CS & IT ENGINEERING

GRAPH THEORY

Lecture No. 2

By- SATISH YADAV SIR

Degree Sequence in Graphs

01 Degree Sequence

02 Graphical sequence

03 Havell-Hakimi Thm

04 Inequalities Thm

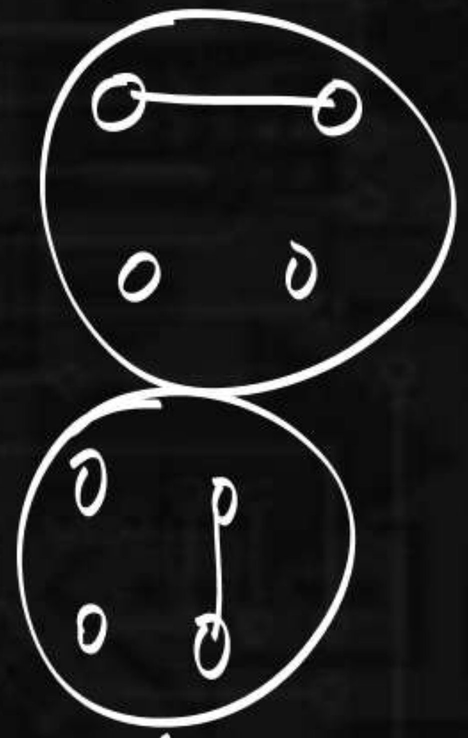
05 Theorem no . 7

Degree Sequence

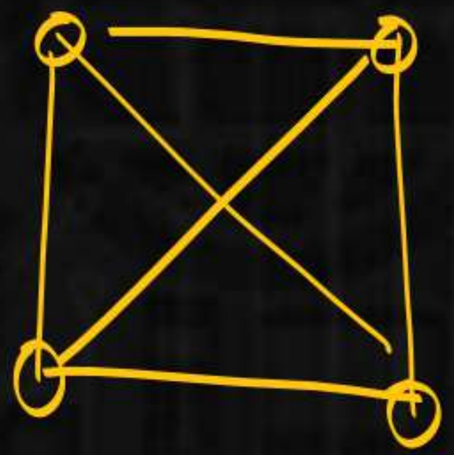
$n = 4$

$e = \frac{n(n-1)}{2} = \frac{4 \cdot 3}{2} = 6.$

$n = 4 \quad e = 0$ $\qquad n = 4 \quad e = 1$



$n = 4$ ✓
$e = 6$ ✓



R.S

no simple Graph.

Degree Sequence

How many graphs are possible with 4 vertices?

$$= 2^{\frac{4\times 3}{2}}$$ no. of edges. ↓ 2 choices.

# of graphs are possible with n vertices

$$2^{\frac{n(n-1)}{2}}$$

2 ways 2 ways.

w1	w2
0	0
0	1
1	0
1	1

2 ways 2 ways.

2^6.

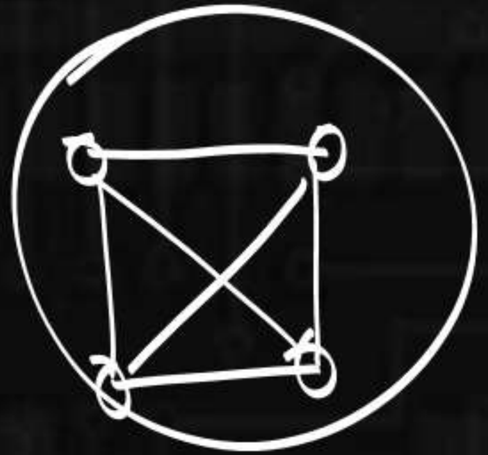
w1	w2	w3	w4	w5	w6.
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0

