CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Different Types of Functions

Lecture No. 4

By- SATISH YADAV SIR

01 ONTO Function

02 BIJECTIVE FUNCTION

03 INVERTIBLE FUNCTION

04 TOTAL NUMBER OF BIJECTIVE FUNCTIONS

05 IDENTITY FUNCTION

06 Different Types of Fuctions

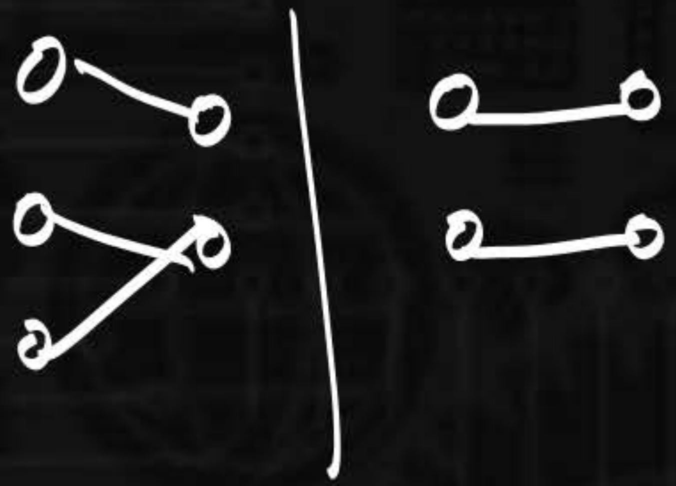
Functions

$f : A \to B$

$|A| = m \quad |B| = n.$

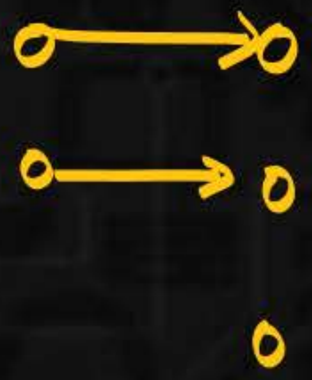
$|A| \geq |B|.$

$A > B$ OR $A = B$



Q.1. $f : A \to B \quad |A| = 2 \quad |B| = 3.$

Total no. of onto functions $= 0$.



Functions

$|A| = m \quad |B| = n$

$$\sum_{i=0}^{n} (-1)^i * {}^nC_i * (n-i)^m$$

GATE. $|A| = n \quad |B| = 2.$

$f : A \to B$

$$\sum_{i=0}^{2} (-1)^i * {}^2C_i \, (2-i)^n$$

$${}^2C_0 (2-0)^n - {}^2C_1 (2-1)^n + {}^2C_2 (2-2)^n$$

$$1 \cdot 2^n - 2 \cdot 1^n = \underline{\underline{2^n - 2}}$$

Q.2: $|A| = 7 \quad |B| = 4.$

Functions

$f : A \to B$

$|A| = n \quad |B| = 2$

$$\text{onto} = \text{Total} - \underbrace{\text{non onto}}_{2}$$

$$= (R.S)^{L.S} - 2$$

$$= 2^n - 2.$$

Functions

$m = |A| = 7 \quad |B| = 4 = n$

$$\sum_{i=0}^{4} (-1)^i * {}^nC_i \, (n-i)^m$$

$$\boxed{8400} = 4^7 - 4 \cdot 3^7 + 6 \cdot 2^7 - 4$$

$$= 4\left(\underline{4^6 - 3^7 + 6 \cdot 2^5 - 1}\right)$$

$${}^4C_0(4-0)^7 - {}^4C_1(4-1)^7 + {}^4C_2(4-2)^7 - {}^4C_3(4-3)^7 + \underline{{}^4C_4(4-4)^7}$$

$$4^7 - 4 \cdot 3^7 + 6 \cdot 2^7 - 4 \cdot 1^7 + 0$$

Functions

How many ways we can arrange 7 people to 4 Room such that none of the rooms are empty?

Ans: 8400

onto

rooms contains atleast 1 person.

