CS & IT ENGINEERING

Graph Theory

Types of Graphs
Part 2

Lecture No. 4

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TOPICS TO BE COVERED

01 Bipartite graph

02 Star Graph

03 Line graph

04 Complement Graph

05 Isomorphic Graph

Types of graph

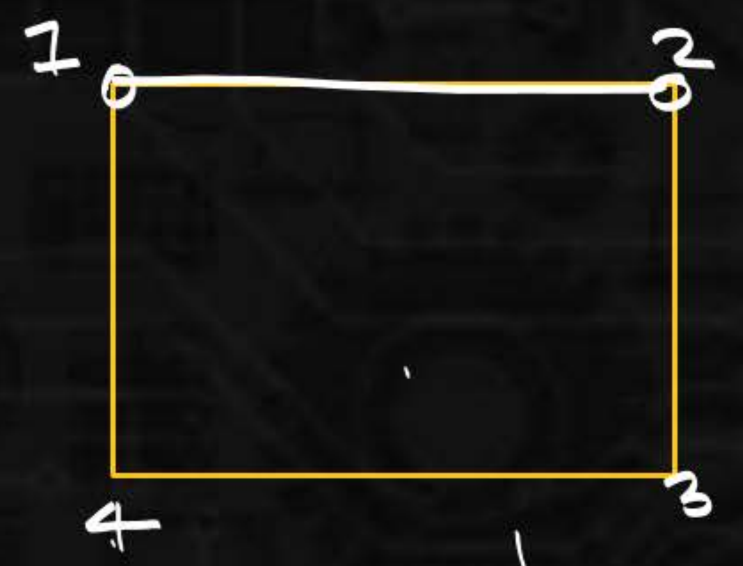
Bipartite Graph :

Graph $G = (V, E)$

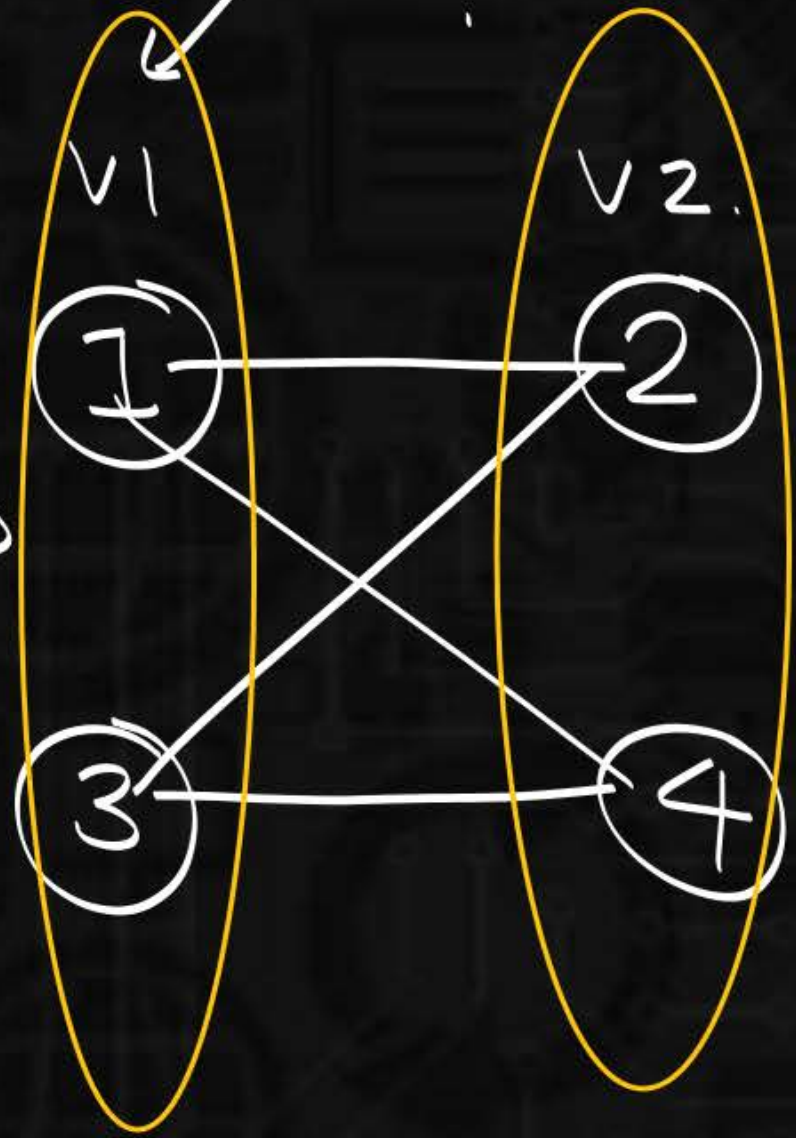
V can be divided into 2 sets

V1, V2.

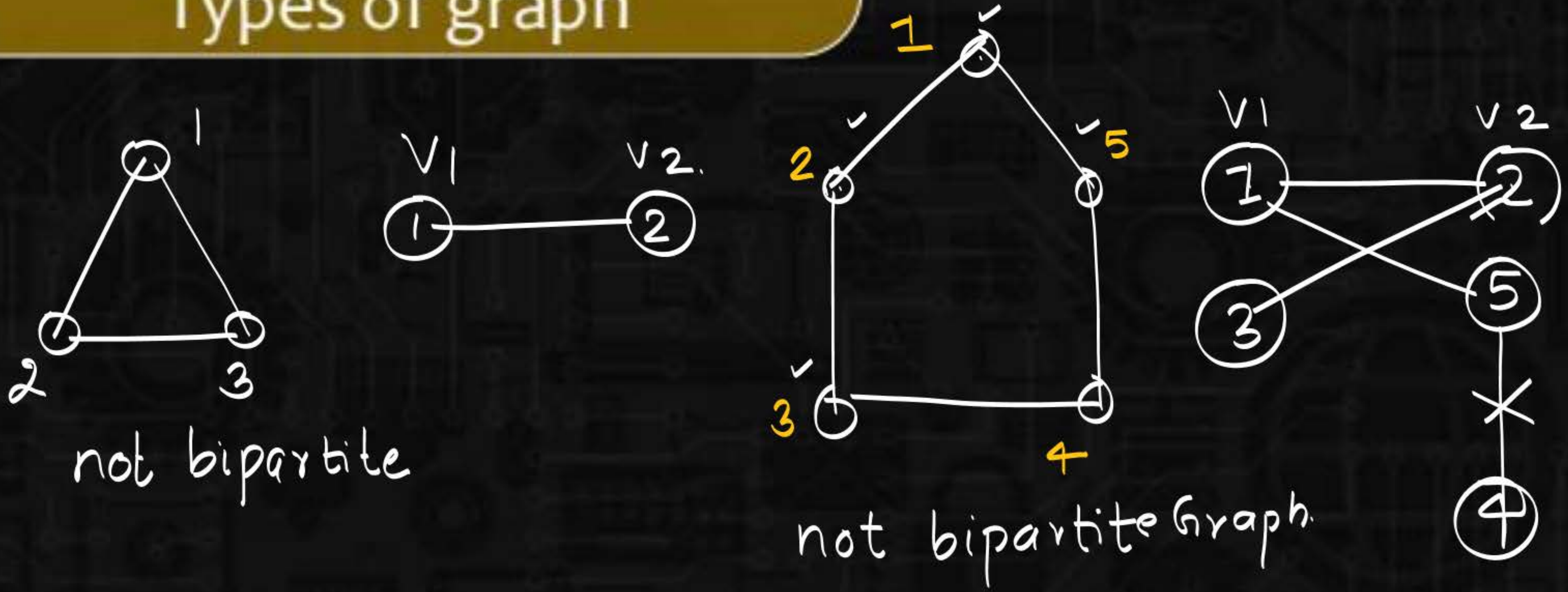
all edges will be from one set to another set but not in same set.



