CS & IT ENGINEERING

GRAPH THEORY
Types of Graphs
Part 3

Lecture No. 5

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TOPICS TO BE COVERED

01 Isomorphic Graph

02 Hypercube Graph

03 Operation in graph 1

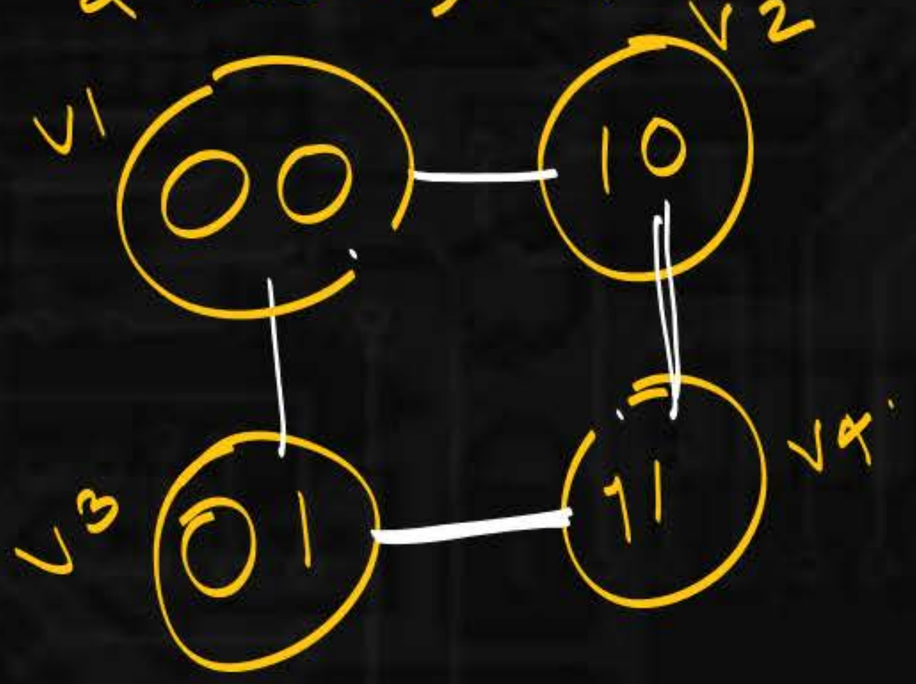
04 Operation in graph 2

05 Operation in graph 3

Types of graph

Consider a graph where vertices are represented as n-bit signal. Two vertices are connected, when between them 1 bit changes, what will be total no. of vertices in a graph?

$n = 2$ 2 bit signal



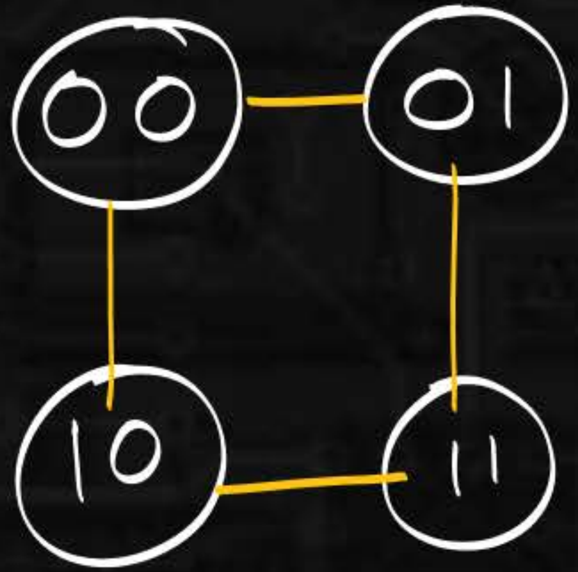
Types of graph

$n = 1$ (1 bit)

Total no. of vertices $= 2^1 = 2$

Q_1.

