CS & IT ENGINEERING

Graph Theory

Connectivity in Graphs part 2

Lecture No. 7

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TOPICS TO BE COVERED

01 Annalysis In Connectivity

* 02 Various definition in Connectivity

03 Edge Connectivity

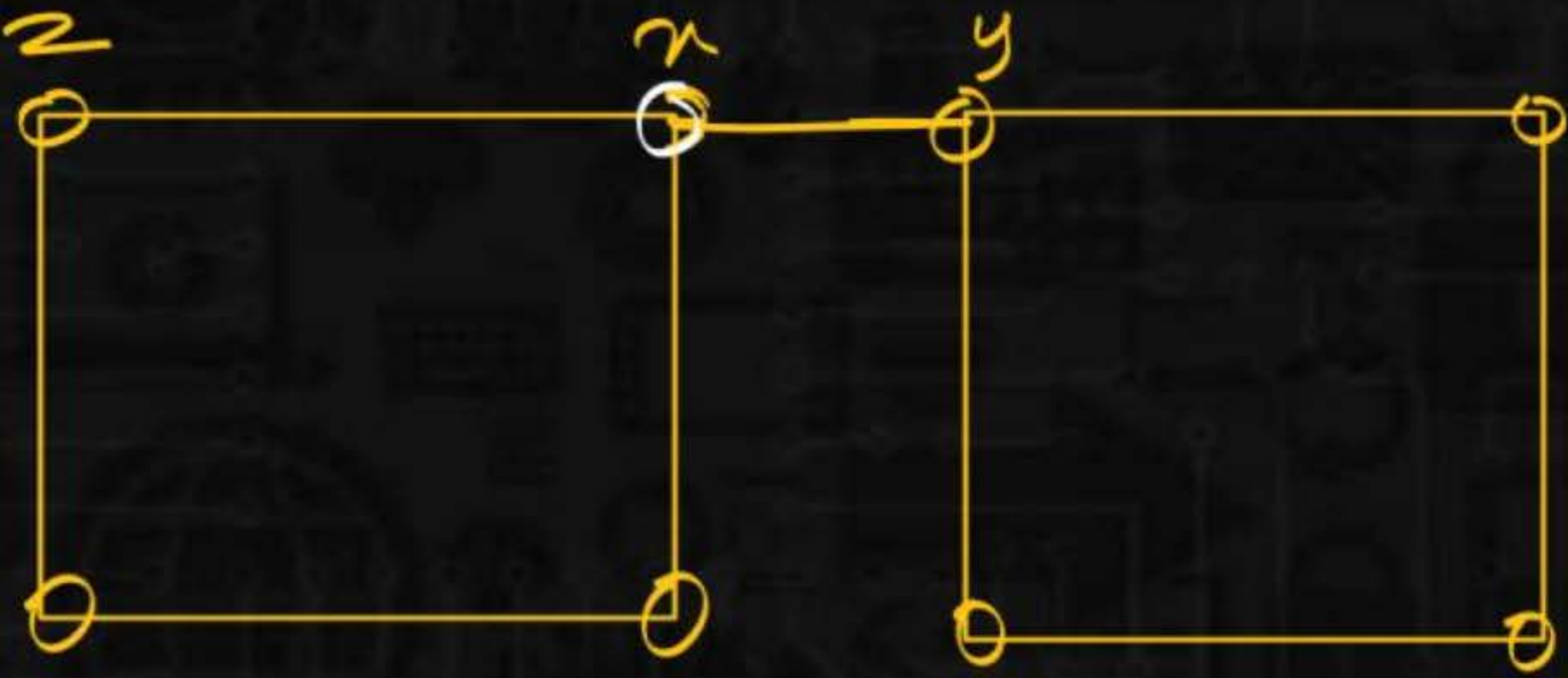
04 Vertex Connectivity

05 Largest inequality theorem

Connectivity in Graphs

Cut edge / Bridge :

Removal of $\boxed{\text{single edge}}$ from a Graph will make graph as a disconnected graph.



$\{xy\}$ is bridge.

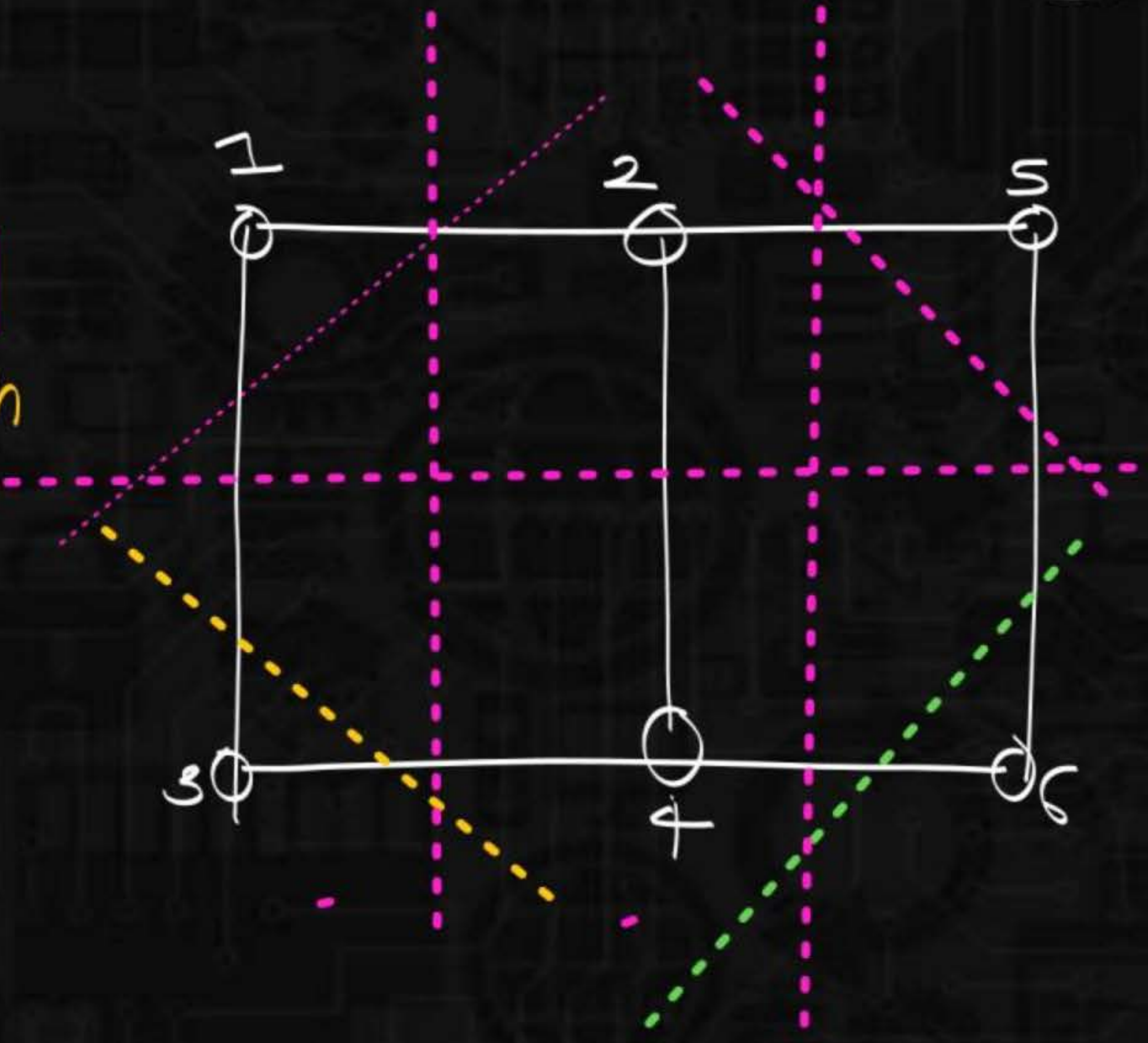
** if cut edge exist then it does not belongs to cycle.

Connectivity in Graphs

cut edge set / cut set :

Removal of $\boxed{\text{set of edges}}$ from a graph will make graph as a disconnected graph.

