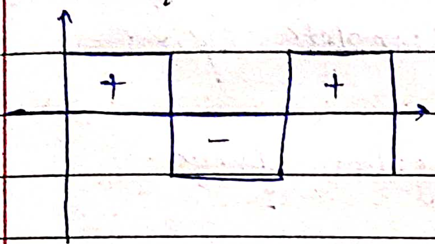
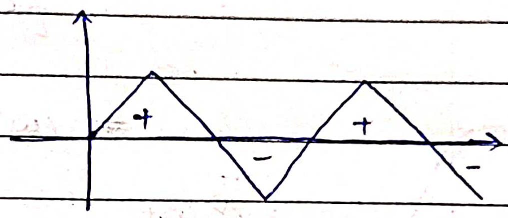
Module - 2 AC Fundamentals

"AC Quantity" The Quantity like (V, I) whose magnitude are directional change w.r.t (Polarity) time.

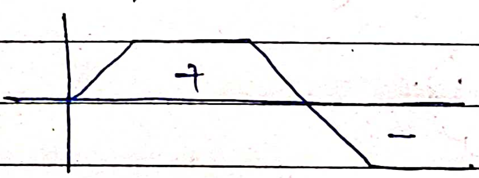
Square Wave



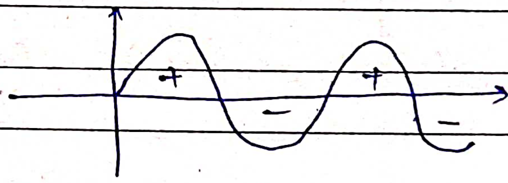
Triangular Wave



Trapazoidal waves

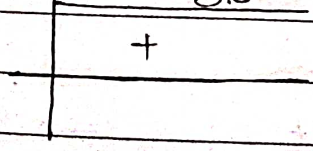


Sin Waves

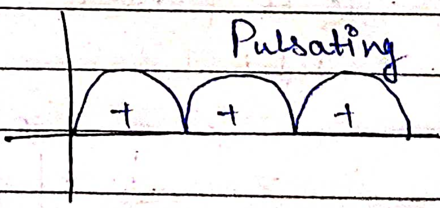


"DC Quantity" whose magnitude may or maynot change, DC Quantity is defined with one which may change its magnitude but does not change the dirⁿ"

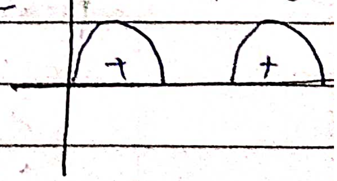
Pure D.C.



Pulsating D.C.

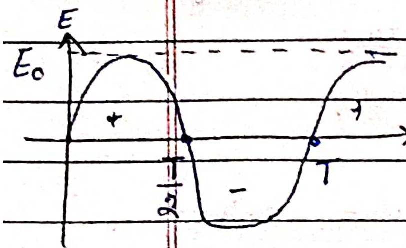


H.W. Rectifi.



The Shape of Curve of V, I when plotted against time as abssicca called waveform.

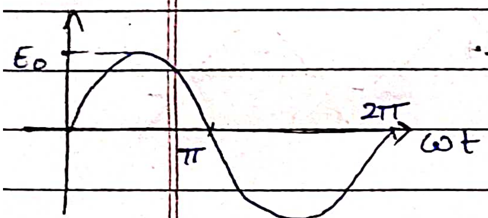
→ alternating voltage and Current means sinusoidal voltage and current otherwise Started



$e = e_0 \sin \omega t$ Instantanious Eqn

Amplitude → Max (+) or (-)ve Value

(T) Time period → Time taken by AC Quantity for one Complete Cycle.



(ω) → Angular freqn.

Frequency (f) → Cycle / sec

→ 50 Hz ⇒ 50 Cycle / sec

$$\boxed{T = \frac{1}{f}}$$

Average value → Average value of A.C. is equal to value of Direct Current which transfers accross any circuit. the same charge as it tranfered by A.C.

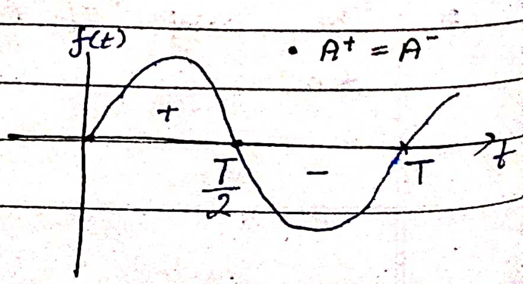
→ let f(t) is a periodic fxn with fondamental period = (T)

$$f_{avg} = \frac{1}{T} \int_0^T f(t)\,dt$$

Waveforms →

a) Symmetrical Waveforms →

$\boxed{F_{avg} = 0}$



□ Symm → avg, rms → half Cycle
Unsym → " " → full "

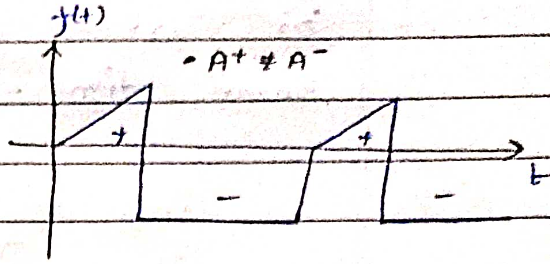
→ We always Consider "half Cycle" for Calculating F_{avg} in "Symmetrical" Waveform.

b) Unsymmetrical Waveform →

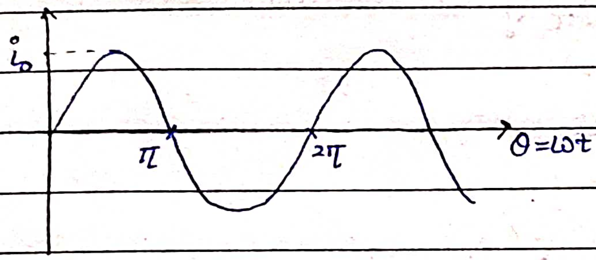
$$\boxed{F_{avg} = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\,d\theta}$$

'OR'

$$\boxed{F_{avg} = \frac{1}{T}\int_0^{T} f(t)\,dt}$$



Sinusoidal Quantity



$$i = i_0 \sin(\omega t)$$

$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i\,d\theta$$

$$i_{avg} = -\frac{i_0}{2\pi}\left[\cos\omega t\right]_0^{2\pi}$$

$$= -\frac{i_0}{2\pi}[1-1]$$

$$\boxed{i_{avg} = 0}$$

⇒ for Half Cycle →

$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\,d\omega t$$

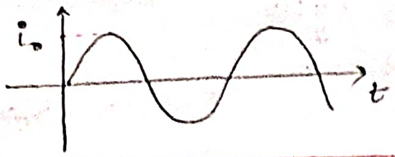
$$i_{avg} = \frac{2 i_0}{\pi} = 0.636\, i_0$$

→ R.M.S. value (root mean square value) →
(Heat Dependent)

(Effective Value)

let $f(t)$ is a periodic fxn with Time period = T

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^{T} f^2(t)\,dt}$$



Full $i_{avg} = 0$ Half $i_{avg} = \frac{2i_o}{\pi}$

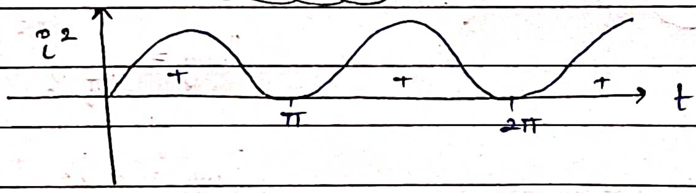
RMS → The rms value is that value of current which produces same amount of heat for a given time as when alternating voltage when or current applied to some circuit for same time.

