# **COMPUTER SCIENCE**



Computer Organization and Architecture

Floating Point Representation



Vijay Agarwal sir

Lecture\_02





# Floating Point Representation



Signed 4 unsigned

1's 2 2's combrement

Why 2's Complement are used.

Floating Point Representation Linky its USED?



# Floating point Representation

M: Mantissa.

e: exponent Actual Exponent

WHY Biog Exponent is need?

2's complement = 
$$\begin{bmatrix} v-1 \\ -2 \end{bmatrix}$$
 to  $+2^{n-1}$   
4bit =  $-2^{n-1}$  to  $+2^{n-1}$   
 $\Rightarrow (-8)$  to  $+7$ 

E=4bit

Bias = 2

is Exponent is k bit
then biae =  $2^{k-1}$ 

# Floating-Point Representation



16 bit fixed point data format then

Range = 
$$-2^{16-1}$$
 to  $+(2^{16-1}-1)$ 

$$\Rightarrow$$
 -(2<sup>15</sup>) to + (2<sup>15</sup> - 1)

If we want to store 61,000 then we cannot store

Because range [-32k to + 32 k -1]

So floating point representation is to represent very large data and very small fraction and consume less memory

$$+ 8.56410000000000.... [\Rightarrow \infty]$$

$$+ 0.000000000007892 \Rightarrow [\Rightarrow 0]$$

# Floating-Point Representation

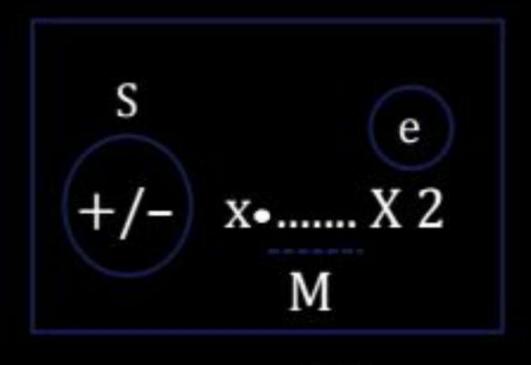




S: sign bit 
$$0 + ve$$
  
1 - ve

E: exponent

M: Mantissa



. ..... X2e

6.5 in Binary  $\Rightarrow$  110.1

$$+6.5$$

$$6.5 = (110.1)_2$$

$$\frac{0.1101}{S} \times \frac{2^3}{2^e}$$

$$S = 0 (+)$$

$$M = 1101$$

$$e = 3 = (11)_2$$

S	e	M
0	11	1101



### Very. Imp

$$6.5 = 110.1$$

$$=.1101 \times 2^{3}$$

$$= [.2^{-1} + 2^{-2} + 2^{-4}] \times 2^{3}$$

$$= [2^2 + 2^1 + 2^{-1}]$$

$$= 6.5$$

Q. 
$$2 + 4.5$$



100.1

 $0.1001 \times 2^{3}$ 

S = 0 (+ve)

M = 1001

e = 3[11]

S e M

0 11 1101



100.11

$$.10011 \times 2^{3}$$

$$S = 0$$

M: 10011

$$e = 3 \Rightarrow (11)_2$$

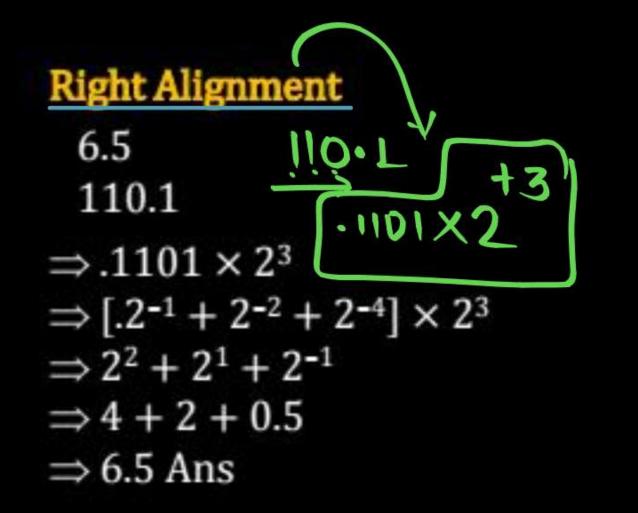
#### NOTE:



Mantissa alignment process is used to adjust the decimal point; in this process right alignment increments the exponent and left alignment decrements the exponent.

 $2^{+\text{shift}}$  power(+) = Right alignment  $\Rightarrow$  Increment the exponent

2-shift power (-) = Left alignment ⇒ Decrease the exponent



### Left Alignment

Data: 0.0000000101 × 2+5

$$[1.01 \times 2^{+5-8}]$$

$$+1.01 \times 2^{-3}$$

(Align to use upto 8 times)

Q. 4 + 0.00101



$$0.101 \times 2^{-2}$$

$$M = 101$$

$$E = -2$$

$$S = 0$$

S	E(4bit)	M(5 bit)
0	1110	10100
	E	М

$$E = -2 = (1110)_2$$
 2's complement

Biasing: is method in which we convert the negative number into

the positive number

Bit	Bit	Bit
S	E	М

Pw

$$S = Sign$$

$$E/BE = Exponent or$$

$$E = e + bias$$

Bias = 
$$2^{K-1}$$

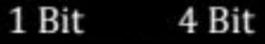
where K is exponent bits

### Example

If 
$$K = 4$$
 bits

Exponent 
$$= 4$$
 bit then

bias = 
$$2^{K-1} = 2^{4-1} = 8$$



S	Е	M

Bias = 
$$2^{K-1} = 2^{4-1}$$

$$bias = 8$$

$$E = e + bias$$

$$E = e + 8$$

#### E = 4 bit

or

#### Excess 8 code

$$2^{K-1} = 8$$

$$2^{K-1} = 23$$

$$K - 1 = 3$$

$$K = 4$$

### E = 4 bit

e [original exponent]	Stored exponent [BE] E
-8 etbiast	2) 0
-7	1,
<b>-</b> 6	2,
<b>-</b> 5	3_
-4	4
-3	5
-2	6
-1	į į
0	8
1	9
2	10
3	11
4	12
5	13
6	14
G) 77+6	15

