

# CS & IT ENGINEERING

## GRAPH THEORY

Chromatic Number and Independent set



**Lecture No.9**



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# TOPICS TO BE COVERED

01 Properly coloring

02 Chromatic number

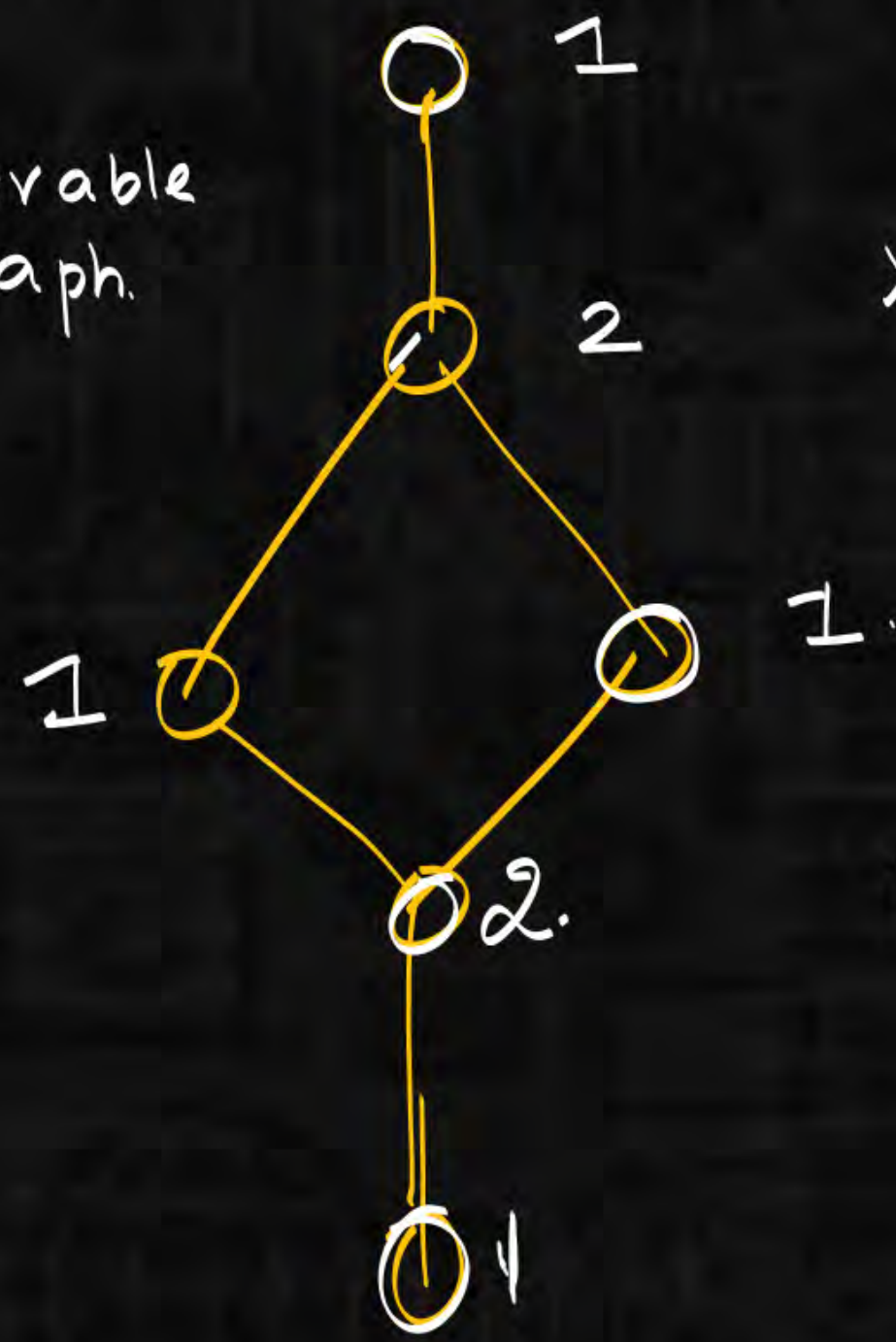
03 Chromatic Number in Graphs

04 ~~Subgraphs~~ / Independent set

05 ~~Graph operations~~ / MIS.



2-colorable  
Graph.



$$\chi(G) = 2.$$

$\chi(G) = k$   
k-colorable Graph.

chromatic no( $\chi(G)$ )

