CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Types of Relation Part 1

Lecture No. 7

By- SATISH YADAV SIR

Total no. of Relations = Total no. of subsets.

$|A| = m \quad |B| = n. \qquad \underline{2^{m \cdot n}}$

$|A| = n \qquad \underline{2^{n^2}}$

$\# R. \rightarrow 2^{n^2 - n}$

$\# \text{sym} \rightarrow 2^n \cdot 2^{\frac{n^2 - n}{2}}$

$\rightarrow$ no problem with same element

$\rightarrow$ demands the flipping

Irreflexive. : $A \to$ non empty set

$\forall a \in A \ (a,a) \notin R$ $A = \{1, 2, 3\}$ $A \times A \longrightarrow \square \to R_1$

↳ hates same elements.

$R_1 = \{(1,1)(1,2)\} \to$ not Irreflexive

$\forall a \in A \ (a,a) \notin R$

not Reflexive

$$= \text{Irreflexive}$$

~~wrong~~

$$R_1 = \{ (1\,2) \; (2\,1) \; \underline{(2\,2)} \}$$

not
Irreflexive (X)

$$R_2 = \{ (1\,3) \; (2\,3) \; (3\,3) \}$$

not Irreflexive

not Reflexive

$$R_3 = \{ \boxed{(1\,1)(2\,2)(3\,3)} \; (1\,2) \}$$

Reflexive

not Irreflexive

Reflexive :

needs diagonal
       elements.

$R_1 = \{$ $\boxed{11 \quad 22 \quad 33}$ (13) $\}$

$R_2 = \{$ $\boxed{11 \quad 22 \quad 33}$ (12) $\}$

$R_3 = \{$ $\}$ $+$

Irreflexive.:

hates same elements.

$R_1 = \{$ (11) $\}$ $\times$

$R_2 = \{$ $\}$ $\checkmark$

Irreflexive.

$R_1 = \{ (1\,2) \} \checkmark$

$R_2 = \{ (1\,2)(2\,3) \} \checkmark$

$R_6 = \{ (1\,1)(2\,2)(3\,3)(1\,2) \} \times$

$\forall a \in A \; (a,a) \notin R.$

$R_3 = \{ \quad \} \checkmark$

$R_4 = \{ (1\,2)(2\,3) \boxed{(3\,3)} \} \times$
$\qquad\qquad\qquad\quad \times$

$R_5 = \{ (2\,1)(2\,3)(3\,2) \boxed{(2\,2)} \} \times$
$\qquad\qquad\qquad\qquad\qquad\quad \times$

$|A| = n$

$A = \{1, 2, 3\}$

$A \times A:$

Diagonal elements $= n$.

Total elements $= n^2$

non diagonals $= n^2 - n$.

(11) (12) (13)

(21) (22) (23)

(31) (32) (33)

non diagonal

↓

2 choices:

→ 1 choice
(absent)

Total no. of Irreflexive $= 2^{n^2 - n}$

$$A = \{1, 2\}$$

$$A \times A =$$

$$\begin{array}{cc} 11 & 12 \\ 21 & 22 \end{array}$$

$\rightarrow$ 1 choice (absent)

