CS & IT ENGINEERING

Graph Theory

Planarity Part-1

Lecture No. 11

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TOPICS TO BE COVERED

01 Matching set

02 Maximal matching set

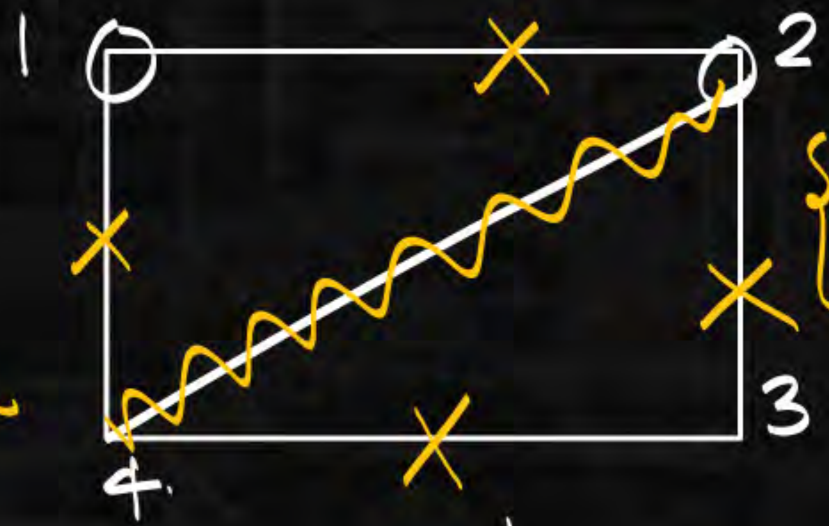
03 Matching no.

04 Covering set

05 Covering number

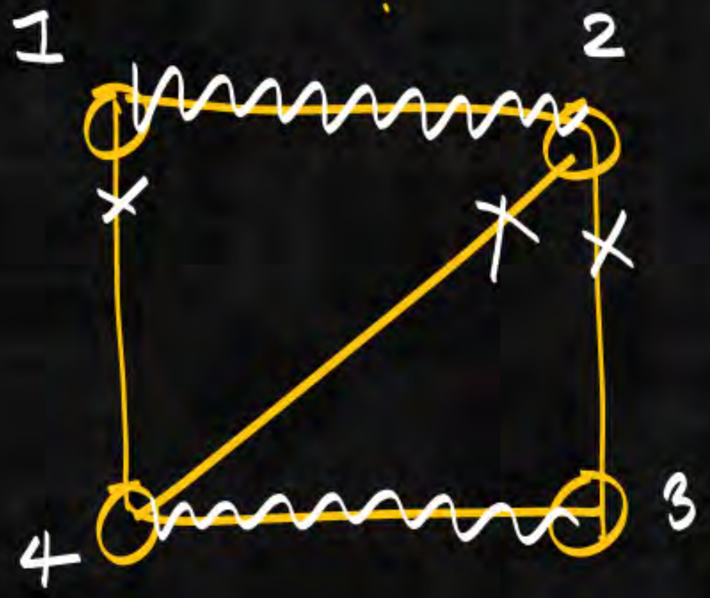
matching set :

set of non adjacent edge.



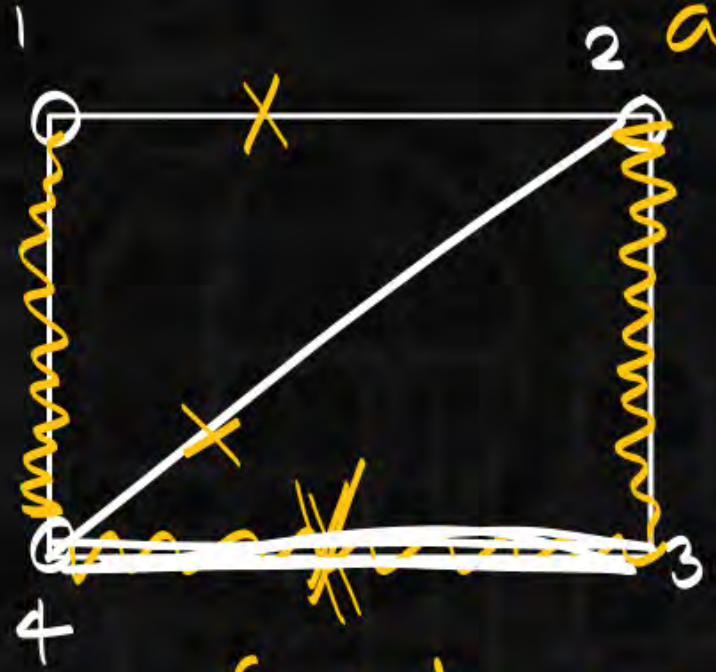
{24}

maximal: can not add new



{12}- matching set

{12, 43}- matching set
maximal matching set



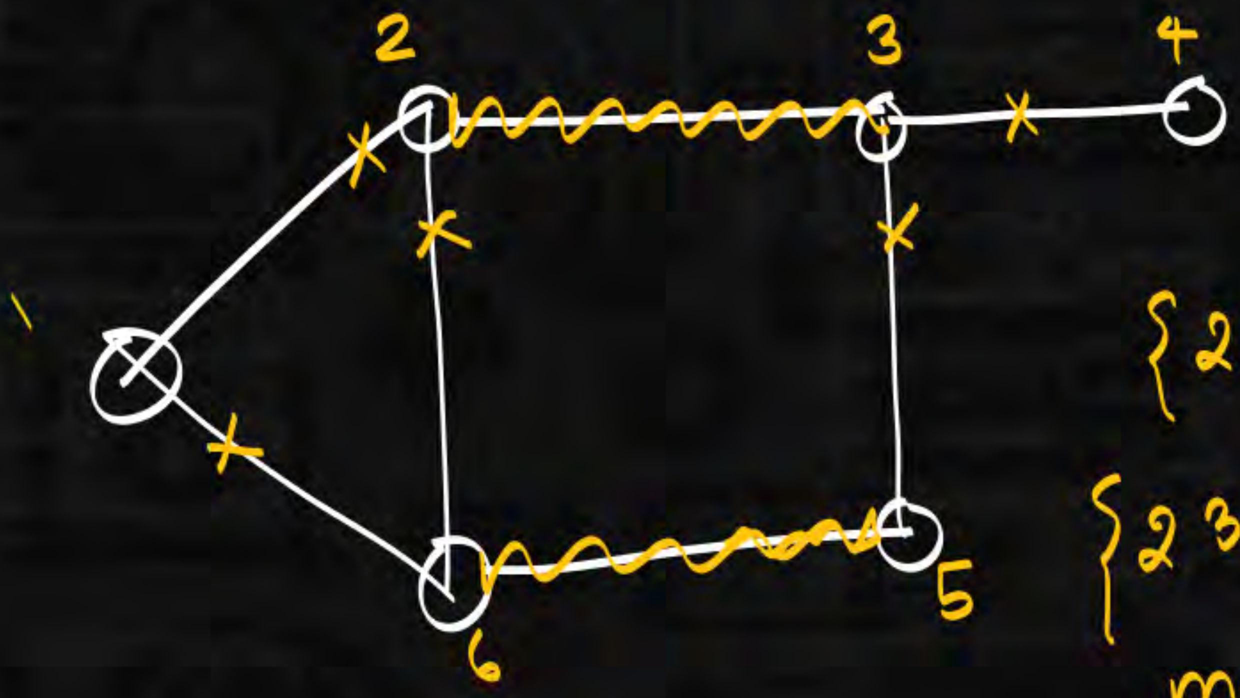
{14}- ms

{14, 23} → mms.

