COMPUTER SCIENCE

Database Management System

FD's & Normalization

Properties of Decomposition

part - 1

Lecture_07

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TOPICS TO BE COVERED

01 Lossy and Lossless Join

02 Dependency preserving

RDBMS Concept

FD Concept & its type

Attribute closure $\{X\}^+$

Super key

Candidate key

Finding Multiple C.K

Membership set

✓ Equality b/w 2 FD set

✓ Minimal Cover.

Minimal Cover.

Step1 : R.H.S contain single Attribute.

Step2 : L.H.S find extra Attribute

Step3 : Find Redundant FD & Delete them.

$$[AB]^+ = [AB \ldots\ldots]$$

Closure of Attribute $(X)^+$

All possible Attributes

$R(ABCDE)$ $\{A \to B,\ B \to C,\ C \to D,\ D \to E\}$

$(A)^+ = \{ABCDE\}$

$(B)^+ = \{BCDE\}$

$(C)^+ = \{CDE\}$

$(D)^+ = \{DE\}$

$(E)^+ = \{E\}$

Closure of FD Set $[F]^+$

Set of all possible FD which can be derived from given FD Set.

$\boxed{R(AB)}$

Component

n

$\boxed{(n-1)\times n+1}$

$3\times 4+1=13$

ϕ
A
B
AB

$\phi \rightarrow \phi$

$\phi \rightarrow A$
$\phi \rightarrow B$
$\phi \rightarrow AB$

$A \rightarrow \phi$
$A \rightarrow A$
$A \rightarrow B$
$A \rightarrow AB$

$B \rightarrow \phi$
$B \rightarrow A$
$B \rightarrow B$
$B \rightarrow AB$

$AB \rightarrow \phi$
$AB \rightarrow A$
$AB \rightarrow B$
$AB \rightarrow AB$

Closure of FD Set $[F]^+$

Set of all possible FD's which can be derived from given FD set is called closure of FD set. $[F]^+$

$[F]^+$ Closure of FD

R(AB)

$\phi \rightarrow \phi$

ϕ	$A \rightarrow \phi$	$B \rightarrow \phi$	$AB \rightarrow \phi$
A	$A \rightarrow A$	$B \rightarrow A$	$AB \rightarrow A$
B	$A \rightarrow B$	$B \rightarrow B$	$AB \rightarrow B$
AB	$A \rightarrow AB$	$B \rightarrow AB$	$AB \rightarrow AB$

$[F]^+ = 13$ Ans

(16 No FD Set is given)

$R(ABC)$

- ϕ $\qquad \phi \to \phi$
- A
- B
- C
- AB
- BC
- AC
- ABC

$A \to \phi$

$A \to A$

$A \to B$

$A \to C$

$A \to AB$

$A \to BC$

$A \to AC$

$A \to ABC$

$R(ABC)$

then $n = 8$ Component

$(n-1) \times n + 1$

$7 \times 8 + 1 = 57$

2^3

R(ABC)

$\phi \to \phi$

$7 \times 8 + 1 = 57$ FD's

Total(F)

57 Ans

ϕ	$A \to \phi$	$B \to \phi$	$C \to \phi$	$AB \to \phi$	$BC \to \phi$	$AC \to \phi$	$ABC \to \phi$
A	$A \to A$	$B \to A$	$C \to A$	$AB \to A$	$BC \to A$	$AC \to A$	$ABC \to A$
B	$A \to B$	$B \to B$	$C \to B$	$AB \to B$	$BC \to B$	$AC \to B$	$ABC \to B$
C	$A \to C$	$B \to C$	$C \to C$	$AB \to C$	$BC \to C$	$AC \to C$	$ABC \to C$
AB	$A \to AB$	$B \to AB$	$C \to AB$	$AB \to AB$	$BC \to AB$	$AC \to AB$	$ABC \to AB$
BC	$A \to BC$	$B \to AC$	$C \to BC$	$AB \to BC$	$BC \to BC$	$AC \to BC$	$ABC \to BC$
AC	$A \to AC$	$B \to BC$	$C \to AC$	$AB \to AC$	$BC \to AC$	$AC \to AC$	$ABC \to AC$
ABC	$A \to ABC$	$B \to ABC$	$C \to ABC$	$AB \to ABC$	$BC \to ABC$	$AC \to ABC$	$ABC \to ABC$

Case I: Note: This is the case when Any FD is Not given & We Find $(F)^+$

Case II: When FD set is given then Find $[F]^+$ closure of FD set.

