CS & IT ENGINEERING

GRAPH THEORY

Lecture No. 1

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TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Discrete maths: (8 – 14)

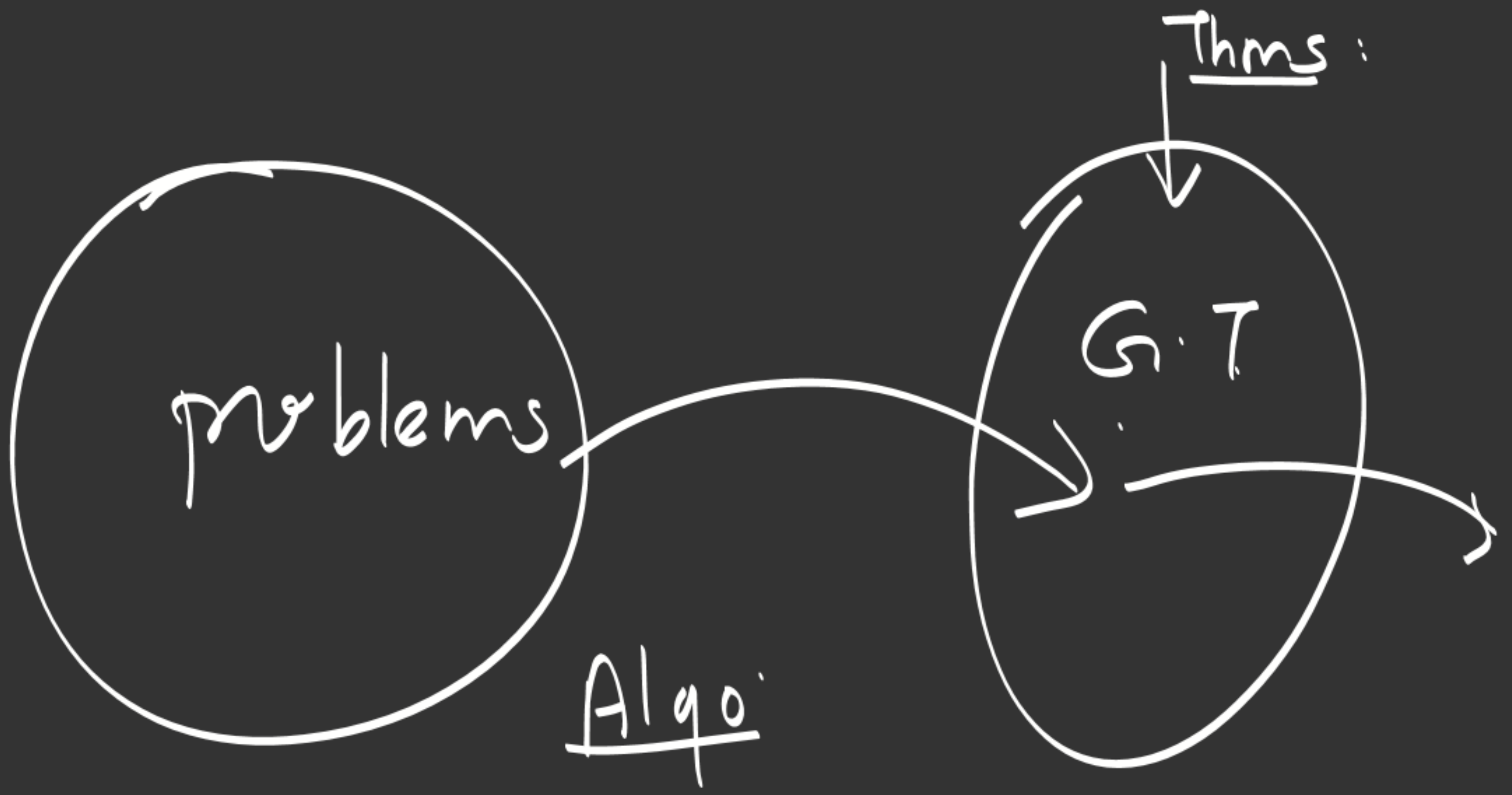
* Graph Theory (4 - 6)

  Set theory (2 - 4)

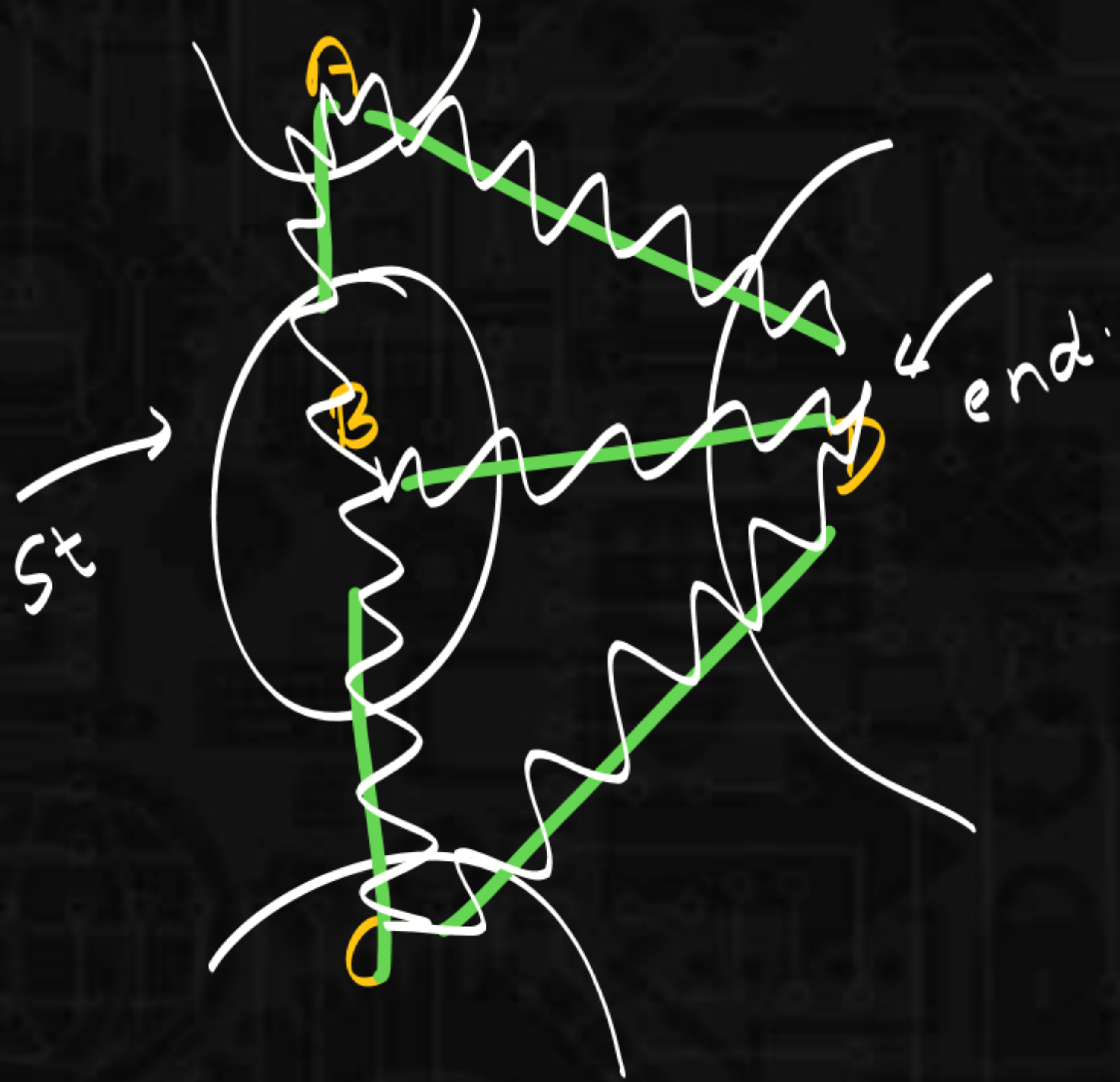
  logic (2 – 4)