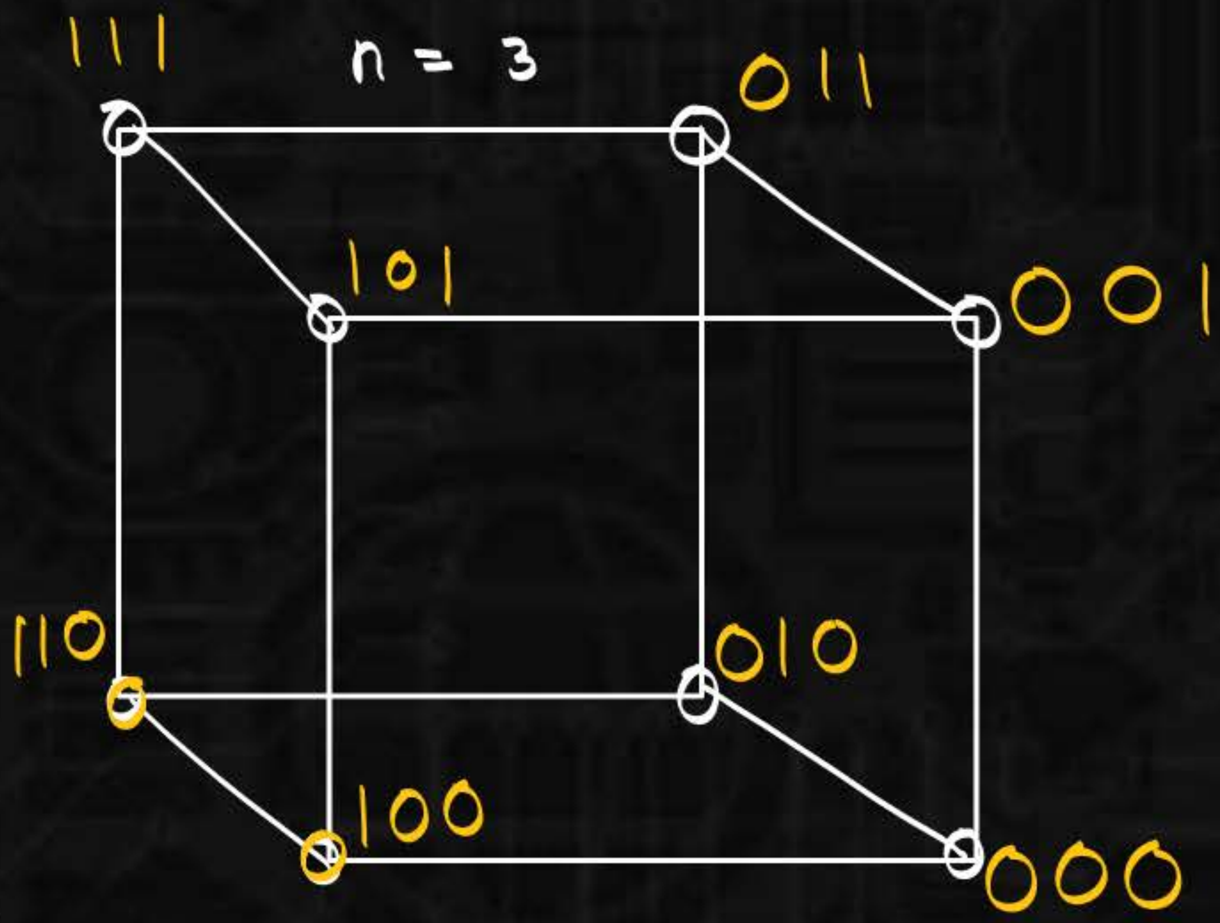
$n = 2$ bit



Total no. of vertices $= 2^2 = 4$.

Q_2

$n = 3$



Total vertices $= 2^3 = 8$

Q_3

Types of graph

n - bit signal.

Total no of vertices $= 2^n$.

Degree of each vertex $= n$.

$$\sum d(v_i) = 2e$$

$$n \times 2^n = 2e$$

$$\boxed{e = n \times 2^{n-1}}$$

$$\frac{n \times 2^n}{2} = e$$

Hypercube (Q_n)

$\downarrow$

n-bit signal.

Total vertices $= 2^n$.

Types of graph

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2} \quad (n - \text{Total vertices})$$

<u>Hypercube:</u>

$$e(G) + e(\bar{G}) = \frac{v(v-1)}{2} \qquad v = \text{Total vertices} = 2^n$$

$\downarrow$

$$n \times 2^{n-1} + e(\bar{G}) = \frac{2^n(2^n-1)}{2}$$

$$\boxed{e(\bar{G}) = \frac{2^n(2^n-1)}{2} - n \cdot 2^{n-1}}$$

Types of graph

K_V $\quad V-1 \quad V-1 \quad V-1.$

K_{2^n} $\quad 2^n-1 \quad 2^n-1 \quad 2^n-1$

G $\quad [n, n, n, n, n \ldots\ldots -]$

$\overline{G}$ $\quad 2^n-1-n,\ 2^n-1-n,\ 2^n-1-n \ldots\ldots$

Total vertices $= 2^n = V$

$V = 2^n$

1. all Hypercubes are regular Graph.
2. all Hypercubes are bipartite Graph.

