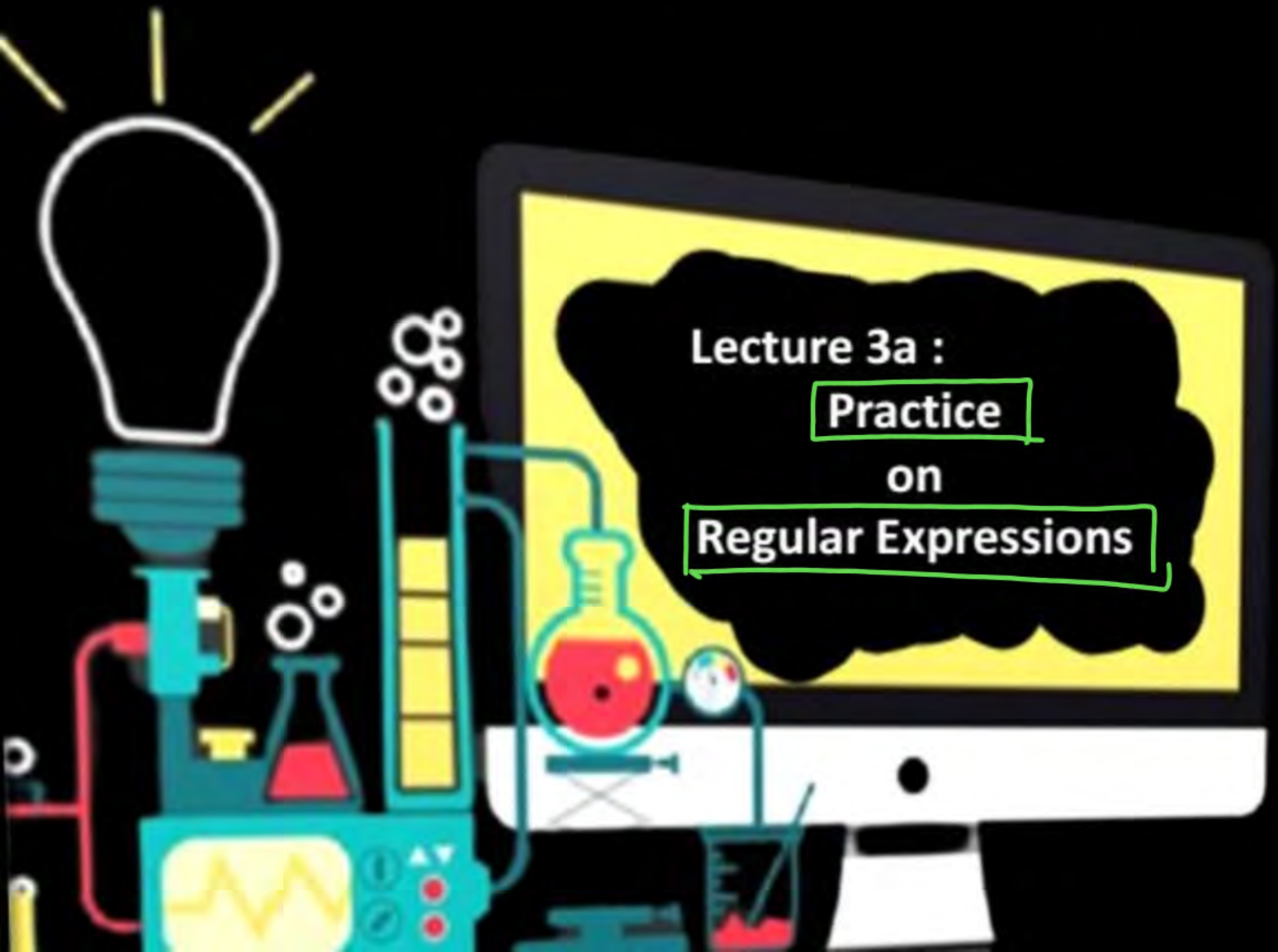
CS & IT Engineering



Lecture 3a :