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d\omega t}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} (i_o \sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{i_o^2}{2\pi}\int_0^{T} (1-\cos 2\omega t)\, d\omega t}$$

$$i_{rms} = \frac{i_o}{\sqrt{2}}$$



Form factor (K_f) = $\frac{\text{rms value}}{\text{average value}}$

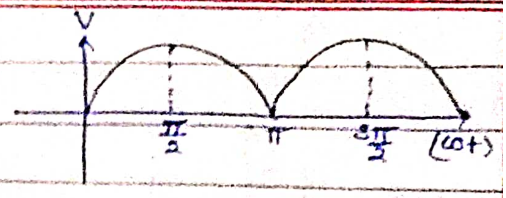
Peak factor (K_p) = $\frac{\text{Peak value}}{\text{rms value}}$

Sinusoidal →

$$K_f = \frac{e_o/\sqrt{2}}{2e_o/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$K_p = \frac{e_o}{e_o/\sqrt{2}} = \sqrt{2} = 1.414$$

Full wave rectifier Output →



$V = |V_0 \sin\omega t|$

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V\, d\omega t$$

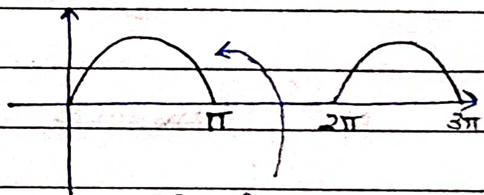
$$\boxed{V_{avg} = \frac{2V_0}{\pi}}$$

& $$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V^2\, d\omega t}$$

$$= \sqrt{\frac{1}{\pi}\left(\frac{V_0^2}{2}\right)\int_0^{\pi}(1-\cos 2\omega t)\, d\omega t}$$

$$\boxed{V_{rms} = \frac{V_0}{\sqrt{2}}}$$

Half Wave rectifier Output



$i = i_0 \sin\omega t$

$$i_{avg} = \frac{1}{2\pi}\int_0^{\pi} i\, d\omega t + \int_{\pi}^{2\pi} 0 \cdot d\omega t$$

$$\boxed{i_{avg} = \frac{i_0}{\pi}}$$

& $$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi}(i_0\sin\omega t)^2 d\omega t + \int_{\pi}^{2\pi}(0)\, d\omega t}$$

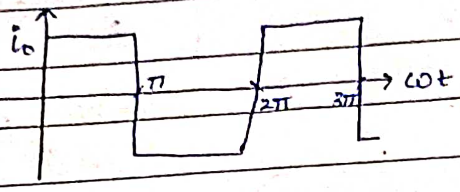
$$= \sqrt{\frac{i_0^2}{4\pi}\left[\omega t - \frac{\sin\omega t}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{i_0^2}{4\pi} \times \pi}$$

$$\boxed{i_{rms} = \frac{i_0}{2}}$$

$$K_p = \frac{i_0}{i_0/2} = 2 \qquad K_f = \frac{i_0/2}{i_0/\pi} = \frac{\pi}{2} = 1.57$$

'Square Wave'



$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d\omega t$

$= \frac{1}{\pi}\int_0^{\pi} i_0\, d\omega t$

$= \frac{i_0}{\pi}\left[\omega t\right]_0^{\pi}$

$= \frac{i_0}{\pi} \times \pi$

$= i_0$

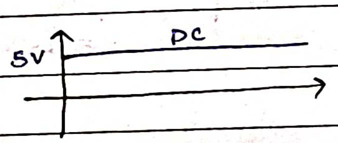
$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d\omega t}$

$= \sqrt{\frac{i_0^2}{\pi}\left[\omega t\right]_0^{\pi}}$

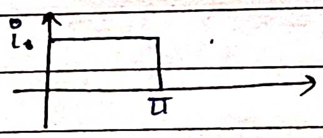
$= i_0$

$K_f = 1$ $\quad K_p = 1$

(c) Constant Waveform → avg = rms value



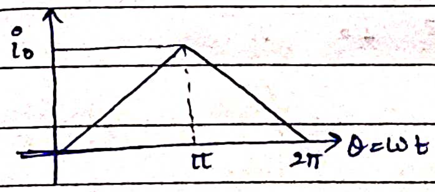
avg = 5 = rms



$avg = \frac{area}{length} = \frac{\pi \times i_0}{\pi} = i_0$

d) Triangular Waveform →

$m = \frac{i_0}{\pi}$



→ let us assume it is symmetrical waveform →
→ half Cycle (2π)
→ Quater Cycle (π)

$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d\omega t$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{i_0}{\pi}\,\theta\, d\theta$$

$$= \frac{1}{\pi^2}\, i_0 \left[\theta^2/2\right]_0^{\pi}$$

$$i_{avg} = i_0/2$$

& $$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d\theta} = \sqrt{\frac{1}{\pi}\times\frac{i_0^2}{\pi^2}\int_0^{\pi}\theta^2\, d\theta}$$

$$i_{rms} = \frac{i_0}{\sqrt{3}}$$

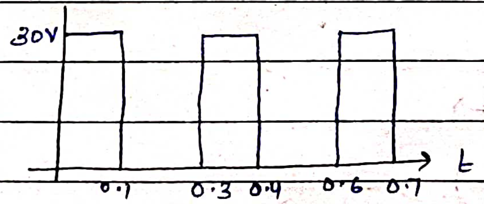
$$K_f = \frac{i_0/\sqrt{3}}{i_0/2} = \frac{2}{\sqrt{3}} = 1.15$$

$$K_p = \frac{i_0}{i_0/\sqrt{3}} = \sqrt{3} = 1.732$$

→ ∵ Area $= i_0\pi$, length $= 2\pi$

$$avg = \frac{i_0\pi}{2\pi} = \frac{i_0}{2}$$

Qus:



$$V_{avg} = \frac{1}{0.3}\int_0^{0.3} f(t)\,dt$$

$$= \frac{30\times 0.1}{0.3}$$

$$= 10$$

$$V_{rms} = \sqrt{\frac{1}{0.3}\int_0^{0.1}(30)^2\,dt}$$

$$= \sqrt{\frac{30\times 30\times 0.1}{0.3}}$$

$$= \sqrt{300} = 10\sqrt{3} = 17.32$$

$$K_f = \frac{10\sqrt{3}}{10} = 1.732$$

$$K_p = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

Page No.:
Date: / /

Waveform	rms	avg	K_f	K_p
Sin wave	$i_0/\sqrt{2}$	$2i_0/\pi$	1.11	1.41
full wave rec.	$i_0/\sqrt{2}$	$2i_0/\pi$	1.11	1.41
Half wave rec.	$i_0/2$	i_0/π	1.57	2
Square wave	i_0	i_0	1	1
Triangular	$i_0/\sqrt{3}$	$i_0/\sqrt{2}$	$2/\sqrt{3}$	$\sqrt{3}$

Qus# A sinusoidally varying alternating current of frequency 60 Hz with Max value of 15 amp.

i) What is instantanious Eqⁿ
ii) Find value of Sinusoidal after $\frac{1}{200}$ sec
iii) Find time taken to reach Current $i = 10$ amp.
iv) Find the average.

i) $i = 15 \sin 120\pi t$

ii) $i = 15 \sin 120\pi \times \frac{1}{200} = 15 \sin \frac{3\pi}{5} = 14.27$

iii) $i = 15 \sin 120\pi t$

$10 = 15 \sin (120\pi t)$

$t = \frac{1}{120\pi} \sin^{-1}(10/15)$

(iv) $avg = \frac{2i_0}{\pi} = \frac{2 \times 15}{3.14} = 9.54$

Qus: # An AC Current has rms value of 40 amp and 50 Hz freq. write instantanious eqⁿ and find its value 2×10^{-3} sec after passing through max (+) value