Types of graph

n-bit signal (Q_n) $\Rightarrow$ Total vertices $= v = 2^n$

$v = 2^n$

K_v	$v-1$	$v-1$	$v-1$	$v-1$	$\ldots v-1$
G	n	n	n	$n \ldots$	$\ldots n$

$\overline{G}$ $v-1-n, v-1-n$

$2^n-1-n, 2^n-1-n \ldots \ldots$

Types of graph

Graph operations:

$G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

Union ($\cup$)

Intersection ($\cap$)

Ring sum ($\oplus$)

$G_3 = G_1 \cup G_2$ $G_3 = (V_3, E_3)$

$V_3 = V_1 \cup V_2$

$E_3 = E_1 \cup E_2$

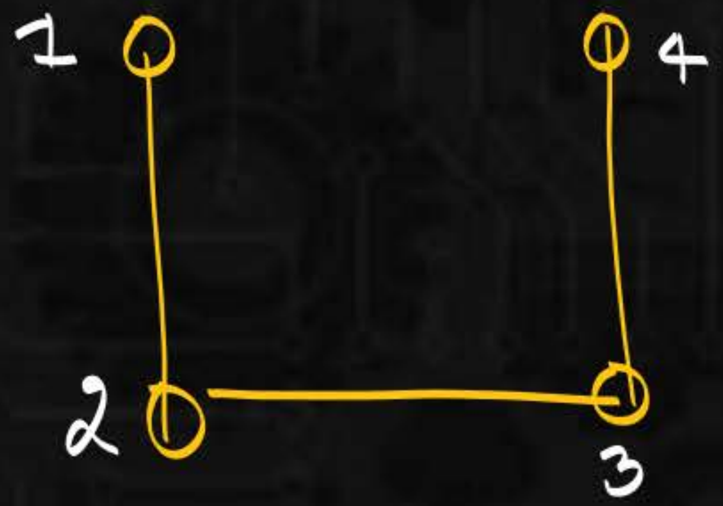
$G \cup G = G$

$G \cap G = G$

$G \oplus G =$ null Graph.

