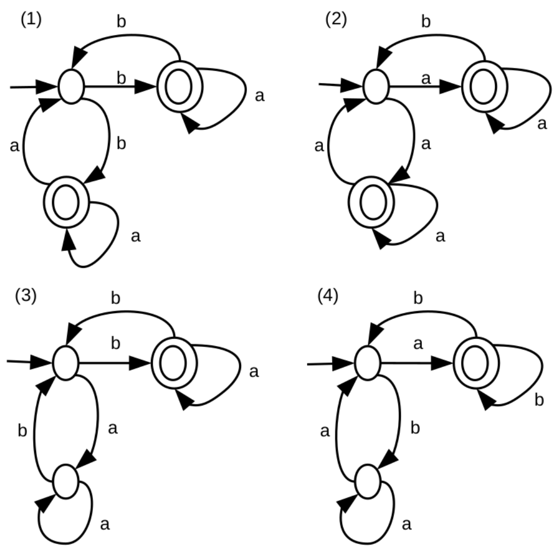
Theory of Computation (CS306)

End Semester Exam, Semester II, 2020-21

May 12, 2020

Instructor: Anil Shukla
Time: 2 : 00 pm–5 : 00 pm Total Marks: 20

(1) Match each NFA with an equivalent regular expression: (2 marks)



(i) $(ab^*b + ba^*a)^*ab^*$

(ii) $(ba^*a + ba^*b)^*ba^*$

(iii) $(aa^*b + aa^*a)^*aa^*$

(iv) $(ba^*b + aa^*b)^*ba^*$

(2) Give minimum state DFA for the set of all strings over $\{0, 1\}$ containing at most one pair of consecutive 0's and at most one pair of consecutive 1's. Note: Strings like 000, 111, 0011, 1100, 00111, 11000 **belongs** to the language. The strings 0000, 1111, 001111 does not belong to the language. (2 marks)

(3) PDA $P = (\{q_0, q_1, q_2, q_3, f\}, \{a, b\}, \{Z_0, A, B\}, \delta, q_0, Z_0, \{f\})$ has the following rules defining δ:

$$\begin{array}{lll}
\delta(q_0, a, Z_0) = (q_1, AAZ_0) & \delta(q_0, b, Z_0) = (q_2, BZ_0) & \delta(q_0, \epsilon, Z_0) = (f, \epsilon) \\
\delta(q_1, a, A) = (q_1, AAA) & \delta(q_1, b, A) = (q_1, \epsilon) & \delta(q_1, \epsilon, Z_0) = (q_0, Z_0) \\
\delta(q_2, a, B) = (q_3, \epsilon) & \delta(q_2, b, B) = (q_2, BB) & \delta(q_2, \epsilon, Z_0) = (q_0, Z_0) \\
\delta(q_3, \epsilon, B) = (q_2, \epsilon) & \delta(q_3, \epsilon, Z_0) = (q_1, AZ_0) &
\end{array}$$

(3a) Give the contents of the stack after P has read b^7a^4 from its input. (1.5 marks)

(3b) Starting from the initial ID $(q_0, baabbb, Z_0)$ show all the reachable ID's. Is the string $baabbb \in L(P)$? (1.5 marks)

(3c) Informally describe $L(P)$. (3 marks)

Note: Just to clarify: here q_0 is the start state and f is the final state.

(4) Consider the Turing machine $M = (\{s, q_0, q_1, q_2, t, r\}, \{0, 1\}, \{0, 1, \sqcup\}, \vdash, \sqcup, \delta, s, t, r)$, where s, t, and r are the start state, accept state, and the reject state, respectively. Whenever M stucks, it automatically goes to the reject state. Once M enters the accept or the reject states, it remains in the same state.
Clearly describe the languge $L(M)$ if δ consists of the following rules:

(4a) $\delta(s, \vdash) = (q_0, \vdash, R)$; $\delta(q_0, 0) = (q_1, 1, R)$; $\delta(q_1, 1) = (q_2, 0, L)$; $\delta(q_2, 1) = (q_0, 1, R)$; $\delta(q_1, \sqcup) = (t, \sqcup, R)$. (2 marks)

(4b) $\delta(s, \vdash) = (q_0, \vdash, R)$; $\delta(q_0, 1) = (q_1, 0, R)$; $\delta(q_1, 0) = (q_0, 1, R)$; $\delta(q_2, 0) = (q_2, 1, L)$; $\delta(q_2, 1) = (q_0, 0, L)$; $\delta(q_1, \sqcup) = (t, \sqcup, R)$. (2 marks)

(5) Design a Turing machine M which first checks if its input is of the form $a^+b^+c^+$. If no, reject. If yes, M shifts its input one cell to the right and halts. For example, for the input $\vdash aabbc \sqcup \sqcup \sqcup \ldots$, M halts with the following tape content: $\vdash \sqcup aabbc \sqcup \sqcup \ldots$.
You must present transition diagram for M. (2 marks)

(6) Let $D \subseteq \Sigma^*$ be a DCFL (deterministic context-free language). One of the following sets is always a DCFL, the other is not necessarily. Which is which? Give proof for both. (1 + 1 marks)

(6a) $L_{6a} = \{x \mid \exists a \in \Sigma, xa \in D\}$

(6b) $L_{6b} = \{x \mid \exists a \in \Sigma, ax \in D\}$

(7) Let M range over Turing machine descriptions. Using Rice's theorem show that neither the set

$$REG = \{M \mid L(M) \text{ is a regular set}\}$$

nor its complement is recursive enumerable. (1 + 1 marks)