CS & IT ENGINEERING

Graph Theory

Planarity Part -03

Lecture No.13

By- SATISH YADAV SIR

01 Inequalities thms in planarity part 1

02 Inequalities thms in planarity part 2

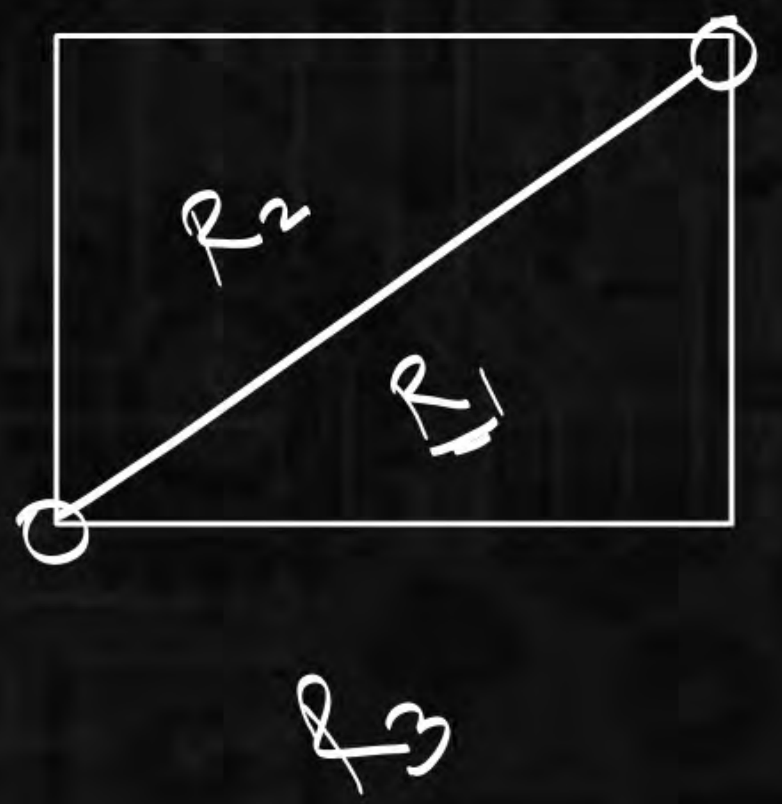
03 ~~Sub Graphs~~ Covering set

minimal covering

04 ~~Graph Operations part 1~~

covering no.

05 ~~Graph Operation Part 2~~



Draw planar Graph $\rightarrow$ Plane.

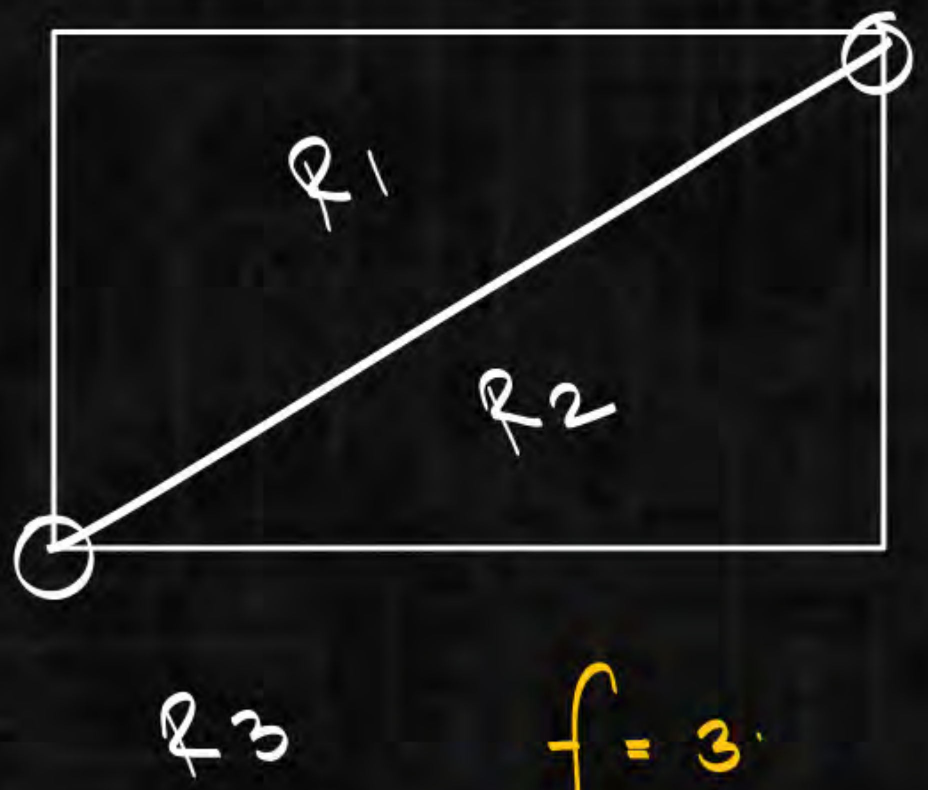
creates the Region

$deg(R_1) = 3$

$deg(R_2) = 3$

$deg(R_3) = 4$

$$\boxed{\sum deg(R_i) = 2e.}$$



$f = 3$

$$\begin{cases} \deg(R_1) = 3 \\ \deg(R_2) = 3 \\ \deg(R_3) = 4 \end{cases} \longrightarrow \left\{ \begin{array}{ll} \deg(R_1) \geq 3 & - \mathrm{I} \\ \deg(R_2) \geq 3 & - \mathrm{II} \\ \deg(R_3) \geq 3 & - \mathrm{III} \end{array} \right.$$