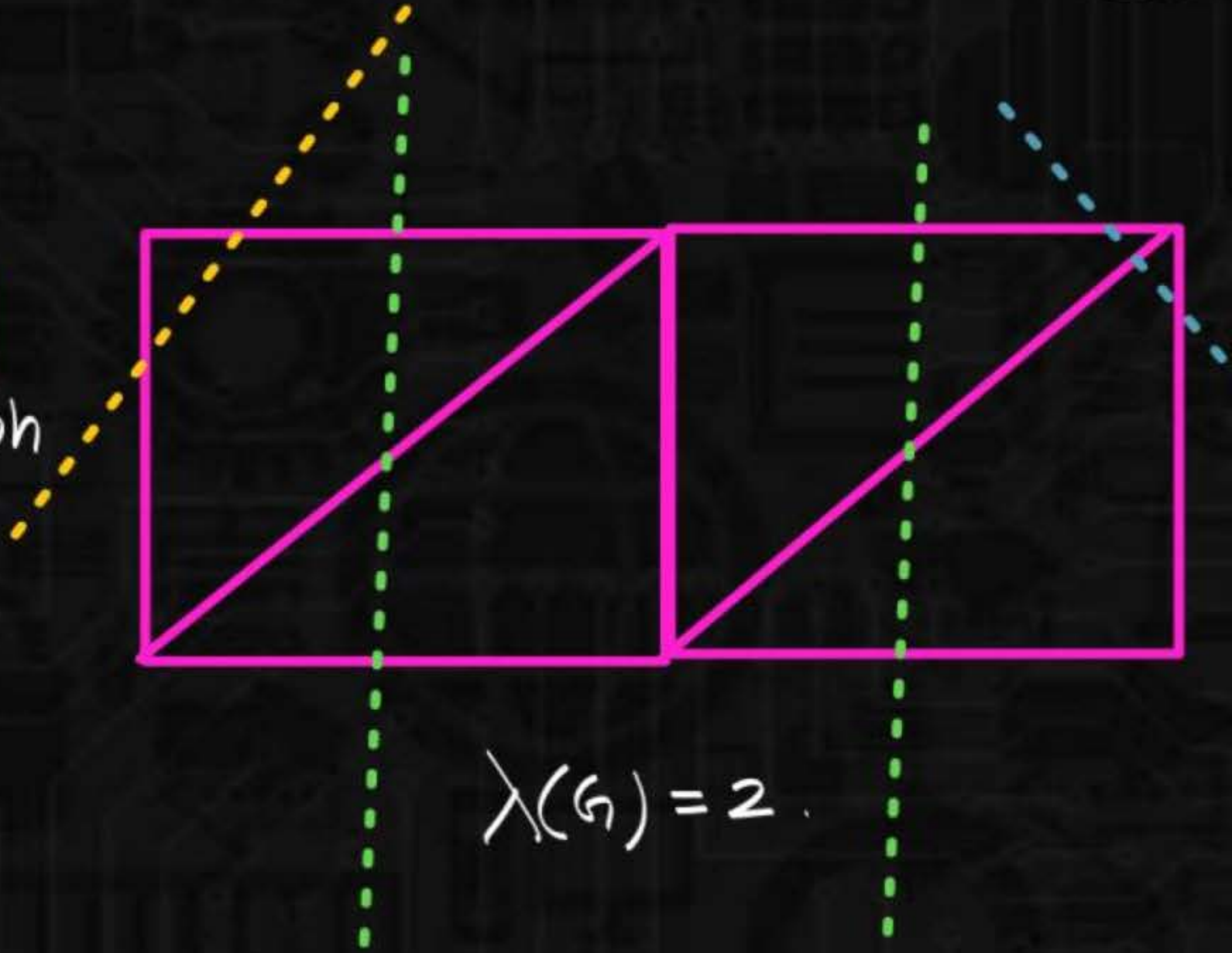
$\{12, 13\}$ cut set.



Connectivity in Graphs

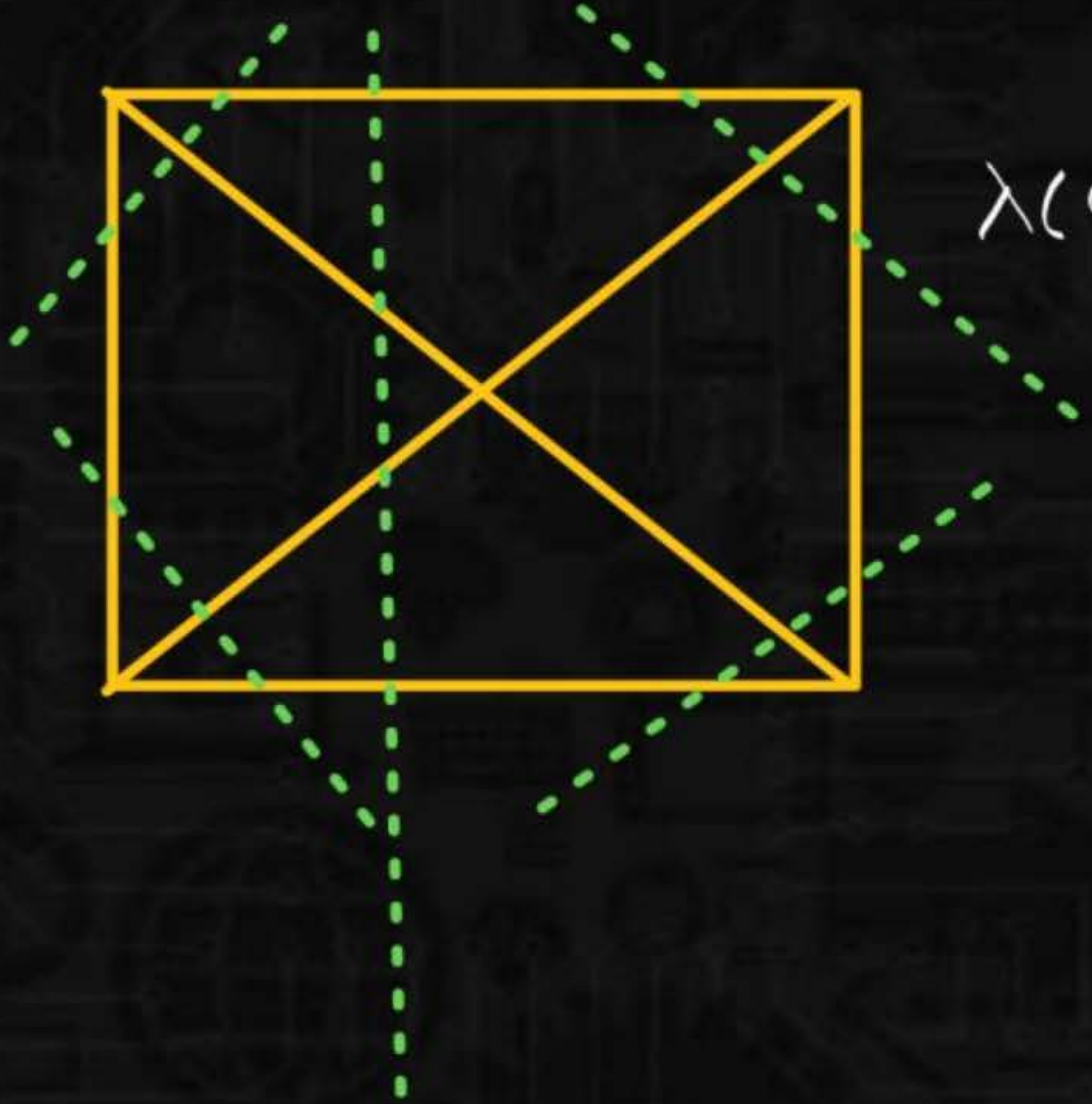
edge connectivity ($\lambda(G)$)

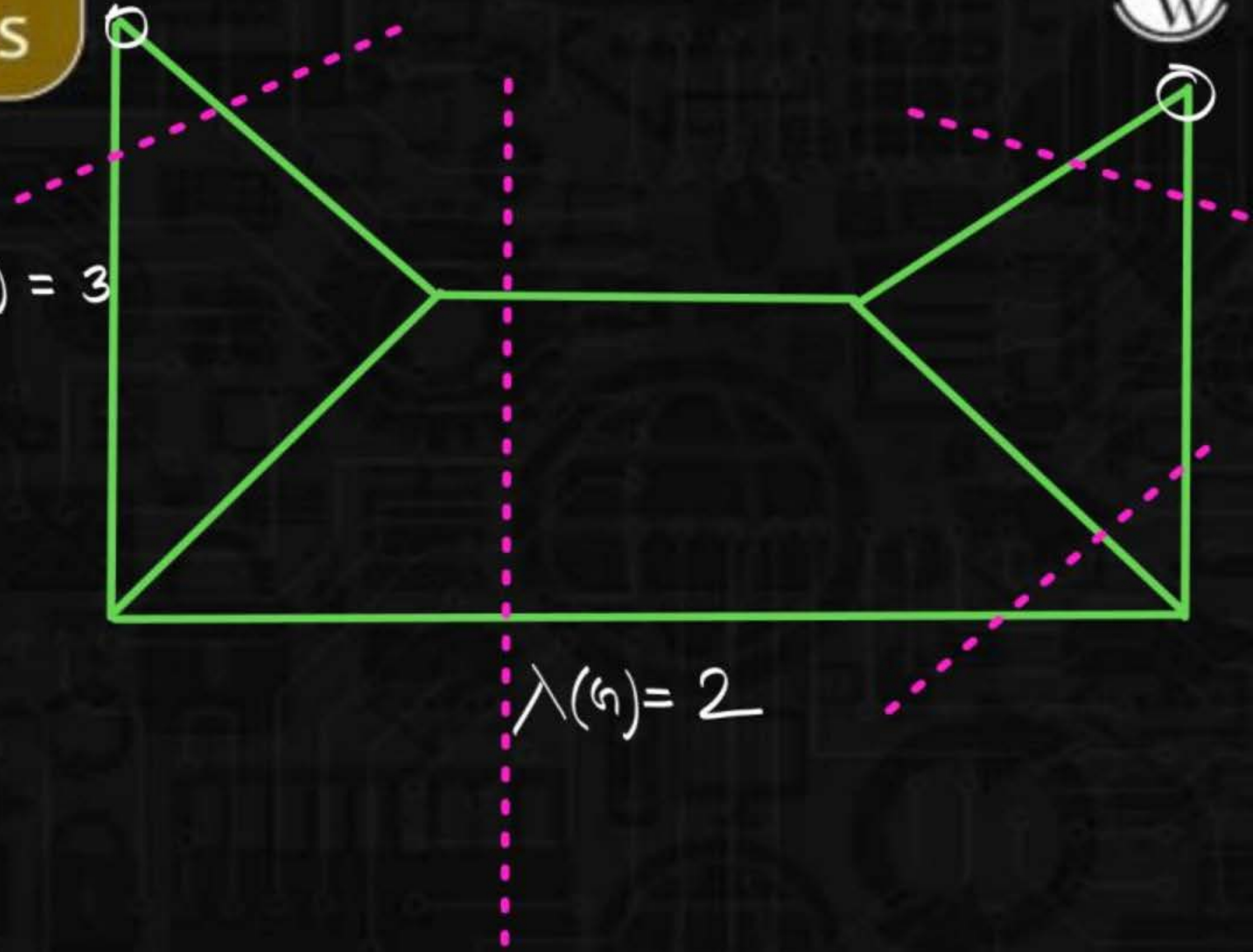
Removal of $\boxed{\text{min. no. of edges}}$ from a graph will make graph as a disconnected graph.



