2) Verify the

$B = \{(2,3,4,2),(4,0,-4,28),(3,4,5,5)\}$ is Linearly independent or not.

Solu.

Let $c_1\ c_2\ c_3$ be scalars consider

$$c_1(2,3,4,2) + c_2(4,0,-4,28) + c_3(3,4,5,5) = (0,0,0,0)$$

$$(2c_1, 3c_1, 4c_1, 2c_1) + (4c_2, 0c_2, -4c_2, 28c_2) + (3c_3, 4c_3, 5c_3, 5c_3) = (0,0,0,0)$$

$$\left.\begin{aligned} 2c_1 + 4c_2 + 3c_3 &= 0 \\ 3c_1 + 0c_2 + 4c_3 &= 0 \\ 4c_1 + (-4c_2) + 5c_3 &= 0 \\ 2c_1 + 28c_2 + 5c_3 &= 0 \end{aligned}\right\} \text{②}$$

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 0 & 4 \\ 4 & -4 & 5 \\ 2 & 28 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 3 \\ 3 & 0 & 4 \\ 6 & 6 & 8 \\ 2 & 28 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 3 \\ 3 & 0 & 4 \\ 0 & -8 & -1 \\ 0 & 24 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 0 & 4 \\ 0 & -8 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$\rho(A) = 3 =$ No. of unknowns.

$\therefore$ The system has unique solu which is a trivial solu.

Hence the vectors are linearly independent vectors.

PART - B

Q1) Show

Solu:

$B = \{(3,3,3), (3,-3,3), (0,3,3)\}$

i) To show that the elements of B are Linearly independent, consider

$C_1(3,3,3) + C_2(3,-3,3) + C_3(0,3,3) = (0,0,0)$

$(3C_1, 3C_1, 3C_1) + (3C_2, -3C_2, 3C_2) + (0C_3, 3C_3, 3C_3)$
$= (0,0,0)$

$3C_1 + 3C_2 + 0C_3 = 0$
$3C_1 + (-3C_2) + 3C_3 = 0$

$3C_1 + 3C_2 + 3C_3 = 0$

$$|A| = \begin{vmatrix} 3 & 3 & 0 \\ 3 & -3 & 3 \\ 3 & 3 & 3 \end{vmatrix} = 3(-9-9) - 3(9+9)$$
$$= 3\times(-18) - 3\times(18)$$
$$= -54 - 54 = -108$$

$|A| = -108$

$\therefore \rho(A) = 3$ = No. of unknowns.

The system ① has unique solu.

ii) To show that B spans $V_3(R)$ consider $(x, y, z) \in V_3(R)$

$$(x, y, z) = (3c_1 + 3c_2 + 0c_3, 3c_1 - 3c_2 + 3c_3, 3c_1 + 3c_2 + 3c_3)$$

$$3c_1 + 3c_2 + 0c_3 = x$$
$$3c_1 + (-3c_2) + 3c_3 = y$$
$$3c_1 + 3c_2 + 3c_3 = z$$

Augmented matrix :-

$$[A : B] = \begin{bmatrix} 3 & 3 & 0 & : & x \\ 3 & -3 & 3 & : & y \\ 3 & 3 & 3 & : & z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & : & x/3 \\ 1 & -1 & 1 & : & y/3 \\ 1 & 1 & 1 & : & z/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & : & x/3 \\ 1 & -1 & 1 & : & y/3 \\ 2 & 0 & 2 & : & z/3 + y/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & : & x/3 \\ 2 & 0 & 1 & : & y/3 + x/3 \\ 2 & 0 & 2 & : & z/3 + y/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & : & x/3 \\ 2 & 0 & 1 & : & y/3 + x/3 \\ 0 & 0 & 1 & : & (z/3 + y/3 - y/3 - x/3) \end{bmatrix}$$

$\rho[A] = \rho[[A : B]] = 3 =$ No. of unknowns

$c_3 = \frac{z}{3} - \frac{x}{3}$ $\quad \Rightarrow 3c_3 = z - x$

$2c_1 + c_3 = \frac{y}{3} + \frac{x}{3}$

$2c_1 = \frac{y}{3} + \frac{x}{3} - \frac{z}{3} + \frac{x}{3}$

$2c_1 = \frac{y}{3} + \frac{2x}{3} - \frac{z}{3}$

$c_1 + c_2 = \frac{x}{3}$

$c_2 = \frac{x}{3} - c_1 = \frac{x}{3} - \frac{y}{6} - \frac{x}{3} + \frac{z}{6}$

$c_2 = \frac{z}{6} - \frac{y}{6}$

i.e every vector of $V_3(R)$ is generated by the elements of B

as,

$$(x, y, z) = \left(\frac{y}{6} + \frac{x}{3} - \frac{z}{6}\right)(3, 3, 3) + \left(\frac{z}{6} - \frac{y}{6}\right)(3, 3, \ldots) + \left(\frac{z}{3} - \frac{x}{3}\right)(0, 3, 3)$$

$\therefore$ B spans $V_3(R)$

i.e $V_3(R) = L[B]$

$\therefore$ B form a basis for $V_3(R)$

PART C

Q7) Determine

Sol: Consider 3 possible vectors namely:

$\vec{v_1} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$ $\vec{v_2} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$ $\vec{v_3} = \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix}$

To have $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ as orthogonal we must have

