

CS & IT ENGINEERING



DISCRETE MATHS
COMBINATORICS
Basics of Combinatorics
Lecture No. 1



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TOPICS TO BE COVERED

01 Sum Rule

02 Product Rule

03 Sum Rule & Product Rule

COMBINATORICS

Sum Rule/product Rule ✓

P/C → Reptn.

pigeon hole principle.

Inclusion-Exclusion.

Euler- ϕ -function ✓

* Generating function (2024)

Recurrence Relation.

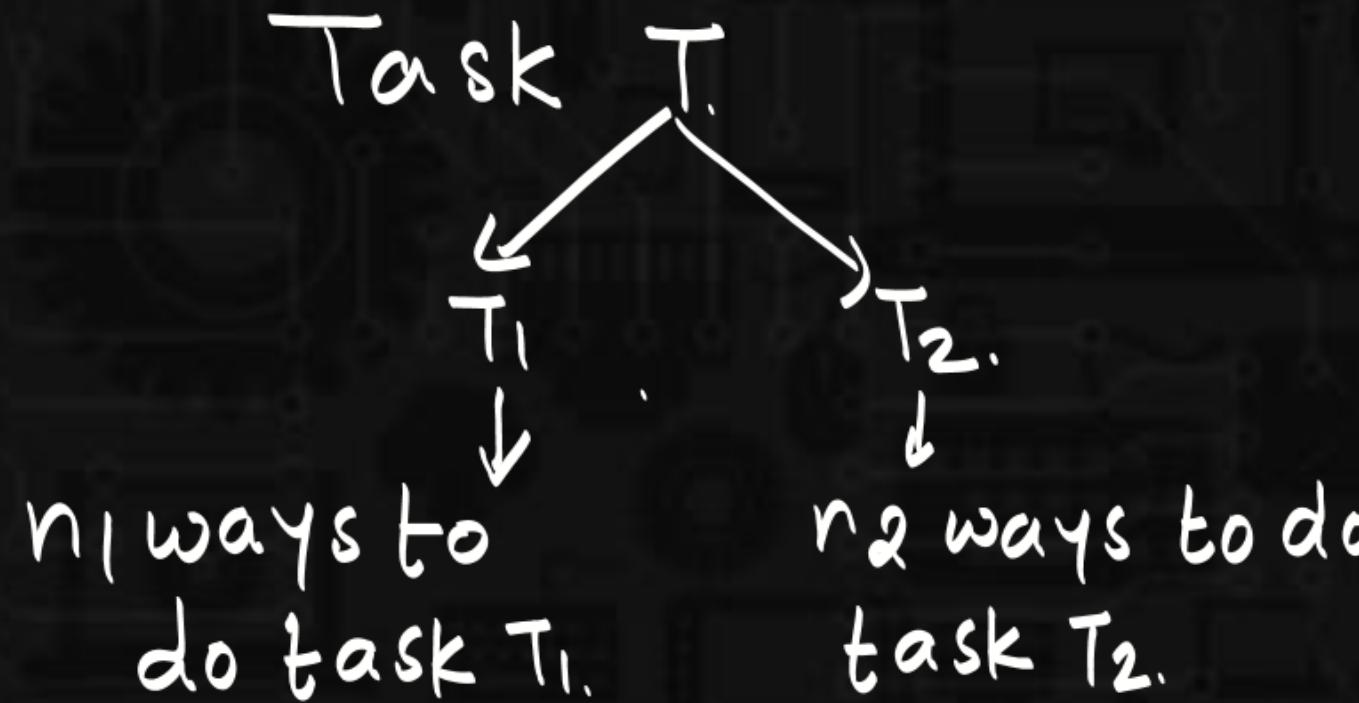
Derangement

Binomial coefficient

Extended binomial coefficient

COMBINATORICS

Product Rule: (x)

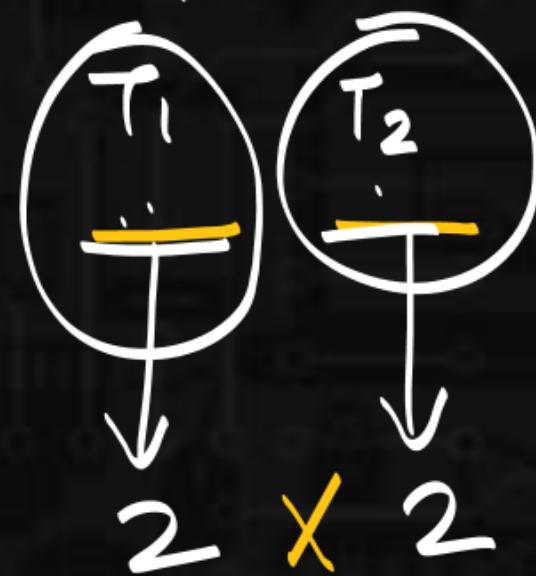


- * both tasks are happening simultaneously.
- * for each of $\underline{n_1}$ ways we can perform $\underline{n_2}$.

