COMPUTER SCIENCE

Database Management System

FD's & Normalization

Minimal Cover-2

Lecture_06

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TOPICS TO BE COVERED

01 Equality between FD Set

02 Minimal Cover

RDBMS Concept

FD Concept

FD types

Attribute closure $[X]^+$

key Concept

Super key

Candidate key

Finding Multiple Candidate key.

Membership set.

Membership Set

$X \rightarrow Y$ is member of FD Set F.

iff $X \rightarrow Y$ logically implied in F

Logically implied means $[X]^+ = [\ldots\ldots Y]$ then

$X \rightarrow Y$ is logically implied / Member of / Valid FD.

nC.k

→ 1 P.K

→ (n-1) A.K / S.K

(a) cc with enjoying

(b) cc

(c) c

(d) Doubt.

Member Ship Set

$F = [\ldots]$

$X \to y$ is Member of FD Set or Not?

$X \to y$ is a member iff. $X \to y$ Logically Implied in F.

$[X]^+ = [\ldots y]$

then $X \to y$ is Member or Logically implied or Valid FD.

Q. In a schema with attributes A, B, C, D and E following set of functional dependencies are given

$A \rightarrow B$

$A \rightarrow C$

$CD \rightarrow E$ $\quad (CD)^+ = [CDEA]$

$B \rightarrow D$ $\quad [BC]^+ = [BCD \ldots\ldots$

$E \rightarrow A$ $\quad BC \rightarrow CD$ is logically Implied,

Which of the following functional dependencies is NOT implied by the above set

[MCQ: GATE - 2M]

(C) $CD \rightarrow AC$ Implied

(A) $BD \rightarrow CD$

(D) $BC \rightarrow CD$

(B) $AC \rightarrow BC$

Q.

Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S and T:

$$P \rightarrow QR$$

$$RS \rightarrow T$$

Which of the following functional dependencies can be inferred/implied from the above functional dependencies?

[MSQ: 2021 - 2M]

A) $PS \rightarrow T$

B) $R \rightarrow T$

C) $P \rightarrow R$

D) $PS \rightarrow Q$

Equality between Two FD set

$F = [\ldots\ldots]$ $G = [\ldots\ldots\ldots]$

let F & G are the given 2 FD set

$(F)^+ = (G)^+$ F and G equal $(F \equiv G)$

iff (i) F Cover G : True

(ii) G Cover F : True.

F Cover G : F Covers all the FD's of G.

(OR)

ALL the functional Dependency (FD's) of G. Should be logically implied by F FD Set.

G Cover F : G Covers all the FD's of F.

(OR)

ALL the Functional Dependency's (FD's) of F, Should be logically implied by G FD Set.

F cover G	True	False	True	False
G cover F	False	True	True	False
	$F \supset G$	$G \supset F$	$F \equiv G$	Uncomparable

$F : [AB \to CD, B \to C, C \to D]$

$G : [AB \to CD, AB \to D, C \to D]$

$\rightarrow F: [\overset{①}{AB \to CD}, \overset{②}{B \to C}, \overset{③}{C \to D}]$

F Cover G

$AB \to CD$ ✓ $[AB]^+ = [ABCD]$

$AB \to D$ ✓ $[AB]^+ = [ABCD]$

$C \to D$ ✓ $[C]^+ = [CD]$

True

$\rightarrow G: [\overset{①}{AB \to CD}, \overset{②}{AB \to D}, \overset{③}{C \to D}]$

G Cover F

$AB \to CD$ ✓ $[AB]^+ = [ABCD]$

$B \to C$ ✗ $[B]^+ = [B]$

$C \to D$ ✓ $[C]^+ = [CD]$

False.

$\therefore \boxed{F \supset G.}$

Q.

Consider relation schema A(P Q R S) with two set of FD's

F : [P → Q, PQ → R, PR → S, Q → R, Q → P]

G : [PQ → S, PR → Q, Q → S, QS → R]

Which of the following is correct?