6ios = 2

C: Actual exponent

E=e+bias

BE = AE + biog

#### From previous question

0.00101

$$0.101 \times 2^{-2}$$

$$M = 101$$

Bias = 
$$2^{5-1}$$

$$Bias = 16$$

$$e = -2$$

$$E = e + bias$$

$$E = -2 + 16$$

$$E = 14$$

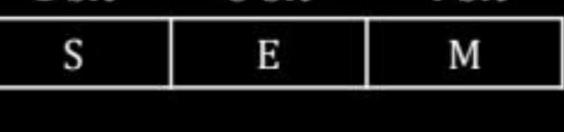
$$E = (011110)_2$$



$$bios = 2$$

$$biog = 2^k$$

1 bit	5 bit	4 bit
S	Е	M



Ans

Formula:  $(-1)^S \times 0.M \times 2^e$ 

$$(-1)^0 \times 0.101 \times 2^{E-bias}$$

$$0.101 \times 2^{14-16} = 0.101 \times 2^{-2}$$
  
 $0.000101 \text{ Ans}$ 



# Mantissa.

0.0001010

Solution is 101011.0110101
Normalized Montrissa
the.

3 100.11101

(b) 0.11010(1101

BE= AE + bias.

- 1 Impliet -> 1. game thing.
- ② Expericit → O·L···







### **Explicit Normalized**

Syntax

Formula to get number

[value formula]

$$(-1)^{s} \times 0.M \times 2^{e}$$

$$(-1)^s \times 0.M \times 2^{E-bias}$$



### Implicit Normalized

Syntax

Formula to get number [value formula]

$$(-1)^s \times 1.M \times 2^e$$

$$(-1)^S \times 1.M \times 2^{E-bias}$$

### **Explicit**



0.1 After the point,

Immediate first bit should be 1

$$0.10111 \times 2^{3}$$
  $e = +3$ 

$$M = 10111$$
,

$$e = 3$$

$$E = e + bias$$

## Implicit



Before the point 1 means 1 ......

### Example

$$1.01111 \times 2^{2}$$

$$M = 0111$$
,

$$e = 2$$

$$E = e + bias$$

# Floating-Point Representation

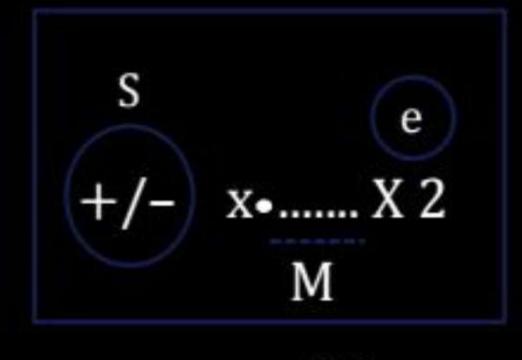


S: sign bit 
$$0 + ve$$
 $1 - ve$ 

E:Biased exponent

M: Mantissa

$$E = e + bias$$
  
or  
 $BE = AE + bias$ 



Explicit

O. Innn

Implicit

I.v.