$\lambda(G) = 2$.

Connectivity in Graphs

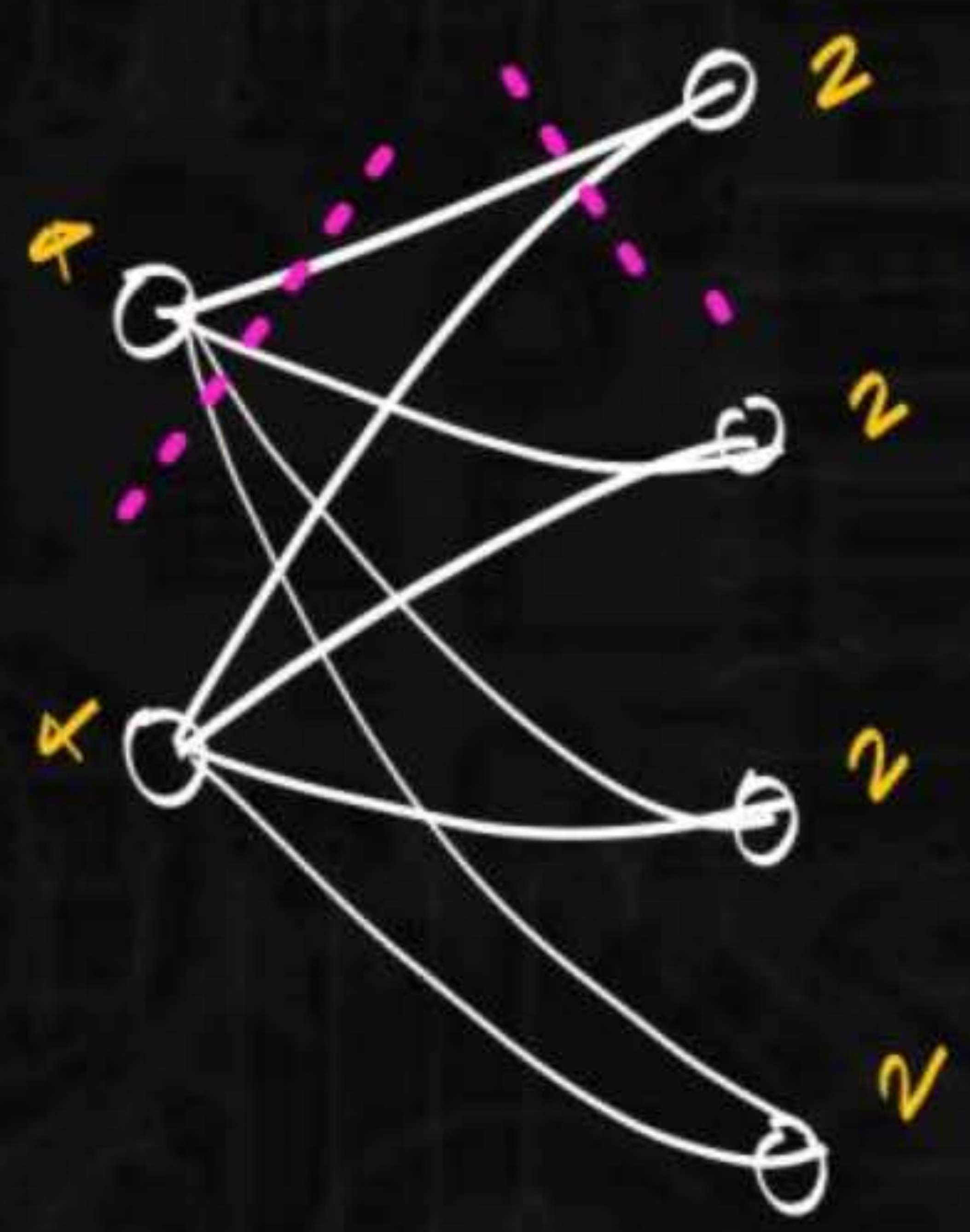




Connectivity in Graphs

$\lambda(K_{m,n}) =$

$K_{2,4}$



$\lambda(K_{2,4}) = 2.$

$\lambda(K_{4,2}) = 2.$

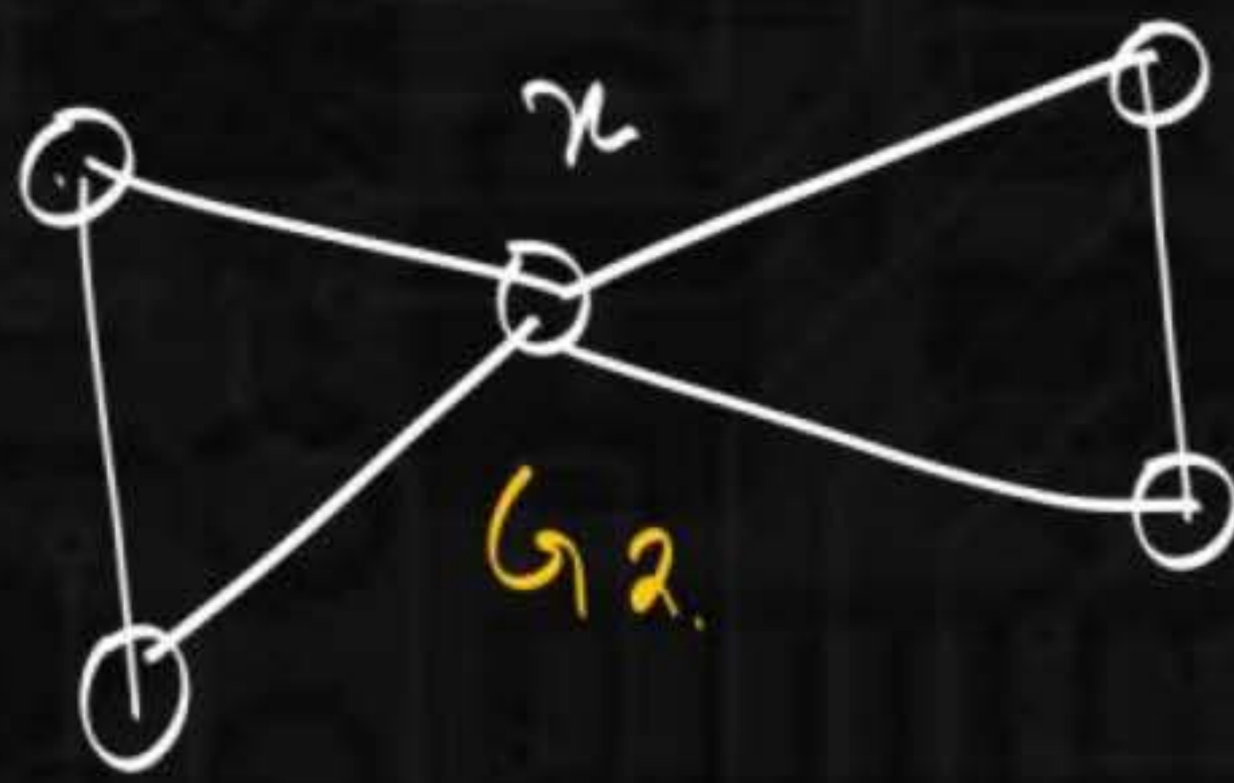
$$\boxed{\lambda(K_{m,n}) = \min(m,n)}$$

$\lambda(Q_n) = n.$

Connectivity in Graphs

Cut vertex / cut point / Articulation point

Removal of $\boxed{\text{single vertex}}$ from a graph will make graph as a disconnected graph.



cut vertex.

