CS & IT ENGINEERING

Graph Theory

Matching and Covering

Lecture No.10

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TOPICS TO BE COVERED

01 Independent set

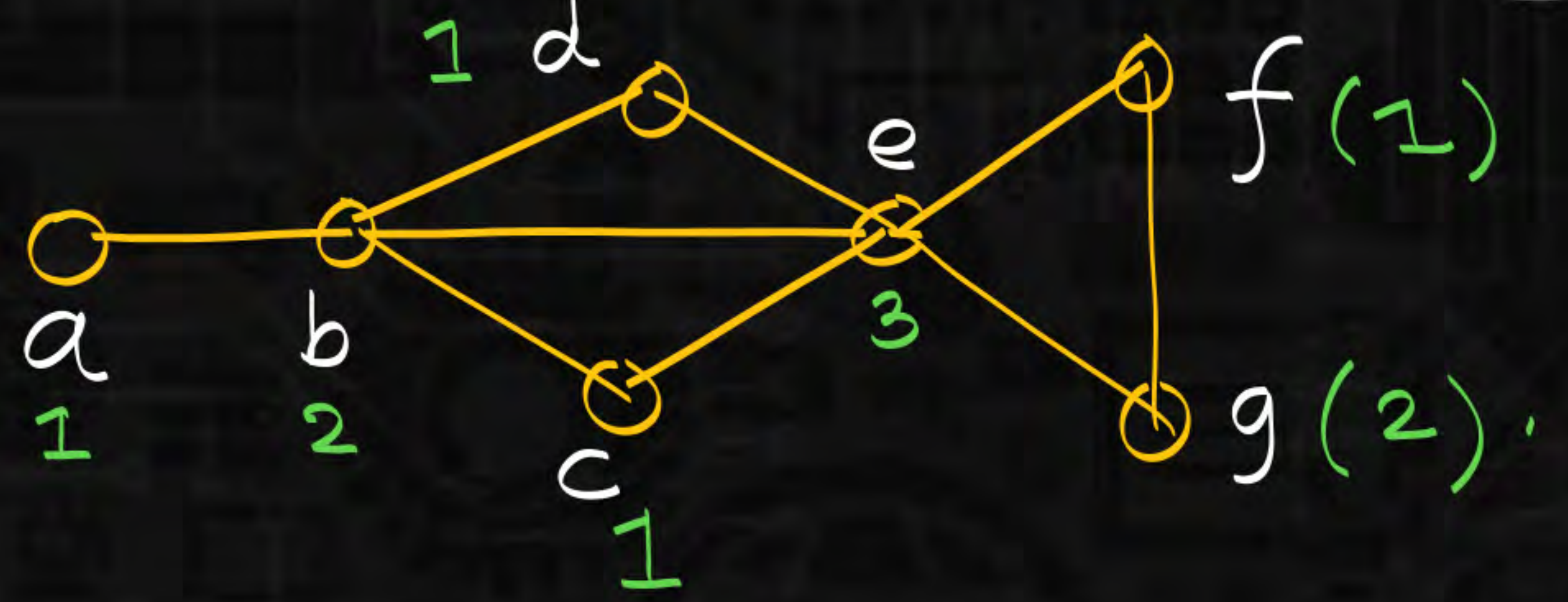
02 Maximal Independent set

03 Dominance set

04 Minimal dominating set

05 Domination number

$\chi(G) = 3.$

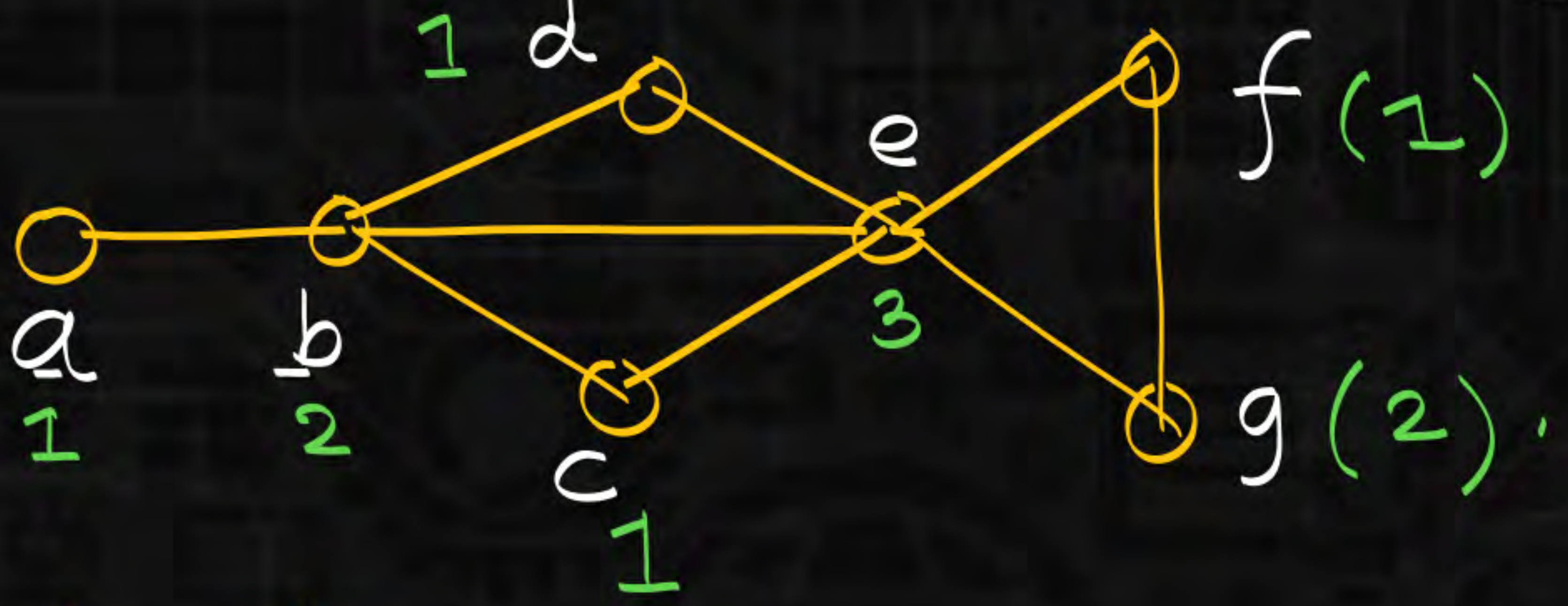


