CS & IT ENGINEERING

Graph Theory

Planarity Part-2

Lecture No.12

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TOPICS TO BE COVERED

$\chi(G)$

$\beta(G)$

$\alpha(G)$

$m(G)$

$\varepsilon(G)$

01 covering set

02 Covering number

03 Planar Graph

04 Euler's Formula In planarity

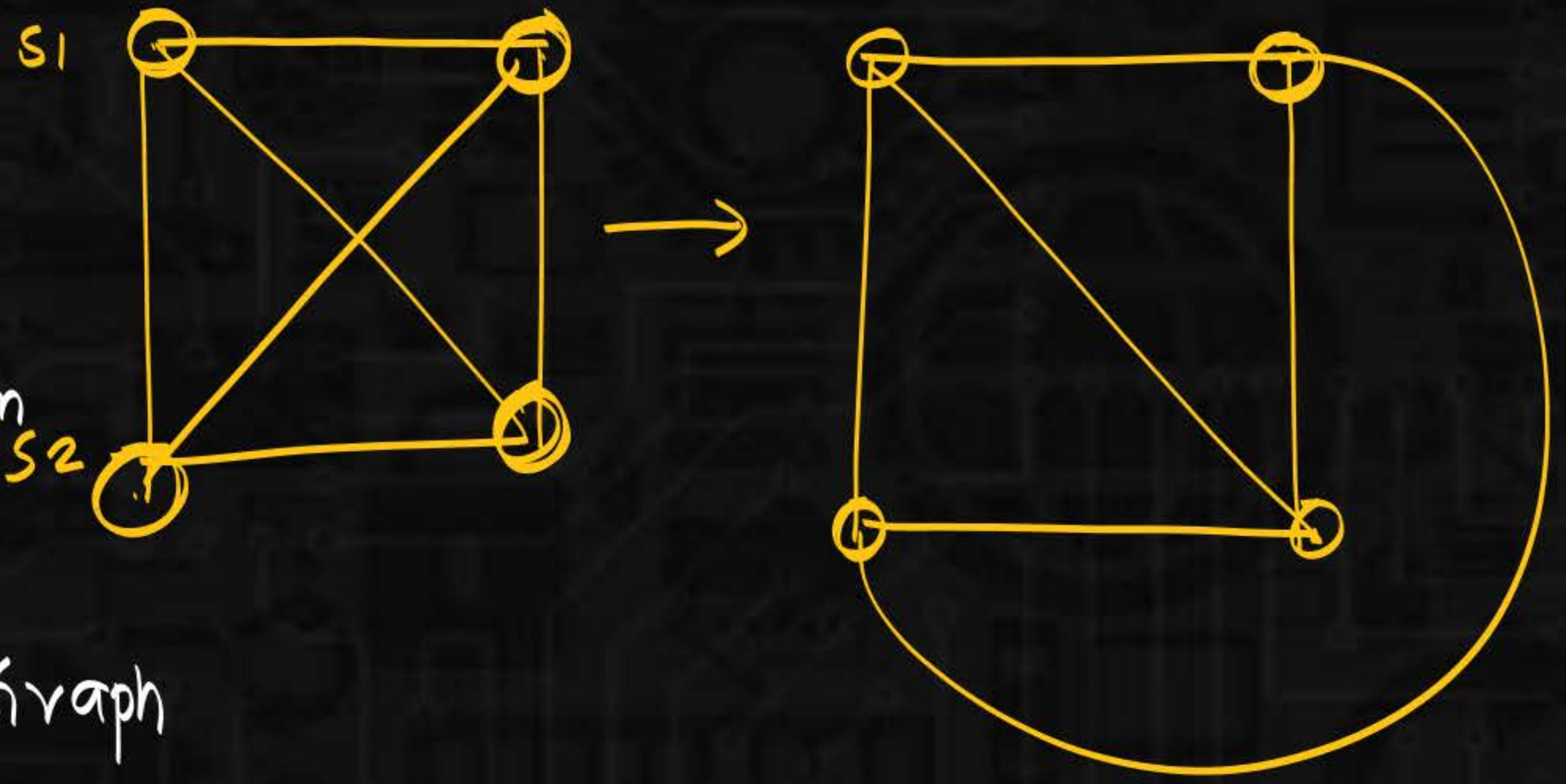
05 Sum of Degrees in Region

Planarity:

Planar Graph:

Embedding.

