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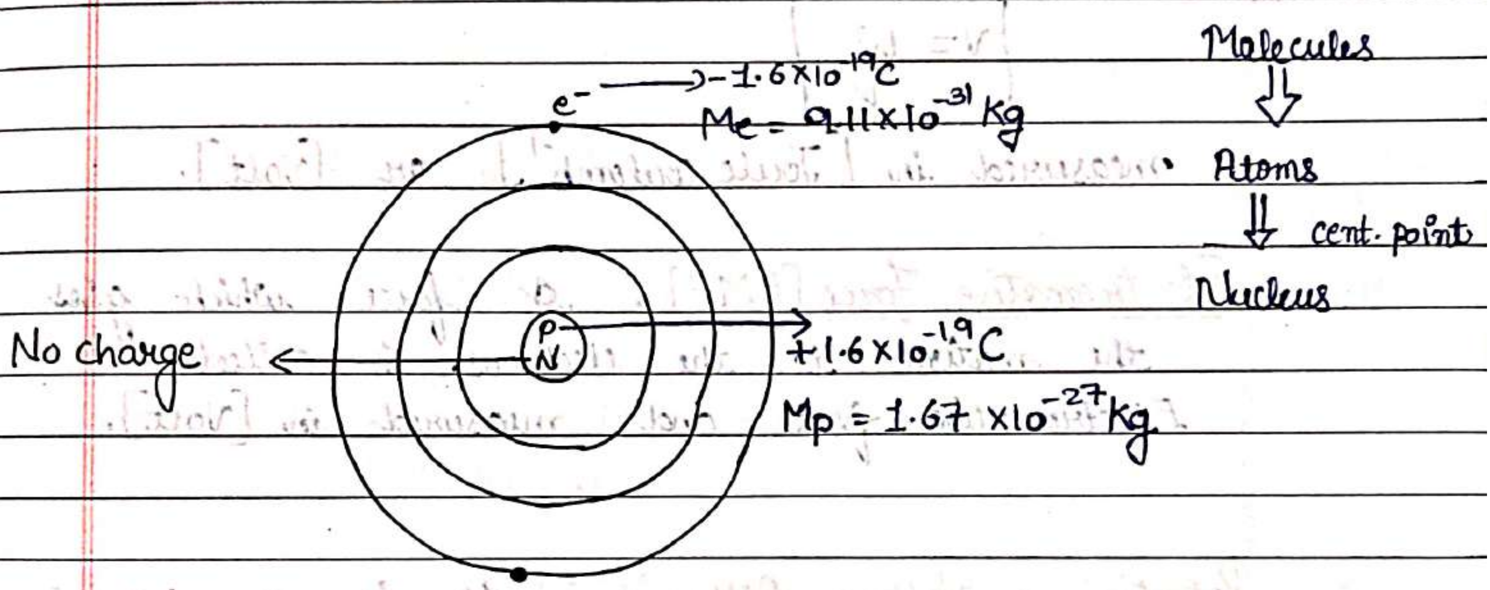
10/Nov/2020

MODULE - 1

DC CIRCUIT ANALYSIS

- Basic Definitions related to electrical circuit:



1. **Charge:** It is an imaginary phenomenon by which any substance experiences a force of attraction and repulsion, is called charge. and denoted by 'q'. and measured in Coulomb.

2. **Electric Current:** The rate of flow of charge through a conductor is called electric current.

$$i = \frac{q}{t}$$

$$\boxed{i = \frac{dq}{dt}}$$

and measured in $\boxed{\text{Coulomb sec}^{-1}}$ or $\boxed{\text{Ampere}}$.

3. <u>Voltage</u> : It is the workdone to bringing a unit positive charge from infinity to at any point.

$$V = \frac{\text{Workdone } (W)}{\text{charge } (q)}$$

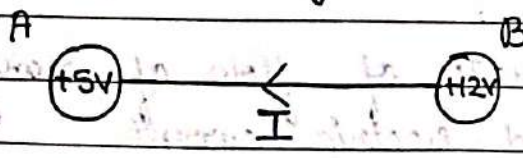
$$\boxed{V = \frac{W}{q}}$$

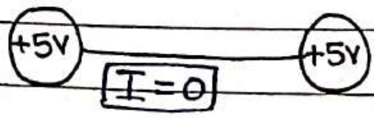
: measured in [Joule coulomb^{-1}] or [Volt].

4. <u>Electromotive Force [EMF]</u> : A force which gives the motion to the electrons, is called Electromotive force and measured in [Volt].

5. <u>Potential or Voltage Difference</u> ; It is the potential difference between two charge bodies.

[higher voltage to lower voltage].





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6. **Power**: The rate at which work is done in an electric circuit is called electrical Power.

$$\text{Power}(P) = \frac{\text{Workdone}(W)}{\text{Time}\ (t)}$$

$$P = \frac{W}{t} = \frac{V.q}{t}$$

$\left\{ V = \frac{W}{q},\ W = V.q \right\}$

$\{q = i.t\}$

$$P = \frac{V.i.\cancel{t}}{\cancel{t}}$$

$\therefore \boxed{P = VI}$

$P = I^2R$

$P = \frac{V^2}{R}$

measured in $\left[\frac{\text{Joule}}{\text{sec}}\right]$ or [Watt].

7. **Electrical Energy**: The total workdone in an electrical circuit is called electrical energy.

Electrical Energy = Electric Power X Time.

$\therefore$ electrical energy $= VI \times T$
$= \boxed{VIT}$

measured in $\boxed{KWh}$

K → kilo, W → Watt, h → hour

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8. **Active elements and Passive elements:**

The elements which are capable of delivering energy are called active elements.

e.g. Voltage Source (Batteries), current source etc.

Passive elements: The elements which will receive the energy and dicipiate / dissipate or store it are called passive elements.

e.g. Resistors (R) ~~are~~, ~~conductors~~ inductors (L) and capicitors (C)

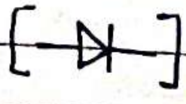
NOTE:- Inductors and capicitors are energy storing elements while the resistance dissipate the energy in the form of heat.

9. **Bilateral and Unilateral elements:**

Bilateral elements are those elements when the voltage and current relation are same irrespective of direction of flow of current.

e.g. Resistors (R)

However, unilateral elements are those elements when the voltage and current relation are different for two possible direction of flow of current.

e.g. Diode 

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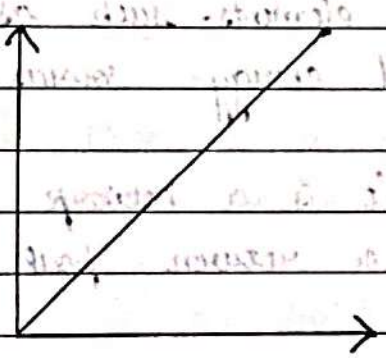
10. Linear and Non-Linear elements :-

A Linear element is one which follows the principle of superposition and homogenpousity.

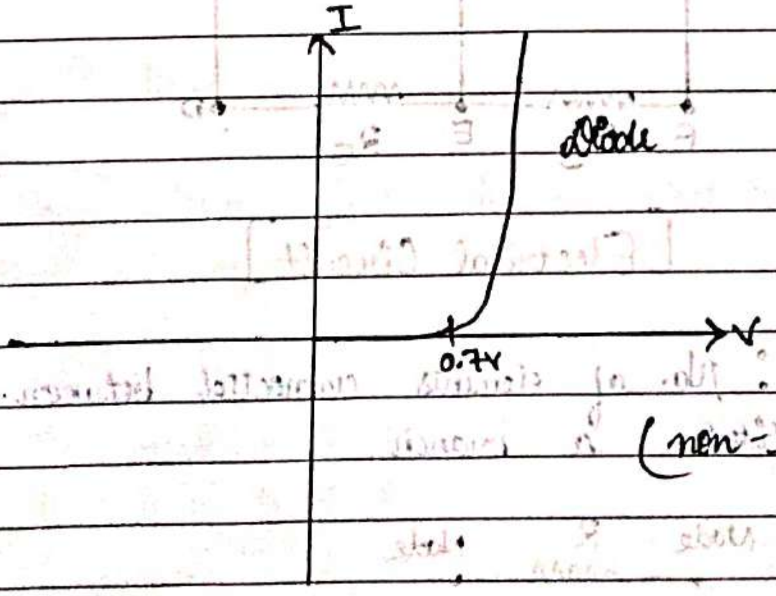
e.g. Resistor (R), Inductor (L), Capicitor (C).

OR A circuit is linear if and only if its input and output can be related by a straight line.



A non-linear elements is one which does not follows the principle of super position and homogenity.

e.g. Diode and Transistors.



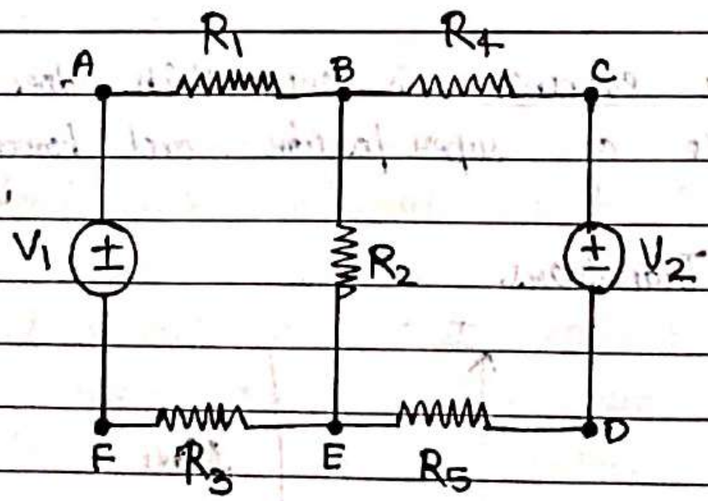
(non-linear relation)

Note: The ohm's law is appliable only for linear elements.

★ Concept of Network or Circuit

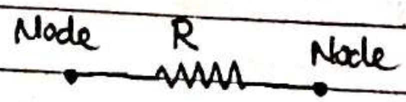
- **Network:** An electrical network is an interconnection of active and passive elements. such as Resistance, Inductance, Capicitance and energy sources.

- **Circuit:** An electrical circuit is a network that has a closed path, giving a return path for the current.



[Electrical Circuit]

(i) **Branch:** No. of elements connected between two nodes. constitute a branch



$\boxed{1 = \text{Branch}}$

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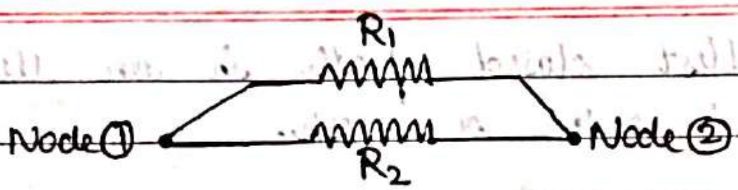
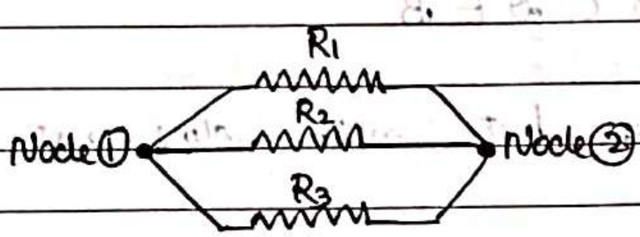


2 – Branch



3 – Branch

• (ii) Node :- Node is a junction where two or more than two branches meet.

• Junction is the point where more than two branches meet.

Node ⇒ Ending points of an element.

Node ⇒ A, B, C, D, E, F

Junction ⇒ B, E

• (iii) Loop :- Any closed path in an electric circuit is called a loop.

Loop ① ⇒ A B C D E F A
Loop ② ⇒ A B E F A
Loop ③ ⇒ B C D E B

(iv) **Mesh** ÷ Smallest closed path in an electric circuit is called a mesh.

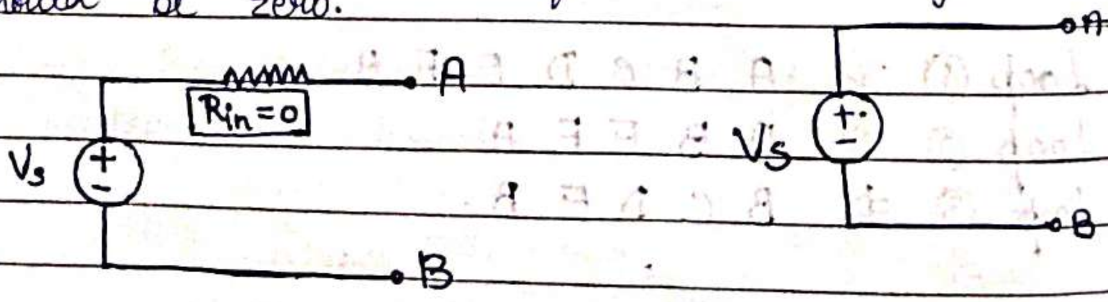
Mesh ① ⇒ A B E F A
Mesh ② ⇒ B C D E B

Note :- Loop contain meshes but mesh does not contain any loop.

(v) **Energy Sources** :- [Independent energy source only]

Energy Sources
- Voltage Source
  - Ideal voltage Source (Imp.)
  - Practical voltage Source
- Current Source
  - Ideal current Source
  - Practical Current Source

- **Ideal Voltage Source** :- [Voltage Source]
- It gives constant voltage accross its terminal.
- The internal resistance of an ideal voltage source should be zero.



[Symbol]

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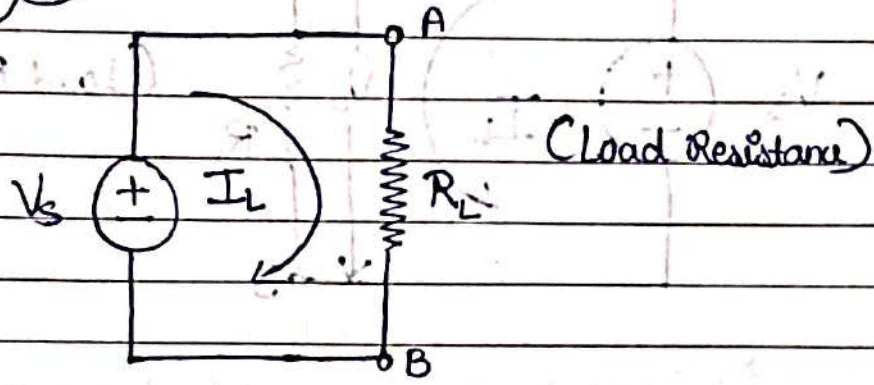
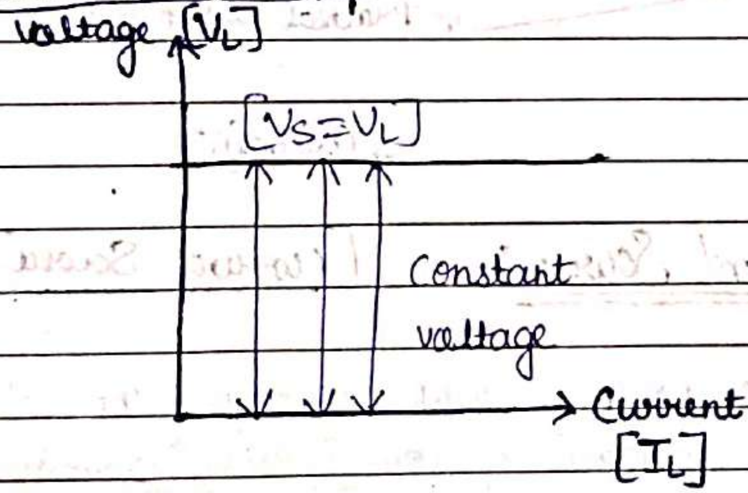
Circuit



I_L ⇨ Current through load resistance.

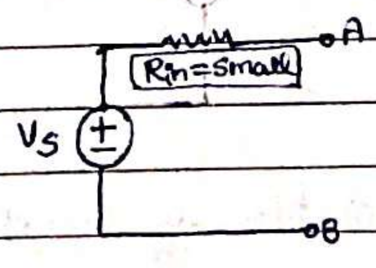
V_L ⇨ Voltage through load resistance.

Characteristics :-

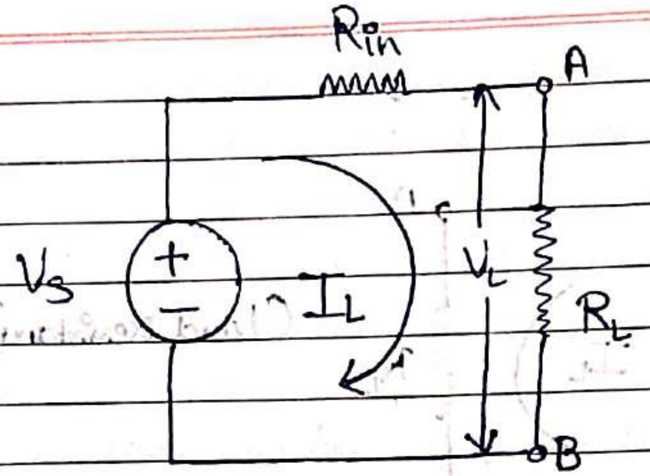


- **Practical Voltage Source :-**

In Practical, ideal voltage Source does not exist. Every voltage source has small internal resistance connected in series with it.

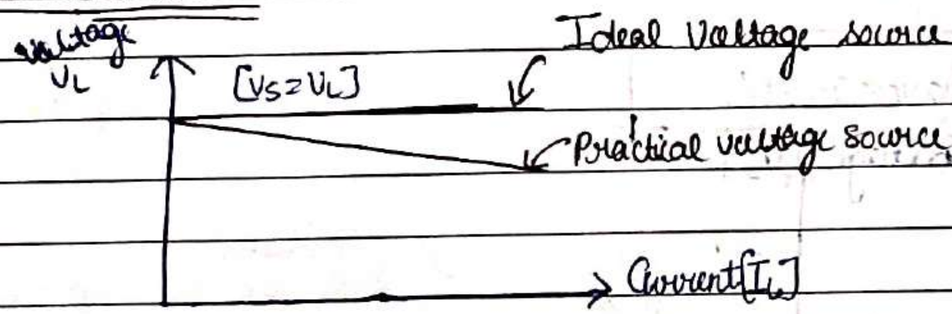


Circuit :



I_L ⇨ Current through load resistance.
V_L ⇨ Voltage across load resistance.
R_{in} ⇨ internal resistance

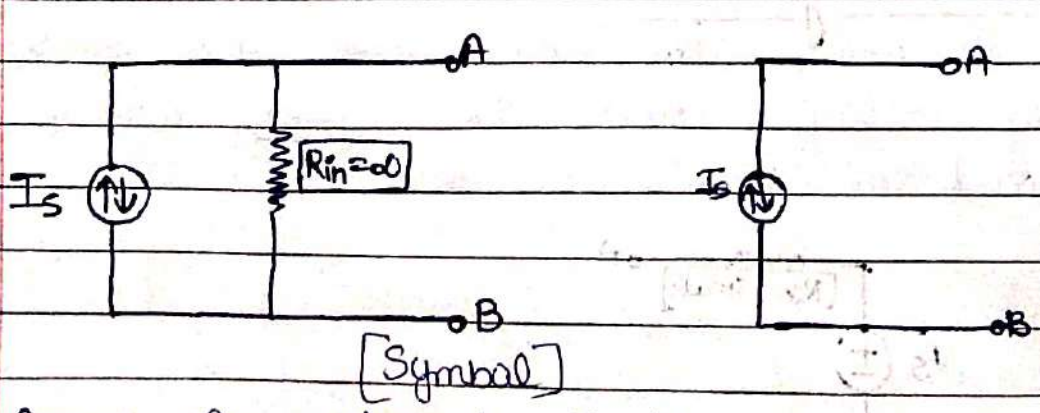
- **Characteristics :**



⇨ Ideal Current Source : [current Source]

- It gives constant current through the circuit.

Imp
- The internal resistance of an ideal current source should be infinite.



[Symbol]

Arrow sign represents direction of current.

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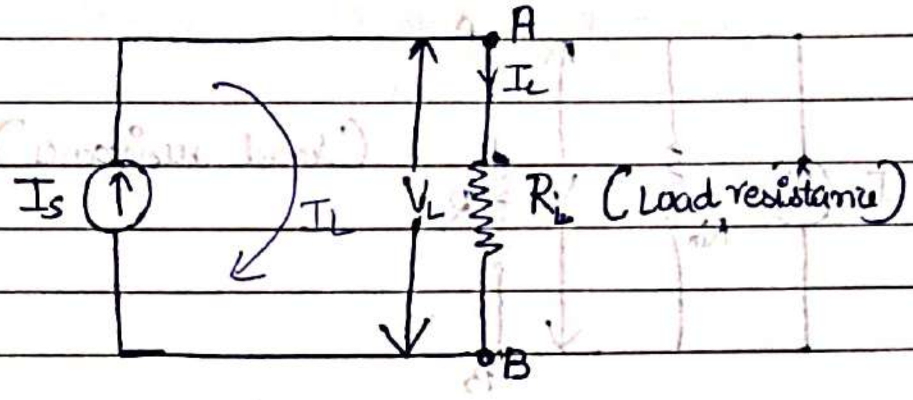
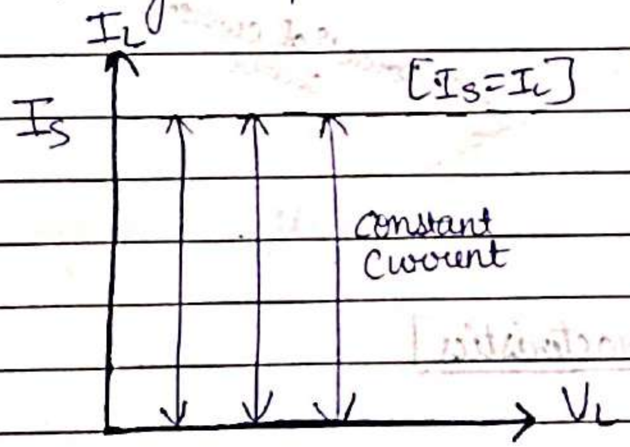


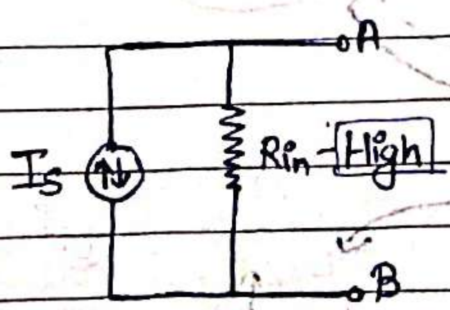
[Circuit]

$I_S \Rightarrow$ Supply current
$I_L \Rightarrow$ load current
$V_L \Rightarrow$ voltage drop across load resistor.

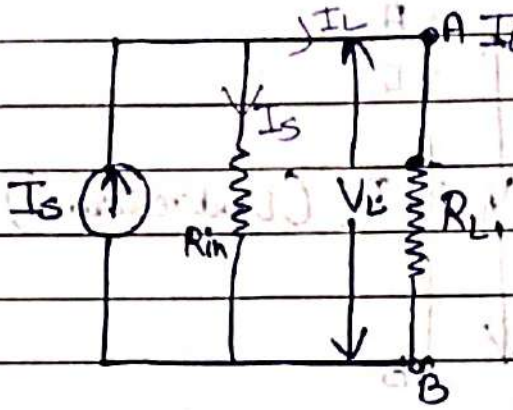


[Characteristics]

- **Practical Current Source:-** In Practical ideal current source does not exist. Every current source has high value resistance connected in parallel with it



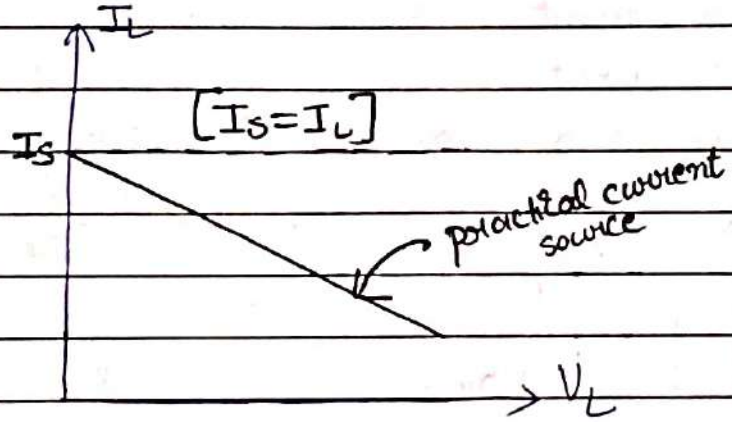
[Symbol]



(load resistance)

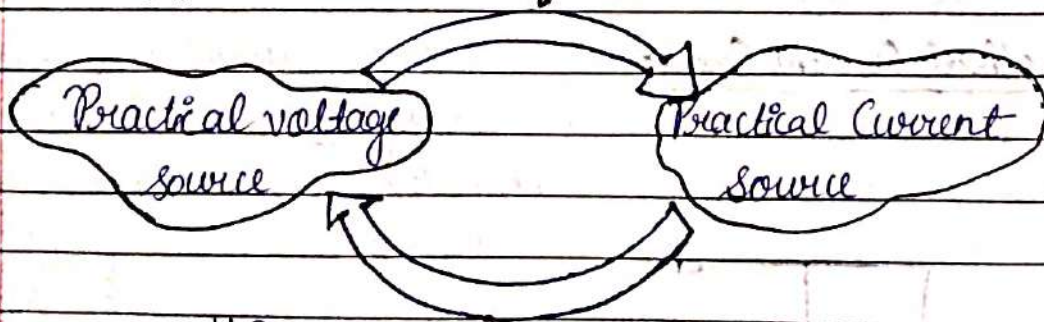
[circuit]

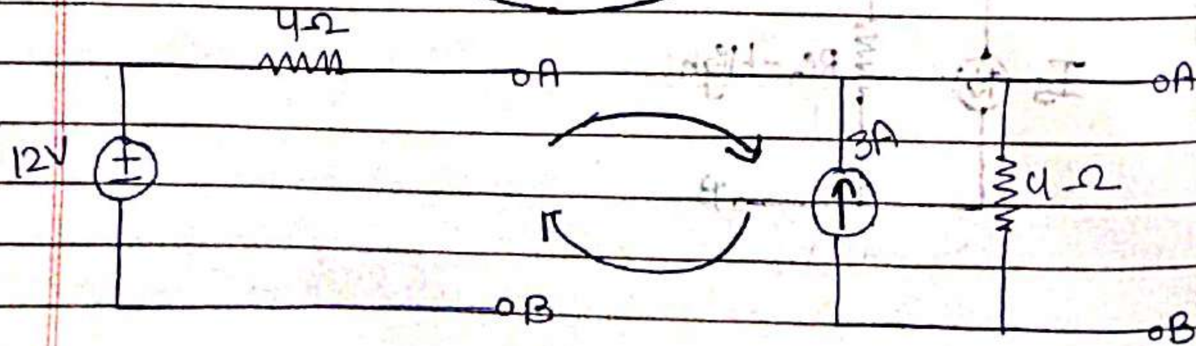
R_{in} ⇒ internal resistance



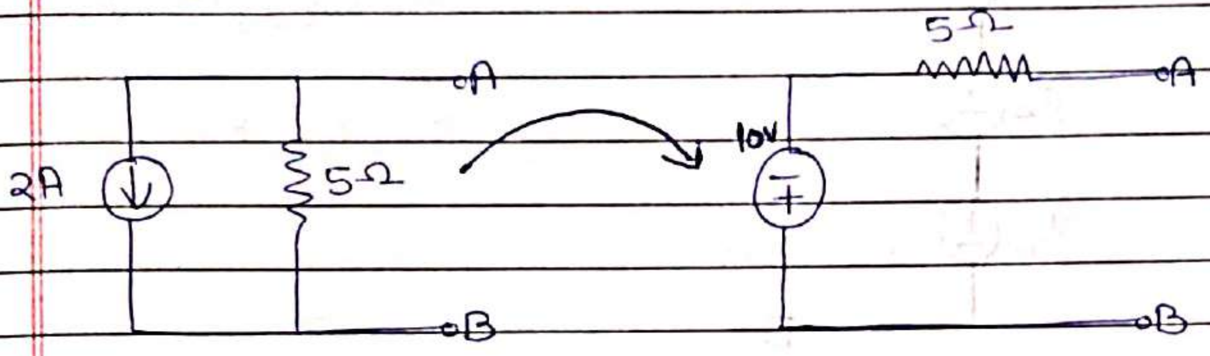
[Characteristics]

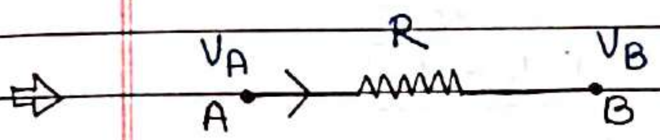
⇒ **Energy Source Transformation :-**



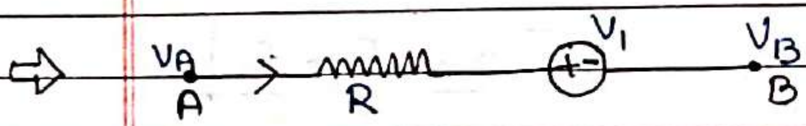


$$I = \frac{V}{R} \Rightarrow I = \frac{12}{4} \Rightarrow \boxed{I = 3\,\text{Amp}}$$

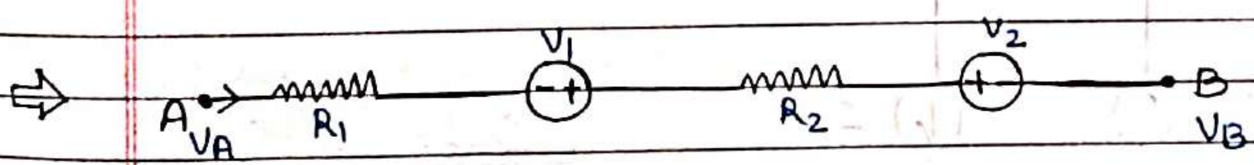




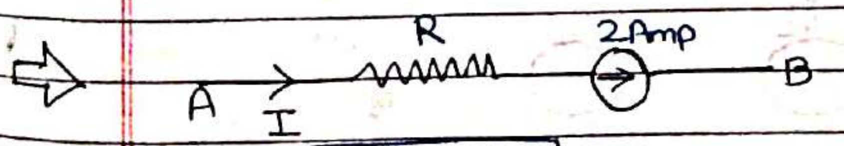
$$I = \frac{(V_A - V_B)}{R}$$



$$I = \frac{V_A - (+V_1) - V_B}{R}$$

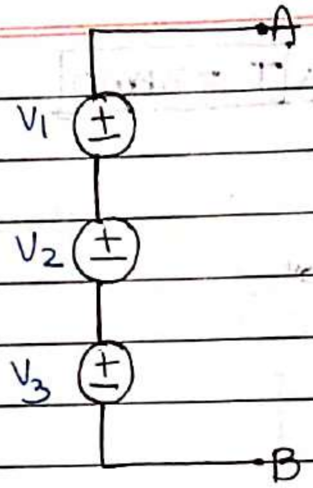


$$I = \frac{V_A - (-V_1) - (+V_2) - V_B}{R_1 + R_2}$$



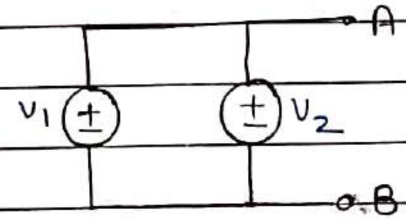
$$\boxed{I = +2\,\text{Amp}}$$

⇒



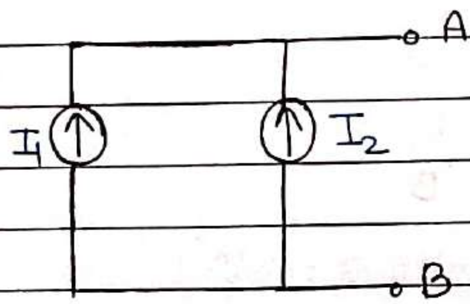
$$\boxed{V = V_1 + V_2 + V_3}$$

⇒



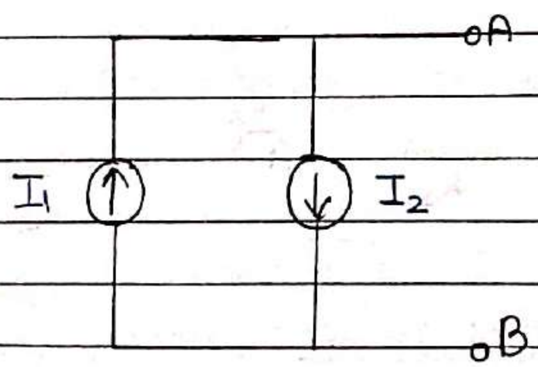
$$\boxed{V_1 = V_2}$$

⇒



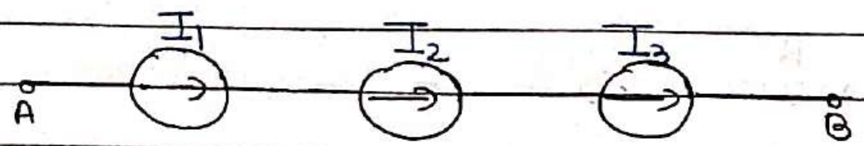
$$\boxed{I_{AB} = I_1 + I_2}$$

⇒



$$\boxed{I_{AB} = I_1 - I_2}$$

⇒



$$\boxed{I}$$

Nodal Analysis

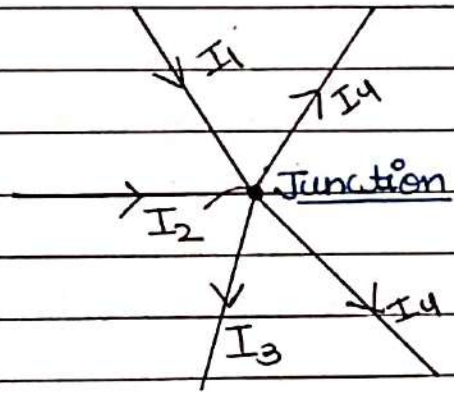
⇒ Kirchoff's Law :-

Junction or Node

1. Kirchoff's current Law :[KCL]

The algebric sum of all currents is zero, at any junction point.

$$\boxed{\Sigma I = 0}$$



$$I_1 + I_2 = I_3 + I_4 + I_5$$

$$\boxed{I_1 + I_2 - I_3 - I_4 - I_5 = 0}$$

+ ⇒ for incoming current

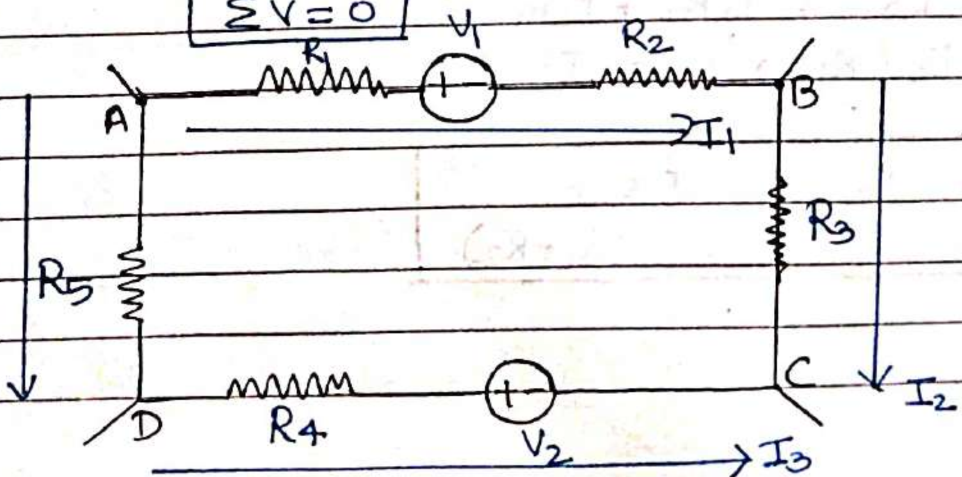
− ⇒ for outgoing current

Note-: The algebric sum of incoming current is equals to algebric sum of outgoing current, at any junction point.

Kirchoff's voltage Law :- [KVL]

The algebric sum of all Emf or voltage is zero, in a closed loop

$$\boxed{\Sigma V = 0}$$



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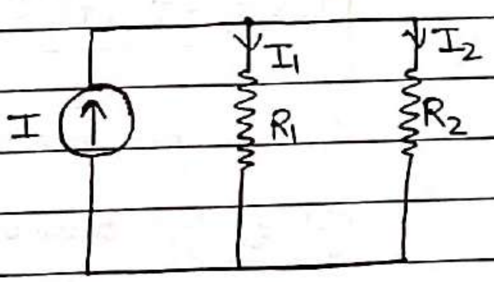
If we travel from $\boxed{A \to B \to C \to D \to A}$

$$\boxed{+I_1R_1 + V_1 + I_1R_2 + I_2R_3 - V_2 - I_3R_4 - I_4R_5 = 0}$$

<u>Note</u> :→ With the direction of current, consider voltage drop across resistance will be always positive and taking voltage source with sign occurs in the way

Current Divider Rule :→



$$I = I_1 + I_2 \text{ ------ } ①$$

$$I_1 R_1 = I_2 R_2$$

$$I_1 = \frac{I_2 R_2}{R_1} \text{ ---- } ②$$

from ① and ②

$$I = \frac{I_2 R_2}{R_1} + I_2 \text{ ---}$$

$$IR_1 = I_2R_2 + I_2R_1$$

$$I_2 (R_1 + R_2) = IR_1$$

$$\boxed{I_2 = \frac{I \times R_1}{(R_1 + R_2)}}$$

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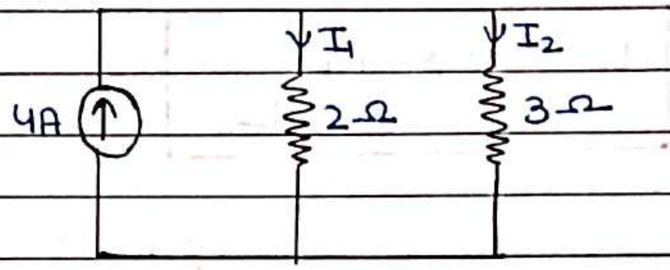
Similarly,

$$\boxed{I_1 = I \times \frac{R_2}{(R_1+R_2)}}$$

Example:



By using current divider rule:

$$I_1 = I \times \frac{R_2}{(R_1+R_2)}$$

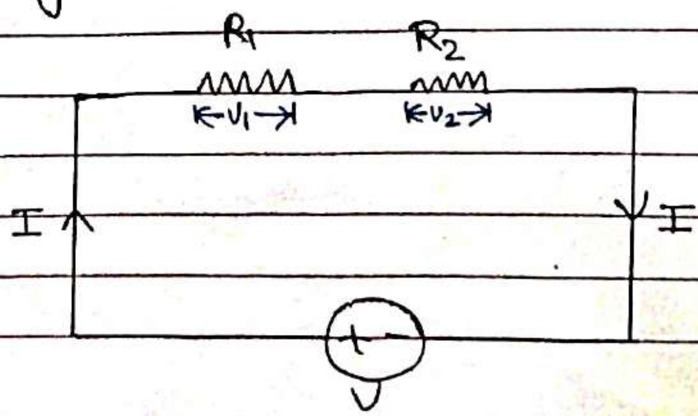
$$I_1 = 4 \times \frac{3}{(2+3)}$$

$$\boxed{I_1 = \frac{12}{5} \text{ Amp}}$$

$$I_2 = 4 \times \frac{2}{(2+3)}$$

$$\boxed{I_2 = \frac{8}{5} \text{ Amp}}$$

Voltage Divider Rule:



$$I = \frac{V}{(R_1 + R_2)} \quad \text{------} \quad (1)$$

voltage drop across resistor $(R_1) = IR_1$

$$\boxed{V_1 = V \times \frac{R_1}{(R_1 + R_2)}}$$

Similarly,

$$\boxed{V_2 = V \times \frac{R_2}{(R_1 + R_2)}}$$

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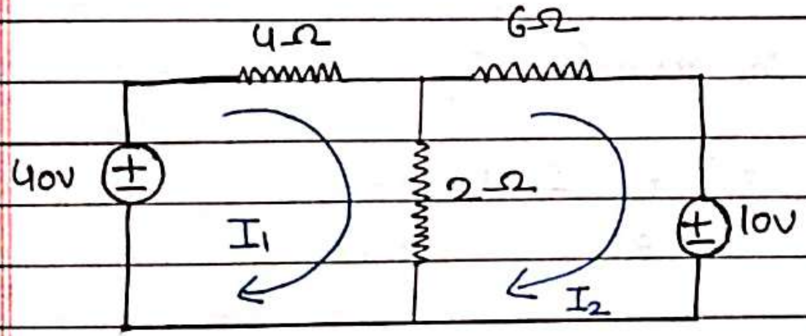
Mesh Analysis

i. Assume meshes in given circuit.
ii. Consider a current in each mesh.
iii Applying KVL in each mesh.
iv If a mesh having current source no need to apply KVL in that loop.
v) Write the meshes equations and solving it.
vi) Calculate the current in desired resistance.



mesh.
↑
KVL
↑

Find the current in 2 Ω using mesh analysis.

Applying KVL in mesh ①

$$+4I_1 + 2(I_1 - I_2) - 40 = 0$$

$$+6I_1 - 2I_2 = 40 \quad \text{---} ①$$

Applying KVL in mesh ②

$$+6I_2 + 10 + 2(I_2 - I_1) = 0$$

$$-2I_1 + 8I_2 = -10 \quad \text{---} ②$$

$\boxed{I_1 = 6.81 \text{ Amp}}$ $\boxed{I_2 = 0.45 \text{ Amp}}$

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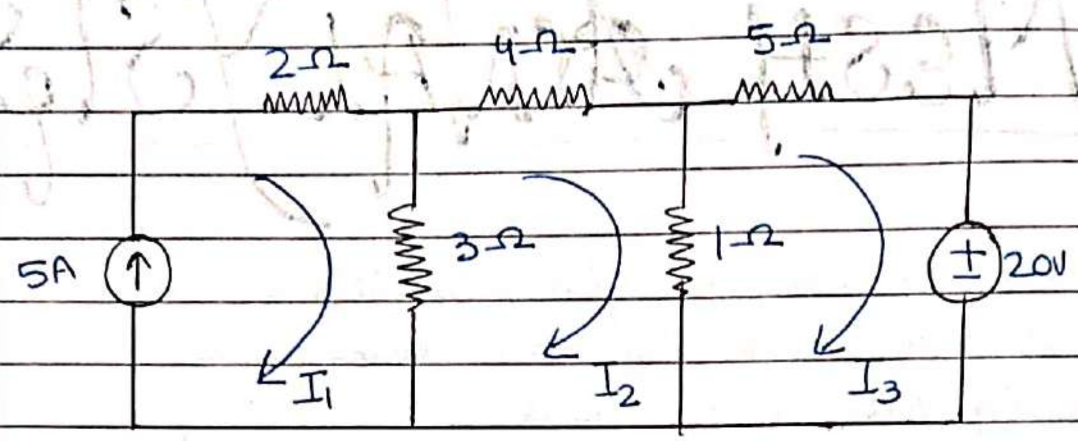
Ques-1.



find the current in 4Ω, using mesh analysis.

from mesh ①

$$I_1 = +5 \text{ Amp} \quad \text{------} \quad (1)$$

Applying KVL in mesh ②

$$+4I_2 + 1(I_2 - I_3) + 3(I_2 - I_1) = 0$$

$$-3I_1 + 8I_2 - I_3 = 0 \quad \text{------} \quad (II)$$

Applying KVL in mesh ③

$$+5I_3 + 20 + 1(I_3 - I_2) = 0$$

$$-I_2 + 6I_3 = -20 \quad \text{------} \quad (III)$$

$I_1 = 5$ Amp

$I_2 = 1.48$ Amp

$I_3 = -3.08$ Amp

Current in 4Ω

$$\boxed{I_2 = I_{4\Omega} = 1.48 \text{ Amp}}$$

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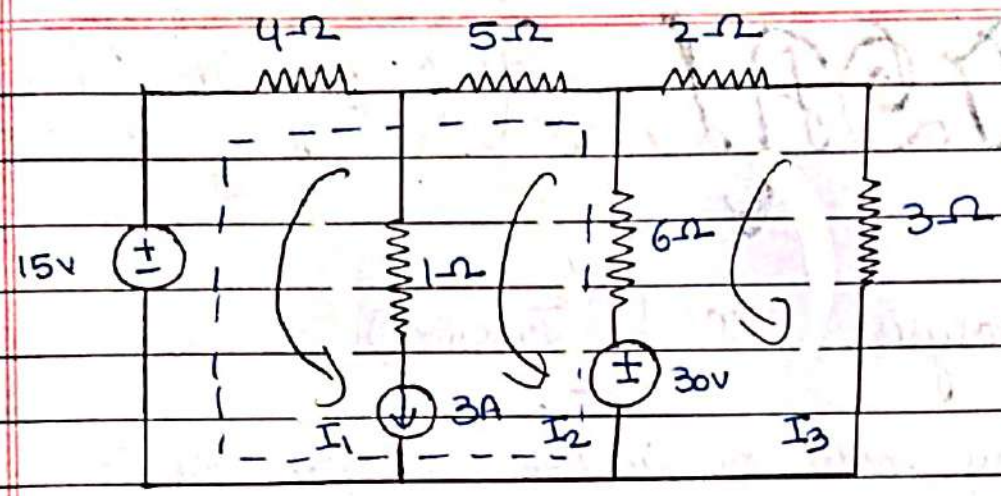

find the current in 5Ω

$$3 = I_2 - I_1 \quad \text{---- (I)}$$

$$+4I_1 + 15 - 30 + 6(I_2 - I_3) + 5I_2 = 0$$

$$4I_1 + 11I_2 - 6I_3 = +15 \quad \text{---- (II)}$$

$$+2I_3 + 6(I_3 - I_2) + 30 + 3I_3 = 0$$

$$-6I_2 + 11I_3 = -30 \quad \text{---- (III)}$$

$$I_1 = -2.09 \text{ Amp}$$

$$I_2 = 0.90 \text{ Amp}$$

$$I_3 = -2.23 \text{ Amp}$$

Current in 5Ω

$$I_{5\Omega} = I = 0.90 \text{ Amp.}$$

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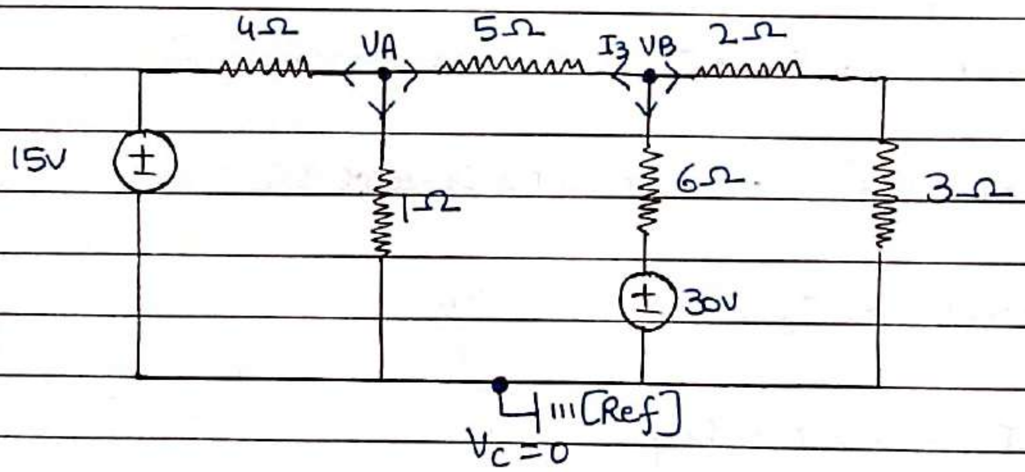
NODAL ANALYSIS

Nodal Analysis [KCL] (Junction)

1. Identifying nodes or junction in given circuit
2. Assign a voltage at each node or junction
3. Assume a reference node or junction, the voltage of reference node is zero.
4. Applying KCL at each node remaining the reference node
5. Write the node equation and find the current in desired resistance.



Applying KCL in junction (A)

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_A - V_C - (+15)}{4} + \frac{V_A - V_C}{1} + \frac{V_A - V_B}{5} = 0$$

$$\frac{V_A - 15}{4} + \frac{V_A}{1} + \frac{V_A - V_B}{5} = 0 \quad \{V_C = 0\}$$

$$0.25(V_A - 15) + V_A + 0.2(V_A - V_B) = 0$$

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$$+1.45 V_A - 0.2 V_B = 3.75 \quad \text{--------} (1)$$

Applying KCL at Junction (B)

$$I_3 + I_4 + I_5 = 0$$

$$\frac{V_B - V_A}{5} + \frac{V_B - (+30)}{6} + \frac{V_B}{5} = 0$$

$$0.2V_B - 0.2V_A + 0.16V_B - 5 + 0.2V_B = 0$$

$$-0.2V_A + 0.56 V_B = 5 \quad \text{———} (2)$$

$$V_A = 4.01 \text{ volt}$$

$$V_B = 10.36 \text{ volt}$$

Current in 5ohm

$$I_{5\Omega} = I_3 = \frac{(V_A - V_B)}{5} = \frac{4.01 - 10.36}{5}$$

$$\boxed{I_{5\Omega} = I_3 = -1.27 \text{ Amp.}}$$

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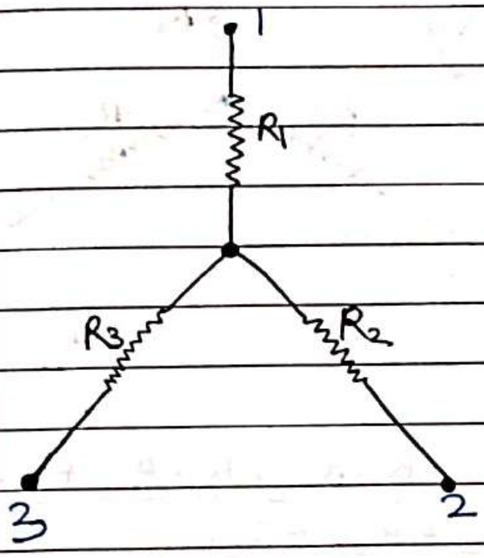
Imp for 10

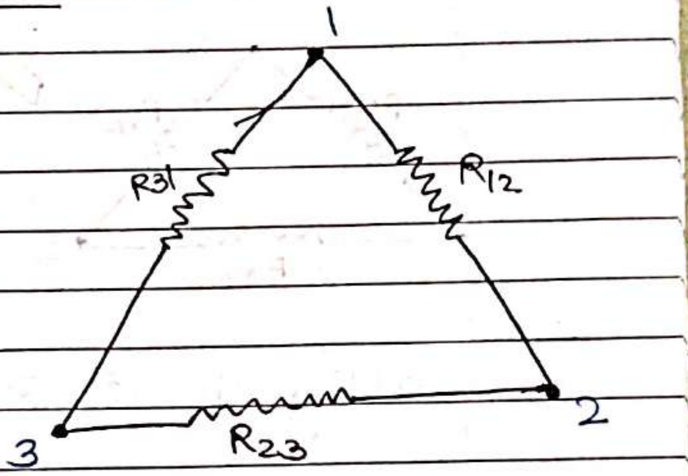
⇒ **Star-Delta (λ-Δ) Transformation :-**



Equivalent resistance b/w 1 and 2 $= R_1 + R_2$

Equivalent resistance b/w 2 and 3 $= R_2 + R_3$

Equivalent resistance b/w 3 and 1 $= R_3 + R_1$



Equivalent resistance b/w 1 and 2

$$= \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

Equivalent resistance b/w 2 and 3

$$= \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

Equivalent resistance b/w 3 and 1

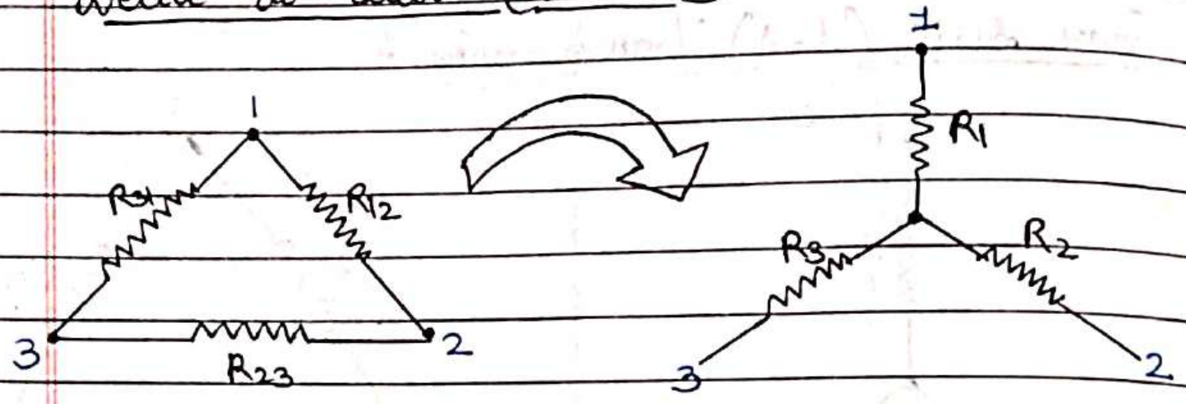
$$= \frac{R_{31}(R_{23}+R_{12})}{R_{12}+R_{23}+R_{31}}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{------} \quad (1)$$

$$R_2 + R_3 = \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{------} \quad (2)$$

$$R_3 + R_1 = \frac{R_{31}(R_{23}+R_{12})}{R_{12}+R_{23}+R_{31}} \quad \text{------} \quad (3)$$

⇒ Delta to Star ($\Delta \to \curlywedge$)



- adding ①, ②, ③ eqⁿ.

$$2(R_1+R_2+R_3) = \frac{R_{12}R_{23} + R_{12}R_{31} + R_{23}R_{31} + R_{23}R_{12} + R_{31}R}{R_{12}+R_{23}+R_{31}}$$

$$\cancel{2}(R_1+R_2+R_3) = \frac{\cancel{2}(R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12})}{R_{12}+R_{23}+R_{31}}$$

$$R_1+R_2+R_3 = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12}+R_{23}+R_{31}} \quad \text{---- (4)}$$

now eqⁿ (4) − eqⁿ (2)

$$R_1+\cancel{R_2}+\cancel{R_3}-\cancel{R_2}-\cancel{R_3} = \frac{R_{12}\cancel{R_{23}}+\cancel{R_{23}R_{31}}+R_{31}R_{12}-\cancel{R_{23}R_{31}}-\cancel{R_{23}R_{12}}}{R_{12}+R_{23}+R_{31}}$$

$$\boxed{R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}} \quad \text{---- (5)}$$

Similarly

$$\boxed{R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}} \quad \text{---- (6)}$$

$$\boxed{R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}} \quad \text{---- (7)}$$

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⇒ Star to Delta ($\lambda \to \Delta$) :

from eqⁿ (5), (6), (7)

$$R_1(R_{12}+R_{23}+R_{31}) = R_{12}R_{31} \quad \text{----} \; (8)$$

$$R_2(R_{12}+R_{23}+R_{31}) = R_{23}R_{12} \quad \text{-----} \; (9)$$

$$R_3(R_{12}+R_{23}+R_{31}) = R_{31}R_{23} \quad \text{------} \; (10)$$

eqⁿ (8) ÷ eqⁿ (9)

$$\frac{R_1}{R_2} = \frac{R_{31}}{R_{23}}$$

$$R_{23} = \frac{R_2}{R_1} R_{31} \quad \text{-----} \; (a)$$

eqⁿ (9) ÷ eqⁿ (10)

$$\frac{R_2}{R_3} = \frac{R_{12}}{R_{31}}$$

$$R_{12} = \frac{R_2}{R_3} R_{31} \quad \text{-----} \; (b)$$

from eqⁿ (8), (a), (b)

$$R_1\left(\frac{R_2 \cancel{R_{31}}}{R_3} + \frac{R_2}{R_1}\cancel{R_{31}} + \cancel{R_{31}}\right) = R_{12}\cancel{R_{31}}$$

$$R_{12} = \frac{R_1R_2}{R_3} + R_2 + R_1$$

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}}$$

Similarly

$$\boxed{R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}}$$

$$\boxed{R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2}}$$

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1-Dec-2020

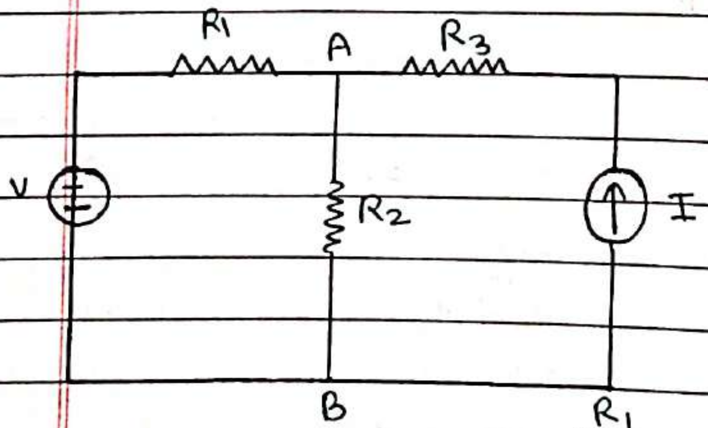
NETWORK THEOREM

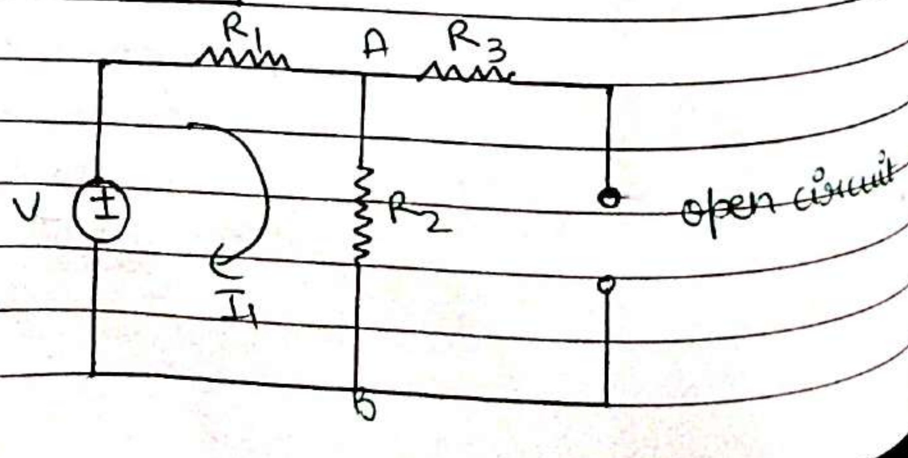
ⓐ Superposition Theorem
ⓑ Thevenin's Theorem
ⓒ Norton's Theorem.

1. **Super-Position Theorem**: This theorem states that, "Any linear bilateral D.C. network containing more than one energy sources (voltage source or current source), the overall current in any branch is the algebraic sum of currents produced by the each source acting alone, while the other energy sources are inoperative."

Note:
- Current Source → open circuit
  → internal resistance infinite
- Voltage Source → short circuit
  → internal resistance zero.

e.g.





