CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-3

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TOPICS TO BE COVERED

- Minimum Hamming distance For error correction

Min. Hamming Distance for Error Correction :

min H.D = 2

Valid code word —1 bit error— Invalid code word —1 bit error— Valid code word

can detect only one bit error

min H.D = 2

Valid code: 0000 —1 bit error— Invalid code: 0010 —1 bit error— Valid code: 0011

0010

Ex1 :

Valid code word

1001 }
1010 } minimum Hamming distance = 2

① sent 1001 —1 bit error— Rcvd 1011 Invalid code word

can detect one bit error but it can't correct one bit error

② sent 1010 —1 bit error— Rcvd 1011 Invalid code word

can detect one bit error but it can't correct one bit error

Ex2 :

Valid code word

0000
0011

minimum Hamming distance = 2

sent: 0000 — 1 bit error → Rcvd: 0010

Invalid code word

Can detect one bit error but It can't correct one bit error

0011 — 1 bit error → 0010

Invalid code word

Can detect one bit error but It can't correct one bit error

Ex3 :

Valid code word

$\left.\begin{matrix}000\\111\end{matrix}\right\}$ minimum Hamming distance = 3

$000 \xrightarrow{\text{1 bit error}}$ $\left.\begin{matrix}100\\010\\001\end{matrix}\right\}$ Invalid code word

$111 \xrightarrow{\text{1 bit error}}$ $\left.\begin{matrix}011\\101\\110\end{matrix}\right\}$ Invalid code word

① sent $\boxed{000}$ 1 bit error Rcvd $\boxed{100}$ Invalid code word

It can detect and correct one bit error

$\boxed{1}00$
$\boxed{0}00$
1 bit ✓

$1\boxed{00}$
$1\boxed{11}$
2 bit

(2)

111 —— 1 bit error —— 110

Invalid code word

Receiver can detect and correct one bit error

1 1 0	1 1 0
0 0 0	1 1 1
2 bit	1 bit ✓

(3)

000 —— 2 bit error —— 0 1 1

Invalid code word

It can detect 2 bit error but It can't correct 2 bit error

0 1 1	0 1 1
0 0 0	1 1 1
2 bit	1 bit ✓

Ex4 :

Valid code word

0000
1111

minimum Hamming distance = 4

① Sent: 0000 —— 1 bit error —— Rcvd: 1000

Invalid code word

It can detect and correct one bit error

1000	1000
0000	1111
1 bit ✓	3 bit

② 0000 —— 2 bit error —— 0011

Invalid code word

It can detect 2 bit error but it can't correct 2 bit error

0011	0011
0000	1111
2 bit	2 bit

Ex5 :

Valid code word

00000
11111

$\left.\begin{matrix} 00000 \\ 11111 \end{matrix}\right\}$ minimum Hamming distance = 5

$\boxed{00000} \xrightarrow{\text{2 bit error}} \boxed{00011}$

Invalid code word

It detect and correct 2 bit error

000 11	00011
000 00	11111
2bit ✓	3bit

Note

1. To correct one bit error min. Hamming distance required $= 3 = 2*1+1$
2. To correct two bit error min. Hamming distance required $= 5 = 2*2+1$
3. To correct 'd' bit error min. Hamming distance required $= 2*d+1 = 2d+1$

Problem Solving on Hamming Distance

Q.1 Consider a binary code that consists of only four valid code words as given below:

00000, 01011, 10101, 11110

a b c d

$$\begin{array}{r} 01011 \\ \text{EX-OR} \\ 11110 \\ \hline 10101 \end{array} \qquad \begin{array}{r} 10101 \\ \text{EX-OR} \\ 11110 \\ \hline 01011 \end{array}$$

Let the minimum Hamming distance of the code be p and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are

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(A) p = 3 and q = 1

(B) p = 3 and q = 2

(C) p = 4 and q = 1

(D) p = 4 and q = 2

$$\left.\begin{array}{l} d(a,b) = 3 \\ d(a,c) = 3 \\ d(a,d) = 4 \\ d(b,c) = 4 \\ d(b,d) = 3 \\ d(c,d) = 3 \end{array}\right\} \text{min. Hamming distance} = 3\ (P)$$

minimum Hamming distance required to correct 'd' bit error $= 2d+1$

$$2d+1 = 3$$
$$2d = 3-1$$
$$2d = 2$$
$$d = 1 \ (2)$$

Q.2 What is the distance of the following code

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$\overset{a}{000000}, \overset{b}{010101}, \overset{c}{000111}, \overset{d}{011001}, \overset{e}{111111}$?

A. 2

B. 3

C. 4

D. 1

$d(a,b) = 3$
$d(a,c) = 3$
$d(a,d) = 3$
$d(a,e) = 6$
$d(b,c) = 2$
$d(b,d) = 2$
$d(b,e) = 3$
$d(c,d) = 4$
$d(c,e) = 3$
$d(d,e) = 3$

Min. Hamming distance = 2

(b) 010101
EX-OR
(c) 000111
010010 → 2

(2m)

(b) 010101
EX-OR
(d) 011001
001100 → 2

Q.3 An error correcting code has the following code words:

00000000, 00001111, 01010101, 10101010, 11110000.

a b c d e

What is the maximum number of bit errors that can be corrected ?