(A) F Cover G

(B) G Cover F $\boxed{\text{Ans (A)}}$

(C) F and G are equivalent

(D) None of these

F Cover G

✓ $PQ \to S$ — $(PQ)^+ = [PQRS]$

✓ $PR \to Q$ — $(PR)^+ = [PRSQ]$

✓ $Q \to S$ — $(Q)^+ = [QRPS]$

✓ $QS \to R$ — $(QS)^+ = [QSRP]$

True

$F \supseteq G$

G Cover F

✗ $P \to Q$ — $(P)^+ = [P]$

✓ $PQ \to R$ — $(PQ)^+ = [PQSR]$

✓ $PR \to S$ — $(PR)^+ = [PRQS]$

✓ $Q \to R$ — $(Q)^+ = [QSR]$

✗ $Q \to P$ — $(Q)^+ = [QSR]$

False

Q. Consider relation schema R(A C D E H) with two set of FD's

[MSQ]

F : [$A \to C$, $AC \to D$, $E \to AD$, $E \to H$]

G : [$A \to CD$, $E \to AH$]

Which of the following is correct?

(A) F Cover G

(B) G Cover F

(C) F and G are equivalent

(D) None of these

Ans (A), (B) & (C).

F Cover G

$A \to CD$ $\quad [A]^+ = [ACD]$

$E \to AH$ $\quad [E]^+ = [EADHC]$

True

G Cover F

$A \to C$ $\quad [A]^+ = [ACD]$

$AC \to D$ $\quad [AC]^+ = [ACD]$

$E \to AD$ $\quad [E]^+ = [EAHCD]$

$E \to H$

True

$\boxed{F \equiv G}$

Membership.

↳ Logically Implied

$X \rightarrow Y$ Logically Implied

$\{X\}^+ = \{ \cdot - \cdot Y \}$

Minimal cover

extra FD
or

The objective of Minimal Cover is elimination of Redundant FD (RFD)

Redundant FD. : R.F.D or extra FD is a FD, if we Delete (R.F.D) that FD then after Deletion does not effect the Power of FD Set.

eg F: [$A \rightarrow B$, $B \rightarrow C$, $A \rightarrow C$]

Lets Assume $A \rightarrow C$ is Redundant (Extra) FD.

New FD set G: [$A \rightarrow B$, $B \rightarrow C$]

eg) F: [$A \to B$, $B \to C$, $A \to C$]

Lets Assume $A \to C$ is R.F.D ; After Deletion of $A \to C$ from original FD set F

getting New FD set $\boxed{G: [A \to B, \ B \to C]}$

$[A]^+ = [ABC]$ from $[A]^+$ getting C

So $A \to C$ is R.F.D. Ans

$F = [A \to B,\ B \to C,\ A \to C]$

$\boxed{F \equiv G}$? F Cover G : True, G Cover F : True.

① Assume $A \to B$ is R.F.D

$G : [B \to C,\ A \to C]$

F Cover G : ✓ $B \to C$ $(B)^+ = [BC]$

$A \to C$ $(A)^+ = [ABC]$

True.

G Cover F

$A \to B$ ✗ $(A)^+ = [AC]$

$B \to C$

$A \to C$

False

$\therefore$ $A \to B$ is Not R.F.D

② Assume $B \to C$ is R.F.D

$G : [A \to B,\ A \to C]$

F Cover G ✓ $A \to B$ $(A)^+ = [ABC]$

✓ $A \to C$

True

G Cover F

$A \to B$ ✓ $(A)^+ = [ABC]$

✗ $B \to C$ ✗ $(B)^+ = [B]$

$A \to C$ ✓

False

$\therefore$ $B \to C$ is Not R.F.D

③ Assume $A \to C$ is R.F.D

$G : [A \to B,\ B \to C]$

F Cover G : ✓ $A \to B$ $(A)^+ = [ABC]$

✓ $B \to C$ $(B)^+ = [BC]$

True.

G Cover F

✓ $A \to B$ $(A)^+ = [ABC]$

✓ $B \to C$ $(B)^+ = [BC]$

✓ $A \to C$ $(A)^+ = [ABC]$

True.

$\boxed{F \equiv G}$

$\therefore$ $A \to C$ is extra (R.F.D) FD

Minimal Cover : $A \to B,\ B \to C$.

Minimal Cover /

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

$A \rightarrow C$ is R.F.D.

