ALL BRANCHES

ME,CE,EC,EE,CS

Probability and Statistics

Lecture No- 02

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Topics to be Covered

01 Conditional Probability

02 Multiplication Theorem

03 Theorem of Total Probability

04 Bayes' Theorem

Revision

Experiments $\rightarrow$ Deterministic
$\rightarrow$ Random
$\downarrow$
Sample space 'S'

Any Subset of 'S' is E.

$$P(E) = \frac{n(E)}{n(S)}$$

For any Event 'E'.

$$0 \leq P(E) \leq 1$$

$$P(S) = 1$$

Mutually Exclusive $\rightarrow A \cap B = \phi$
$\Rightarrow P(A \cap B) = 0$

Mutually Exhaustive $\rightarrow A \cup B = S$
$\Rightarrow P(A \cup B) = 1$

Independent Events $\rightarrow P(A \cap B)$
$= P(A) \cdot P(B)$

Addition Theorem: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

If A & B are mutually exclusive, then $P(A\cup B) = P(A) + P(B)$.

$$P(A^c \cap B^c) = 1 - P(A\cup B)$$

$$P(A^c \cap B) = P(B) - P(A\cap B)$$

→ When two dice are rolled once, the sample space is

$$S = \left\{\begin{array}{llllll}
(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\
(2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\
(3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\
(4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\
(5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\
(6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)
\end{array}\right\}$$

1 2, 2 3, 3 4, 4 5, 5 6, 6 7

8 5

9 4

10 3

11 2

12 1

$$P(\text{Sum}=k) = \begin{cases} \dfrac{k-1}{36}; & \text{if } k \le 7 \\[2mm] \dfrac{13-k}{36}; & \text{if } k > 7 \end{cases}$$

Pack of cards (52).

52 Cards
- Red cards
  - Hearts (♡)
  - Diamonds (♢)
- Black cards
  - Spades (♠)
  - Clubs (♣)

} → 4 Suits.

52 Cards → 4 Suits. (4×13)

Each Suit Contains Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, $\underbrace{\text{Jack, Queen, King}}_{\text{Face Cards.}}$

∴ Total No. of face cards in a Pack = 12.

→ The Probability that a leap year has 53 Sundays is ___.

1 leap year → 366 days → $364 + 2$

$$= \underbrace{(52 \times 7)}_{52 \text{ Sundays}} + 2$$

Total Possibilities → (Su, Mo), (Mo, Tu), (Tu, We), (We, Th), (Th, Fr), (Fr, Sa), (Sa, Su).

$\therefore P(53^{rd} \text{ sunday}) = \frac{2}{7} = 0.2857$.