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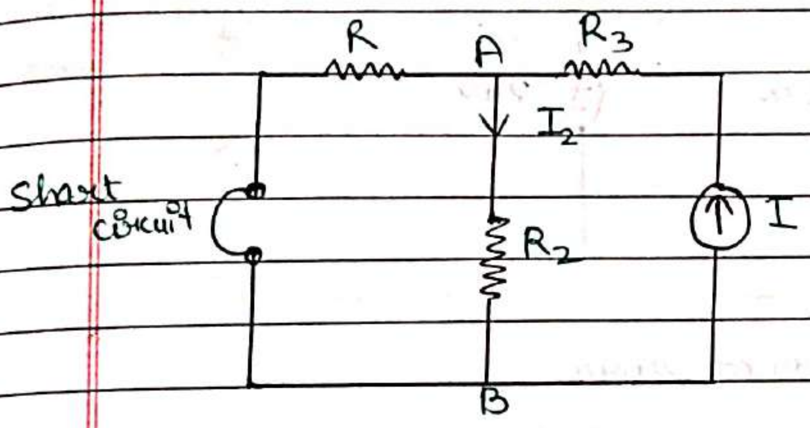
Current in $R_2 = I_1$ [A→B]



Current in $R_2 = I_2$ [A→B]

Total current in R_2 =

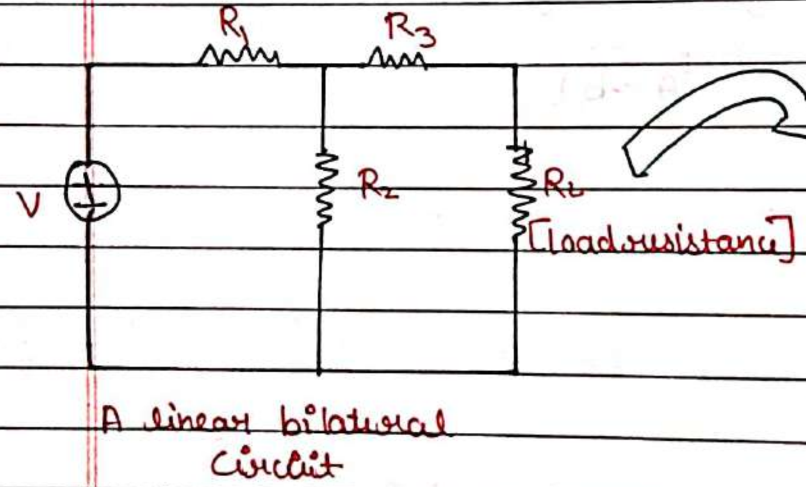
$$\boxed{I_T = \underset{[A\to B]}{I_1} + \underset{[A\to B]}{I_2}}$$

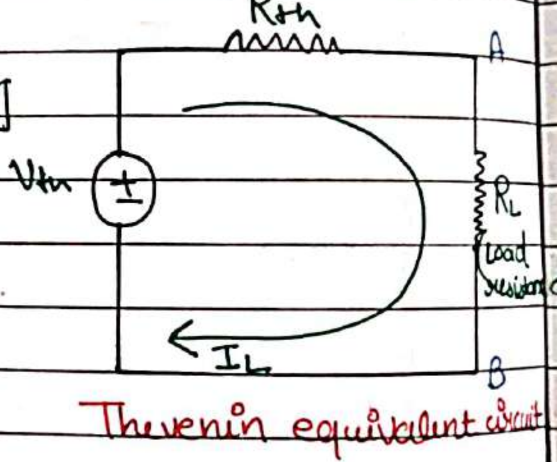
Limitations of Superposition Theorem :-

1. Applicable only for Linear and Bilateral circuit.
2. Circuit containing more than one energy sources
3. Can not used for power calculation.

Thevenin's Theorem :

In a linear bilateral complex circuit converted into a thevenin's equivalent circuit which contain a voltage source known as Thevenin's equivalent voltage ~~(VTH)~~ (V_{TH}) and an internal resistance known as thevenin's equivalent resistance (R_{TH}) connected in series with load resistance (R_L).





$$\boxed{I_L = \frac{V_{th}}{R_{th} + R_L}}$$

where, V_{th} = Thevenin voltage
Voltage across load terminal

R_{th} = Thevenin Resistance
Equivalent resistance b/w load terminal

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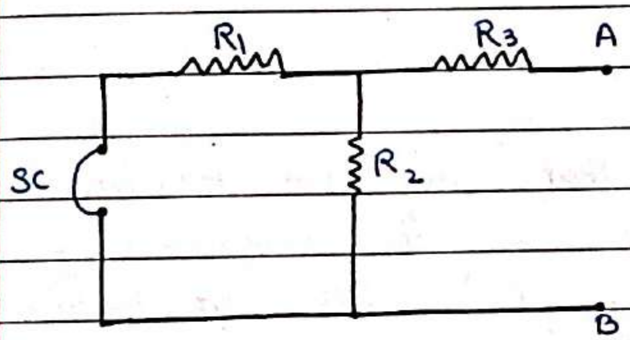
Steps for R_{th} :

1. Remove the load resistance in given network.
2. Repeace all energy sources into their internal resistances

Note: Voltage source ⇒ $R_{in}=0$ ⇒ SC
Current source ⇒ $R_{in}=\infty$ ⇒ OC

3. Find the equivalent resistance between load terminal.



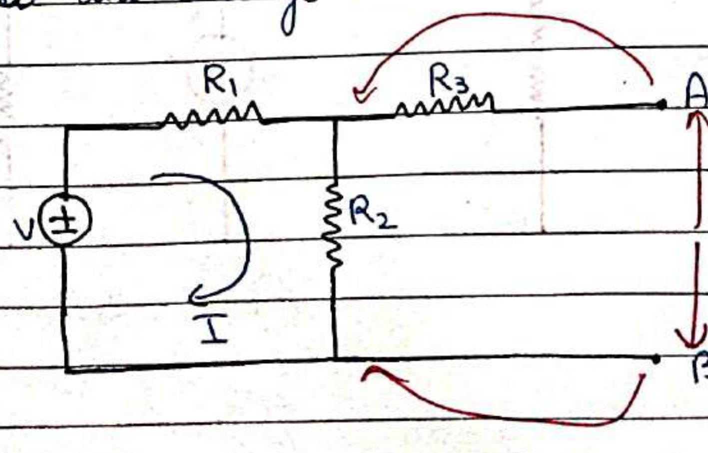
$R_{AB} = R_{th}$

$$R_{th} = [R_1 \parallel R_2] + R_3$$

$$\boxed{R_{th} = \frac{R_1 R_2}{R_1+R_2} + R_3} \quad [\Omega]$$

Steps for V_{th} :

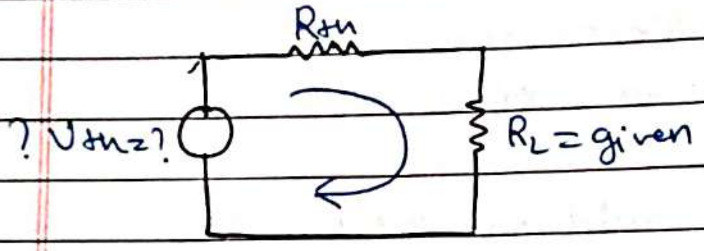
1. Remove the load resistance in given network.
2. Find the voltage across load terminal.



$V_{AB} = V_{th}$

$$\boxed{I = \frac{V}{R_L + R_2}}$$

V_{th} = Voltage drop across (R_2) = $\boxed{IR_2}$

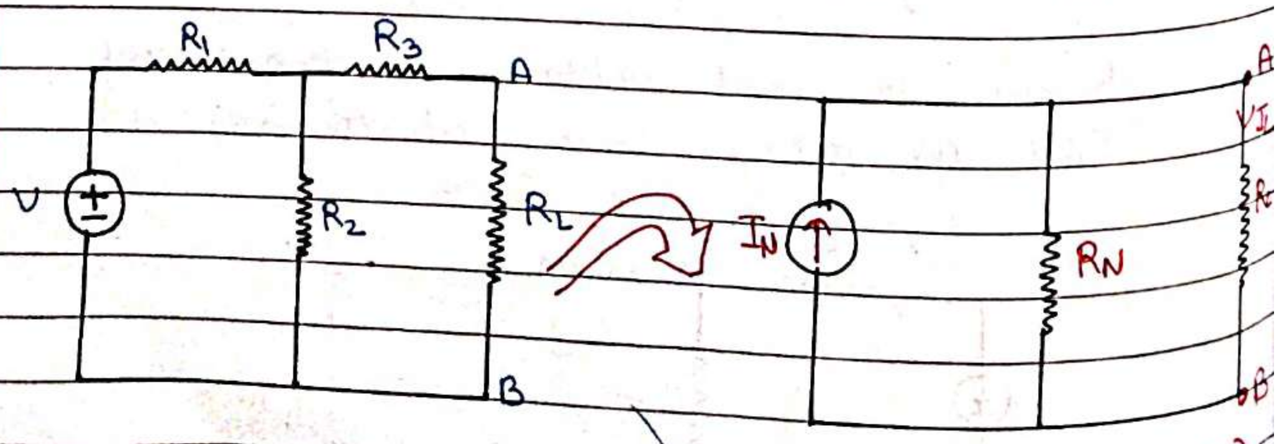


Norton's Theorem :-

This theorem states that, " any two terminal linear bilateral DC network can be replaced by a single current source (I_N) with a parallel equivalent resistance (R_N)
where,

I_N = Norton Current
= Short circuit current through Load terminal

R_N = Norton Resistance
= Equivalent resistance between load terminal.



(Norton equivalent circuit)

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$$\boxed{I_L = I_N \times \left(\frac{R_L}{R_N + R_L}\right)}$$

[given]

(By using current divider rule)

2) <u>Steps for R_N</u> :

$$\boxed{R_{th} = R_N}$$

Both are calculated as same.

2) <u>Steps for I_N</u> :

1. Remove the load resistance and shortcircuited the load terminal through a wire and find the short circuit current.

$$\boxed{I_{SC} = I_N}$$

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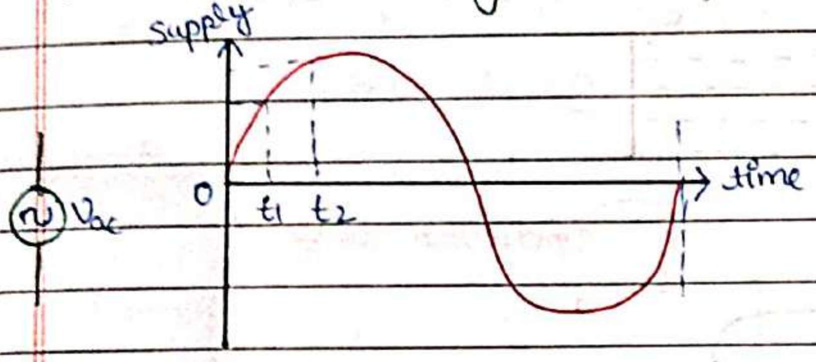
Module - 2

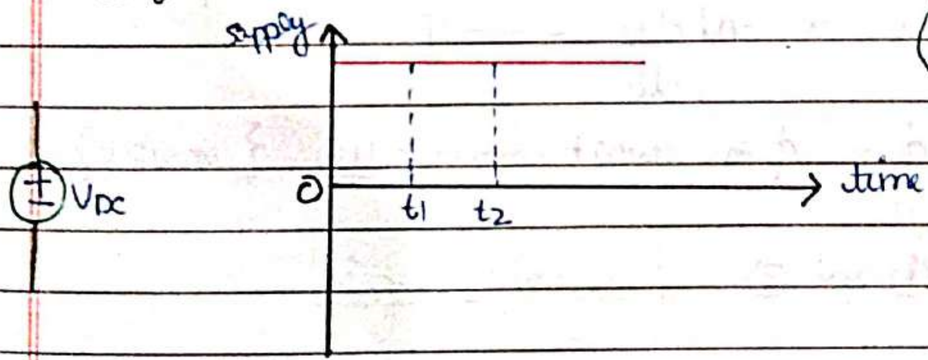
[A.C. Circuit Analysis]

AC supply [Alternating current]



magnitude and direction both are changes.

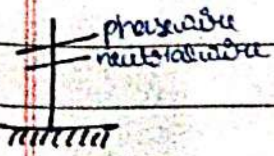
DC supply
↳ direct current



magnitude and direction both are constant.

AC supply:
- → 1 phase (1-Φ) AC supply
- → 3-phase (3-Φ) A.C. supply

1-phase: phase wire [Red / yellow / Blue]
Neutral wire [Black]



$\boxed{220V}$ $\boxed{\text{Line to Neutral}}$

phase to neutral

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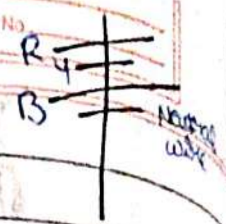
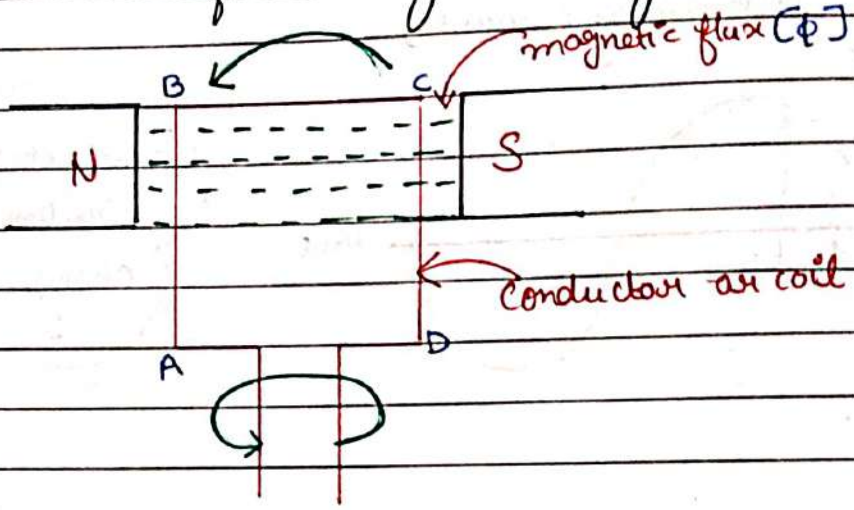


3-phase wire : $\boxed{\text{line to line}}$

$\boxed{\text{phase to phase}}$ $\boxed{440V}$

Generation of Alternating Quantity :-



According to Faraday's Law

$$e = -N\frac{d\phi}{dt} \text{ --- (1)}$$

$$\phi = \phi_m \cos\omega t \text{ --- (II)} \quad \{\theta = \omega t\}$$

from eqn (1) and (2)

$$e = -N\frac{d}{dt}(\phi_m \cos\omega t)$$

$$e = -N\phi_m \omega(-\sin\omega t)$$

$$e = -N\phi_m \omega \sin\omega t \text{ --- (1)}$$

for maximum value of voltage

$\sin\omega t = 1$

$$e_m = N\phi_m \omega \text{ --- (3)}$$

flux: magnetic lines of forces.

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$\theta = 0^\circ$
$\phi = \phi_m$

$\theta = 90$
$\phi =$

from eq^n (iii) and (iv)

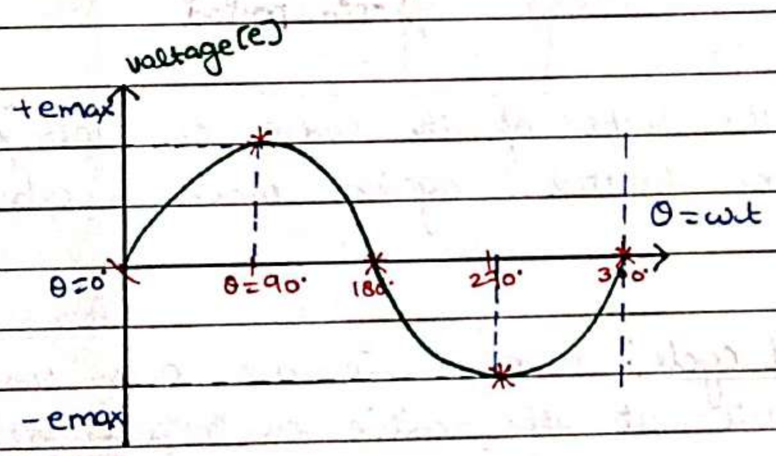
$$\boxed{e = e_m \sin\omega t}$$

OR

$$\boxed{V = V_m \sin\omega t}$$

Where, e = instantaneous value of voltage
e_m = maximum value of voltage
ω = angular velocity in radian.
t = time in sec.

Sine wave form



Similarly, alternating current

$$\boxed{i = i_m \sin\omega t}$$

* Advantages of Sinusoidal wave form :-

1. The sin wave can be expressed in simple mathematical form. It is very easy to write the equation for sin wave form.
2. Sinusoidal voltage and current produce less interference (noise) on telephone lines.
3. Sinusoidal voltage and current produce minimum disturbances in an electric circuit during operation.

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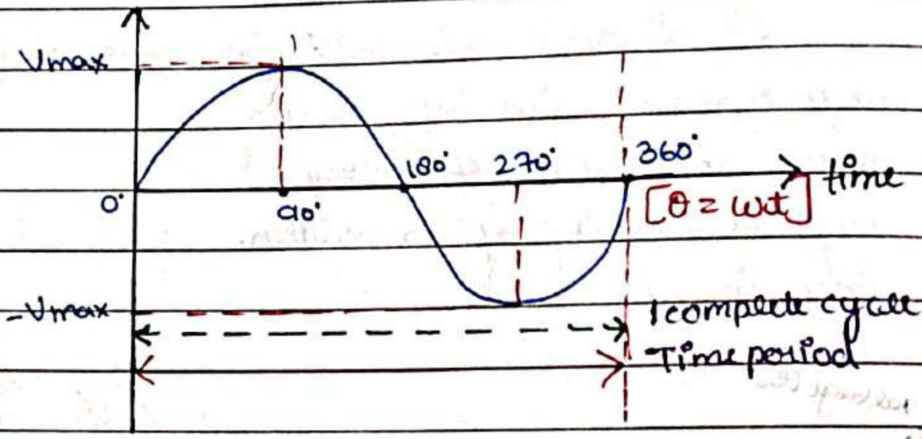
4 In A.C. machines of sinusodial voltage and current produce less iron and copper loses.

Important terms related to sin wave forms :-

Sine wave form.



1. Wave form : The shape of the curve of voltage or current when plotted against time is called wave form.

2. Alternation and cycle : When a sinewave goes through the one complete set of positive or negative values, it complete one alternation.

When a sine wave goes through the one complete set of positive and negative values, it complete one alternation.

3. Time Period :- The time taken by an alternating Quantity to complete its one cycle is known as its time period and denoted by (T) in Sec

4 Frequency : The number of cycle completed by an alternating quantity per second is known as its frequency. It is denoted by 'f' (f) in Hertz

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$$f = \left[\frac{1}{\text{Time period}}\right] \left[\frac{\text{cycle}}{\text{per second}}\right]$$

5. <u>Angular frequency</u>: It is the frequency expressed in electrical radian per second as one cycle of an alternating quantity corresponds to 2π radian, the angular frequency can be expressed as angular frequency $[\omega] = \frac{2\pi}{T}$

$$\boxed{\omega = 2\pi f} \quad [\text{rad/sec}]$$

A. velocity

<u>Note</u>: After every T sec the cycle of alternating Quantity must repeats.

6. <u>Peak Value</u> :- The maximum value attains by an alternating Quantity during positive or negative half cycle is called its Peak value / maximum value / <u>amplitude</u>.

<u>Values of an Alternating Quantity</u>:

1. Instantaneous Value
2. Maximum Value
3. Average or mean value
4. Root Mean Square [RMS] or Effective value

} Imp. real life

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Average or Mean Value: The average value of an alternating quantity is defined as the value which is obtained by averaging the all the ~~instantaneous~~ instantenous value over a period of half cycle.

practical importance of average value:

1. The average value is used for applications like rectifier circuit and battery charging.

2. The DC ammeter and voltmeter indicates the average or mean value.

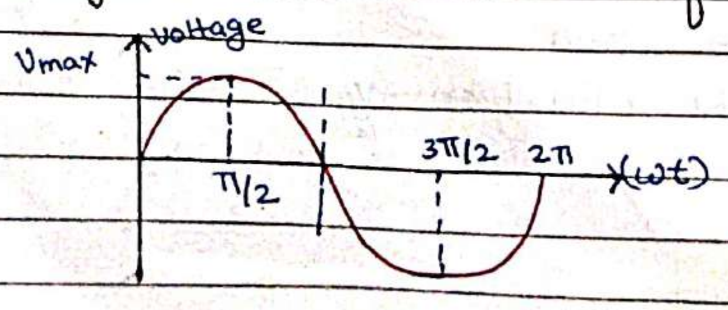
3. The average value of pure sinwave form is always zero.

Note: "For a symmetrical wave form, "the average value over a complete cycle is always zero. Hence the average value is define for half cycle only."

$$\boxed{f(t)_{avg.} = \frac{1}{T}\int_0^T f(t)\cdot dt}$$

where T= time period.

Average Value or mean value for a pure sinewave



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$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$V_{avg} = \frac{V_m}{\pi}\int_0^{\pi} \sin\omega t \cdot d(\omega t)$$

$$V_{avg} = \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi} d\omega t$$

$$V_{avg} = \frac{V_m}{\pi}\left[-\cos(\pi) + \cos(0)\right]$$

$$V_{avg} = \frac{V_m}{\pi}\left[+1+1\right]$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

$$\boxed{V_{avg} = 0.63 V_m.}$$

Similarly for alternating current

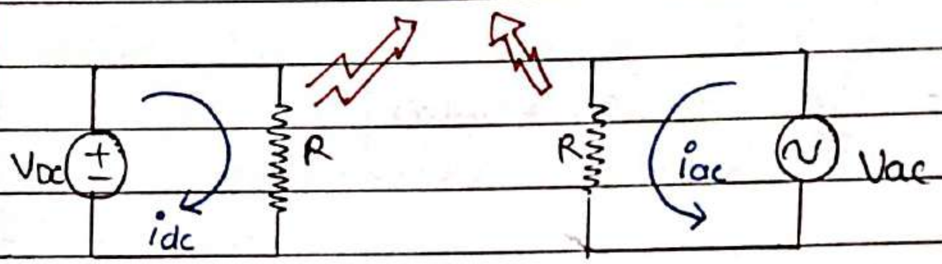
$$\boxed{i_{avg} = \frac{2i_m}{\pi}}$$

$$\boxed{i_{avg} = 0.63 V_m.}$$

Root Mean Square Value :-

The Rms value of an alternating current is, by definition is equal to that "value of direct current (DC) which will produce the same heat in the same time in the same resistor."

Same heat



$$\boxed{i_{dc} = [i_{ac}]_{RMS}}$$

$$\boxed{f(t)_{RMS} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\, dt}}$$

where T = time period
$T = t_2 - t_1$

RMs value or effective value for Pure Sine wave

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m \sin\omega t)^2\, d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} [\sin^2\omega t]\, d(\omega t)}$$

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$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\omega t)\cdot d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\omega t)\cdot d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\left[\pi - 0\right]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\cancel{\pi}} \times \cancel{\pi}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

$$\boxed{V_{rms} = 0.707 V_m}$$

Similarly for alternating current,

$$\boxed{i_{rms} = \frac{i_m}{\sqrt{2}}}$$

$$\boxed{i_{rms} = 0.707\, i_m}$$

For an

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Imp.

Form factor :- for an alternating quantity "the ratio of RMS value to the average value, is called Form factor."

$$\text{form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

$$\text{form factor} = \frac{V_{rms}}{V_{avg}} = \frac{\left(\frac{V_m}{\sqrt{2}}\right)}{\left(\frac{2V_m}{\pi}\right)}$$

$$\text{form factor} = \frac{V_m}{\sqrt{2}} \times \frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}} = \frac{3.14}{2\sqrt{2}}$$

$$\boxed{\text{form factor} = 1.11}$$

Imp.

Peak factor : for an alternating quantity, "the ratio of maximum value to the RMS value is called Peak factor".

$$\text{Peak factor} = \frac{\text{Maximum Value}}{\text{RMS value}}$$

$$\text{peak factor} = \frac{V_m}{\left(\frac{V_m}{\sqrt{2}}\right)} = \cancel{V_m} \times \frac{\sqrt{2}}{\cancel{V_m}}$$

$$\boxed{\text{Peak factor} = 1.41}$$

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Ques The equation of an alternating current is
$i = 141.4 \sin 314t$ Amp.
What is the RMS value of current and frequency?

Ans $i = i_m \sin \omega t$

i) RMS value of current $= \frac{i_{max}}{\sqrt{2}}$

$= \frac{141.4}{\sqrt{2}}$

$\boxed{i_{rms} = 100 \text{ Amp.}}$

(ii) $\omega = 314 = 2\pi f$

$f = \frac{314}{2\pi}$

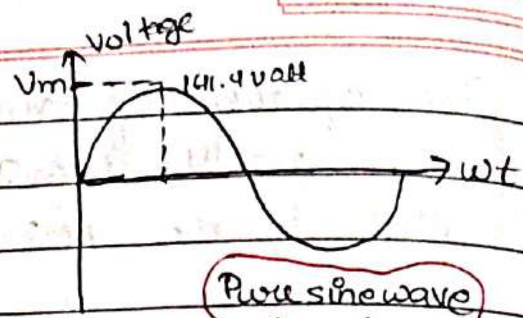
$= \frac{314}{2 \times 1.41}$

$\boxed{f = 50 Hz}$

Ques An alternating voltage is given by $v = 141.4 \sin 314 t$ volt
(i) find frequency
(ii) RMS value
(iii) Average value
(iv) The instantaneous value of voltage when t is 3ms.
(v) the time taken for the voltage to reach 100 volt. for the first time after passing through 0 value.

Ans frequency (f) =
$\omega = 314 = 2\pi f$

$\boxed{f = 50 Hz}$



Pure sinewave

(ii) $V_{RMS} = \dfrac{V_{max}}{\sqrt{2}}$

$= \dfrac{\cancel{1.41}\ 141.4}{1.41}$

$\boxed{V_{RMS} = 100\ \text{volt}}$

(iii) $V_{avg.} = \dfrac{2V_m}{\pi}$

$V_{avg.} = \dfrac{2 \times 141.4}{3.14} = 90.06\ \text{volt.}$

→ form factor $= \dfrac{V_{rms}}{V_{avg}}$

$= \dfrac{100}{90.06}$

form factor $= 1.11$

→ peak factor $= \dfrac{V_{max}}{V_{rms}}$

peak factor $= \dfrac{141.4}{100} = 1.41$

(iv) Instantaneous of voltage, when $t = 3ms = 3 \times 10^{-3} s$

$= 0.003 sec$

$v = 141.4 \sin(314 \times 0.003)$

$v = 141.4 \sin(0.94)$

$v = 141.4 \times 0.80$

$\boxed{v = 114.35\ \text{volt.}}$

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(v) $V = 100$ volt , time $(t) = ?$

$$100 = 141.4 \sin(314 \times t)$$

$$\frac{100}{141.4} = \sin(314t)$$

$$\sin^{-1}(0.70) = 314t$$

$$\frac{0.785}{314} = t$$

$$0.0025 = t$$

$$\boxed{2.5ms = t}$$

Ques A sinosodal alternating current of frequency 25 Hz has a maximum value of 100 Amp. How long will it take for the current to attains value of 20 and 100 Amp

An
$f = 25Hz$
$I_{max} = 100$ Amp

$f = \frac{1}{\text{time period}}$ $\qquad \omega = 2\pi f = \frac{25}{2 \times 3.14}$

$\omega = 2\pi 25$
$\omega = 50\pi$

$$i = i_m \sin \omega t$$
$$i = 100 \sin(50\pi)t$$

When value of current is 20 Amp

$$20 = 100 \sin 50\pi t$$

$$\frac{1}{5 \sin 50\pi} = t$$

$$t = 0.00128 \text{ sec.}$$

When value of current is 100 Amp.

$$100 = 100 \sin 50\pi t$$

$$\frac{100}{100 \sin 50\pi} = t$$

$$\boxed{t = 0.01 \text{ sec}}$$

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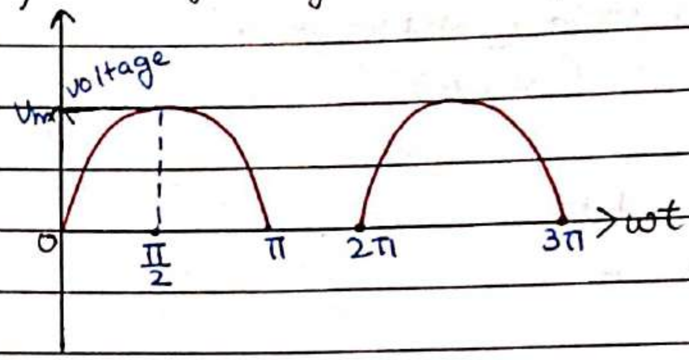
Ques find the average value, RMS value, form factor and peak factor for given wave form



Half wave rectifier

$T = t_2 - t_1$

$T = 2\pi - 0$ $\boxed{T = 2\pi}$

$$f(t)_{avg} = \frac{1}{T}\int_0^T f(t).dt$$

$$V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \; d(\omega t)$$

$$V_{avg} = \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$V_{avg} = \frac{V_m}{2\pi}\left[-\cos\pi + \cos 0\right]$$

$$V_{avg} = \frac{V_m}{2\pi}\left[1+1\right]$$

$$\boxed{V_{avg} = \frac{V_m}{\pi}} \qquad \boxed{V_{avg} = 0.318 V_m}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}\left[V_m \sin\omega t\right]^2 d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\sin^2\omega t \; d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi \times 2}\int_0^{\pi}(1-\cos 2\omega t)\, d(\omega t)}$$

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$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}\left[\pi - 0\right]}$$

$$\boxed{V_{rms} = \frac{V_m}{2}}$$

$$\text{form factor} = \frac{V_{rms}}{V_{avg}} \Rightarrow \frac{0.5\cancel{V_m}}{0.318\cancel{V_m}}$$

$$\boxed{\text{form factor} = 1.57}$$

$$\text{peak factor} = \frac{V_{max}}{V_{rms}} = \frac{V_m}{V_{rms}}$$

$$= \frac{\cancel{V_m}}{0.5\cancel{V_m}}$$

$$\boxed{\text{peak factor} = 2}$$

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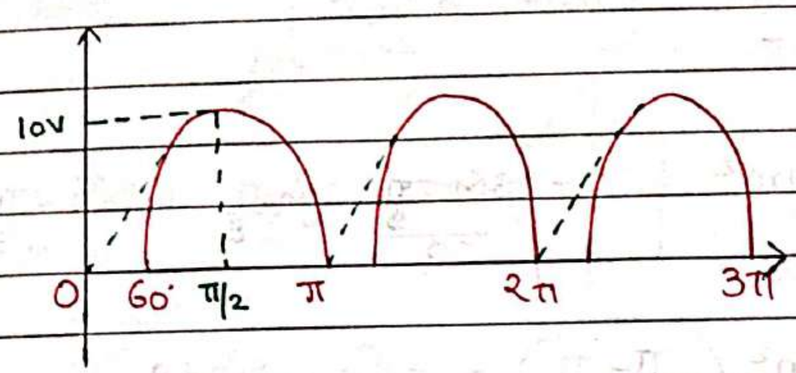
Ques



Find the average value, RMS value, form factor, peak factor

$$f(t)_{avg} = \frac{1}{T}\int_0^T f(t)\cdot dt$$

$$V_{avg} = \frac{1}{\pi}\int_{\pi/3}^{\pi} 10\sin\omega t\, d(\omega t)$$

$$V_{avg} = \frac{10}{\pi}\left[-\cos\omega t\right]_{\pi/3}^{\pi}$$

$$V_{avg} = \frac{10}{\pi}\left(-\cos\pi + \cos\frac{\pi}{3}\right)$$

$$V_{avg} = \frac{10}{\pi}\left(1+\frac{1}{2}\right)$$

$$V_{avg} = \frac{\cancel{30}^{15}}{\cancel{2}\pi} \Rightarrow 4.77\,\text{volt}$$

$$V_{rms} = \sqrt{\frac{100}{\pi}\int_{\pi/3}^{\pi}\left[V_m\sin\omega t\right]^2 d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{100}{\pi}\int_{\pi/3}^{\pi}\sin^2\omega t\, d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{100}{\pi\times 2}\int_{\pi/3}^{\pi}(1-\cos 2\omega t)\, d\omega t}$$

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$$V_{rms} = \sqrt{\frac{100}{2\pi} \left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\frac{\pi}{3}}^{\pi}}$$

$$V_{rms} = \sqrt{\frac{50}{\pi} \left[\pi - \frac{\pi}{3} - \frac{\sin 2\pi}{2} + \frac{\sin \frac{2\pi}{3}}{2}\right]}$$

$$V_{rms} = \sqrt{\frac{50}{\pi} \left(\pi - \frac{\pi}{3}\right) - \left(0 - \frac{\sin \frac{2\pi}{3}}{2}\right)}$$

$$V_{rms} = \sqrt{\frac{50}{\pi}\left[\left(\frac{2\pi}{3}\right) + 0.43\right]}$$

$$V_{rms} = \sqrt{\frac{50}{\pi}\left[2.09 + 0.43\right]}$$

$$V_{rms} = \sqrt{\frac{50}{\pi}\left[2.52\right]}$$

$$\boxed{V_{rms} = 6.33 \text{ volt.}}$$

$$\text{form factor} = \frac{6.33}{4.77}$$

$$\boxed{\text{form factor} = 1.32}$$

$$\text{peak factor} = \frac{10}{6.33}$$

$$\boxed{\text{peak factor} = 1.57}$$

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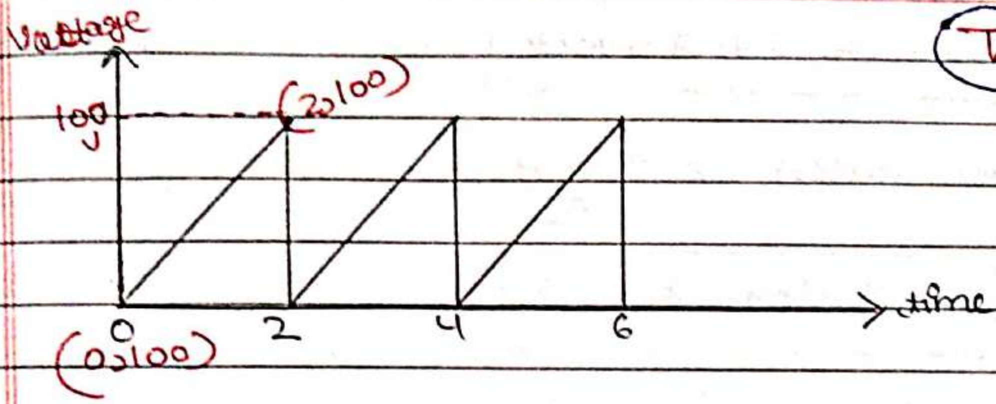
Q



Triangular waveform

Find Average value, RMS value, form factor and peak factor for the given wave form.

$$f(t)\,avg = \frac{1}{T}\int_0^T f(t)\cdot dt$$

$$V_{avg} = \frac{1}{2}\int_0^2 50t\,.\,d(t)$$

$$V_{avg} = \frac{50}{2}\int_0^2 t\cdot dt$$

$$V_{avg} = \frac{50}{2\times 2}\left[t^2\right]_0^2$$

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{100-0}{2-0}(x-0)$$

$$V_{avg} = \frac{50}{2\times 2}\left[4-0\right]$$

$$y = 50x$$
$$v = 50t$$

$$\boxed{V_{avg} = 50 \text{ volt}}$$

$$V_{rms} = \sqrt{\frac{1}{2}\int_0^2 (50t)^2 dt}$$

$$V_{rms} = \sqrt{\frac{50\times 50}{2\times 3}\left[t^3\right]_0^2}$$

$$V_{rms} = \sqrt{\frac{50\times 50}{2\times 3}\left[8-0\right]}$$

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$$\boxed{V_{rms} = 57.73\ \text{volt}}$$

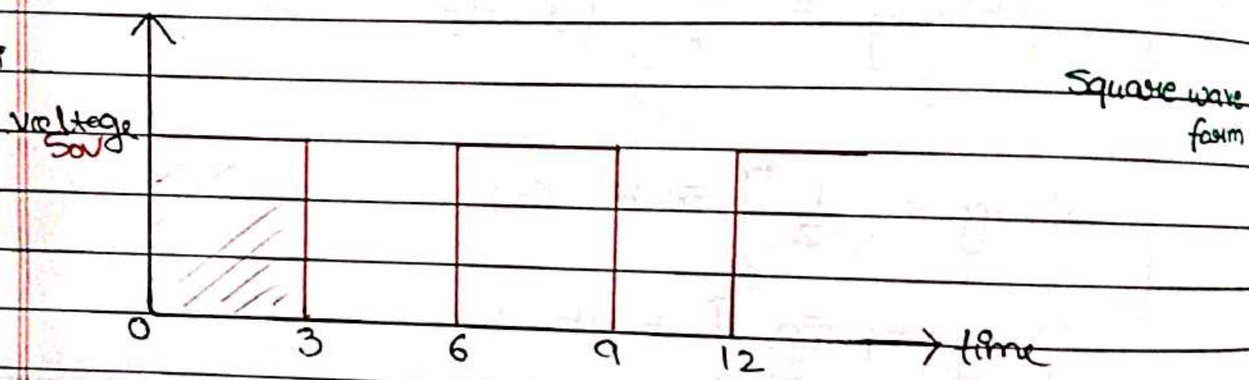
$$\text{form factor} = \frac{57.73}{50}$$

$$\boxed{\text{form factor} = 1.15}$$

$$\text{peak factor} = \frac{100}{57.73}$$

$$\boxed{\text{peak factor} = 1.73.}$$

Ques 2.



T = 6

$$f(t)_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$V_{avg} = \frac{1}{6}\int_0^3 50.dt$$

$$V_{avg} = \frac{50}{6}\int_0^3 1.dt$$

$$V_{avg} = \frac{50}{6}\left[t\right]_0^3$$

$$V_{avg} = \frac{\cancel{50}^{25}}{\cancel{6}\times\cancel{3}_{2}}\left[\cancel{3}-0\right]$$

$$\boxed{V_{avg} = 25\ \text{volt}}$$

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$$V_{rms} = \sqrt{\frac{1}{6}\int_0^3 (50)^2 \cdot dt}$$

$$= \sqrt{\frac{50 \times 50}{6}\int_0^3 dt}$$

$$= \sqrt{\frac{50 \times 50}{6}[3]}$$

$$\boxed{V_{rms} = 35.35 \text{ valt}}$$

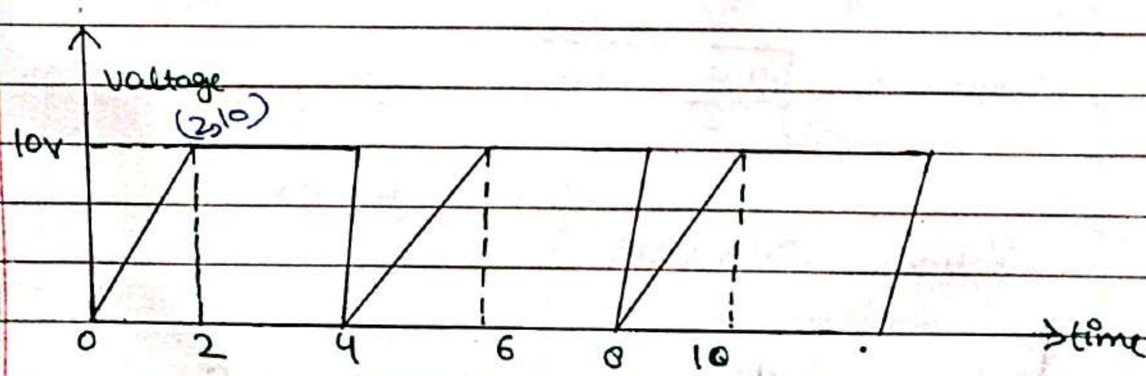
$$\text{form factor} = \frac{35.35}{25}$$

$$\boxed{\text{form factor} = 1.41}$$

$$\text{peak factor} = \frac{50}{35.35} \Rightarrow 1.41$$

$$\boxed{\text{peak factor} = 1.41}$$

Ans



$$V_{avg} = \frac{1}{4}\left[\int_0^2 5t \cdot dt + \int_2^4 10 \cdot dt\right]$$

$$V_{avg} = \frac{1}{4}\left[\frac{5}{2}\left[t^2\right]_0^2 + 10\left[t\right]_2^4\right]$$

$$V_{avg} = \frac{1}{4}\left[\frac{5}{2}[4] + 10[4-2]\right]$$

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$$V_{avg} = \frac{1}{4}[10 + 20]$$

$$= \frac{1}{4} \times 30$$

$$\boxed{V_{avg} = 7.5 \text{ volt}}$$

$$V_{rms} = \sqrt{\frac{1}{4}\left(\int_0^2 5.t\,dt + \int_2^4 10.dt\right)^2 .dt}$$

$$V_{rms} = \sqrt{\frac{1}{4} \times 5 \times 5 \int_0^2 t^2 .dt + 10[t]_2^4}$$

$$V_{rms} = \sqrt{\frac{25}{4 \times 2} \times 4 + 10[4-2]}$$

$$V_{rms} = \sqrt{\frac{25}{2} + 10 \times 2}$$

$$V_{rms} = \sqrt{\frac{25}{2} + 20}$$

$$V_{rms} = 8.16 \text{ volt}$$

$$\text{form factor} = \frac{8.16}{7.5} = 1.08$$

$$\text{peak factor} = \frac{10}{8.16} = 1.22$$

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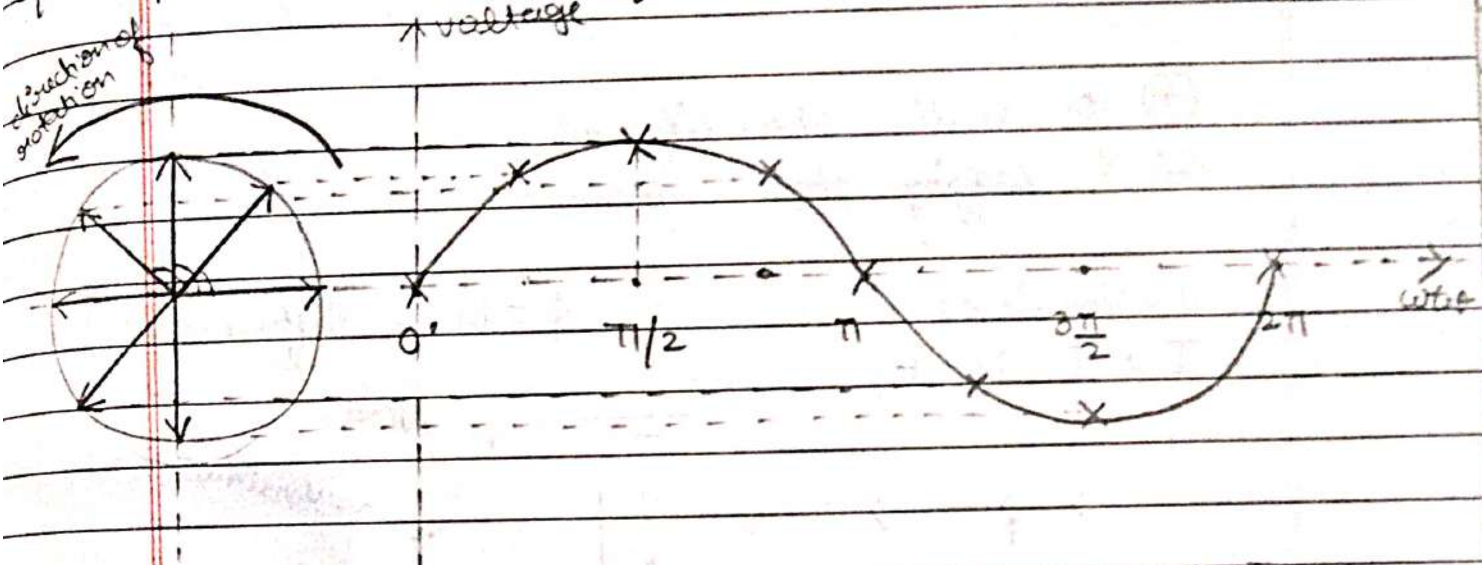
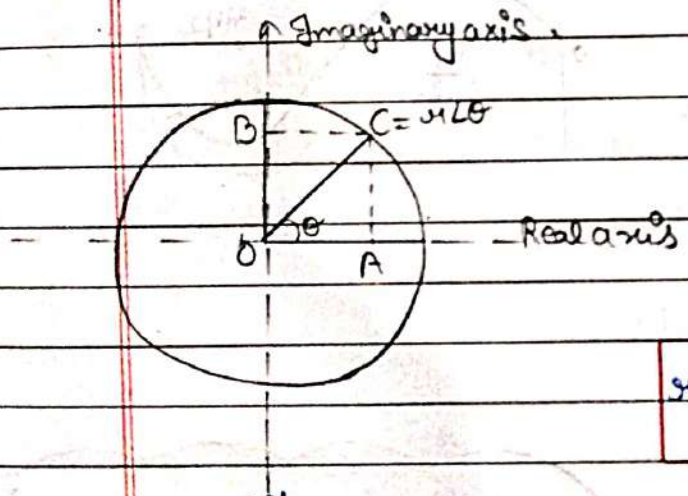
phasor representation of an alternating Quantity :





$OA = r\cos\theta$

$OB = r\sin\theta$

$OC = OA + j\,OB$

$OC = r\cos\theta + j\,r\sin\theta$

$OC = a + jb$ rectangle form

$$\boxed{r\angle\theta = OC = r(\cos\theta + j\sin\theta)}$$

Where,

$r\angle\theta \Rightarrow$ Polar form

$r(\cos\theta + j\sin\theta) \Rightarrow$ Rectangle form

alternating voltage or current can be written as

$V = V_m \sin\omega t$

ωt → phase angle

V_m → magnitude

$$\boxed{V = V_m \angle 0^\circ}$$

$$\boxed{V = V_m \sin(\omega t + \phi)}$$

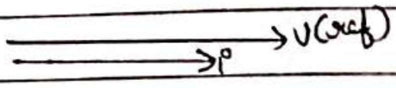
(+) ⇒ Leading phase difference

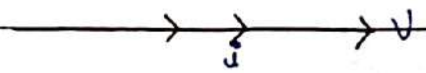
(−) ⇒ Lagging phase difference.

$V = V_m \sin \omega t$

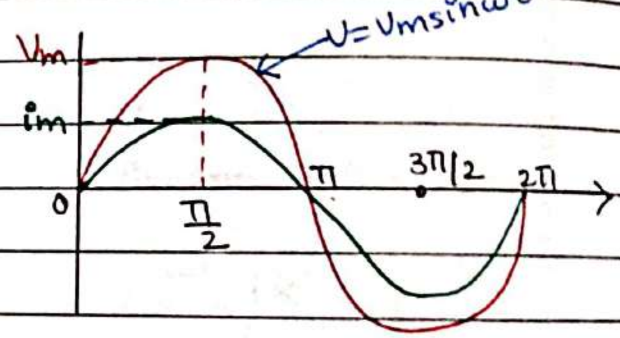
$I = I_m \sin \omega t$

[ϕ = phase difference = 0]

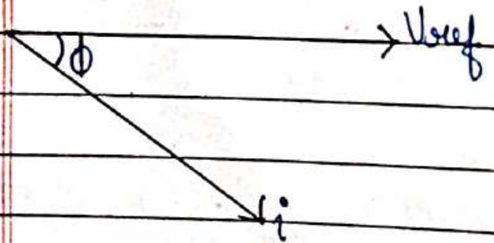




zero phase difference

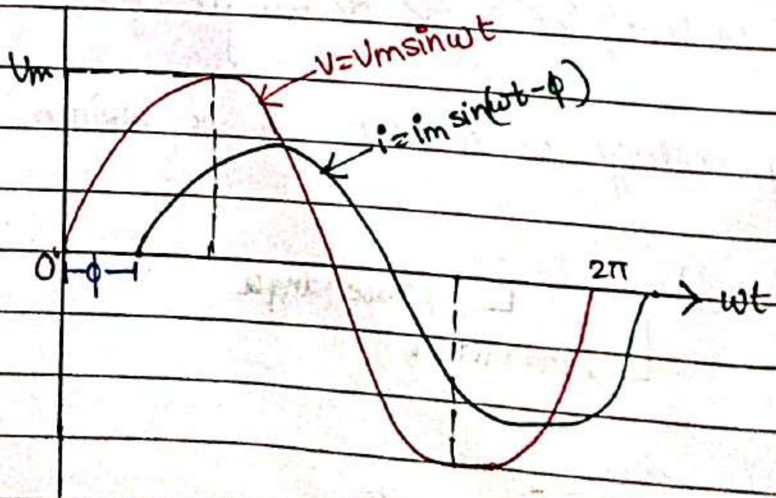


$$\boxed{\begin{array}{l} V = V_m \sin \omega t \\ I = I_m \sin(\omega t - \phi) \end{array}}$$



Current lags voltage by ϕ

Lagging phase difference

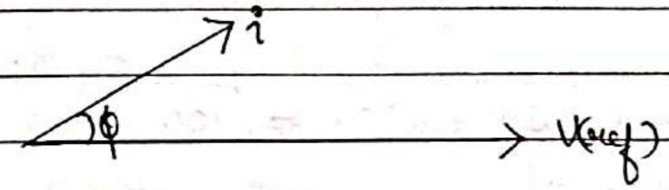


Leading phase difference

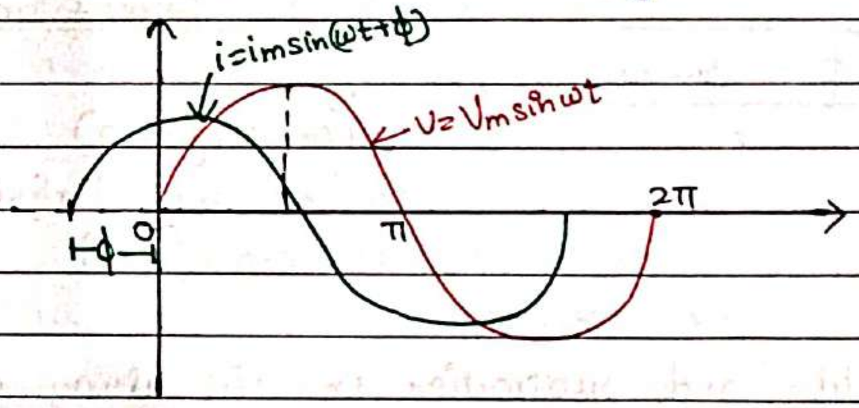
$V = V_m \sin \omega t$

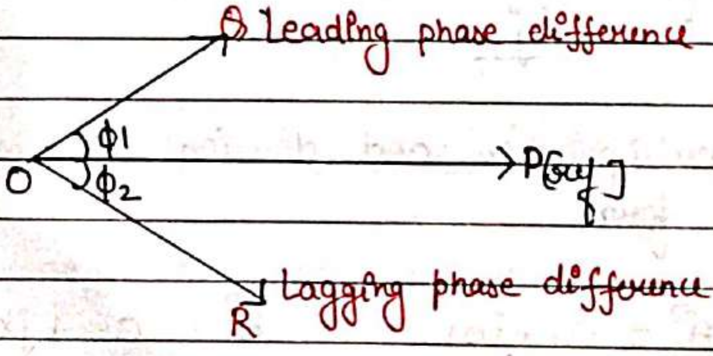
$I = I_m \sin(\omega t + \phi)$

Leading current



Current leads voltage by ϕ°
or voltage lags current by ϕ°





Polar form ⇄ Rectangle form

$r \angle \phi \to$ phase angle
$r \downarrow$ magnitude

$r(\cos\phi + j\sin\phi)$
$x + jy \to$ imaginary value
$x \downarrow$ real value

$\boxed{x = \sqrt{x^2+y^2}}$

$\tan\phi = \frac{y}{x}$

$\boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$

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Rectangle form : $(8+j6)$

$\Downarrow$

polar form $[r \angle \phi]$

$$r = \sqrt{(8)^2+(6)^2} = \sqrt{100}$$

$$\boxed{r = 10}$$

$$\phi = \tan^{-1}\left(\frac{6}{8}\right)$$

$$\boxed{\phi = 36.86}$$

$$\boxed{10 \angle +36.86^\circ}$$

Polar form

$r(\cos\phi + j\sin\phi)$

$10[\cos(36.86) + j\sin(36.86)]$

$10[0.8 + j0.6]$

Note :- • For addition and subtraction we will always use rectangle form.
$x+jy$

• For multiplication and division we will always use polar form.

Example : $\vec{A} = a_1 + jb_1$ $\quad \vec{B} = a_2 + jb_2$

(i) $A+B = (a_1+a_2) + j(b_1+b_2)$

(ii) $A-B = (a_1-a_2) + j(b_1-b_2)$

(iii) $A \times B = r_1 \angle \phi_1 \times r_2 \angle \phi_2 = r_1 r_2 \angle \phi_1 \cdot \phi_2$

(iv) $A \div B = \dfrac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \dfrac{r_1}{r_2} \angle \dfrac{\phi_1}{\phi_2}$

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Example: AC voltages are given as

$v_1 = 10\sin\omega t$

$v_2 = 20\sin(\omega t - 60^\circ)$

$v_3 = 5\sin(\omega t + 45^\circ)$

Find the RMS value of resultant voltage and draw phasor diagram also.

Soln

$v_1 = 10\sin\omega t = r\angle\phi = 10\angle 0^\circ$

$v_2 = 20\sin(\omega t - 60^\circ) = 20\angle -60^\circ$

$v_3 = 5\sin(\omega t + 45^\circ) = 5\angle +45^\circ$

$V_T = V_1 + V_2 + V_3$

$= 10\angle 0^\circ + 20\angle -60^\circ + 5\angle 45^\circ$

$V_T = 10 + j0 + 10 - j17.32 + 3.53 + j3.53$

$\boxed{[V_T = 25.53 - j13.79] \text{ volt}}$ rectangle form

$V_T = r\angle\phi$

$\boxed{V_T = 27.27\angle -30.37^\circ \text{ volt}}$ polar form

RMS value of resultant voltage

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$V_{rms} = \frac{27.27}{\sqrt{2}}$$

$$\boxed{V_{rms} = 19.28 \text{ volt.}}$$

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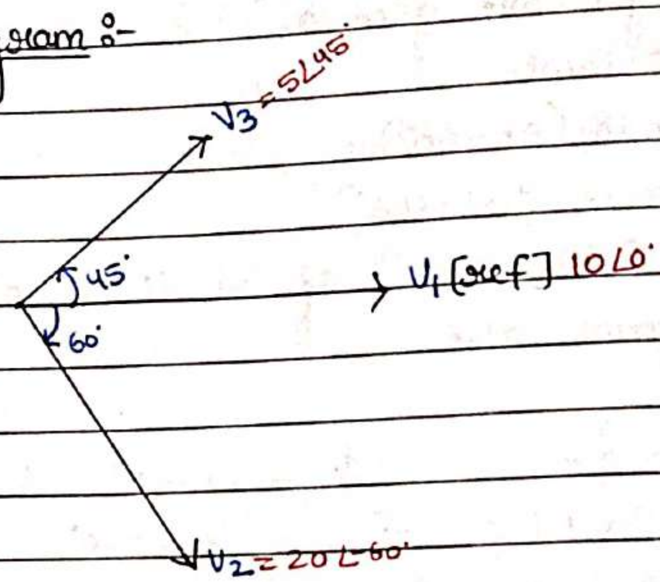
Phasor diagram :-



Standard form :-

$$V = 27.27 \sin(\omega t - 30.37°)$$

Operator j :

(i) An alternating voltage or current is a phasor quantity but since the instantaneous value are changing continuously, it must be represented by a rotating vector phasor.

ii) A phasor is a vector rotating at a constant angular velocity.

iii) "j" is define as an operator which turns a phasor by 90° counter clock wise direction without changing the magnitude of phasor.

$$\boxed{j = 1\angle 90°}$$

$$rj = r\angle 90°$$

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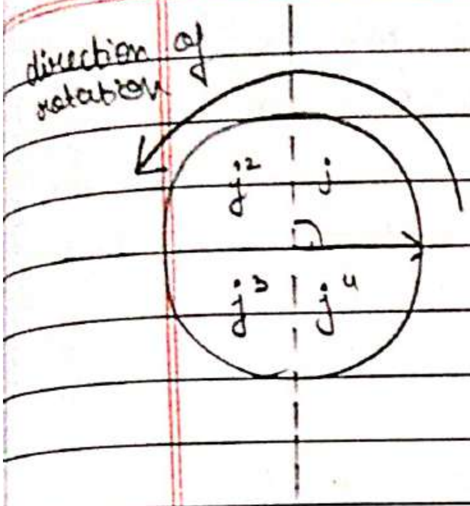

$j = \sqrt{-1}$

$j^2 = (\sqrt{-1})^2 = 1$

$j^3 = j^2 \cdot j = -1\sqrt{-1}$

$j^4 = j^2 \cdot j^2 = 1$

Ques Draw the phasor diagram showing the following voltages and find the rms value of resultant voltage

$v_1 = 100 \sin 500t$

$v_2 = 200 \sin\left(500t + \frac{\pi}{3}\right)$

$v_3 = -50 \cos(500t)$

$v_4 = 150 \sin\left(500t - \frac{\pi}{4}\right)$

Solⁿ

$v_1 = 100 \sin 500t \quad = 100 \angle 0^\circ$

$v_2 = 200 \sin\left(500t + \frac{\pi}{3}\right) = 200 \angle 60^\circ$

$v_3 = -50 \cos\left(\frac{\pi}{2} + 500t\right)$

$v_3 = -50 \sin 500t \quad = -50 \angle 90^\circ$

$v_4 = 150 \sin\left(500t - \frac{\pi}{4}\right) = 150 \angle -45$

$V_R = v_1 + v_2 + v_3 + v_4$

$= 100 \angle 0^\circ + 200 \angle 60^\circ + 150 \angle -45^\circ - 50 \angle 90^\circ$

$V_R = (100 + 0j) + (100 + 173.20j) - (0 + 50j)$
$\quad + (106.06 - 106.06j)$

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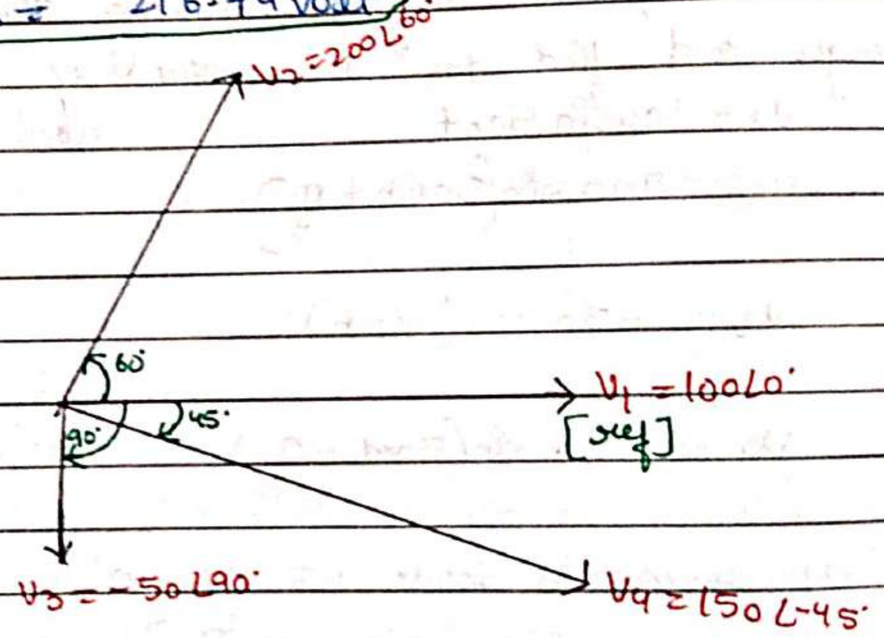
$V_R = 306.06 + 17.14j$

$$\boxed{V_R = 306.53 \angle +3.20^\circ}$$

Rms value of voltage

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$\boxed{V_{rms} = 216.74 \text{ volt}}$$



$V = V_m \sin(\omega t \pm \phi)$ (Instantaneous value of voltage)

$$\boxed{V = 306.53 \sin(500t + 3.2^\circ)}$$

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Que Find the rms value of resultant current

$i(t) = 14.14 \sin 120t + 7.07 \cos(120t + 30^\circ)$

$I_1 = 141.14 \sin 120t$

$I_2 = 7.07 \cos(120t + 30^\circ)$

$I_2 = 7.07 \cos\left(120t + \frac{\pi}{2} + \frac{\pi}{6}\right)$

$I_2 = 7.07 \cos\left[120t + \frac{4\pi}{6}\right]$

$I_2 = 7.07 \cos[120t + 120^\circ]$

$I_T = I_1 + I_2$

$= 141.14 \angle 0^\circ + 7.07 \angle 120^\circ$

$\boxed{I_R = 10.61 + 6.12j \text{ Amp}}$ (Rectange form)

$\Downarrow$

polar form

$\boxed{i(t) = 12.24 \angle 30^\circ \text{ Amp.}}$

$i_{rms} = \frac{i_{max}}{\sqrt{2}} = \frac{12.24}{\sqrt{2}}$

$\boxed{i_{rms} = 8.65 \text{ Amp}}$

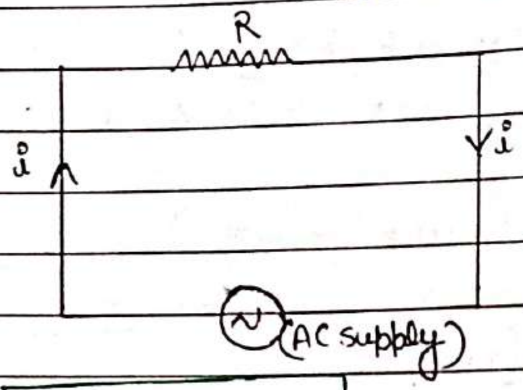
instantaneous value of current

$i(t) = i_m \sin(\omega t \pm \phi)$

$i(t) = \overset{12.24}{\cancel{i_m \sin}}\ \cancel{(\omega t \pm}\ \sin(120t\ 30^\circ)$ Amp.