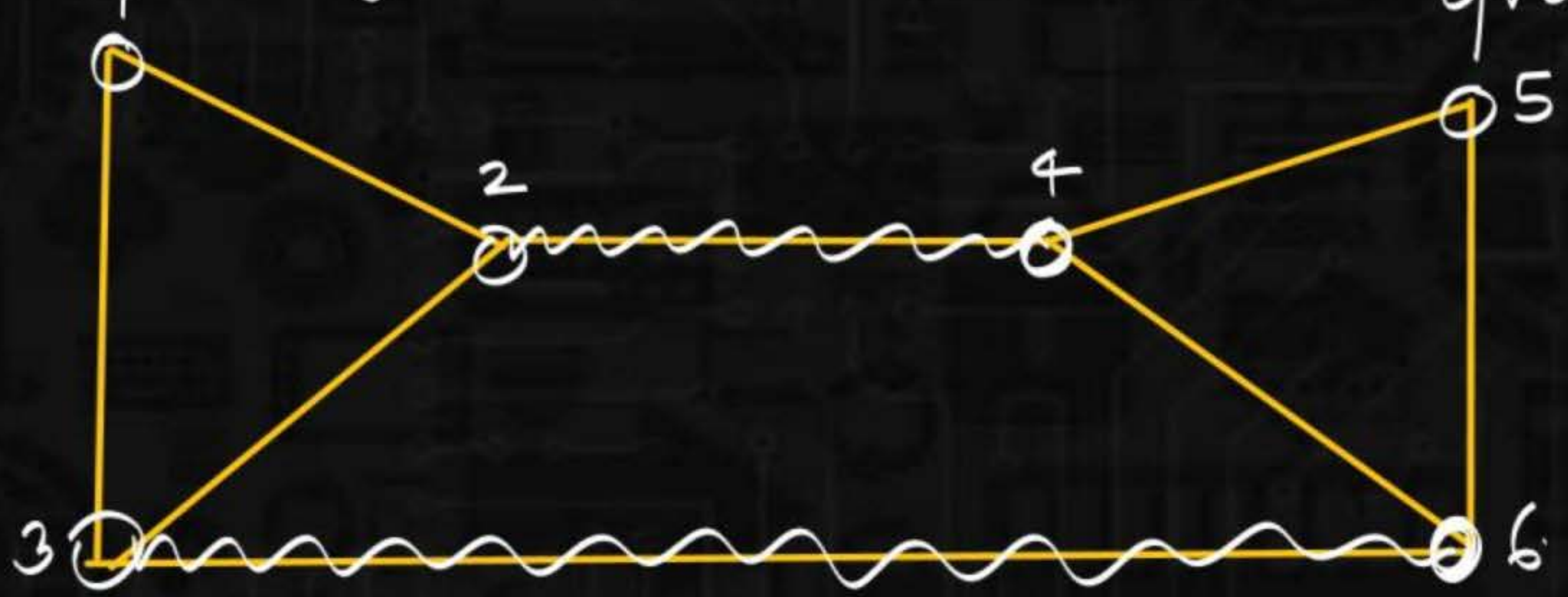
$G_1 \to \{x\}$

$G_2 \to \{x\}$

Connectivity in Graphs

Cut vertex set

Removal of set of vertices from a graph will make graph as a Disconnected graph.



$K(G) = 2.$

Connectivity in Graphs

vertex connectivity ($K(G)$)

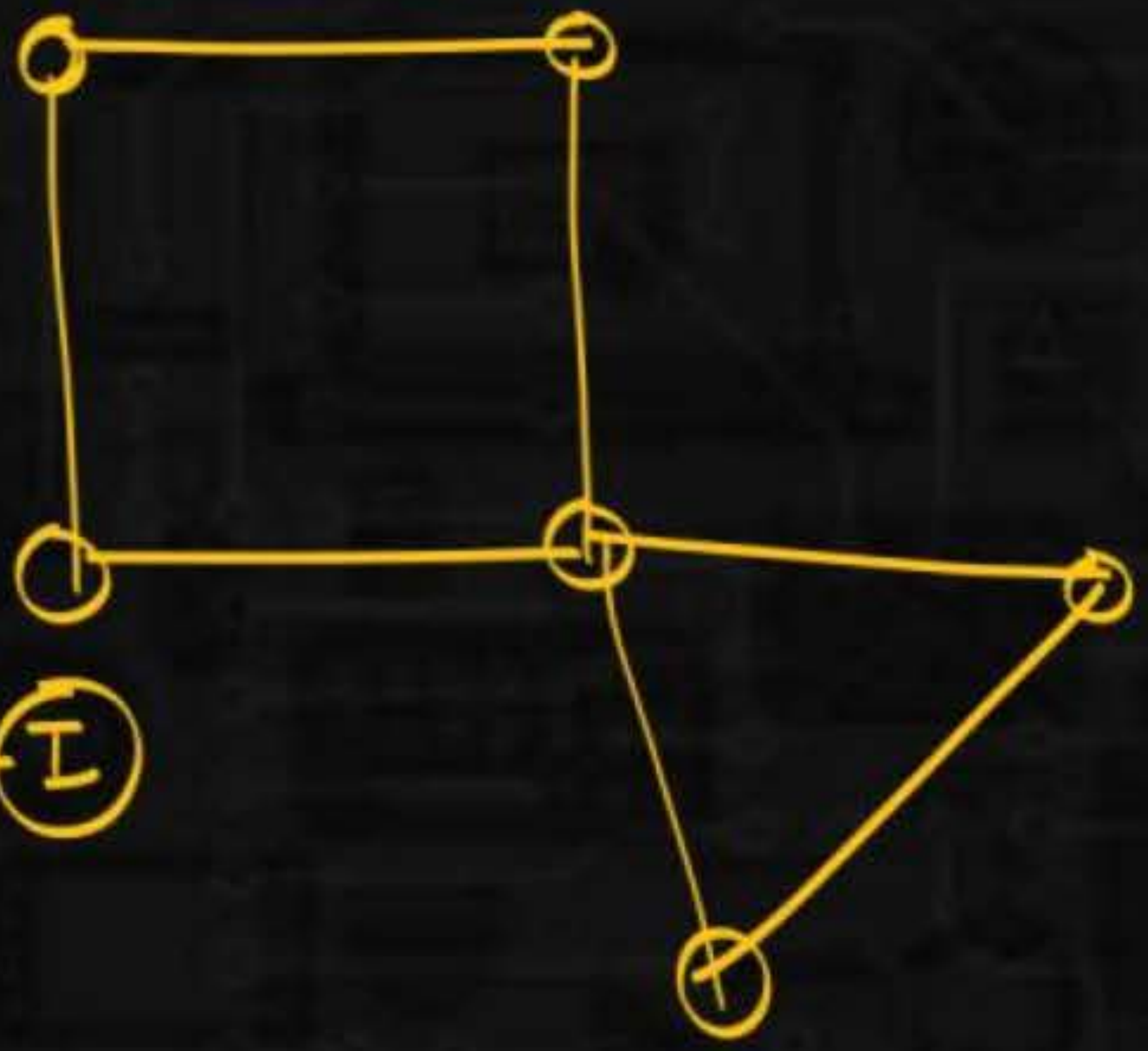
Removal of min. no. of vertices from a Graph will make graph as a disconnected graph.

