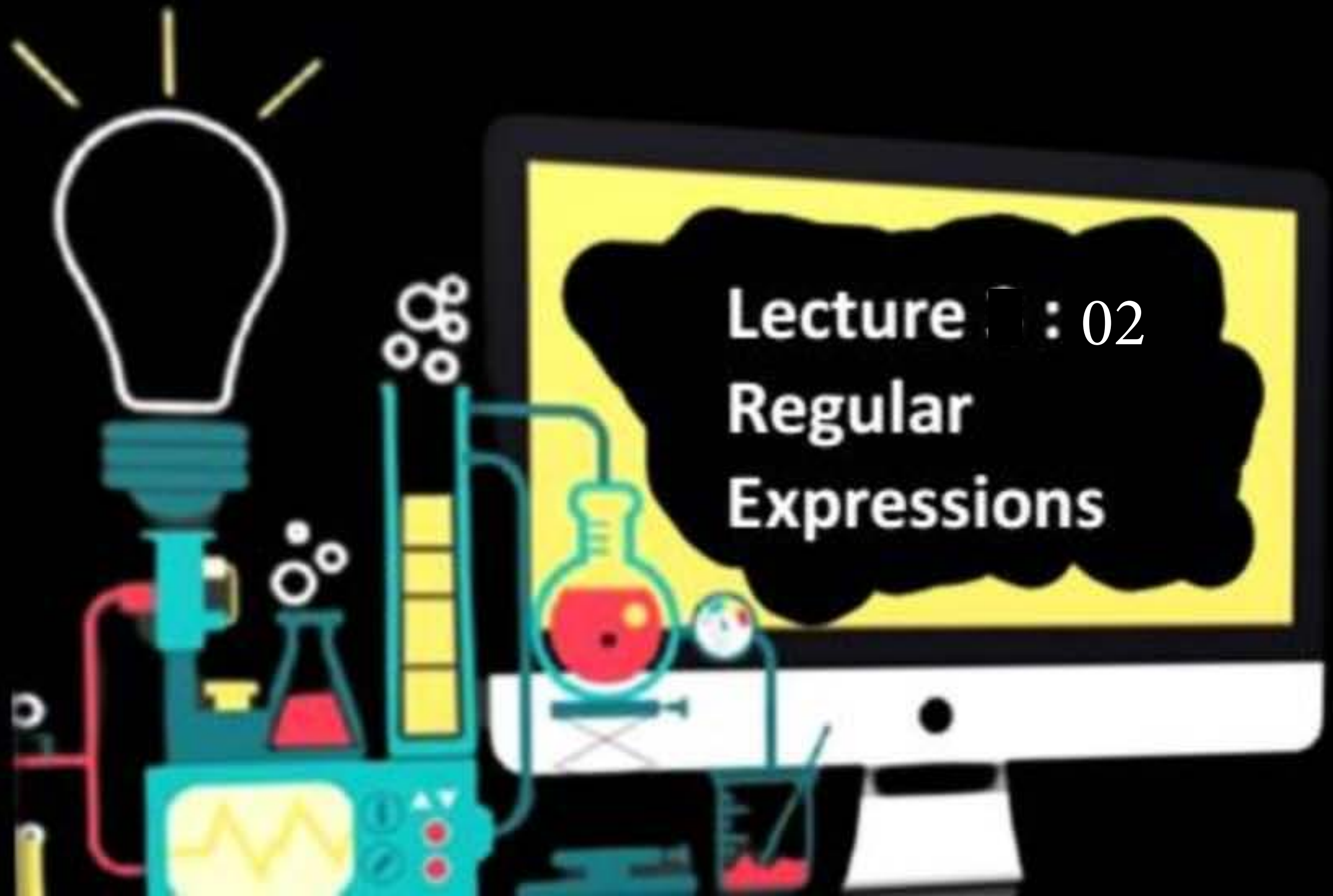
CS & IT

Engineering



Deva sir

Topics: To be covered

Regular Expressions

1) Simplify

2) How to write ?