A.C. Circuit Analysis:

A.C. circuit containing pure resistance — mmm



$$\boxed{V = V_m \sin \omega t}$$

The applied voltage is

$$V = V_m \sin \omega t \quad \text{------ (1)}$$

current through the resistance

$$i = \frac{V}{R} \quad \text{———— (2)}$$

from (1) and (2)

$$i = \frac{V_m \sin \omega t}{R} \quad \text{———— (3)}$$

for maximum value of current

$$\sin \omega t = 1$$

$$i_m = \frac{V_m}{R} \quad \text{———— (4)}$$

from (3) and (4)

$$i = i_m \sin \omega t$$

$$V = V_m \sin \omega t$$

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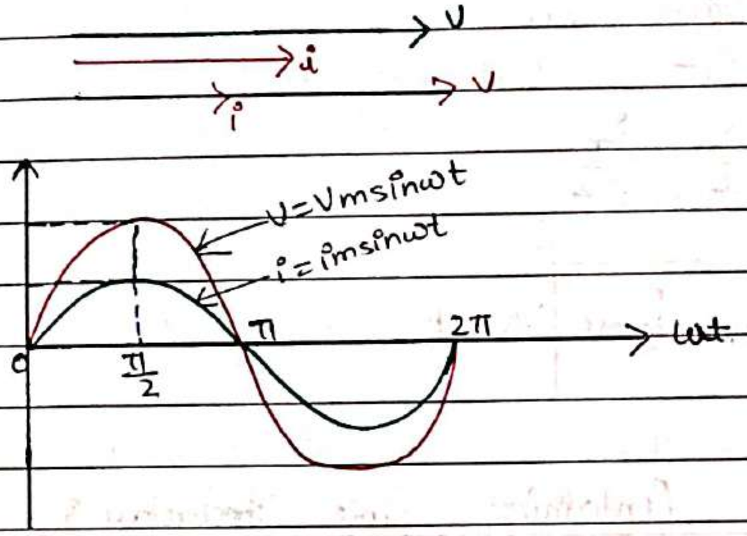
phase difference between current and applied voltage is zero
$[\phi = 0^\circ]$



Instantenous power (P) :

$$(P) = v \cdot i$$

$$P = V_m \sin\omega t \cdot i_m \sin\omega t$$

$$P = V_m i_m \sin^2\omega t$$

$$P = \frac{V_m i_m}{2}(1 - \cos 2\omega t)$$

$$P = \underbrace{\frac{V_m i_m}{2}}_{\text{constant power}} - \underbrace{\frac{V_m i_m \cdot \cos 2\omega t}{2}}_{\text{fluctuating power}}$$

for a complete ac cycle, the average of power

$$P_{avg} = \frac{V_m i_m}{2} - \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m i_m}{2}(\cos 2\omega t \cdot d\omega t)$$

$$P_{avg} = \frac{V_m i_m}{2} - \frac{V_m i_m}{8\pi}\left[t \sin 2\omega t\right]_0^{2\pi}$$

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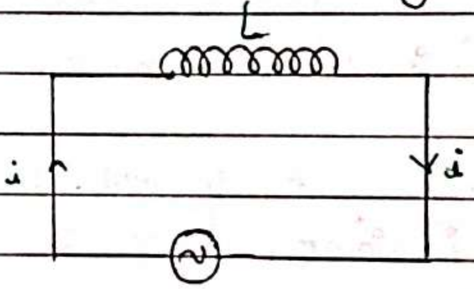
$$P_{avg} = \frac{V_m i_m}{2} - \frac{V_m i_m}{8\pi}\left[+\sin 2(2\pi) - \sin 2(0)\right]$$

$$P_{avg} = \frac{V_m i_m}{2} - 0$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}}$$

$$\boxed{P_{avg} = V_{rms} \cdot i_{rms}} \text{ watt}$$

(b) A.C. Circuit Containing Pure Inductor :



$$V = V_m \sin\omega t \quad \text{——— (1)}$$

$$V = L\frac{di}{dt} \quad \text{——— (2)}$$

from eq^n (1) and eq^n (2)

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t \cdot dt$$

integrating on both sides

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$$i = \frac{V_m}{L}\int \sin\omega t \cdot dt$$

$$i = \frac{V_m}{\omega L}\left[-\cos\omega t\right]$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{——— (3)}$$

for maximum value of current

$$\sin\left(\omega t - \frac{\pi}{2}\right) = 1$$

$$i_m = \frac{V_m}{\omega L} \quad \text{——— (4)}$$

The term $[\omega L]$ is called inductive Reactance and denoted by X_L

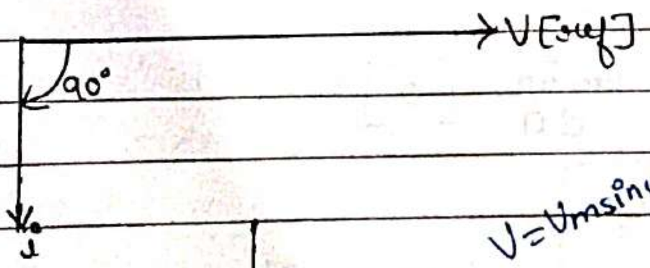
$$\boxed{X_L = \omega L = (2\pi f)L} \quad [\Omega]$$

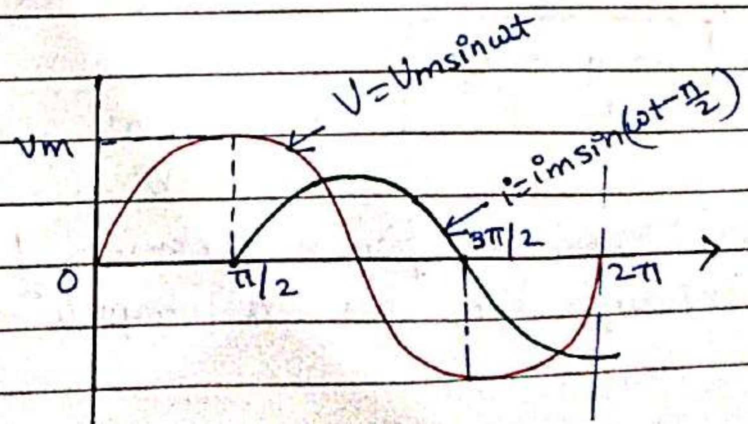
from eq^n (ii) and (4)

$$\boxed{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

Current lags voltage by 90°

$$\boxed{V = V_m \sin\omega t}$$





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Avg. power (P) = v.i. is zero in case of pure inductor. ~~Pove~~ Prove that

$$\text{power } (P) = v.i.$$
$$p = V_m \sin\omega t \cdot i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$p = -V_m i_m \sin\omega t \cos\omega t$$
$$p = \frac{-V_m i_m}{2}\left[2\sin\omega t \cos\omega t\right]$$

$$p = \frac{-V_m i_m}{2}\left[\sin 2\omega t\right] \qquad \{2\sin A\cos A = \sin 2A\}$$

average value of power, for a complete cycle

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m i_m}{2}\sin 2\omega t \cdot d(\omega t)$$

$$P_{avg} = \frac{-V_m i_m}{8\pi}\left[-\cos 2\omega t\right]_0^{2\pi}$$

$$P_{avg} = \frac{-V_m i_m}{8\pi}\left[-\cos 2(2\pi) + \cos^2(0)\right]$$

$$P_{avg} = \frac{-V_m i_m}{8\pi}\left[-1+1\right]$$

$$P_{avg} = \frac{-V_m i_m}{8\pi}\left[0\right]$$

$$\boxed{P_{avg} = 0}$$

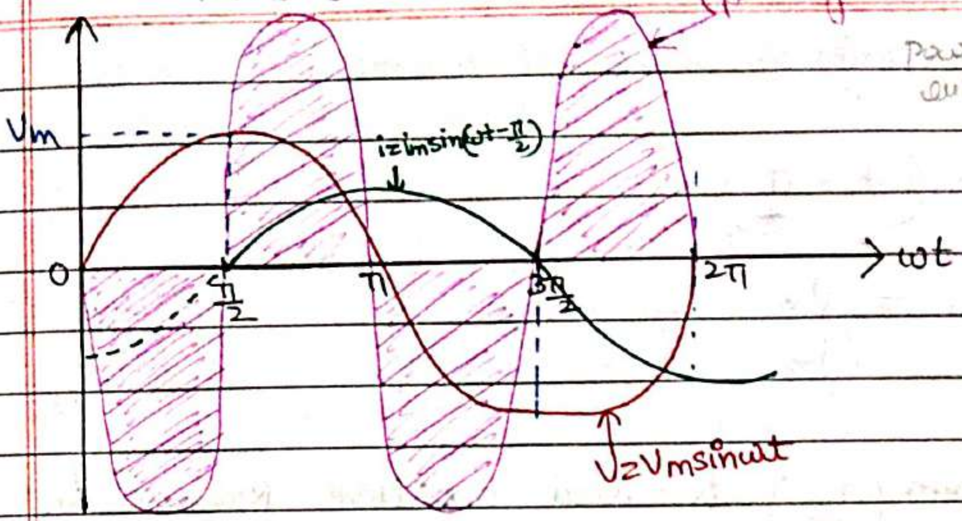
"Hence average power consumed by a pure inductor is always zero for a complete ac cycle".

wattless → Voltage nhi hoga

$P = V_r I_r \cos\phi$

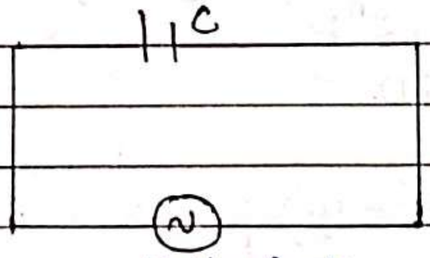
$Z = R + jX_L$

R and C are wattless component



Power consume karega wattless component

39) A.C. circuit containing only pure capacitor:



$V = V_m \sin\omega t$ ——— (1)

Current through the capacitor

$$i = C\frac{dv}{dt} \quad \text{——— (2)}$$

from (1) and (2)

$$i = C\frac{d}{dt}(V_m \sin\omega t)$$

$$i = V_m \omega C \cos\omega t$$

$$i = V_m \omega C \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{——— (iii)}$$

for maximum value of current

$$\sin\left(\omega t + \frac{\pi}{2}\right) = 1$$

$$i_m = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \text{ ——— (iv)}$$

The term $\left(\frac{1}{\omega C}\right)$ is called capicitive Reactance and denoted by X_C.

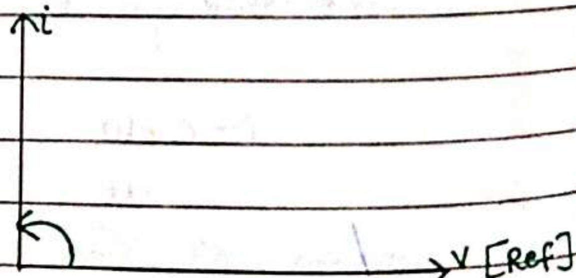
$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} \quad [\Omega]$$

from (iii) and (iv)

$$\boxed{i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$\boxed{V = V_m \sin \omega t}$$

[Current leads voltage by 90°]



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Average power (P) is zero in case of pure capacitor. Prove that:

$$P = V_m \sin\omega t \cdot i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$P = V_m i_m \sin\omega t \cos\omega t$$

$$P = \frac{+V_m i_m}{2}\left[2\sin\omega t \cos\omega t\right]$$

$$P = \frac{+V_m i_m}{2}\left[\sin 2\omega t\right]$$

Average value of power, for a complete cycle

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{+V_m i_m}{2} \sin 2\omega t \, d(\omega t)$$

$$P_{avg} = \frac{+V_m i_m}{8\pi}\left[+\cos 2\omega t\right]_0^{2\pi}$$

$$P_{avg} = \frac{+V_m i_m}{8\pi}\left[-[\cos 2(2\pi) + \cos^2(0)\right]$$

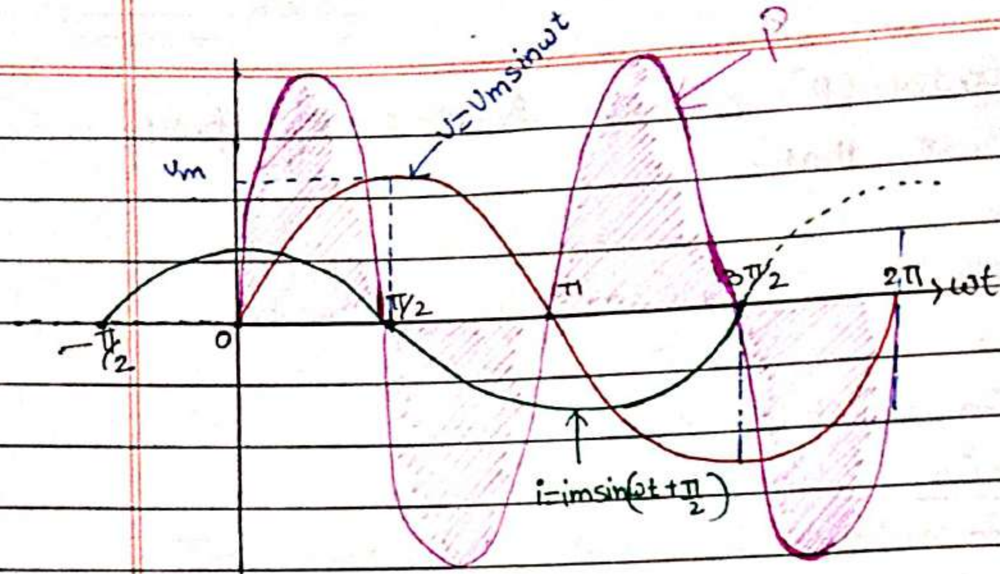
$$P_{avg} = \frac{+V_m i_m}{8\pi}\left[-1+1\right]$$

$$P_{avg} = \frac{+V_m i_m}{8\pi}\left[0\right]$$

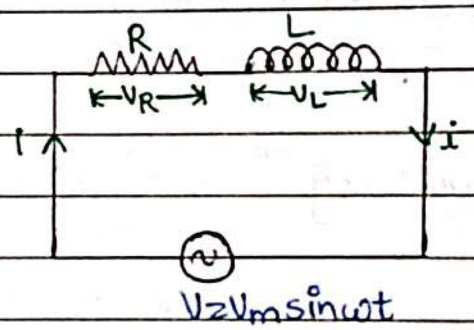
Both are energy storing elements first they store Power and then they release after some time.

$$\boxed{P_{avg} = 0}$$

"Hence average power consumed by a pure capacitor is always zero for a complete "cycle""



R-L series ac circuit :-



where V = supply voltage
i = Resulting current

V_R = Voltage drop across resistance

$\boxed{V_R = iR}$ volt

V_L = Voltage drop across inductor

$\boxed{V_L = iX_L}$ volt

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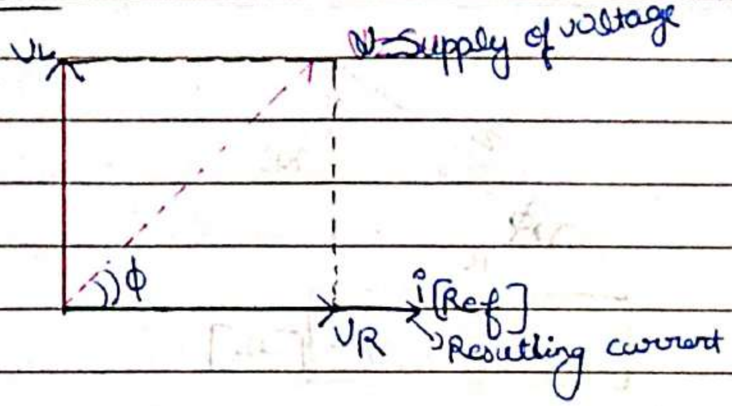
Phasor diagram :



ϕ = Phase angle between supply voltage and resulting current

from phasor diagram

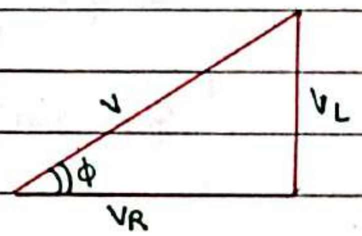
$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(iR)^2 + (iX_L)^2}$$

$$V = i\sqrt{(R)^2 + (X_L)^2}$$

$$\frac{V}{i} = \sqrt{(R)^2 + (X_L)^2}$$

[From phasor diagram]



[Voltage Triangle]

The ratio $\left(\frac{V}{i}\right)$ in an ac circuit is known as impedance and denoted by z

$$\boxed{Z = \frac{V}{i} = \sqrt{(R)^2 + (X_L)^2}} \quad [\Omega]$$

Current in R-L series ac circuit

$$\boxed{i = \frac{V}{Z} = \frac{V}{\sqrt{(R)^2 + (X_L)^2}}} \quad [Amp]$$

Inductance: jo current k change ka oppose karta hai.

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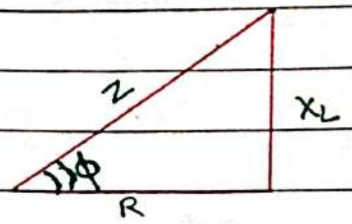
Impedence Triangle

[from voltage triangle]



$$\boxed{Z = \sqrt{R^2 + X_L^2}} \ [\Omega]$$

- **Phase angle : [ϕ]**

From impedence triangle

$$\tan\phi = \frac{X_L}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)} \quad [\text{degree}]$$

- **Power factor : [$\cos\phi$] [0.8] (lagging)**

$$\boxed{\cos\phi = \frac{R}{Z}}$$

Hint : (Nature of power factor) :
"The position of resultant current with respect to supply voltage."

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Power in R-L series A.C. circuit :

$$P = V.i$$

$$P = V_m \sin\omega t \cdot i_m \sin(\omega t - \phi)$$

$$P = V_m i_m \overset{\sin\omega t}{} \sin\omega t\,(\omega t - \phi)$$

$$P = \frac{V_m i_m}{2}\left[2\sin\underbrace{\omega t}_{A} \cdot \sin\underbrace{(\omega t - \phi)}_{B}\right]$$

Hint : $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$P = \frac{V_m i_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$P = \frac{V_m i_m}{2}\cos\phi - \underbrace{\frac{V_m i_m}{2}\cos(2\omega t - \phi)}_{\Rightarrow 0}$$

$$P = \frac{V_m i_m}{2}\cos\phi$$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cdot \cos\phi$$

$$\boxed{P = V_{rms} \cdot I_{rms} \cos\phi} \text{ watt}$$

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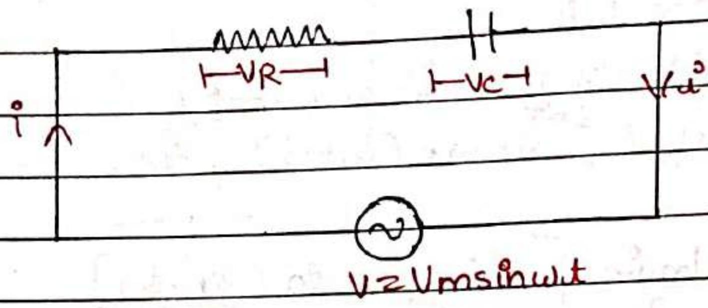
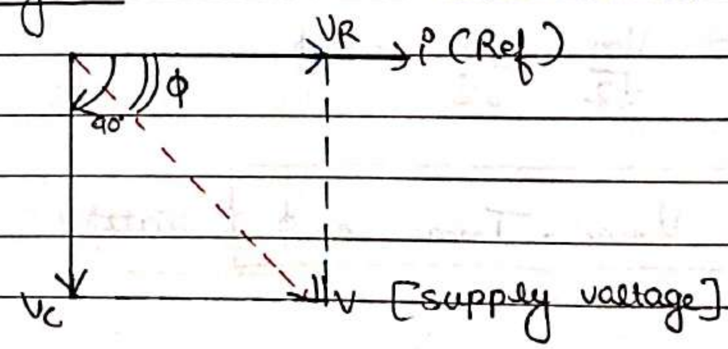
power in R-C series AC circuit



Where V = supply voltage
i = Resultant current
V_R = Voltage drop across resistor
$\boxed{V_R = iR}$ volt
V_C = Voltage drop across capacitor
$\boxed{V_C = iX_C}$ volt

phasor diagram :-



ϕ = phase angle b/w supply voltage and resultant current

from phasor diagram

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$V = \sqrt{(iR)^2 + (iX_C)^2}$$

$$V = i\sqrt{(R)^2 + (X_C)^2}$$

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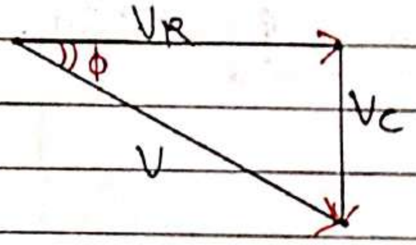
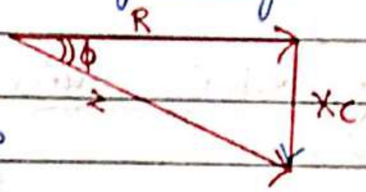
impedence $\boxed{Z = \frac{V}{i} = \sqrt{R^2 + (X_C)^2}}$



from phasor diagram

voltage triangle



impedence Δ

phase angle [ϕ]

$$\tan\phi = \left[\frac{-X_C}{R}\right]$$

$$\boxed{\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)}$$ [degree]

power factor [$\cos\phi$]

$$\boxed{\cos\phi = \frac{R}{Z}}$$ [leading]

Power (P) = V.i.

$$P = V_m \sin\omega t \,.\, i_m \sin(\omega t + \phi)$$

$$P = V_m i_m \sin\omega t \,.\sin(\omega t + \phi)$$

$$P = \frac{V_m i_m}{2}\left[2\sin\omega t \,.\, \sin(\omega t + \phi)\right]$$

$$P = \frac{V_m i_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

$$P = \frac{V_m i_m}{2}\cos\phi - \frac{V_m i_m}{2}\cos(2\omega t + \phi)$$

$$P = \frac{V_m i_m}{2}\cos\phi$$

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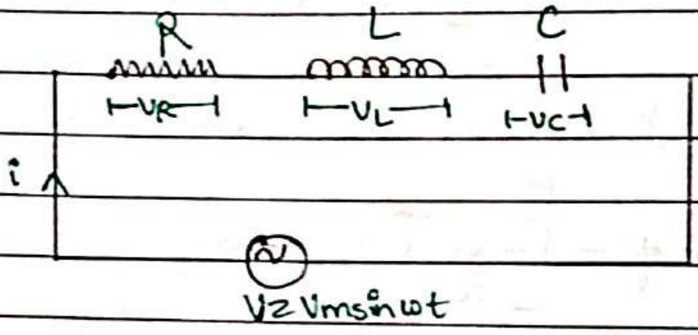
$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P = V_{rms}\, i_{rms} \cos\phi} \quad \text{(watt)}$$

RLC series A-C circuit :



where,

V = supply voltage
i = resultant current

V_R = Voltage drop across resistor
$\boxed{V_R = iR}$ volt

V_L = Voltage drop across ~~capacitor~~ inductor
$\boxed{V_L = iX_L}$ volt

V_C - Voltage drop across capacitor
$\boxed{V_C = iX_C}$ volt

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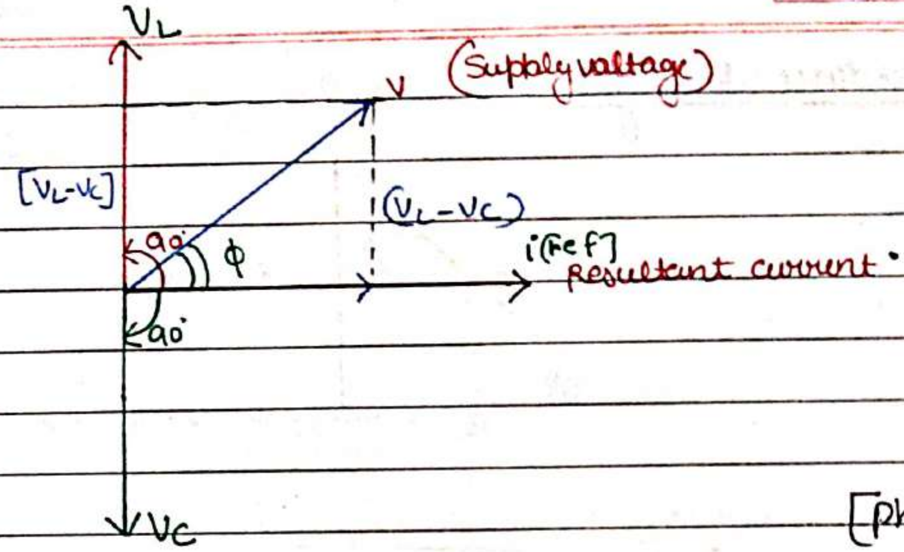
$X_L > X_C$

$V_L > V_C$

highly inductive



[Phasor diagram]

from phasor diagram

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(iR)^2 + (iX_L - iX_C)^2}$$

$$V = i\sqrt{(R)^2 + (X_L - X_C)^2}$$

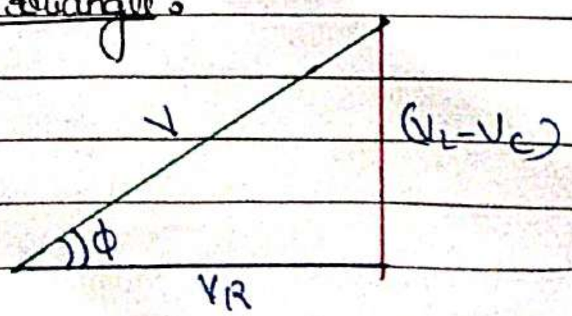
Impedence,

$$\boxed{Z = \frac{V}{i} = \sqrt{(R)^2 + (X_L - X_C)^2}}$$

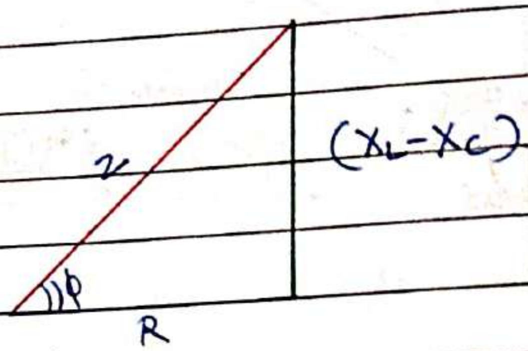
Current in RLC A.C. Circuit :-

$$\boxed{i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}}$$ Amp

Voltage triangle :



⇒ Impedence triangle :



⇒ Phase angle :

$$\tan\phi = \frac{(X_L - X_C)}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$ degree

⇒ Power factor :

$$\boxed{\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}}$$ [lagging]

⇒ Power :

$$P = vi$$

$$P = \cancel{V_{rms}}\, V_m \sin\omega t \,.\, i_m \sin(\omega t - \phi)$$

$$\boxed{P = V_{rms} I_{rms} \cos\phi}$$ watt

(highly inductive)

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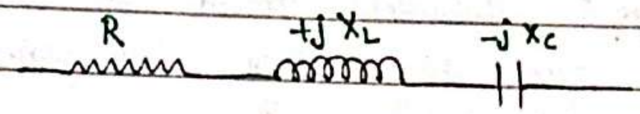
Impedence in Complex Form for a RLC series AC circuit.



$$\boxed{Z = R + jX_L - jX_C}$$ in complex form

$$\boxed{|Z| = \sqrt{(R)^2 + (X_L - X_C)^2}}$$ magnitude

(1) $X_L > X_C$, highly inductive, Power factor [$\cos\phi$] is lagging.

[2] $X_C > X_L$, highly capicitive, power factor [$\cos\phi$] is leading.

[3] $X_C = X_L$, purely resistive, power factor ($\cos\phi$) is unity [1.]

Hint:

$\boxed{R}$

$V = V_m \sin\omega t = V_m \angle 0^\circ$

$i = i_m \sin\omega t = i_m \angle 0^\circ$

$Z = \frac{V}{i} = \frac{V_m \angle 0^\circ}{i_m \angle 0^\circ} = \boxed{R = Z}$

$\boxed{L}$

$V = V_m \sin\omega t = V_m \angle 0^\circ$

$i = i_m \sin(\omega t - 90^\circ) = i_m \angle -90^\circ$

$Z = \frac{V}{i} = \frac{V_m \angle 0^\circ}{i_m \angle -90^\circ}$

$Z = X_L \angle 90^\circ$

$\boxed{Z = +jX_L}$ Ω

$\boxed{C}$

$V = V_m \sin\omega t = V_m \angle 0^\circ$

$i = i_m \sin(\omega t + 90^\circ) = i_m \angle +90^\circ$

$Z = \frac{V}{i} = \frac{V_m \angle 0^\circ}{i_m \angle 90^\circ} = X_C \angle -90^\circ$

$\boxed{Z = -jX_C}$ Ω

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Imp.

Apparent Power, Active Power, Reactive Power and Power Factor:

- **Apparent Power:** Apparent power is the product of RMS value of voltage and current.
  The power given by the source.

  Apparent Power = $V_{rms} \cdot I_{rms}$

  $$\boxed{(S) = V_{rms} \cdot I_{rms}}$$

It is measured in $\boxed{VA}$ or $\boxed{KVA}$

- **Active Power:** It is the product of apparent power and cosine of the angle between voltage and current.

power consumed by the user

Active power = $V_{rms} I_{rms} \cos\phi$

$$\boxed{P = V_{rms} I_{rms} \cos\phi}$$

It is measured in $\boxed{\text{watt}}$ or $\boxed{\text{Kilowatt KW}}$

- It is also called "True Power" or "Actual Power" or "Real Power".

- **Reactive Power:** It is the product of apparent power and sine of the angle between voltage and current.

waste power

Reactive Power = $V_{RMS} I_{RMS} \sin\phi$

$$\boxed{Q = V_{RMS} I_{RMS} \sin\phi}$$

It is measured in $\boxed{VAR}$ (Volt Ampere Reactive) OR $\boxed{KVAR}$

Same source se jada energy utilize kar ske factor of the power which utilize.

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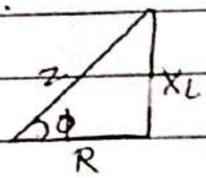
- **Power Factor :- [$\cos\phi$]** It is defined as cosine of the angle between voltage and current.

OR

It is the ratio of Resistance to Impedence.

$$\boxed{\cos\phi = \frac{R}{Z}}$$ [from impedence Δ]



OR

It is the Ratio of Active Power to Apparent power.

always high

$$\boxed{\cos\phi = \frac{KW}{KVA}}$$

$(V_A)\cos\phi$ = watt

Note : Standard value of $\cos\phi = 0.8$

power factor is always lies between 1 to 0 $\rightarrow \phi = 90^\circ$ / $\rightarrow \phi = 0^\circ$

Ques An A.C. voltage $V(t) = 141.4 \sin 120 t$ is applied to a series R-C. circuit. The current through the circuit is obtained as $I(t) = 14.14 \sin 120 t + 7.07 \cos(120 t + 30^\circ)$

(i) Determine Impedence

(ii) Value of Resistance and Capacitance

(iii) Power factor

(iv) Power

Ans

$$\boxed{V_t = 141.4 \angle 0^\circ \text{ volt}}$$

$$i(t) = 14.14 \angle 0^\circ + 7.07 \sin(120t + 120)$$

$$i(t) = 14.14 \angle 0^\circ + 7.07 \angle +120^\circ$$

$$i(t) = (14.14 + 0j) - 3.53 + 6.12j$$

$$i(t) = (10.61 + 6.12j)$$

$$\boxed{i(t) = 12.24 \angle 30^\circ} \text{ Amp}$$

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(i) impedence $(z) = \frac{v}{i}$

$$= \frac{\left(\frac{141.4}{\sqrt{2}}\right)\angle 0^\circ}{\left(\frac{12.24}{\sqrt{2}}\right)\angle 30^\circ}$$

$$\boxed{z = 11.55 \angle -30^\circ} \; [\Omega]$$

(ii) Value of resistance (R) and Capicitance (C)

$z = 11.55 \angle -30 \; [\Omega]$

in rectange form

$z = (10 - 5.77j) \; [\Omega]$

$\boxed{z = R \pm jX}$ $\{X = X_L - X_C\}$ $\boxed{R = 10\Omega}$ $\boxed{X_C = 5.77\Omega}$

$5.77 = \frac{1}{120 \times C} \Rightarrow X_C = \frac{1}{\omega C}$

$\boxed{C = 0.001}$ farad

(iii) power factor $\cos\phi = \cos(-30)$

$\boxed{\cos\phi = 0.86}$ leading

(iv) Power (P) = $V_{rms} I_{rms} \cos\phi$

$= 100.2 \times 10.02 \times \frac{\sqrt{3}}{2}$

$\boxed{P = 744.2}$ watt

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Ques A load having impedence of $(1+1j)\ \Omega$ is connected to an A.C. voltage represented as $v(t) = 20\sqrt{2}\cos(\omega t + 10^\circ)$ volt. Find the current in load and expressed in the form of $i(t) = i_m \sin(\omega t \pm \phi)$
Find the apparent power, active power, reactive power and draw power triangle also.

Ans
$$Z = 1 + j$$
$$Z = \frac{V}{i} \qquad i = \frac{V}{Z} = \frac{20\sqrt{2}}{\sqrt{2}} \angle 100^\circ = 20\angle 100^\circ$$

$$i = \frac{20\angle 100}{1.41\angle 45^\circ}$$

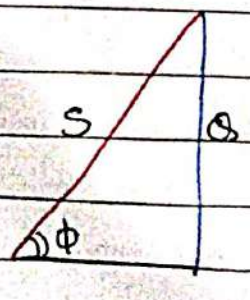
$$\boxed{i = 14.18\angle 55 \text{ Amp}}$$

$S = V_{rms} I_{rms}$ (Apparent power)
$$= 20 \times 14.18$$
$$\boxed{S = 283.6 \text{ VA}}$$

Reactive power $(Q) = V_{rms} I_{rms} \sin\phi$
$$= 283.6 \sin 45^\circ$$
$$\boxed{Q = 200.5 \text{ VAR}}$$

Active power $(P) = V_{rms} I_{rms} \cos\phi$
$$= 20 \times 14.18 \cos 45^\circ$$
$$\boxed{P = 200.5 \text{ watt}}$$

power triangle



$$S = \sqrt{Q^2 + P^2}$$

$$KVA = \sqrt{(KW)^2 + (KVAR)^2}$$

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Resonance in Series RLC ac circuits :-

A series RLC AC circuit is said to be in Resonance, "When the power factor of circuit is unity."

$\boxed{\cos\phi = 1}$ $[\phi = 0]$

OR

A series RLC AC circuit is said to be in Resonance "When the supply voltage and Resultant current are in same phase".

OR

A series RLC AC circuit is said to be resonance, "When the inductive reactance $[X_L]$ is equal to the capacitive reactance $[X_C]$".

$$\boxed{X_L = X_C}$$ {frequency equal}

We know that, the impedence of RLC series AC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

at series Resonance $[X_L = X_C]$

$$Z = \sqrt{R^2}$$

$$\boxed{Z = R} \rightarrow \text{minimum}$$

Reactive value depends on frequency X_L and X_C

This is the condition for series Resonance.

The current flow in RLC series circuit at resonance

$$\uparrow i = \frac{V}{Z\downarrow}$$

maximum

minimum

Radar, walky-talkey {frequency match and then cond^n is in resonance}

Expression for Resonance Frequency :

At series Resonance

$$\boxed{X_L = X_C}$$

$$\omega L = \frac{1}{\omega C}$$

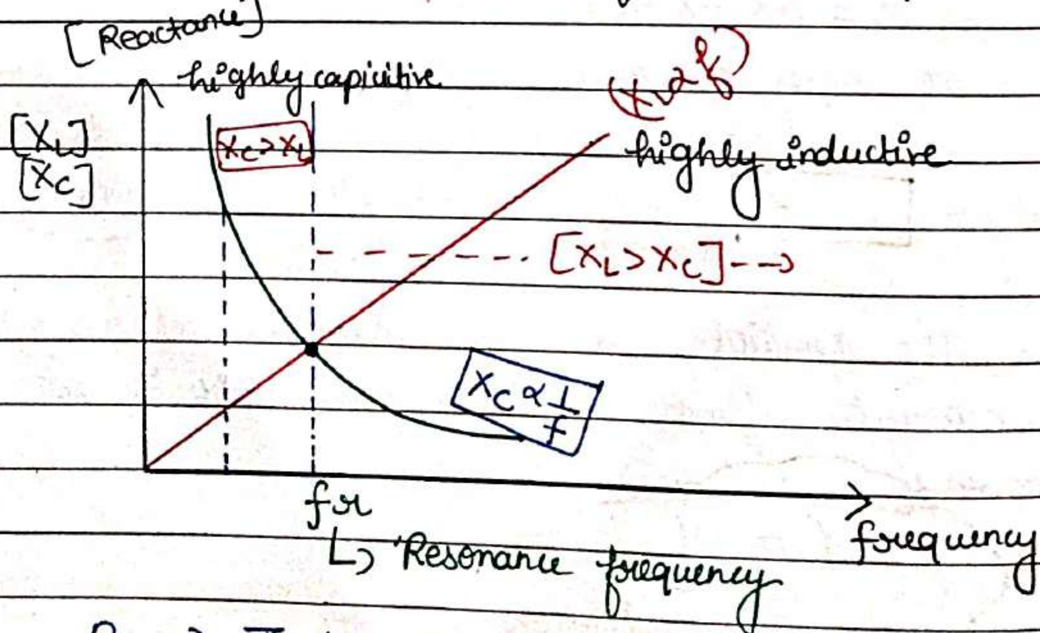
$$\boxed{\omega_r^2 = \frac{1}{LC}}$$

$$(2\pi f_r)^2 = \frac{1}{LC}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \quad [Hz]$$

Graphical Representation of Resonance frequency :



$R \rightarrow$ Independent from frequency

$L \rightarrow$ Inductive reactance $[X_L] = \omega L$

$$X_L = 2\pi f L$$

$$\boxed{X_L \propto f}$$

Selectivity → it's a tendency to select desired frequency
Quality factor ∝ selectivity

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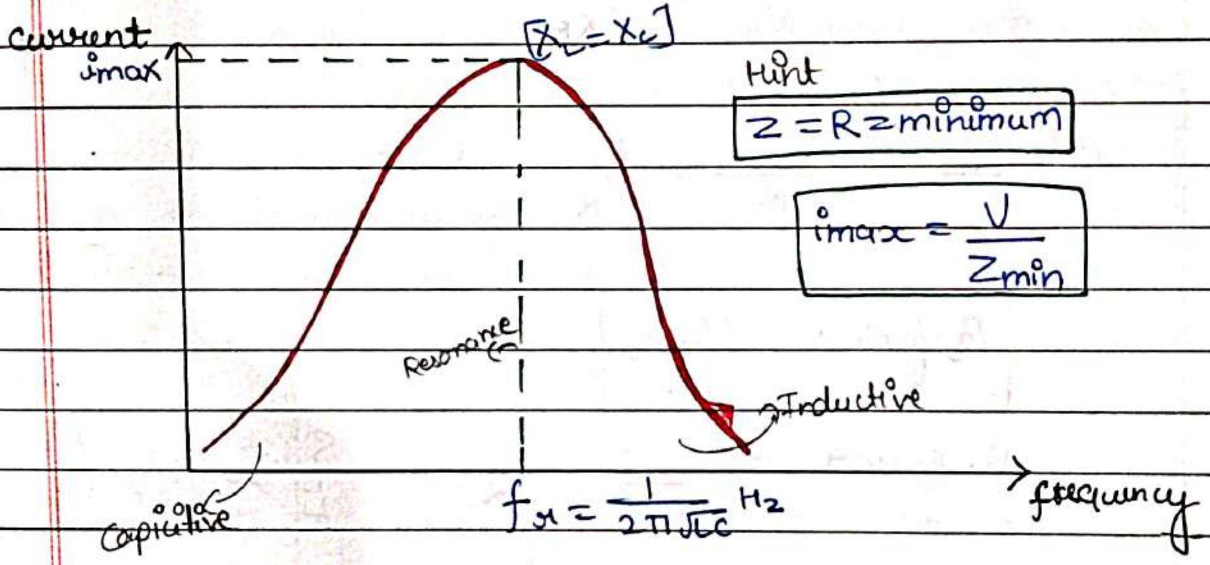
C → Capacitive Reactance $\left[X_C = \frac{1}{\omega C}\right]$

$$X_C = \frac{1}{(2\pi f)C}$$

$$\boxed{X_C \propto \frac{1}{f}}$$

Imp # **Resonance Curve :-** The curve between current and frequency is called Resonance Curve, at series resonance condition.



Resonance is the phenomenon in the electrical circuit where the output of the circuit is maximum at one particular frequency and that frequency is known as resonant frequency at resonant fr. X_C and X_L are equal.

→ to establish a condⁿ of stable fr. in circuit

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Quality Factor : [Voltage magnification]

At series resonance, "the ratio of voltage drop across inductor (L) or capacitor to the voltage drop across resistor is called Quality factor."

$$Q \text{ factor} = \frac{\text{Voltage drop across L and C}}{\text{Voltage drop across R}}$$ (at series resonance)

$$Q \text{ factor} = \frac{V_L}{V_R} = \frac{iX_L}{iR}$$

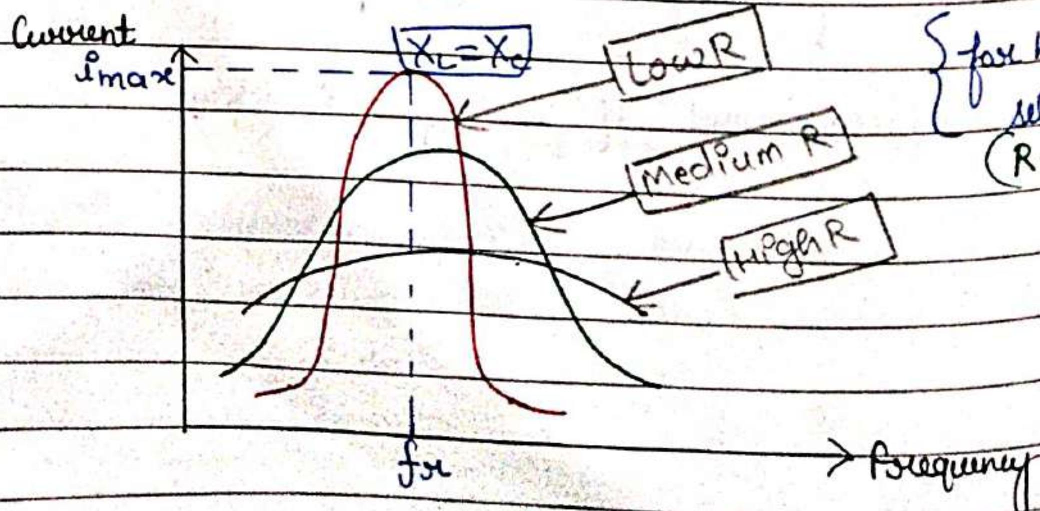
$$Q.\text{ factor} = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$\boxed{Q \text{ factor} = \frac{\omega_r L}{R}}$$

$$Q.\text{ factor} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} \qquad \left\{ \omega_r^2 = \frac{1}{LC} \right\}$$

$$Q.\text{ factor} = \frac{1}{R} \frac{\cancel{L}.\sqrt{L}}{\cancel{\sqrt{L}.\sqrt{L}}.\sqrt{C}}$$

$$\boxed{Q.\text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$



{for highly selective}
(R<<<)

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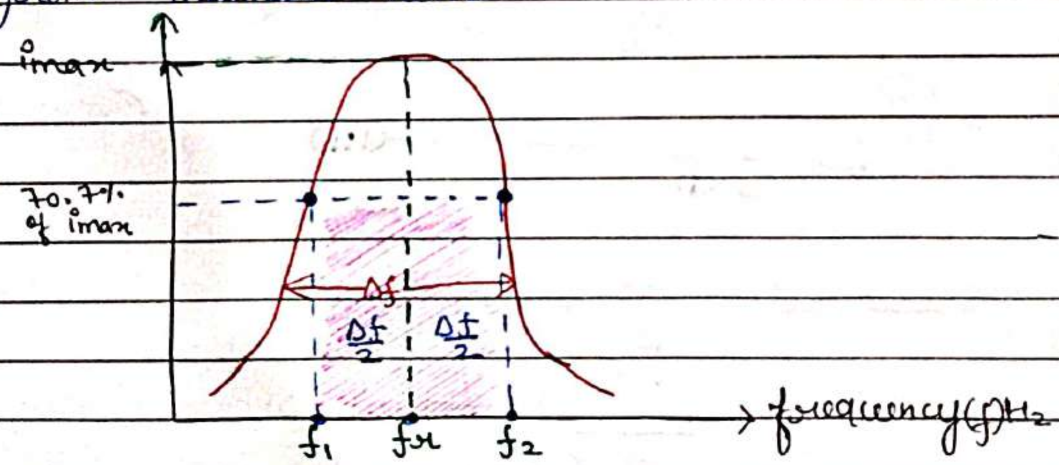
Series Resonance
↓
Voltage Resonance
↓ OR

Bandwidth of a series-Resonant circuit :

The Band width of a series resonant circuit is defined as, " The range of frequencies over which circuit current is equal to or greater than $\geq 70.7\%$ of maximum current

from resonance curve



$$\boxed{\text{Bandwidth } (\Delta f) = f_2 - f_1 \text{ Hz}} = \frac{R}{2\pi L}$$

Where,

f_r = resonant frequency
f_1 = lower cut off frequency
f_2 = upper cut off frequency

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Note: Sometimes f_1 and f_2 also known as half power frequencies.

Current in series RLC ac circuit

$$i = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}} \quad \text{——— (i)}$$

At resonance condition

$$i_{max} = \frac{V}{R} \quad \text{——— (ii)}$$

Current at f_1 and f_2

$$i = \frac{i_{max}}{\sqrt{2}} \quad \left\{ i_{max} = \frac{V}{R} \right\}$$

$$i = \frac{V}{\sqrt{2}R} \quad \text{——— (iii)}$$

for resonance $\quad P = (i_{max})^2 R$

$$P = \left(\frac{i_{max}}{\sqrt{2}}\right)^2 R$$

$$\boxed{P = \frac{i_{max}^2 R}{2}}$$

from (i) and (iii)

$$\frac{\cancel{V}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\cancel{V}}{\sqrt{2}R}$$

$$\sqrt{(R)^2 + (X_L - X_C)^2} = \sqrt{2}R$$

Squaring on both sides.

$$(R)^2+(X_L-X_C)^2 = 2R^2$$
$$(X_L-X_C)^2 = R^2$$
$$\boxed{X_L - X_C = \pm R}$$

$$\omega_1 L - \frac{1}{\omega_1 C} = +R \quad \text{——— (a)}$$
$$\omega_2 L - \frac{1}{\omega_2 C} = -R \quad \text{——— (b)}$$

Imp

Relation between f_1, f_2 and f_r :

adding equation (A) and (B)

$$(\omega_1+\omega_2) L - \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$

$$(\omega_1+\omega_2) - \frac{1}{LC}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right) = 0 \quad \text{(divide by } L \text{ both sides)}$$

$$(\omega_1+\omega_2) - \omega_r^2\left(\frac{\omega_1+\omega_2}{\omega_1\times\omega_2}\right) = 0$$

$$\cancel{(\omega_1+\omega_2)} = \omega_r^2 \frac{\cancel{(\omega_1+\omega_2)}}{(\omega_1\times\omega_2)}$$

$$\omega_r^2 = (\omega_1 \times \omega_2)$$

$$\boxed{\omega_r = \sqrt{\omega_1 . \omega_2}} \quad \text{(in terms of angular frequency)}$$

$$(2\pi f_r)^2 = 2\pi f_1 . 2\pi f_2$$
$$\cancel{4\pi^2} f_r^2 = \cancel{4\pi^2} f_1 . f_2$$
$$f_r^2 = f_1 . f_2$$
$$\boxed{f_r = \sqrt{f_1 . f_2}} \quad \text{(in terms of frequency)}$$

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Calculation of f_1 and f_2 :-

eq^n (A) $-$ eq^n (B)

$$(\omega_1-\omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right) = 2R$$

$$(\omega_1-\omega_2) - \frac{1}{LC}\left(\frac{\omega_2-\omega_1}{\omega_1\cdot\omega_2}\right) = \frac{2R}{L} \quad \left\{ \begin{array}{l} \omega_r^2 = \frac{1}{LC} \\ \omega_r^2 = \omega_1\omega_2 \end{array} \right\}$$

$$(\omega_1-\omega_2) - \cancel{\omega_1\omega_2}\left(\frac{\omega_2-\omega_1}{\cancel{\omega_1\omega_2}}\right) = \frac{2R}{L}$$

$$\omega_1-\omega_2-\omega_2+\omega_1 = \frac{2R}{L}$$

$$\cancel{2}(\omega_1-\omega_2) = \frac{\cancel{2}R}{L}$$

$$(\omega_1-\omega_2) = \frac{R}{L}$$

OR $\boxed{\omega_2-\omega_1 = \frac{R}{L}}$

$$2\pi f_2 - 2\pi f_1 = \frac{R}{L}$$

$$(f_2-f_1) = \frac{R}{2\pi L}$$

$$\boxed{\text{Bandwidth} = \Delta f = f_2-f_1 = \frac{R}{2\pi L}} \text{ Hz}$$

$$f_1 = f_r - \frac{\Delta f}{2} \quad \text{(from resonance curve)}$$

$$\boxed{f_1 = f_r - \frac{R}{4\pi L}} \text{ Hz}$$

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$$f_2 = f_r + \frac{\Delta f}{2}$$ (from resonance curve)

$$\boxed{f_2 = f_r + \frac{R}{4\pi L}}\ Hz$$

Relation between Quality factor, Resonant frequency and Bandwidth :-

We know that,

$$Q\ factor = \frac{\omega_r L}{R}$$

$$Q\ factor = \frac{(2\pi f_r) L}{R}$$

$$Q\ factor = \frac{f_r (2\pi L)}{R} \qquad \left\{\Delta f = \frac{R}{2\pi L}\right\}$$

$$\boxed{Q\ factor = \frac{f_r}{\Delta f}}$$

Ques Why series resonance is called as voltage resonance? (voltage magnification)

Ans At series resonance the voltage across inductance and capacitance becomes quality factor (Q) times of applied voltage. So it is called voltage resonance.

mathematically,

$$V_L = i X_L$$

At series resonance, $\boxed{Z = R}$ minimum

$$i_{max} = \frac{V}{R \downarrow}$$

$$V_L = \frac{V}{R}(\omega_r L)$$

$$V_L = \left(\frac{\omega_r L}{R}\right) V$$

$$\boxed{V_L = QV}$$ (Voltage magnification or voltage resonance)

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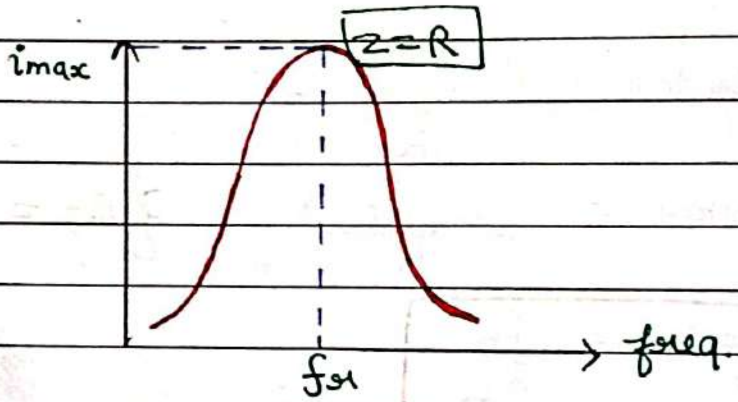
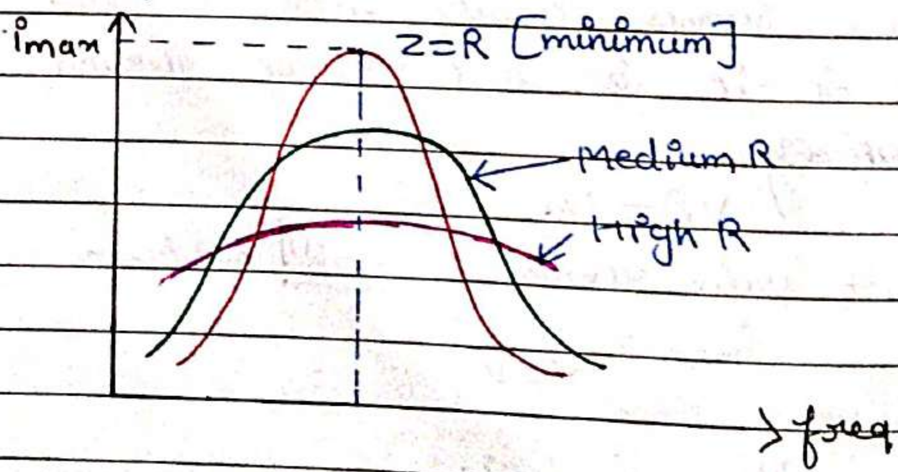
Ques Why series resonance circuit is also known as 'acceptor circuit'?

Ans At series resonance, the impedence of the circuit is minimum, it accept frequencies close to resonance frequency. So, it is called as acceptor circuit.

at series resonance
$Z \rightarrow$ minimum
$i \rightarrow$ maximum



Ques What is selectivity?
Ans Sharpness of the resonance curve.



$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\downarrow R \quad \uparrow S$

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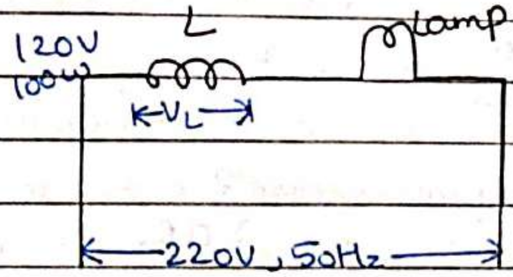
Numericals:

Ques-29 A 120watt, 100watt lamp is to be connected to a 220V, 50Hz AC supply. What value of pure Inductance should be connected in series. In order to run the lamp on rated voltage.

Ans



$$V = \sqrt{(V_{lamp})^2 + (V_L)^2}$$

$P = Vi$
$100 = 120 i$
$i = \frac{100}{120}$
$\boxed{i = 0.83}$ Amp

$$220 = \sqrt{(120)^2 + (V_L)^2}$$

$\boxed{V_L = 184.39}$ volt

$V_L = 184.39 = iX_L = 0.83\,(2\pi f L)$

$\Rightarrow (0.83 \times 2 \times 3.14 \times 50L) = 184.39$

$\boxed{L = 0.7046 \text{ Henry}}$

Ques A series RLC circuit $R = 10$ ohm, $L = 0.1H$, $C = 8\mu f$
Determine (i) Resonance frequency
(ii) Quality factor of the circuit
(iii) Half power frequencies

A given $R = 10$ ohm $L = 0.1H$ $C = 8\mu f$

Resonance frequency $f_R = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{0.1 \times 8}} \Rightarrow \frac{1}{2 \times 3.14\sqrt{0.8 \times 10^{-6}}}$

$= 177.94$

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(b) Quality factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{10}\sqrt{\frac{0.1}{8\times10^{-6}}}$

$= 0.1\sqrt{}$

$\boxed{Q = 11.17}$

(c) half power frequencies $= \frac{R}{2\pi l}$

$= \cancel{\frac{10}{2\times3.14\times0.1}}$

$= 15.92$

$\Rightarrow f_1 = f_r - \frac{R}{4\pi l}$

$\Rightarrow 177.94 - \left(\frac{10}{4\times3.14\times0.1}\right)$

$= 177.94 - 7.96$

$\boxed{f_1 = 169.99 \text{ Hz}}$

$f_2 = f_r + \frac{R}{4\pi l}$

$= 177.94 + 7.96$

$\boxed{f_2 = 185.90 \text{ Hz}}$

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Que A series circuit consisting of a resistance 100 Ω, L = 0.2H and a capacitance of 20mf is connected to a 240V, 50 cycle per second supply. determine
1. Total impedence
2. Total current
3. Voltage across each component
4. Power factor
5 Power consumption in the circuit
6. Frequency at which resonance will occur.
Draw the complete vector diagram.

