CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Combination With Repetition

Lecture No. 2

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Topics to be Covered

01 Combination With Repetition

02

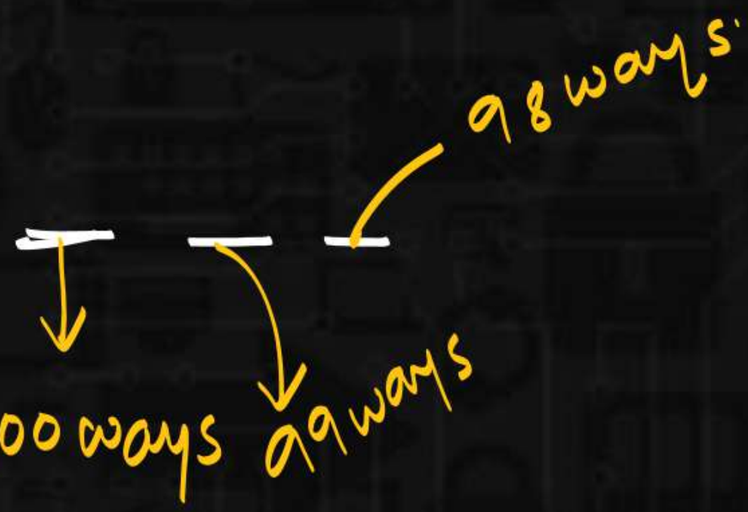
03

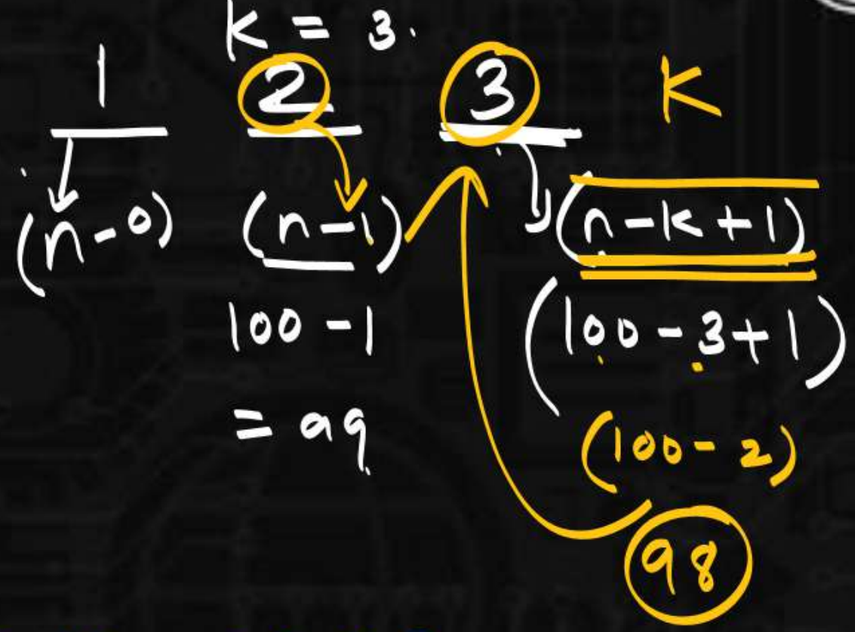
04

COMBINATORICS

$n = 100$

100 prizes, how many ways we can distribute 3 prizes among 3 students?