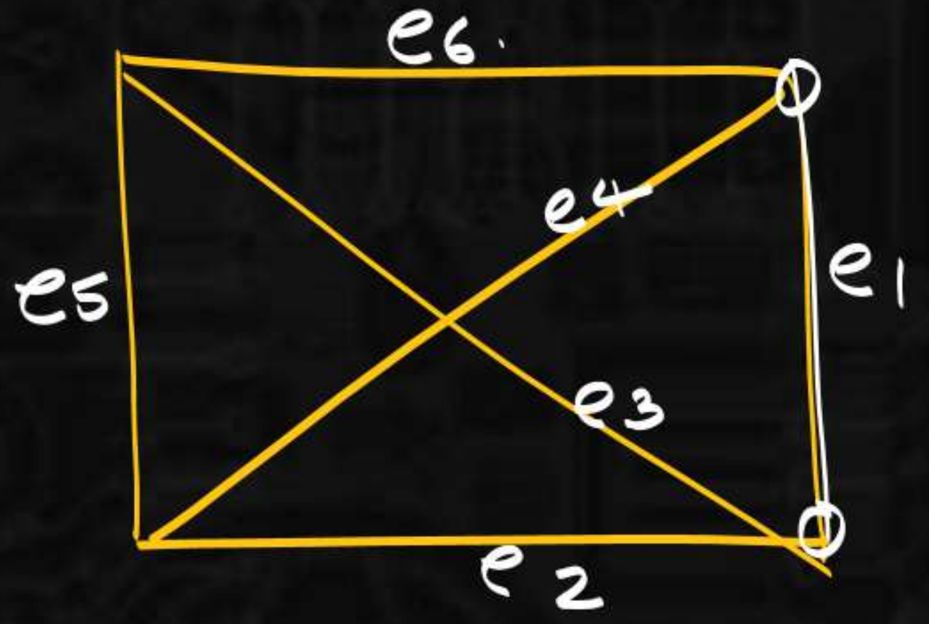
Degree Sequence

2^6

e_1	e_2	e_3	e_4	e_5	e_6
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
		1			
			1		
				1	
					1
1	1	1	1	1	1