Types of graph

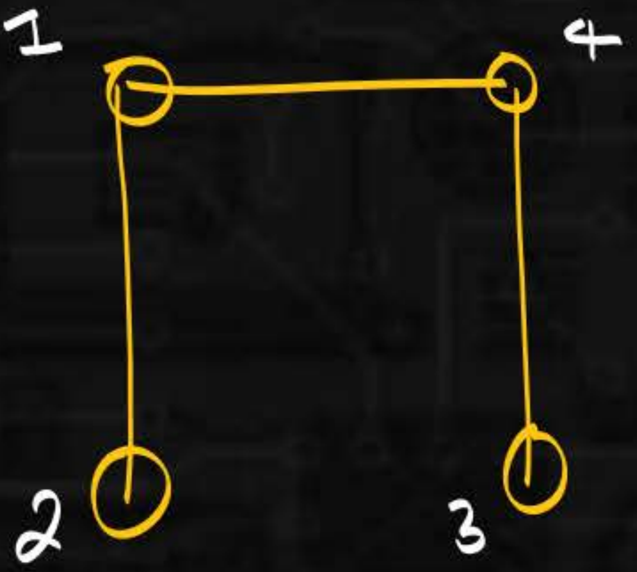
G_1



$V_1 = \{1, 2, 3, 4\}$

$E_1 = \{12, 23, 34\}$

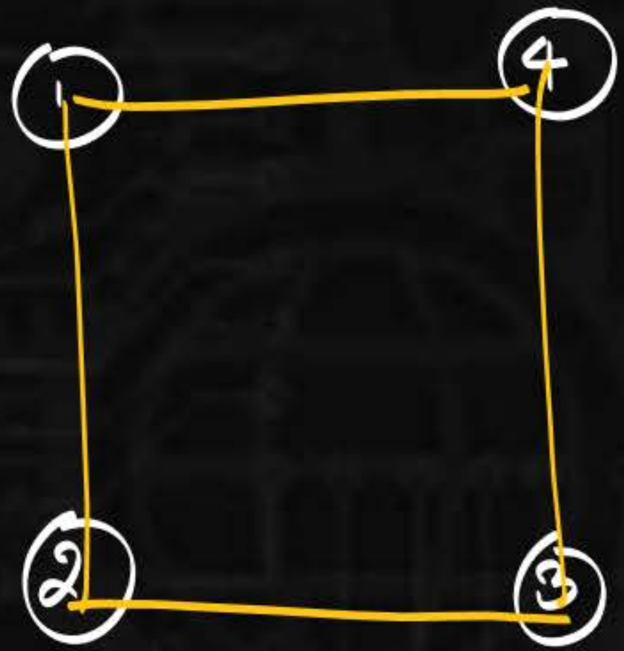
G_2



$V_2 = \{1, 2, 3, 4\}$

$E_2 = \{12, 14, 34\}$

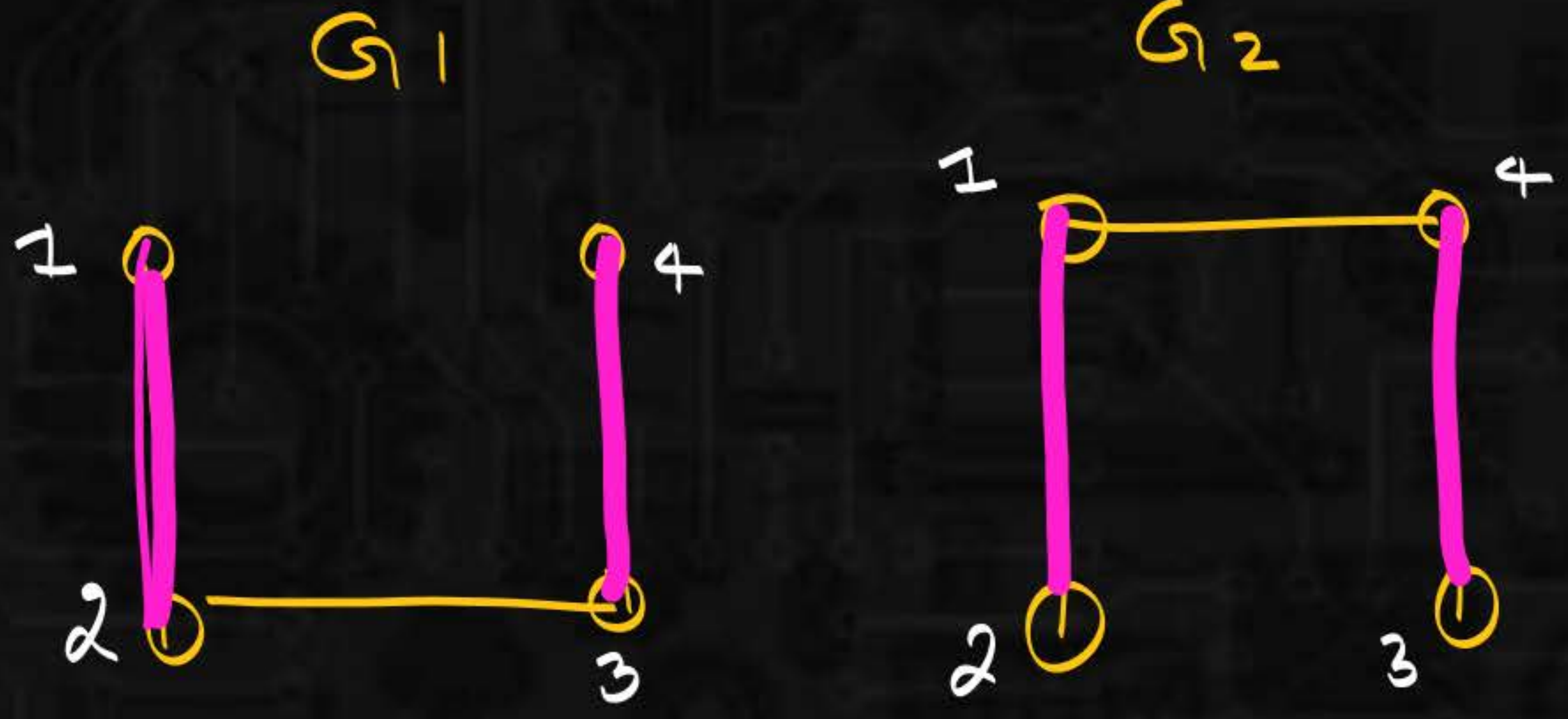
$G_3 = (V_3, E_3)$



$V_3 = V_1 \cup V_2 = \{1, 2, 3, 4\}$

$E_3 = E_1 \cup E_2 = \{12, 23, 34, 14\}$

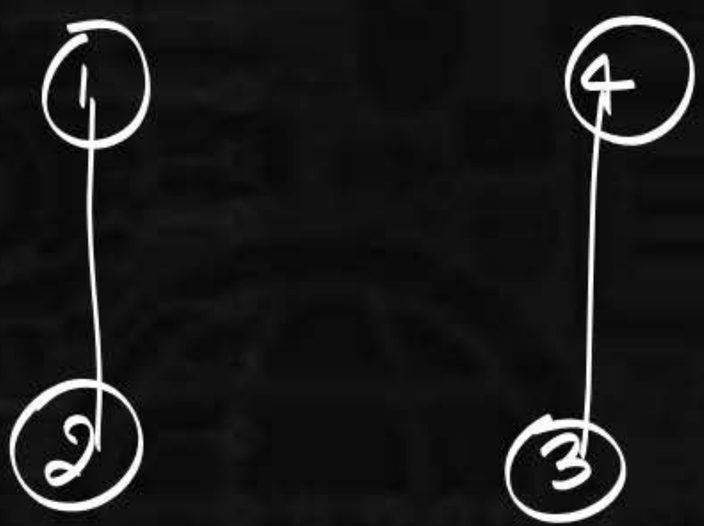
Types of graph



$V_1 = \{1, 2, 3, 4\}$ $\quad$ $V_2 = \{1, 2, 3, 4\}$