$$\text{Total ways} = n_1 * n_2.$$

COMBINATORICS

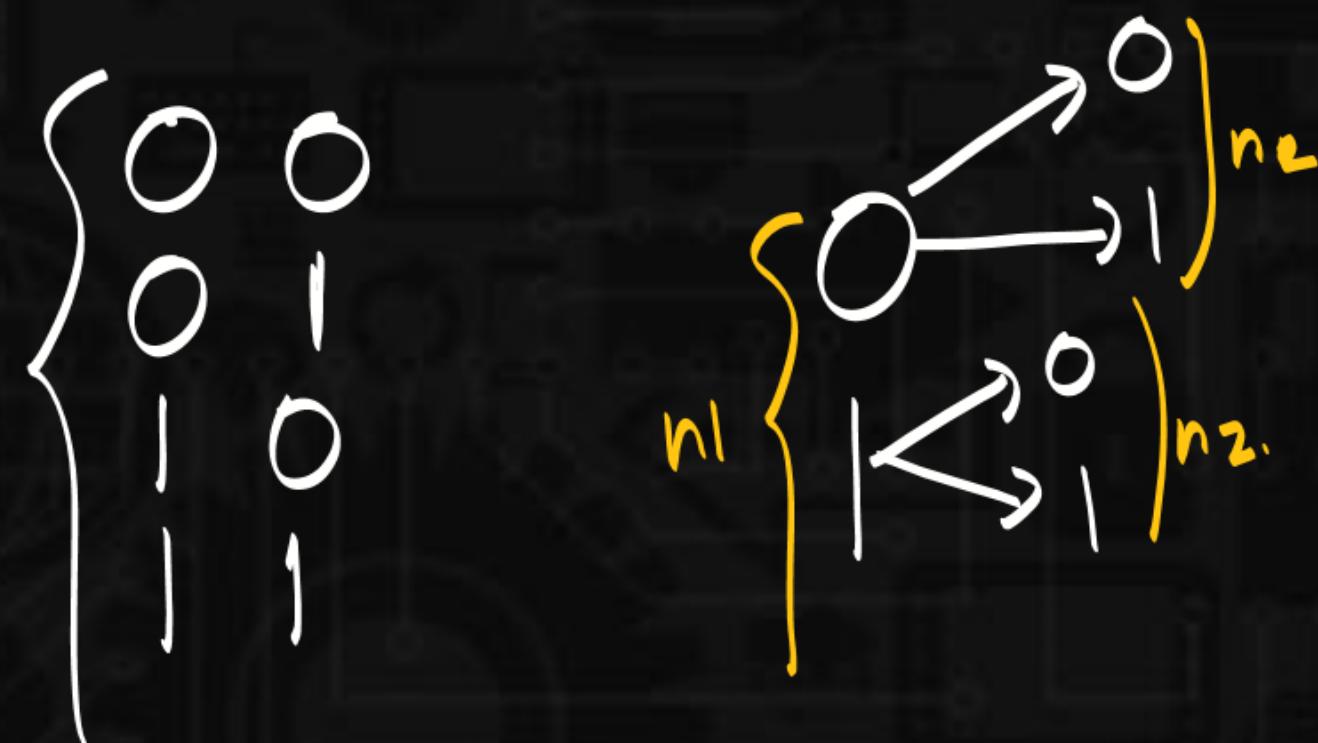
T: how many ways we can generate signal if we have 2 bit code ?



2 ways $\rightarrow T_1 \begin{cases} 0 \\ 1 \end{cases}$

2 ways $\rightarrow T_2 \begin{cases} 0 \\ 1 \end{cases}$

$$\text{Total ways} = 2 \times 2 = 2^2 = 4.$$



COMBINATORICS

$$\left\{ \begin{array}{l} T_1: f \text{ or } i^o = 1 \rightarrow n_1 \\ T_2: f \text{ or } j^o = 1 \rightarrow n_2. \\ K = K + 1. \end{array} \right.$$

$\begin{matrix} i^o = 1 & j^o = 1 \rightarrow n_2. \\ i^o = 2 & j^o = 1 \rightarrow n_2 \\ \vdots & \vdots \\ i^o = n_1 & j^o = 1 \rightarrow n_2. \end{matrix}$