${}^6C_1$



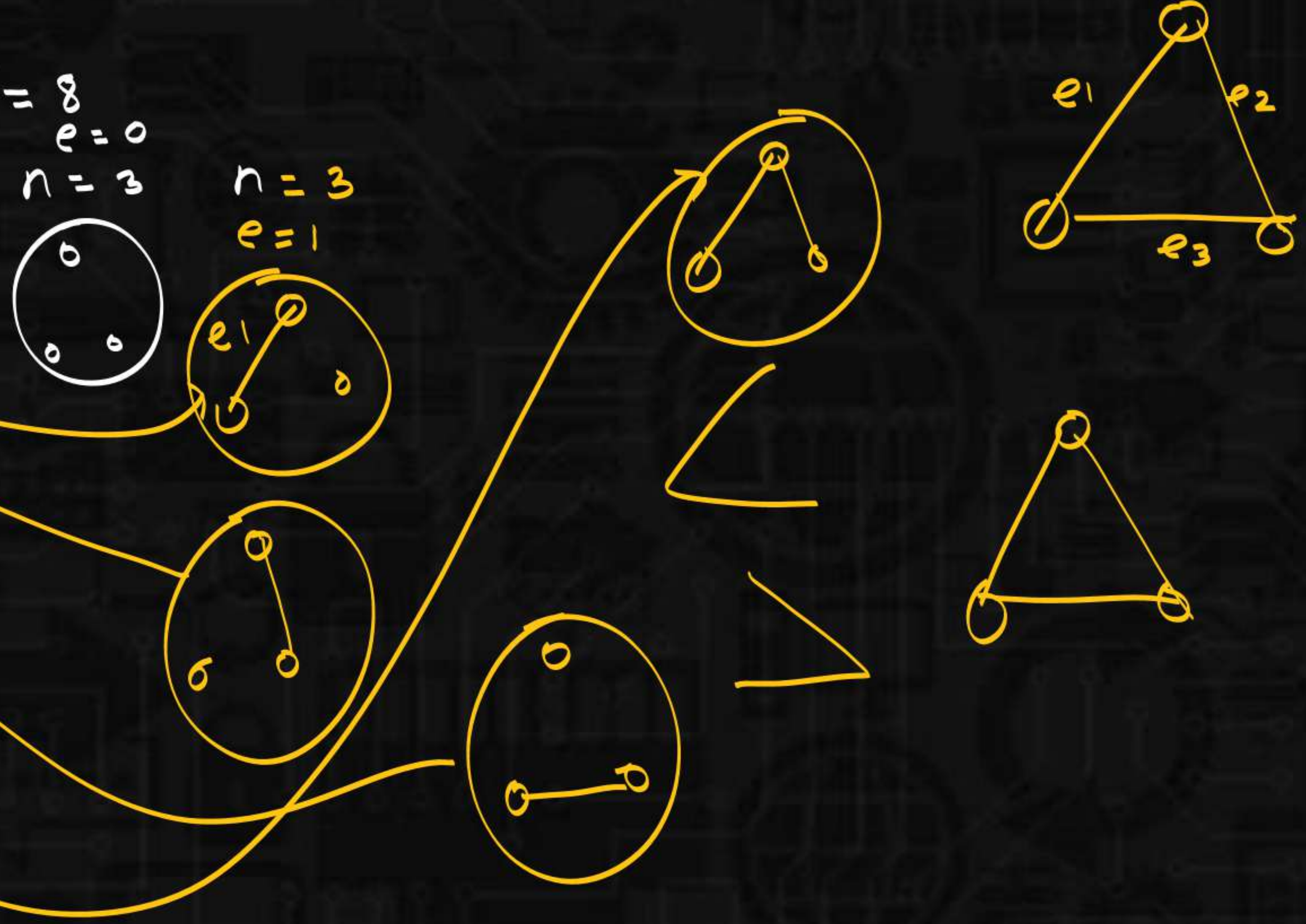


Degree Sequence

$n = 3 \quad 2^{\frac{3\times 2}{2}} = 2^{3} = 8$

2^3

e_1	e_2	e_3
0	0	0
1	0	0
0	1	0
1	1	0
0	1	1
1	0	1
1	1	1



Degree Sequence

How many graphs are possible with 4 vertices & 1 edge.

4 vertices max we can have 6 edges.

Choosing 1 edge $\rightarrow {}^{6}C_{1}$. $\left({}^{\frac{4\times 3}{2}}C_{1}\right)$

(**) How many graphs are possible with n vertices & exactly (n) edges.

$${}^{\frac{n(n-1)}{2}}C_{n}.$$

Degree Sequence

of graphs with 4 vertices & at least 2 edges.

method1 : $^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$.

method2: $2^6 - {}^6C_0 - {}^6C_1$

$^6C_0 + {}^6C_1 + \boxed{{}^6C_2 + {}^6C_3 \cdots {}^6C_6} = 2^6$.

$= 2^6 - {}^6C_0 - {}^6C_1$

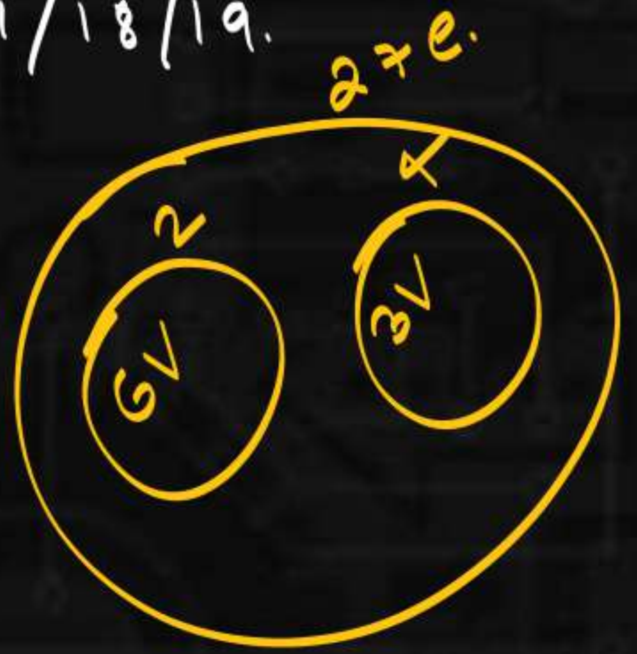
Degree Sequence

Consider a Graph 27 edges.

Total Vertices = 6 vertices of degree 2 (GATE)
+
3 vertices of degree 4
+
x $\boxed{\text{remaining vertices}}$ will have degree 3.

What will be total no. of vertices?

10/11/18/19.



$$\Sigma\, d(v_i) = 2e$$

$$6v \times 2 + 3v \times 4 + x \times 3 = 2 \times 27$$

$$12 + 12 + 3x = 54$$

$$3x = 30$$

$$\boxed{x = 10}$$

Total vertices $= 6 + 3 + 10$

$= \underline{\underline{19}}$

Degree Sequence

$n = 10$

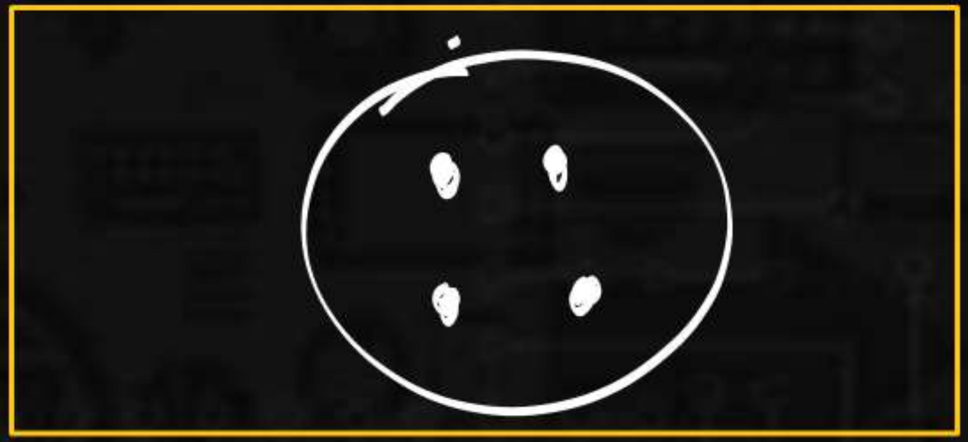
how many max no. vertices will be possible when 15 edges & degree of each vertex is atleast 3?

