

DC circuit

* (A) Types of circuit elements

(1) Linear and non-linear circuit elements

Linear circuit element \rightarrow if across the particular element the excitation is directly proportional to response

ex : Resistance

$$V \propto I$$

Inductance

$$V = L \frac{dI}{dt}$$

Capacitance

$$V = C \frac{dq}{dt}$$

These elements follows homogeneity and additivity for linearity.

Homogeneity \rightarrow if $n \rightarrow y = f(x)$
 $\alpha n \rightarrow \alpha y$

the change is same

Additivity $\rightarrow n_1 \rightarrow y_1$

$$n_2 \rightarrow y_2$$

$$(n_1 + n_2) \rightarrow (y_1 + y_2)$$

Non-linear element \rightarrow excitation is not directly proportional to response.
 e.g. diode, transistor. Do not follow homogeneity and additivity both

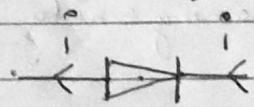
(2) Active and passive element

Active element \rightarrow deliver net power or energy to other element present in circuit ex \rightarrow source voltage and current.

Passive element \rightarrow consumer power and energy when connected in a circuit.
ex \rightarrow Resistance Inductance Capacitance

(3) Unilateral & bilateral

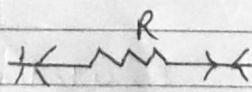
Unilateral element \rightarrow Accross which V-I relation depends on direction of flow of current



$$R = 0$$

$$R = \infty$$

Bilateral element \rightarrow independent of direction of current V-I relation



$$\frac{V}{I} = R$$

R L C are bilateral.

Type of source

Current source

Independent

Ideal

Practical

dependent

Voltage

Current

Voltage source

Independent

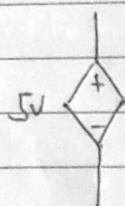
Ideal

Practical

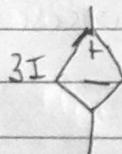
Dependent

Voltage

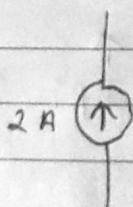
Current



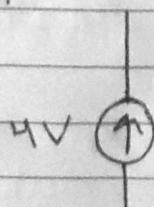
Voltage dependent Voltage source



current dependent voltage



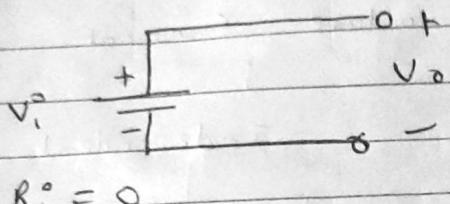
current dependent current source



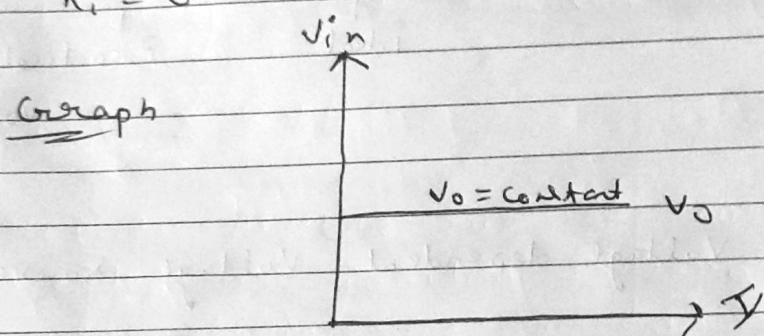
Voltage dependent current source

Independent voltage source

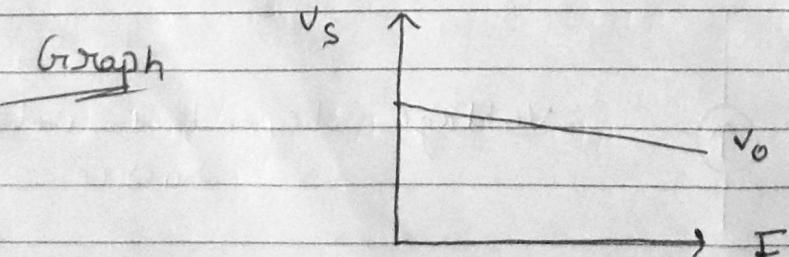
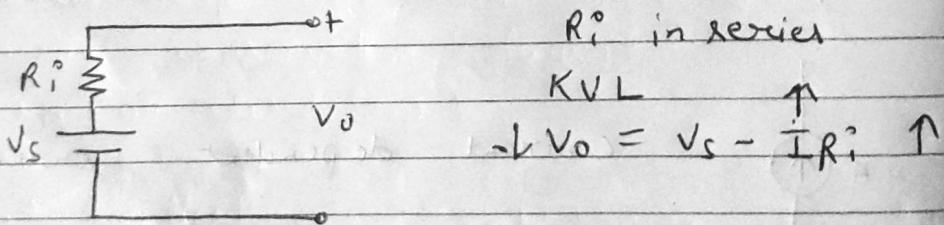
- ① Ideal \rightarrow it is capable of delivering constant output voltage irrespective of current taken from it. $R_i = 0$



ideal source voltage

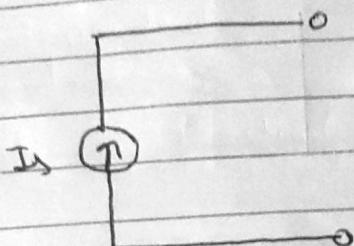


- ② Practical \rightarrow terminal voltage decrease with current taken.



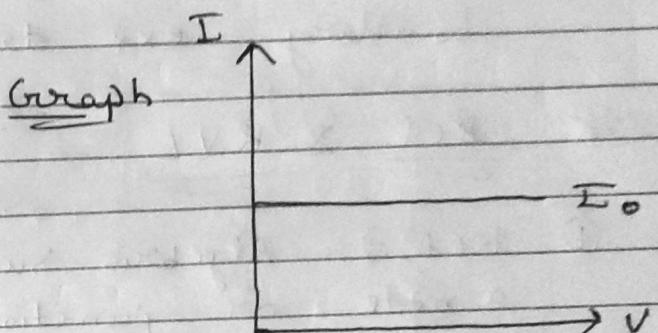
Independent Current source

- ① Ideal \rightarrow delivers constant current irrespective of its voltage across terminal

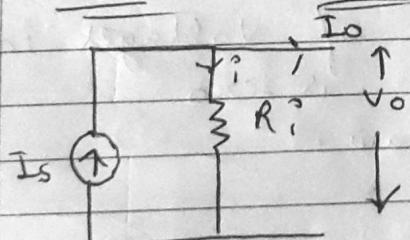


$R_i^o = \infty$ replaced by open ckt

$$I_o = I_s$$



- ② Practical current source \rightarrow I_o decrease with increasing Voltage.



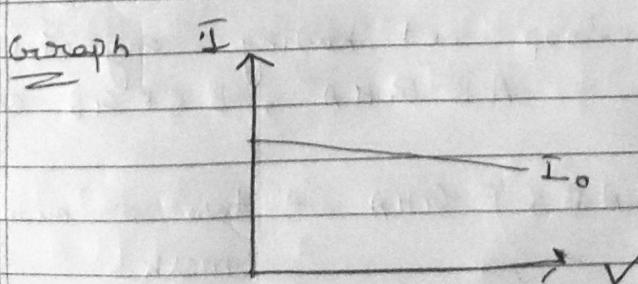
i \rightarrow internal current

KCL

$$I_s = i + I_o$$

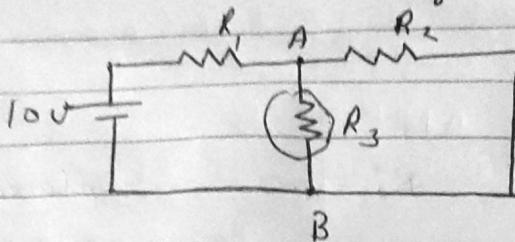
$$I_o = I_s - i$$

$$I_o = I_s - \frac{V_o}{R_i^o}$$



* A (ii) \rightarrow lumped and distributed circuit

(i) lumped \rightarrow are those where the physical boundaries are specified.



(ii) distributed \rightarrow cannot specify the physical boundary ex \Rightarrow transmission line.