  Combinatorics (2 - 4)

  count + properties


Thms :
problems
G.T
Algo :

Basics of Graph



1) A

2) cross all bridges exactly once.

Basics of Graph

point/joint → vertex/vertices.

line/branch → edge/edges.

Graph $G = (V, E)$

E → set of edges.

lines.

V → set of vertices

set of point

$V = \{ v_1, v_2, v_3 \dots \}$

$E = \{ e_1, e_2, e_3 \dots \}$

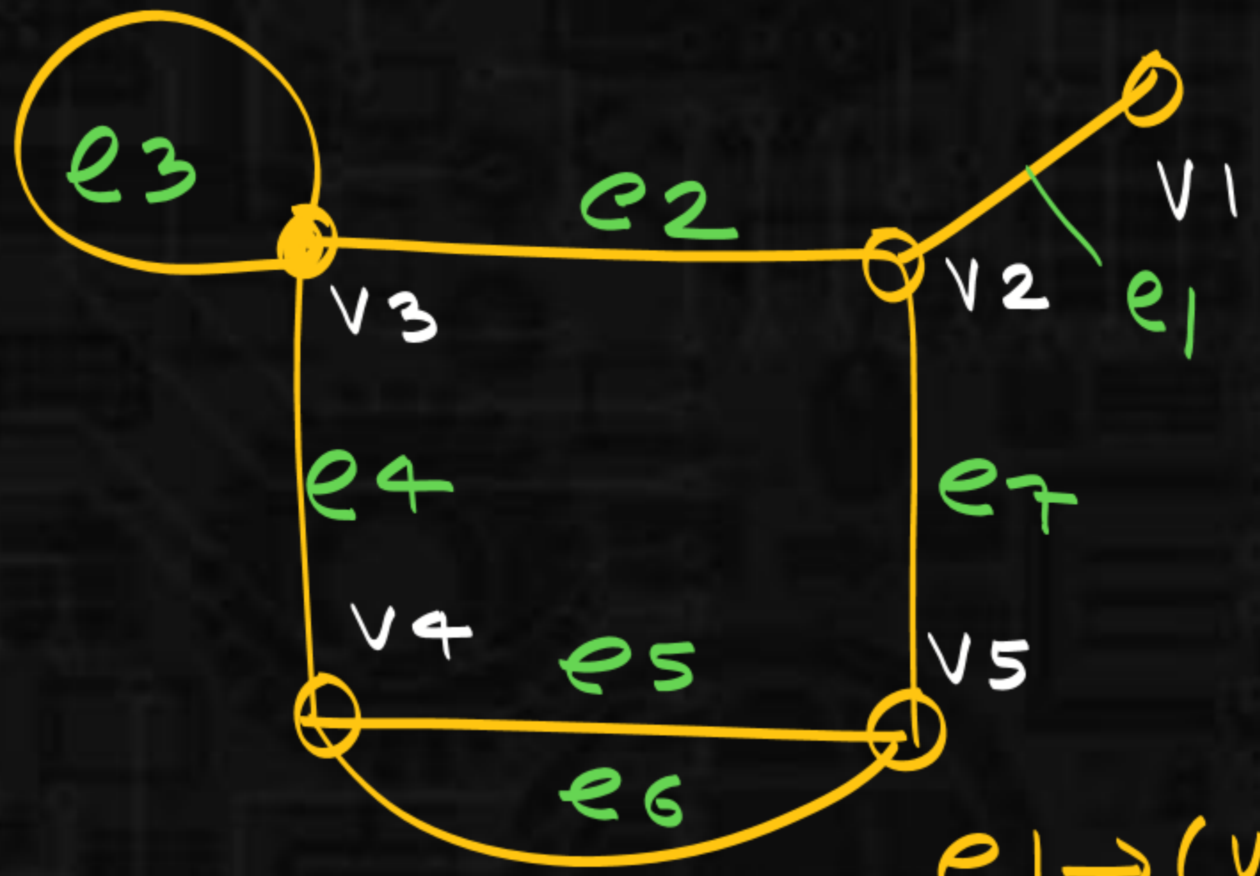
Basics of Graph

$G = (V, E)$

$V = \{v1, v2, v3, v4, v5\}$

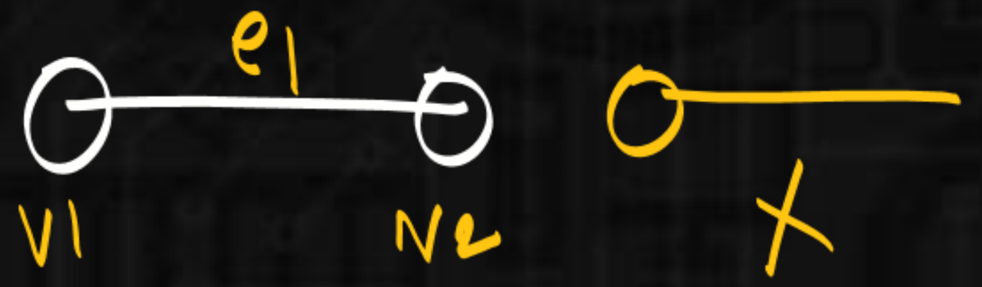
