CS & IT ENGINEERING

Graph Theory

Connectivity in Graphs

Lecture No. 6

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TOPICS TO BE COVERED

01 Definition In Connectivity

02 Connected vs Disconnected

03 Range of Edges

04 Concepts of tree

05 Connectivity theorem

Connectivity in Graphs

Walk: R.V | R.E $\quad 1\ e_2\ \underbrace{2\ \overbrace{e_3\ 5\ e_3}^{R.E}\ 2}_{R.V}$

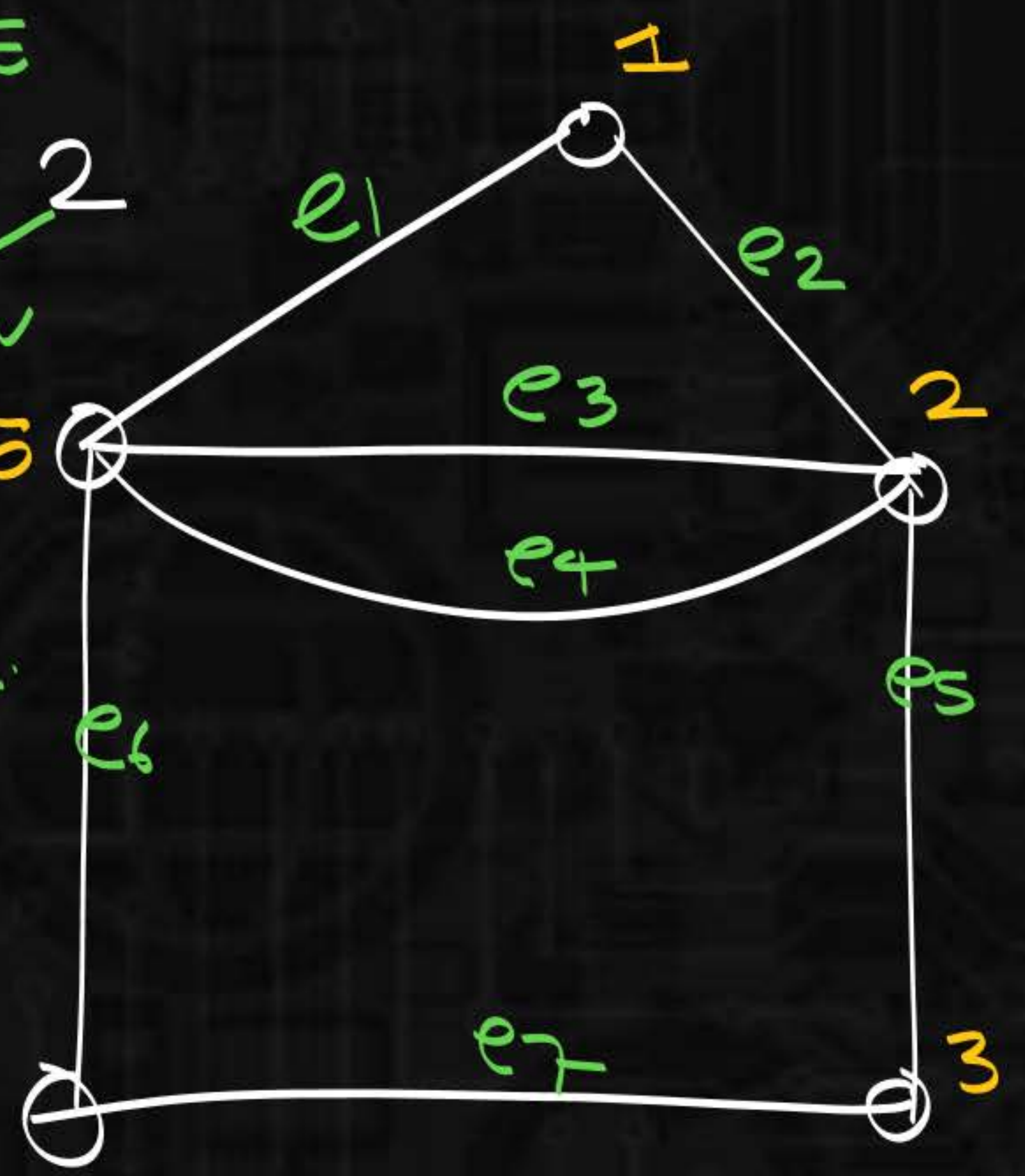
alternate sequences of vertices & edges.

Trail: R.V | ~~R.E~~ $\quad 1\ e_2\ \overbrace{2\ e_3\ 5\ e_4\ 2}^{R.V}$

alternate sequences of vertices & edges

Path: alternate sequences of vertices & edges.