* KCL & KVL

(1) KCL \rightarrow Algebraic sum of current across a node or junction is zero

$$\text{Sum of incoming current} = \text{Sum of outgoing current}$$

(Node or junction)

(2) KVL \rightarrow Algebraic sum of voltages in a closed loop or mesh is zero.

Energy conservation law.
(Mesh or closed loop)

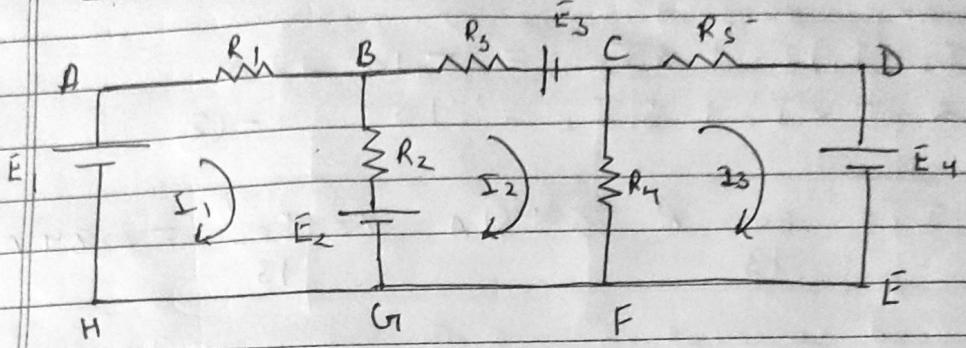
Mesh \rightarrow smallest loop not having any closed loop in it. ABGHA, BCFGGB etc

Closed loop \rightarrow ABCFGHA \rightarrow two or more mesh

Example

Numerically

(Mesh Analysis)



Mesh A B G H A :-

$$I_1 R_1 + (I_1 - I_2) R_2 = -E_2 + E_1$$

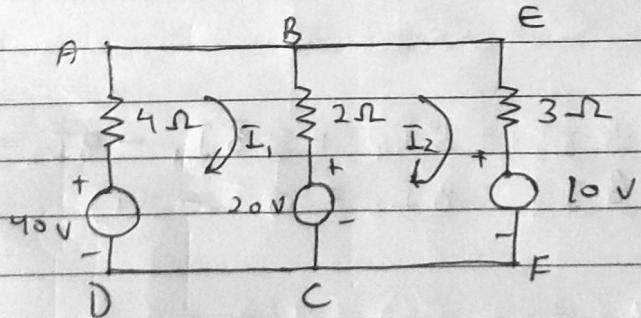
Mesh B C F G B :-

$$I_2 R_3 + (I_2 - I_1) R_2 + (I_2 - I_3) R_4 = -E_3 + E_2$$

Mesh C D E F G

$$I_3 R_5 + (I_3 - I_2) R_4 = -E_4$$

★



find current
in 2 Ω

Mesh A B C D A

$$4I_1 + (I_1 - I_2) 2\Omega = -20 + 40$$

$$4I_1 + 2I_1 - 2I_2 = 20$$

$$\therefore 6I_1 - 2I_2 = 20 \quad \text{--- (1)}$$

Mesh B E F C B

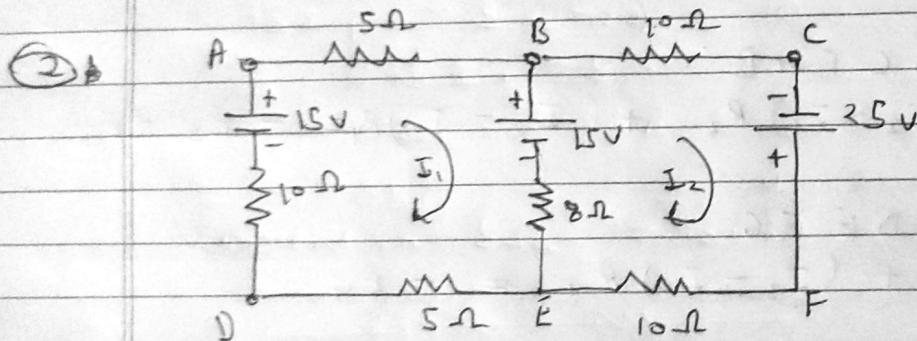
$$(I_2 - I_1)2 + 3I_2 = -10 + 20$$

$$\therefore -2I_1 + 5I_2 = 10 \quad \text{--- (2)}$$

$$I_1 = \frac{60}{13} A = 4.61 A \quad V = \frac{50}{13} = 3.84 A$$

$$\text{Current in } 2\Omega = I_1 - I_2$$

$$= 0.769 A \text{ B to C}$$



Mesh A B E D A

$$10I_1 + 5I_1 + 5I_1 + 8I_1 - 8I_2 = 15 - 0 \quad \text{--- (1)}$$

$$28I_1 - 8I_2 = 0 \quad \text{--- (1)}$$

Mesh B C F E B

$$8I_2 - 8I_1 + 10I_2 + 10I_2 = 15V + 25V$$

$$-8I_1 + 28I_2 = 40 \quad \text{--- (2)}$$

$$I_1 = \cancel{-2I_1} \quad 4/9$$

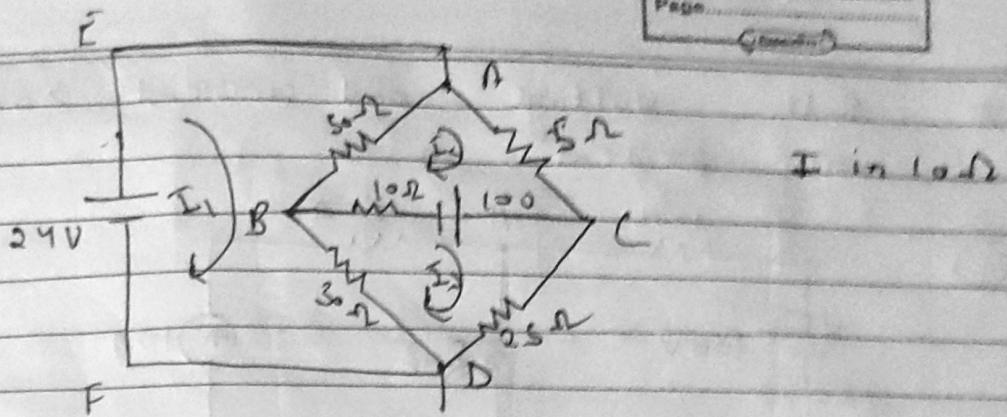
$$= 0.445 A$$

$$I_{12} \text{ in } 8\Omega = I_2 - I_1 = 1.55 - 0.445$$

$$I_2 = \cancel{50/13} \quad 14/9$$

$$= 1.55 A$$

$$= 0.967 A \quad \text{BE}$$



Meth EABDEE

$$(I_1 - I_2)5\Omega + (I_1 - I_3)3\Omega = 24$$

$$5\Omega I_1 - 5\Omega I_2 + 3\Omega I_1 - 3\Omega I_3 = 24$$

$$8\Omega I_1 - 8\Omega I_2 = 24 \quad 5\Omega I_2 - 3\Omega I_3 = 24 \quad (1)$$

Meth ACBA

$$(I_2 - I_1)$$

$$5\Omega(I_2 - I_1) + 5I_2 + 10I_2 = -10V$$

$$-5\Omega I_1 + 65I_2 - 10I_3 = -10 \quad (2)$$

Meth BCDAB

$$10(I_3 - I_2) + 30(I_3 - I_1) + 25I_3 = +10$$

$$10I_3 - 10I_2 + 30I_3 - 30I_1 + 25I_3 = 10$$

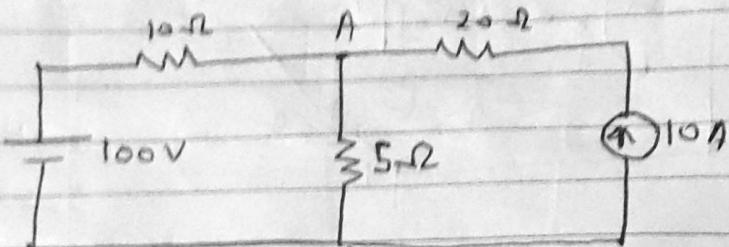
$$-30I_1 - 10I_2 + 65I_3 = 10 \quad (3)$$

$$I_1 = 1.11 \quad I_2 = 0.82 \quad I_3 = 0.794$$

$$\text{in } 10\Omega = I_2 - I_3 = 0.825 - 0.794 \\ = 0.031 A$$

* Both voltage and current source are present.