Practice

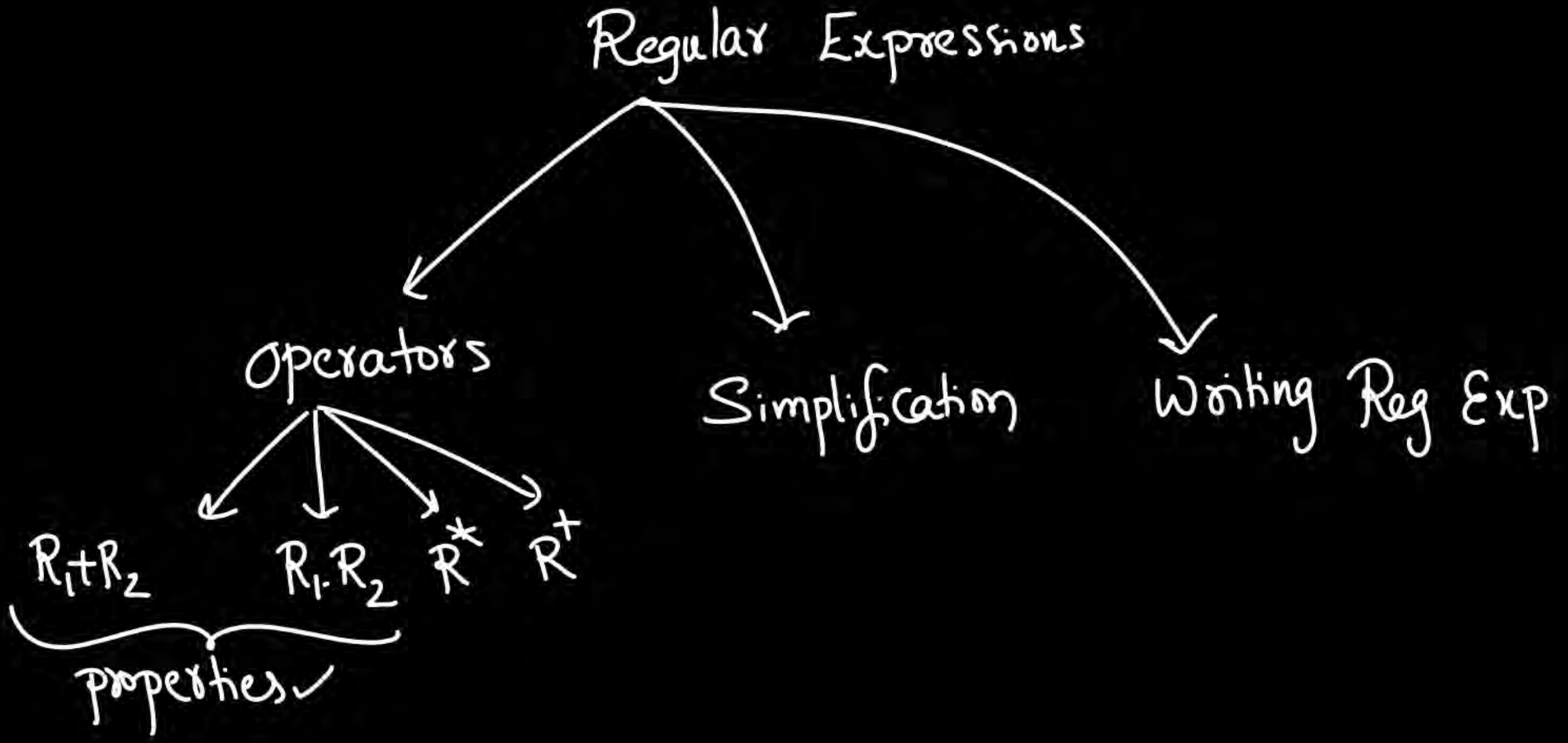
on

Regular Expressions

Deva sir

Topics:

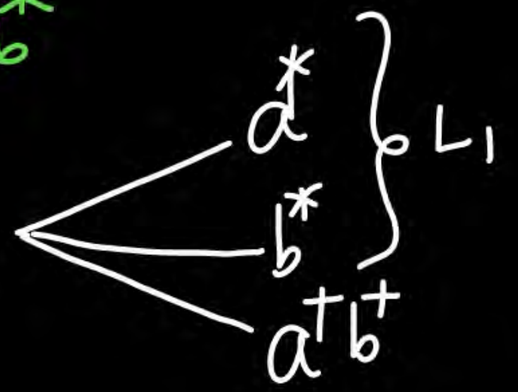
- practice on Regular Exp.
- Doubts clearing
- Conclusion on Reg Exp
- Relation with other topics