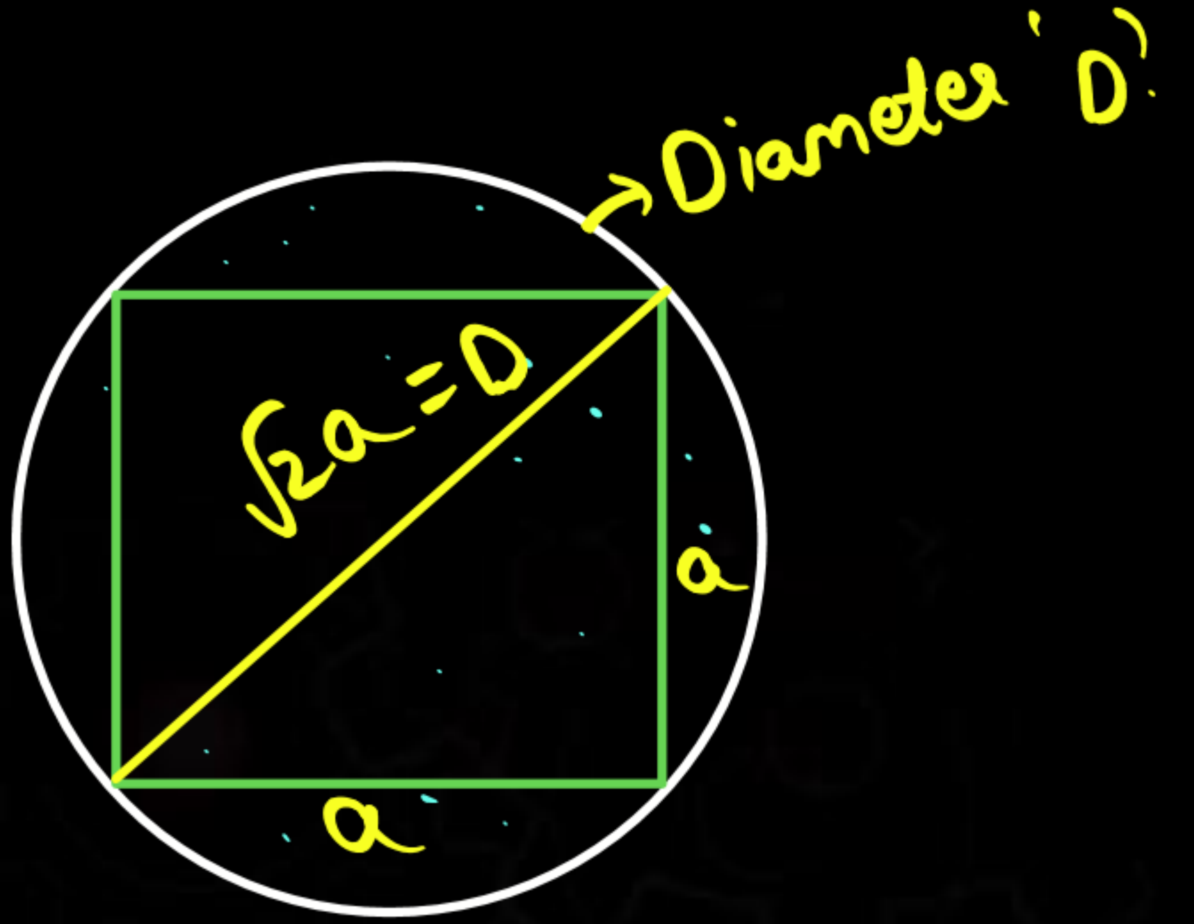
→ If two cards are picked randomly from a pack, the probability that one card is a spade and other card is a face card in Diamonds.

$$P(\text{Spade, face card in Diamond}) = \frac{{}^{13}C_1 \times {}^{3}C_1}{{}^{52}C_2}$$

Spades → 13 ; No. of face cards in Diamonds = 3.

$$\therefore \text{Probability} = \frac{\cancel{13} \times \cancel{3}}{{}_{2}\left(\frac{\cancel{52} \times \cancel{51}}{\cancel{2} \times 1}\right)_{17}} = \frac{1}{34}.$$

→ If a Point is Plotted at random in the circle shown below, the Probability that it lies inside the Square is ___.



$$P(\text{Point in square}) = \frac{a^2}{\frac{\pi}{4}D^2} = \frac{4a^2}{\pi D^2}$$

$$\sqrt{2}\,a = D \Rightarrow \frac{a}{D} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \left(\frac{a}{D}\right)^2 = \frac{1}{2}.$$

$$\therefore P(\text{Point lies in square}) = \frac{\cancel{4}^{\,2}}{\pi}\cdot\frac{1}{\cancel{2}} = \frac{2}{\pi} = 0.6366 \checkmark$$

→ A number 'n' is randomly selected from a set of 1st 100 Natural numbers, the Probability that $\int_{-\pi/2}^{\pi/2} \sin^n x \, dx = 0$ is $\underline{0.5}$

For $\int_{-\pi/2}^{\pi/2} \boxed{\sin^n x}\, dx = 0$; $n \to$ odd Number.

$\sin^n x \to$ odd function.