$= 100 \cdot 99 \cdot 98$

COMBINATORICS

$500 \times 499 \times 498 \times 497$

COMBINATORICS

n ways $(n-1)$ way $\cdots$ $(n-k+1)$

$$= n(n-1)(n-2)\cdots(n-k+1)\,\frac{(n-k)!}{(n-k)!}$$

$$= \frac{n!}{(n-k)!} = {}^{n}\text{Babita}_{k}$$

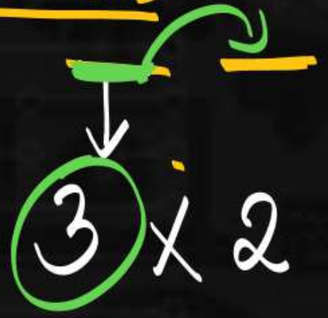
COMBINATORICS

$\{a, b, c\}$

$$^3P_2 = \frac{3!}{(3-2)!} = \frac{3 \cdot 2 \cdot 1}{1!}$$

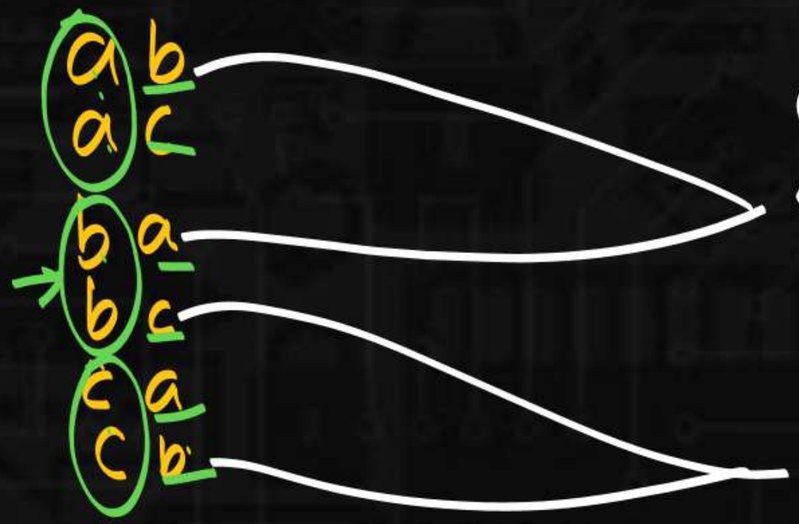
how many ways we can arrange 2 letters?

