COMPUTER SCIENCE

Database Management System

FD's & Normalization

Lecture_10

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TOPICS TO BE COVERED

01 Normal Forms Concept

02 2NF, 3NF

RDBMS Concept

FD Concept & its type

Attribute Closure

Keys Concept

- Super key
- Candidate key
- Finding Multiple C.K

~~At most 1 Primary key~~

Membership set

Equality b/w 2 FD set

Minimal Cover.

Finding # Super keys & # Maximum C.K

$^{n}C_{\lfloor n/2 \rfloor}$

n: # Attribute

Properties of Decomposition

- Lossless Join
- Dependency Preserving.

Normal Form :

Redundancy level.

1NF > 2NF > 3NF > BCNF

$X \to Y$

Redundancy Too High.

0% Redundancy.

But Suffer from M.V.F.D.

$[X \to\to Y]$

Normal Forms

First Normal Form (1NF) : A Relational Schema R is in 1NF if all attribute of R are atomic.

Note

Default RDBMS is in 1NF.

Possible Non Trivial FD which Cause Redundancy :

CASE I

Proper subset of Candidate key → Non key Attribute

Violation of 2NF (or) Eliminated by 2NF

CASE II

Non key Attribute → Non key Attribute

Violation of 3NF (or) El. By 3NF.

CASE III

Proper subset of one Candidate key → Proper subset of another Candidate key

Violation of BCNF (or) El. By BCNF.

Note : 2NF does not allow Case I, but allow Case II, Case III.

Note 3NF does not allow Case I, Case II but allow Case III.

Note BCNF does not allow Case I, Case II & Case III.

Note BCNF has 0% Redundancy.

Case I :

Proper subset of Candidate key $\longrightarrow$ Non key Attribute

2NF Not allow this case.

eg $R(ABCDE)$ $[AB \to C,\ C \to D,\ B \to E]$

AB

Proper subset

A B

Candidate key : $[AB]$

Non key Attributes $= [C, D, E]$

$B \longrightarrow E$

Proper subset of C.K

Non key Attribute.

Not in 2NF.

Case II :

Non key Attribute $\longrightarrow$ Non key Attribute

3NF Not allow this case.

Eg) $R(ABC)\ \{A \rightarrow B,\ B \rightarrow C\}$

Candidate key = [A]

Non Prime / Non key Attribute = [B, C]

$$\underset{\text{Non key Attribute}}{B} \longrightarrow \underset{\text{Non key Attribute}}{C}$$

Not in 3NF.

Case III

Proper subset of One Candidate key $\longrightarrow$ Proper subset of another C.K

BCNF Not allow this case.

eg R(ABCD) [AB → CD, D → A]

Candidate key = [AB, DB]

D → A

Proper subset of One Candidate key

Proper subset of another Candidate key.

Not in BCNF.

Normal Forms

2NF

3NF

BCNF.

Normal Forms

<u>second Normal Form</u> (2NF)

Relation R is in 2NF iff

① R is in 1NF

② R does not Contain $\boxed{\text{Proper subset of C.K}} \longrightarrow \boxed{\text{Non key Attribute.}}$

Wrong

Partial Dep.

Proper Subset of C.K → Non Key Attribute

This is Not P.D, its Only Violation of 2NF.

$2+2=4$

$2*2=4$

$3+3=6$

$3*3=9$

Partial Dependency

$X \rightarrow Y$ is Partial FD if $\forall B \in X$

$\underline{(X - B)} \longrightarrow Y$

eg $AB \rightarrow C$ is Partial FD if either $A \rightarrow C$ or $B \rightarrow C$

Q R(ABC)

[AB → C , B → C]   Which one is P.F.D ?

Soln

$\boxed{AB \longrightarrow C \text{ is Partial FD}}$

Because Here $B \longrightarrow C$.

A is extra

$X \rightarrow y$ is Partial

$\forall B \in X$

$(X - B) \longrightarrow y$

R(ABC) {AB → C} is full FD

if $A \not\rightarrow C$ A does not determine C.

$B \not\rightarrow C$ B does not determine C.