$\therefore$ Probability $= \dfrac{\text{No. of odd Numbers in 1 to 100}}{100}$

$= \dfrac{50}{100} = 0.5$

→ A bag contains 40 tokens numbered 01, 02, 03, ... 40. Four tokens are selected at random simultaneously and arranged in ascending order. The Probability that 3^{rd} Number is 25 is ___.

Sol:

00, 01, 02, ... 24, 25, 26, ... 40.

$$\text{Probability} = \frac{{}^{24}C_2 \times {}^{1}C_1 \times {}^{15}C_1}{{}^{40}C_4}$$

$\boxed{<25}$ $\underset{3^{rd}}{25}$ $\boxed{>25}$

<25: 24 tokens

25: 1

>25: 15 tokens

$$= \frac{\left(\frac{\cancelto{6}{24} \times 23}{\cancel{2} \times 1}\right) \times 1 \times \cancelto{3}{15}}{\left(\frac{\cancelto{5}{40} \times \cancelto{13}{39} \times \cancelto{19}{38} \times 37}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}\right)} = \frac{18 \times 23}{(13 \times 19 \times 37)} = 0.0453$$

Conditional Probability

If 'A' and 'B' are two events with in a sample space 'S', the conditional probability of happening of event 'A' given that event 'B' has already happened is denoted by $P(A/B)$ and it is given by

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{n(A\cap B)}{\cancel{n(S)}}}{\frac{n(B)}{\cancel{n(S)}}} = \frac{n(A\cap B)}{n(B)}$$

$$\boxed{\therefore P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

$$\rightarrow P\left(\text{Even number} / \geq 4\right) = \frac{P(\text{Even Number} \cap \geq 4)}{P(\geq 4)}$$

$$= \frac{\left(2/\cancel{6}\right)}{\left(3/\cancel{6}\right)} = \frac{2}{3} = 0.667$$

We have $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Case (i): If A, B are Independent Events,

then $P(A \cap B) = P(A) \cdot P(B)$.

$$\Rightarrow P(A/B) = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A).$$

Case (ii): If A, B are mutually Exclusive Events,

then $P(A \cap B) = 0$

$$\therefore P(A/B) = 0.$$

Multiplication Theorem of Probability:

If A, B are two events with in a sample space 's',

then $$P(A/B) \cdot P(B) = P(B/A) \cdot P(A).$$

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = P(A/B) \cdot P(B) \longrightarrow (1)$$

$$P(B/A) = \frac{P(B\cap A)}{P(A)} \Rightarrow P(B\cap A) = P(B/A) \cdot P(A) \longrightarrow (2)$$