matching no (m(G))

no. of edges present in largest maximal matching set

$m(G) = 3$

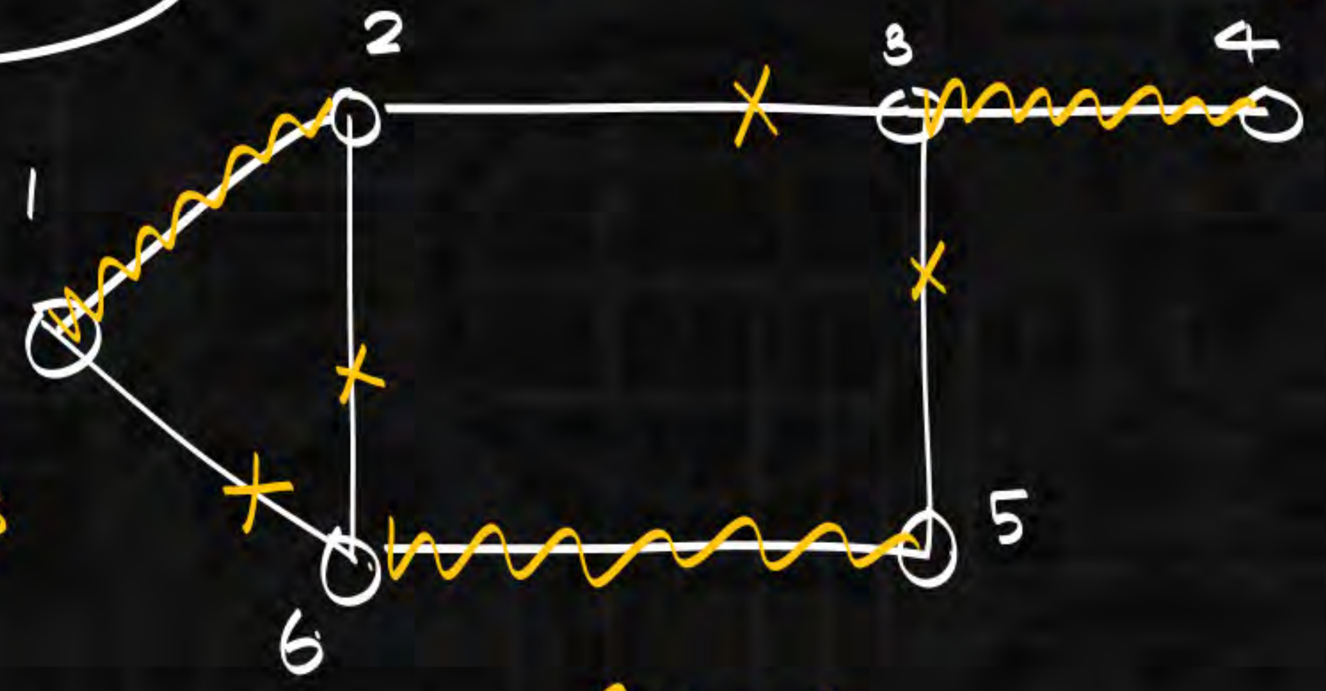


{23} - ms

{23, 65} - ms

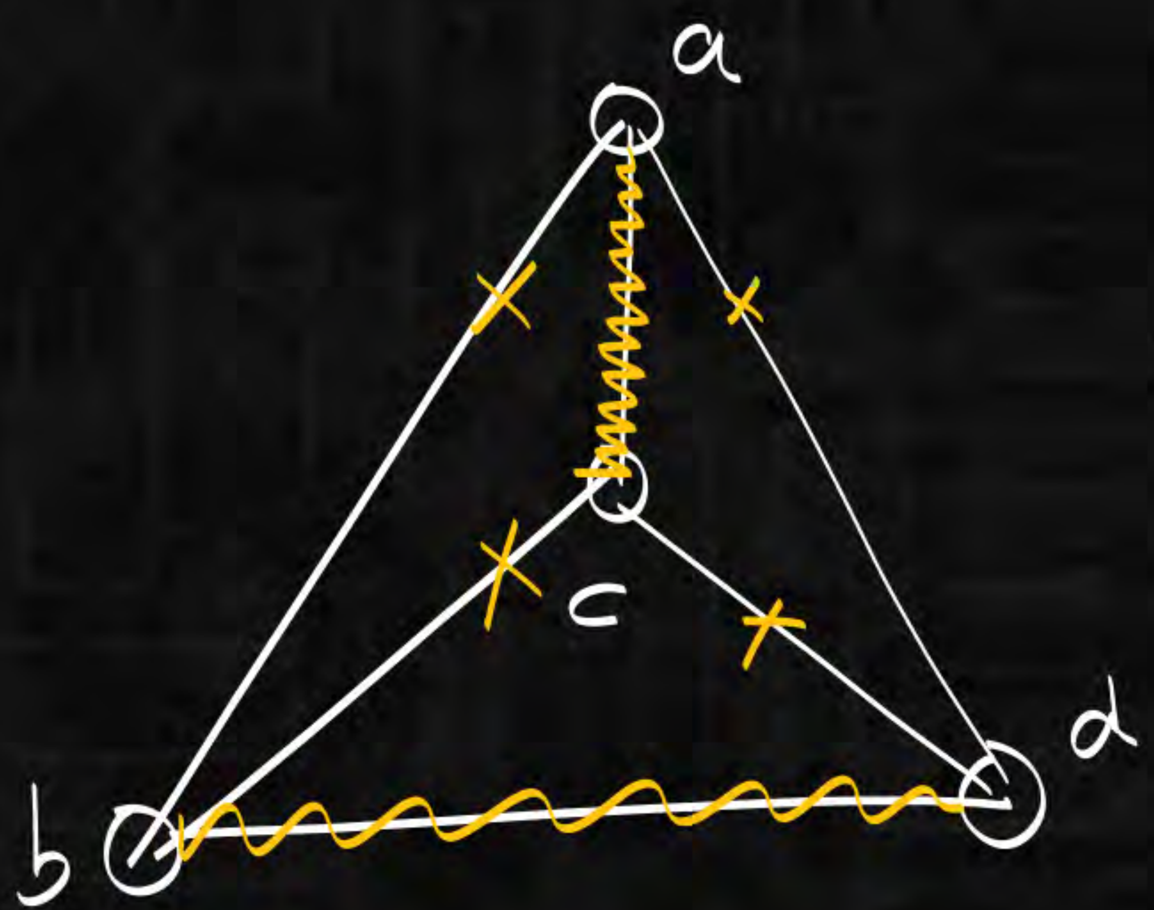
maximal matching set

{34, 65, 12} - maximal ms.



{34}

{34, 65}



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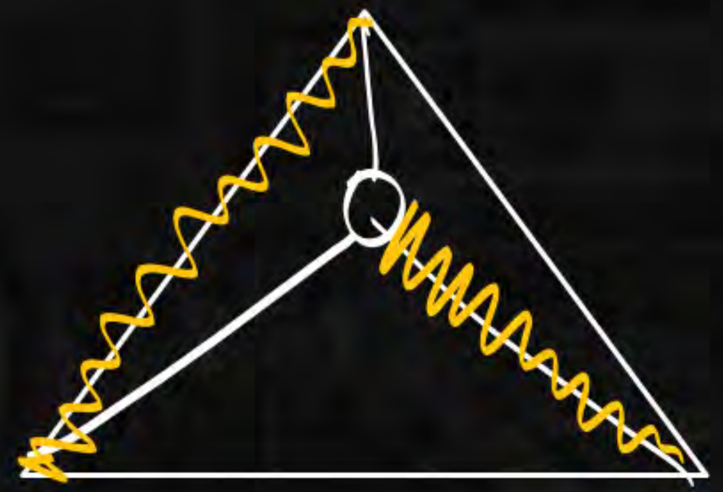
1. Total no. of matching set