Q) $R(xyz)$

$(xy \rightarrow z, \quad x \rightarrow z]$

$xy \rightarrow z$ is Partial, $\because$ Here if we Remove y then

also we getting $\underline{x \rightarrow z}$

$xy \rightarrow z$ is Partial if

either $x \rightarrow z$

OR

$y \rightarrow z$

α is Attribute set, $\gamma \in \alpha$

$\boxed{\alpha \longrightarrow \beta \text{ is Partial}}$

$(\alpha - \gamma) \longrightarrow \beta$

FULL FD.

$\boxed{AB \rightarrow C \text{ is FULL FD}}$

iff $A \not\rightarrow C$

$B \not\rightarrow C$

$\boxed{\alpha \longrightarrow \beta \text{ is Full FD}}$

$(\alpha - \gamma) \not\longrightarrow \beta$

Partial F.D

$\boxed{AB \rightarrow C \text{ is a Partial FD}}$

if either $\underline{A \rightarrow C}$

OR

$\underline{B \rightarrow C}$

$\boxed{\alpha \longrightarrow \beta \text{ is Partial FD}}$

if $(\alpha - \gamma) \longrightarrow \beta$

Q) $R(abcde)$ $[ab \to cd,\ cd \to ab,\ c \to e]$

(i) $ab \to c$ is Full or Partial ?

(ii) $cd \to ab$ is Full or Partial ?

(ii) $cd \to e$ is Full or Partial ?

(iii) $bc \to e$ is Full or Partial ?

Sol^n] $ab \to c$?

× $a \to c$ $[a]^+ = [a]$

× $b \to c$ $[b]^+ = [b]$

Neither $a \to c$ nor $b \to c$ $\left[\begin{array}{l} a \not\to c, \\ b \not\to c \end{array}\right]$

$ab \to c$ is Full FD.

Sol^n (ii) $cd \to e$?

$d \to e$ $[d]^+ = (d)$

$\underline{c \to e}$ $[c]^+ = (ce)$

$\therefore cd \to e$ is Partial FD.

(8) R(abcde) [ab → cd, cd → ab, c → e]

(i) $ab \rightarrow c$ is Full or Partial?

(ii) $cd \rightarrow e$ is Full or Partial?

(iii) $bc \rightarrow e$ is Full or Partial?

(iv) $cd \rightarrow ab$ is Full or Partial?

(iii) $bc \rightarrow e$?

Here $c \rightarrow e$

$\boxed{\text{So } bc \rightarrow e \text{ is Partial FD.}}$

$\because (bc - b) \longrightarrow e$

$\boxed{\alpha \rightarrow \beta \text{ is P.F.D}}$

if $(\alpha - \gamma) \longrightarrow \beta$

(iv) $cd \rightarrow ab$?

$c \rightarrow ab$ $\quad (c)^+ = [ce]$ ×

$d \rightarrow ab$ $\quad (d)^+ = (d)$ ×

$cd \rightarrow ab$ is Full FD.

Normal Forms

Navathe Book.

15.3.5 Second Normal Form

Second normal form (2NF) is based on the concept of *full functional dependency*. A functional dependency $X \to Y$ is a **full functional dependency** if removal of any attribute A from X means that the dependency does not hold any more; that is, for any attribute $A \varepsilon X$, $(X - \{A\})$ does *not* functionally determine Y. A functional dependency $X \to Y$ is a **partial dependency** if some attribute $A \varepsilon X$ can be removed from X and the dependency still holds; that is, for some $A \varepsilon X$, $(X - \{A\}) \to Y$. In Figure 15.3(b), {Ssn, Pnumber} → Hours is a full dependency (neither Ssn → Hours nor Pnumber → Hours holds). However, the dependency {Ssn, Pnumber} → Ename is partial because Ssn → Ename holds.

Definition. A relation schema R is in 2NF if every nonprime attribute A in R is *fully functionally dependent* on the primary key of R.

$\boxed{X \to y \text{ is Full FD}}$

if $(X - A) \not\to y$

Not Hold.

$\boxed{X \to y \text{ is Partial FD}}$

$A \in X$

$(X - A) \to y$.

