CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-5

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TOPICS TO BE COVERED

CRC

Cyclic Code

Cyclic Code

Cyclic code :

- Cyclic code are special Linear Block codes with one extra property.
- In Cyclic code, if a codeword is cyclically shifted (rotated), the result is another codeword.

Suppose, C is a Code Word given as

$C = [C_1, C_2, C_3 \ldots\ldots C_{n-1}]$

Then after cyclic shifts

Right Shift

$C = [C_1, C_2, C_3 \ldots\ldots C_{n-1}]$

$C^0 = [C_{n-1}, C_1, C_2 \ldots\ldots C_{n-2}]$

$C^1 = [C_{n-2}, C_{n-1}, C_1\ C_2, \ldots\ldots C_{n-3}]$

Or

$C_1\ C_2\ C_3\ C_4$

$C_4\ C_1\ C_2\ C_3$

$c_1 c_2 c_3 c_4$

$c_4 c_1 c_2 c_3$

Left Shift

$C = [C_1, C_2, C_3 \ldots\ldots C_{n-1}]$

$C^0 = [C_2, C_3 \ldots\ldots C_{n-1,} C_1]$

Or

$C_1\ C_2\ C_3\ C_4$

$C_2\ C_3\ C_4\ C_1$

$c_1 c_2 c_3 c_4$

$c_2 c_3 c_4 c_1$

Linear Block codes :

- A Linear block code is a code in which the XOR (⊕) of two valid code words create another valid code word.
- Today all most all error detecting codes are linear block codes: Non Liner block codes are difficult to implement.
- It is simple to find the minimum Hamming distance for linear block code the minimum Hamming distance is the number of 1's in a Non zero valid code word with the smallest Number of 1's

Ex1 :

Valid code word

(a) 0 0 0

(b) 0 1 1

(c) 1 0 1

(d) 1 1 0

XOR (a, b) = 011 (valid code word)

XOR (a, c) = 101 (valid code word)

XOR (a, d) = 110 (valid code word)

XOR (b, c) = 110 (valid code word)

XOR (b, d) = 101 (valid code word)

XOR (c, d) = 011 (valid code word)

So above code word is Liner block code.

Min Hamming distance = 2 (min. no. of 1's in the non zero code word)

Cyclic Code

Ex :