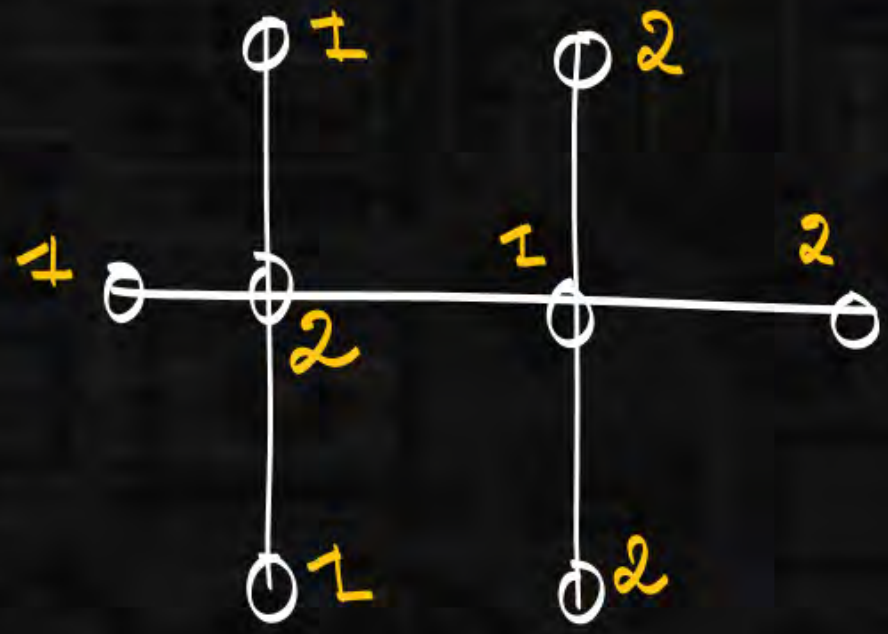
properly  
coloring.

3) min no. of colors.  
+  
adjacent should not  
... have same clr.



2. Tree:

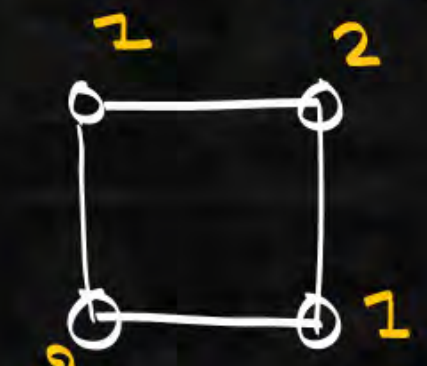
$\chi(\text{Tree}) = 2.$



3. Cycle Graph  $(C_n)$  ( $n \geq 3$ )



$\chi(C_3) = 3$



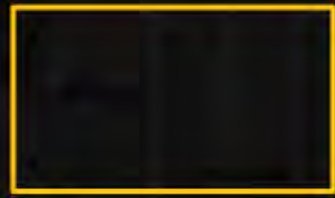
$\chi(C_4) = 2$

$$\left. \begin{aligned} \chi(C_n) &= 2 & n \text{ is even} \\ \chi(C_n) &= 3 & n \text{ is odd.} \end{aligned} \right\}$$



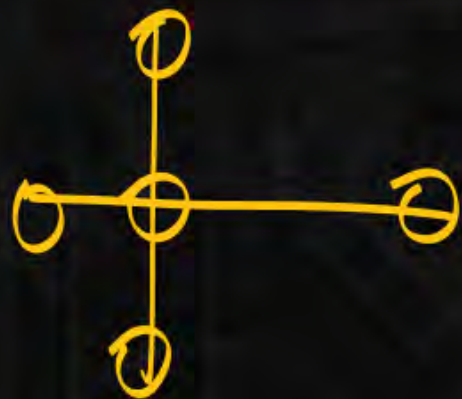
→ Every Tree will always be 2-colorable (True)

→ Every 2-colorable Graph will always be Tree (false)

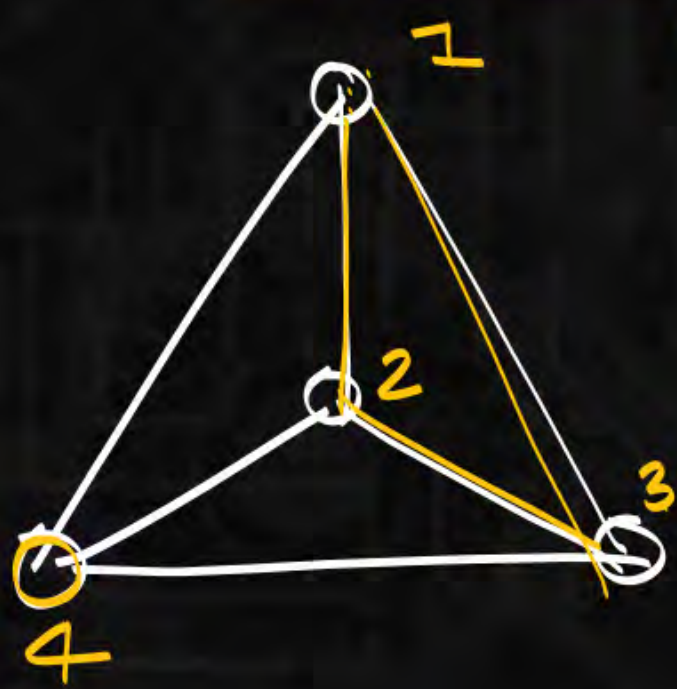


→ Every Even length cycle will always be 2-colorable (True)