$E = \{e1, e2, \ldots\ldots e7\}$

→ every edge must be associated with $\boxed{\text{unordered}}$ pair of vertices



$e1 \rightarrow (v1, v2)$

$e1 \rightarrow (v2, v1)$



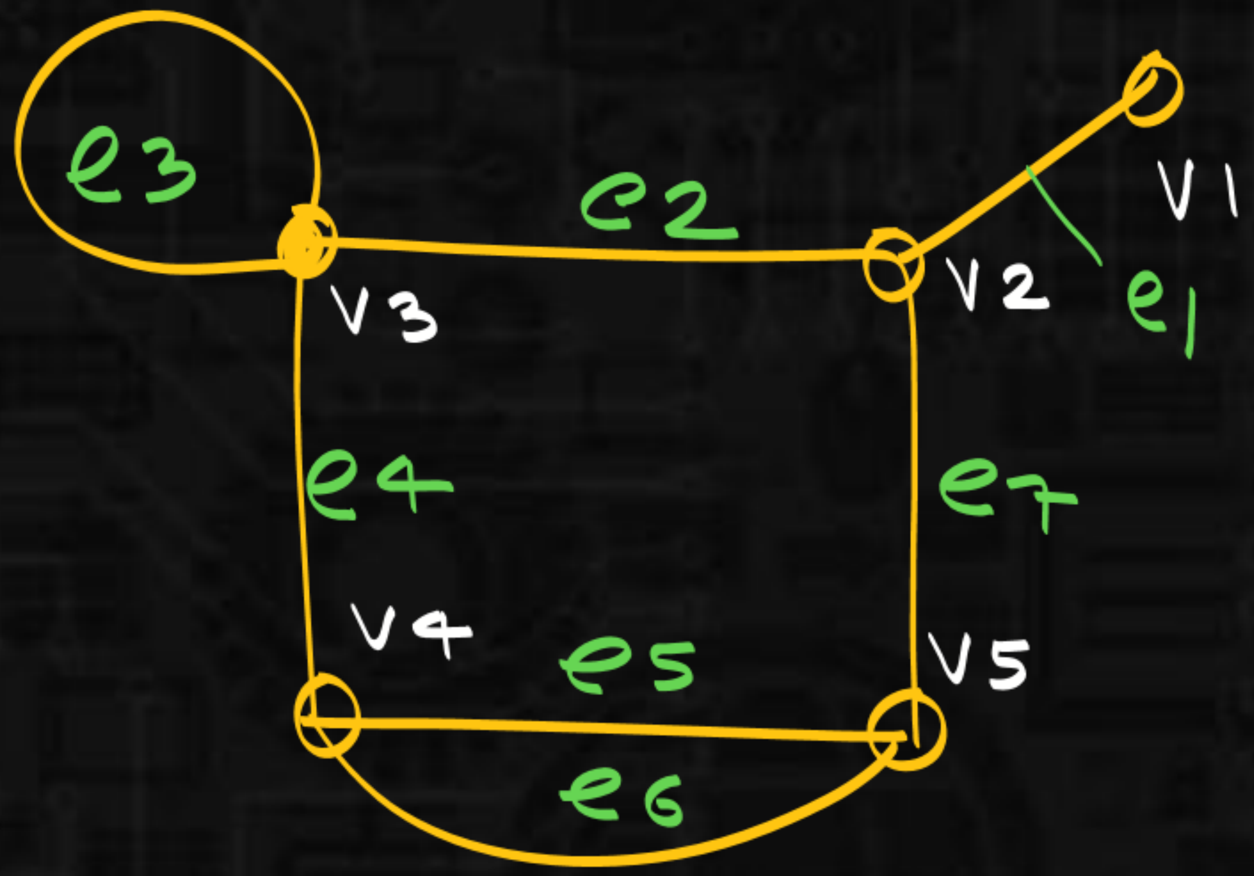
Basics of Graph

$\psi : E \rightarrow V \times V$

$e_1 \rightarrow (v_1, v_2)$

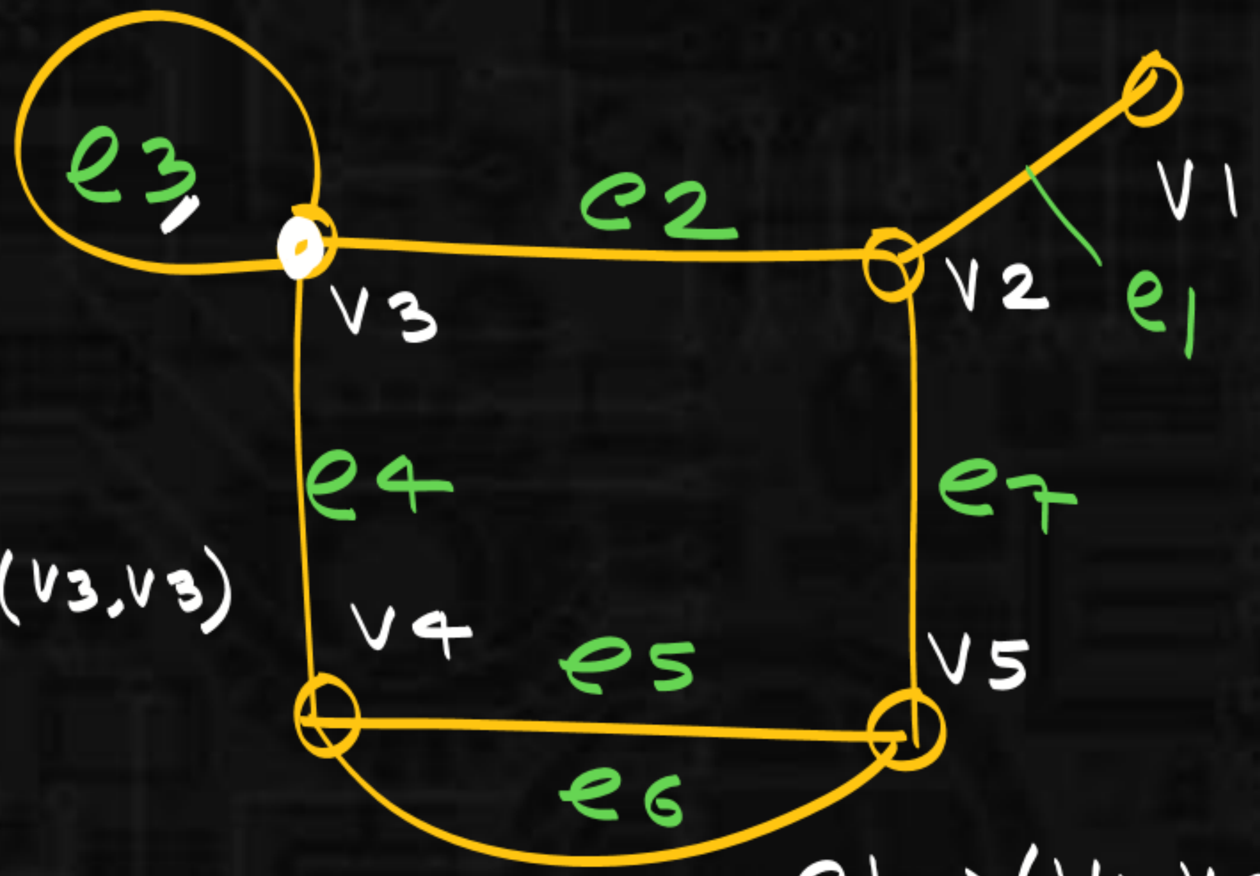
$e_2 \rightarrow (v_2, v_3)$

$e_3 \rightarrow (v_3, v_3)$



$$\boxed{\begin{gathered} G = (V, E, \psi) \\ \psi : E \rightarrow V \times V \end{gathered}}$$

Basics of Graph



end vertices:

unorder pair vertices are called end vertices.

$e_3 \rightarrow (v_3, v_3)$

$e_1 \rightarrow (v_1, v_2)$

$$\boxed{\begin{array}{l} e_5 \rightarrow (v_4, v_5) \\ e_6 \rightarrow (v_4, v_5) \end{array}}$$

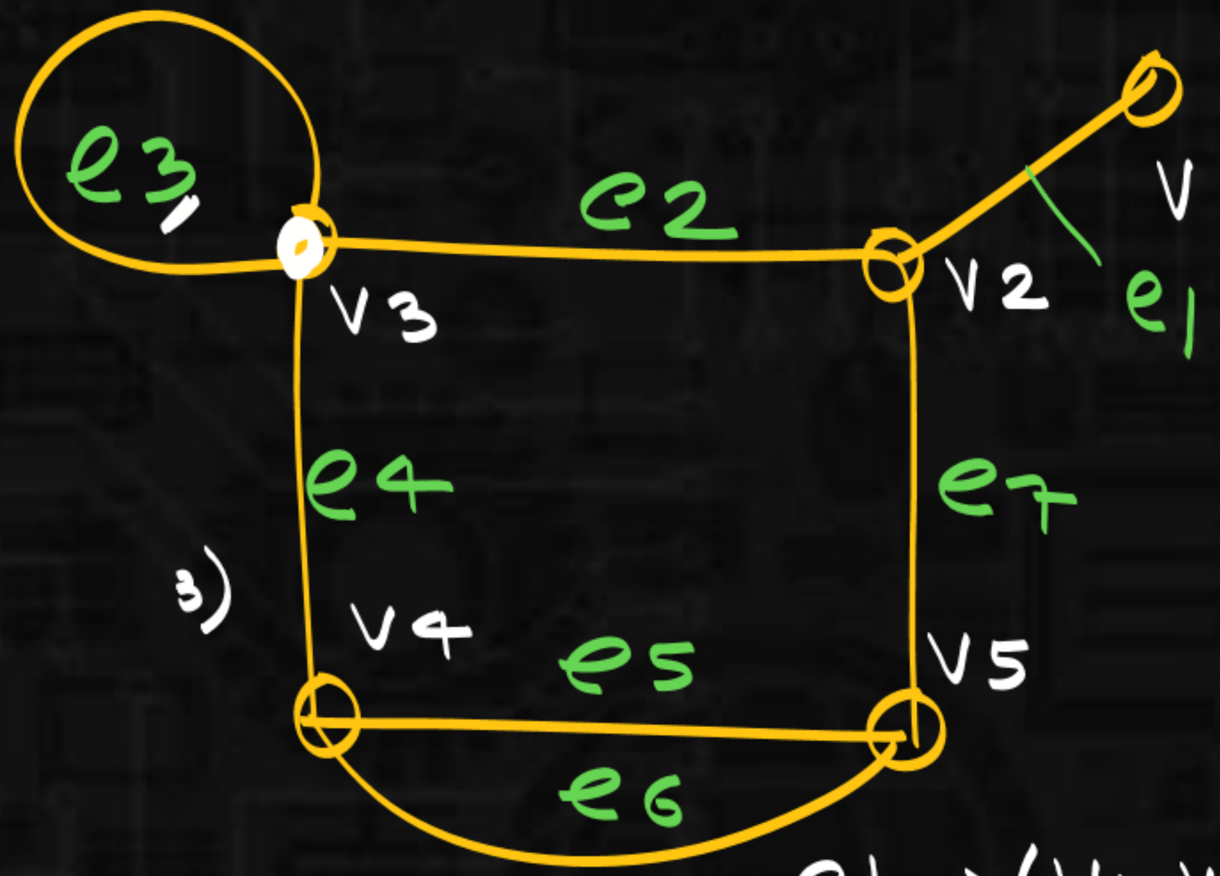
|| edges:

2 or more edges associated with same end vertices.

Self loop / loop:

edge associated with same end vertices.

Basics of Graph



incident point : 

meeting point is called as incident point.

Degree/valency $d(v_i)$

no. of edges incident on a vertex.