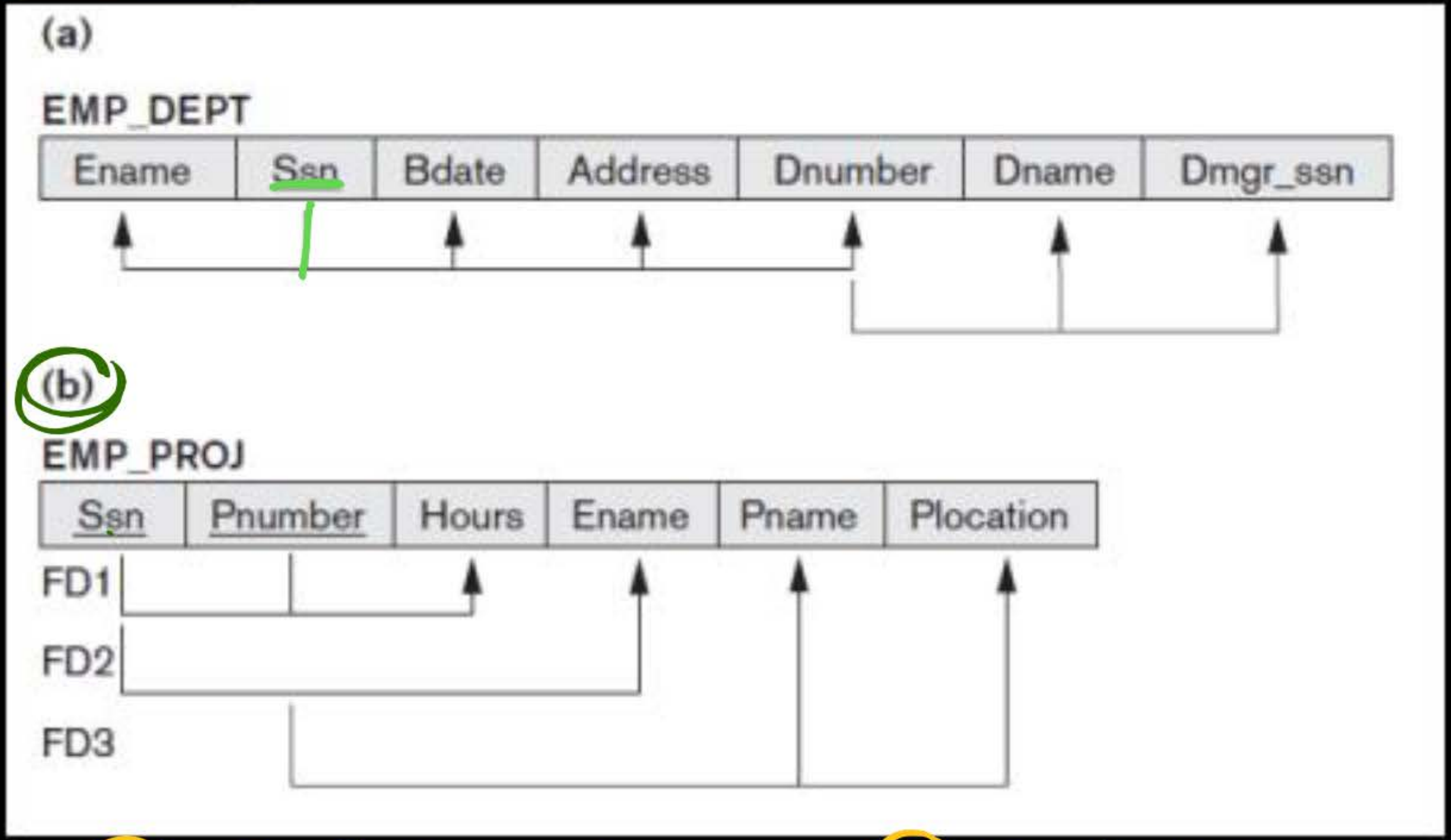
Hold.

Ssn Pnumber $\rightarrow$ Hours is Full FD

Bcz Ssn $\not\rightarrow$ Hours

Pnumber $\not\rightarrow$ Hours

Figure 15.3



Ssn → ename Bdate, Address, Dnumber

Dnumber → Dname, Dmgr no

FD1: Ssn Pnumber → Hours

2: Ssn → ename

3: Pnumber → Pname Plocation

Q.1 Ssn Pnumber → Hours

Full FD.

Q.2 Ssn Pnumber → ename

is Partial Bcz Ssn → ename.