Ans $R = 100\,\Omega$, $L = 0.2H$, $C = 20mf$, $V = 240$, $f = 50Hz$

$$X_L = \omega l$$

$$X_L = (2\pi f) l \Rightarrow (2 \times 3.14 \times 50 \times 0.2)$$

$$\boxed{X_L \Rightarrow 62.8\,\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f) C}$$

$$= \frac{1}{2 \times 3.14 \times 50 \times 20 \times 10^{-6}}$$

$$\boxed{X_C = 159\,\Omega}$$

(i) $$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(100)^2 + (62.8 - 159)^2}$$

$$Z = \sqrt{10000 + 9254.4}$$

$$\boxed{Z = 138.87.}\ \Omega$$

(ii) $I = \frac{V}{Z}$

$= \frac{240}{138.87} \Rightarrow \boxed{1.72}$ Amp

(iii) $\boxed{V_R = iR = 172.98}$ volt

$\boxed{V_L = iX_L = 108.02}$ volt

$\boxed{V_C = iX_C = 273.48}$ volt

(iv) $\cos\phi$ = Power factor

$\cos\phi = \frac{R}{Z} = \frac{100}{138.87} = \boxed{0.72}$ [leading]

(v) $P = V_i i \cos\phi$

$P = 240 \times 1.72 \times \cancel{\cos}\, 0.72$

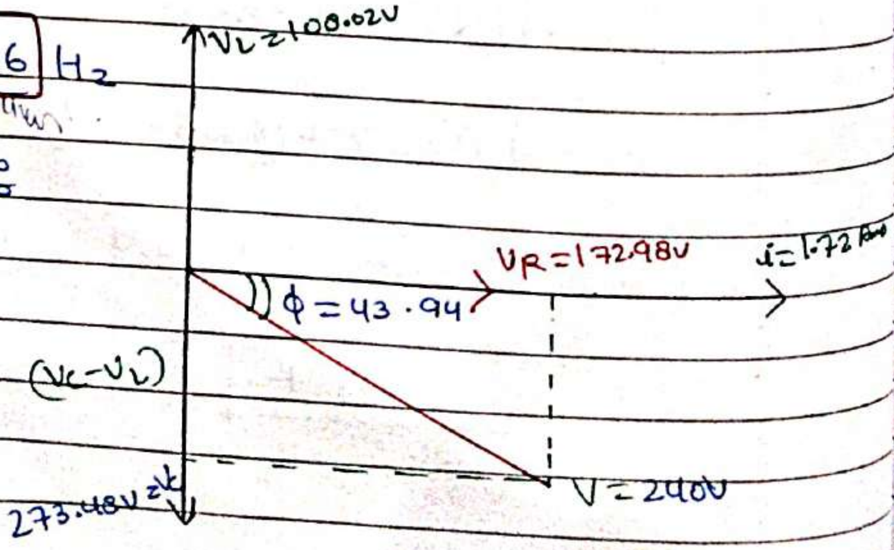
$\boxed{P = 297.21}$ watt

(vi) $f_r = \frac{1}{2\pi\sqrt{LC}}$

$f_r = \frac{1}{2 \times 3.14 \sqrt{0.2 \times 20 \times 10^{-6}}}$

$\boxed{f_r = 79.6}$ Hz

(vii) Vector diagram :



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Ans-38 Valtage across R, L, C connected in series are 5, 8, 10 valt respectively. Calculate the supply valtage at 50 Hz. Also find the frequency at which the circuit would resonate

A $V_R = 5$ valt $\quad V_L = 8$ valt $\quad V_C = 10$ valt

$f = 50 Hz$

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(5)^2 + (8-10)^2}$$

$$V = \sqrt{25+4} = \sqrt{29} = \boxed{5.38} \text{ valt}$$

$f_r = ?$

$$\frac{V_L}{V_C} = \frac{iX_L}{iX_C} = \frac{\omega L}{\frac{1}{\omega C}} = \omega^2 LC = \frac{8}{10}$$

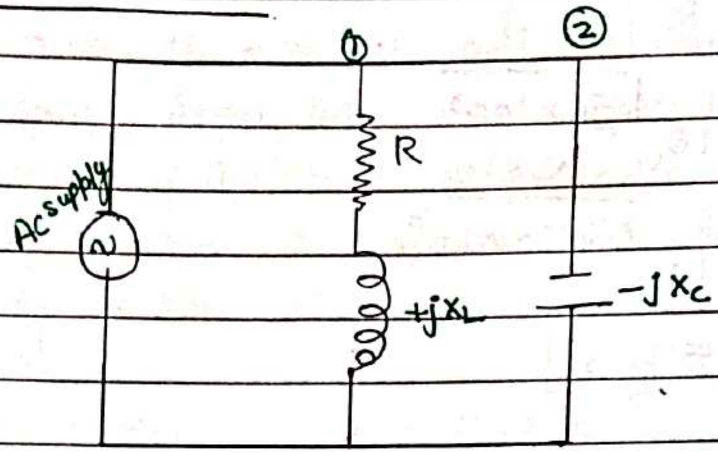
$$LC = \frac{8}{10 \times 314 \times 314}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1 \times 7}{2 \times 22 \times \sqrt{\frac{8}{10 \times 314 \times 314}}}$$

$$f_r = \frac{7\sqrt{10 \times 314 \times 314}}{2 \times 22 \times \sqrt{8}}$$

$$\boxed{f_r = 55.85}$$

Parallel circuit



Admittance Method

↳ Reciprocal of impedence

$$\boxed{[Y] = \frac{1}{[Z]}} \quad [\mho]$$

impedence of branch ① $z_1 = (R + jX_L)\ \Omega$

Impedence of branch ② $z_2 = -(jX_c)\ \Omega$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$$

$$\frac{1}{z} = \frac{1}{(R+jX_L)} \quad \frac{1}{jX_c}$$

$$\frac{1}{z} = \frac{R - jX_L}{R^2 + X_L^2} + j\omega C$$

$$\frac{1}{z} = \frac{R}{R^2 + X_L^2} - \frac{jX_L}{R^2 + X_L^2} + j\omega C$$

$$\frac{1}{z} = \frac{R^2}{R^2 + X_L^2} + j\left(\omega C - \frac{X_L}{R^2 + X_L^2}\right)$$

$$Y = \frac{R^2}{R^2 + X_L^2} + j\left(\omega C - \frac{X_L}{R^2 + X_L^2}\right)$$

$$\boxed{Y = G + jB}$$

$\boxed{Z = R + jX}$

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Where Y = admittance $= \frac{1}{Z}$

G = conductance $= \frac{R}{R^2 + X_L^2}$

B = susceptance $= \left(\omega C - \frac{X_L}{R^2 + X_L^2}\right)$

Parallel Resonance Frequency :

reciprocal of reactance

Susceptance $(B) = 0$

$Z = R + jX_L$
$X_L > 0$

$Y = G + jB$
$B > 0$

$$\left(\omega C - \frac{X_L}{R^2 + X_L^2}\right) = 0$$

$$\omega C = \frac{X_L}{R^2 + X_L^2}$$

$$\cancel{\omega} C = \frac{\cancel{\omega} L}{R^2 + (\omega L)^2}$$

$$C(R^2 + (\omega L)^2) = L$$

$$R^2 + \omega_r^2 L^2 = \frac{L}{C}$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Condition for parallel resonance :
or
Impedence in parallel resonance :

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$$Y = G = \frac{R}{R^2 + X_L^2}$$

$$Z = \frac{R^2}{R} + \frac{X_L^2}{R^2}$$

$$Z = R + \frac{\omega_{or}^2 L^2}{R}$$

$$Z = R + \frac{L^2}{R}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)$$

$$Z = R + \frac{L^{\cancel{2}}}{R\cancel{L}C} - \frac{\cancel{L^2}}{\cancel{R}} \times \frac{R^2}{\cancel{L^2}}$$

$$Z = \cancel{R} + \frac{1}{RC} - \cancel{R}$$

$$\boxed{Z = \frac{L}{RC}}\ [\Omega] \quad \text{(Dynamic Impedence)}$$

$$\boxed{Z_d = \frac{L}{RC}}\ [\Omega]$$

{ Value of Resistance is less than dynamic impedence in parallel resonance }

$$\boxed{\downarrow i = \frac{V}{Z_d \uparrow}}$$

<u>Resonance Curve</u>: Curve between current and frequency

Value of current minimum

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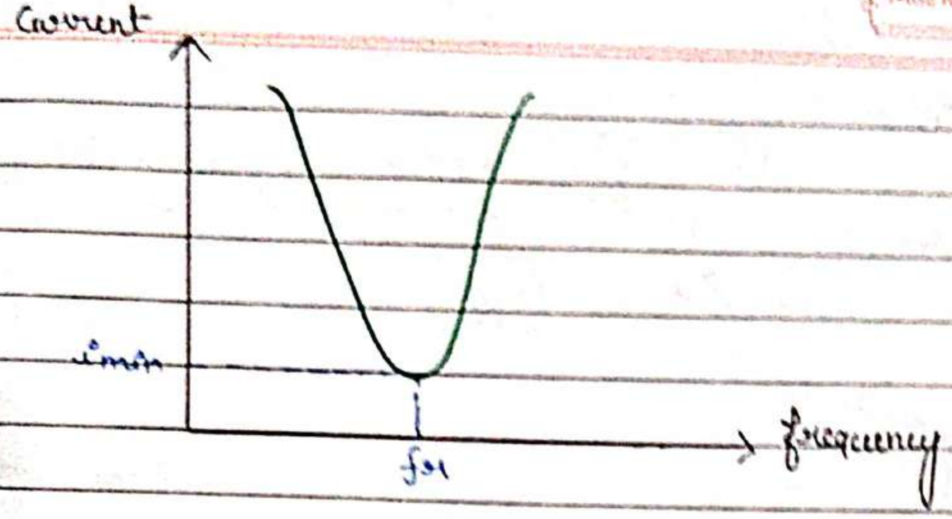
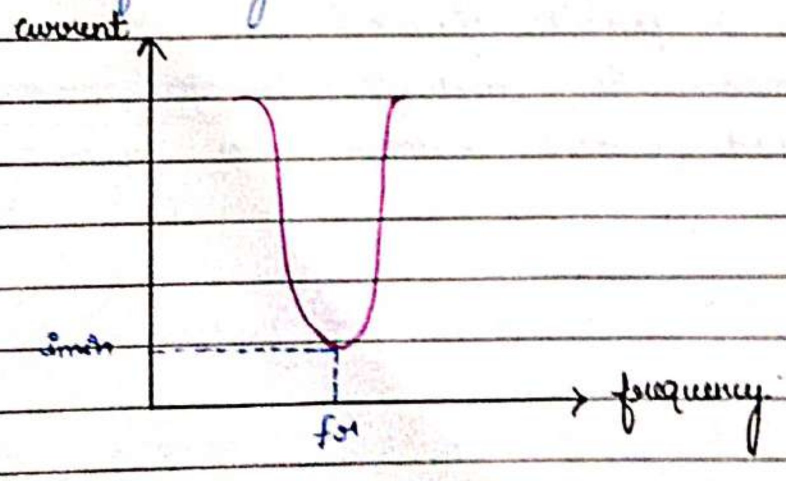


$$\boxed{\min\Downarrow\ i = \frac{V}{Z_d\ \Uparrow\max}}$$

Que Why a parallel resonance ~~is~~ circuit is called rejector circuit.

An A parallel resonant circuit is also called a rejector circuit since the current at parallel resonance is minimum due to impedence is maximum.
It almost rejects the frequencies closed to the resonance frequency.



Ques Why a parallel resonance is called current magnification

A $i = \frac{V}{Z}$

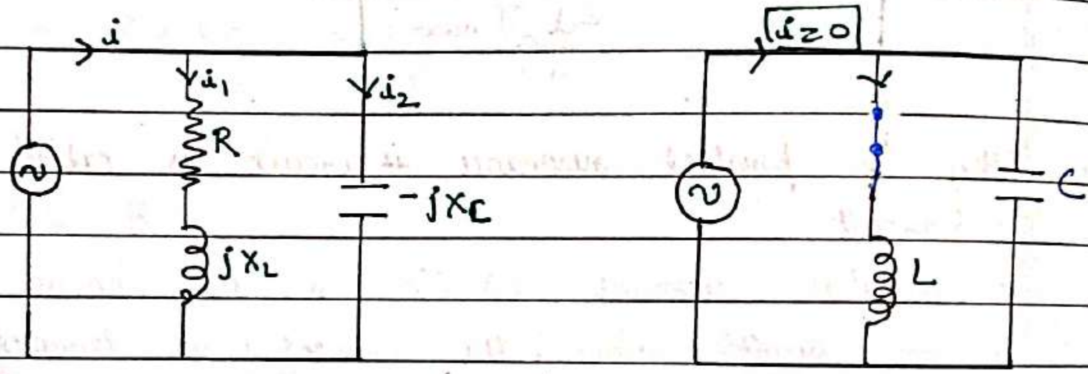
At parallel resonance

$$Z_d = \frac{L}{RC} \, [\Omega]$$

$$\text{Current}(I) = \frac{V}{Z_d} = \frac{V}{\left(\frac{L}{RC}\right)}$$

$$\boxed{i = \frac{VRC}{L}}$$

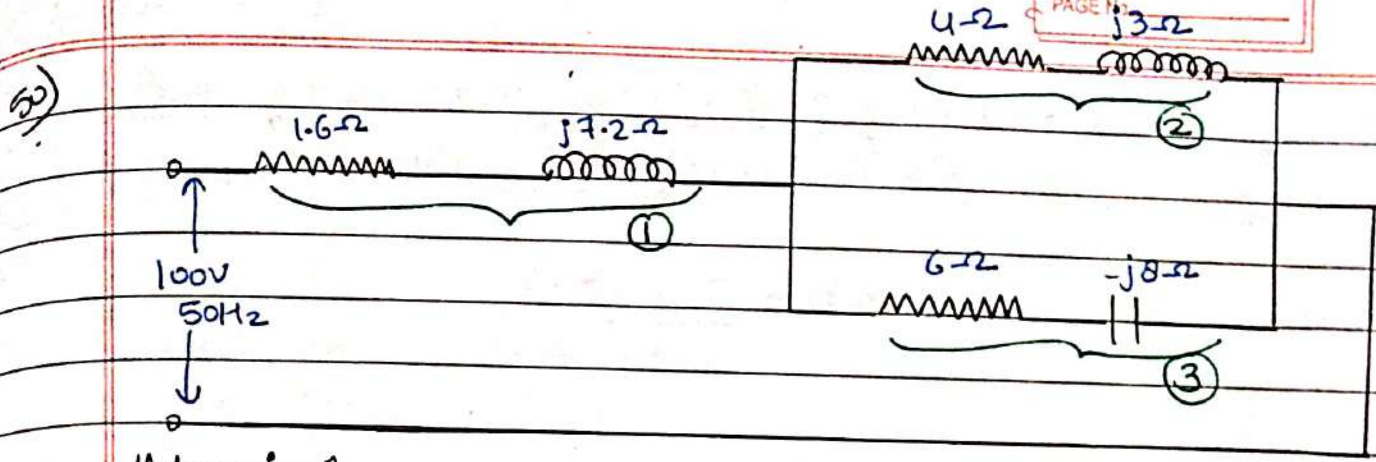
$R = 0$, $\boxed{i = 0}$



Tank circuit

The supply current is 0.
when R=0 large current
circulates in parallel circuit (L and C)
at resonance So sometimes parallel resonance is
also called current resonance or current magnification

50)



Determine:

(i) admittance of each branch
(ii) Total circuit impedence
(iii) Supply current and power factor
(iv) Total power supplied

(i) impedence of branch ① $Z_1 = (1.6 + j7.2)\,\Omega$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{(1.6+j7.2)} = \frac{1}{7.37\angle 77.47}$$

$$\boxed{Y_1 = 0.13\angle -77.47} \;(\mho)$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{4+j3}$$

$$\boxed{Y_2 = 0.2\angle -36.86^\circ} \;(\mho)$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{6-j8}$$

$$\boxed{Y_3 = 0.1\angle 53.13^\circ} \;[\mho] \longrightarrow (\text{moh})$$

(ii) Total impedence $(Z_T) = Z_1 + \left(\frac{Z_2 Z_3}{Z_2+Z_3}\right)$

$$Z_T = (1.6+j7.2) + \frac{(5\angle 36.86^\circ)(10\angle -53.13^\circ)}{(4+j3)+(6-j8)}$$

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$$Z_T = (1.6 + j7.2) + \frac{50 \angle -16.27}{(10 - j5)}$$

$$Z_T = (1.6 + j7.2) + \frac{50 \angle -16.27}{11.18 \angle -26.56^\circ}$$

4.39 will be correct otherwise answer will vary

$$Z_T = (1.6 + j7.2) + 4.47 \angle +10.29^\circ$$

$$Z_T = (1.6 + j7.2) + (4.47 + j0.79)$$

$$\boxed{Z_T = (6.07 + 7.99j)} \qquad (5.99 + 7.99j)$$

$$R + jX_L$$

$$\boxed{Z_T = 10.03 \angle +52.77^\circ} \; [\Omega] \qquad 9.98 \angle 53.14$$

(iii) I $i_T = \frac{V}{Z_T}$

$$= \frac{100}{10.03 \angle 52.77^\circ} \rightarrow 10.02$$

$$\boxed{i_T = 9.97 \angle -52.77^\circ} \text{ amp} \qquad 10.02 \angle -52.77$$

(iv) Power factor ($\cos\phi$) $= \frac{R}{Z}$

$$\cos\phi = \frac{R}{Z} = \frac{6.07}{10.03} \qquad \frac{5.99}{9.98}$$

$$\boxed{\cos\phi = 0.60} \; [\text{lagging}] \qquad 0.60$$

(v) $P = \cancel{P} V_{rms} I_{rms} \cos\phi$

$$= 100 \times 9.97 \times \cancel{\cos} \, 0.60$$

$$\boxed{P = 598.2} \text{ watt} \qquad \boxed{601.2 \text{ watt}}$$

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Power factor, causes of low Power factor, disadvantages of low power factor and how to improve it :-

Power factor is defined as "the ratio of active power to apparent power."

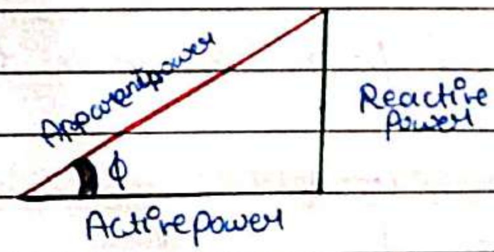
$$\boxed{\cos\phi = \frac{KW}{KVA}}$$

$$\cos\phi = 0.8$$

"cos ϕ varies from $1(\phi=0)$ to $0(\phi=90°)$"

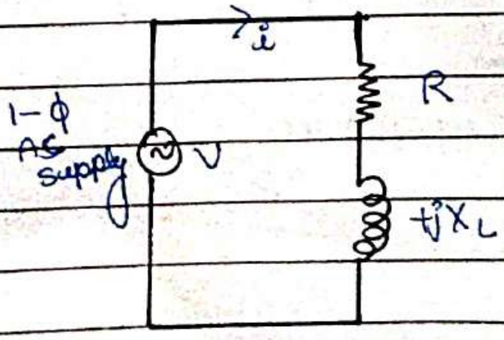
→ **Importance of power factor :**

The power factor of the circuit is measure of its effectiveness in utilising the apparent power drawn by it. The greater a power factor of a circuit, the greater its ability to utilise the apparent power.



cos ϕ ↑ increases then ϕ ↓ decreases.

→ **Causes of Low Power factor :-**



highly inductive loads

higher inductive load leads to low power factor.

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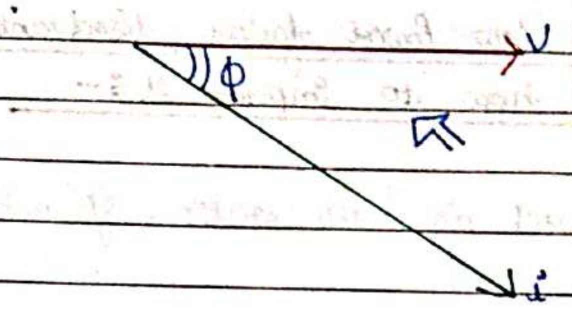

$\phi >>>$

$\cos\phi <<<$ (Low Power factor)

1. The most of the AC motors are of induction type, which have low lagging power factor.
2. The arc lamp, electric discharge lamp and heating furnaces operates at low lagging power factor.
3. The load on the power system is varying, being high during morning and evening and low at other times.

Disadvantages of Low Power factor :

We know that the power in single phase A.C. circuit.

$$P = Vi\cos\phi$$

$$i = \frac{P}{V\cos\phi}$$

1. Greater conductor size
2. Large copper loss
3. Poor efficiency
4. poor voltage regulation.
5. Large KVA rating of equipments.

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$$\cos\phi = \frac{KW}{KVA}$$

$$KVA = \frac{kW}{\cos\phi} \quad \cos\phi = \boxed{1}$$

$$\boxed{KVA = kW}$$

$$KVA = \frac{kW}{\cos\phi} = \boxed{0.5}$$

$$\boxed{KVA = 2kW}$$

$$KVA = \frac{kW}{\cos\phi} = \boxed{0.2}$$

$$\boxed{KVA = 5kW}$$

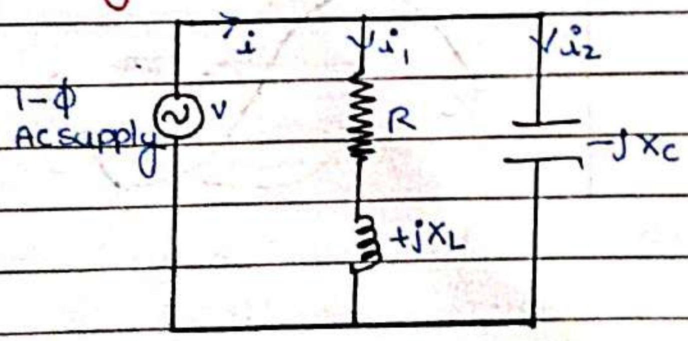
Power factor improvement :-

Power factor can be improved by using two factors:

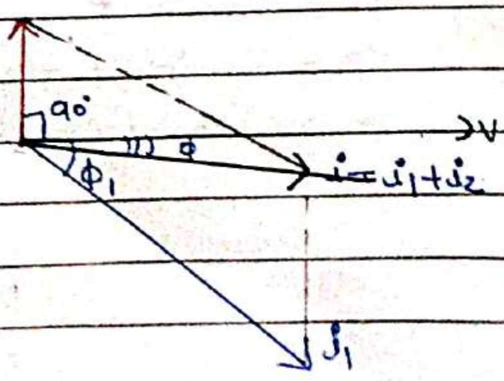
(a) by using capacitor or condenser method

(b) By using synchronous motors. (→ run at constant speed. after applying any load.)

By using capacitor or and condenser method



$i_1 + i_2 = i$

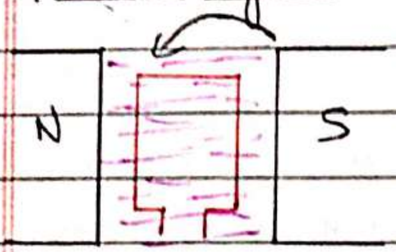


$\boxed{\phi <<<}$

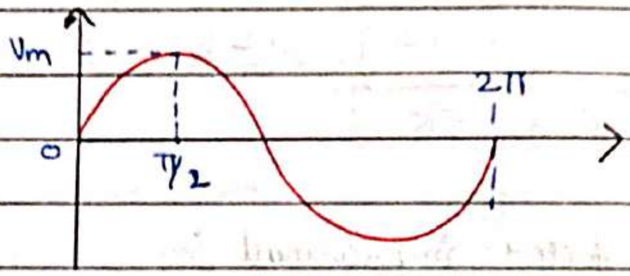
$\boxed{\cos\phi >>>}$

Three Phase A.C. Circuit

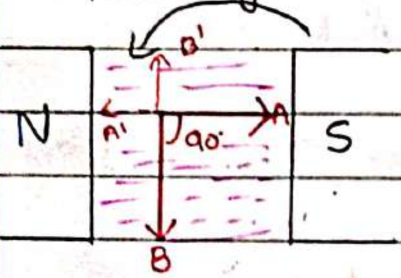
1-ϕ AC system



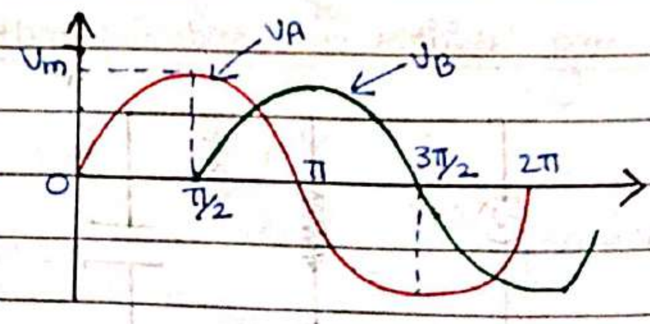
$V = V_m \sin\omega t$



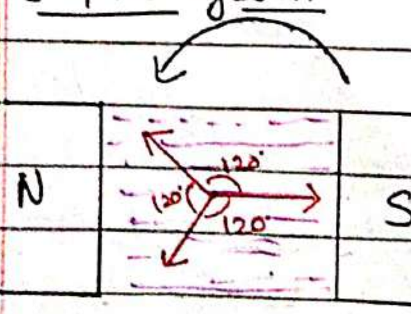
2-ϕ AC system



$V_A = V_m \sin\omega t$
$V_B = V_m \sin(\omega t - 90^\circ)$



3-ϕ AC System



$V_A = V_m \sin\omega t \qquad = V_m \angle 0^\circ$
$V_B = V_m \sin(\omega t - 120^\circ) \qquad = V_m \angle -120^\circ$
$V_C = V_m \sin(\omega t - 240^\circ) \qquad V_m \angle -240^\circ$

$\frac{360}{P}$ → how much phase difference we have to put (valid for 3 phase or more than 3 phase)
→ no. of phases.

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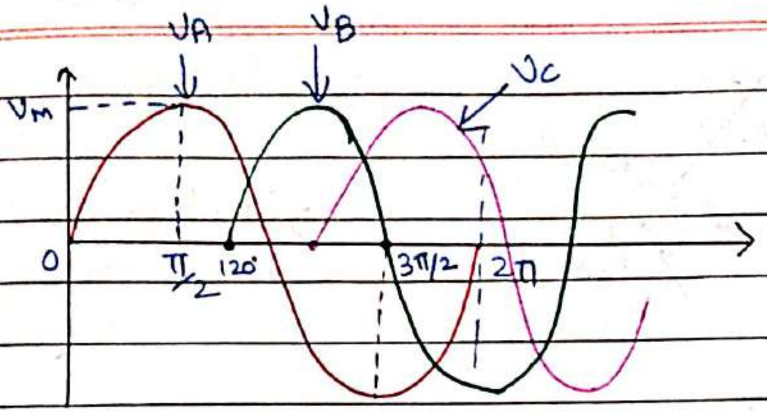


$\theta = \omega t$

$\frac{2\pi}{t} \times \frac{t}{4} \Rightarrow \frac{\pi}{2}$

$$\boxed{\text{phase displacement} = \frac{360^\circ}{3} = 120^\circ = \frac{360^\circ}{P} \quad P = \text{no. of phase}}$$

$V_A = V_m \sin\omega t = V_m \angle 0^\circ$

$V_B = V_m \sin(\omega t - 120^\circ) = V_m \angle -120^\circ$

$V_C = V_m \sin(\omega t - 240^\circ) = V_m \angle -240^\circ$

$V_{resultant} = \vec{V_A} + \vec{V_B} + \vec{V_C}$

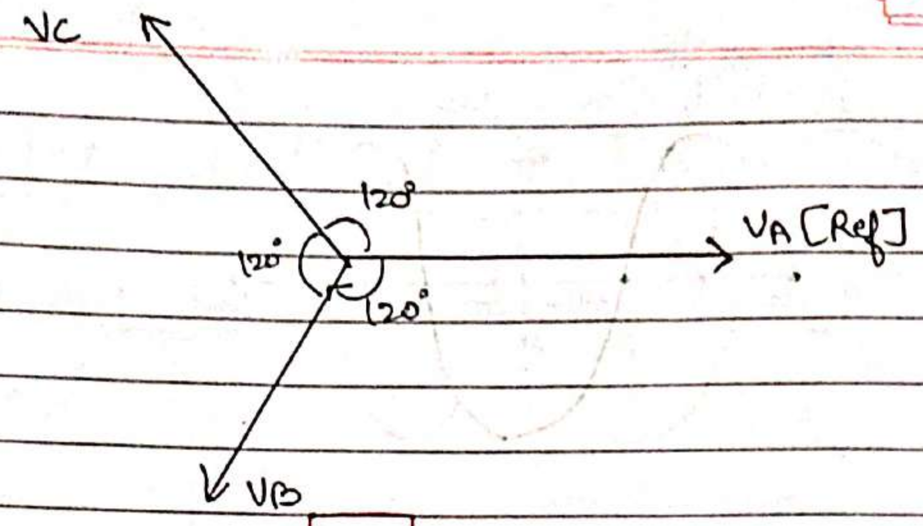
$= V_m \angle 0^\circ + V_m \angle -120^\circ + V_m \angle -240^\circ$

$V_{resultant} = V_m [1 \angle 0^\circ + 1 \angle -120^\circ + 1 \angle -240^\circ]$

$V_{resultant} = V_m [0]$

$$\boxed{V_{resultant} = 0}$$

"The phasor sum of all the emf and voltage is equal to zero at any instant"



A
B
C

Positive Phase Sequence

Colour code:

Red ← R
Yellow ← Y
Blue ← B
Black ←

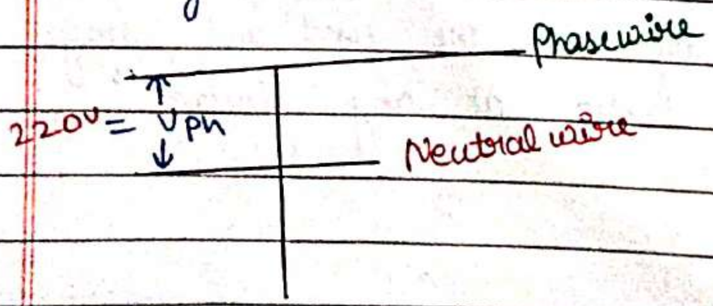
Positive phase sequence

Basic Terminologies: related to three phase AC system:

- **Phase Voltage**: [V_{ph} | E_{ph}]
The voltage between a phase to neutral wire.

OR

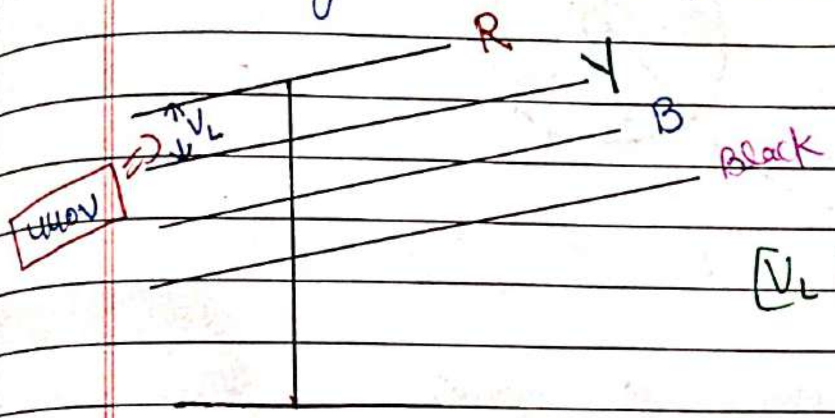
- The voltage across a phase is also called phase voltage.



(1-ϕ AC supply)

- <u>Line Voltage</u> : $[V_L / E_L]$

The voltage between two phases is called line voltage.

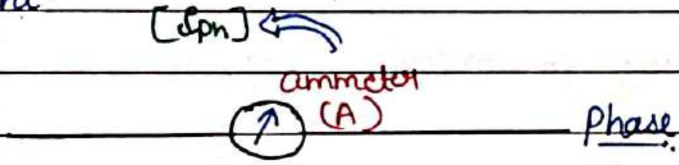


(3-φ AC supply)

$[V_L \mid E_L]$

- <u>Phase Current</u> : $[i_{ph}]$

The current flows through a phase is called phase current.



- <u>Line Current</u> : $[i_L]$

The current flows through a line is called line current.

<u>Balanced supply</u> : [3 − φ]

$$V_R = V_m \angle 0^\circ$$
$$V_Y = V_m \angle -120^\circ$$
$$V_B = V_m \angle -240^\circ$$

<u>Balanced Load</u>

$$Z_1 = |z| \angle 0^\circ$$
$$Z_2 = |z| \angle -120^\circ$$
$$Z_3 = |z| \angle -240^\circ$$

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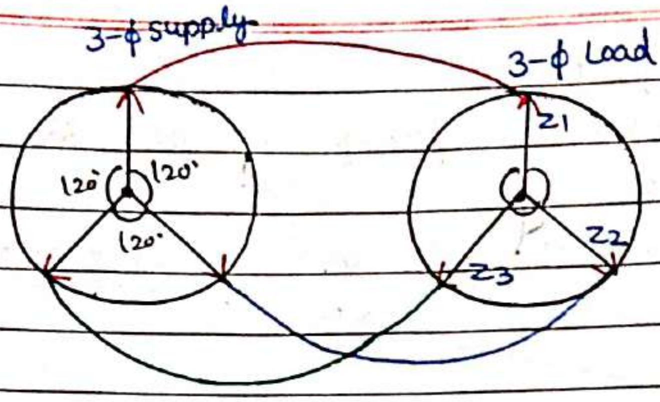

Imp Advantages of 3-phase AC system:

(i) Three phase machines are smaller in size for same output as compared to single phase.

(ii) There is saving in material [copper]

(iii) Efficiency is better.

(iv) power factor is high

(v) Single phase motors have no starting torque, whereas three phase motors are self starting.

(vi) The performance of three phase machines is much better than single phase machine.

(vii) Three phase system of energy transmission is more economical than single phase.

(viii) The power in three phase AC circuit is constant at every instant.

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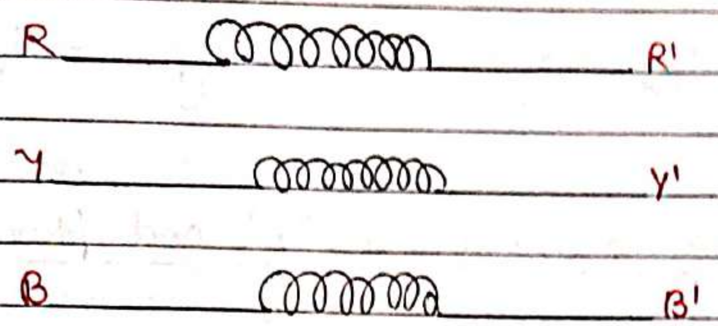
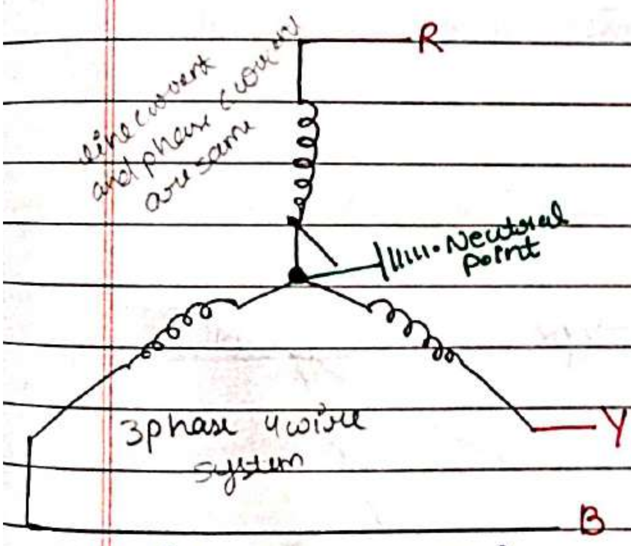
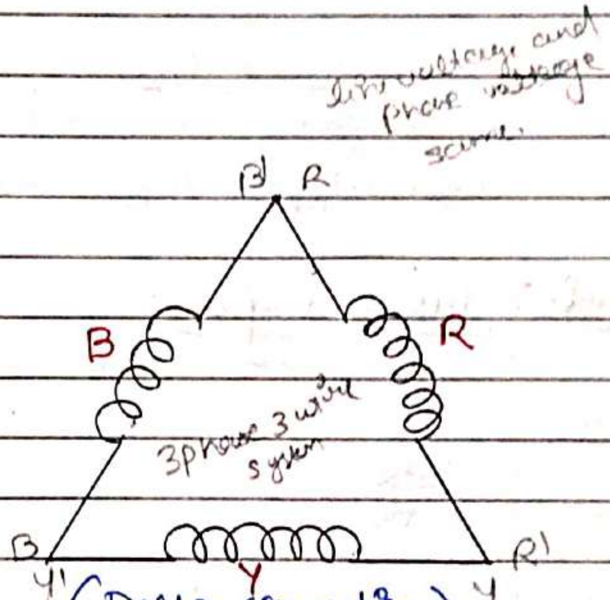
Interconnection of three phases :



- Star connection [⅄]
- Delta connection [△]

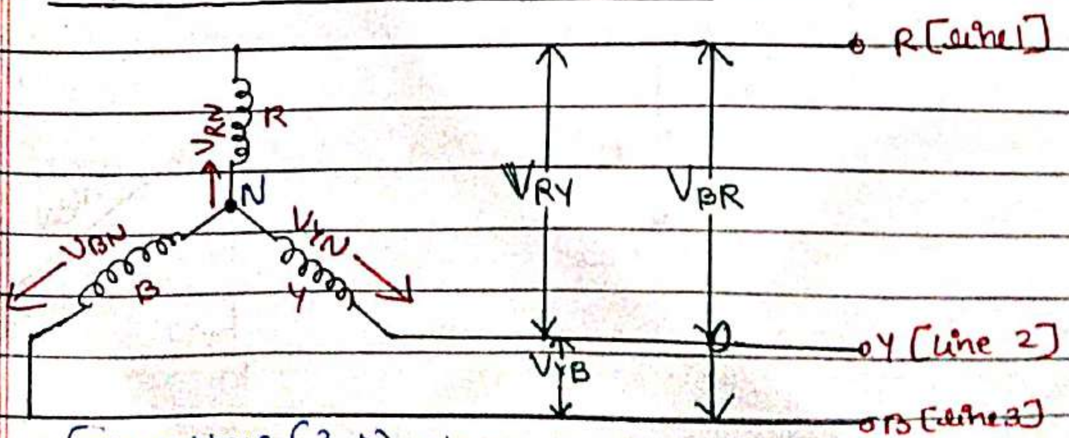


(Star connection)



(Delta connection)

Star Connection :



[Three phase (3-ϕ) star connected balanced supply]

Double subscript notation $V_{AB} = -V_{BA}$

where V_{RN}, V_{YN}, V_{BN} = Phase Voltage $\Rightarrow V_{Ph}$

V_{RY}, V_{YB}, V_{BR} = line Voltage $\Rightarrow V_L$

Relation between line Voltage (V_L) and phase voltage (V_{Ph})

(Star connection)

$V_{RN} - V_{YN} = V_{RY}$ ---- (1)

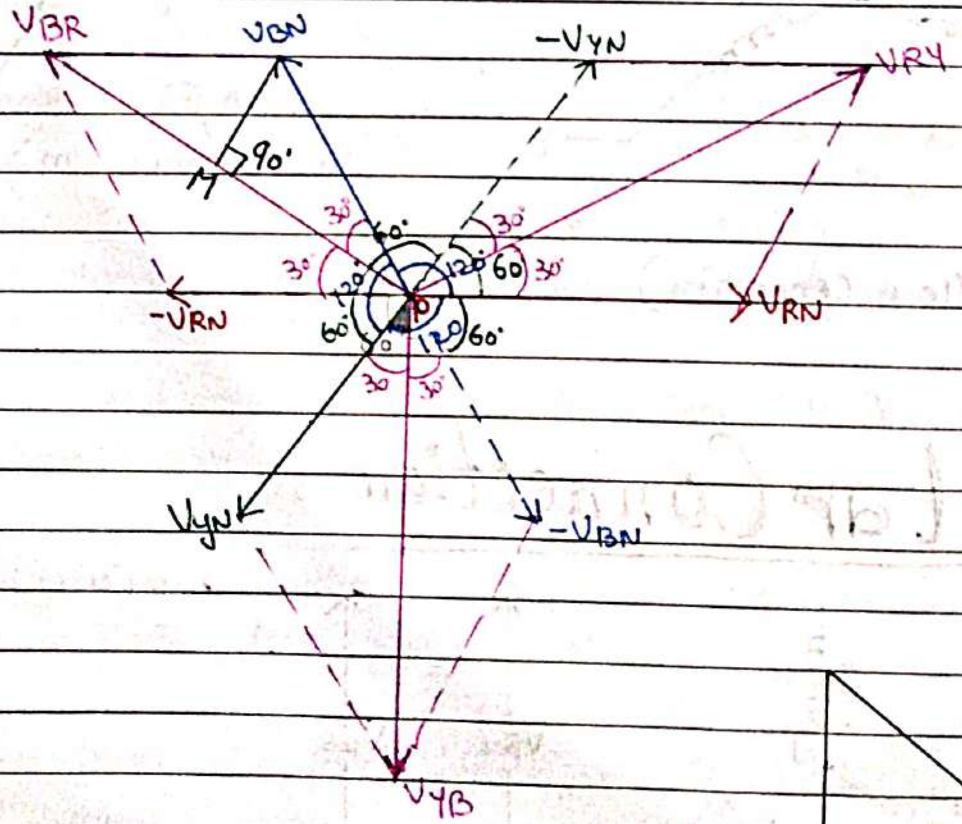
$V_{YN} - V_{BN} = V_{YB}$ ---- (2)

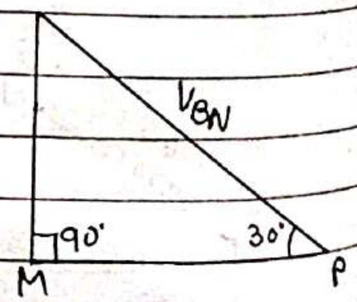
$V_{BN} - V_{RN} = V_{BR}$ ---- (3)

$V_{RN} + V_{NY} = V_{RY}$

$V_{RN} - V_{YN} = V_{RY}$

Imp. Phasor diagram :





Hint: $\cos 30° = \frac{MP}{V_{BN}}$

$MP = V_{BN} \cos 30°$

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$$V_{BR} = 2MP$$

$$V_{BR} = 2 \times V_{BN} \cos 30^\circ$$

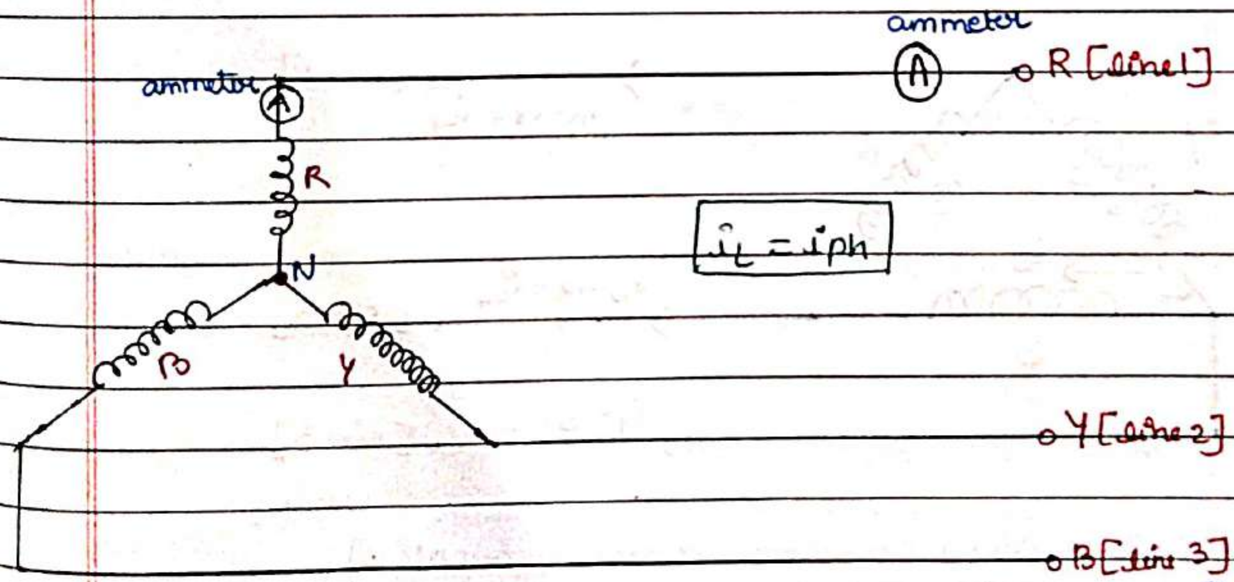
$$V_{BR} = \cancel{2} \times V_{BN} \times \frac{\sqrt{3}}{\cancel{2}}$$

$$V_{BR} = \sqrt{3}\, V_{BN}$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

"In three phase star connection, line voltage V_L is $\sqrt{3}\, V_{ph}$ times of phase voltage."

Relation b/w line current and phase current :-



$$\boxed{i_L = i_{ph}}$$

[Three phase (3-ϕ) star connected balanced Ac supply]

Power in (3-ϕ) AC circuit :

$P = V_{ph}\, i_{ph} \cos\phi$ [1-ϕ]

Active Power $\boxed{P = 3 V_{ph}\, i_{ph} \cos\phi}$ [3-ϕ]

in terms of phase voltage (V_{ph}) and phase current (i_{ph})

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$$P = 3\left(\frac{V_L}{\sqrt{3}}\right) i_L \cos\phi$$

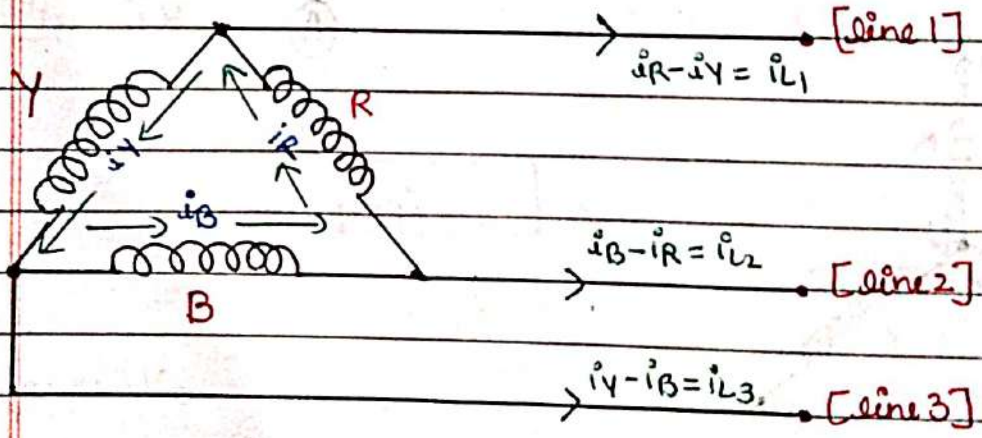
$$\boxed{P = \sqrt{3}\, V_L\, i_L \cos\phi \text{ watt}}$$

in terms of line voltage (V_L) and line current (i_L)

$\boxed{\text{Reactive power} = 3V_{ph} i_{ph} \sin\phi}$ KVAR

$\boxed{\text{Apparent power} = 3V_{ph} i_{ph}}$ KVA

DELTA Connection :



[A 3-ϕ. Delta connected balanced supply]

where i_R, i_Y, i_B is Phase Current is i_{ph}

i_{L1}, i_{L2}, i_{L3} is Line Current is i_L

Relation b/w Line current and phase current

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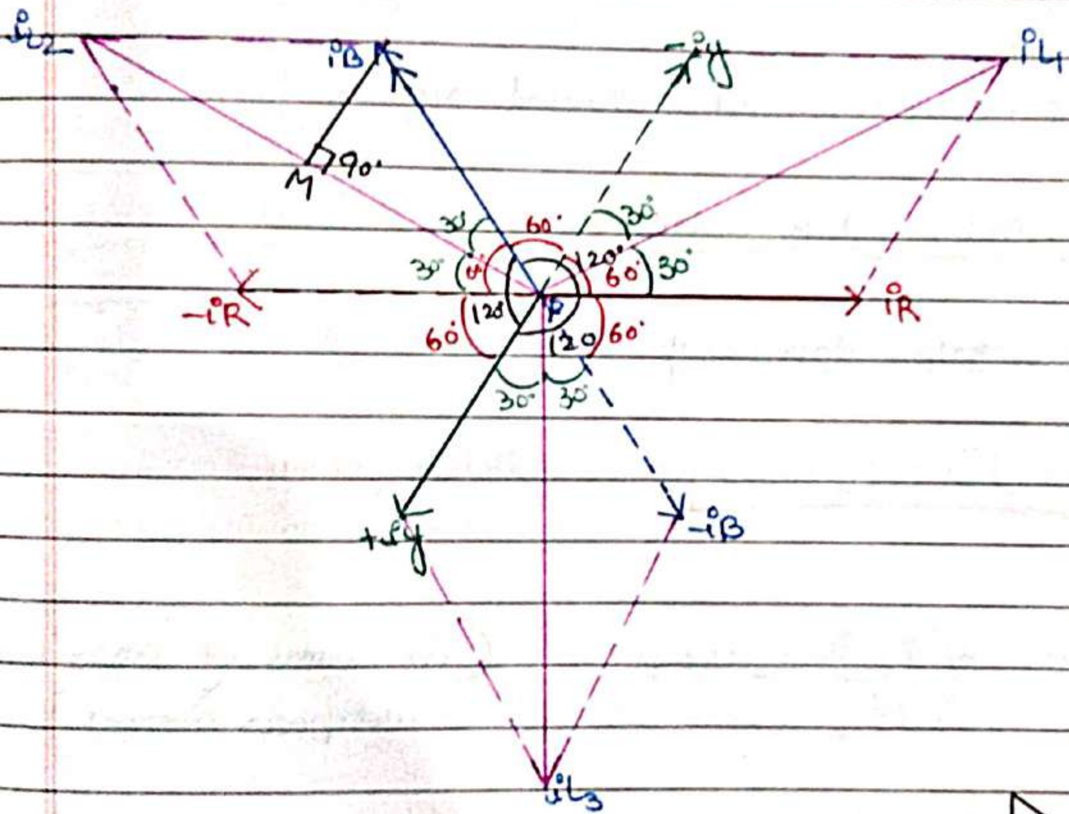
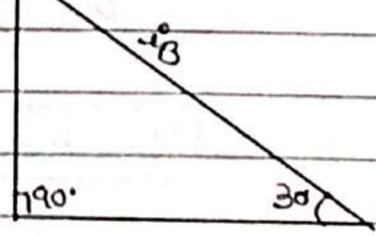
$\bar{i}_R - \bar{i}_Y = \bar{i}_{L_1}$ ------------ (1)

$\bar{i}_B - \bar{i}_R = \bar{i}_{L_2}$ ------------ (2)

$\bar{i}_Y - \bar{i}_B = \bar{i}_{L_3}$ ------------ (3)

phasor diagram :-





$i_{L_2} = 2MP$

$i_{L_2} = 2\, i_B \cos 30°$

$i_{L_2} = \cancel{2}\, i_B \dfrac{\sqrt{3}}{\cancel{2}}$

$i_{L_2} = \sqrt{3}\, i_B$

$\cos 30° = \dfrac{MP}{i_B}$

$MP = i_B \cos 30°$

$\boxed{i_L = \sqrt{3}\, i_{ph}}$

in (3-φ) delta connection line current is $\sqrt{3}$ times phase current.

wattmeter → used to measure power

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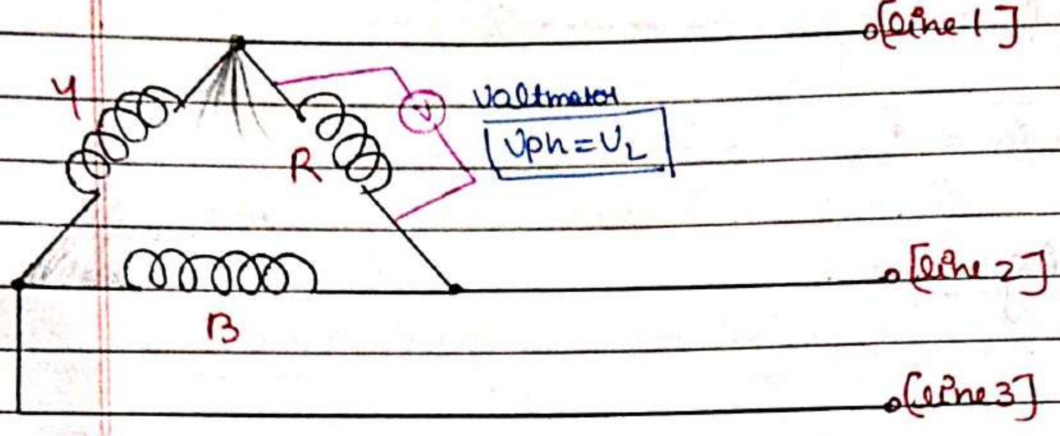
Relation b/w line voltage and phase voltage :-



[A 3-ϕ Delta connected balanced supply]

power in (3-ϕ) Ac circuit

$$P = 3 V_{ph} \, i_{ph} \cos\phi$$

$$\boxed{P = \sqrt{3} \, V_L \, i_L \cos\phi} \quad = \text{Active power.}$$

$$P = 3\left(\frac{i_L}{\sqrt{3}}\right) V_L \cos\phi$$ (in terms of phase voltage and phase current)

$$P = \sqrt{3} \, V_L \, i_L \cos\phi$$

$\boxed{\text{Reactive power} = 3 V_{ph} \, i_{ph} \sin\phi}$ KVAR

$\boxed{\text{Apparent power} = 3 V_{ph} \, i_{ph}}$ KVA

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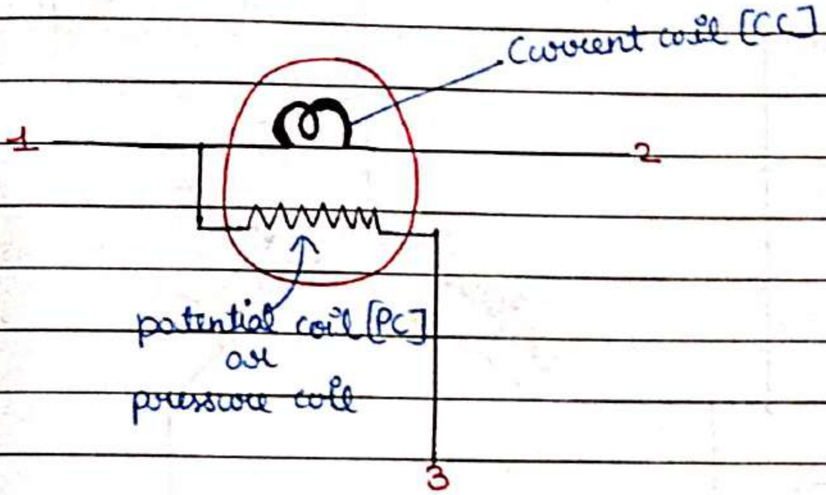
Measurement of power in (3-ϕ) AC circuit :

[$P = 3 V_{ph} I_{ph} \cos\phi$]

power = Wattmeter



$\uparrow R = \rho \frac{l}{A\downarrow}$

Current coil = Connected in series with phase or line

potential coil = Connected in parallel b/w two phase or line.