$e1 \rightarrow (v1, v2)$

$e5 \rightarrow (v4, v5)$
$e6 \rightarrow (v4, v5)$

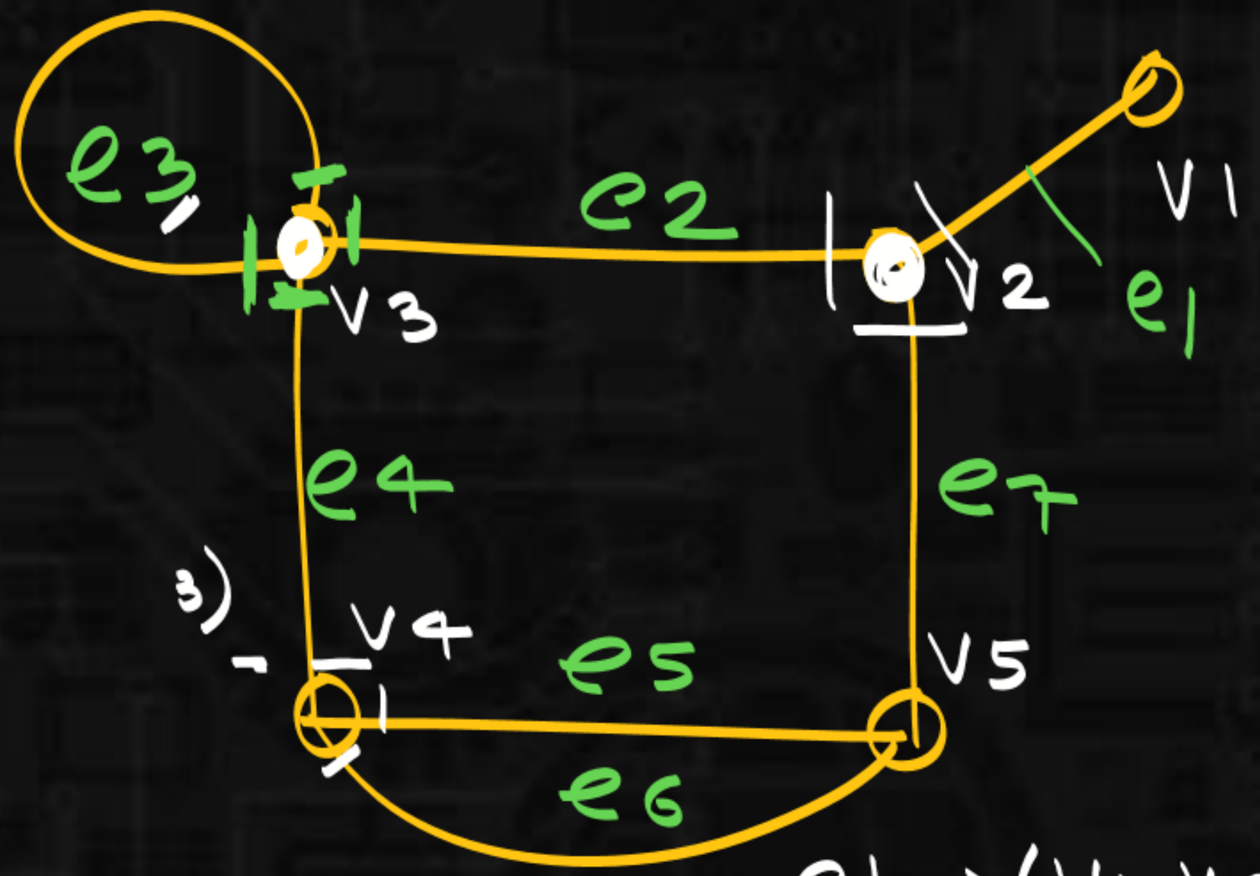
Basics of Graph

$d(v_1) = 1$. $d(v_3) = 4$.

$d(v_2) = 3$

$d(v_4) = 3$

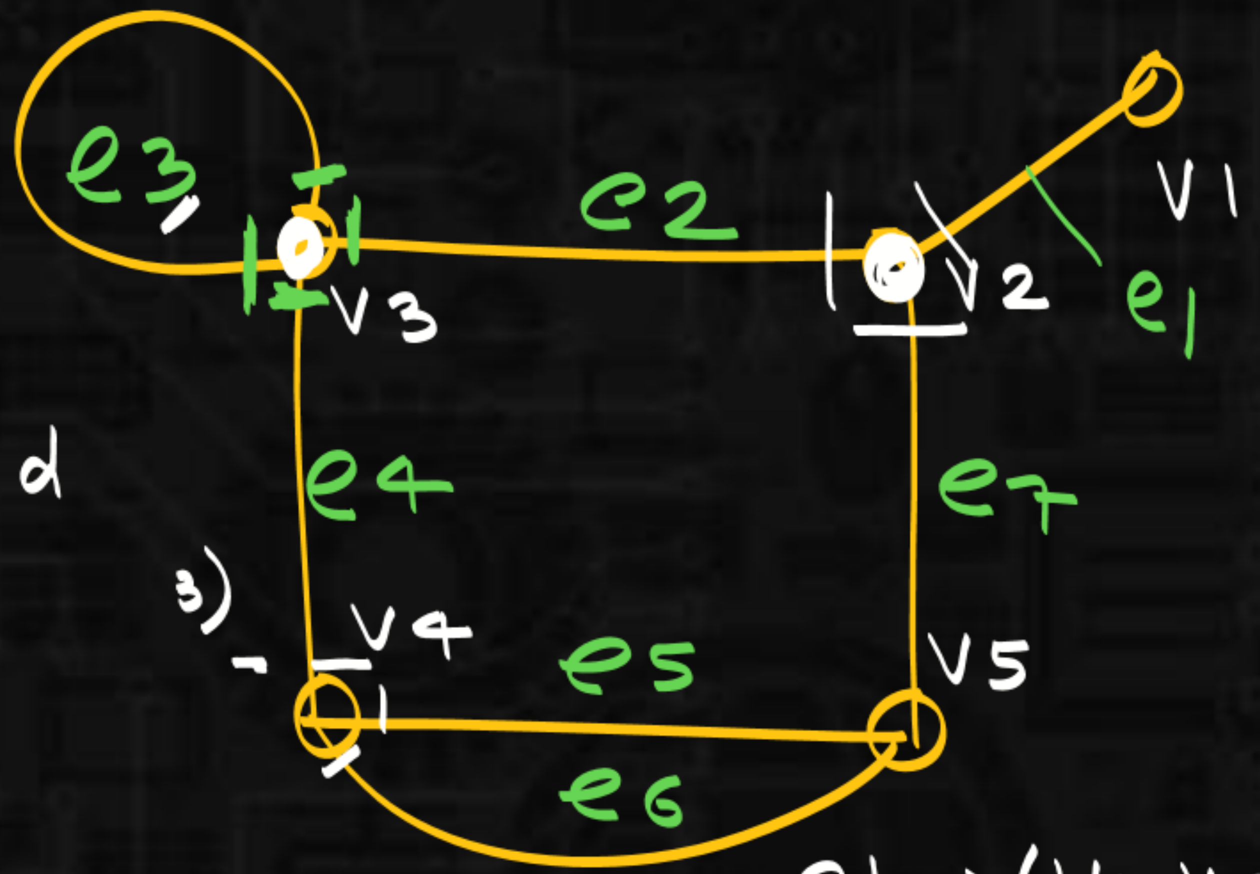
$d(v_5) = 3$



$e1 \rightarrow (v1, v2)$

$e5 \rightarrow (v4, v5)$
$e6 \rightarrow (v4, v5)$

Basics of Graph



isolated vertex:

Degree 0 vertex is called isolated vertex.

pendant vertex:

Degree 1 vertex is called pendant vertex.