$1 \rightarrow \{a, c, d, f\}$

$2 \rightarrow \{b, g\}$

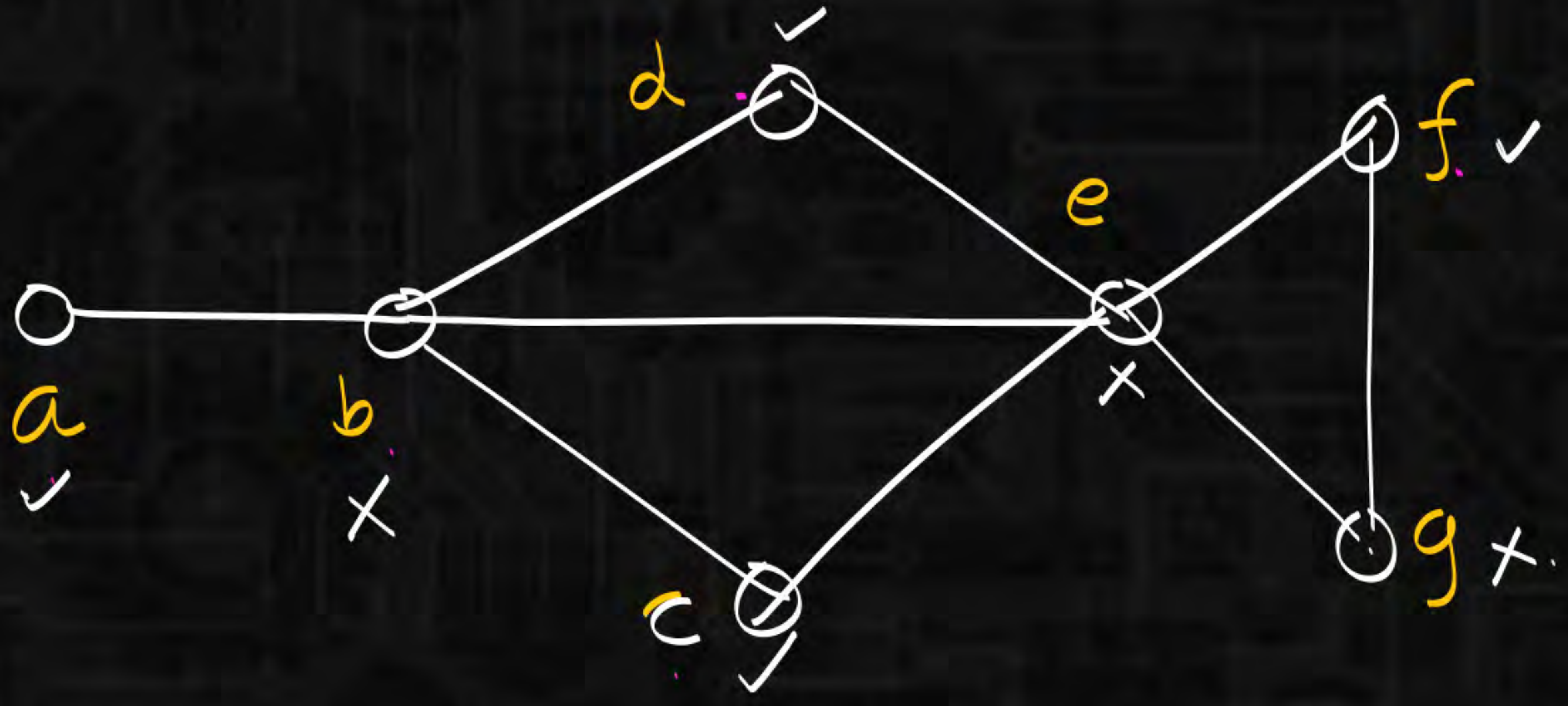
$3 \rightarrow \{e\}$

$\{a, c, d, f\}$



Set of non adjacent vertices.

Independent set:



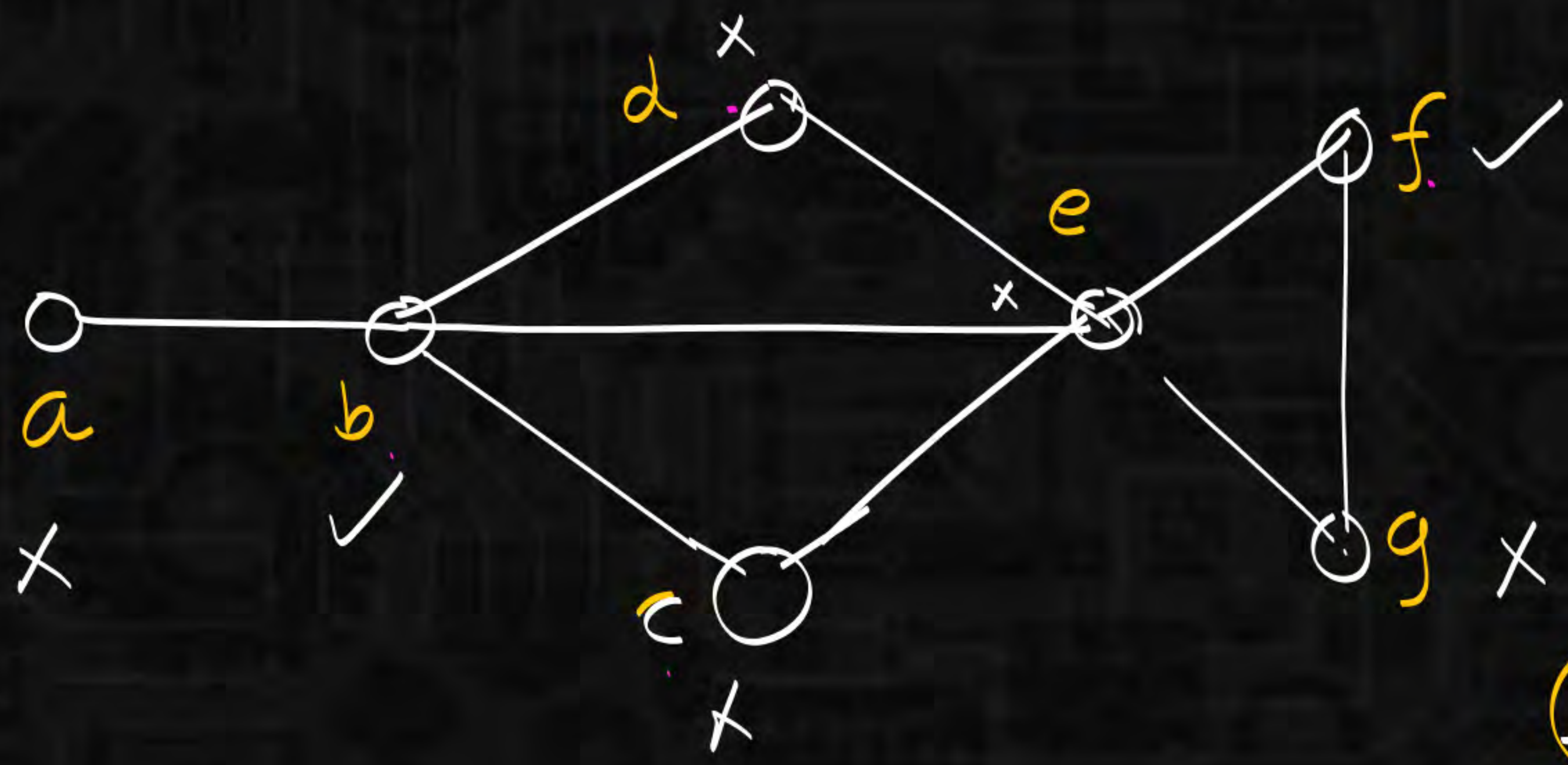
$\{a\}$

$\{a\}$ – IS.

$\{a, c\}$ – IS.

$\{a, c, d\}$ – IS.

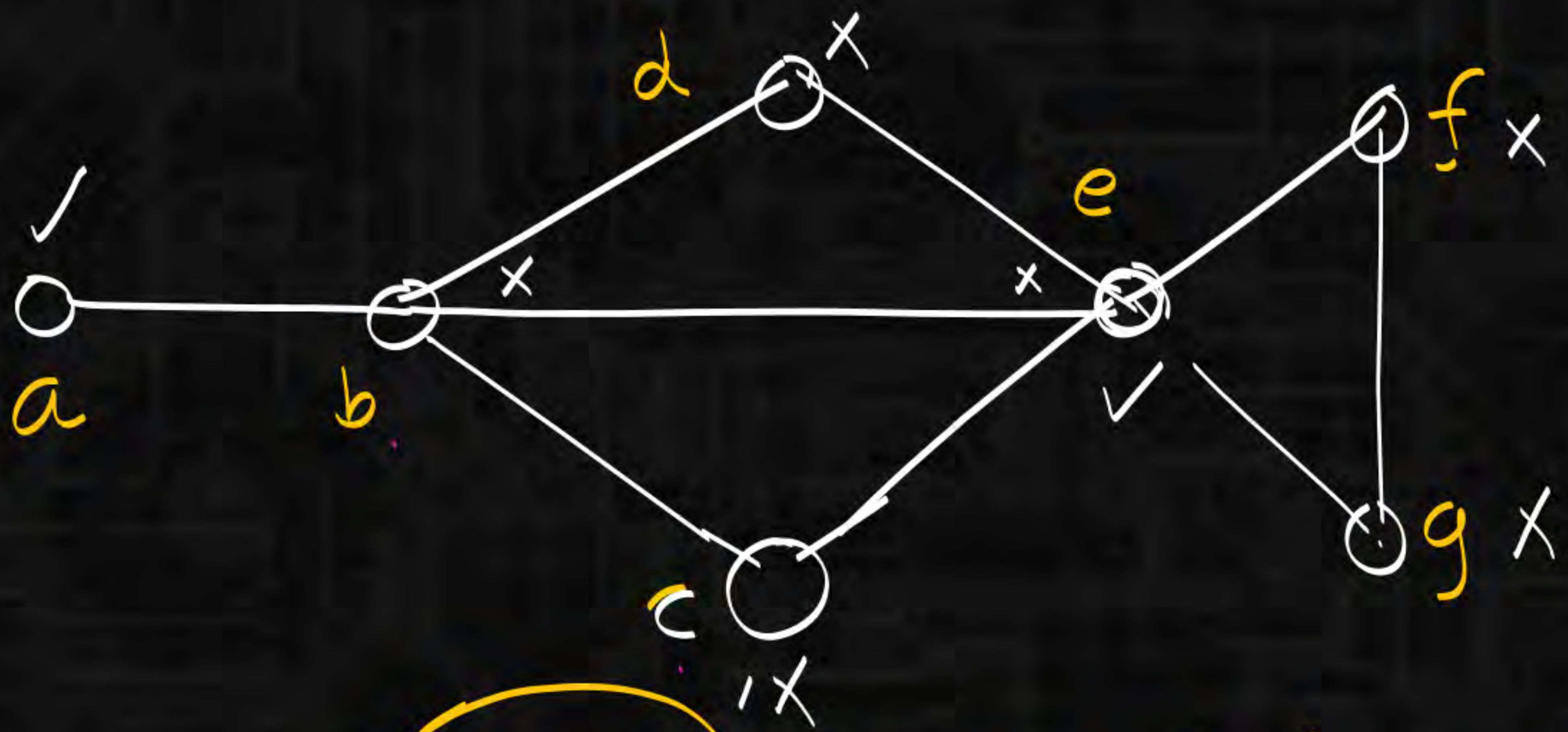
$\{a, c, d, f\}$ – IS.