(1)



find current in 5Ω by mesh analysis.

$$10I_1 + 5(I_1 - I_2) = 100$$

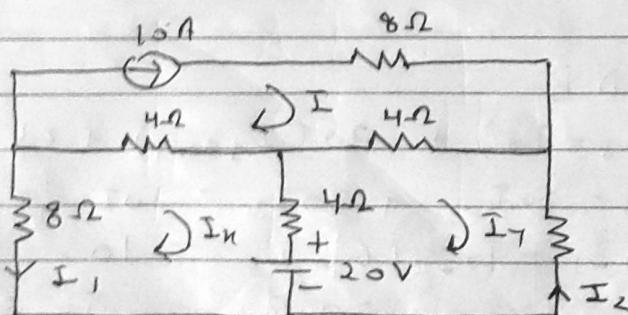
$$15I_1 - 5(-10) = 100$$

$$15I_1 = 50$$

$$I_1 = 3.33 \text{ A}$$

Current in $5\Omega = 13.33 \text{ A}$ to B

(2)



$$I = 10 \text{ A}$$

$$8I_n + 4(I_n - I_x) + 4(I_n - I_y) = -20$$

$$16I_n - 4I_y - 40 = -20$$

$$16I_y - 4I_y = 20$$

$$4(I_y - I) + 8I_y + 4(I_y - I_n) = 20$$

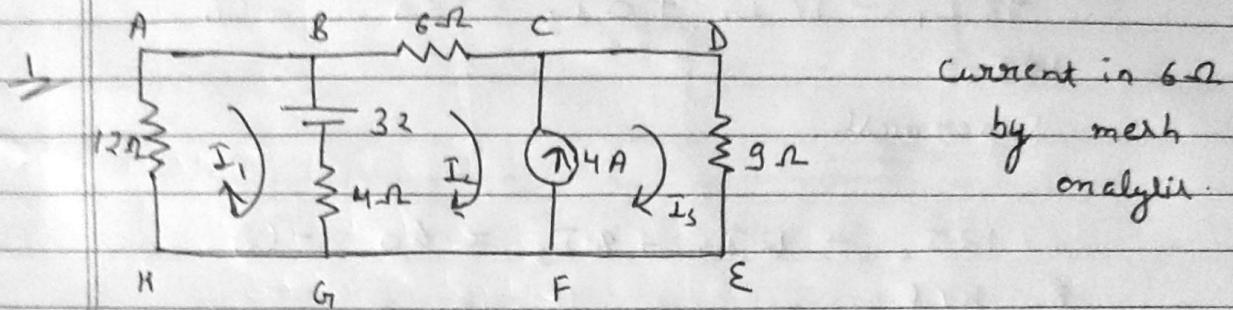
$$-4I_n + 16I_y = 60$$

$$I_n = 2.333 \text{ A} \quad I_y = 4.33 \text{ A}$$

$$\text{Current } I_1 = -2.333$$

$$I_2 = -4.333$$

* Super mesh analysis for common current source



Current in 6Ω

by mesh analysis.

By KVL in ABGHA

$$+10I_3 + 16I_1 - 4I_2 = -32$$

In Supermesh BCD EFGB

$$6I_2 + 9I_3 + 4(I_2 - I_1) = 32$$

$$-4I_1 + 6I_2 + 9I_3 = 32$$

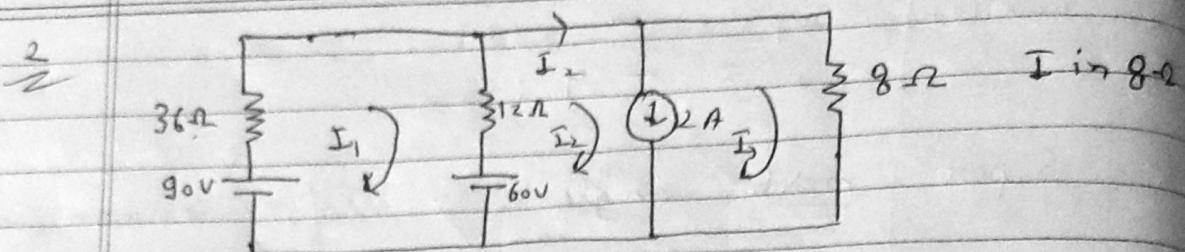
By KCL at Node C

$$I_2 + 4 = I_3 \quad \text{--- (3)}$$

$$0I_1 + I_2 + I_3 = -4$$

$$I_1 = -2.166A \quad I_2 = -0.666A \quad I_3 = 3.33A$$

$$\text{Current in } 6\Omega = 0.666 \text{ A} - 3$$



$$48I_1 - 12I_2 + 0I_3 = 30 \quad \text{---(1)}$$

~~1st~~Supermesh

$$12I_2 - 12I_1 + 8I_3 = 60 \quad \text{---(2)}$$

By KCL:

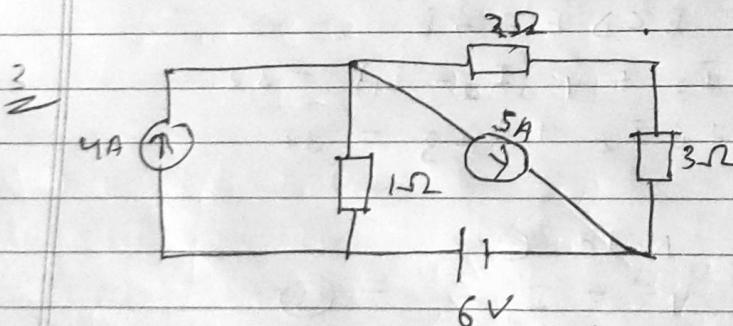
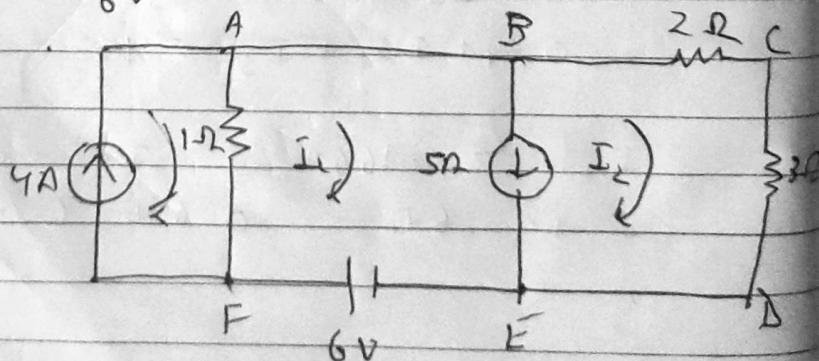
$$I_2 - 2 = I_3$$

$$0I_1 + I_2 - I_3 = 2 \quad \text{---(3)}$$

$$I_1 = 1.85A$$

$$I_2 = 4.91A$$

$$\underline{I_3 = 2.91A}$$

Simplified

$$1(I_1 - 4) + 2I_2 + 3I_3 = 6$$

$$I_1 + 5I_2 = 10 \quad \text{---(1)}$$

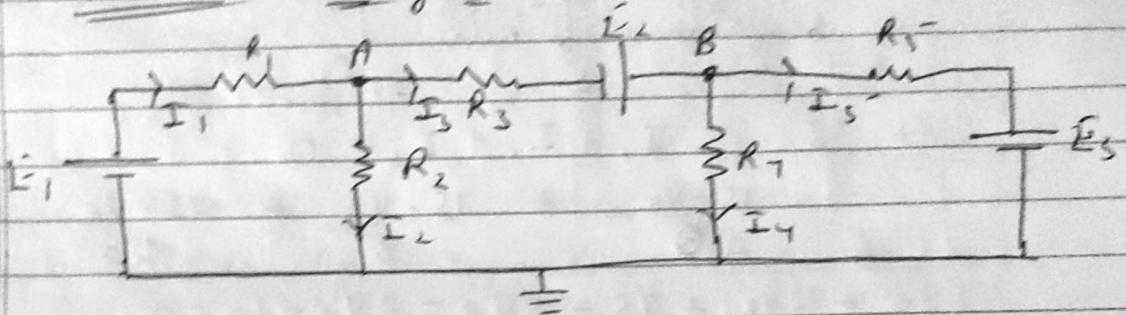
By KCL at B

$$I_1 = 5 + I_2 \quad \text{--- (1)}$$

$$I_1 = 5 - 833 \quad I_2 = 0.833$$

$$V = I_2 R = 0.833 \times 3 = \underline{\underline{2.499 \text{ V}}}$$

Nodal analysis



Nodes \rightarrow 3 or more elements are connected.