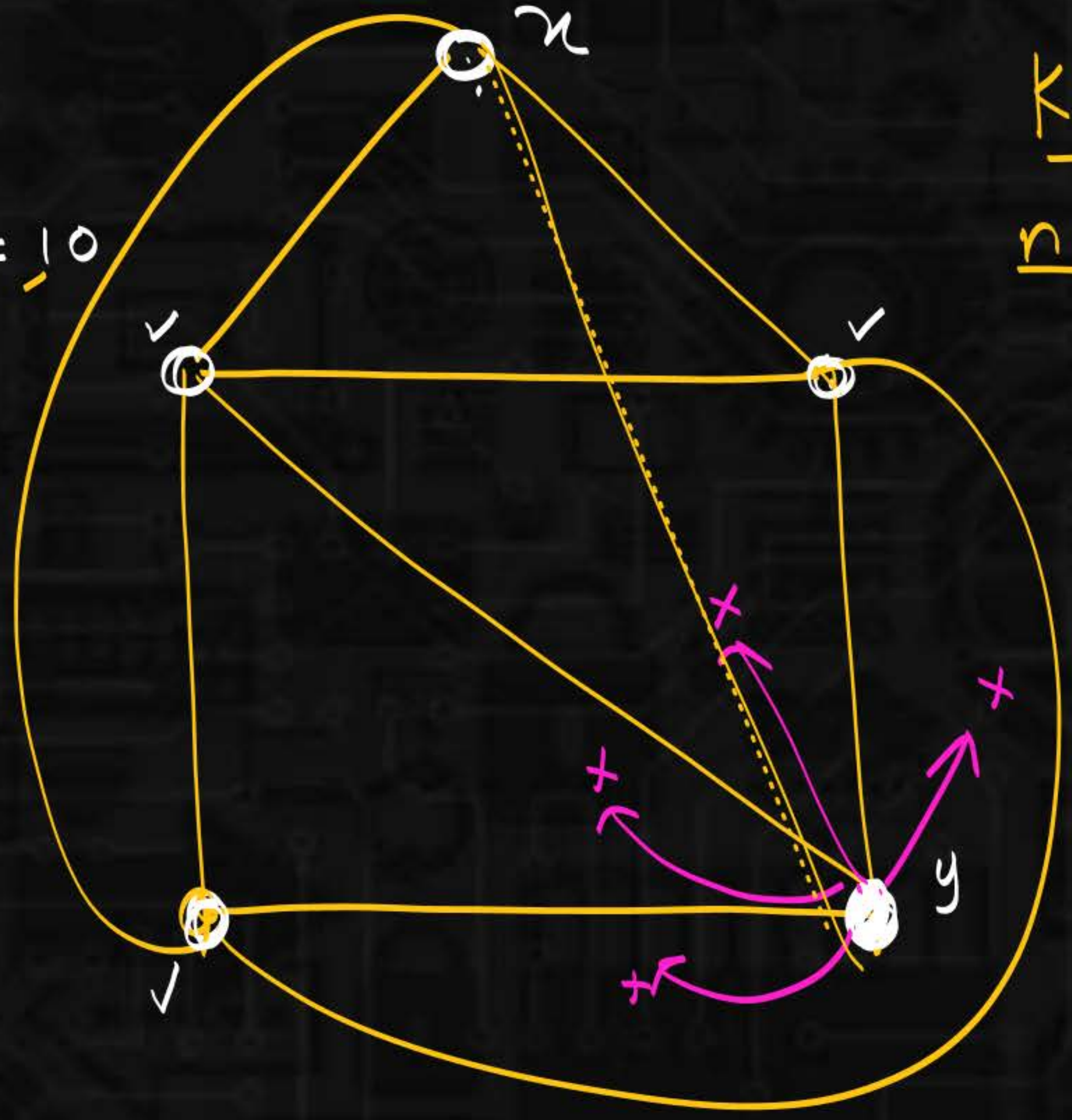
if we can draw graph without intersectⁿ of its edges then the graph is called planar Graph otherwise it is nonplanar.



K5

Total vertices = 5

Total edges = $\frac{5 \times 4}{2} = 10$



K5 is nonplanar.