$e1 \rightarrow (V1, V2)$

$$\boxed{\begin{array}{l} e5 \rightarrow (V4, V5) \\ e6 \rightarrow (V4, V5) \end{array}}$$

Null Graph: set of isolated vertices

Basics of Graph

$d(v_1) = 1$. $d(v_3) = 4$.

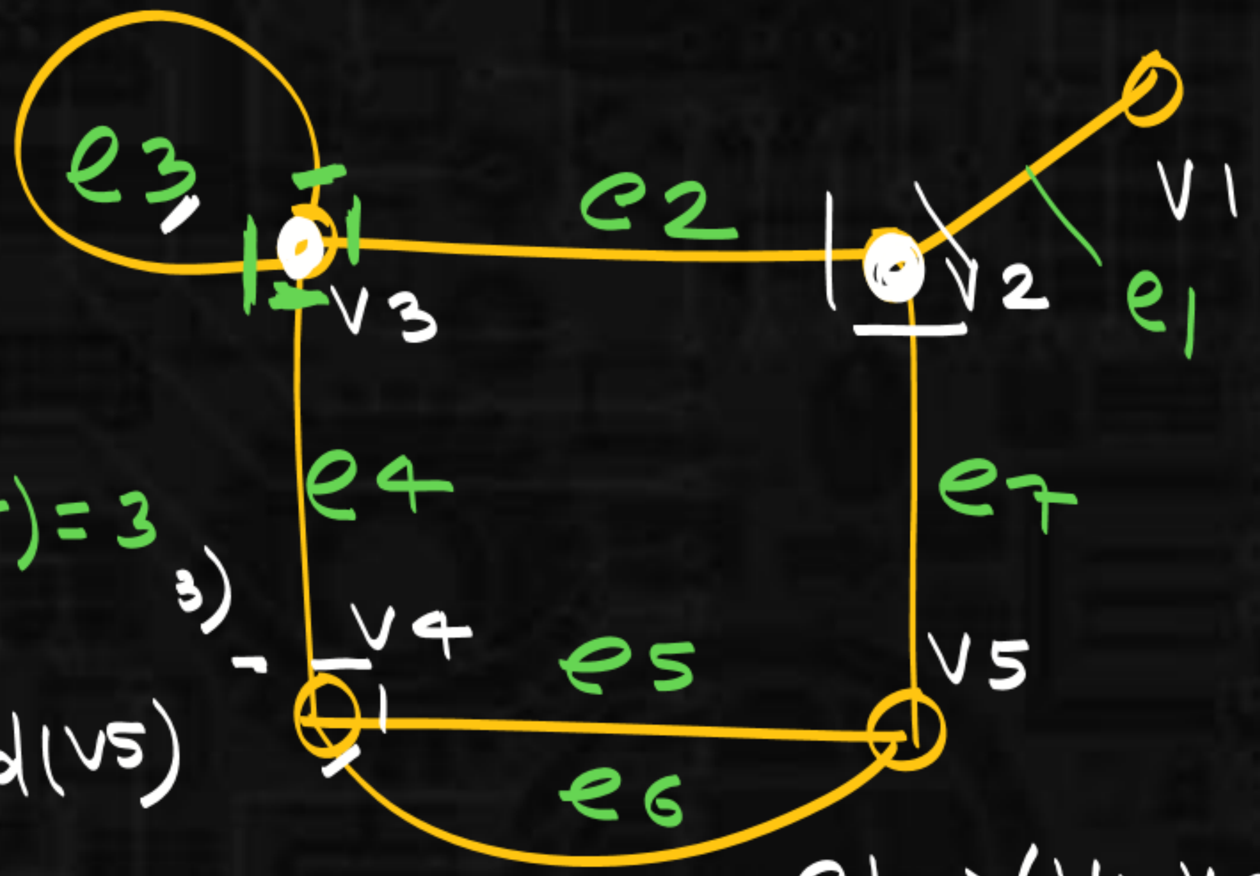
$d(v_2) = 3$ $d(v_4) = 3$ $d(v_5) = 3$

odd odd even

$d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$

$= 1 + 3 + 4 + 3 + 3$

$= 14 = 2 \times 7 \rightarrow$ no. of edges.



$e1 \rightarrow (v1, v2)$

$\boxed{\begin{array}{l} e5 \rightarrow (v4, v5) \\ e6 \rightarrow (v4, v5) \end{array}}$

Basics of Graph

Thm 1: sum of degrees of all the vertices is equals to twice the no. of edges.

Handshaking Thm

even

$$\boxed{\sum d(v_i) = 2e}$$

$$\boxed{\sum d(v_i) = 17}$$

not possible.

note:

Sum of degrees of all vertices will be even.

Graph may have 7 edges

Basics of Graph

L.H.S		R.H.S
Degrees	=	2 × no.of edges
2	=	2×(1)
2+2	=	2(1+1)
2+2+2+2	=	2(1+1+1+1)

Basics of Graph

$\sum d(v_i) = 2e = \text{even}.$

$d(v_1) + d(v_2) + d(v_3) + \ldots d(v_n) = \text{even}$

odd

$1 + 3 = \text{even}$

$1 + 3 + 5 = \text{odd}$

odd degree | even degree

$\boxed{O, O, O} + \boxed{} = \text{even}$

$0 + 0 = \text{even}$

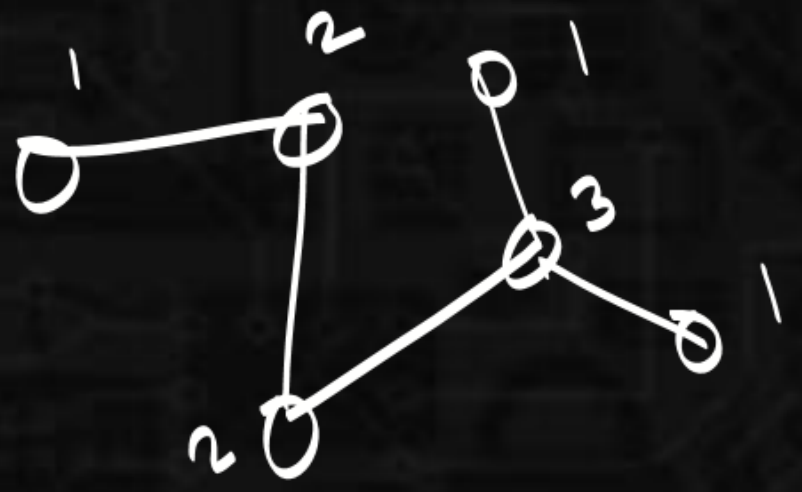
$0 + 0 + 0 = \text{odd}.$

$\begin{array}{l} \text{odd} + \text{even} \\ \hline \text{odd} \neq \text{even}. \end{array}$

