CS & IT ENGINEERING

Graph Theory

Types of Graphs

Lecture No. 3

By- SATISH YADAV SIR

01 Complete Graph

02 Cycle graph

03 Wheel graph

04 Bipartite graph

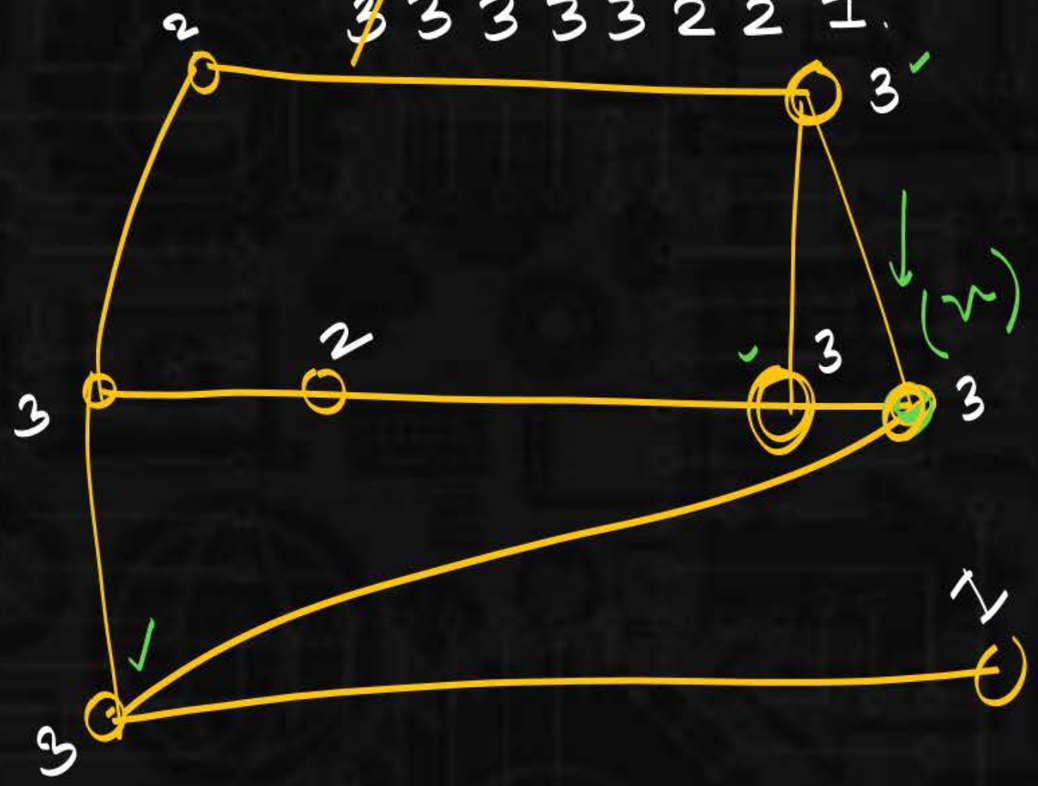
05 Line graph

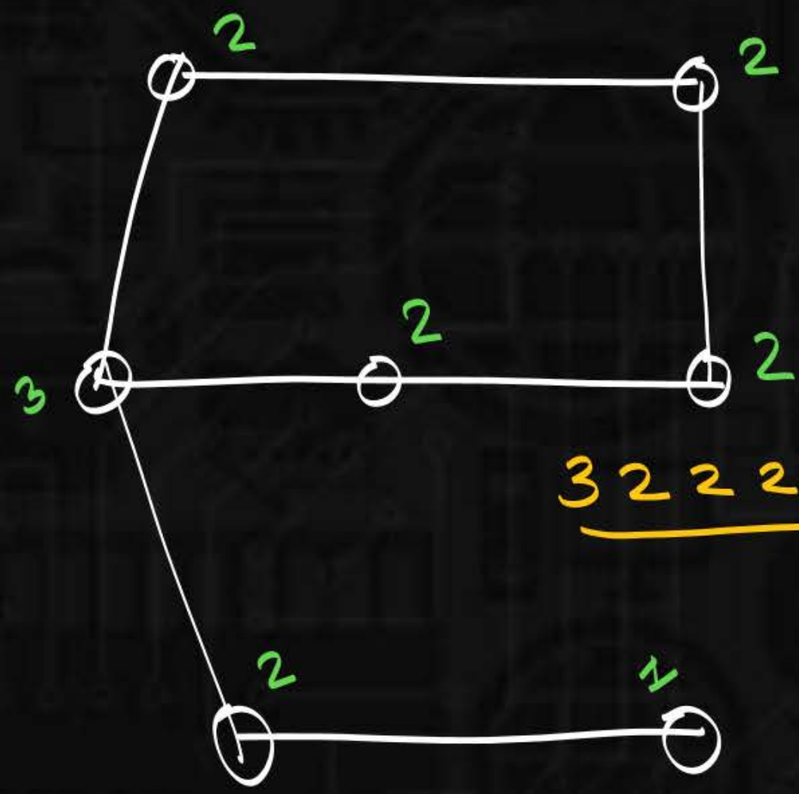
Types of graph

Havell-Hakimi:

It checks wheather Degree sequence is graphical.