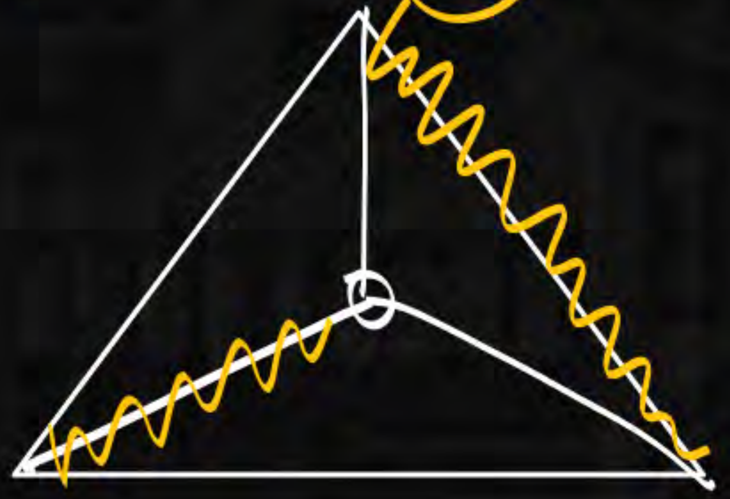
0 edge/single empty set → 1 matching

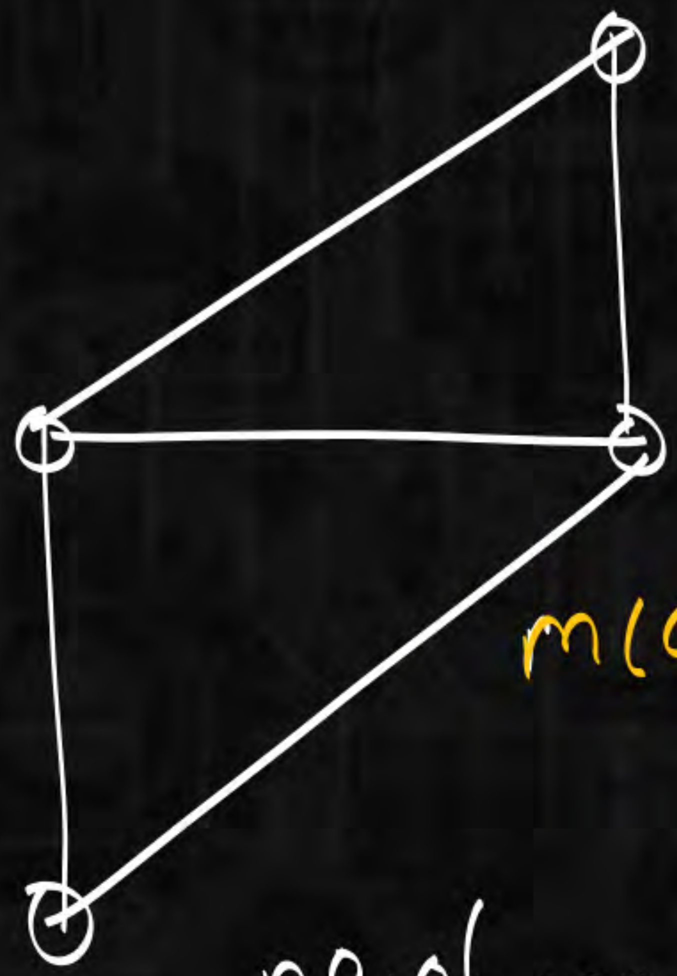
single edge → 6 matching set

2 edges → 3 matching set

10



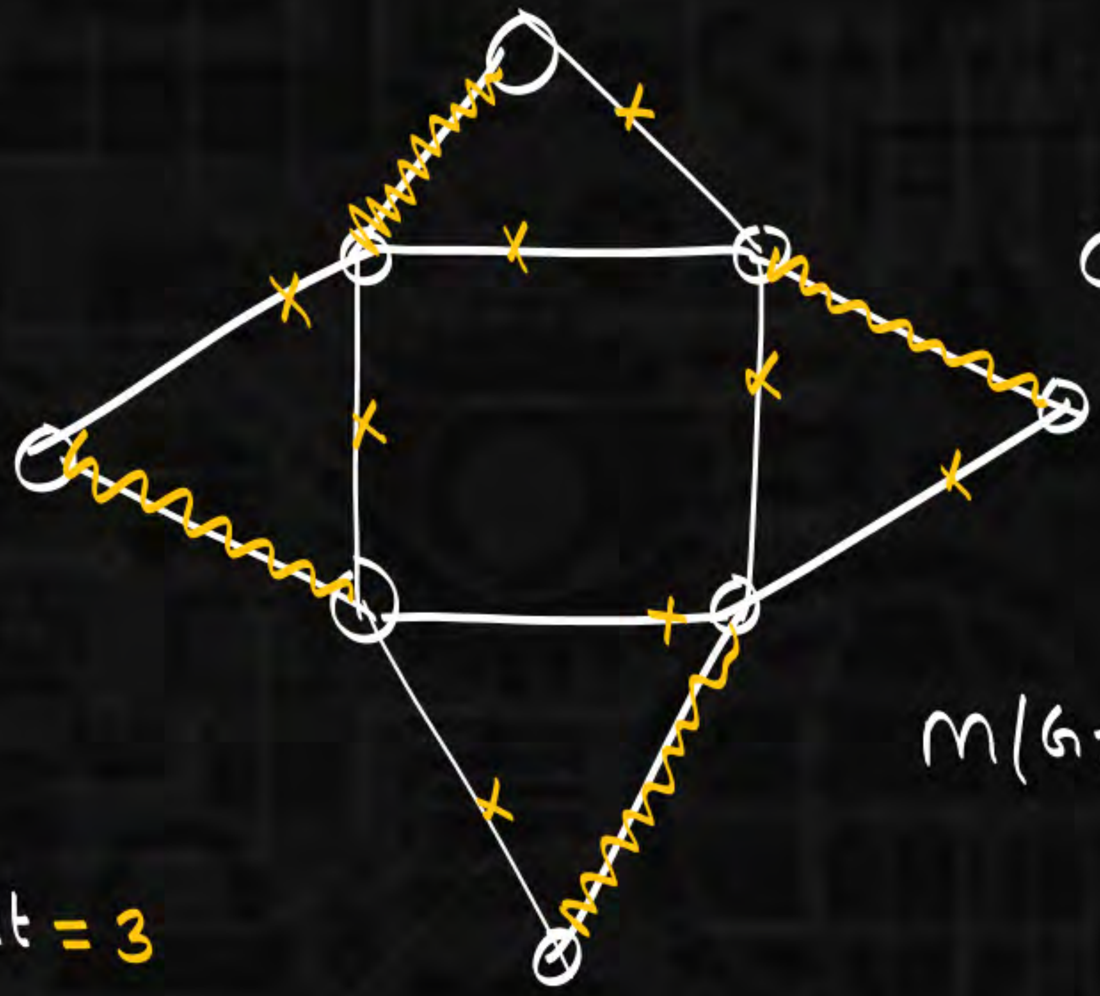




G_1

$m(G_1) = 2$

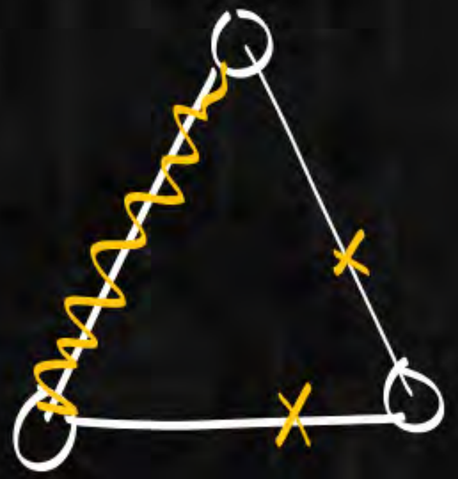
no. of maximal matching set = 3



G_2

$m(G_2) = 4$

Cycle Graph $(C_n)\ (n \geq 3)$

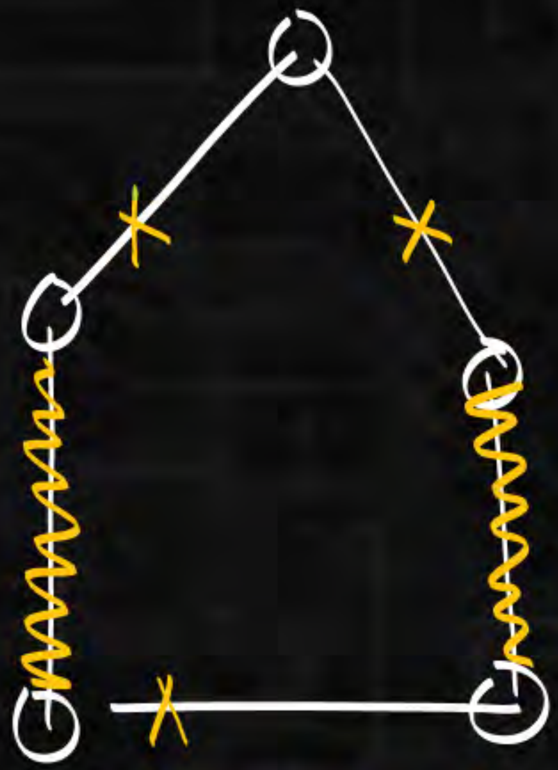


$m(C_3) = 1$

$\left\lfloor \frac{3}{2} \right\rfloor = 1.5$

$m(C_4) = 2$

$\left\lfloor \frac{4}{2} \right\rfloor = 2$



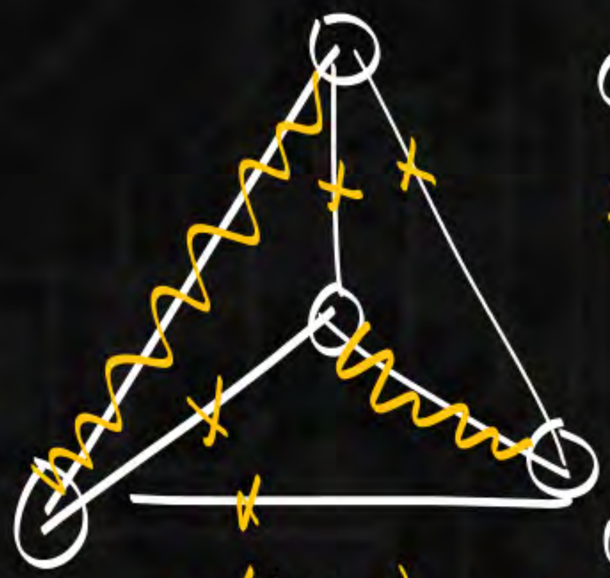
$m(C_5) = 2$

$\left\lfloor \frac{5}{2} \right\rfloor = 2.5$