Note : 1. Current coil has less number of turns and large cross sectional area. (thick wire)

2. potential coil has more number of turns and less cross-sectional area. (thin wire)

Methods of power measurements :- [3-ϕ]

1. One wattmeter method [Balanced]

$$\boxed{P = 3 \text{ wattmeter reading}}$$

Universal method

2. Imp Two wattmeter method [Balanced and unbalanced]

3. Three wattmeter method [Balanced and unbalanced]

$$\boxed{P = W_1 + W_2 + W_3}$$

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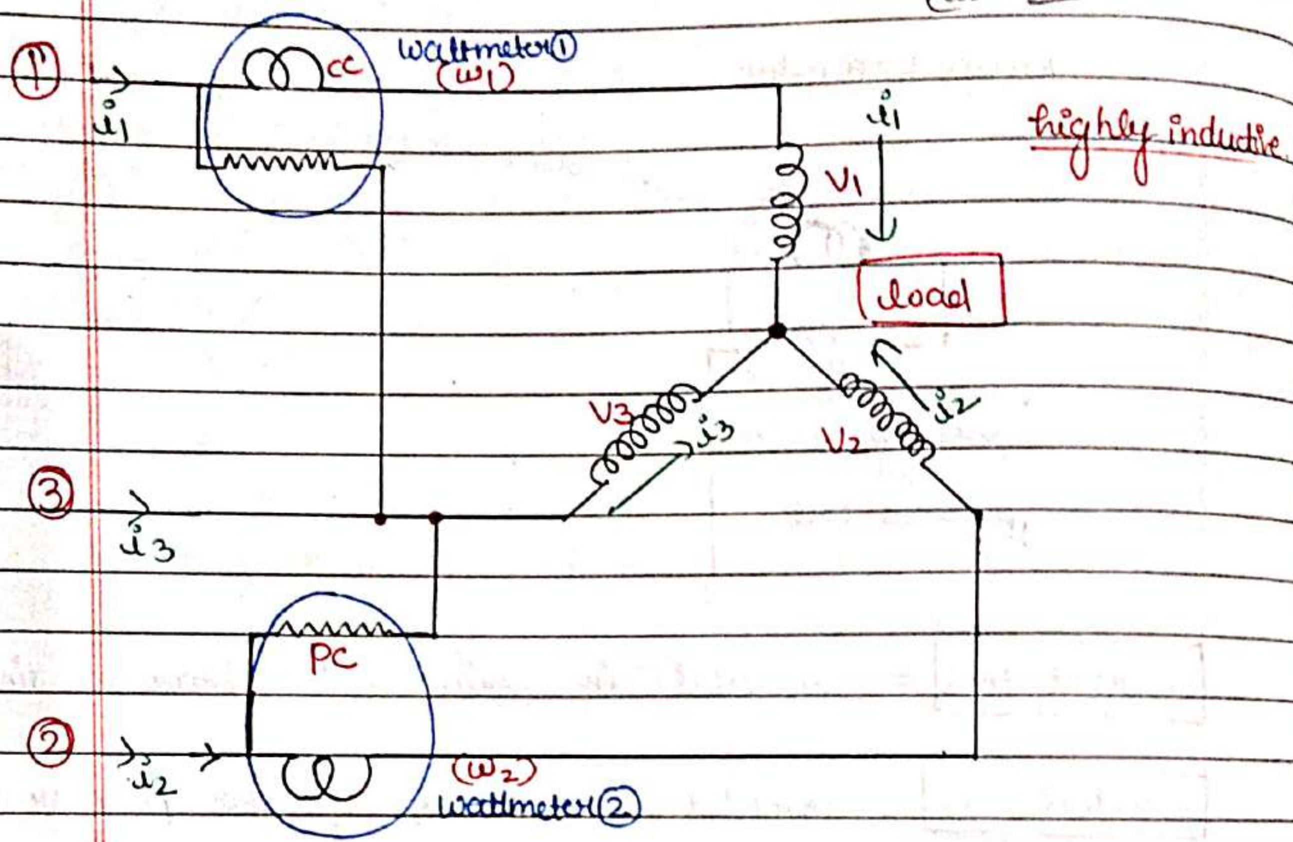
TWO WATTMETER METHOD : (for power calculation of 3-phase) (balanced and unbalanced)



[3-phase star connected balanced load]

Instantaneous power

$$p = V_1 i_1 + V_2 i_2 + V_3 i_3 \quad \text{---} \quad (1)$$

KCL at (N) point

$$i_1 + i_2 + i_3 = 0$$

$$i_3 = -(i_1 + i_2) \quad \text{---} \quad (ii)$$

from eq^n (i) and eq^n (ii)

$$p = V_1 i_1 + V_2 i_2 - \{V_3(i_1 + i_2)$$

$$p = V_1 i_1 + V_2 i_2 - V_3 i_1 - V_3 i_2$$

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$$P = (V_1 - V_3)\, i_1 + (V_2 - V_3)\, i_2$$

$$\boxed{P = V_{13}\, i_1 + V_{23}\, i_2}$$

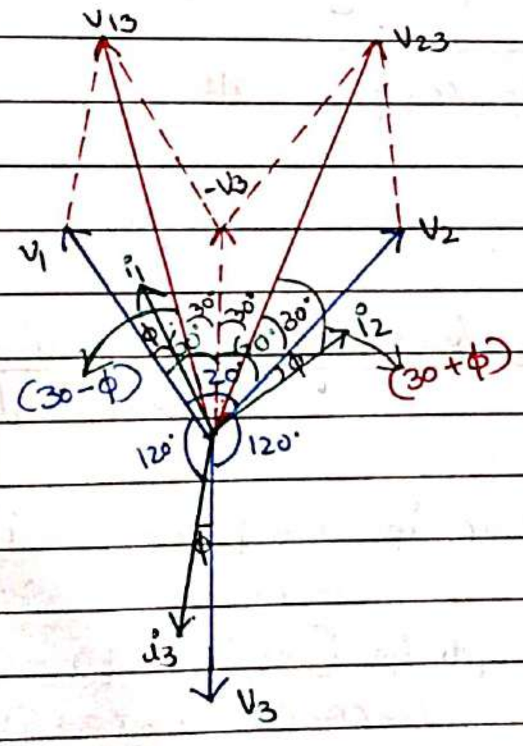
Reading of wattmeter (W_1) — Reading of wattmeter (W_2)

phasor diagram :-

(highly inductive load)

$(V_1 - V_3) = V_{13}$

$(V_2 - V_3) = V_{23}$



from phasor diagram :

reading of wattmeter (W_1) = $V_{13}\, i_1 \cos(30° - \phi)$

& reading of wattmeter (W_2) = $V_{23}\, i_2 \cos(30 + \phi)$

$$\boxed{\text{Total power } (P) = W_1 + W_2}$$

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$$P = V_{13}\, i_1 (\cos 30 - \phi) + V_{23}\, i_2 \cos(30 - \phi)$$

Star connection
$V_{13} = V_1 - V_3$
V_L = line voltage
$\boxed{V_L = \sqrt{3} V_{ph}}$
i_1 = phase current $\boxed{i_{ph}}$
$\boxed{i_{ph} = i_L}$

$$P = V_L\, i_L (\cos 30 - \phi) + V_L i_L \cos(30 - \phi)$$

$$P = \sqrt{3}\, V_{ph}\, i_{ph} \left[\cos(30 - \phi) + \cos(30 + \phi)\right]$$

$\{\cos(A-B) + \cos(A+B) = 2\cos A \cos B\}$

$$P = \sqrt{3}\, V_{ph}\, i_{ph} \left[2 \cos 30^\circ \cos\phi\right]$$

$$P = \sqrt{3}\, V_{ph}\, i_{ph} \left[2 \times \frac{\sqrt{3}}{2} \cos\phi\right]$$

$$\boxed{P = 3 V_{ph}\, i_{ph} \cos\phi} \text{ watt}$$

Determination of power factor by using two wattmeter method :

$$W_1 + W_2 = 3 V_{ph}\, i_{ph} \cos\phi \quad \text{———} (i)$$

$$W_1 - W_2 = \sqrt{3}\, V_{ph}\, i_{ph} \left[\cos(30^\circ - \phi) - \cos(30^\circ + \phi)\right]$$

$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$

$$W_1 - W_2 = \sqrt{3}\, V_{ph}\, i_{ph} \left[2 \sin 30^\circ \sin\phi\right]$$

$$W_1 - W_2 = \sqrt{3}\, V_{ph}\, i_{ph} \sin\phi \quad \text{———} (ii)$$

from eq^n $\frac{(ii)}{(i)}$

$$\frac{(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}}{3} \tan\phi$$

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$$\frac{(W_1-W_2)}{(W_1+W_2)} = \frac{1}{\sqrt{3}} \tan\phi$$

$$\tan\phi = \sqrt{3}\frac{(W_1-W_2)}{(W_1+W_2)}$$

$$\boxed{\phi = \tan^{-1}\sqrt{3}\frac{(W_1-W_2)}{(W_1+W_2)}}$$

power factor $\cos\phi = \boxed{\cos\left[\tan^{-1}\sqrt{3}\frac{(W_1-W_2)}{(W_1+W_2)}\right]}$

Important Cases:

$W_1 = V_{13}\, i_1 \cos(30\mp\phi)$ $\qquad$ $W_2 = V_{23}\, i_2 \cos(30+\phi)$

$W_1 = V_L\, i_L \cos(30\mp\phi)$ $\qquad$ $W_2 = V_L\, i_L \cos(30+\phi)$

$\boxed{W_1 = \sqrt{3}\, V_{ph}\, i_{ph} \cos(30-\phi)}$ $\qquad$ $\boxed{W_2 = \sqrt{3}\, V_{ph}\, i_{ph} \cos(30+\phi)}$

Case I: $\boxed{W_1 = W_2}$ power factor $\boxed{\cos\phi}$ $\boxed{\phi = 0}$
$\Rightarrow$ unity power factor $\boxed{\cos\phi = 1}$

Case II: $\boxed{W_1 = -W_2}$ power factor $\cos\phi$ $\phi = 90^\circ$
$\Rightarrow$ zero power factor

Case III: $\boxed{W_1 = +ve \text{ reading}}$

$\boxed{W_2 = 0}$

$\cos\phi = \frac{1}{2} \Rightarrow 0.5$

$\boxed{\phi = 60^\circ}$

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Steps for Numerical problems:

1. The given voltage is always line voltage (V_L)

2. Calculate the phase voltage (V_{ph}) depending on three phase connections ($\curlywedge / \Delta$) star/delta.

   Star, $V_L = \sqrt{3} V_{ph}$

   delta $V_L = V_{ph}$

3. Determine the phase current [i_{ph}]

$$\boxed{i_{ph} = \frac{V_{ph}}{Z_{ph}}} \quad \text{where } \boxed{Z_{ph} = R \pm JX}$$

4. Calculate the line current (i_L)

   $\boxed{\text{star}}$, $i_L = i_{ph}$

   $\boxed{\text{delta}}$, $i_L = \sqrt{3} . i_{ph}$

5. Power factor [$\cos\phi$]

$$\boxed{\cos\phi = \frac{R}{|Z|}}$$

6. Calculate the three phase power.

$$\boxed{P = 3 V_{ph} i_{ph} \cos\phi} \text{ (watt)}$$

$$\boxed{Q = 3 V_{ph} i_{ph} \sin\phi} \text{ (VAR)}$$

$$\boxed{S = 3 V_{ph} i_{ph}} \text{ (VA)}$$

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Numericals:

Ques1 A balanced star connected load of $(8+J6)$ ~~Ω~~ Ω per phase is connected to a three phase & 230V supply. (line voltage)
find the i) Phase voltage
ii) line current
iii) Power factor
(iv) Total volt ampere (apparent power)
(v) Active power
(vi) Reactive power

Ans given $Z_{ph} = (8+j6)\Omega$ $|z| = \sqrt{(8)^2+(6)^2}$
$V_L = 230V$ $|z| = \sqrt{64+36} = \sqrt{100}$
$|z| = 10$

$V_L = \sqrt{3} V_{ph}$

$\frac{230}{\sqrt{3}} = V_{ph}$

$\boxed{V_{ph} = 132.79}$ volt

(ii) $i_L = i_{ph}$

$i_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{\sqrt{8^2+6^2}} = \frac{132.79}{10} = 13.279$ Amp

$i_L = i_{ph} \Rightarrow \boxed{13.279 \text{ Amp}}$

(iii) power factor $\cos\phi = \frac{R}{|z|}$

$= \frac{8}{10} \Rightarrow \boxed{0.8}$ [lagging]

(iv) $S = 3V_{ph} \cdot i_{ph}$

$S = 3 \times 132.79 \times 13.279$

$\boxed{S = 5.28}$ $\cancel{\times 10^3}$ KVA

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(v) Active power $= 3 V_{ph} i_{ph} \cos\phi$

$= 3 \times 132.79 \times 13.279 \times \cos\phi$

$= 5.28 \times 0.8$

$\boxed{P = 4.22}$ KW

(vi) Reactive power $= 3 V_{ph} i_{ph} \sin\phi$

$= 5.28 \times 0.6$

$\boxed{Q = 3.16}$ KVAR

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MODULE → 3

- → Magnetic circuit
- → Single Phase [1-ϕ] Transformer
- → Auto transformer
- → Three phase (3-ϕ) transformer connection

Magnetic materials : There are three types of magnetic material

1. Paramagnetic
2. Diamagnetic
3. Ferromagnetic

- **Paramagnetic material :**
The materials which are not strongly attract by a magnet such as tin, platinum, aluminium, etc. are known as paramagnetic materials.
"The relative permeability (μ_r) is small but positive."

Paramagnetic materials magnetised when placed in strong magnetic field and act in the direction of magnetic field. These materials have little application in the field of electrical engineering.

- **Diamagnetic material :**
The materials which are repelled by magnet such as zinc, led, wood, copper etc. are known as diamagnetic material.
"Their relative permeability is less than unity"

Diamagnetic materials are unimportant from the point of view application in the field of electrical engineering.

3. Ferromagnetic material :

The materials which are strongly attracted by a magnet such as iron, steel, cobalt, nickel etc are known as ferromagnetic material.

"Their relative permeability is very high."

There are two types of ferromagnetic materials.

1. Soft ferromagnetic material

2. Hard ferromagnetic material

- Soft ferromagnetic material : They have high relative permeability and low coersive force, easily magnetised and demagnetised and have small hysteresis loop. [B-H loop]

- Hard ferromagnetic material : They have also high relative permeability and very high coersive force. They are difficult to magnetise and demagnetised.

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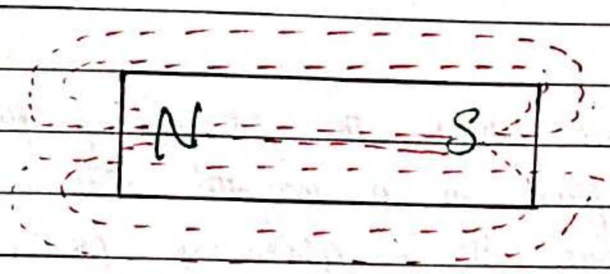
Some Basic Definitions related to magnetic circuit :

1. Magnetic field : The area around a magnet is called magnetic field.



2. Magnetic flux : It is the amount of magnetic field or the number of magnetic lines of force produced by the magnetic source.
It is denoted by [ϕ] and measured in [Weber][Wb]

3. Magnetic flux density : The flux per unit area is called magnetic flux density.
It is denoted by [B].

Magnetic flux density $\boxed{[B] = \frac{\phi}{A}}$ [Wb/m²] or [Tesla]

4. Magnetic field strength : This gives quantative measurement of strongness or weakness of the magnetic field. It is denoted by [H].

magnetic field strength $[H] = \frac{NI}{l}$ $\left[\frac{AT}{m}\right]$

Where, N = Number of turns in coil.

I = Current through the coil.

l = length of the coil.

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5. Magneto Motive Force (MMF):

We know that the flow of electrons is known as current which is basically due to electromotive force (EMF)

Similarly the force behind the set up of flux or production of flux in a magnetic circuit is called magneto motive force. It is given by product of the number of turns of magnetising coil and the current passing through it.

$$\boxed{MMF = NI} \quad [\text{Amp-Turn}]\ [AT]$$

where, N = number of turns in coil
I = Current through the coil

$$\boxed{H = \frac{NI}{l}}$$

$$\boxed{MMF = Hl = NI}$$

6. Reluctance :

In an electric circuit current is opposed by the resistance of the material.
Similarly, there are opposition by the material to production of flux. is called reluctance
It is denoted by ['S'].

$$\text{Reluctance } [S] = \boxed{\frac{MMF}{FLUX} = \frac{NI}{\phi}} \left[\frac{AT}{Wb}\right]$$

$$(MMF = \phi . S)$$

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$$S = \frac{Hl}{B \cdot a}$$

$$S = \frac{l}{\left(\frac{B}{H}\right) a} \qquad \left(B = \frac{\phi}{A}\right)$$

$$\left(\frac{B}{H} = \mu_0 \mu_r\right)$$

$$\boxed{S = \frac{l}{\mu_0 \mu_r a}}$$

******

7. **Permeance :-**

The permeance of the magnetic circuit is defined as the reciprocal of the reluctance (resistance). It can be defined as the property of magnetic circuit due to which allows production of magnetic flux through it.

$$\text{permeance} = \frac{1}{\text{Reluctance}}$$

$$\boxed{\text{Permeance} = \frac{\mu_0 \mu_r a}{l}} \quad \left[\frac{Wb}{AT}\right]$$

8. **Permeability :**

It is defined as the ability of a material to carry the magnetic lines of force or magnetic flux [ϕ].

There are two types of permeability.

1. Absolute permeability. [μ_0]
2. Relative Permeability. [μ_r]

permission to set up the magnetic flux in a material.

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- **Absolute permeability :**

The magnetic field strength decided the magnetic flux density to be produced by the magnet around it, in a given medium.

The ratio of magnetic flux density (B) in a particular medium to the magnetic field strength (H) producing that magnetic flux density, is called 'absolute permeability'.

$$\text{Absolute permeability} = \frac{B}{H}$$

$$\boxed{\mu_0 = \frac{B}{H}} \quad [\text{henry / meter}]$$

The ratio of B to H is constant for free space, air, vacuum which is $4\pi \times 10^{-7}$ H/m

$$\boxed{\mu_0 = \frac{B}{H} = \text{Constant} = 4\pi \times 10^{-7}} \text{ H/m.}$$

- **Relative Permeability :**

It is defined as the ratio of magnetic flux density [B] in a medium to the flux density produced in a free space under the same magnetic field strength and under identical condition

$$\text{Relative permeability} = \frac{B}{H} =$$

$$\boxed{\mu_0 \mu_r = \frac{B}{H}}$$

Where μ_0 = absolute permeability
μ_r = Relative permeability

"For free space, air, vacuum the relative permeability is unity."

"The relative permeability of metals like iron, steel varies from 100 to 10^5.

E.g.

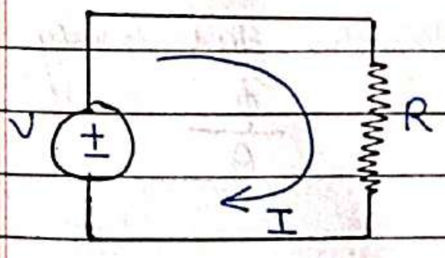
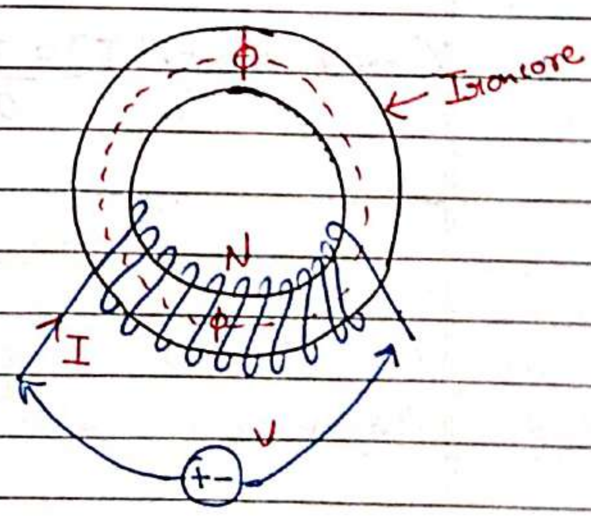
"If the relative permeability of iron is 1000. means, it is 1000 times more magnetic than the free space."

Analogy between electric and magnetic circuit.

OR

Similarities and dissimilarities between electric and magnetic circuit.

Similarities :

	Electric circuit	Magnetic circuit
1.		1. 
2.	The closed path followed by the electric current, is called the electric circuit	2. The closed path followed by the magnetic flux is called the magnetic circuit.

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S.R.No	Electrical circuit	Magnetic circuit
3.	Emf is a driving force in the electric circuit and measured in volt.	3. MMF is a driving force in magnetic circuit and measured in AT.
4.	The flow of electron decides the current in the conductor.	4. The number of magnetic lines of force decided the magnetic flux.
5.	Resistance opposes the flow of current.	5. Reluctance opposes the set up of magnetic flux
6.	$I = \frac{EMF}{R}$	6. $Flux[\phi] = \frac{MMF}{S}$
7.	$R = \rho \frac{l}{a}$ $[\Omega]$	7. $S = \frac{l}{\mu_0 \mu_r a}$ $\left[\frac{AT}{Wb}\right]$
8.	Current density $[J] = \frac{I}{a}$	Magnetic flux density $B = \frac{\phi}{A}$

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Dissimilarities :

S.R.No	Electric circuit	Magnetic circuit.
1.	The electric current flow in the electric circuit	1. Magnetic flux does not flow [set up in any core]
2.	Electric current cannot pass through the air gap.	2. Magnetic flux can passes through the air gap.
3.	Resistance of electric circuit is practically constant for constant temperature.	3. Reluctance is not constant for given material.

B-H curve :

The curve between magnetic flux density (B) and magnetic field strength (H) is called [B-H] curve.

→ B-H Curve for non-magnetic material :

The B-H Curve for non magnetic material (air, wood, rubber) is shown in the figure. The relation between B and H is given by

non magnetic material:

$$\boxed{\mu_0 \mu_r = \frac{B}{H}}$$

$\mu_0 =$ constant $= 4\pi \times 10^{-7}$ [H/m]

$\mu_r =$ relative permeability

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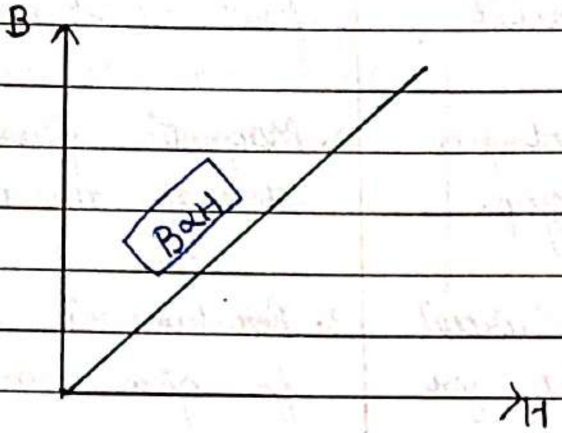
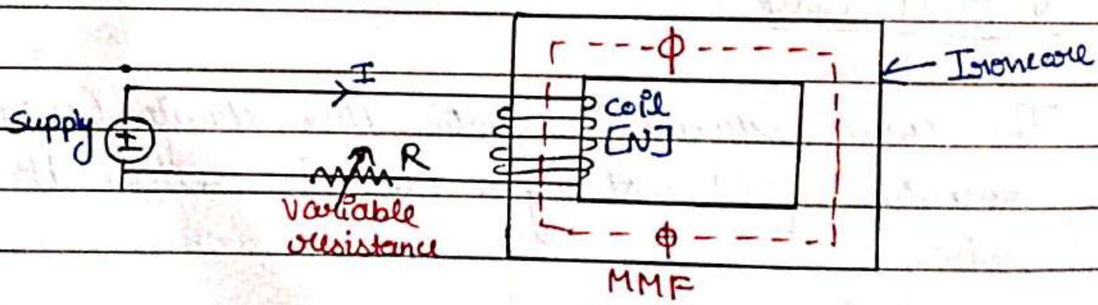
$\boxed{\mu_r = 1}$ [non-magnetic material]

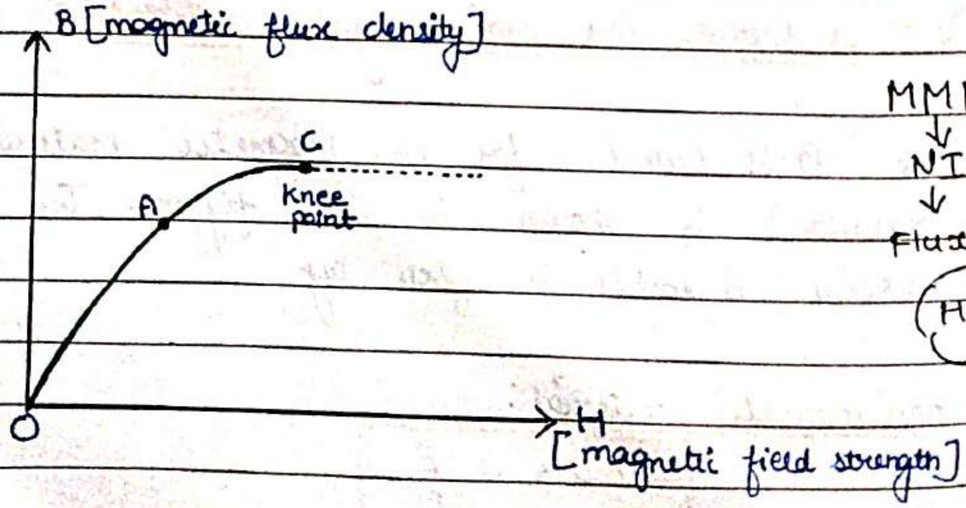
$$\boxed{\text{constant} = \mu_0 = \frac{B}{H}}$$

$\boxed{B \propto H}$



B-H Curve for magnetic material :





MMF
↓
NI
↓
Flux [ϕ]

$H = \frac{NI}{l}$

The graph can be analysed into three regions

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(a) Initial region
(b) middle region
(c) saturation region

(a) Initial region : Near the origin for low value of H, magnetic flux density does not increase rapidly. This is represented by curve OA.

(b) Middle region: In this region H increase, as the magnetic flux density 'B' increase rapidly. This is almost straight line at point C it starts bending. The point C where the curve bends is called knee point or saturated point.

(c) Saturation region: After the knee point, the magnetic flux density 'B' is almost constant and the core is said to be saturated. and the region is called saturation region

Hysteresis Loop [B-H] loop :

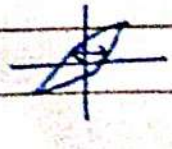
When a magnetic material is subjected to a cycle of magnetisation and demagnetisation.
(It is magnetise first in one direction and demagnetise in other direction). larger the area of hysteresis loop more loss of hysteresis loop.

"It is found that the magnetic flux density in the material lags behind the applied magnetic field strength, this is called hysteresis."

Hysteresis loss
⇓
area of hysteresis loop

Hysteresis loss $\propto$ area of hysteresis loop

"stainless steel" core



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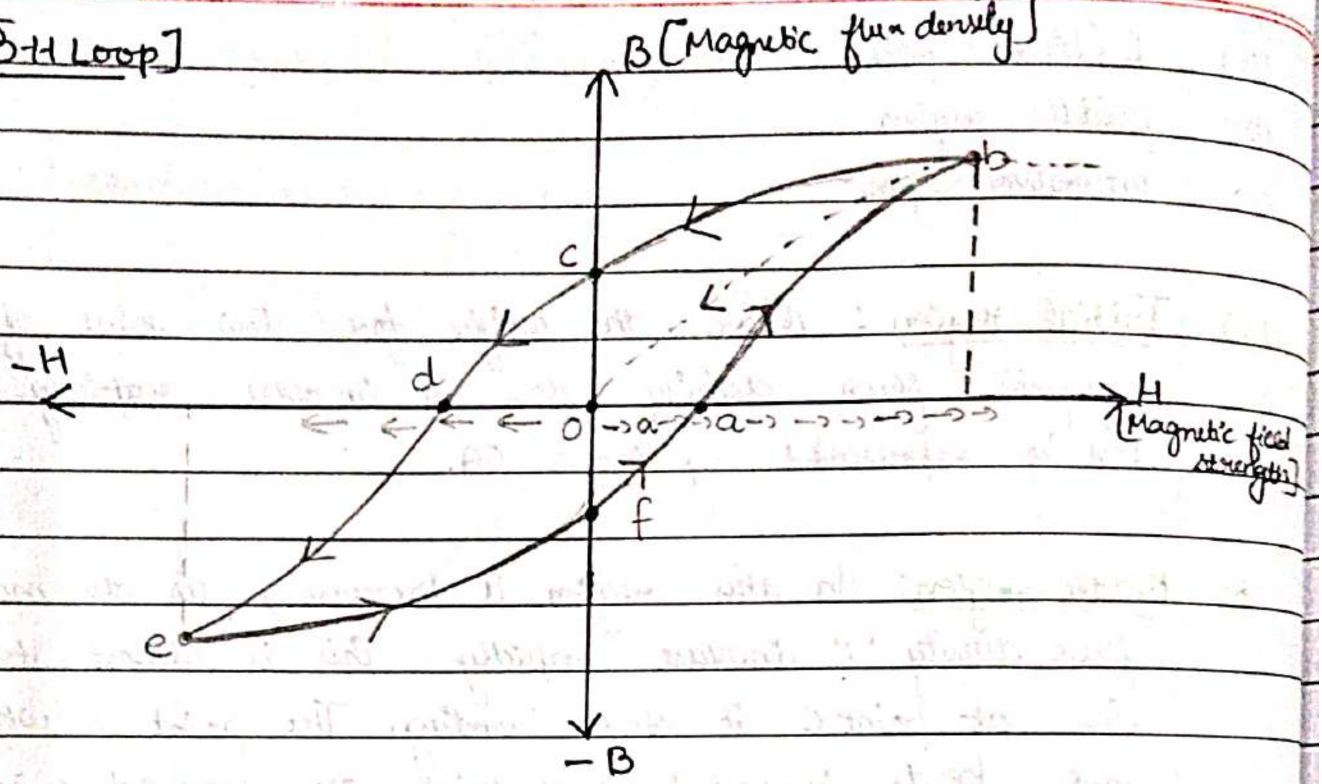
[B-H Loop]



- **Residual magnetic field :**
When magnetic field strength (H) gradually reduced, it is found that the magnetic flux density does not decrease along [b-0] but it follows the path [b-c]. At point c magnetic field strength [H] is zero but magnetic flux density [B] has a finite positive value which is called "Residual Magnetic field."

- **Coercive force :**
The value of magnetic field strength or magnetising force [H] required to wipe out or remove residual magnetic field is known as "coercive force."

- **Retentivity :**
It is the power which is used to remove the residual magnetism.

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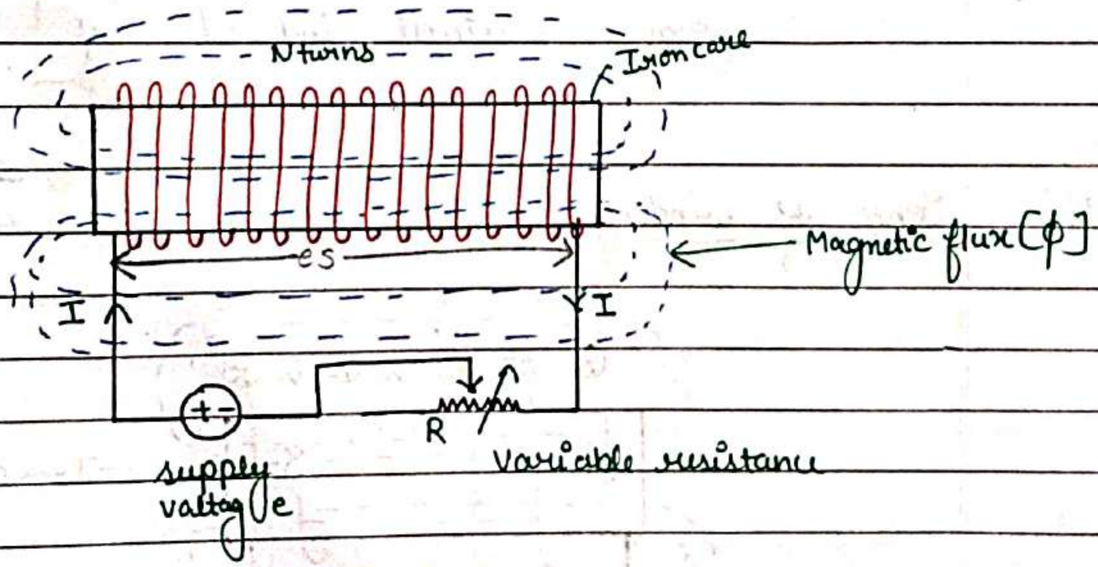
Induced Emf :

Nature of Induced Emf

- Dynamic induced EMF (rotating machine) (motor)
- Statically induced Emf (no rotating part by getting voltage)
  - Self induced EMF
  - Mutual induced EMF

• Self-induced EMF :

MMF
⇓
NI
⇓
Flux(ϕ)



Where,

e_s = self induced emf.

$$\boxed{e = -\frac{d\phi}{dt}}$$ → faraday's law of electromagnetic induction.

(the minus sign) → Lenz's law

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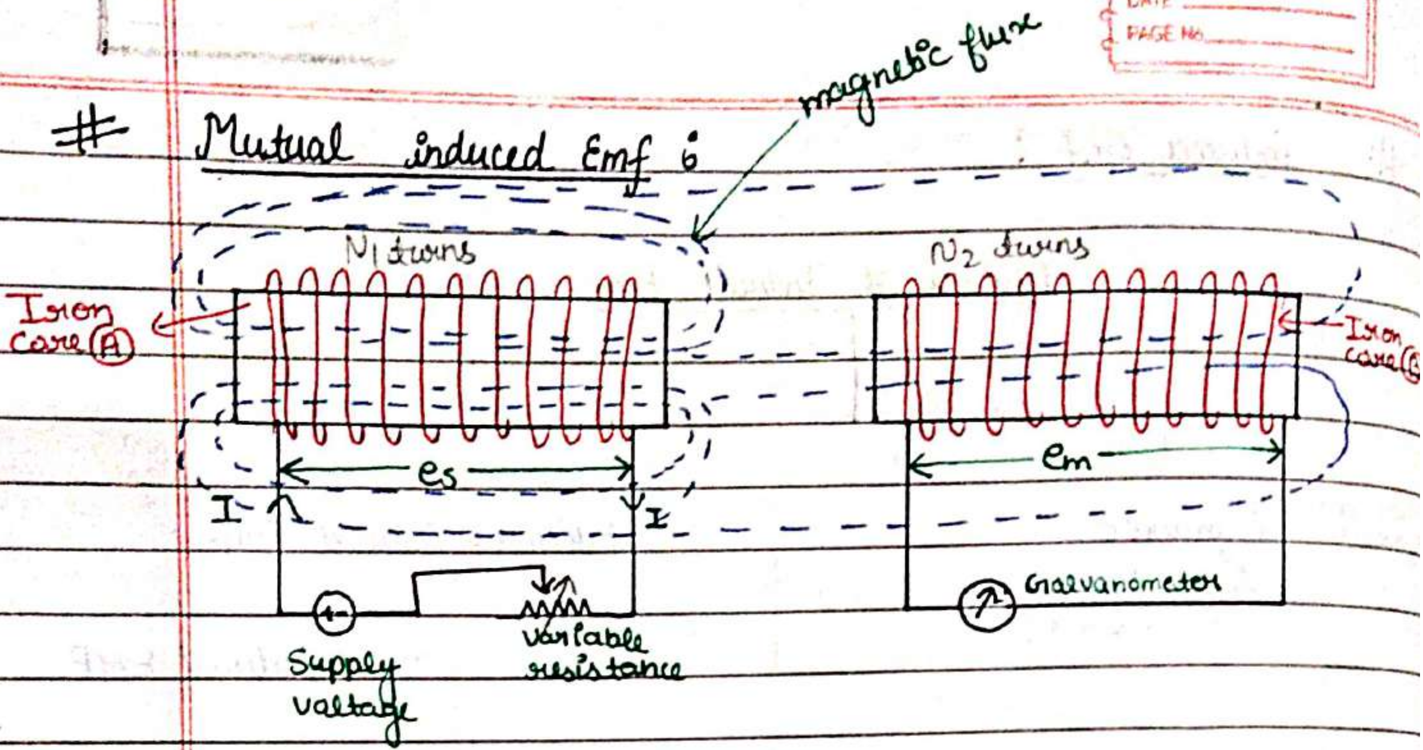
Mutual induced Emf :

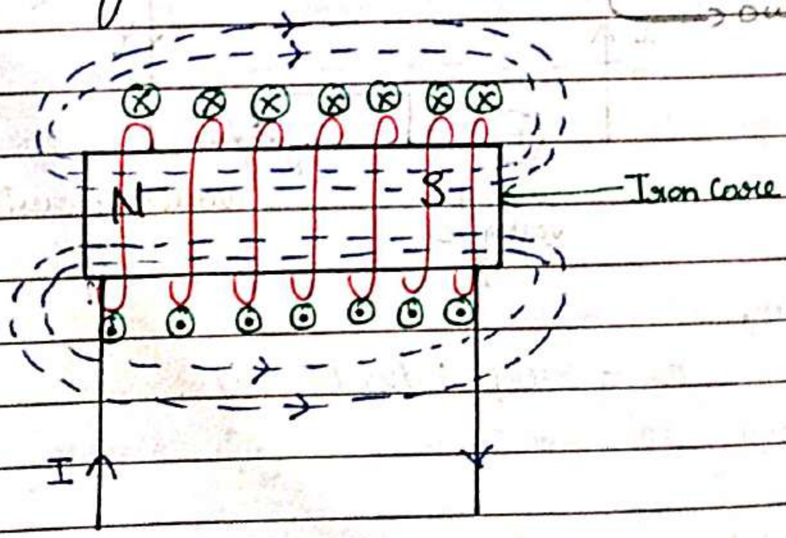


Where, $\boxed{e_s = \text{self induced emf}}$

$\boxed{e_m = \text{mutual induced emf}}$

Sense of winding :

⊗ clockwise direction
⊙ Anticlockwise dir.
→ Inwards
→ outwards



Fleming's Rule :

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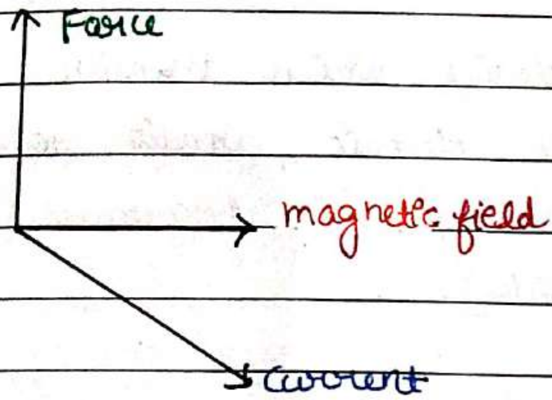
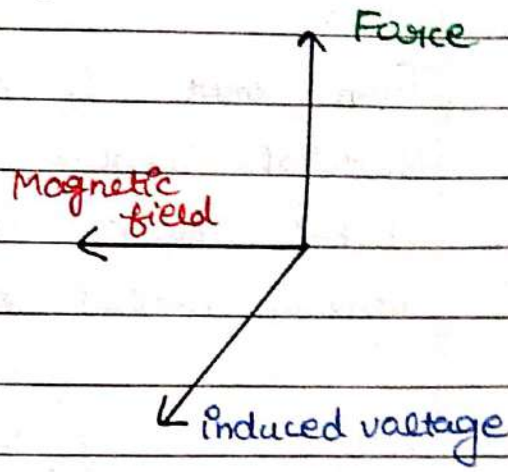
Left hand Rule	Right hand rule
	
It is used in motor action.	It is used in Generator action.

Single Phase (1 − ϕ) transformer : (distortion)

Transformer is a static device which transfer electrical energy from one electric circuit to another electric circuit, without changing its frequency. (frequency remains constant on both sides).

Advantages of Transformer :

1. It is economical to transmit the electric energy at high voltages.
2. For distribution purpose.
3. Efficiency of transformer is very high.
4. Isolate one circuit from another circuit.
5. The resistance between primary windings and secondary windings is always infinite.

Construction of Transformer :

There are three main parts of a transformer.

i) Magnetic circuit
ii) Electric circuit
iii) Dielectric circuit.

Types of Transformer :

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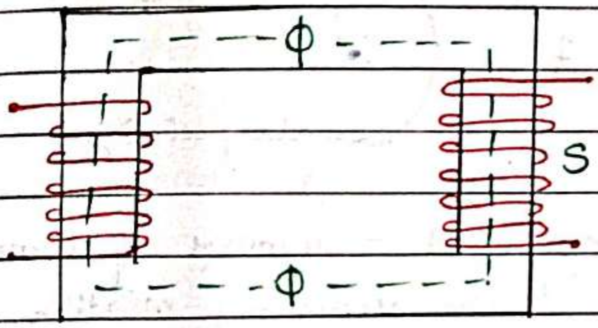
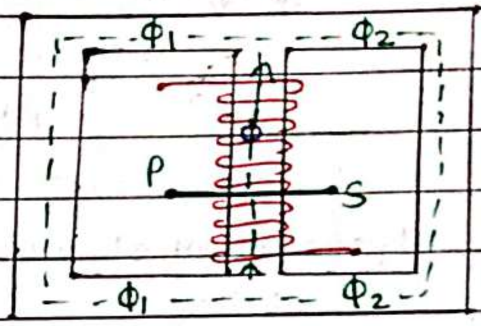
There are two types of transformer according to construction.

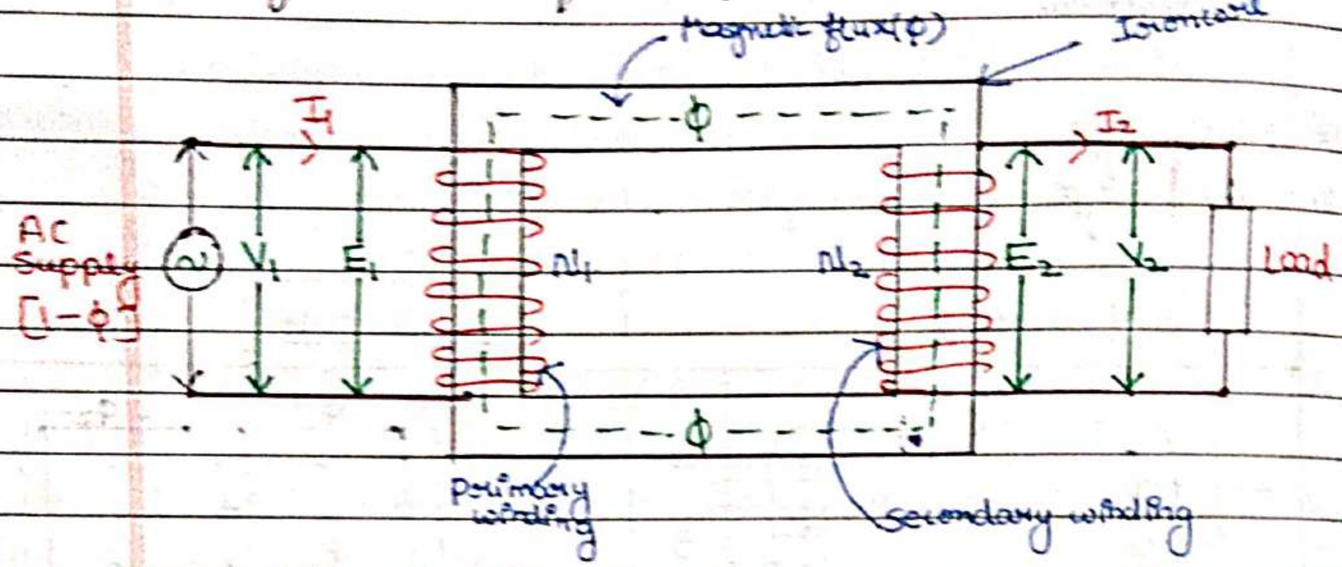
1. Core type
2. Shell type

	Core type	Shell type
		
1.	It has a single mangnetic circuit.	1. Magnetic circuit is divided into two or more parts.
2.	Core is rectangular in shape of uniform cross section. It consist of two vertical limbs and two horizontal lines.	2. It consist of three vertical limbs.
3.	Low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil.	3. High voltage coils are placed between low voltage coils.
4.	Generally used for low and medium voltage level.	4. Used for high voltage level.

Faraday's law of electromagnetic mutual induction principle

$$e = -N\frac{d\phi}{dt}$$

Working principle of Transformer :



N_1 = number of turns in primary winding	N_2 = number of turns in secondary winding
I_1 = Current in primary winding	I_2 = current in secondary winding.
V_1 = Supply voltage	V_2 = Voltage across load
and $\boxed{E_1 = \text{Self induced emf}}$	and $\boxed{E_2 = \text{Mutual induced EMf}}$

→ Working Steps :

Step-1 When the primary winding is exited by an alternating voltage (V_1).

Step-2 This voltage circulates alternating current in primary winding.

This alternating current provides an alternating magneto motive force (mmf) [$N_1 I_1$] in primary winding.

Step-3 Due to this alternating (mmf) an alternating flux will be set up in the magnetic core.

(E_1 and E_2 oppose the supply voltage acc^n to lenz law)

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Step 4 Due to this time varying magnetic field or flux emf will be induced in the primary and secondary winding. (according to faraday's law of electromagnetic induction).

#(19) Emf equation of a transformer :

According to Faraday's Law of electromagnetic induction

$$e = -N\frac{d\phi}{dt} \quad \text{——— (i)}$$

the nature of flux is also alternating

$$\phi = \phi_m \sin \omega t \quad \text{——— (ii)}$$

from eq^n (i) and (ii)

$$e = -N\frac{d}{dt}(\phi_m \sin \omega t)$$

$$e = -N\phi_m \frac{d}{dt}(\sin \omega t)$$

$$e = -N\phi_m \cdot \cos \omega t \cdot \omega$$

$$e = -N\phi_m \omega \cos \omega t$$

$$e = N\phi_m \omega \sin(\omega t - 90^\circ) \quad \text{——— (iii)}$$

for maximum value of induced emf (e)

$$\sin(\omega t - 90^\circ) = 1$$

$$e_{max} = N\phi_m \omega \quad \text{——— (iv)}$$

for RMS value of induced emf

$$E_{RMS} = \frac{e_{max}}{\sqrt{2}} = \frac{N\phi_m \omega}{\sqrt{2}}$$

Two-winding — high voltage
— low voltage

$$E_{rms} = \frac{N\phi_m(2\pi f)}{\sqrt{2}}$$

$$\boxed{E_{RMS} = 4.44\,\phi_m\,f\cdot N}$$ [standard]

$$\boxed{E_1 = 4.44\,\phi_m f N_1}$$ volt [for primary winding]

$$\boxed{E_2 = 4.44\,\phi_m f N_2}$$ volt [for secondary winding]

Note: The induced [E_1 and E_2] will always oppose the supply voltage V_1

Ques-1 What will happen when the primary winding is connected to DC supply? of a transformer?

Ans Due to DC supply, the flux produced in the core is constant in nature. and according to faraday's law of electromagnetic induction, there are no emf induced in secondary winding.

$$e = -N\frac{d\phi}{dt}$$

ϕ = constant [due to DC]

$$\boxed{e = 0}$$

and the primary winding represents the low impedence and due to high current the primary winding burns out.

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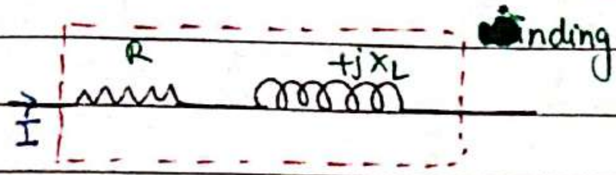
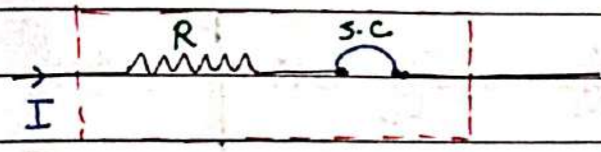


$$\boxed{I = \frac{V}{Z}}$$

DC Supply [f = 0]



Ratio of a transformer :-

We know that

$$E_1 = 4.44\,\phi_m f N_1 \text{ volt}$$
$$E_2 = 4.44\,\phi_m f N_2 \text{ volt}$$

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$ ratio

$\boxed{N_2 = N_1}$ $\boxed{K = 1}$

$\boxed{E_2 = E_1}$

$\boxed{N_2 > N_1}$ $\boxed{E_2 > E_1}$ step up transformer

$\boxed{N_2 < N_1}$ $\boxed{E_2 < E_1}$ step down transformer

Current Ratio of a transformer :

For an Ideal transformer,

input volt ampere = output volt ampere

$$V_1 I_1 = V_2 I_2$$

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$$\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

$\boxed{K \Rightarrow \text{current transformation ratio.}}$

$$\boxed{\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$ v. Imp

Hint:

Primary

$\left(\frac{2200}{200}\right)$ volt volt

secondary

$$\left[K = \frac{220}{2200}\right]$$

Condition for an Ideal transformer:

(i) Total losses are zero

(ii) Winding resistance should be zero.

(iii) permeability of the core should be infinite.

(iv) Reluctance should be zero of the magnetic material.

(v) 100% efficiency.

(vi) No leakage flux

Load increases
↓
current increases
↓
mmf increases
↓
flux increases
and
E1 in primary decreases

$I_1 = \frac{V_1 - E_1}{R}$

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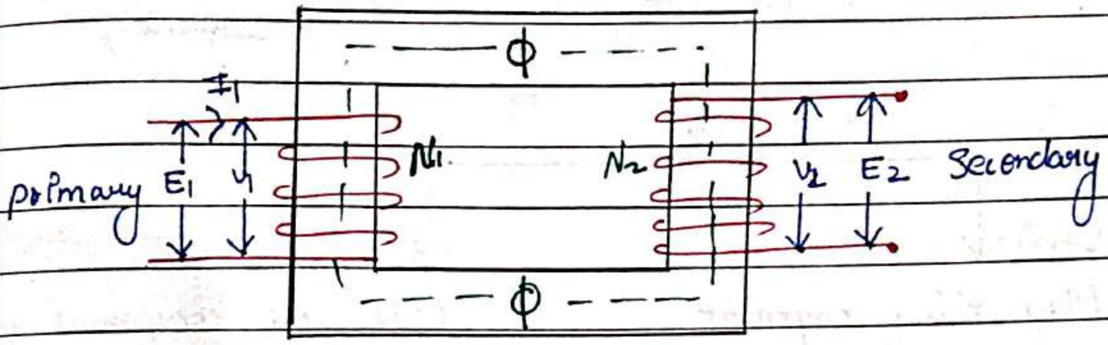
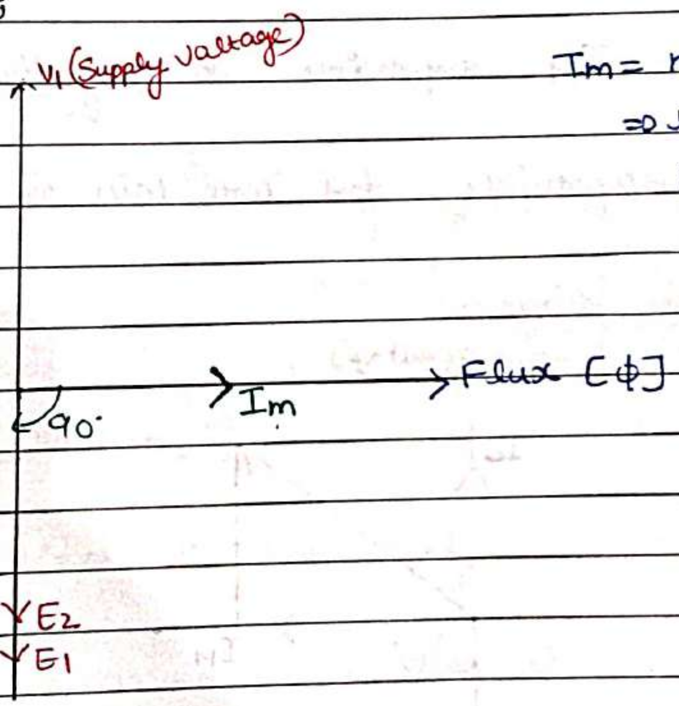
Ideal transformer : - [K=1] [Total loses = 0]



phase diagram :

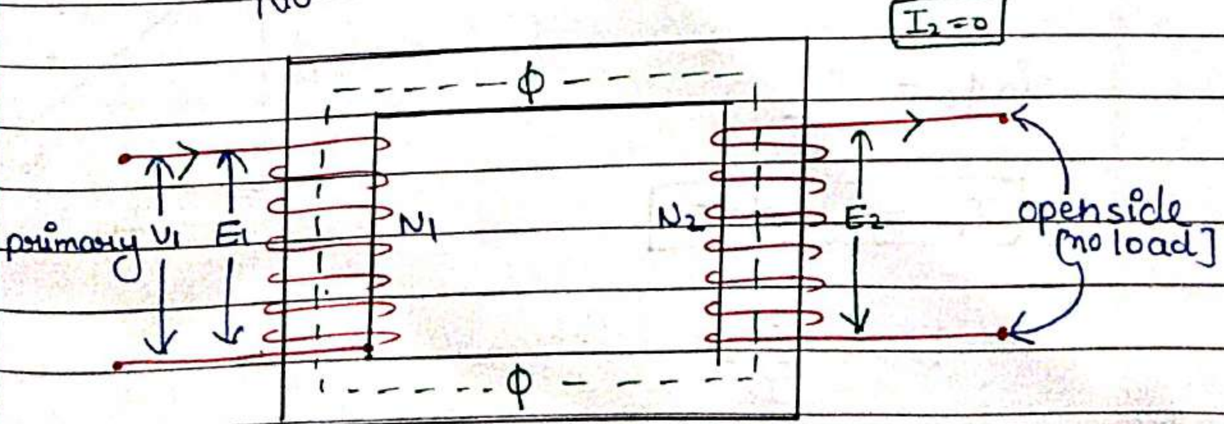
Im = magnetising current
⇒ to setup flux in the core

Flux Φ
⇓
mmf
⇓
NI
⇓
Im



Practical ~~Ideal~~ transformer at no load :

No



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No Load Current ⇒ Very small current (2 to 5% of full load current) 10%

$[I_1 = I_0]$

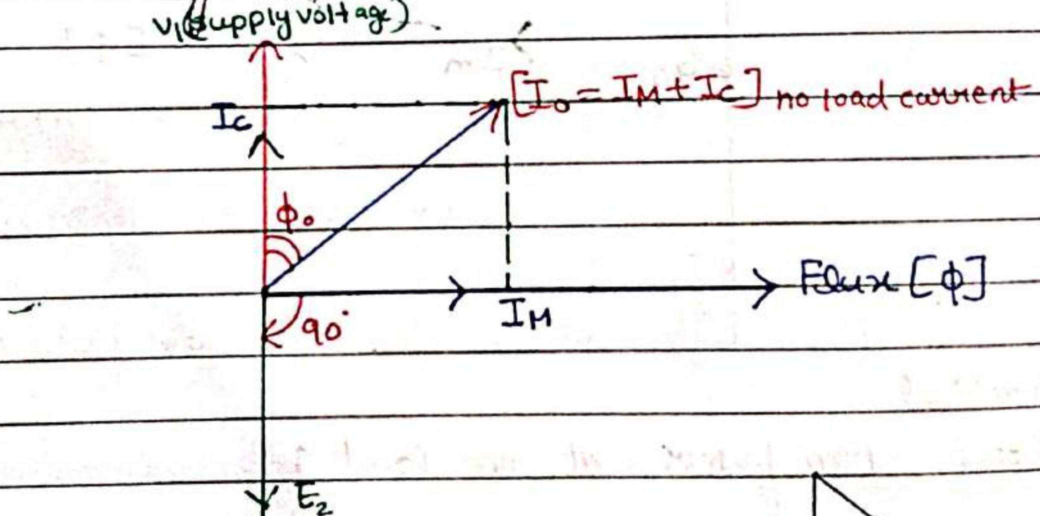
$[I_M]$ Magnetising component of [no load current]

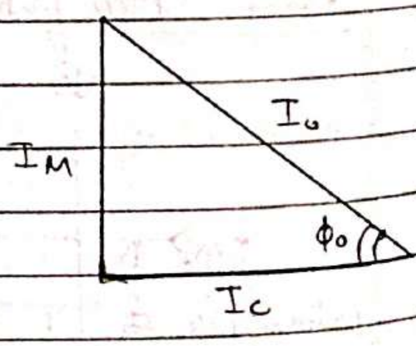
$[I_c]$ Core loss component of No load current

Where I_M responsible to magnetise the Iron core

I_c responsible for core loses or iron loses

Phasor diagram:





$$\sin\phi_0 = \frac{I_M}{I_0}$$

$$\boxed{I_M = I_0 \sin\phi_0}$$

$$\cos\phi_0 = \frac{I_c}{I_0}$$

$$\boxed{I_c = I_0 \cos\phi_0}$$

$$\boxed{I_0 = \sqrt{I_M^2 + I_c^2}}$$

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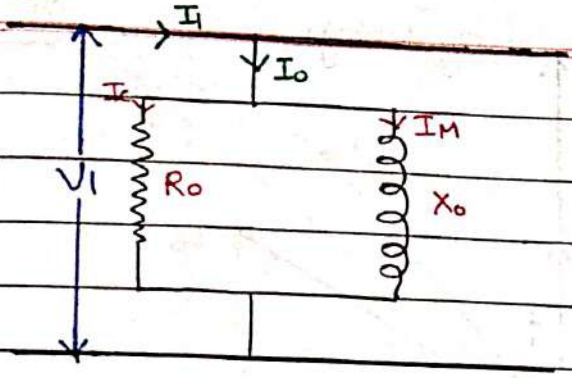
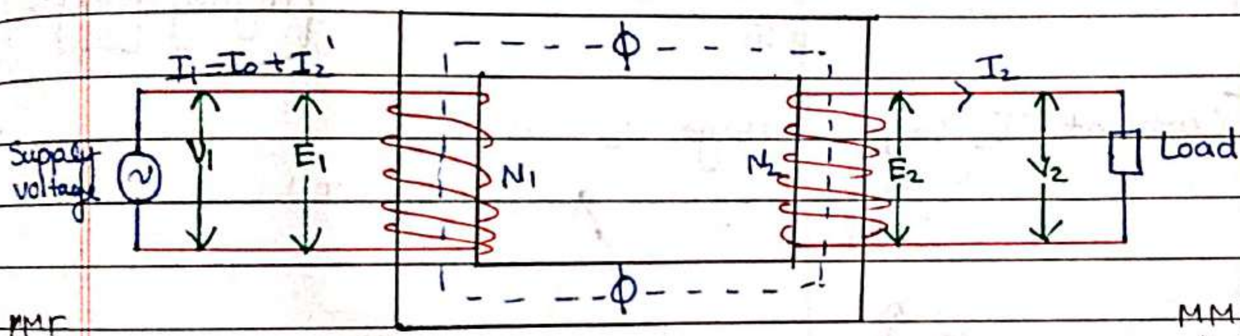


$$I_C = \frac{V_1}{R_0} \qquad I_M = \frac{V_1}{X_0}$$

Practical Transformer on load:



MMF
↓
$N_1 I_0$
⇩
$N_1 I_0 \rightarrow$ MMF produced by primary side

MMF
⇩
$N_2 I_2$
$N_2 I_2 \rightarrow$ MMF ~~produced in~~ secondary side

I_2' neutralizes the effect of I_2

$N_1 I_0 \rightarrow$ MMF produced by primary side

$N_2 I_2 \rightarrow$ MMF produced in secondary side

"Flux [ϕ_2] & flux [ϕ_2']" both are equal and opposite in direction.