$i_{rms} = 40A$

$= i_0/\sqrt{2} \Rightarrow i_0 = 40\sqrt{2}$

$\therefore \; i = 40\sqrt{2}\, \sin(100\pi t)$

$f = 50\ Hz \Rightarrow T = \frac{1}{f} = 0.02\ sec$

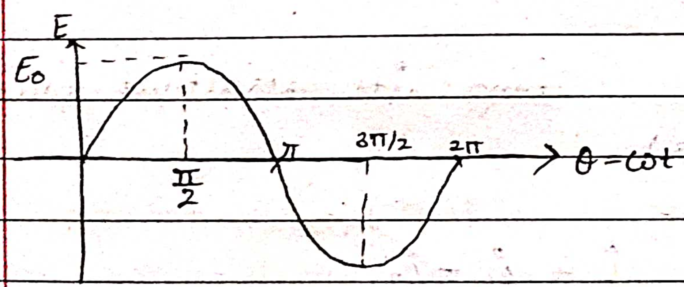
$\Rightarrow$ Time to Complet half cycle = 10 mili sec
" " have Max value = 5 mili sec

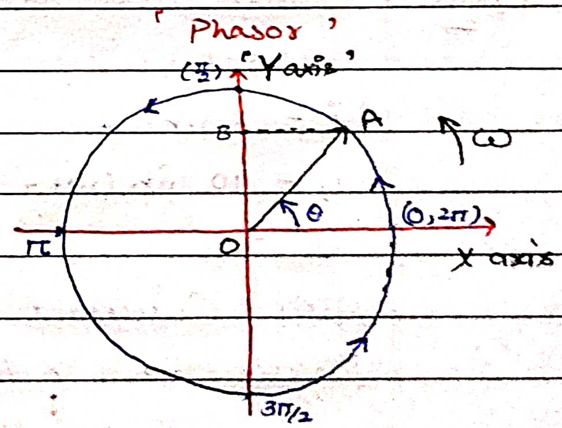
$\Rightarrow t = 5\times10^{-3} + 2\times10^{-3} = 7\times10^{-3}$

So $i = 40\sqrt{2}\, \sin(100\pi \times 7\times10^{-3})$
$= 45.76$ Amp.

$\Rightarrow$ Phaser representation of AC Quantity

For the Soln of AC problems it is good to represent a sinusoidal Quantity either V or I by the line of definite length rotating Counter Clockwise dirn with same angular velocity as that of sinusoidal Quantity Such a rotating line is Called phasor.





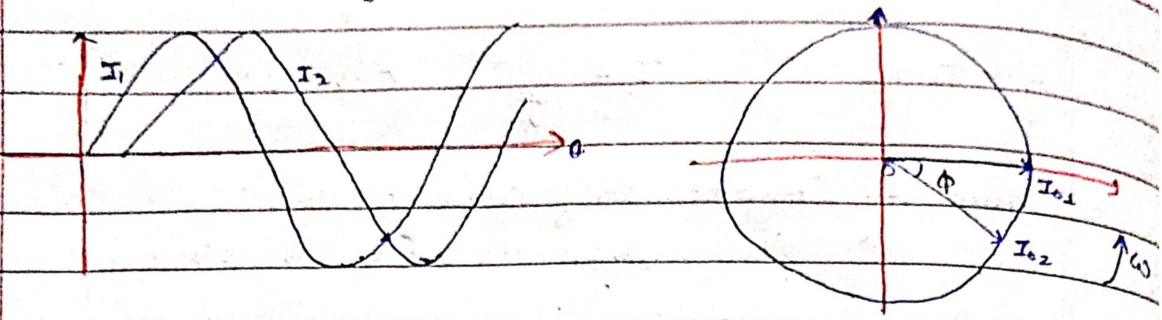
$OB = OA \sin\theta$

$\boxed{E = E_0 \sin\omega t}$

- Phasor is always drawn from phasor value.

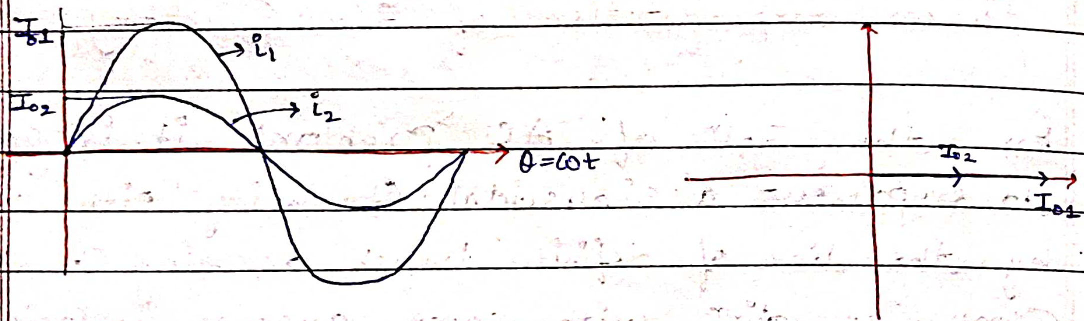
Page No.:
Date: / /

I_1 lead I_2 by angle ϕ →



⇒ ϕ ⇒ Constant

I_1 and I_2 are in same phase →



Imp

Ques: Two alternating Quantities

$i_1 = 7 \sin \omega t$ find its resultant Current
$i_2 = 10 \sin(\omega t + \pi/3)$

A.C. Current $i = i_1 + i_2$

Phasor → $r \angle \theta$ ⇒ (Polar form) $= r(\cos\theta + j\sin\theta)$
↓ Magnit. Angle

$=$ $a + j(b)$ ⇒ (Rectangular form) ; $j = \sqrt{-1}$
↓ Real ↓ Imaginary

$\overline{I_1}$ = Phase of i_1

$= \frac{7}{\sqrt{2}} \angle 0° = 4.94 \angle 0°$

$= 4.94(\cos 0° + j \sin 0°)$

$\overline{I_2}$ = Phase of i_2

$= \frac{10}{\sqrt{2}} \angle 60^\circ = 7.07 (\cos 60 + j \sin 60)$

$= (3.535) + j(6.1226)$

$\overline{I} = \overline{I_1} + \overline{I_2}$

$= 4.94 + 3.535 + j(6.1226)$

$= 8.475 + j(6.1226)$ → Rect form

$= 10.455 \angle 35.84^\circ$ → Polar form

→ $I_o = (10.455)\sqrt{2} \sin(\omega t + 35.84^\circ)$

Ques: $V_1(t) = 30 \sin(314t + 45^\circ)$

$V_2(t) = 60 \sin(314t + 60^\circ)$

$\overline{V_1} = \frac{30}{\sqrt{2}} \angle 45^\circ = 14.99 + 14.99j$

$\approx 15 + 15j$

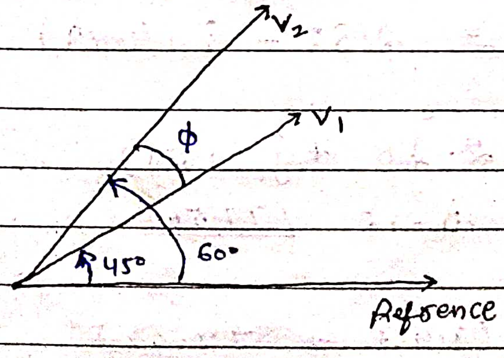
$\overline{V_2} = \frac{60}{\sqrt{2}} \angle 60^\circ = 21.21 + j(36.736)$

Inst $\overline{V} = \overline{V_1} + \overline{V_2} = (36.21) + j(51.736)$

$= 63.14 \angle 55^\circ$

$V = V_o \sin \omega t$

$= 63.14\sqrt{2} \sin(314t + 55^\circ)$



$\phi = 60^\circ - 45^\circ$

$= 15^\circ$

Page No.:
Date: / /

Ques:

$V_1 = 100 \sin 500t$

$V_2 = 200 \sin (500t + \pi/3)$

$V_3 = -50 \cos (500t) = 50 \sin (500t - \pi/2)$

$V_4 = 150 \sin (500t - \pi/4)$

Find rms of Resultant.