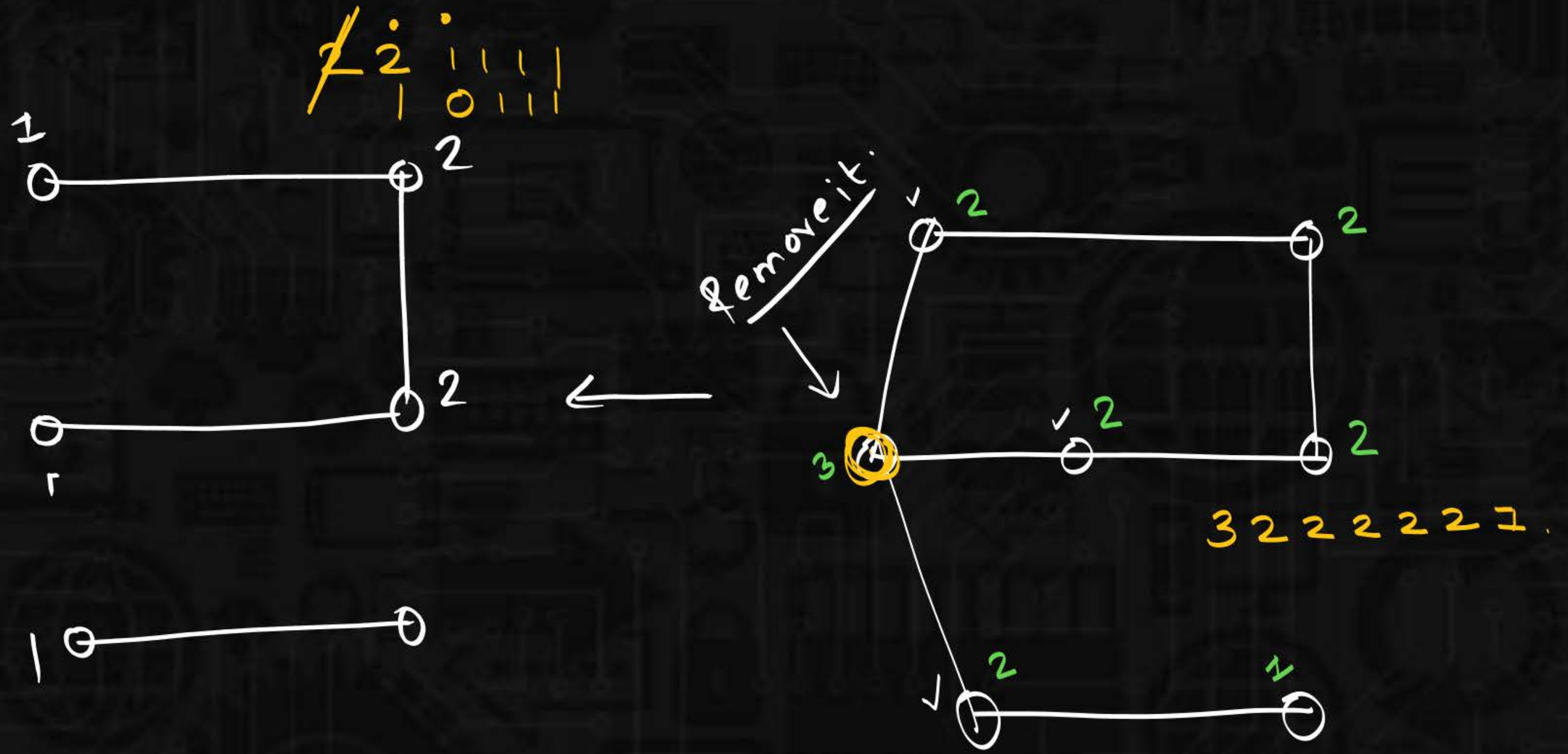
3 3 3 3 3 2 2 1.