~~R.V~~ | ~~R.E~~

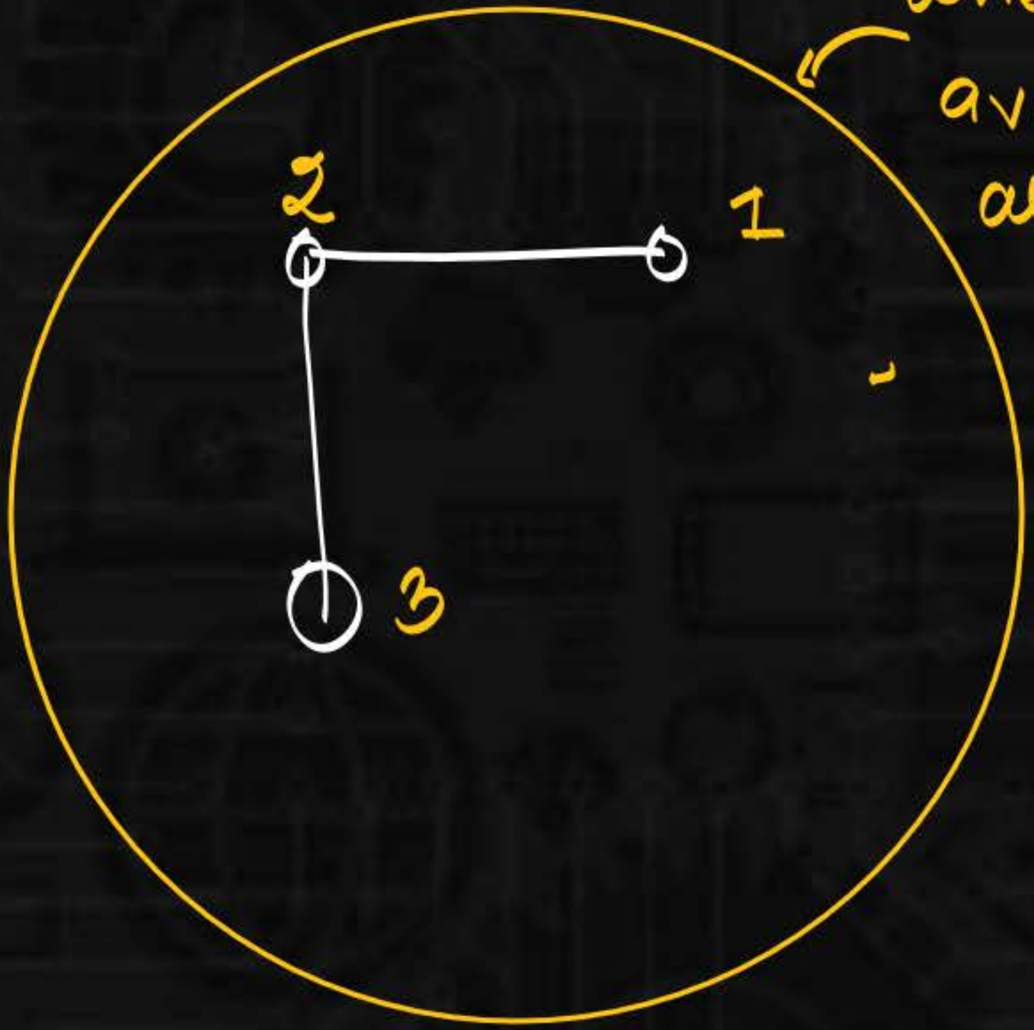


Connectivity in Graphs

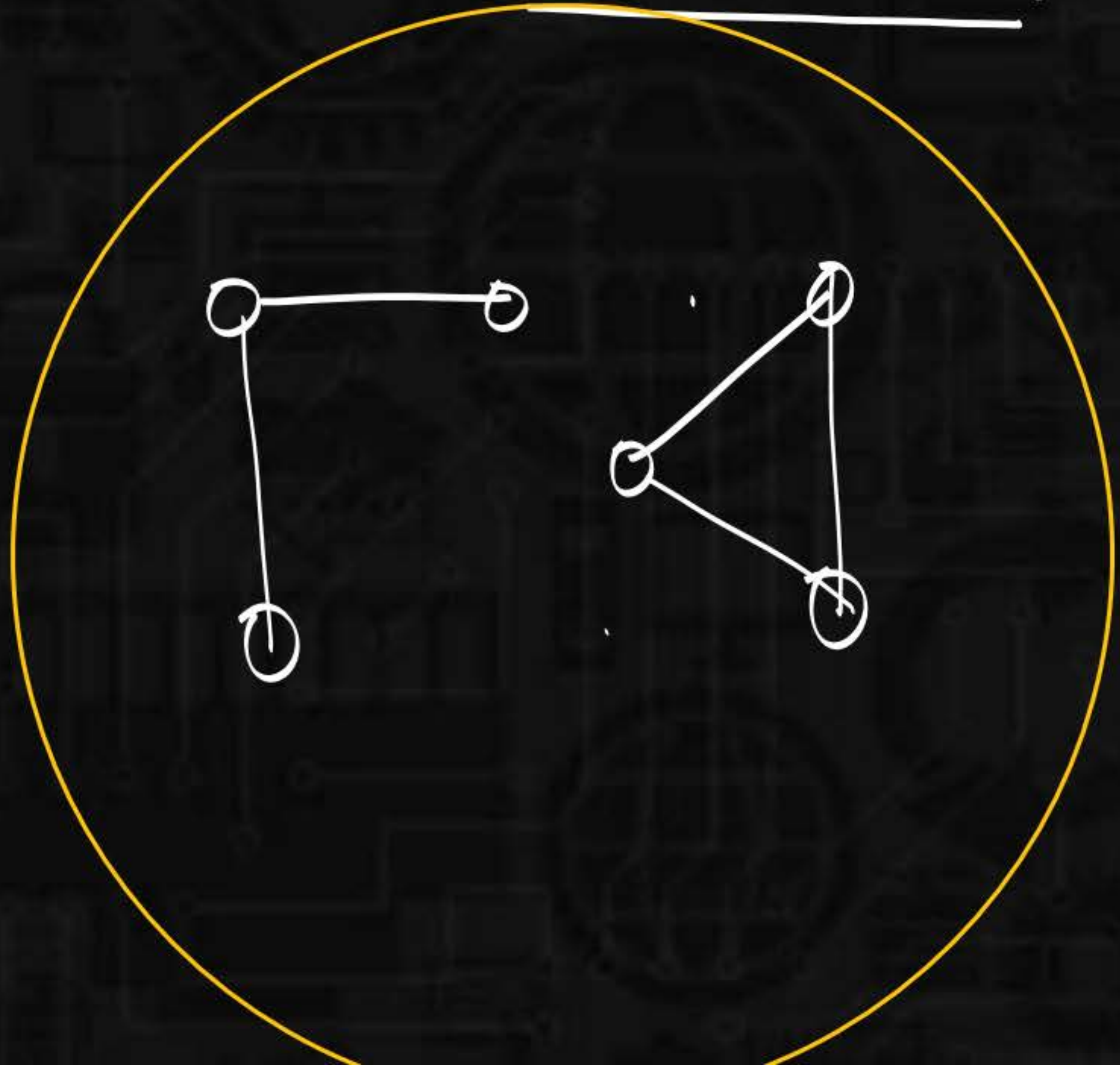
Graphs:

Connected: (K=1)

when Path is available bet^n all pair of vertices.



Disconnected:



Connectivity in Graphs

k=2

Graphs

$(k \geq 2)$

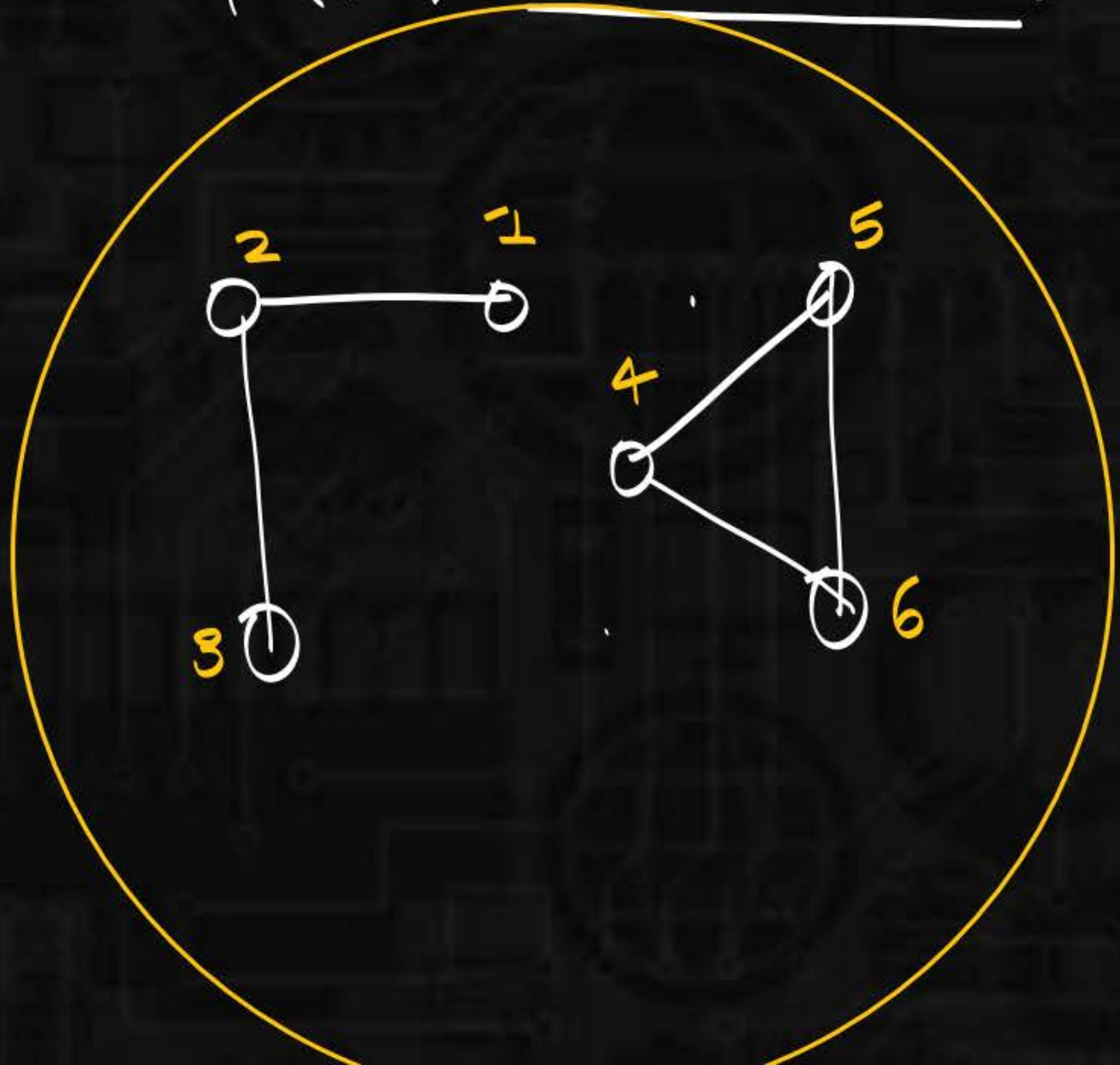
$G=(V,E)$ Disconnected:

