CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Basics Of Relations

Lecture No. 6

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TOPICS TO BE COVERED

01 Basics of relations

02 Types of relations

03 Number Of relations

Relations

$P_1 \; R \; P_2$ (w_1, w_2)

$e_1 \; R \; e_2$ $5 > 3$

$\text{Set}_1 \; R \; \text{set}_2$ $A \subseteq B$

$G_1 \; R \; G_2$ $G_1 \equiv G_2$

$\vdots$

Set

$A = \{1, 2, 3\}$

$B = \{2, 3\}$

Relations

$A = \{1, 2, 3\}$ $B = \{2, 3\}$

$A \times B = \{(1, 2)\ (1, 3)\ (2, 2)\ (2, 3)\ (3, 2)\ (3, 3)\}$

$\boxed{a = b}$

$R_1 = \{(2, 2)\ (3, 3)\}$ ✓

Relations

<u>Relation</u>: subsets of crossproduct of 2 set's A, B.

$|A| = m \quad |B| = n. \quad |A \times B| = \underline{m \cdot n}.$

Total no. of Relations $= 2^{m \cdot n}$

eg: no. of Relations $= 2^{n^2}$

$|A| = n \quad |A \times A| = n^2$

Relations

$A = \{1, 2, 3\} \quad B = \{2, 3\}$

$$A \times B = \{(1,2)\ (1,3)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$$

$$A \times B \longrightarrow \boxed{a < b} \longrightarrow R_3 = \{(1,2)\ (1,3)\ (2,3)\}$$

$R_1 = \{(a,b) \mid a = b\}$

$R_1 =$

$$A \times B \longrightarrow \boxed{a = b} \longrightarrow \{(2,2)\ (3,3)\}$$

$$A \times B \longrightarrow \boxed{a > b} \longrightarrow R_2 = \{(3,2)\}$$

$$R_2 = \{(a,b) \mid a > b\}$$

Relations

$Z = \{ \dots\dots\dots \}$ $Z = \{ \dots\dots\dots \}$ $R_1 = \{(a,b) \mid a = b\}$

$Z \times Z$

$(0,0), (1,1)\ (1,2), (1,3)$

$(2,1)\ (2,2) \dots\dots$

$\dots\dots$

c_1 $a = b$

c_2 $a \geq b$

$R_2 = \{(a,b) \mid a \geq b\}$

Relations

$A = \{1, 2\} \quad B = \{2, 3\}$

$A \times B = \{(1, 2)\ (1, 3)\ (2, 2)\ (2, 3)\}$

$|A \times B| = 4$

$2^4 \begin{cases} \phi \\ S_1 \\ S_2 \\ S_3 \\ \vdots \\ A \times B \end{cases}$

$A \times B \longrightarrow \boxed{a+b=1} \longrightarrow R_1 = \phi$

$a+b=3 \quad \boxed{R_2 = \{(1, 2)\}}$

$a+b=4 \quad R_3 = \{(1\ 3)\ (2\ 2)\}$

$a+b=5 \quad R_4 = \{(2\ 3)\}$

$a+b \leq 100$

$R_5 = \{\ldots$

Relations

Symmetric Relation :

A → non empty set.

$$A \times A = \{ \ldots\ldots \}$$

$$\forall a \forall b \left[(a, b) \in R \longrightarrow (b, a) \in R \right]$$

$$R_1 = \{ \overset{a,\ b}{(1,\ 2)} \}$$

not symmetric.

$$(a, b) \in R \rightarrow (b, a) \in R.$$

$$\underline{(1, 2) \in R} \longrightarrow (2, 1) \in R$$

$$T \longrightarrow F$$

$a = 1$ $b = 2$

Relations

$$\left\{ \underline{(a,b)\in R} \longrightarrow \underline{(b,a)\in R} \right\}$$

$A = \{1, 2, 3\}$

$A \times A = \{\ldots\ldots\ldots\}$

Symmetric

$R_2 = \{(\underline{1,2})(2,1)\}$

$(a,b)\in R \longrightarrow (b,a)\in R$

$\underset{T}{\underline{(1,2)\in R}} \longrightarrow \underset{T}{\underline{(2,\cancel{1})\in R}}$

it is symmetric Relation

$R_3 = \{\ \ \}$

$\underset{F}{\underline{(a,b)\in R}} \longrightarrow (b,a)\in R$

$\longrightarrow$

Relations

$$R_3 = \{ (\overset{\checkmark}{1}, 1) (\overset{\checkmark}{2}, 2) \}$$

* Symmetric Relation.