$\{b\}$ – is. ✓

✗ $\{b, f\}$ – is. ✓

maximal Independent set.

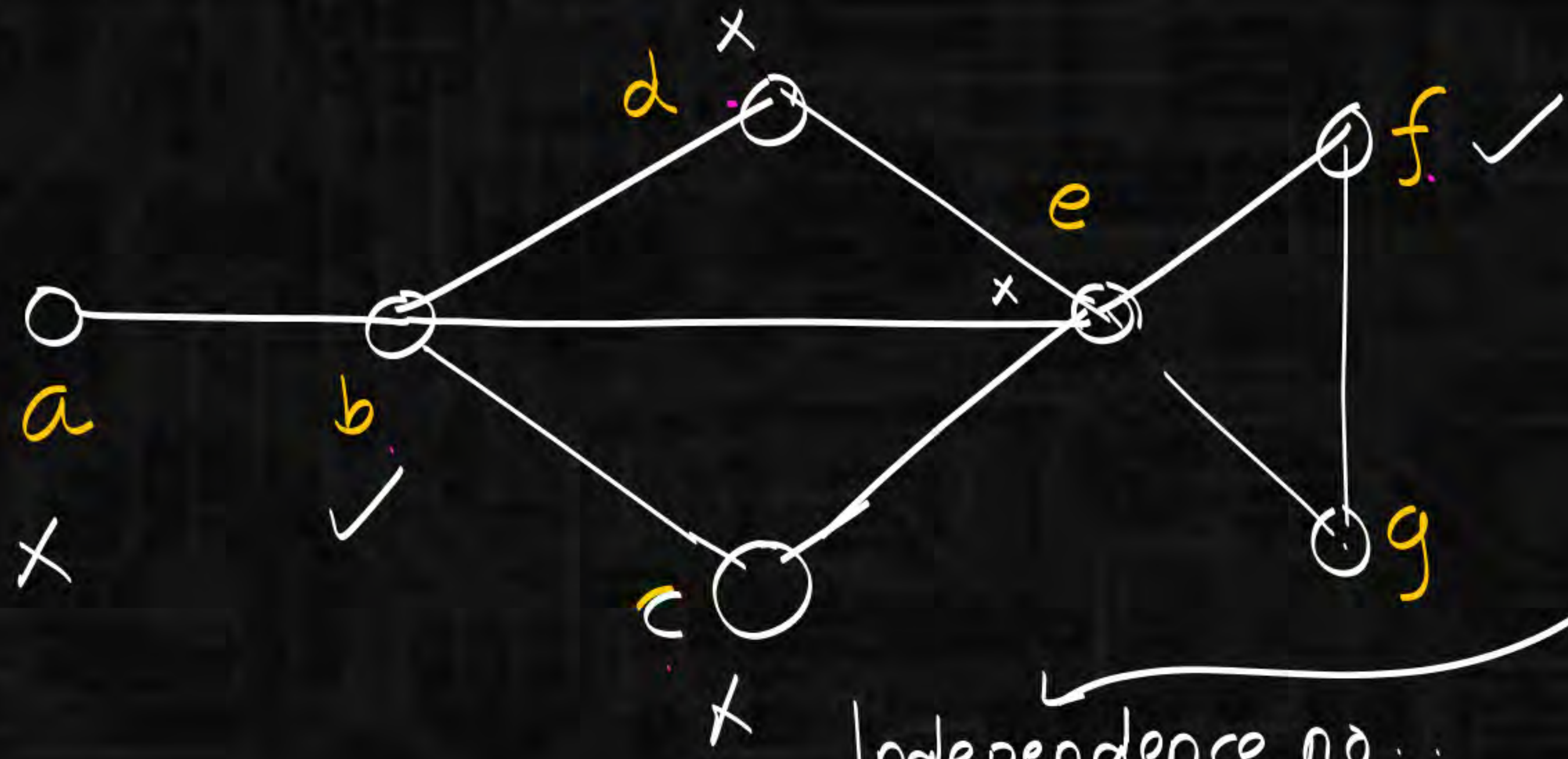


{a} – IS.

{a, e} (MIS)

maximal Independent set : Independent set such that we cannot add new vertex into this.

maximal → not Related to size but property.



maximal Independent set:

$\{a, c, d, f\}$

$\{b, f\}$

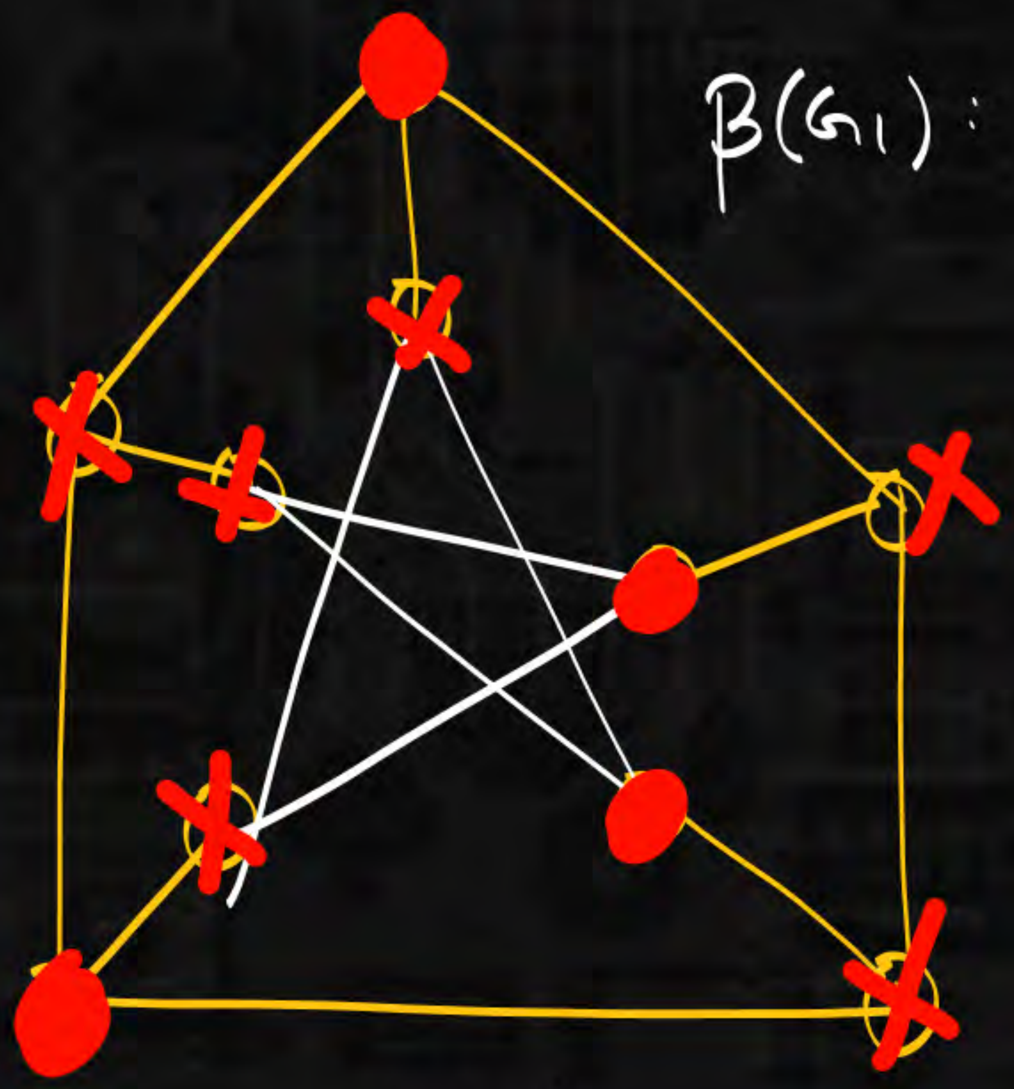
$\{b, g\}$

$\{a, e\}$

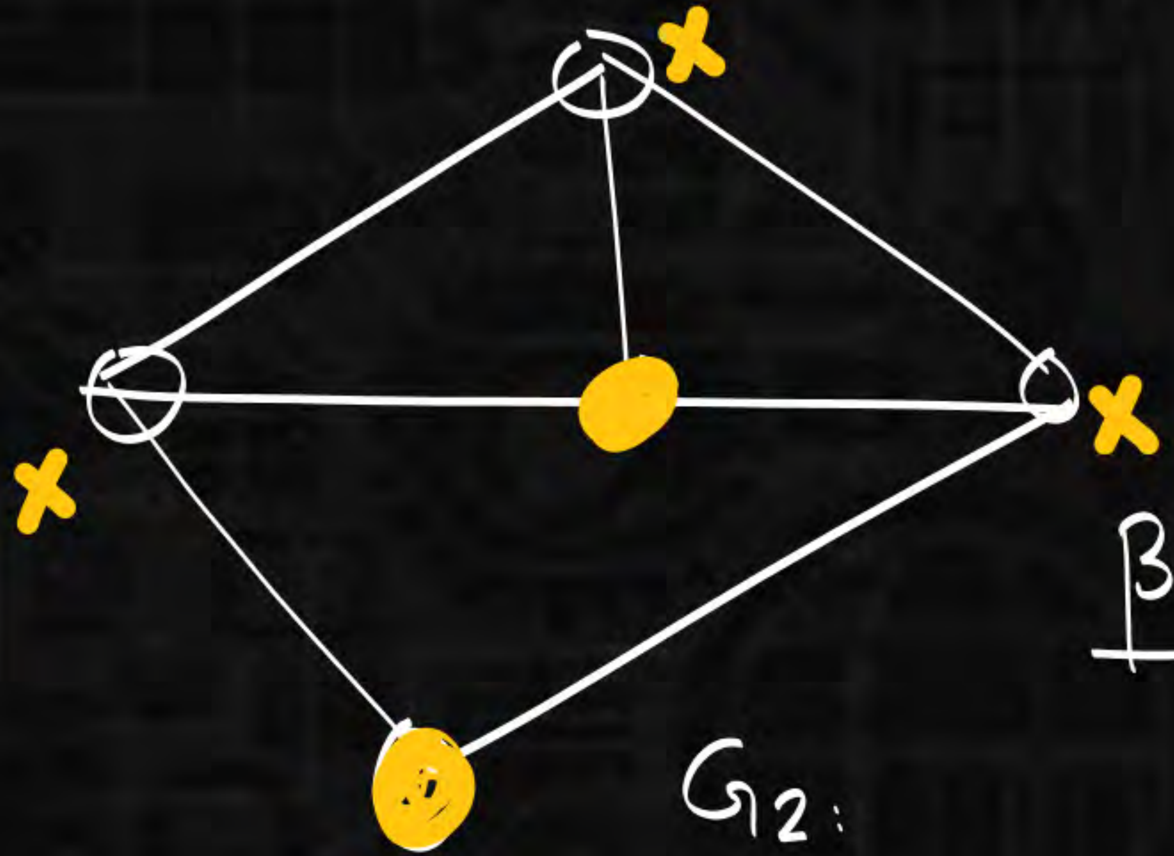
Independence no:

$\beta(G): 4$ no of vertices present in largest MIS.

$\beta(G_1)$:

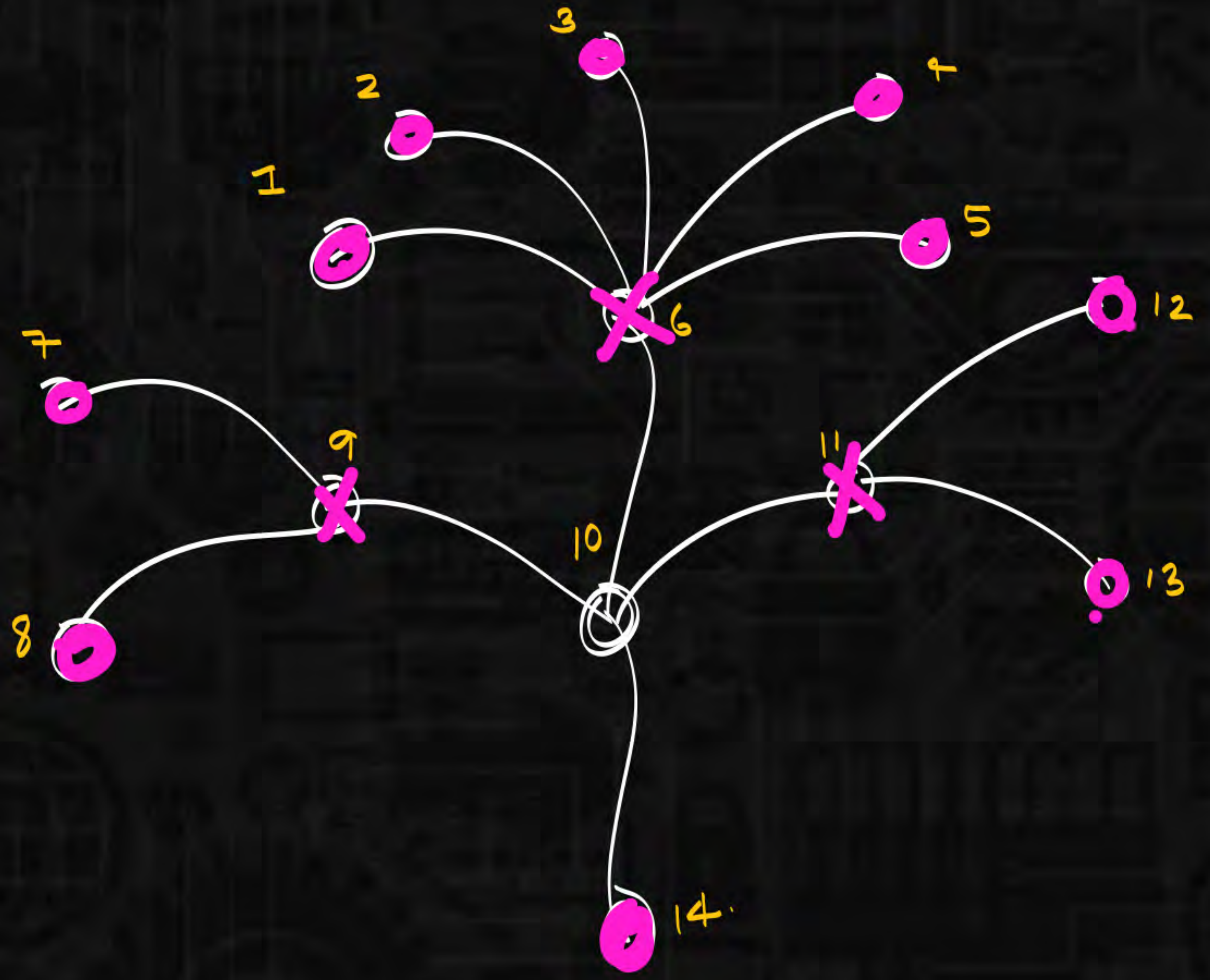


$\beta(G_1) = 4$



$\beta(G_2) : 2$

G_2 :