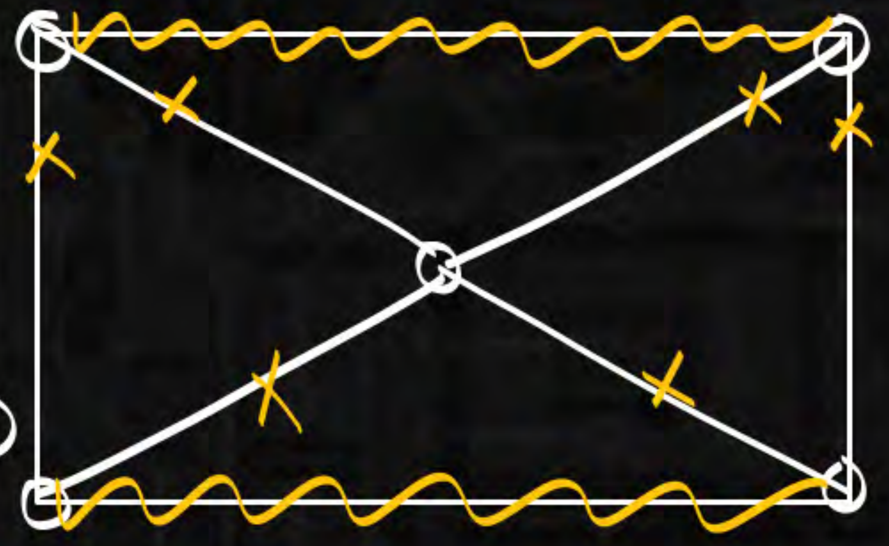
$$m(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$$\underline{m(W_n)} = \lfloor n/2 \rfloor$$

$$\underline{m(K_n)} = \lfloor n/2 \rfloor$$



$m(W_4) = 2$



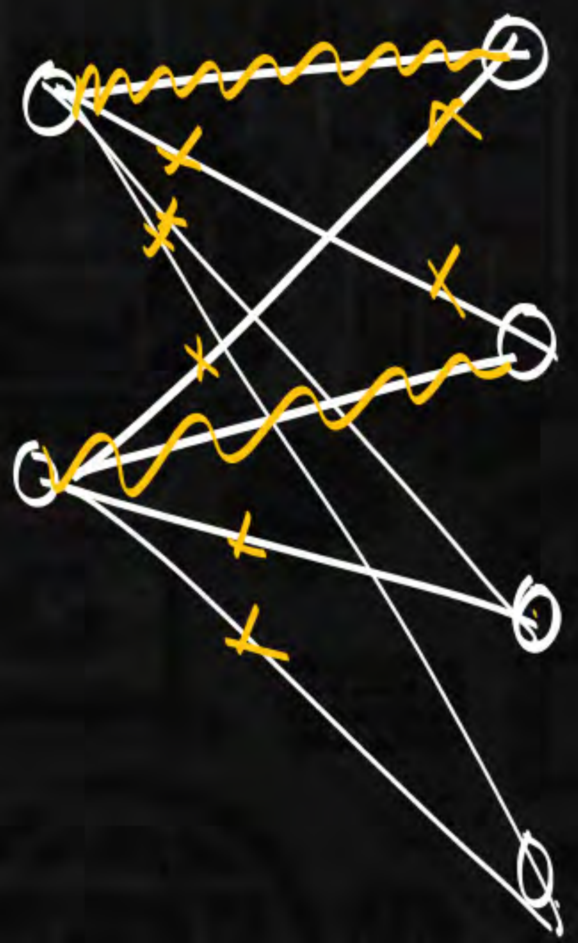
$m(W_5) = 2.$

$m(W_n) = \lfloor n/2 \rfloor$

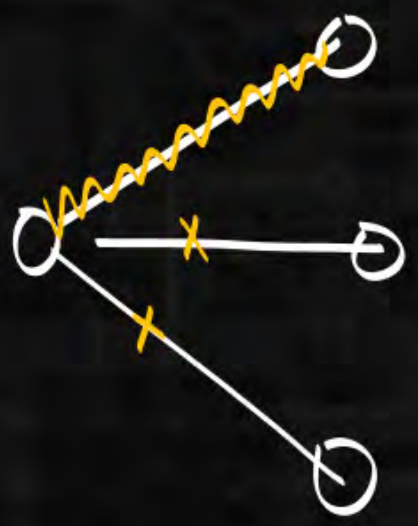
$m(K_n) = \lfloor n/2 \rfloor$

$m(W_n) = m(K_n) = m(C_n) = \lfloor n/2 \rfloor$

$m(K_{2,4}) = 2.$



$m(K_{1,3}) = 1.$



$$\boxed{m(K_{m,n}) = \min(m,n)}$$

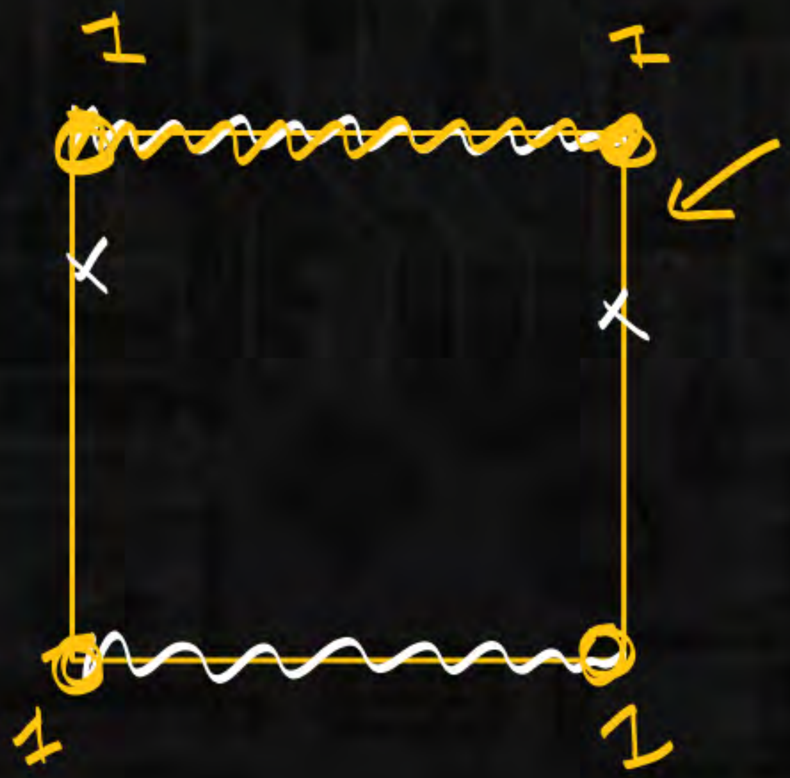
Consider a Disconnected Graph.
of 11 vertices & maximum edges:

$K_1 \quad K_{10} \longrightarrow m(\underline{K_{1,10}}) = \min(1,10)$

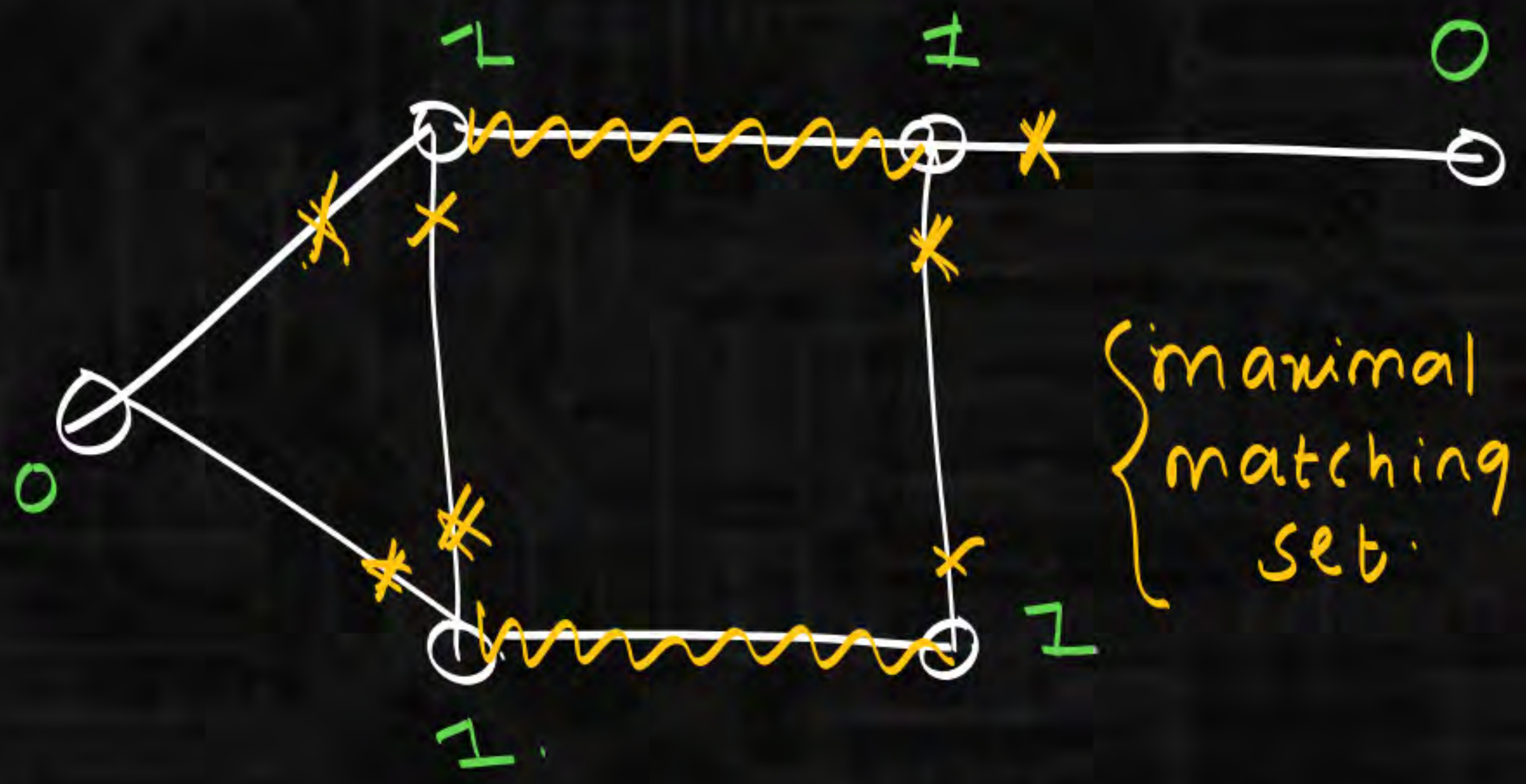
$= 1.$

take a complement of above graph
& find out matching no.

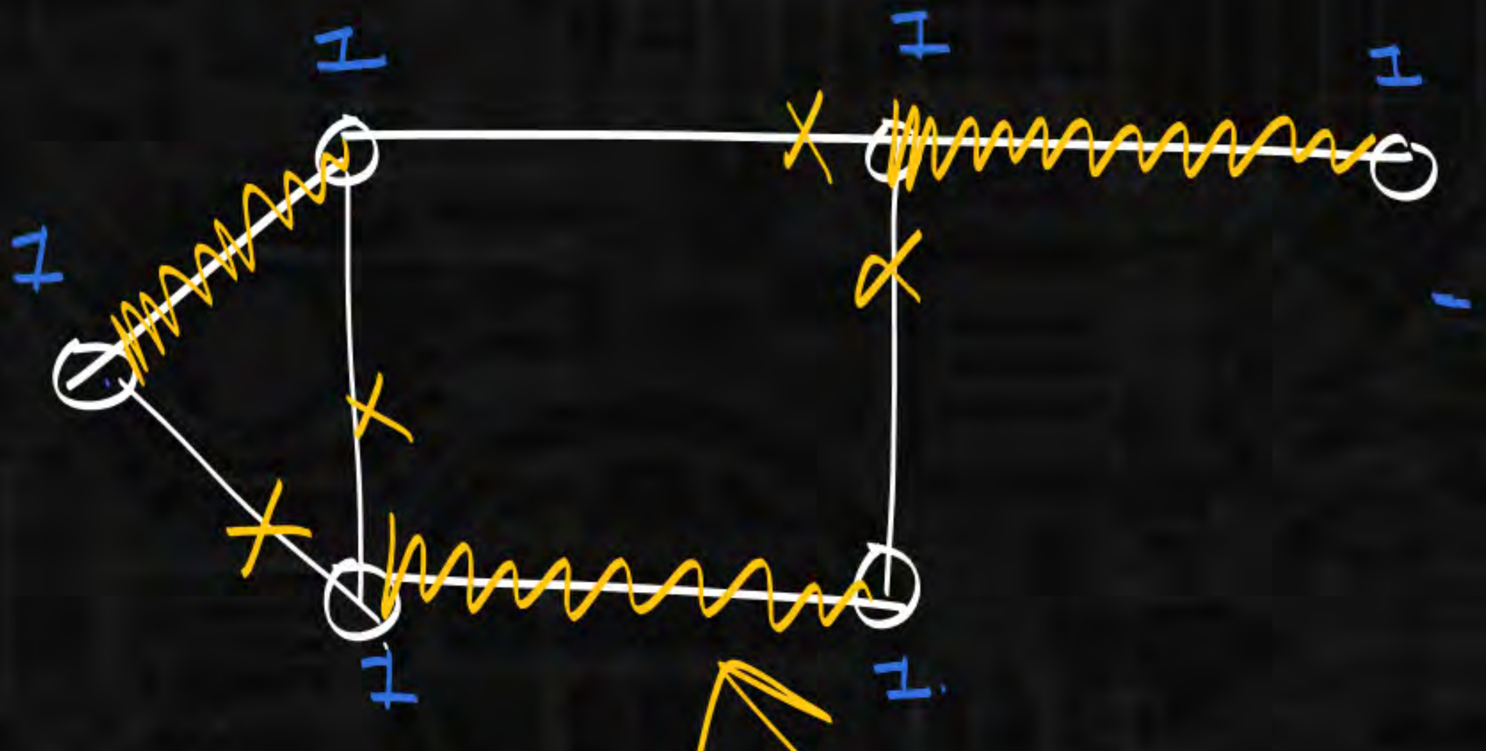
Perfect matching :