Applying KCL at A

$$I_1 = I_2 + I_3$$

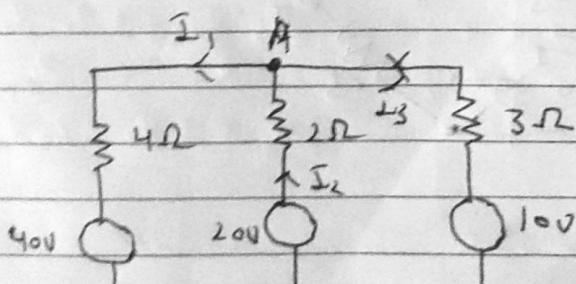
At B

$$I_3 = I_4 + I_5$$

$$0 + \frac{E_1 - V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A - V_B + E_2}{R_3}$$

$$\text{for } B \quad \frac{V_A + E_2 - V_B}{R_3} = \frac{V_B}{R_4} + \frac{V_B - E_3}{R_5}$$

*



By KCL

$$I_2 = I_1 + I_3$$

$$20 - V_A = V_A - 4 + \frac{2}{4}$$

$$\frac{V_B - 1}{3}$$

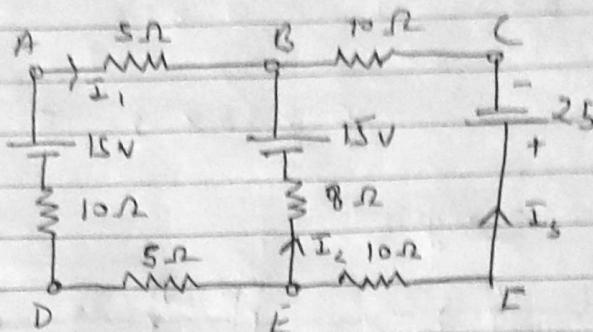
$$12 - 6V_A = 3V_A - 12 + 4V_A - 4$$

$$\therefore V_A = 28 \quad V_A = 21.588 \text{ Volt}$$

$$I_2 = \frac{20 - 21 - 538}{2} = -0.763 A$$

Copper
diameter

★



At B $I_1 + I_2 + I_3 = 0$

$$\frac{15 - V_B}{2} + \frac{15 - V_B}{8} + \frac{-25 - V_B}{20}$$

$$30 - 2V_B + 75 - 5V_B - 2V_B - 50 = 0$$

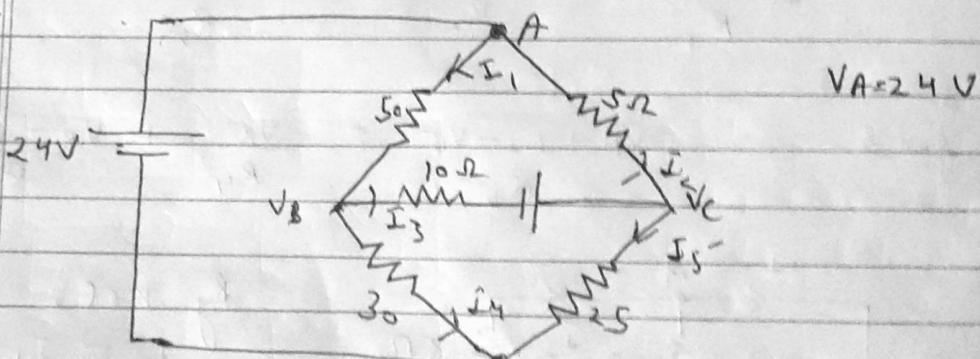
$$9V_B = 55$$

$$V_B = 6.111 \text{ Volt}$$

I in 8Ω

$$I_2 = \frac{15 - 6.111}{8} = 1.111 A \rightarrow A \rightarrow B$$

★



at B

$$I_1 = I_4 + I_3$$

$$\frac{24 - V_B}{5} = \frac{V_B}{30} + \frac{V_B + 10 - V_C}{10}$$

$$72 - 3V_B = 5V_B + 15V_C + 150 - 15V_C$$

$$23V_B - 15V_C = -78 \quad - (1)$$

A + C $I_3 + I_2 = I_1$

$$\frac{V_B + 10 - V_C}{10} + \frac{24 - V_C}{5} = \frac{V_C}{25}$$

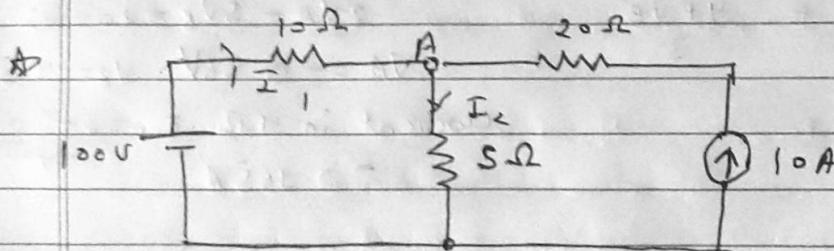
$$5V_B + 50 - 5V_C + 240 - 12V_C = 2V_C$$

$$5V_B - 17V_C = -290 \quad - (2)$$

$$V_B = 9.569V \quad V_C = 19.873V$$

$$I \text{ in } 10\Omega \quad \text{in} \quad I_3 = \frac{V_B + 10 - V_C}{10} = -0.030A$$

C-to-B



A + A $I_1 + 10 = I_2$

$$\frac{100 - V_A}{10} + 10 = \frac{V_A}{5}$$

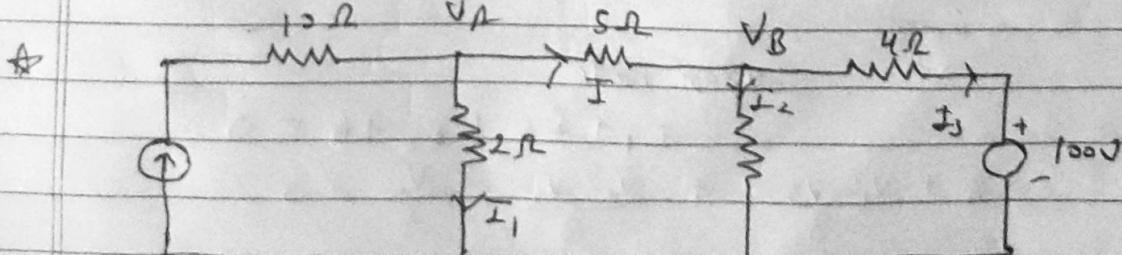
$$100 + V_A + 100 = 2V_A$$

$$3V_A = 200$$

$$V_A = 66.66V$$

$$I \text{ in } 5\Omega$$

$$I_2 = \frac{V_A}{5} = 13.33$$



A + A

$$2 = I_1 + I$$

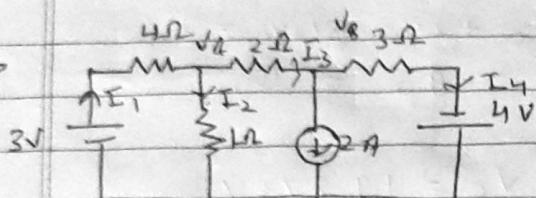
$$2 = \frac{V_A}{V_2} + \frac{V_A - V_B}{5} \quad \text{--- (1)}$$

A + B

$$I = I_2 + I_3$$

$$\frac{V_A - V_B}{5} = \frac{V_A}{2} + \frac{V_B - 10}{4} \quad \text{--- (2)}$$

