**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS: B.0.2676**

since theservice manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off ,

Therefor the mean = 10+45=55

S.D = 8

X= 60

P(x>60)= 1-stats.norm.cdf(60,55,8) (calculated in python)

**=** **0.266**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS:**

**A. FALSE**

Mean = 38 , SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  that is  84.13 %

People above 44 age = 100 - 84.13 =  15.87% that is  63 out of 400

Z score for 38  = (38 - 38)/6 = 0 that is 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % that is 137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

**B. TRUE**

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  % that is 36 out of 400.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS:** We know that if X∼N(µ1,σ1^2 ) and Y ∼ N(µ2, σ2^2 ) are two independent random variable then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

Similarly if z= aX+bY, then Z is linear combination of X and y, Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore :

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N(0µ,2 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS:**

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99

So the Probability of going wrong, or the Probability outside the a and b area is 0.

The Probability towards left from a = -0.005 (ie. 0.01/2). 01 (ie. 1-0.99)

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z, we can calculate the X values.

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

**ANS is D.**

1. For Probability 0.005 the Z Value is -2.57 (from Z Table).Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANS: (99.00810347848784, 980.9918965215122)**

1. Specify the 5th percentile of profit (in Rupees) for the company

**ANS: 169.875**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS:** Division1: 0.0477

Division2: 0.04005

Therefore **Division1** has the larger probability of making a loss in a given year.