$$(a, b) \in R \longrightarrow (b, a) \in R.$$

$$\underline{(1, 1) \in R} \longrightarrow \underline{(1, 1) \in R}.$$

$a = 1 \quad b = 1$

$R_1 = \{ \ \}$ ✓

$R_2 = \{ (1\ 2) \}$ ×

$R_3 = \{ (1\ 2) (2\ 1) \}$ ✓

$R_4 = \{ (1, 1)$

Relations

$R_1 = \{ \ \} \checkmark$

$R_2 = \{(1,3)\ (3,1)\} \checkmark$

$R_3 = \{(1,2)\ (2,1)\ (1,1)\} \checkmark$

$R_4 = \{(2,3)\ \overset{\checkmark}{(2,2)}\}$

$(a,b) \in R \rightarrow (b,a) \in R.$

$\underbrace{(2,3) \in R}_{T} \rightarrow \underbrace{(3,2) \in R}_{F}$

<u>note</u>:

- → Symmetric demands flipping.
- → no problem with same element

Relations

$$R_1 = \left\{ \overset{\checkmark}{\underset{e_1}{(1,1)}}\ \underset{e_2}{(1,2)}\ \overset{e_3}{(2,1)}\ \overset{\checkmark\ e_4}{\underline{(2,3)}}\ \overset{e_5}{(3,2)} \right\}$$

$$R_1 = \left\{ \overset{\times}{\boxed{(1,2)}}\ \overset{\checkmark}{\boxed{(1\ 1)}}\ \overset{\checkmark}{\boxed{(2\ 3)}}\ \overset{\checkmark}{\boxed{(3,2)}} \right\}$$

$\rightarrow$ not symmetric.

$$\left\{ \overset{\checkmark}{\boxed{(1,1)}}\ \overset{\checkmark}{(1,2)}\ \overset{\checkmark}{(2,3)}\ \overset{\checkmark}{\underline{(2,1)}}\ \overset{\checkmark}{\underline{(3,2)}} \right\} \checkmark$$

Relations

$$R_1 = \left\{ \underset{P}{\boxed{(1,1)}} \ \underset{P}{\boxed{(1,2)\ (2,1)}} \right\}$$

how many symmetric Relations are possible?

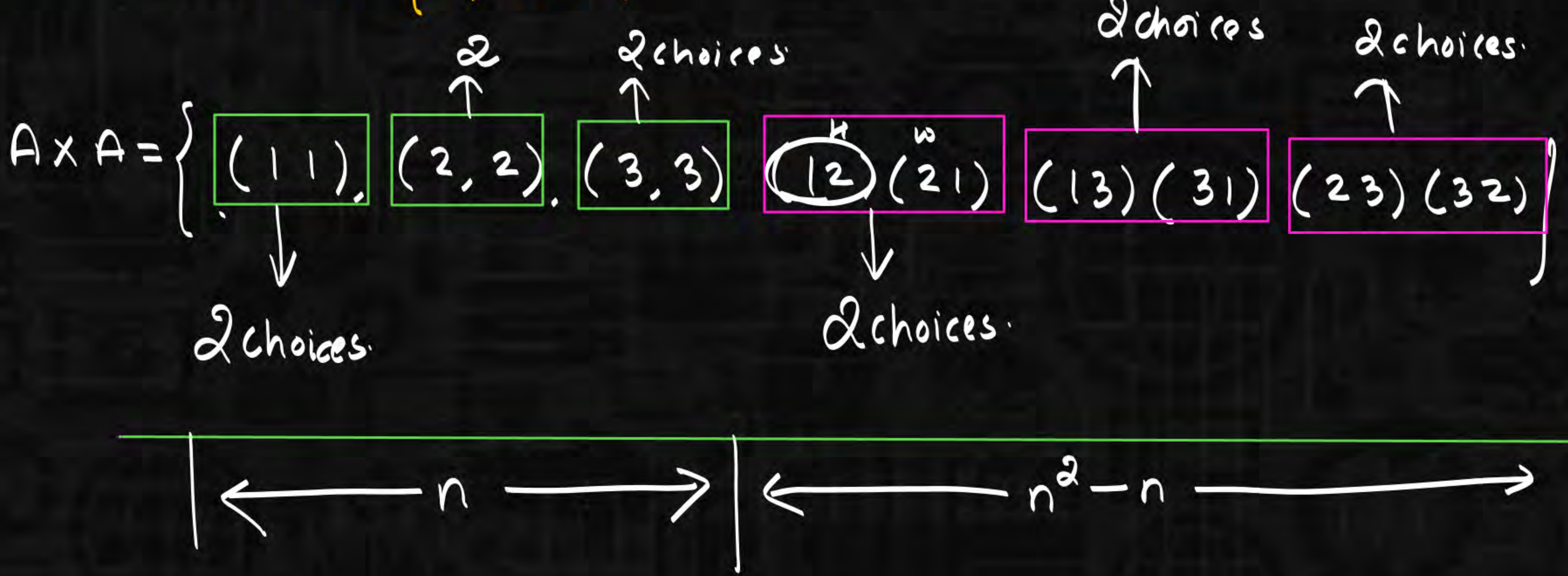
$$R_2 = \left\{ \underset{P}{\boxed{(12)\,(21)}} \right\} \checkmark$$