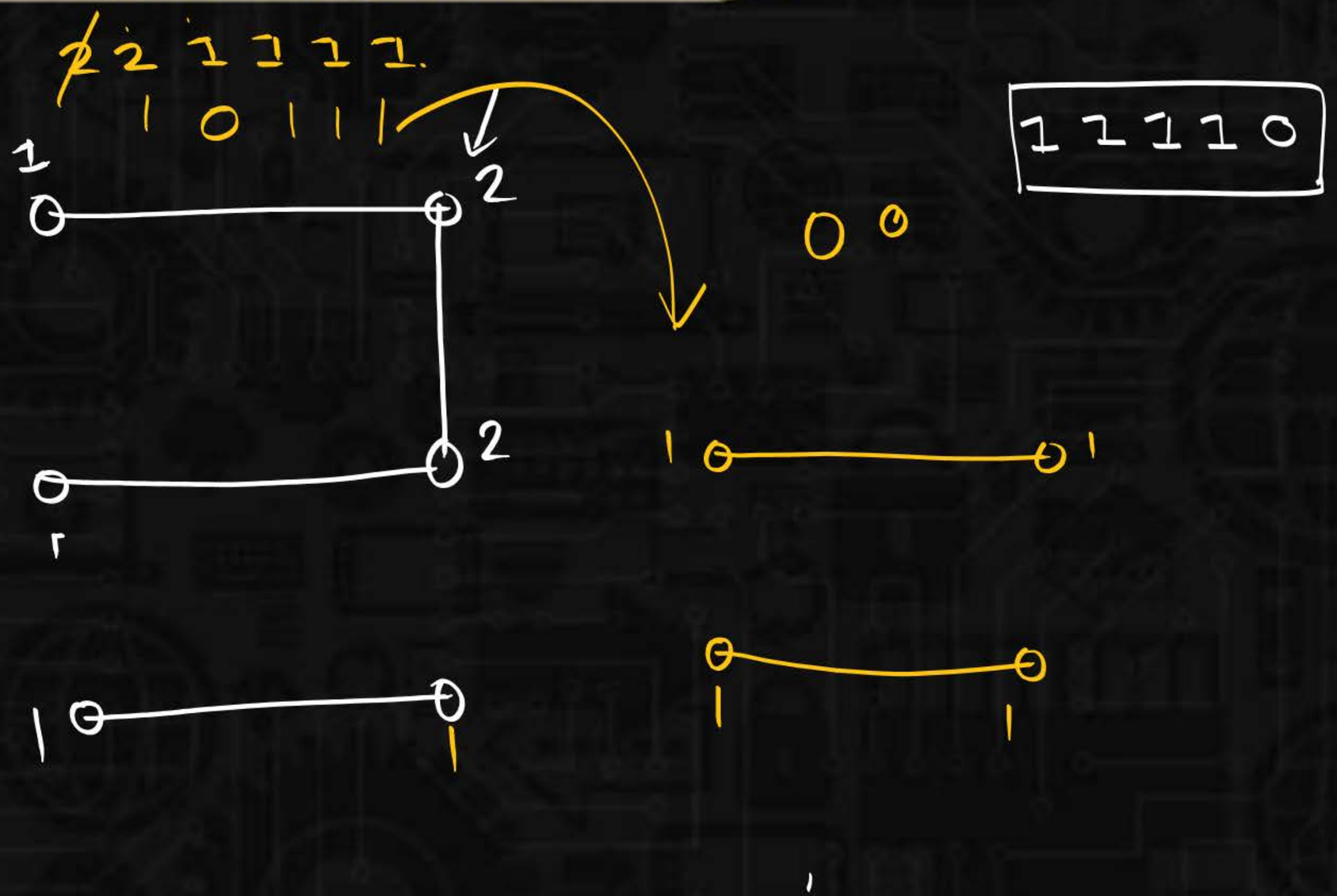


3 2 2 2 2 2 1.

Types of graph



Types of graph



Types of graph

1 1 1 1 0

0 1 1 0

1 1 0 0

3 3 3 3 3 2 2 1.

2 2 2 3 2 2 1.

(ordering)

3 2 2 2 2 2 1.

1 1 1 2 2 1

2 2 1 1 1 1.

1 0 1 1 1.

1 1 1 1 0

cut

count

mark it

delet it

order it

Types of graph

GATE : Graphical ?

7 6 5 4 4 3 2 1 ✓

6 6 6 6 3 3 2 2 ✗

7 6 6 4 4 3 2 2 ✓

8 7 7 6 4 2 1 1 ✗

7 6 5 4 4 3 2 1

5 4 3 3 2 1 0

3 2 2 1 0 0

1 1 0 0 0

6 6 6 6 3 3 2 2

5 5 5 2 2 1 2

ordering.