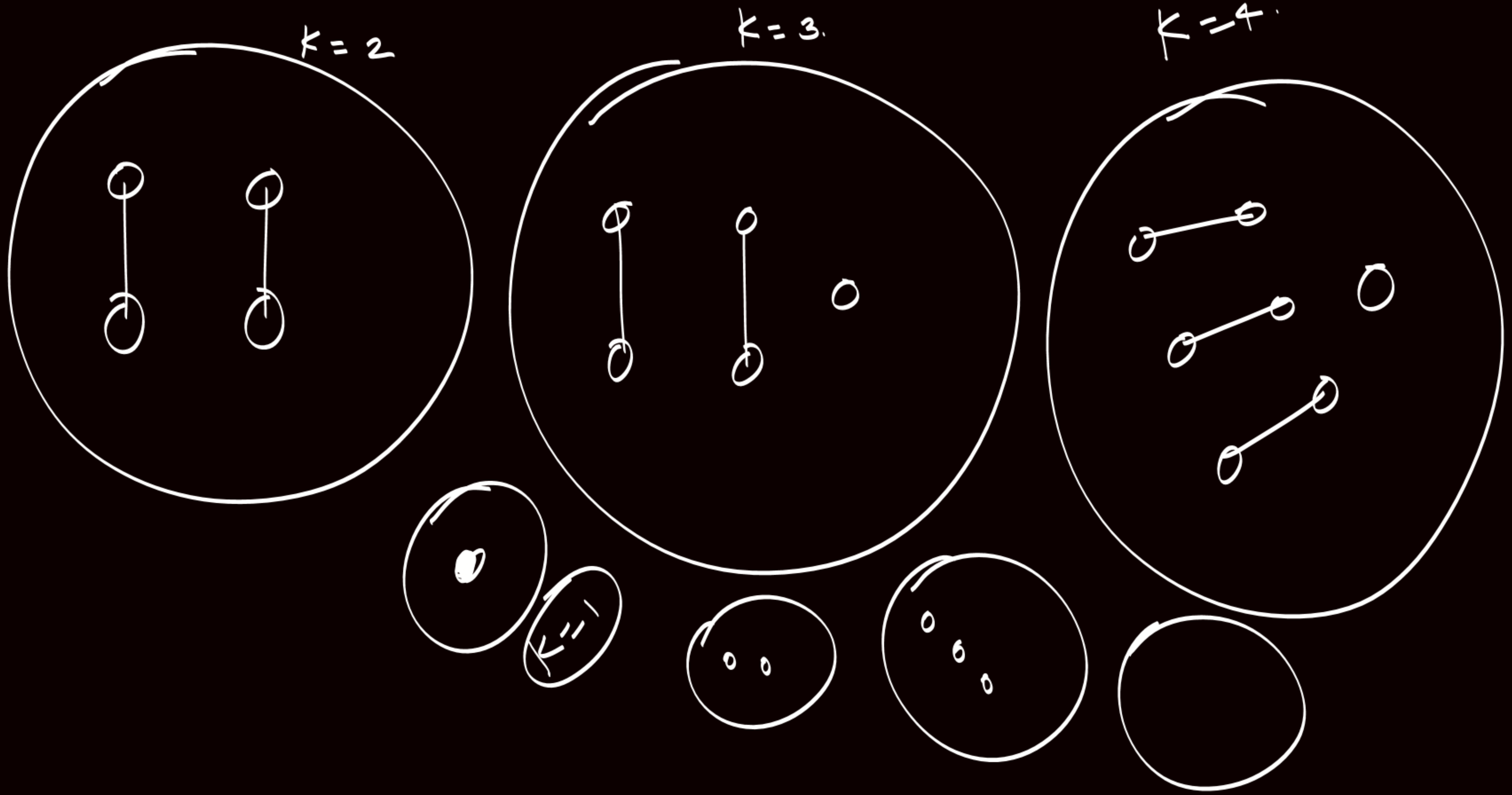
Path is not available for atleast 1 pair of vertices.

Disconnected graph contains

$\boxed{\text{connected subparts}}$

component(k)