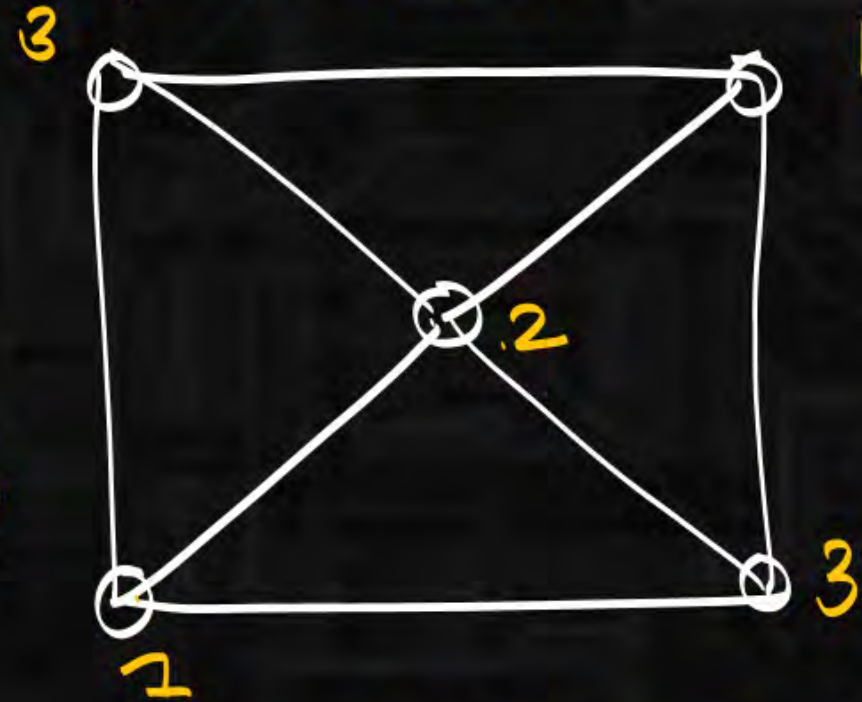
→ Every 2-colorable Graph will be even length cycle (false)



# Wheel Graph $(W_n)$ ( $n \geq 4$ )



$$\chi(W_4) = 4$$



$$\chi(W_5) = 3$$

$$\begin{aligned} \chi(W_n) &= 3 & n \text{ is odd} \\ \chi(W_n) &= 4 & n \text{ is even.} \end{aligned}$$