Normal Forms

Korth Book

8.17 A functional dependency $\boxed{\alpha \to \beta \text{ is called a partial dependency}}$ if there is a proper subset γ of α such that $\gamma \to \beta$. We say that β is *partially dependent* on α. A relation schema R is in second normal form (2NF) if each attribute A in R meets one of the following criteria:

- It appears in a candidate key.
- It is not partially dependent on a candidate key.

$\alpha \to \beta$ is Partial.

$\gamma \to \beta$

$AB \to C$

$B \to C$ is Partial ✗

But In actually $AB \to C$ is Partial.

Q) $R(ABCDEF)$ $[ABC \to DE,\ DE \to ABC,\ DE \to F,\ AB \to D,\ E \to C]$

(i) $ABC \to D$

Sol^n 1 $ABC \to D$ is Partial $\because AB \to D$.

(ii) $AE \to C$

Sol^n 2 $AE \to C$ is Partial $\because E \to C$.

(iii) $AF \to D$

(iv) $AB \to D$

Sol^n 3 $AF \to D$

? $A \to D$ $[A]^+ = [A]$

? $F \to D$ $[F]^+ = [F]$

$AF \to D$ is Full FD.

(v) $AC \to D$

(vi) $BC \to D$.

(vii) $DE \to C$

(viii) $AB \to F$

$R(ABCDEF)$ $\{ABC \to DE,\ DE \to ABC,\ DE \to F,\ AB \to D,\ E \to C\}$

(i) $ABC \to D$

(ii) $AE \to C$

(iii) $AF \to D$

(iv) $\underline{AB \to D}$

(v) $AC \to D$

(vi) $BC \to D$.

(iv) $AB \to D$

$A \to D$ ✗ $(A)^+ = (A)$

$B \to D$ $(B)^+ = (B)$

$AB \to D$ is Full FD.

(v) $AC \to D$.

$(A)^+ = (A)$

$(C)^+ = (C)$

$AC \to D$ is Full.

(vii) $DE \to C$

(viii) $AB \to F$

(vi) $BC \to D$

$(B)^+ = (B)$

$(C)^+ = (C)$

$BC \to D$ is Full FD

$R(ABCDEF)$ $[ABC \to DE,\ DE \to ABC,\ DE \to F,\ AB \to D,\ E \to C]$

(i) $ABC \to D$

(ii) $AE \to C$

(iii) $AF \to D$

(iv) $AB \to D$

(v) $AC \to D$

(vi) $BC \to D$.

(vii) $DE \to C$

(viii) $AB \to F$

Sol^n 7 $DE \to C$

is Partial FD

$\because$ $E \to C$.

Sol^n 8 $AB \to F$

$(A)^+ = [A]$

$(B)^+ = [B]$

$AB \to F$ is Full FD.

$R(ABCDEF)$ $\{ABC \to DE,\ DE \to ABC,\ DE \to F,\ AB \to D,\ E \to C\}$

(i) $ABC \to D$

(ii) $AE \to C$

(iii) $AF \to D$

(iv) $AB \to D$

(v) $AC \to D$

(vi) $BC \to D$.