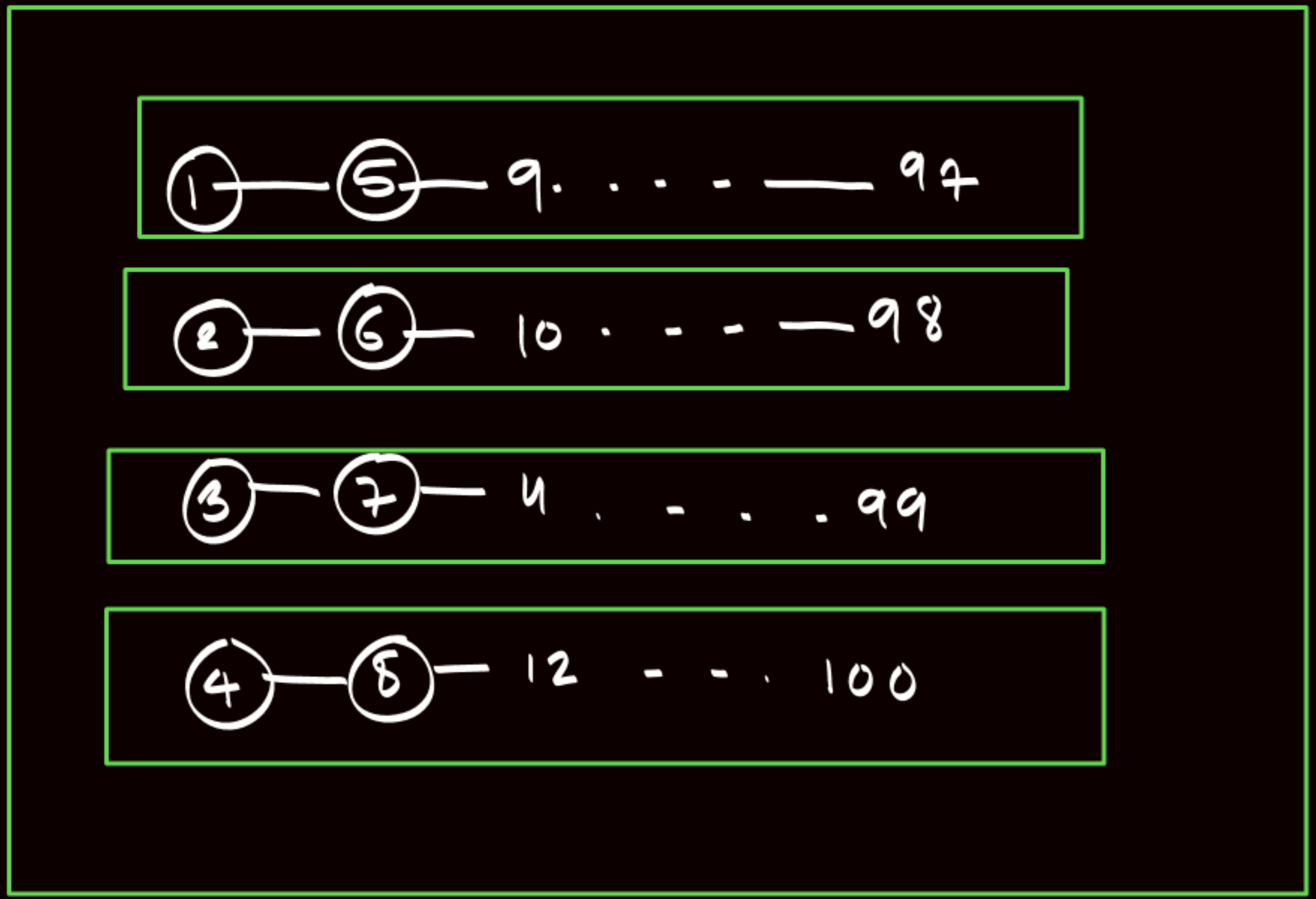

K = 2
K = 3.
K = 4.
K = 1

$G = (V, E)$

$V = \{1, 2, 3, \ldots, 100\}$

$|x - y| = 4$

How many components?



$G = (V, E) \quad K = 4.$

Connectivity in Graphs

1 ≡ 2 ≡ 3

Range of edges ($K = 1$) Connected Graph

Tree

Connected + min. no of edges

$$n-1 \le e \le \frac{n(n-1)}{2}$$

K_n (max no. of edges)

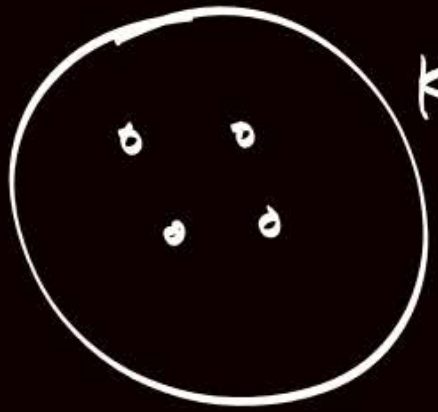
1. Graph does not contains cycle.

2. unique path available bet.^{n} all pair of vertices.

3. minimally connected Graph.