$R(AB)$

find $(F)^+ = 13$ Ans

ϕ

A

B

AB

$3 \times 4 + 1 = 13$

$\phi \to \phi$

$A \to \phi$	$B \to \phi$	$AB \to \phi$
$A \to A$	$B \to A$	$AB \to A$
$A \to B$	$B \to B$	$AB \to B$
$A \to AB$	$B \to AB$	$AB \to AB$

Case II : when FD set is given then find $[F]^+$

$R(ABC)$ $[A \to B, B \to C]$ $[F]^+ = 43$ Ans.

ϕ 0 attribute = $\phi \to \phi$

A 1Attribute = $[A]^+ = [ABC] = 2^3 = 8FD$

$A \to \phi$, $A \to A$, $A \to B$, $A \to C$
$A \to AB$, $A \to BC$, $A \to AC$, $A \to ABC$

B $[B]^+ = [BC] = 2^2 = 4FD$ $[B \to \phi, B \to B, B \to C, B \to BC]$

C $[C]^+ = [C] = 2^1 = 2FD$ $[C \to \phi, C \to C]$

AB 2Attribute = $[AB]^+ = [ABC] = 2^3 = 8FD$ $[AB \to \phi, AB \to A, AB \to B, AB \to C, AB \to AB, AB \to BC, AB \to AC, AB \to ABC]$

BC $[BC]^+ = [BC] = 2^2 = 4FD$ $(BC \to \phi, BC \to B, BC \to C, BC \to BC)$

AC $[AC]^+ = [ABC] = 2^3 = 8FD$ $(AC \to \phi, AC \to A, AC \to B, AC \to C, AC \to AB, AC \to BC, AC \to AC, AC \to ABC)$

ABC 3 Attribute= $[ABC]^+ = [ABC] = 2^3$ $8FD$ $[ABC \to \phi, ABC \to A, ABC \to B, ABC \to C, ABC \to AB, ABC \to BC, ABC \to AC, ABC \to ABC]$

$$[A]^+ = \begin{bmatrix} A\to\phi, A\to A, A\to B, A\to C \\ A\to AB, A\to BC, A\to AC, A\to ABC \end{bmatrix}$$

$$[B]^+ = [B \to \phi, B \to B, B \to C, B \to BC]$$

$$[C]^+ = [C \to \phi, C \to C]$$

$$[AB]^+ = \begin{bmatrix} AB\to\phi, AB\to A, AB\to B, AB\to C \\ AB\to AB, AB\to BC, AB\to AC, AB\to ABC \end{bmatrix}$$

43 Ans

$$[BC]^+ = [BC \to \phi, BC \to B, BC \to C, BC \to BC]$$

$$[AC]^+ = \begin{bmatrix} AC\to\phi, AC\to A, AC\to B, AC\to C \\ AC\to AB, AC\to BC, AC\to AC, AC\to ABC \end{bmatrix}$$

$$[ABC]^+ = \begin{bmatrix} ABC\to\phi, ABC\to A, ABC\to B, ABC\to C \\ ABC\to AB, ABC\to BC, ABC\to AC, ABC\to ABC \end{bmatrix}$$

$R(AB)$ $[A \to B]$

$[F]^+$

FD

0 Attribute : $\phi \to \phi$ 1 $\phi \to \phi$

ϕ

A

B

AB

1 Attribute : $(A)^+ = (\underline{AB}) = 2^2$ 4 $[A \to \phi,\ A \to A,\ A \to B,\ A \to AB]$

$(B)^+ = (B) = 2^1$ 2 $[B \to \phi,\ B \to B]$

2 Attribute $(AB)^+ = (AB) = 2^2 =$ 4 $(AB \to \phi,\ AB \to A,\ AB \to B,\ AB \to AB$

11

$(F)^+ = 11$ Ans

$R(AB)\ [A \rightarrow B]$

ϕ	0 attribute = 1		1
A	1 Attribute =	$[A]^+\ [\underline{AB}] = 2^2$	= 4
B		$[B]^+ = [B] = 2^1$	= 2
AB	2 Attribute =	$[AB]^+ = [AB] = 2^2$	= 4
			11

ϕ	1	$\phi \rightarrow \phi$
A	$\underline{4}$	$(A\rightarrow\phi, A\rightarrow A, A\rightarrow B, A\rightarrow AB)$
B	2	$(B\rightarrow\phi, B\rightarrow B)$
AB	4	$\begin{pmatrix} AB\rightarrow\phi, AB\rightarrow A \\ AB\rightarrow B, AB\rightarrow AB \end{pmatrix}$

11 Ans.

Finding Number of super keys

Let R be the Relational Schema with n Attributes $(A_1 A_2 A_3 \ldots\ldots A_n)$

How many Super keys are possible ?