$\bar{V_1} = 70.7 \angle 0° = 70.7$

$\bar{V_2} = 141 \angle 60° = 70.5 + j(122.10)$

$\bar{V_3} = 35.35 \angle -90° = -j(35.35)$

$\bar{V_4} = 100.06 \angle -45° = 75 - j(75)$

$\bar{V} = 216.12 + j(12.14)$

$= 216.750 \angle 3.2°$

Resultant $V = 216.75\sqrt{2} \sin (500t + 3.2°)$

Ques:

$i_1 = 10 \sin (314t + \pi/4)$ find Phase diff. btw i_1 & i_2

$i_2 = 8 \sin (314t - \pi/3)$

Find difference of these Current.

$\bar{I_1} = \frac{10}{\sqrt{2}} \angle 45° = 5 + j(5)$

$\bar{I_2} = 5.65 \angle -60° = 2.82 - j(4.89)$

$I = \bar{I_1} - \bar{I_2} = 2.18 + j(9.89)$

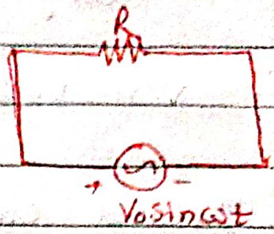
$= 10.12 \angle 77.56$

Ans ⇒ $I = 10.12\sqrt{2} \sin (314t + 77.62)$

⇒ & Phase diff. btw i_1 & i_2 $= \frac{\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{7\pi}{12}$

$(\phi)_{\text{Purely Resist.}} = 0$ btw V & I

Purely Resistive Circuits (lamps, bulbs)



(Circuit Diagram)

$\because$ Inst. power $\Rightarrow P = VI$

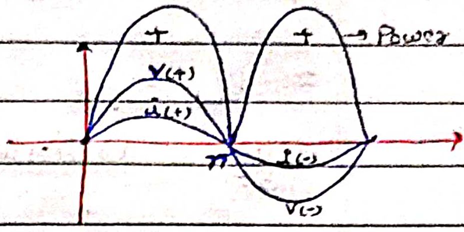
$= V_0 \sin\omega t \times I_0 \sin\omega t$

$= V_0 I_0 \sin^2\omega t$

$= \frac{V_0 I_0}{2} \left[2\sin^2\omega t\right]$

$= \frac{V_0 I_0}{2} \left[1 - \cos 2\omega t\right]$

$= \underbrace{\frac{V_0 I_0}{2}}_{\text{Const. Part}} - \underbrace{\frac{V_0 I_0}{2} \cos 2\omega t}_{\text{Pulsating Part}}$



Wave diagram

+ Ins. power in Purely Resistive circuit is pulsating at double freqn –

So we take avg. Power –

$P_{avg} = \frac{1}{\pi}\int_0^{\pi} p\, d\omega t = \frac{1}{\pi}\int_0^{\pi}\left[\frac{V_0 I_0}{2} - \frac{V_0 I_0}{2}\cos 2\omega t\right]$

$= \frac{1}{\pi}\frac{V_0 I_0}{2}\left[\omega t\right]_0^{\pi}$

$= \frac{V_0 I_0}{2}$

$= \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$

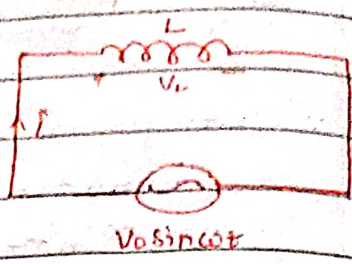
$$\boxed{P_{avg} = V_{rms} \cdot I_{rms}}$$

Power factor $\rightarrow$ It is the Cosine of angle btw V & I.

for Pur. Resi. $\Rightarrow \cos\phi = \cos 0^\circ$

$= \underline{1}$

Purely Inductive Circuit

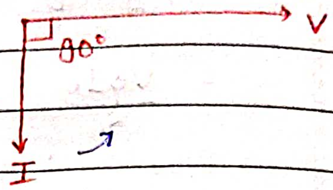


- $V = V_0 \sin\omega t$

& Induced emf $= L\dfrac{di}{dt} = V_L$

$\because V_L = V$

$L\dfrac{di}{dt} = V_0 \sin\omega t$



$\int di = \int \dfrac{V_0}{L} \sin\omega t \, dt + C$ } ; C is eventually zero so →

$i = \dfrac{V_0}{L}\left(-\dfrac{\cos\omega t}{\omega}\right)$

$= -\dfrac{V_0}{\omega L} \cos\omega t$

$i = \dfrac{V_0}{\omega L} \sin(\omega t - \pi/2)$

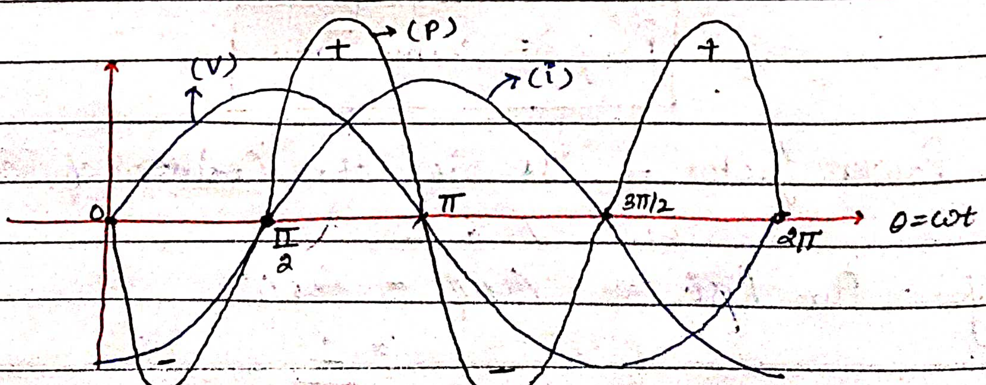
$i = i_0 \sin(\omega t - \pi/2)$

⇒ Current Lags behind Voltage by $\pi/2$ / $90°$.

⇒ $(\phi)_{\text{Purely Ind.}} = 90°$

& $\because \dfrac{V_0}{\omega L} = i_0$

⇒ $\omega L = \dfrac{V_0}{i_0} = X_L$



+ Power ⇒ Power Consumed by Inductor
− Power ⇒ Power Returned to Source

* In Pure inductive Circuit net Power Consumed by inductor = 0.

Inst. Power →

$$P = VI$$

$$= V_0 \sin \omega t \, I_0 \sin(\omega t - \pi/2)$$

$$= -V_0 I_0 \sin \omega t \sin(\pi/2 - \omega t)$$

$$= -V_0 I_0 \sin \omega t \cos \omega t$$

$$= \underbrace{-\frac{V_0 I_0}{2} \sin 2\omega t}_{\text{Pulsative in nature}}$$

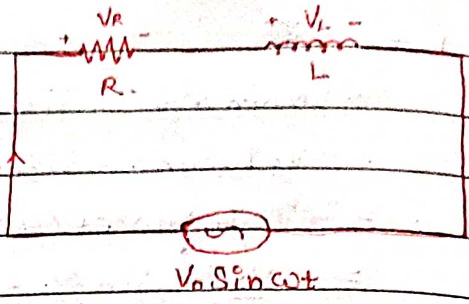
$$P_{avg} = \frac{1}{\pi} \int_0^{\pi} P \, d\omega t$$

$$= 0$$

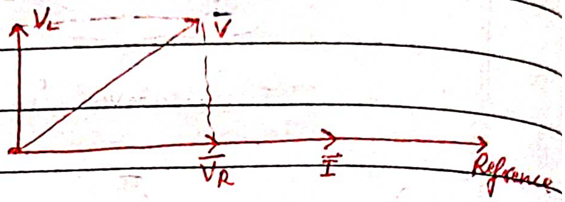
$Z \rightarrow$ impedence is like complex so use bar on it.

Page No.:
Date: / /

"Series R-L Circuit"



* In series circuit – Current is constant so it is usually taken as Refrence.