5 5 5 2 2 2 1

4 4 1 1 1 1

3 0 0 0 1

3 1 0 0 0

→ no simple Graph.

Types of graph

Thm 6 : In Simple Graph atleast 2 vertices will have same degree ($n \geq 2$)

Assumption : all degrees are distinct

$\{1, 2, 3, 4, \ldots, n-1\}$ Total no. of vertices = n.

Total distinct degrees = n-1.

Types of graph

Thm 6: In Simple Graph atleast 2 vertices will have same degree ($n \geq 2$)

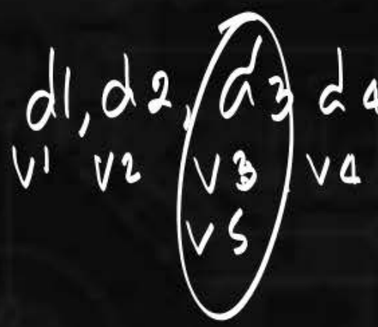
Assumption: all degrees are distinct

Case 1:

{ 1, 2, 3, 4, ... n-1 }

Total no. of vertices = n.

Total distinct degrees = n-1.



V1 V2 V3 V4 V5

Types of graph

Thm 6: In Simple Graph atleast 2 vertices will have same degree ($n \geq 2$)

Assumption: all degrees are distinct

Cased:

$\{0, 1, \ldots n-2\}$

Total distinct degrees = n-1.

Total no. of vertices = n.

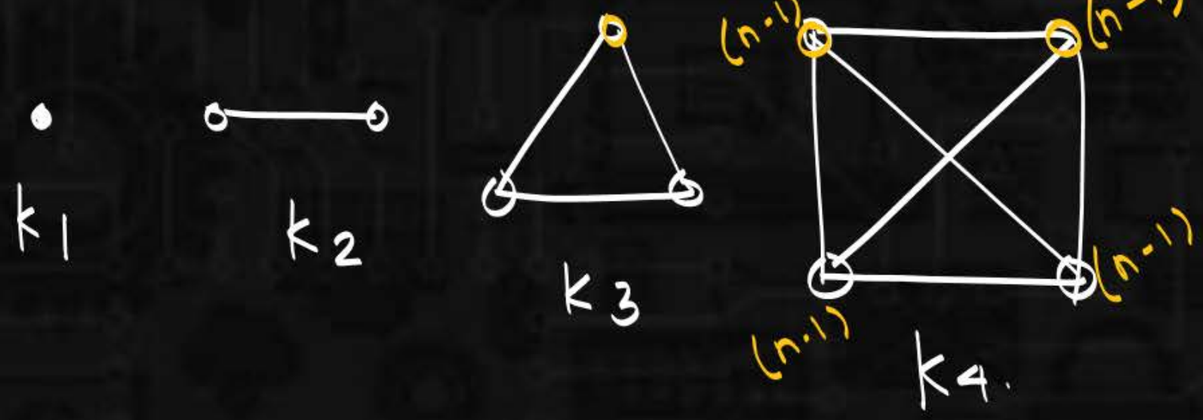
Types of graph

Steps for Degree sequence.

1. no of odd degree vertices should be even (Thm 2)
2. max degree $\leq n-1$ (Thm 3)
3. $n-1, n-1 \ldots\ldots 1$ (Raavan's Rule) (no simple Graph)
4. all degrees are distinct graph is not possible (Thm 6)
5. Havell-Hakimi

Types of graph

Complete Graph (K_n) ($n \geq 1$)



Degrees of all vertices are $n-1$.

$$e = \frac{n(n-1)}{2}$$

$$\delta(G) = \frac{2e}{n} = \Delta(G) = n-1.$$

Types of graph

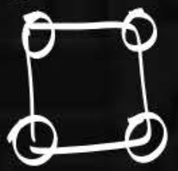
Regular Graph :

if degrees of all vertices are same, it is called Regular.