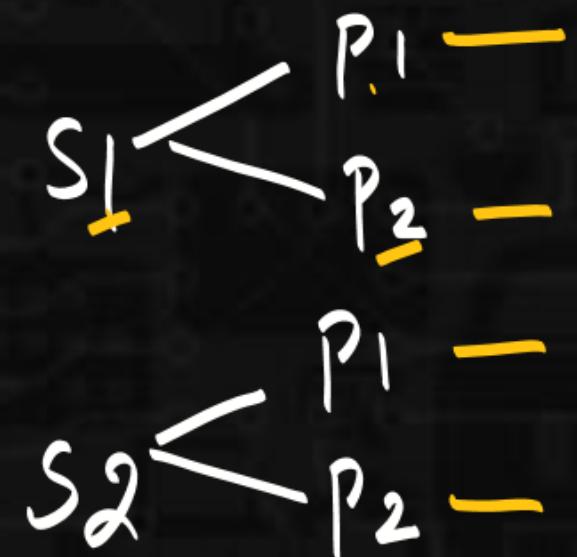
$K = n_1 \times n_2.$

COMBINATORICS

how many ways i can dress up
if i have 2 shirts & 2 pants?

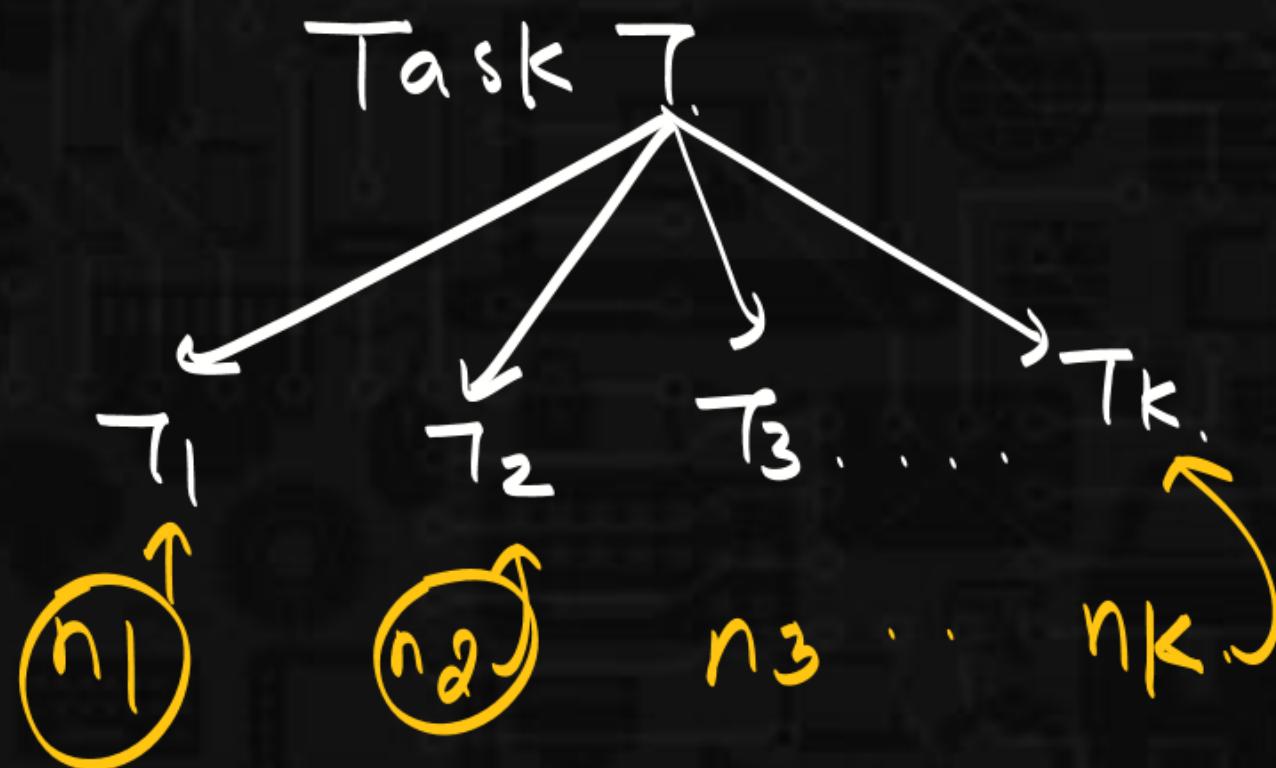
$$\text{Total ways} = 2 \cdot 2 = 4$$

$$\begin{array}{c} 2 \\ \hline S_1 \\ S_2 \end{array} \quad \begin{array}{c} 2 \\ \hline P_1 \\ P_2 \end{array}$$



COMBINATORICS

Extended product Rule. :



all tasks are happening
Simultaneously.

Total ways
 $= n_1 \times n_2 \times n_3 \dots n_k$

COMBINATORICS

$$k = 0$$

$$K = n_1 \times n_2 \times \dots \times n_k.$$

for $i_1 = 1 \rightarrow n_1$

for $i_2 = 1 \rightarrow n_2$

for $i_3 = 1 \rightarrow n_3$.

⋮

for $i_k = 1 \rightarrow n_k$.