$n = 5$ $e = 9$ $(K5 - \{e\})$

↳ planar Graph

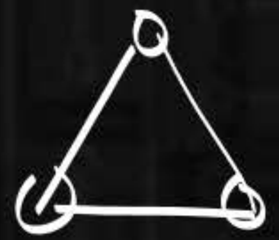
$n = 5$ $e = 10 (K5)$.

nonplanar

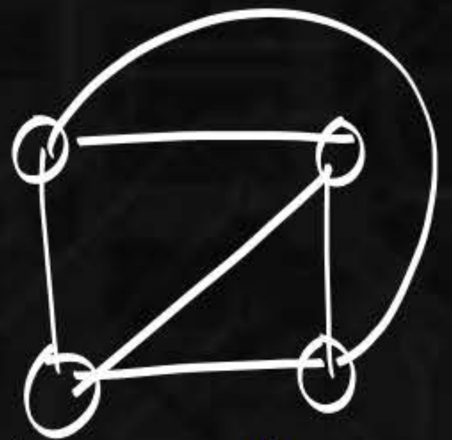
$n = 1$

$n = 2$

$n = 3$



$n = 4$



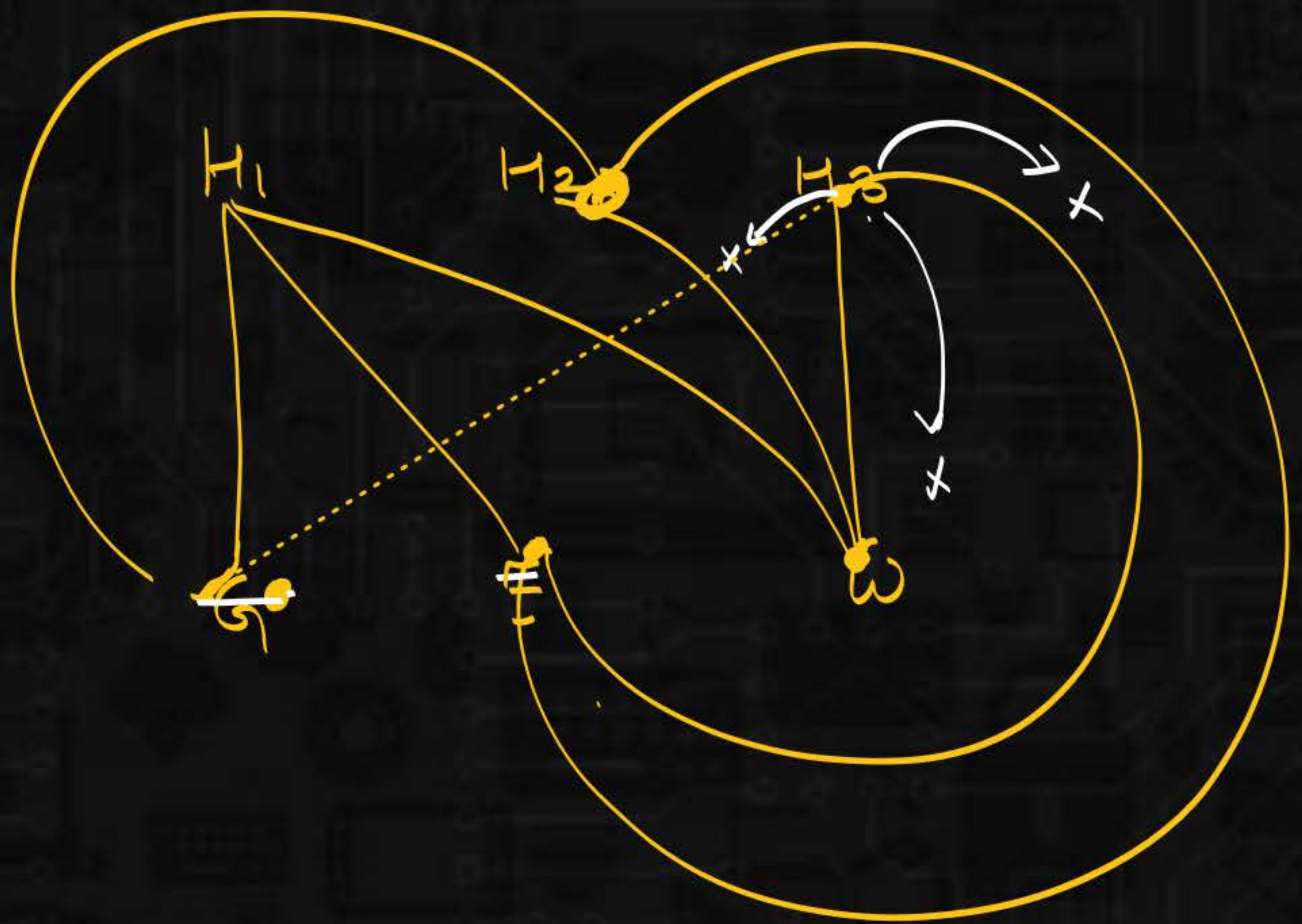
$n = 5$

planar

$n = 5$
$e = 9$

$K5$ → non planar.