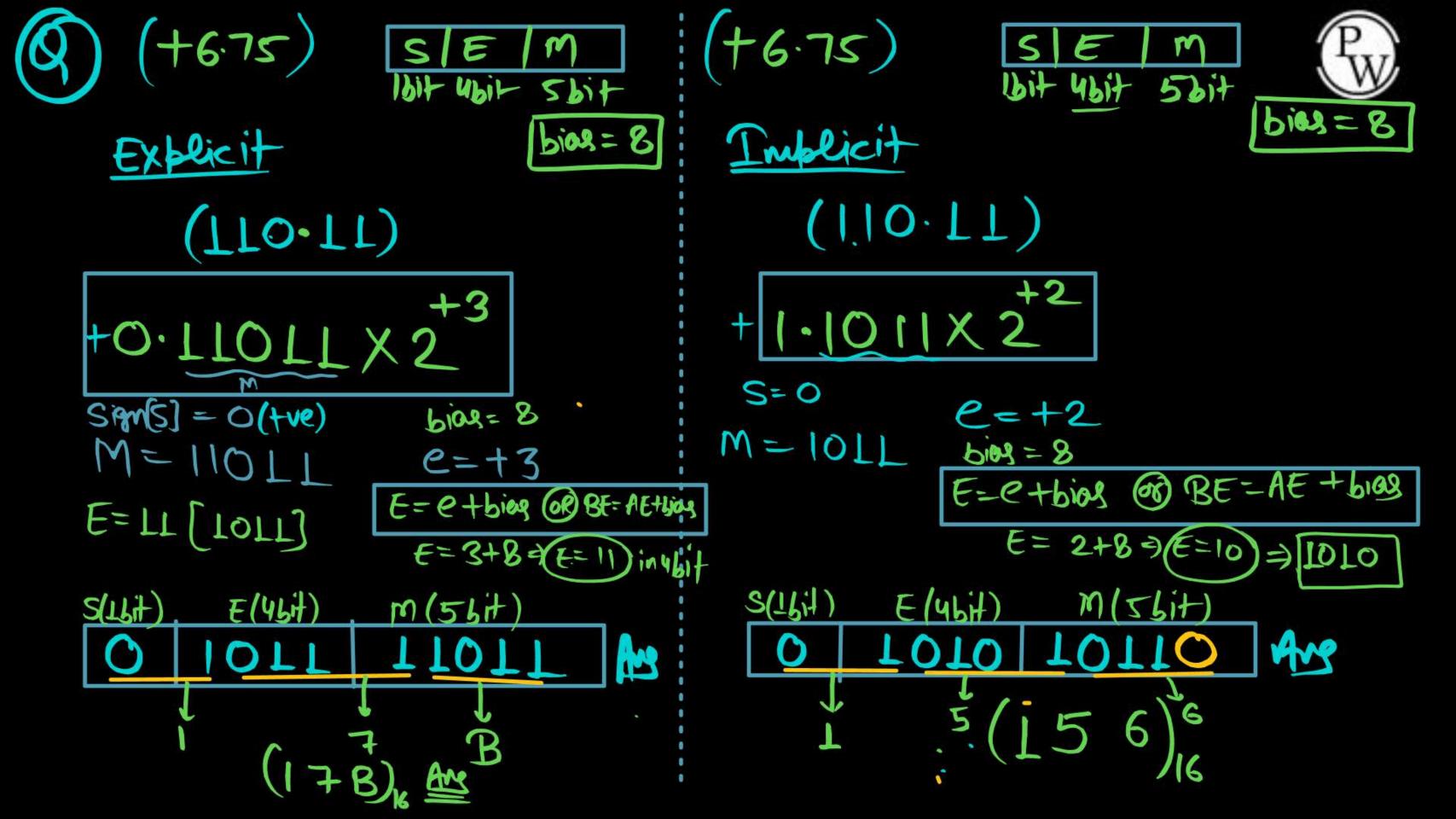
Q. 1 +(6.75) format

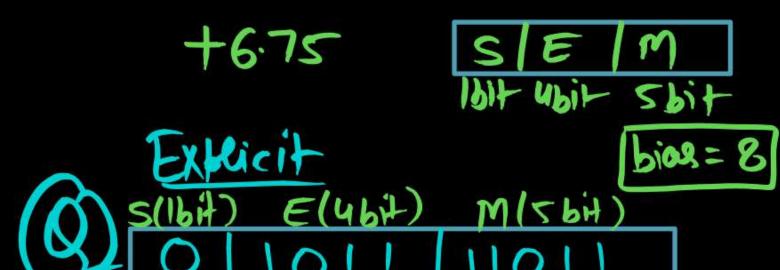
ib exponent iskbit

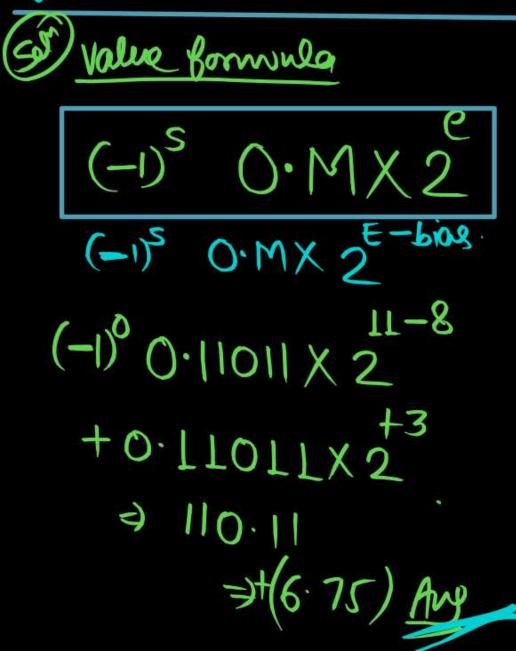
Then do explicit and implicit normalization