(vii) $DE \to C$

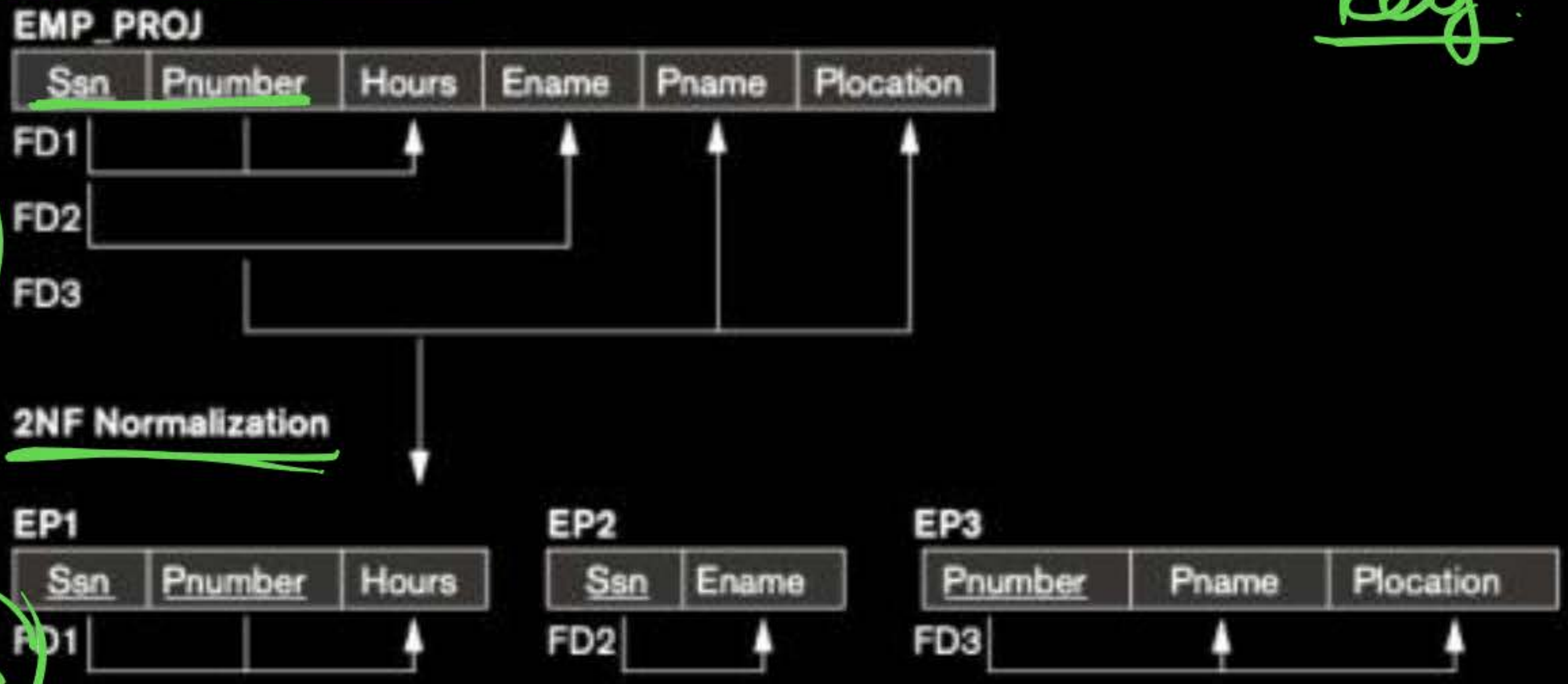
(viii) $AB \to F$

Normal Forms

Second Normal Form

Definition: A relation schema R is in 2NF if every nonprime attribute A in R is fully functionally dependent on the primary key of R.

key



ssn → ename

Pnumber → Pname location

Violation of 2NF

Proper subset of C.K → Non key Attribute

2NF Violation

Proper Subset of Candidate Key $\longrightarrow$ Non Key / Non Prime Attribute

if this FD exist then Relation R is Not in 2NF.

Q. Let R(A, B, C, D, E, P, G) be a relational schema in which the following functional dependencies are known to hold:

$AB \rightarrow CD, \ DE \rightarrow P, \ C \rightarrow E, \ P \rightarrow C$ and $B \rightarrow G$.

(GATE 2marks)

The relational schema R is

Candidate key = AB

Non key Attribute = (C, D, E, F, G)

(A) In BCNF

(B) In 3NF, but not in BCNF

(C) In 2NF, but not in 3NF

(D) Not in 2NF

$B \longrightarrow G$

Proper subset of C.K

Non key Attribute.

Not in 2NF

3NF

R(ABC) [A → B, B → C]

Candidate key = [A]

Non key Attribute = [B, C]

B ⟶ C

Non key Attribute → B

Non key Attribute → C

Violation of 3NF.

Not in 3NF

$X \to y$, $y \to z$

$X \to z$ is Trisitive.

OR

$\boxed{X \to z \text{ is Transitive}}$

X ⟶ y ⟶ z

If A → B, B → C then

$\boxed{A \to C \text{ is Transitive.}}$

Normal Forms

RDBMS

E.F Codd

Third Normal Form

Definition: I According to Codd's original definition, a relation schema R is in 3NF if it satisfies 2NF and no nonprime attribute of R is transitively dependent on the primary key.

Definition: II A relation schema R is in third normal form (3NF) if, whenever a nontrivial functional dependency $X \rightarrow A$ holds in R either (a) X is a superkey of R, or (b) A is a prime / key attribute of R.