Q. (i) With only Candidate key A_1 ?

Q. (ii) With only Candidate key A_1, A_2 ?

Q. (iii) With Only Candidate key $A_1 A_2$, $A_3 A_4$?

Q. (iv) With Only Candidate key $A_1 A_2$, $A_2 A_3$?

Q. (v) With Only Candidate key A_1, A_2, A_3 ?

Sol^n I) With only candidate key A_1.

Super key: Total n Attribute.

A_1

$A_1 A_2$

$A_1 A_2 A_3$

$A_1 A_2 A_3 A_4$

$A_1 A_2 A_3 A_4 A_5$

⋮

$A_1 \underbrace{A_2 A_3 A_4 A_5 \ldots A_n}_{(n-1)\text{ attribute}}$

$$\boxed{\#\text{ super keys} = 2^{n-1}}$$

n is # of Attribute.

Q) R(ABCD) C.K [A] Total # Super keys?

$$\boxed{\#\text{ Super keys} = 2^{n-1}} \Rightarrow 2^{4-1} = 2^3 = 8 \text{ super key}$$

OR

$R(\underline{A}\underbrace{BCD})$

$\Downarrow$

2^3

$= 8$ Super keys Ans

OR

$R(\underline{A} B C D)$

A	B	C	D	
	0	0	0	→ A
	0	0	1	→ AD
	0	1	0	→ AC
	0	1	1	→ ACD
	1	0	0	→ AB
	1	0	1	→ ABD
	1	1	0	→ ABC
	1	1	1	→ ABCD

8 Super keys.

Solⁿ 2 With only Candidate key A_1, A_2?

A_1

$A_1 A_2$

$A_1 A_2 A_3$

⋮

$A_1 \underbrace{A_2 A_3 A_4 \ldots A_n}_{(n-1)}$

A_2

$A_2 A_1$

$A_2 A_1 A_3$

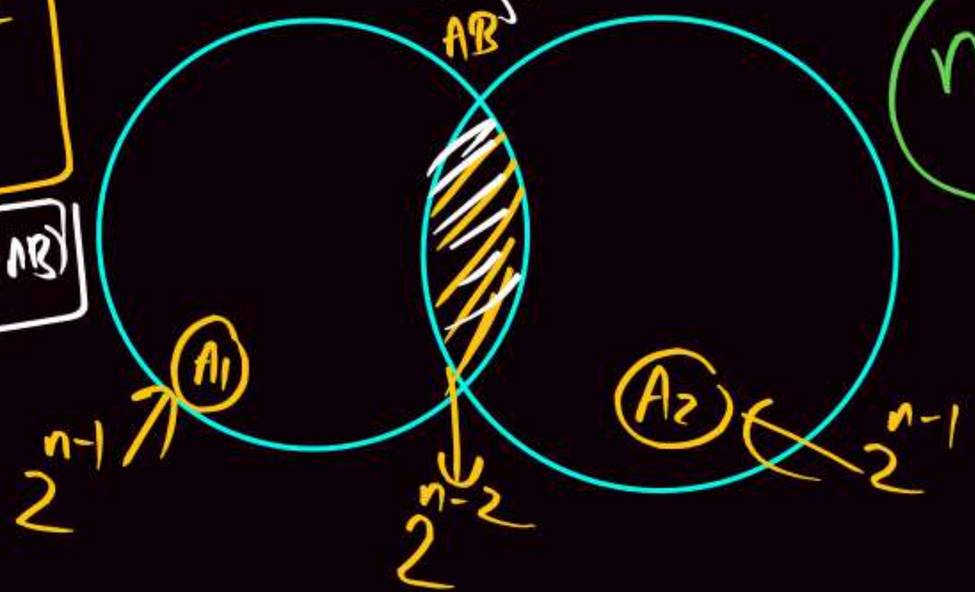
⋮

$A_2 \underbrace{A_1 A_3 \ldots A_n}_{n-1}$

$$\boxed{\#\text{Super keys} \Rightarrow 2^{n-1} + 2^{n-1} - 2^{n-2}}$$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Venn Diagram.



n: # of Attributes

Eg R(ABCDE) c.k [A, C].