$$\boxed{1 \ 2} \qquad \boxed{2 \ 1}$$

$$\downarrow \qquad \downarrow$$

$$2 \qquad 2$$

0	0
0	1
1	0
1	1

$\rightarrow$

$$R_1 = \{ \} \checkmark$$

$$R_2 = \{(21)\} \checkmark$$

$$R_3 = \{(12)\} \checkmark$$

$$R_4 = \{(12)(21)\} \checkmark$$
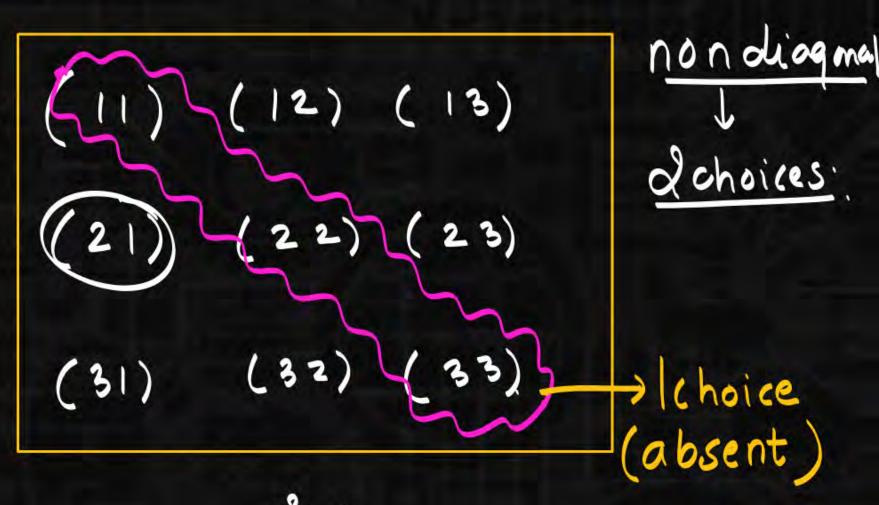
Antisymmetric:

$$\forall a \forall b \left((a,b) \in R \land (b,a) \in R \longrightarrow a = b \right)$$

$$R_1 = \{ (1\ 2) \}$$

$(a,b) \in R \land (b,a) \in R \longrightarrow a = b$

$\underset{T}{(1,2) \in R} \land \underset{F}{(2,1) \in R} \longrightarrow$

F $\longrightarrow$

T.

$$R_1 = \{ (1 2) \} \quad \text{Anti} \checkmark$$

$$(a, b) \in R \wedge (b, a) \in R \longrightarrow a = b$$

$$\underbrace{(1, 2) \in R}_{T} \wedge \underbrace{(2, 1) \in R}_{F} \longrightarrow a = b$$

$$\underbrace{ }_{F} \longrightarrow a = b$$

$$\text{True}$$

$$R_2 = \{ \ (\ 1 \ 1 \ ) \ \} \ \text{Anti} \checkmark$$

$$(a, b) \in R \wedge (b, a) \in R \longrightarrow a = b.$$

$$(1, 1) \in R \wedge (1, 1) \in R \longrightarrow 1 = 1.$$

$$a = 1$$
$$b = 1$$

$$\left\{ \begin{array}{l} \underline{\text{Anti}^\circ} \\ \text{no problem} \\ \text{with same} \\ \text{element} \end{array} \right.$$

$$R_3 = \{ (1\,2) \; (2\,1) \} \quad \text{not Anti}$$

$$(a,b) \in R \wedge (b,a) \in R \longrightarrow a = b$$

$a = 1$
$b = 2$

$$\boxed{(1,2) \in R} \wedge \boxed{(2,1) \in R} \longrightarrow 1 = 2$$

T $\qquad \wedge \qquad$ T $\qquad\qquad\qquad\qquad$ F

$\underline{\qquad\qquad\qquad\qquad\qquad\qquad}$

T $\qquad\qquad\qquad\qquad \longrightarrow$

$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

false

Anti $\rightarrow$ hates flipping.

$\rightarrow$ no problems
with same element.

$$R_1 = \{ (23)(32)(21) \} \ \times$$

$$R_2 = \{ (22)(12) \} \ \checkmark$$

$$R_3 = \{ (12)(11)(22) \}$$

$$R_1 = \{ (1\,1) \, (1\,2) \} \checkmark$$

$$R_2 = \{ (1\,2) \} \checkmark$$

$$R_3 = \{ (2\,3) \} \checkmark$$

$$R_4 = \{ (3\,2) \} \checkmark$$

$$R_5 = \{ \quad \} \checkmark$$

$$A = \{1, 2, 3\}.$$

absent

present

$$A \times A = \{ (1\,1) \quad (2\,2) \quad (3\,3) \quad (1\,2) \quad (2\,1) \quad (2\,3)(3\,2) \quad (1\,3)(3\,1) \}$$

2   2   2.

present

$$\{ (2\,1) \} \checkmark$$

$$\{ (1\,2) \} \checkmark$$

$$\{ \} \checkmark$$

Diagonal elements.

$$= 2^n.$$

non diagonal $= n^2 - n$.

bones $= \dfrac{n^2 - n}{2}.$

Total Anti $= 2^n \cdot 3^{\frac{n^2-n}{2}}$

(1 1)   (1 2)   (1 3)

(2 1)   (2 2)   (2 3)

