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DESIGN AND ANALYSIS OF ALGORITHM

01CT0512

* Assignment - 3

Q Calculate Order of Complexity using Master's Theorem :

(1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3$

$b = 2$

$k = 2$

$p = 0$

→ case 3 : $a < b^k$

$3 < 2^2$

(a) $p \geq 0$, $T(n) = \Theta(n^k \log^p n)$

$T(n) = \Theta(n^2 \log^0 n)$

$= \Theta(n^2)$

(2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4$ $k = 2$

$b = 2$ $p = 0$

case 2 : $a = b^k$

(a) $p > -1$ $T(n) = \Theta(n^{\log_b^a} \log^{p+1} n)$

$= \Theta(n^{\log_2^4} \log^1 n)$

$= \Theta(n^2 \log n)$

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(3) $T(n) = T\left(\frac{n}{2}\right) + n^2$ $a = 1$ $k = 2$

$b = 2$ $p = 0$

case 3 : $a < b^k$

(a) $p \geqslant 0$; $T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2)$

(4) $T(n) = 16\,T\left(\frac{n}{4}\right) + n$ $a = 16$ $k = 1$

$b = 4$ $p = 0$

case 1 : $a > b^k$

$T(n) = O(n^{\log_b^a}) = O(n^{\log_4^{16}})$

$= O(n^2)$

(5) $T(n) = 2\,T\left(\frac{n}{2}\right) + n\log n$ $a = 2$ $k = 1$

$b = 2$ $p = 1$

case 2 : $a = b^k$

(a) $p > -1$; $T(n) = \Theta(n^{\log_b^a} \log^{p+1} n)$

$= \Theta(n^{\log_2^2} \log^2 n)$

(6) $T(n) = 2\,T\left(\frac{n}{2}\right) + \frac{n}{\log n}$ $a = 2$ $k = 1$

$b = 2$ $p = -1$

case 2 : $a = b^k$

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(b) $p = -1$; $T(n) = \Theta(n^{\log_b^a} \log \log n)$

$= \Theta(n^{\log_2^2} \log \log n)$

$= \Theta(n \log \log n)$

(7) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2$ $k = 0.51$

$b = 4$ $p = 0$

case 2 : $a = b^k$

(a) $p > -1$; $T(n) = \Theta(n^{\log_b^a} \log^{p+1} n)$

$= \Theta(n^{\log_4^2} \log^1 n)$

$= \Theta(n^{0.60} \log n)$

(8) $T(n) = 8T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 8$ $k = 2$

$b = 3$ $p = 1$

case 3 : $a < b^k$

(a) $p \geq 0$; $T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2 \log n)$

(9) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7$ $k = 2$

$b = 3$ $P = 0$

case 3 : $a < b^k$

(a) $p \geq 0$; $T(n) = \Theta(n^2 \log^0 n)$

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(10) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$ $\quad a = 4 \quad k = 0$

$b = 2 \quad p = 1$

case 1 : $a > b^k$

$T(n) = O(n^{\log_b a})$

$= O(n^{\log_2 4}) = O(n^2)$

(11) $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$ $\quad a = 16 \quad k = n$

$b = 4 \quad p = 0$

case 3 : $a < b^k$

(a) $p \geq 0$; $T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^n)$

(12) $T(n) = 3T\left(\frac{n}{2}\right) + n$ $\quad a = 3 \quad k = 1$

$b = 2 \quad p = 0$

case 1 : $a > b^k$

$T(n) = O(n^{\log_b a})$

$= O(n^{\log_2 3})$

(13) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$ $\quad a = 3 \quad k = 1$

$b = 4 \quad p = 1$

case 3 : $a < b^k$

(a) $p \geq 0$; $T(n) = \Theta(n^k \log^p n)$

$= \Theta(n \log n)$

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(14) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3 \quad k = -2$

$b = 3 \quad p = 0$

case 1 : $a > b^k$

$T(n) = O(n^{\log_b a})$

$= O(n^{\log_3 3}) = O(n)$

(15) $T(n) = 2T\left(\frac{n}{2}\right) + n$

$a = 2 \quad k = 1$

$b = 2 \quad p = 0$

case 2 : $a = b^k$

(a) $p > -1$; $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$= \Theta(n^{\log_2 2} \log^1 n)$

$= \Theta(n \log n)$