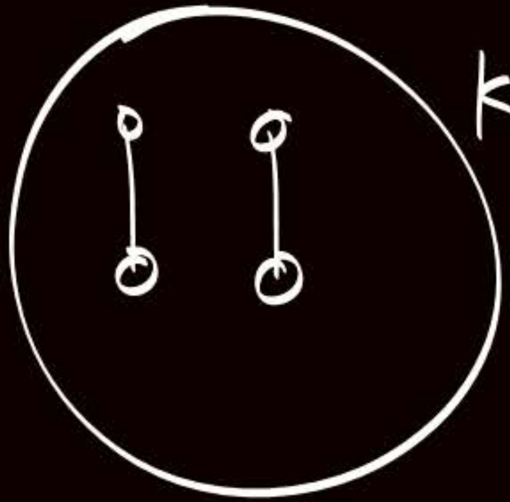
$n = 4 \quad e = 0$

$k = 4.$

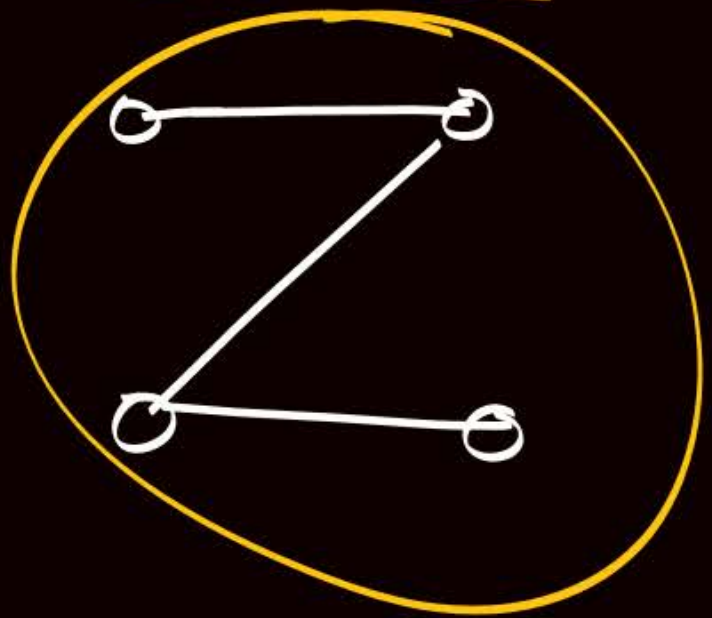


$n = 4 \quad e = 2.$

$k = 2.$

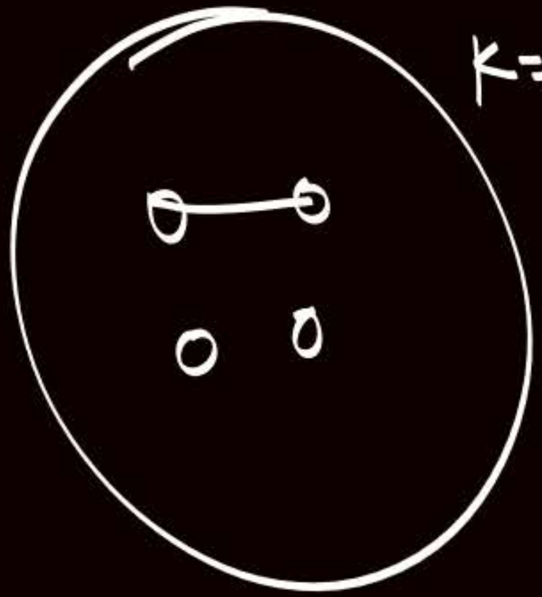


$n = 4 \quad e = 3 \quad k = 1.$



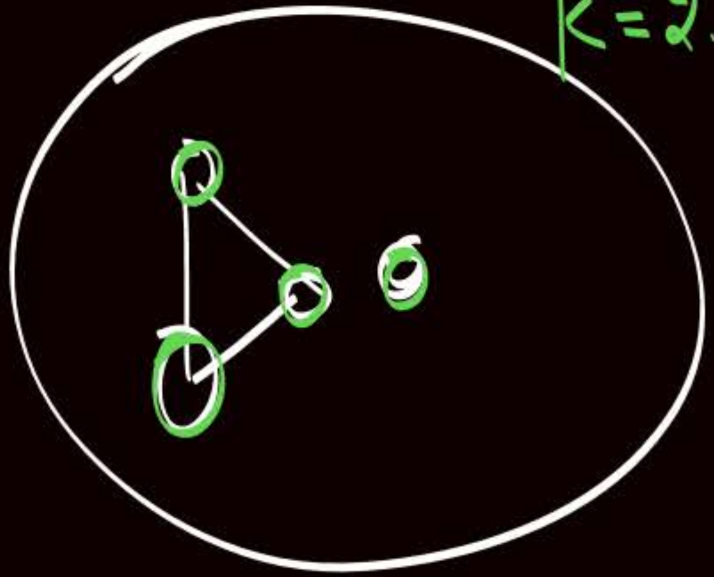
$n = 4 \quad e = 1$

$k = 3$



$n = 4 \quad e = 3$

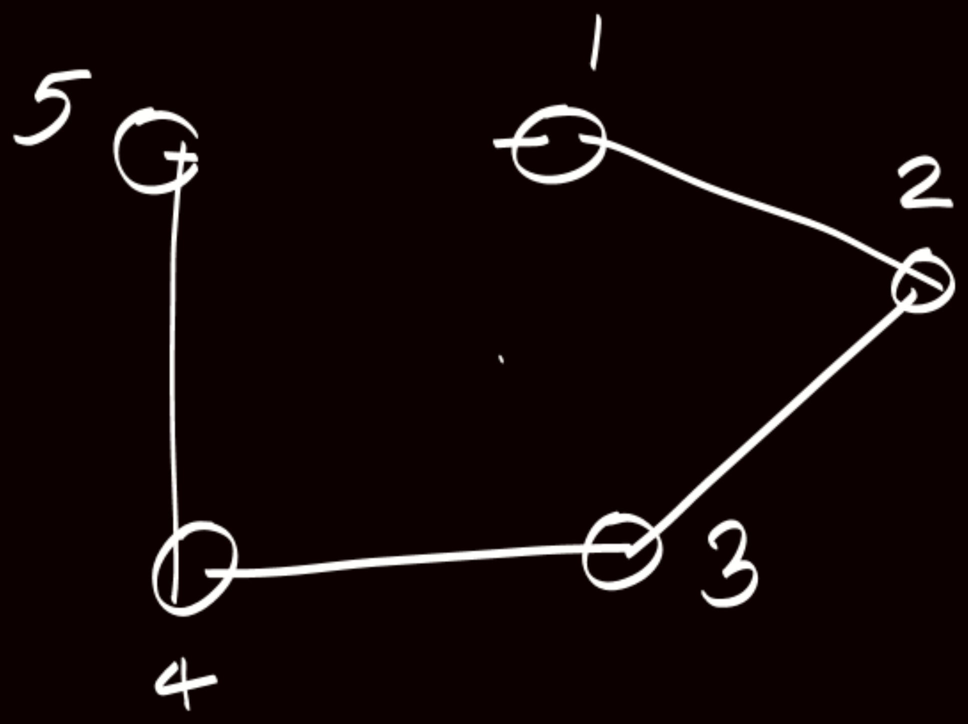
$k = 2.$



$n = 5 \quad e = 4$ ✓

$n = 6 \quad e = 5.$

Total vertices = n   Total no. of edges = $n-1$ (connected)



→ no cycle is possible.

→ unique path is available bet^n all pair of vertices.