$$X(c_n) + X(w_n) = 6 \quad n \text{ is even.}$$

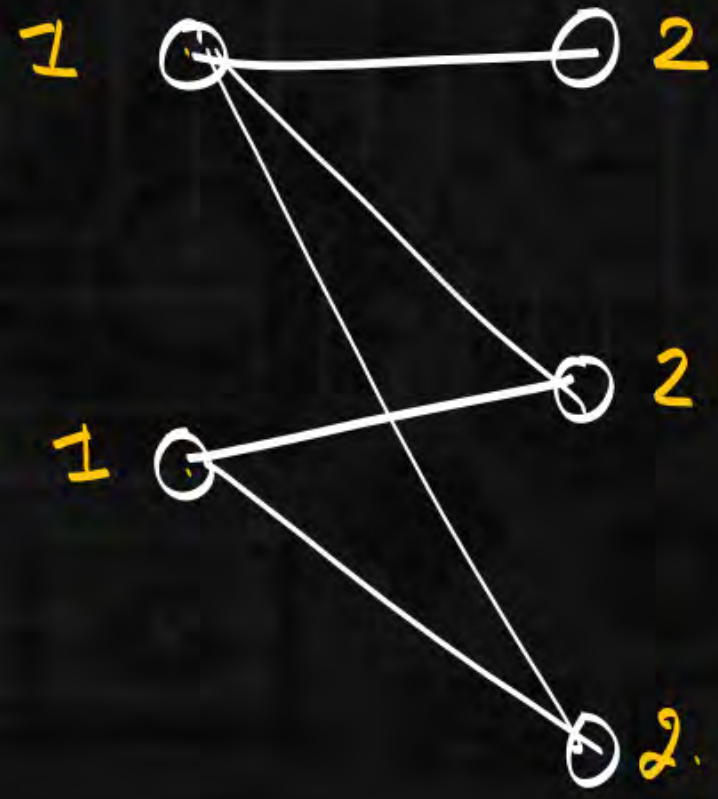
$$\downarrow$$

$$2 + 4 = 6.$$

$$X(c_n) + X(w_n) = 6 \quad (n \text{ is odd})$$

$$\downarrow \quad \downarrow$$

$$3 + 3 = 6$$

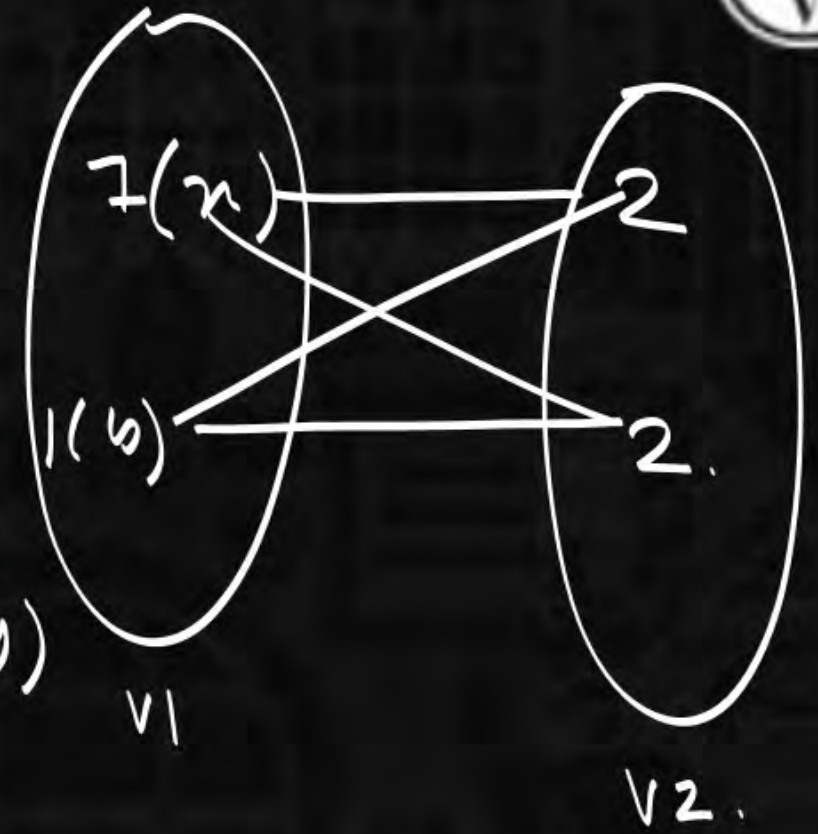
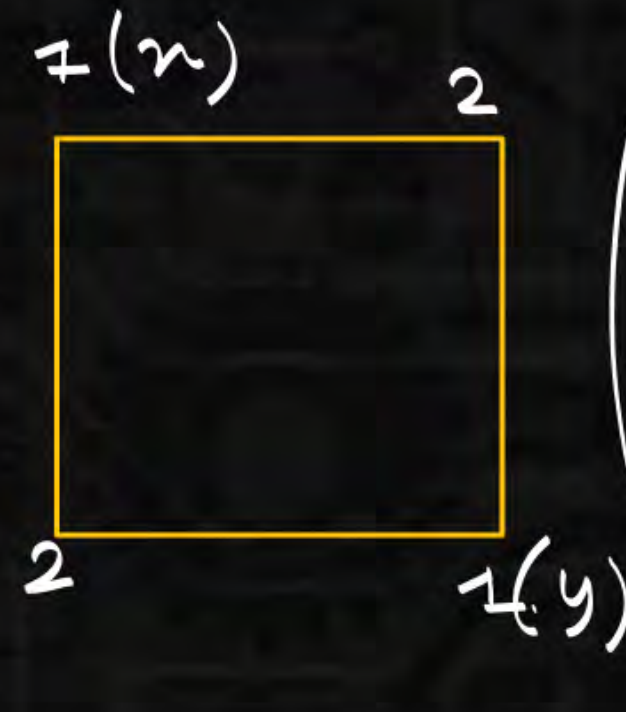
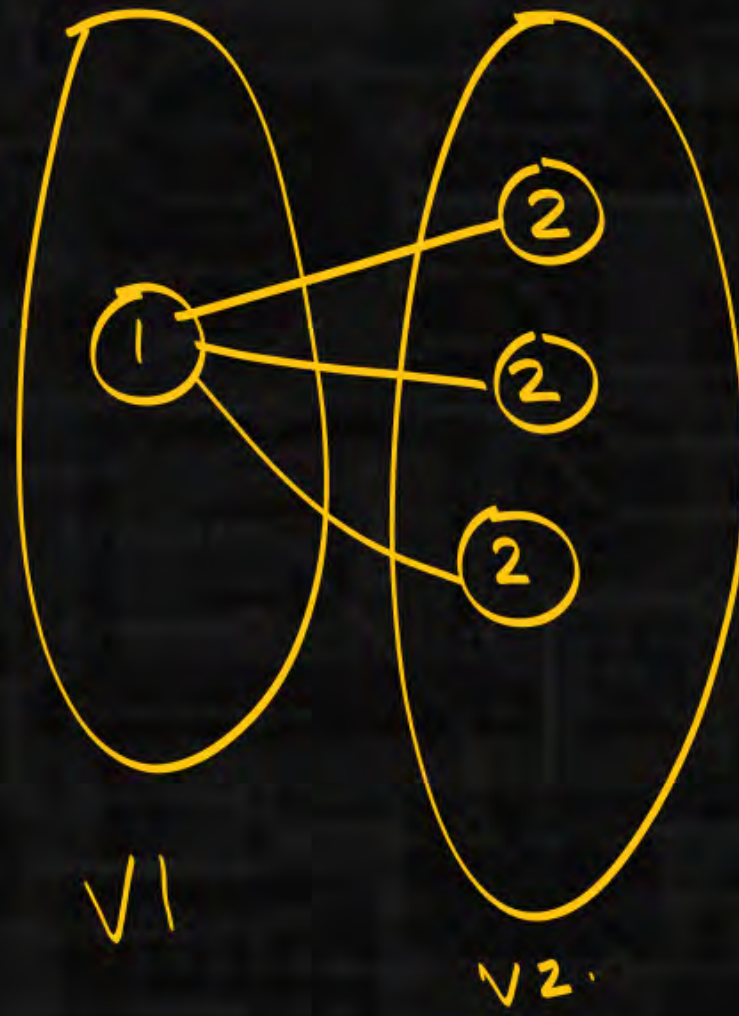
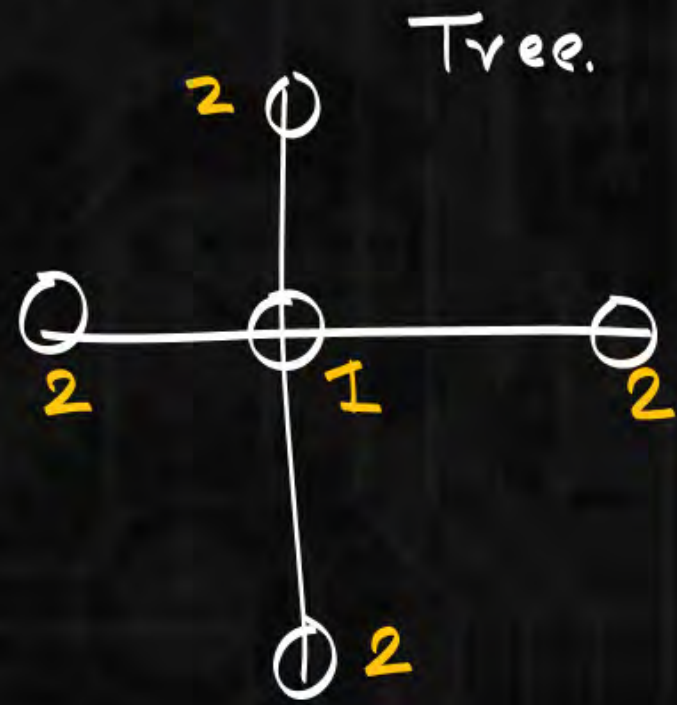