② $e = 15$

$\delta(G) = 3$

$n = ?$

$e = 15$



$$\delta(G) \le \frac{2e}{n}$$

$$3 \le \frac{2(15)}{n}$$

$$n \le \frac{30}{3}$$

$$\boxed{n \le 10}$$

① $(\ldots\ldots 10) \le 10$

max

Degree Sequence

Thm5 :

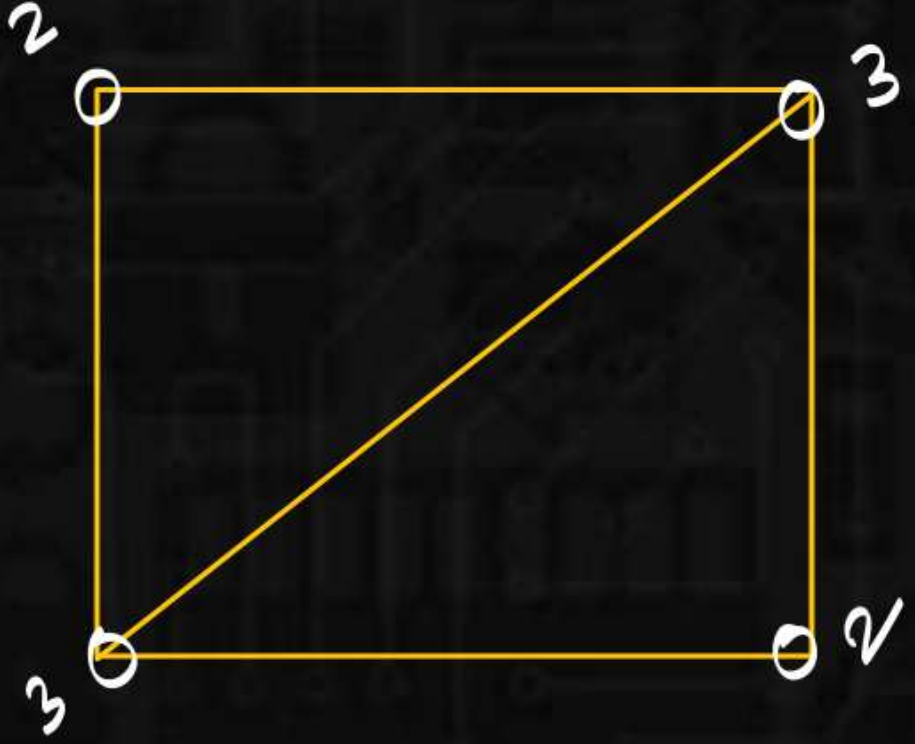
maximum degree ($\underline{\Delta(G)}$)

minimum degree ($\delta(G)$)

Case1:

$\Delta(G) = 2$

$\delta(G) = 2$



$\Delta(G) = 3$

$\delta(G) = 2.$

Degree Sequence

Totalvertices = n = 4.

Case I:

$\Delta(G) = 2$ $\quad \frac{2e}{n} = 2$

$\delta(G) = 2.$

$$\boxed{\delta(G) = \frac{2e}{n} = \Delta(G) - \text{(I)}}$$

$\Sigma d(v_i) = 2e$

$$\text{avg. degree} = \frac{\text{degrees of all vertices}}{\text{total no. of vertices}} = \frac{2+2+2+2}{4} = \frac{2e}{n} = 2.$$

Degree Sequence

$\Delta(G) = 3$

$\delta(G) = 2$

$$\delta(G) < \frac{2e}{n} < \Delta(G) \text{ — (II)}$$

$$\frac{2e}{n} = \frac{2+3+3+2}{4} = \frac{10}{4} = 2.5$$

Degree Sequence

Thm 5 :

atmost

(Thus relates degrees)

$$\delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1.$$

atleast

$16 \le (n \ldots\ldots\ldots)$

$16 \le 16$

$16 \le 17$

16

Consider a graph having 40 edges & degree of each vertex is at most 5. what will be min no of vertices