$$biss = 2$$

$$bios = 2^3$$

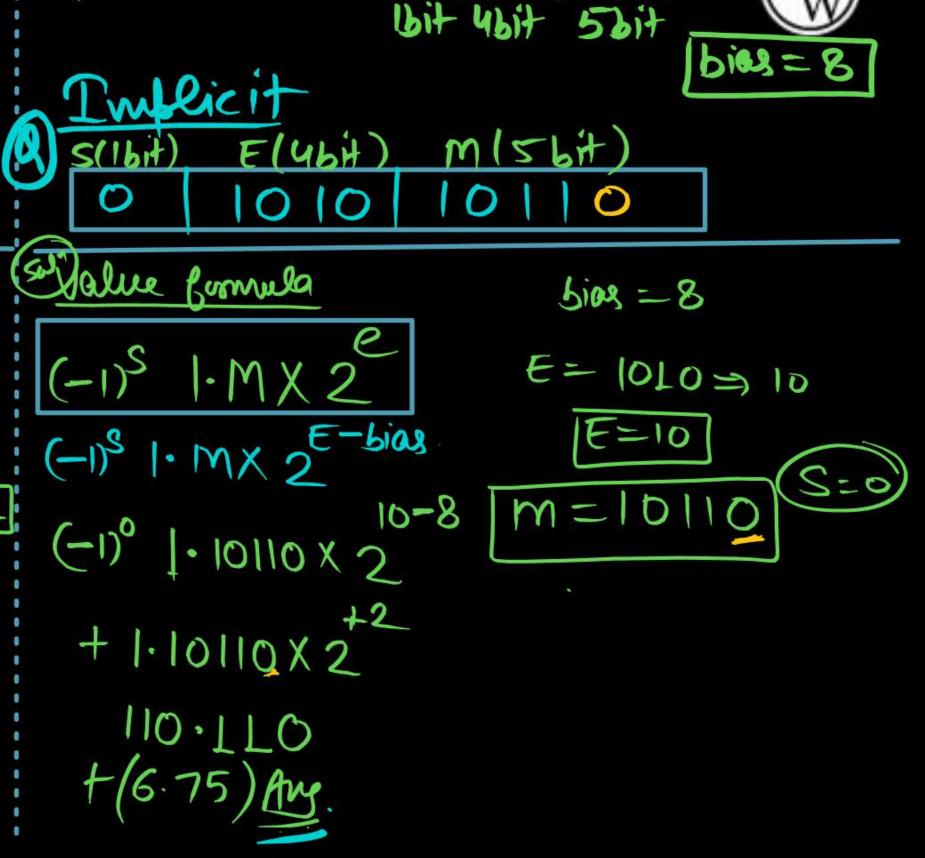


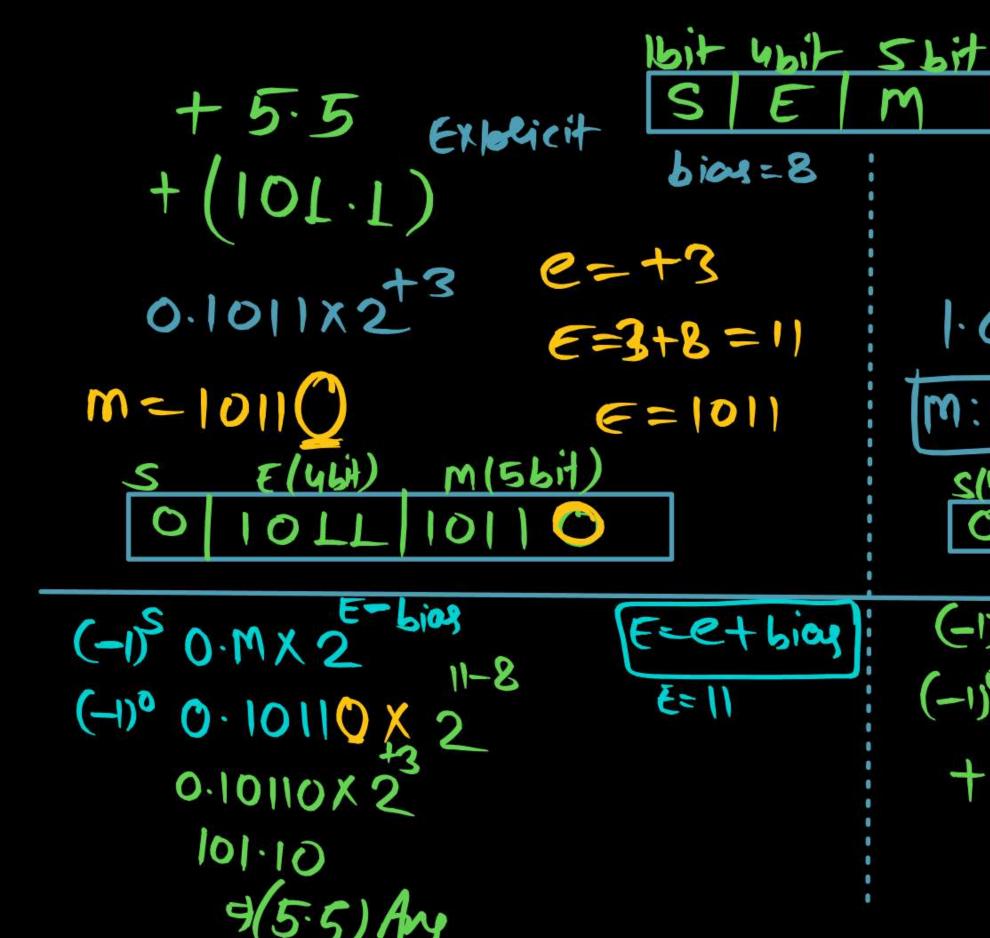


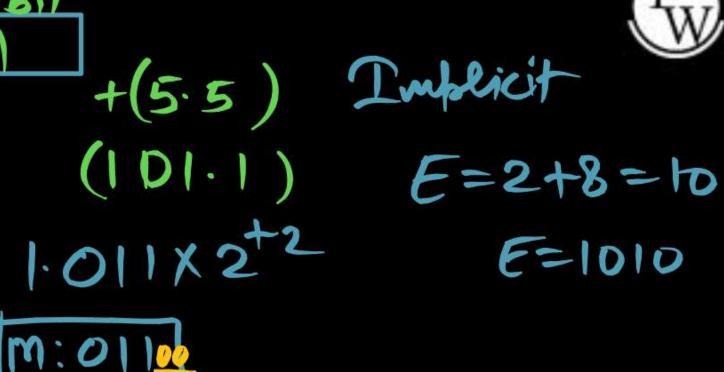


```
bios - 8
E= 1017 = E=11
S=0, M=11011
```

+6.75







M (56H)

S(16ir) E(46it)

(-1)° 1.01100 x2. +1.01100 x2+2 101.100 > 165.5) Ang S E M H- 1. ARANX 2 S(1(-ve) Implicit

E=e+bias (-1)<sup>S</sup> 1·M X2

BE = AE + bias

(-1)S 1. MX 2 F-bias

4.875

### +(4.875) format



### Then do explicit and implicit normalization

### Explicit

(+4.875)

100.111

 $0.100111 \times 2^{3}$ 

M = 100111

e = 3, bias =  $2^{4-1}$ 

E = 3 + 8

E = 11

E = 1011

1 bit	4 bit	5 bit
0	1011	10011

Value Formula:  $(-1)^S \times 0.M \times 2^e$ 

 $(-1)^0 \times 0.10011 \times 2^{11-8}$ 

 $0.10011 \times 2^3$ 

100.11

4.75

(Not getting very accurate)

### Implicit

$$(+4.875)$$

100.111

$$1.00111 \times 2^{2}$$

$$M = 00111$$

$$e = 2$$
, bias =  $2^{4-1}$ 

$$E = 2 + 8$$

$$E = 10$$

$$E = 1010$$





### Value Formula: $(-1)^S \times 1.M \times 2^e$

$$(-1)^0 \times 1.00111 \times 2^{10-8}$$

$$1.00111 \times 2^{2}$$

(Getting very accurate)



Cither Increase the bits in Montissa

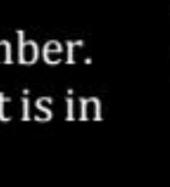
Using Implicit Normalization.