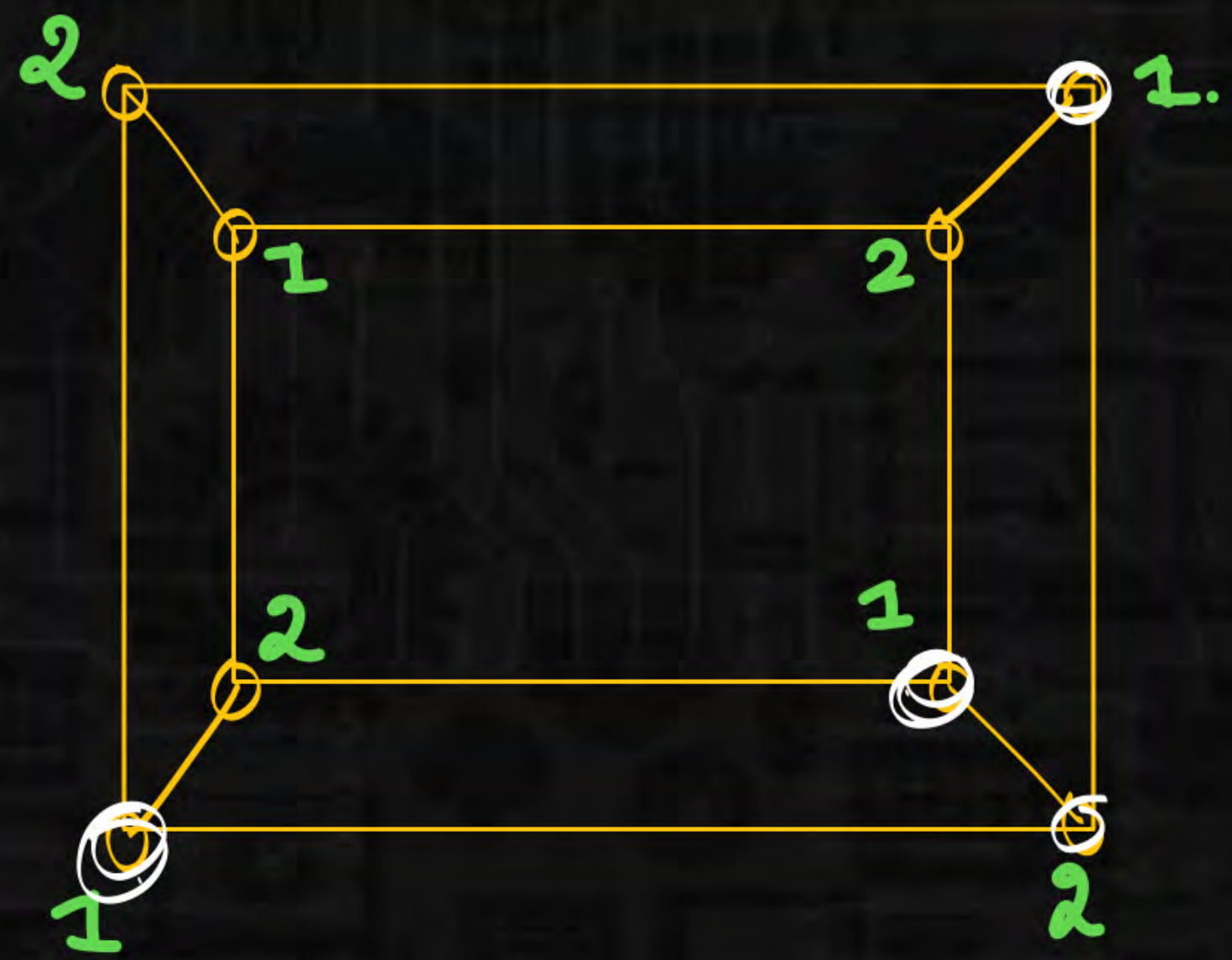
{ Every Bipartite Graph will be 2-colorable Graph.  
 → Every 2-colorable Graph will be bipartite Graph.