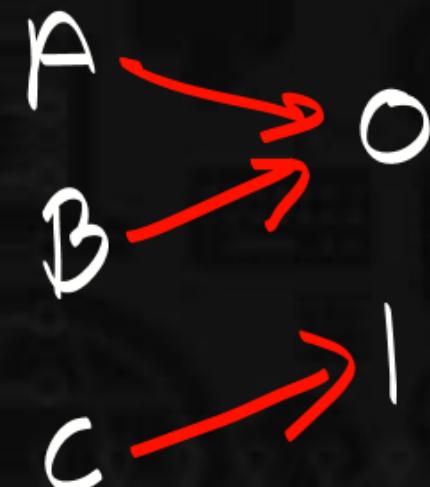
$$k = k + 1.$$

COMBINATORICS

 $f: A \rightarrow B$

$|A| = 3 \quad |B| = 2$

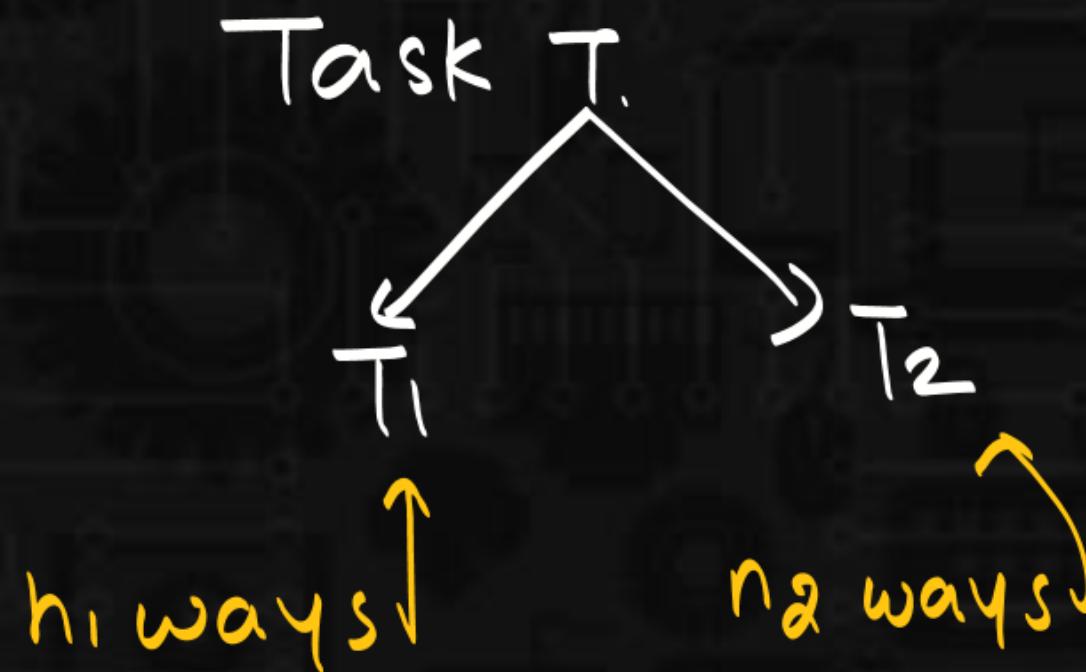
$\text{Total no. of functions} = (r \cdot s)^{|s|} = 2^3$



$$\frac{A}{2} \cdot \frac{B}{2} \cdot \frac{C}{2} = 2^3$$

COMBINATORICS

Sum Rule:



- * both the tasks are **not** happening simultaneously.