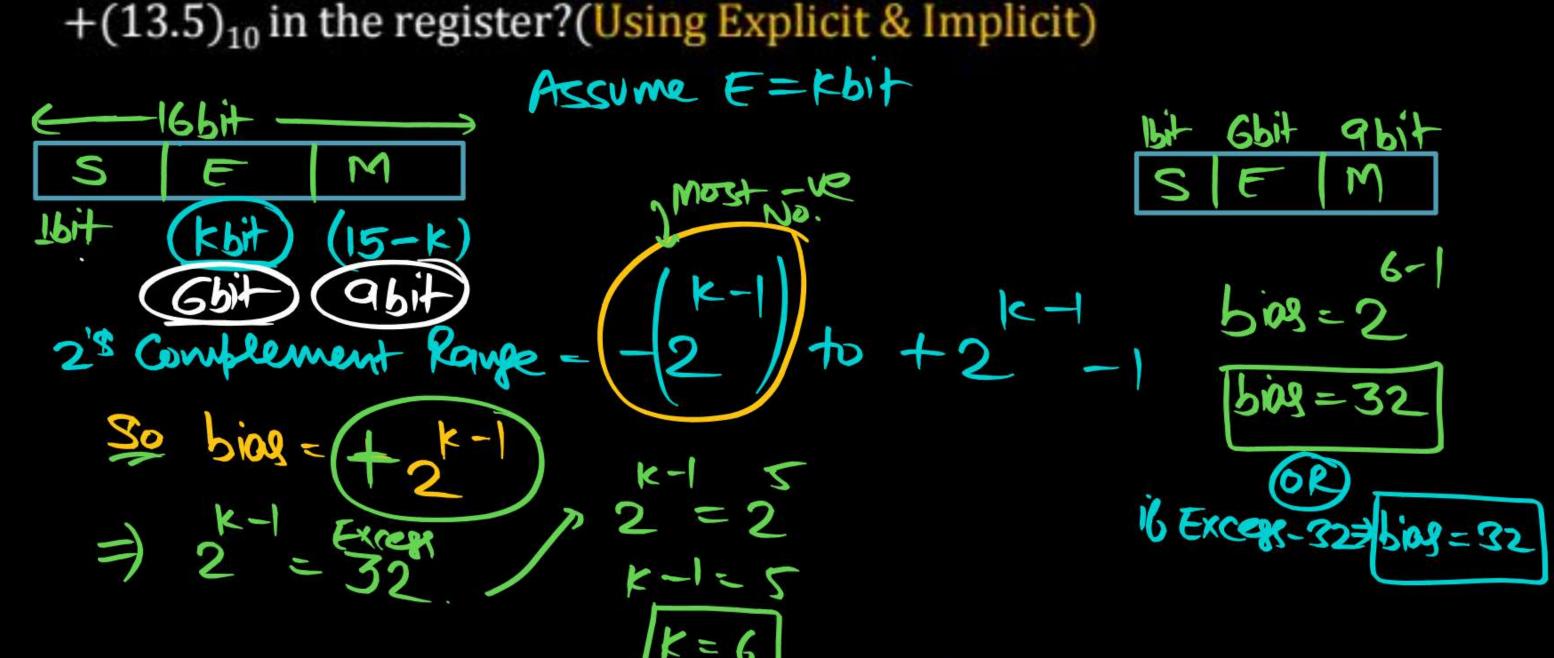
Mantisso: Givier Becision. (More & More bit in Mantissa.

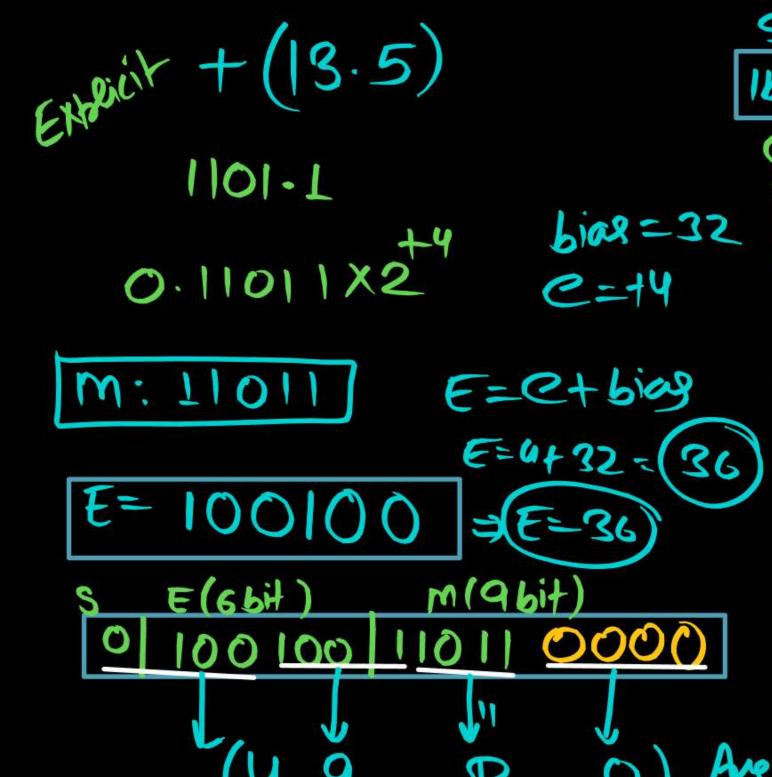
Typoment: gives the Roye.

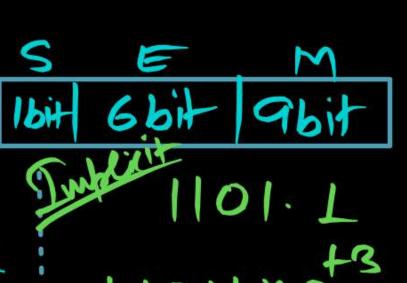
(More bit in Exponent means large larger Number).