$$\deg(R_1) + \deg(R_2) + \deg(R_3) \geq 3 + 3 + 3$$

$$(\mathrm{I} + \mathrm{II} + \mathrm{III})$$

$$\sum \deg(R_i) \geq 3 \cdot 3 \qquad (f = 3)$$

$$\boxed{2e \geq 3 \cdot f}$$

$2e \geq 3f$

$n - e + f = 2$

$f = 2 + e - n$

$2e \geq 3(2 + e - n)$

$2e \geq 6 + 3e - 3n$

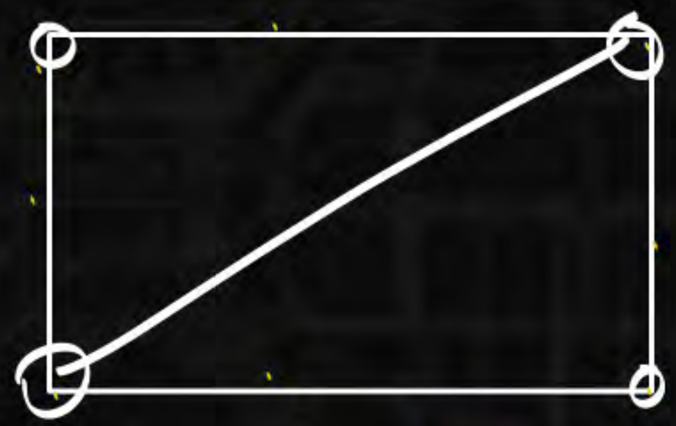
$\boxed{e \leq 3n - 6}$

$3n - 6 \geq 3e - 2e$

$3n - 6 \geq e$

<u>Thm</u>:

if G is Planar then $e \leq 3n - 6$.



$n = 4$

$e = 5$

$e \leq 3n - 6$

$5 \leq 3(4) - 6$

$5 \leq 6$

if G is Planar Graph then $e \leq 3n-6$

(Region is made up of at least 3 ~~Regions~~ Degrees ($n \geq 3$))

OR

if $e > 3n-6$ then the Graph is non planar.

(viceversa)

if $e \leq 3n-6$ then G is Planar (false)

$K_{3,3}$ $n=6$ $e=9$

$e \leq 3n-6$

$9 \leq 3(6)-6$

$9 \leq 12$ (True)

$\deg(R_1) = 4.$ $\quad \deg(R_1) \geq 4$

$\deg(R_2) = 4.$ $\quad \deg(R_2) \geq 4$

$\deg(R_3) = 6$ $\quad \deg(R_3) \geq 4.$

$$\underline{\deg(R_1) + \deg(R_2) + \deg(R_3)} \geq 4 + 4 + 4.$$

$$2e \geq 4 \cdot 3$$

$$\boxed{2e \geq 4 \cdot f}$$

$2e \geq 4f$

$n - e + f = 2$

$f = 2 + e - n$

if G is Planar then $e \leq 2n - 4$

$\left(\underline{\deg(R_i) \geq 4}\right)$

$2e \geq 4(2 + e - n)$

$\boxed{e \leq 2n - 4}$

$2e \geq 8 + 4e - 4n$

$\div$ by 2

$e \geq 4 + 2e - 2n$

$2n - 4 \geq 2e - e$

$\deg(R_1) \geq 5$

$\deg(R_2) \geq 5$

$\deg(R_3) \geq 5$

$\deg(R_4) \geq 5$

$$\deg(R_1) + \deg(R_2) + \deg(R_3) + \deg(R_4) \geq 5+5+5+5$$
$$\geq 5 \cdot 4$$
$$\geq 5 \cdot f$$

$$2e \geq 5f$$

$$2e \geq 5(2+e-n)$$

$$2e \geq 10 + 5e - 5n$$

$$n - e + f = 2$$

$$f = 2 + e - n$$

$$5n - 10 \geq 5e - 2e$$

$$5n - 10 \geq 3e$$

$$e \leq \left(5/3\right)n - \left(10/3\right)$$

if G is Planar then $e \leq \left(\frac{5}{3}\right)n - \frac{10}{3}$. $\left(\deg(R_i) \geq 5\right)$

$$K(G) \le \lambda(G) \le \boxed{\delta(G) \le 2e/n} \le \Delta(G) \le n-1.$$

if G is Planar Graph then $\delta(G) \le 5$.

