A REPORT ON

ANALYSIS OF COVID-19 SPREAD IN INDIA USING MATHEMATICAL MODELS

BY:

Shashank Agrawal **2020A7PS0073P**

Yugal Joshi **2020A7PS0002P**

UNDER

Dr. Tanu Shukla , Deepanjana Chakraborty, V Mounika Prasanthi

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (Rajasthan)

(January 2022-April,2022)

TABLE OF CONTENTS

ACKNOWLEDGMENT

We would like to express our special gratitude to our professor Dr Tanu Shukla and our TAs, V Mounika Prasanthi mam and Deepanjana Chakraborty mam. They gave us the golden opportunity to research the topic of Analysis of COVID-19 spread in India using Mathematical Models.

We are thankful for their assistance in every step of the research.

1. ABSTRACT

The end of 2019 left us with one of the deadliest havoc the human race had ever faced, the Novel coronavirus, popularly known as COVID19. The first case was discovered in Wuhan. Since then, it has spread to more than 200 countries worldwide, infecting almost 49.1 crore people worldwide and causing the death of nearly 61.5 lakh people.(Source:(Ritchie, 2020a))
India is one of the pandemic's biggest hotspots, with 4.3 crore cases and 5.21 Lakh reported deaths (Ritchie, 2020b). This virus is known to mutate several times, leading to several variants. Due to the very capability of coronavirus to mutate, there were three major waves of COVID 19:

- wave 1(N440K ***variant***)
- wave 2(delta variant)
- wave 3(omicron variant)

The new omicron variant is still causing havoc in India. The major problem with this virus was that it was constantly mutating, due to which we had to suffer from 3 deadly waves of covid.

So, it is crucial to design prediction and control strategies to restrain its exploding. In this study, a model is proposed to simulate the spreading of COVID-19. Simulation results show that the proposed model has a good predictive ability. This study provides a reliable model to predict cases of infection and death and reasonable suggestions to control COVID-19. We have also presented the validity of our model by comparison with the trend and patterns observed. Predicting the cases will prepare us to face future waves or similar conditions. It will also help us devise our policies adequately and wisely distribute our resources(economic and health).

Keywords: Modelling Strategies; COVID-19; Heuristic model; prediction; verification; SARS-CoV-2; Epidemiology.

2. RATIONALE BEHIND THE TOPIC

The best way to cope with any disaster is preparation to mitigate the disaster. Proper prediction of calamity gives us time and an idea of the level of preparation. COVID 19 is also a biological calamity, so preparedness is the biggest key to controlling its spread and effects.

When COVID 19 first struck the world, no one was prepared for it. We had almost no clue about the transmission mode, longevity and deadliness of the virus. Also, there was very little information and accurate data about casualties and characteristics of the virus, so most of the predictions about the spread of the virus went wrong. Thus, we were not well prepared for it. Due to less efficient prediction, we had inaccurate information about the requirement of beds, oxygen cylinders, health centres, quarantine centres, etc.

Proper preparation for corona includes the preparation of Government (economics, policies), medical facilities (beds, quarantine centres, oxygen cylinders) and preparation on the individual scale(stocks of food, medicines, use of sanitisers, social distancing).

The three waves of Covid resulted in many deaths and increased the unemployment rate, decreased GDP growth rate, etc.

We have seen three deadly waves of COVID, and we have got enough data for a proper accurate prediction. The accurate prediction will help the government devise policies to control unemployment and provide economic stability to poor and needy people. It will also help us prepare on our community level like following social distancing more sincerely, wearing, mask etc.

3. INTRODUCTION

In December 2019, in Wuhan city in Hubei province of the People's Republic of China (PRC), some presumed pneumonia patients were reported (as per Wu et al. 2020). However, it was later discovered that the standard medical treatment routine for pneumonia performed on these individuals was ineffective and that several of their illnesses progressively deteriorated. As a result, a novel virus known as SARS-CoV-2 has been identified as the causative. The virus was probably given because of the 'corona'-like arrangement of its spike proteins. Because the virus that caused the current outbreak belonged to the same family as SARS, it was given the designation SARS-CoV-2 (as per Zou et al., 2020).

According to the Chinese lunar calendar, the first epidemic began with the new year. The virus spreads swiftly in China due to considerable people's mobility due to the festival season. Because humans are the primary carriers of this virus, it spread invisibly over the globe before being discovered and controlled. The virus's success is linked to its accidental ability to exploit human movement patterns. When the infection is primarily restricted to the upper respiratory tract, the infected individual frequently misinterprets the symptoms as mild flu and becomes infectious.

Containment of the illness by clinical treatments is still an unsolved problem due to the lack of any clinical preventive mechanism (such as vaccination) or efficient medications to cure sick patients. As a result, the only way to slow the rate of disease spread in communities is to identify and isolate these types of carriers by clinical diagnosis, which WHO refers to as "Test, Test, Test". Countries like South Korea and Singapore have successfully implemented such a policy. However, there are operational, clinical, infrastructural, and budgetary barriers to applying this method in an extensive and densely populated nation like India.

Massive biological and medical research is being conducted to produce a vaccine for this "unstoppable" outbreak. However, in this anti-epidemic war, theoretical research, which employs statistical and mathematical modelling, may be a crucial instrument in addition to medical and

biological study. It may map the epidemic characteristics and anticipate the peaks and end times. Several efforts have been made to calculate the various critical factors such as doubling time, reproduction rate, inflexion point, etc.
Mathematical modelling based on dynamic equations using time series data is highly suitable for such a situation. The susceptible, exposed infectious recovered model (SEIR) is an extensively used model. The current paper is based on theoretical research that uses the extended SEIR model, an improved extension of the conventional SEIR model.

We have predicted the outbreak of COVID19 in India between 30th Jan 2020 and 25th Nov 2020, using the accurate data from 24th Jan 2020 to 27th March 2020. The peak occurrence in the predicted curve of total active cases closely matches the natural curve of active cases with a difference of only one week, which shows the validity of our model.

4. LITERATURE REVIEW

4.1. SPREAD OF CORONA:

4.1.1 The Initial Phase (spread in China and the USA, Declaration of Medical Emergency):

(source: *Coronavirus: The First Three Months as It Happened*, 2020)

This deadly virus started its cruise from a Huanan Seafood Wholesale Market in Wuhan. Its first active case was reported from Wuhan, China, on December 31 2019. Soon till January 21, 15 healthcare workers were confirmed infected with this virus in Wuhan, which led to the meeting of the World health organisation on January 22 about the declaration of the medical emergency[. The first U.S. case was confirmed on January 21.

FIG.1 shows the seven day rolling average of confirmed cases at the end of Jan (Ritchie, 2020b)

Due to the rapid outburst of the case in Wuhan, the Chinese government Closed off Wuhan and Huanggang on January 23. Chinese authorities suspended all travels in and out of Wuhan. Soon. By the end of January 30 Jan, China confirmed 7,736 active cases and 180 death.
After several delays, on January 30 2020, WHO declared COVID 19 a Public Health Emergency of International Concern.

4.1.2 Rapid spread in European countries and the United States, Lockdown In china (27 JAN - MARCH 27):

China remained a hotspot of covid cases till Feb end. But by the start of March 2020[FIG.2], Europe and the United States saw the immense rise of this deadly virus; countries like Italy, France, the U.S., and Spain surpassed China for the first time in the total number of covid cases and total deaths in a single date.

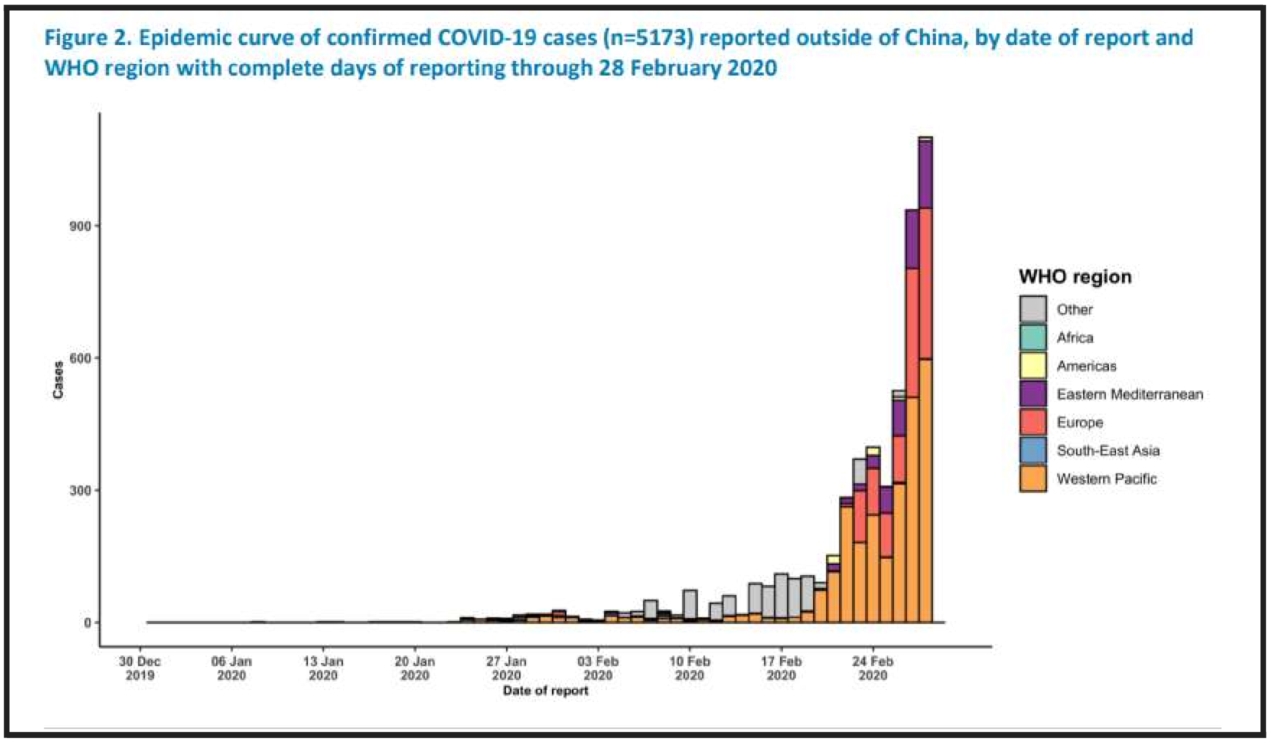


FIG.2 Shows a Rapid increase of cases in Europe and the US From Mid-Feb(*Coronavirus: The First Three Months as It Happened*, 2020, Nature)

The first case of COVID 19 in India was reported in Kerala on January 27 2020.
In this period, the covid cases increased exponentially, making the base of the most deadly phase of the pandemic the world witnessed. by March 27, the global covid cases had crossed the half-million mark. It took only three days to reach the 5 million mark from 4 million cases **(source: *Coronavirus: The First Three Months as It Happened*, 2020, Nature)**.