$\{\vec{v_1}, \vec{v_2}\}$, $\{\vec{v_2}, \vec{v_3}\}$ & $\{\vec{v_1}, \vec{v_3}\}$ be orthognal

$\therefore \vec{v_1} \cdot \vec{v_2} = (-1) \times 5 + 4 \times 2 + (-3) \times 1$

$= -5 + 8 - 3 = 0$

$\vec{v_2} \cdot \vec{v_3} = 5 \times 3 + 2 \times (-4) + 1 \times (-7)$

$= 15 - 8 - 7 = 0$

$\vec{v_1} \cdot \vec{v_3} = (-1) \times 3 + 4 \times (-4) + (-3) \times (-7)$

$= -3 + (-16) + 21 = 2$

Since $\vec{v_1} \cdot \vec{v_3} \neq 0$

$\therefore \{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ is not an orthogonal set.

ii) $\{(2, -7, -1)', (-6, -3, 9)', (3, 1, -1)\}$

Let 3 possible vectors be

$\vec{v_1} = \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}$ $\vec{v_2} = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}$ $= v_3 \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

$\vec{v_1} \cdot \vec{v_2} = 2 \times (-6) + (-7) \times (-3) + (-1) \times 9) = -12 + 21 - 9$

$= 0$

$\vec{v}_2 \cdot \vec{v}_3 = (-6 \times 3) + (-3 \times 1) + (9 \times -1)$

$= -18 - 3 + (-9)$

$= -30$

Since $\vec{v}_2 \cdot \vec{v}_3 \neq 0$.

$\therefore$ the give $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is not orthogonal.

iii) $\{ (3, -2, 1, 3)', (-1, 3, -3, 4)', (3, 8, 7, 0)' \}$

Let the 3 possible ~~set~~ vector be

$$\vec{v}_1 = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 3 \\ -3 \\ 4 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 3 \\ 8 \\ 7 \\ 0 \end{bmatrix}$$

Consider 3 diff. pairs $\{\vec{v}_1, \vec{v}_2\}, \{\vec{v}_1, \vec{v}_3\} \{\vec{v}_2, \vec{v}_3\}$

$\vec{v}_1 \cdot \vec{v}_2 = -3 - 6 - 3 + 12$
$= 0$

$\vec{v}_1 \cdot \vec{v}_3 = 9 - 16 + 7 = 0$

$\vec{v}_2 \cdot \vec{v}_3 = (-1 \times 3) + (3 \times 8) + (-3 \times 7)$
$= -3 + 24 - 21 = 0$

Since each pair of distinct vector is orthogonal and $\neq 0$

$\therefore \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set.

~~PART~~

PART - D

Q2)

$$A = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 1 & 0 \\ 0 & -6 & 1 \end{bmatrix}$$

Solu:

Let $\vec{x_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $\vec{x_2} = \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}$ $\vec{x_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\vec{v_1} = \vec{x_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\vec{v_2} = \vec{x_2} - \frac{\vec{x_2} \cdot \vec{v_1}}{\vec{v_1} \cdot \vec{v_1}} \vec{v_1} = \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix} - \frac{10}{5} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

$$\vec{v_2} = \begin{bmatrix} 6 \\ -3 \\ -6 \end{bmatrix}$$

$$\cancel{v_2' = 4 \vec{v_2} = 4 \begin{bmatrix} 6 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ -24 \end{bmatrix}}$$

$$\cancel{\vec{v_3} = x_3 - \frac{\vec{x_3} \cdot \vec{v_1}}{\vec{v_1} \cdot \vec{v_1}} \vec{v_1} - \frac{\vec{x_3} \cdot \vec{v_2}'}{\vec{v_2} \cdot \vec{v_2}} \vec{v_2}'}$$

$$\cancel{= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 0 - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

$$\cancel{= \begin{bmatrix}0\\0\\1\end{bmatrix} - 0 - \frac{(-24)}{5}\begin{bmatrix}24\\-12\\-24\end{bmatrix}}$$

$$\vec{v_2} = \begin{bmatrix}6\\-3\\-6\end{bmatrix} = 3\begin{bmatrix}2\\-1\\-2\end{bmatrix}$$

$$\vec{v_3} = \vec{x_3} - \frac{\vec{x_3}\cdot\vec{v_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} - \frac{\vec{x_3}\cdot\vec{v_2}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2} =$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix} - 0 - \frac{(-6)}{81}\begin{bmatrix}6\\-3\\-6\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix} + \frac{2}{27}\begin{bmatrix}6\\-3\\-6\end{bmatrix}$$

$$=$$

Now normalize the vectors $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ to obtain $\cancel{\{\vec{x_1}, \vec{x_2}, \vec{x_3}\}}$ $\{\vec{u_1}, \vec{u_2}, \vec{u_3}\}$

$$\vec{u_1} = \frac{\vec{v_1}}{\|\vec{v_1}\|} = \begin{bmatrix}1/\sqrt{5}\\2/\sqrt{5}\\0\end{bmatrix}$$

$$\vec{u_2} = \frac{\vec{v_2}}{\|\vec{v_2}\|} = \begin{bmatrix}6/9\\-3/9\\-6/9\end{bmatrix}$$

$$\vec{U_3} = \frac{\vec{V_3}}{\|\vec{V_3}\|} = \begin{bmatrix} 0.59 \\ -0.2981 \\ 0.7453 \end{bmatrix}$$

$$Q = [\vec{U_1} \ \vec{U_2} \ \vec{U_3}]$$

$$R = Q^T A$$