Connectivity in Graphs

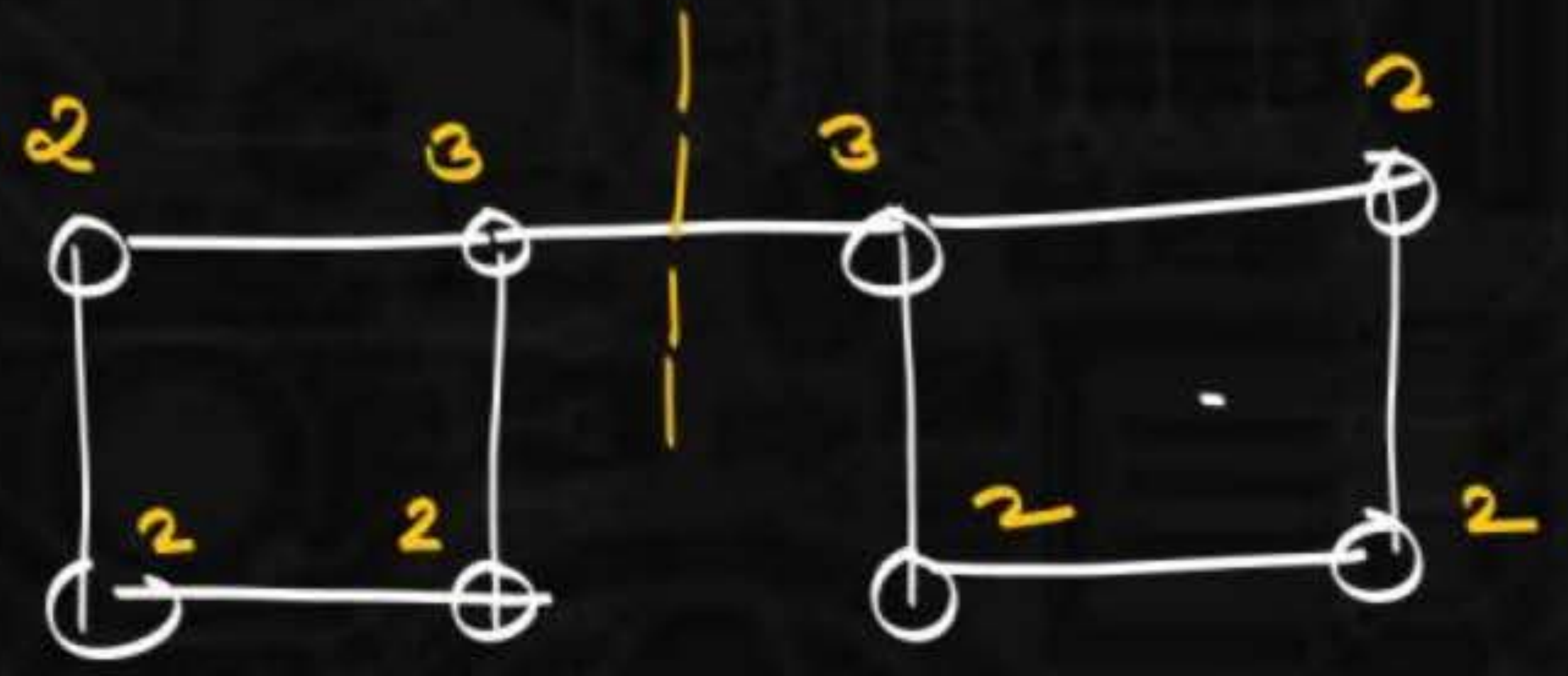
$$\boxed{\lambda(G) \leq \delta(G)}$$

$\delta(G) = 2$

$\lambda(G) = 2$



$\lambda(G) = \delta(G)$ — I



$\delta(G) = 2.$

$\lambda(G) = 1.$

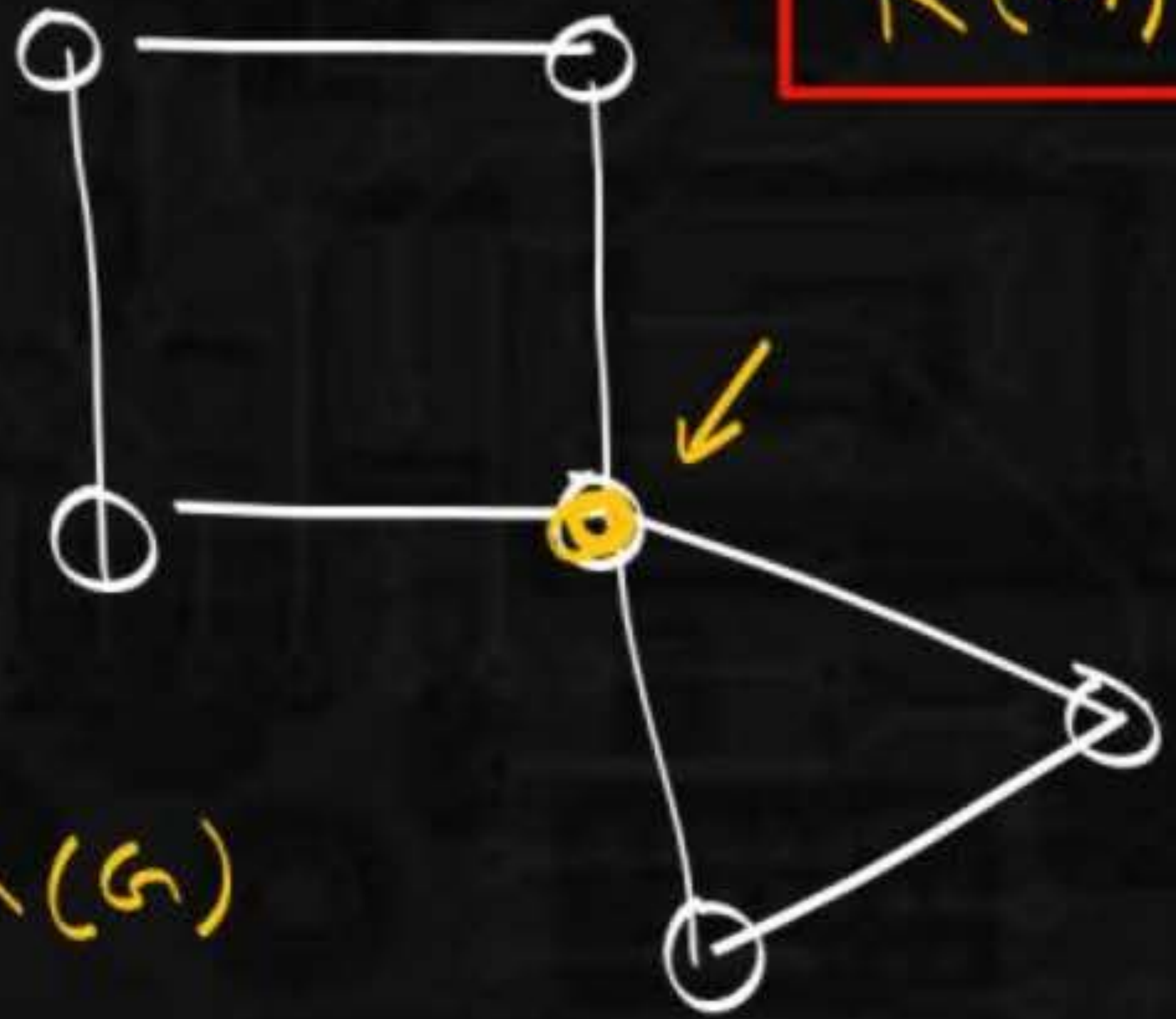
$\lambda(G) < \delta(G)$ — II

Connectivity in Graphs

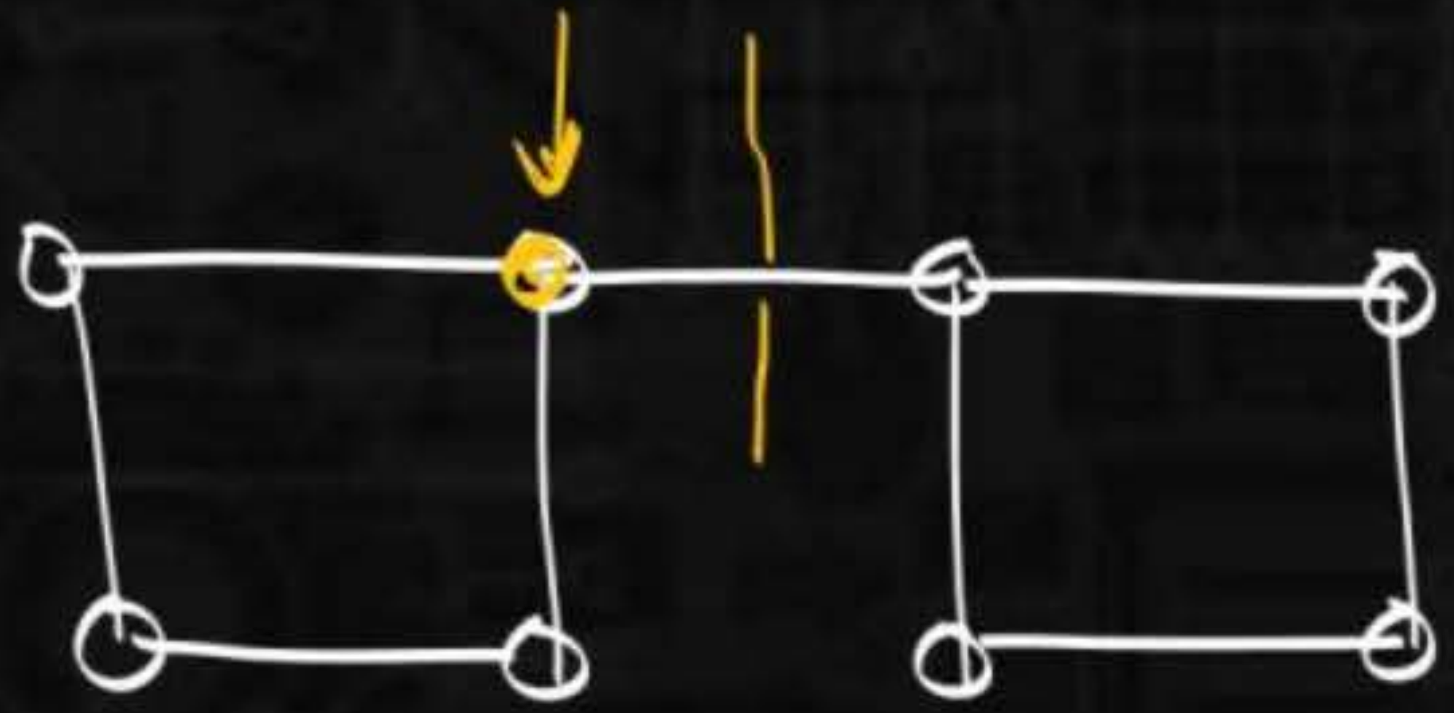
$$\boxed{K(G) \leq \lambda(G)}$$

$\lambda(G) = 2$

$K(G) = 1$



$K(G) < \lambda(G)$



$K(G) = 1$

$\lambda(G) = 1$

$K(G) = \lambda(G)$

Connectivity in Graphs

Consider a Graph having vertex connectivity 3.
& it is having 30 edges then what will be maximum value n?