$R(ABC)$ $\{A \to B,\ B \to C\}$

Candidate key = $[A]$

Non key / Non Prime Attribute = $[B, C]$

$A \to C$ is Transitive

$$\begin{tikzcd} A \arrow[r] \arrow[rr, bend left, "\text{Transitive}"] & B \arrow[r] & C \end{tikzcd}$$

Here Non Prime Attribute C is Transitively Dependent on Primary key so R Not in 3NF.

Defination III

R is in 3NF if Not allowed Both Case I & II.

Subset key

$X \longrightarrow Y$

R.H.S key Attribute

Case I : Proper Subset of C.K $\longrightarrow$ Non key Attribute ✗

Case II Non key Attribute $\longrightarrow$ Non key Attribute. ✗

Non Trivial FD

3NF

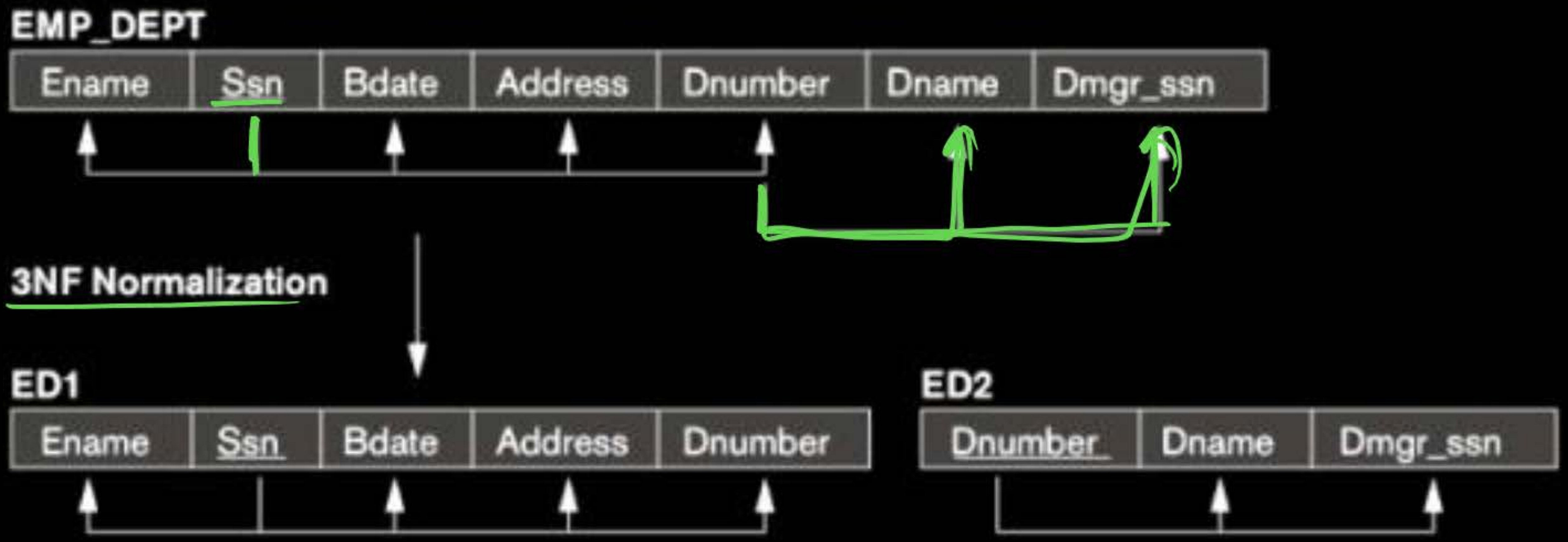
$X \rightarrow Y$ is in 3NF

if either X : Super key

OR

Y : key/Prime Attribute.

Candidate key = {Ssn}



Boyce – Codd Normal Form

Definition: A relation schema R is in BCNF if whenever a nontrivial functional dependency $X \rightarrow A$ holds in R, then X is a superkey of R.

BCNF : R is in BCNF if Not allowed Case I, II & III.