Super keys?

$$\#S.k = 2^{n-1} + 2^{n-1} - 2^{n-2}$$

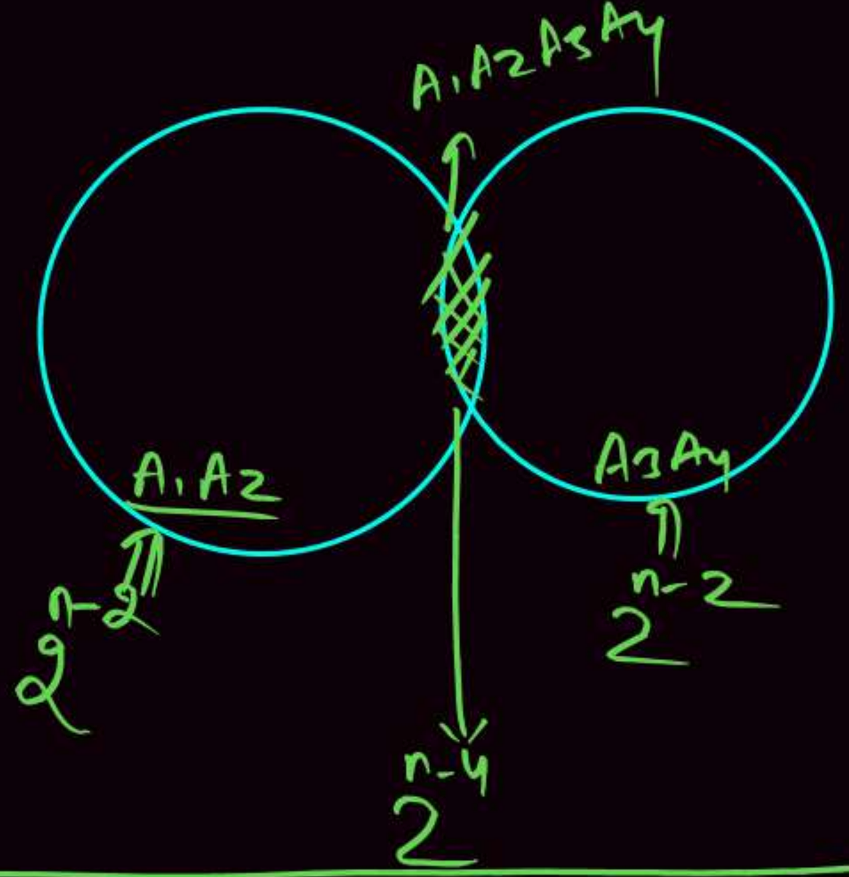
$$\Rightarrow 2^{5-1} + 2^{5-1} - 2^{5-2}$$

$$= 2^4 + 2^4 - 2^3$$

$$= 16 + 16 - 8$$

$$= 24 \text{ Super keys}$$ Ans

(iii) With only Candidate key A_1A_2, A_3A_4 ?



$$\boxed{\#\text{Superkeys} = 2^{n-2} + 2^{n-2} - 2^{n-4}}$$

Q) R(ABCDE) C.K (AB, DE)

\# Super keys ?