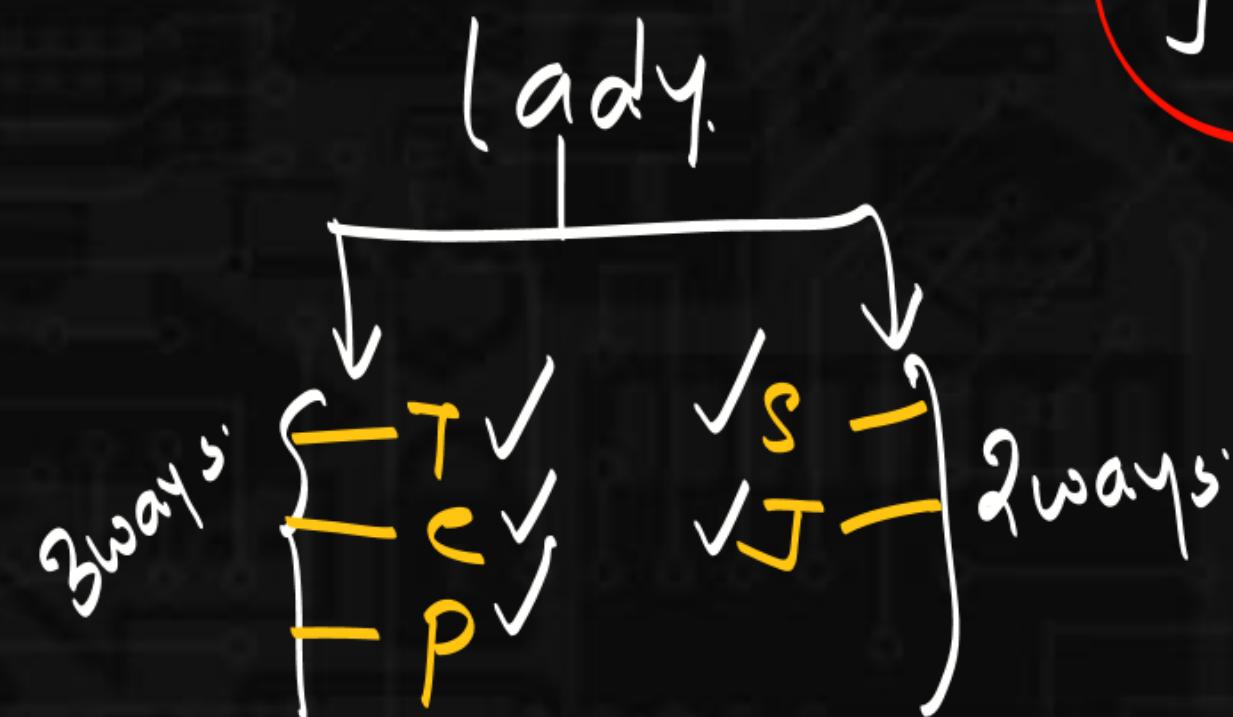
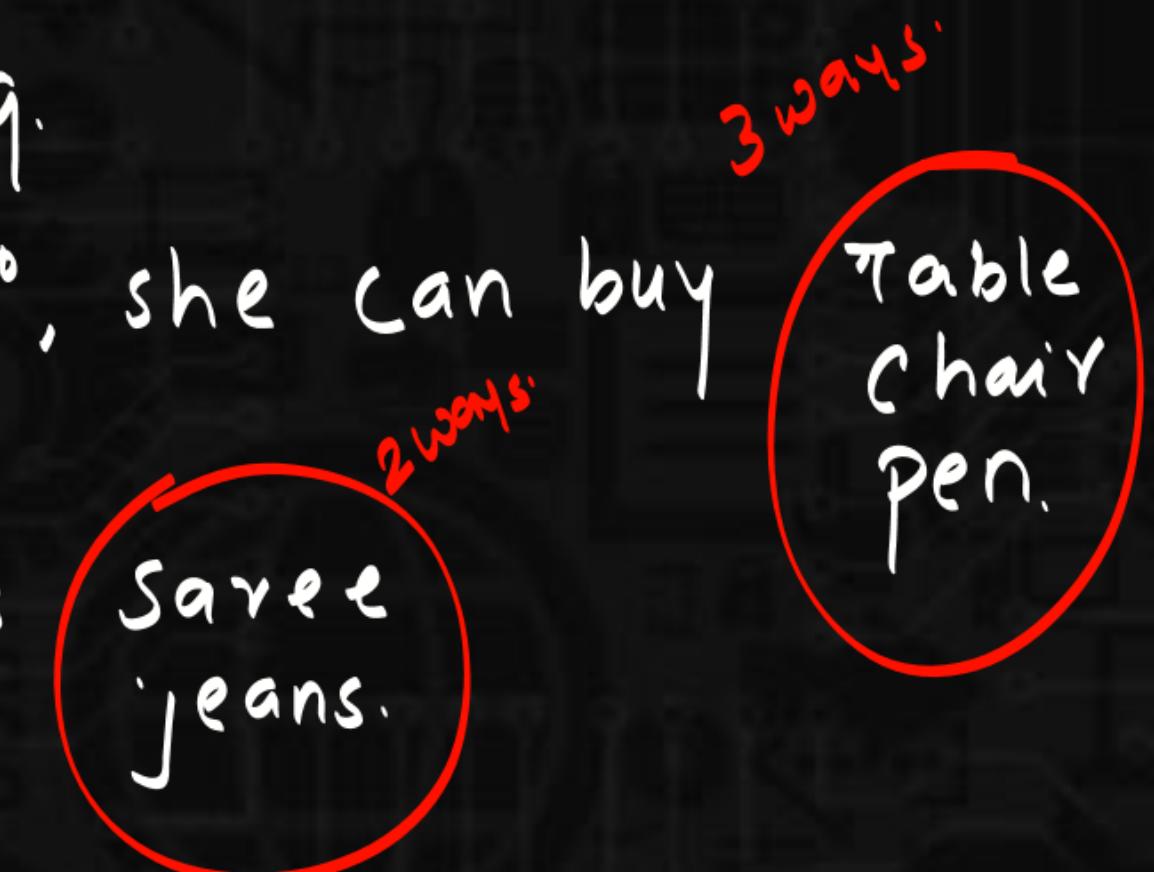
$$\text{Total ways} = n_1 + n_2.$$

COMBINATORICS

Babitaji is going for shopping.