p.m. { maximal matching set + Drawing degrees of all vertices are 1



maximal matching set

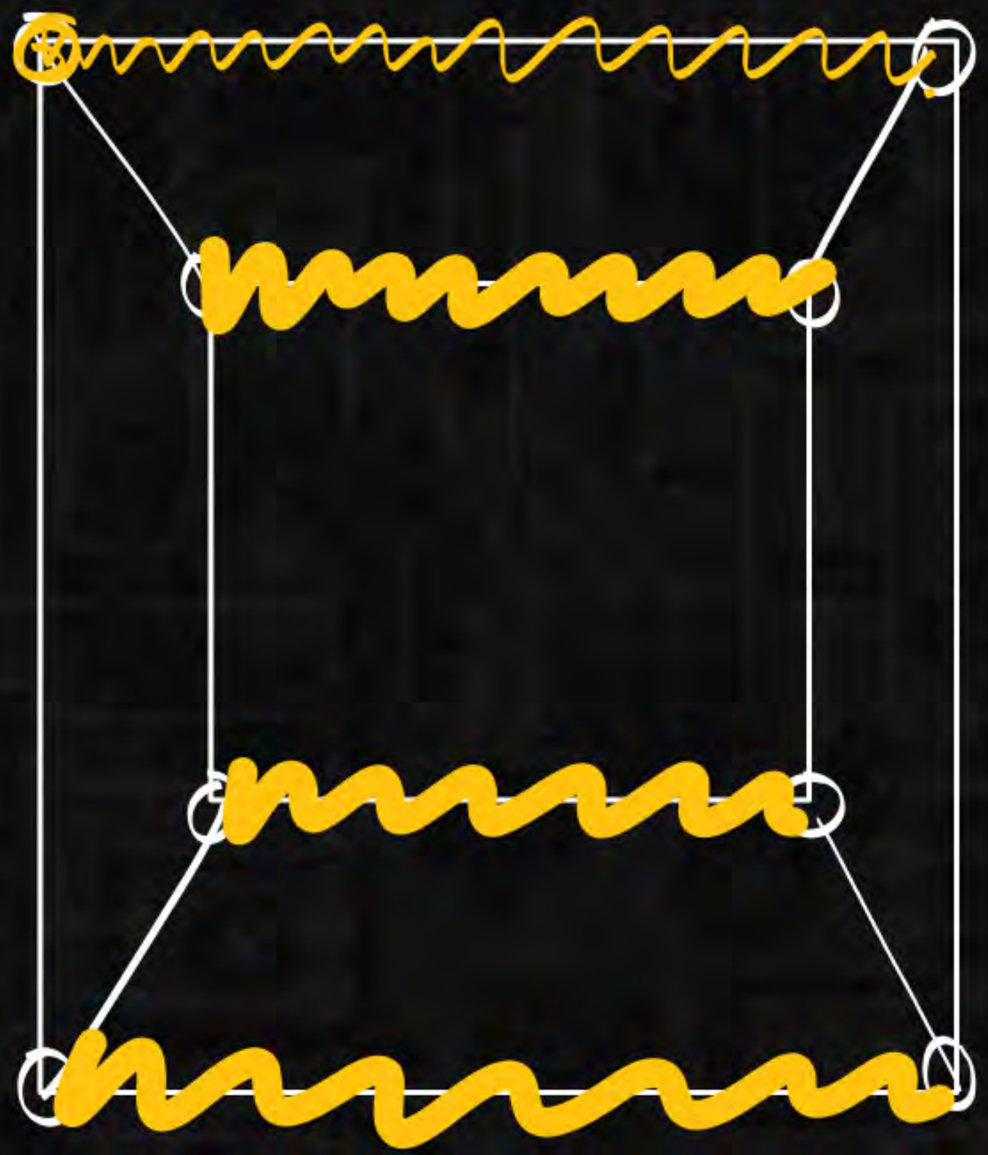


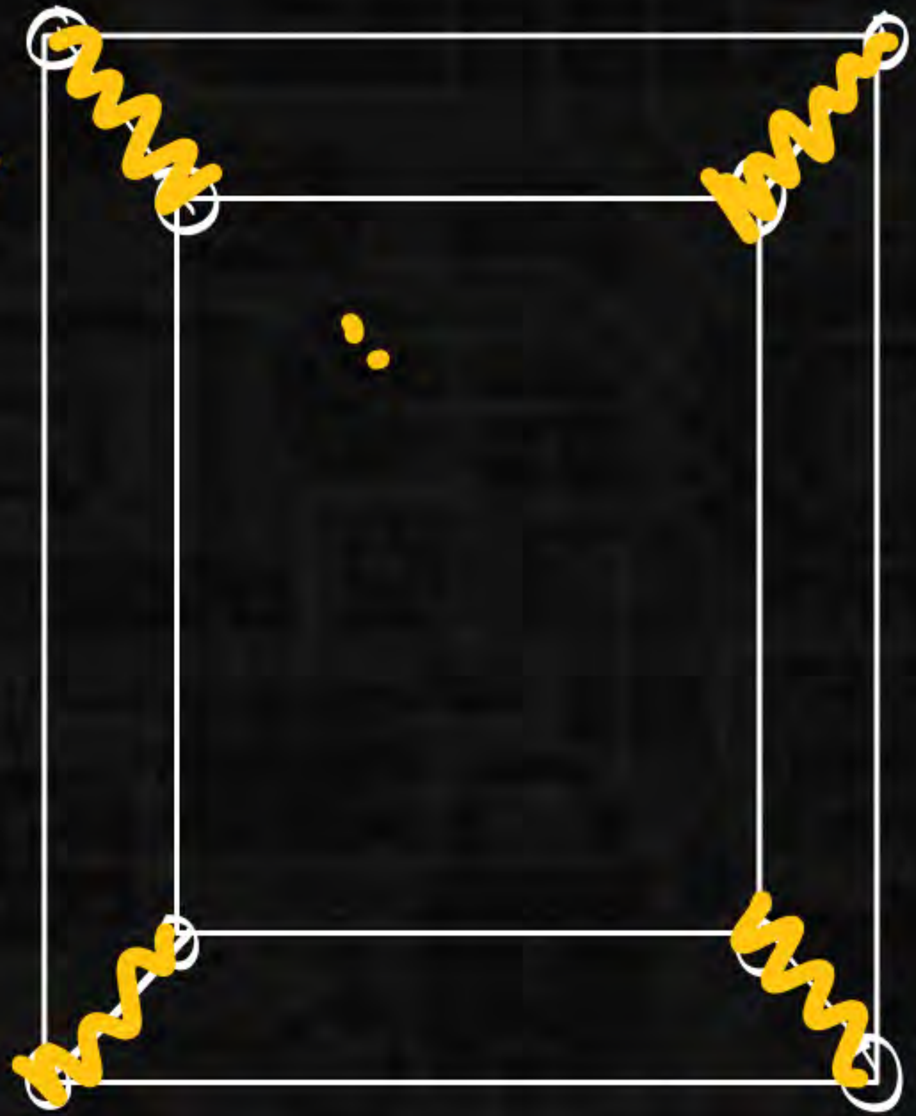
Perfect matching

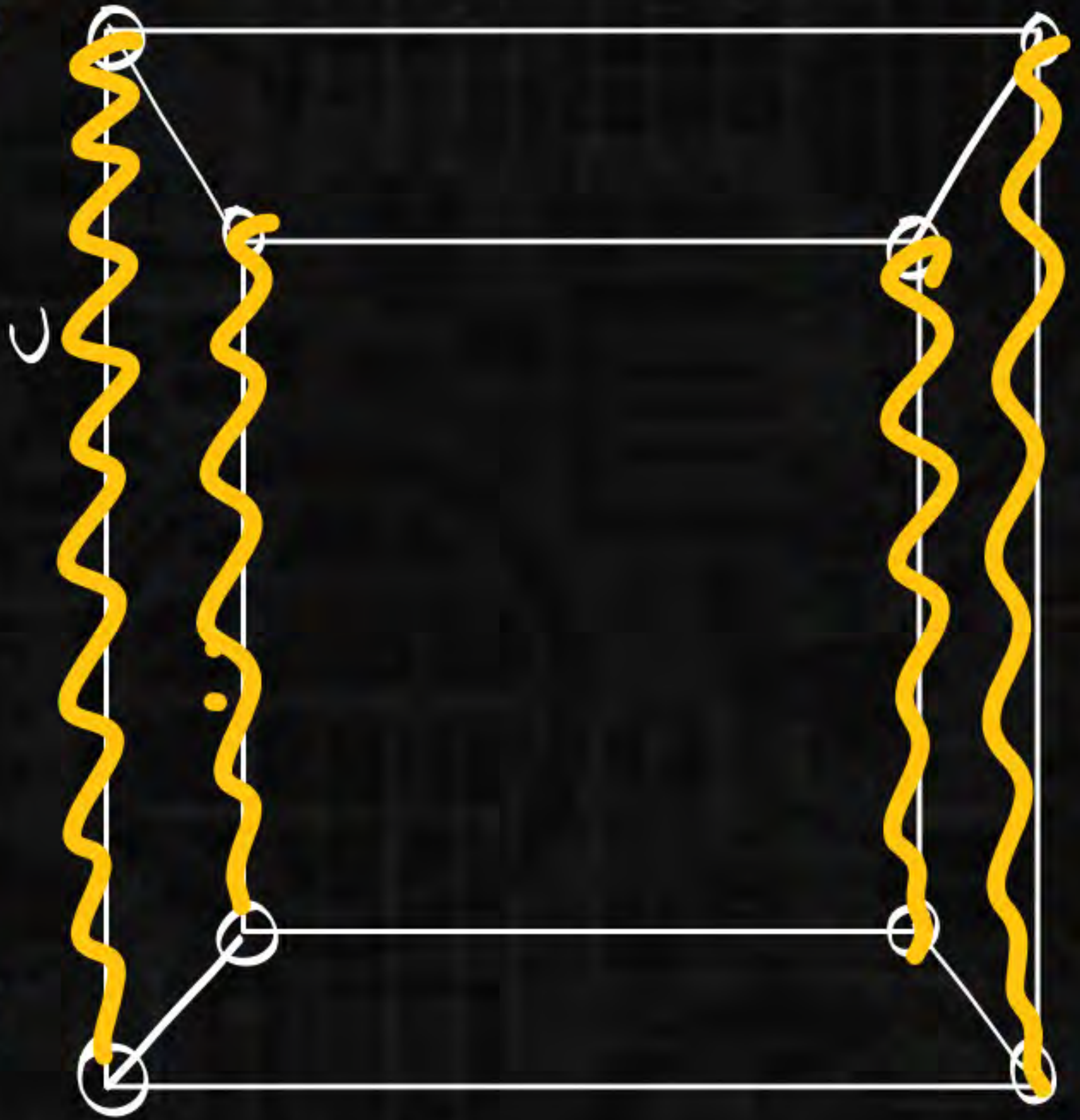
if Perfect matching exist then no. of vertices will be even. (True)

if Graph contains even no. of vertices then P.M exist (false)

$n = 6$







Total no. of perfect matching in K_{2n} $=$ $\boxed{\dfrac{(2n)!}{2^n \cdot n!}}$