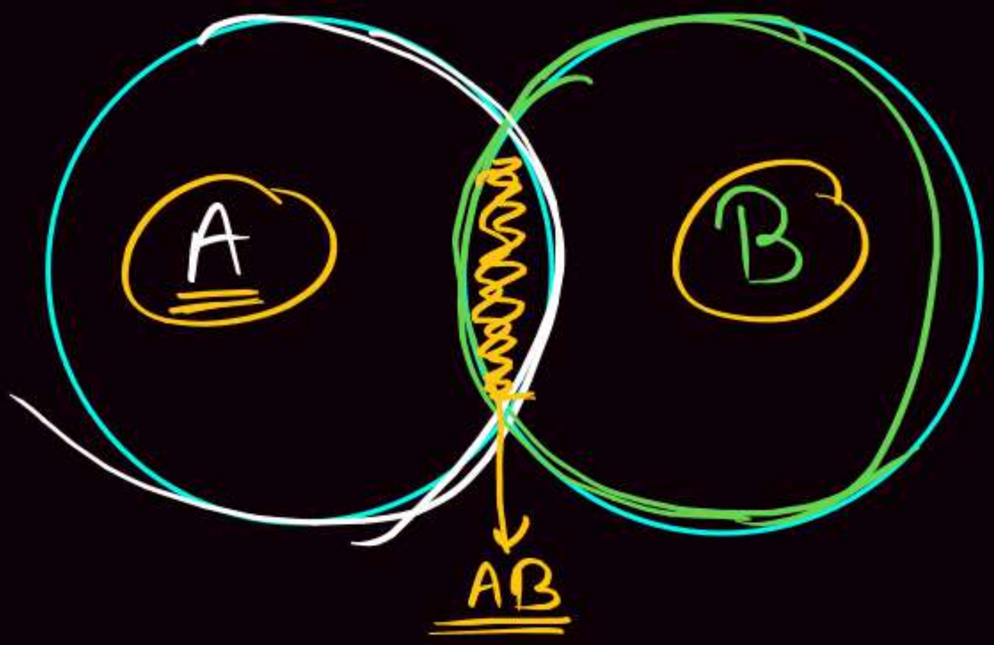
AB, DE

$$\#S.K = 2^{5-2} + 2^{5-2} - 2^{5-4}$$

$$\Rightarrow 2^3 + 2^3 - 2^1$$

$$= 8 + 8 - 2$$

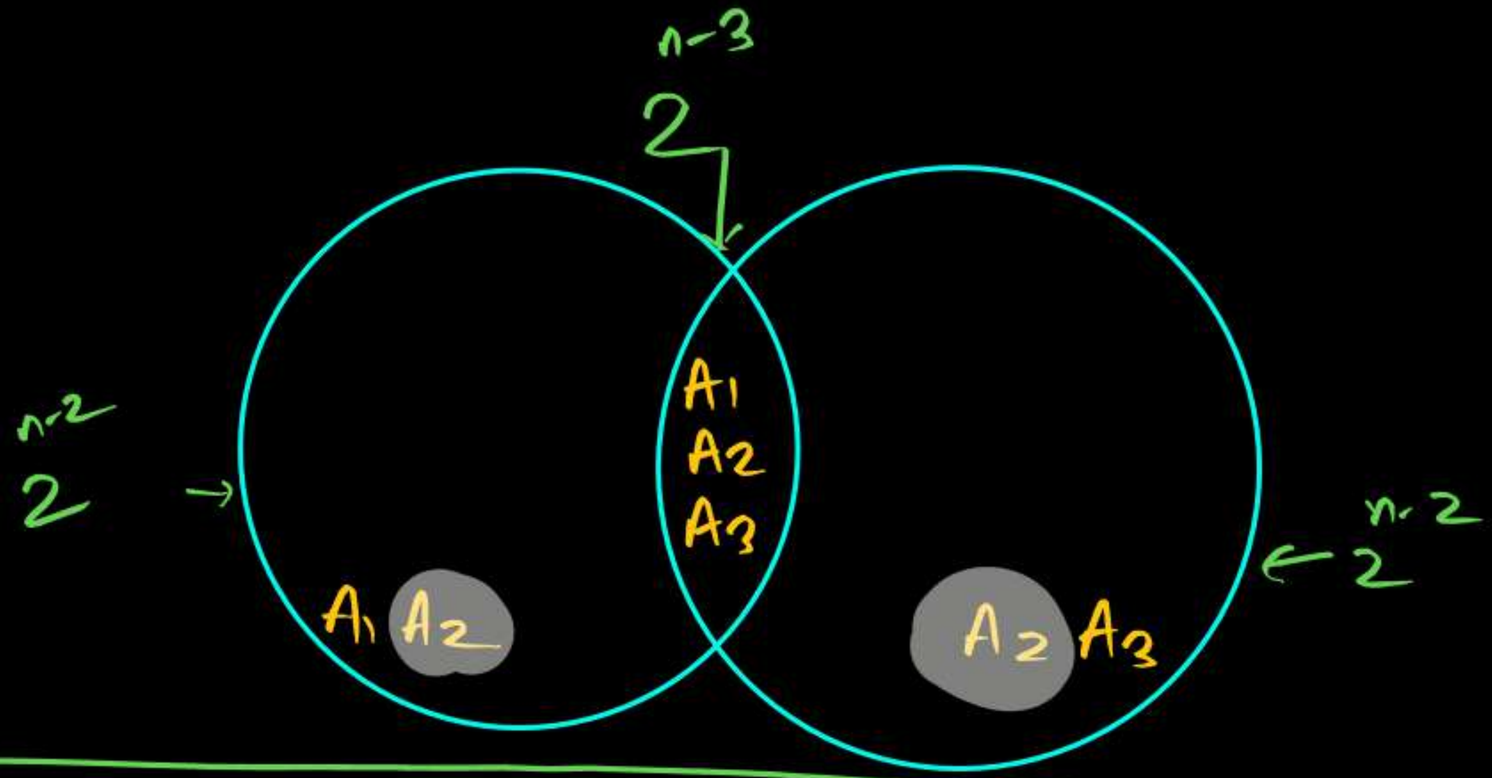
$$= 14 \quad \text{Ans}$$



$$n(A \cup B) = n(A) + n(B) - \underline{n(A \cap B)}$$

$$2^{n-1} + 2^{n-1} - 2^{n-2}$$

(iv) with only candidate key A_1A_2, A_2A_3 ?



$$\boxed{\#\text{super key} = 2^{n-2} + 2^{n-2} - 2^{n-3}}$$

(Q) $R(ABCDE)$ c.k $[AB, BC]$

\# super keys ?

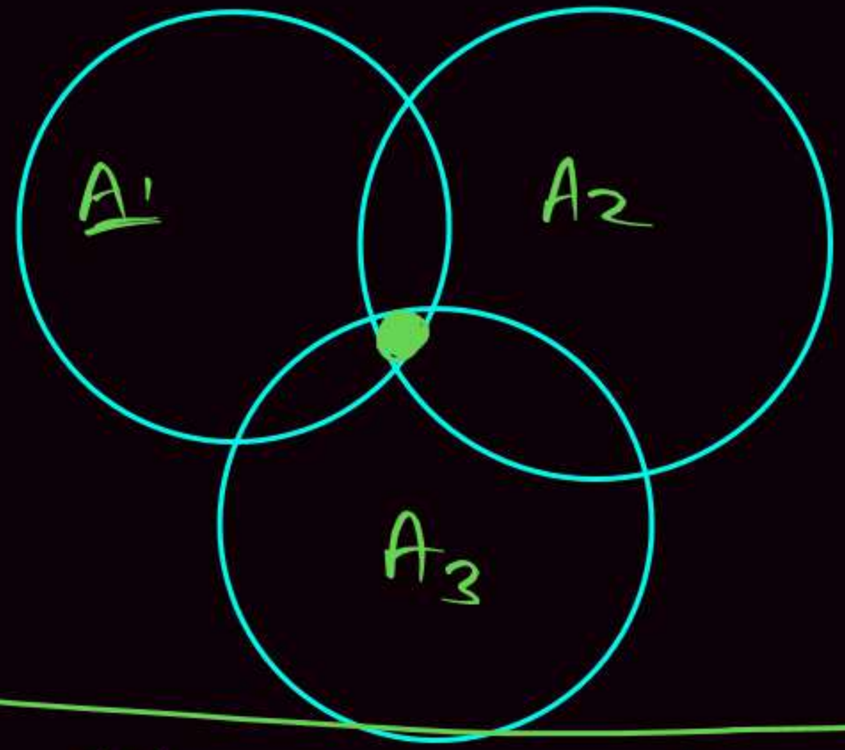
$$\#S.K = 2^{5-2} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^3 + 2^3 - 2^2$$

$$= 8 + 8 - 4$$

$$= 12 \text{ super keys } \underline{\text{Ans}}$$

(v) with only C.K A_1, A_2, A_3 ?



$$\boxed{\begin{aligned} n(A\cup B\cup C) = {} & n(A)+n(B)+n(C) \\ & -n(A\cap B)-n(B\cap C)-n(C\cap A) \\ & +n(A\cap B\cap C) \end{aligned}}$$

$$\boxed{\#\ \text{Superkeys} = 2^{n-1}+2^{n-1}+2^{n-1}-2^{n-2}-2^{n-2}-2^{n-2}+2^{n-3}}$$

GATE 2013

R(EFGH) with Candidate key = E.