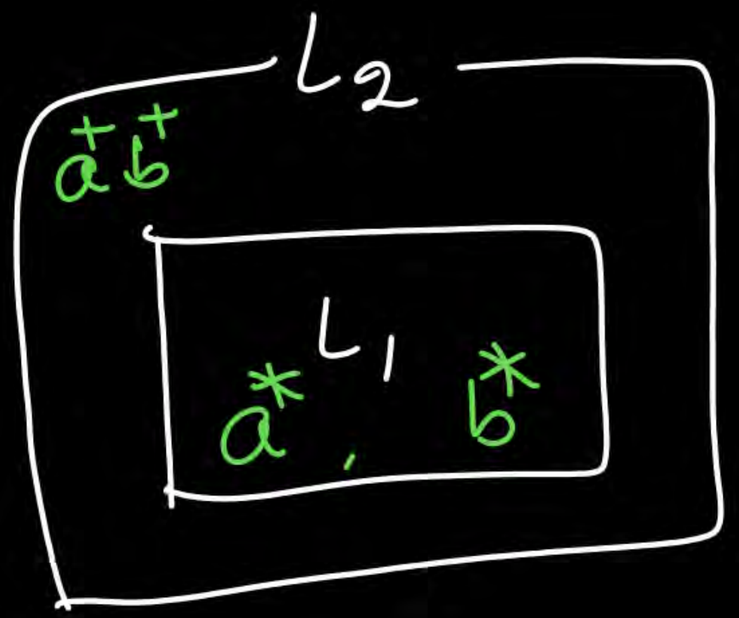

Regular Expressions
Operators
Simplification
Writing Reg Exp
R_1+R_2
$R_1.R_2$
R^*
R^+
properties

Let L1 = a* + b*, and L2= a*b*. Then find relation between L1 and L2.