★



At node A

$$I_1 = I_2 + I_3$$

$$\frac{3 - V_A}{4} = \frac{V_A}{2} + \frac{V_A - V_B}{2}$$

A + B

$$I_3 = 2 + I_4$$

$$\frac{V_A - V_B}{2} = 2 + \frac{4 + V_B}{3}$$

$$7V_A - 2V_B = 3 \quad \text{--- (1)}$$

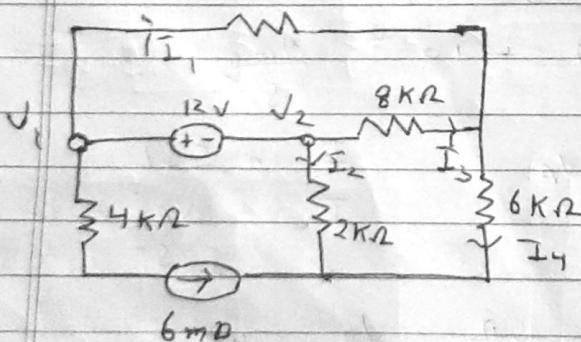
$$3V_A - 5V_B = 20 \quad \text{--- (2)}$$

$$V_A = -0.86V \quad V_B = -4.57V$$

$$\text{current in } 4\Omega \quad I_1 = \frac{3 - (-0.86)}{4}$$

$$= 0.965A$$

★

Super node

Node 3

$$I_1 + I_3 = I_4$$

$$\frac{V_1 - V_3}{8} + \frac{V_2 - V_3}{8} = V_3$$

$$3V_1 + 3V_2 - 10V_3 = 0 \quad \text{(1)}$$

Super node

$$I_1 + I_3 + I_2 + 6 = 0$$

$$6 + \frac{V_1 - V_3}{8} + \frac{V_2 - V_3}{8} + \frac{V_2}{2} = 0$$

$$48 + V_1 - V_3 + V_2 - V_3 + 4V_2 = 0$$

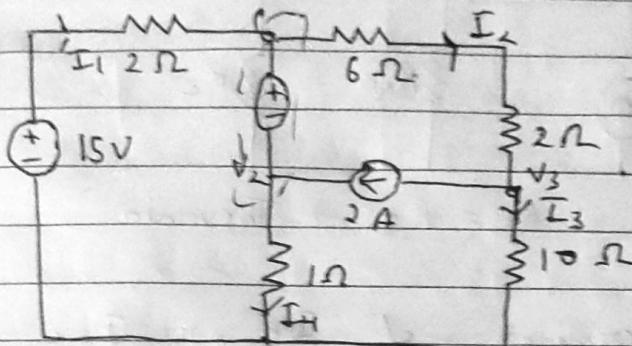
$$V_1 + 5V_2 - 2V_3 = -48 \quad (3)$$

KVL in supernode $V_1 = V_2 + 12$

$$V_1 = 1 \quad V_2 = -11 \quad V_3 = -3$$

$$I_{in} 2\Omega K\Omega = \frac{V_2}{2} = -11/2 = -5.5mA$$

★



At Supernode

$$I_1 + I_2 = I_2 + I_4$$

$$\frac{15 - V_1}{2} + I_2 = \frac{V_1 - V_3 + V_2}{8}$$

$$5V_1 + 8V_2 - V_3 = 76 \quad (3)$$

At (3)

$$I_2 = I_3 + I_4$$

$$\frac{V_1 - V_3}{8} = 2 + \frac{V_3}{10}$$

$$5V_1 + 0V_2 - 9V_3 = 80 \quad (3)$$

$$V_1 - V_2 + 0V_3 = 30 \quad (3)$$

$$V_1 = 24.678V$$

$$V_2 = -5.321V$$

$$V_3 = 4.821V$$

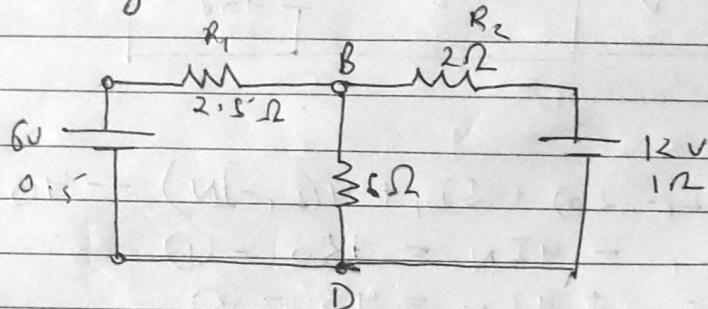
$$I_{in} 1\Omega = 5.321A$$

PTO

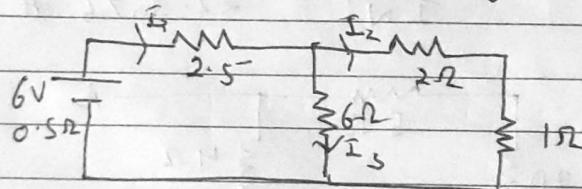
Superposition Theorem

linear active

In any linear bilateral network consisting of two or more sources the current flowing in any branch is algebraic sum of currents that would flow in the same direction branch when each current source is considered separately with all other sources replaced by internal resistance.



Step I considering 6 V



At B

$$I_1' = I_2' + I_3'$$

$$\frac{6 - V_B}{3} = \frac{V_B}{3} + \frac{V_B}{6}$$

$$I_1' = 1.2 \text{ A} \quad A \rightarrow B$$

$$12 - 2V_B = 2V_B + V_B$$

$$I_2' = 0.8 \text{ A} \quad B \rightarrow C$$

$$V_B = 2.4 \text{ Volts}$$

$$I_3' = 0.4 \text{ A} \quad B \rightarrow D$$

$$I = \frac{\text{Total incoming current}}{R_{\text{in Parallel Sum}}} \times R_{\text{of branch}}$$

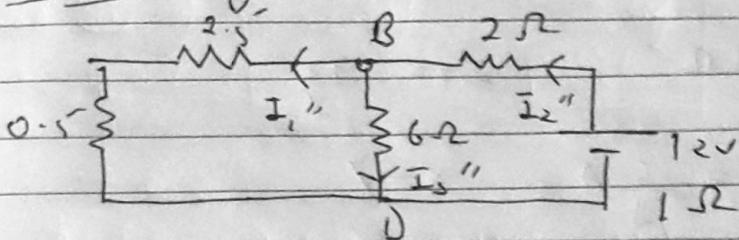
$$R_T = 2.5 + \frac{3 \times 6}{9} + 0.5 = 5 \Omega$$

$$I_1' = \frac{6}{5} = 1.2 \text{ A}$$

$$I_2' = \frac{I_1'}{6+3} \times 6 = \frac{1.2}{9} \times 6 = 0.8$$

$$I_3' = \frac{1.2}{6+3} \times 3 = 0.4 \text{ A}$$

Considering $\underline{12V}$



A + B

$$\underline{I_2''} = \underline{I_1''} + \underline{I_3''}$$

$$\frac{12 - V_B}{3} = \frac{V_B}{3} + \frac{V_B}{6}$$

$$V_B = \frac{27}{3} = 4.8 \text{ Volt}$$

$$I_1'' = 1.6 \text{ A} \quad B \rightarrow A$$

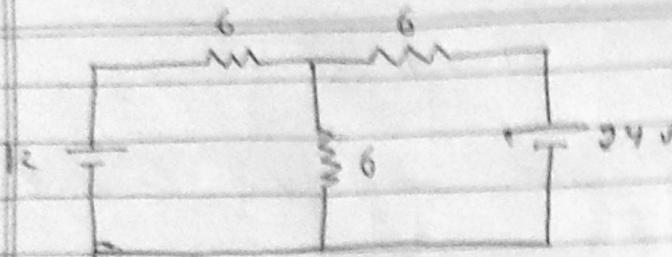
$$I_2'' = 2.4 \text{ A} \quad C \rightarrow B$$

$$I_3'' = 0.8 \text{ A} \quad B \rightarrow D$$

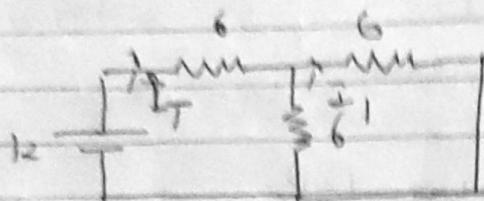
Step 3 Net current $I_1 = \underline{I_1'} + \underline{I_1''}$
 $= -1.2 + 1.6 = 0.4$
 B to A