products



$= 6$ ways

visualization

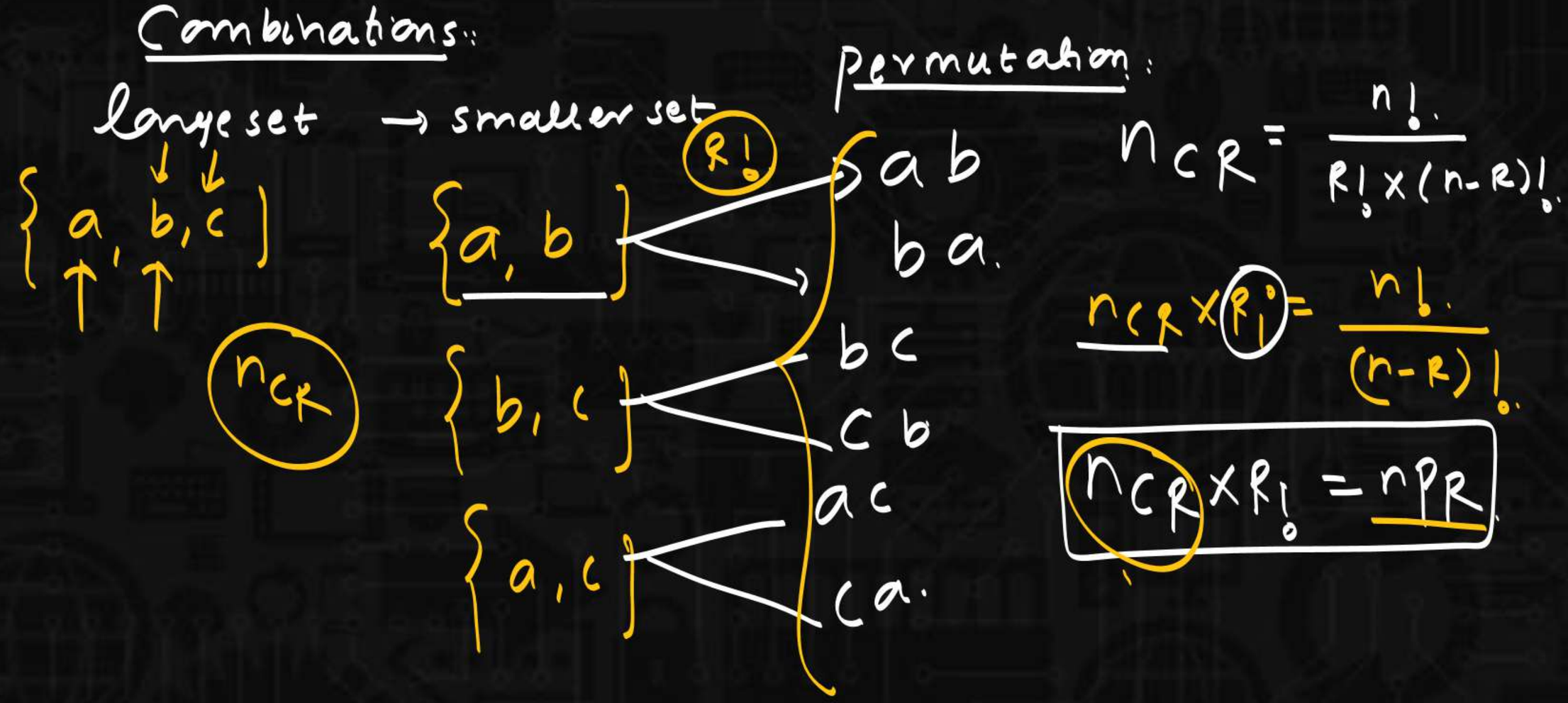


Combination:

$\{a, b\}$

$\{c, b\}$

COMBINATORICS



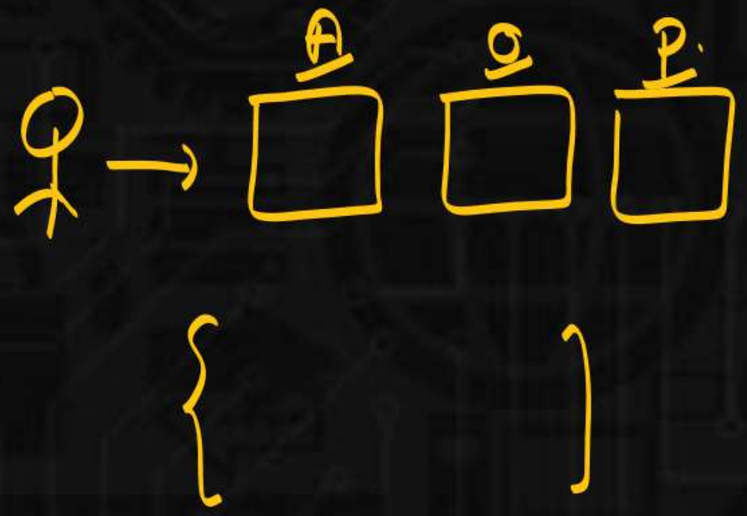
COMBINATORICS

Combination with Repetn. :

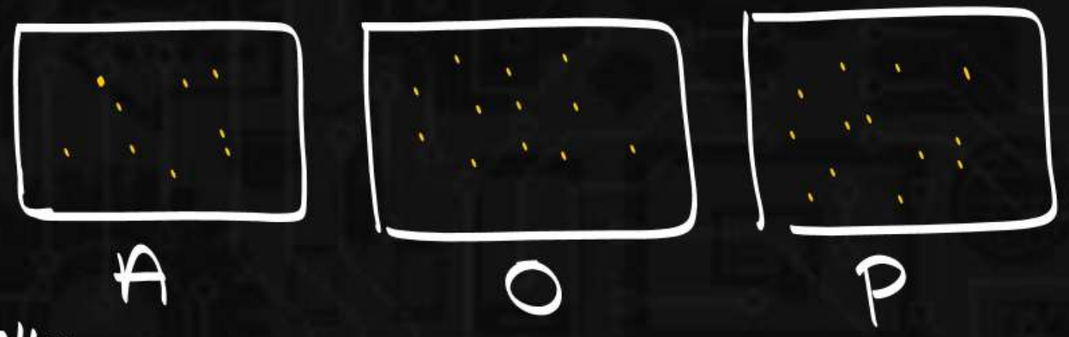
lady, she goes to mall, 3 containers.