Viewing the disease's fatality, When the cases in India were only 500, the Government of India Declared a 21 day National Lockdown on March 24 2020, after a 14-hour Voluntary Curfew on March 22. **(source: "COVID-19 Lockdown in India," 2021)**

By March 27, the U.S. and Europe had become new hotspots of Covid, reporting 12,174.1 cases on a 7-day roll average, followed by Spain(6,472.71), Italy(5,639.57), Germany(4,047.29) and the united kingdom(2,332.71) **(source: Ke et al., 2020).**

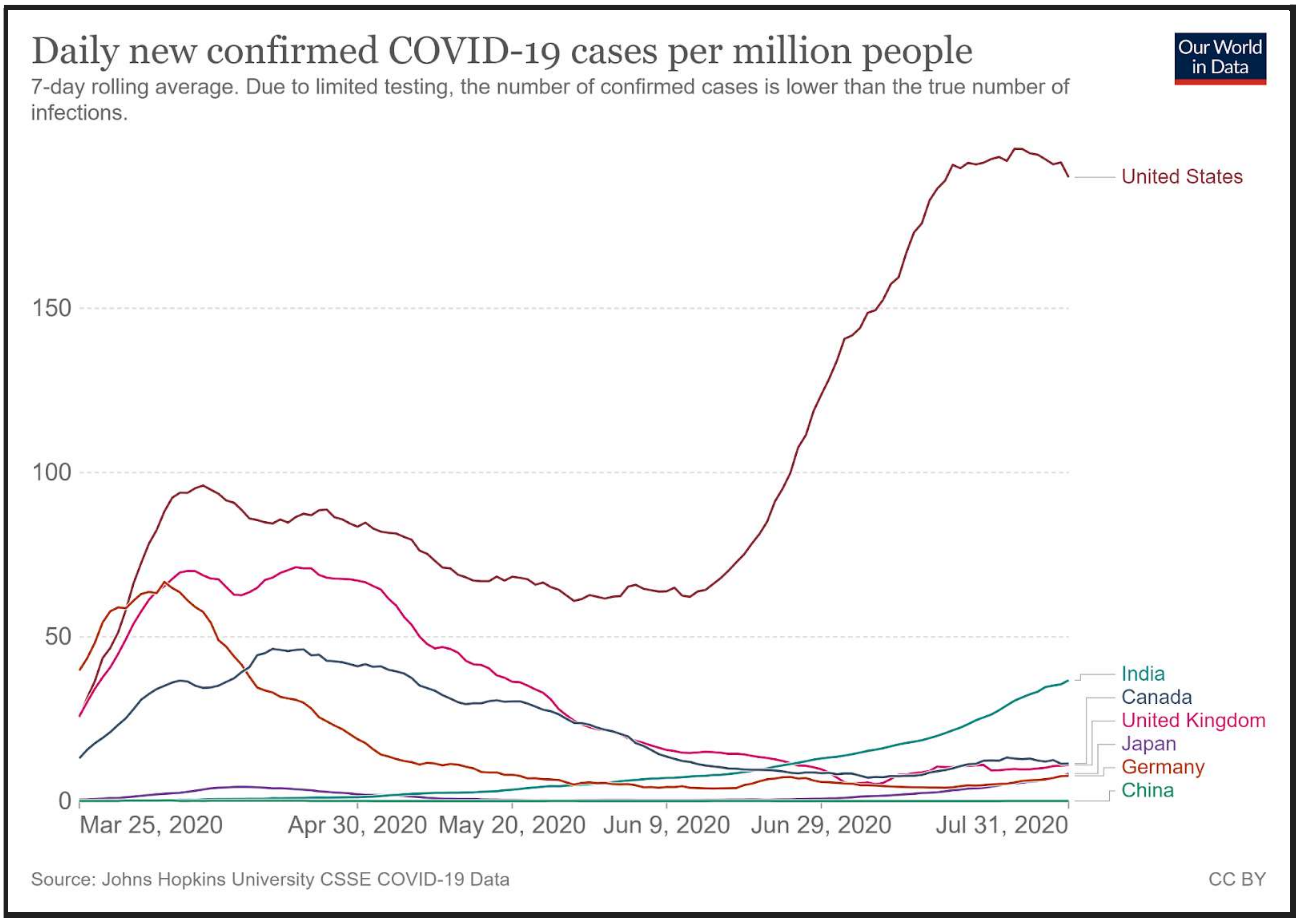
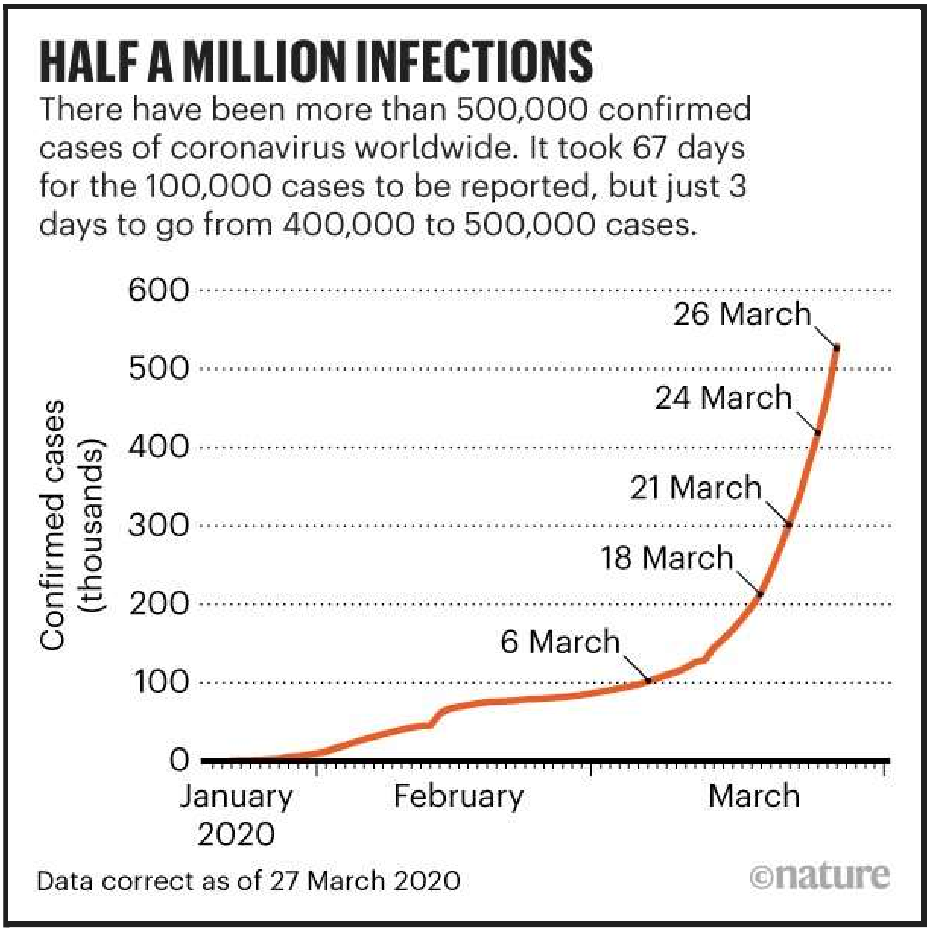


FIG.3 COVID Stats of India and Other Countries During Lockdown In India (Relative to population)(Ritchie, 2020b)

By the end of the day on 26 March, the United States had overtaken China for the highest number of confirmed cases. Italy is also poised to surpass China in the coming days. Italy and Spain now have the two highest death tolls, with Italy accounting for more than one-third of the global total.COVID-19 has claimed the lives of nearly 23,000 people. More than 120,000 have recovered from the disease. **(source:(Ritchie, 2020b)**



\

FIG4: Showing the first 1 million Global Cases (2 April 2020)(*Coronavirus: The First Three Months as It Happened*, 2020, Nature)

On April 2, 2020, The number of active cases reached one million, taking just a week for another half-million infections.

4.1.3 Covid spread in India (Lockdown in India) [Fig 3]:

As the world reached 1 million active cases on April 2, gaining 5 million global cases in just seven days, India's infection rate remained under control due to a well-timed Lockdown.

FIG 5: Break the Chain(*Break the Chain of Transmission*, 2020)

The infection continued worldwide at a tremendous rate, crossing 2 million infections by April 15. The United States turned out to be the most significant hotspot during this period, having steeply increasing new cases of infection. The rate of infection in India was steady due to Lockdown.