$G = (V, E)$

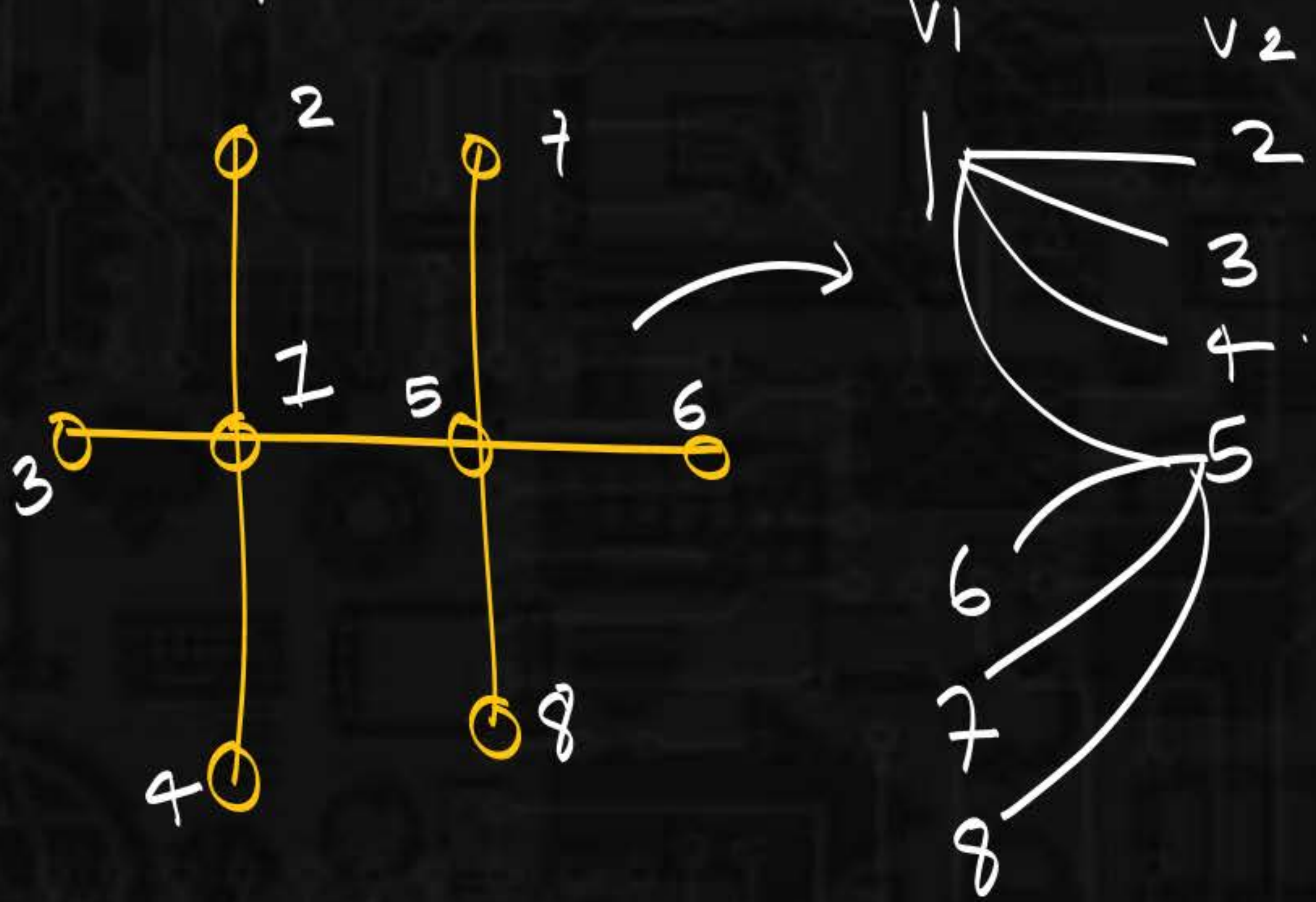


Types of graph



Types of graph

Thm7: Bipartite Graph does not contains odd length cycle.



→ even length cycle

→ no cycle at all.

Types of graph

Bipartite Graph having n vertices what will be maximum no. of edges ?

$n = 6.$

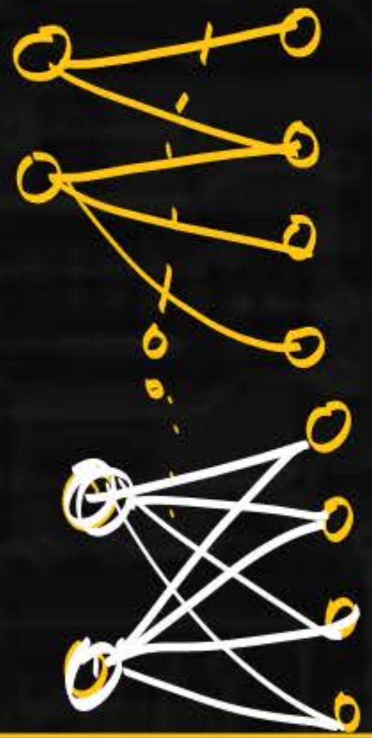
case 2

$V_1 = 1$ $V_2 = 5$

$K_{1,5}$

$e = 5$

$V_1 = 2$ $V_2 = 4$

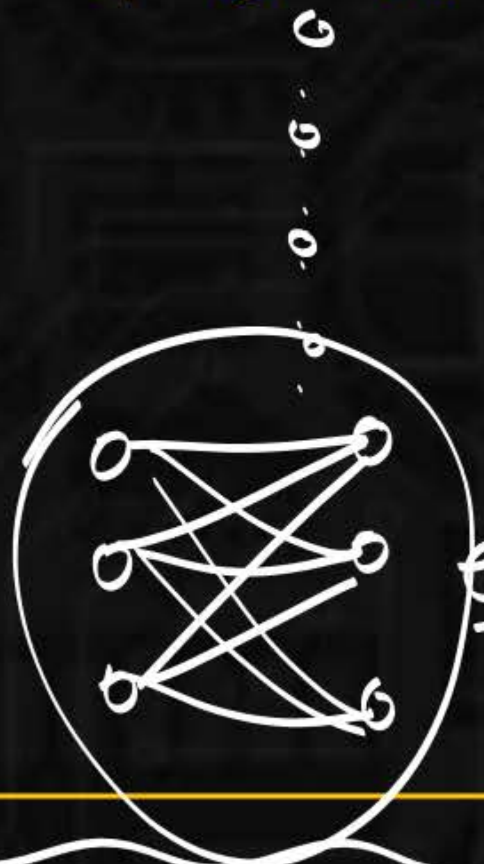


$e = 5$

$K_{2,4}$

$e = 8$

$V_1 = 3$ $V_2 = 3.$



$K_{3,3}$

$e = 9.$

$e = 9$

Types of graph

Thm 8: maximum no. of edges in bipartite Graph $\leq \left\lfloor \frac{n^2}{4} \right\rfloor$

n = Total no of vertices.

$n = 7.$

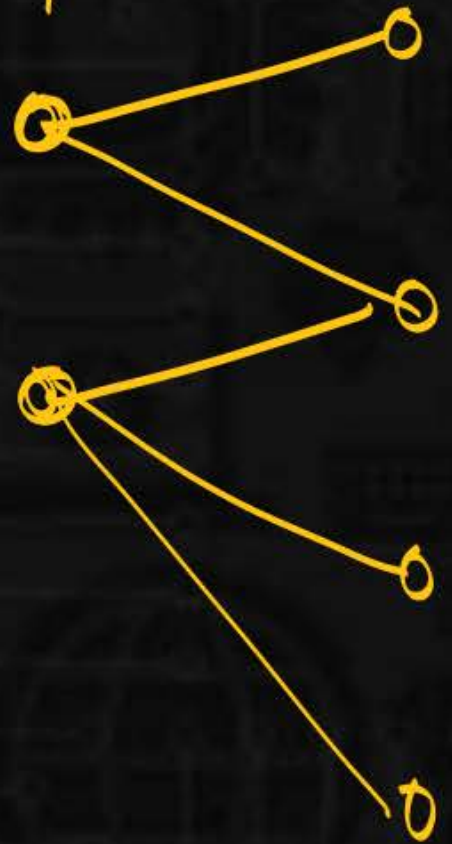
1. 6.	2 5	3 4	4 3.
6.	10	12	12
$K_{1,6}$	$K_{2,5}$	$K_{3,4}$	$K_{4,3}$

$e = 12$

Types of graph

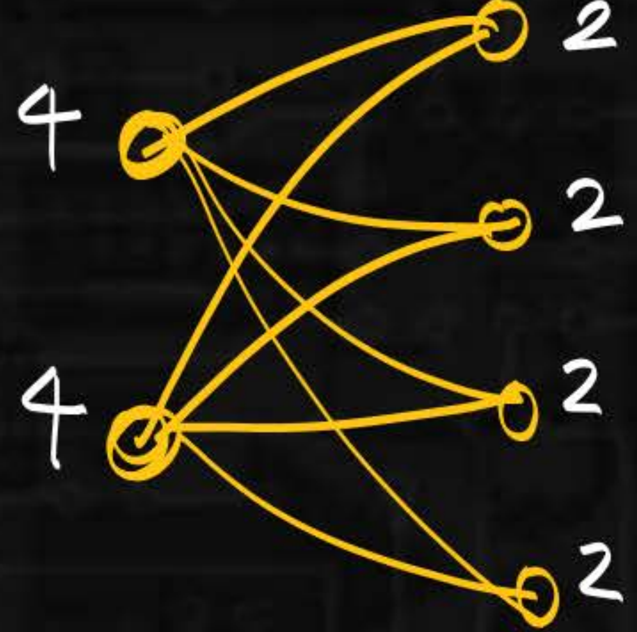
Complete bipartite Graph. $(K_{m,n})$ $|V_1| = m$ $|V_2| = n$.

bipartite Graph.