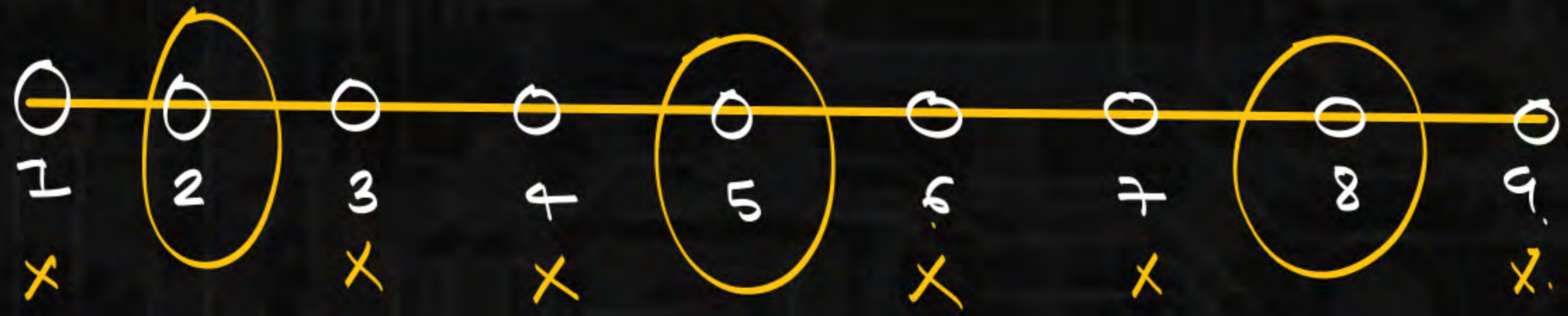


$\beta(G)$:

largest MIS

{1, 3, 5, 7, 9} – mis

{2, 4, 6, 8} – mis



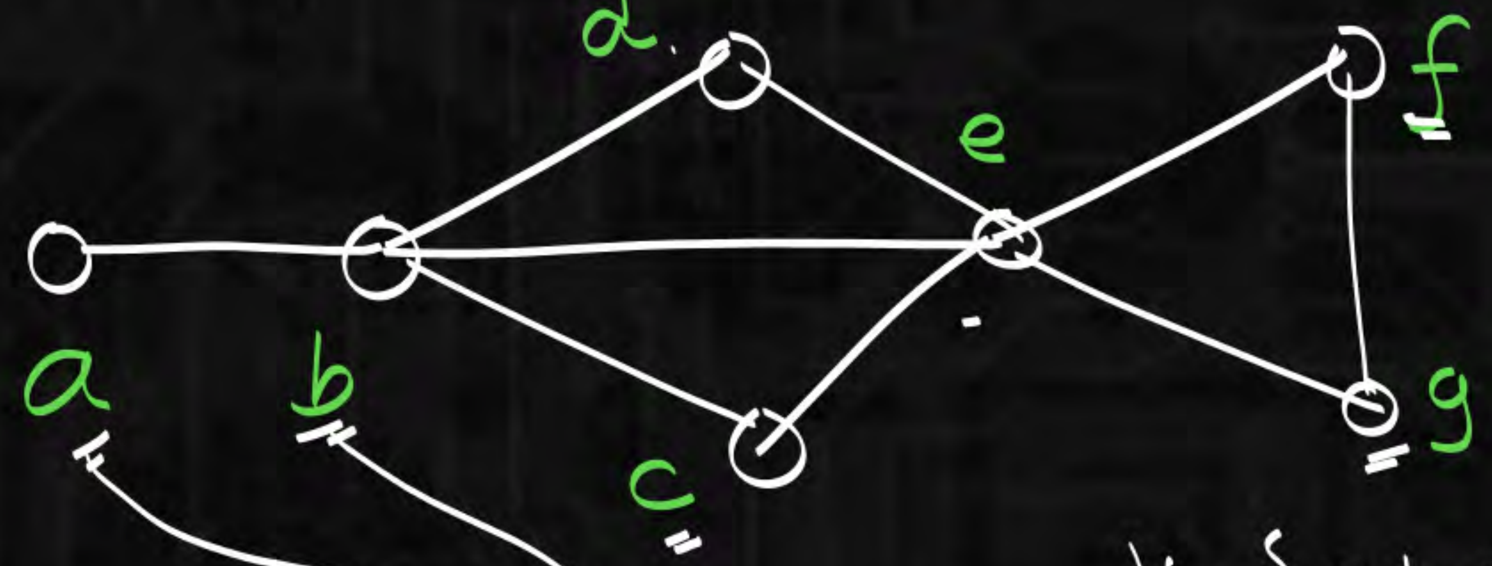
$\{2, 5, 8\}$ Smallest mis size is 3.

Dominating set :

Graph $G=(V,E)$ Dominating set D

$D \subseteq V$

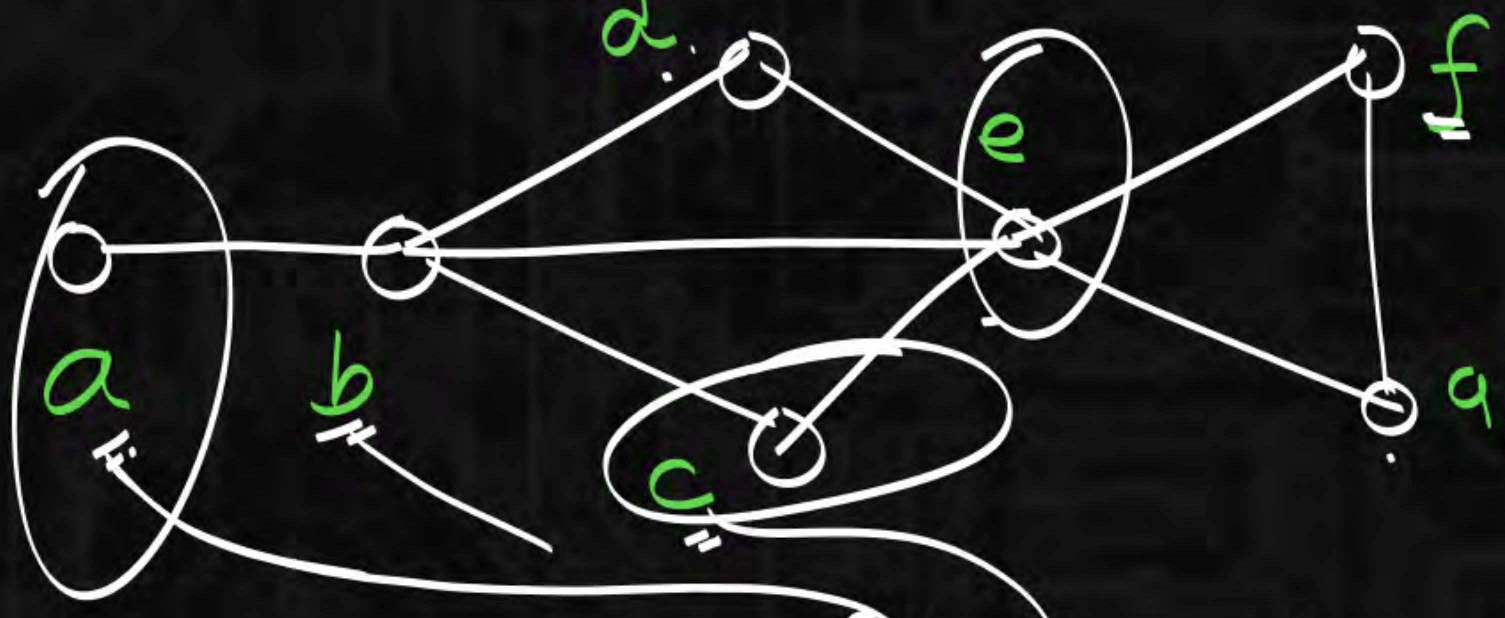
if we take any vertex from G either that vertex belongs to D or its adjacent will belongs to D.



