CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-6

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TOPICS TO BE COVERED

- CRC

Cyclic Code Analysis

Cyclic Code Analysis

- Data word = $d(x)$
- Codeword = $c(x)$
- Generator = $g(x)$
- Syndrome = $s(x)$
- Error = $e(x)$

In Cyclic code

1. If $\boxed{S(x) \neq 0,}$ one or more bit is corrupted.

2. If $\boxed{S(x) = 0,}$ either

a) No bit is corrupted

b) Some bits are corrupted, but decoder fails to detect them

$\boxed{\text{Received code word = sent code word + error}}$

Received code word = c(x) + e(x)

I. If there is no error, $\boxed{e(x) = 0}$ then

Received code word = c(x)

$$\boxed{\frac{c(x)}{g(x)} = 0}$$

II. If their is error, $\boxed{e(x) \neq 0}$ then

Received code word = $\boxed{c(x) + e(x)}$

(1)

Sent code word

1	0	1	1	0	1	0	1

$$
\begin{array}{r}
c(x) = 10110101 \\
e(x) = \quad +1000 \\
\hline
10111101 \\
\hline
\end{array}
$$

Received code word

1	0	1	1	1	1	0	1

x^3

c(x)

1	0	1	1	0	1	0	1

$e(x) = x^3 = 1\cdot x^3 + 0\cdot x^2 + 0\cdot x^1 + 0\cdot x^0 = 1000$

1	0	0	0

②

sent code word $c(x)$

1	1	1	1	0	1	0	1

$$\begin{array}{r} 11110101 \\ +\ 1010 \\ \hline 11111111 \\ \hline \end{array}$$

Received code word

1	1	1	1	1	1	1	1
				x^3		x^1	

$e(x) = x^3 + x^1 = 1\cdot x^3 + 0\cdot x^2 + 1\cdot x^1 + 0\cdot x^0 = 1010$

$c(x)$

1	1	1	1	0	1	0	1

$+$

1	0	1	0

$\boxed{\text{Received code word} = C(x) + e(x)}$

$$\frac{c(x)+e(x)}{g(x)}$$

Received code word $= \dfrac{C(x)}{g(x)} + \dfrac{e(x)}{g(x)}$

$$\frac{c(x)}{g(x)} + \frac{e(x)}{g(x)}$$

$\boxed{\dfrac{C(x)}{g(x)} = 0}$ According to the definition of CRC. So Syndrome is actually the remainder of $\dfrac{e(x)}{g(x)}$

$\boxed{\dfrac{e(x)}{g(x)} = 0}$

(i) Either $\boxed{e(x) = 0}$ [No error]CRC scheme is working fine

(ii) $\boxed{e(x) \neq 0,}$ but we are getting $\boxed{\dfrac{e(x)}{g(x)} = 0}$. it means e(x) is divisible by g(x)

dataward = 1101001

Divisor = 1001

Sender

```
1001 ) 1101001000
       1001
       ----------
       0100001000
        1001
        ---------
        000101000
           1001
           ------
           001100
             1001
             ----
             0101
```

CRC or Remainder

Sent codeword = 1101001 101

Received codeword = 1101000 100

x^3 x^0

[2 bit error]

error $e(x) = x^3 + x^0$

$= 1\cdot x^3 + 0\cdot x^2 + 0\cdot x^1 + 1\cdot x^0$

$= 1001$

Receiver

```
1001 ) 1101000100
       1001
       ----------
       0100000100
        1001
        ---------
        000100100
           1001
           ------
           000000
```

⇒ syndrom = 0 means No error dataword Accepted

$$\boxed{\frac{e(x)}{g(x)} = \frac{1001}{1001} = 0}$$

CRC scheme Fails to Detect the error

Note:

1. CRC is not perfect scheme if e(x) is divisible by g(x) then that error can't be detected.
2. Probability of such error is very less, Hence error detection Probability of CRC is very high.

$$\boxed{\frac{\text{Received cod word}}{g(x)} = \frac{c(x)}{g(x)} + \frac{e(x)}{g(x)}}$$

Syndrome = $s(x)$

1. $\boxed{\text{If } s(x) \neq 0,}$ then code word is rejected and CRC scheme is working fine

2. $\boxed{\text{If } s(x) = 0}$ and $\boxed{e(x) = 0}$ then codeword is accepted & CRC scheme is working Fine.

3. $\boxed{\text{If } s(x) = 0}$ and $\boxed{e(x) \neq 0}$ [$e(x)$ is divisible by $g(x)$] then codeword Accepted and scheme failed to detect the error.