Apple, orange, papaya.

how many ways she can choose 4 fruits?



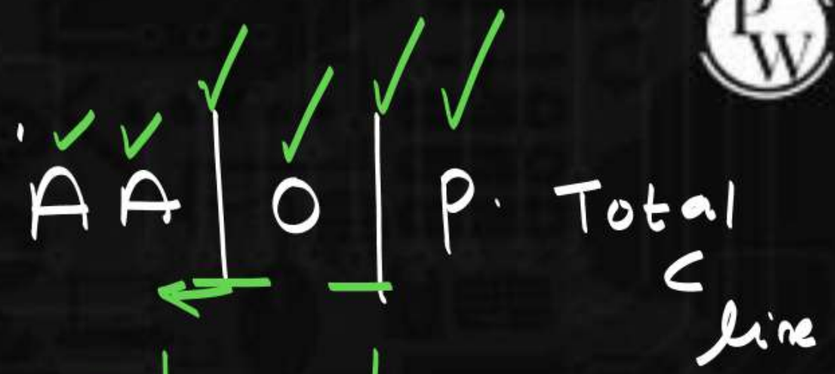
COMBINATORICS



manually:

4A	3A1O	2A2O	2A1O1P
4O	3A1P	2O2P	2O1A1P
4P	3O1A	2P2A	2P1A1O
3	3O1P	3	3
	3P1A		
	3P1O		
	6		

15 ways



A | OO | P → $6C_2$

A | OOO |

$6C_2 = \frac{6 \cdot 5}{2 \cdot 1} =$ 15 ways

COMBINATORICS

{ a, b, c } selecting 1 element = shifting 1 line

how many ways choose 1 element ?.

{a} → { | - - }
{b} → { - | - }
{c} → { - - | }

$^3C_1$

3 line

Total 3 line.

{ a, b, c } → $^3C_2$

how many ways we can choose 2 elements ?.

{ a b } → { | | - }
{ a c } → { | - | }
{ b c } → { - | | }

$^3C_2$

COMBINATORICS

How many ways we can distribute 5 coins among 3 children ?.

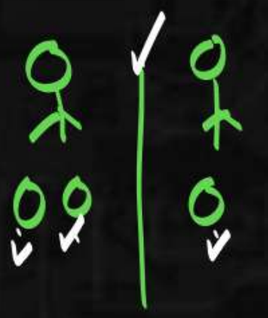
5 coins Similar

Total ways $= {}^{7}C_{2} = \frac{}{}$.

how many ways we can distribute 3 coins among 2 children ?.

4 ways.

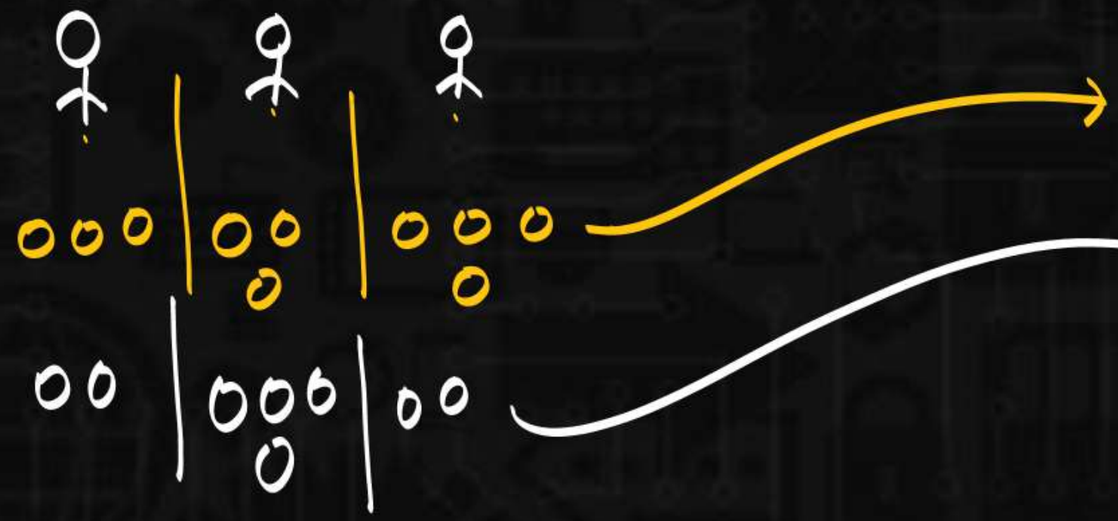
3	0 → ooo |
0	3 → | ooo
2	1 → oo | o
1	2 → o | oo