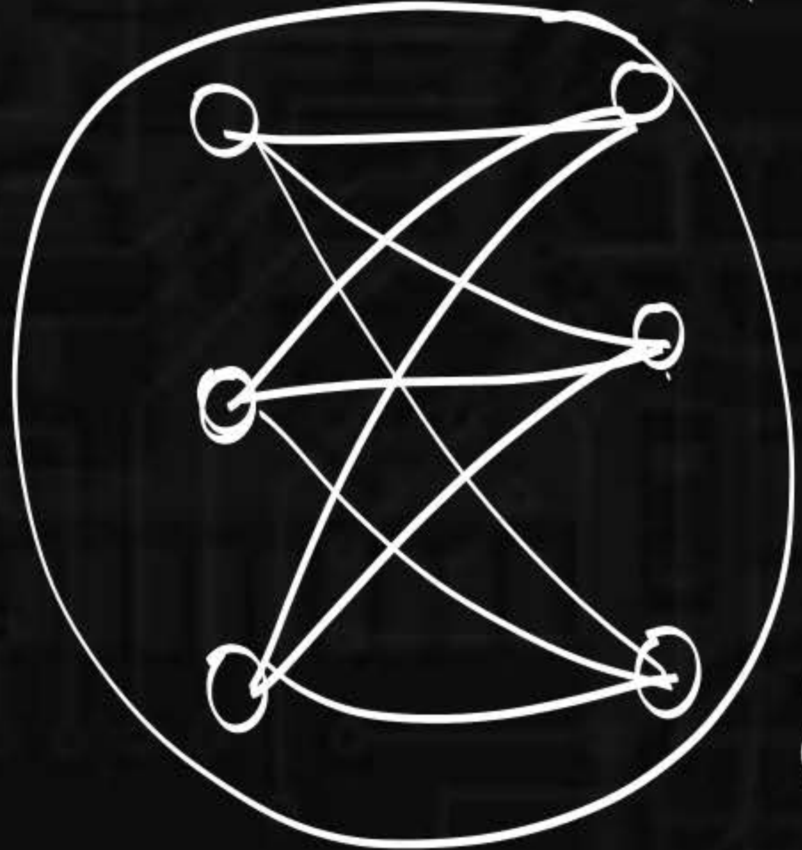
$\boxed{K_{2,4}}$

$e = 2 \times 4 = 8$



$K_{3,3}$.

$e = 3 \times 3 = 9$.



$e(K_{m,n})$

$= m \times n$

Total vertices

$= m + n$.