Functions

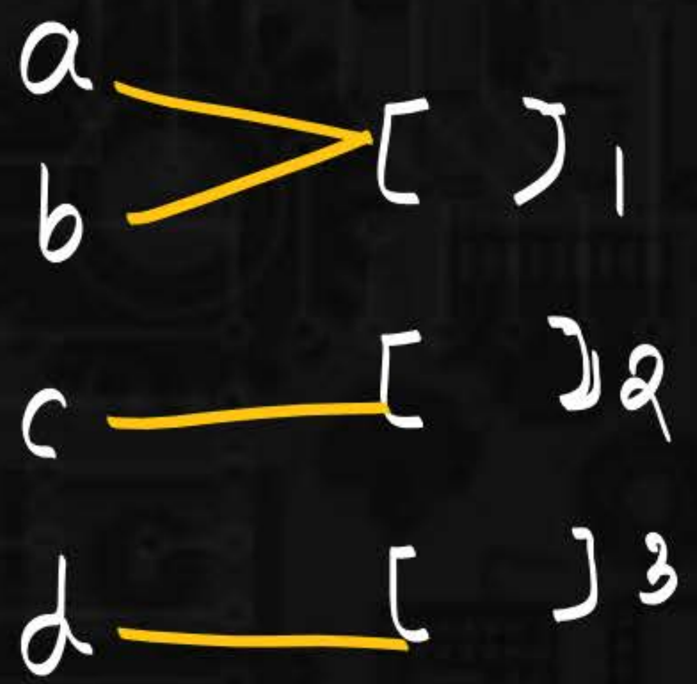
$|A| = 4 \quad |B| = 3.$

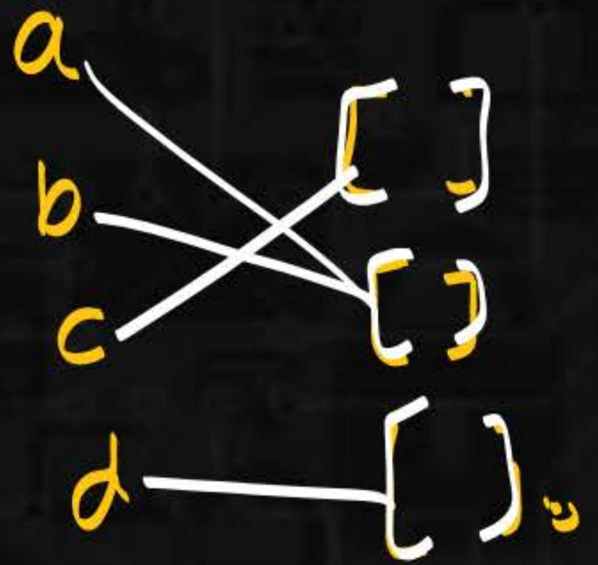
$$^3C_0(3-0)^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4 - {}^3C_3(3-3)^4$$

$$= 3^4 - 3\cdot 2^4 + 3\cdot 1^4 - 0$$

$$= 81 - 3\cdot 16 + 3 = 84 - 48 = 36$$

onto





$\underbrace{[ab]_1\,[c]_2\,[d]_3}_{[ab]\,[c]\,[d]} = \underbrace{[ab]_2\,[c]_1\,[d]_3}_{[ab]\,[c]\,[d]}$

Functions

onto :

objects $\longrightarrow$ distinguishable boxes. (36)

4 people $\longrightarrow$ 3 diff rooms. (onto)

4 people $\longrightarrow$ 3 identical rooms

$$\frac{\text{onto}}{3!}$$

Functions

a

b

c

$[\ ]_1$

$[\ ]_2$.

onto = 6

diff boxes:

$[ab]_1\ [c]_2$

$[ab]_2\ [c]_1$

$[ac]_1\ [b]_2$.

$[ac]_2\ [b]_1$

$[bc]_1\ [a]_2$.

$[bc]_2\ [a]_1$.

identical boxes.