Total ways $= {}^{4}C_{1} = 4$

COMBINATORICS

how many ways we can distribute 10 similar coins among 3 students ?



$x_1 + x_2 + x_3 = 10$

$x_1 = 3 \quad x_2 = 3 \quad x_3 = 4$

$x_1 = 2 \quad x_2 = 4 \quad x_3 = 2$

$x_1 + x_2 + x_3 = 10 \quad x_i \geq 0$

how many non negative integer solutions are possible ?

$^{12}C_2$

COMBINATORICS

Objects containers

10 similar coins → 3 students

S1 S2 S3
10 0 0

Object	container → maybe empty
Same	diff.

C.R

Onto

cannot empty.

Objects distinct

a1
a2
a3
a4
:
a10

containers distinct

n1
n2
n3

COMBINATORICS

How many ways we can distribute 10 similar coins among 3 students ?.

$= \quad x_1 + x_2 + x_3 = 10 \quad x_i \geq 0$

$^{10+2}C_2$

$^{12}C_2$.

how many ways we can distribute 10 similar coins among 3 students such that each student will get atleast 1 coin?.

COMBINATORICS

$x_1 + x_2 + x_3 = 10 \quad x_i \geq 1$

how many ways we can distribute 10 similar coins among 3 students such that each student will get atleast 1 coin?

COMBINATORICS

10 coins 3 stnds.

$10+2C_2 = 12C_2$

0 0 0 $\quad 10-3=7.$

0 0 0

7 coins

$7+2C_2 = 9C_2.$

COMBINATORICS

$x_1 + x_2 + x_3 = 10 \quad x_i \geq 1.$

$x_i \geq 1.$

$x_i - 1 \geq 0$

y_i

$y_i = x_i - 1$

$y_i + 1 = x_i$

$y_1 + 1 = x_1$

$y_2 + 1 = x_2$

$y_3 + 1 = x_3.$

$y_1 + 1 + y_2 + 1 + y_3 + 1 = 10$

$y_1 + y_2 + y_3 = 10 - 3 \quad y_i \geq 0.$

$y_i \geq 0$

$y_1 + y_2 + y_3 = 7 \quad y_i \geq 0$

$^{7+2}C_2$

COMBINATORICS

Q: how many ways we can distribute 10 similar coins among 3 students such that each students must get at least 2 coins ?.

=

$x_1 + x_2 + x_3 = 10$

$x_i \geq 2$.

Ans: $6C_2$.

COMBINATORICS

$x_1 + x_2 + x_3 = 20$

$x_1 \geq 2 \quad x_2 \geq 3 \quad x_3 \geq 4.$

$\underbrace{x_1 - 2 \geq 0}_{y_1} \quad \underbrace{x_2 - 3 \geq 0}_{y_2} \quad \underbrace{x_3 - 4 \geq 0}_{y_3}$