$\Delta(K_{m,n}) =$

$\max(m,n)$

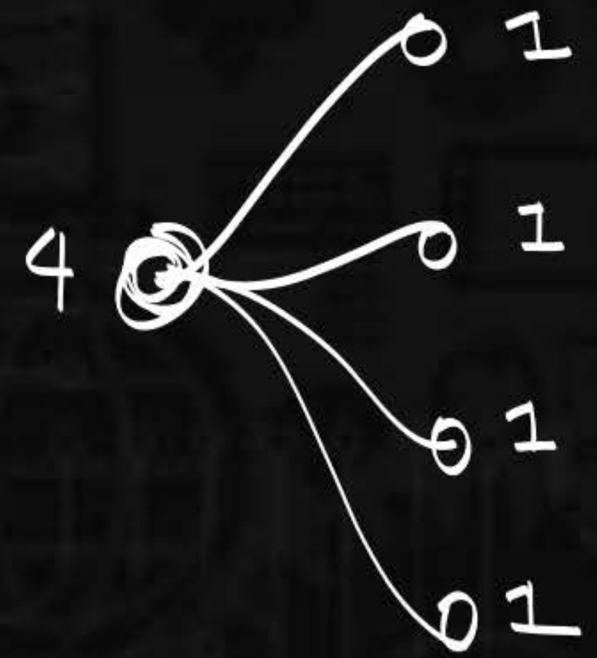
$\delta(K_{m,n}) = \min(m,n)$

Types of graph

<u>Star Graph.</u> $(K_{1,n-1})$ Total vertices $= 1 + n - 1 = n.$

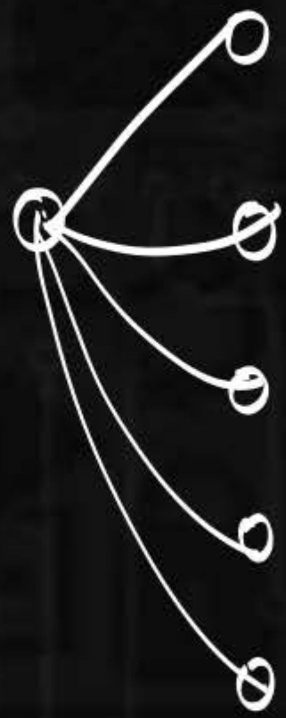
Draw star Graph of 5 vertices

$K_{1,4}$

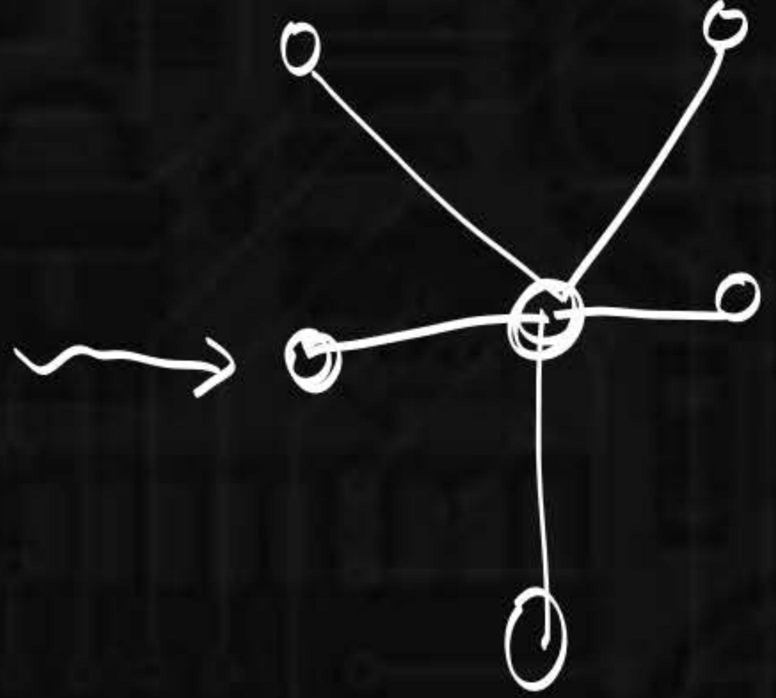


$K_{1,5}$

G_1



G_2



$G_1 = G_2$

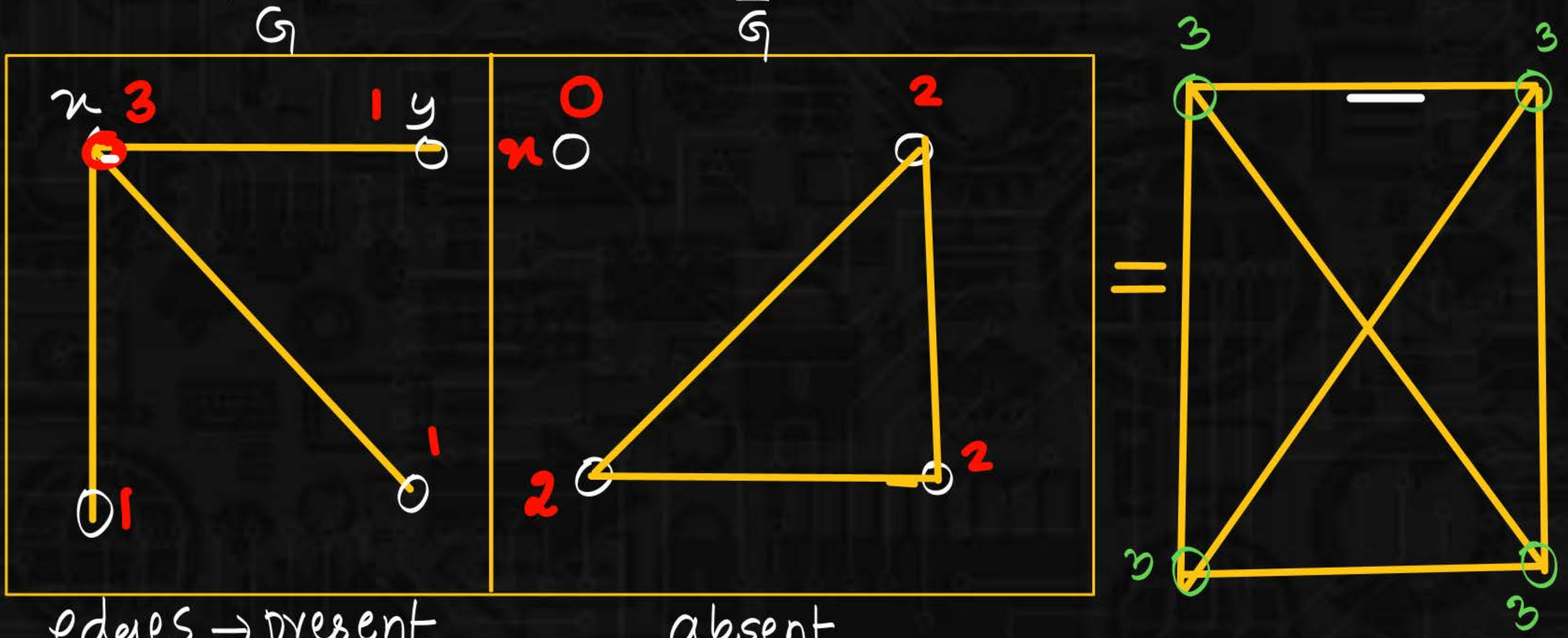
$e(K_{1,n-1}) = n-1$

$\Delta(K_{1,n-1}) = n-1.$

$\delta(K_{1,n-1}) = 1.$

Types of graph

Complement Graph ($\bar{G}$)



edges → present

absent

Types of graph

$G + \bar{G} = K_n$ $\qquad e(G) = x$

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

1. $\boxed{e(\bar{G}) = \frac{n(n-1)}{2} - x}$

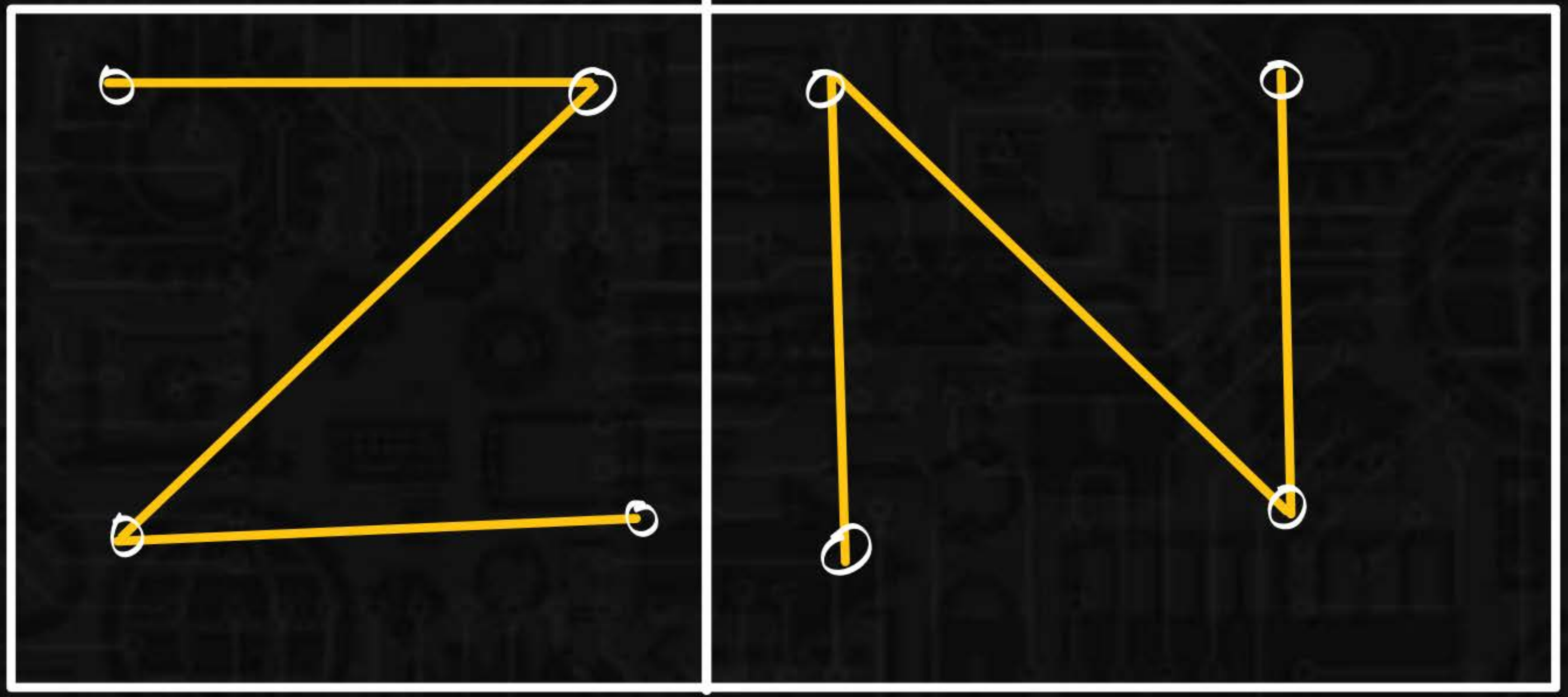
2.

$$\begin{array}{llllll} K_n & n-1 & n-1 & n-1 & \cdots & n-1 \\ G & d_1 & d_2 & d_3 & \cdots & d_n \\ \hline \bar{G} & n-1-d_1 & n-1-d_2 & n-1-d_3 & \cdots & n-1-d_n \end{array}$$

Types of graph

Self-complement $(G \equiv \bar{G})$

Graph which is isomorphic to its own complement.



Types of graph

$G + \bar{G} = K_n$ $\qquad e(G) = e$

$$\underbrace{e(G)}_{\downarrow} + \underbrace{e(\bar{G})}_{\downarrow} = \frac{n(n-1)}{2}$$

$$e + e = \frac{n(n-1)}{2}$$

$$2e = \frac{n(n-1)}{2}$$

$$\boxed{e = \frac{n(n-1)}{4}}$$

$$\frac{(\ )\times(\ )}{4}$$

$n = 4 \quad e = \frac{n(n-1)}{4} = \frac{\cancel{4}\,3}{\cancel{4}} = \underline{\underline{3}}$