$|\varepsilon| = 0$

string

$|\{\ \}| = 0$

set

$L(\varepsilon) = \{\varepsilon\}$

$L(\phi) = \{\ \}$

ε ϕ

$L(\varepsilon) = \{\varepsilon\}$

empty string

set with one element

$L(\phi) = \{\ \} = \phi$

empty expression

empty set

ε

→ string $\quad |\varepsilon| = 0$ → string

→ exp

→ not set

ϕ

→ not string

→ empty expression

→ empty set $\quad |\phi| = 0$ → set

1. $\varepsilon + \varepsilon = \varepsilon$
2. $\varepsilon \cdot \varepsilon = \varepsilon$
3. $\phi \cdot \phi = \phi$
4. $\phi + \phi = \phi$
5. $\phi \cdot \varepsilon = \phi$
6. $\varepsilon + \phi = \varepsilon$

* 7. $a + \varepsilon = a + \varepsilon$
8. $a + \phi = a$
9. $a \cdot \varepsilon = a$
10. $a \cdot \phi = \phi$

$|\phi| = |\{\ \}| = 0$

$|\{a\}| = 1$

$|\{a, b\}| = 2$

(11) $\underbrace{a}_{\text{must}} \cdot a^* = a^+$

(12) $a + a^* = a^*$

(13) $\underbrace{a^*}_{a^0} \cdot \underbrace{a^*}_{} = a^*$

(14) $a^* + a^* = a^*$

(15) ** → a is not possible

$\underbrace{a}_{\text{must}} \cdot \underbrace{a^+}_{\text{atleast one a}} = a^2 + a^3 + \ldots = aa^+ = a^+a$

$= aaa^* = aa^*a$

$= a^*aa$

(16) $a + a^+ = a^+$

(17) $a^+ + a^+ = a^+$

** (18) $\underbrace{\underbrace{a^+}_{\geq 1a} \cdot \underbrace{a^+}_{\geq 1a}}_{\geq 2 a's} = (15)$

(19) $(a^+)^+ = a^+$

(20) $(a^*)^* = a^*$

$$(a^+)^+ = (a^+)^1 \cup (a^+)^2 \cup (a^+)^3 \cup \cdots$$

$$= a^+$$

$$(a^*)^* = \underbrace{(a^*)^0} \cup (a^*)^1 \cup \underbrace{(a^*)^2 \cup \cdots\cdots}$$

$$= a^*$$

aa^+

$a\ a = a^2$

$a \cdot a^2 = a^3$

$a \cdot a^3 = a^4$

$a \cdot a^4 = a^5$

$= aaa^*$

$= aa^*a$

$= a^*aa$

$= a^+a$

$= a^+a^+$

$$a \cdot a^*$$

always

$$\begin{array}{ccc} \varepsilon & \longrightarrow & a \\ a & \longrightarrow & a^2 \\ a^2 & \longrightarrow & a^3 \\ & & \vdots \end{array} \Bigg\} \, a^+$$

21 $(a^+)^* = a^*$

22 $(a^*)^+ = a^*$

23 $(a+\varepsilon)^* = a^*$

*** 24 $(a+\varepsilon)^+ = a^*$

25 $(a+\underbrace{aa}+\underbrace{aaa})^* = a^*$

26 $\phi^* = \varepsilon$

27 $\phi^+ = \phi$

28 $\varepsilon^* = \varepsilon$

29 $\varepsilon^+ = \varepsilon$

30 $\underbrace{\phi^*}_{\varepsilon} + \underbrace{\phi . a^*}_{\phi} = \varepsilon$

$$(a+\varepsilon)^* = (\quad)^0 + (a+\varepsilon)^1 + \underbrace{(a+\varepsilon)^2} + \cdots$$

$$R^* = R^0 + R^1 + R^2 + \cdots$$

$$= \varepsilon + a + \varepsilon + \underbrace{\varepsilon + a + aa} + \cdots$$

$$= \varepsilon + a + a^2 + \cdots$$

$$= a^*$$

$$\boxed{(a+\varepsilon)^2 = (a+\varepsilon)^1 (a+\varepsilon)^1 = \varepsilon + a + a^2}$$

$$(a+\varepsilon)^+$$

$$R^+ = R^1 + R^2 + \cdots = (a+\varepsilon)^1 + \underbrace{(a+\varepsilon)^2} + \cdots$$