$E_1 = \{12, 23, 34\}$ $\quad$ $E_2 = \{12, 14, 34\}$

$G_4 = (V_4, E_4)$



$G_4 = G_1 \cap G_2$

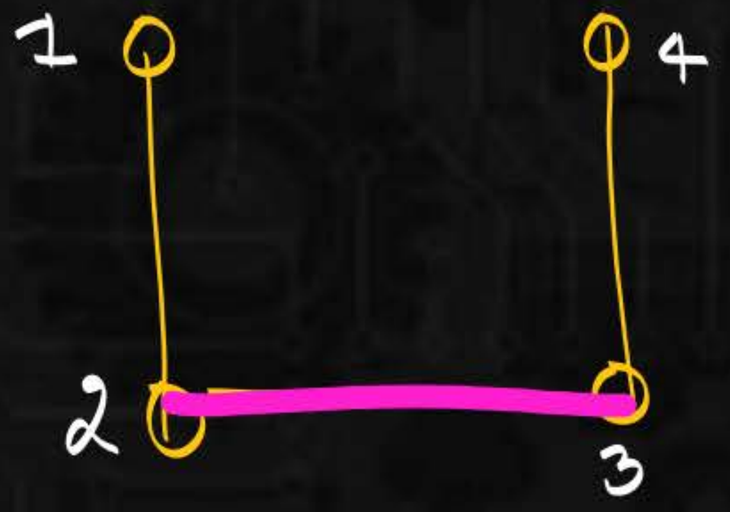
$V_4 = V_1 \cap V_2 = \{1, 2, 3, 4\}$

$E_4 = E_1 \cap E_2 = \{12, 34\}$

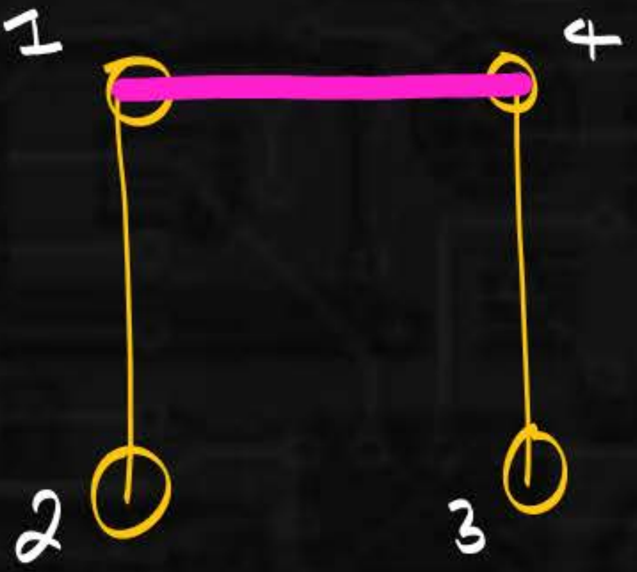
Types of graph

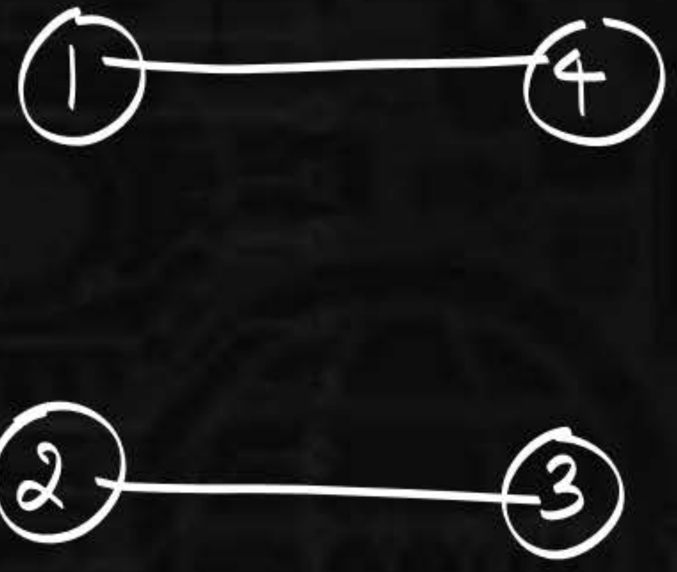
$G_5 = (V_5, E_5)$

G_1



G_2





$V_1 = \{1, 2, 3, 4\}$ $\quad V_2 = \{1, 2, 3, 4\}$

$E_1 = \{12, 23, 34\}$ $\quad E_2 = \{12, 14, 34\}$