$$R_3 = \left\{ \boxed{(11)} \right\} \checkmark$$

Relations

$n = 3.$ $\quad$ boxes $= n^2 - n/2.$

$A = \{1, 2, 3\}.$



$A \times A = \{ \boxed{(1\,1)}, \boxed{(2,2)}, \boxed{(3,3)}, \boxed{(12)\,(21)}\ \boxed{(13)\,(31)}\ \boxed{(23)\,(32)} \}$

Relations

A = n
A = {1, 2, 3}

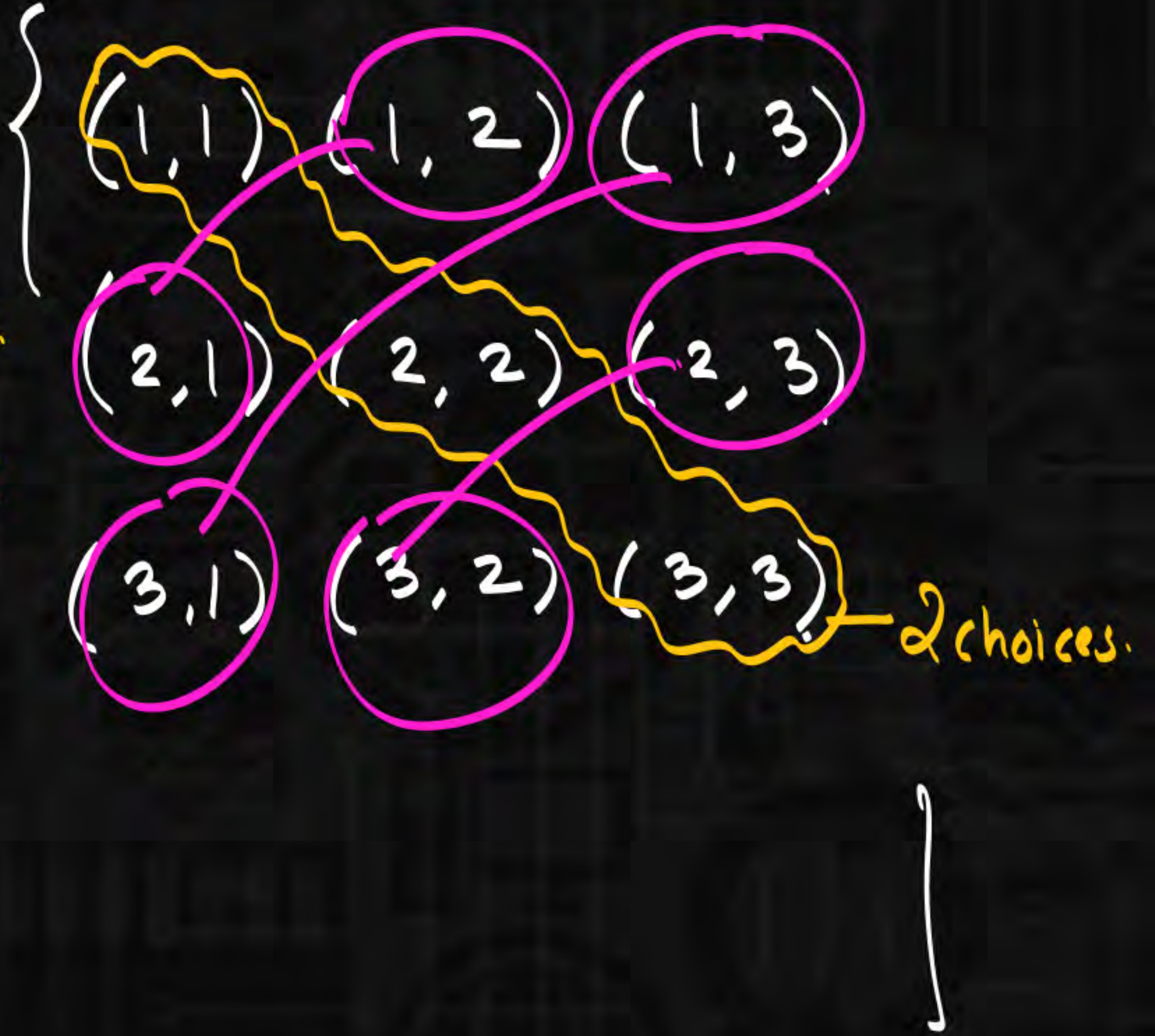
n = 3

$A \times A = n^2$.

Diagonal elements = 3 = n.

non diagonal elements
= $n^2 - n$.

boxes = $\frac{n^2-n}{2}$



$$A \times A = \left\{ \begin{array}{ccc} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{array} \right\}$$

Relations

Diagonal boxes:

$$2^n \qquad 2^{\frac{n^2-n}{2}}$$

# of symmetric Relations = $2^n \cdot 2^{\frac{n^2-n}{2}}$

OR

$$2^{n(n+1)/2}$$

$$= 2^n \cdot 2^{\frac{n^2-n}{2}}$$

$$2^{n+\frac{n^2-n}{2}} = 2^{\frac{2n+n^2-n}{2}}$$

$$= 2^{\frac{n^2+n}{2}} = 2^{\frac{n(n+1)}{2}}$$

Relations

A → non empty set

Reflexive :

$\underline{\forall a \in A}\ (a,a) \in R$

$A = \{1, \textcircled{2}, 3\}$

$A \times A : \{\ldots\ldots\}$

a

$R_1 = \{\underbrace{(1,1)}_{P\checkmark}(1,2)(2,1)(2,3)\}$ ✗

⋮

Relations

$A = \{1, 2, 3\}$