$V = \{a\ b\ c\ d\ e\ f\ g\}$

$\{a, b, f\} \rightarrow$ Dominating set

Directly OR adjacent } belongs



$\{a, c, e\}$

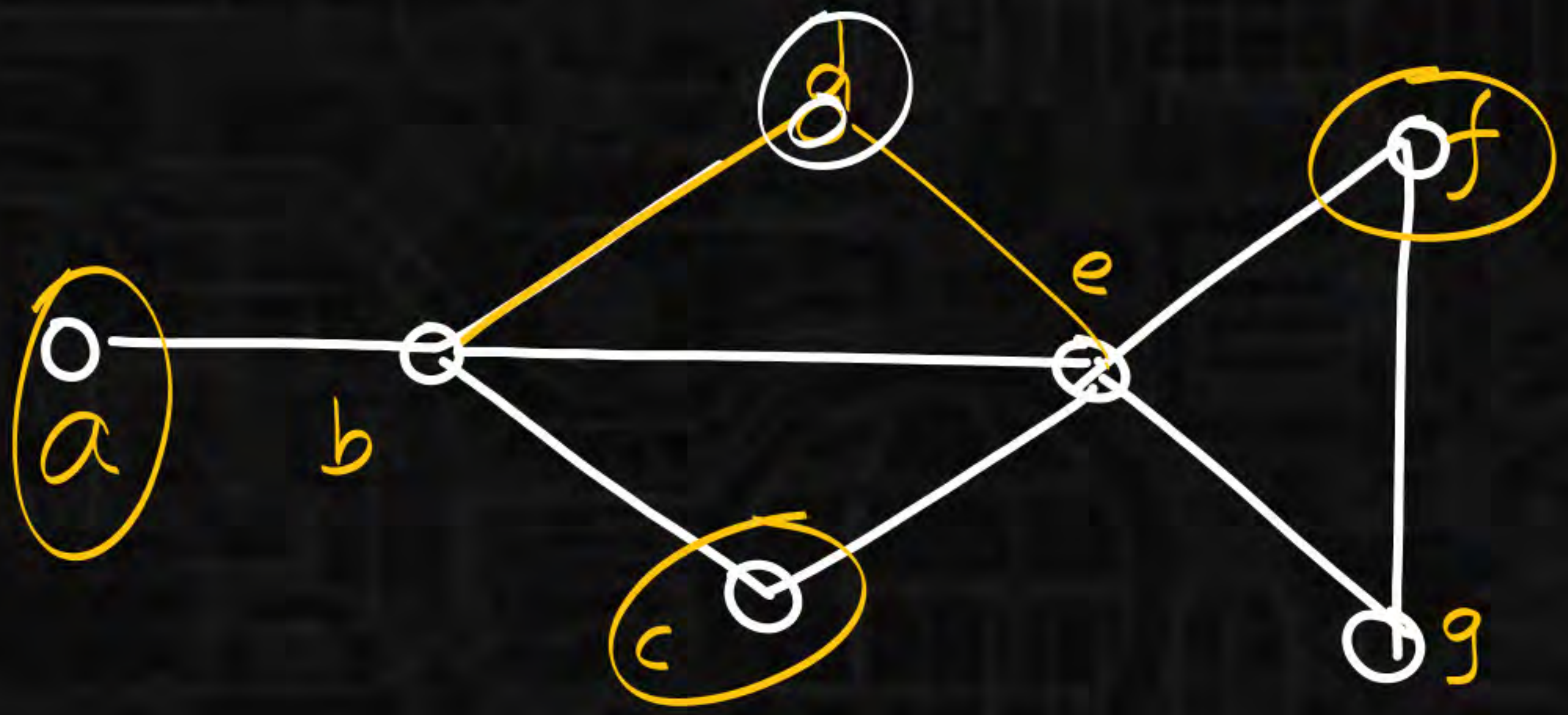
$\{b, f\}$

$\{a\,b\,c\,d\,e\,f\,\cancel{g}\} \rightarrow D.S.$

$\{a\,b\,c\,d\,\cancel{e}\,f\} \rightarrow D.S.$

$\{a\,\cancel{b}\,c\,d\,f\} \rightarrow D.S.$

$\{a\,c\,d\,f\} \rightarrow D.S.$



$\{a\ b\ c\ d\ e\ f\ \cancel{g}\}$ – DS

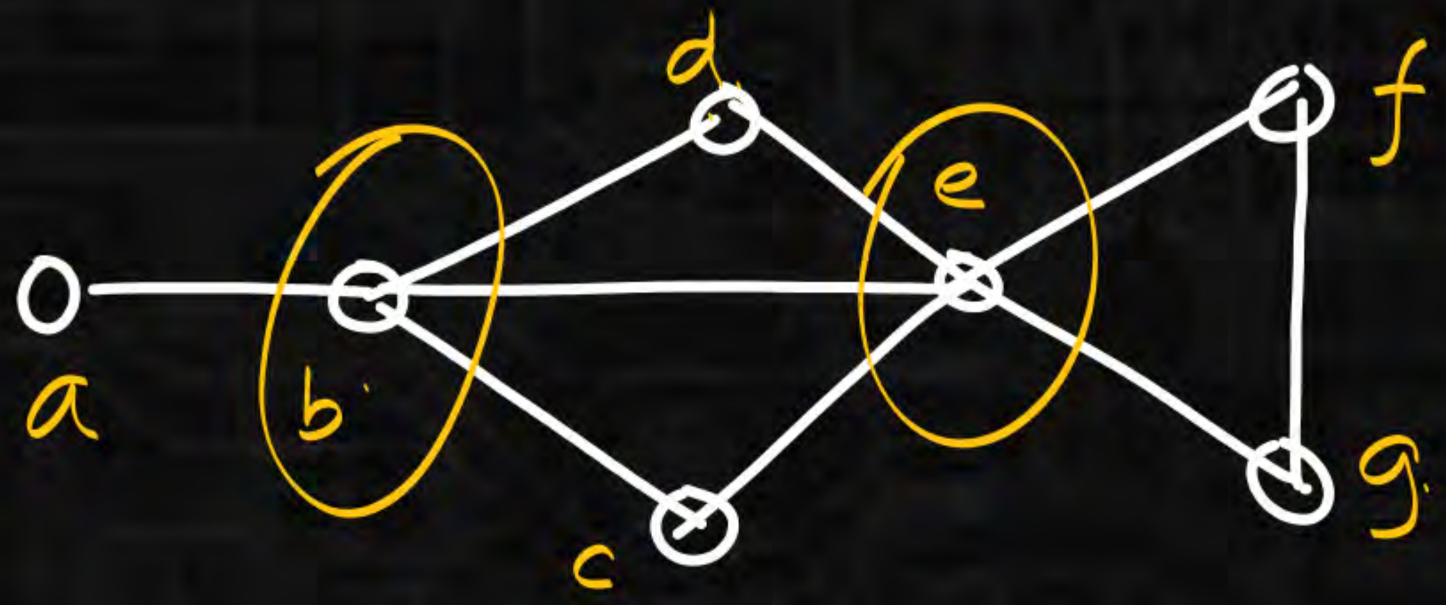
$\{a\ b\ c\ d\ e\ \cancel{f}\}$ – DS

$\{\cancel{a}\ b\ c\ d\ e\}$ – D.S

$\{b\ \cancel{c}\ \cancel{d}\ e\}$ – D.S

$\{b\ e\}$ → DS

minimally Dominating set



* maximal → can not add

* minimal → can not Remove.

minimal Dominating set :

Dominating set such that we cannot remove new vertex from it.

$\{a\ c\ d\ f\}$

$\{b, f\}$

$\{b, g\}$

$\{b, e\}$

$\{a, e\}$

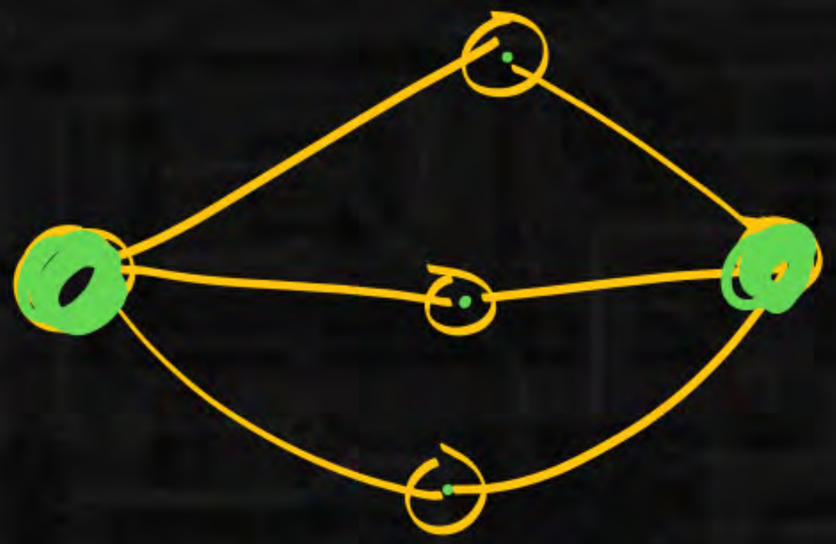
$\alpha(G) = 2$

Domination no. $(\alpha(G))$

no. of vertices present in smallest minimal Dominating set.