$V_5 = V_1 \cup V_2$

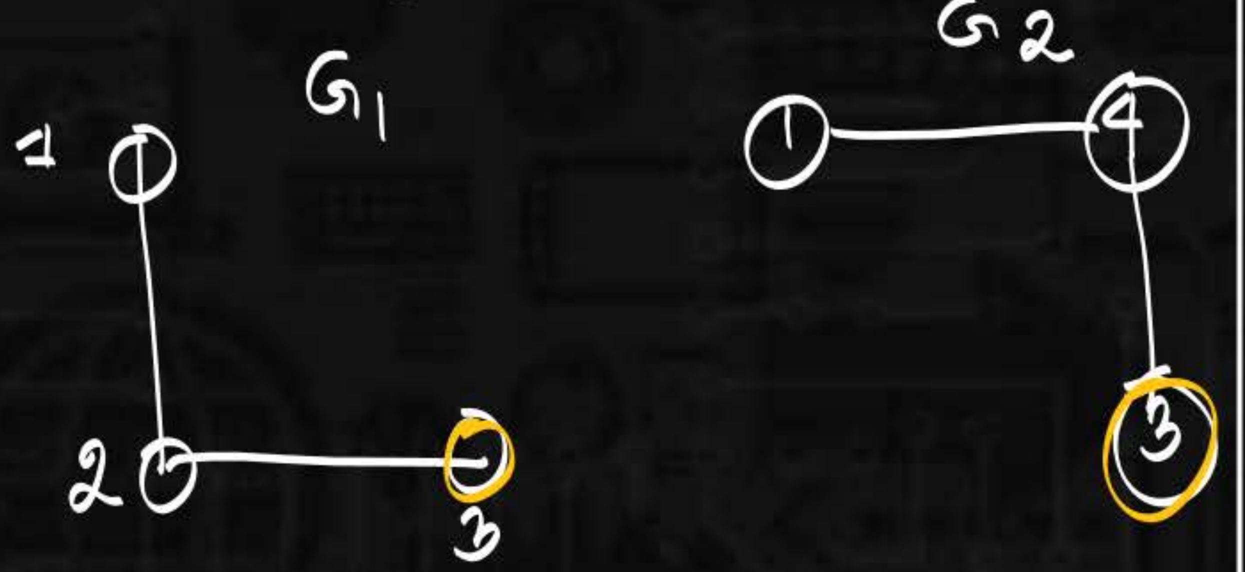
$E_5 = (E_1 \cup E_2) - (E_1 \cap E_2)$

not in both.

Types of graph

edge disjoint Graphs:

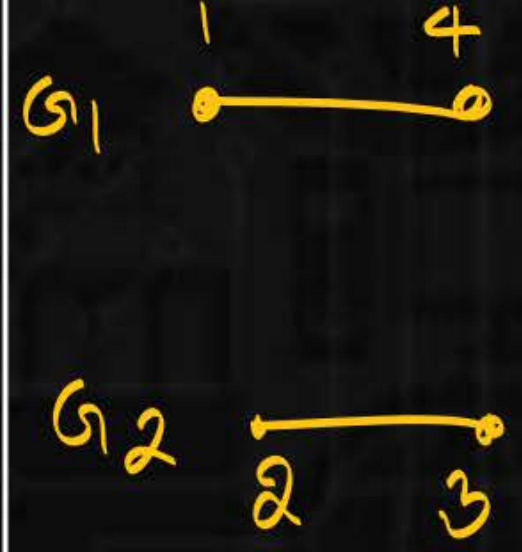
G_1, G_2 such that they are not having any common edges.



edge disjoint Graphs may have common vertex but vertex disjoint Graph will not have any common vertex/edge

vertex disjoin Graphs:

G_1, G_2 are two graphs such that they are not having any common vertices.