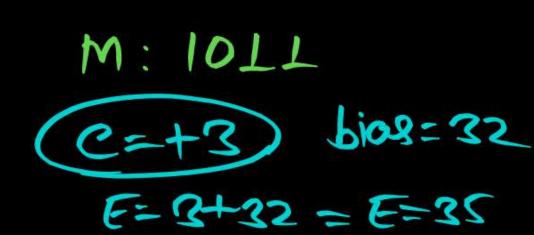
Consider a 16 bit register used to store floating point number. Mantissa is normalized signed fraction number. Exponent is in Excess-32 form then what is 16-bit for











lias=32

m: 1011

E= 100011

```
m (9bit)
E(Gbit)
          101100000
```

E=361 ENDRICH 100100 11011 0000 (-1) S O.M X 2 (-1° 0.110110000 X 2 0.110110000 X 2+4 1101. 7000 (13.5) Any

Implict 00011 bios=32 (-1)S 1.M X 2 32-35 (-1)° 1-101100000X2 1.101100000X2+3 1101.100000 (13.5) Am

Q. +21.75 1 bit 7 bit Implicit? S E

Pw

10101.11

$$1.0101111 \times 2^4$$

$$M = 010111$$

$$e = 4$$
, bias =  $2^{7-1}$ 

$$E = 4 + 64$$

$$E = 68 = (1000100)_2$$

#### Value Formula:

8 bit

M

$$(-1)^{S} \times 1.M \times 2^{e}$$

$$(-1)^0 \times 1.0101111 \times 2^{68-64}$$

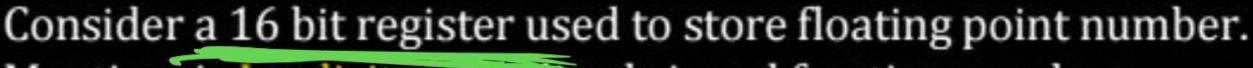
$$1.0101111 \times 2^{4}$$

$$10101.11 = (21.75)_{10}$$

Ans

S(1bit)	E(7bit)	M(8 bit)
0	1000100	01011100

Hexadecimal = 
$$(445C)_{16}$$





Mantissa is Implicit normalized signed fraction number. Exponent is in Excess-64 form then

(i) what is the First Smallest Positive number?

(ii) what is the Second Smallest Positive number?

(iii) what is the Difference between First Smallest & Second Smallest Positive number?

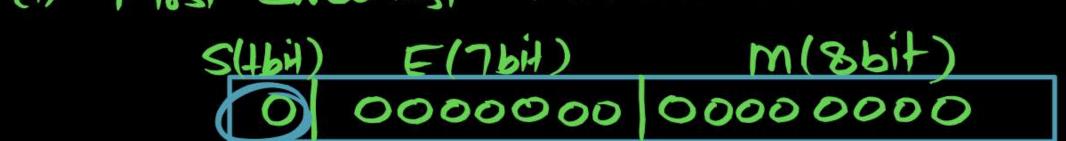
$$\frac{k-1}{2} = 2^{6} [64]$$

## We Can not Represent '0' in cither.

(i) Implict 1. nnn) => 1.0 (Not zero)

(ii) Explict 0.1.n.r... => 0.1 (Not Zerro)