$\#\text{Super keys} = 2^{n-1}$

$\Rightarrow 2^{4-1}$

$= 2^3$

= 8 super key.

OR

$(\underline{E}\underbrace{FGH})$

$\Downarrow$

$\#\text{S.k} = 2^3$

= 8 Super keys

OR

$R(\underline{E}FGH)$

$\underline{E}$

$\underline{E}F$

$\underline{E}G$

$\underline{E}H$

$\underline{E}FG$

$\underline{E}GH$

$\underline{E}FH$

$\underline{E}FGH$

8 Super key

$R(\underline{E}FGH)$

$\underline{E}\ F\ G\ H$

000	$\rightarrow$ E
001	$\rightarrow$ EH
010	$\rightarrow$ EG
011	$\rightarrow$ EGH
100	$\rightarrow$ EF
101	$\rightarrow$ EFH
110	$\rightarrow$ EFG
111	$\rightarrow$ EFGH.

8 Super keys.

GATE 2022

R(ABCDE) Number of Super key with only Candidate key: AB ?

$\# \text{Super key} = 2^{n-2}$

$\Rightarrow 2^{5-2}$

$= 2^3$

= 8 Super keys

Ans

R(AB CDE)

$\Downarrow$

2^3

$\Rightarrow$ 8 Super key.

R(AB CDE)

AB
AB C
AB D
AB E
AB CD
AB DE
AB CE
AB CDE

8 Super keys. Ans

ISRO 3Marks

$R(\underbrace{ABCDE}_{\text{5 Attribute}})$ Candidate keys $\{A, BC\}$ Total # super keys ?

$$\# \text{ super key} = 2^{5-1} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^4 + 2^3 - 2^2$$

$$\Rightarrow 16 + 8 - 4$$

$= 20$ super keys.

IInd Approach.

R(ABCDE) Candidate keys {A, BC} Total # Super keys?

Number of super key

$R(\cancel{A}\,B\,C\,D\,E) = 2^4 = \underline{16}$

{A, AB, AC, AD, AE,
ABC, ACD, ADE, ABE
ABCD, ACDE, ABDE
ABCE . . . . .

$\underline{BC}\,D\,E = 2^2 = 4$

BC, BCD, BCE, BCDE

20 Super keys.

R(ABCDE)   C.K [A, B, CD]   # Super keys

A: $\cancel{A}\underbrace{BCDE}$ $\Rightarrow 2^4 = 16$

B: $\cancel{B}\underbrace{CDE} = 2^3 = 8 \rightarrow$ (B, BC, BD, BE, BCD, BDE, BCE, BCDE)

CD $\underbrace{CD\ E} = 2^1 = 2$

⇓

CD, CDE

26 Super key

R(ABCD) C.k [A, B, C. D]

$\cancel{A}\ \underline{BCD} = 2^3 = 8$

$\cancel{B}\ \underline{CD} = 2^2 = 4$

$\cancel{C}\ \underline{D} = 2^1 = 2$

$\underline{D} = 2^0 = 1$

15.

Total # Superkeys $= 2^n - 1$

$\Rightarrow 2^4 - 1$

$= 16 - 1$

$= 15$ Superkeys

Note

Total Maximum # of Super keys $= 2^n - 1$

But under the Assumption if every Attribute is Candidate keys.

Note

Total Maximum # Candidate keys $= {}^{n}C_{\lfloor \frac{n}{2} \rfloor}$

n: # Attributes.

When FD is Not given.

eg) If Relation R(ABCDEF) with 6 Attribute then maximum Number of Candidate keys.