$L_1 = a^* + b^*$

$L_2 = a^* b^*$





A. $L_1 = L_2$

B. $L_1 \subseteq L_2$ ✓

C. $L_1 \supseteq L_2$

D. None of these

$$L_1 = L(a^* + b^*)$$

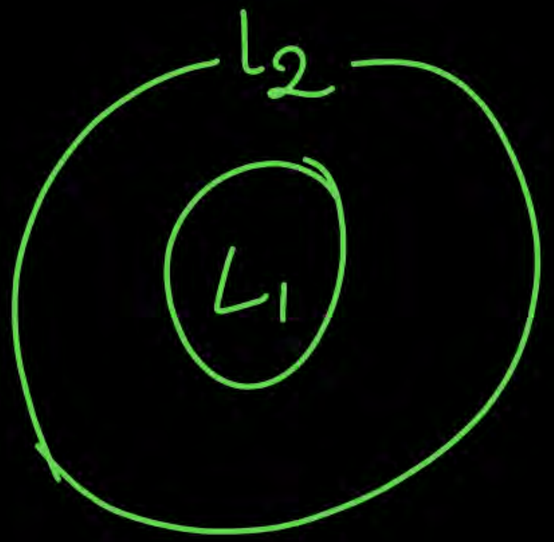
$$L_2 = L(a^* b^*)$$

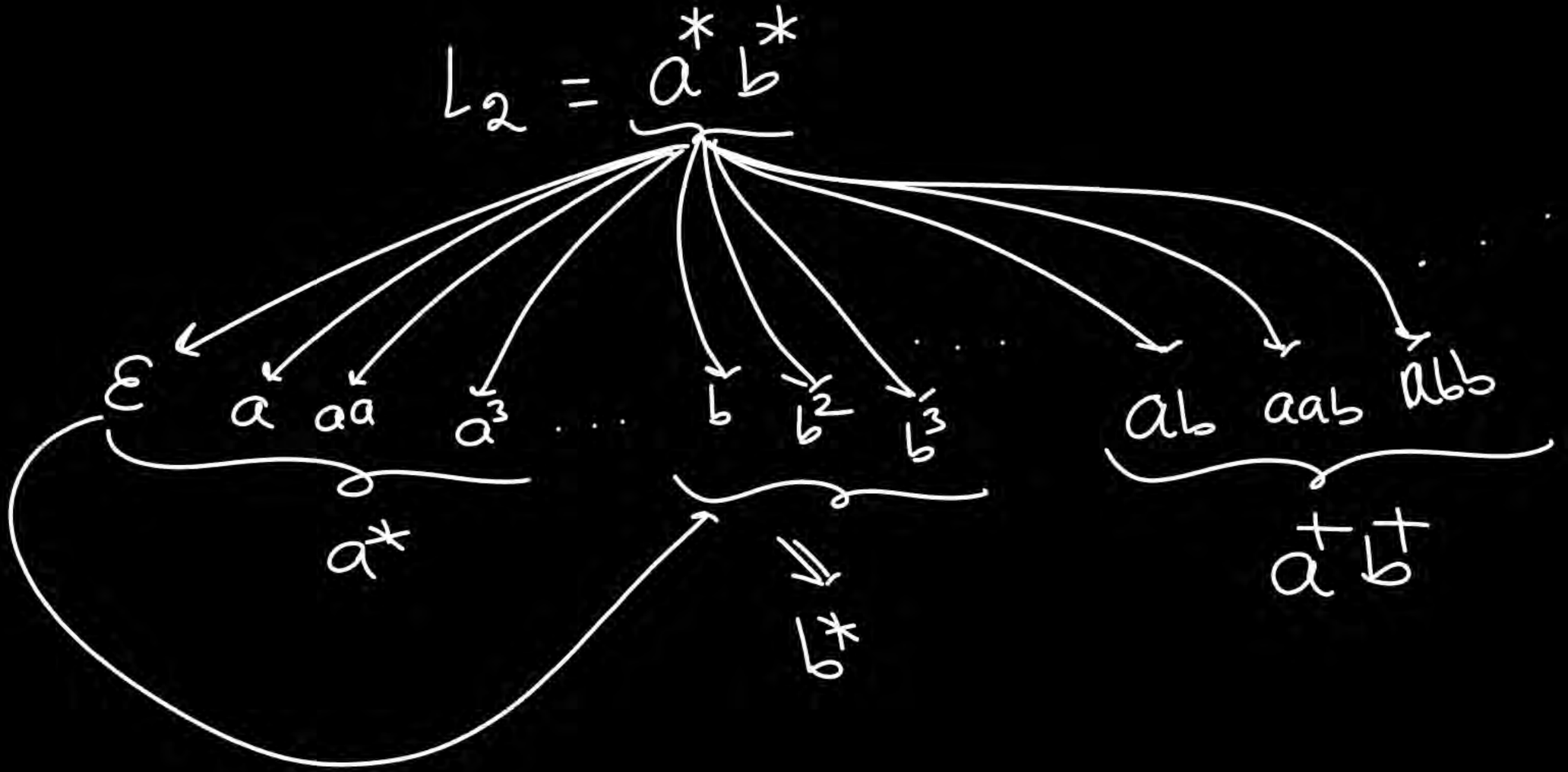
$L_1 - L_2 = \phi$

$L_2 - L_1 = a^+ b^+$

$L_1 \cup L_2 = L_2$

$L_1 \cap L_2 = L_1$




$L_2 = a^* b^*$
ε a aa a^3 . . . b b^2 b^3 . . . . ab aab abb
a^*
b^*
$a^+ b^+$

$= \varepsilon + a^+ + b^+ + a^+ b^+$

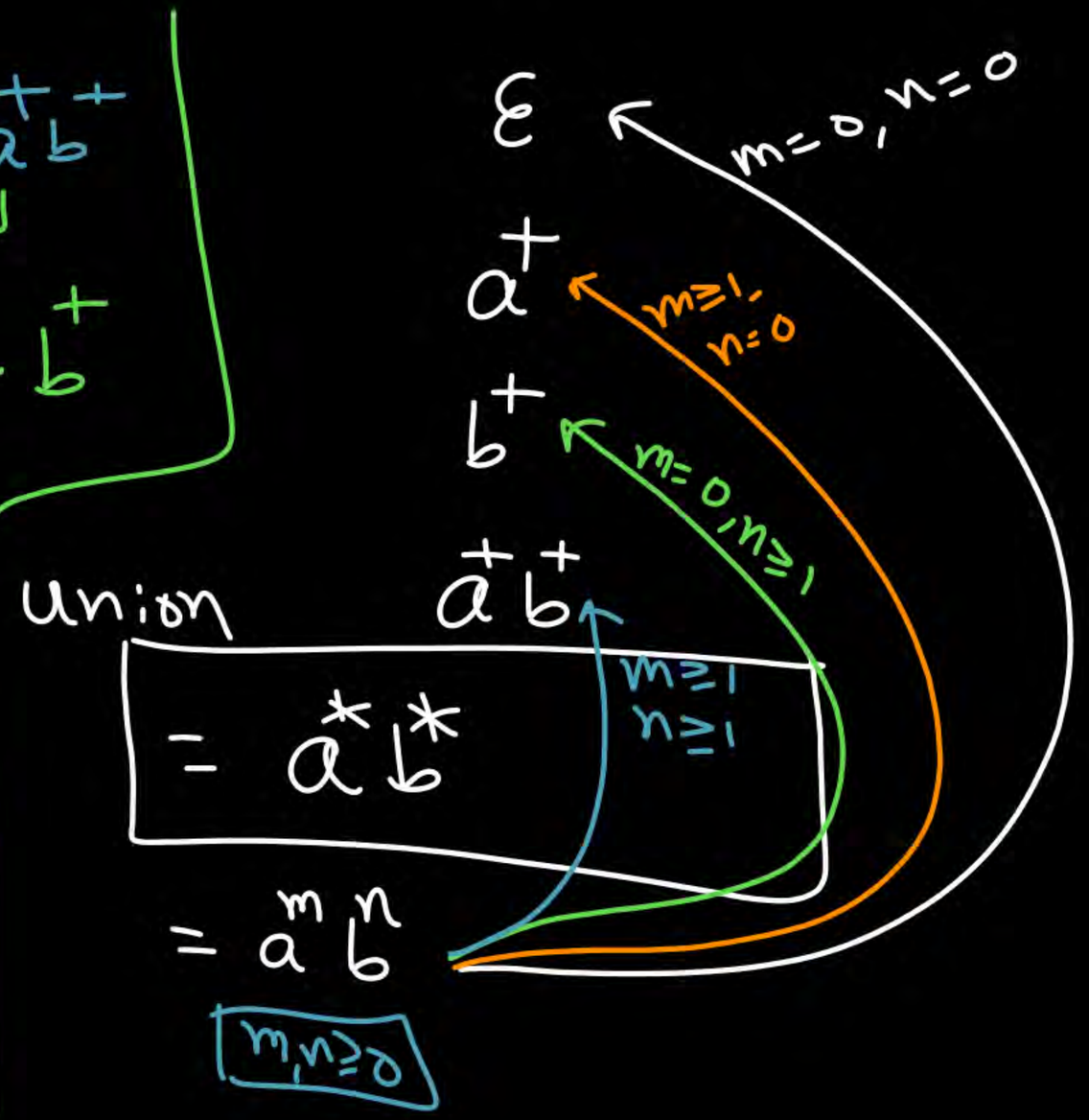
$= \varepsilon + a^+ (\varepsilon + b^+) + b^+$