Net $I_2 = \underline{I_2'} + \underline{I_2''}$
 $= -0.8 + 2.4 = 1.6$ C to B

Net $I_3 = \underline{I_3'} + \underline{I_3''} = 0.4 + 0.8$
 $= 1.2 \text{ A}$ B to D



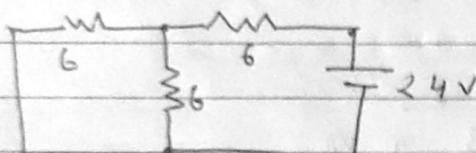
for 12V



$$R_T = 9 \Omega$$

$$I_T = \frac{12}{9}$$

$$I_1 = \frac{6}{9} A \quad X - >$$

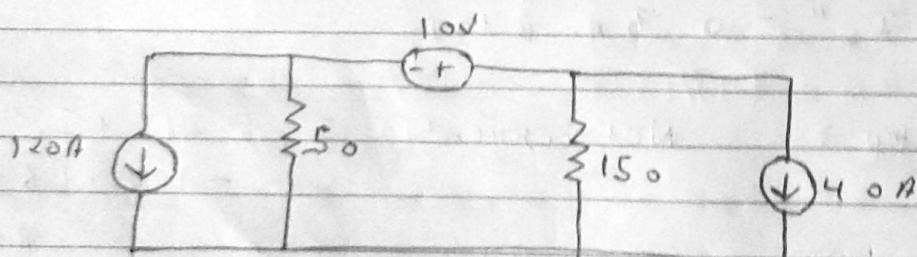
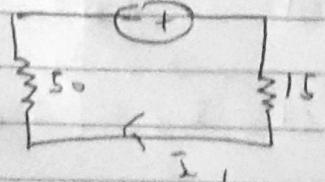


$$R_T = 9 \Omega$$

$$I_2 = I_T = \frac{24}{9} \quad Y - 1.0 A$$

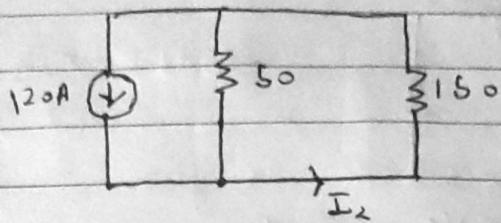
$$\text{Total} = I_1 + I_2 = \frac{6}{9} - \frac{24}{9}$$

$$= -2A$$

Considering 10V

$$I_1 = \frac{10}{20} A$$

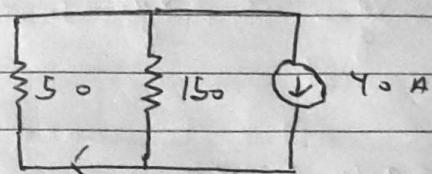
Considering 120A source



$$I_L = \frac{120}{20} \times 50$$

$$= I_L = -30A$$

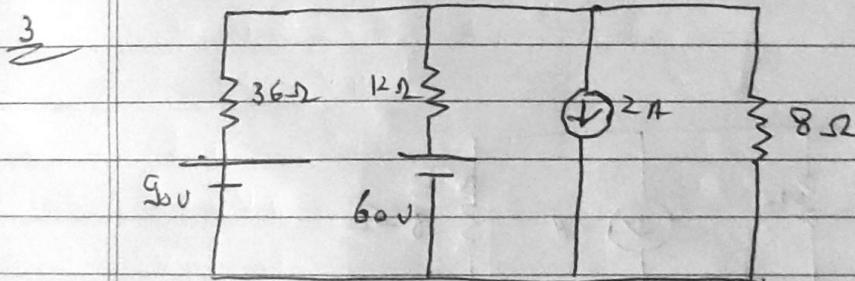
considering 40A



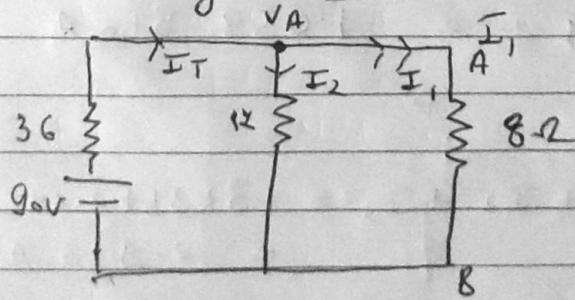
$$I_3 = \frac{20^2}{40} \times 15$$

$$= 30A$$

Net current = $I_1 + I_L + I_3 = \frac{10}{200} A$



Considering 90V



At Node $I_T = I_2 + I_1$

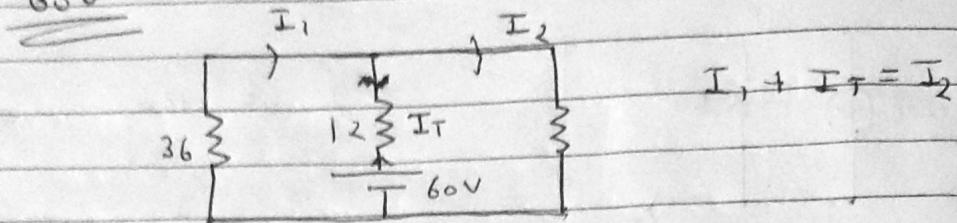
$$\frac{90 - V_A}{36} = \frac{V_A}{12} + \frac{V_A}{8}$$

$$180 - 2V_A = 6V_A + 9V_A$$

$$17V_A = 180$$

$$V_A = 10.588$$

$$I_1 = 1.3235 A \text{ and } I_T = 1.3235 A + 0.588 A = 1.9115 A$$

60V

$$\frac{0 - V_A}{36} + \frac{60 - V_A}{12} = \frac{V_A}{8}$$

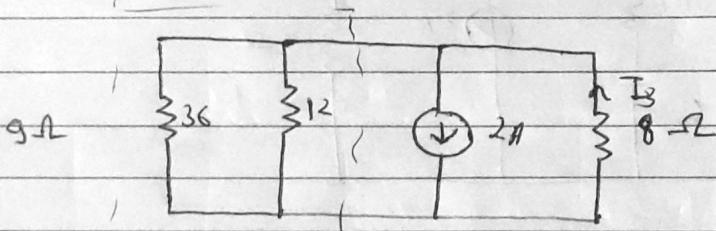
$$\frac{60 - V_A}{12} = \frac{V_A}{8} + \frac{V_A}{8}$$

$$360 - 6V_A = 9V_A + 2V_A$$

$$17V_A = 360$$

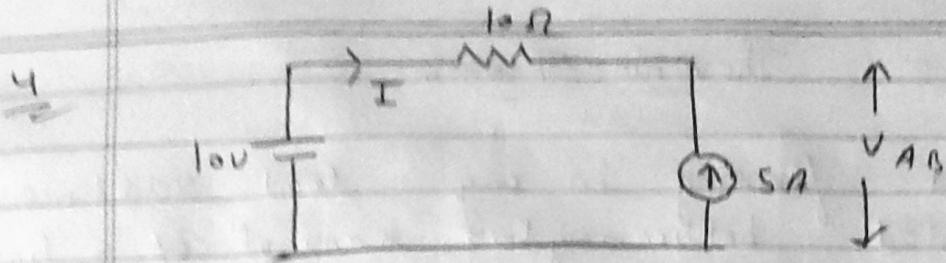
$$V_A = 21.176V$$

$$I_2 = 2.647A \quad A - B$$

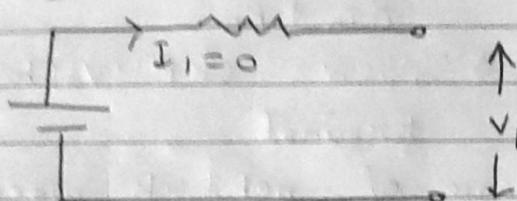
for 2A

$$I_3 = \frac{2}{(9+8)} \times 9 = 1.058 \text{ A}$$

$$\text{Total} = I_1 + I_2 + I_3 = 1.3235 + 2.647 - 1.058 \\ = 2.910 \text{ A}$$

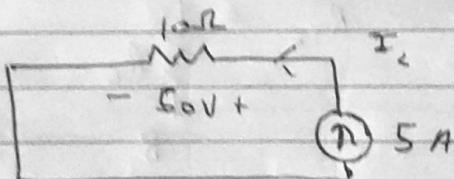


for 10V



$$V_1 = 10V$$

for 5A



$$I_2 = -5A$$

$$\xrightarrow{\text{Net}} \text{Net } I = I_1 + I_2 = -5A$$

$$\xrightarrow{\text{Net } V} \text{Net } V \Rightarrow V_{AB} = V_1 + V_2 = 10 + 5 = 60V$$