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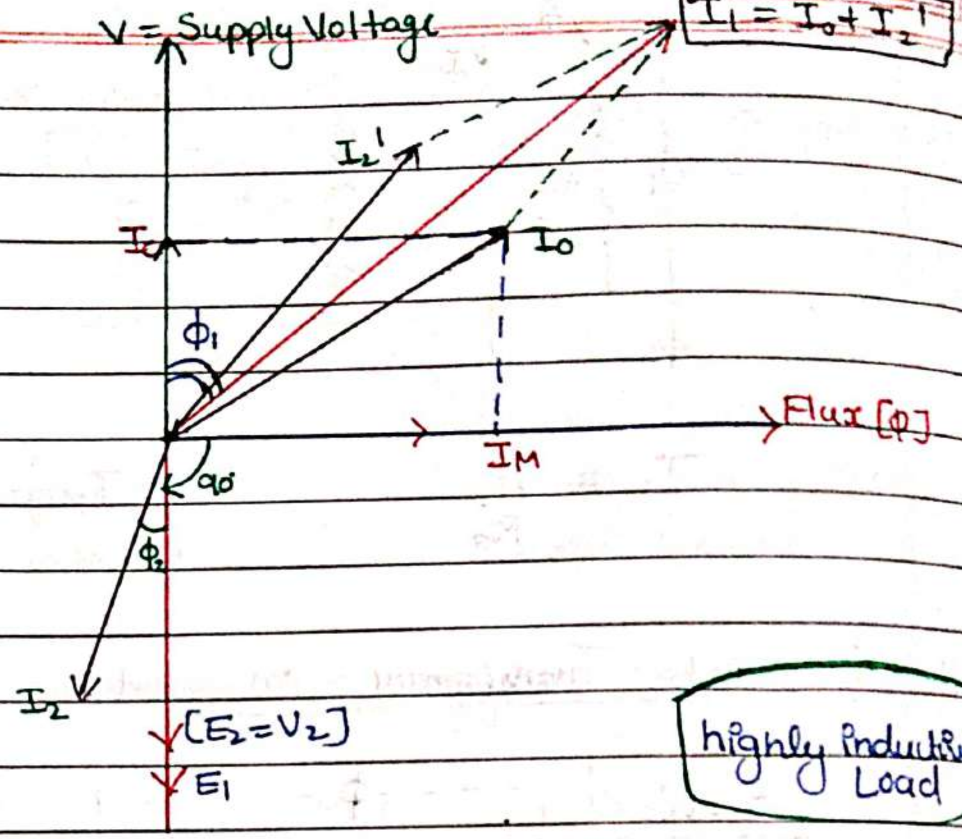
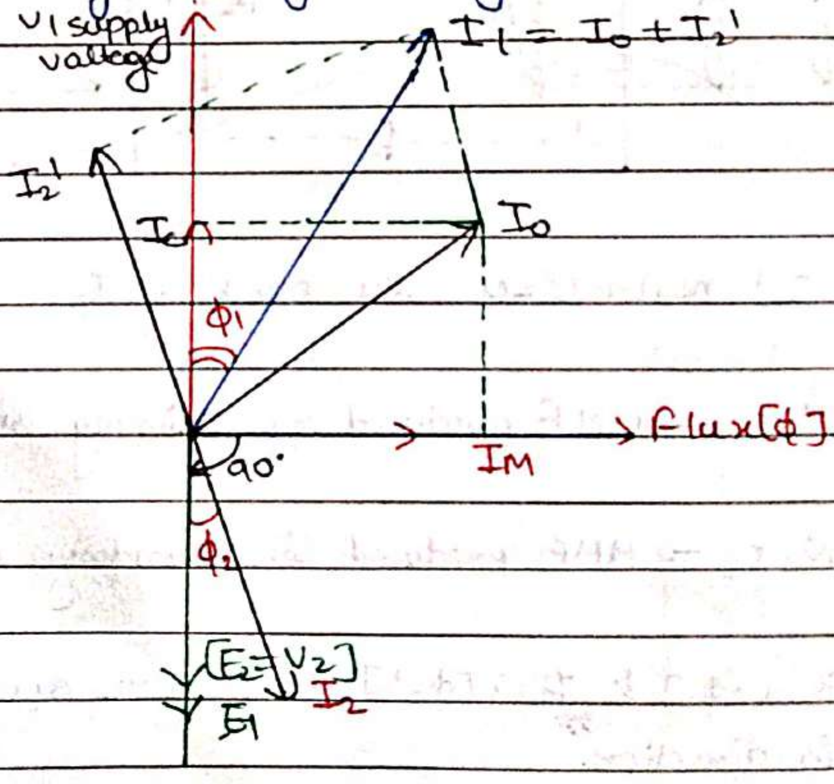
Case-I



highly inductive Load

[Current I_2 lag voltage V_2 by ϕ_2]

Case II



Current I_2 leads voltage V_2 by ϕ_2

OR

Highly capacitive load.

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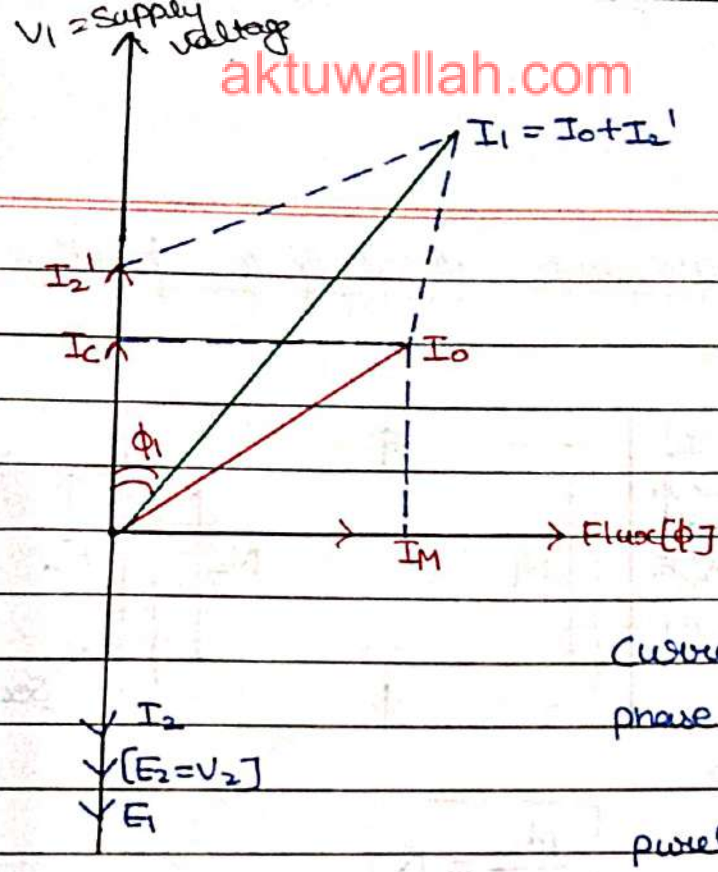
Case III



Current I_2 in same phase with voltage V_2

OR

purely resistive load

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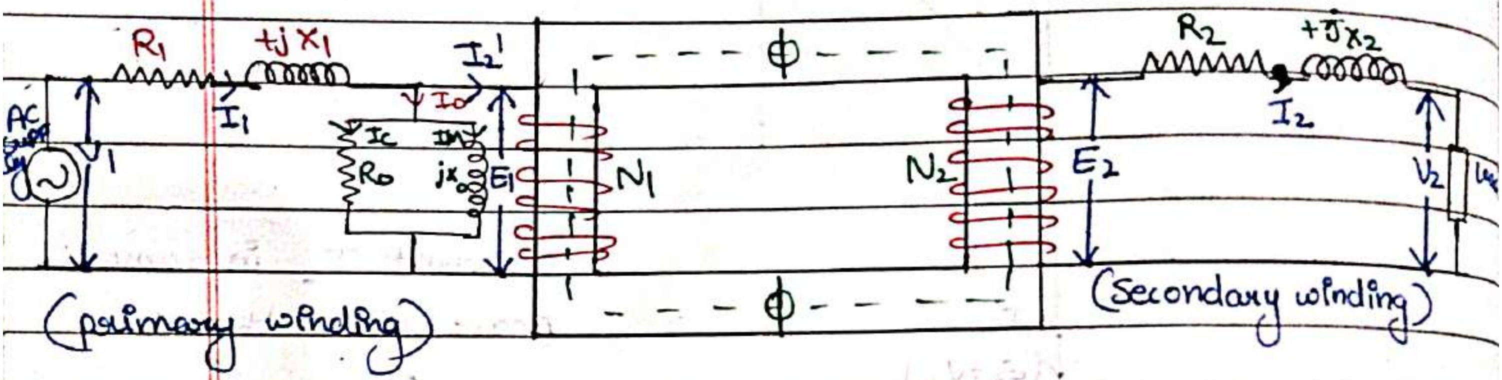
Actual equivalent circuit of a practical transformer:



$$\boxed{\frac{N_2}{N_1} = K}$$

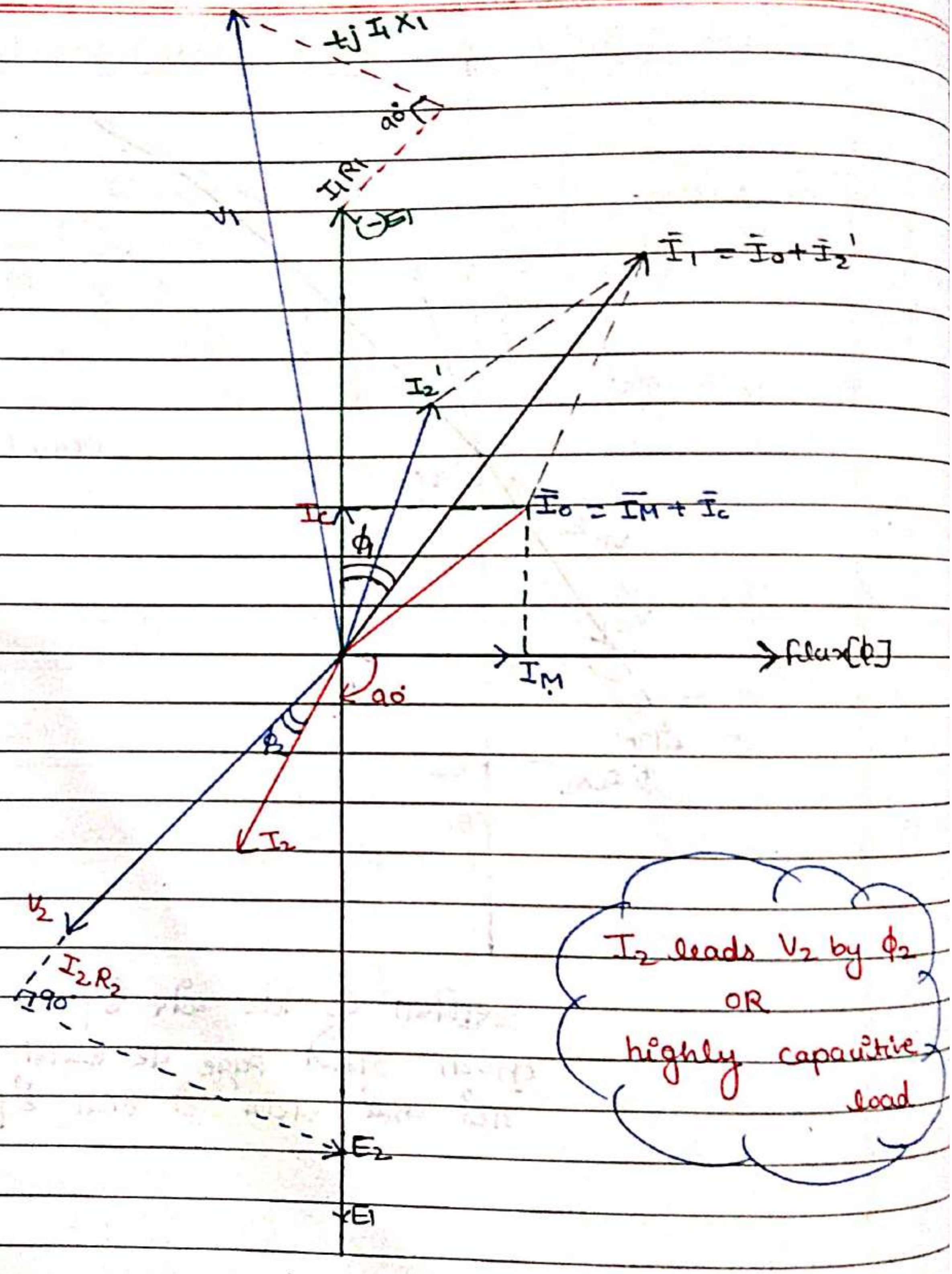
$$\boxed{[E_2 = V_2 + I_2R_2 + jI_2X_2]}$$

$$\boxed{[\bar{I}_0 = \bar{I}_m + \bar{I}_c]}$$

"The phase angle b/w V_2 and I_2 depends on nature of the load."

$$\boxed{[\bar{I}_1 = \bar{I}_0 + \bar{I}_2']}$$

$$\boxed{[V_1 = -(E_1) + I_1R_1 + jI_1X_1]}$$



$+jI_1X_1$
$90°$
I_1R_1
V_1
$(-E_1)$
$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$
I_2'
I_c
$\bar{I}_0 = \bar{I}_M + \bar{I}_c$
ϕ_1
flux[ϕ]
I_M
$90°$
ϕ_2
I_2
V_2
I_2R_2
$90°$
E_2
E_1
I_2 leads V_2 by ϕ_2
OR
highly capacitive load

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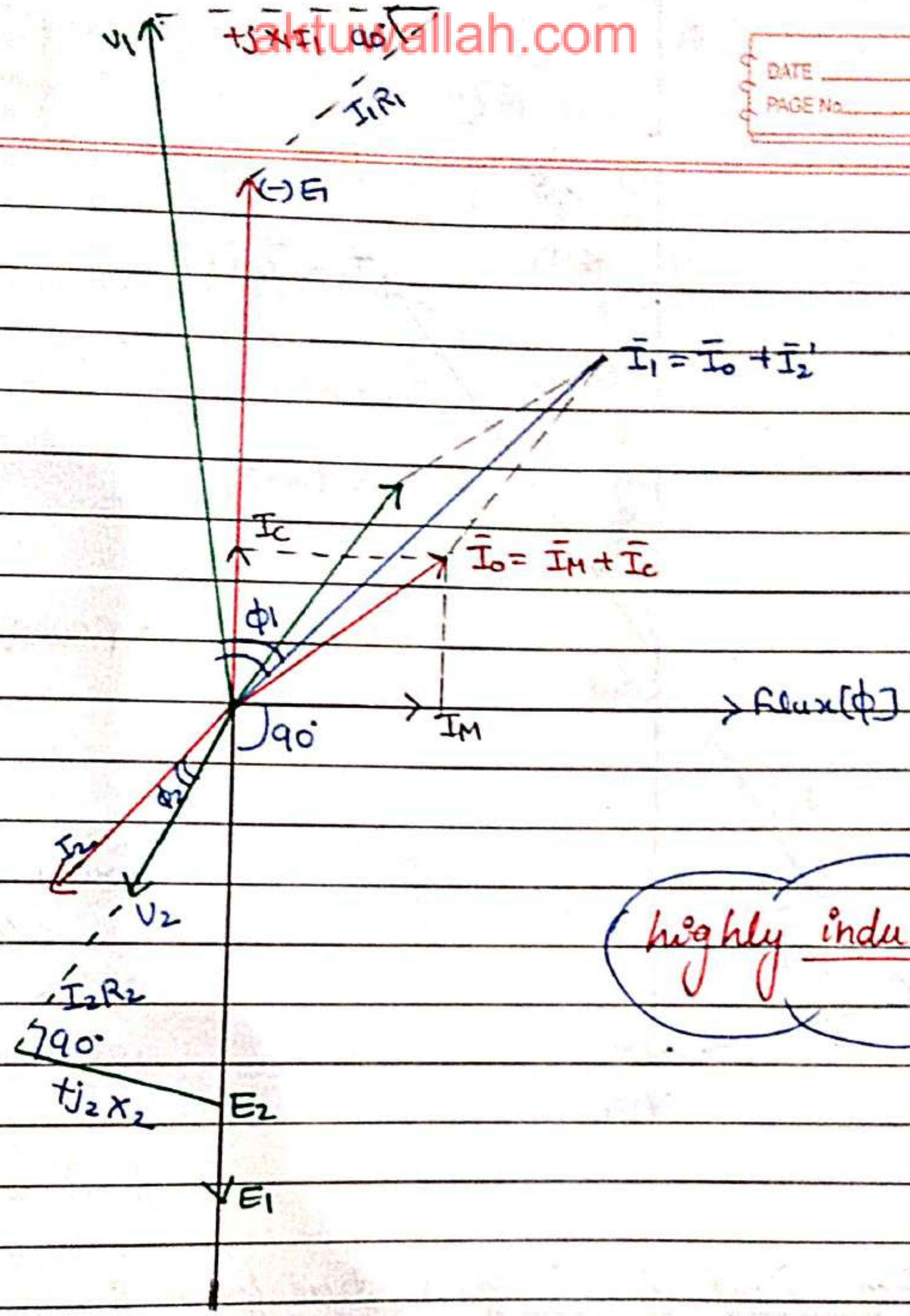
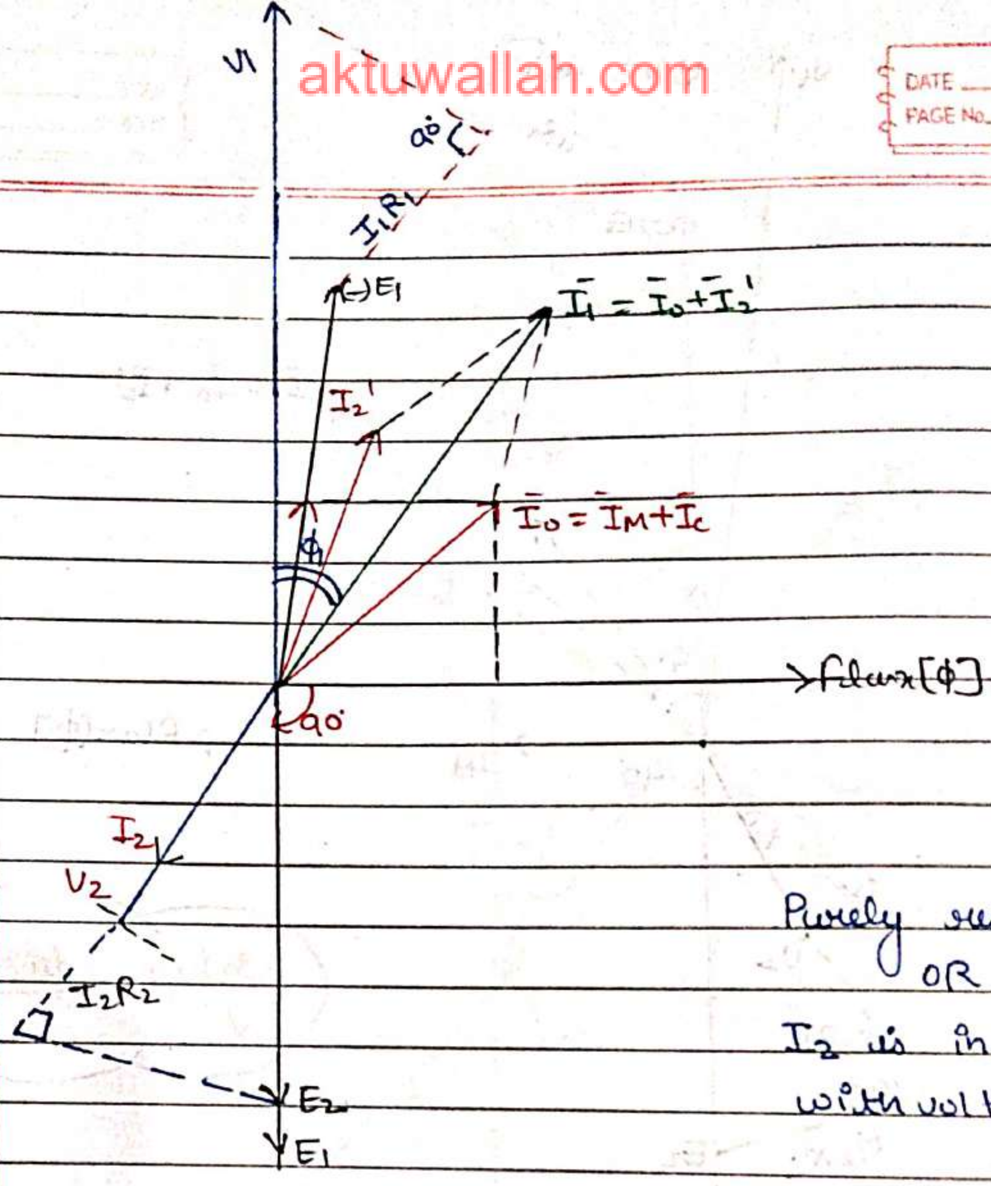


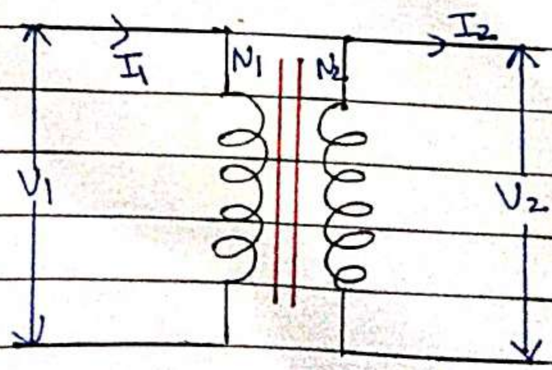
highly inductive load



Purely resistive load
OR
I_2 is in same phase with voltage V_2

Equivalent circuit of a transformer :

1. To determine circuit parameters.
2. To determine the total copper losses.
3. Efficiency calculation.
4. To find voltage regulation.



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$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$

Impedence of primary coil

$$z_1 = \frac{V_1}{I_1}$$

Impedence of secondary coil

$$z_2 = \frac{V_2}{I_2}$$

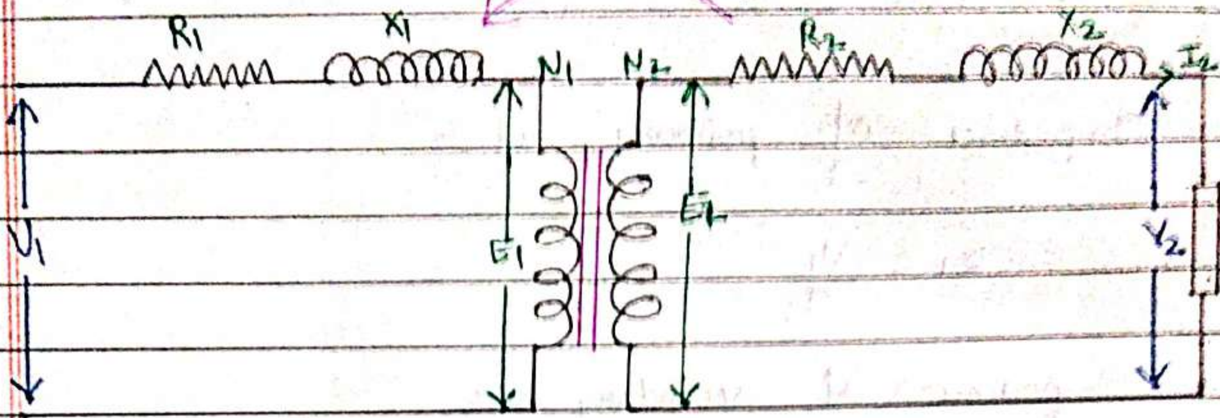
$$\frac{z_2}{z_1} = \frac{\left(\frac{V_2}{I_2}\right)}{\left(\frac{V_1}{I_1}\right)} = \frac{V_2}{I_2} \times \frac{I_1}{V_1}$$

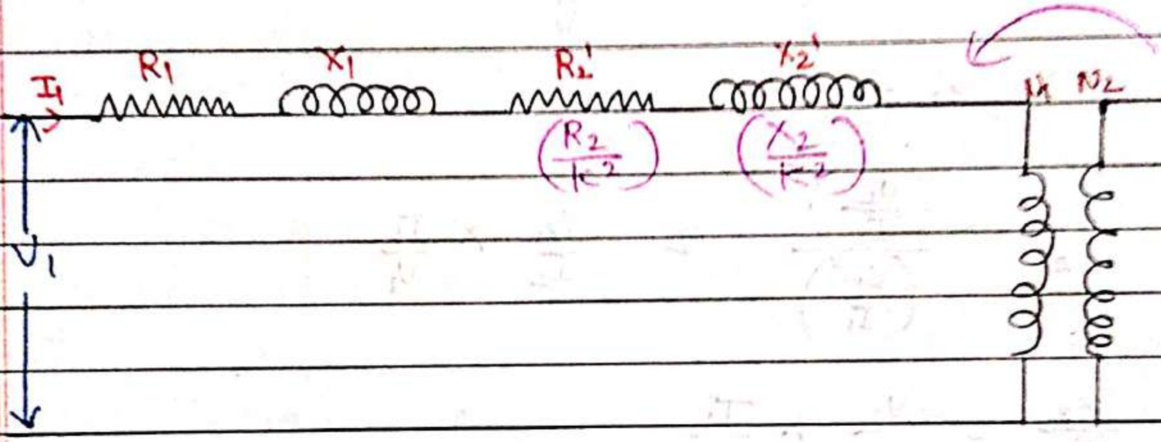
$$\frac{z_2}{z_1} = \frac{V_2}{V_1} \times \frac{I_1}{I_2}$$

$$\frac{z_2}{z_1} = k \times k$$

$$\boxed{\frac{z_2}{z_1} = K^2}$$ (Shifting rule)

Equivalent circuit refer to primary side :





(i) Equivalent resistance refer to primary side

$$R_{1e} = R_1 + R_2'$$

$$\boxed{R_{1e} = R_1 + \frac{R_2}{K_2^2}} \quad [\Omega]$$

(ii) equivalent reactance refer to primary side

$$\boxed{X_{1e} = X_1 + \frac{X_2}{K^2}} \quad [\Omega]$$

(iii) equivalent impedence refer to primary side

$$\boxed{Z_{1e} = \sqrt{(R_{1e})^2 + (X_{1e})^2}} \quad [\Omega]$$

(iv) Total copper loss (losses ohmic),

$$\boxed{P_{cu} = I_1^2 R_{1e}} \quad [watt]$$

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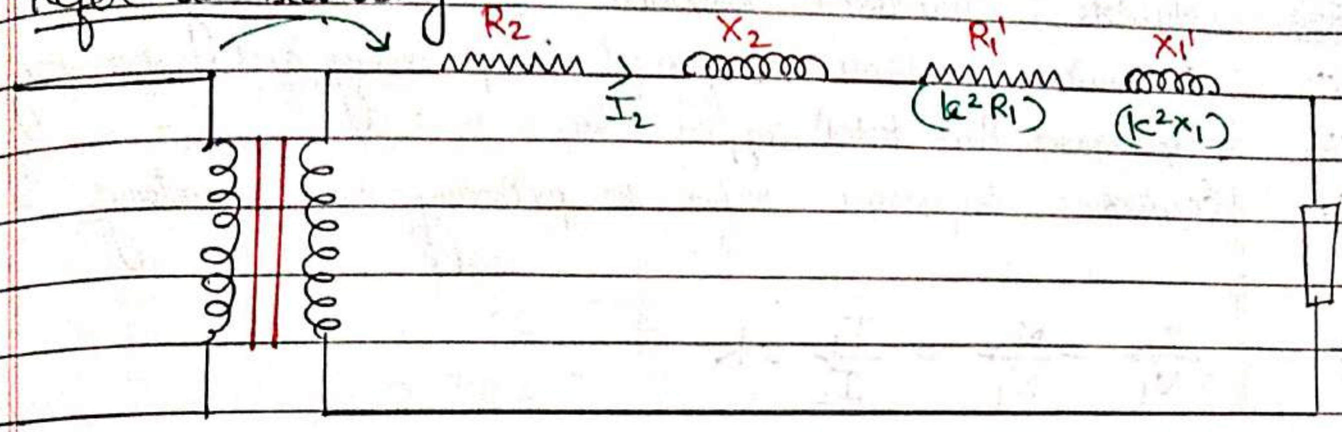
Refer to secondary



(i) Equivalent resistance refer to secondary side

$$R_{2e} = R_2 + R_1'$$

$$\boxed{R_{2e} = R_2 + K^2 R_1} \; [\Omega]$$

(ii) equivalent reactance refer to secondary side

$$\boxed{X_{2e} = X_2 + k^2 X_1} \; [\Omega]$$

(iii) equivalent impedence refer to secondary side

$$\boxed{Z_{2e} = \sqrt{(R_{2e})^2 + (X_{2e})^2}} \; [\Omega]$$

$P_{oh} = I_1^2 R_{1e} + I_2^2 R_{2e}$

$I_1^2 \left[R_1 + \left(\frac{I_2}{I_1}\right)^2 R_2\right]$

I_1

(iv) Total copper losses. (ohmic losses)

$$\boxed{P_{cu} = I_2^2 R_{2e}} \; (\text{watt})$$

Ques: A 30 KVA, $\left(\frac{2000}{200}\right)$ volt, 50Hz single phase transformer has following parameters:

$R_1 = 3.5\Omega$

$X_1 = 4.5\Omega$

$X_2 = 1.0.02\Omega$.

$R_2 = 3.5\Omega$

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Ques Calculate : Equivalent resistance refer to primary and secondary
(ii) Equivalent reactance referred to primary and secondary
(iii) ~~Equivalent~~ find total copper losses.
(iv) Equivalent impedence refer to primary and secondary

Ans

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

$$\frac{V_2}{V_1} = K \Rightarrow \frac{200}{2000} \Rightarrow 0.1$$

$$\boxed{K = 0.1}$$

(i) $$Z_{1e} = \sqrt{(R_{1e})^2 + (X_{1e})^2}$$

$$\boxed{Z_{1e} = 353.55} \ \Omega$$

$$R_{1e} = R_1 + \frac{R_2}{K^2}$$

$$\boxed{R_{1e} = 353.5} \ \Omega$$

$$X_{1e} = X_1 + \frac{X_2}{K^2}$$

$$\boxed{X_{1e} = 6.5} \ \Omega$$

$$Z_{2e} = \sqrt{(R_{2e})^2 + (X_{2e})^2}$$

$$R_{2e} = R_2 + K^2 R_1$$

$$\boxed{R_{2e} = 3.53} \ \Omega$$

$$X_{2e} = X_2 + K^2 X_1$$

$$\boxed{X_{2e} = 0.065} \ \Omega$$

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(ii) $Z_{2e} = \sqrt{(R_{2e})^2 + (X_{2e})^2}$

$Z_{2e} = \sqrt{(3.5)^2 + (0.065)^2}$

$\boxed{Z_{2e} = 3.53}\ \Omega$

(iii) $P_{cu} = I_1^2 R_{1e}$

$P_{cu} = (15)^2 \times 353.5$

$\boxed{P_{cu} = 79.53}$ kW

$P_{cu} = I_2^2 R_{2e}$

$P_{cu} = (150)^2 \times 3.53$

$\boxed{P_{cu} = 79.42}$ kW

Que A 25 KVA, $\left(\frac{2200}{220}\right)$V, 50Hz single phase transformer has following parameter: $R_1 = 1.75\,\Omega$, $R_2 = 0.0045\,\Omega$, $X_1 = 2.6\,\Omega$, $X_2 = 0.0075\,\Omega$. Calculate:

(i) equivalent resistance reffered to secondary

(ii) equivalent impedence reffered to primary side.

(iii) Total copper loss.

Ans $R_1 = 1.75\,\Omega$ $R_2 = 0.0045\,\Omega$ $X_1 = 2.6\,\Omega$ $X_2 = 0.0075\,\Omega$

(i) $Z_{1e} = \sqrt{(R_{1e})^2 + (X_{1e})^2}$ —(i)

$(R_{1e}) = R_1 + \frac{R_2}{K^2} \Rightarrow 2.20\,\Omega$

$(X_{1e}) = X_1 + \frac{X_2}{K^2} \Rightarrow 3.35\,\Omega$

$Z_{1e} = \sqrt{(2.20)^2 + (3.35)^2}$

$\boxed{Z_{1e} = 4\,\Omega}$

$$Z_{2e} = \sqrt{(R_{2e})^2 + (X_{2e})^2}$$

$$\boxed{Z_{2e} = 0.039\ \Omega}$$

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(ii) $R_{2e} = R_2 + k^2 R_1$

$R_{2e} = 0.022\ \Omega$

$X_{2e} = X_2 + k^2 X_1$

$\boxed{X_{2e} = 0.0335}\ \Omega$

(iii) Total copper lose $P_{cu} = I_1^2 R_{1e}$

$= (11.36)^2 \times 2.20$

$25\ kVA = V_1 I_1$

$\boxed{P_{cu} = 283.9\ \text{watt}}$

$25 \times 1000 = V_1 I_1$

$I_1 = \frac{25000}{2200}$

$P_{cu} = I_2^2 R_{2e}$

$= 283.9$ watt.

$I_1 = 11.36$ Amp

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Losses in Transformer

- Constant losses: **Core Losses or Iron Losses**
  - Hysteresis losses
  - Eddy current lose
- Variable losses: **Copper losses**
  - Copper loss in primary $[I_1^2 R_1]$
  - Copper losses in secondary $[I_2^2 R_2]$

Total copper losses in transformer

$$\boxed{P_{cu} = I_1^2 R_1 + I_2^2 R_2} \ [\text{watt}]$$

Core Loss or Iron Loss :

There are two types of iron losses. The iron losses occurs in the magnetic material [iron core]. Sometimes the iron loses are also known as constant losses.

- **Hysteresis losses :**

When a magnetic material is subjected to a cycle of magnetisation, an energy loses take place due to molecular friction in the magnetic material.
These losses are in the form of heat and are known as Hysteresis losses. The hysteresis losses depend on the following factors:

* The hysteresis loss is directly proportional to the area under the hysteresis loop.

* It is directly proportional to the frequency of magnetisation.

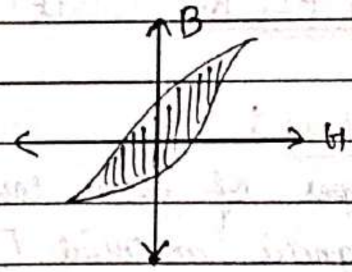
* Directly proportional to the volume of the material

The Hysteresis losses can be expressed as

$$\boxed{P_h = K_h B_M^{1.6} f \cdot V.} \text{ watt}$$

where, K_h = hysteresis constant
B_m = magnetic flux density
f = frequency of magnetisation
V = volume of the core

Ques How to minimize Hysteresis Losses?
Ans The Hysteresis losses can be minimised by using "silicon steel core."



Note: Hysteresis losses also occur when an iron part rotates in a constant magnetic field.

Eddy Current Losses :-

The changing flux induced emf in the core, according to faraday's law. this induced emf circulate current within the conducting material, these circulating currents are known as eddy currents. and losses due to these currents are known as eddy current losses. and given by the formula

$$\boxed{P_e = K_e B_m^2 f^2 t^2} \text{ watt / unit volume.}$$

where K_e eddy current constant.

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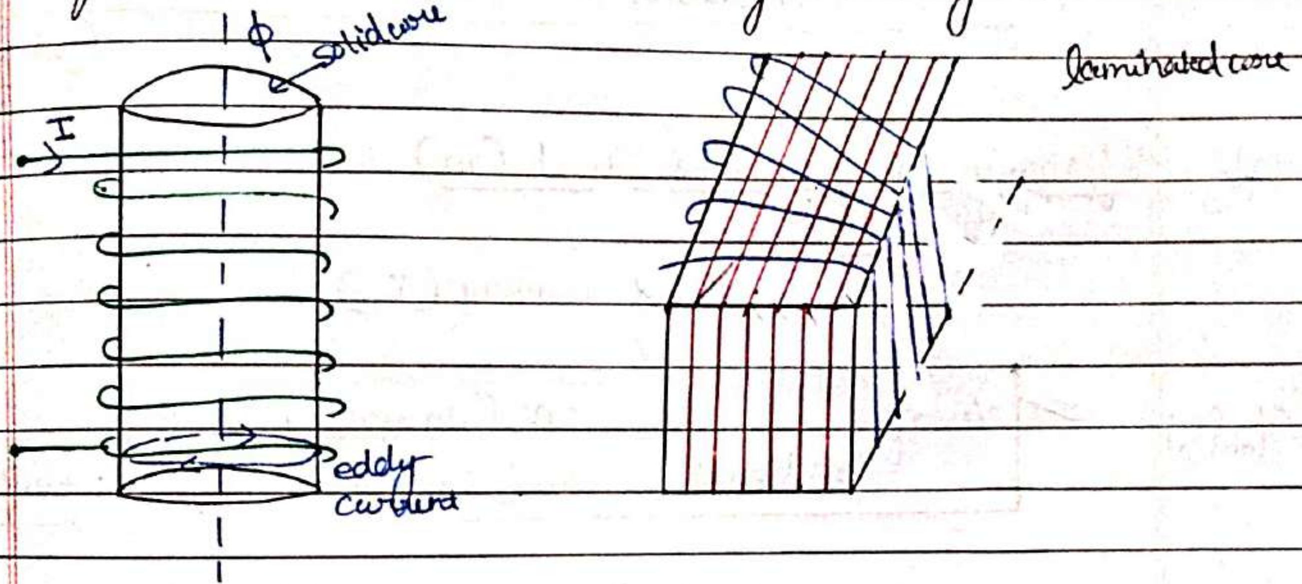
B_m = maximum flux density
f = frequency of induced emf
t = thickness of the core.

Note: Eddy current losses can be reduced by laminating the core.



efficiency of transformer :

The ratio of output power to the input power is called efficiency of a transformer.
It is denoted by 'η'

$$\boxed{\%\eta = \frac{\text{output power}}{\text{input power}} \times 100}$$

$P_{cu} = [I^2 R]$
Load ∝ I_2
$m = \frac{1}{2}$
$I = \frac{1}{4}$

Input power = output power + Total losses

Input power = output power + P_i + P_{cu}

$$\boxed{\text{Input power} = V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

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$$\%\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}} \times 100$$

$$\boxed{\%\eta = \frac{(\text{VA rating}) \cos\phi_2}{(\text{VA rating})\cos\phi_2 + P_i + I_2^2 R_{2e}} \times 100}$$

(at full load)

Efficiency at fractional load (m) :

$$\text{load}(m) \propto \text{current}(I_2)$$

(at any load)

$$\boxed{\%\eta_{\text{fractional}} = \frac{m(\text{VA rating})\cos\phi}{m(\text{VA rating})\cos\phi + P_i + m^2(P_{cu})_{fl}} \times 100}$$

m = fractional load

$\boxed{\text{full load} = 1}$ $\boxed{\text{half load} = \frac{1}{2}}$

$$\boxed{\text{fractional load } (m) = \frac{\text{Actual load}}{\text{full load}}}$$

Condition for maximum efficiency :

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\eta = \frac{V_2 \cos\phi_2}{\left[V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 R_{2e}\right]} \Rightarrow A$$

$P_i + P_{cu} = 0 \cos\theta$

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$$\frac{dA}{dI_2} = 0$$

$$\frac{d}{dI_2}\left[V_2\cos\phi_2 + \frac{P_i}{I_2} + I_2 R_{2e}\right] = 0$$

$$0 - \frac{P_i}{I_2^2} + R_{2e} = 0$$

$$\cancel{-} P_i = \cancel{-} I_2^2 R_{2e}$$

$$P_i = I_2^2 R_{2e}$$

Imp

$$\boxed{\text{Iron Loss} = \text{Copper Loss}}$$ (maximum efficiency)

$$\boxed{\text{Constant loss} = \text{Variable loss.}}$$

Current at maximum efficiency :

$$P_i = I_2^2 R_{2e}$$

$$I_2^2 = \frac{P_i}{R_{2e}}$$

$$\boxed{I_2 = \sqrt{\frac{P_i}{R_{2e}}} = I_{2m}}$$ (at maximum condition)

(in terms of P_i and R_{2e})
R_{2e} → equ. resistance refer to secondary

$$\frac{I_2}{I_{(2)fl}} = \frac{1}{I_{(2)fl}}\sqrt{\frac{P_i}{R_{2e}}}$$

$$I_2 = I_{2m} = I_{(2)fl}\sqrt{\frac{P_i}{I_{(2)fl}^2 . R_{2e}}}$$

$\frac{1}{4} = \frac{100}{4}$ 25

$$\boxed{I_{2m} = I_2 = I_{(2)fl}\sqrt{\frac{P_i}{P_{(cu)fl}}}}$$

$I_{2m} = I_2$

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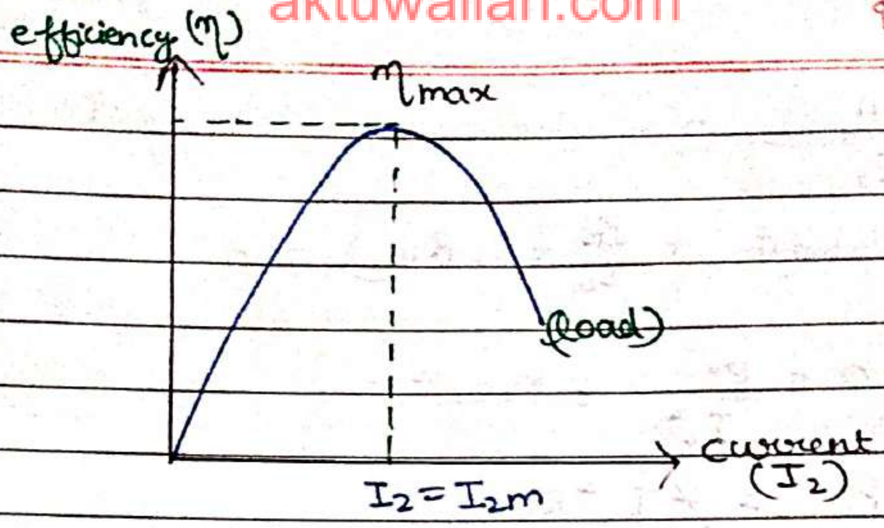

KVA supplied at maximum efficiency :-
OR
Load corresponding to maximum efficiency :

KVA supplied at $\eta_{max} = V_2 I_{2m}$

$$(KVA)_{\eta max} = V_2 I_{(2)f.l.} \sqrt{\frac{P_i}{P_{cu}}}$$

imp

$$\boxed{(KVA)_{\eta max} = (KVA)\text{ rating} \sqrt{\frac{P_i}{P_{cu}}}}$$

Maximum Efficiency (η_{max})

$$\boxed{\%\eta = \frac{(KVA)_{\eta max} \cos\phi}{(KVA)_{\eta max} \cos\phi + 2P_i} \times 100}$$

imp

Ques. Why the rating of a transformer is in KVA?

Ans. If the load on a transformer will change then the copper losses will change because current from the transformer will change

$$P_{(cu)} \propto \text{load} \propto I_2$$

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It is seen that the iron losses depends on the supply voltage while the copper losses depends on the current. The losses are not dependent on the phase angle between voltage and current. Hence the rating of the transformer is in KVA.

Ques A 200 KVA, 1-ϕ transformer has 1000 watt iron losses and 2000 watt copper losses at full load. Calculate
(i) efficiency at full load on 0.8 power factor (lagging).
(ii) efficiency at half load on 0.8 power factor (lagging)
(iii) Load at which its efficiency is maximum.
(iv) maximum efficiency at 0.8 power factor (lagging).

(i) $$\%\eta_{fl} = \frac{(\text{VA rating})\cos\phi}{(\text{VA rating})\cos\phi + P_i + (P_{cu})_{fl}} \times 100$$

$$= \frac{(200 \times 1000)\,0.8}{(200\times1000)\times0.8 + 1000 + 2000} \times 100$$

$$\%\eta_{fl} = 98.16\%$$

(ii) % η at 0.8 p.f lagging [(half load) $m = \frac{1}{2}$]

$$\%\eta_{hl} = \frac{0.5\,(200\times1000)\,0.8}{0.5\,(200\times1000)\,0.8 + 1000 + (0.5)^2 \times 2000} \times 100$$

$$\%\eta_{hl} = 98.15\%$$

(iii) KVA supplied at η_{max}

$$(\text{KVA})_{\eta\,max} = (\text{KVA})\text{ rating}\sqrt{\frac{P_i}{P_{cu\,fl}}}$$

$P_i = m^2 P_{(cu)fl}$

$m = \sqrt{\frac{P_i}{P_{(cu)fl}}}$

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$$(kVA)_{\eta max} = (200)\sqrt{\frac{1000}{2000}}$$

$$(kVA)_{\eta max} = 141.42 \text{ kVA}$$

(iv) $$\%\eta_{max} = \frac{141.42 \times 1000 \times 0.8}{141.42 \times 1000 \times 0.8 + 2 \times 1000} \times 100$$

$$\%\eta_{max} = 98.26\%$$

$$P_i = m^2 P_{(cu)fl}$$

$$m = \sqrt{\frac{P_i}{P_{(cu)fl}}} = \sqrt{\frac{1000}{2000}} = 0.707$$

$$\boxed{m = 70.7\%}$$

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Imp:- Voltage Regulation :

The change in secondary terminal voltage from no load to full load with primary voltage and frequency held constant is called voltage regulation of a transformer.

$$\boxed{\% \text{ voltage regulation (VR)} = \frac{E_2 - V_2}{V_2} \times 100}$$

where,

E_2 = secondary terminal voltage at no load
V_2 = secondary terminal voltage at full load.

"The voltage regulation of an ideal transformer is always 0."

$$\boxed{\% VR = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100}$$

- (−) → for leading power factor
- (+) → for lagging power factor

for unity power factor [$\cos\phi = 1$]

$$\boxed{\% VR = \frac{I_2 R_{2e}}{V_2} \times 100}$$

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Ques Calculate voltage regulation of a transformer in which ohmic losses are 1% and reactive drop is 5% of the voltage when the power factor is 0.8 (lagging)
(ii) 0.8 (leading)
(iii) unity power factor.

(i) <u>for lagging</u>

$$\% VR = \left[\frac{I_2 R_{2e}}{V_2}\cos\phi + \frac{I_2 X_{2e}}{V_2}\sin\phi\right] \times 100$$

$$\% VR = [\% R \cos\phi + \% X \sin\phi] \times 100$$

$$\% VR = [0.01 \times 0.8 + 0.05 \times 0.6] \times 100$$

$$\boxed{\% VR = +3.8\%}$$ (lagging power factor)

<u>for leading</u>

$$\% VR = \left[\frac{I_2 R_{2e}}{V_2}\cos\phi - \frac{I_2 X_{2e}}{V_2}\sin\phi\right] \times 100$$

$$\% VR = [\% R \cos\phi - \% X \sin\phi] \times 100$$

$$\% VR = [0.01 \times 0.8 - 0.05 \times 0.6] \times 100$$

$$\boxed{\% VR = -2.2\%}$$

<u>for unity</u>

$$\% VR = \frac{I_2 R_{2e}}{V_2} \times 100$$

$$= 0.01 \times 100$$

$$\boxed{\% VR = 1\%}$$

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Q. Calculate the voltage regulation of transformer whose ohmic drop is 1.5% and leakage reactance drop is 5% at 0.8 power factor (lagging)

A. for lagging

$$\%VR = \left[\frac{I_2 R_{2e} \cos\phi}{V_2} + \frac{I_2 X_{2e}}{V_2} \sin\phi\right] \times 100$$

$$\%VR = [0.015 \times 0.8 + 0.05 \times 0.6] \times 100$$

$$\%VR = [0.12 + 0.3] \times 100$$

$$\%VR = 0.42 \times 100$$

$$\boxed{\%VR = 4.2\%}$$

Q.35 In a 25 kVA, 2000/200 V transformer, the constant and variable losses are 350 W and 400 W respectively. Calculate the efficiency on unity power factor (i) full load (ii) half load.

Ans.

$P_{iron} = 350$ watt

$P_{cu} = 400$ watt

$\cos\phi = 1$

(i)
$$\%\eta = \frac{m(\text{VA rating})\cos\phi}{m(\text{VA rating})\cos\phi + P_i + m^2 P_{cu}} \times 100$$

full load (m) = 1

$$\%\eta = \frac{1 \times 25 \times 1000}{1 \times 25 \times 10^3 + 350 + 400} \times 100$$

$$\%\eta = \frac{25000}{(25000 + 750)} \times 100 \Rightarrow 97.08\%$$

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(ii) $m = \frac{\text{Actual load}}{\text{full load}} = \frac{\cancel{50}}{\cancel{100}} \frac{1}{2}.$

$m = \frac{1}{2} \Rightarrow 0.5$

$$\% \eta = \frac{0.5 \times 25000}{0.5 \times 25000 + 350 + (0.5)^2 \times 400} \times 100$$

$$\% \eta = \frac{12500}{\cancel{0.5\times} 12500 + 350 + 100} \times 100$$

$$\boxed{\% \eta = 96.52\%}$$

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Auto Transformer :

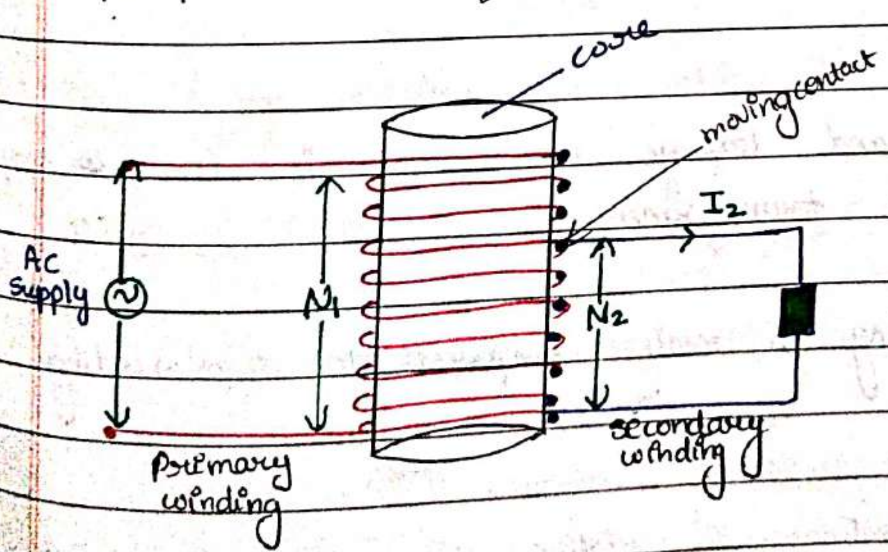
An auto transformer is a special type of transformer in which a part of winding is common to both the primary and secondary. The operating principle and general construction of an autotransformer is the same as that of two winding transformer.

In a Two winding transformer the primary and secondary windings are electrically isolated, but in an auto transformer the two winding are not electrically isolated.

There are two types of auto transformer

(i) Step down auto transformer

(ii) Step up auto transformer.



(Variable voltage on secondary side)
(Single winding transformer)

$$\boxed{N_1 > N_2}$$

[step down transformer]

$$\boxed{\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$

Note: It provides variable AC voltage and a single winding transformer.

* ## Advantages of an Auto transformer :

(i) An auto transformer requires less winding material (copper) than a two winding transformer.

(ii) The efficiency is higher comparable to two winding transformer.

(iii) An auto transformer is smaller in size and cheaper than two winding transformer of the same output.

(iv) Since there is a reduction in conductor material and core material, the copper losses and iron losses are small.

(v) The resistance and leakage reactance is less the compared to two winding transformer.

(vi) Volt ampere rating is more compared to two winding transformer.

(vii) A smooth and continuous variation of voltage is possible.

* ## Limitations of an Auto transformer :

(i) The secondary winding is not insulated with primary winding.

(ii) The short circuit current in an autotransformer is large

I — N

N ∝ length
I ∝ Cross section

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than that for the two winding transformer.

(iii) No electrical seperation between primary and secondary which is risky in case of high voltage levels.

(iv) If a section of winding, common to primary and secondary is opened, full primary voltage appears across the secondary resulting in higher voltage on secondary and danger for accident.

Applications of an Auto transformer :

1. Auto transformer is used as a variable AC or variac in laboratory.

2. It can be used as regulating transformer.

3. For safely starting the machines like Induction motor.

Saving of Copper in an auto transformer :

Weight of copper $\propto NI$

where N = number of turns in coil
I = Current through the coil

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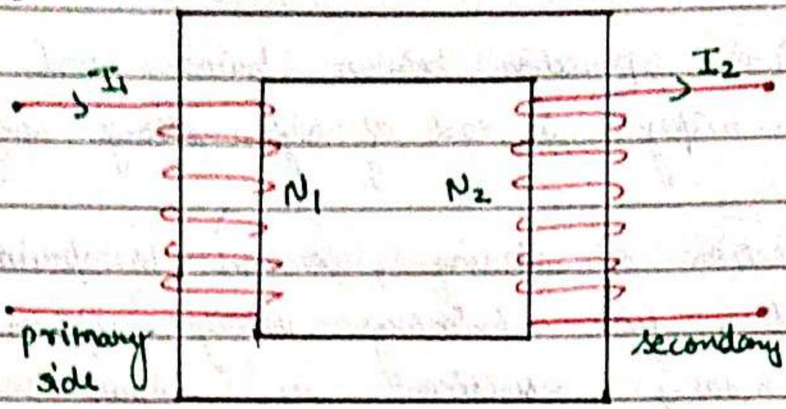
* 2 winding Transformer :



Copper weight in primary $\propto N_1 I_1$

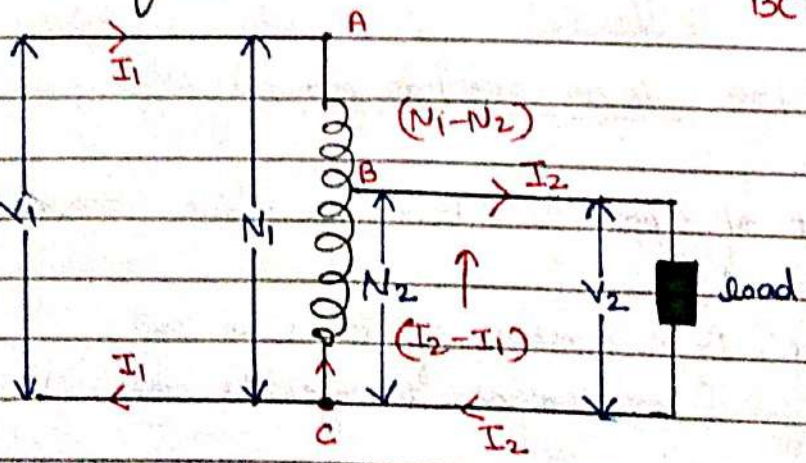
Copper weight in secondary $\propto N_2 I_2$

Tatal Copper weight in 2-winding Transformer

$$W_{2w} \propto (N_1 I_1 + N_2 I_2) \quad \text{------} \quad (1)$$

* Auto Transformer :

BC → common in primary and secondary



section AB, copper weight $\propto I_1 (N_1 - N_2)$

section BC, copper weight $\propto (I_2 - I_1) N_2$

$$W_{auto} \propto I_1 (N_1 - N_2) + (I_2 - I_1) N_2 \quad \text{------} \quad (2)$$

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from eqⁿ ① and ②

$$\frac{W_{2w}}{W_{auto}} = \frac{N_1 I_1 + N_2 I_2}{I_1(N_1 - N_2) + (I_2 - I_1) N_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$

$$\frac{W_{2w}}{W_{auto}} = \frac{1}{1-K}$$

$$W_{auto} = (1-K) W_{2w} \quad \text{-----} ③$$

<u>Total copper saving in an autotransformer</u>

$$\text{Total cu saving} = W_{2w} - W_{auto}$$

$$\text{Total cu saving} = W_{2w} - (1-K) W_{2w}$$

$$\text{Total cu saving} = \cancel{W_{2w}} - \cancel{W_{2w}} + K W_{2w}$$

$$\boxed{\text{Total cu saving} = K W_{2w}}$$

K = Transformation ratio
K = Close to unity [1]

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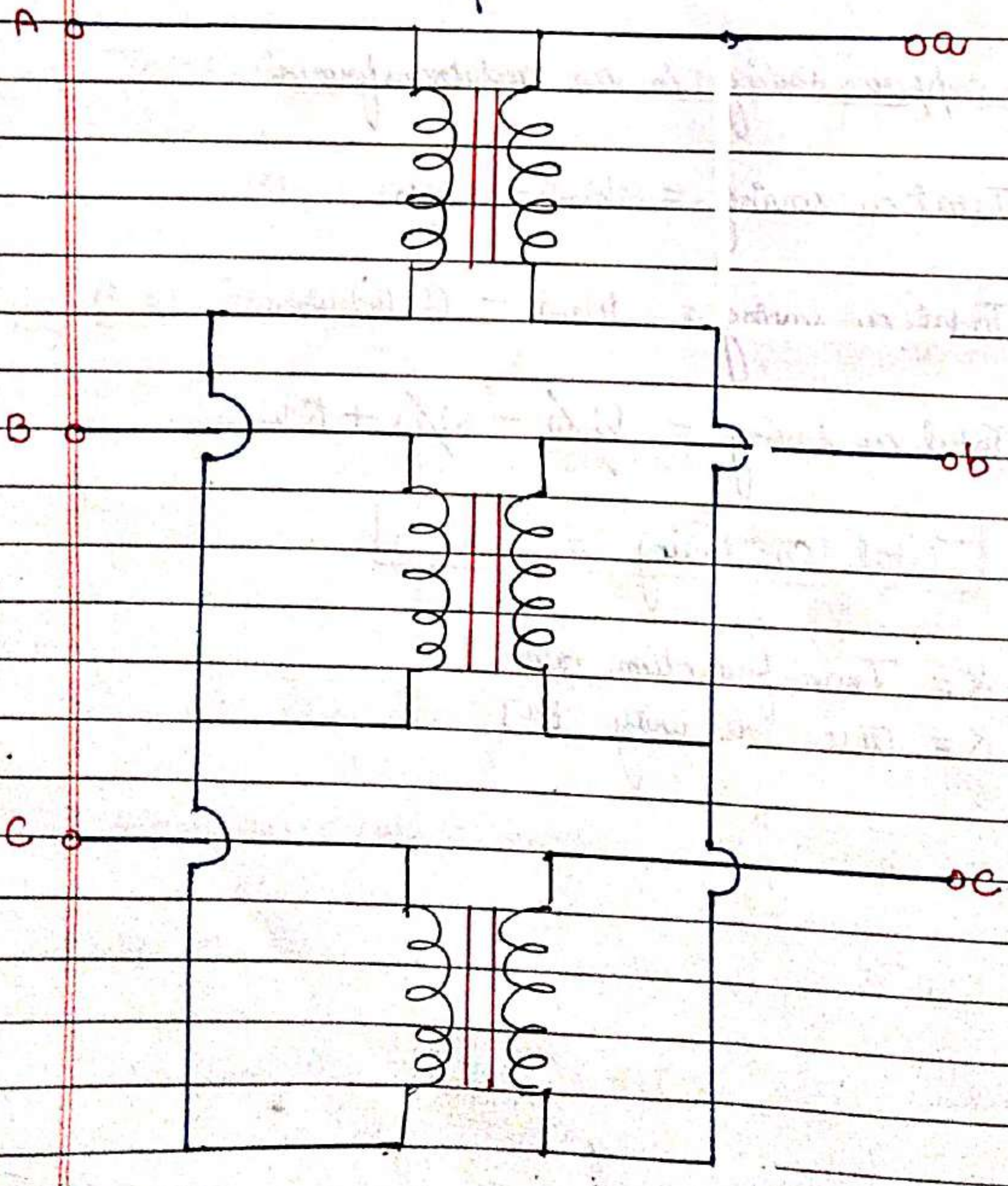
Three Phase (3-ϕ) Transformer Connections :-

* Star-star Connection
* delta-delta connection
* star-delta connection
* delta-star connection

Star-star connection :

It gives the line voltage $\sqrt{3}$ times of phase voltage. It is economical for small rating, high voltage transformer as the number of turns per phase and the amount of insulation required is less.