$= \varepsilon + a^+ b^* + b^+$

$= a^+ b^* + b^*$

$= (a^+ + \varepsilon) b^*$

$= a^* b^*$



Let L1 = a* + b*, and L2= a*b*. Then find relation between L1* and L2*.

$$L_1 = a^* + b^*$$

$$L_2 = a^* b^*$$

$$L_1^* = (a^* + b^*)^* = (a+b)^*$$

$$L_2^* = (a^* b^*)^* = (a+b)^*$$

Find the length of shortest string generated by the following regular expression R.

R= a(ab*a+aab)+

NAT

$R = a(ab^*a + aab)^+$

min

$a(ab^0a + aab)^1$

$a(aa + aab)$

aaa, ~~aaab~~

min

$= 3$

Find the length of shortest string not generated by the following regular expression R.

R= $(a^*b+ba+aa+bb)^+$

$R = (a^*b + ba + aa + bb)^+$

min b

min string = b

ε not generated

a

b → generated

aa

ab

ba

bb

Note :

$R = (a^*b + ba + aa + bb)^*$

min string generated = ε

min string not generated = a

Identify equivalent expressions from the following.

R1 = a(a+b)* → a X
R2 = (a+b)* → X
R3 = (ab*)+ → a X
R4 = (ba*)+ → b X

A. R_1 and R_2
B. R_1 and R_3
C. R_1 and R_4
D. R_2 and R_3
E. R_2 and R_4
F. R_3 and R_4

$(ab^*)^+$ — all strings starts with a

$(ba^*)^+$ — all strings starts with b

$(ab^*)^+$

$(ab^0)^1 = \rightarrow a$

$(ab^0)^2 = a^2 = \rightarrow aa$

$(ab^1)^1 = \rightarrow ab$

$(ab^0)^3 = a^3 = \rightarrow aaa$

$(\quad)^2 \Rightarrow (ab^1)(ab^1) \rightarrow aaab$

$\rightarrow abab$

$(ab^2)^1 = \rightarrow abb$

$\vdots$

$\vdots$

εx

bx

bax

bbx

$\vdots$

$\vdots$

$\vdots$

When you are in live class ?

→ 1st : Listen

2nd : write notes (if you have understood)

3rd : At the end of session

Revise video, maintain notes

not clear → Note it / ask doubt

Total 3 revisions

1. Listen
2. Write notes
3. Revise

Regular Expressions

Important

Identify equivalent expressions from the following.