$\bar{V}$ = Supply Voltage
$= \bar{V}_R + \bar{V}_L$

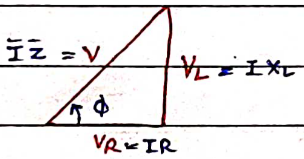
$$|\bar{V}|_{Magn.} = \sqrt{V_R^2 + V_L^2}$$

$$\bar{I}\bar{Z} = \bar{I}R + \bar{I}(jX_L)$$

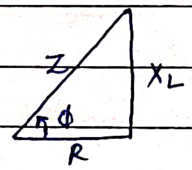
↳ impedence

$$\bar{Z} = R + jX_L \qquad |Z|_{mag} = \sqrt{R^2 + X_L^2}$$

★ Voltage Triangle — Impedence Triangle

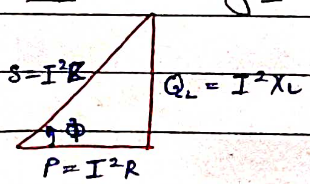


$\div I \rightarrow$



$\downarrow \times I$

Power Triangle



Power Factor
$$\cos\phi = R/Z$$
&
$$\cos\phi = P/S$$

Avg Power $= P =$ Active Power / Real Pow. / True Power $= I^2R$

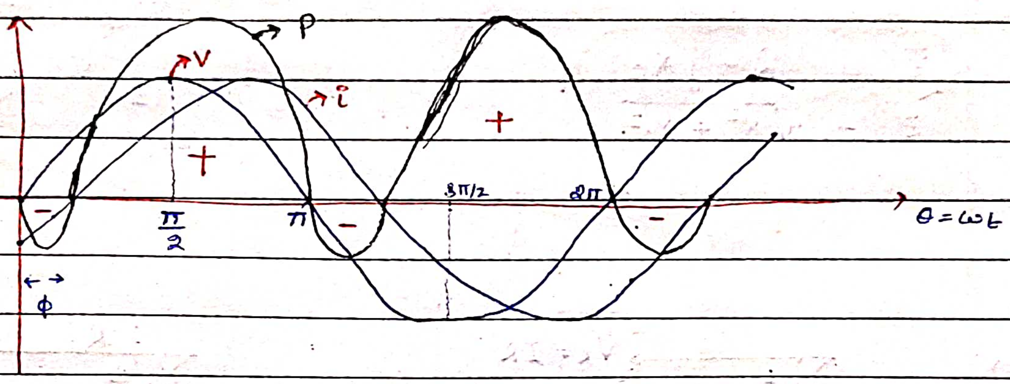
$$\boxed{P = I^2R = VI\cos\phi}$$

↳ Power Consumed in Resistant of the Circuit

Unit → watt, Kw., Mega watt

- Q_L = Reactive Power = $I^2 X_L = VI \sin\phi$
  ↳ Power Consumed in reactance of the Circuit
  Unit = VAR , KVAR , MVAR

- S = Apperent Power = $I^2 Z = VI$
  ↳ Power consumed by the impedence (z) of Circuit.
  Unit = VA , KVA



$P+ >> P-$

Instantanious Power →

$$P = V \times I$$
$$= V_0 \sin\omega t \times I_0 \sin(\omega t - \phi)$$
$$= \frac{V_0 I_0}{2}\left[2\sin\omega t \sin(\omega t - \phi)\right]$$
$$= \frac{V_0 I_0}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$
$$P = \underbrace{\frac{V_0 I_0}{2}\cos\phi}_{\text{Fixed Part}} - \underbrace{\frac{V_0 I_0}{2}\cos(2\omega t - \phi)}_{\text{Pulsating Part}}$$

Avg Power → $P_{avg} = \frac{1}{\pi}\int_0^{\pi} P \, d\omega t$

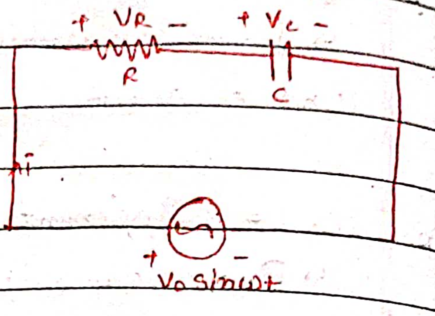
$$= \frac{V_0 I_0}{2}\cos\phi = V_{rms} I_{rms} \cos\phi$$

* R-L $\Rightarrow$ $Z = R + jX_L$
* R-C $\Rightarrow$ $Z = R - jX_C$

Page No.:
Date: / /

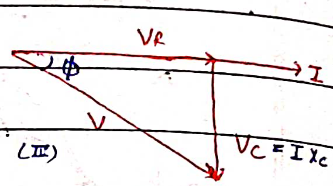
"R-C Series Circuit"

($\bar{I}$ leads $\bar{V}$ by ϕ)



$$\bar{V} = \bar{V}_R + \bar{V}_C$$

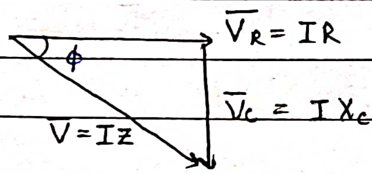
Mag $V = \sqrt{V_R^2 + V_C^2}$



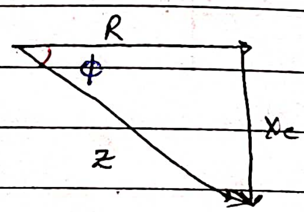
$$\bar{Z} = R - jX_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$\Rightarrow$



÷ I →

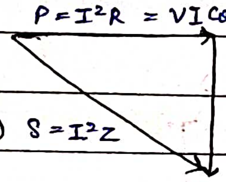


Impedence Triangle

$P_{avg} = VI\cos\phi$

$\downarrow \times I$

(True Power) $P = I^2R = VI\cos\phi$



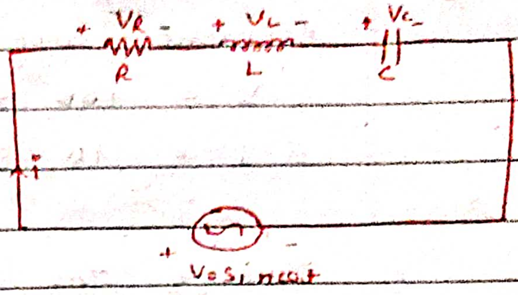
(Apperent Power) $S = I^2Z$

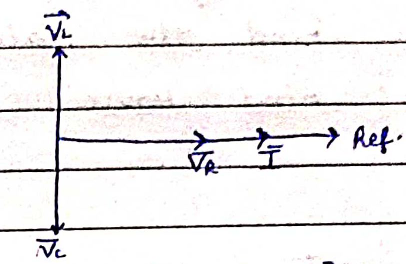
$Q_c = I^2X_C = VI\sin\phi$

↳ Reactive Power (Capacitive)

(Power Triangle)

"Series R-L-C Circuit"

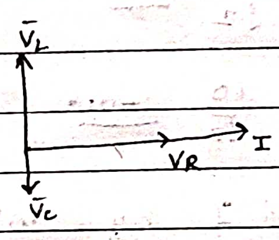




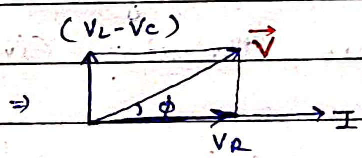
Phase Diagram

Case 1 : if $(X_L > X_C)$

$V_L > V_C$



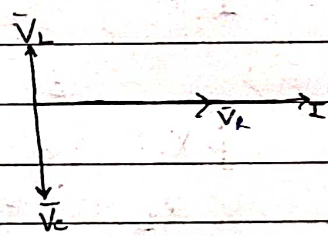
then it'll behave like R-L Circuit



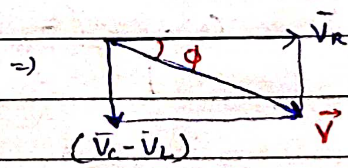
$\phi > 0$

Case 2 : if $(X_L < X_C)$

$V_L < V_C$



then it'll behave like R-C Circuit.



