favourable elements of E in Sample space 'S', then Probability of event 'E' to happen is denoted by $P(E)$ and it is given by

$$P(E) = \frac{n(E)}{n(S)}$$

Ex: If RE is Rolling a dice, then P{Getting a Prime number} = ?

$$S = \{1, 2, 3, 4, 5, 6\}$$

Prime Number $= \{2, 3, 5\}$

$$P(\text{Getting a Prime Number}) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

→ Axioms of Probability:

① For any Event 'A' with in a Sample space 'S',

$$0 \le P(A) \le 1$$

$n(S)$

$n(A)$

$$\frac{\overset{0}{\cancel{0}}}{\cancel{n}(S)} \le \overset{P(A)}{\boxed{\frac{n(A)}{n(S)}}} \le \frac{\overset{1}{\cancel{n(S)}}}{\cancel{n}(S)}$$

② $P(S) = 1 \Rightarrow$ Every Random Experiment will have a result.

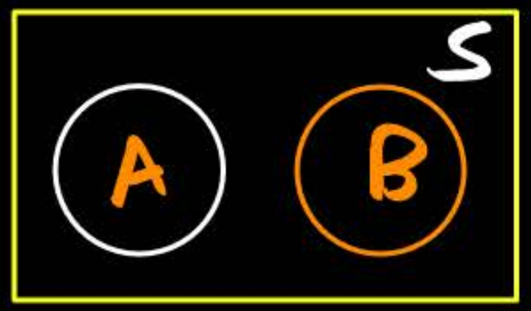
→ Venn-Diagrams:

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\text{Area of circle}}{\text{Area of rectangle}}$$

Types of Events:

① **Mutually Exclusive Events:** $\rightarrow P(A\cap B)=0$



If 'A' and 'B' are two events in a Sample space 'S', they are said to be mutually Exclusive if $A\cap B=\phi=\{\ \}$

i.e Events A and B doesn't have any element in common.

Ex: If Rolling a dice is Random Experiment, then

$A \rightarrow$ Getting a Prime Number

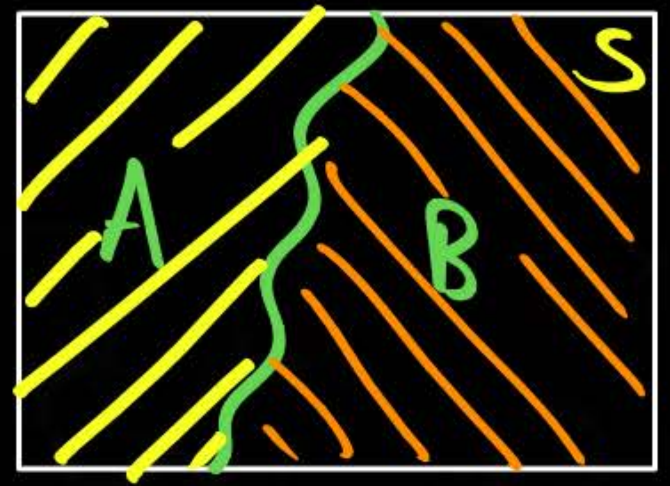
$B \rightarrow$ Getting a Composite Number

$S=\{1,2,3,4,5,6\}$

$\left.\begin{array}{l} A=\{2,3,5\} \\ B=\{4,6\} \end{array}\right\} A\cap B=\phi$

(2) Mutually Exhaustive Events:

Two events A and B with in a Sample space 'S' if $A \cup B = S$. (i.e When the Random Experiment is conducted, definitely either 'A' or 'B' comes as an outcome).



Ex: If RE is Rolling a dice, then

A → Getting an odd Number

B → Getting an Even Number

$\boxed{\therefore P(A \cup B) = P(S) = 1}$

$S = \{1,2,3,4,5,6\}$; $A = \{1,3,5\}$, $B = \{2,4,6\} \Rightarrow A \cup B = S$ & $A \cap B = \phi$

(3) Independent Events:

Two Events E_1 and E_2 with in a Same Sample Space (or) different Sample Spaces are Said to be Independent events if Happening of one event doesn't Impact the happening of other.

Ex: $E_1 \rightarrow$ Getting a Head when a coin is flipped.

$E_2 \rightarrow$ Getting an Odd Number when a dice is Rolled.