R1 = (a+b)*
R2 = (ab*)*
R3 = b*(ab*)*
R4 = a*(ba*)*

$R_1 = (a+b)^*$

Every string generated

$R_2 = (ab^*)^*$

$(ab^*)^+$

↳ starting with 'a'

$(ab^*)^0$

↳ ϵ

$\epsilon + aX$

$R_3 = b^*(ab^*)^*$

$\epsilon \quad aX \quad bX$

$= (a+b)^*$

$R_4 = a^*(ba^*)^*$

$\epsilon \quad aX \quad bX$

$= (a+b)^*$

R_1, R_3, R_4 equivalent

$b^*(ab^*)^*$

b^0 starts with a

$(ab^*)^+$

starts with 'a'

$b^+(ab^*)^*$

starts with b

$b^+(ab^*)^*$

starts with b

$b^0(ab^*)^0$

Σ

$aX = (ab^*)^+$

$bX = (ba^*)^+$

$Xa = (b^*a)^+$

$Xb = (a^*b)^+$

$(a+b)^*$

$= b^*(ab^*)^*$

$= a^*(ba^*)^*$ Imp

$= (b^*a)^* b^*$

$= (a^*b)^* a^*$

Identify relation between every two regular expressions from the following.

R1 = a*b*
R2 = a*+b*
R3 = b*a*
R4 = (a+b)*

$R_1 = a^* b^*$ → ε, a^+, b^+, a^+b^+

$R_2 = a^* + b^*$ → ε, a^+, b^+

$R_3 = b^* a^*$ → ε, b^+, a^+, b^+a^+

$R_4 = (a+b)^*$

$R_1 \subset R_4$
$R_2 \subset R_4$
$R_3 \subset R_4$

$R_2 \subset R_3$

$R_2 \subset R_1$

Identify equivalent expressions from the following.

R1 = (a+b)*
R2 = (b*a)*b*
R3 = (a*b)*a*
R4 = a*(ba*)*

$R_5 = b^*(ab^*)^*$ H.W.

Equivalent

$L = \{a^m b^n \mid m + n = \text{even}, m, n >= 0\}$

$= \boxed{(aa)^*(bb)^* + a(aa)^* b(bb)^*}$

$a^m b^n \mid m+n = \text{even}$

Both m and n are even

OR

Both m and n are odd

I) $m = \text{even}, n = \text{even} \Rightarrow a^{\text{even}} b^{\text{even}} \Rightarrow (aa)^*(bb)^*$

II) $m = \text{odd}, n = \text{odd} \Rightarrow a^{\text{odd}} b^{\text{odd}} \Rightarrow a(aa)^* b(bb)^*$

= even

even + even

odd + odd

$a^{2n} = a^{even} \Rightarrow (aa)^*$

$a^{2n+1} = a^{odd} \Rightarrow a(aa)^* = a(aa)^*$

$L = \{a^m b^n \mid m + n = \text{odd}, m, n >= 0\}$

I) m = odd, n = even

OR

II) m = even, n = odd

$$= a^{odd}\, b^{even} + a^{even}\, b^{odd}$$

$$= [a(aa)^*(bb)^*] + [(aa)^*\, b(bb)^*]$$

Assignment Questions :

(23) $\{w \mid w \in \{a,b\}^*, n_a(w) \leq 2\}$

$n_a(w) = 0/1/2$

$$= \boxed{b^* + b^*ab^* + b^*ab^*ab^*} = b^*(a+\varepsilon)b^*(a+\varepsilon)b^*$$

$n_a(w) = 0 \Rightarrow b^*$

$n_a(w) = 1 \Rightarrow b^*ab^*$

$n_a(w) = 2 \Rightarrow b^*ab^*ab^*$

Atmost 2 a's

$\Sigma = \{a, b\}$

$$= b^* (a+\varepsilon)\ b^*\ (a+\varepsilon)\ b^*$$

No a
I) $b^* \varepsilon\ b^* \varepsilon\ b^* \longrightarrow b^*$

1 a
II) $b^*\ a\ b^*\ \varepsilon\ b^* \longrightarrow b^* a b^*$

2 a's
III) $b^*\ a\ b^*\ a\ b^*$

$a+\varepsilon$

atmost 1 a

$\Sigma = \{a\}$

(24) $n_a(w) \geq 2$ $\Sigma = \{a, b\}$

= X a X a X

$= [(a+b)^* \underbrace{a}_{\text{must}}][(a+b)^* \underbrace{a}_{\text{must}}](a+b)^*$

$= [(a+b)^* a]^2 (a+b)^*$

$\Sigma = \{a\}$

aaa^*

aa^*a

aa^+

a^+a

***

25

$n_a(w) = even$

$\Sigma = \{a, b\}$

$= b^* (b^* a b^* a b^*)^* b^*$

$= (b^* a b^* a b^*)^* + b^*$

$= b^* (b^* a b^* a b^*)^*$ optimal

$= (b^* a b^* a b^*)^* b^*$

$= b^* (a b^* a b^*)^*$

$= (b^* a b^* a)^* b^*$

$(b^* a b^* a b^*)^*$ ⊂ Even no. of a's

It is not equivalent to "even no. of a's"

