

All Borel Graphs of degree d in n variables

In this notebook you can get all the Borel Graphs of d degree and n variables

First, fetch the Borel matrix for the n th variable d . For example, if you want all the Borel graphs of monomials of degree 4 in 3 variables, you should fetch the Borel matrix for c^4 , since c is the third variable and the desired degree is 4.

Note that when you declare the ring, declare it only up to the maximum variable. For example, if you want 3 variables, declare the ring $S = \mathbb{Q}\mathbb{Q}[a,b,c]$; don't declare something like $S = \mathbb{Q}\mathbb{Q}[a,b,c,d]$

MainMat = (* Enter the matrix here! *)

Now run the following code:

```

BorelGraph[K_] :=
Module[{R = Transpose[K],
  M = ConstantArray[0, {Dimensions[K][[2]], Dimensions[K][[2]]}], mon},
  For[i = 0, i < Dimensions[R][[1]], i++ ×
    For[j = 0, j < Dimensions[R][[1]], j++
      ×
      If[(EuclideanDistance[R[[i, All]], R[[j, All]]])2 == 2 && SelectFirst[
        R[[i, All]] - R[[j, All]], # != 0 &] == -1, M[[i, j]] = 1, M[[i, j]] = 0]
    ]
  ] ×
  M
];

BorMats = Table[Select[Transpose[MainMat][[1 ;; i, All]],
  #[[1]] ≥ Transpose[MainMat][[i, 1]] &], {i, 1, Dimensions[MainMat][[2]]}];

BorMats = Transpose[#] & /@ BorMats;

AdMats = BorelGraph[#] & /@ BorMats;

AdMats = % /. Null → 1;

BorList =
Table[Table[FromCoefficientRules[{Transpose[BorMats[[j]]][[i, All]] → 1},
  Alphabet[]][[1 ;; Dimensions[Transpose[BorMats[[j]]][[2]]]],
  {i, 1, Dimensions[Transpose[BorMats[[j]]][[1]]}],
  {j, 1, Dimensions[BorMats][[1]]}];

options = Sequence[VertexLabelStyle → Directive[20, FontFamily → "Helvetica"],
  GraphLayout → "SpringElectricalEmbedding", ImageSize → 500, EdgeStyle → Blue];

AlltheBorels =
AdjacencyGraph[AdMats[[#]], DirectedEdges → True, VertexLabels → Table[
  i → BorList[[#]][[i]], {i, 1, Dimensions[BorList[[#]][[1]]}], options,
  VertexStyle → {1 → Blue, Dimensions[BorList[[#]][[1]]] → Red},
  VertexSize → Automatic] & /@ Range[Dimensions[AdMats][[1]]];

```

If you ran all the preceding code, your Borel graphs are ready. You have two options. Either you can view them [here](#) or you can export them to your working directory

To view the graphs [here](#), run the code on the next line.

AlltheBorels

To export them to your directory, run the following code instead. To view your directory, you can use the `Directory[]` function. To set your directory, you can use the `SetDirectory[]` function.

```
SetDirectory[(* Put the directory path here *)]
Table[
  Export[ToString[InputForm[Last[BorList[[i]]]]] <> ".png", AlltheBorels[[i]]],
  {i, 1, Dimensions[BorList][[1]]}
]
```