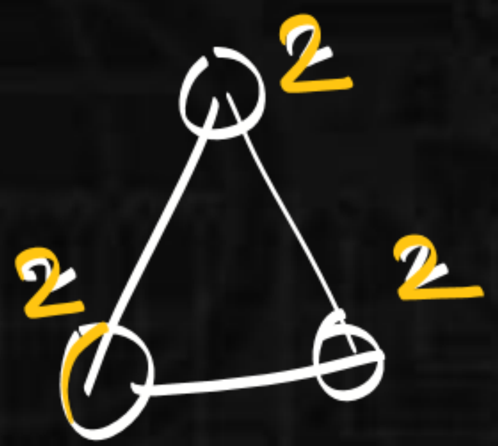
Basics of Graph

<u>Thm 2</u>: No. of odd degree vertices in a graph will always be even.

no. of odd degree = 2

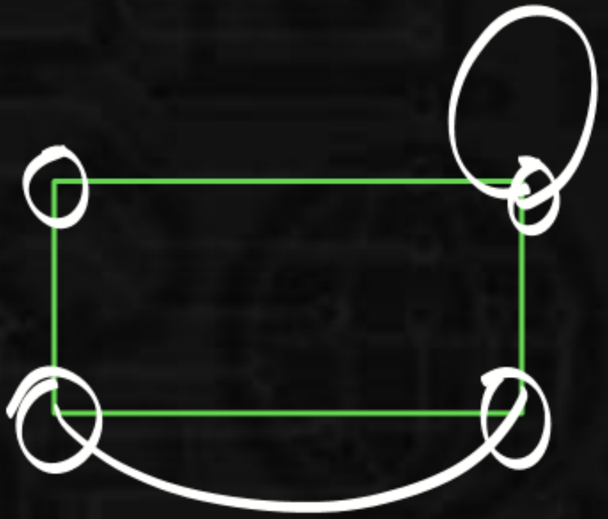


no. of odd degree = 4



no. of odd degree vertices = 0

Basics of Graph



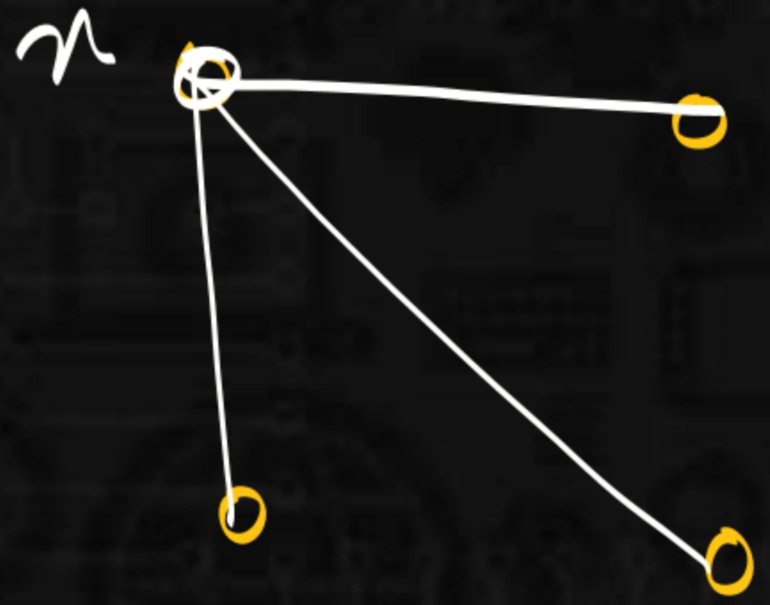
	loop	∥ edges.
Simple Graph	✗	✗
multigraph	✗	✓
Pseudograph.	✓	✓

Basics of Graph

Thm3 :

maximum degree in Simple graph of n vertices $\leq n-1$.

$n = 4$

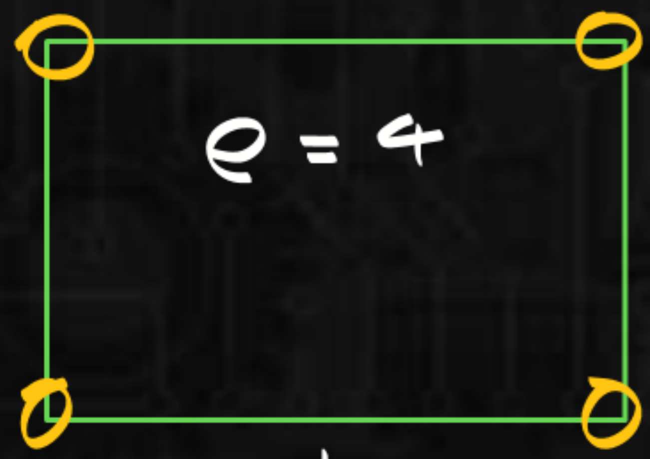


Basics of Graph

≤ 3

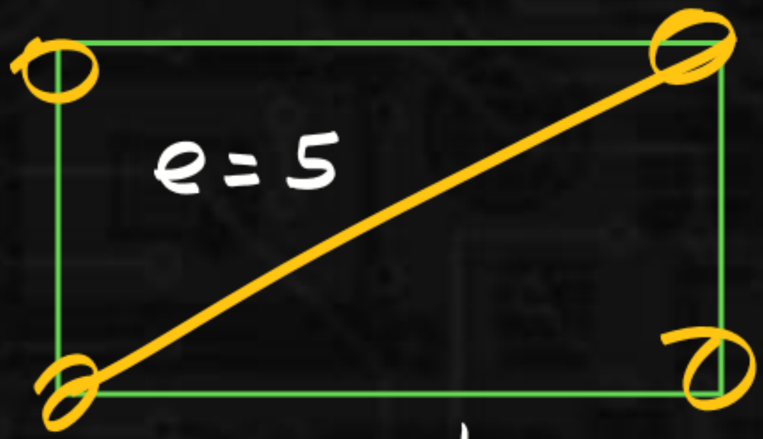
$n = 4$.