$K(G) = 3 \quad e = 30$

$K(G) \leq 2e/n$

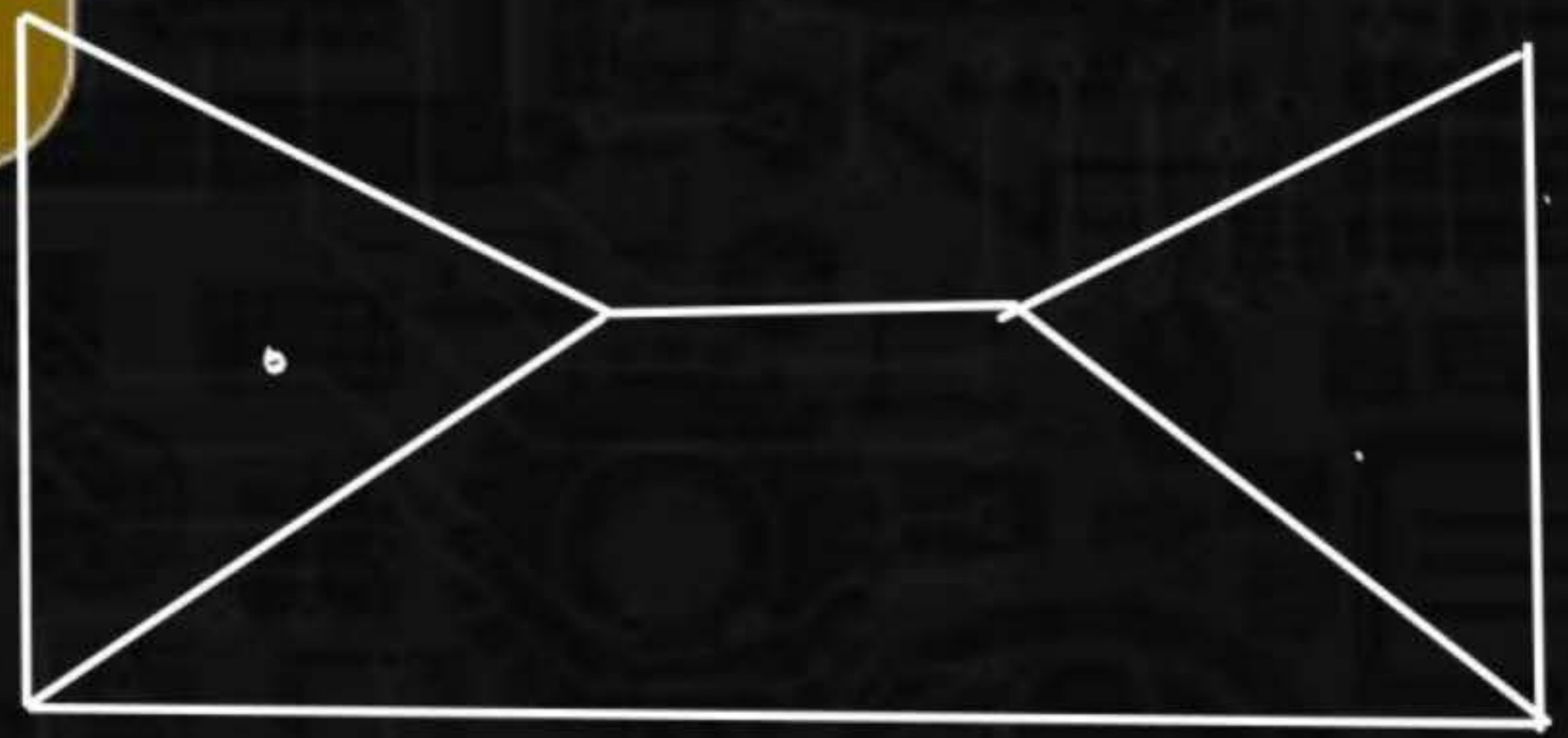
$3 \leq \frac{2 \times 30}{n}$

$n \leq \frac{2 \cdot \cancel{30}^{10}}{\cancel{3}}$

$n \leq 20$

$\left(\underline{\qquad\qquad\qquad 20} \right) \leq 20$

Connectivity in Graphs



$\delta(G) = 2$

$K(G) \leq \lambda(G) \leq 2$

$\cancel{1}\ 2$

$K(G) = 2 \quad \lambda(G) = 2.$

Connectivity in Graphs

NAT

Consider a Graph having 10 vertices (disconnected) and maximum no. of edges then take the complement of this Graph & findout addition of (vertex & edge connectivity) ________.

$$\lambda(K_{m,n}) = \underline{\min(m,n)}$$
$$K(K_{m,n}) = \min(m\ n)$$
$$\lambda(K_{1,n-1}) = \min(1, n-1) = 1.$$
$$K(K_{1,n-1}) = \min(1, n-1) = 1.$$

D.C + max edges $\rightarrow$

$\xrightarrow{\bar{G}}$ $K_{1,9}$

$$K(K_{1,9}) = 1.$$
$$\lambda(K_{1,9}) = 1.$$
$$1 + 1 = \underline{2}.$$

Connectivity in Graphs

Disconnected

Consider a Graph having 10 vertices and maximum no. of edges

what will edge/vertex connectivity?

$\delta(G) = 3$

K_4.

$\lambda(K_4) = 3$

$K(K_4) = 3$.

Disconnected + maximum no. → 2 components

K_1 K_9

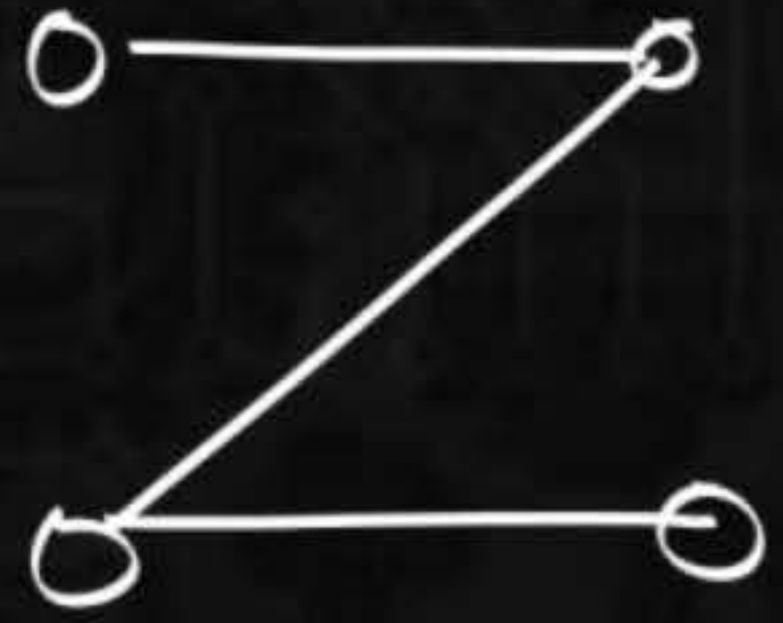
$\lambda(G) = 8$

$K(G) = 8$

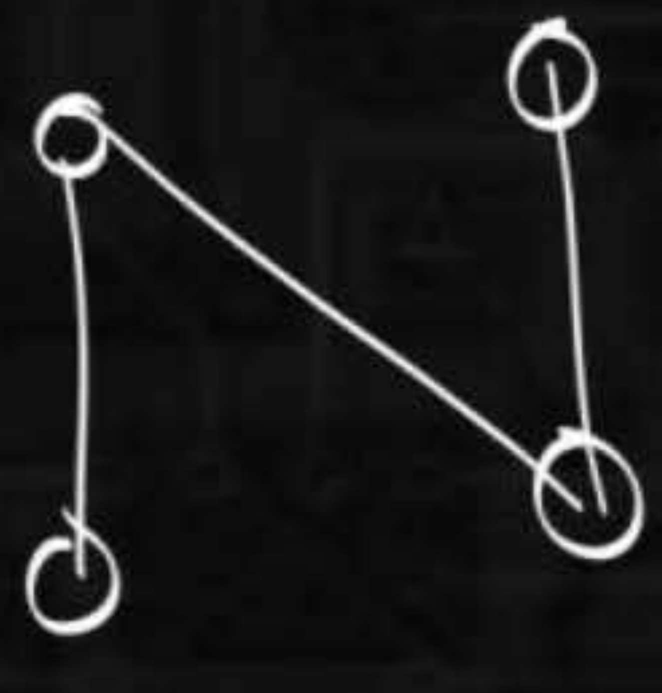
Connectivity in Graphs

if G is connected then $\overline{G}$ will be disconnected.