$K3,3$.



$\underline{K_{3,3}}$: is non planar

$\underline{n = 6}$ $\underline{e = 9}$.

* K5 is 1st nonplanar Graph. (Kuratowski's 1st Graph)
* K3,3 is second non planar Graph (Kurowtski's 2nd Graph)
* Both Graphs are Regular Graph.
* if we remove single edge from both the graphs, both graphs will become planar Graph.
* 1st nonplanar Graph having min. no of vertices.
  2nd ——— " ——— having min no of edges.

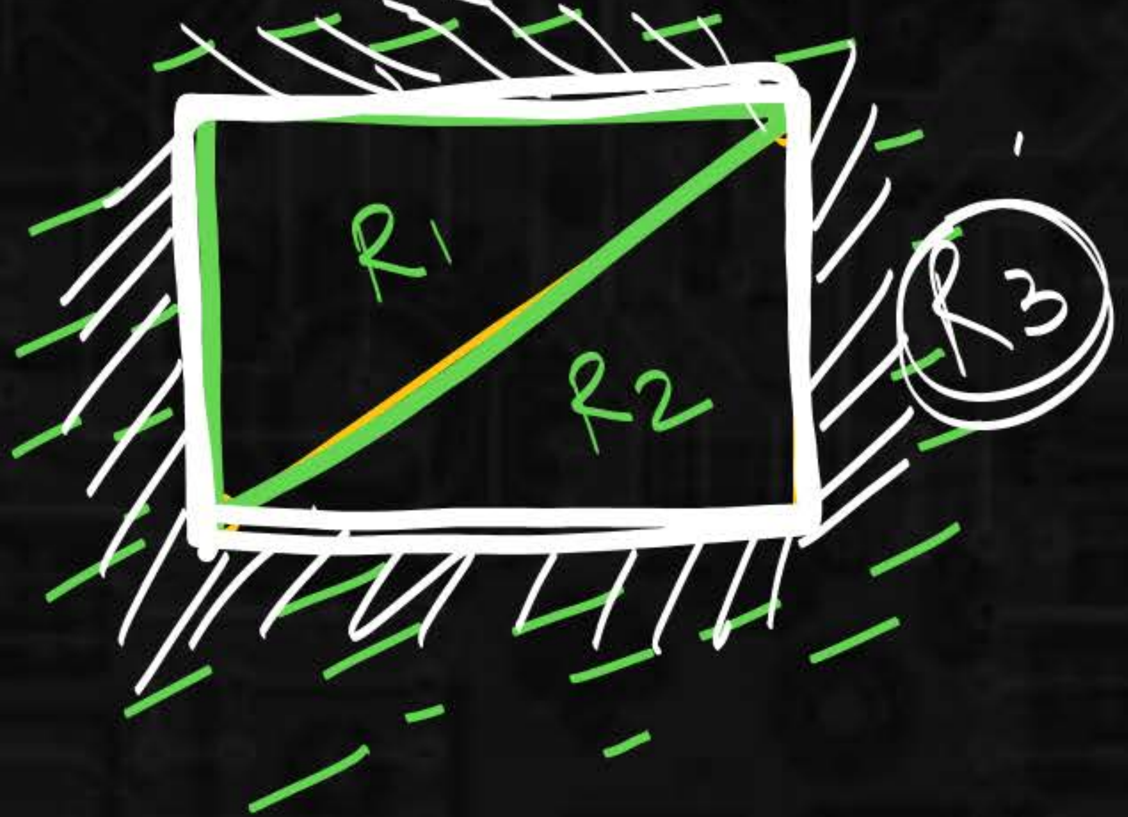
nonPlanar

	min.	
K_5	$n = 5$	$e = 10$
$K_{3,3}$	$n = 6$	$e = 9$
		min.