$X \longrightarrow Y$

- **CASE I**: Proper subset of Candidate key → Non key or Non Prime Attribute — X Violation of 2NF
- **CASE II**: Non key Attribute → Non key Attribute — X Violation of 3NF
- **CASE III**: Proper subset of one C.K → Proper subset of another C.K — X Violation of BCNF

X: SUPER KEY

OR

$X \rightarrow y$ every Non Trivial FD is in BCNF

if

$\boxed{X : \text{SUPER KEY.}}$

Important Points

A Relation R with only one Candidate key, then R is in 1NF But May or May Not in 2NF, 3NF & BCNF.

Q) R(ABCDE) {AB → C, C → D, B → E}

Candidate key - {AB}

Non key Attribute = {C, D, E}

B → E

Proper subset of CK — Non key Attribute

} Not in 2NF.

Important Points [2NF]

① If Every Candidate key (of Relation R) is a Simple (Single Attribute) [Not Composite]
then Relation R is in always 2NF But May or May Not in 3NF & BCNF.

eg R(ABCDE) [A→B, B→C, C→D, D→E, C→A]

Soln Candidate key = [A, C, B]

All Candidate keys are simple (single Attribute).

So R is in 2NF But Not in 3NF.

D → E

D: Non key Attribute, E: Non key Attribute

D → E

D: Not Subel key

or

E: Non key Attribute

Important Points [3NF]

① If Every Attribute of Relation R is a key/Prime Attribute then R is in 3NF But may or may Not in BCNF.

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ex R(ABCD) [A→B, B→C, C→A]

Candidate keys = [AD, BD, CD]

Prime/key Attribute = [A, B, C, D]

Here all attribute of R is key Attribute

So R is in 3NF But Not in BCNF

BCNF

$X \to Y$

$A \to B$
$B \to C$
$C \to A$

X is Not Super key

Important Points [BCNF]

① If Every Candidate key is a Simple [NotComposite] & if R is in 3NF then Relation R is in BCNF.

Single Attribute

eg R(ABC) [A→B, B→C, C→A]

Candidate keys = [A, B, C]

$\underbrace{A \to B,\ B \to C,\ C \to A}_{X \to Y}$

X is super key so R is in BCNF.

Important Points [BCNF]

② A Relation with 2 Attribute (Binary Relation) is in BCNF

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eg) $R(AB)$ $[A \to B]$

Candidate key = $[A]$

$A \to B$

A ← super key

$\therefore$ R is in BCNF.

$R(AB)$ $[B \to A]$

Candidate key = $[B]$

$B \to A$

B ← super key

$\therefore$ R is in BCNF.

$R(AB)$ $[A \to B, B \to A]$

Candidate key = $[A, B]$

$A \to B$, $B \to A$

A ← super key, B ← super key

$\therefore$ R is in BCNF.

Important Points [BCNF]

3. Relation R with No Non Trivial FD is in BCNF.

OR Trivial FD always in BCNF.

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eg) $XYZ \rightarrow XY$

superkey

$\therefore$ R is in BCNF.

$X \rightarrow Y$ is Trivial

$X \supseteq Y$

Q. In a relational data model, which one of the following statements is TRUE?

(A) A relation with only two attributes is always in BCNF.

(B) If all attributes of a relation are prime attributes, then the relation is in BCNF.

(C) Every relation has at least one non-prime attribute.

(D) BCNF decompositions preserve functional dependencies.

Q. Which of the following statement is/are true?

(A) Second normal form (2NF) have transitive dependency.

(B) No relation can be in both BCNF and 3NF.

(C) Second normal form(2NF) does not have partial dependency.

(D) In BCNF lossless join & dependency - preserving decomposition is always possible.

Q. Let R(A, B, C, D, E, P, G) be a relational schema in which the following functional dependencies are known to hold:

$AB \rightarrow CD$, $DE \rightarrow P$, $C \rightarrow E$, $P \rightarrow C$ and $B \rightarrow G$.

The relational schema R is

A. In BCNF

B. In 3NF, but not in BCNF

C. In 2NF, but not in 3NF

D. Not in 2NF

Q

Consider the following statements: [MSQ]

S_1: If every attribute is prime attribute in R, then Relation R will always be in BCNF.

S_2: Any Relation with two Attribute is in 3 NF and 2 NF.

S_3: If every key of relation R is a simple candidate key (No composite key) then the relation R not always in NF.

S_4: In BCNF there is always a lossless join and Dependency Preserving Decomposition.