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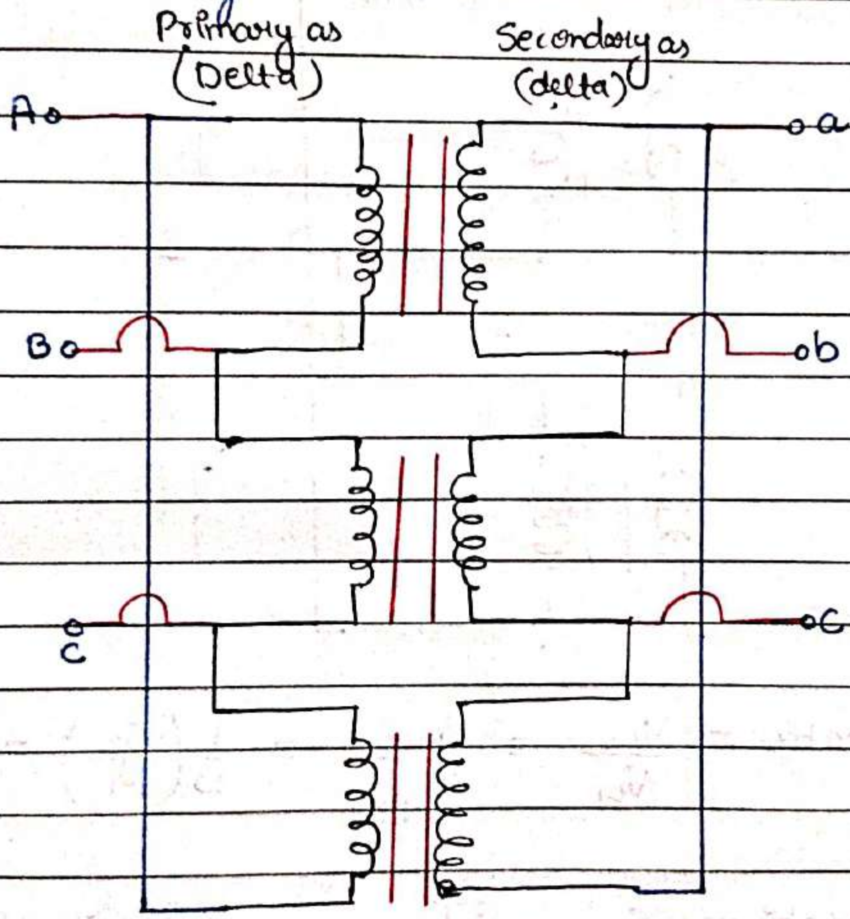
$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{\sqrt{3}V_{ph_2}}{\sqrt{3}V_{ph1}} = \frac{N_2}{N_1} = K$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{I_{ph2}}{I_{ph1}} = \frac{N_1}{N_2} = \frac{1}{K}$$

Delta-Delta Connection :

This arrangement is generally used in system which carries large currents.



$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{V_{ph2}}{V_{ph1}} = \frac{N_2}{N_1} = K$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{\sqrt{3}I_{ph2}}{\sqrt{3}I_{ph1}} = \frac{N_1}{N_2} = \frac{1}{K}$$

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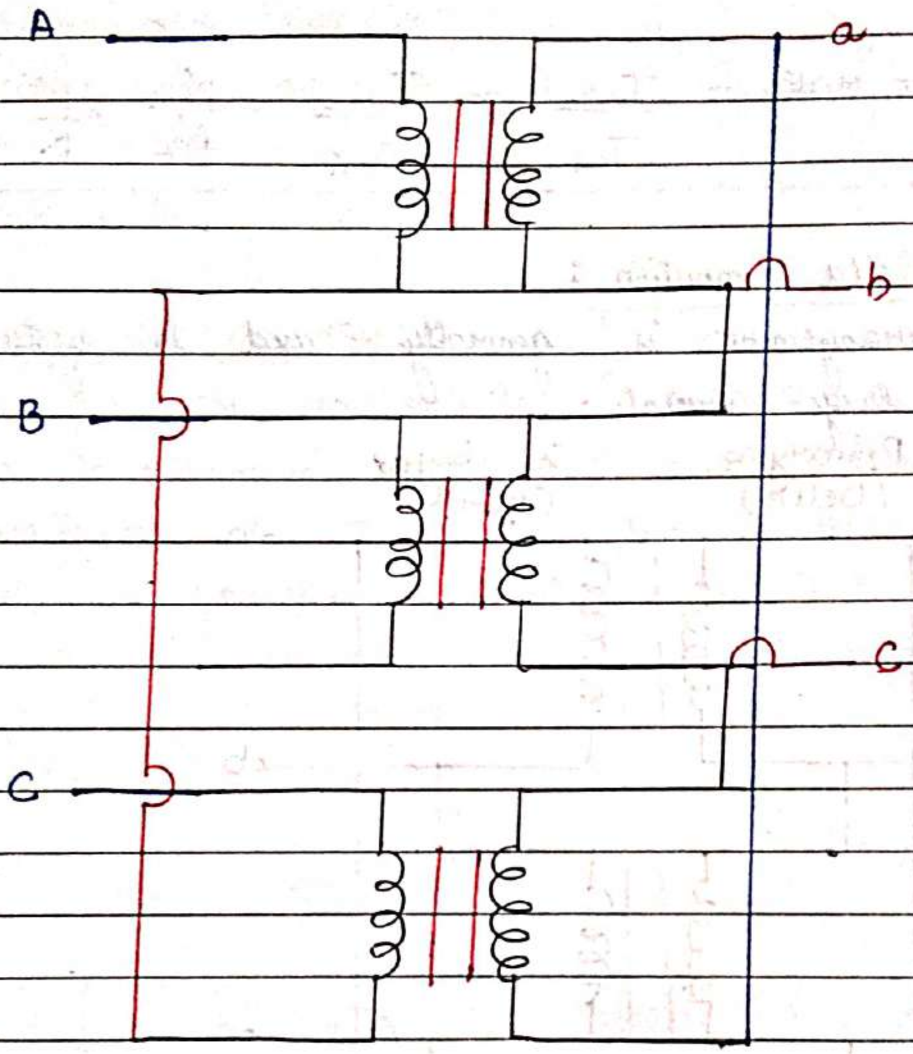
Star-Delta Connection: Such connection are used where it is necessary to step up the voltage

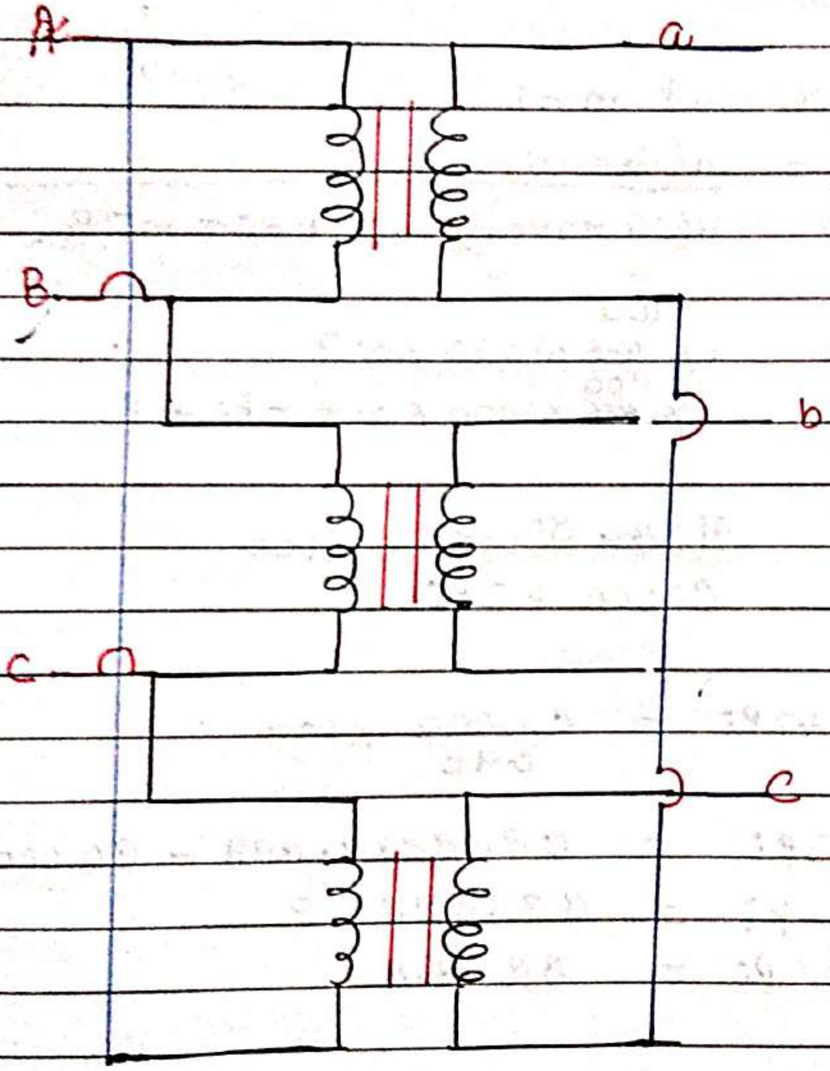


$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{V_{ph2}}{\sqrt{3}V_{ph1}} = \frac{1}{\sqrt{3}}\left(\frac{N_2}{N_1}\right) = \frac{1}{\sqrt{3}}(K)$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{\sqrt{3}\,I_{ph2}}{I_{ph1}} = \sqrt{3}\left(\frac{N_1}{N_2}\right) = \sqrt{3}\left(\frac{1}{K}\right)$$

Delta-Star Connection:

This arrangement is very popular in distribution system because it can be used to serve both the (3-ϕ) power equipment and the single phase lighting circuit.



$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{\sqrt{3}V_{ph_2}}{V_{ph_1}} = \sqrt{3}\left(\frac{N_2}{N_1}\right) = \sqrt{3}K$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{I_{ph_2}}{\sqrt{3}I_{ph_1}} = \frac{1}{\sqrt{3}}\left(\frac{N_1}{N_2}\right) = \frac{1}{\sqrt{3}}\cdot\frac{1}{K}$$

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MODULE - 4

Electrical machines

- DC machines
  - DC generator
    - seperately excited (rarely)
    - DC series
    - DC shunt
    - DC compound (rarely)
  - DC motor
    - seperately excited
    - DC series
    - DC shunt
    - DC compound
- AC machines
  - AC motor
    - 1-ϕ induction motor
    - 3-ϕ I/M
    - 3-ϕ synchronous motor
  - AC generator or alternator

I/M → Induction motor

Input (electrical) ⟶ Magnetic conductor ⟶ output (mechanical)

[Motor]

input (Electrical energy) ⟹ Electrical machine ⟹ output (mechanical energy)

input [mechanical energy] ⟹ Electrical machine (generator) ⟹ output [electrical energy]

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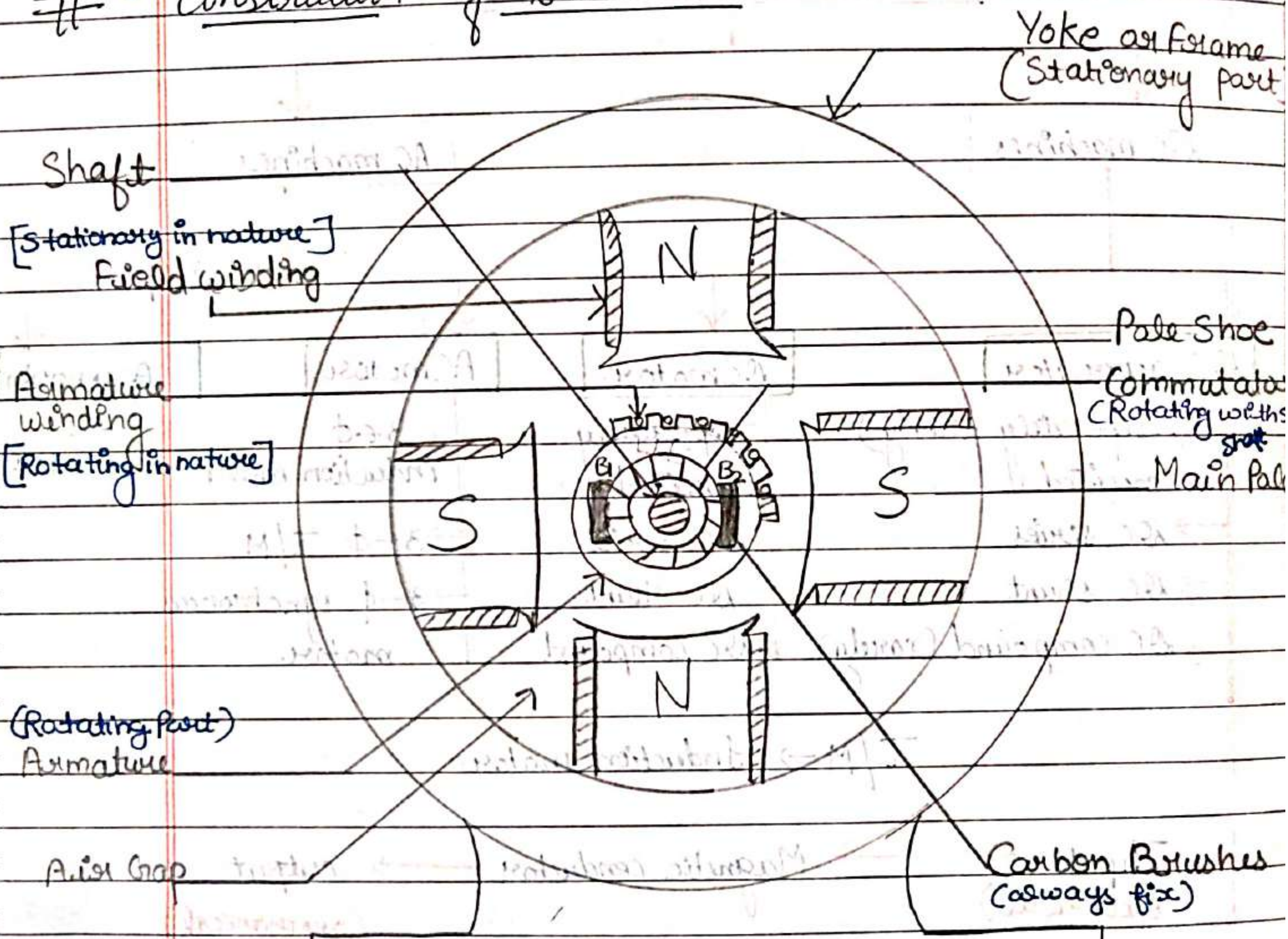
DC MACHINES

Construction of DC Machine :



A 4- POLE DC MACHINE

(except induction motor)

Field winding : To produce magnetic field in machines
working field produce

Armature winding : in which working emf induced
that works with working field

Carbon brushes collect produced electricity

If air gap is more problem in motion of machine

Stator
Rotor
Air gap
field winding (Induction)
Armature winding (AC)
Shaft

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The construction of DC machine remains same whethere it is a motor or generator.
~~Eg~~ Any DC generator can be run as a DC motor.
All DC machines has four main parts.

1. Yoke or Frame :

It is the outer cover of DC machine in which main poles are fixed. The insulating material get protected from harmful atmosphere like as moisture and dust. The Yoke or frame serve the following function

• Functions :

(i) It provide mechanical support to the inner part of the machine.

(ii) It provide a low reluctance path for the magnetic flux.

2. Magnetic field system :

It is the stationary part of the machine it produce the main magnetic flux in which armature, conductors rotates.
Field system consist of pole core and pole shoe.
pole core basically carries a field winding which is necessary to produce the main flux.
Each pole core has a pole shoe. The pole shoe having a curve surface as shown in the figure. The pole shoe serve the following function.

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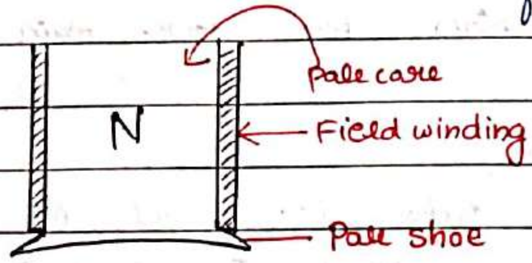
(i) It support the field winding.

(ii) It increase cross sectional area of magnetic flux.

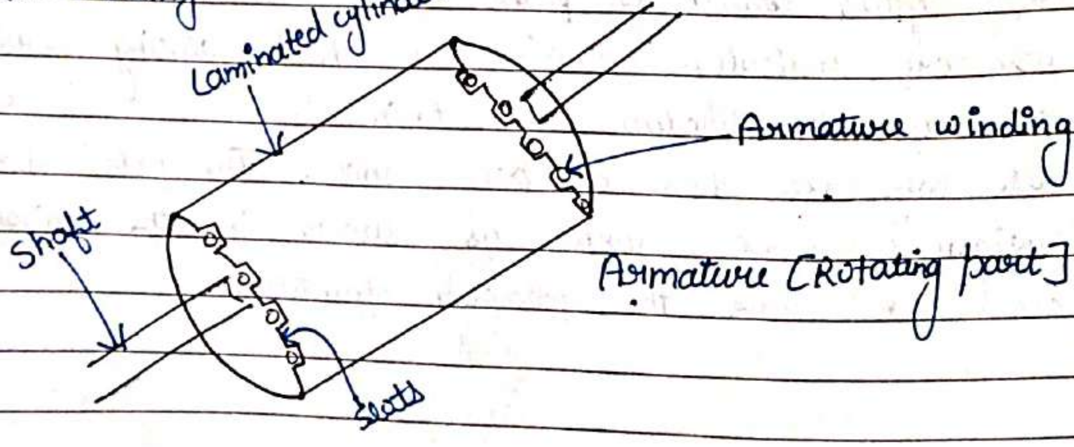


(iii) Armature:

It is the rotating part of a DC machine. The purpose of armature is to rotate the armature conductor in the uniform magnetic field.
The armature consist of a shaft upon which a laminated cylinder called armature core is mounted.
The armature core has slots on its outer phori-phori or surface on the armature and armature conductors is placed on slots.

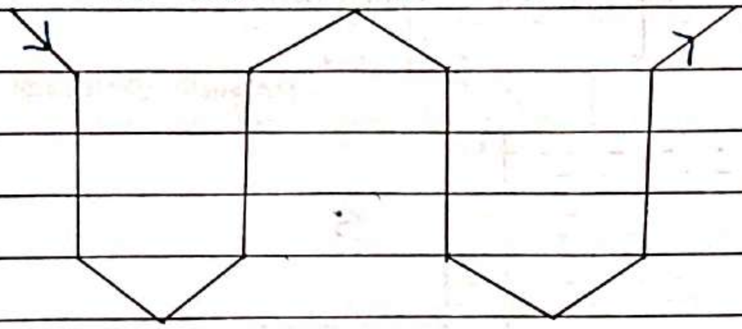
"The working EMf is induced in the armature conductors or winding"

The armature winding is of two types:
(a) Wave winding
(b) LAP winding.



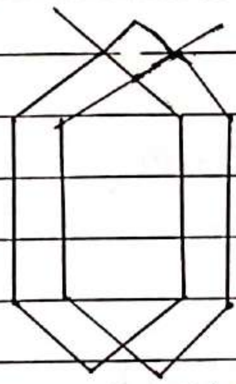
Armature [Rotating part]

Low current high voltage wave winding <500 Amps



$\boxed{A=2}$ = no. of parallel path.

Wave winding



LAP winding

$\boxed{A=P}$

P = number of poles.

low voltage and high current lap winding >500

Imp. (only in dc machine)

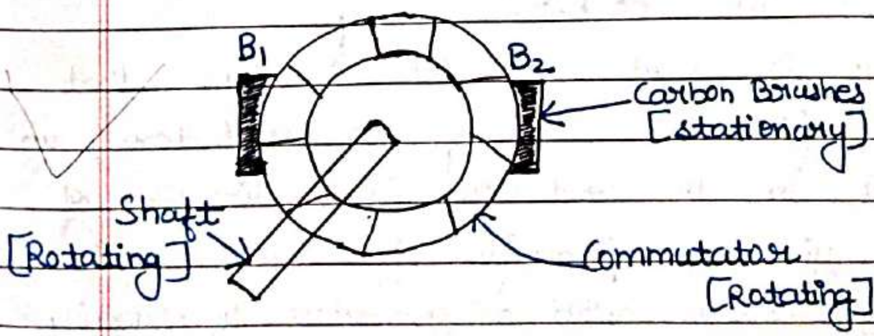
(iv). Commutator and Carbon brushes :

The basic nature of induced emf in DC generator is alternating (AC)

A commutator is a mechanical rectifier which convert alternating voltage (AC) into direct voltage (DC).

The carbon brushes are stationary and resting on the surface of the commutator.

The current is collected from the armature winding with the help of two or more carbon brushes.



When a current carrying conductor is placed in a magnetic field it exp a force that force is given by Flemming's left hand rule.

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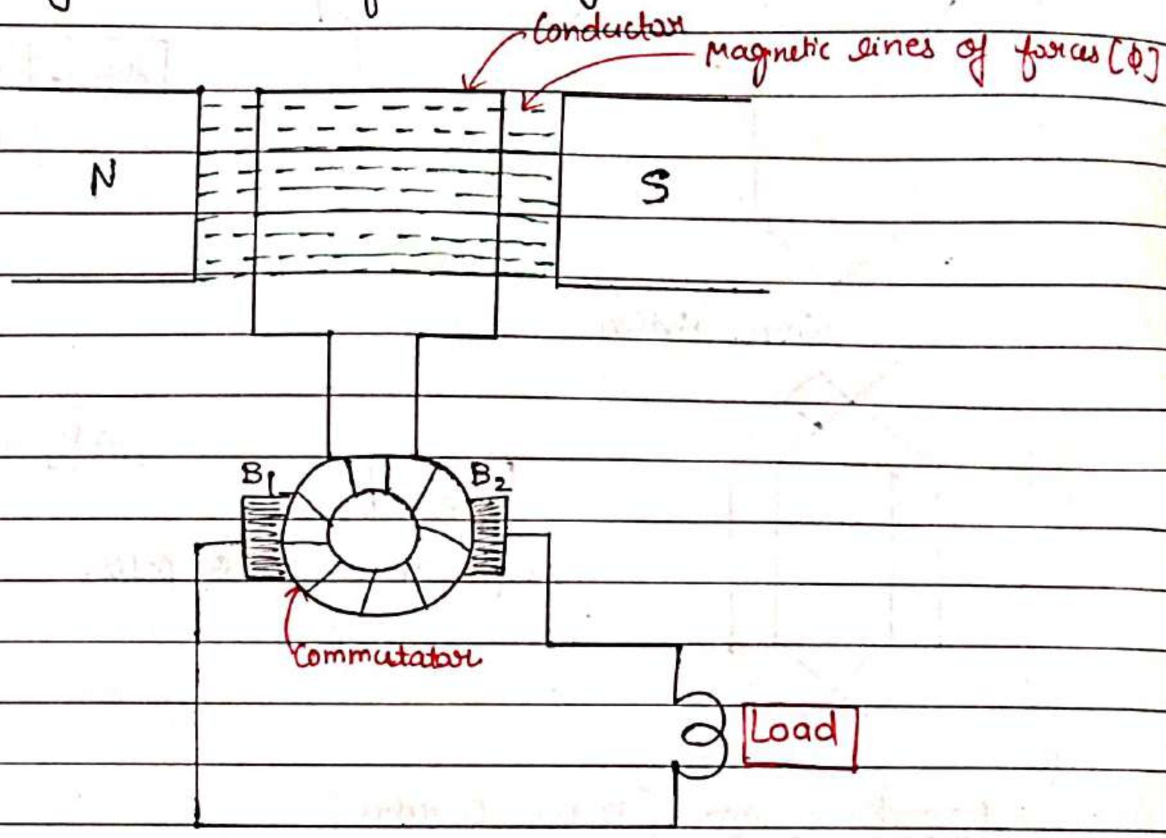
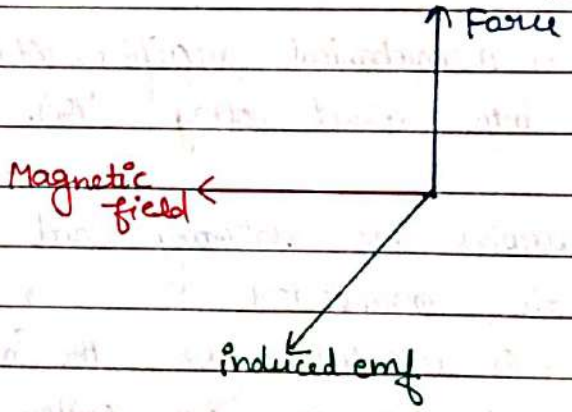
Working Principle of a DC generator :



Direction of induced emf ⇒ Flemming's Right hand rule



- An electrical generator is a machine which convert mechanical energy into electrical energy.
A DC generator is based on the principle that whenever magnetic flux is cut by a conductor, an emf is induced in the conductor. The direction of induced emf is given by flemming right hand rule.
(DC machine can work both as motor or generator by changing its input supply)

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The induced emf is given by

$$\boxed{\text{induced emf } (e) = Bvl} \text{ volt.}$$

where B = magnetic flux density
v = velocity of the conductor
l = length of the conductor.

Imp. Emf equation of a DC generator :

$$E_g = \frac{P\phi Z N}{60\,A} \text{ volt}$$

where E_g = generated emf
P = no. of poles
Z = Total number of conductors
N = Armature speed [rpm]
A = No. of parallel path
ϕ = Flux per pole [wb]

According to faraday's law of electromagnetic induction ~~the~~ an emf is induced in the armature conductor hence average value of induced emf in each armature conductor

$$e = \frac{d\phi}{dt} \quad \text{———} \quad (1)$$

Magnetic flux cut by one conductor in one revolution

$$d\phi = P\phi \quad \text{———} \quad (2)$$

time taken to complete one revolution

$$dt = \frac{60}{N} \quad \text{———} \quad (3)$$

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from equation ② and ③

$$\underset{\text{(per conductor)}}{e} = \frac{d\phi}{dt} = \frac{P\phi}{\left(\frac{60}{N}\right)} = \frac{PN\phi}{60}$$

This is the induced emf per conductor.
The induced emf for a DC generator,

$$E_g = \left[e_{conductor}\right] \times \left[\frac{Z}{A}\right]$$

$$E_g = \left(\frac{P\phi N}{60}\right) \times \left(\frac{Z}{A}\right)$$

Total no of conductors in series.

$$\boxed{E_g = \frac{P\phi Z N}{60\,A}} \text{ volt} \qquad K = \frac{PZ}{2\pi A} \quad \sim K_a \phi \omega$$

Case-I For wave wound DC generator [A=2]

$$\boxed{E_g = \frac{P\phi Z N}{120}} \text{ volt}$$

Case-II For LAP wound DC generator [A=P]

$$\boxed{E_g = \frac{\phi Z N}{60}} \text{ volt}$$

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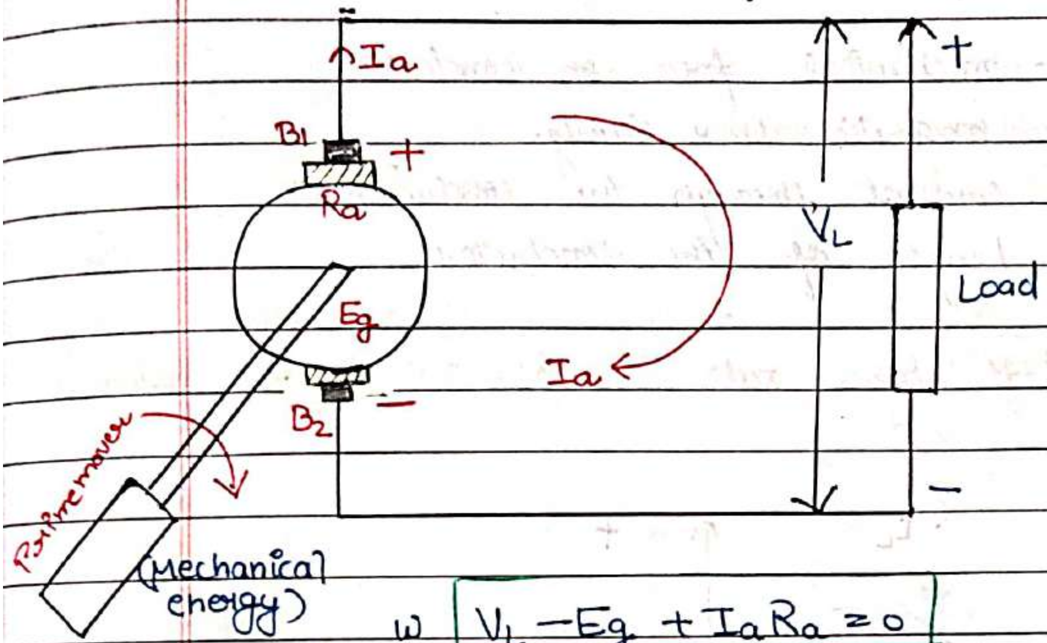
Equivalent electric circuit of a DC generator:



$$\boxed{V_L - E_g + I_a R_a = 0}$$

$$\boxed{V_L = E_g - I_a R_a} \Rightarrow \text{DC generator}$$

$$\boxed{E_g = V_L + I_a R_a} \Rightarrow \text{DC generator}$$

Where, R_a = armature winding resistance
I_a = armature winding current
E_g = Generated emf in armature
V_L = Voltage across load or terminal voltage

DC Motor:

A machine which convert electrical energy into mechanical energy.

(Input) Electrical energy → (output) Mechanical energy

Principle:

When a current carrying conductor is placed in a magnetic field. It experience a mechanical force. and the mechanical force is given as

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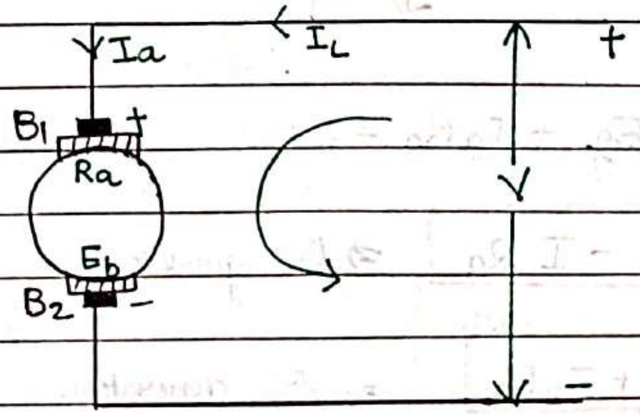
$\boxed{\text{Magnetic force } (F) = BIl}$ Newton

where F = mechanical force on conductor.
B = magnetic flux density.
I = Current through the conductor
l = Length of the conductor.

(Flemming's left hand rule is applied in DC motor)



$+E_b - V + I_a R_a = 0$

$$\boxed{V = E_b + I_a R_a}$$

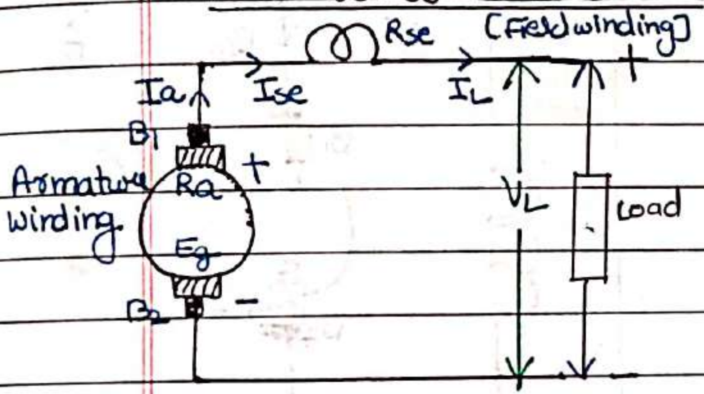
$$\boxed{E_b = V - I_a R_a}$$

$$\boxed{E_g = \frac{P\phi ZN}{60A} = E_b}$$ (Back EMF)

Types of DC Machines:

In DC series machine, the field winding is connected in series with with armature winding.

DC series Generator



(i) $\boxed{I_a = I_{se} = I_L}$

$+V_L - E_g + I_a R_a + I_{se} R_{se} = 0$

(ii) Terminal voltage $V_L = E_g - I_a r_a - I_{se} R_{se}$

$$\boxed{V_L = E_g - I_a(R_a + R_{se})}$$

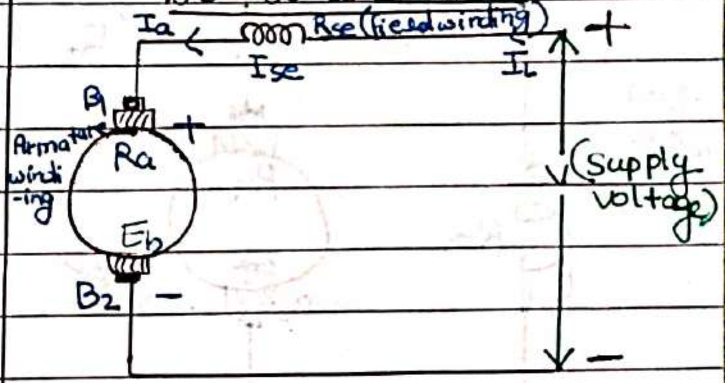
(iii) Mechanical Power in armature

$\boxed{P_{mech} = E_g I_a}$ watt

(iv) power delivered to load

$\boxed{P_{load} = V_L I_L}$ watt

DC Series Motor



(i) $I_L = I_{se} = I_a$ [Armature current]

(ii) $V_L = E_b + I_a(R_a + R_{se})$

$$\boxed{E_b = V_L - I_a(R_a + R_{se})}$$

(iii) Mechanical power developed in armature

$\boxed{P_{mech} = E_b I_a \text{ watt}}$

(iv) Power developed in Input

$\boxed{P_{load} = V_L I_L \text{ watt}}$

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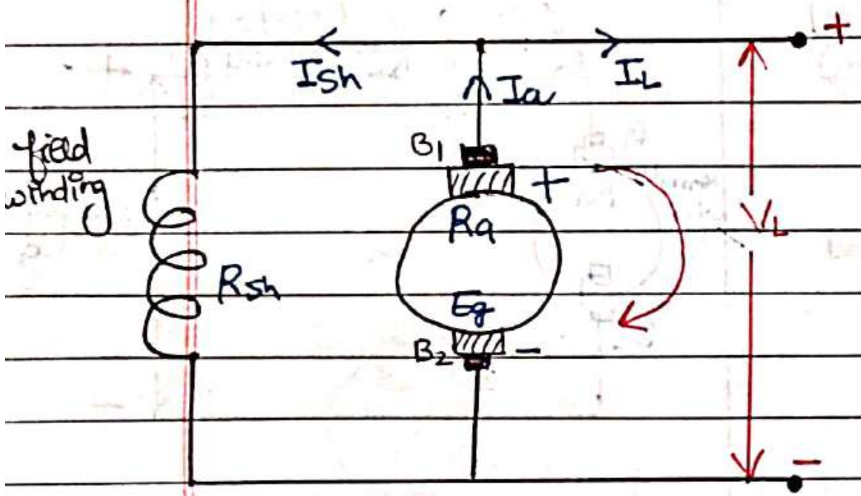
DC shunt machine :

In DC shunt machine, the field winding is connected in parallel or shunt with armature winding.

DC shunt generator



(i) Armature current $[I_a] = I_{sh} + I_L$

$$\boxed{I_{sh} = \frac{V_L}{R_{sh}}}$$

(ii) $+V_L - E_g + I_a R_a = 0$

$$\boxed{E_g = V_L + I_a R_a}$$ generated emf in armature

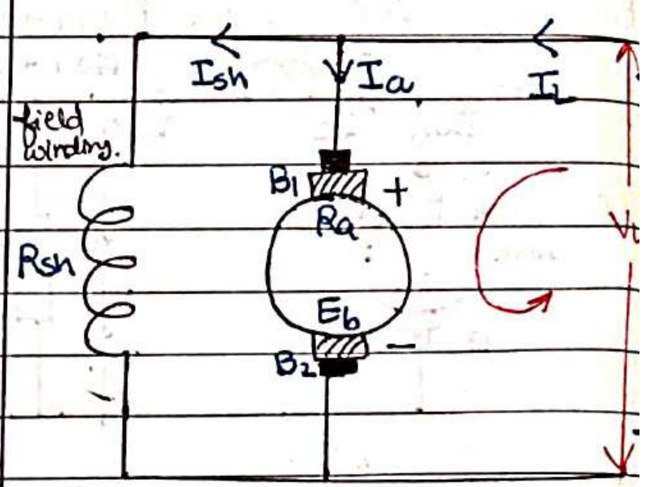
(iii). Mechanical power supplied in armature.

$$\boxed{P_{mech} = E_g \cdot I_a}$$

(iv) Armature power deleivered to the load.

$$\boxed{P_{load} = V_L I_L}$$

DC shunt motor



(i) Line current $[I_L] = I_a + I_{sh}$

Armature current $[I_a]$

$= I_L - I_{sh}$

$$\boxed{I_{sh} = \frac{V_L}{R_{sh}}}$$

(ii) Back emf induced in armature

$E_b = V_L + (-I_a) R_a$

$$\boxed{E_b = V_L - I_a R_a}$$

(iii) Mechanical power supplied in armature

$$\boxed{P_{mech} = E_b \times I_a}$$

$P_{mech} = (V_L - I_a R_a) I_a$

(iv) power delivered to the load

$$\boxed{P_{load} = V_L I_L}$$

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⇒ **Power Equation in a DC Motor :**

$$E_b = V - I_a R_a$$

multiplying by I_a on both sides

$$\boxed{E_b I_a = V I_a - I_a^2 R_a}$$

$E_b I_a$ = Power developed in Armature

$V I_a$ = Input power supplied

$I_a^2 R_a$ = Copper losses in armature winding.

Condition for maximum power developed in armature :

Mech power $= V I_a - I_a^2 R_a$

$$\frac{dP_{mech}}{dI_a} = 0$$

$$\frac{d}{dI_a}(V I_a - I_a^2 R_a) = 0$$

$$V - 2 I_a R_a = 0$$

$$V = 2 I_a R_a$$

$$\boxed{\frac{V}{2} = I_a R_a} \text{ ——— } (1)$$

$$E_b = V - \frac{V}{2} \qquad [E_b = V - I_a R_a]$$

$$\boxed{E_b = \frac{V}{2}}$$

Role of Back Emf in $\boxed{DC\ motor}$:

$$E_b = \frac{P \phi Z N}{60 A}$$

P, Z, 60, A are constants

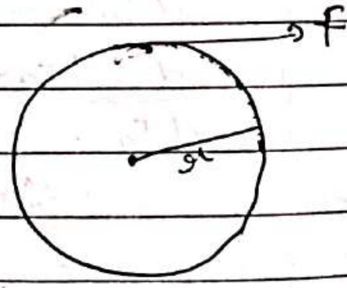
$$E_b \propto \phi N$$

Armature speed (N) $\propto \frac{E_b}{\phi}$ —— ①

$$E_b = V - I_a R_a$$

Armature current (I_a) = $\frac{V - E_b}{R_a}$ —— ②

Mechanical force of Conductor [F] = $B I_a l$ —— ③



load increases

$N \downarrow$

$\boxed{E_b \downarrow}$

Armature current $[I_a] \uparrow$

Force [F] = $B I_a l \uparrow$

Force [F] $\uparrow$

$$\underset{\uparrow}{T_a} = F \times r$$

Armature torque $[T_a] \uparrow$

With the help of back emf, a DC motor adjust the armatu

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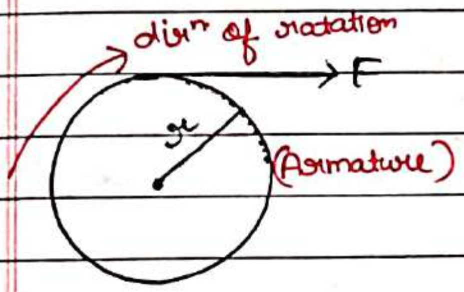
current according to the load and motor runs as as self regulating machine.

4) Torque equation of a DC motor:

(Anticlock wise torque) Torque is the turning force about an axis



Torque is the product of force and the radius at which this force is act.

$$T_a = F \times r \quad ----- ①$$

The angular speed of the armature

$$\omega = 2\pi n \text{ (radian/sec)}$$

$$\omega = \frac{2\pi N}{60} \quad ----- ②$$

Workdone in one revolution

$$W = F \times \text{distance travel in one revolution.}$$

$$W = 2\pi r \quad ------ ③$$

mechanical power developed

$$P_{mech} = \frac{\text{Workdone } [W]}{\text{time taken in one revolution}}$$

$$P = \frac{F \times 2\pi r}{\left(\frac{60}{N}\right)}$$

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$$P = \frac{(F \times r)\, 2\pi N}{60}$$

$$P = T_a \cdot \omega \qquad \text{—— (IV)}$$

Mechanical power developed in armature $= E_b I_a$ —— (V)

$$E_b I_a = T \times \omega$$

$$\frac{\phi \cancel{N} Z P}{\cancel{60} A} \cdot I_a = T \times \frac{2\pi \cancel{N}}{\cancel{60}}$$

$$\boxed{T = \frac{1}{2\pi} \frac{\phi Z P I_a}{A}}$$

$$\boxed{T_a \propto \phi I_a}$$

where Z, P and A are constant.

Torque Equation : DC series motor

$$\phi \propto I_a$$

$$T_a \propto \phi I_a$$

$$T_a \propto I_a^2$$

(ii) Torque equation of DC shunt motor

$$\phi \propto I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$T_a \propto \phi I_{sh}$$

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$\phi \propto I_{sh} \propto \text{constant}$

$\phi = \text{constant}$

$\boxed{T_a \propto I_a}$

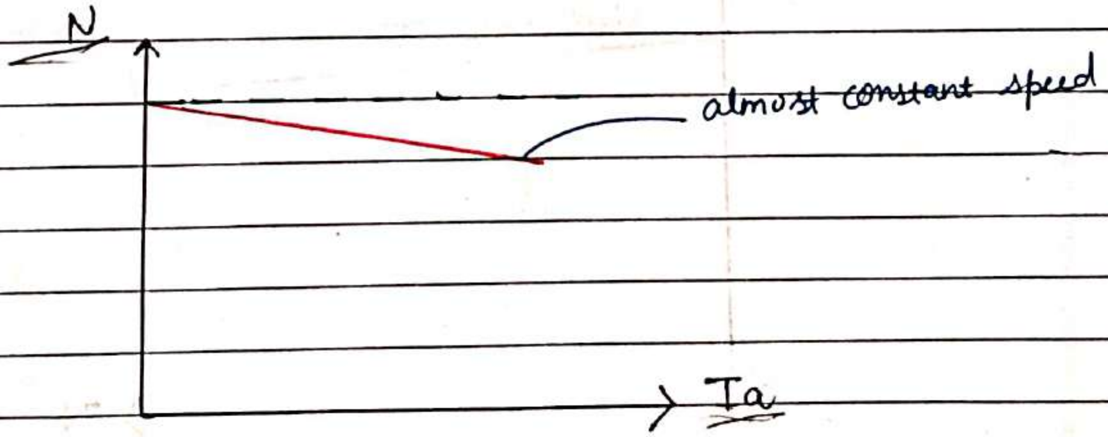
Characteristics of DC Equations :

(i) Speed (N) – Armature current [Ia] characteristic

(ii) Torque (Ta) – Armature current [Ia]

(iii) Speed (N) – Torque (Ta) characteristics

- DC shunt motor :



speed equation $\boxed{N \propto \frac{E_b}{\phi}}$

$K_a \phi N = V_t - I_a R_a$

$E_b = V - I_a R_a$

$N = \frac{V_t}{K_a \phi} - \frac{I_a R_a}{K_a \phi}$

constant

$N \propto \frac{V - I_a R_a}{\phi}$

$\phi = \text{constant}$

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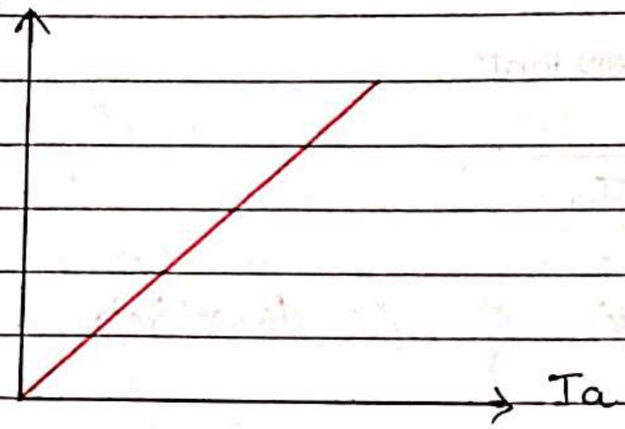
2. Torque and armature current characteristics :

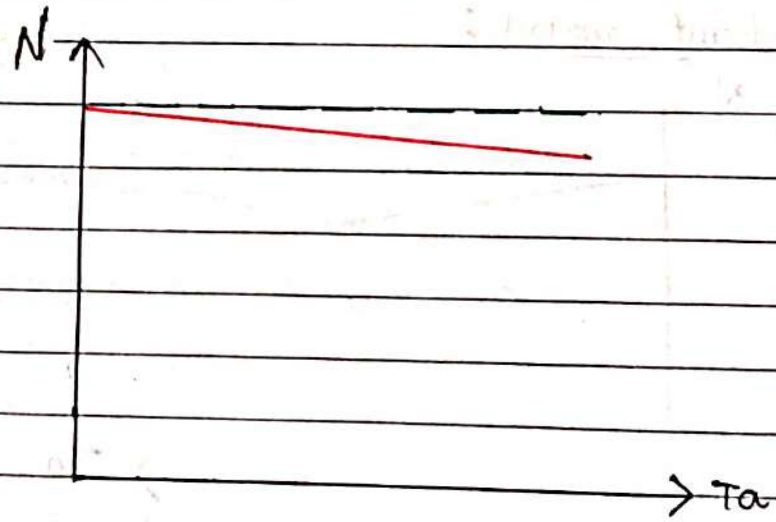


Torque equation $\boxed{T_a \propto \phi I_a}$

ϕ = constant

$\boxed{T_a \propto I_a}$

3. Speed Torque characteristics :



$T \propto I_a$

$T = K_c I_a$

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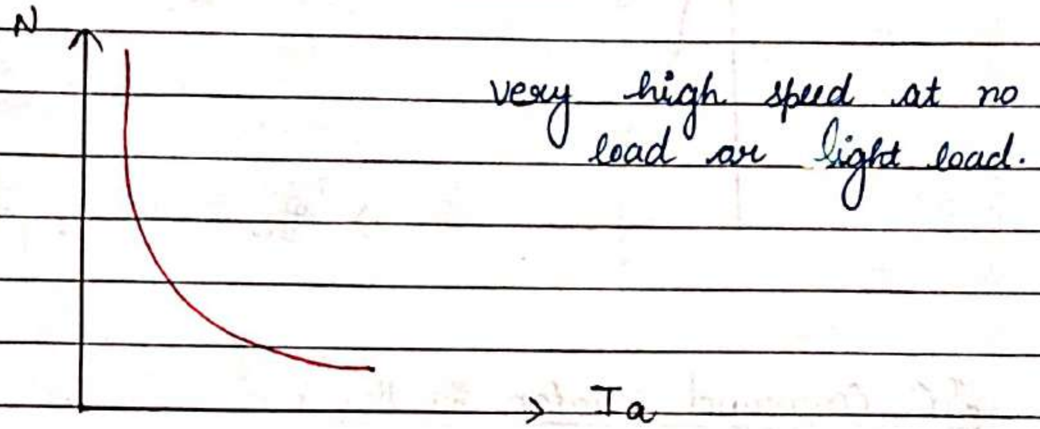
Characteristics of a DC series motor:

1. Speed armature current characteristics:

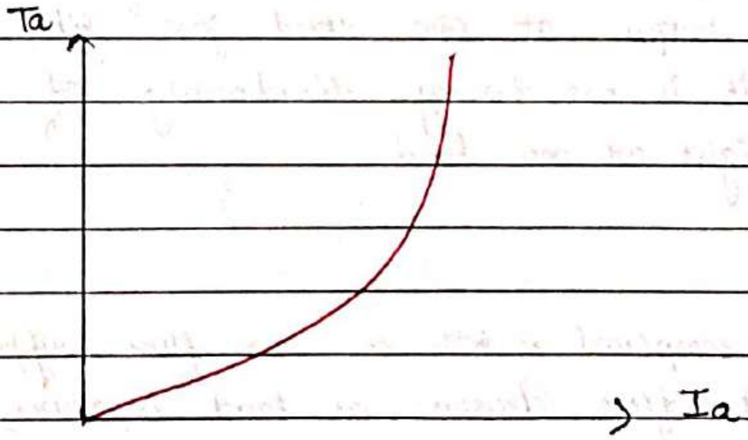


Speed equation $N \propto \frac{E_b}{\phi}$

$E_b = V - I_a R_a$

$\boxed{\phi \propto I_a}$

2. Torque armature current characteristics:



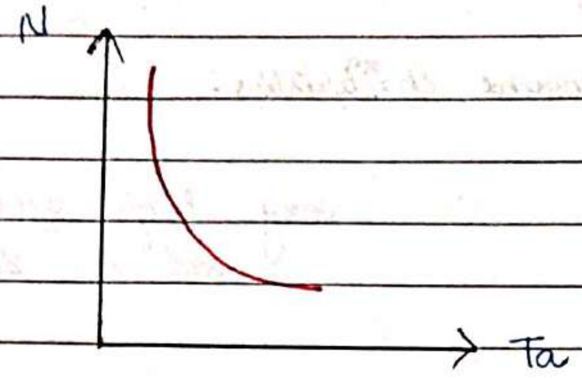
Torque equation $T_a \propto \phi I_a$

$\phi \propto I_a$

$T_a \propto I_a^2$

(iii) Speed - Torque characteristics.

$T \propto I_a^2$



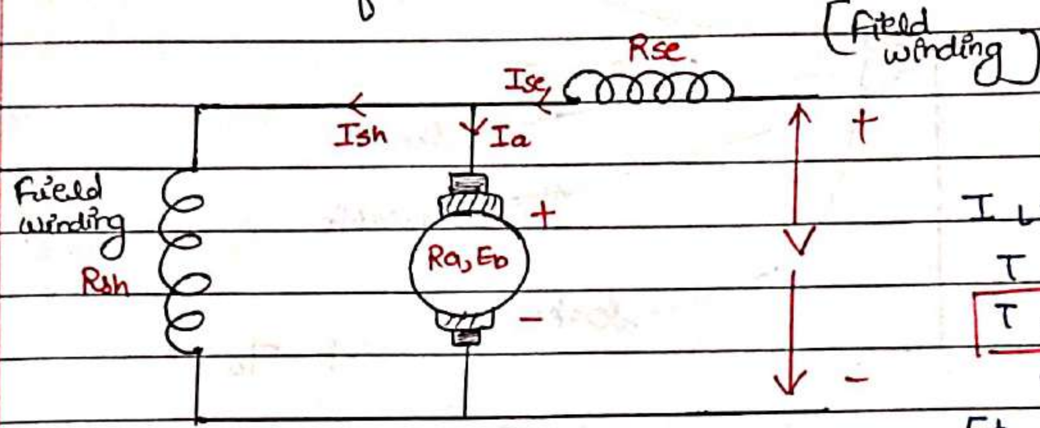
DC Compound Motor :

A DC compound motor has both the series and shunt winding. so compound motor characteristics depends on the fact whether the motor is commulative compound or differential compound.

Note :(i) Comulative compound motor is capable of developing large amount of torque at low speed just like series motor. However, it is not having disadvantage of series motor even at light or no load.

(ii) Differential compound motor, as two flux opposes each other the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load The motor may try to run with very high speed. So, differential compound motor is generally not used.

Characteristics of DC compound motor :



$E_b = V - I_a(R_a + R_{se})$

$I_L = I_a + I_{sh}$

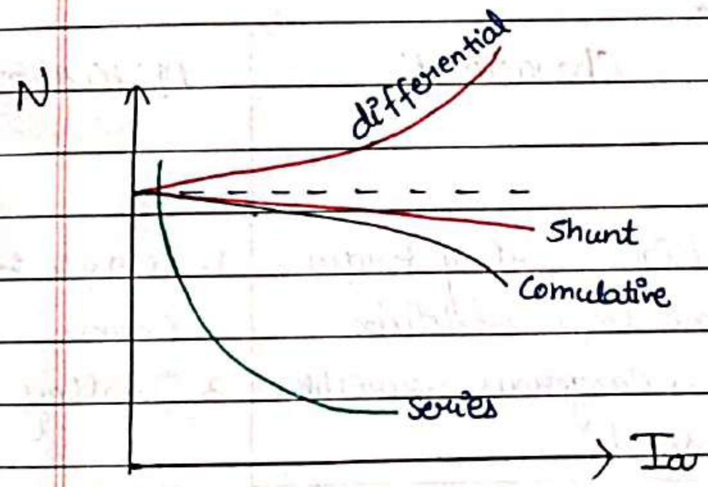
$T \propto \phi I_a$

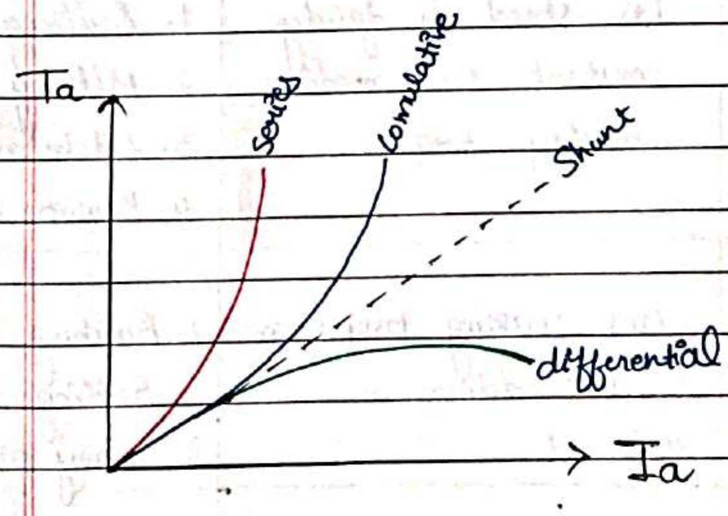
$\boxed{T \propto (\phi_{se} + \phi_{sh}) I_a}$

$\phi \propto I_a$

$[\phi_{se} + \phi_{sh}] \propto I_a$

(Comulative Compound DC motor)





$p = V_t I_L - I_a^2 R_a = E_b I_a \Rightarrow P_m = T \times N$

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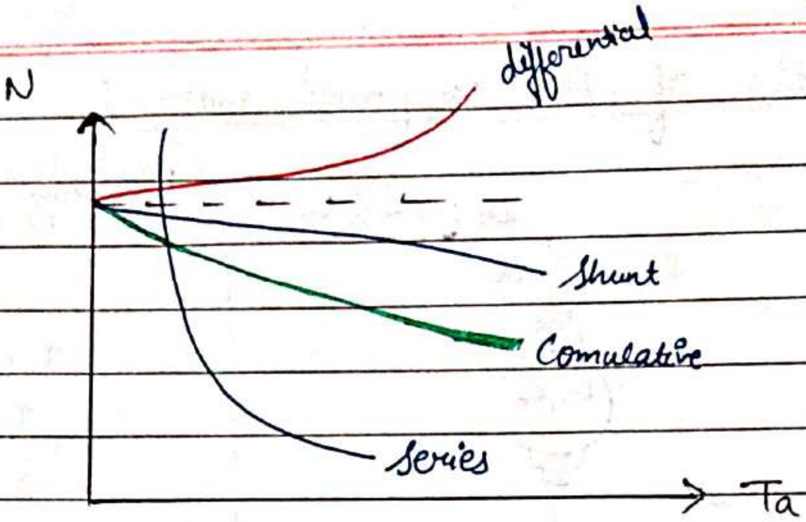

Imp 2 marks Applications of DC motor :

S.R.No.	Types of DC motors	Charactoristics	Application
1.	DC series motor (don't start on no load)	High starting torque, no load condition is dangerous, variable speed	1. Electric locomotives 2. Cranes 3. Trolleys
2.	DC shunt motor	It's speed is fairly constant and medium starting torque.	1. Drilling machines 2. Milling machine 3. Latche machine 4. Blowers and fans
3.	Cumulative Compound	High starting torque, no load condition is allowed	1. Punching machine 2. Rolling Mills 3. heavy planers
4.	Differential Compound	Speed increase as load increase	1. Not suitable for any practicle applications.

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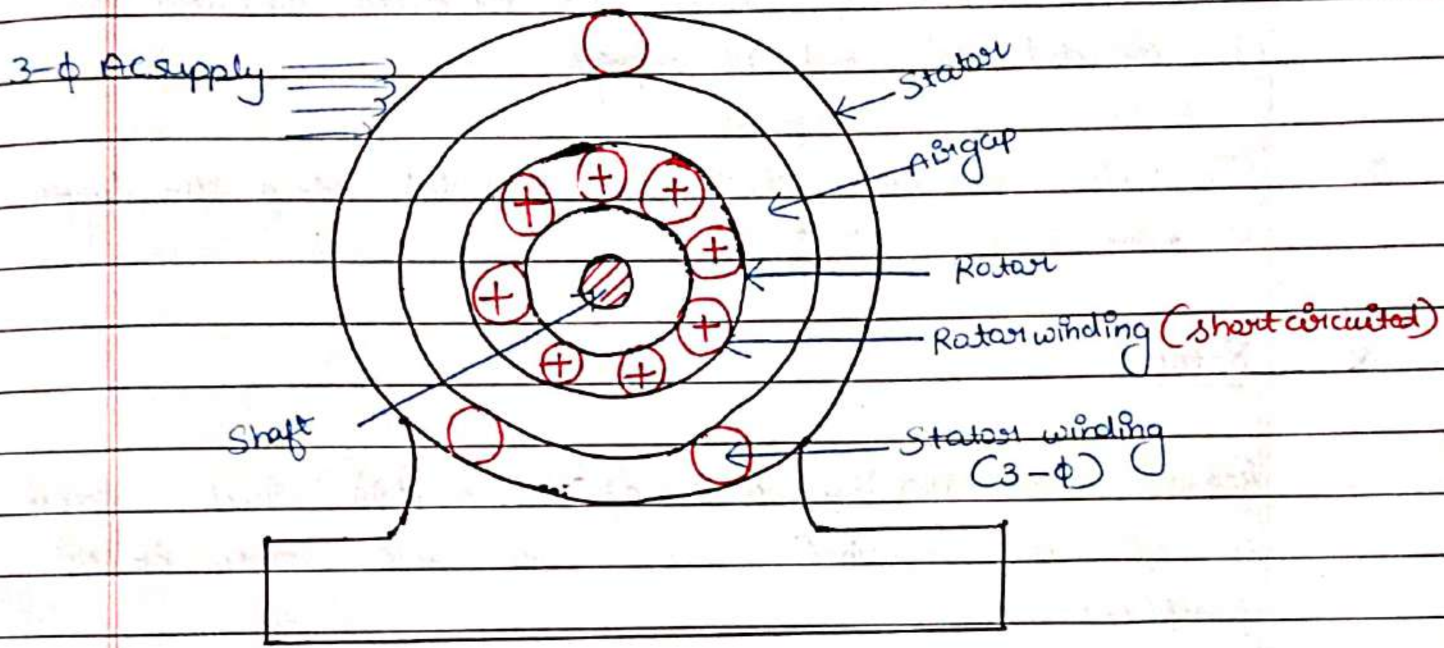
AC Machines :

Three phase [3-ϕ] Induction Motor : (self starting)



- Self starting
- Single excited motor

Note : Induction motor cannot runs at synchronous speed. [N_s]

$$\boxed{N_s = \frac{120 f_s}{P}}$$

Construction :-

An induction motor consist of mainly two parts

* Stator
* Rotor

- Stator : Stator core is made of laminated steel (stampling) and has slots and teeth on its inner Phoriphery to house stator winding.

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2. Stator carries a three phase windings having phase displacement of 120° electrical.

3. The 3-ϕ winding is either star or delta connected and is connected to 3-ϕ AC supply.

4. The radial ventilating ducts are provided along the length of stator core.

* **Rotor:**

1. Rotor is the rotating part of the machine which comprises a cylindrical laminated iron core with slots on outer periphery.

2. Like stator, Rotor lamination are punched in one piece for small machine.

3. In larger machine the laminations are segmented.

4. If there are ventilating ducts on the stator core, an equal number of such ducts is provided on rotor core.

- According to rotor windings 3-ϕ induction motor are of two types:

a) Squirrel cage rotor type.

b) Slip ring or wound rotor.