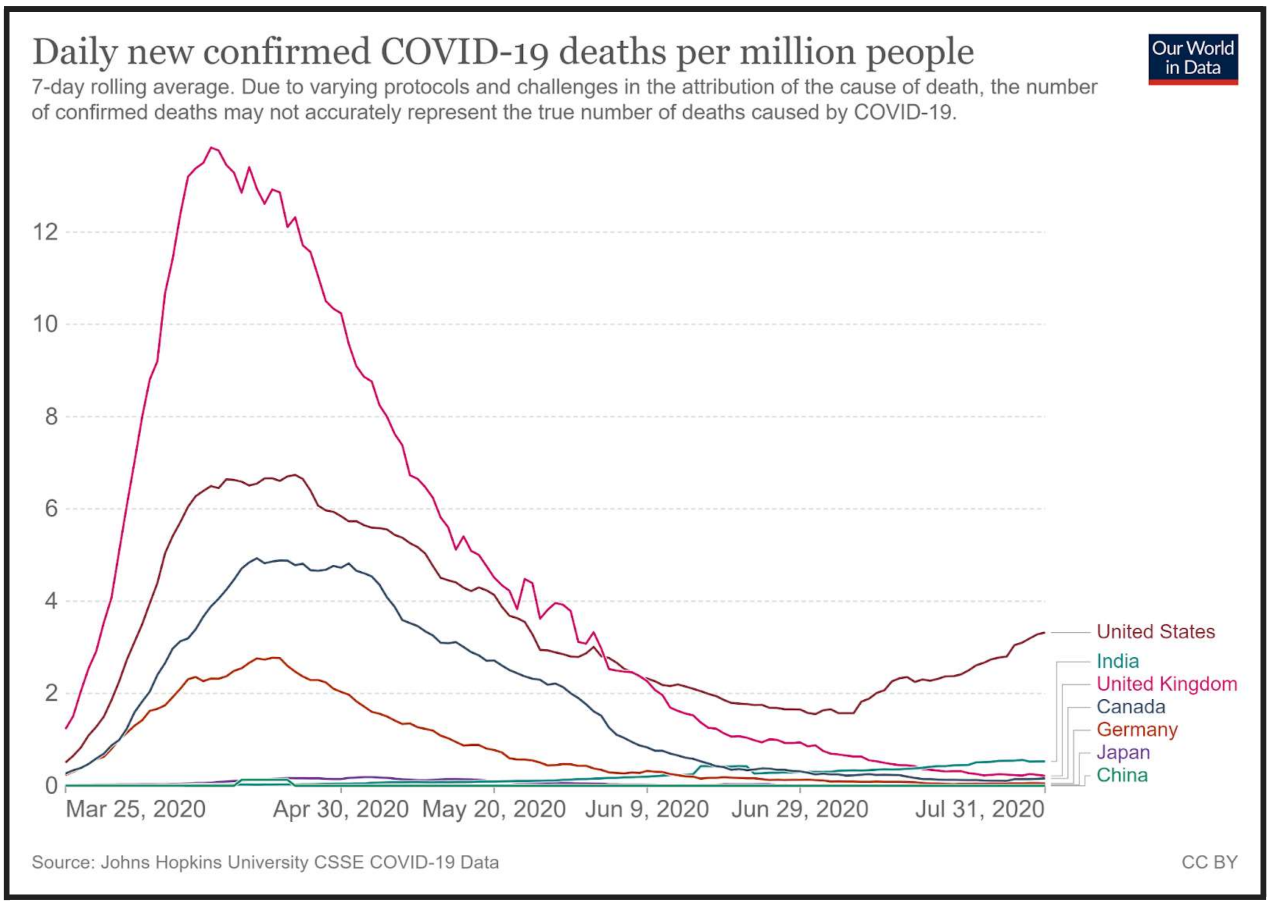


FIG 6: Showing COVID Stats of Deaths per million [relative to population](Ritchie, 2020b)

Since the first Lockdown in India on March 24, there have been four lockdowns from March 25 to May 31 2020 (Lockdown 1.0: March 25 to April 14; Lockdown 2.0: April 15 to May 3, Lockdown 3.0:4 May to May 17 and Lockdown 4.0: May 18 to May 31 2020) and two unlock periods June 1 to July 31 2020 (UL 1.0: June 1 2020, to June 30 2020, and UL 2.0: July 1 to July 31). **(“COVID-19 Lockdown in India,” 2021)**

4.1.4 Effects of Lockdown on Growth Of Covid Cases

Five thousand two hundred seventy-four people were affected in India till April 8 2020 (Ministry of Health and Family welfare), out of which 419 patients were recovered, and 149 died. While in other countries, due to untimed Lockdown, the covid cases were increasing exponentially. In India, the variation of Covid cases increased in trajectory, making smaller angles than in the USA, France, Spain, Italy, etc.

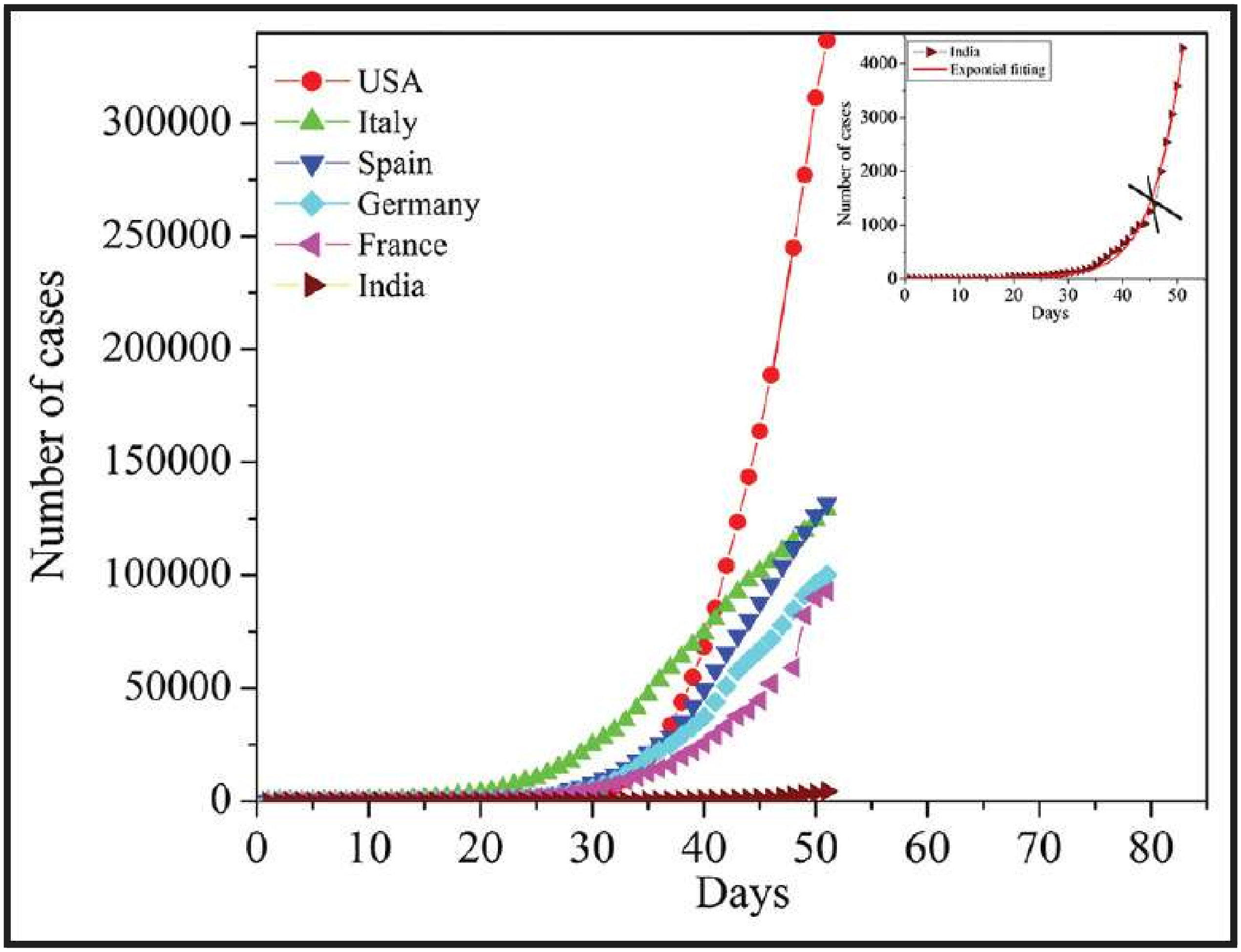


FIG 7: Variation of total SARS-CoV-2 infection cases vs days during Lockdown (“COVID-19 Lockdown in India,” 2021)

Strong growth of infection has been found in India as the flexible curve shown is firmly entrenched. The coefficient of regression R2 is 0.99544, which is very close to 1 and indicates the best fitting. In FIG 5, a cross symbol is shown where a significant increase in the number of virus cases was observed. The slow growth rate of COVID-19 infection is due to the active pace of social isolation, Janata Curfew, and closure. Following the emergence of the first case of SARS-CoV-2 infection, GoI has put in place various procedures to prevent SARS-CoV-2. SARS-CoV-2 infections in the United States of America, Italy, Spain, Germany, India and France started about the same time, around January 30, 2020, and India was largely successful in preventing corona infection due to the implementation of all preventive measures and appropriate public monitoring. **(“COVID-19 Lockdown in India,” 2021)**

4.1.5 Second Wave of Covid:

_It was caused due to The **Delta variant** (B.1.617.2). It started at the beginning of February and peaked in India and the world in July 2021.