(3 1)   (3 2)   (3 3) $\longrightarrow$ 2 choices.

$$R_1 = \{ (a,b) \mid a+b \leq 3 \}$$

$$\underbrace{}_{\text{not Anti}^\circ}$$

$$R_2 = \{ (a,b) \mid a = b+1 \}$$

$$aRb \land bRa \longrightarrow a=b$$

$$a+b \leq 3 \land b+a \leq 3 \longrightarrow a=b$$

$$1+2 \leq 3 \land 2+1 \leq 3$$

$$\underbrace{(1,2) \in R}_{T} \land \underbrace{(2,1) \in R}_{T} \longrightarrow 1=2$$

$$\longrightarrow F$$

$$aRb \land bRa \longrightarrow a=b$$

$$\underline{a=b+1} \land \underline{b=a+1} \longrightarrow a=b$$

$$\text{cas1: } \underline{T} \land \underline{F} \longrightarrow$$

$$\text{Cag: } F \land T \longrightarrow b$$

z

$z \times z$

$(0,0) \quad (0,1)$

$(1,0)$

$(1,1)$

$(2,1)$

$(1,2)$

$a = b + 1 \longrightarrow R = \{$

$2 = 1 + 1$

$1 = 2 + 1$

$(2,1)$

$(3,2)$

$(4,3)$

Asymmetric :

$$\forall a \, \forall b \left((a,b) \in R \longrightarrow (b,a) \notin R \right)$$

$R_1 = \{ \quad \} \checkmark$

$(a,b) \in R \longrightarrow (b,a) \in R$
$$\underline{F} \longrightarrow$$
$$\underline{T.}$$
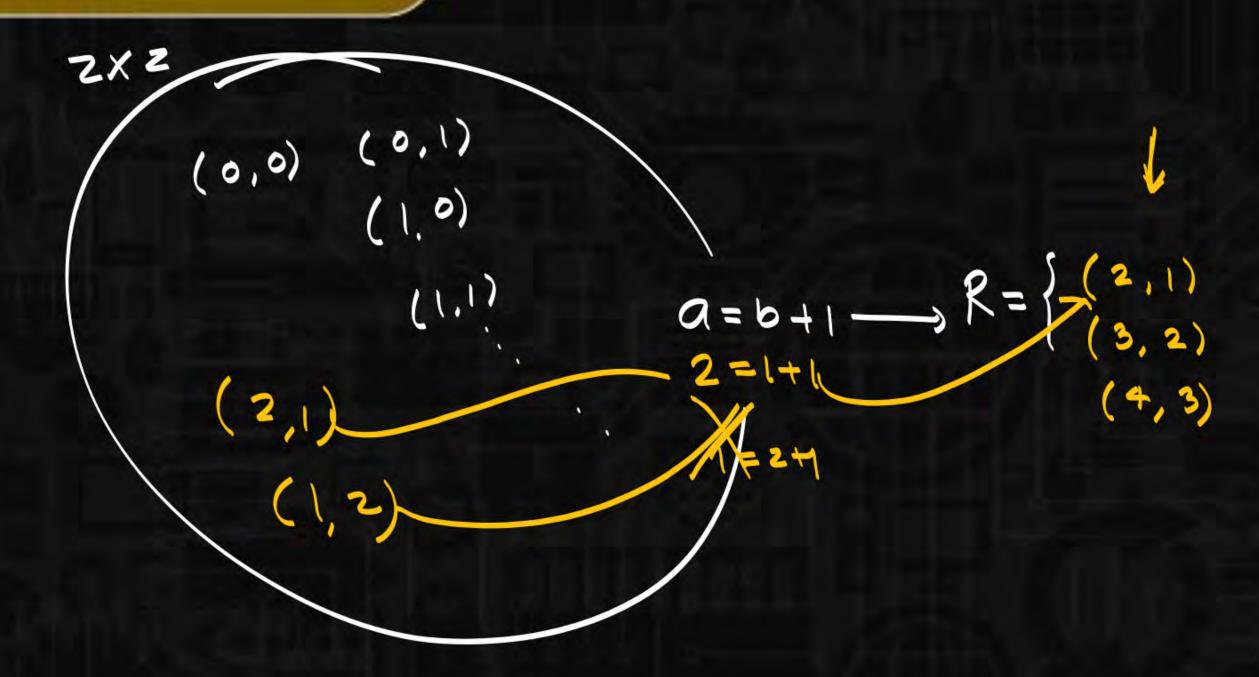
$R_2 = \{ (1 \, 2) \} \checkmark$

$(a,b) \in R \longrightarrow (b,a) \notin R.$
$(1,2) \in R \longrightarrow (2,1) \notin R.$
$$\underline{T} \longrightarrow \underline{T.}$$

$R_3 = \{ (2\,3)\,(3\,2) \}$ not Asymmetric.