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A. 0

B. 1

C. 2

D. 3

(b) 00001111

(c) 01010101

01011010 → No. of 1's = 4 (Hamming distance)

Minimum Hamming distance = 4

min. Hamming distance required to correct 'd' bit error = 2d+1

$2d+1=4$

$2d=3$

$d=\frac{3}{2}$

$d=\lfloor 1.5 \rfloor = 1$

'd' bit error correction

min HD = $2d+1$

$d=2$

$2*2+1 = 5$ (min Hamming distance)

Q.4 The minimum Hamming distance to correct upto 5 bit error successfully is ____

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min. Hamming distance required to correct 'd' bit error $= 2d+1 = 2*5+1 = \underline{11}$

Q.5 The minimum Hamming distance to detect upto 10 bit error successfully is ____

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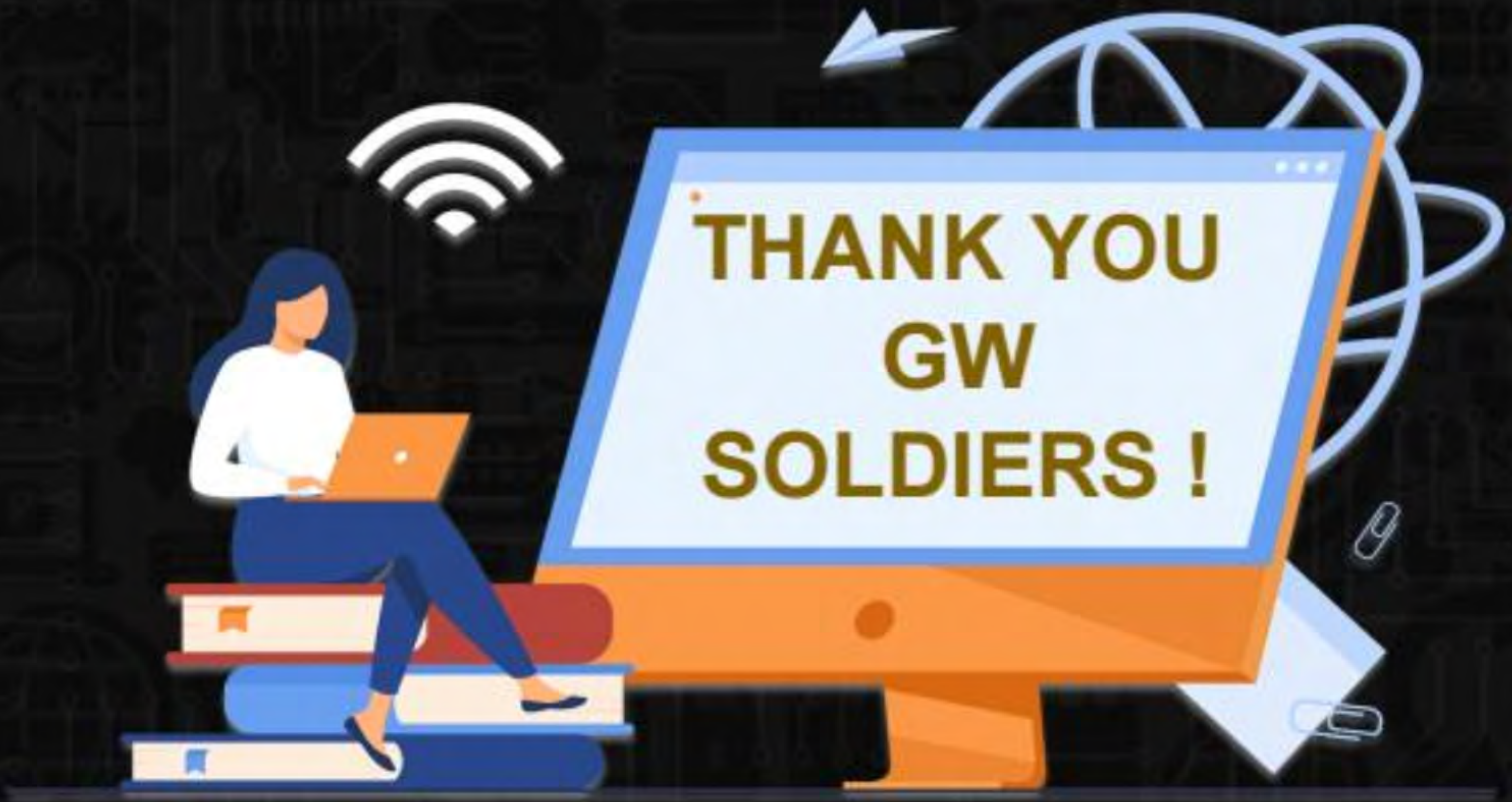
minimum Hamming distance required to detect 'd' bit error $= d+1 = 10+1 = 11$


THANK YOU
GW
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