Total vertices $= 2n$

$$= (2n-1)(2n-3)(2n-5)\ldots\ldots$$

$$\frac{2n}{2n}(2n-1)\frac{(2n-2)}{(2n-2)}(2n-3)\frac{2n-4}{2n-4}(2n-5) = \frac{(2n)!}{2n\cdot(2n-2)(2n-4)}$$

$\downarrow$ ① (take 2 common)

$$= \frac{(2n)!}{2^n \cdot n(n-1)(n-2)\cdots}$$

$$= \frac{(2n)!}{2^n \cdot n!}$$

$$(2n)(2n-2)(2n-4)(2n-6)\cdots\cdots\cdots-1.$$

tak 2 common.

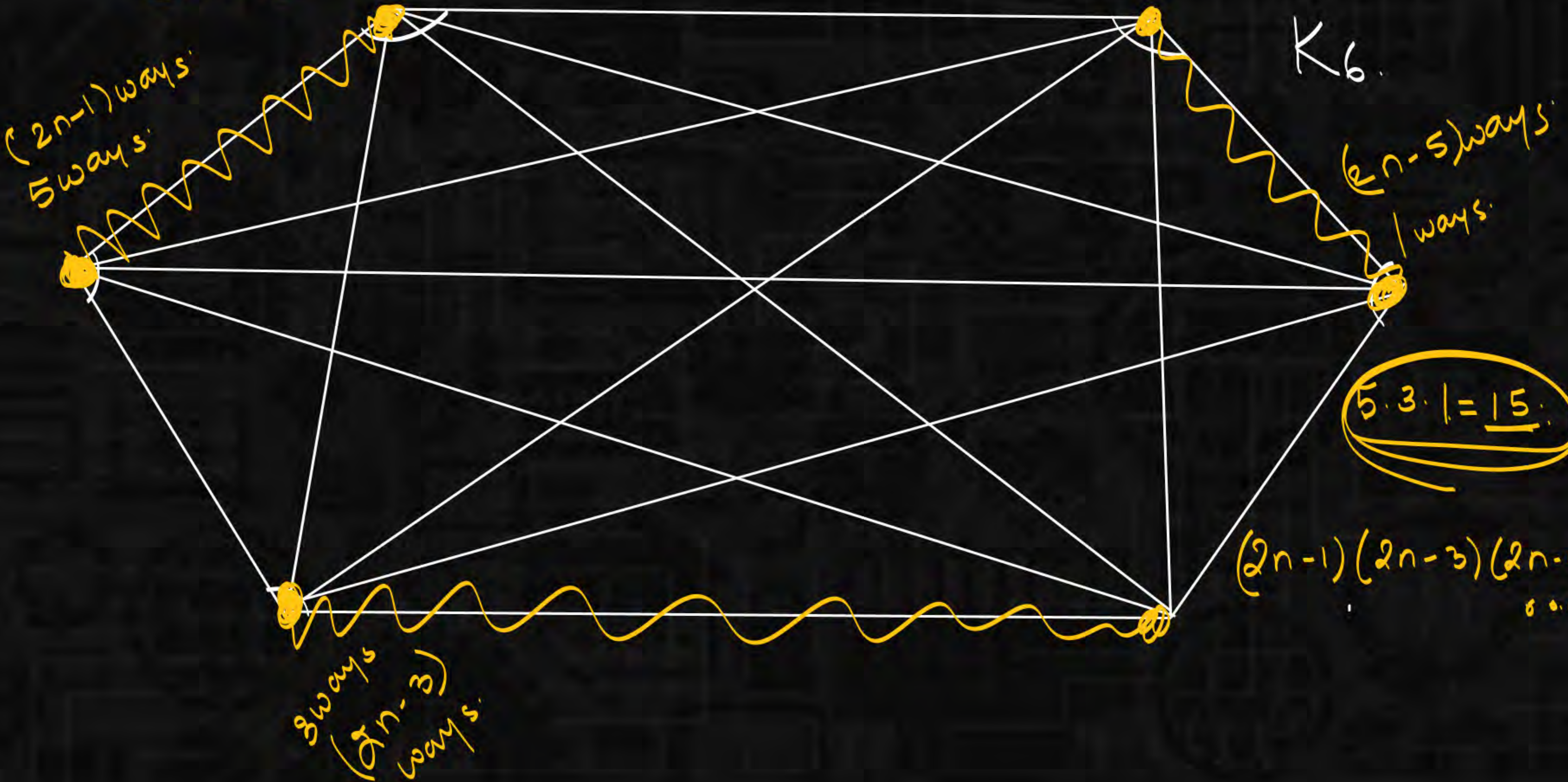
$$2(n)\times 2(n-1)\times 2(n-2)\times 2(n-3)\cdots\cdots$$