$\forall a \in A \ (a, a) \in R$

$R_2 = \{(2,1)\ (2,3)\ \underset{P\checkmark}{\underline{(2,2)}}\ (1,3)\ \underset{P\checkmark}{\underline{(1,1)}}\ (3,1)\ \underset{P\checkmark}{\underline{(3,3)}}\}$ Reflexive

$R_3 = \{\ \}$ not Reflexive

$R_4 = \{(\overset{\checkmark}{1}\,1)\ (\overset{\checkmark}{2}\,2)\}$

$A = \{1, 2, \boxed{3}\}$

a

Relations

$R_1 = \{ \boxed{11 \quad 22 \quad 33} \}$ ✓

$R_3 = \{ (11)\ (22)\ (33)\ (12)\ (21) \}$ ✓

$R_2 = \{ \boxed{11 \quad 22 \quad 33}\ (12) \}$ ✓

Relations

$A = \{1, 2, 3\}$

$|A| = n = 3$

Diagonal elements $= n$.

non diagonal elements $= n^2 - n$.

$$A \times A = \begin{vmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{vmatrix} \rightarrow$$

present (1 choice)

$$1^n \cdot 2^{n^2 - n}$$

Relations

Total no. of reflexive Relations = 2^{n^2-n}.

$$\overbrace{\overbrace{\underset{\substack{\downarrow\\ \text{present}}}{(1\,1)\ (2\,2)\ (3\,3)}}^{n}\ \Big|\ \overbrace{(1\,2)\ (2\,1)\ (1\,3)\ (3\,1)\ (2\,3)\ (3\,2)}^{n^2-n}}^{n^2}$$

# of Reflexive = 2^{n^2-n}

Relations

$|A| = n$

$|A \times A| = n^2$

Symmetric = $2^{n} \, 2^{n^2-n/2}$.