$\Delta(G) = 5 \quad e = 40$

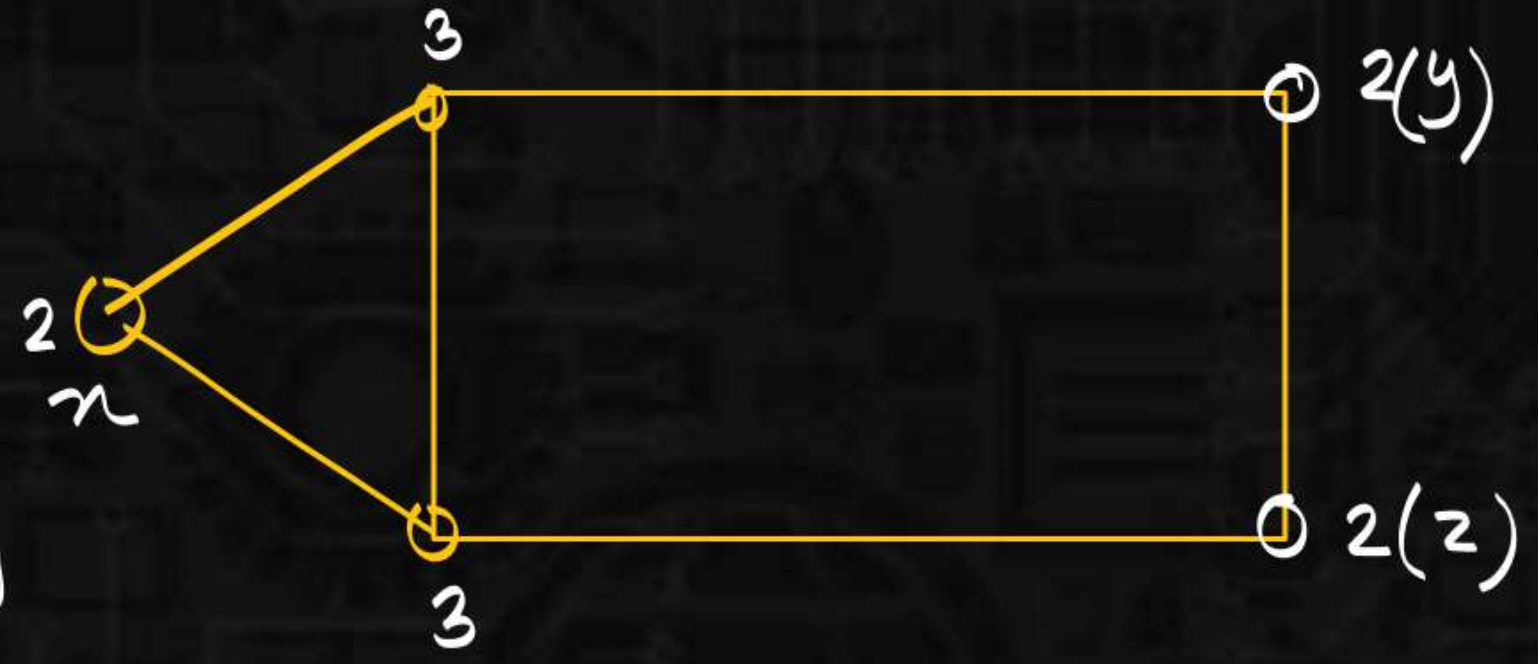
$\frac{2e}{n} \le \Delta(G) \quad \frac{2 \times 40}{n} \le 5$

$\frac{80}{5} \le n$

Degree Sequence

Degree sequence :

writing degrees of all vertices either in increasing or decreasing order.



3, 3, 2, 2, 2

OR

2, 2, 2, 3, 3.

Degree Sequence

How many edges will be present

5, 2, 2, 2, 2, 1 ?

m1:

$\sum d(v_i) = 2e$

$5+2+2+2+1 = 2e$

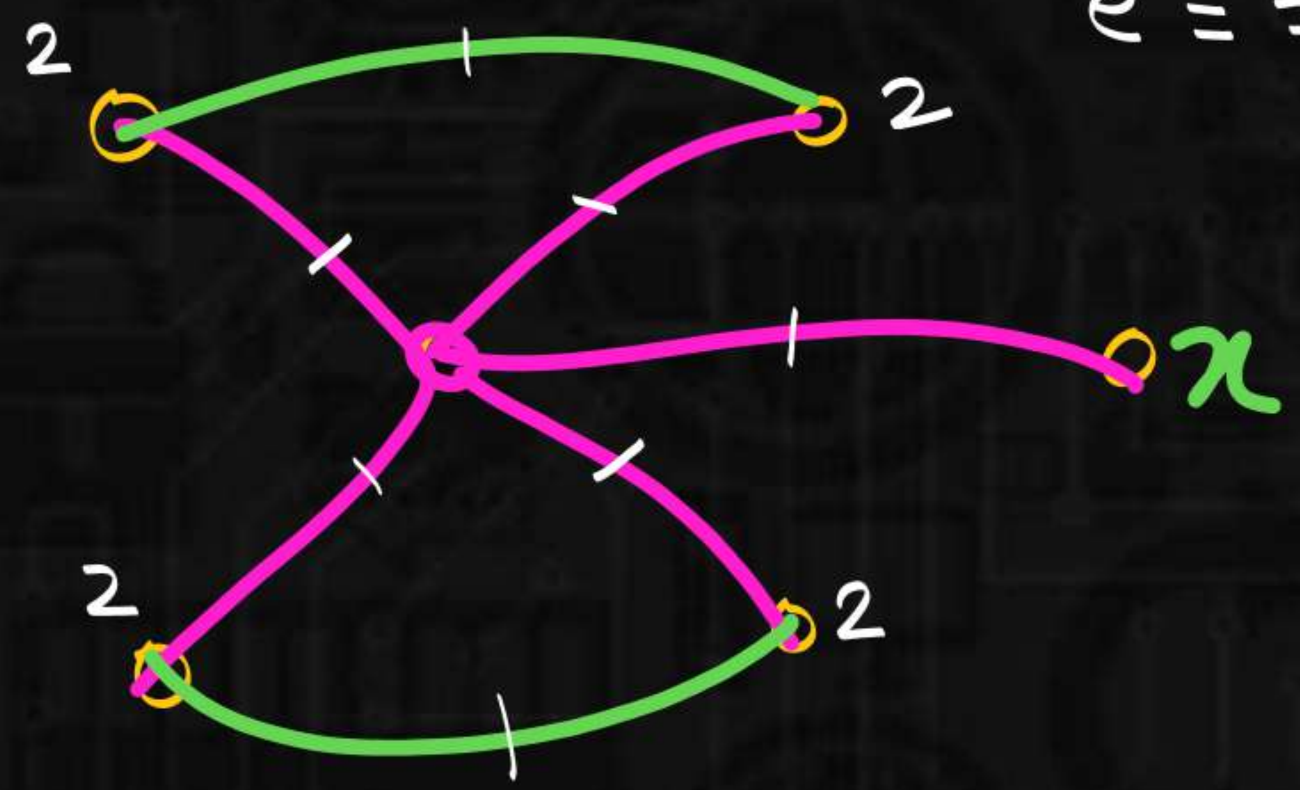
$14 = 2e$

$\boxed{e=7}$

$n = 6$

m2: 5, 2, 2, 2, 2, 1

$e = 7$



Degree Sequence

How many edges will be possible in 3, 3, 3, 1 ?