Mallesham Devasane

$$= a + \varepsilon + \underbrace{\varepsilon + a + aa} + \cdots$$

$$= \varepsilon + a + a^2 + \cdots$$

$$= a^*$$

$$(a+\varepsilon)^+ = \varepsilon, a, a^2, a^3, \ldots$$

$$= a^*$$

$$\Downarrow$$

$$a^+ \cup \{\varepsilon\} = a^*$$

(31) $(a+a^+)^* = a^*$

(32) $(a+a^*)^+ = a^*$

(33) $(a^+ + a^+)^+ = a^+$

(34) $(a^+ + a^+)^* = a^*$

(35) $(\underbrace{a.a^*})^* = (a^+)^* = a^*$

$aa^* = a^+$

(36) $(aa^+)^* = \varepsilon + aa^+$

(37) $(aa^+)^+ = aa^+$

(38) $(a^+ + a^*)^* = a^*$

(39) $(a^+ + a^*)^+ = a^*$

(40) $\underset{\varepsilon}{a^*}\,\underset{\varepsilon}{a^*}\,\underset{\varepsilon}{a^*}\,\underset{a^*}{a^*} = a^*$

DEVA Sir

$$(aa^+)^* =$$

$$\boxed{\varepsilon + aa^+}$$

$$(\ )^0 \longrightarrow \varepsilon$$

$$(\ )^1 \longrightarrow \boxed{aa^+} \longrightarrow \text{min 2 a's}$$

$$(\ )^2 \longrightarrow aa^+aa^+ \longrightarrow \text{min 4 a's}$$

$$(\ )^3$$

$$\boxed{a^{+} + \varepsilon = a^{*}}$$

$$a^* \Rightarrow \varepsilon, a, a^2, a^3, \ldots$$

$$a^*a^* \Rightarrow \varepsilon, a, a^2, a^3, \ldots$$

ε: a^0a^0

a: a^1a^0 OR a^0a^1

a^2: a^2a^0 OR a^1a^1 OR a^0a^2

$$a^*a^*a^* \Rightarrow \varepsilon, a, a^2, \ldots$$

ε: $a^0a^0a^0$

a: $a^0a^0a^1$

$$a^*a^*a^*a^* = a^*$$

$$a^*a^*a^+ = a^+$$

$$a^*a^*a^+a^+ = a^+a^+ = aa^+$$

$$\underbrace{a^+_{\geq 1}\,a^+_{\geq 1}\,a^+_{\geq 1}}_{\geq 3} = \underbrace{a_{1}\,a_{1}\,a^+_{\geq 1}}_{\geq 3}$$

41 $(a+b)^*$ = every string exist over a's and b's

42 $(a+b+\varepsilon)^* = (a+b)^*$

43 $(\boxed{a}+\boxed{b}+\underbrace{\boxed{aa}+\boxed{ab}+\boxed{ba}+\boxed{aaab}}_{\text{not required}})^* = (a+b)^*$

44 $(a+b+\underbrace{(ab)^*}_{\text{not needed}})^* = (a+b)^*$

45 $(a+b+a^*b^*)^* = (a+b)^*$

Equivalent

a^* $\Rightarrow$ Universal set over $\Sigma = \{a\}$

$\not\Rightarrow$ Not generates universal set over $\Sigma = \{a, b\}$

$(a+b)^*$ $\Rightarrow$ Universal set over $\Sigma = \{a, b\}$

$\not\Rightarrow$ Not universal over $\Sigma = \{a, b, c\}$

$(a+b+c)^*$ $\Rightarrow$ Universal over $\Sigma = \{a, b, c\}$

$(a+b+c+d)^*$ $\Rightarrow$ Universal over $\Sigma = \{a, b, c, d\}$

$$(a+b)^* = (a+b)^0 + (a+b)^1 + (a+b)^2 + (a+b)^3 + \dots$$

It generates Universal Set over $\Sigma = \{a, b\}$

$(a+b)^0$	$(a+b)^1$	$(a+b)^2$	$(a+b)^3$
ε ✓	a ✓	aa ✓	aaa ✓
	b ✓	ab ✓	aab ✓
		ba ✓	aba ✓
		bb ✓	abb ✓
			baa ✓
			bab ✓
			bba ✓
			bbb ✓