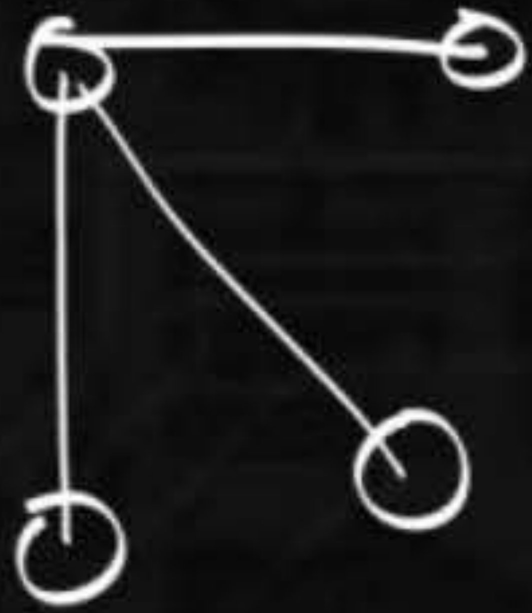
(false)



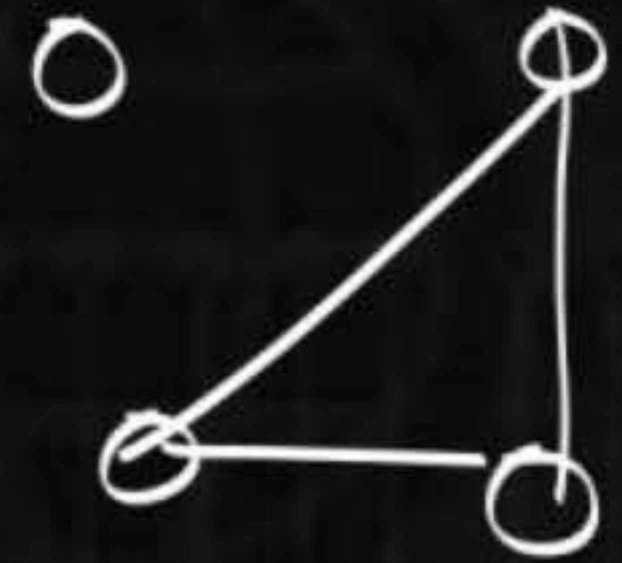
G



$\overline{G}$



G

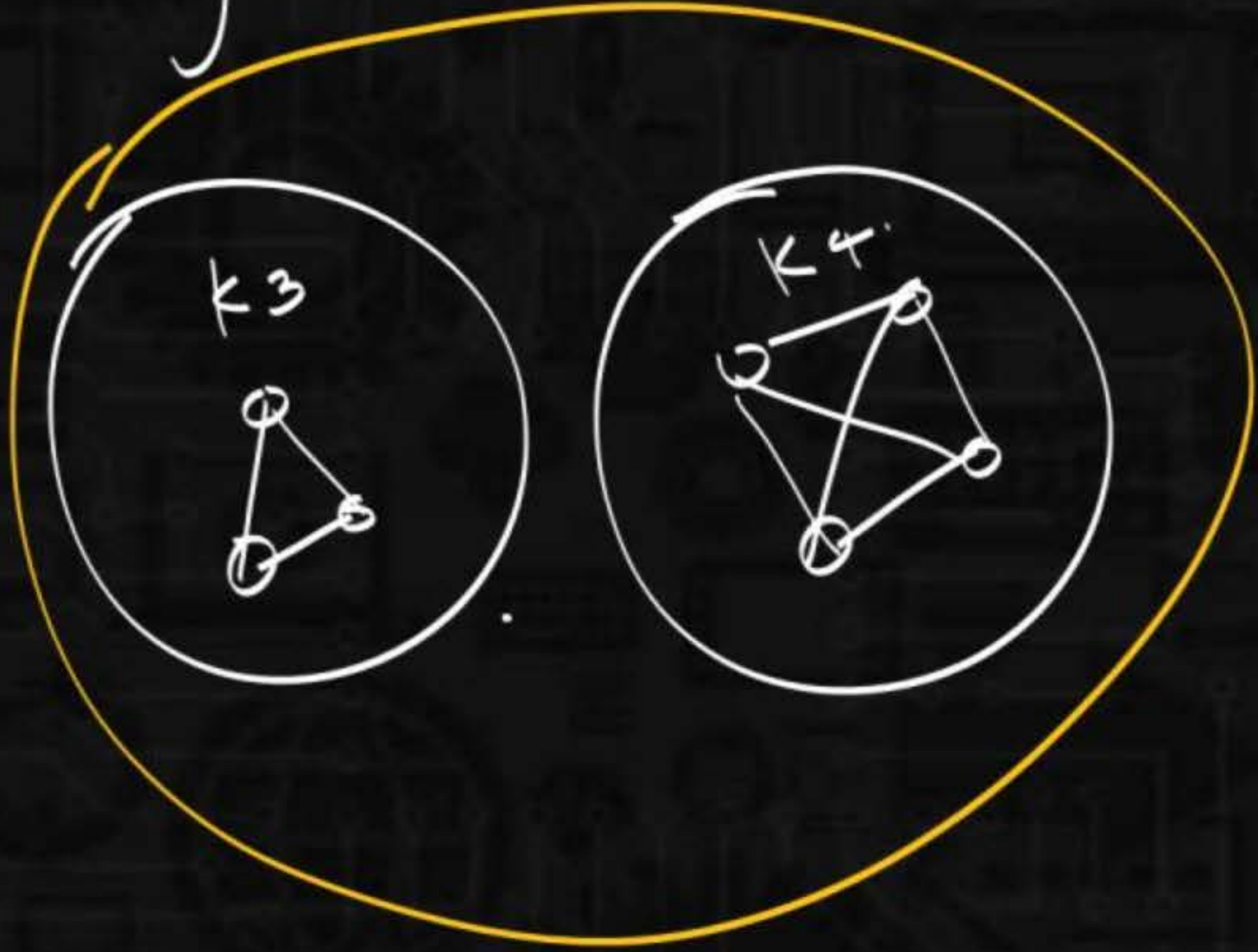


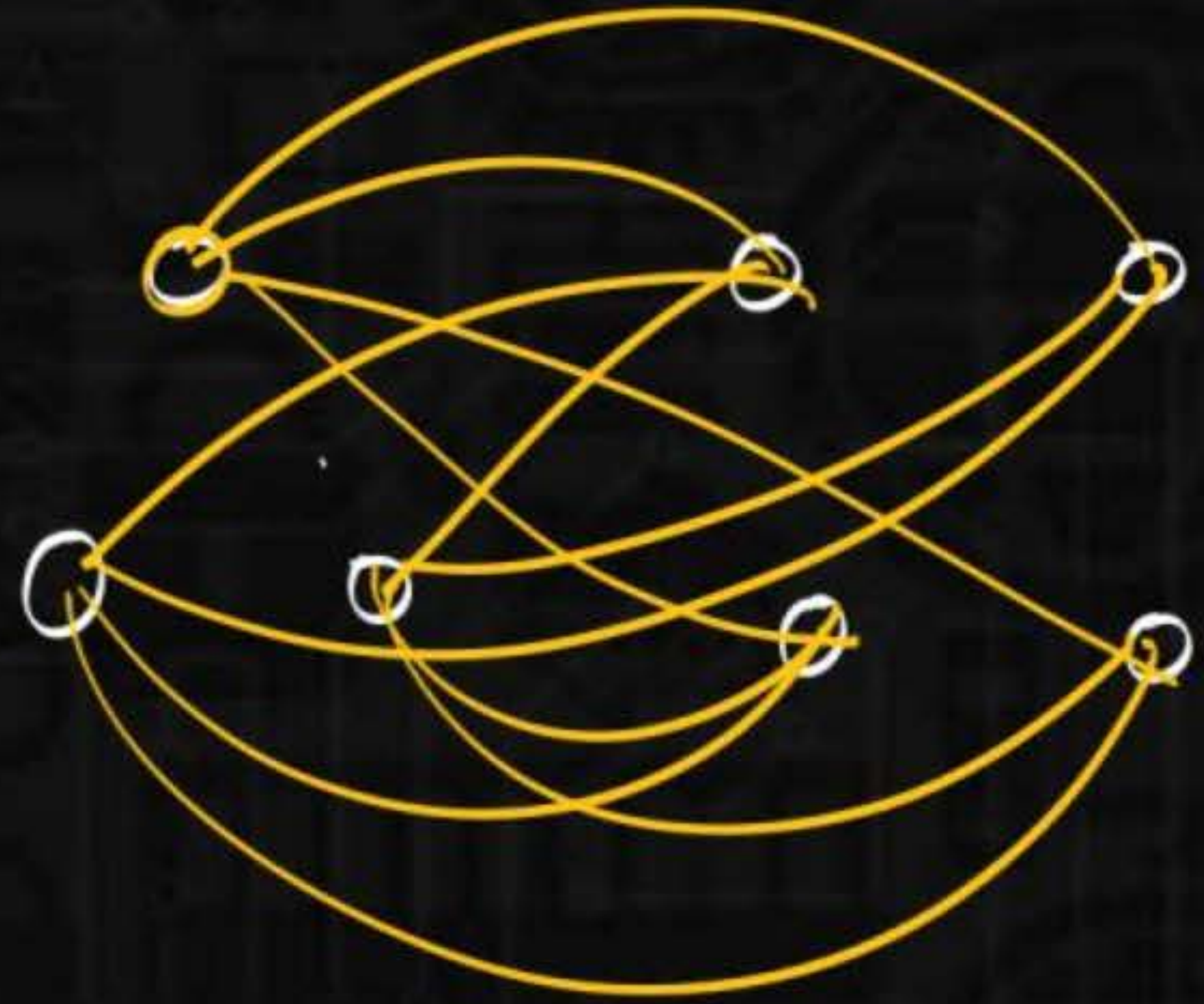
$\overline{G}$

Connectivity in Graphs

if G is Disconnected then $\overline{G}$ will be connected. ✓

(True) ✓





Connectivity in Graphs

Consider a Graph having 10 vertices & $\underline{\delta(G) \geq 5}$

Check it is connected OR