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$P(E_1) = \frac{1}{2}; \quad P(E_2) = \frac{1}{2}.$$

$$P(E_1 \cap E_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Ex: If $E_1 \rightarrow$ Getting a Head in a 1^{st} toss.
$E_2 \rightarrow$ Getting a Tail in 2^{nd} Toss.

$$\boxed{P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)} \rightarrow \text{Independent.}$$

(4) Impossible Event: The Event which Can't happen when a Random Experiment is Conducted.

$$\Rightarrow P(\text{Impossible Event}) = 0.$$

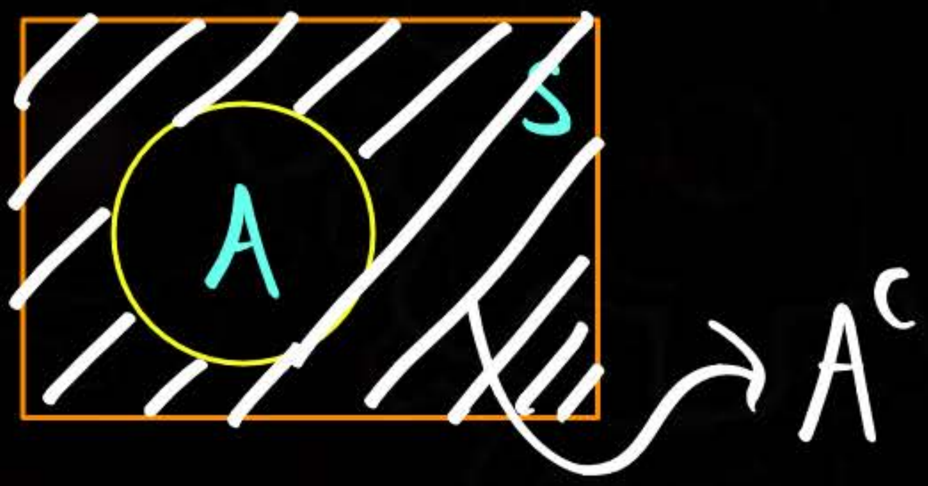
→ Compliment of an Event:

If 'A' is any event with in a sample space 'S' and $P(A)$ is the Probability of happening of event A, then A^c (or) $\bar{A}$ denotes not happening of 'A' and $P(A^c)$ (or) $P(\bar{A})$ denotes the Probability of not happening of event 'A'.

$$n(A)+n(A^c)=n(S)$$

$$\Rightarrow \overbrace{\frac{n(A)}{n(S)}}^{P(A)}+\overbrace{\frac{n(A^c)}{n(S)}}^{P(A^c)}=\overbrace{\frac{n(S)}{n(S)}}^{1}$$

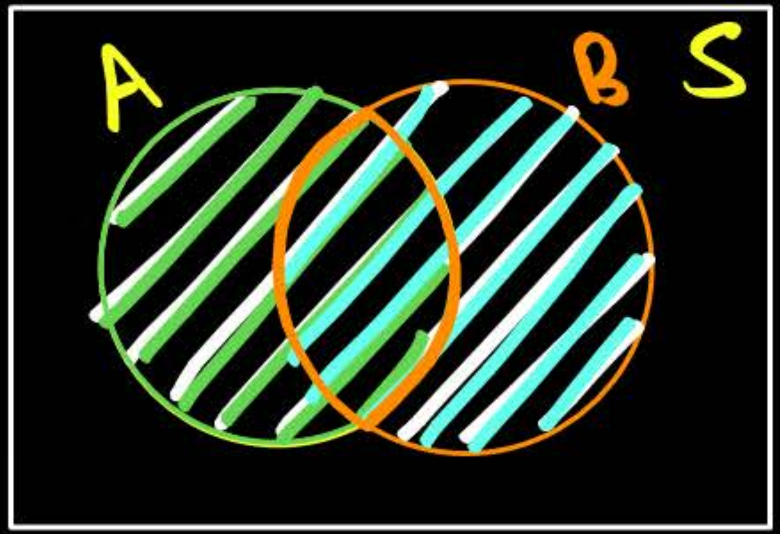
$$\boxed{\Rightarrow P(A^c)=1-P(A)}$$



Basic Theorems on Probability

① Addition Theorem of Probability:

If 'A' and 'B' are two Events within a Sample Space 'S', then



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cup B \rightarrow$ Atleast one of 'A' or 'B'.

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

Case (i): If A and B are mutually Exclusive, then $A \cap B = \phi$

$$\Rightarrow P(A \cap B) = 0.$$

$$\Rightarrow P(A \cup B) = P(A) + P(B).$$

If $E_1, E_2, E_3, \ldots E_n$ are 'n' collectively Exclusive Events in a Sample Space 's', then

$$\boxed{P(E_1 \cup E_2 \cup E_3 \cup \ldots E_n) = P(E_1) + P(E_2) + \ldots + P(E_n)}$$

Case (ii): If 'A' and 'B' are Independent Events, then

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= P(A) + P(B) - (P(A)\cdot P(B))$$
$$= P(A) + P(B)(P(A^c))$$

Since $P(A\cup B) = P(A) + P(B) - \{P(A)\cdot P(B)\}$
$$= P(A)\cdot P(B^c) + P(B)$$

$$\boxed{\therefore P(A\cup B) = P(A) + P(B)\cdot P(A^c) = P(B) + P(A)\cdot P(B^c)}$$