m1: $\Sigma d(v_i) = 2e$

$3+3+3+1 = 2e$

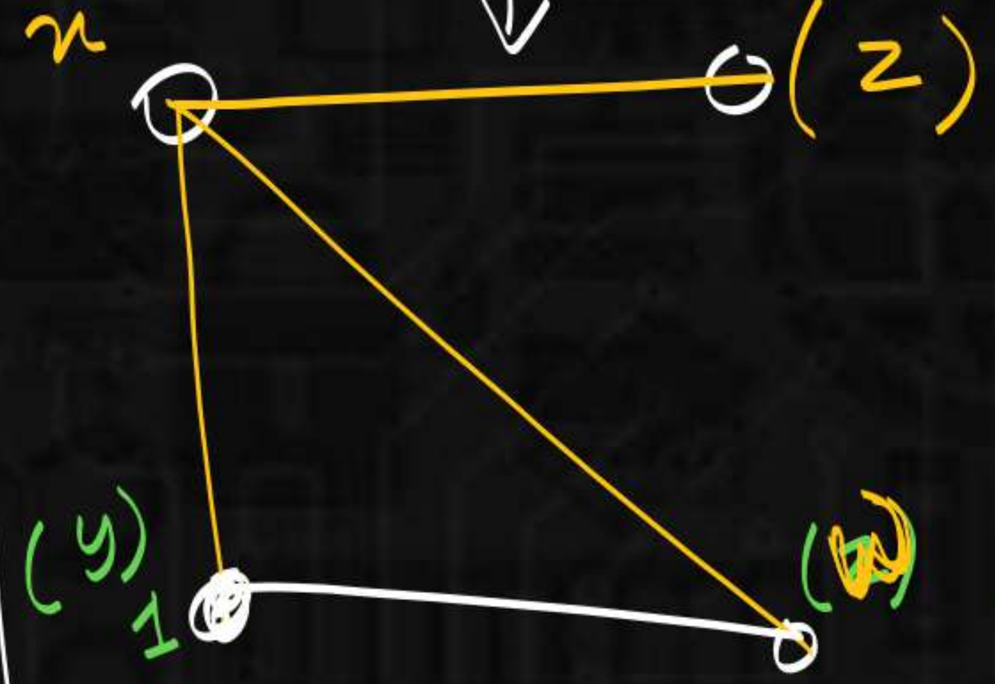
$10 = 2e$

$\boxed{e = 5}$

$3 \quad \boxed{3, \; 3} \quad 1$ Total vertices = 4.

z

no simple graph is possible.



$\underline{y}$

demands $\rightarrow$ 2 edges.

$y \not\rightarrow x$

Degree Sequence

Degree sequence $\xrightarrow{\text{Draw}}$ simple Graph.

Graphical sequence.

3, 2, 2, 2, 2, 1

Degree sequence $\longrightarrow$ not simple Graph.

Graphical sequence (Graphical)

we can draw a graph for any degree sequence then the sequence is called graphical.

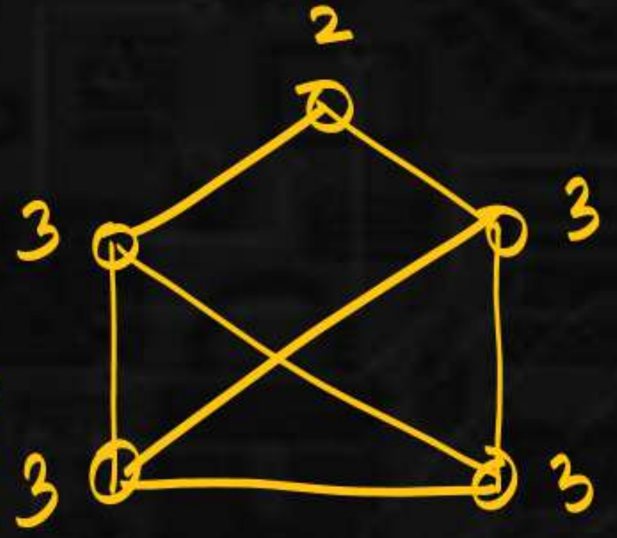
Degree Sequence

Which of the following is graphical sequence?

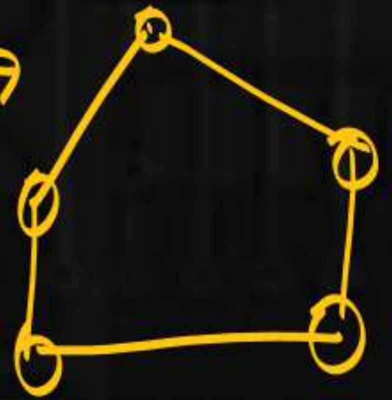
1) 5, 4, 3, 2, 1.

2) 4, 4, 3, 2, 1.

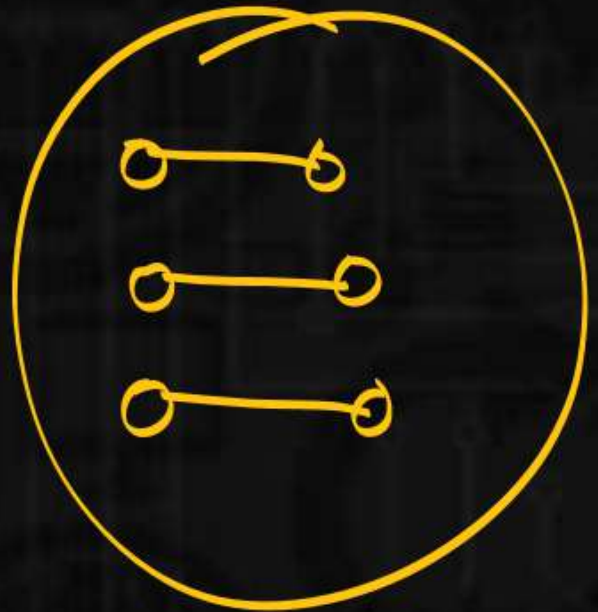
3) 3, 3, 3, 3, 2



4) 2, 2, 2, 2, 2



5) 1, 1, 1, 1, 1, 1.