$\phi < 0$

Page No.:
Date: / /

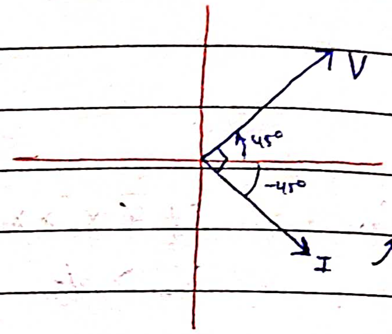
Ques: The Voltage and Current through a Circuit element are -

$$V = 100 \sin(314t + 45^\circ)$$
$$I = 10 \sin(314t + 315^\circ)$$

i) Identify the Circuit element
ii) Find the value of element
iii) Find Power Consumed

i) $\bar{V} = \frac{100}{\sqrt{2}} \angle 45^\circ$

$\bar{I} = \frac{10}{\sqrt{2}} \angle -45^\circ$



$\because$ V lead I by 90°
$\Rightarrow$ Pure Inductor Circuit

ii) $Z = \frac{V}{I} = \frac{100/\sqrt{2}\ (\angle 45^\circ)}{10/\sqrt{2}\ (\angle -45^\circ)}$

$= 10\ (\angle 90^\circ)$

$j X_L = j(10)$

$X_L = 10$

$\omega L = 10$

$L = \frac{10}{314} = 0.03$ Henry

iii) $P_{avg} = 0$

$P_{ins.} = -\frac{V_0 I_0}{2} \sin 2\omega t$

$= -500 \sin(628t)$

While addⁿ & Sub. use Rect. & by Mult. & Div. use Polar form → $\frac{e_1}{e_2} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$

Qus: Find Circuit element by given V & I

$V(t) = 9 \sin t$
$i(t) = 3 \cos t$

⇒ $\omega = 1$ rad/sec

$i(t) = 3 \sin(\omega t + \pi/2)$
$= 3 \sin(t + \pi/2)$ ∵ I leads V by $90°$ ⇒ Pure Capac.

Phasor →

$\bar{V} = \frac{9}{\sqrt{2}} \angle 0°$

$\bar{I} = \frac{3}{\sqrt{2}} \angle 90°$

$Z = \frac{\bar{V}}{\bar{I}} = 3 \angle -90°$
$= -j(3)$

$Z = -jX_c = -j(3)$

$X_c = 3$

$\frac{1}{\omega C} = 3$

$C = 1/3$ Farad

Qus: A 50 Hz 230 V is applied on 26.5 μF Capacitor -
i) write time expressions of volt Current.
ii) Find Max energy stored in Capacitor

⇒ $\omega = 2\pi f = 100\pi$
$V_{rms} = 230$ V
$C = 26.5 \times 10^{-6}$ F
$V = 230\sqrt{2} \sin(100\pi t)$

Page No.:
Date: / /

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 26.5 \times 10^{-6}}$$

$$= 120.1\ \Omega$$

$$\bar{I} = \frac{\bar{V}}{Z}$$

$$= \frac{230 \angle 0^\circ}{120.1 \angle -90^\circ}$$

$$= 1.9148 \angle 90^\circ$$

$$I = 1.9148\sqrt{2} \sin(100\pi t + \pi/2)$$

iii) $$W = \frac{1}{2} C V_0^2$$

$$= \frac{1}{2} \times 26.5 \times 10^{-6} (230\sqrt{2})^2$$

$$= 1.4\ J$$

Ques: $V = 200\sqrt{2} \cos 500t$

$P = 250\ W$

$pf = \cos\phi = 0.7$ (lag)

$Q = ?$

$P = VI\cos\phi$

$$I = \frac{P}{V\cos\phi} = \frac{250}{200 \times 0.7} = 1.7857\ \text{Amps}$$

$\because Q = VI\sin\phi$

$= 200 \times 1.7857 \sqrt{1-(0.7)^2}$

$= 255.03$

Ques: If we apply 10 volt dc to RL Circuit, we'll get 5 amp Current. Now if we apply 10V AC to the same RL Circuit, we will get 3 amp Current, find inductance of inductor. ($f = 50\,Hz$)

$\because \quad R = \frac{V}{I}$ for D.C.

$$R = \frac{10}{5} = 2\,\Omega$$

$Z = \frac{V}{I}$ for AC

$$\sqrt{R^2 + X_L^2} = \frac{10}{3}$$

$$X_L^2 = \frac{100}{9} - R^2 = \frac{100}{9} - 4$$

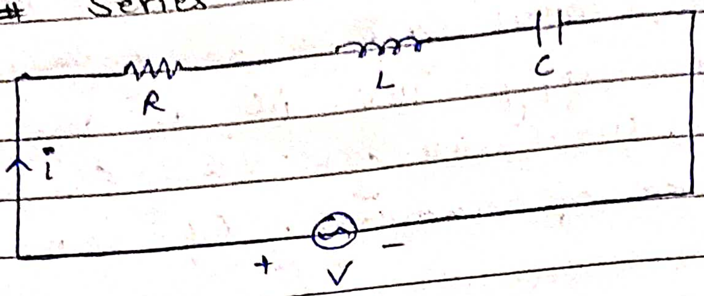
$$X_L = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

$$\omega L = \frac{8}{3}$$

$$L = \frac{8}{3} \times \frac{1}{2\pi f} = \frac{8}{3 \times 100\pi} = 8.4 \text{ m Henry}$$

(Voltage Resonance)

Series Resonance



$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z_{min} = R$

Conditions for Resonance

→ for Some freq^n $X_L = X_C$;

i) Net Reactance $X = X_L - X_C = 0$

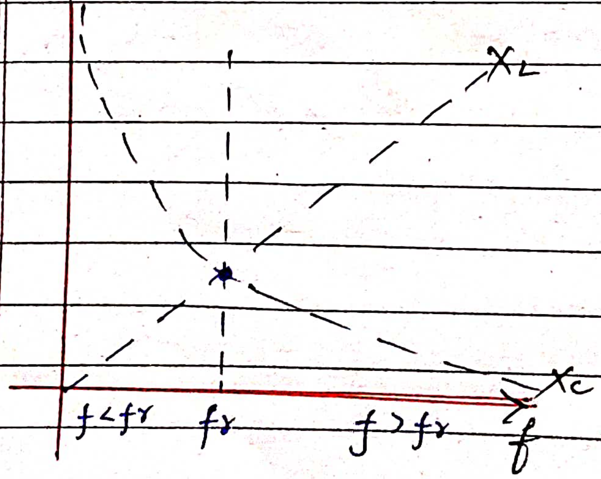
ii) impedence $Z = R$

iii) $I_{max} = V/R$

iv) $\cos\phi = 1$

f_r = resonant freq^n = freq^n at which resonance occurs.

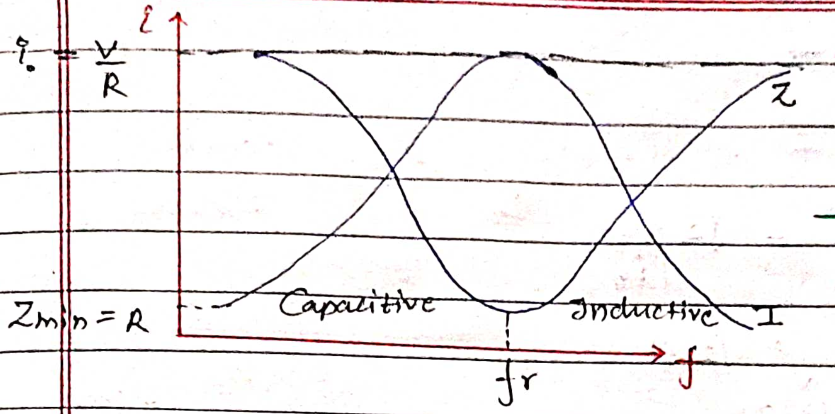
$X_L = X_C \Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$; $\omega_r = \frac{1}{\sqrt{LC}}$