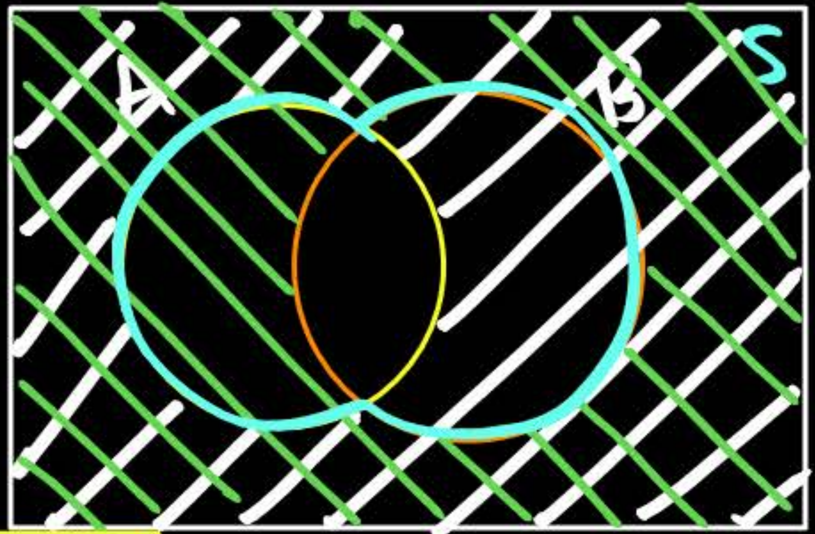
$\rightarrow P(A^c \cap B^c) = 1 - P(A \cup B)$

$= P((A \cup B)^c)$

$A^c \rightarrow S - A$

$B^c \rightarrow S - B$

$A^c \cap B^c = S - (A \cup B)$



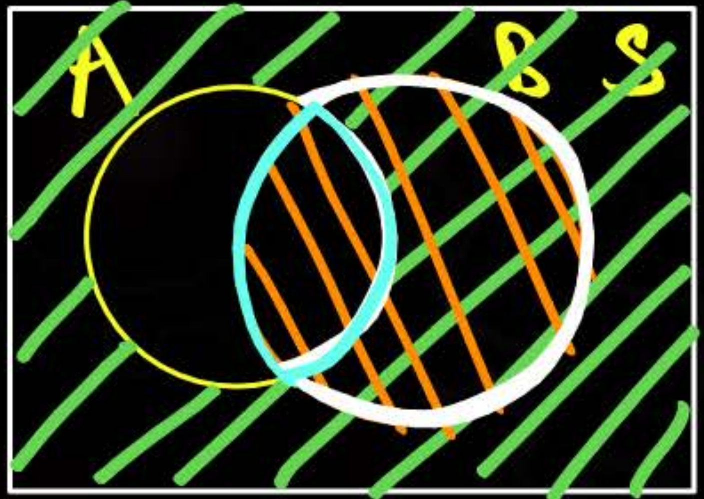
$$\boxed{\Rightarrow P(A^c \cap B^c) = 1 - P(A \cup B)}$$

$\rightarrow P(A^c \cap B) = B - (A \cap B)$

$$\boxed{\Rightarrow P(A^c \cap B) = P(B) - P(A \cap B)}$$

$A^c \rightarrow S - A$

$B \rightarrow B$



→ $\boxed{\text{Permutations}}$ & $\boxed{\text{Combinations:}}$

Arrangements.

Selection.

Selecting 'r' things from 'n' different things $= {}^nC_r = \dfrac{n!}{(n-r)!\cdot r!}$

$1, 2, 3, 4 \rightarrow 3$

4 ways
$$\left\{\begin{array}{l} 1, 2, 3 \\ 1, 2, 4 \\ 1, 3, 4 \\ 2, 3, 4 \end{array}\right.$$

$${}^4C_3 = \frac{4!}{(4-3)!\,3!} = \frac{4!}{1!\cdot 3!} = \frac{4\times \cancel{3!}}{1\times \cancel{3!}}$$

$= 4$

$^nP_r \rightarrow$ Selecting 'r' objects from 'n' different objects and arranging the objects in 'r' Places.

$$^nP_r = {^nC_r} \times r!$$

$$= \frac{n!}{(n-r)!\,\cancel{r!}} \times \cancel{r!}$$

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$

$0! = 1$

$$^nC_n = 1$$

$$\Rightarrow \frac{\cancel{n!}}{(n-n)!\,\cancel{n!}} = 1$$

$$\Rightarrow \frac{1}{0!} = 1$$

$$\boxed{\Rightarrow 0! = 1}$$

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$

1, 2, 3, 4, 5

Thank You!

GW Soldiers