Degree Sequence

5, 4, 3, 2, 1. it is not Graphical.

Reason 1: no. of odd degree vertices will be even.

Reason 2: max degree $\leq n-1$. $n = 5$

max degree ≤ 4

Degree Sequence

4, 4, 3 2. 1.

((n-1) (n-1) . . . . . . 1)

Reason: Raavan Rule.

Thm 2 ✓

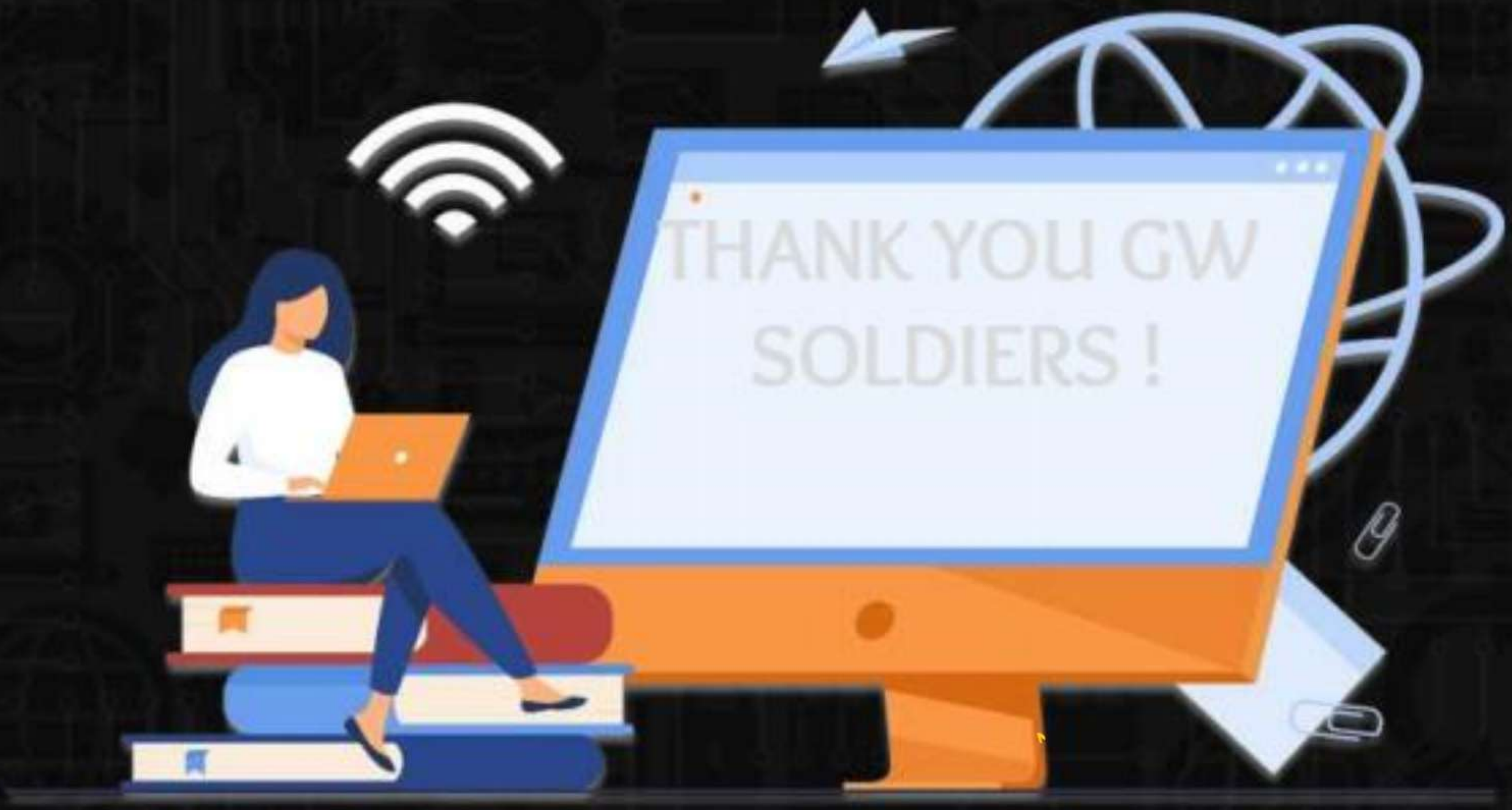
Thm 3: $\Delta(G) \leq n-1$. ✓

Degree Sequence

$n-1, \; n-1 \; \ldots\ldots\ldots \; 1$

3, 3, 3, 1

4, 4 (. .) 1


THANK YOU GW
SOLDIERS !