$^{n}C_{\lfloor n/2 \rfloor}$

$^{6}C_{\lfloor 6/2 \rfloor} = {^{6}C_{3}}$

$= 20$

Soln

(i) If every single Attribute is C.K $\Rightarrow$ $^{6}C_{1}$ $= 6$

(ii) If every 2 Attribute is C.K form $\Rightarrow$ $^{6}C_{2}$ $= 15$

(iii) If every 3 Attribute form CK $= {^{6}C_{3}}$ $= 20$ $\quad ^{6}C_{3}$

(iv) If every 4 Attribute form C.K $= {^{6}C_{4}}$ $= 15$

(v) If every 5 Attribute form a C.K $= {^{6}C_{5}}$ $= 6$

(vi) If every 6 Attribute form a C.K $= {^{6}C_{6}}$ $= 1$

$$\frac{\cancel{6} \times 5 \times 4}{\cancel{3 \times 2}} = 20$$

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$\boxed{^{n}C_{\lfloor n/2 \rfloor}}$$

Q) R(ABCDE) C.K [AB, BC]

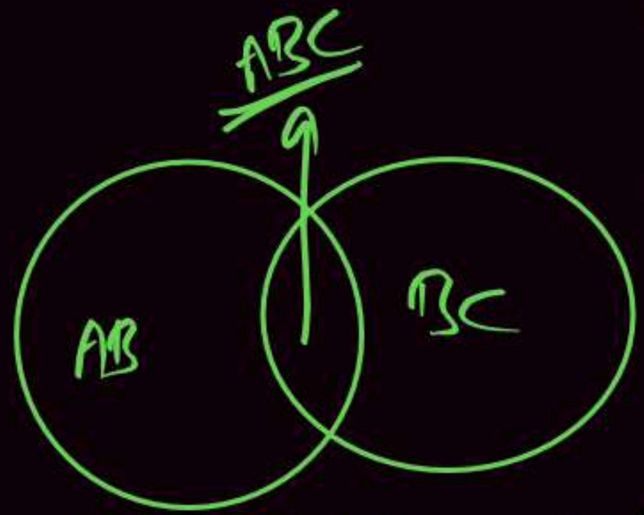
Super keys ?

Super keys = $2^{5-2} + 2^{5-2} - 2^{5-3}$

$\Rightarrow 2^3 + 2^3 - 2^2$

$= 8 + 8 - 4$

= 12 Super keys



$2^{n-2} + 2^{n-2} - 2^{n-3}$

OR

R(ABCDE) [AB, BC]

~~AB~~ CDE $= 2^3 = 8$

BC DE $= 2^2 = 4$

12 Ans

AB CDE [AB, ABC, ABD, ABE, ABCD, ABDE, ABCE, ABCDE]

8 ⇒

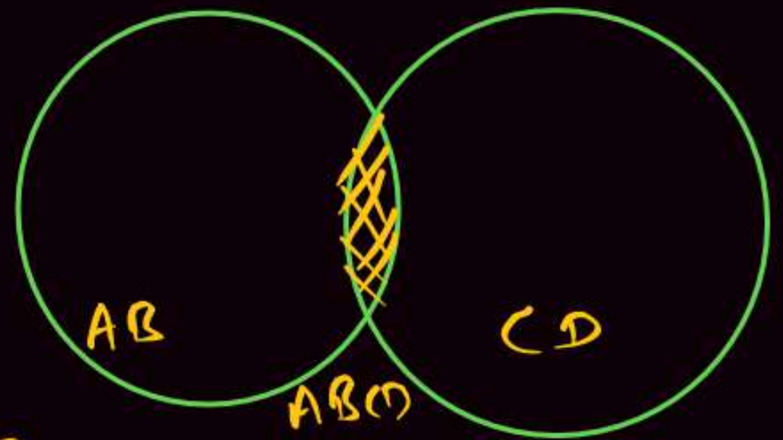
BC DE (BC, BCD, BCE, BCDE)

4 ⇒

12 Ans

Q. R(ABCDE) C.K [AB, CD]

Superkey



$$2^{n-2} + 2^{n-2} - 2^{n-4}$$

$$\Rightarrow 2^{5-2} + 2^{5-2} - 2^{5-4}$$

$$= 2^3 + 2^3 - 2^1$$

$$= 8 + 8 - 2$$

= 14 Superkey

R(ABCDE) Superkey

$\cancel{AB}\ CDE = 2^3 = 8$

$CD\ \cancel{E} = 2^1 = 2$

$\times 10$

Total = 14 Ans

1. AB
2. ABC
3. ABD
4. ABE
5. ABCD
6. ABDE
7. ABCE
8. ABCDE
9. CD
10. CDE
11. CD A
12. CD B
13. ACDE
14. BCDE

CD A

CD B

CD AE

CD BE

Any Doubt ?

THANK YOU!