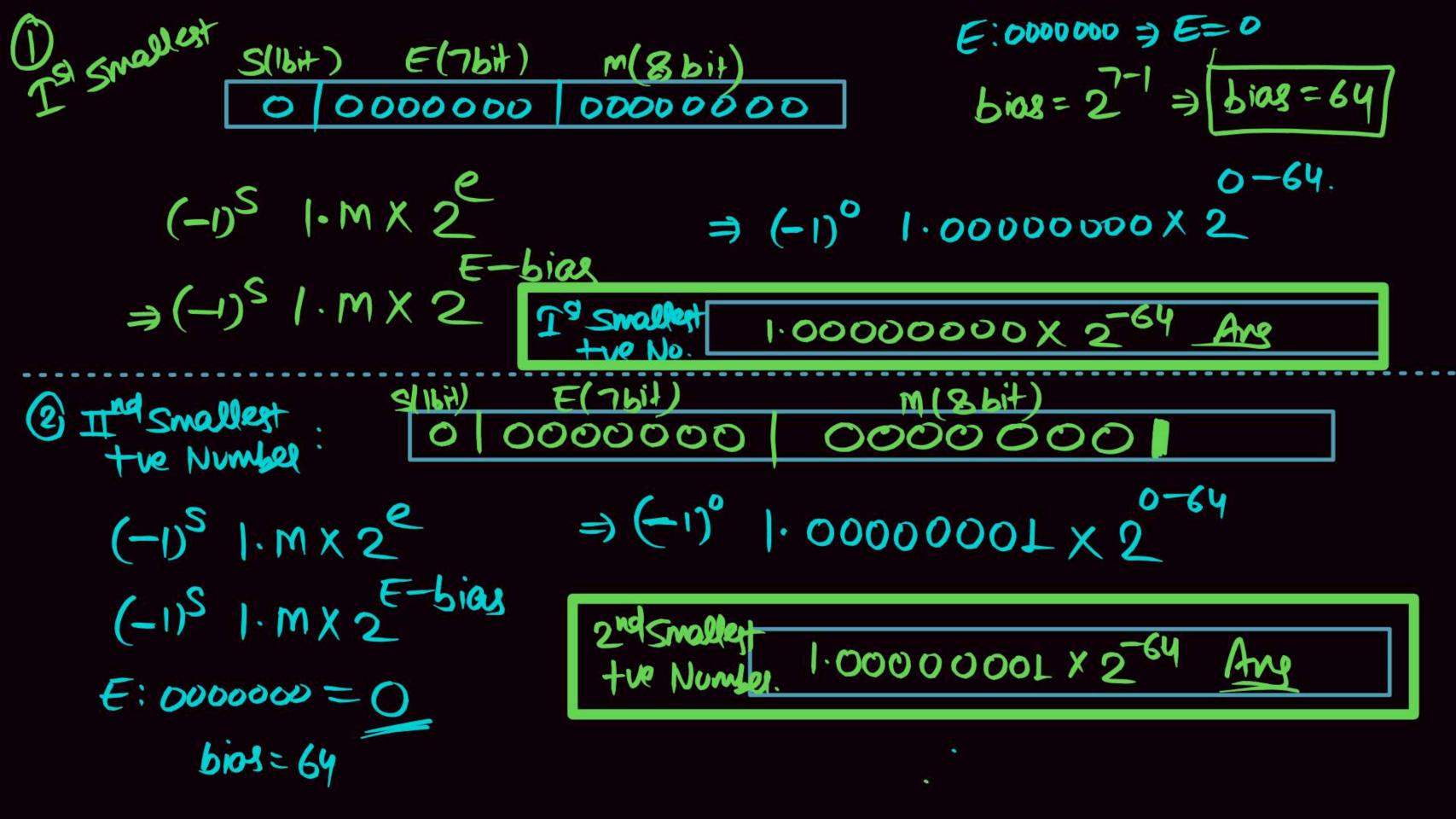




(100 In the explicit Representation we connot Represent '0'
Because 0.1....

(Note) In the Implicit we can not worte/Represent any O Because 1.0 required "So we use Single Precision of Double Presion IFTE 754 Formation

But there for first smallest we but all 0's in Ed Mantissa. Here exactly value is Not Zeeno (10.....)





1665 Ory

(3) Dilbertence the Ist smallest 4 2nd smallest.

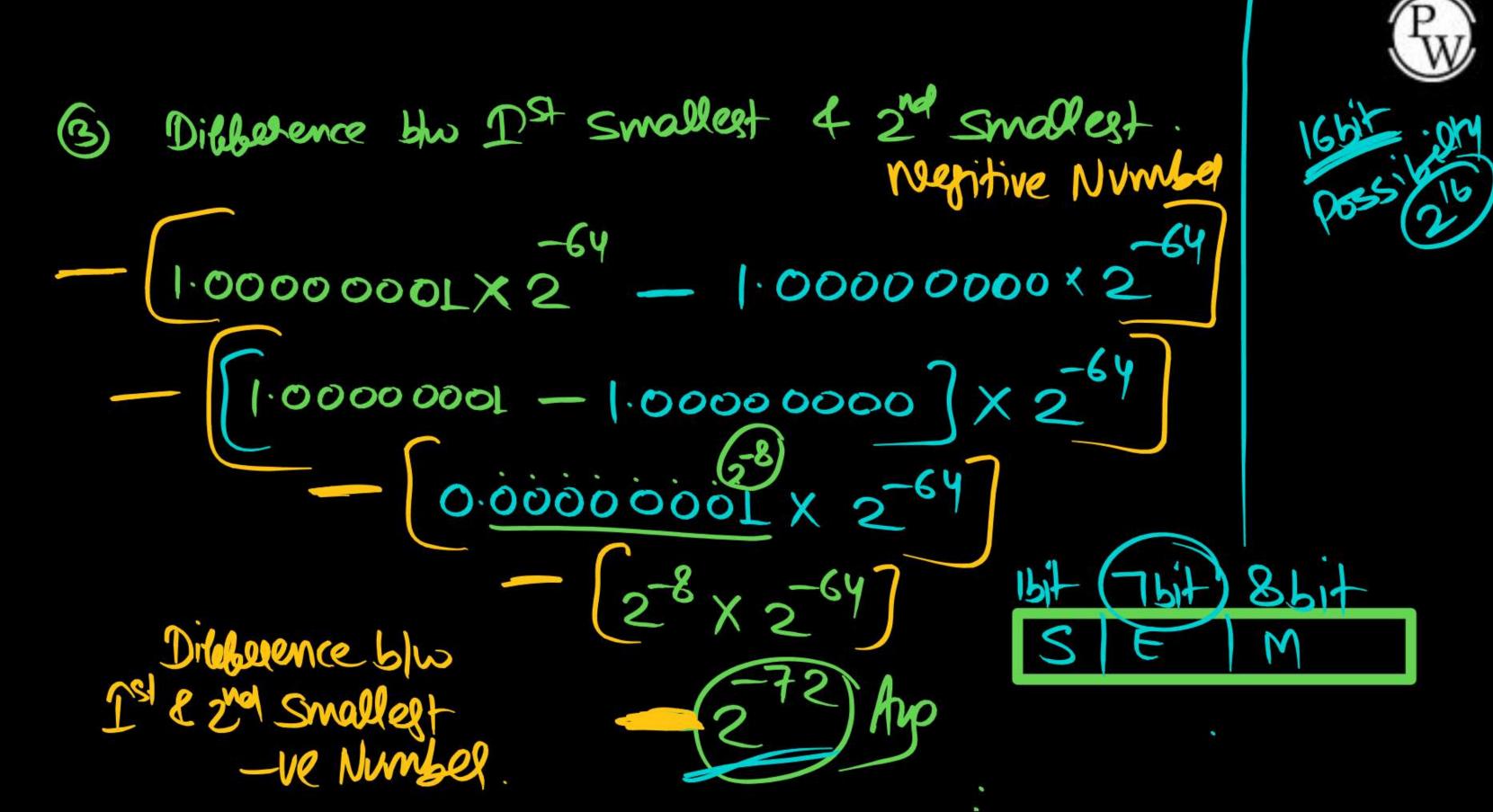
1.0000000LX2 - 1.00000000 × 2

 $[1.000000001 - 1.000000000] \times 2^{-64}$ 

2-8 X 2-64

Positive Di66 (272) App

15t (Tbit) 8bit SEM We want the Answer Bro Neghive Number then PLZ (P.T.O)





Consider a 16 bit register used to store floating point number. Mantissa is Implicit normalized signed fraction number.