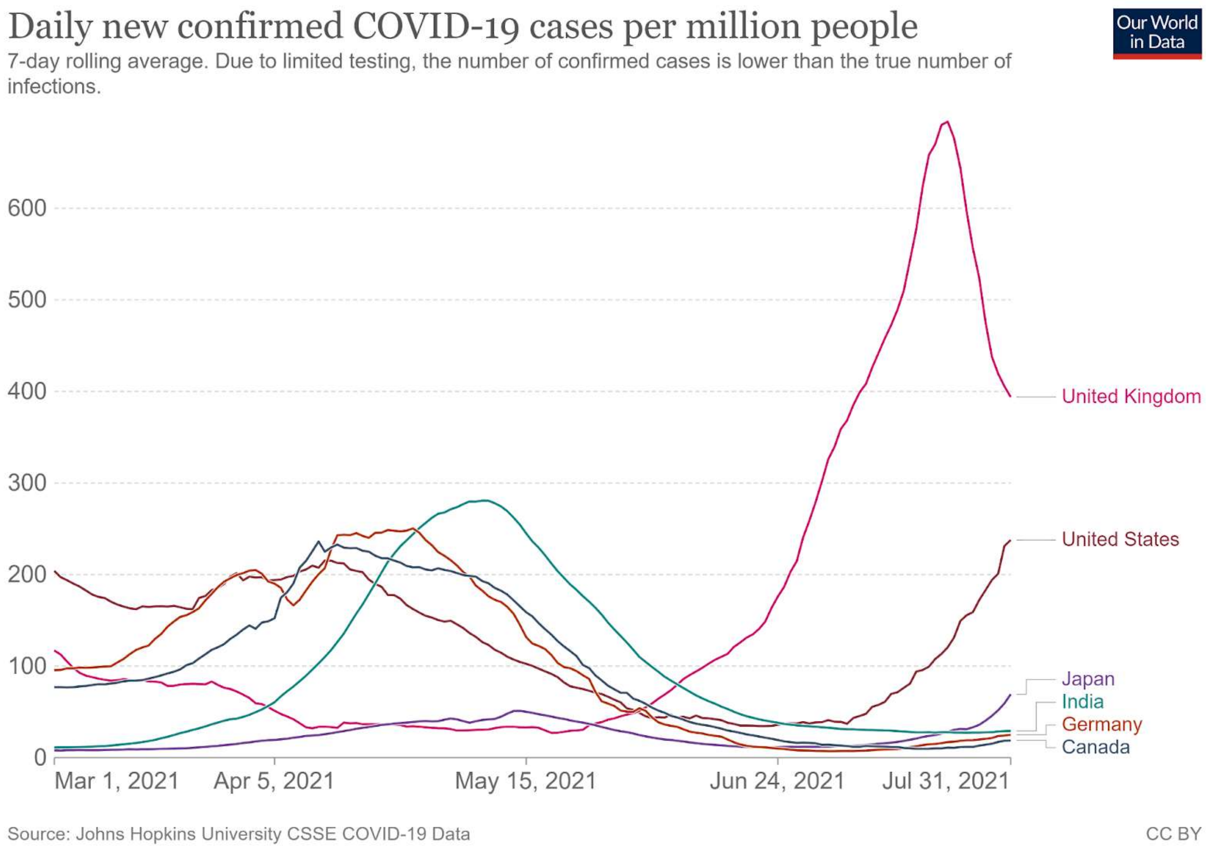


FIG 8: Stats of Covid19 Cases during Second Wave of Corona(Mar- July)(Ritchie, 2020b)

Its first case in the world was discovered in India on October 5 2020, so it was named Indian Variant. Since the first wave had consumed most of India's health resources, it resulted in a shortage of vaccines, hospital beds, oxygen cylinders and other essential medical supplies.
On April 9, India surpassed 9 million active cases, soon overtaking Brazil on April 12 as the second most covid cases worldwide. India led the tally of new and active cases worldwide by the end of April. India became the first country to Surpass 4 Lakh active cases every day(24 hrs) on April 30 2020.

4.1.6 Implications of Second-wave and Cases of Black Fungus :

During the second wave in India, multiple cases of mucormycosis, also known as black fungus, were reported in diabetics and patients with COVID19 and patients recovering from the infection. Excessive use of steroids in treating COVID19 and viral immunosuppression has led to the emergence of this opportunistic fungal infection. Although few cases of black fungus were reported in the first wave, the case of the second wave has become more prominent in many cities. In India, the state governments also declared this an epidemic. As of June 7, 2021, the Indian Ministry of Health has recorded 28,252 cases of ringworm(Bhatia 2021). The risk of aspergillosis, a white fungus, was thought to be even more deadly than black fungus, increasing, with some cases being reported in parts of India. 6 people with comorbidities had a higher risk of death. However, young people appear to be more susceptible to infection in this latest cycle.

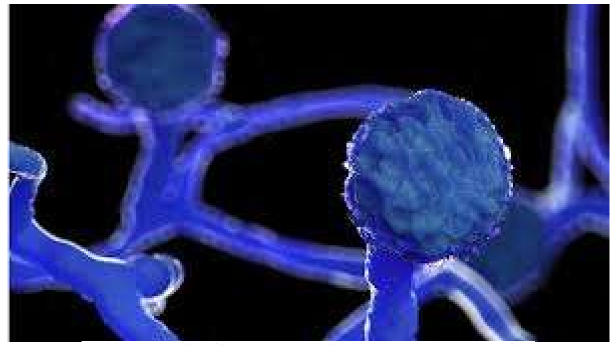


FIG 8: Black Fungus (BBC Image, n.d.)

Many patients have died at a young age, including patients between the ages of 25 and 50. However, as the situation evolves, an important observation is that every individual appears to be equally at risk of contracting the virus. Still, the ability to maintain and overcome the infection varies between individuals. Some people with supposedly suboptimal immune responses may survive, and some, despite apparently stronger immunity, may not get over the infection as quickly. Another observation at the peak of the second wave was the sudden drop in oxygen saturation of some patients, even when they were recovering well, leaving them with less time to receive appropriate ventilatory support. This situation makes patients and family members fear and panic because it is uncertain whether the patient will survive the virus infection even if they show signs of recovery. There is no answer to why individuals respond differently to SARSCoV2 infection. There are several possible explanations for this finding. One possible explanation is the presence of different strains of SARSCoV2 infecting individuals simultaneously, with some variants causing more disease than others. India's poor air quality index could be a potential factor in why the spread of the disease has been increasingly severe across the country. Community and colleagues report that an increase in fine particles (<2 5 µm) is associated with an increased risk of COVID-19 infection.

Given that nine of the 15 most polluted cities in the world are in India, the capacity of the Indian people to fight COVID19 may be weak as their lungs are heavily affected by air pollution. Another reason may be the presence of different immune responses between individuals. A healthy-looking person may not have a strong enough or strong enough response to immunity. However, the lack of scientific studies on individual immune responses has limited our rationale for this hypothesis. (**Bhatia, M. (2021). The rise of mucormycosis)**

4.1.7 Third Wave: (THREE YEARS OF PANDEMIC):

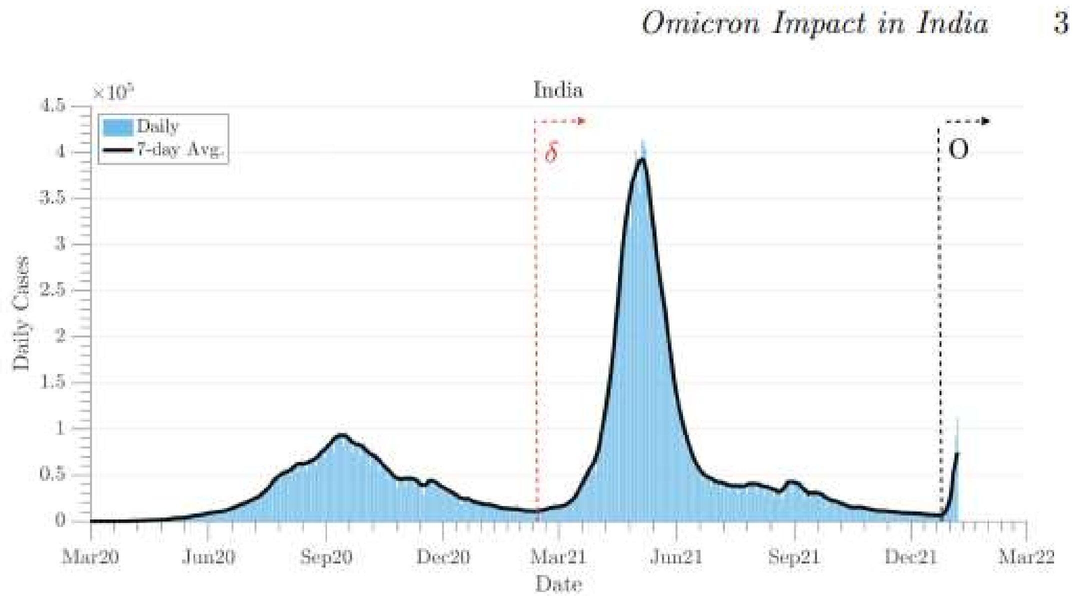


FIG 10: Showing the Peak of All three Waves of Covid (Yadav, A.)

In late 2021, as the delta variant continues to wreak havoc in parts of the world, a new variant. B.1.1.529, or "Omicron", was discovered in South Africa (SA). Soon after, it was declared a variant of concern by the World Health Organisation (WHO)

(WHO) on November 26, 2021. due to several mutations, including 15 mutations in the mutant protein's receptor binding (RBD) and increased transmissibility. Omicron quickly displaced the Delta strain in SA to produce new daily peaks above 37,000 cases in a record time of just 20 days compared to daily highs of 26,000 infections in all previous waves.

By 2021, Omicron has spread to more than 100 countries, including the United Kingdom (UK), United States (US), France, Italy and India, causing new waves. The number of global daily infections shot at about 2.8 million on January 7, 2022. The United States reported more than 1 million infections on January 4, 2022. **(Kavitha et al., 2021)**

4.2. Approach to Modelling and forecast

Forecasting models have proved useful in estimating the future number of confirmed cases, casualties, and recovery figures.
There are many approaches to forecasting, such as statistical analysis, time series analysis, Mathematical analysis and ensemble learning. Metrics such as R-squared value, root mean squared error (RMSE), mean squared error (MSE), mean absolute errors (MAE), and mean absolute percentage error are used to assess the accuracy of trained models (MAPE). The suggested forecasting models may be used to track the growth of COVID-19 instances, allowing government authorities to make necessary modifications to their system. Some of the approaches are discussed here.