$$\boxed{\begin{aligned}(a+b)^2 &= (a+b).(a+b)\\ &= aa+ab+ba+bb\end{aligned}}$$

$$= \varepsilon + a + b + aa + ab + ba + bb + \dots$$

46. $(a+b)^+ + \varepsilon = (a+b)^*$

*** 47. $(a+b+\varepsilon)^+ = (a+b)^*$

$$\boxed{(R+\varepsilon)^+ = R^*}$$

48. $(a+b+\boxed{a^*})^+ = (a+b)^*$

just take ε

49. $(a+b+\boxed{b^*})^+ = (a+b)^*$

50. $(a+b+(ab)^*+aaa)^+ = (a+b)^*$

$$\boxed{R^+ + \varepsilon = R^*}$$

$$(a+b)^+ + \varepsilon = (a+b)^*$$

$$a^+ + \varepsilon = a^*$$

$$(a+b+\varepsilon)^+ = (a+b)^*$$

$$(a+b)^+ \cup \{\varepsilon\}$$

only ε is missin

$$\boxed{(a+b)^* = \ ?}$$

ba ① $a^* b^*$

ab ② $a^* + b^*$

ε ③ $a + b$

*** ④ $(a^* b^*)^*$

⑤ $(a^* b^*)^+$

ε ⑥ $a^+ b^+$

a ⑦ $(a^+ b^+)^*$

⑧ $(a^* + b^*)^*$

⑨ $(a^* + b^*)^+$

⑩ $(a^* + b)^+$

⑪ $(a + b^*)^+$

⑫ $(a + b + \varepsilon)^+$

⑬ $(b^* a^*)^*$

⑭ $(a^* b^* a^*)^*$

⑮ $(a^* b^* a^* b^*)^*$

$$(a^+ b^+)^* \to \varepsilon \checkmark \quad (\ )^0$$

$$\to a \times$$

Single a is impossible (always there)

$$\left(\boxed{a^+ \, b^+}\right)$$

$$\underbrace{\text{must} \quad \text{must}}$$

ab

aab

abb

⋮

Variable name

↳ pattern

↳ Regular exp

```
ch=getchar()
if(ch=='l'||ch=='_')
    f();
```

→ starts with letter or underscore

→ uky l, d, _

$$\boxed{(l+\_)\,(l+\_+d)^*}$$

$$(\underline{a^*} + b)^+$$

$$(\boxed{\varepsilon} + \boxed{a} + aa + \cdots + \boxed{b})^+$$

$$(\varepsilon + a + b)^+ = (a+b)^*$$

$$(a^* + b^*)^*$$

$$(\varepsilon + \boxed{a} + aa + \cdots + \varepsilon + \boxed{b} + bb + \cdots)^*$$

$$(a + b)^*$$

$(a^{*\geq 0} b^{*\geq 0})^{*\geq 0} \longrightarrow \varepsilon \longleftarrow (\quad)^0$