Exponent is in Excess-64 form then

- (i) what is the First Highest Positive number?
- (ii) what is the Second Highest Positive number?
- (iii) what is the Difference between First Highest & Second

Highest Positive number?

(Jbit) E(7bit) m(8bit)
0 | 111111 | 1111 | 1

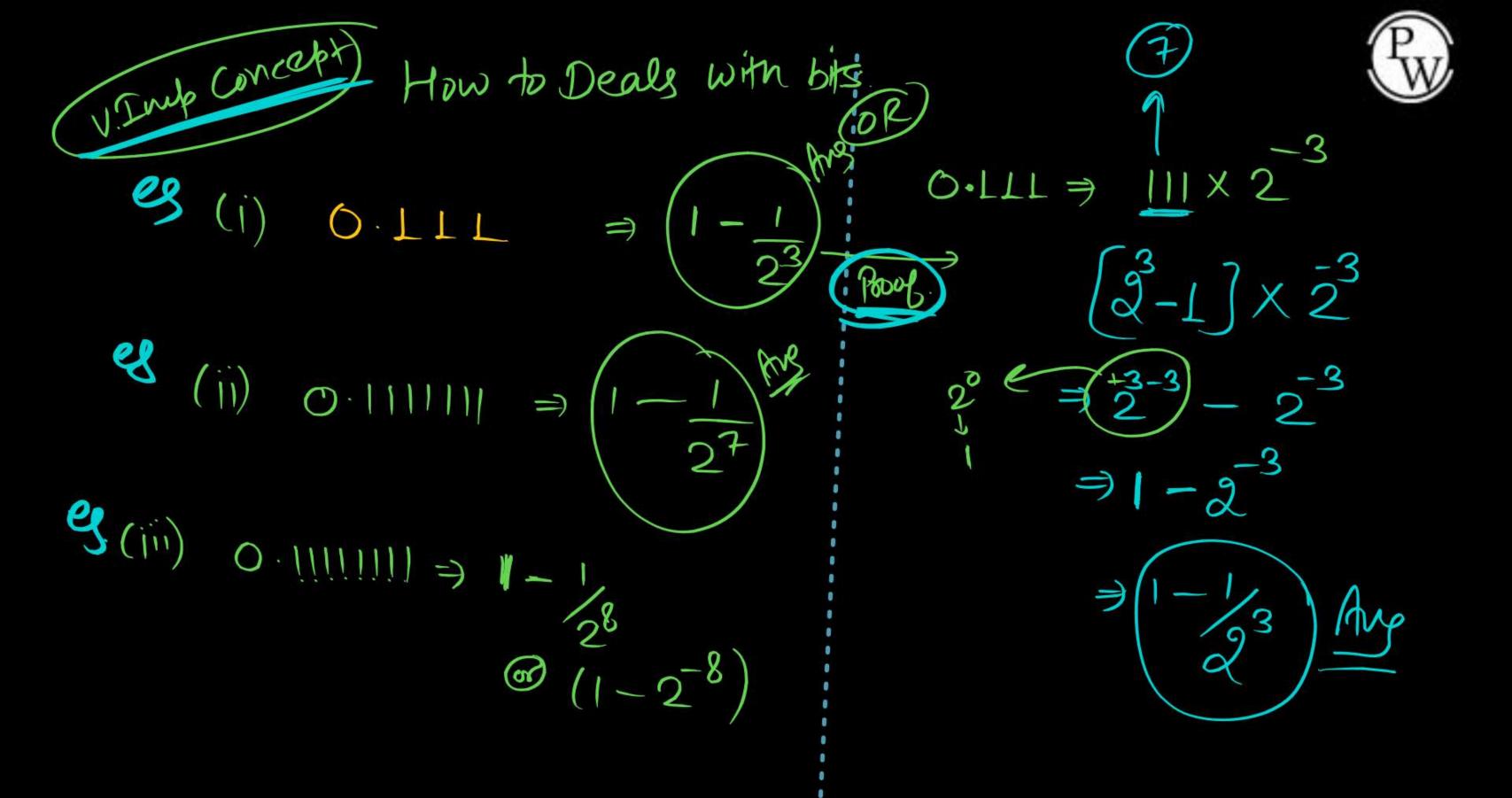
Pw

2nd Highest

Ist Nignest

S(4) E(76) M(86)
0 1111111 11110

Home work



G.P. Series 
$$a = \frac{1}{2}$$

$$\sigma = \frac{1}{2}$$

$$\frac{a(1-r)}{1-r}$$

$$\frac{b}{b}(1-b)$$

$$\frac{b}{b}(1-b)$$

$$\frac{b}{b}(1-b)$$

$$\frac{b}{b}(1-b)$$

$$\frac{b}{b}(1-b)$$

$$153(2^{4}-1)$$
 $153(2^{4}-1)$ 
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12 /2 /3 /24 G.P Series a (1-8") 12 (1-1/4) \* x (1-/34)



Consider a 16 bit register used to store floating point number. Mantissa is Explicit normalized signed fraction number. Exponent is in Excess-32 form then what is 16-bit for –(29.75)<sub>10</sub> in the register?



## Solution

1 bit	6 bit	9 bit
S	Е	M

-29.75

-11101.11

 $0.11101111 \times 2^{5}$ 

M: 1110111

e = 5

bias =  $2^{6-1}$ 

bias = 32

 $E = 5 + 32 = 37 = (100101)_2$ 

S(1 bit)

E(6 bit)

M(9 bit)

1 100101 111011100



Q. +21.75 1 bit 7 bit Implicit? S E

Pw

10101.11

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8 bit

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Ans

S(1bit)	E(7bit)	M(8 bit)
0	1000100	01011100

 $Hexadecimal = (445C)_{16}$ 