$n = 5 \quad e = \frac{5 \cdot \cancel{4}}{\cancel{4}} = 5$

$\times\ \underline{n = 6} \quad e = \frac{6 \cdot 5}{4} = \frac{15}{2} = 7.5$ ✗

$\times\ n = 7 \quad e = \frac{7 \cdot 6}{4} =$

Types of graph

$\frac{n}{4}$ OR $\frac{n-1}{4}$

$\frac{n-0}{4}$ OR $\frac{n-1}{4}$

$n \equiv 0 \pmod 4$ OR $n \equiv 1 \pmod 4$

$$\boxed{n \equiv 0 \text{ OR } 1 \pmod 4}$$

$1 \equiv 5 \pmod 4$

$\frac{1-5}{4} = -\frac{4}{4} = -1 \in Z$

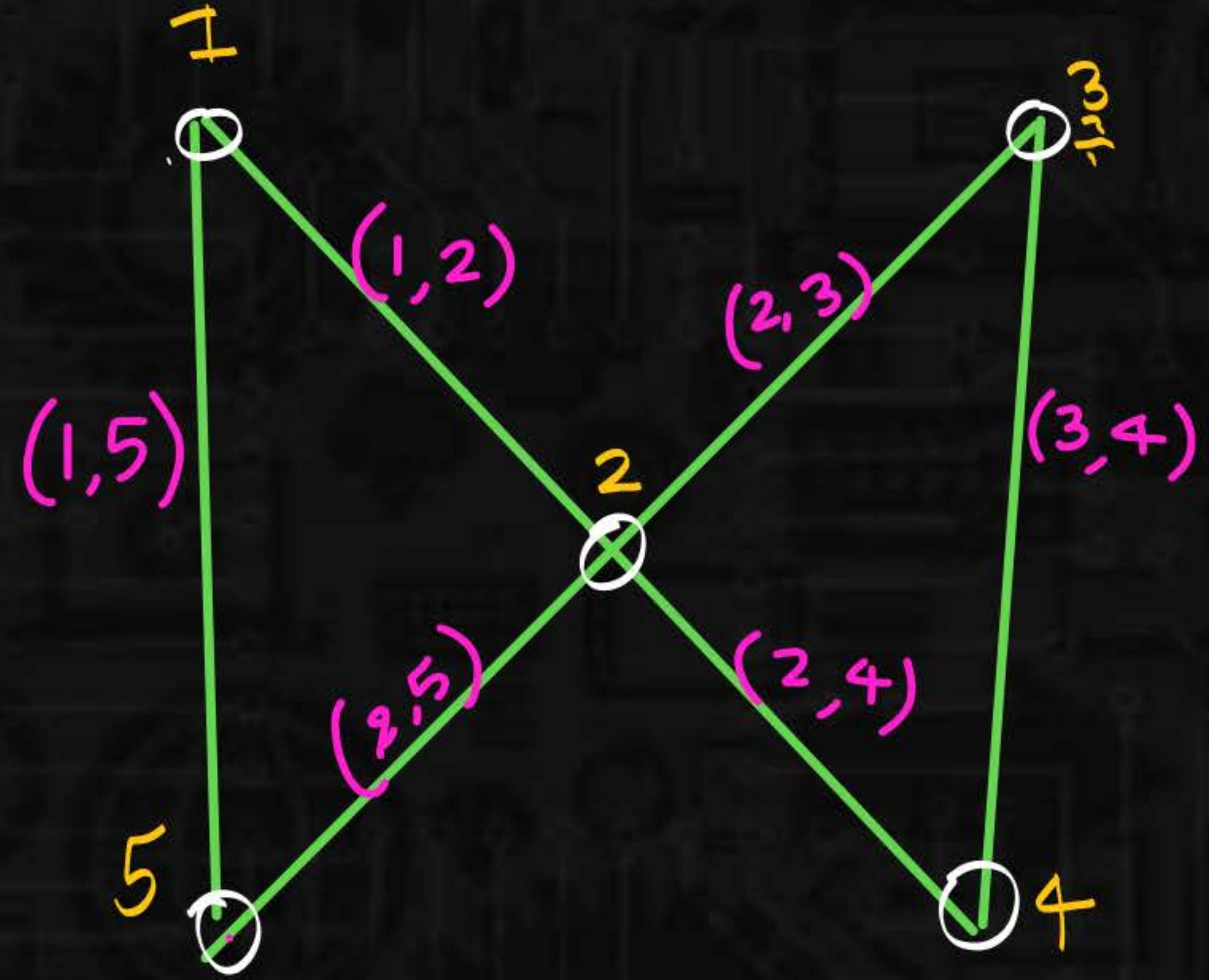
$a \equiv b \pmod n$

$\frac{a-b}{n} \in Z$ OR a, b are having same remainder w.r.t n.

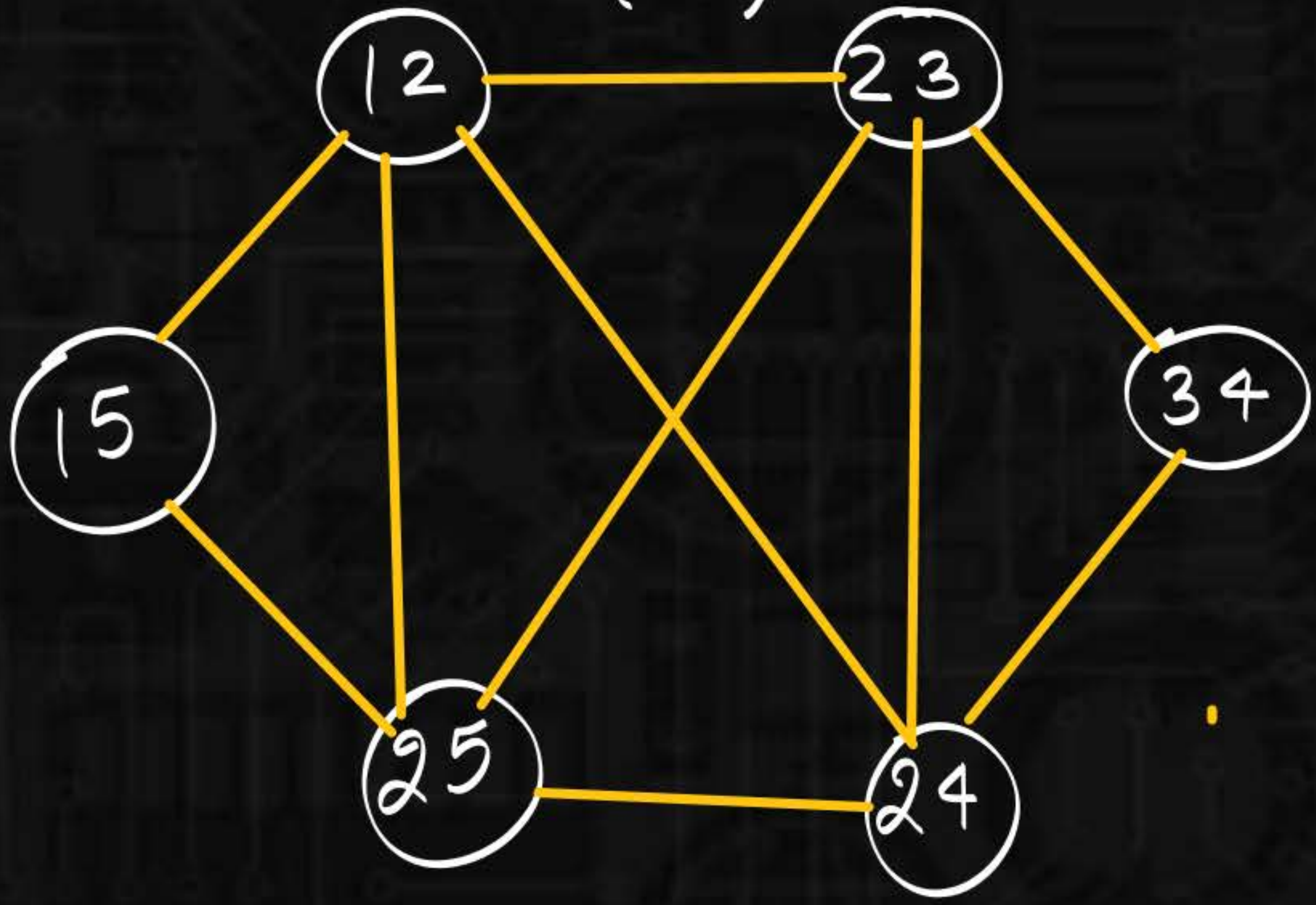
Types of graph

Line Graph (L(G))