4.2.1 Statistical Approach:

In this paper, the prediction of COVID-19 cumulative confirmed cases and deaths for India is analysed based on various statistical models such as
(a) time-series
(b) machine-learning ETC.
For making predictions, Autoregressive integrated moving average (ARIMA) and Holt-Winters exponential smoothing in time series; support vector regression (SVR) and linear regression (LR) in machine learning (ML), and random forest regression in ensemble learning (EL) have been implemented.
The accuracies of the trained models are evaluated using metrics such as R-squared value, root mean squared error (RMSE), mean squared error (MSE), mean absolute errors (MAE), and mean absolute percentage error (MAPE). The proposed forecasting models can be used to monitor the rise in COVID-19 cases, which can thereby be helpful for government officials to make necessary changes to their system. **(Vespignani et al., 2020)**

4.2.2 Time-series Forecasting:

According to new research… (Mirza, 2020). It is a method of forecasting future events based on past events. It assumes that future events will follow the same trend as past events.
It analyses the pattern, recurrence, or relationship between past events and gives an idea about future events' frequency, expected time, and consequences. It is done by model fitting historical data.
In it, Analysts examine past occurrences to check patterns of time decomposition. The most commonly looked at patterns are trends, seasonal, cyclical, and regularity. These patterns help decide which forecasting algorithms and data preprocessing should be used for predictive modelling.

Time Series Decomposition:
It is of two types
1. Based on rates of change
2. Based on predictivity

4.2.3. Machine Learning ETC:

Machine learning is a large field which encompasses ideas from various related fields like statistics, artificial intelligence, deep learning etc. Its focus is to observe the past patterns and trends and learn from them to give the best output that describes the patterns.
It uses supervised, unsupervised or reinforced learning to observe the trends in data by linear regression or other complex mathematical models.
It can be inductive, deductive, or Transductive. It can consider multiple factors at a time, which is impossible for humans to do.

4.2.4. MATHEMATICAL MODELS

The rapid spread of the COVID-19 Coronavirus has renewed scientific and political interest in epidemic mathematical models. Many scholars are working hard to develop more improved models to assess the situation and anticipate probable future scenarios. Mathematical Compartmental models such as SIR and SIER are continuously being analysed and modified to best fit the dynamics of covid. Solving systems of differential equations has been made very comfortable by using python libraries. Results can be compared by plotting charts of different model parameters vs time.

In the research article by Cooper et al. (2020), the SIR(susceptible, infectious and research) model was used for the same purpose, and an almost accurate peak analysis was done for some communities where covid spread was almost homogeneous. Fig11 shows the results of the analysis using SIR.

Similarly, in the research article by Saikia et al. (2021), the SIER model is used with variable time-dependent parameters and analysis is done for many countries with the available disease data up to the end of August 2020, with a projection of 42 days into September and October 2020, and results were seemingly very accurate. The results fit predicted results and agree well with the data up to 42 days beyond the end of the data used in the modelling. The results were also compared with logistic regression. This might not be useful to analyse the long term predictions, and it is helpful to gain insights into the pandemic as a single function can be used to model the pandemic in diverse circumstances. In contrast to the compartmental models, the logistic modelling gets better and better as more and more data becomes available, making them less helpful in making predictions. The ineffectiveness of Logistic regression models is shown by making comparisons between predicted curves for both models.

In the recent research article (Gopal et al., 2022), a modified version of the SEIR model and almost accurate prediction of the peak is made for many states in India, as well an analysis of government actions is provided. Variable parameters are used, and their plot analysis is used.

Fig 12 below shows the results obtained from the analysis.

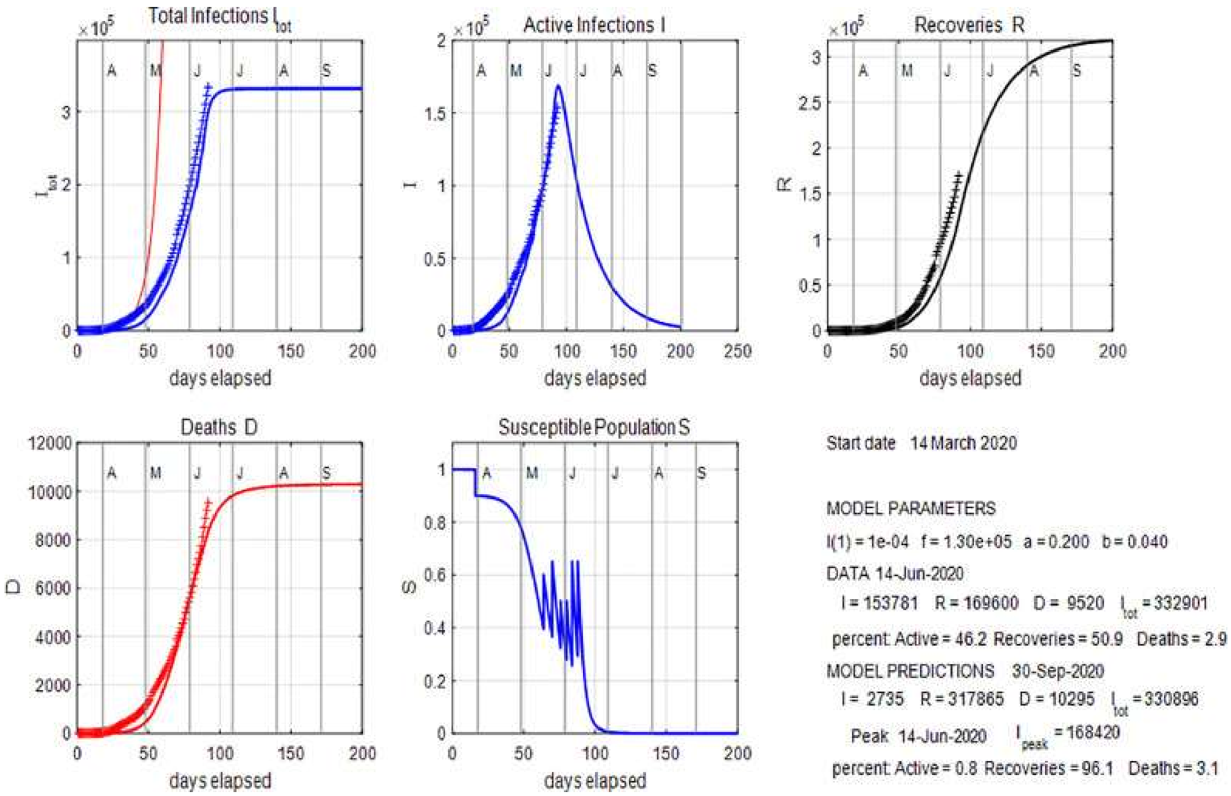


Fig 11 India: Model predictions from 14 March to 30 September 2020 with data from March to June 2020. (Using SIR Model)

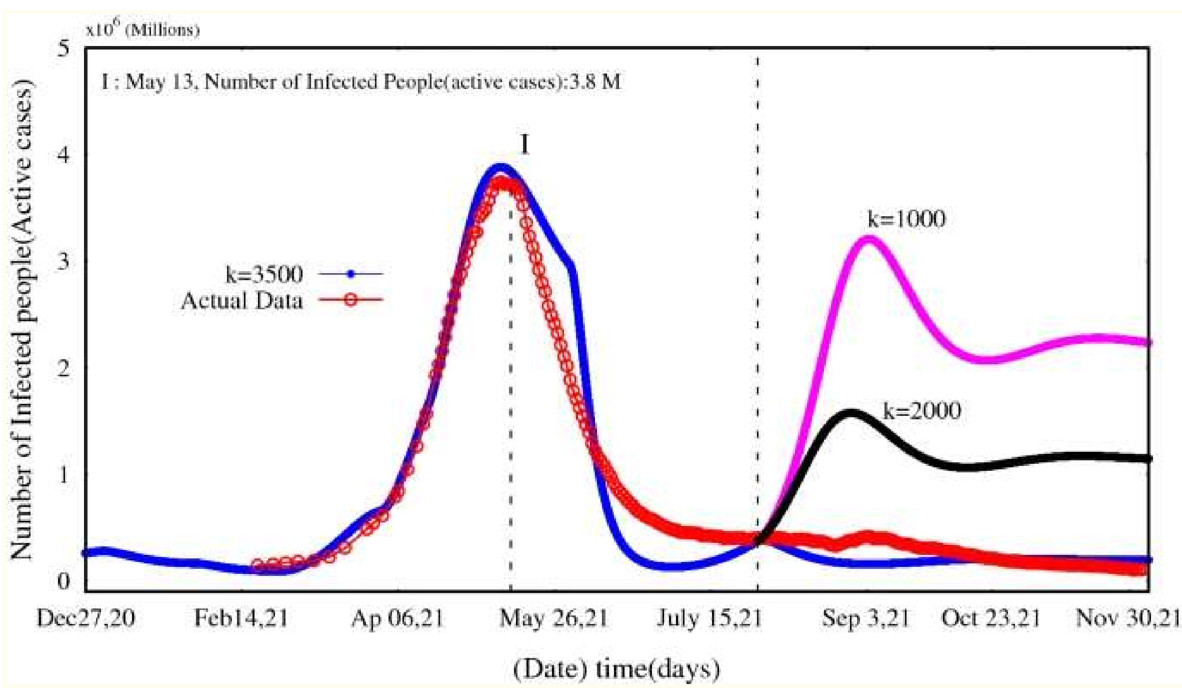


Fig 12 India: Model predictions using SEIR model.

4.3. SUMMARY OF LITERATURE REVIEW

4.3.1. Transmission of Virus:

The Corona Virus is a highly infectious communicable virus; It spreads through droplets and tiny airborne particles. When an infected person sneezes or coughs, this virus transmits to air via droplets, and when an average person gets in contact with the air containing the droplet, he gets infected by the virus.

The virus started in Wuhan, China, and first came to the scene when it caused havoc in china. Soon, it spread to Europe and the US, causing loss of life and material. It proved to be the worst time ever in the 21st century, making thousands of children orphans, thousands of youths jobless and many more. After causing havoc in Europe and the US, it created havoc in India.

The rate of transmission in India while Lockdown was under control compared to other big countries. There are a total of 3 waves of corona in India, with the second wave caused by the delta variant, the most deadly wave.