Which of the above statement are incorrect

(A) S_1

(B) S_2

(C) S_3

(D) S_4

Q

The relation scheme student Performance (name, courseNO, rollNo, grade) has the following functional dependencies:

[2004: 2 Marks]

$$\text{name, courseNo} \rightarrow \text{grade}$$

$$\text{RollNo, courseNo} \rightarrow \text{grade}$$

$$\text{name} \rightarrow \text{rollNo}$$

$$\text{rollNO} \rightarrow \text{name}$$

The highest normal form of this relation scheme is

(A) 2 NF

(B) 3 NF

(C) BCNF

(D) 4 NF

Normal Form Decomposition

Q.

Consider the following FD Set: **[MSQ]**

$\{P \rightarrow QR, Q \rightarrow PR, R \rightarrow PQ\}$ which of the following is/are the minimal cover for the above FD set?

(A) $P \rightarrow Q, Q \rightarrow R, R \rightarrow P$

(B) $P \rightarrow R, Q \rightarrow R, R \rightarrow PQ$

(C) $Q \rightarrow P, P \rightarrow R, R \rightarrow Q$

(D) $P \rightarrow QR, Q \rightarrow P, R \rightarrow P$

$P \rightarrow Q$ $P \rightarrow R$ $Q \rightarrow P$ $Q \rightarrow R$ $R \rightarrow P$ $R \rightarrow Q$

Minimal Cover

Step 1 Split the FD R.H.S Single Attribute

Step 2 find the Redundant FD & Delete

Step 3 Remove Redundant Attribute on L.H.S Side

$P \to QR,\ Q \to PR,\ R \to PQ$

Step1 $\quad P \to Q \quad P \to R \quad Q \to P \quad Q \to R \quad R \to P \quad R \to Q$

Step2 $\quad$ ① $\cancel{P \to Q}$ $\quad$ ② $P \to R$ $\quad$ ③ $\cancel{Q \to P}$ $\quad$ ④ $Q \to R$ $\quad$ ⑤ $R \to P$ $\quad$ ⑥ $R \to Q$

$(P)^+ = [PRQ] \quad (P)^+ = [P] \quad (Q)^+ = [QRP] \quad (Q)^+ = (Q) \quad (R)^+ = (RQ) \quad (R)^+ = [RP]$

$P \to R \quad Q \to R \quad R \to P \quad R \to Q$

$P \to R \quad Q \to R \quad R \to PQ$

b

$P \to QR,\ Q \to PR,\ R \to PQ$

② $P \to Q$ ① $\cancel{P \to R}$ ③ $\cancel{Q \to P}$ ④ $Q \to R$ ⑤ $R \to P$ ⑥ $\cancel{R \to Q}$

$(P)^+ = (P)$ $(P)^+ = (PQR)$ $(Q)^+ = (QRP)$ $(Q)^+ = (Q)$ $(R)^+ = (RQ)$ $(R)^+ = (RPQ)$

$P \to Q \quad Q \to R \quad R \to P$

a

$P \to QR,\ Q \to PR,\ R \to PQ$

③ $\cancel{P \to Q}$ ④ $P \to R$ ✓ ⑤ $Q \to P$ ✓ ⑥ $\cancel{Q \to R}$ ① $\cancel{R \to P}$ ② $R \to Q$ ✓

$(P)^+ = \{PRQ\}$ $(P)^+ = \{P\}$ $(Q)^+ = \{QP\}$ $(Q)^+ = \{QPR\}$ $(R)^+ = \{RQP\}$ $(R)^+ = \{R\}$

$P \to R \quad Q \to P \quad R \to Q$

C

$P \to QR, \quad Q \to PR, \quad R \to PQ$

(5) $P \to Q$ (6) $P \to R$ (2) $Q \to P$ (1) $\cancel{Q \to R}$ (4) $R \to P$ (3) $\cancel{R \to Q}$

$(P)^+ = (PR)$ $(P)^+ = (PQ)$ $(Q)^+ = (Q)$ $(Q)^+ = (QPR)$ $(R)^+ = (R)$ $(R)^+ = (RPQ)$

$P \to Q \quad P \to R \quad Q \to P \quad R \to P$

$$\boxed{P \to QR \quad Q \to P \quad R \to P}$$

d

Any Doubt ?

THANK YOU!