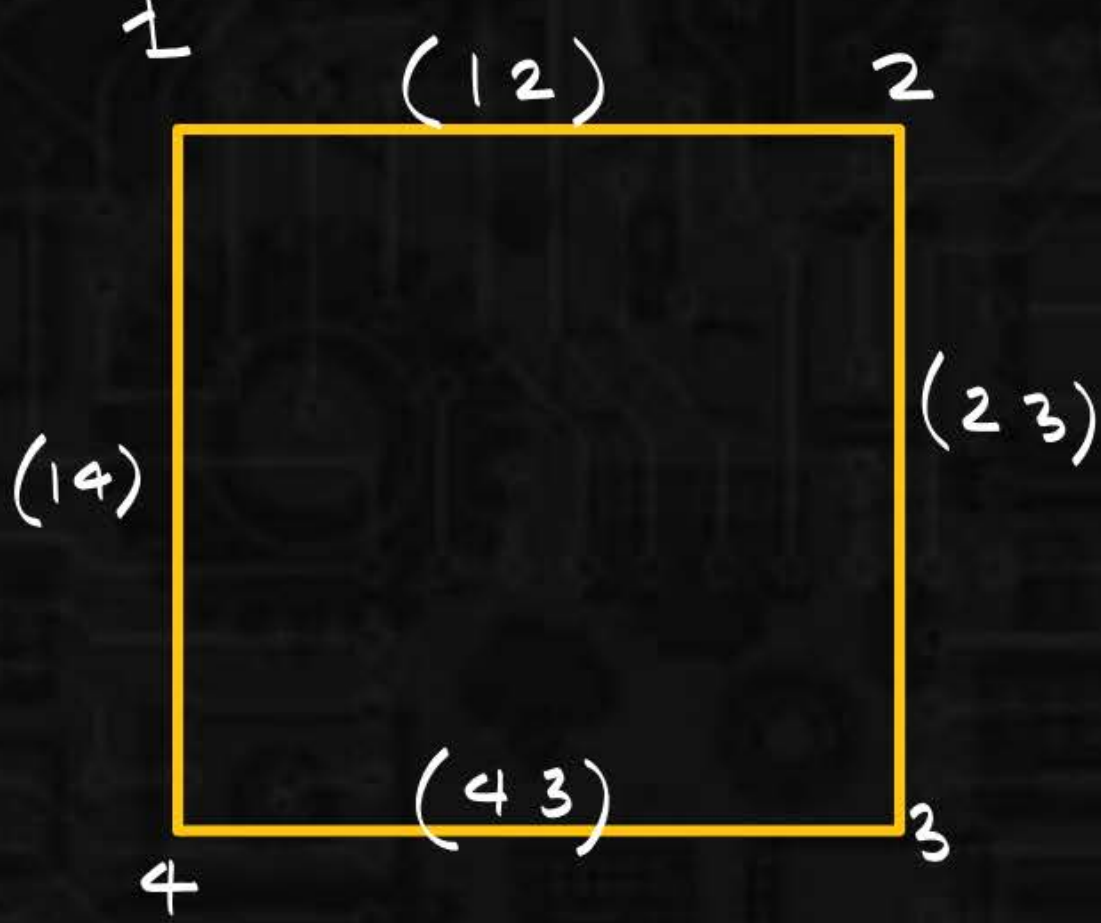
line → edge

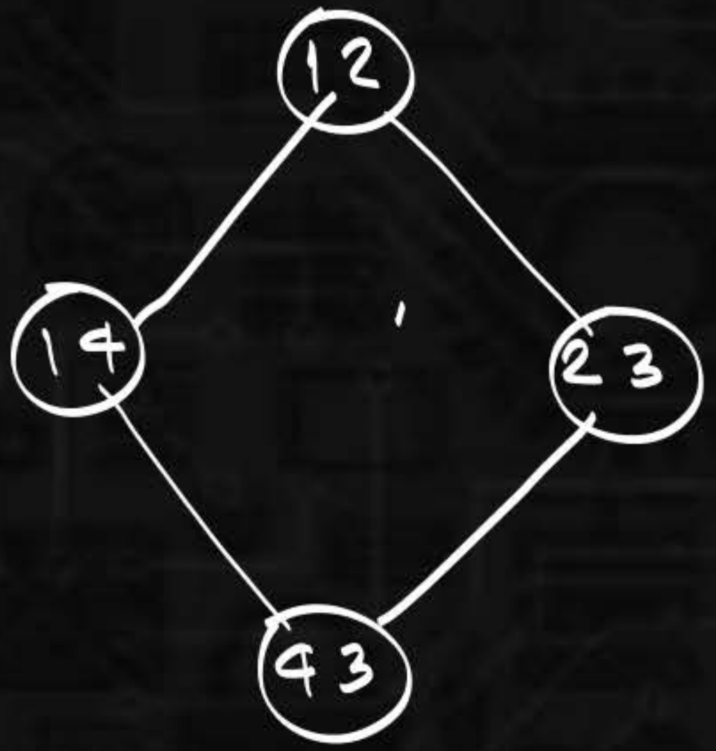


L(G)



Types of graph





Types of graph

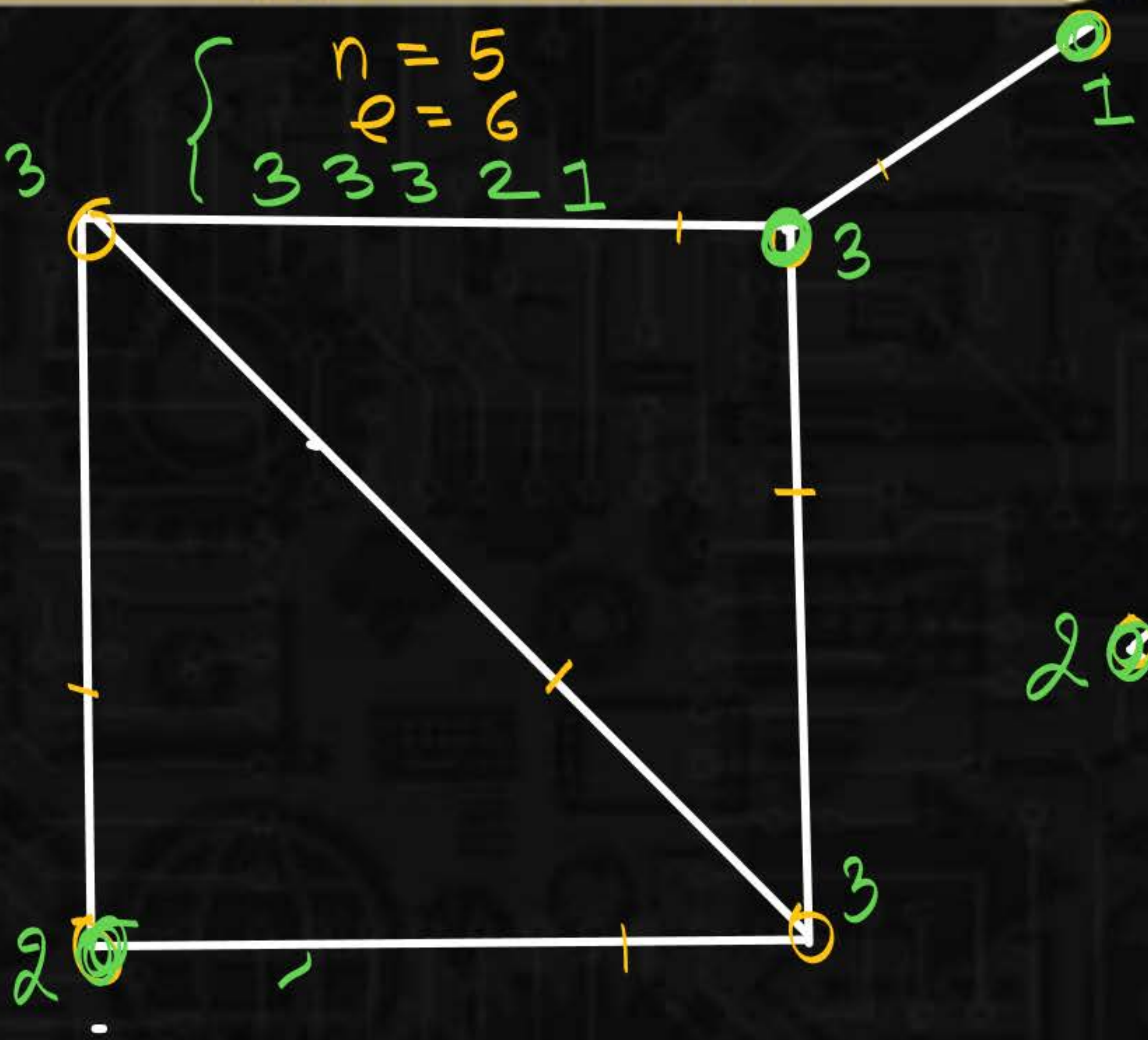
Isomorphic :

Two Graphs G_1, G_2 are isomorphic to each other when they have same incident property.