$[ab]\ [c]$

$[ac]\ [b]$

$[bc]\ [a]$

$\frac{6}{2!} = 3$.

$$S(m,n) = \frac{\text{onto}}{n!} = \frac{1}{n!}\sum_{i=0}^{n} (-1)^i * {}^nC_i * (n-i)^m$$

m people $\longrightarrow$ n identical boxes

Onto:

people $\rightarrow$ labelled Room

Unidentical room

diff room

Functions

Bijective function (1:1 correspondance)

1:1 correspondance = $\underset{\downarrow}{\underline{1:1}}$ + $\underline{\text{onto}}$

$A = B$

$\underline{A \leq B \wedge A \geq B} \longrightarrow A = B$

Functions

$$2^3 \begin{cases} TTT \longrightarrow 111 \\ TFT \longrightarrow 101 \\ FFT \longrightarrow 001 \end{cases} \quad 2^3$$

$$2^3 \begin{cases} \begin{array}{ccc} a & b & c \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \end{cases} \begin{array}{c} \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{array} \begin{array}{c} \underline{\text{Subset}} \\ \phi \\ \{a\} \\ \{b\} \\ \{c\} \end{array} \quad 2^3$$

Functions

$|A| = |B| = n.$

Total no. of 1:1 c $\rightarrow n!.$

$\{a\ b\ c\}$ $\{1\ 2\ 3\}$

$f : \overset{m}{A} \rightarrow \overset{n}{B}.$

$1:1 \longrightarrow B^{A}$

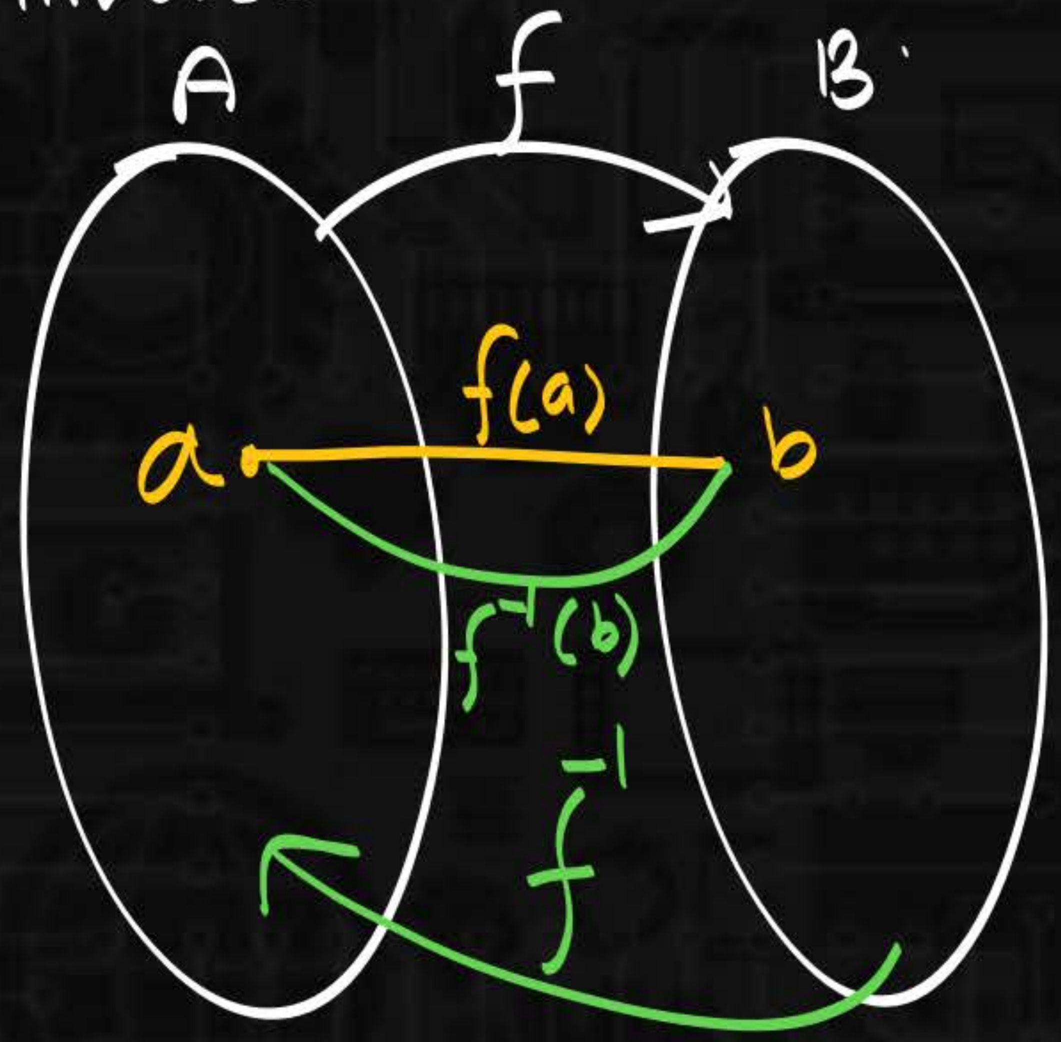
onto $\sum_{i=0}^{n} (-1)^{i} \cdot {}^{n}C_{i} \cdot (n-i)^{m}.$

1:1 c $\rightarrow m = n$ $n!$

Functions

Invertible function.