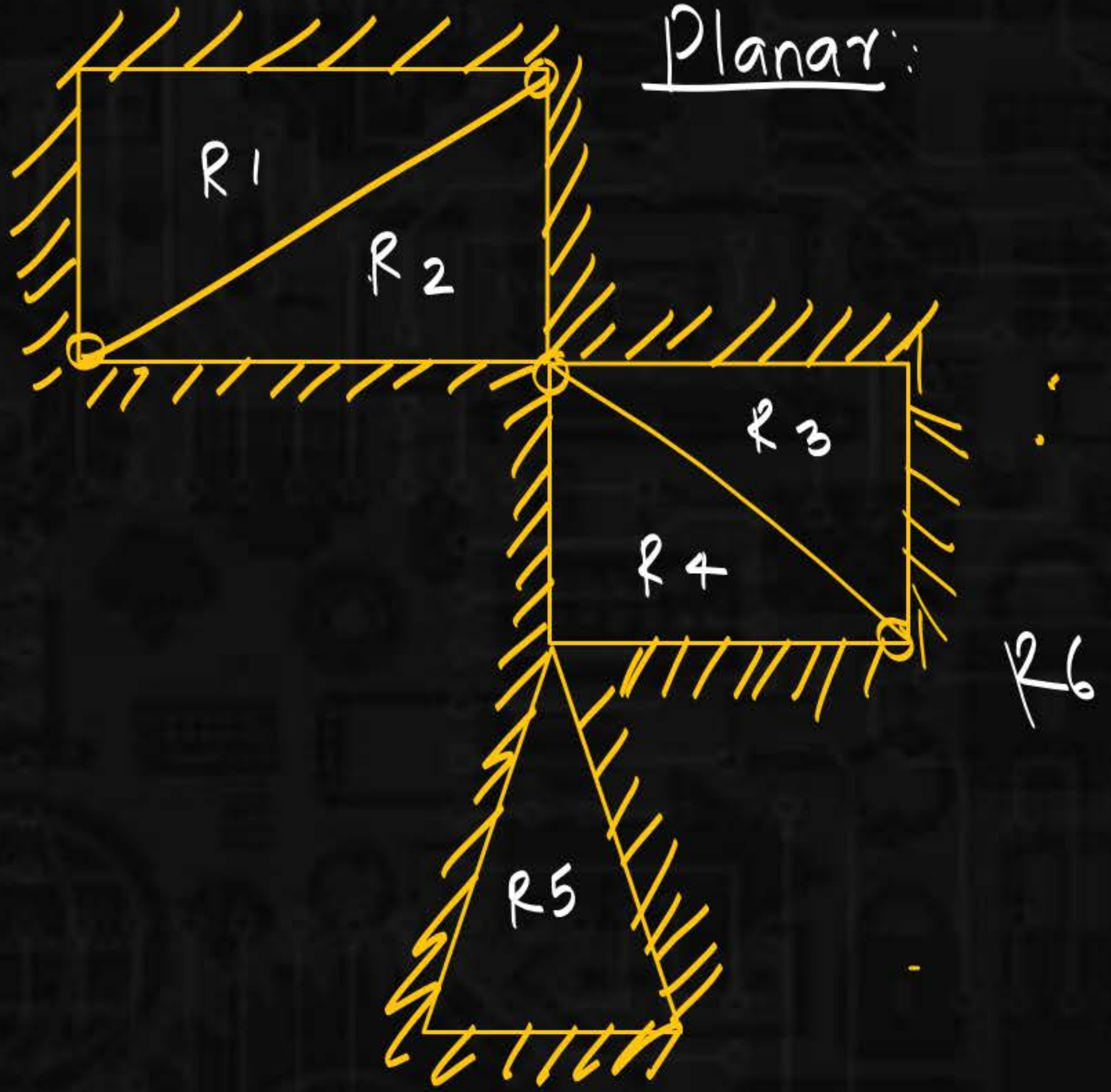
$\rightarrow$ nonplanar + min. no of vertices

$\rightarrow$ nonplanar + min. no of edges

When we Draw Planar Graph on a Plane, it creates Regions / faces.
(R) (f)



- finite OR bounded OR Closed — (R1, R2)
- Infinite OR unbounded OR open — (R3)



Planar:

Finite Closed	Infinite: open
R_1	R_6.
R_2	
R_3	
R_4	
R_5	

Q2

C.R : R_1

O.R : R_2.