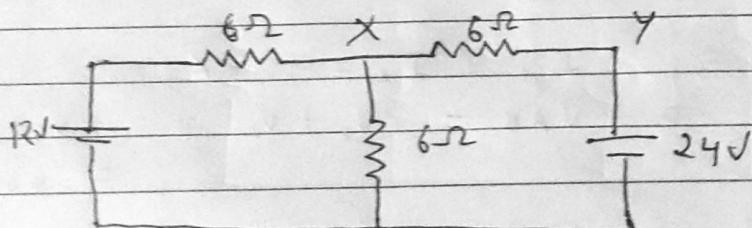
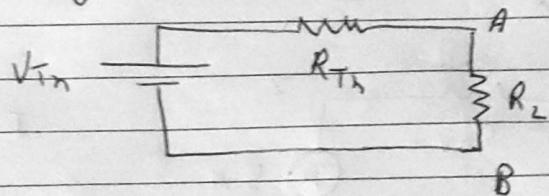
Thevenin Theorem

Current flowing in any load resistance R_L connected between any two terminal of a linear active bilateral network is given by

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

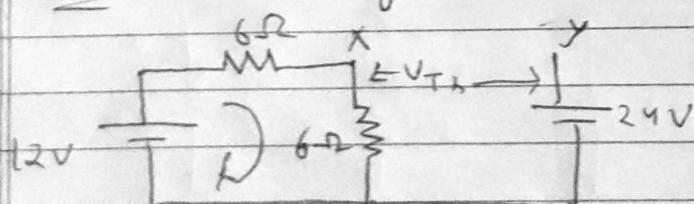
where $V_{Th} \rightarrow$ open ckt voltage across terminals

$R_{Th} \rightarrow$ Resistance of network when looked back into the network with all sources are replaced by their internal resistance



Step I Det of V_{Th}

(load open)



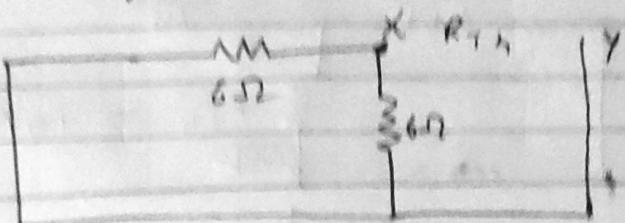
$$V_{Th} \text{ in } 6\Omega = 6 \times 1 = 6V$$

$$V_{Th} = 6 - 24 = 18V$$

II

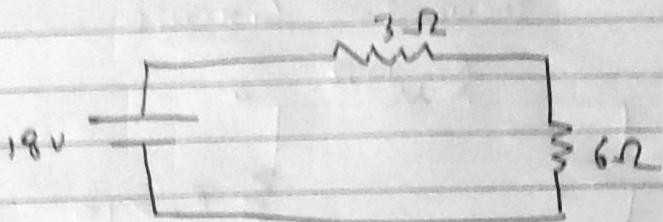
Det. of R_{Th}

3 Ward batteries



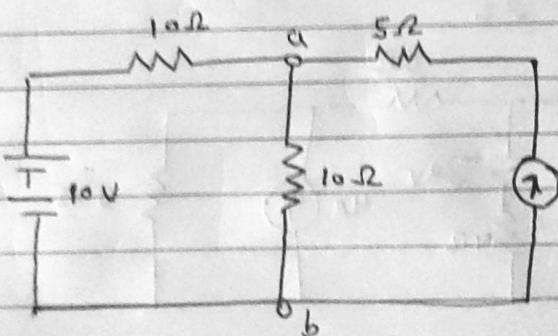
$$R_{Th} = 6 \parallel 6 = 3\Omega$$

III

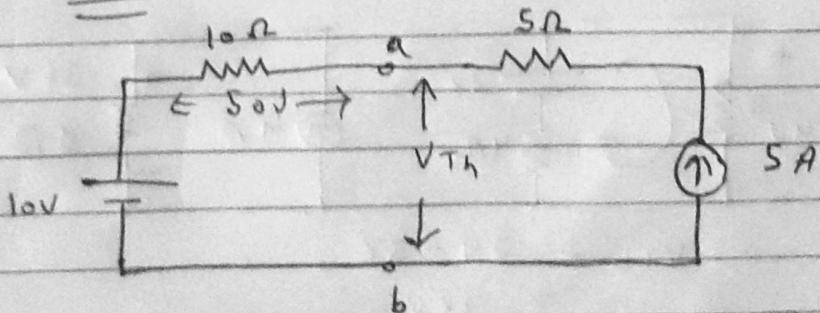


$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{2A}{3\Omega + 6\Omega}$$

IV

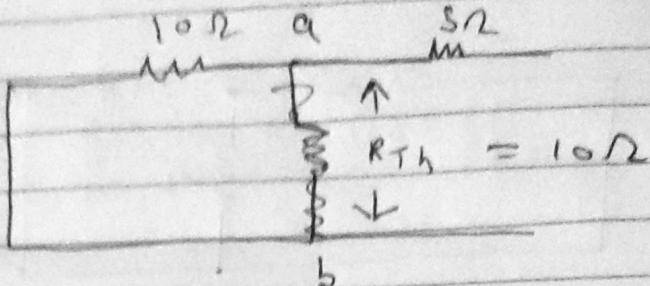


(C)