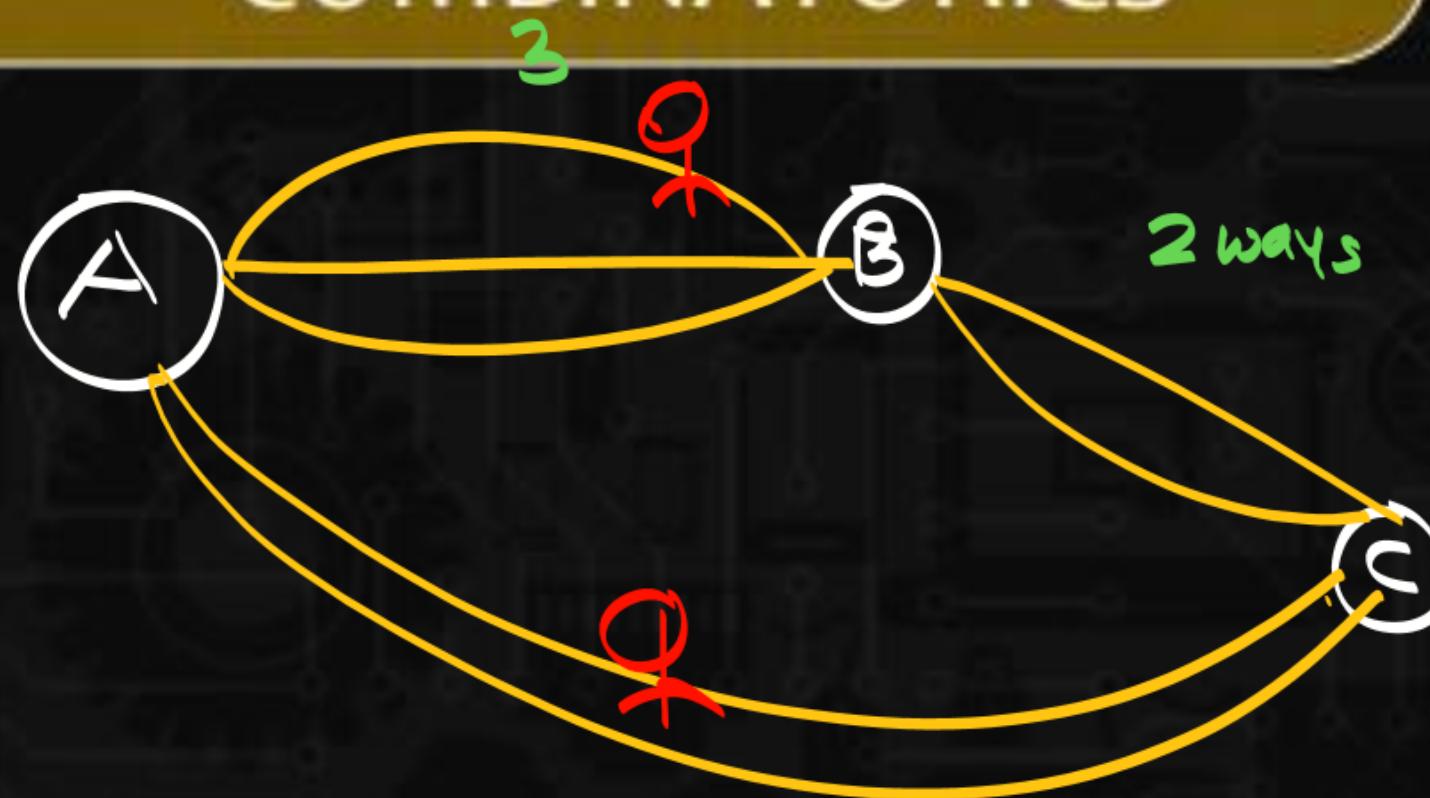
if she does the shopping in Delhi, she can buy

She does shopping in Mumbai she buys



$$\begin{aligned} \text{Total ways} &= 3 + 2 \\ &= \underline{5 \text{ ways}} \end{aligned}$$