Every MIS

will always be MDS.



maximal Independent set :

cannot add new vertex

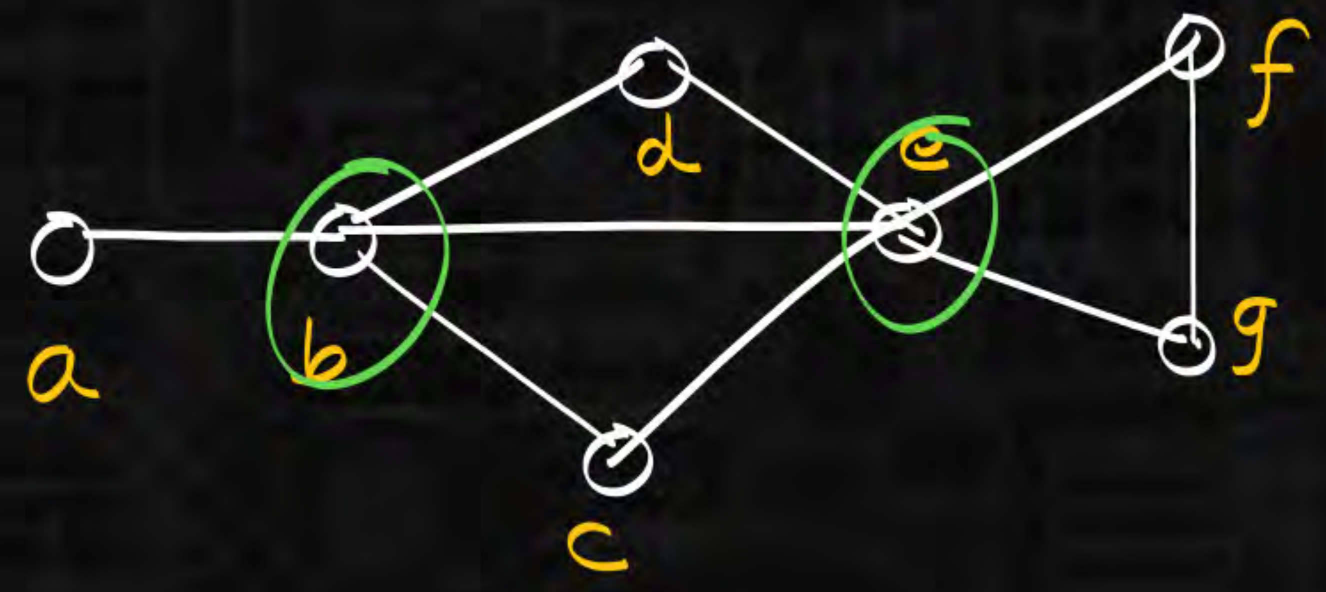
non adjacent.

MIS	MDS
acdf	acdf
ae	ae
bf	bf
bg	bg
	be

$\beta(G) = 4$

$\alpha(G) = 2$

l:



every MIS will always be MDS.

but Every MDS need not be MIS.