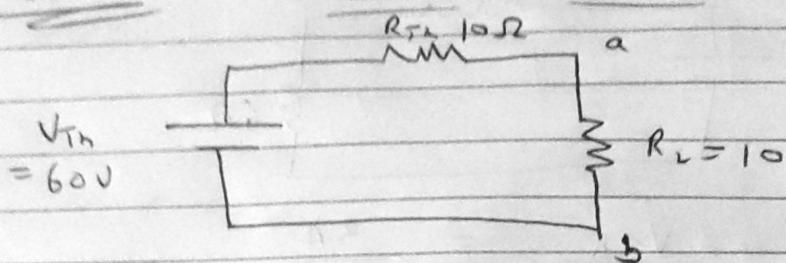
Det. V_{Th} 

$$V_{Th} = 10 + 5 = 60V$$

II

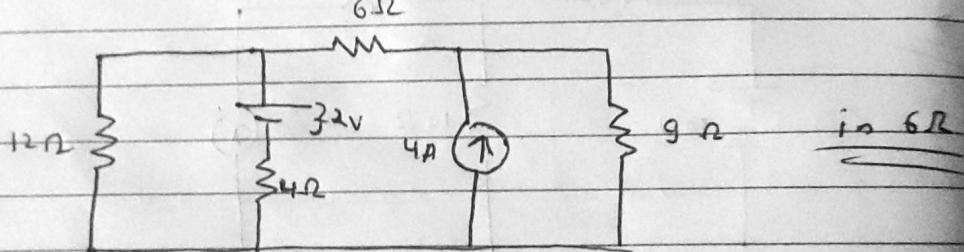
 R_{Th} 

III

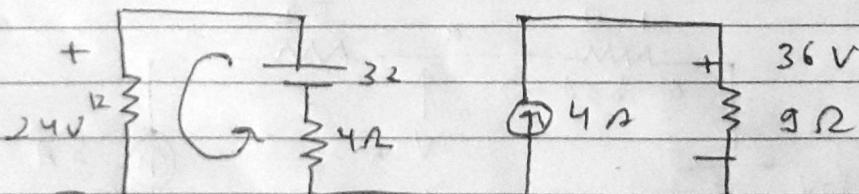
DrawThévenin Circuit

$$I_L = \frac{60}{20} = 3 \text{ A}$$

3



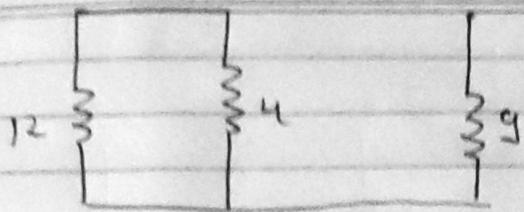
I

Find V_{Th} 

$$V_{Th} = 24 - 36$$

$$= -12 \text{ V}$$

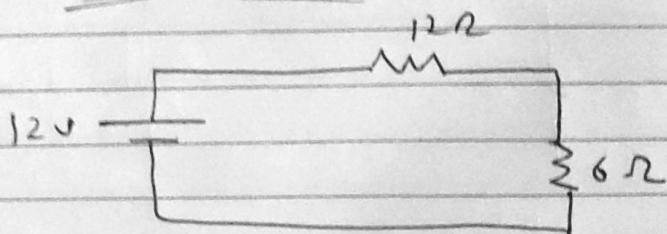
II



$$R_{Th} = \frac{(2 \times 4)}{12 + 4} \times 9 = 12\Omega$$

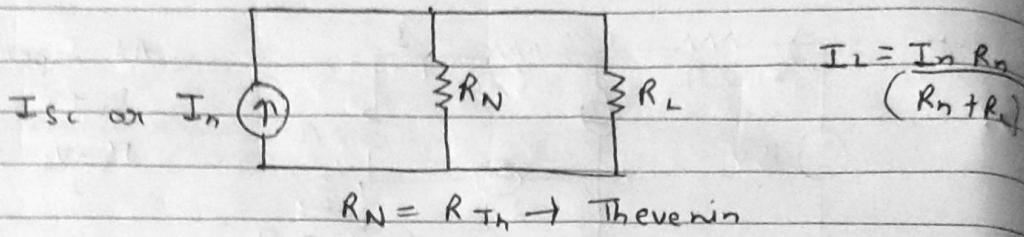
III

Draw Thevenin



$$I_L = \frac{12}{18} = 0.666A \quad B + 0A$$

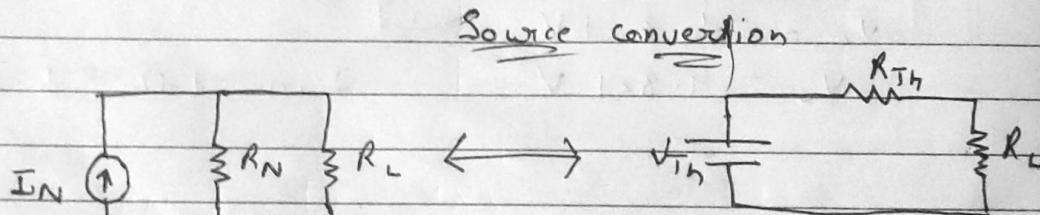
★ Norton's Theorem



Step 1 \rightarrow determination of I_n or I_{sc} short ckt.

Step 2 \rightarrow determination of R_n

Step 3 \rightarrow draw Norton Ckt.

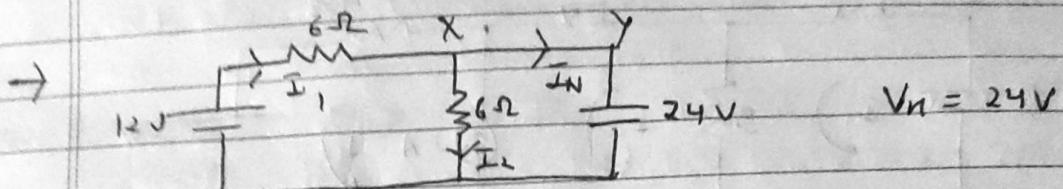
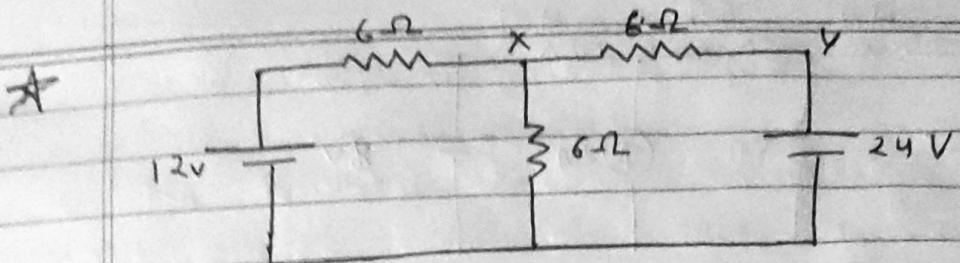


$$I_{in} = \frac{V_{Th}}{R_{Th}}$$

$$R_n = R_{Th}$$

$$V_{Th} = I_n R_N$$

$$R_N = R_{Th}$$



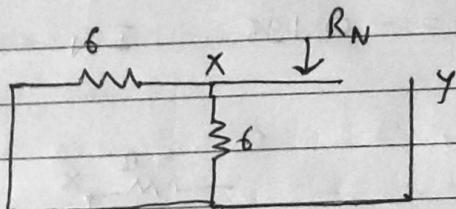
$$I_1 = I_2 + I_N$$

$$\frac{12 - V_n}{6} = \frac{V_n}{6} + I_N$$

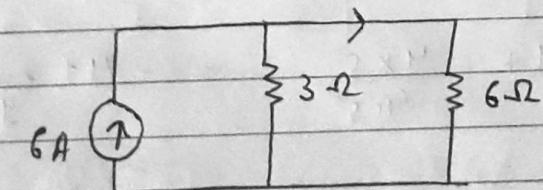
$$-2 = 4 + I_N$$

$$I_N = -6 \text{ A}$$

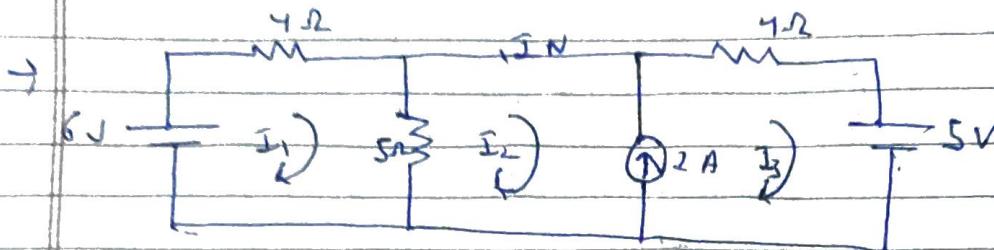
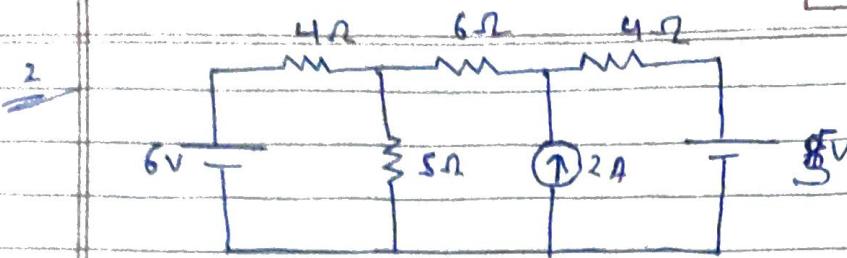
Det of $\underline{\underline{R_N}}$



$$R_N = 6 // 6 = 3 \Omega$$



$$I_L = 2 \text{ A}$$



$$9I_1 - 5I_2 + 0I_3 = 6 \quad (1)$$

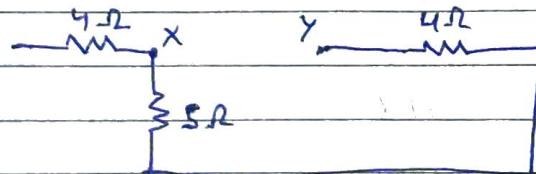
$$-5I_1 + 5I_N + 4I_3 = -5 \quad (2)$$

$$0I_1 + I_N - I_3 = -2 \quad (3)$$

$$I_1 = -0.196 \quad I_N = -1.55 \quad I_3 = 0.446$$

Ytux

(ii) R_N



$$R_N = 4 + \frac{4 \times 5}{4 + 5} = 4 + \frac{20}{9} = 6.222\Omega$$