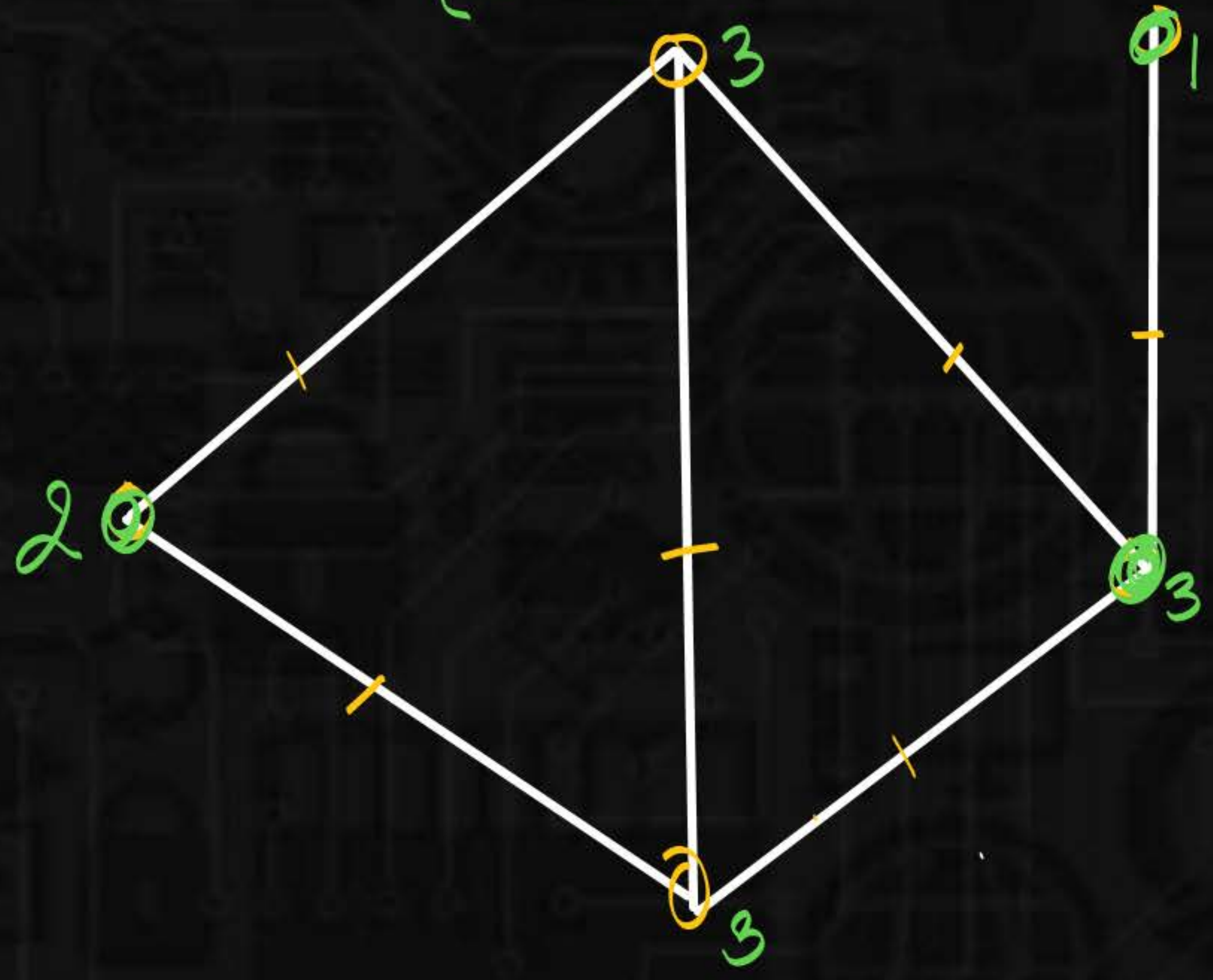
- → no. of vertices
- → no. of edges
- → no. of degree sequence