$(a,b) \in R \longrightarrow (b,a) \notin R.$

$(2,3) \in R \longrightarrow (3,2) \notin R.$

$\underline{T} \longrightarrow \underline{F}$

false.

$$R_3 = \{ (11) \}. \quad \text{not Asymmetric}$$

$$(a, b) \in R \longrightarrow (b, a) \notin R$$

$$\underbrace{(1,1) \in R}_{T} \longrightarrow \underbrace{(1,1) \notin R}_{F}$$

$a = 1$

$b = 1$

$$\underline{\text{Asymmetric}}$$

$\longrightarrow$ no flipping.

$\longrightarrow$ no same element.

(11) (12) (13)

(21) (22) (23)

(31) (32) (33) $\rightarrow$ 1 choice (absent)

$(12)\ (21)$ $(23)\ (32)$ $(13)\ (31)$

P   P   absent

$$bones = \frac{n^2 - n}{2}$$

$$Asymmetric = 3^{\,n^2 - n/2}$$

Symmetric	Antisymmetric	Asymmetric
$(a,b) \in R \longrightarrow (b,a) \in R$	$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$	$(a,b) \in R \rightarrow (b,a) \notin R$
$\rightarrow$ Demands flipping.	$\rightarrow$ no flipping.	$\rightarrow$ no flipping.
$\rightarrow$ allows same element.	$\rightarrow$ allows same element.	$\rightarrow$ no same element.
$2^{n} \cdot 2^{\frac{n^2-n}{2}}$	$2^{n} \cdot 3^{\frac{n^2-n}{2}}$	$3^{\frac{n^2-n}{2}}$

$$R = \{ (x\,y)\,(x\,z)\,(z\,x)\,(z\,y) \} \quad (GATE-09)$$

↓ no $y\,x$ ↓ not Anti

not Symmetric → no flipping of $(x\,y)$

not Anti → $(x\,z)\,(z\,x)$

Any X.

Anti X.

$$R = \{ (11) \ (21) \ (22) \ (23) \ (24) \ (31) \ (32) \ (33) \ (34) \}$$

$$A = \{1, 2, 3, 4\}$$

G-98

1. Ref ✗ $(a,a) \in R$.

2. Sym $(1,2) \notin R$

3. Anti

4. Asym.

Set: A

C.P   A × A ⟶ ▢ ⟶ Result

3 R 5
5 R 7

Set: A^2

C.P $A^2 × A^2$ ⟶ ▢

(3.5) R (6.5)

⟶ Result

$$Z^2 \times Z^2$$

$$aRb \rightarrow bRa.$$

$$L.SRR.S \rightarrow R.S \, R_{L.S}.$$

$$R: \left\{ (a,b) \, R \, (c,d) \; \middle| \; ad = bc \; \middle| \; Symm \right.$$

$$\underset{LS}{(a,b)} \; \underset{R}{R} \; \underset{RS}{(c,d)} \longrightarrow \underset{RS}{(c,d)} \; R \; \underset{LS.}{(a,b)}$$

$$ad = bc$$

$$\longrightarrow cb = da.$$

h.w.

$A \longrightarrow$ set

$A \times A \longrightarrow c.p.$

R_1, R_2 are reflexive relation of A, $R_1 \cup R_2 = $ Reflexive?

$R_1 \cap R_2 = $ Reflexive.

$\left. \begin{array}{l} R_1, R_2 \\ R_1, R_2 \\ R_1, R_2 \\ R_1, R_2 \end{array} \right\} \begin{array}{l} \text{Sym} \checkmark \\ \text{Anti} \\ \text{Asy.} \\ \text{I.w.} \end{array}$

THANK YOU GW SOLDIERS !