COMBINATORICS



$$3_0 \left\{ \begin{array}{c} A \xrightarrow{\text{via } B} C \xrightarrow{\text{via } B} A \\ \downarrow \qquad \downarrow \end{array} \right.$$

$$4 \left\{ \begin{array}{c} A \xrightarrow{\quad} C \xrightarrow{\quad} A \\ \downarrow \qquad \downarrow \end{array} \right.$$

$$1. \ A \xrightarrow{\quad} B \xrightarrow{\quad} C$$

3×2 ways

$$2. \ A \xrightarrow{\quad} C$$

via B

without B

$$3 \times 2 + 2 \text{ ways}$$

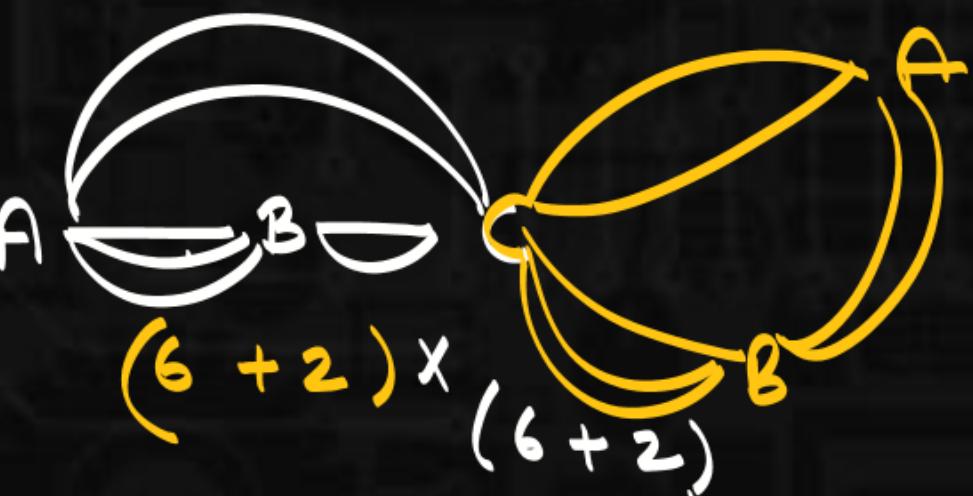
COMBINATORICS



3ways \times 2ways \times 2ways \times 3ways.

= 36 ways.

Q.4.



$$8 \times 8 = 64 \text{ ways.}$$



$$3 \times 2 + 2$$

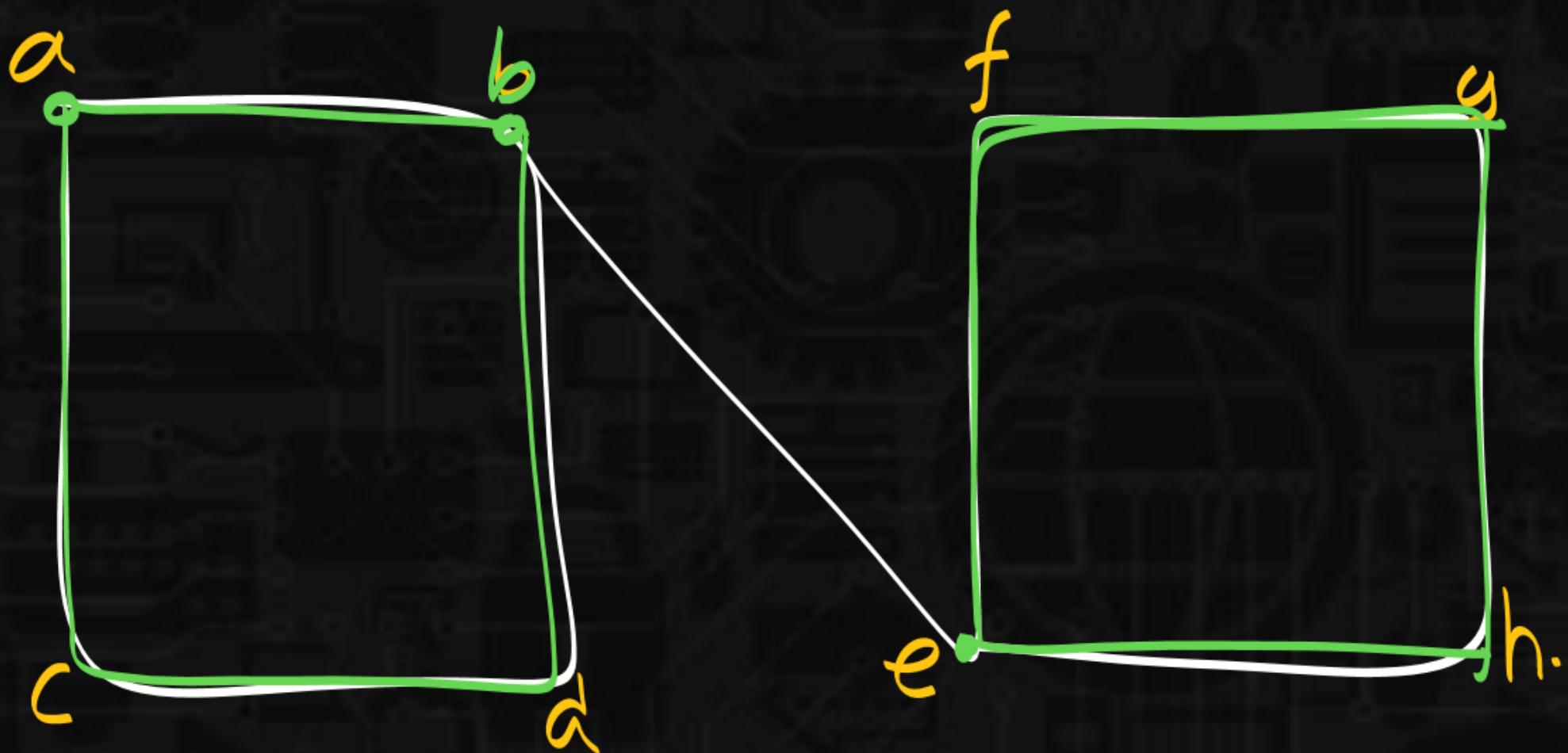
$$= 8 \text{ ways} \times 6 \text{ ways} + 8 \text{ ways} \times 2 \text{ ways}$$

