Predator Prey Modeling in Cellular Automaton

Using 2D cellular automata to visualize Lotka-Volterra equations that describe the simplest predator prey model.

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***Abstract*—this project aims to visualize predator prey model using two dimensional cellular automaton and study the changes in results with varying birth rate and death rates.**

Keywords—predator-prey model, cellular automata

I. INTRODUCTION

A cellular automaton is a collection of "colored" cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells. The rules are then applied iteratively for as many time steps as desired. Here we model Lotka Volterra equations that describe predator-prey dynamics in their simplest case (one predator population, one prey population).

II. PREDATOR PREY DYNAMICS

A. The Model

The predator-prey was developed independently by Alfred Lotka and Vito Volterra [1] in the 1920's, and is characterized by oscillations in the population size of both predator and prey, with the peak of the predator's oscillation lagging slightly behind the peak of the prey's oscillation. The model makes several simplifying assumptions:

- the prey population will grow exponentially when the predator is absent
- the prey has infinite amount of food
- the predator population will starve in the absence of the prey population (as opposed to switching to another type of prey)
- predators can consume infinite quantities of prey and
- there is no environmental complexity (in other words, both populations are moving randomly through a homogeneous environment).

B. Methods

We begin by looking at what happens to the predator population in the absence of prey; without food resources, their numbers are expected to decline exponentially, as described by the following equation:

$$\frac{dY}{dt} = -dY$$

This equation uses the product of the number of predators (Y) and the predator mortality rate (d) to describe the rate of decrease (because of the minus sign on the right-hand side of the equation) of the predator population (Y) with respect to time (t). In the presence of prey, however, this decline is opposed by the predator birth rate, cXY, where the product XY represents the interaction between the two species and 'c' is the predator birth rate. The equation describing the predator population dynamics becomes

$$\frac{dY}{dt} = cXY - dY$$

Turning to the prey population, we would expect that without predation, the numbers of prey would increase exponentially. The following equation describes the rate of increase of the prey population with respect to time, where *a* is the growth rate of the prey population, and X is the abundance of the prey population:

$$\frac{dX}{dt} = aX$$

In the presence of predators, however, the prey population is prevented from increasing exponentially. The term for consumption rate from above (bXY) describes prey mortality, where *b* is the prey death rate, and the population dynamics of the prey can be described by the equation

$$\frac{dX}{dt} = aX - bXY$$

III. CELLULAR AUTOMATA

A stochastic cellular automata based computer simulation model has been developed to approximate a simple prey–predator system. It is a two-dimensional CA model which has three different cell states: empty, prey and predator.

A. Cell States and Neighborhood

A square grid of with 70 cells on one side is made. The total number of cells, thus, are 4900. Each cell has 3 possible states out of which a cell can exhibit only one of the three states at a particular time. Each cell can either be empty, prey or predator.

The neighborhood [1] of a cell is the nearby, usually adjacent, cells. The two most common types of neighborhoods are the von Neumann neighborhood and the Moore neighborhood. The former, named after the founding cellular automaton theorist, consists of the four orthogonally adjacent cells. The latter includes the von Neumann neighborhood as well as the four remaining cells surrounding the cell whose state is to be calculated.

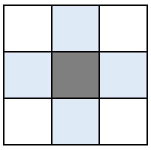

Figure 1: The blue cells are the Von Neumann neighborhood of the gray cell.

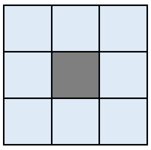

Figure 2: The blue cells are the Moore neighborhood of the gray cell.

For the purpose of the predator-prey model, we have used the Moore neighborhood to calculate a cell's new state in the next generation. For such a cell and its Moore neighborhood, there are 19683 (= 3^9) possible patterns. For each of the 19683 possible patterns, the rule table would state whether the center cell will be empty, prey or predator on the next time interval.

B. Rules of the Automaton

The evolutions for each cell *(i,j)* are based on cell current state, $S_{i,j}^{t}$, the number of its neighboring cells occupied by prey, *X*, and the number of its neighboring cells that are occupied by predator, *Y*.

Thus, the new state of a cell can be expressed by the expression:

$$S_{i,j}^{t+1} = f(S_{i,j}^{t}, X, Y)$$

We first calculate the values of $\frac{dX}{dt}$ and $\frac{dY}{dt}$ using the equations given above and substituting the values of *X,Y,a,b,c, and d.*

The function *f* can be defined as:

1. If $S_{i,j}^{t}$ is empty:
   - if $\frac{dX}{dt} > 0$ and $\frac{dX}{dt} > \frac{dY}{dt}$ then, $S_{i,j}^{t+1}$ will contain a prey.
   - if $\frac{dY}{dt} > 0$ and $\frac{dY}{dt} > \frac{dX}{dt}$ then, $S_{i,j}^{t+1}$ will contain a predator.
   - in all other cases, $S_{i,j}^{t+1}$ will remain empty.
2. If $S_{i,j}^{t}$ is prey:
   - if $\frac{dX}{dt} < 0$ and $\frac{dY}{dt} < 0$ then $S_{i,j}^{t+1}$ will be empty.
   - if $\frac{dX}{dt} < 0$ and $\frac{dY}{dt} > 0$ then $S_{i,j}^{t+1}$ will be a predator.
   - in all other cases, $S_{i,j}^{t+1}$ remains a prey.
3. If $S_{i,j}^{t}$ is predator:
   - if $\frac{dY}{dt} \leq 0$ then $S_{i,j}^{t+1}$ will be empty.
   - in all other cases, $S_{i,j}^{t+1}$ remains a predator.

C. Patterns

Interesting patterns - waves, spirals, and symmetric patterns and can be generated by providing different initial conditions and different values to parameters a, b, c, and d. This cellular automata exhibits 3 kinds of results:

- Both species get extinct. In this case prey gets extinct due to low birth rate, thus, predator dies due to lack of food.
- Only preys survive. In this case predator gets extint due to low interaction with the prey. Prey survives as per the assumption that it has infinite food.
- Both survive. In this case the model cycles around the equilibrium point, and is referred as the ideal case. None of the species get extinct.

In the following figures, red represents the prey population, blue the predator population, and white empty space.

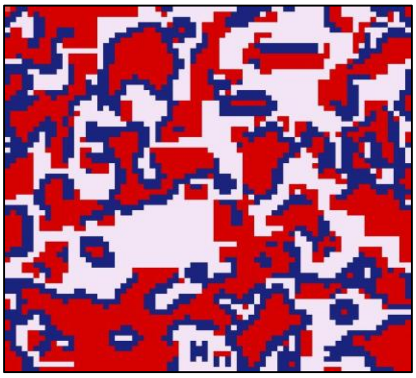

Figure 3: Both species survive

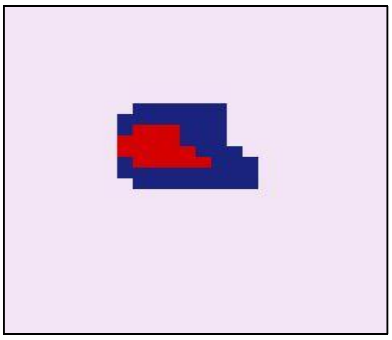

Figure 4: Both species will get extinct

REFERENCES

[1] "Lotka Volterra Equation," Wikipedia, [Online]. Available: http://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equation.

[2] "Predator-Prey Dynamics," [Online]. Available: http://www.tiem.utk.edu/~gross/bioed/bealsmodules/predator-prey.html.