Choosing Divisor in CRC

I. Sent code word $= 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0$

Received code word $= 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1$

x^4

$x^5 = 100000\ (32)$

$x^6 = 1000000\ (64)$

error $e(x) = x^4 = 1\cdot x^4 + 0\cdot x^3 + 0\cdot x^2 + 0\cdot x^1 + 0\cdot x$

$= 10000\ (16)$

II. Sent code word $= 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0$

Received code word $= 1\ 0\ 1\ 0\ 0\ 0\ 1\ 0$

x^3

error $e(x) = x^3 = 1\cdot x^3 + 0\cdot x^2 + 0\cdot x^1 + 0\cdot x^0$

$= 1000\ (8)$

III. Sent code word $= 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0$

Received code word $= 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1$

x^0

error $e(x) = x^0 = 1 \cdot x^0 = 1$

Note:

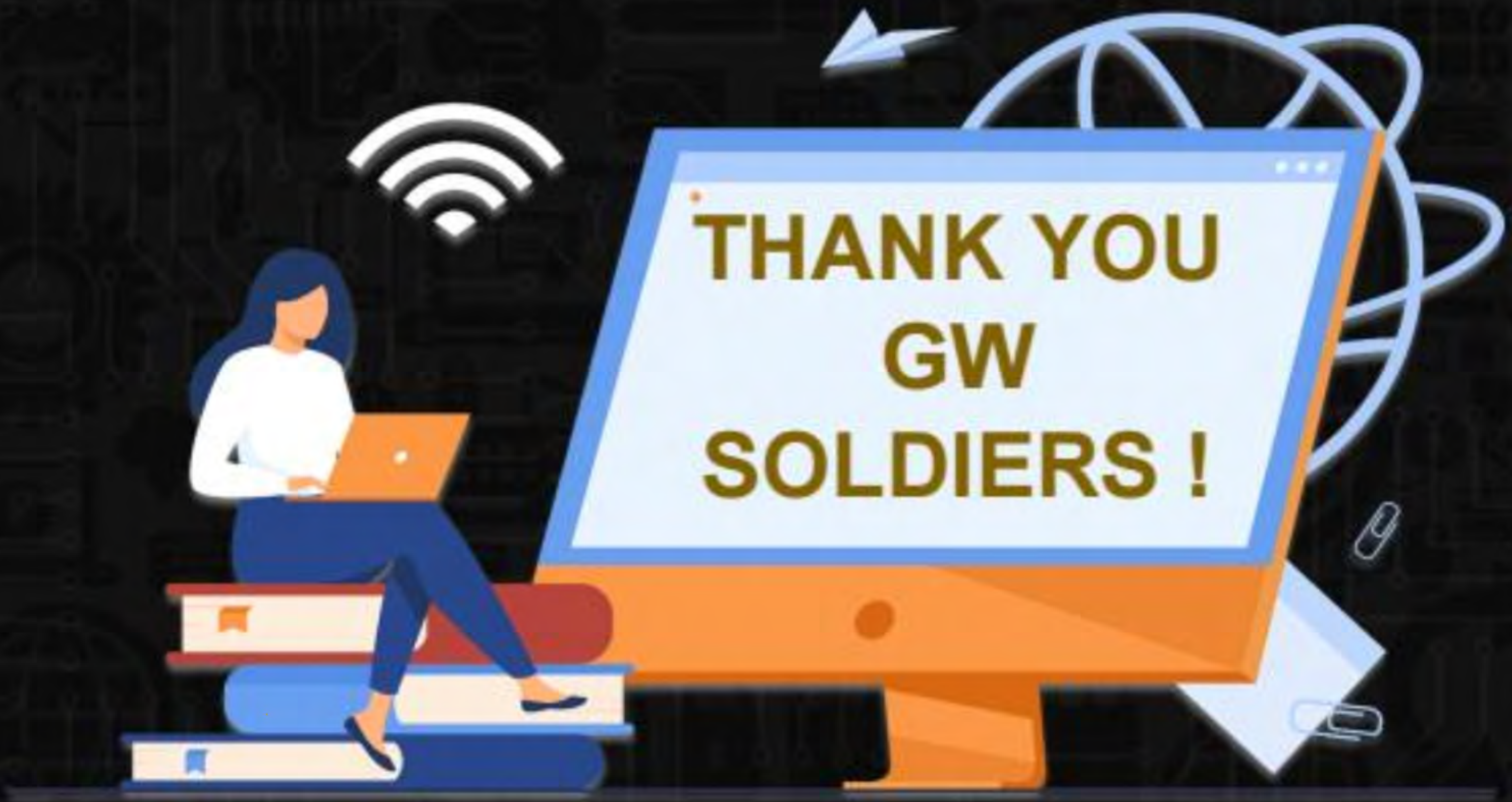
1. Except for last bit error, single bit error gives even number so divisor must not be even. If divisor is also even then remainder will be zero , so we can not caught single bit error

2. To detect all single bit error the last bit of the divisor must be 1. so that divisor becomes an odd number and hence all single bit error detected.

1. If the generator has more than one term and coefficient of x° is 1, all single bit error can be detected.
2. If a generator cannot divide $x^t + 1$ (t between 0 and n – 1) then all isolated Double error can be detected
3. A generator that contains a Factor of x + 1 and detect all odd numbered errors.

A good polynomial generator needs to have the following characteristics:

1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $x^t + 1$, for t between 2 and n - 1.
4. It should have the factor $x + 1$.


THANK YOU
GW
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