eg: {be}

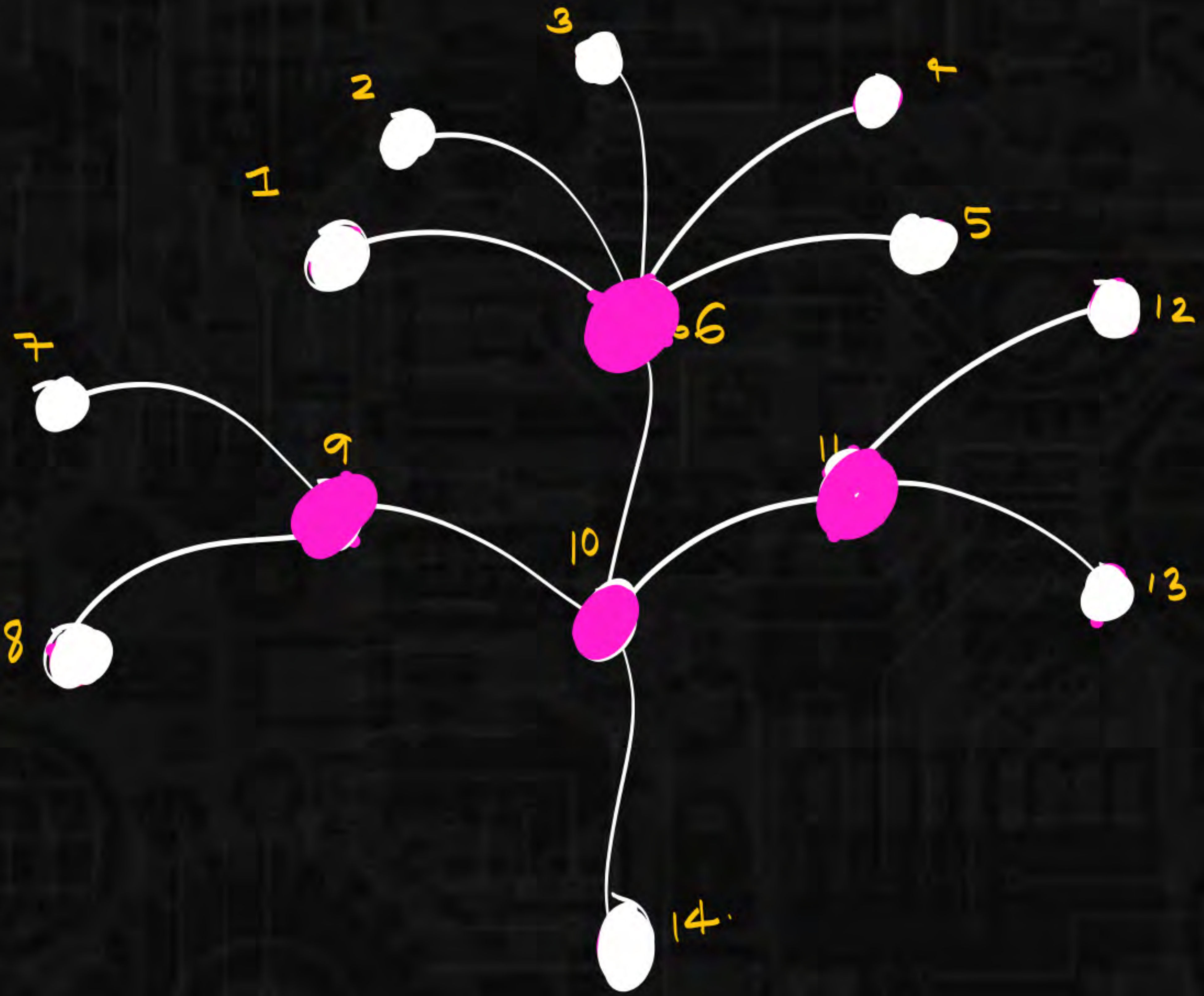
MIS $\longrightarrow$ MDS

$\beta(G)$	$\alpha(G)$
5	5
4	4
3	3

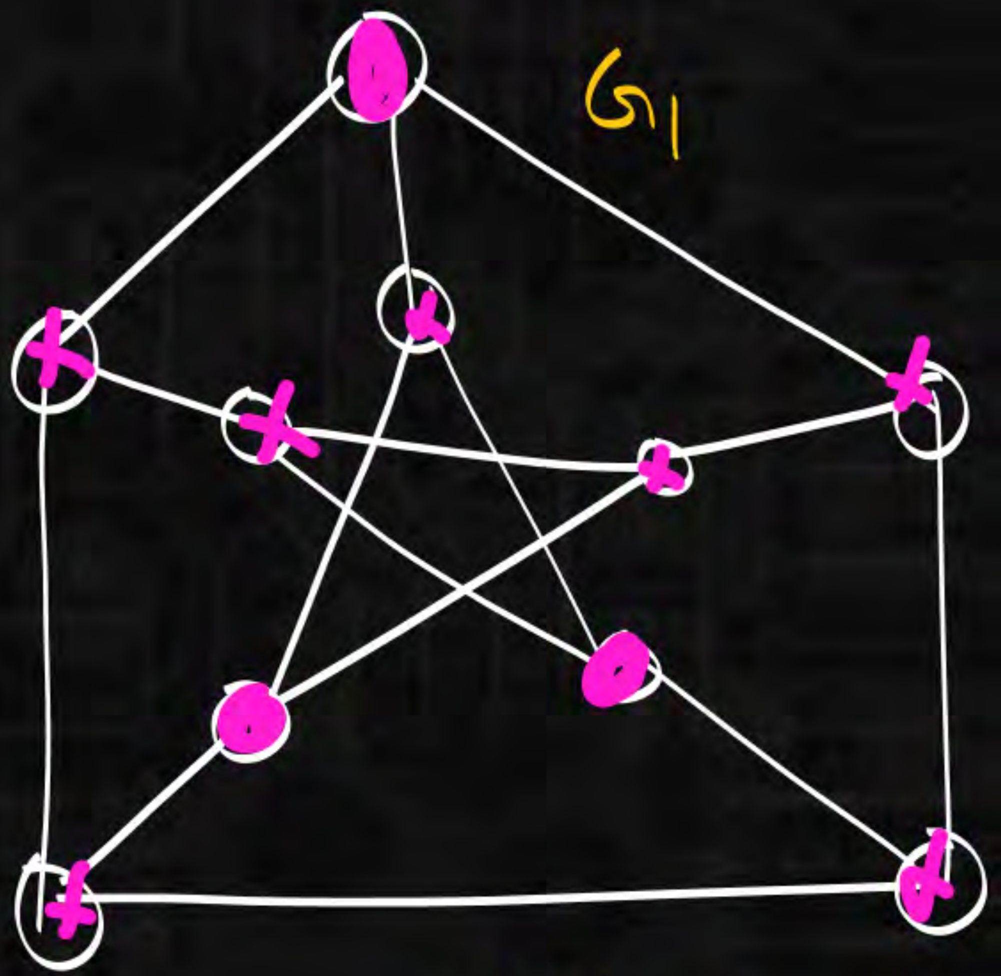
Scan largest

scan smallest

$$\boxed{\alpha(G) \le \beta(G)}$$

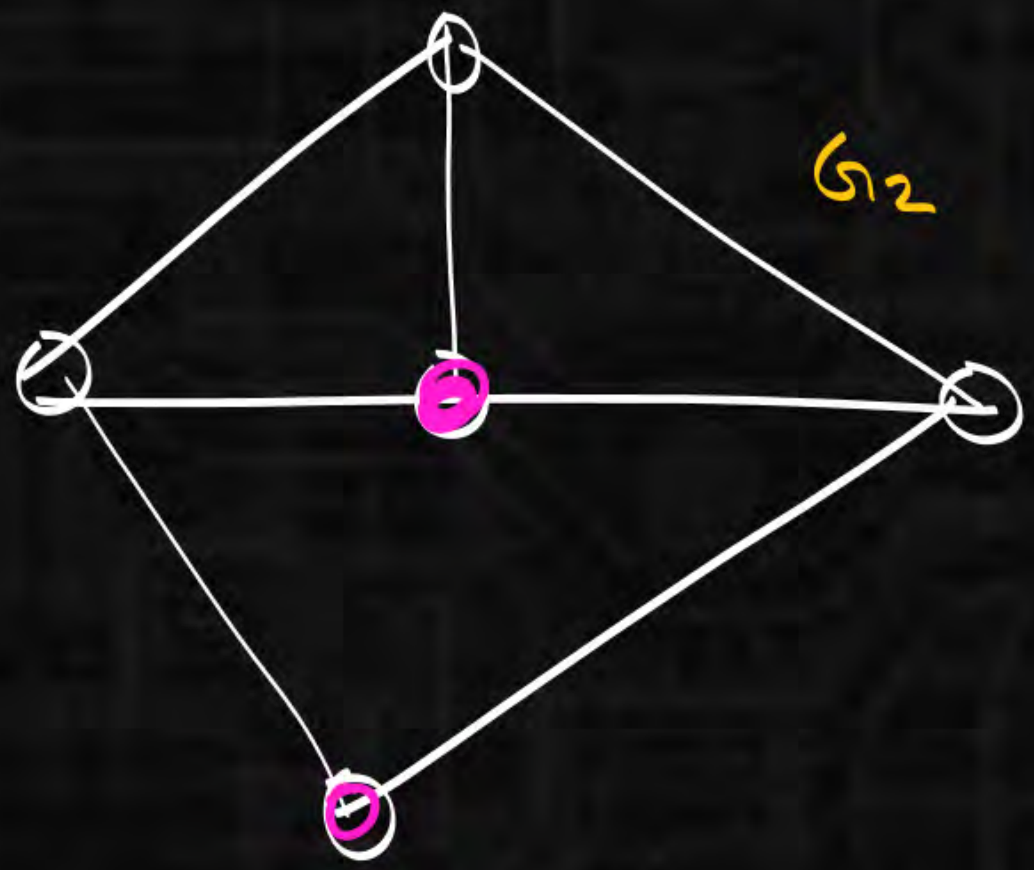


$\alpha(G) = 4$



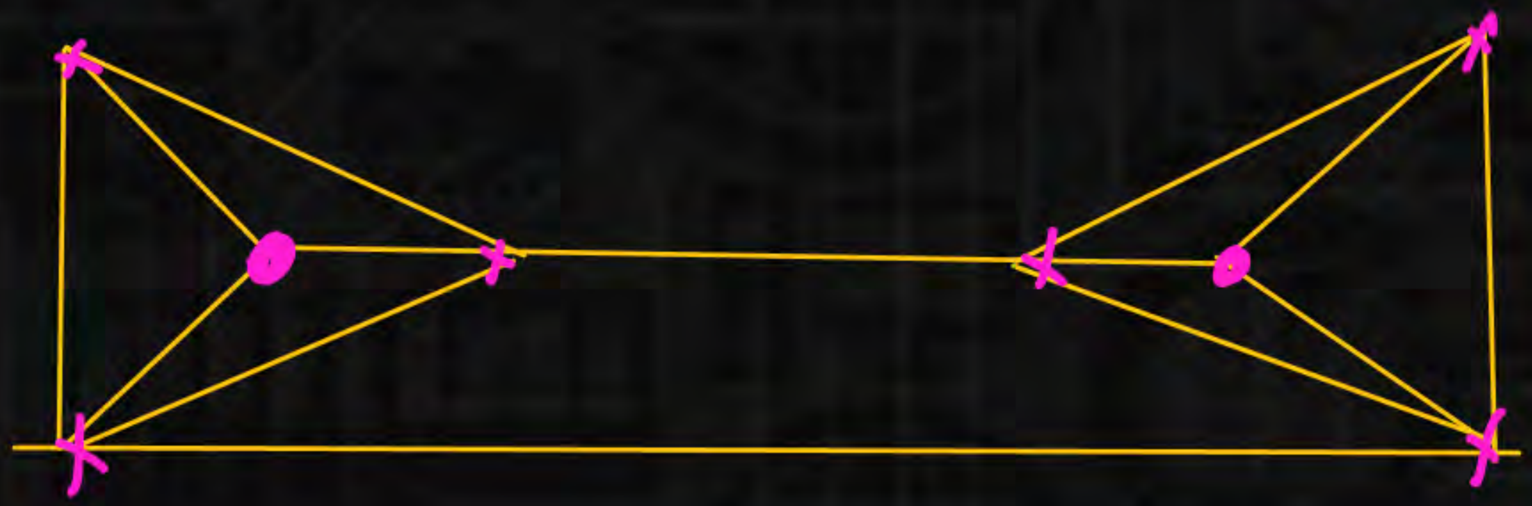
G_1

$\alpha(G) = 3$



G_2

G_3




THANK YOU GW
SOLDIERS !