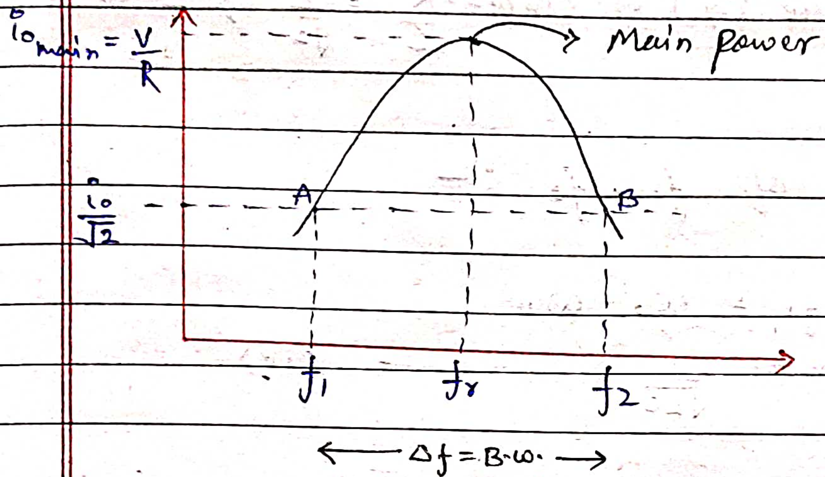
$X_C > X_L$	$X_L > X_C$
$f\uparrow$ $X_C\downarrow$ $Z\downarrow$	$f\uparrow$ $X_L\uparrow$ $Z\uparrow$
P.f. lead	Pf lag
$\phi < 0$	$\phi > 0$

* P.f. if $I > V$ then ~~xxx~~ (leads)
  if $I < V$ then ~~xxx~~ (lags)

∘ Cutoff freqⁿ → point at which $i \to \frac{i_0}{\sqrt{2}}$



→ Resonance Curve



→ Main power $P_0 = i_0^2 R$
↓
Resonance Current

□ Band width = Band of frequencies which lies btw 2 points on either sides of resonant frequency (f_r).

$$\Delta f = f_2 - f_1 \; ;$$

$$f_2 = f_r + \frac{\Delta f}{2}$$

$$f_1 = f_r - \frac{\Delta f}{2}$$

★ f_1 = Lower Cutoff freqⁿ
OR
Lower half power freqⁿ

★ f_2 = Upper Cutoff freqⁿ

at f_1, f_2 (half power points)

$i = i_0/\sqrt{2}$

at A, B

$$P = \frac{1}{2} i_0^2 R$$

$$= \frac{P_0}{2}$$

Page No.:
Date: / /

at A, B →

i) $i = \frac{i_0}{\sqrt{2}}$

ii) $Z = \frac{V_0}{i_0/\sqrt{2}} = \frac{\sqrt{2}\,V_0}{i_0}$

$= \sqrt{2}\,R$

iii) $P_A = P_B = P_0/2$

iv) Phase angle (ϕ) ⇒ $\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}} \Rightarrow \pm 45° = \phi$ (+ lead, − lag)

□ Calculation of f_1, f_2 :-

B.W. $= f_2 - f_1 = \omega_2 - \omega_1$

$\omega_r = \frac{1}{\sqrt{LC}}$ ——— (iv)

at $f_1, f_2 \rightarrow i = i_0/\sqrt{2}$

$\frac{i_0}{\sqrt{2}} = \frac{V}{Z} \Rightarrow \frac{i_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}}$ $\{\because i_0 = \frac{V}{R}$

$\sqrt{2}\,R = \sqrt{R^2 + (X_L - X_C)^2}$

$X_L - X_C = \pm R$ ——— (i)

★ At half power points net reactance of the circuit is equal to resistance of circuit

$\omega_1 L - \frac{1}{\omega_1 C} = -R$ ——— (ii)

$\omega_2 L - \frac{1}{\omega_2 C} = R$ ——— (iii)

eq^n (ii) + (iii)

$$\omega_1 L - \frac{1}{\omega_1 C} + \omega_2 L - \frac{1}{\omega_2 C} = 0$$

$$L(\omega_1 + \omega_2) - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$$

$$(\omega_1+\omega_2) L - \frac{(\omega_1+\omega_2)}{\omega_1\omega_2 C} = 0$$

$$1 - \frac{1}{\omega_1\omega_2 LC} = 0$$

$$\omega_1\omega_2 = \frac{1}{LC} \quad \text{- - - - (v)}$$

Comparing 4 & 5 →

$$\omega_r^2 = \omega_1\omega_2$$

$$f_r^2 = f_1 f_2$$

Subtrating 3 (iii) − 2 (ii) →

$$\left(\omega_2 L - \frac{1}{\omega_2 C}\right) - \left(\omega_1 L - \frac{1}{\omega_1 C}\right) = R - (-R)$$

$$L(\omega_2 - \omega_1) + \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{1}{C} = 2R$$

$$(\omega_2 - \omega_1)\left[1 + \frac{1}{\omega_1\omega_2 LC}\right] = \frac{2R}{L}$$

$$(\omega_2 - \omega_1)\left[1 + \frac{(LC)}{LC}\right] = \frac{2R}{L} \quad \left\} \because \omega_1\omega_2 = \frac{1}{LC}\right.$$

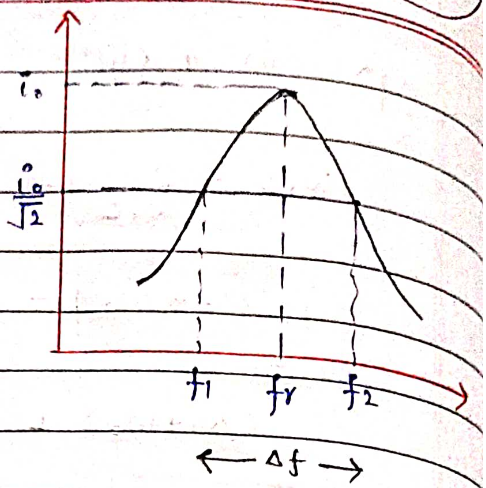
★★ $\boxed{\Delta\omega = \frac{R}{L}}$ ⟹ Bandwidth

★★ $\boxed{\Delta f = \frac{R}{2\pi L}}$

Page No.:
Date: / /

$\Rightarrow \omega_1 = \omega_r - \frac{\Delta\omega}{2}$ — ①

$\Rightarrow \omega_2 = \omega_r + \frac{\Delta\omega}{2}$ — ②



$$\omega_1 = \omega_r - \frac{R}{2L}$$

$$\omega_2 = \omega_r + \frac{R}{2L}$$

$$f_1 = f_r - \frac{R}{2L(2\pi)}$$

$$f_2 = f_r + \frac{R}{2L(2\pi)}.$$

"At Resonance" ↝

* Quality factor (Q_r) ⇒ Voltage Magnification factor.

$Q_r \uparrow$
$\rightarrow V_L \uparrow\ V_C \uparrow$

it is ratio of V_L or V_C to Source Voltage.

$Q_r = \frac{V_L}{V}$ or $Q_r = \frac{V_C}{V}$

$= \frac{I X_L}{I R}$ $= \frac{I X_C}{I R}$

* $\boxed{Q_r = \frac{\omega_r L}{R}}$ * $\boxed{Q_r = \frac{1}{\omega_r C R}}$

$\Rightarrow \frac{R}{L} = \frac{\omega_r}{Q_r}$

$\Rightarrow \boxed{Q_r = \frac{\omega_r}{\Delta\omega}}$

Similarly $\boxed{Q_r = \frac{f_r}{\Delta f}}$

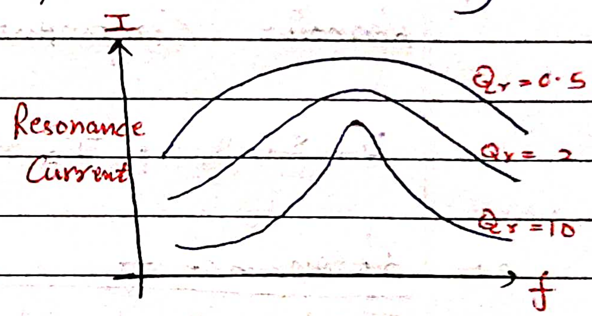
$\because \quad Q_r = \omega_r \frac{L}{R}$

$= \frac{1}{\sqrt{LC}} \times \frac{L}{R}$

$$\boxed{Q_r = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

\# Significance of Q.f. → it is measure of Sharpness or Selectivity

∵ Selectivity: It is the ability of RLC Circuit to Select any perticular Signal from group of Signals.