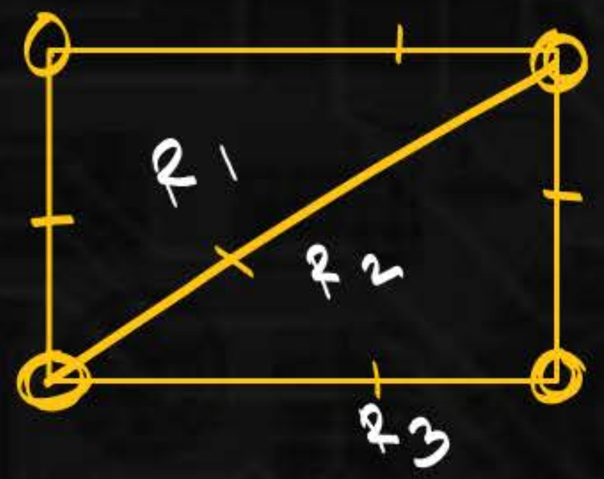
Euler's formula :

$$n - e + f = 2$$

n = Total no. of vertices.

e = Total no. of edges.

f = face / Region.



$n = 4 \quad e = 5$

$n - e + f = 2$

$4 - 5 + f = 2$

$$\boxed{f = 2 + 1 = 3.}$$

bounded : 2

Unbounded : 1.

$n = 2 \quad e = 1.$

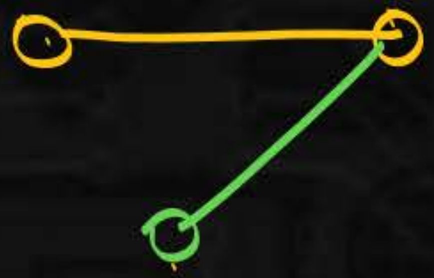
$n - e + f = 2$

$2 - 1 + f = 2$

$f = 1.$

Case1:

n	e
$+1$	$+1.$



$n - e + f = 2$

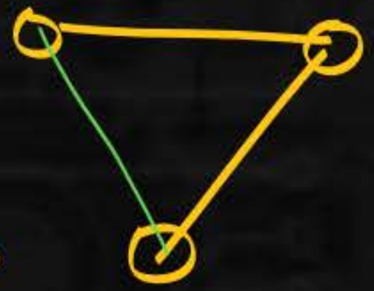
$n = 3 \quad e = 2.$

$3 - 2 + f = 2.$

$f = 1.$

Case2:

e	f
$+1$	$+2$



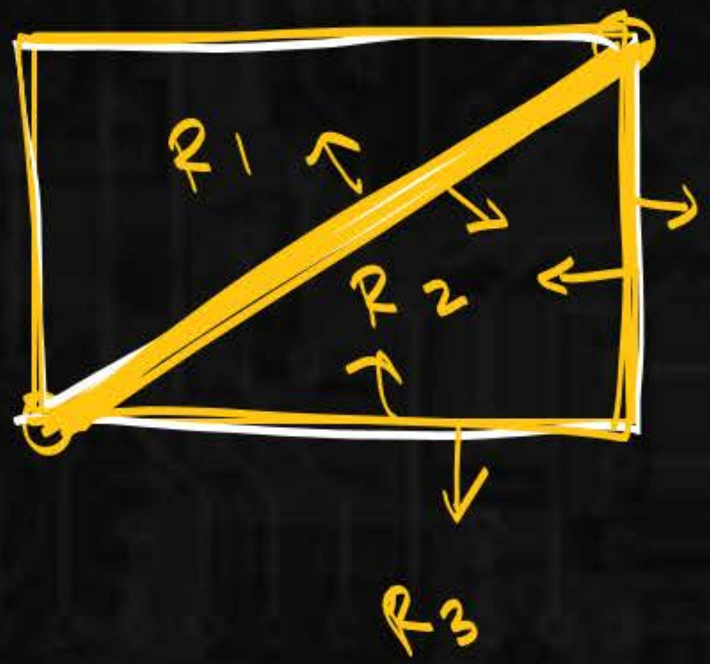
$n = 3$

$e = 3$

$n - e + f = 2$

$3 - 3 + f = 2$

$f = 2.$



$deg(R_i)$ = no. of edges involved into Region's.

$deg(R_1) = 3$

$deg(R_2) = 3$

$deg(R_3) = 4.$

$deg(R_1) + deg(R_2) + deg(R_3)$

$= 3 + 3 + 4.$

$= 10$

$= 2 \times 5 \rightarrow$ no. of edges.

$$\boxed{\sum d(R_i) = 2e.}$$


THANK YOU GW
SOLDIERS !