Minimal Cover : $[A \rightarrow B, B \rightarrow C]$

Q. $AB \rightarrow C$, $D \rightarrow E$, $E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C$, $D \rightarrow E$, $AB \rightarrow E$, $E \rightarrow C$.

[GATE: 2 Marks]

Minimal Cover.

F: $[AB \rightarrow C,\ D \rightarrow E,\ AB \rightarrow E,\ E \rightarrow C]$

G: $[AB \rightarrow C,\ D \rightarrow E,\ E \rightarrow C]$

F Cover G.

✓ $AB \rightarrow C$ $\quad [AB]^+ = [ABCE]$

✓ $D \rightarrow E$ $\quad (D)^+ = (DEC)$

✓ $E \rightarrow C$ $\quad [E]^+ = [EC]$

True.

G Cover F

✓ $AB \rightarrow C$ $\quad [AB]^+ = [ABC]$

✓ $D \rightarrow E$ $\quad (D)^+ = [DEC]$

✗ $AB \rightarrow E$ $\quad [AB]^+ = [ABC]$

✓ $E \rightarrow C$ $\quad [E]^+ = [EC]$

False.

Not a Minimal Cover.

Procedure to bind minimal cover

Step1 : Split the FD Such that R.H.S (Right hand Side) Contain Single Attribute.

eg $X \rightarrow yz \Rightarrow X \rightarrow y,\ X \rightarrow z$.

Step 2 : Find the Redundant (Extra) Attribute on L.H.S & Remove from the FD Set.

eg $AB \longrightarrow C$; A is extra if $(B)^+ = [\ldots\ldots A]$ B^+ Contain A; then A extra.

B is extra if $[A]^+ = [\ldots\ldots B]$ A^+ Contains B; then B extra.

Step 3 : Find the Redundant (Extra) FD, & Delete/Remove from FD set F.

eg $A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C$; Here $A \rightarrow C$ is Redundant FD So Delete/Remove then

$\boxed{A \rightarrow B,\ B \rightarrow C}$.

Procedure to find minimal set

Step (1)

Split the FD such that RHS contain single Attribute.

Ex. $A \rightarrow BC, \quad \Rightarrow \quad A \rightarrow B$ and $A \rightarrow C$

Step (2)

Find the redundant attribute on L.H.S and delete them.

Ex. $AB \rightarrow C,$

A – Can be deleted $[B]^{+}=[A]$ $\quad B^{+}$ Contains 'A'

extra

OR

B can be delete if A^{+} contain 'B' $\quad [A]^{+}=[...B]$

Step

(3)

Find the redundant FD and delete them from the set

Ex. $\{A \rightarrow B,\ B \rightarrow C,\ \underbrace{A \rightarrow C}_{R.F.D.}\}$

$\{A \rightarrow B,\ B \rightarrow C\}$

Example1:

$[AB \to CD, A \to E, E \to C]$

Step1: Split the FD such that R.H.S Contain Single Attribute.

$AB \to C$, $AB \to D$ $A \to E$, $E \to C$

①	②	③	④
$AB \to C$,	$AB \to D$.	$A \to E$,	$E \to C$.

Step2: On L.H.S Find Redundant (Extra) Attribute

$AB \to C$

A is extra if $[B]^+$ Contain A.

B is extra if $[A]^+$ contain B.

$[A]^+ = [AEC]$ getting Not b.

so B is Not extra.

$[B]^+ = [B]$ Not getting a: So A is Not Extra.

Minimal cover.

$AB \to D$

$A \to E$

$E \to C$

Step3: Find the Redundant FD & Delete from FD set.

$AB \to C$ extra

① $\cancel{AB \to C}$ ② $AB \to D$ ③ $A \to E$ ④ $E \to C$

$[AB]^+ = [ABDEC]$ $[AB]^+ = [ABEC]$ $[A]^+ = [A]$ $[E]^+ = [E]$

Example 2:

$$[A \to C,\ AC \to D,\ E \to AD,\ E \to H]$$

Step1: R.H.S contain single Attribute.