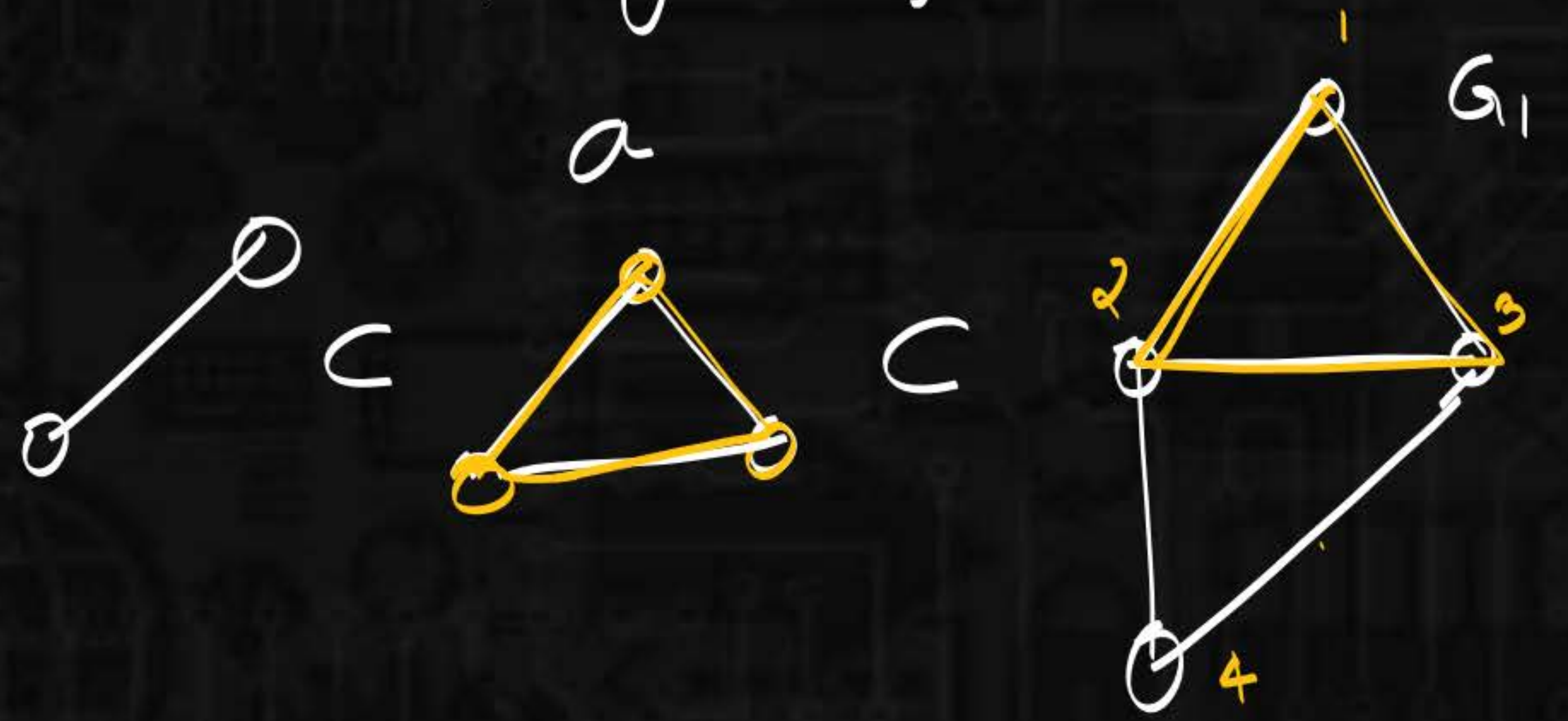


Types of graph

<u>Subgraphs</u> : ($\subset$)

$a \subset G$

a is subgraph of G, if all vertices & edges of a are in G.



Types of graph

1) $G \subset G$.

2) single vertex $\subset G$.

3) single edge $\subset G$.

4) $a \subset b \subset G$ $\qquad$ $a \subset G$.