4.3.2. Research Gap:

As we look at the trends of the Coronavirus, we observed that timely Lockdown in India proved to be deciding factor in controlling the spread of it. As the Lockdown ended, the COVID cases in India boomed. So it is evident that the proper prediction was the key to controlling this virus. However, researchers initially had *significantly less data about the infections,* so their predictions were less accurate.

Most of the articles used interpolation of the data, which does not take count of the **virus's mutation rate**. So they were not able to accurately predict the covid cases.

It proved to be very dangerous as in the peak of Covid we suffered from a lack of oxygen cylinders, quarantine centres and health-care facilities etc.

5. Objectives Of Research

- Developing a model for time series analysis of COVID 19 CASES.
- Verification of this model with the actual trend observed.
- How this model could help in future in addressing a pandemic situation.

6. METHODOLOGY - Modelling of Covid 19 Cases

In the present section, we have tried to predict the pattern of the number of active, recovered deaths and total cases of covid19 using generalised models.

Prediction of spreading patterns of the covid19 virus in India is a complicated and tedious process due to its diverse demography and meteorological data distribution. Many researchers have developed various models to correlate the data with patterns.
We consider deterministic models based on a system of initial values problems of Ordinary Differential Equations (ODEs). This theory was widely studied(for about a century) by W.O. Kermack and A. G. MacKendrick in their research("A Contribution to the Mathematical Theory of Epidemics," 1927)

6.1. SIR model

In the SIR models, the population is considered to be "closed" and composed of three states
1.)Susceptibles - These include all the people capable of being sick from the infection.
2.)Infectious - People infected from the disease or potentially spread the disease.
3.)Recovered - Those who have developed immunity and are not susceptible to the disease.

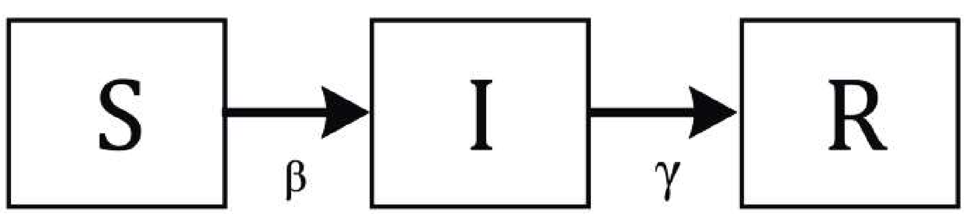


Fig 13 A SIR Model representative diagram

Susceptibles go to infectious; infectious goes to Recovered. Hence, transitions are happening between these states. The Infected group's evolution is influenced by a critical parameter called R0, representing the introductory reproduction rate. The value of R0 can be calculated using available data or extrapolated from epidemiological studies or statistical data from the literature. We use the available data to determine the value of R0 that best fits the data in this paper.

Parameters

- β is the transmission parameter which is the average number of individuals that one infected individual will infect in unit time. It is determined by the contact chance and probability of disease transmission.
- γ is the rate of recovery in a specific period.
- D is the average time period during which an infected individual remains infectious(1/γ).
- R_o basic reproduction number = $\frac{\beta}{\gamma}$

Equations :

$$\frac{dS}{dt} = -\frac{\beta}{N}SI \qquad (1)$$

$$\frac{dI}{dt} = \frac{\beta}{N}SI - \gamma I \qquad (2)$$

$$\frac{dR}{dt} = \gamma I \qquad (3)$$

Explanations:

1.) The cause of more people getting into the infectious category is contact between Susceptible and infectious people. Hence the number of susceptible will go on decreasing with unit time as βIS.

2.) These people will hence come in infectious categories. Also some people will get recovered over time leaving Infectious category(γI).

3.) The number of recovered individuals will increase by the number of Infectious people who got recovered.

To find the peak of infectious cases one can infer from eqn (2) I(t) becomes maximum when $dI(t)/dt = 0$, or $N = \frac{S\beta}{\gamma} = R0*S$

To lower the infectious cases(N0), one should follow proper protocols(that will in turn reduce the transmission rate or β), similarly one has to increase γ , i.e. the rate of recovery by taking proper treatment protocols.

This is the most basic model and can be easily expandable to accommodate more dynamics. For example the birth and the death rate can be accommodated using respective rates, hence also considering the fact that the population is not constant and changing.

6.2. SEIR model

This model is the refinement of the previous SIR model. In this model a new state is introduced, "Exposed" along with the three existing states(Susceptible, Infectious and Recovered).

The exact definition is shown as follows:

1.) Susceptible (S) stands for the number of people that were never infected, but are capable of being sick from infection.

2.) Exposed (E) stands for the number of infected people, but not yet infectious (i.e. in incubation period).

3.) Infected (I) stands for the number of infected people that are also infectious.

4.) Removed (R) stands for the number of people that recovered or deceased.

Likewise the SIR model, the interactions between these states are expressed as a system of Nonlinear ODE's.

Equations

$$\frac{dS}{dt} = -\frac{\beta}{N} SI$$

$$\frac{dE}{dt} = \frac{\beta}{N} SI - \alpha E$$

$$\frac{dI}{dt} = \alpha \, E - \gamma \, I$$

$$\frac{dR}{dt} = \gamma \, I$$

Here α represents the incubation rate. The difference between the exposed (E) and infected (I) is that the former have contracted the disease but are not infectious, and the latter can spread the disease.

A variation of the SEIR model is when we also consider the interactions between Susceptibles and Exposed people. This is important because, although infected people spread more than Exposed people, some Exposed people could also be potential spreaders. We could include that factor too in our equations as follows.

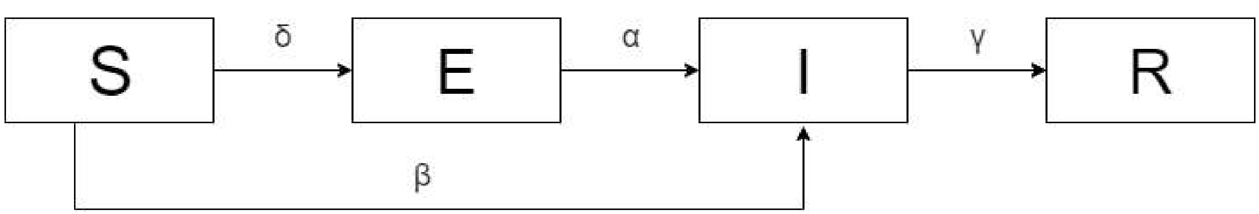


Fig 14 A SEIR Model representative diagram

Though this increases the complexity of the model(by introducing another non-linearity in the differential equations), it could be quite valuable.

Modified Equations

$$\frac{dS}{dt} = -\frac{\beta}{N}SI - \frac{\delta SE}{N} \qquad (1)$$

$$\frac{dE}{dt} = \frac{\beta}{N}SI - \alpha E + \frac{\delta SE}{N} \qquad (2)$$

$$\frac{dI}{dt} = \alpha\, E - \gamma\, I \qquad (3)$$

$$\frac{dR}{dt} = \gamma\, I \qquad (4)$$

6.3 SEIRD model

This is a further modification of the SEIR model, in which the death rate is also included. In the above model, the Total population(N) was assumed constant. However, that will be somewhat inaccurate if the death ratio is sufficiently large(as in Britain and Italy).
In this model, the death rate is assumed to be constant.

6.4 SEIRDQ model

This is a modified model that considers the elimination rate of susceptible, exposed and infected individuals in Quarantine. The primary assumption is that this conversion is under a constant elimination parameter(by time, i.e. 1/times), and after the conversion, the individual becomes "recovered" and will no longer transmit the disease. The new system is written as:

$$\frac{dS}{dt} = -\beta SI - \gamma SE - \omega S \tag{1}$$

$$\frac{dE}{dt} = \beta SI - \alpha E + \gamma ES - \omega E \tag{2}$$

$$\frac{dI}{dt} = \alpha\, E - (\zeta + \delta + \omega)\, I \tag{3}$$

$$\frac{dR}{dt} = \zeta\, I + \omega(S + E + I) \tag{4}$$

$$\frac{dD}{dt} = \delta\, I \tag{5}$$

6.5 Parameters :

ω is the conversion rate parameter for susceptible, Exposed and infected individuals that become recovered due to removal to a quarantine. γ here is the interaction parameter between Susceptibles and Exposed, telling rate of susceptibles which get exposed to virus.

ζ is the average time of Recovery of covid(which is assumed to be constant 14 days for the population). δ is the death rate which is also constant.

7. Results and Analysis

In this section, we present our results based on SIR, SIER, and SIERDQ models implementations applied to covid19 case data of India. The data for use is collected from

- https://www.kaggle.com/imdevskp/corona-virus-report
- https://www.kaggle.com/fernandol/countries-of-the-world

Data for finding parameters is from 24th Jan 2020 to 27th March 2020. A prediction of the next 300 days is made using each of the above models. The '0' in the graph represents the first case in India that was observed on the 27th of Jan 2020. We modelled the differential equations in python and solved them using ode_int and other methods.