→ minimally connected

Connectivity in Graphs

Range of edges $(k \geq 2)$ (Disconnected graph)

$$\left(\begin{array}{c}\text{min no. of}\\\text{edges}\end{array}\right) \; n-k \leq e \leq \frac{(n-k)(n-k+1)}{2} \left(\begin{array}{c}\text{max no. of}\\\text{edges}\end{array}\right)$$

$k = 1$ (connected)

$$\frac{(n-1)(n-\cancel{1}+\cancel{1})}{2}$$

$$= \frac{n(n-1)}{2}$$

Connectivity in Graphs

Total vertices $= n$ $\quad k = 4$. forest

$\underline{n} = n_1 + n_2 + n_3 + n_4$.

$e_1 = n_1 - 1$. $\quad e_2 = n_2 - 1$ $\quad e_3 = n_3 - 1$ $\quad e_4 = n_4 - 1$.

n_1 Tree $\quad n_2$ Tree $\quad n_3$ Tree $\quad n_4$ Tree.

min no. of edges $= n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1$

$= \underline{(n_1 + n_2 + n_3 + n_4)} - 4$.

$n - 4 \equiv \underline{n - k}$ (forest)

Connectivity in Graphs

Graph G having 17 vertices & 5 components

what will be minimum & max no. of edges ?

$n = 17 \quad k = 5$

Connectivity in Graphs

$n = 7 \quad k = 2$

min no of edges $= n - k = 7 - 2 = \underline{\underline{5e}}$

Case1: $2v \quad 5v$

Case2: $1v \quad 6v$

Case3: $3v \quad 4v$.

$e = 2-1 = 1$ ($2v$) $\quad e_2 = 5-1 = 4$ ($5v$) $= 5e$.

$0e$ ($1v$) $+$ $5e$ ($6v$) $= 5e$

$2e$ ($3v$) $\quad 3e$ ($4v$) $= \underline{\underline{5e}}$.

Connectivity in Graphs

$n = 7 \quad k = 2$

max no. of edges

$$= \frac{(n-k)(n-k+1)}{2}$$

$$= (7-2)(7-2+1)/2$$

$$= 5 \cdot 6/2 = 15e$$

Case 1 → $2V \qquad 5V$

$K_2 \qquad K_5$

$$\frac{2\times 1}{2} = 1e + \frac{5\times 4}{2} = 10e = 11 \text{ edges.}$$

Case 2 → $1V \qquad 6V$

$$0e + \frac{6\times 5}{2} = 15e = 15e$$

Case 3 → $3V \qquad 4V$

$$\frac{3\times 2}{2} = 3 + \frac{4\times 3}{2} = 6e = 9 \text{ edges.}$$

Connectivity in Graphs

$n = 11 \quad k = 3$

min no. of edges $= 11 - 3 = 8$ ✓

Case 1: 5v + 3v + 3v

$5-1 = 4e$ + $2e$ + $2e$

$= 8$ edges.

Case 2: 3v + 7v + 1v

$2e$ + $6e$ + $0e$

$\therefore = 8$ edges.

$n = 11 \quad k = 3 \quad e = (n-k)(n-k+1)/2 = (11-3)(11-3+1)/2 = 8 \cdot 9/2 = 36$ edges.

Case 1:

3v, 3v, 5v

K_3: 3e, K_3: 3e, K_5: $\frac{5 \times 4}{2} = 10$

$= 16e$

Case 2:

4v, 4v, 3v

6e, 6e, 3e

$= 15e$

Case 3

1v, 1v, 9v

0e, 0e, 36e

$= 36e$

Connectivity in Graphs

Consider a Graph having 4 components & 100 vertices.

1st component → 25V

2nd component → 30V

3rd component → 20V

4 —"— → 25V

max no. of edges ?

$(n-k)(n-k+1)/2$

25V, 30V, 20, 25V

$$\frac{25\times 24}{2}\ K_{25} + \frac{30\times 29}{2}\ K_{30} + \frac{20\times 19}{2}\ K_{20} + \frac{25\times 24}{2}\ K_{25} = \cdot\ \cdot\ \cdot\ K_{97}$$

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Connectivity in Graphs

K1, K9

Consider disconnected of 10 vertices & what will be maximum no. of edges it can have?

$$e = \frac{(n-k)(n-k+1)}{2}$$

K = 2   K = 3   K = 4 . . . . .