$a \longleftarrow (a^1 b^0)^1$

$b \longleftarrow (a^0 b^1)^1$

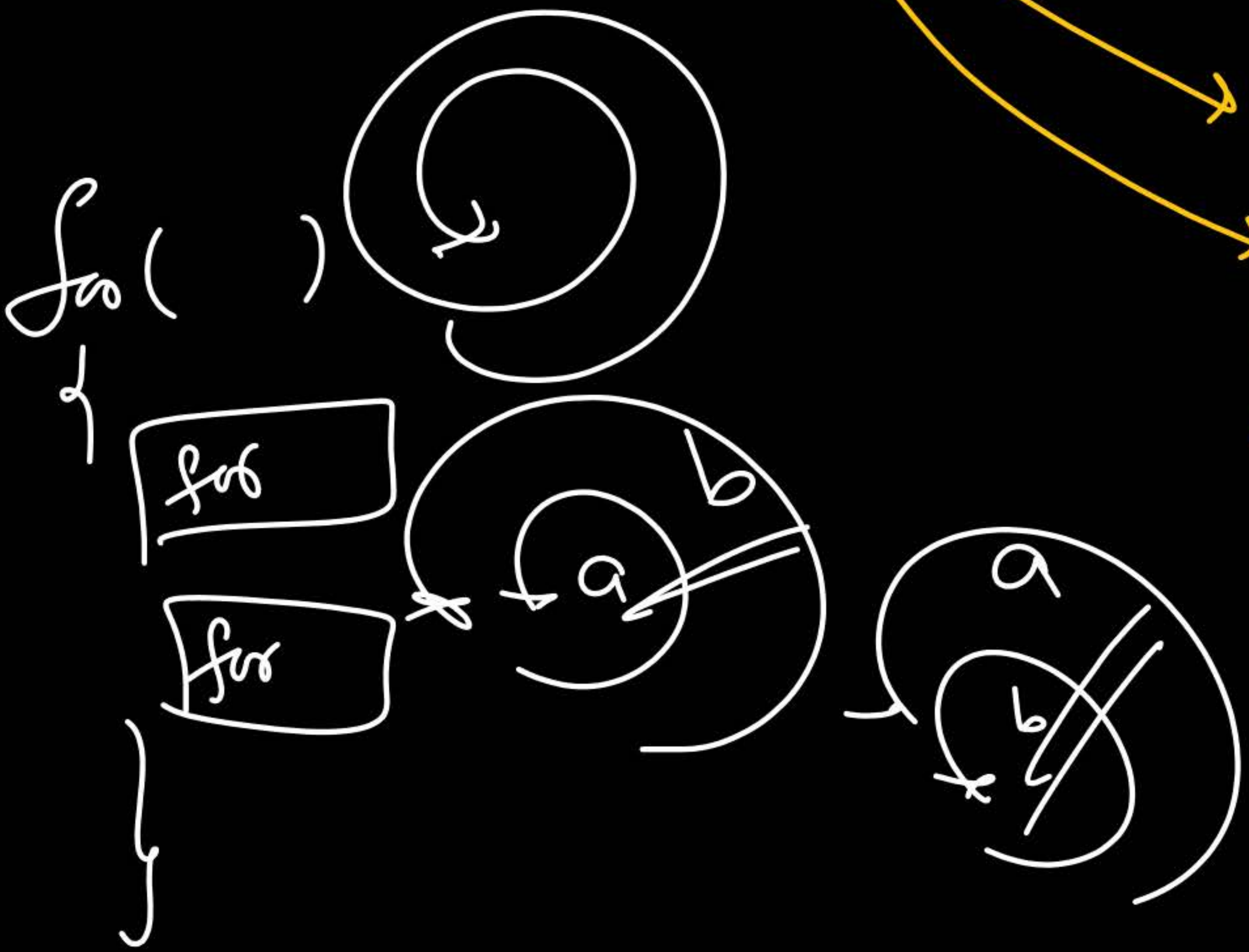
$aa \longleftarrow (\quad)^2 = (a^1 b^0)(a^1 b^0)$

$ab \longleftarrow$

$ba \longleftarrow (a^* b^*)^2 = \underbrace{(a^1 b^0)}_{a}\underbrace{(a^0 b^1)}_{b}$

$bb \longleftarrow$

$\vdots$



a^* for $i=0 \longrightarrow \infty$

a^+ for $i=1 \longrightarrow \infty$

abaa

$(a^1b^0)^1(a^0b^1)^1(a^1b^0)^1(a^1b^0)^1$

$= \underline{\underline{abaa}}$

$$(a+b)^*$$

$$(a^*b^*)^*$$

$\to a = (a^1b^0)^1$

$\to b = (a^0b^1)^1$

a^*b^*

ϵ ✓
a ✓
b ✓
aa ✓
ab ✓
ba

ϵ
a
b
aa
ab
ba
bb

$(a^*b^*)^*$

$(\quad)^2 = \underbrace{(a^0b^1)}_{b}\underbrace{(a^1b^0)}_{a}$

$= ba$ ✓

How to write Regular Expression ?