$\boxed{A \to C \quad AC \to D, \quad E \to A, \quad E \to D, \quad E \to H}$

Step2: find Redundant Attribute on L.H.S.

$AC \to D$; A is extra if $[C]^+$ Contain A

C is extra if $[A]^+$ Contain C.

$(A)^+ = [AC \ldots]$ C is extra Bcz $(A)^+$ contain C in $A\cancel{C} \to D$

$\boxed{A \to C,\ A \to D,\ E \to A,\ E \to D,\ E \to H}$

$AC \to D$

$[C]^+ = [C]$; A is Not extra Bcz $[C]^+$ Not Contain A.

Minimal Cover.

$\boxed{A \to C \quad A \to D \quad E \to A \quad E \to H}$ Ans

OR

$\boxed{A \to CD,\ E \to AH}$ Ans

Step3: Find the Redundant FD & Delete from FD set

1. $A \to C$ — $(A)^+ = [AD]$
2. $A \to D$ — $(A)^+ = [AC]$
3. $E \to A$ — $(E)^+ = [EDH]$
4. ~~$E \to D$~~ — $(E)^+ = [EHACD]$
5. $E \to H$ — $(E)^+ = [EACD]$

$\underline{F}$

$[A \to C,\ AC \to D,\ E \to AD,\ E \to H]$

$\underline{\underline{G}}$ <u>Minimal cover.</u>

$A \to CD,\ E \to AH$

F Cover G : True

G Cover F : True

$(F \equiv G)$

(Already Done in equality b/w 2 FD Set)

Note How to check Extra FD (Assume $A \to C$) Directly?

Soln Hide that FD & take closure of L.H.S of that FD in all the Remaining FD Set [except $A \to C$], if we are getting R.H.S of that FD then we can say ($A \to C$) that FD is Extra FD (R.F.D)

Note Minimal Cover May or May Not Unique. ie we can have More than One Minimal Cover.

Example3:

Q.1 HW

$$[B \rightarrow A,\ D \rightarrow A,\ AB \rightarrow D]$$

Example 4:

Q.2

HW. $[A \to BC,\ CD \to E,\ E \to C,\ D \to AEH,\ ABH \to BD,\ DH \to BC]$

Q.

Given the following two statements:

S1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF.

Not Done in Previous slide.

S2: $AB \rightarrow C, D \rightarrow E, E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C,\ D \rightarrow E, AB \rightarrow E, E \rightarrow C$.

Which one of the following is CORRECT? [MCQ: 2014: 2M]

A. S1 is TRUE and S2 is FALSE.

B. Both S1 and S2 are TRUE.

C. S1 is FALSE and S2 is TRUE.

D. Both S1 and S2 are FALSE.

Q. 3

HW

The following functional dependencies hold true for the relational schema R{V, W, X, Y, Z}:

$$V \rightarrow W$$
$$VW \rightarrow X$$
$$Y \rightarrow VX$$
$$Y \rightarrow Z$$

Which of the following is irreducible equivalent for this set of functional dependencies? [MCQ:2017]

(A)
$V \rightarrow W$
$V \rightarrow X$
$Y \rightarrow V$
$Y \rightarrow Z$

(B)
$V \rightarrow W$
$W \rightarrow X$
$Y \rightarrow V$
$Y \rightarrow Z$

(C)
$V \rightarrow W$
$V \rightarrow X$
$Y \rightarrow V$
$Y \rightarrow X$
$Y \rightarrow Z$

(D)
$V \rightarrow W$
$W \rightarrow X$
$Y \rightarrow V$
$Y \rightarrow X$
$Y \rightarrow Z$

Q.4

H.W

Consider the following FD Set: $\{P \rightarrow QR, Q \rightarrow PR, R \rightarrow PQ\}$ which of the following is/are the minimal cover for the above FD set?

[MSQ]

(A) $P \rightarrow Q, Q \rightarrow R, R \rightarrow P$

(B) $P \rightarrow R, Q \rightarrow R, R \rightarrow PQ$

(C) $Q \rightarrow P, P \rightarrow R, R \rightarrow Q$

(D) $P \rightarrow QR, Q \rightarrow P, R \rightarrow P$

[Home Work]

Any Doubt ?

THANK YOU!