Inverse.



$f : A \to B$

$$\left\{ \begin{array}{l} f^{-1} : B \to A \\ f^{-1}(b) = a. \end{array} \right.$$

Functions

$a \longrightarrow 1$

$b \longrightarrow 2$

$c \longrightarrow 3$

$f(a) = 1$

$f(b) = 2$

$f(c) = 3$

$f^{-1}(1) = a$

$f^{-1}(2) = b$

$f^{-1}(3) = c$

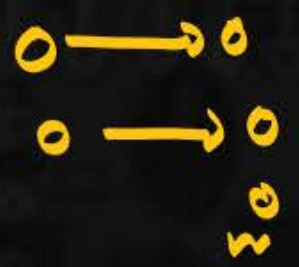
$f^{-1} : B \to A$

$1 \longrightarrow a$

$2 \longrightarrow b$

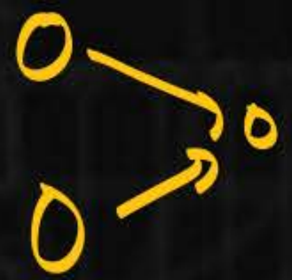
$3 \longrightarrow c$

1:1



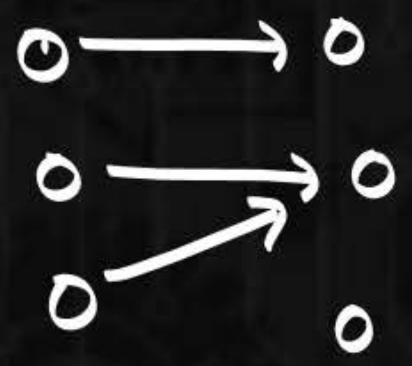
not function

onto

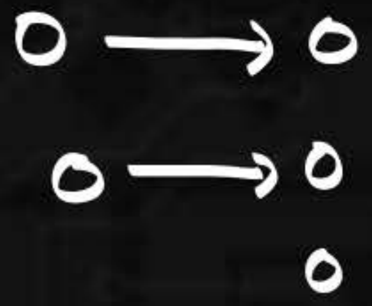


not functⁿ

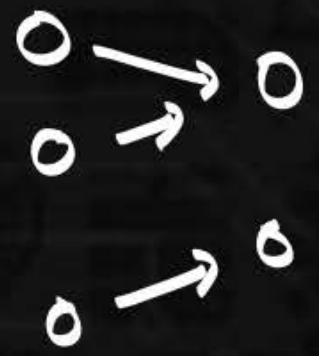
Functions



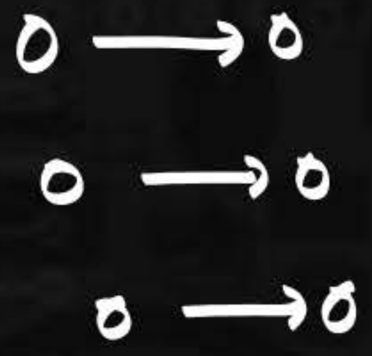
not 1:1.
not onto.



1:1
not onto



not 1:1
but onto.



1:1 ✓
onto ✓

1:1 C.