Valid code word

(a) 0 0 0

(b) 0 1 1

(c) 1 0 1

(d) 1 1 0

Linear Block code

Right shift	Left shift
0 1 1	0 1 1
1 0 1	1 1 0
1 1 0	1 0 1
0 1 1	0 1 1

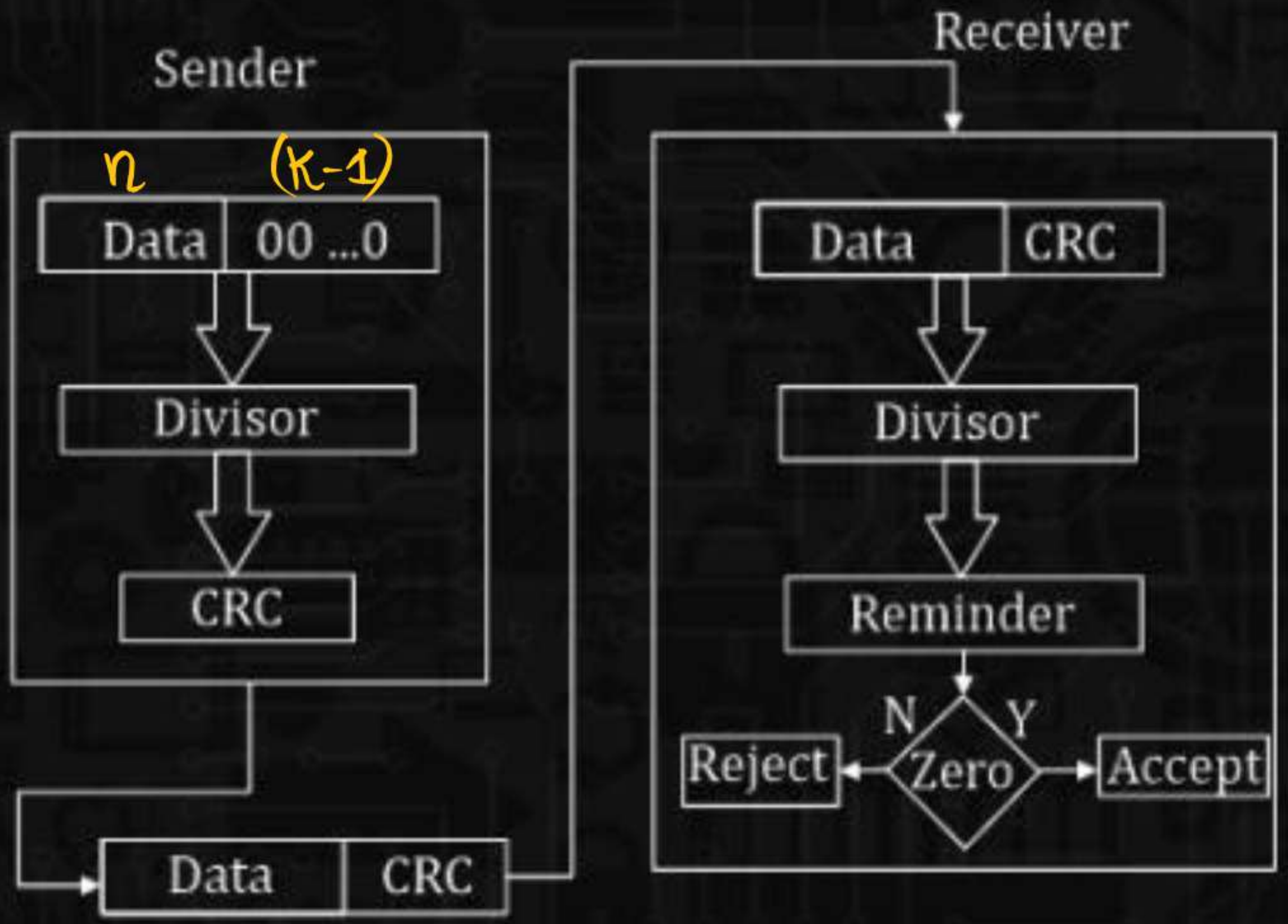
Introduction To CRC

Introduction to CRC :

- Length of the dataword=n
- Length of the divisor=k
- Append (k-1) Zero's to the original message
- Perform modulo 2 division
- Remainder of division = CRC
- Code word =(n+k-1) bits

Note: CRC must be (k-1) bits

- Codeword = dataword with appended (k-1) Zeros+ CRC



Sender
n
(K-1)
Data
00 ...0
Divisor
CRC
Data
CRC
Receiver
Data
CRC
Divisor
Reminder
N
Y
Reject
Zero
Accept

Ex- Data=1001001 , n=7

Divisor or CRC generator=1101 , k=4

sender

```
1101 ) 1001001000
       1101
       0100001000
        1101
        010101000
         1101
         01111000
          1101
          0010000
            1101
            01010
             1101
             0111
```

111 → Remainder OR CRC

Codeword = 1001001 111

Transmitted data = 1001001111

or

Codeword = 1001001 000
+111

1001001111

Codeword = (n+k−1) bits
= 7+4−1 = 10 bits

If Receiver Received uncorrupted data

Receiver

```
1101 ) 1001001111
       1101
       0100001111
        1101
        010101111
         1101
         01111111
          1101
          0010111
            1101
            01101
             1101
             0000
```

Syndrom = 0 OR Remainder = 0

Dataword Accepted (1001001)

If Receiver Received corrupted data

Receiver

```
1101 ) 1011001111
       1101
       ----------
       0110001111
        1101
        ---------
        000101111
           1101
           ------
           011011
            1101
            -----
            00001
```

$\Rightarrow$ syndrom $\neq 0$ OR Remainder $\neq 0$

Dataword Rejected by the Receiver

Polynomial Notation In CRC

Polynomial Notation in CRC

- Data word=d(x)
- Codeword=c(x)
- Generator=g(x)
- Syndrome=s(x)
- Error=e(x)

Polynomial Notation in CRC

How to apply the CRC step by step :