→ b is not generated

bb

bbb

→ It can't generate all strings which are having even a's

(27) 2nd Symbol from begin is 'a'

$$= \underbrace{(a+b)}_{1^{st}} \underbrace{a}_{2^{nd}} \underbrace{(a+b)^*}_{\text{any sequence}}$$

1st: any symbol

2nd: must 'a'

$$= (aa+ba)(a+b)^*$$

$$= aa(a+b)^* + ba(a+b)^*$$

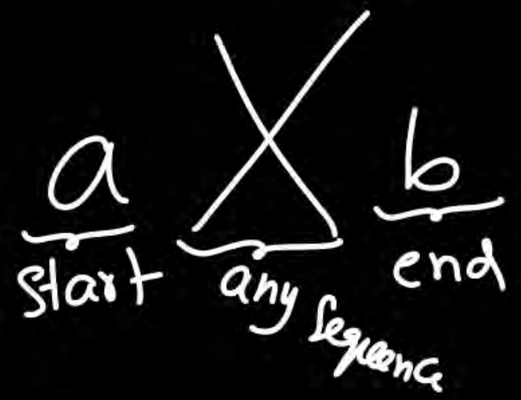
(28) 2nd symbol from end is 'a'

$$= \underbrace{(a+b)^*}_{\text{any sequence}} \underbrace{a}_{\substack{2^{nd} \\ \text{last} \\ \text{must 'a'}}} \underbrace{(a+b)}_{\substack{\text{last} \\ \text{any symbol}}}$$

(29) 2nd symbol is 'a'
OR
4th symbol is 'b'

$$= (a+b)\,a\,(a+b)^* + (a+b)(a+b)(a+b)\,b\,(a+b)^*$$

(30) Starts with 'a' and ends with 'b'



$$= a(a+b)^* b$$

$$= (ab^*)^+ b$$

$$= a(a^*b)^+$$

(31) $L = \{ w \mid w \in \{a,b\}^*,$ w starts and ends with same symbol $\}$

$= \{ \underbrace{a}_{\text{len 1}}, \underbrace{b}_{\text{len 1}}, \underbrace{aa}_{\text{len 2}}, \underbrace{bb}_{\text{len 2}}, \underbrace{a(a+b)a}_{\text{len 3}}, \underbrace{b(a+b)b}_{\text{len 3}}, \underbrace{a(a+b)^2a}_{\text{len 4}}, \ldots \}$