Functions

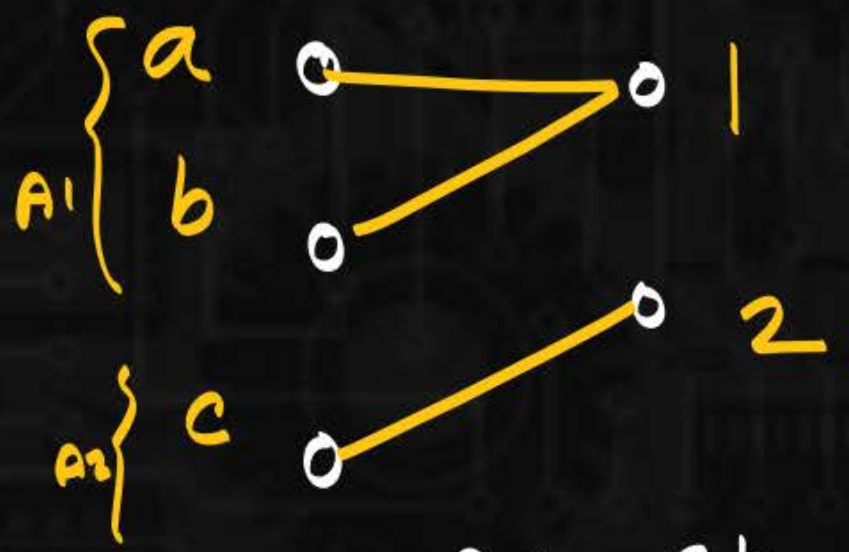
$f : A \longrightarrow B$

$A_1, A_2 \subseteq A$

a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

b) $f\underbrace{(A_1 \cap A_2)}_{\emptyset \quad \subseteq} \subseteq f(A_1) \cap f(A_2)$

Functions



A1 = ab

A2 = c

$f(A_1) = f(a,b) = 1.$

$f(A_2) = f(c) = 2$

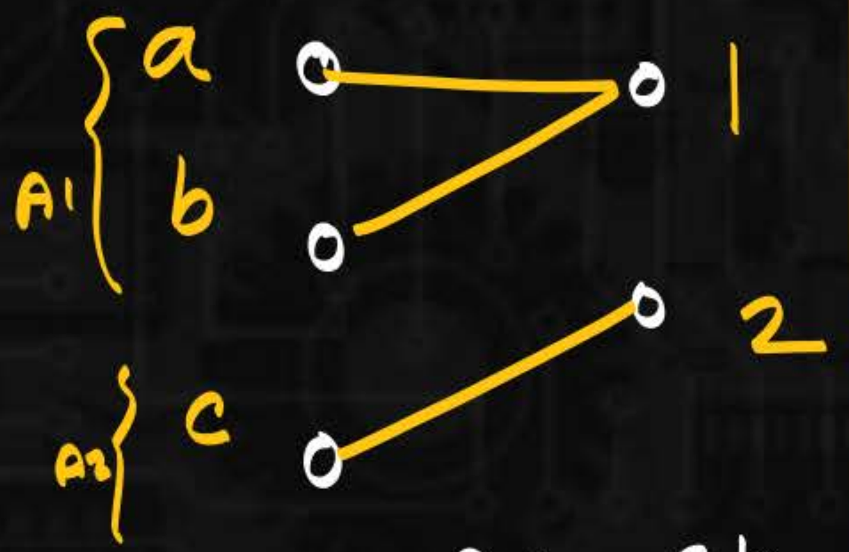
$\underline{f(A_1)} \cup \underline{f(A_2)}$