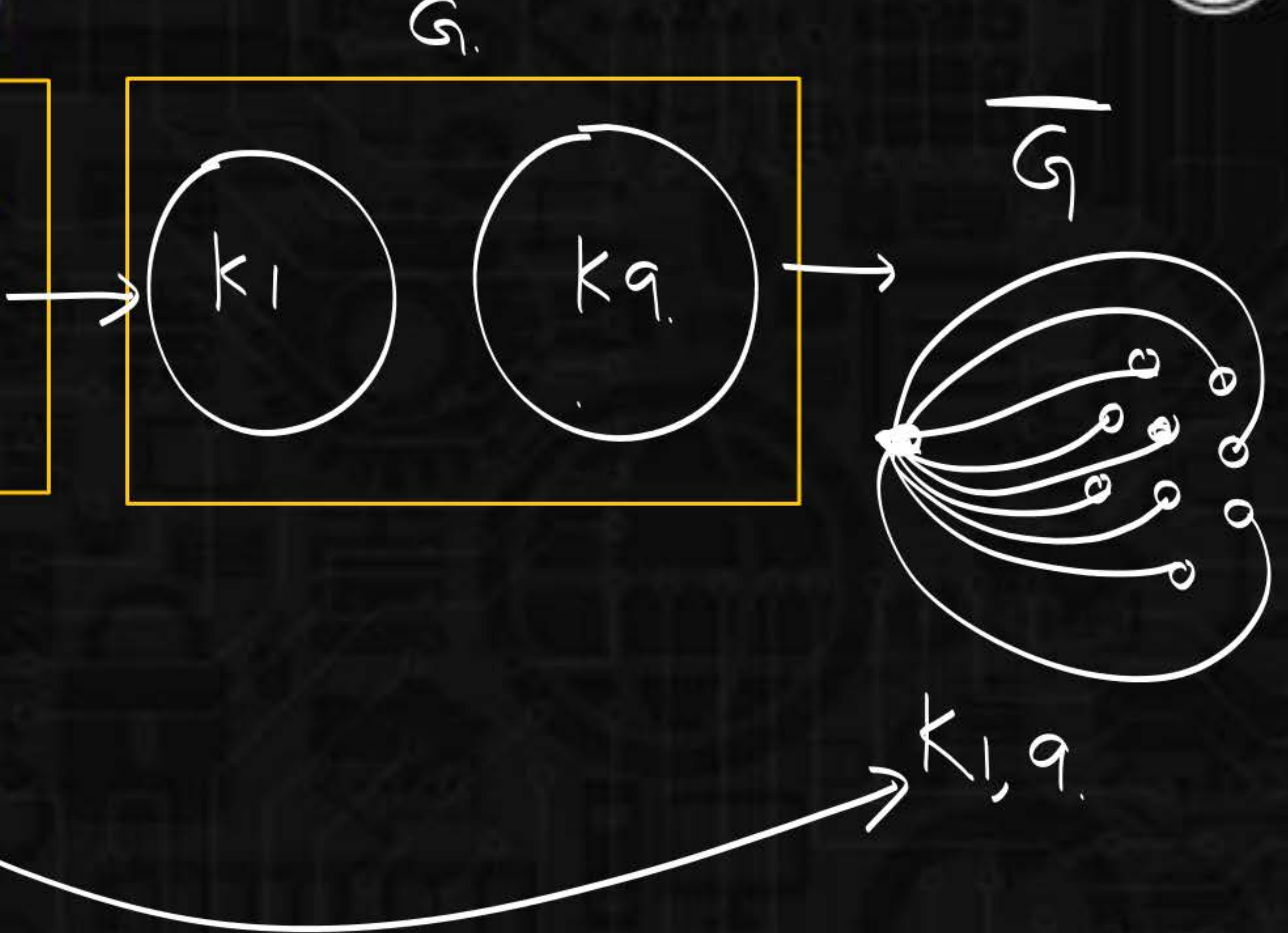
$(10-2)(10-2+1)/2$ $(10-3)(10-3+1)/2$ $(10-4)(10-4+1)/2$

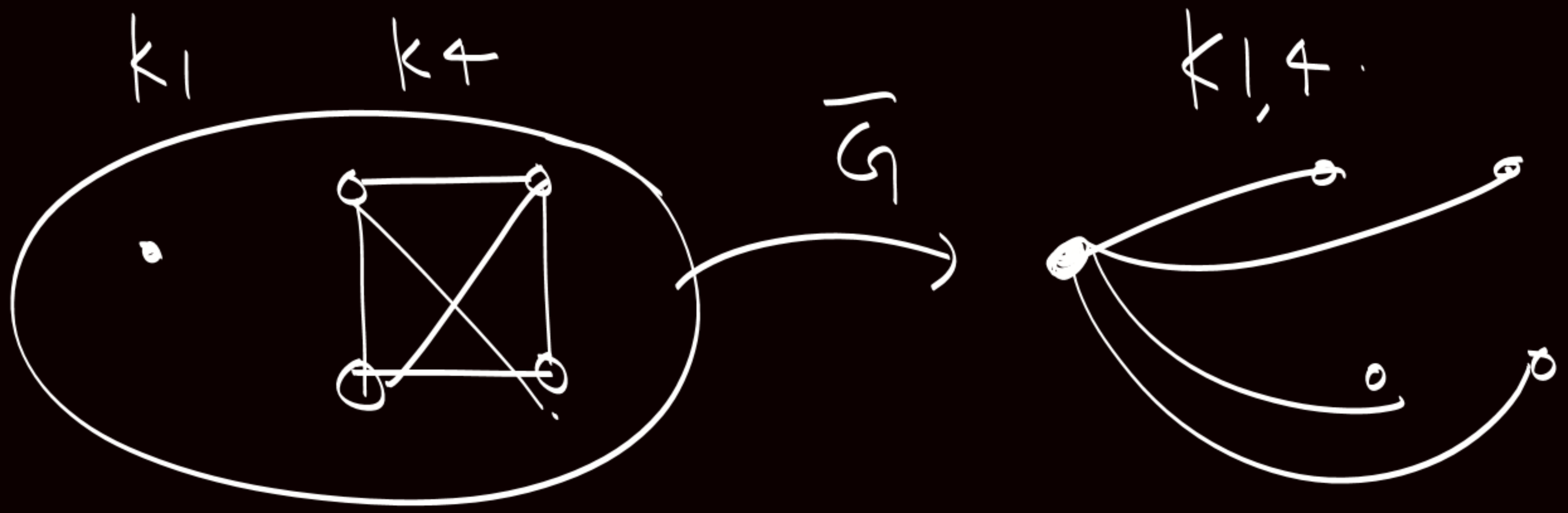
$= \frac{8 \cdot 9}{2}$ $= 7 \cdot 8/2$ $= 6 \cdot 7/2$

$= 36e$ $= 28e$ $= 21e$

Connectivity in Graphs

Consider a disconnected Graph.
10 vertices + maximum no. of edges.




K1
K4
$\overline{G}$
K1,4


THANK YOU GW
SOLDIERS !