$$\boxed{\Rightarrow P(A|B) \cdot P(B) = P(B/A) \cdot P(A)}$$

$P(A/B) = 0.2$; $P(A) = 0.5$; $P(B) = 0.3$; $P(B/A) = ?$

We have $P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$

$$\Rightarrow P(B/A) = \frac{P(A/B) \cdot P(B)}{P(A)}$$

$$= \frac{(0.2)(0.3)}{(0.5)} = 0.12 \text{ //.}$$

Theorem of Total Probability

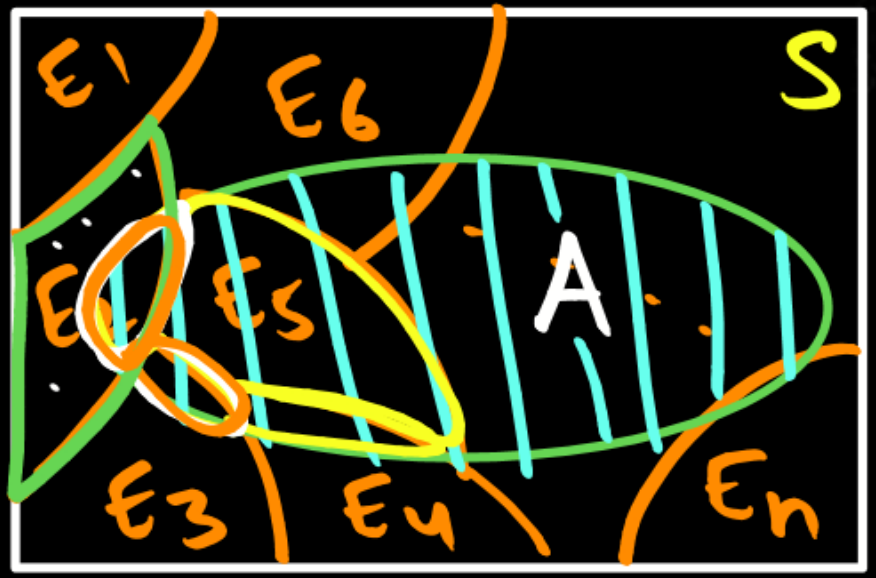
(Implemented when an event has multiple possibilities to occur).

If $E_1, E_2, E_3, E_4, \ldots E_n$ are 'n' mutually exclusive and collectively Exhaustive Events with in a Sample Space 'S', then for any Event 'A' With in the same Sample space 'A'.

$$\boxed{P(A) = \sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)}$$

Given $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive & Collectively Exhaustive Events.

$(E_1 \cup E_2 \cup E_3 \cup \ldots E_n = S)$; $(E_i \cap E_j = 0 \ \forall i \neq j)$



$$A = A \cap S$$

$$\Rightarrow A = A \cap (E_1 \cup E_2 \cup E_3 \cup \ldots E_n)$$

$$\Rightarrow A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup \ldots (A \cap E_n)$$

If $E_1, E_2, \ldots E_n$ are mutually exclusive,

Since $E_1, E_2, \ldots E_n$ are mutually Exclusive,

$A \cap E_1, A \cap E_2, \ldots A \cap E_n$ are also mutually Exclusive

$$\therefore P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

$$\Rightarrow P(A) = P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2) + \cdots + P(A/E_n) \cdot P(E_n)$$

$$\boxed{\Rightarrow P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

→ A Student answers an MCQ. The Probability that he knowns the answer is 0.7 and if he don't know he guesses it. The Probability that the guess becomes correct is 0.25. The Probability that the Student $\boxed{\text{writes correct answer}}$ to an MCQ is <u>0.775</u>

$$\underset{A}{\text{Correct}} \longrightarrow \underset{E_1}{\text{knowns}} + \underset{E_2}{\text{Guess is Correct}}$$

$$\therefore P(\text{correct}) = P(\text{known} \cap \text{correct}) + P(\text{Guess} \cap \text{correct})$$

$$= P(\text{Known}) \cdot P(\text{correct}/\text{know}) + P(\text{Guess}) \cdot P(\text{correct}/\text{Guess})$$

$$= (0.7 \times 1) + (0.3 \times 0.25) = 0.775 /.$$

Bayes' Theorem

If $E_1, E_2, E_3, \ldots E_n$ are 'n' mutually Exclusive and Collectively Exhaustive Events with in a Sample Space 'S', then for any Event 'A' with in the Same Sample Space S,

$$\boxed{P(E_i/A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}}$$

Ex: $P(\text{know}) = 0.7$; $P(\text{Guess}) = 0.3$

$P(\text{correct}/\text{know}) = 1$ $P(\text{correct}/\text{Guess}) = 0.25$.

$$P(\text{Guess}/\text{correct}) = \frac{P(\text{Guess} \cap \text{correct})}{P(\text{known} \cap \text{correct}) + P(\text{Guess} \cap \text{correct})}$$

$$= \frac{P(G) \cdot P(C/G)}{P(K) \cdot P(C/K) + P(G) \cdot P(C/G)}$$

$$= \frac{(0.3) \times (0.25)}{(0.7 \times 1) + (0.3 \times 0.25)} = 0.096 \%$$

Thank You!

GW Soldiers