CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-2

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TOPICS TO BE COVERED

Hamming distance

Hamming Distance

$\oplus \rightarrow$ EX-OR $\rightarrow$ Mod 2 sum or Mod 2 Addition

$$0 \oplus 1 = 1$$

$$0 \oplus 0 = 0$$

$$1 \oplus 0 = 1$$

$$1 \oplus 1 = 0$$

$$1 \oplus 0 \oplus 1 = 0$$

$$1 \oplus 1 \oplus 1 = 1$$

$$0 \oplus 1 = 1$$

Hamming Distance :

Hamming distance between two Binary string of same size is the number of differences between corresponding bits.

Hamming distance between two Binary string is denoted by d(x, y)

sent Rcvd

1. d(000, 011) = 2 (Hamming distance)
2. d(100, 011) = 3 (Hamming distance)
3. d(10101, 11110) = 3 (Hamming distance)

Shortcut

③ 10101
EX-OR
11110
01011 → No. OF 1's = 3 → Hamming distance

Hamming distance can easily be found if we apply XOR operation (⊕) on the two words and count the number of 1's in the result.

minimum
Hamming Distance :

In a set of codewords, the minimum Hamming distance is the smallest Hamming distance between all possible pairs of code words.

Valid code word

(a)	0	1	0
(b)	1	0	1
(c)	1	1	0
(d)	0	0	1

$$\left.\begin{aligned} d(a,b) &= 3 \\ d(a,c) &= 1 \\ d(a,d) &= 2 \\ d(b,c) &= 2 \\ d(b,d) &= 1 \\ d(c,d) &= 3 \end{aligned}\right\}$$ minimum Hamming distance $= 1$

Minimum Hamming distance for Error detection :

① Sent: 010 → Rcvd: 110 (1 bit error)

Valid code word

Receiver can't detect one bit error

② 010 → 011 (1 bit error)

Invaid code word

Receiver can detect one bit error

Note: All one bit error can not be detected

min
H.D = 1

Valid code word —— 1 bit error —— Valid code word

Receiver can't detect all one bit error

Ex2 :

Valid code word

(a) 0 0 0

(b) 0 1 1

(c) 1 0 1

(d) 1 1 0

$$\left.\begin{aligned} d(a,b) &= 2 \\ d(a,c) &= 2 \\ d(a,d) &= 2 \\ d(b,c) &= 2 \\ d(b,d) &= 2 \\ d(c,d) &= 2 \end{aligned}\right\}$$ minimum Hamming distance = 2

sent		Rcvd

① $\boxed{000}$ $\xrightarrow{\text{1 bit error}}$ $\boxed{100}$

Invaid code word

Receiver can detect one bit error

(a) $000 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 100 \\ 010 \\ 001 \end{matrix}\right\}$ Invaid code word

(b) $011 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 111 \\ 001 \\ 010 \end{matrix}\right\}$ Invalid code word

Note: All one bit error detected

(c) $101 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 001 \\ 111 \\ 100 \end{matrix}\right\}$ Invalid code word

(d) $110 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 010 \\ 100 \\ 111 \end{matrix}\right\}$ Invalid code word

2. Sent $\boxed{0\ 0\ 0}$ — 2 bit error → Rcvd $\boxed{1\ 1\ 0}$

Valid code word

Receiver can not detect 2 bit error

Note: All 2 bit error can't be detected

Min H·D = 2

Valid code word —(1 bit error)— Invalid code word —(1 bit error)— Valid code word

Receiver can detect all one bit error

but

Receiver can't detect 2 bit error

Ex3 :

Valid code word

(a) 0 0 0 0 0

(b) 0 1 0 1 1

(c) 1 0 1 0 1

(d) 1 1 1 1 0

$d(a,b) = 3$

$d(a,c) = 3$

$d(a,d) = 4$

$d(b,c) = 4$

$d(b,d) = 3$

$d(c,d) = 3$

minimum Hamming distance = 3

① sent $\boxed{00000}$ —1 bit error→ Rcvd $\boxed{10000}$ Invalid code word

Receiver can detect one bit error

(a) 00000 $\xrightarrow{\text{1 bit error}}$

$$\left.\begin{array}{l}10000\\01000\\00100\\00010\\00001\end{array}\right\}\text{Invalid code word}$$

Note: All one bit error detected

② Sent $\boxed{00000}$ —— 2 bit error ——> Rcvd $\boxed{11000}$

Invalid code word

Receiver can detect 2 bit error

(a) $00000 \xrightarrow{\text{2 bit error}}$

11000
01100
00110
00011
⋮

} Invalid code word

Note: All 2 bit error detected

3. $\boxed{00000}$ 3 bit error $\boxed{11100}$

Invalid code word

Receiver can detect 3 bit error

4. $\boxed{00000}$ 3 bit error $\boxed{01011}$

Valid code word

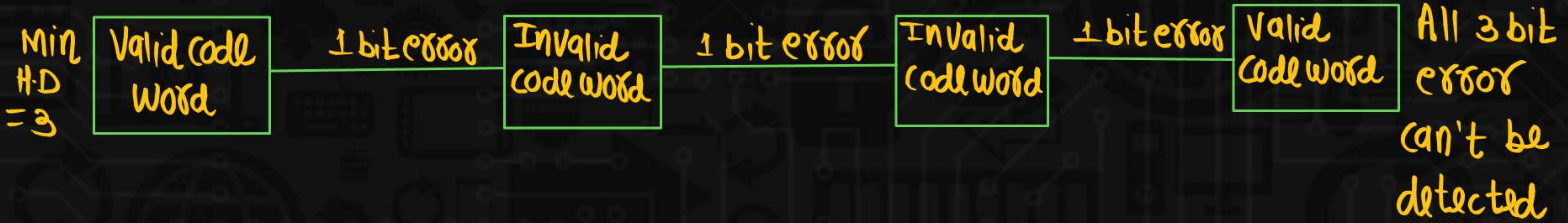
Receiver can't detect 3 bit error

Note: All 3 bit error can not be detected.