Our results and code work can be seen here →
https://deepnote.com/workspace/psindiap-34dcd417-c5fb-4586-a581-5d8477e0202d/project/TSRMCU-dc57ba6e-be15-44f8-a950-3572ffb61812/%2Fnotebook.ipynb

7.1 Results SIR model :

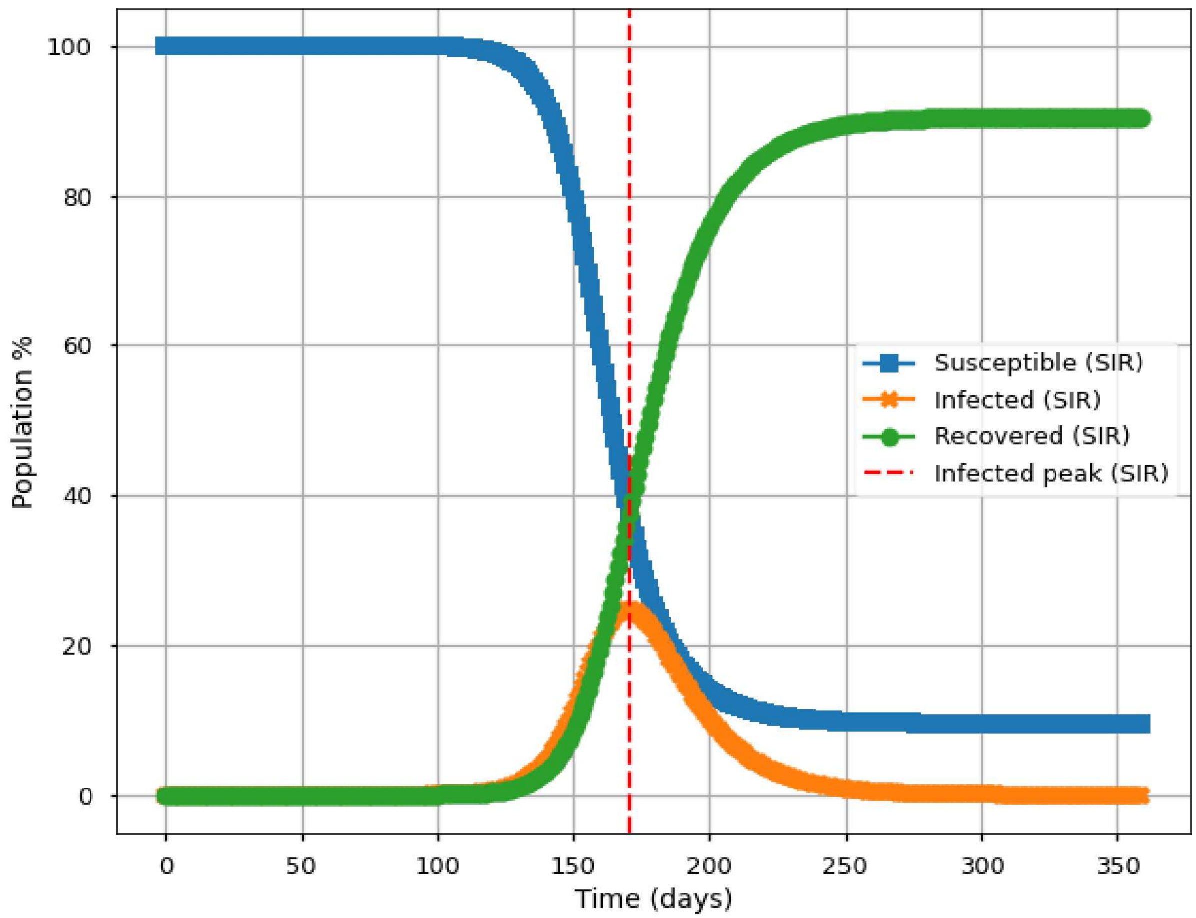


Fig 14 Analysis of Covid-19 patterns using the SIR Model

As depicted in the graph, the peak of infected people is around 170 days ahead of the 27th Jan. Calculated from this model; the date comes out to be **2020-07-18,** i.e. 18th July.

- Max number of infected individuals (SIR model): 271872155
- Population percentage of the max number of infected individuals (SIR model): 24.82%
- Day estimate for max number of infected individuals (SIR model): 171

7.2 Results SEIR model :

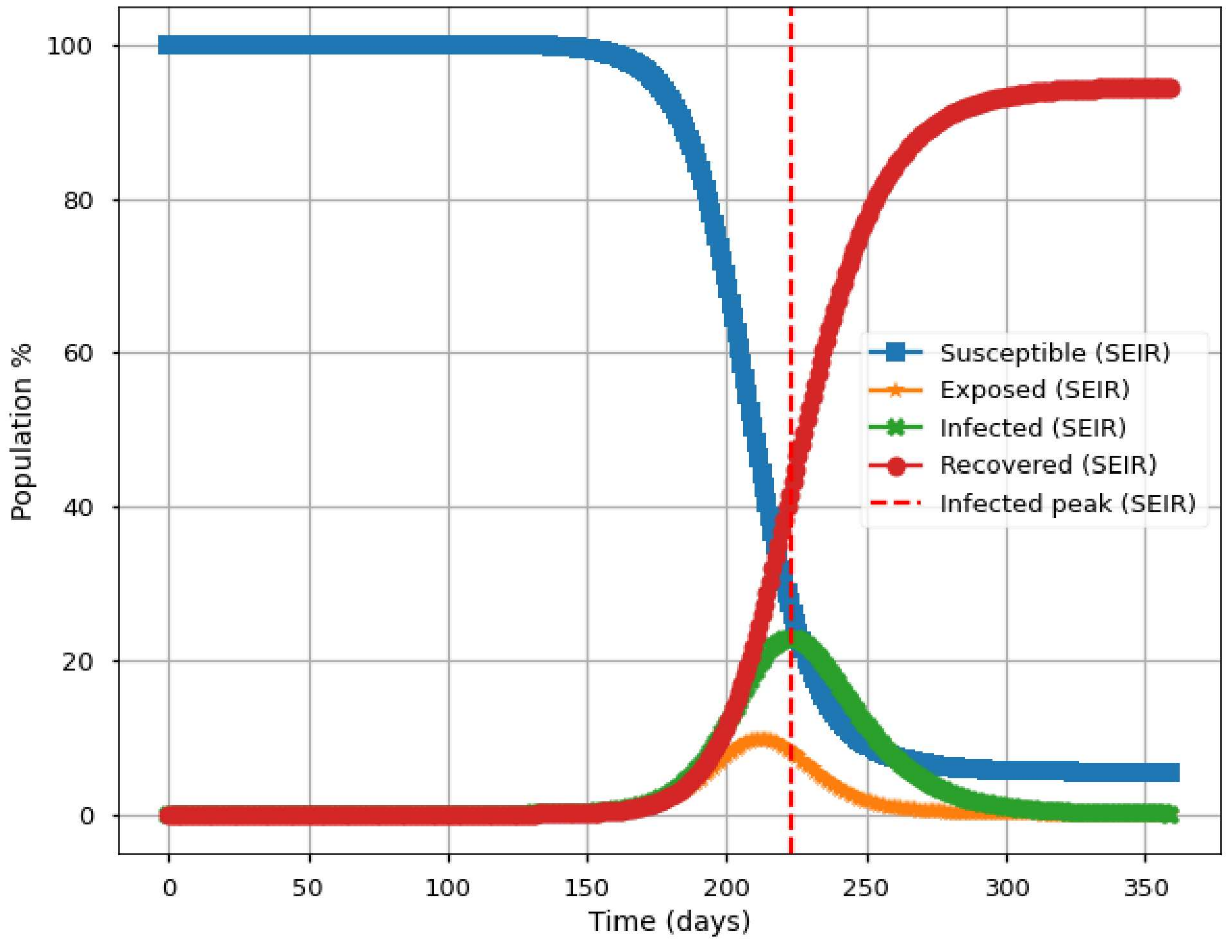


Fig 15: Analysis of Covid19 patterns using SEIR model

The peak of infected people comes out to be around 225 days after 30th Jan. Actual date predicted from the model is **2020-09-08 (8th Sept 2020).**

- Max number of infected individuals (SEIR model):   250260630
- Population percentage of the max number of infected individuals (SEIR model): 22.85%
- Day estimate for max number of infected individuals (SEIR model): 223

7.3 Results SEIRDQ model :

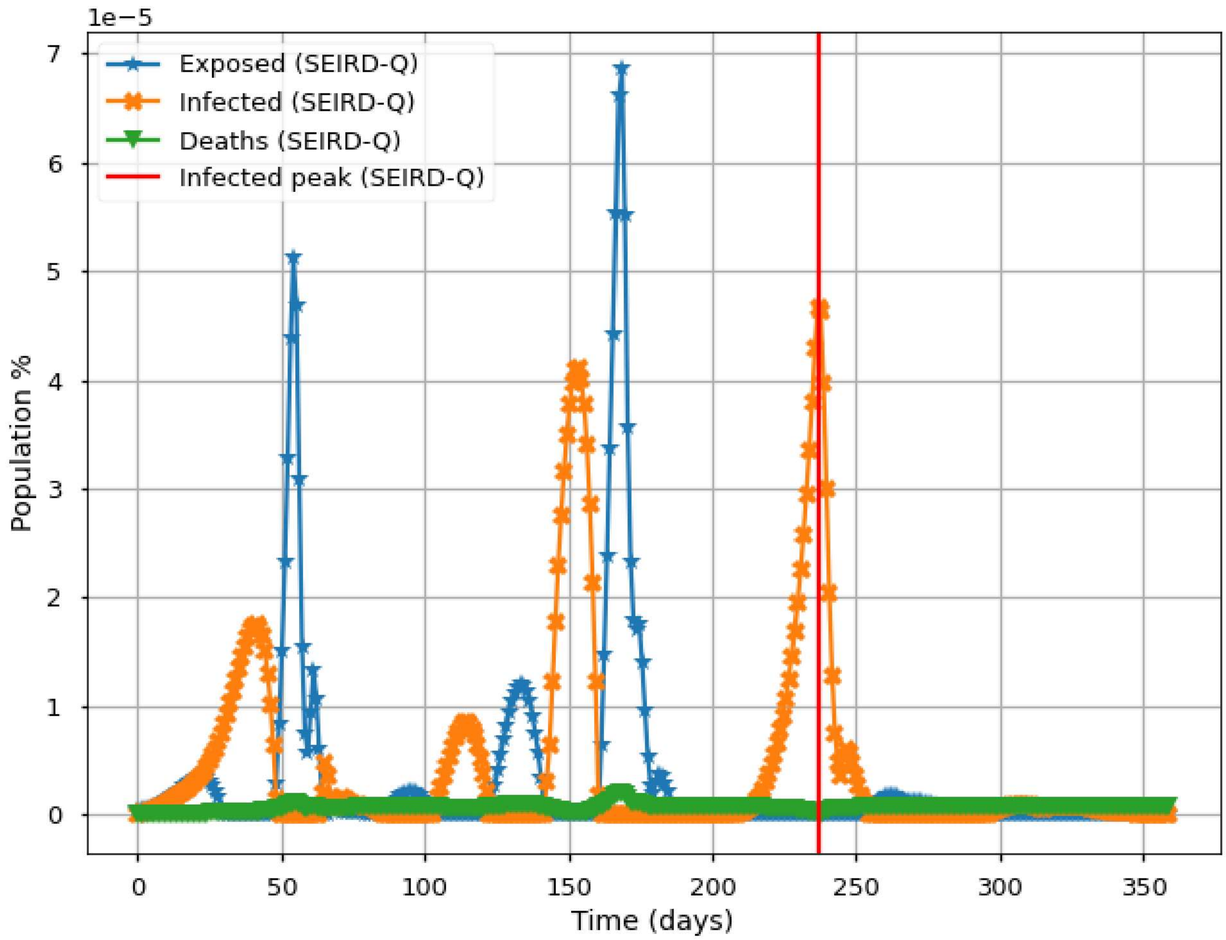


Fig 16: Analysis of Covid19 patterns using the SEIRDQ model

The peak of infected people comes out to be around 240 days after 30th Jan. Actual date predicted from the model is **2020-09-22 (22th Sept 2020).**