Series RLC Circuit → Radio Reciever as accepter Circuit

In series →

$Z = R + j(X_L - X_C)$

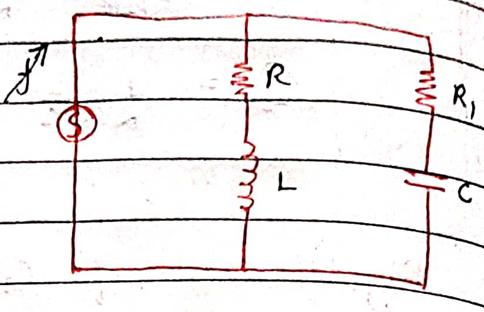
& at Reso →

$\text{ima}[Z] = 0$

"Parallel Resonance"

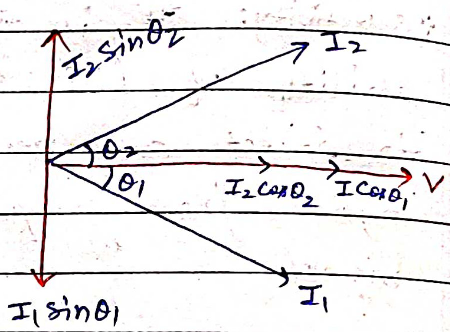
(Anti Reso. OR Current Reso.)

when Resistive Component of Circuit becomes zero this circuit is said to be in Resonance.



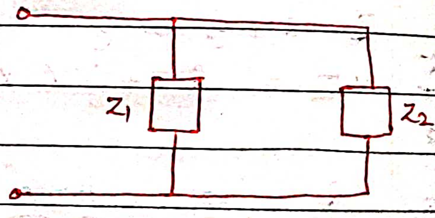
Reactive Component of Current →

$I_1 \sin\theta_1$; $I_2 \sin\theta_2$



at Resonance →

$$\boxed{I_1 \sin\theta_1 = I_2 \sin\theta_2}$$

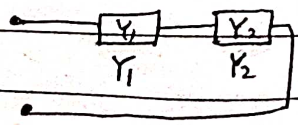


★ $Z_1 = R + jX_L$

$Z_2 = R - jX_C$

□ Calculation of Resonant freq^n →

Admittance → $(Y) = \frac{1}{Z}$



$$Y = Y_1 + Y_2$$

$$Y = \frac{1}{R + jX_L} + \frac{1}{R - jX_C}$$

$$Y = \frac{R - jX_L}{R^2 + X_L^2} + \frac{R + jX_C}{R^2 + X_C^2} \quad \text{——— (1)}$$

$\because \quad Z = R + j(X_L \sim X_C)$

$\Rightarrow \quad Y = \underbrace{G}_{\text{Conductance}} + j\underbrace{B}_{\text{Susceptance}}$

at Reso. in eq^n (1) $\rightarrow$

$\text{im}[Y] = 0$

$$B_C = B_L$$

$$\frac{X_C}{R_1^2 + X_C^2} = \frac{X_L}{R^2 + X_L^2}$$

$$\frac{1}{\omega C\left(R_1^2 + \frac{1}{\omega^2 C^2}\right)} = \frac{\omega L}{R^2 + \omega^2 L^2}$$

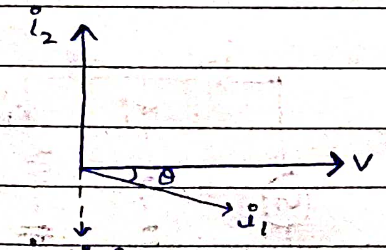
$$\frac{\omega C}{\omega^2 C^2 R_1^2 + 1} = \frac{\omega L}{R^2 + \omega^2 L^2}$$

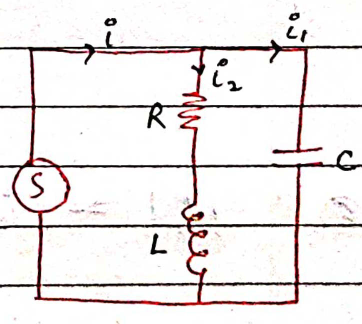
$$R^2 C + \omega^2 L^2 C = \omega^2 R_1^2 C^2 L + L$$

$$\omega^2 = \frac{1}{LC}\left[\frac{R^2 C - L}{R_1^2 C - L}\right]$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{R^2 - L/C}{R_1^2 - L/C}}}$$

- Tank Circuit :-





$$* \quad f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R^2 - L/C}{R_1^2 - L/C}}$$

0

at Reso $\rightarrow$

$I_2 = I_1 \sin\theta_1$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C}$$

$$\Rightarrow Z^2 = X_L X_C$$

$$\boxed{Z = \sqrt{L/C}}$$

$$i = i_1 \cos\theta_1$$

$$i = \frac{V}{Z} \times \frac{R}{Z}$$

$$i = VR/Z^2$$

$$Z^2 = VR/i$$

$$\because \frac{V}{i} = Z_0$$

$$\frac{Z^2}{R} = \frac{L}{RC}$$

$$\Rightarrow \boxed{Z_0 = \frac{L}{RC}}$$ dynamic impedence

imp. Points (parallel Reso.)

① Net Susceptance = 0
($B = 0$)

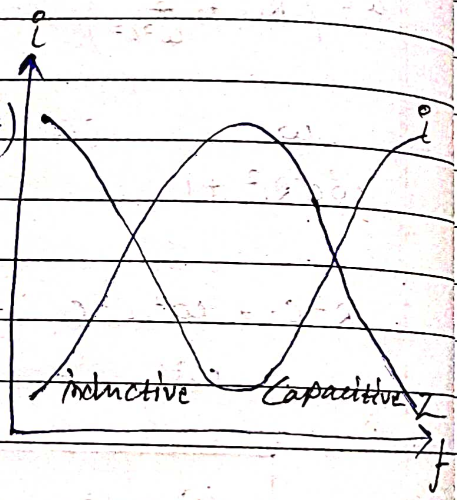
② Admittance = Conductance

③ P.f. = 1

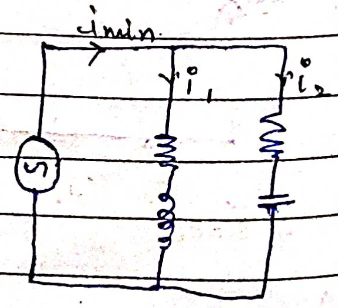
④ Pure Resistive
($i_{min} = V/Z_0$)

⑤ $Y_{min} \to Z_{main} \to I_{main}$
($V/(L/RC)$)

($Z_0 = \frac{L}{RC}$)



⑥ $|i_1| = |i_2| >> i$ ~~supply voltage~~



Current through capacitor and Inductor is greater as compared to supply current thats why Current Resonance.

@ Also called Rejector circuit because at Resonance line current ~~and~~ is minimum and Rejected

Current Magnification factor -

$Q_r = \frac{i_L}{i}$ $\quad$ $Q_r = \frac{i_c}{i}$ $\quad$ $\left\{ \begin{matrix} i_L = i_1 \\ i_c = i_2 \end{matrix} \right\}$

$Q_r = \frac{1}{R}\sqrt{\frac{L}{C}}$ $\quad$ $\left(Q_r = \frac{V_L}{V} = \frac{V_c}{V} \right)$