$$8 \text{ ways} (6+2)$$

$$= 64$$

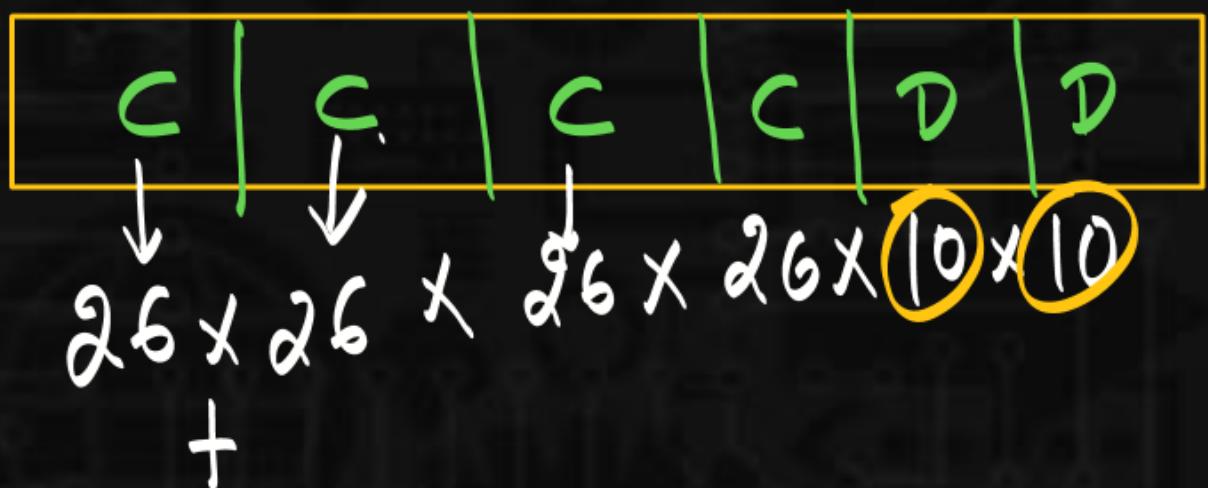
COMBINATORICS

paths are
there
from a to h.



COMBINATORICS

how many number plates
we can generate if = $26^4 \times 10^2$.
4 characters followed by
2 digit ?.



COMBINATORICS

how many number plates
we can generate if
4 characters followed by
2 digit or 3 or 4 digit ?.

$$\begin{aligned} & 26^4 \cdot 10^2 + 26^4 \cdot 10^3 + 26^4 \cdot 10^4 \\ & = 26^4 (10^2 + 10^3 + 10^4) \end{aligned}$$

$$\boxed{C | C | C C \quad D \quad D} + C C C C \underline{D D D} + C C C C \underline{\overline{D D D D}}$$
$$\begin{aligned} & 26^4 \cdot 10^2 \\ & 26^4 \cdot 10^3 \\ & + 26^4 \cdot 10^4 \end{aligned}$$

COMBINATORICS

how many number plates we can generate if 1 OR 2 OR 3 OR 4 characters followed by 1 OR 2 OR 3 OR 4 Digit ?.

$$(26 + 26^2 + 26^3 + 26^4)(10 + 10^2 + 10^3 + 10^4)$$

COMBINATORICS

how many 4 digit integers
are there with digit 0
appearing exactly once?

Ans: 2187

$$|S| = n$$

how many ways we can choose
subsets P and Q

$$P \cap Q = \emptyset$$

- a) 2^n
- b) $3^n - 1$
- c) $3^n + 1$
- d) 3^n

COMBINATORICS

how many integers

$$\{ 1, 2, 3, \dots, 1,00,000 \}$$

Ans: 12600

contains exactly one 3, exactly one 4, exactly one 5?
in decimal representation?