1. Determine the degree 'r' of g(x) (highest power)
   $g(x) = x^6 + x^3 + 1$, $r = 6$
2. Determine $x^r d(x)$
3. Determine the remainder by dividing $x^r d(x)$ by $g(x)$
4. Codeword= $x^r d(x)$+remainder OR CRC

Q: dataword $d(x) = 1001001$

a_6	a_5	a_4	a_3	a_2	a_1	a_0
1	0	0	1	0	0	1
x^6	x^5	x^4	x^3	x^2	x^1	x^0

$$d(x) = 1\cdot x^6 + 0\cdot x^5 + 0\cdot x^4 + 1\cdot x^3 + 0\cdot x^2 + 0\cdot x^1 + 1\cdot x^0$$

$$d(x) = x^6 + x^3 + 1$$

divisor or generator $g(x) = 1101$

a_3	a_2	a_1	a_0
1	1	0	1
x^3	x^2	x^1	x^0

$$g(x) = x^3 + x^2 + 1 \quad , \; r = 3$$

(1) Determine the degree 'r' of $g(x)$

$$g(x) = x^3 + x^2 + 1, \quad r = 3$$

(2) Determine $x^r \cdot d(x)$

$$x^3 \cdot [x^6 + x^3 + 1]$$

$$= x^9 + x^6 + x^3 \xrightarrow{10\text{ bit}} \begin{cases} \text{No. of 1's} = 3 \\ \text{No. of 0's} = 7 \end{cases}$$

$$= 1 \cdot x^9 + 0 \cdot x^8 + 0 \cdot x^7 + 1 \cdot x^6 + 0 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 0 \cdot x^1 + 0 \cdot x^0$$

$$= \underbrace{1001001}_{\text{(data)}}\ \underbrace{000}_{(k-1)\text{ 0's}}$$

(3) Determine the Remainder by dividing $x^r \cdot d(x)$ by $g(x)$

Sender

$$x^3+x^2+1 \,\big)\, \cancel{x^9}+\cancel{x^6}+x^3 \,\big(\, x^6+x^5+x^4+x^3+x+1$$

$$\cancel{x^9}+x^8+\cancel{x^6}$$

$$\cancel{x^8}+x^3$$

$$\cancel{x^8}+x^7+x^5$$

$$\cancel{x^7}+x^5+x^3$$

$$\cancel{x^7}+x^6+x^4$$

$$\cancel{x^6}+\cancel{x^5}+x^4+\cancel{x^3}$$

$$\cancel{x^6}+\cancel{x^5}+\cancel{x^3}$$

$$\cancel{x^4}$$

$$\cancel{x^4}+x^3+x$$

$$\cancel{x^3}+x$$

$$\cancel{x^3}+x^2+1$$

$$x^2+x+1 \rightarrow \text{Remainder OR CRC}$$

$$\text{CRC} = x^2+x+1$$

$$\text{CRC} = 1\cdot x^2+1\cdot x^1+1\cdot x^0$$

$$\text{CRC} = 111$$

4. codeword $= x^r d(x) +$ Remainder

$$x^9 + x^6 + x^3 + x^2 + x + 1$$

$$1 \cdot x^9 + 0 x^8 + 0 \cdot x^7 + 1 \cdot x^6 + 0 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 1 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$$

100 100 1111

If Receiver Received uncorrupted data

Receiver

$$x^3+x^2+1 \overline{\big) \cancel{x^9}+\cancel{x^6}+x^3+x^2+x+1} \quad \big(x^6+x^5+x^4+x^3+x+1$$