max. degree : 3

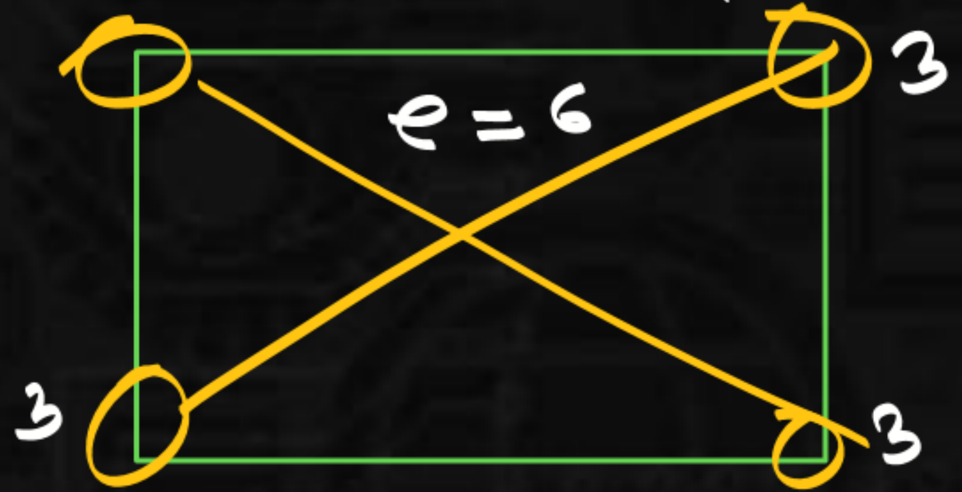


↓

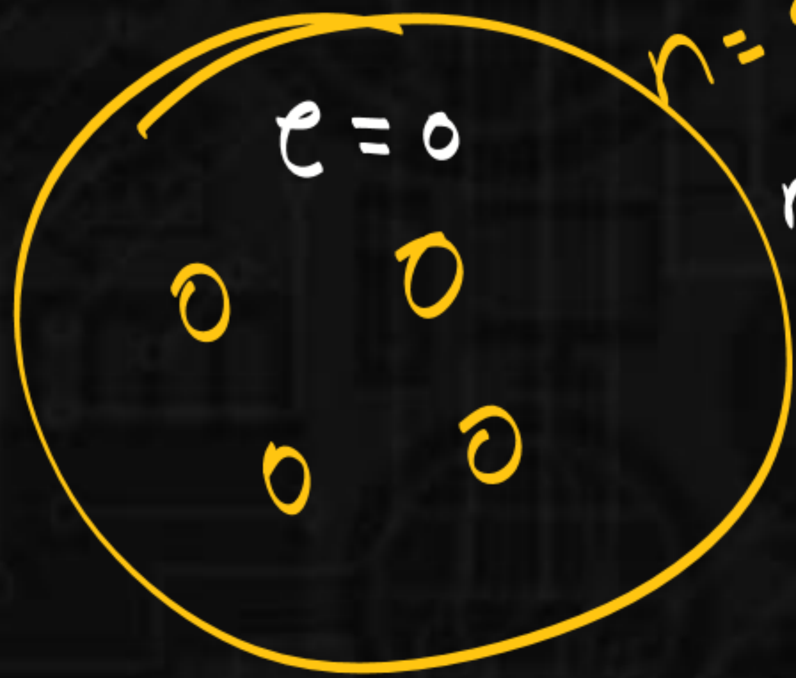
maximum deg: 2.



max. deg: 3



$n = 4$

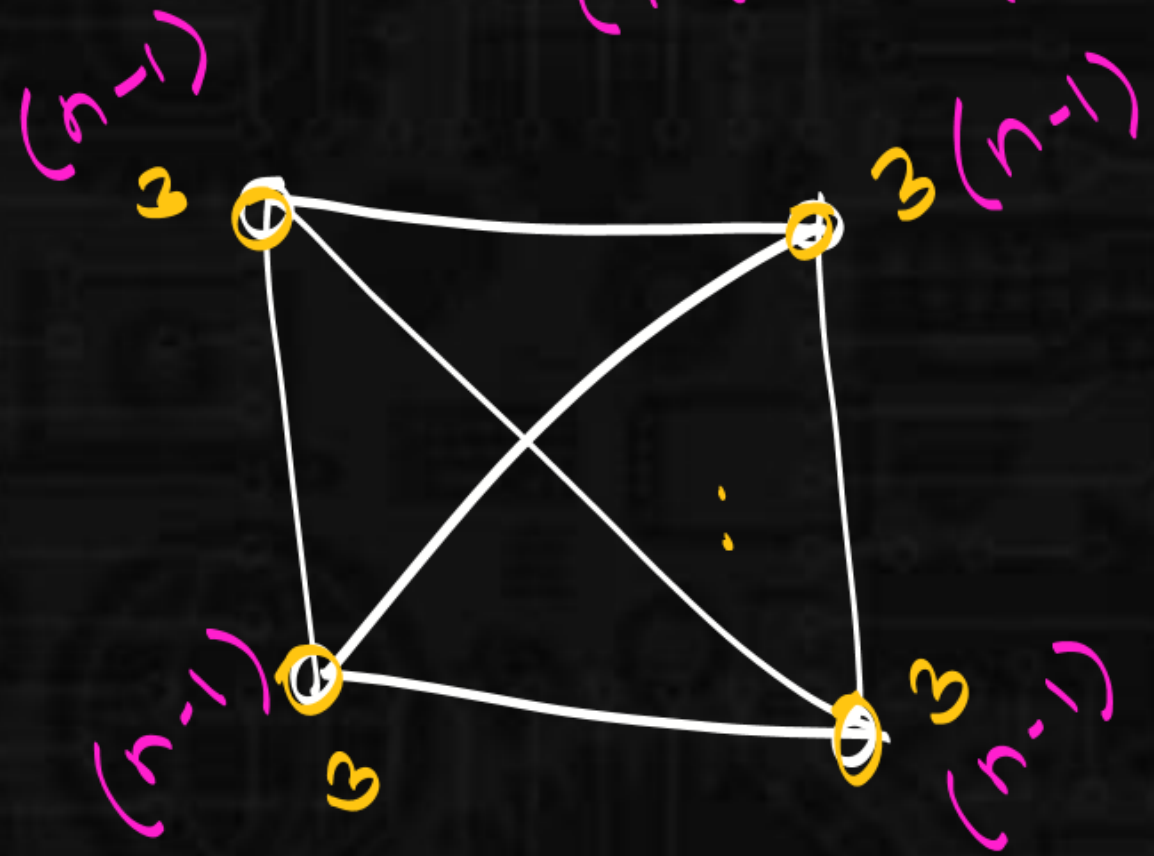


max deg: 0

Basics of Graph

Thm 4: In Simple Graph maximum no of edges $\leq \frac{n(n-1)}{2}$

$n = 4$ (Total vertices)



$n(n-1) = 2e$

$$\boxed{e = \frac{n(n-1)}{2}}$$


THANK YOU GW
SOLDIERS !