- Max number of infected individuals (SEIRD-Q model): 250290000
- Population percentage of the max number of infected individuals (SEIRD-Q model): 23.00%
- Day estimate for max number of infected individuals (SEIRD-Q model): 237
- Percentage of number of death estimate (SEIRD-Q model): 7.030%
- Number of death estimates (SEIRD-Q model): 76210

8. CONCLUSIONS

Theoretical models of mathematical modelling are valuable tools for dealing with the time evolution and patterns of disease outbreaks. They offer us relevant predictions investigating intervention effects on the number of susceptible, infected and recovered rates.

In this work, we have presented the analysis of classical mathematical models (SIR and SEIR) and their modifications and tried to provide insights into the spread of COVID-19 in India. A modified version of the SEIR model, incorporating Quarantined and deceased individuals, was used to analyse the spread of Covid in the Indian population from the end of Jan to November. The Results of peak analysis were almost accurate with the SEIR and modified SEIRDQ. The actual peak was around 15th Sept *(Covid 19 Spread, 2022)*. SEIR predicted the peak to be around 22nd Sept, and the other predicted it to be about 9th Sept. These predictions are pretty accurate and validate the use of such models for different future uses.

Further, the proposed model is highly extendable. We believe it could be easily adapted to monitor various infected areas with different restriction policies and other factors by introducing new States and parameters in the equations as did for including the “quarantined” Parameter.

9. References:

A contribution to the mathematical theory of epidemics. (1927). *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, *115*(772), 700–721. https://doi.org/10.1098/rspa.1927.0118

BBC Image. (n.d.). *Black Fungus* [Diagram]. Mucormycosis. https://images.app.goo.gl/Vmp1mMHwa2dzRK8H6

Bhatia, M. (2021). The rise of mucormycosis in Covid-19 patients in India. *Expert Review of Anti-Infective Therapy*, *20*(2), 137–138. https://doi.org/10.1080/14787210.2021.1960822

Break the chain of Transmission. (2020). [Photograph]. https://images.app.goo.gl/Dgeyc3Nnxbaw1cic7

Centres For Disease Control and Prevention. (2020, September). *Why COVID-19 Forecasting Is Important*. https://www.cdc.gov/coronavirus/2019-ncov/science/forecasting/forecasting.html

Coronavirus Disease 2019 (COVID-19). (2020, February 11). Centres for Disease Control and Prevention. https://www.cdc.gov/coronavirus/2019-ncov/science/forecasting/forecasting-us.html?CDC_AA_ refVal=https%3A%2F%2Fwww.cdc.gov%2Fcoronavirus%2F2019-ncov%2Fcovid-data%2Fforecasting-us.html

Covid 19 spread. (2022, March 22). Times Of India. Retrieved April 20, 2022, from https://timesofindia.indiatimes.com/coronavirus/data/covid-19-spread#:~:text=First%20wave%3A%20The%20first%20case,per%20day%20in%20mid%2DSeptember.

Cooper, I., Mondal, A., & Antonopoulos, C. G. (2020). A SIR model assumption for the spread of COVID-19 in different communities. Chaos, solitons, and fractals, 139, 110057. https://doi.org/10.1016/j.chaos.2020.110057

Coronavirus: the first three months as it happened. (2020, April 22). Nature. https://www.nature.com/articles/d41586-020-00154-w?error=cookies_not_supported&code=c8c25573-e8ee-4868-a3c9-aa8fe3abbfad

COVID-19 lockdown in India. (2021, April 18). In *Wikipedia*. https://en.wikipedia.org/wiki/COVID-19_lockdown_in_India

https://aut.ac.nz.libguides.com/APA7th

https://owl.purdue.edu/owl/research_and_citation/apa_style/apa_formatting_and_style_guide/reference_list_basic_rules.html

https://www.mybib.com

Kavitha, C., Gowrisankar, A., & Banerjee, S. (2021). The second and third waves in India: when will the pandemic culminate? *The European Physical Journal Plus*, *136*(5). https://doi.org/10.1140/epjp/s13360-021-01586-7

Ke, R., Sanche, S., Romero-Severson, E., & Hengartner, N. (2020). The fast spread of COVID-19 in Europe and the US suggests the necessity of early, solid and comprehensive interventions. *Fast Spread of COVID-19 in Europe and the US Suggests the Necessity of Early, Strong and Comprehensive Interventions*. https://doi.org/10.1101/2020.04.04.20050427

Ministry of Health and Family Welfare. (n.d.). *COVID 19 CASES IN INDIA*. MOHFW. https://www.mohfw.gov.in/

Mirza, T. (2020). Prediction of COVID-19 trend in India using time series forecasting. *Indian Journal of Science and Technology*, *13*(32), 3248–3274. https://doi.org/10.17485/ijst/v13i32.1214

National Health Commission (NHC) of the People's Republic of China. (2020, April 22). Coronavirus Cases: Statistics and Charts. Retrieved April 24, 2022, from https://www.worldometers.info/coronavirus/coronavirus-cases/

Nair, S., Ckm, G., Varsha, R., Ghosal, S., Vergin, M., Anbarasi, L.J. (2022). Intelligent Forecasting Strategy for COVID-19 Pandemic Trend in India: A Statistical Approach. Raje, R.R., Hussain, F., Kannan, R.J. (eds) Artificial Intelligence and Technologies. Lecture Notes in Electrical Engineering, vol 806. Springer, Singapore. https://doi.org/10.1007/978-981-16-6448-9_53

Pan American Health Organization. (2020). *Why Predictive Modeling is Critical in the Fight against COVID-19*. World Health Organisation. https://iris.paho.org/bitstream/handle/10665.2/52276/PAHOEIHISCOVID-19200007_eng.pdf?sequence=8

Ritchie, H. (2020, March 5). *Coronavirus Pandemic (COVID-19)*. Our World in Data. https://ourworldindata.org/coronavirus

Ritchie, H. (2020a, March 1). *COVID-19 Data Explorer* [Dataset]. https://ourworldindata.org/explorers/coronavirus-data-explorer?zoomToSelection=true&time=2020-03-01..latest&facet=none&pickerSort=desc&pickerMetric=new_cases_smoothed_per_million&Metric=Confirmed+cases&Interval=7-day+rolling+average&Relative+to+Population=true&Color+by+test+positivity=false&country=IND%7EUSA%7EGBR%7ECAN%7EDEU%7EFRA

Saikia, D., Bora, K., & Bora, M. P. (2021). COVID-19 outbreak in India: an SEIR model-based analysis. *Nonlinear Dynamics*. https://doi.org/10.1007/s11071-021-06536-7

Vespignani, A., Tian, H., Dye, C., Lloyd-Smith, J. O., Eggo, R. M., Shrestha, M., Scarpino, S. V., Gutierrez, B., Kraemer, M. U. G., Wu, J., Leung, K., & Leung, G. M. (2020). Modelling COVID-19. *Nature Reviews Physics*, *2*(6), 279–281. https://doi.org/10.1038/s42254-020-0178-4

Wu, J. T., Leung, K., Bushman, M., Kishore, N., Niehus, R., de Salazar, P. M., Cowling, B. J., Lipsitch, M., & Leung, G. M. (2020). Estimating clinical severity of COVID-19 from the transmission dynamics in Wuhan, China. *Nature Medicine*, *26*(4), 506–510. https://doi.org/10.1038/s41591-020-0822-7

Yadav, A. (2020). Impact of lockdown to control over Novel Coronavirus and COVID-19 in India. *Journal of Family Medicine and Primary Care*, *9*(10), 5142. https://doi.org/10.4103/jfmpc.jfmpc_692_20

Zhu, H., Wei, L., & Niu, P. (2022, March 2). *The novel coronavirus outbreak in Wuhan, China*. Springer.Com. https://www.biomedcentral.com/epdf/10.1186/s41256-020-00135-6?sharing_token=DevG6_KM4D2RbOm2ZffqRW_BpE1tBhCbnbw3BuzI2RMHvvbv8bIKEUIf-POVZNBOU7bxLHW_6h03ozwEiuuvo9CN5_HJEYUjS6ni1_rYBqdruHY0DXKUN8xBkRnX32xLRqFBe2u2QiY9BoHG6leZLwtlvNctM_AUDrqOl01zxeQ%3D

Zou, L., Ruan, F., Huang, M., Liang, L., Huang, H., Hong, Z., Yu, J., Kang, M., Song, Y., Xia, J., Guo, Q., Song, T., He, J., Yen, H. L., Peiris, M., & Wu, J. (2020). SARS-CoV-2 Viral Load in Upper Respiratory Specimens of Infected Patients. *New England Journal of Medicine*, *382*(12), 1177–1179. https://doi.org/10.1056/nejmc2001737