Types of graph

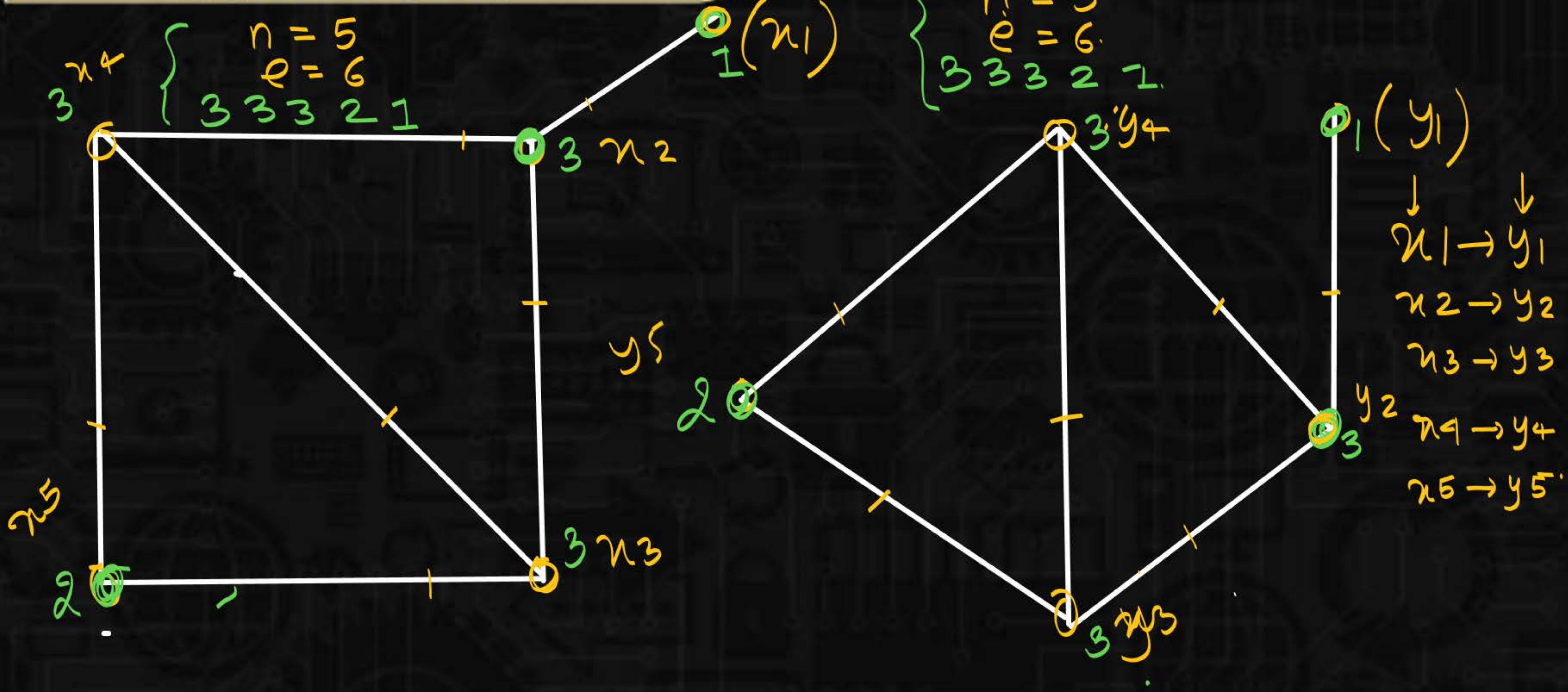
$n = 5$
$e = 6$
3 3 3 2 1



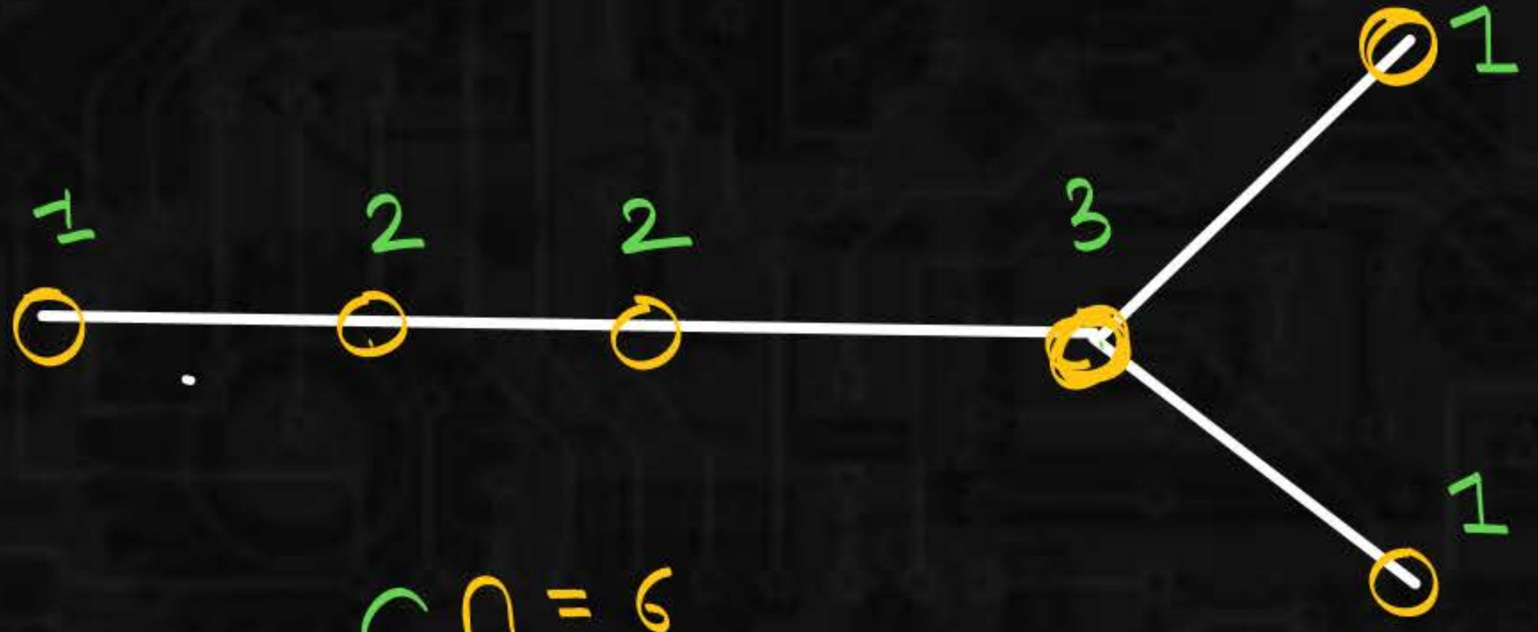
$n = 5$
$e = 6.$
3 3 3 2 1.

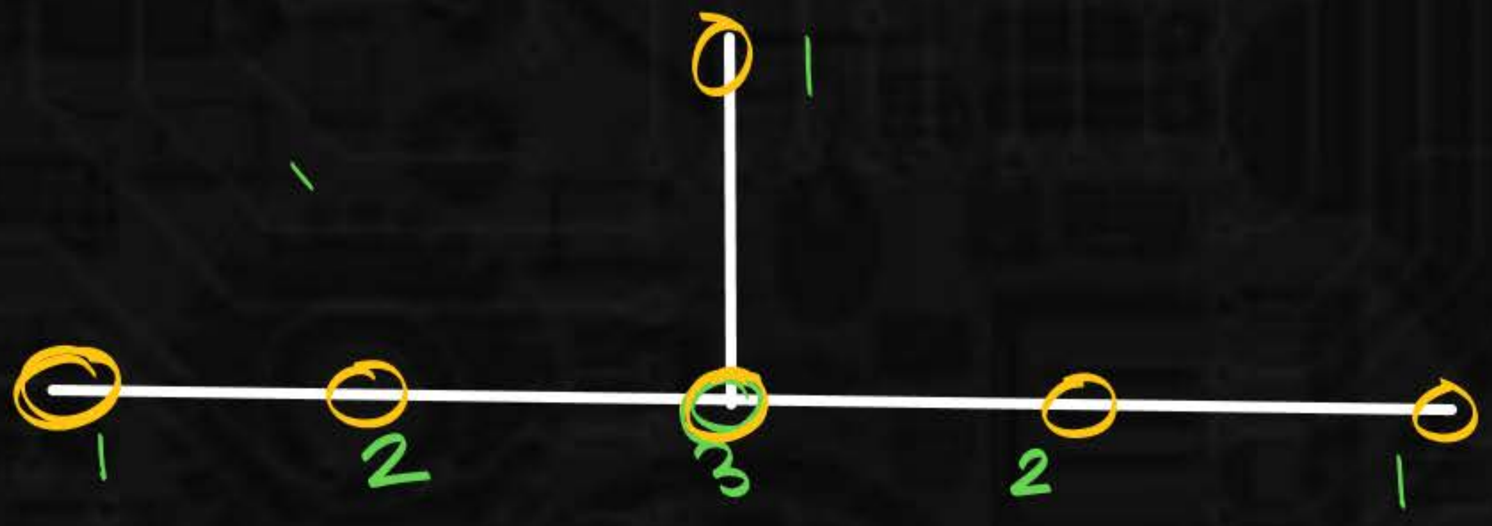


Types of graph



Types of graph





$n = 6$

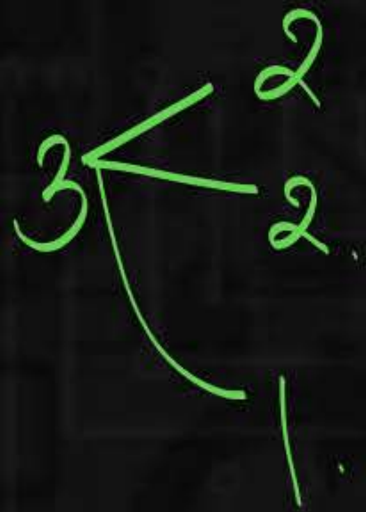
$e = 5$

3 2 2 1 1 1

$n = 6$

$n = 5$

3 2 2 1 1 1



Types of graph

Two Graphs are isomorphic to each other when they have 1:1 correspondance.

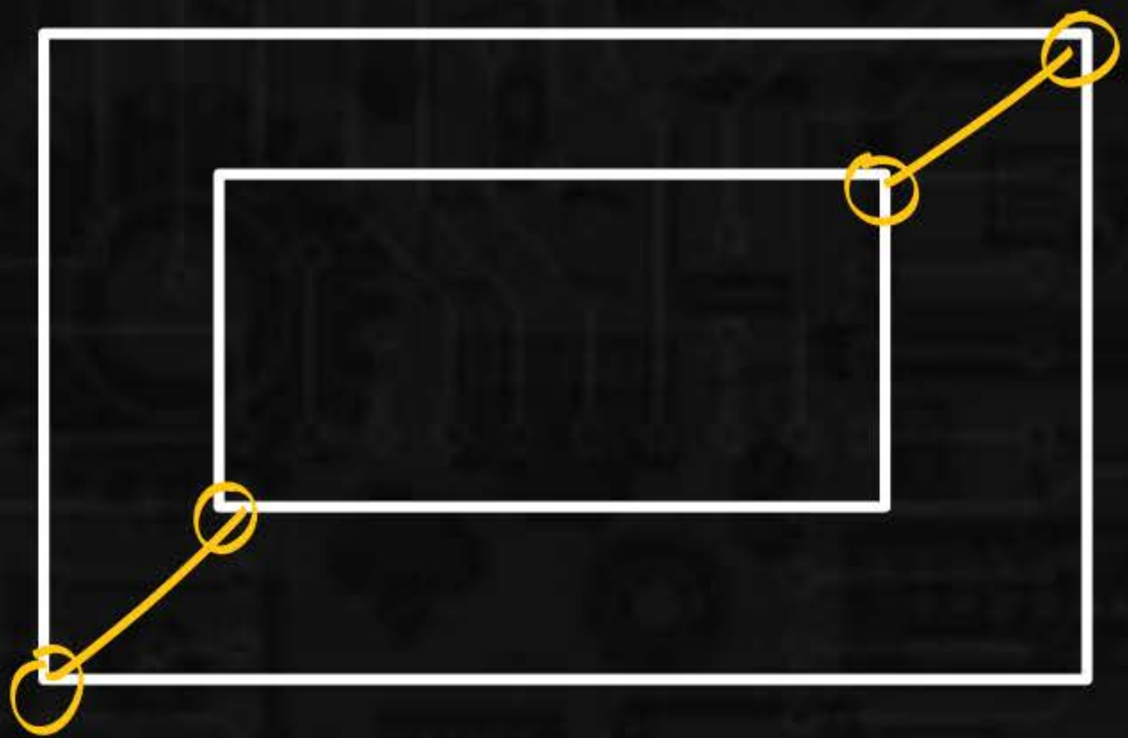
$G_1 = (V_1, E_1, \psi_1)$

$G_2 = (V_2, E_2, \psi_2)$

$f : G_1 \rightarrow G_2$ $\quad f : E_1 \rightarrow E_2$

$f : V_1 \rightarrow V_2$ $\quad f : \psi_1 \rightarrow \psi_2$.

Types of graph




THANK YOU GW
SOLDIERS !