$x_1 - 2 = y_1 \quad x_2 - 3 = y_2 \quad x_3 - 4 = y_3$

$x_1 = y_1 + 2 \quad x_2 = 3 + y_2 \quad x_3 = y_3 + 4.$

$y_i \geq 0$

$\rightarrow \quad y_1 + 2 + y_2 + 3 + y_3 + 4 = 20$

$y_1 + y_2 + y_3 = 20 - (2+3+4)$

$y_1 + y_2 + y_3 = 11$

$11 + 2 C_2 = {}^{13}C_2$

COMBINATORICS

HW

$x_1 + x_2 + x_3 + x_4 = 32.$

a) $x_i \geq 0$

b) $x_i > 0 \quad x_i \geq 1.$

c) $x_1, x_2 \geq 5 \quad x_3, x_4 \geq 7$

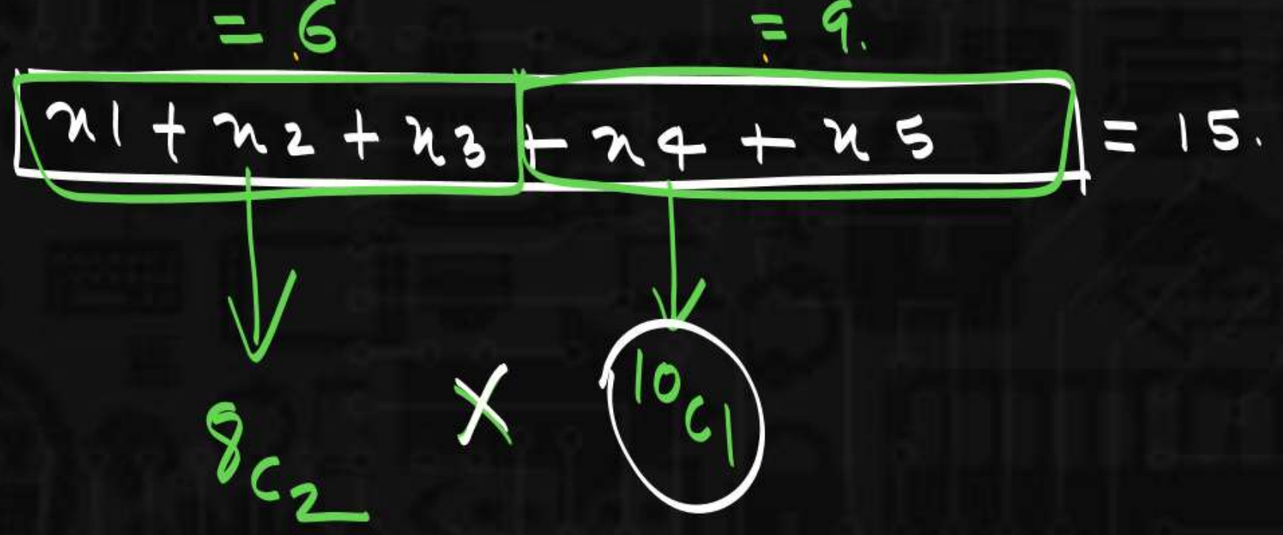
d) $x_i \geq 8$

e) $x_i \geq -2$

COMBINATORICS

$x_1 + x_2 + x_3 + x_4 + x_5 = 15$.

how many nonnegative solutions are possible if $x_1 + x_2 + x_3 = 6$.



COMBINATORICS

(HW) $x_1 + x_2 + x_3 \ldots x_7 = 37$.

$5C_3 \times {}^{34}C_{31}$

$x_1 + x_2 + x_3 = 6$ $\qquad x_i^\circ > 0$

$x_i^\circ \geq 1$.

(HW) how many ways we can toss 100 identical dice so that at least 3 of each face will be showing?

$87C_{82}$

COMBINATORICS

m identical balls, n distinct bags. $m \geq kn$

how many ways can balls be placed each bag contains atleast k balls? (GATE '03)

a. $\binom{m-k}{n-1}$

b. $\binom{m-kn+n-1}{n-1}$

c. $\binom{m-1}{n-k}$

d. $\binom{m-kn+n-2}{n-k}$

COMBINATORICS

Total balls = m.
Total boxes = n. Partitions = n-1.

Total balls = m

Remaining balls = m - nk.

$$m-nk+n-1 \; C_{n-1}$$


THANK YOU GW
SOLDIERS !