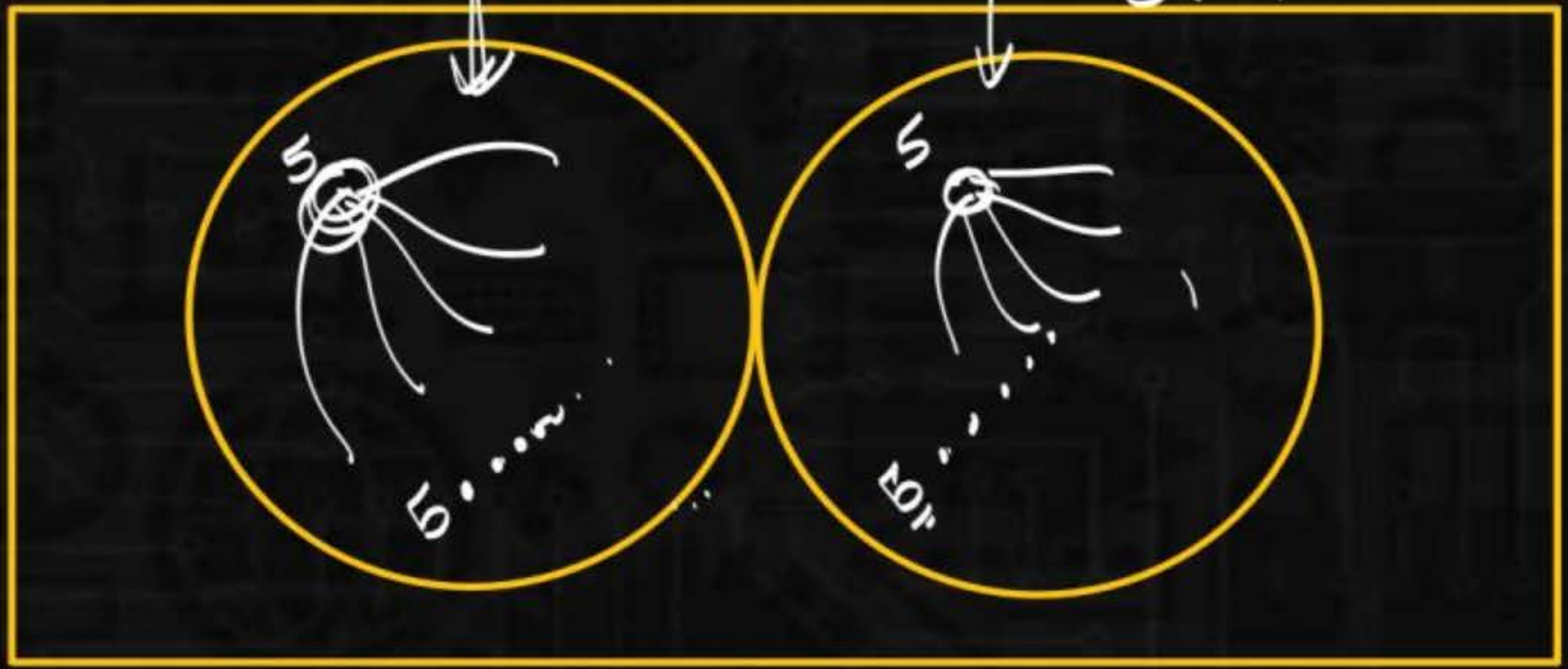
Disconnected ?

Connectivity in Graphs

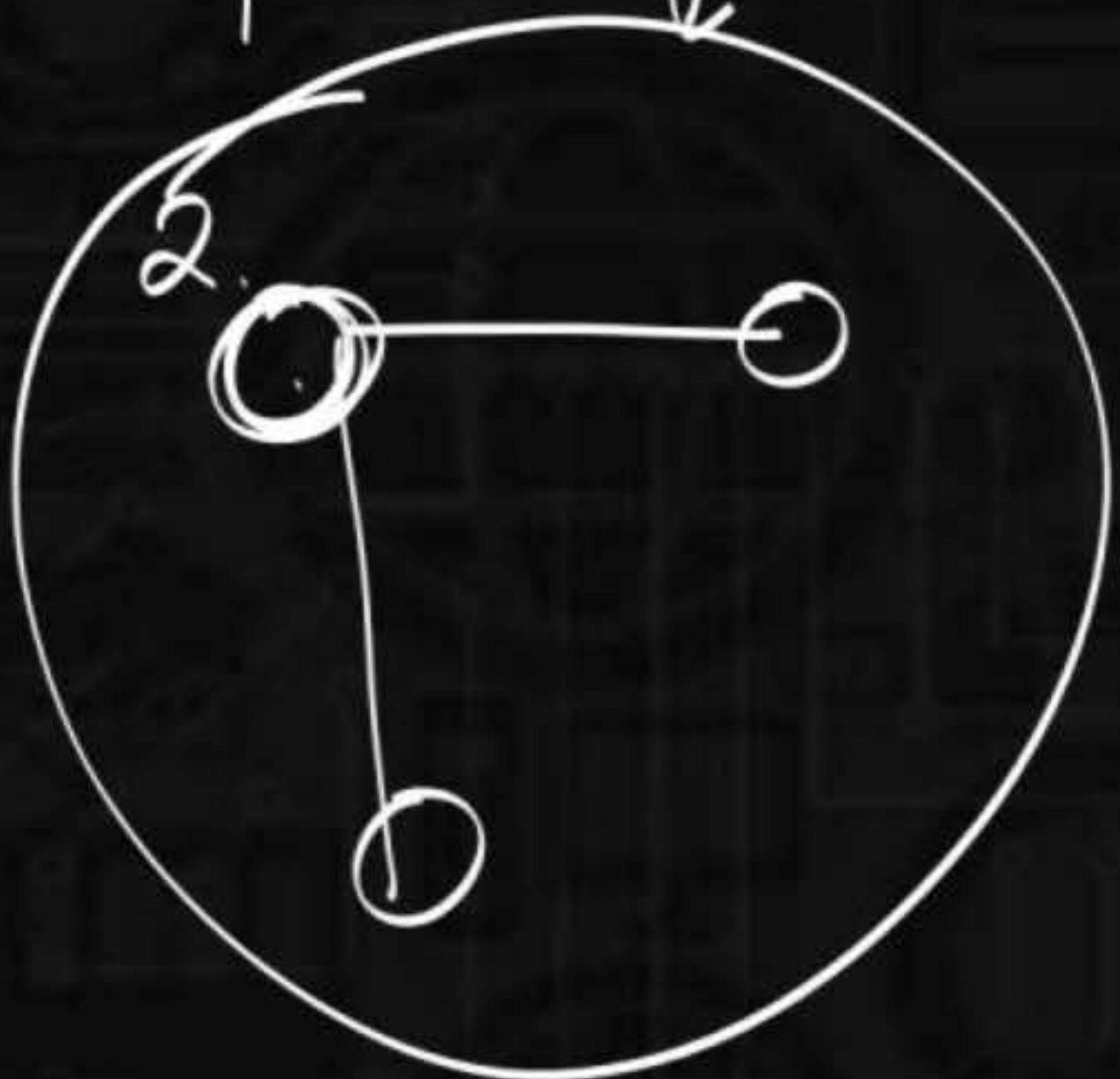
$n = 10$ $\delta(G) \geq 5$

Assumption: Let's take Disconnected Graph.

6vertice 6vert $\delta(G) \geq 5$



3



$n = 10 \quad \underline{\delta(G) \geq 5}$

5 V 4

5 V ..... 4 ✗

7 V 6

3 V 2

$\delta(G) = 2$

6 V 5

4 V 3

$\delta(G) = 3$

✗

Connectivity in Graphs

if $\delta(G) \geq \frac{n-1}{2}$ then it is connected Graph.


THANK YOU GW
SOLDIERS !