$\{1\} \cup \{2\} = \{1\,2\}.$

$f(A_1 \cup A_2) = f(a,b,c) = \{1\,2\}.$

$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

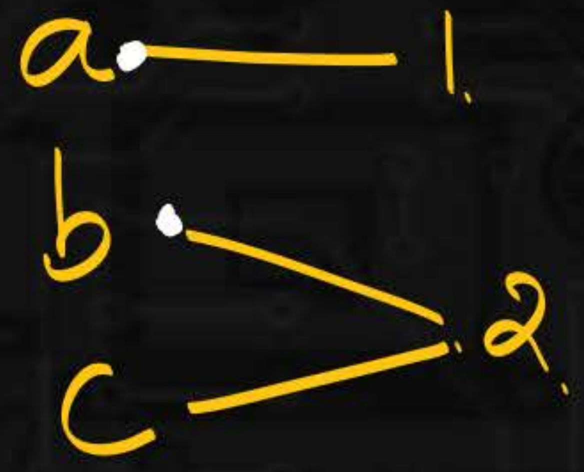
Functions



$A_1 = ab$

$A_2 = c$

$=$



$A_1 = ab$

$A_2 = c$

$f(A_1) = f(ab) = \{1\ 2\}$

$f(A_2) = f(c) = \{2\}$

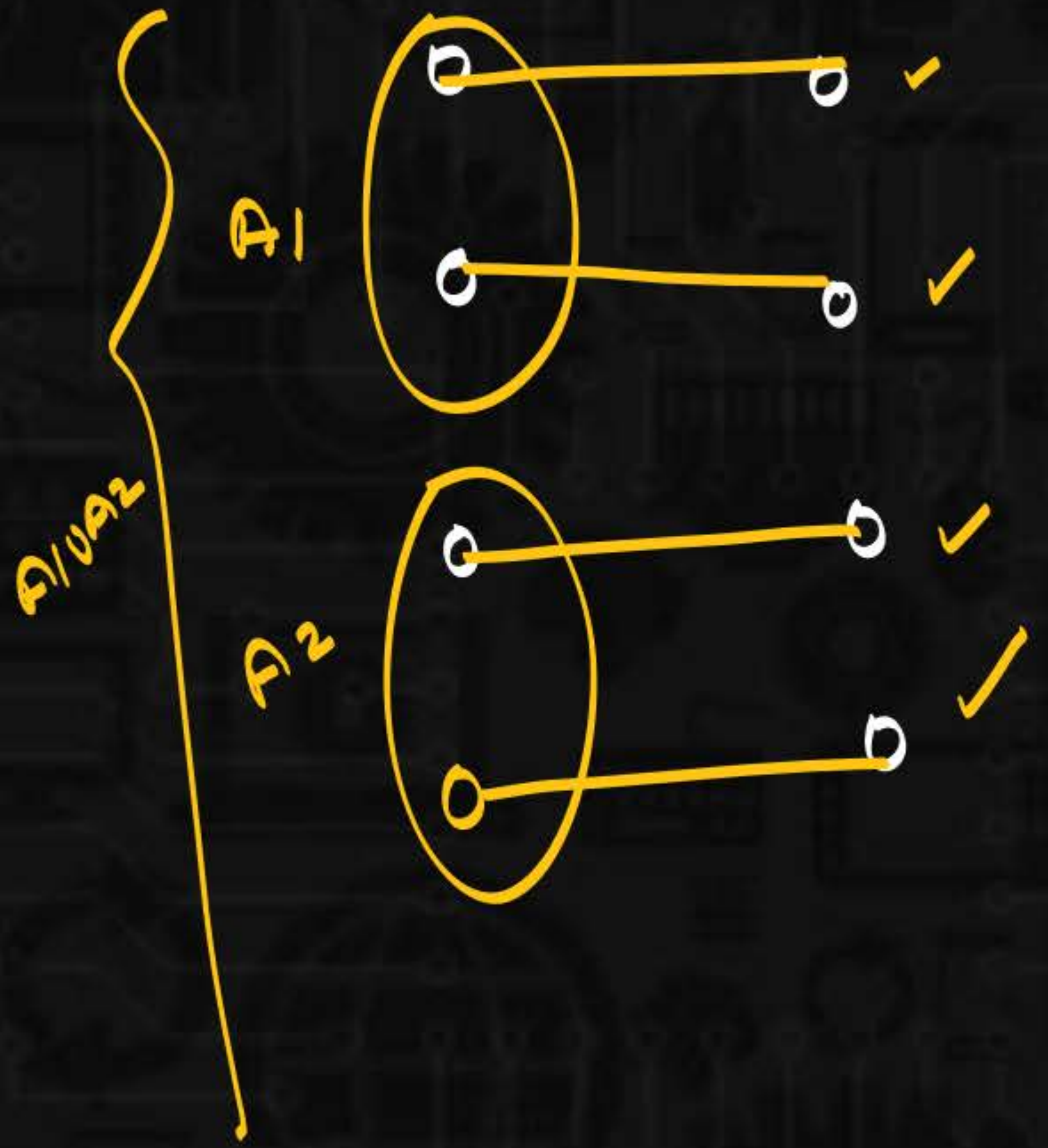
$f(A_1) \cap f(A_2) = \{1\ 2\} \cap \{2\}$

$= \{2\}$

$f(A_1 \cap A_2) = f(\emptyset) = \emptyset$

$\emptyset \subseteq \{2\}$

Functions



$f(A_1) = ✓$

$f(A_2) = ✓$

$f(A_1 \cup A_2) =$


THANK YOU GW SOLDIERS !