$$\delta(G) \le 2e/n$$

$$\boxed{e \le 3n-6.}$$

$$\delta(G) \le 6 - 12/n.$$

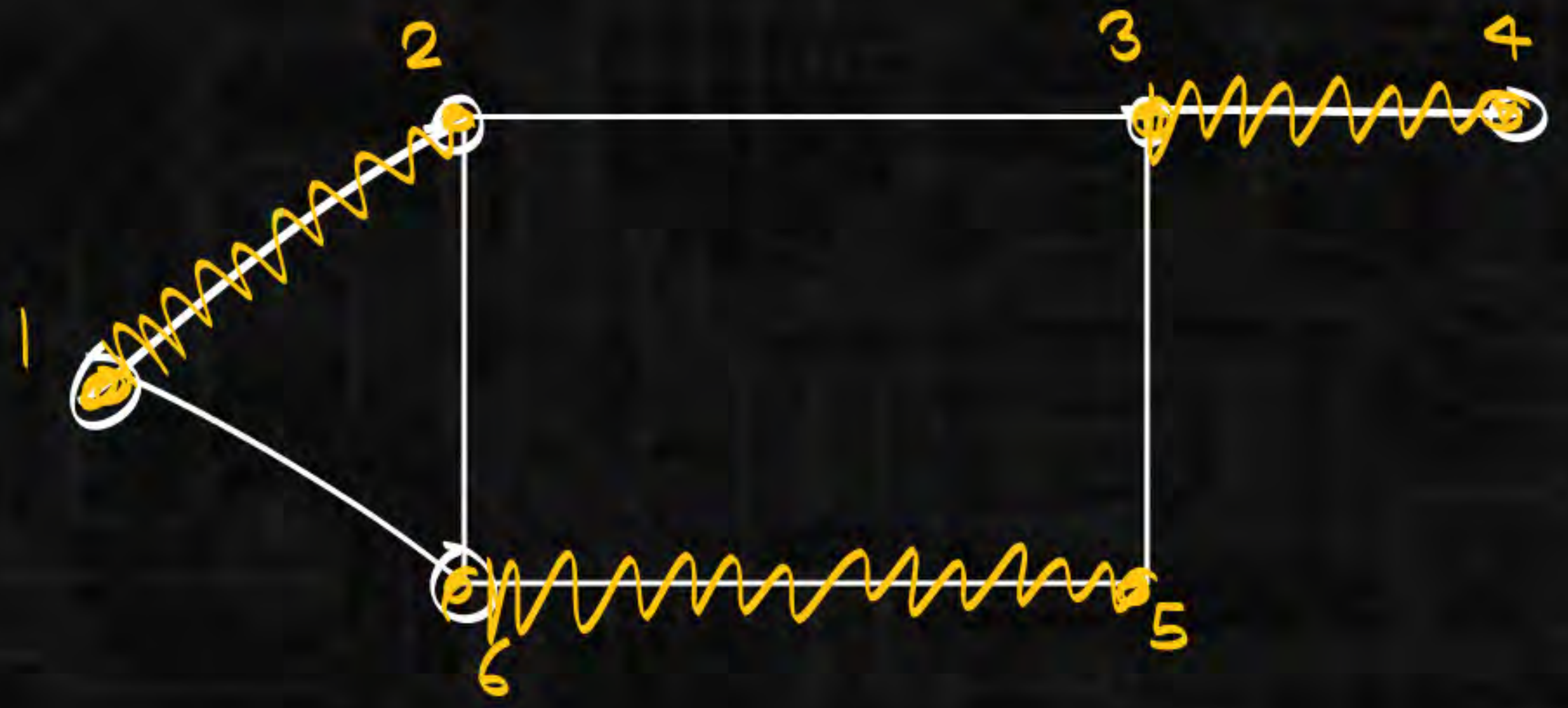
$$\boxed{\delta(G) \le 5.}$$

$$\delta(G) \le \frac{2e}{n} \le \frac{2(3n-6)}{n} \le \frac{6n-12}{n} \le \frac{6n}{n} - 12/n.$$

Covering set : (atleast 1 marriage proposal to all vertices)

Set of edges, such that if take all vertices

all vertices incident on atleast one edge.



all edges → covering set

covering set

{12, 23, 34, ~~35~~, 65, 26, 16}

{12, 23, 34, 65, ~~26~~, 16} covering set

{12, ~~23~~, 34, 65, ~~16~~} covering set

all vertices → at least 1 edge

{12, 34, 65} minimal covering set

minimal covering set :

Set of edges such that we cannot Remove new edge from this.

$c(G) = 3$.

covering no ($c(G)$) :

no. of edges present in smallest minimal covering set.


THANK YOU GW
SOLDIERS !