→ Tree  
 → Even length cycle.

B.P → 2

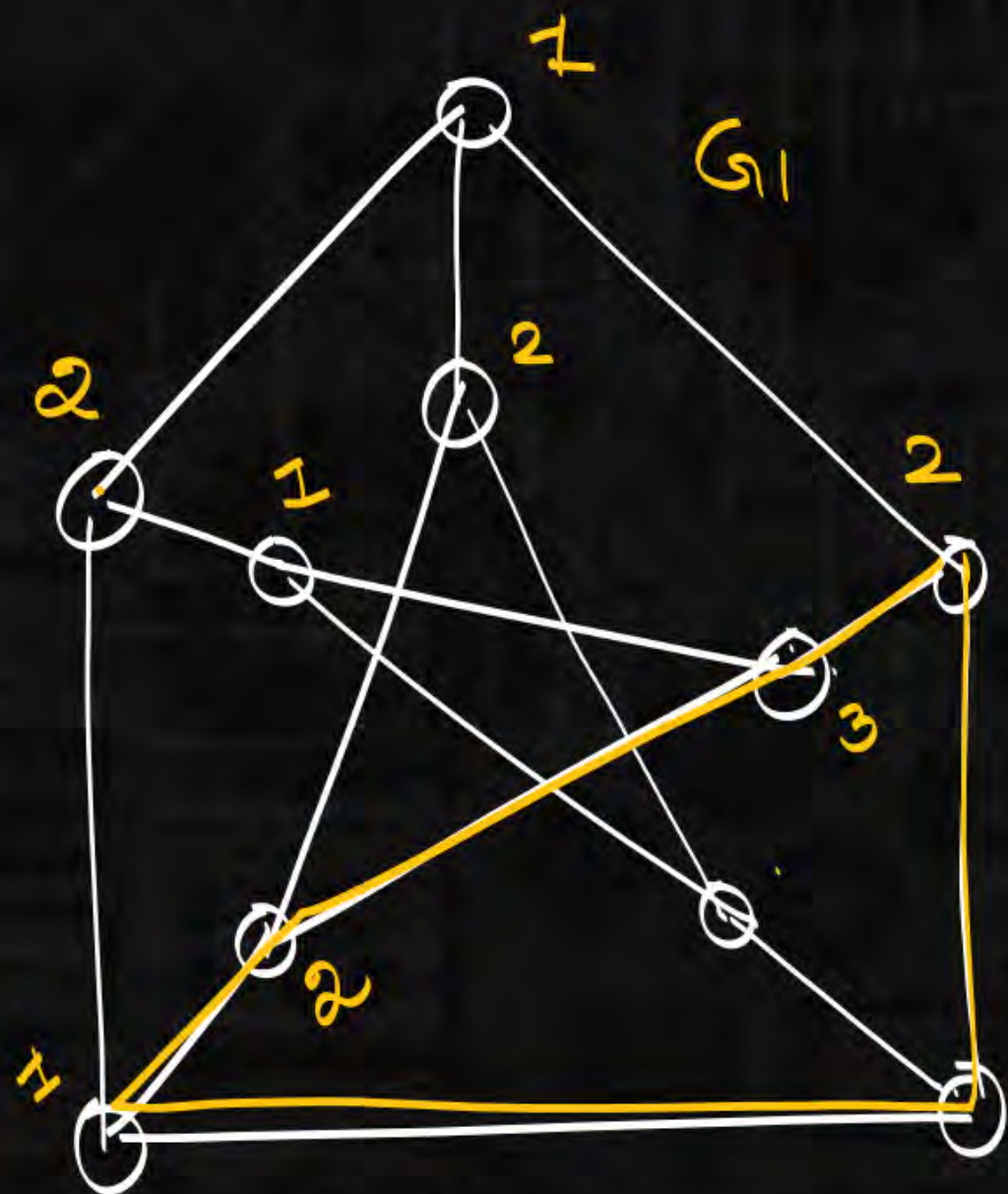






Check its B.P Graph or not ?