ε is not Symbol

$= a(a+b)^*a + b(a+b)^*b + a + b$

$= (ab^*)^*a + (ba^*)^*b$

$= a(b^*a)^* + b(a^*b)^*$

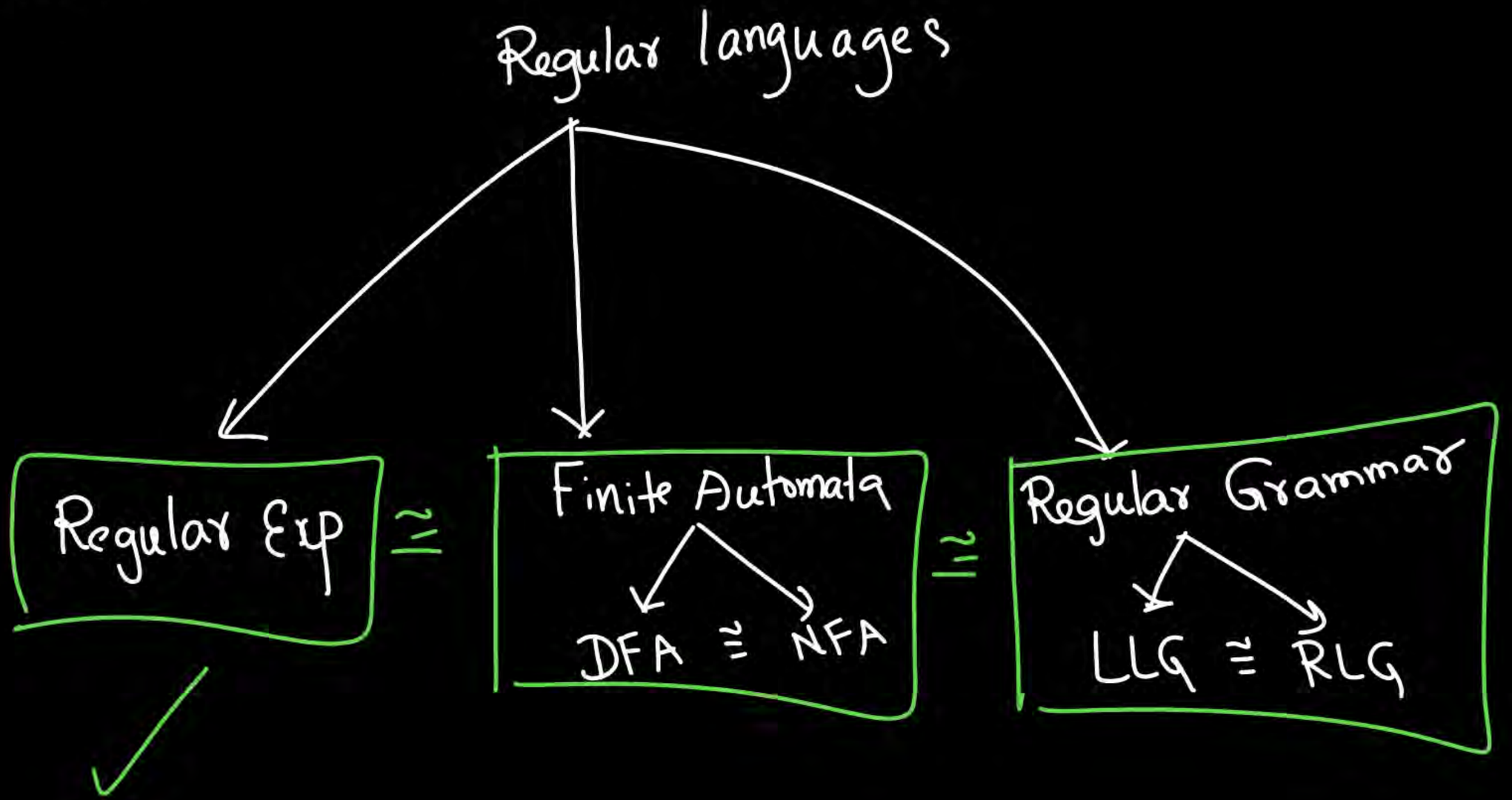
(32) $L = \{w \mid w \in \{a,b\}^*$, w starts and ends with different Symbol $\}$

$$a\,X\,b \;+\; b\,X\,a$$

$$= a(a+b)^*b \;+\; b(a+b)^*a$$

(33)	$(a+aa)^* = a^*$	$\varepsilon, a, a^2, a^3, \ldots$
(34)	$(aa+aaaaa)^* = (aa)^*$	$\varepsilon, a^2, a^4, a^6, \ldots$
(35)	$(aa+aaa)^* = \varepsilon + aa^+$	$\varepsilon, \times a, a^2, a^3, a^4, a^5, a^6, a^7 \ldots$; aa^+
(36)	$(aa+aaaaa)^* = \varepsilon + a^2 + a^4 a^*$	$\varepsilon, \times a, a^2, \times a^3, a^4, a^5, a^6, a^7 \ldots$; a^4a^*

Only a is not possible


Regular languages
Regular Exp
≅
Finite Automata
DFA ≅ NFA
≅
Regular Grammar
LLG ≅ RLG

$X \to aX$

$X \to bX$

$X \to a$

$X \to b$

$X \to aX$

$X \to abX$

$X \to aa$

$X \to X(aa+bb)$

$X \to xaX$

$X \to axx$

$(a+b)^2$

$((a+b)^2)^*$

$x \to (a+b)$

$X \to (a+b)^*$

In class: Reg Exp ✓

What to do now ?

1st : Understand all Concepts

2nd : Make 1 or 2 page Short notes

3rd : practice every GATE Question Only from regular Exp.

I) Important RegExp :

$a^*, b^*, a^*b^*, a^+b^+, (a+b)^*, aX, bX, Xa, Xb, XaX, XbX,$

$xaX, XbX, (a+b)^2, (a+b+\varepsilon)^2, (aa)^*, a(aa)^*,$

$(a^*b^*)^*, (a+b)^*, (a^*b)^+, (ab^*)^+, \ldots$

Revise regularly

II) Some Simplifications

① $\phi^* = \varepsilon = \varepsilon^* = \varepsilon^+ = \varepsilon^2 = a^0 = \phi^0 = b^0 = R^0$

② $\phi^+ = \phi = R.\phi = a.\phi = \phi.a$

③ $aa^* = a^*a = a^+ = (a^+)^+ = a^+a^* = a^*a^+$

④ $(a+b)^* = (a^*b^*)^* = (a^*+b^*)^* \ldots$

Summary

Reg Exp ✓

Next : Finite Automata

Compute

Represent

Thank you