$$\delta(G) = \frac{2e}{n} = \Delta(G)$$

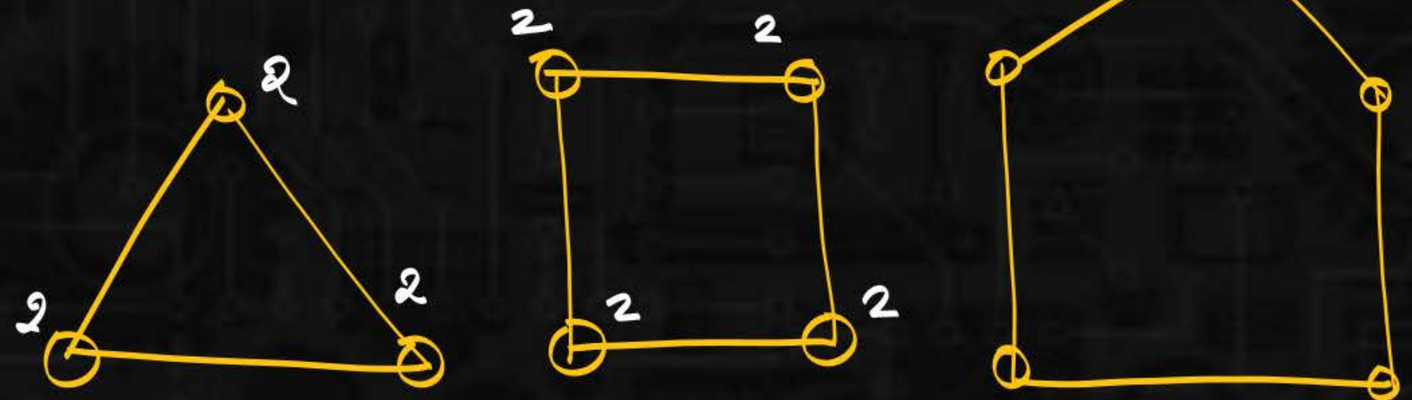
all $K_n \rightarrow$ Regular Graphs.

all Regular Graph $\not\rightarrow K_n$.



Types of graph

Cycle Graph (C_n) $(n \geq 3)$



C_3

$n = e$

C_4

$n = e$

C_5

$n = e$

if G is C_n → Degrees of all vertices are 2.

$\sum d(v_i) = 2e$

$n \times 2 = 2e$

$n = e$

if G is C_n → $n = e$.

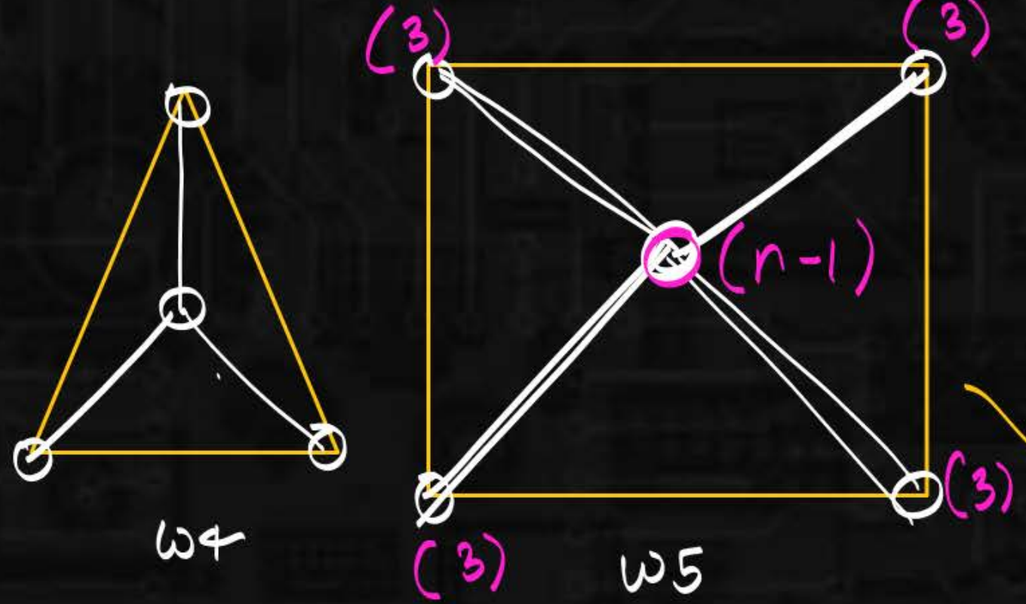
if $n = e$ ↛ G is C_n.

all C_n → Regular Graph. (T)

all Regular Graphs ↛ C_n

Types of graph

Wheel Graph. $(W_n)(n \geq 4)$



W_4 W_5

$e(W_n) = 2(n-1)$

if G is $W_n \longrightarrow e = 2(n-1)$

if G is having $e = 2(n-1) \not\rightarrow$ G is W_n.

W_{100}

1 1st (n-1) 3, 3, 3, 3, 3 3 3

99 vertices.


THANK YOU GW
SOLDIERS !