Reflexive = 2^{n^2-n}

$(R \cap S) =$

Relations

11	12	13
21	22	23
31	32	33

1 choice

R 2^{n^2-n}

S $2^{n} \quad 2^{n^2-n/2}$

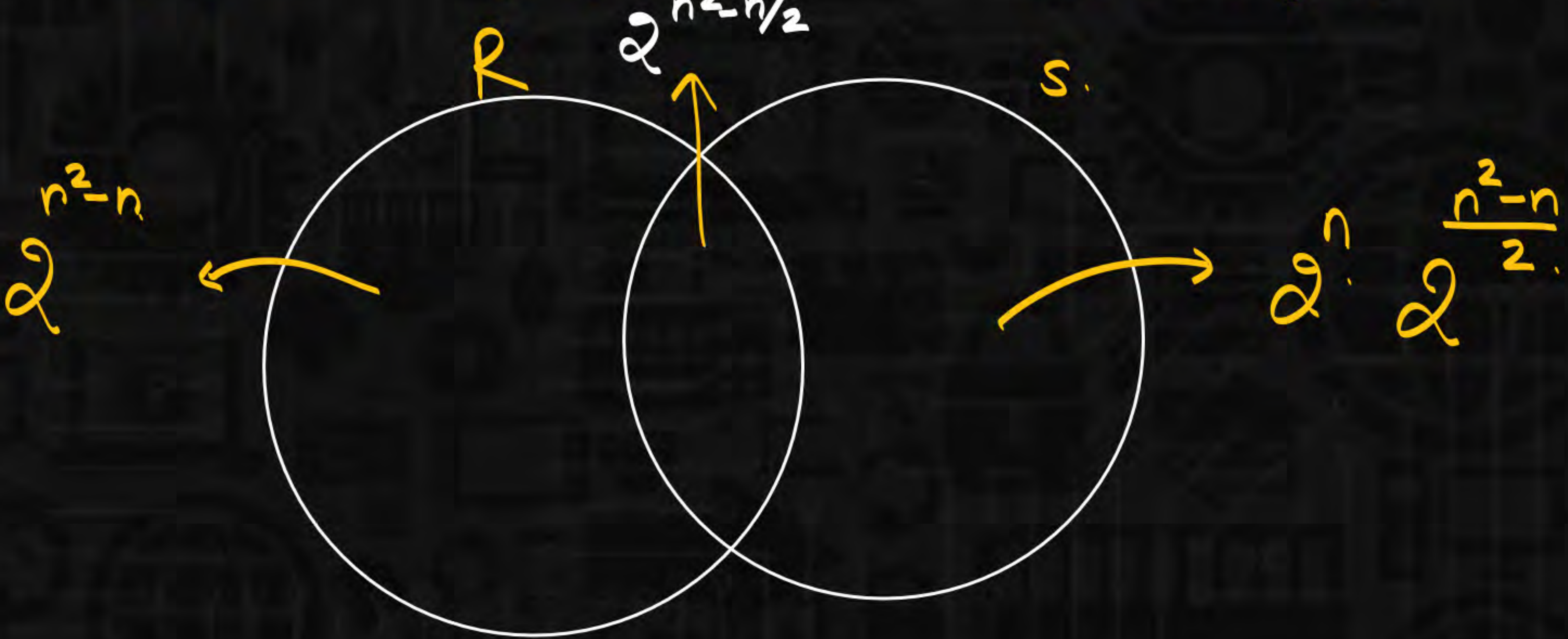
$\downarrow$

1.

$$2^{n^2-n/2}$$

Relations

R → Total no. of Reflexive S → total no. of symmetric.



Relations

$$R \cup S = R + S - R \cap S.$$

$$= 2^{n^2-n} + 2^n \cdot 2^{\frac{n^2-n}{2}} - 2^{\frac{n^2-n}{2}}.$$

Relations

$R_1 = \{(a, b) \mid a + b \leq 3\}$

symm ✓

$aRb \longrightarrow bRa$

$\underline{a+b \leq 3} \longrightarrow \underline{b+a \leq 3}$.

$(1,2) \in R \longrightarrow (2,1) \in R$.

$1+2 \leq 3 \longrightarrow 2+1 \leq 3$

Symmetric: $(a, b) \in R \longrightarrow (b, a) \in R$ (finite)

OR

$aRb \longrightarrow bRa$.

Relations

$R_2 = \{ (a, b) \mid a \leq b \}$.

$\underline{aRb \rightarrow bRa}$.

$a \leq b \rightarrow b \leq a$.

Reflexive: $(a, a) \in R$

OR

$\underline{aRa}$

$R_2 = \{ (a, b) \mid a \leq b \}$ ✓

$a \leq a$ (for all)

Relations

$R_1 = \{ (a,b) \mid a = b+1 \}$

$R_3 = \{ (a,b) \mid a+b \leq 0 \}$

$R_2 = \{ (a,b) \mid \gcd(a,b) = 1 \}$

THANK YOU GW SOLDIERS !