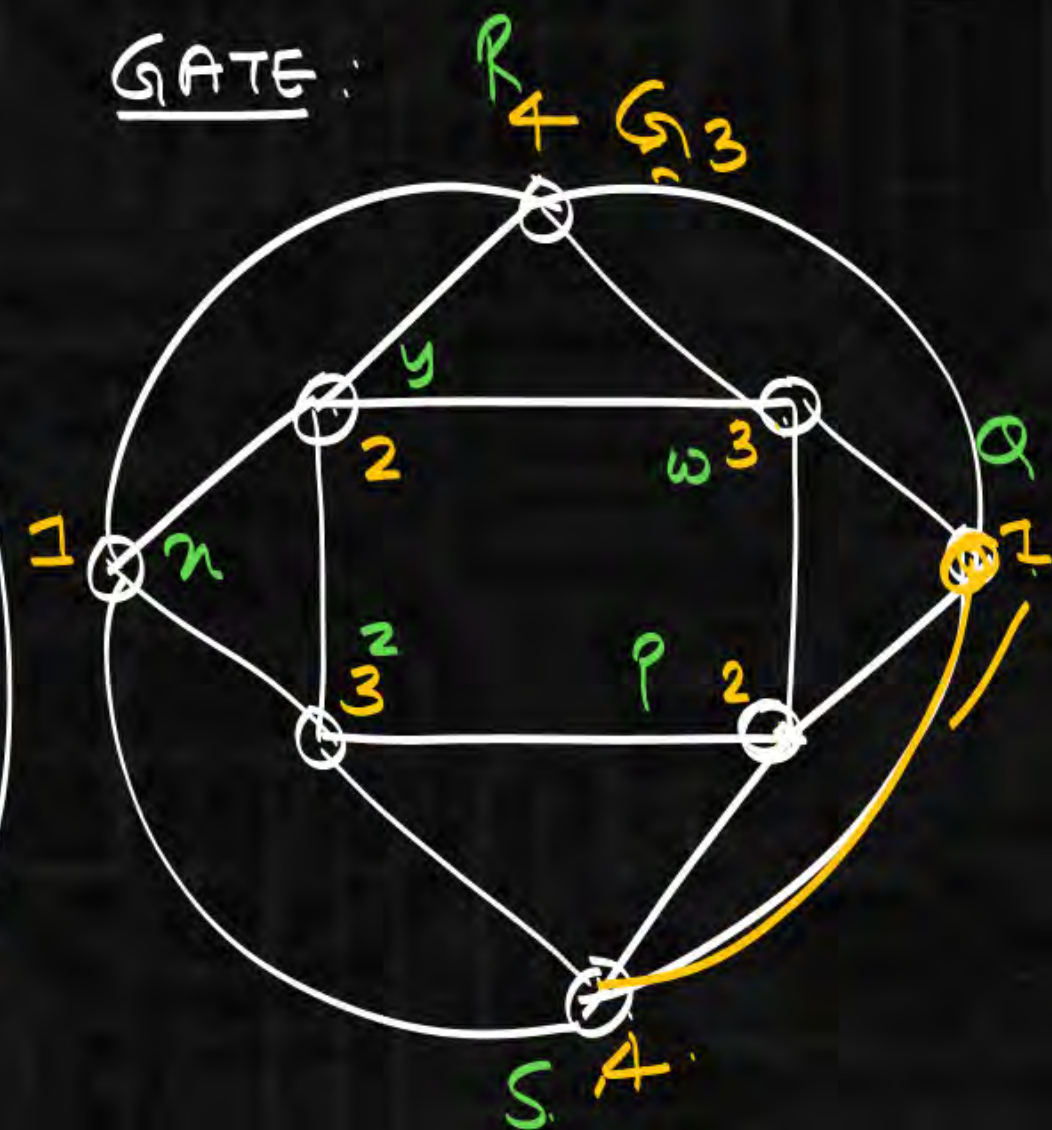




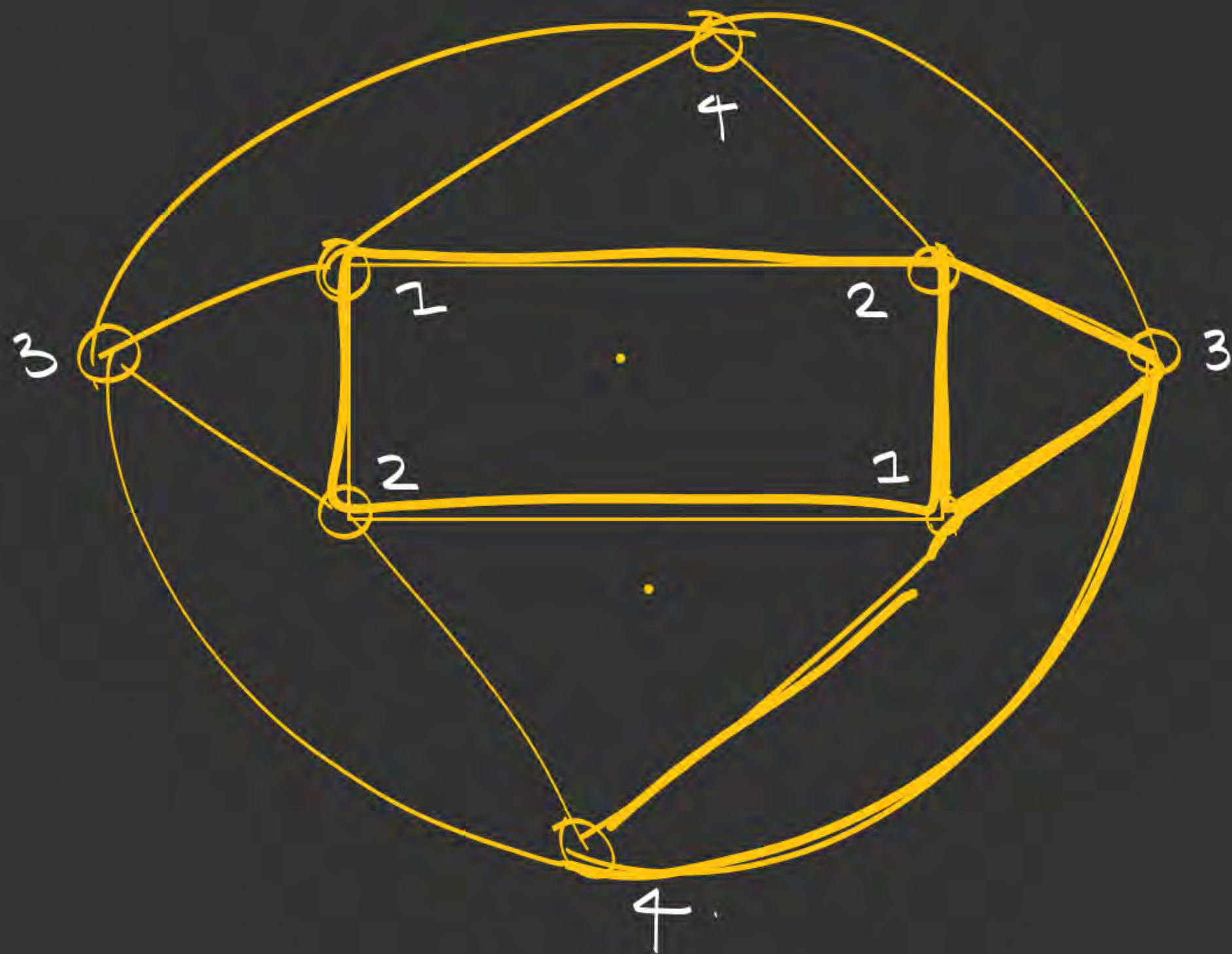
$$\chi(G_1) = 3$$