$\Sigma = \{a\}$

Any no. of a's in sequence $\Rightarrow a^*$

Atleast 1 a in sequence $\Rightarrow a^+$

Atleast 2 a's in sequence $\Rightarrow aa^+ = a^+a = aaa^*$

$= a^*aa$

$= aa^*a$

Zero a's in sequence $\Rightarrow \varepsilon$

$\boxed{a^0 = \varepsilon}$ → String

No string $\Rightarrow \phi$

Q1) Set of all $\underbrace{\text{binary}}$ strings

$\Sigma = \{0, 1\}$
$= \{x, y\}$
$= \{a, b\}$

$= (0+1)^*$

$(x+y)^*$

$(a+b)^*$

2. $L = \{w \mid w \in \{a,b\}^*, \underbrace{|w| = 0}_{\text{zero length string}}\}$

$$\boxed{\varepsilon}$$

3. $L = \{w \mid w \in \{a,b\}^*, \underbrace{|w| \neq 0}_{\text{It means } |w| \geq 1}\}$

It means $|w| \geq 1$ except ε

$$\boxed{(a+b)^+}$$

4. $L = \{w \mid w \in \{a,b\}^*, \underbrace{|w| < 0}_{\text{No string}}\}$

$$\boxed{\phi}$$

(5) $L = \{ w \mid w \in \{a,b\}^*,\ w \text{ starts with } `a' \}$

a is followed by any sequence $\boxed{a(a+b)^*} = (ab^*)^+$

(6) $L = \{ w \mid w \in \{a,b\}^*,\ w \text{ starts with } `b' \}$

$\boxed{b(a+b)^*} = (ba^*)^+$

(5) = (7) $L = \{ w \mid w \in \{a,b\}^+,\ w \text{ starts with } `a' \}$

$\boxed{a(a+b)^*}$

(6) = (8) $L = \{ w \mid w \in \{a,b\}^+,\ w \text{ starts with } `b' \}$

$\boxed{b(a+b)^*}$

(9) What is language generated by $(ab^*)^+$?

$= a(a+b)^*$

All Strings starting with 'a'

aab

$(\ )^2 \Rightarrow \underbrace{(ab^0)^1}_{a} \ \underbrace{(ab^1)^1}_{ab}$

$\underbrace{(ab^*)^+}$

- ε ×
- a ✓
- b ×
- aa ✓
- ab ✓
- ba ×
- bb ×
- aaa ✓
- aab ✓
- aba ✓
- abb ✓
- baa ×
- bab ×
- bba ×
- bbb ×

$\boxed{\text{Starting with } a}$

$$= a(a+b)^*$$

$$= (ab^*)^+ = (a^+ b^*)^+$$

$$= a(a^*b^*)^*$$

$$= a(a^*+b^*)^*$$

ε
a
b
aa
ab
ba
bb
aaa
⋮

(10) $L = \{ w \mid w \in \{a,b\}^*, \ w \text{ ends in } b \}$

$$\boxed{R = (a+b)^* b} = (a^* b)^+$$

(11) $L = \{ w \mid w \in \{a,b\}^*, \ w \text{ starts with } aa \text{ or } bb \}$

$$\boxed{(aa+bb)(a+b)^*} = \boxed{aa(a+b)^* + bb(a+b)^*}$$

(12) $L = \{ w \mid w \in \{a,b\}^*, \ w \text{ ends with } aa \text{ or } bb \}$