Subgraph of a subgraph of a Graph is subgraph of a Graph.

Types of graph

vertex are represented as numbers 1 .... 100.

two vertices are
connected with each other

1 — 5 — 9 — 13

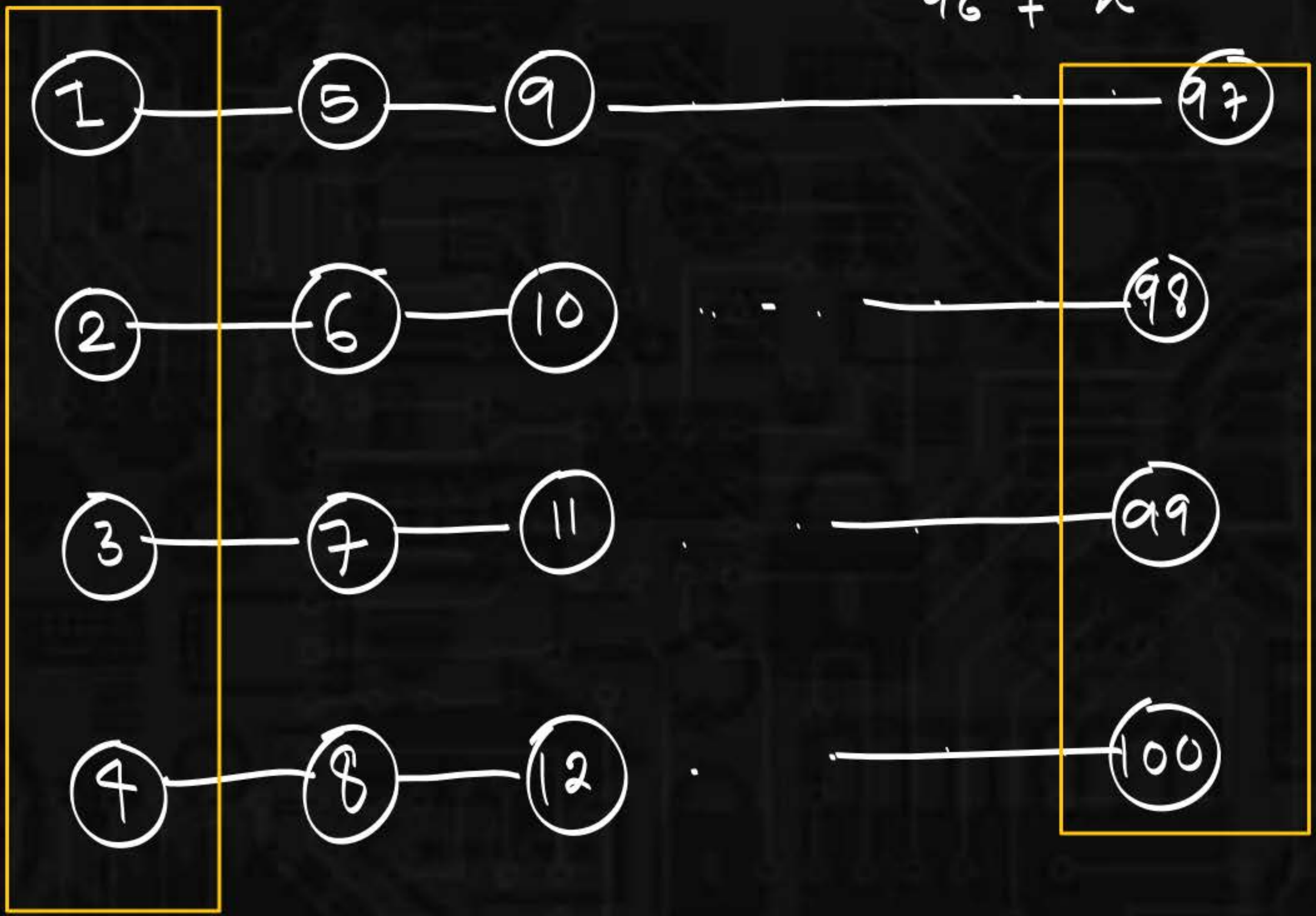
$|x-y| = 4$ x, y are vertices.

$e(G) = ?$ $e(\overline{G}) = ?$

Types of graph

$$e(G) + e(\bar{G}) = \frac{100 \times 99}{2}$$

$$96 + x$$



$$4 \times 1 + 4 \times 1$$

$$+ 92 \times 2 = 2e$$

$$e = 96$$


THANK YOU GW SOLDIERS !