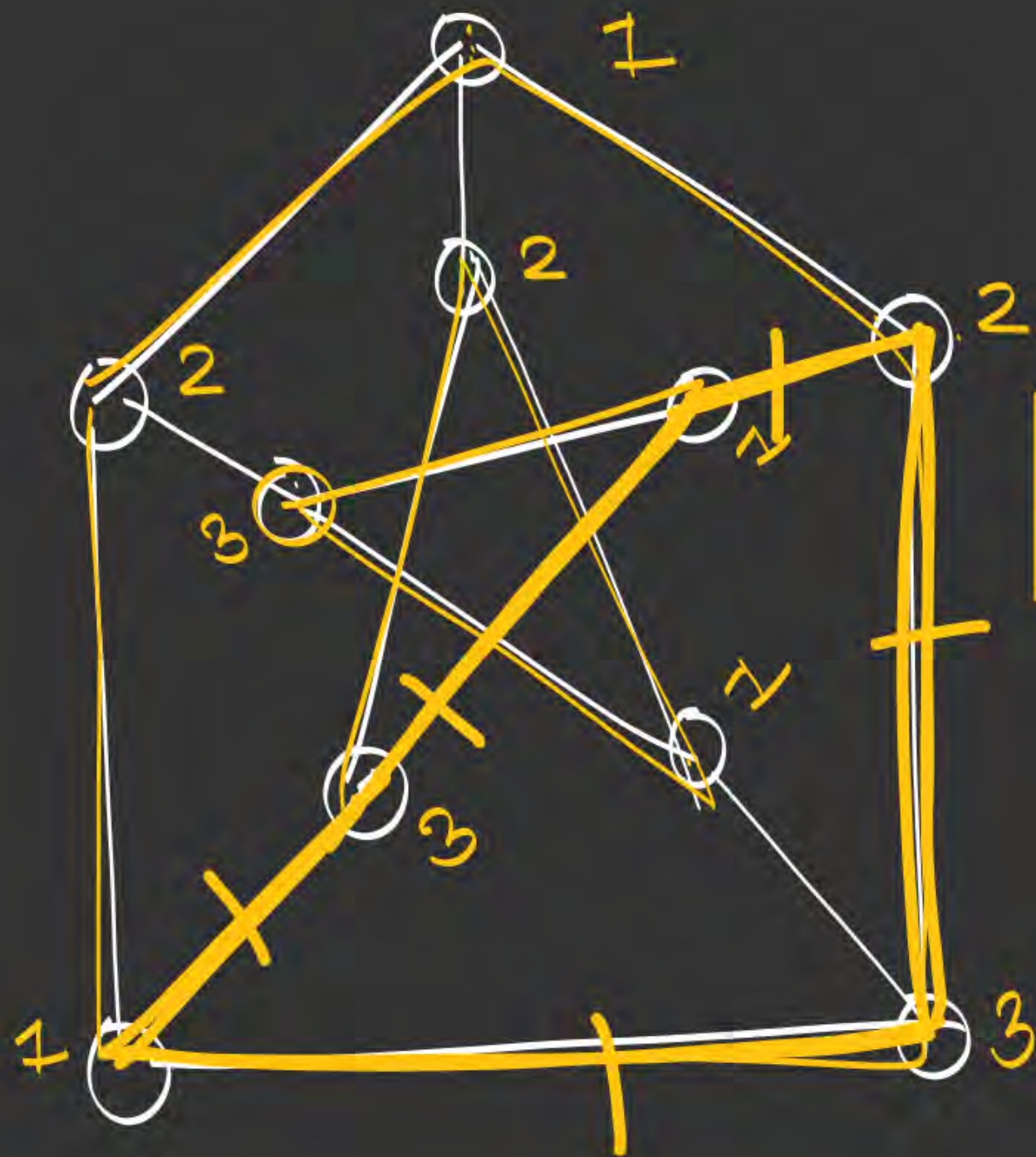
$$\chi(G_2) = 3$$



$$\chi(G_3) = 4$$







$$\chi(G) = 3$$