$$\cancel{x^9}+x^8+\cancel{x^6}$$

$$\cancel{x^8}+x^3+x^2+x+1$$

$$\cancel{x^8}+x^7+x^5$$

$$\cancel{x^7}+x^5+x^3+x^2+x+1$$

$$\cancel{x^7}+x^6+x^4$$

$$\cancel{x^6}+\cancel{x^5}+x^4+\cancel{x^3}+x^2+x+1$$

$$\cancel{x^6}+\cancel{x^5}+\cancel{x^3}$$

$$\cancel{x^4}+x^2+\cancel{x}+1$$

$$\cancel{x^4}+x^3+\cancel{x}$$

$$\cancel{x^3}+\cancel{x^2}+\cancel{1}$$

$$\cancel{x^3}+\cancel{x^2}+\cancel{1}$$

syndrom = 0

Dataword Accepted $= x^6+x^3+1$

1001001

Problem solving on CRC

Q.1 Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial $X^3 + X + 1$. Suppose the message $m_4m_3m_2m_1m_0 = 11000$ is to be transmitted. Check bits $c_2c_1c_0$ are appended at the end of the message by the transmitter using the above CRC scheme. The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the check bit sequence $c_2c_1c_0$ is

generator $= x^3+x+1$

GATE 2021 (2m)

$1\cdot x^3+0\cdot x^2+1\cdot x^1+1\cdot x^0$

1011

message or dataword = 11000

A. 111

B. 100 ✓

C. 101

D. 110

Sender

1011) 11000 000 (

1011

01110000

1011

0101000

1011

000100

$c_2 c_1 c_0$ → CRC or Remainder

Q.2 Given the generator function G(X) and the message function M(X) as follow

$$G(X) = X^4 + X + 1 \quad , \; r = 4$$

$$d(x) = M(X) = X^7 + X^6 + X^4 + X^2 + X$$

Calculate the transmission function T(X) or $c(x)$

A. $X^{11} + X^7 + X^5 + X^4 + X^3 + X$

B. $X^{11} + X^{10} + X^8 + X^6 + X^5 + X^2 + X$

C. $X^{10} + X^7 + X^6 + X^2 + X$

D. $X^{11} + X^{10} + X^8 + X^6 + X^5$

(1) $r = 4$

(2) determine $x^r \cdot d(x)$

$$x^4 \cdot (x^7 + x^6 + x^4 + x^2 + x)$$

$$x^{11} + x^{10} + x^8 + x^6 + x^5$$

(3) Determine the Remainder by dividing $x^r \cdot d(x)$ by $g(x)$

Sender

$$x^4+x+1 \;\big)\; \cancel{x^{11}}+x^{10}+\cancel{x^8}+x^6+x^5 \;\big(\; x^7+x^6+x$$

$$\cancel{x^{11}}+\cancel{x^8}+x^7$$

$$\cancel{x^{10}}+\cancel{x^7}+\cancel{x^6}+x^5$$

$$\cancel{x^{10}}+\cancel{x^7}+\cancel{x^6}$$

$$\cancel{x^5}$$

$$\cancel{x^5}+x^2+x$$

$$x^2+x \longrightarrow \text{Remainder OR CRC}$$

$$\text{Codeword} = x^r d(x) + \text{Remainder}$$

$$= x^{11}+x^{10}+x^8+x^6+x^5+x^2+x$$

Q.3 The message 11001001 is to be transmitted using the CRC polynomial $x^3 + 1$ to protect it from errors. The message that should be transmitted is

GATE 2007

H.W

A. 11001001000

B. 11001001011

C. 11001010

D. 110010010011

Q.4 A computer network uses polynomial over GF(2) for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits.

In this network, the message 01011011 is transmitted as.

GATE 2017

A. 01011011010

B. 01011011011

C. 01011011101 ✓

D. 01011011100

generator $= x^3 + x + 1$

$= 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0 = 1011$

sender

$$
\begin{array}{r|l}
1011 & 01011011000 \\
 & \;\;1011 \\ \hline
 & 00000011000 \\
 & \qquad\quad 1011 \\ \hline
 & \qquad\quad 01110 \\
 & \qquad\quad\; 1011 \\ \hline
 & \qquad\quad\; 0101
\end{array}
$$

CRC OR Remainder

Q.5 Consider the following message M = 1010001101. The cyclic redundancy check (CRC) for this message using the divisor polynomial $x^5 + x^4 + x^2 + 1$ is

GATE 2005

H·W

A. 01110

B. 01011

C. 10101

D. 10110

Q.6 Consider generator polynomial function G(x) is $X^3 + 1$, the data stream at sender end is 10110101110101, then calculate CRC

H.W

A. 100

B. 110

C. 101

D. 010


THANK YOU
GW
SOLDIERS !