Note

If minimum Hamming distance = 3

- All one bit error deted
- All two bit error detected
- All three bit error can not be detected



Note: 1 If minimum Hamming distance = 'd'
So we can detect upto (d-1) bit error

2 minimum Hamming distance required to detect 'd' bit error = d+1

Linear Block codes :

- A Linear block code is a code in which the XOR ($\oplus$) of two valid code words create another valid code word.
- Today all most all error detecting codes are linear block codes: Non Liner block codes are difficult to implement.
- It is simple to find the minimum Hamming distance for linear block code the minimum Hamming distance is the number of 1's in a Non zero valid code word with the smallest Number of 1's

Ex1 :

Valid code word

(a) 0 0 0

(b) 0 1 1

(c) 1 0 1

(d) 1 1 0

XOR (a, b) = 011 (valid code word)

XOR (a, c) = 101 (valid code word)

XOR (a, d) = 110 (valid code word)

XOR (b, c) = 110 (valid code word)

XOR (b, d) = 101 (valid code word)

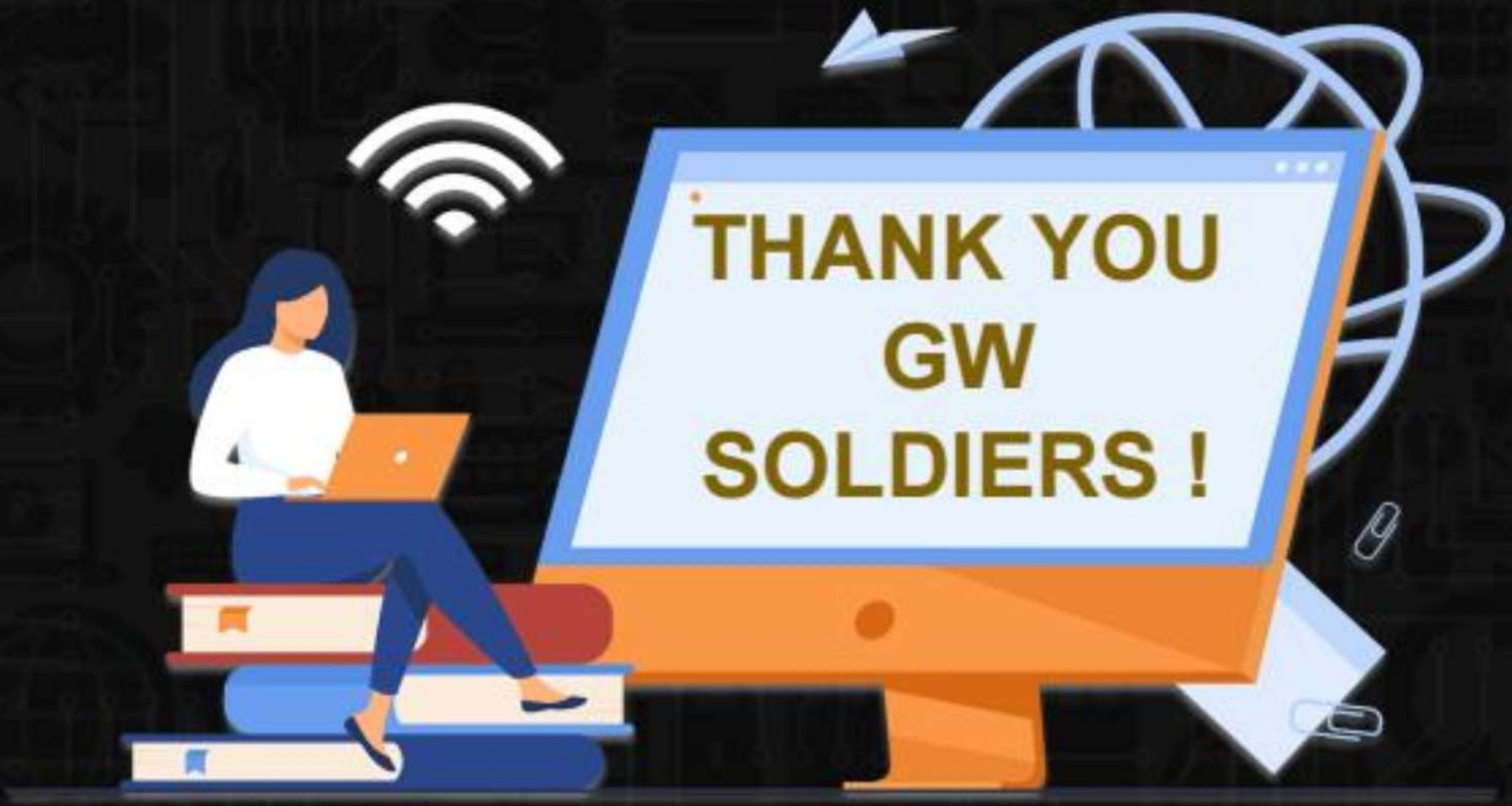
XOR (c, d) = 011 (valid code word)

So above code word is Liner block code.

Min Hamming distance = 2 (min. no. of 1's in the non zero code word)

Assume this is Linear Block code

$$\text{Non zero code}\begin{array}{l} \times\ 000 \\ \left\{\begin{array}{l} 001 \\ 011 \\ 100 \\ 111 \end{array}\right\} \end{array} \text{min No. of 1's} = 1 \longrightarrow \text{Hamming distance} = 1$$


THANK YOU
GW
SOLDIERS !