$$2^n\times(n)(n-1)(n-2)(n-3)\cdots\cdots$$

$$2^n\cdot n!$$

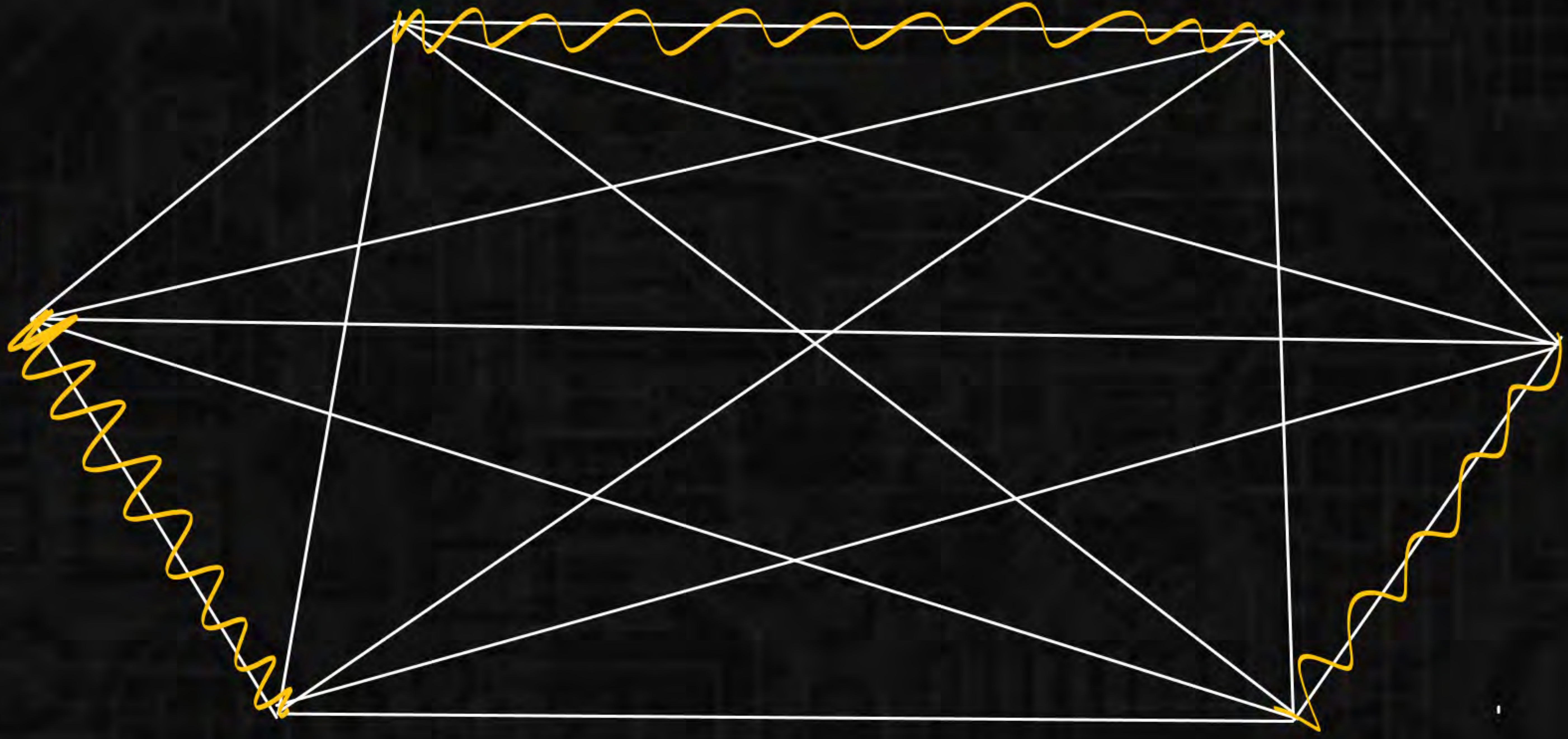
$2n = 6$

K_6



$5 \cdot 3 \cdot 1 = 15$

$(2n-1)(2n-3)(2n-5)\ldots$



GATE:

Total no. of P.M in complete Graph of 6 vertices.

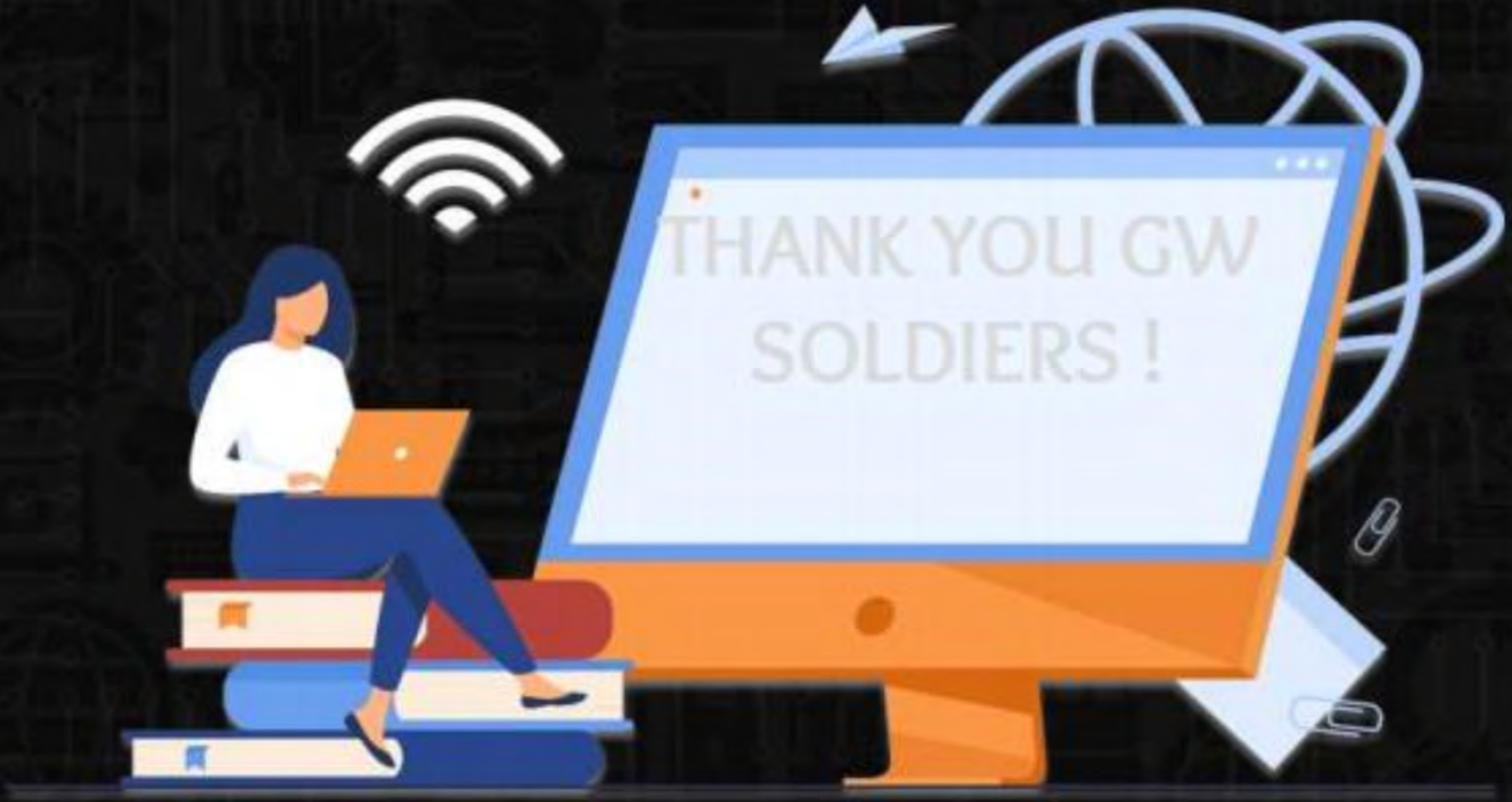
$$\frac{(2n)!}{2^n \cdot n!}$$

$n = 6$ (mistake)

$2n = 6$

$n = 3$

$$\frac{(2\cdot 3)!}{2^3 \cdot 3!} = \frac{6!}{8 \cdot 3} = \underline{\underline{15}}$$


THANK YOU GW
SOLDIERS !