$$\boxed{(a+b)^*(aa+bb)}$$

13 $L = \{ w \mid w \in \{a,b\}^*,\ w \text{ contains `a' as substring} \}$

$$\boxed{(a+b)^* a (a+b)^*} = \boxed{b^* a (a+b)^*}$$

14 $L = \{ w \mid w \in \{a,b\}^*,\ w \text{ Contains `ab' as substring} \}$

$$\boxed{(a+b)^* . ab . (a+b)^*}$$

15 $L = \{ w \mid w \in \{a,b\}^*,\ w \text{ containing `aa' or `bb' as Substring} \}$

$$\boxed{(a+b)^* (aa+bb) (a+b)^*}$$

*** 16 $L = \{ w \mid w \in \{a,b\}^*,\ w$ contains both 'aa' and 'bb' as substrings $\}$

$$\left[(a+b)^* aa (a+b)^* bb (a+b)^*\right]$$

$$+$$

$$\left[(a+b)^* bb (a+b)^* aa (a+b)^*\right]$$

$X\ aa\ X\ bb\ X$

$X\ bb\ X\ aa\ X$

17 $\{w \mid w \in \{a,b\}^*, |w| = 2\}$

$$\boxed{aa+ab+ba+bb} = \boxed{(a+b)^2}$$

18 $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \leq 2}_{\text{atmost 2 length}}\}$

$$\boxed{\varepsilon + a+b+aa+ab+ba+bb} = (\varepsilon+a+b)^2$$

$\varepsilon \to 0$ len

$a+b \to 1$ len

$\varepsilon+a+b \to$ atmost 1 len

19 $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \geq 2}_{\text{atleast 2 length}}\}$

$$\boxed{(a+b)^+(a+b)} = \boxed{(a+b)^2(a+b)^*} = \boxed{(a+b)^*(a+b)^2} = \boxed{(a+b)(a+b)^*(a+b)}$$

$$\underbrace{|w| \le 2}$$

$|w| = 0$ or 1 or 2

$$\underbrace{\underset{\downarrow}{\varepsilon} + \underbrace{a+b}}$$

zero len

one length

almost 1 length

$$\underbrace{\underset{\le 1}{(\varepsilon+a+b)}\underset{\le 1}{(\varepsilon+a+b)}}_{\le 2 \text{ length}}$$

$$= \underline{\varepsilon} + \underline{a+b} + \underline{aa+ab+ba+bb}$$

almost 2 length

20 $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \text{ is even}}\}$

$|w|$ is divisible by 2
$|w|$ is multiple of 2
$|w| = 2n, n \geq 0$

$|w| \% 2 = 0$

$= (\underbrace{aa+ab+ba+bb})^*$

$= [(a+b)^2]^*$

$\{a,b\}$

$\underbrace{\boxed{a} \overset{\text{or}}{+} \boxed{b}}$

one len   one len

any one length

$\underline{aa}$
2 length

H.W.

(21) $\{w \mid w \in \{a,b\}^*,\ |w| = odd\}$

$\boxed{\underbrace{[(a+b)^2]^*}_{even}\ \underbrace{(a+b)}_{one\ len}}$

$\boxed{|w| \% 2 = 1}$

$\boxed{|w| = 2n+1,\ n \geq 0}$

(22) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = 2\} \Rightarrow \boxed{b^* \boxed{a}\, b^* \boxed{a}\, b^*}$

(23) $\{$ " $,\ n_a(w) \leq 2\}$

(24) $\{$ " $,\ n_a(w) \geq 2\}$

(25) $\{$ " $,\ n_a(w) = even\}$

(26) $\{$ " $,\ n_a(w) = odd\}$

H.W.

(27) $\{w \mid w \in \{a,b\}^*,$ 2nd symbol from begin is 'a'$\}$

(28) $\{w \mid$ " , 2nd symbol from end is 'a'$\}$

(29) $\{w \mid$ " , 2nd symbol is 'a' OR

4th symbol is 'b'$\}$

(30) $\{w \mid w \in \{a,b\}^*,$ w starts with a,

w ends with 'b'$\}$

Summary

Simplification ?

Writing Reg Exp. ?

Thank you