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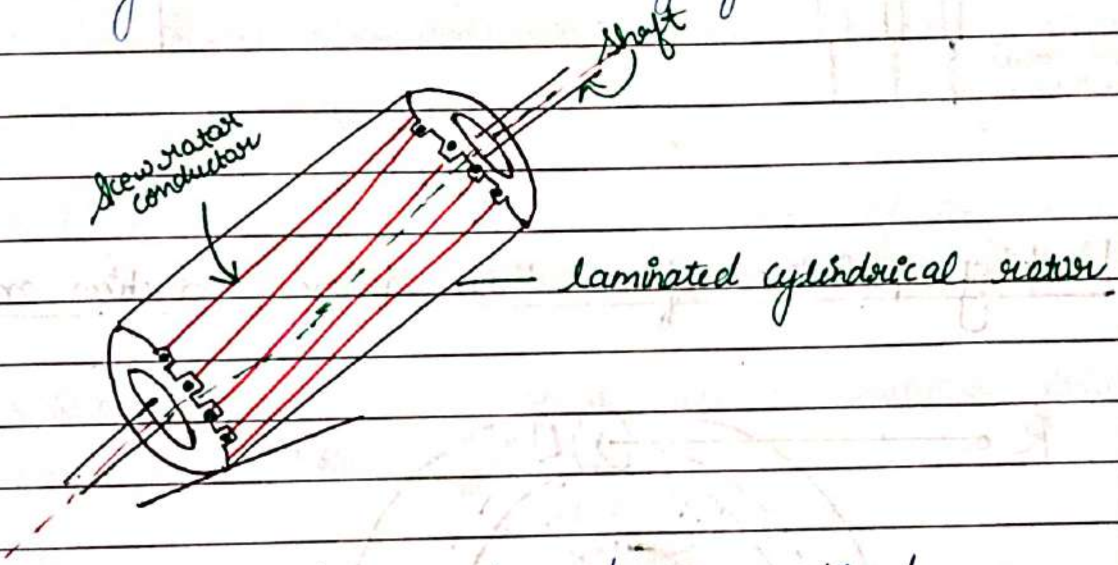
1. Squirrel Cage Rotor :

- This rotor consist of a cylindrical laminated core with parallel slots.

- Rotor slots are usually not quiet parallel to the shaft but for reducing the magnetic humming and locking. tendency rotor slots are slightly skew.



- In Rotor slots heavy copper bars are placed

- Rotor Bars are permanently short circuited at the ends. This limits that no external resistance inserted is possible.

2 Slip ring or Wound Rotor :

1. The Rotor is wound for the same number of poles and number of phases as that of stator.

2. Rotor winding is either star or delta but star connection is preferred.

3. The three star terminals are connected to three brass slip rings mounted on rotor shaft.

4. These slip rings are insulated from rotor shaft.

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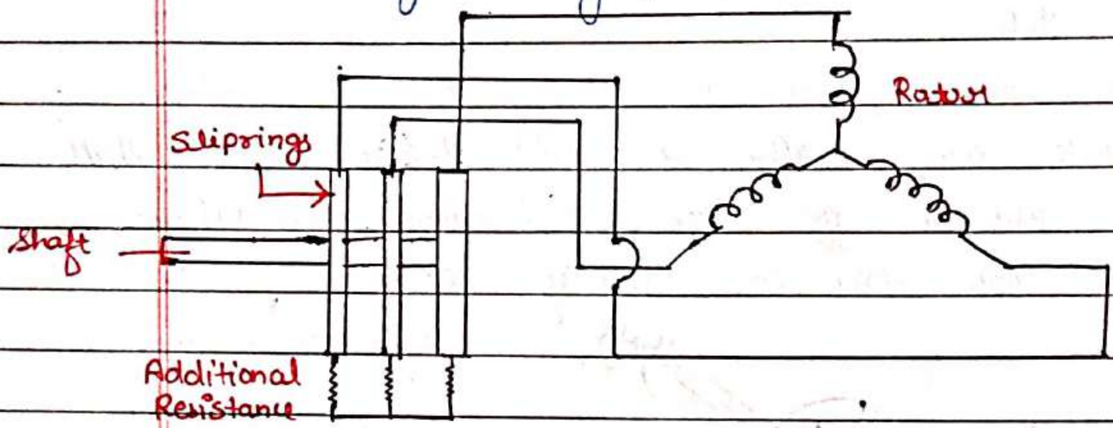
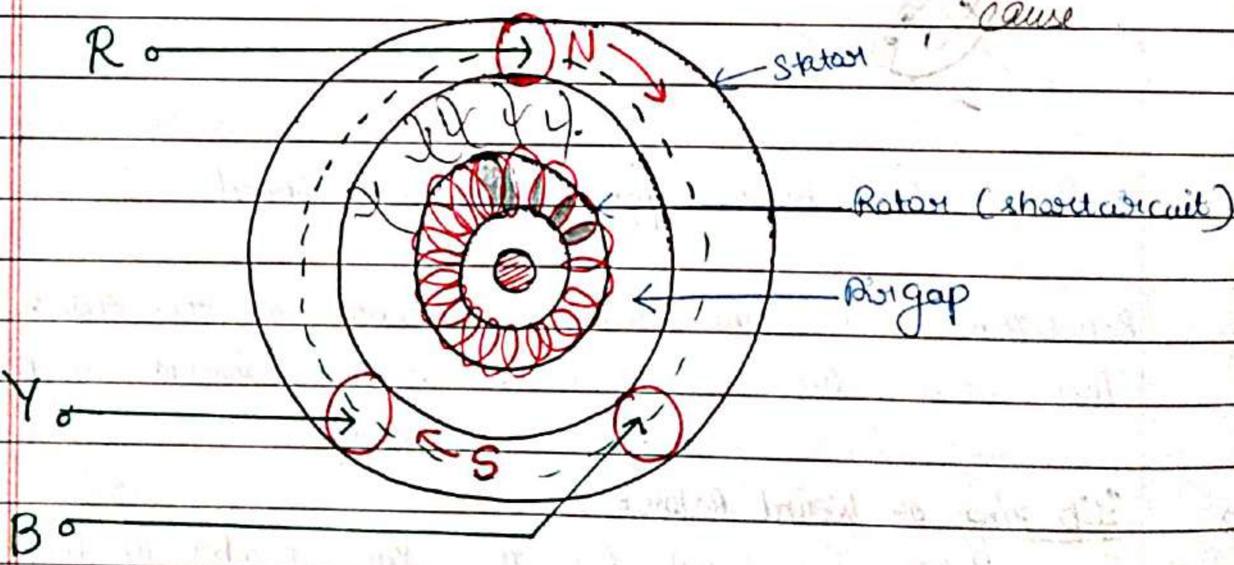
4. This makes possible introduction to external resistance in the rotor circuit during starting period.



Working principle of a three phase induction motor:



Step-1: When a three phase stator winding supplied from a three phase AC supply, a rotating magnetic field is set up in the stator core.

Step-2: This rotating magnetic field rotates with synchronous speed $\left[N_s = \frac{120 f_s}{P}\right]$ rpm, with respect to stationary stator in the air gap

where f_s = supply frequency and P = no of poles.

Cause is the relative speed b/w rotating magnetic field ~~B~~ and the stationary rotor.

$1-\phi \Rightarrow$ pulsating magnetic field

$3-\phi \Rightarrow$ Rotating magnetic field

Its speed ~~se~~ field ghumti hai.

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Step-3 This rotating magnetic field passes through the air gap and cuts the stationary rotor conductor.

Step-4 Due to relative speed b/w the rotating magnetic flux and the stationary rotor, an emf induced in the rotor conductor.

Step-5 If the rotor conductors are short circuited, current start flowing in the rotor conductor.

Step-6 According to Lenz's law the direction of induced current is such that it opposes the cause.

Step-7 Cause is the relative speed b/w the rotating field and the stationary rotor.

Step-8 Hence a rotor has a tendency to reduce the relative speed (rotor has a tendency to move in the dirⁿ of rotating magnetic field)

Step-9 So, rotor begins to move in the direction of rotating magnetic field and continuous towards synchronous speed. and the machine runs at a speed near but blow synchronous speed depending upon load on shaft.

Ques Why an induction motor cannot runs at synchronous speed?

Ans As the speed of rotor reaches to synchronous speed, the relative speed is zero. Hence no emf, no current and therefore no torque at synchronous speed.
Hence, rotor never reaches to synchronous speed.

Relative speed $= N_s - N_r$
$\hookrightarrow$ slip speed

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Slip speed: The relative speed b/w the rotating magnetic field (N_S) and the rotor speed (N_R) is called slip speed.

$$\text{Slip speed} = (N_S - N_R) \text{ rpm.}$$

Slip: percentage changed in slip speed is called slip(s)

$$\boxed{\text{slip}(s) = \frac{N_S - N_R}{N_S} \times 100}$$

Note! Slip at standstill condition: ($N_R = 0$)

$$S = \frac{N_S - N_R}{N_S}$$

$$S = \frac{N_S - 0}{N_S} = \boxed{S = 1}$$

Slip at synchronous speed.

($N_R = N_S$)

$$S = \frac{N_S - N_R}{N_S}$$

$$S = \frac{N_S - N_S}{N_S}$$

$$\boxed{S = 0}$$

N_r - speed of the rotating magnetic field w.r.t. to conductor

$N_S - N_r = \frac{120 f_r}{P}$

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Note: "The value of the slip varies from 1 to zero for an induction motor."

Effect of slip on rotor paramaters in running Condition:

1. Effect of slip on rotor current frequency:

$$N_s = \frac{120 f_s}{P}$$

Where f_s = supply frequency
P = no of poles.

$$f_s = \frac{P N_s}{120} \quad \text{———(i)}$$

When Rotor rotates

$$f_r = \frac{P(N_s - N_R)}{120} \quad \text{———(ii)}$$

from (i) and (ii)

$$\frac{f_r}{f_s} = \frac{\cancel{P} N_s}{\cancel{120}} \times \frac{\cancel{120}}{\cancel{P}(N_s - N_R)}$$

$$\frac{f_r}{f_s} = \frac{N_s - N_r}{N_s}$$

$$\frac{f_r}{f_s} = \text{slip}(s)$$

$$\boxed{f_r = S f_s}$$ Imp

Rotor current frequency is slip(s) times of supply frequency.

$E_s = 4.44 f s \phi_m N$

$E_{sr} = 4.44 s f_s \phi_m N$

DATE $E_{sr} = s f_s$

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Similarly, $\boxed{E_{sr} = S E_s}$

$E_s = \sqrt{2}\pi f_s \phi_m T_s$

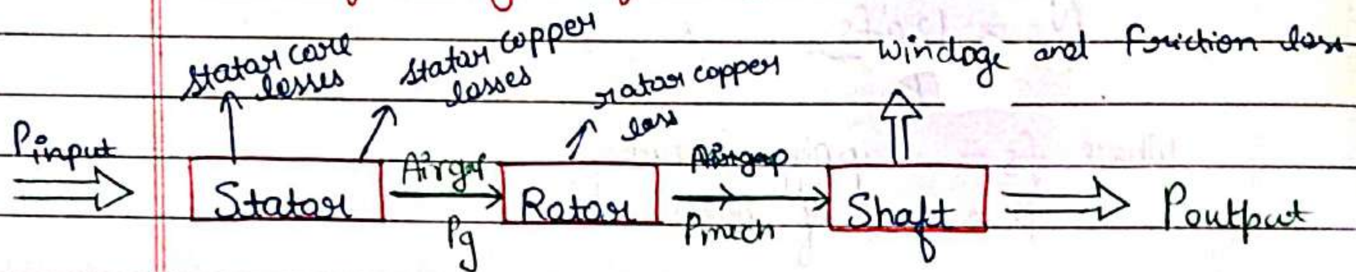
$E_r = \sqrt{2}\pi s f \phi_m T_r$

$\frac{E_s}{E_{sr}} = \frac{1}{s}$

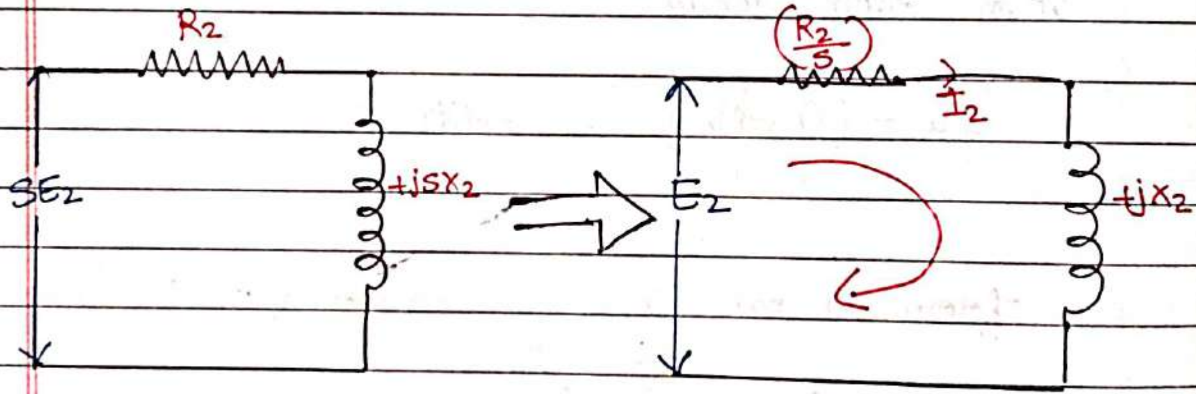
E_{sr} = Emf induced in rotor

E_s = Emf on stator.

Power flow diagram for an induction motor :



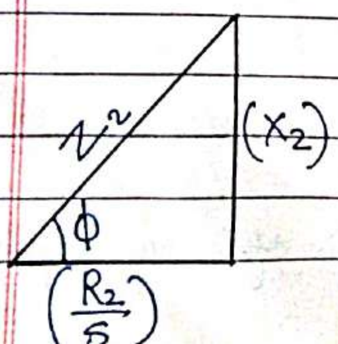
Rotor current, emf and Power in running condition :



Rotor equivalent circuit

(Running condition)

$$\text{Rotor current } (I_2) \text{ Per phase} = \frac{E_2}{\left[\left(\frac{R_2}{S}\right) + jX_2\right]} = \frac{E_2}{\sqrt{\left(\frac{R_2}{S}\right)^2 + (X_2)^2}}$$



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reaches at air gap through rotor P_g

per phase power given to rotor :

$$P_g = E_2 I_2 \cos\phi_2$$

$$P_g = I_2 \cancel{Z_2} I_2 \times \frac{\left(\frac{R_2}{S}\right)}{\cancel{Z_2}}$$

$$\boxed{P_g = \frac{I_2^2 R_2}{S}}$$ per phase.

Mechanical power developed in ~~motor~~ rotor per phase :

$$P_m = P_g + P_{ohmic}$$

$$P_m = \frac{I_2^2 R_2}{S} - I_2^2 R_2$$

$$P_m = \frac{I_2^2 R_2}{S} (1-S)$$

$P_g = I_2^2 R_2 / S$

$P_{mech} = P_g(1-S)$

$P_{ohm} = S P_g$

$$\boxed{P_m = P_g (1-S)}$$

$$\boxed{P_{ohmic} = I_2^2 R_2}$$

$P_{ohmic} = S P_g$

$$\boxed{P_g : P_m : P_{ohmic} :: 1 : (1-S) : S}$$

Gross torque developed in Rotor :

$$P_m = \text{Torque} \times \omega_r$$

$$T = \frac{P_m}{\omega_r}$$

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Hint:

$$T = \frac{P_g(1-\cancel{s})}{\omega_s(1-\cancel{s})}$$

$$\boxed{T = \frac{P_g}{\omega_s}}$$

$$\boxed{T = k P_g} \rightarrow \text{ohmic losses.}$$

$$S = \frac{N_s - N_r}{N_s}$$

$$S = \frac{\omega_s - \omega_r}{\omega_s}$$

$$S\omega_s = \omega_s - \omega_r$$

$$\omega_r = \omega_s - S\omega_s$$

$$\boxed{\omega_r = \omega_s(1-S)}$$

$$\boxed{N_r = N_s(1-S)}$$

Imp. Torque Equation and Torque slip characteristics of a 3-ϕ induction motor 6

We know that,
the torque developed is 3-ϕ induction motor is given by

$$T = \frac{3P_g}{\omega_s}$$

$$T = \frac{3}{\omega_s}(P_g)$$

$$T = \frac{3}{\omega_s}\left(\frac{I_2^2 R_2}{S}\right)$$

$$\boxed{T = \frac{K E_2^2 \left(\frac{R_2}{S}\right)}{\left[\left(\frac{R_2}{S}\right)^2 + (X_2)^2\right]}}$$

$$\boxed{K = \frac{3}{\omega_s} = \text{constant}}$$

$T \propto \cancel{E_2}$ (supply voltage)

This is a torque equation for three phase induction motor.

$E_2 \propto$ supply voltage.

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Condition for maximum torque :

$$T = \frac{K E_2^2 R_2}{\left[\frac{R_2^2}{S} + S (X_2)^2\right]} \Rightarrow A$$

$$\frac{dA}{ds} = 0$$

$$\frac{d}{ds}\left[\frac{R_2^2}{S} + S(X_2)^2\right] = 0$$

$$-\frac{R_2^2}{S^2} + X_2^2 = 0$$

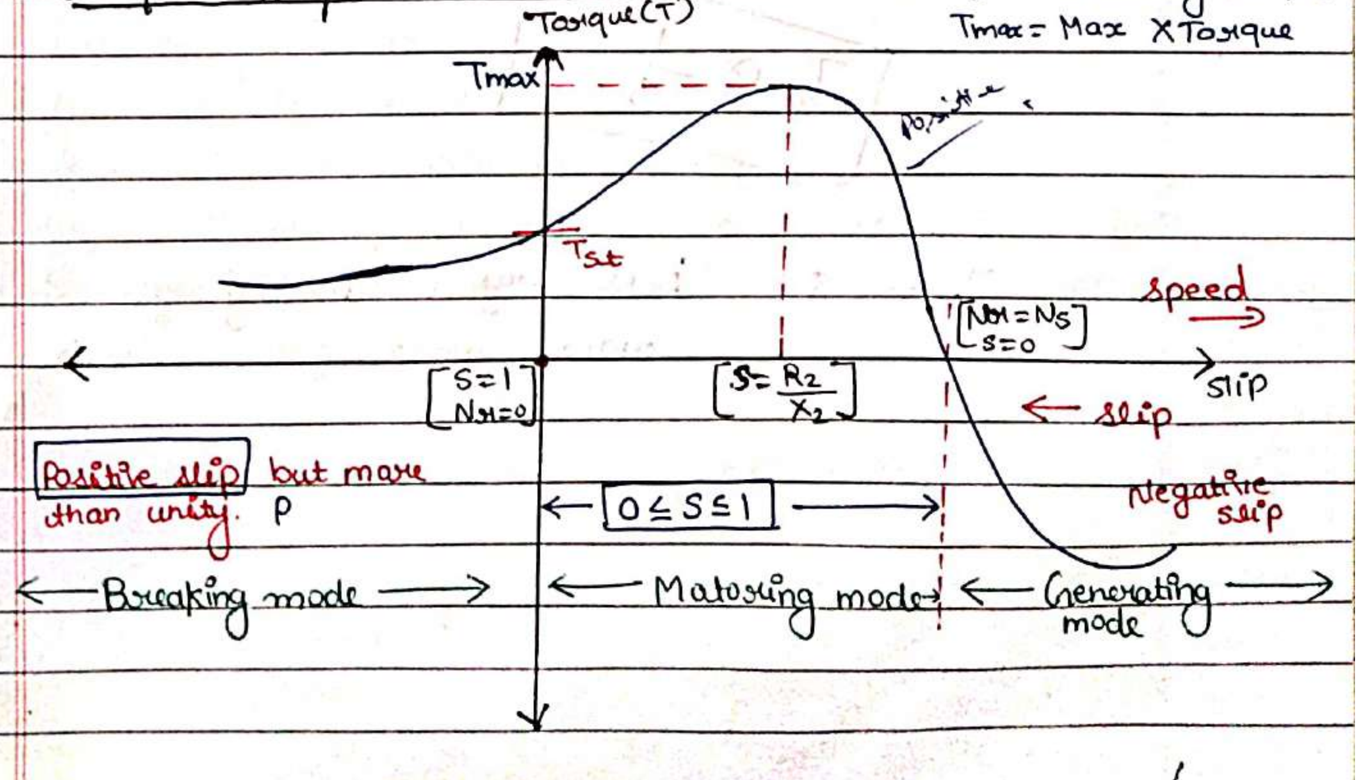
$$\frac{\cancel{-}R_2^2}{S^2} = \cancel{-}X_2^2$$

$f_r = s f_s$
$s = \frac{N_s - N_r}{N_s}$

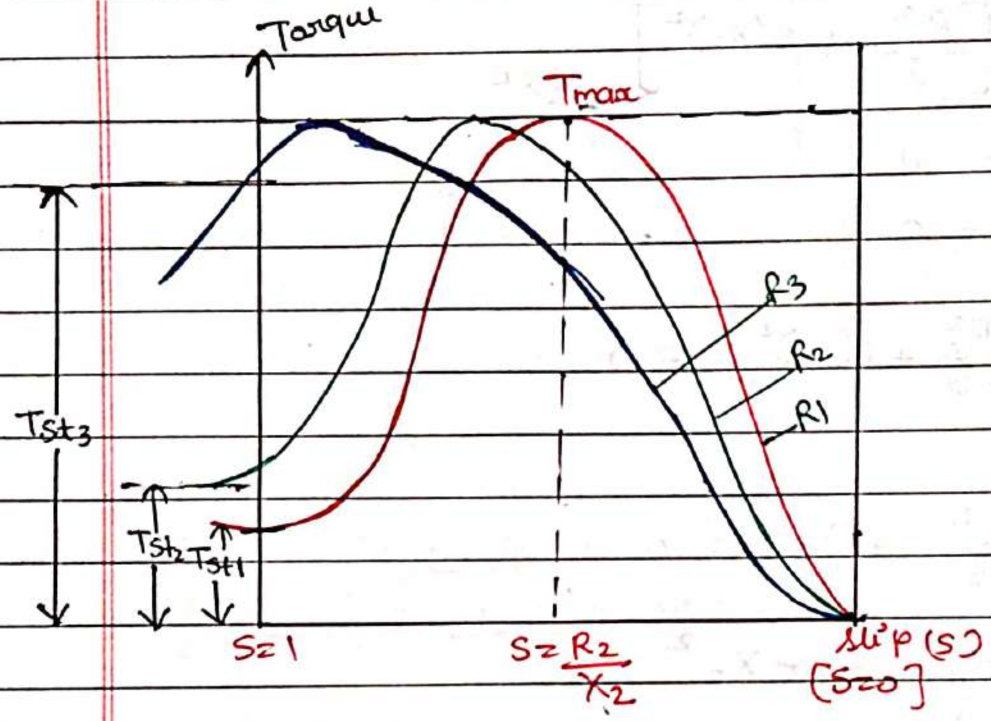
$$\boxed{\text{slip}(S) = \frac{R_2}{X_2}}$$ (This is condition for Torque)

Torque slip characteristics :

T_{st} = starting torque
T_{max} = Max Torque



Torque slip characteristics for different different rotor resistance 6



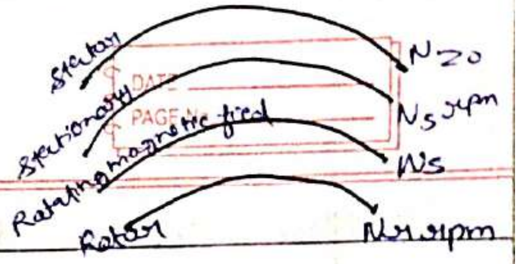
$R_3 > R_2 > R_1$

$T_{st3} > T_{st2} > T_{st1}$

As rotor resistance increases the value of starting torque increases.

$$\boxed{T_e \propto \frac{1}{s}}$$

$T_d \propto s$ (Active region) / working region in which motor works.



Steps for Numerical problems :

(i) Synchronous speed (N_s) = $\frac{120 f_s}{P}$ [rpm]

(ii) % Slip(s) = $\frac{N_s - N_r}{N_s} \times 100$

(iii) Rotor speed / motor speed (N_r) = $N_s(1-s)$ [rpm]

(iv) Rotor current frequency (f_r) = $s f_s$ [Hz]

(v) Slip at maximum torque [S_m] = $\frac{R_2}{X_2}$

(vi) $P_g : P_m : P_{ohmic} :: 1 : (1-s) : s$

Numericals :

Ques1 A three phase induction motor runs at 1140 rpm at full load when supplied with power from a 60 Hz from a 3-ϕ line. Calculate

i) No. of poles
ii) Slip at full load
iii) frequency of rotor voltage
(iv) Speed of rotor field wrt rotor
(v) speed of rotor field wrt stator and stator field
(vi) speed of rotor at a slip of 10%, and rotor frequency at this slip.

Ans = 1140 rpm $f_s = 60$ Hz

$N_s = \frac{120 f_s}{P}$

$P = \frac{120 \times 60}{1200} \Rightarrow \boxed{P = 6}$

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(ii) $S = \frac{N_s - N_r}{N_s} \times 100$

$S = \frac{1200 - 1140}{1200}$

$\boxed{S = 0.05}$

(iii) $f_r = S f_s$

$= 0.05 \times 60$

$\boxed{f_r = 3 Hz}$

(iv) $N_s - N_r = 1200 - 1140$

$= 60 rpm$

(v) $N_s - 0 = 1200 - 0 = 1200$ rpm

$N_s - N_s = 0$

(vi) Speed of rotor $(N_r) = N_s(1-S)$

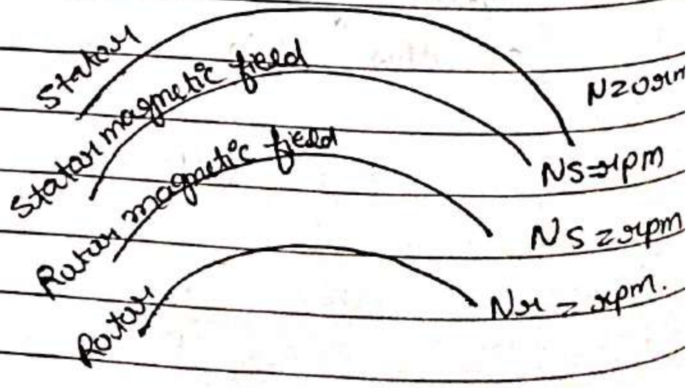
$N_r = 1200(1 - 0.1)$

$\boxed{N_r = 1080 rpm.}$

$f_r = S f_s$

$= 0.1 \times 60$

$\boxed{f_r = 6 Hz}$



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Qu A 6 pole 3-ϕ 60Hz induction motor takes 48KW input power at 1140 rpm. The stator copper loss is 1.4kW, stator core loss is 1.6kW and rotor mechanical losses are 1kW. Find the motor efficiency.

As $p = 6$, $f_s = 60Hz$

$P_{stator\ core} = 1.6kW$

$P_{stator\ cu} = 1.4 kW$

$P_{mech\ loss} = 1kW$

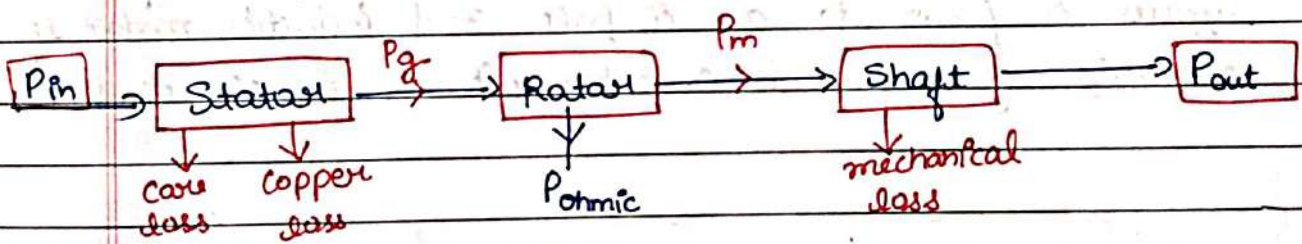
$N_s = 1200$ rpm

$N_r = 1140$ rpm

$$S = \frac{N_s - N_r}{N_s} = \frac{1200 - 1140}{1200} = \boxed{0.05 = S}$$

$$\% \eta = \frac{\text{output power}}{\text{input power}}$$

$P_{input} = 48kW$



$$\% \text{ motor efficiency } (\eta) = \frac{\text{output power}}{\text{input power}} = \frac{41.75}{48} \times 100$$

$$= 86.97\%$$

$P_{in} = P_{stator} + P_g$

$P_g = P_{in} - P_{stator}$

$P_g = 48 - (1.6 + 1.4) \Rightarrow \boxed{P_g = 45kW}$

$$P_m = (1-S)P_g$$

$$P_m = (1-0.05)45$$

$$\boxed{P_m = 42.75\text{ KW}}$$

$$P_m = P_{mech\ loss} + P_{output}$$

$$P_{output} = P_m - P_{mech\ loss}$$

$$P_{output} = 42.75 - 1$$

$$\boxed{P_{output} = 41.75\text{ kw}}$$

Que-3 A 12 pole 3-ϕ alternator driven at a speed of 500 rpm supplies a power to an 8 pole 3-ϕ induction motor if the slip of motor is 0.03 per unit, calculate its speed.

A

Alternator —— 3-ϕ I/M

$P = 12$ $\qquad$ $P = 8$

$N = 500$ rpm $\qquad$ $S = 0.03$

$$N = \frac{120 f_s}{P}$$

$$f_s = \frac{NP}{120}$$

$$f_s = \frac{500 \times 12}{120}$$

$$\boxed{f_s = 50\text{ Hz}}$$

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$$N_r = N_s(1-s) \text{ rpm}$$

$$\downarrow$$

$$N_s = \frac{120 f_s}{P}$$

$$N_s = \frac{120 \times 50}{8}$$

$$\boxed{N_s = 750 \text{ rpm}}$$

$$N_r = 750(1-0.03)$$

$$\boxed{N_r = 727.5} \text{ rpm}$$

Ques-1 Why a 1-ϕ phase starting motor is not self starting Explain with the help of double field revolving theory? Explain 1, 2, 3 methods to self start it

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Single phase (1-ϕ) induction motor :

(i) 1-ϕ induction motor looks similiar to that of a 3-ϕ induction motor.

(ii) Its stator is provided with a single phase winding.

(iii) Rotor construction is identical to that of 3-ϕ induction motor (squirrel cage type)

(iv) Rotor of any single phase induction motor is interchangable with that of a 3-ϕ induction motor.

(v) A simple single phase winding would produce no rotating magnetic field and no starting torque and hence 1-ϕ induction motor is not self starting.

(vi) For making it self start the stator winding is split into two winding

(a) Main winding
(b) Auxilary or starting winding

(vii) 1-ϕ induction motor are built with two phase winding having space displacement of 90° electrical.

(viii) Time phase shift is achieved by connecting resistance, inductance or capitance in series with starting winding and such arrangements in connected in parallel to main winding and then to single phase AC supply

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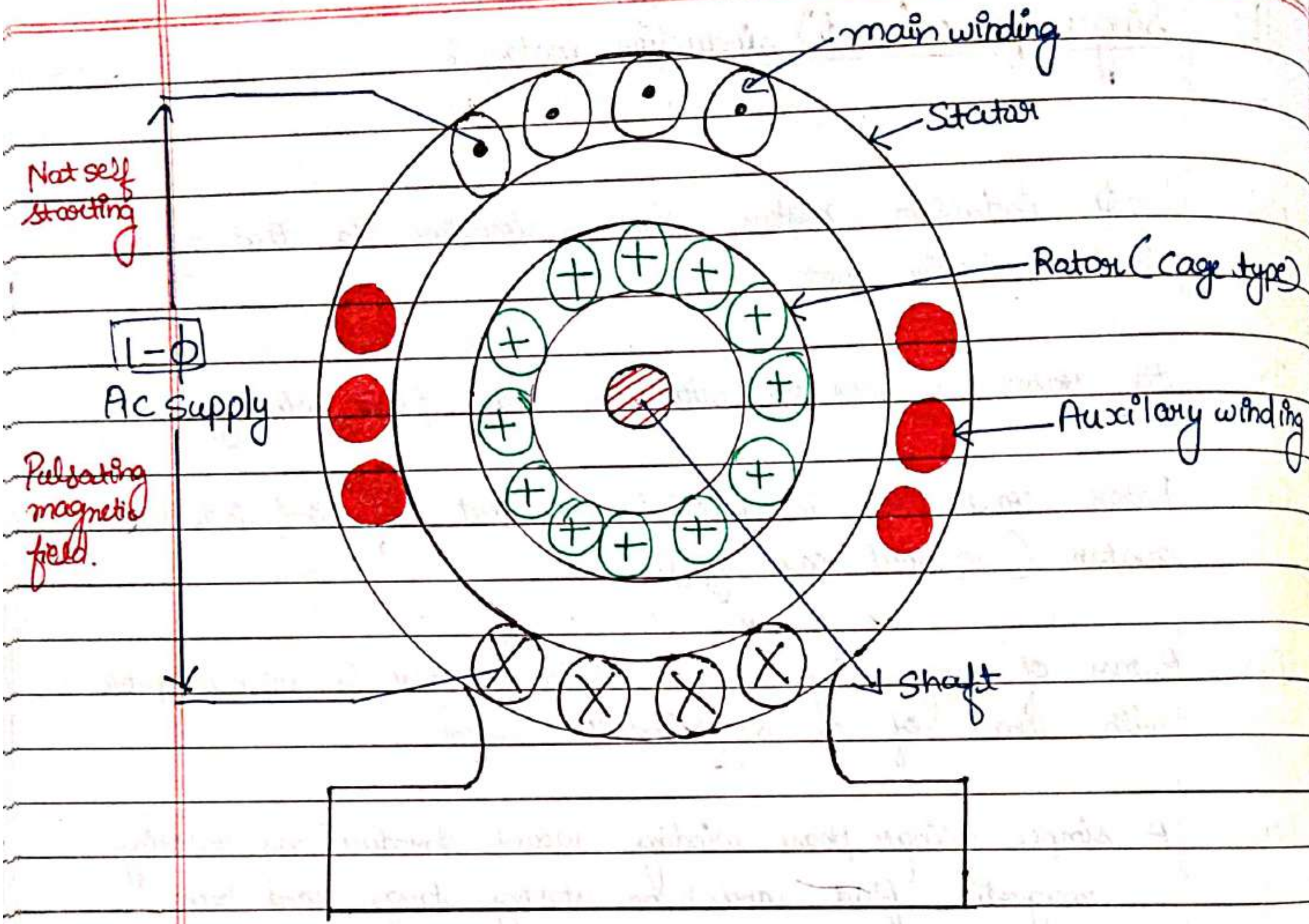
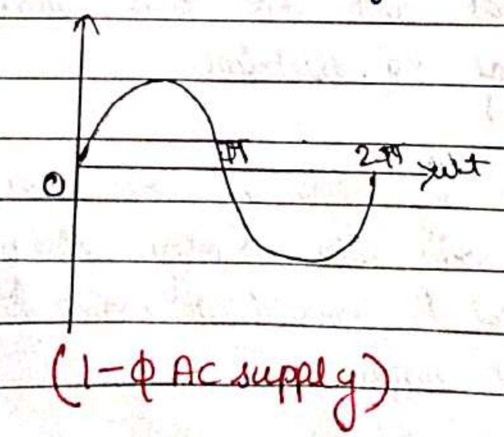


II. Working principle of 1-ϕ induction motors :

(i) When a stator winding is supplied through a single phase

(ii) An alternating (pulsating) magnetic field is produced along one space axis only.



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iii) Due to this pulsating magnetic flux an alternating emf and current is induced in the rotor conductor.

(iv) Now these current carrying conductor experience a force

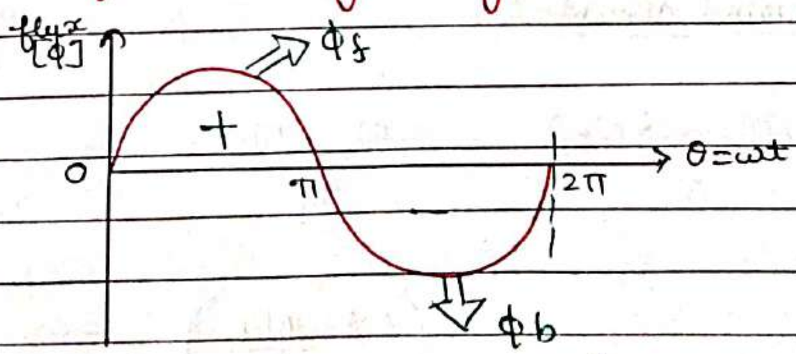
(v) But after each cycle the direction of induced current is changed. Hence the direction of force or torque is changed after each cycle.

(vi) So pulsating flux acting on a stationary squirrel cage rotor cannot produce rotation and therefore 1-ϕ induction motor is not self start.

(vii) However if the rotor of such a machine is given initial start by hand or otherwise in either direction, then immediately a torque arise and a motor accelerates in that direction.

The above behaviour can be explained by
i) double field revolving theory
ii) Cross field theory

Imp. • **Double field revolving theory:**



ϕ_f = flux in forward dir^n.

ϕ_b = flux in backward dir^n.

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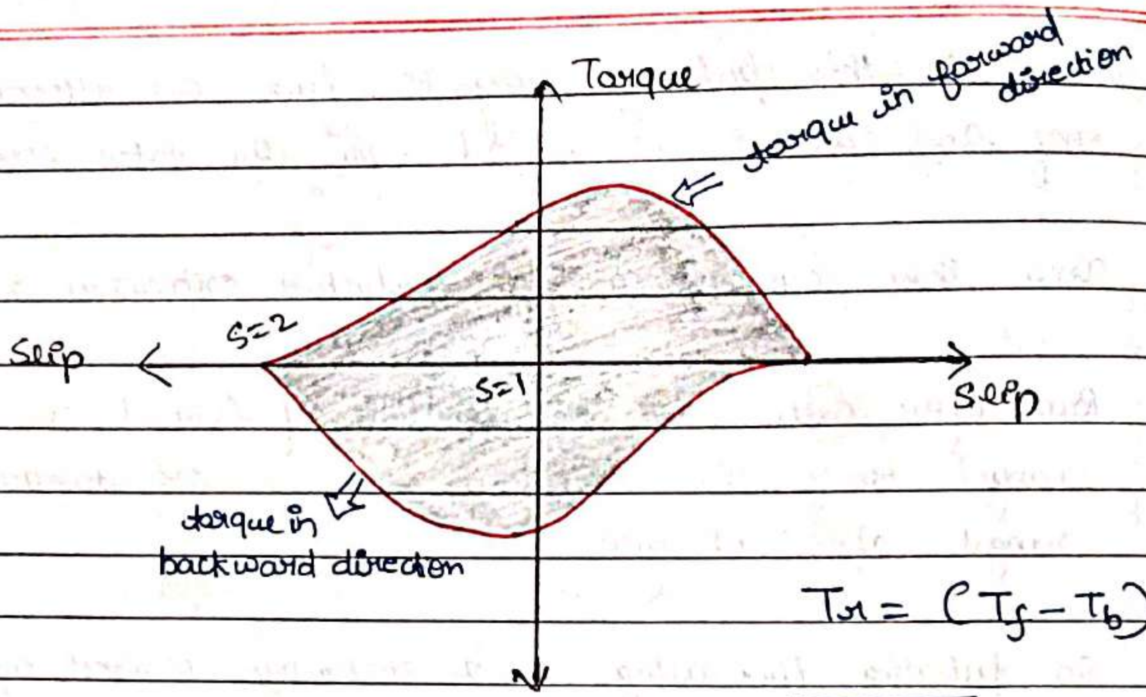

$$T_r = (T_f - T_b)$$

$$\boxed{T_r = 0}$$

where T_f = Torque in forward direction due to forward flux (ϕ_f)

T_b = Torque in backward direction due to backward flux (ϕ_b)

(i) Slip in forward direction:

$\boxed{T_f > T_b}$

$$S_f = \frac{N_s - N_r}{N_s} = \left(1 - \frac{N_r}{N_s}\right) \quad \text{--- (1)}$$

(ii) Slip in backward direction:

$$S_b = \frac{N_s - (-N_r)}{N_s} = \frac{N_s + N_r}{N_s}$$

$\boxed{T_b > T_f}$

$$= \left(1 + \frac{N_r}{N_s}\right) \quad \text{--- (2)}$$

from (i) and (ii)

$$S_f + S_b = 1 - \cancel{\frac{N_r}{N_s}} + 1 + \cancel{\frac{N_r}{N_s}}$$

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$$S_f + S_b = 2$$

$$\boxed{S_b = 2 - S_f}$$

The slip at stand still condition is equal to 1 or unity.

if $S_f = 1$ then $\boxed{S_b = 1}$ (motor is in stand still condn)

"hence a single phase motor is not self starting"

Methods of starting of 1-ϕ phase induction motor :

(i) In a single phase induction motor starting torque is zero because of pulsating single phase magnetic flux.

(ii) To produce starting torque of the motor requires the generation of a rotating magnetic flux. similiar to the rotating flux in a 3-ϕ induction motor.

(iii) Two perpendidular (Electrically) coils having current 90° out of phase can generate the necessarily rotating magnetic field which start the motor.

(iv) Therfore 1-ϕ motors are built with 2-ϕ winding.

The phase shift is achieved by connecting
* resistance
* an inductance or a capicitance.

in series of starting winding.

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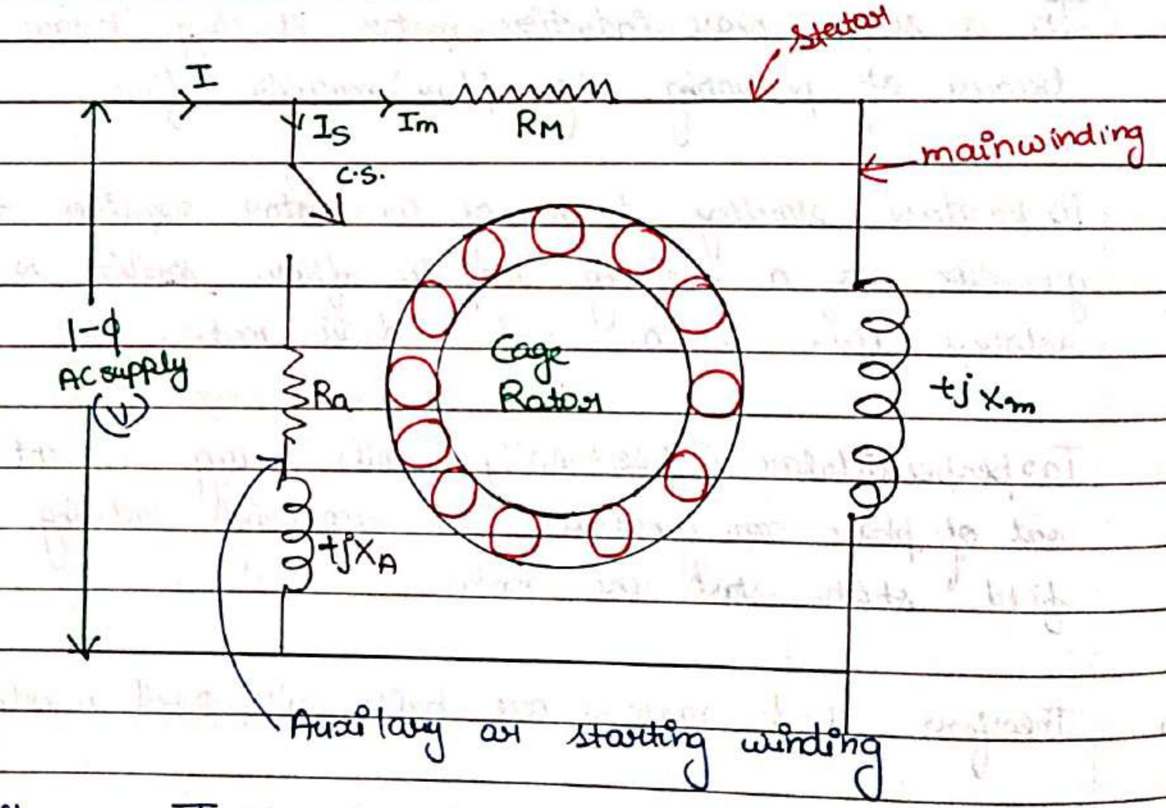
Types of single phase induction motor :

1. phase split method :
   - → Resistance start motor
   - → Capicitor start motor
   - → Capicitor start and capicitor run motor (permanent capicitor start)

2. Shaded pole motor :

3. Reluctance motor

Resistance start motor:



Where, I = supply current

I_m = main winding current

I_s = starting winding current

C_s = Centrifugal switch

Mainwinding → highly inductive and low resistive

starting or auxilary winding → highly resistive and low inductive

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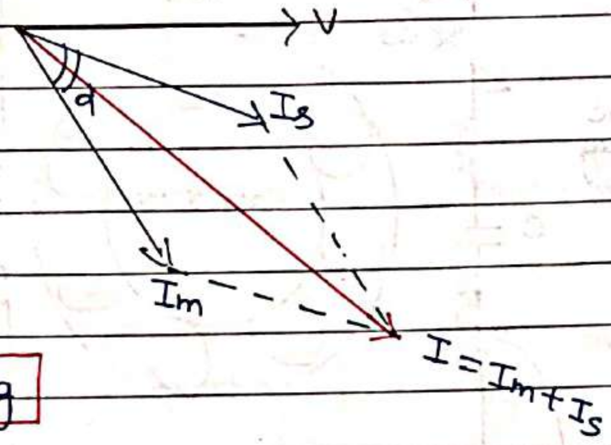
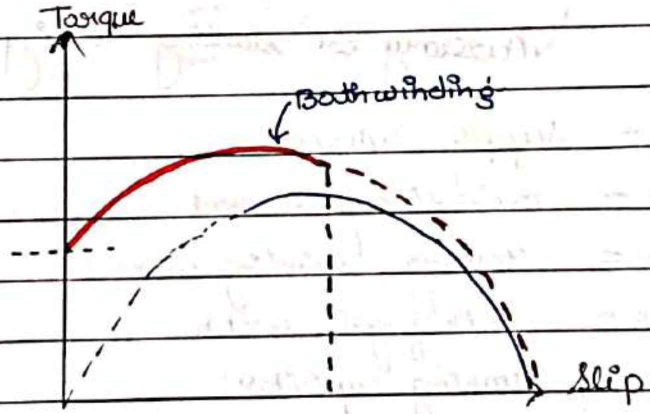
$$\vec{I} = \bar{I}_m + \bar{I}_s$$



$\boxed{T \propto \sin\alpha}$

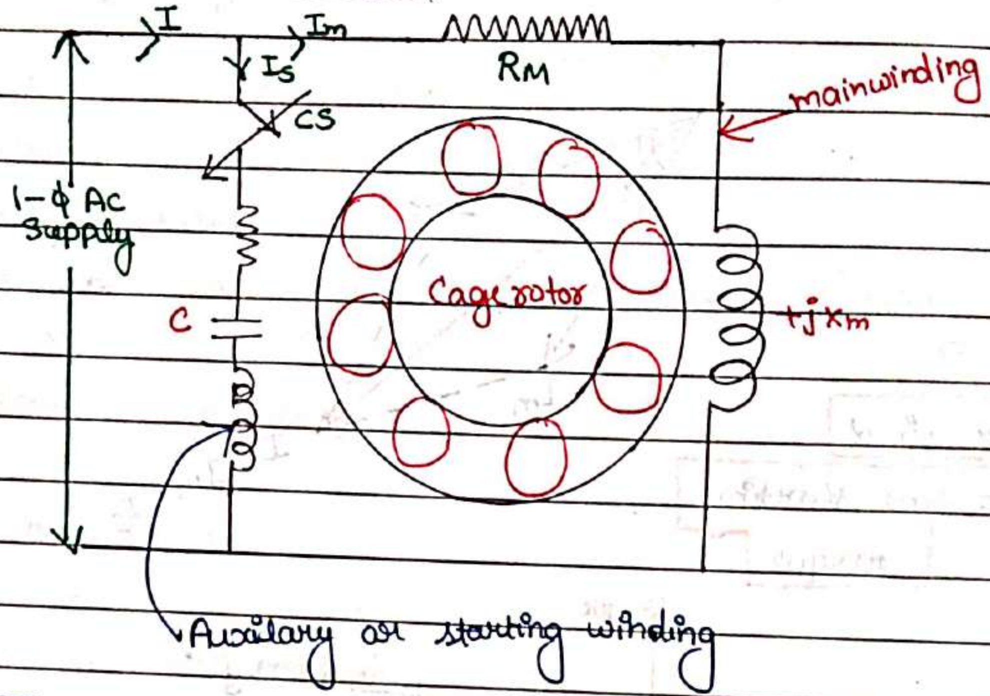
$\boxed{T = \text{low starting torque}}$



Applications of Resistance start motor :

i) Fans
(ii) Washing machines
(iii) Oil burner
(iv) Centrifugal pumps
(v) Small machines

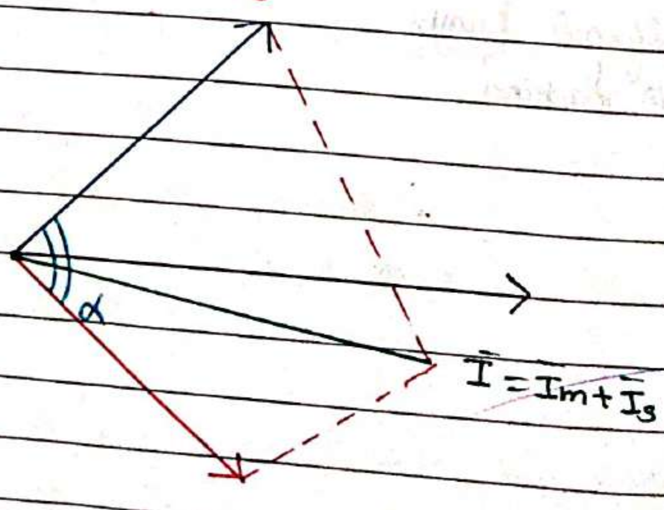
Capicitor start motor :



I = supply current
IM = mainwinding current
Is = starting winding current
Cs = centrifugal switch
C = starting capicitor.

main winding = highly inductive and low resistive
Auxilary winding = highly capicitive and low inductive

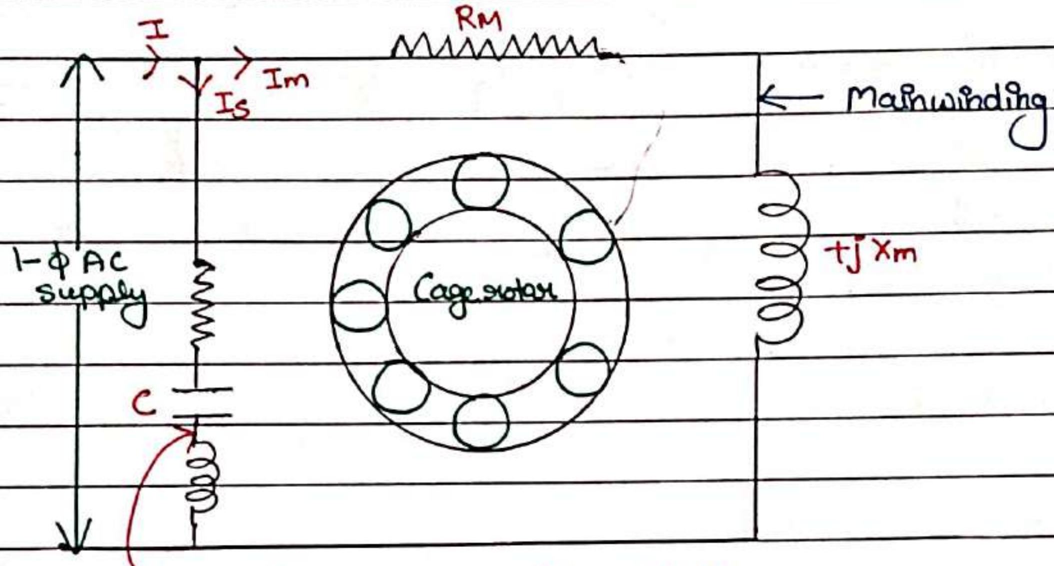
$T \propto \sin \alpha$
T = high starting torque



Capicitor start and capicitor run motor:

or

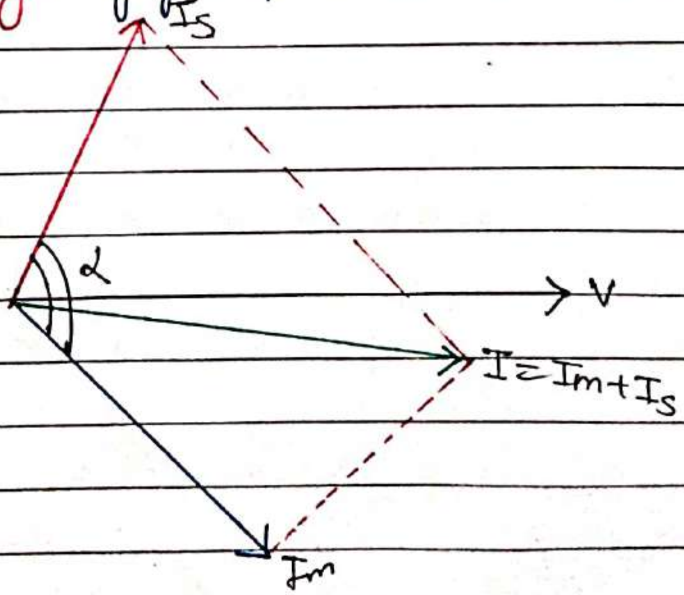
permanent start capicitor motor:



C = starting capicitor
I = supply current
Im = mainwinding current
Is = starting winding current

Main winding : highly inductive and low resistive
Auxilary winding: highly capitive and low inductive

Applications:
Fans, AC.



Q. Explain the working principle of 3-ϕ synchronous motor. Write two applications of it.

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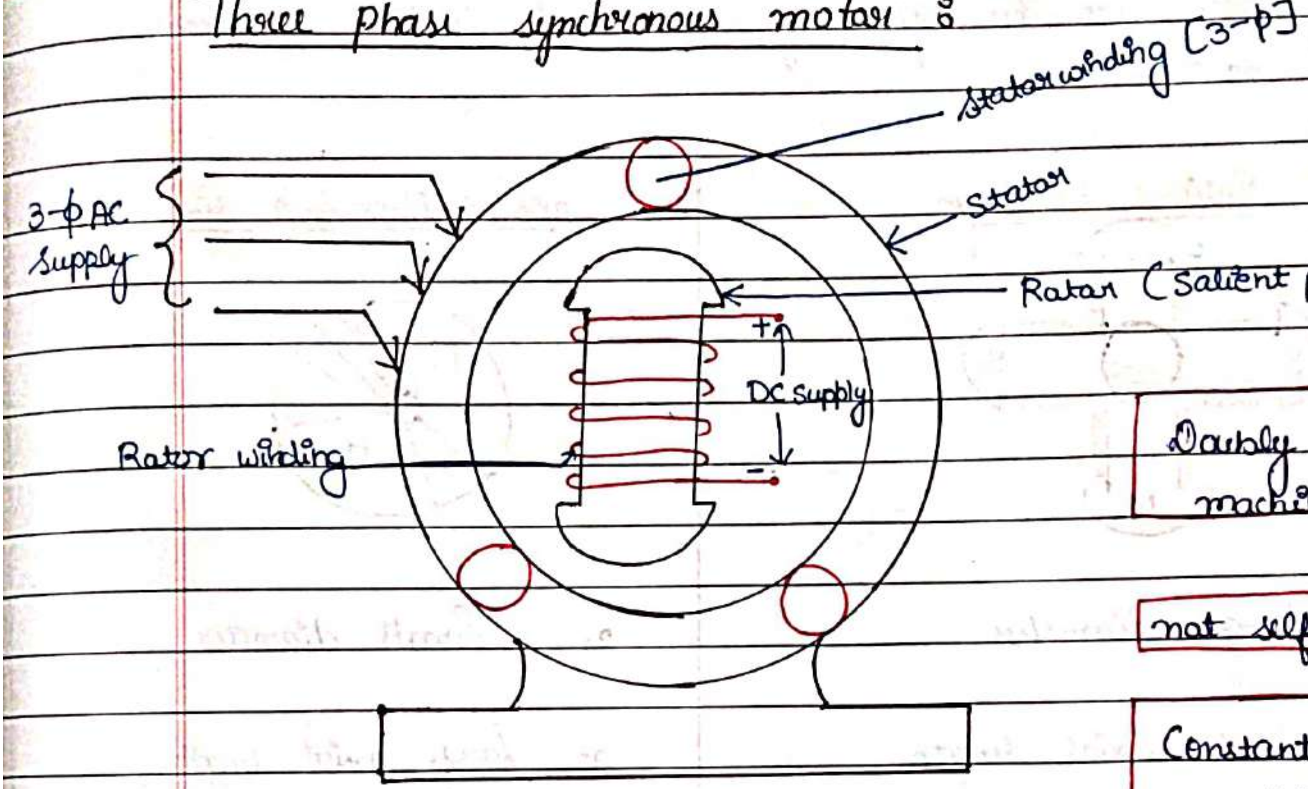
Three phase synchronous motor :



Doubly excited machine

not self starting

Constant speed machine

Construction of a 3-ϕ synchronous motor :

(i) Stator : • Stationary part of the machine

• Three phase winding is placed in the stator. (armature winding)

• For small size, cast iron is used for manufacturing

• For large size ; high grade steel.

(ii) Rotor : Rotating part of the machine.

• Field winding is placed on rotor.

• Feild winding is excited by DC supply

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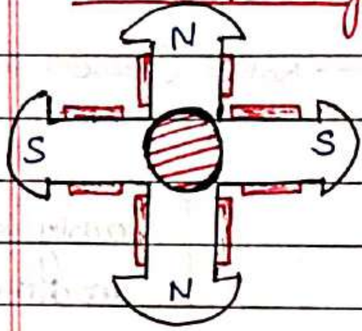
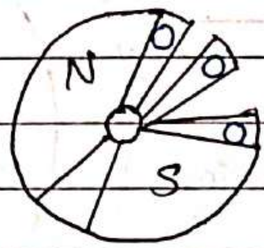
There are two types of rotors used in 3-ϕ synchronous motor :

SN.No	Salient pole type	Non-salient pole type
1.		1. 
2.	Large diameter	2. Small diameter
3.	Small axial length	3. Large axial length
4.	more number of poles	4. less number of poles
5.	Less speed [750 rpm]	5. High speed [750 - 3000] rpm
6.	Hydropower plant (used)	6. Thermal power plant (used)

Working principle of three phase synchronous motor :

Synchronous motor works on the principle of magnetic locking.

where N_s and S_s is magnetic poles of stator
Rotating in nature

N_R and S_R is magnetic poles of rotor
stationary in nature

Hence a 3-ϕ synchronous motor is not self started.

self excited

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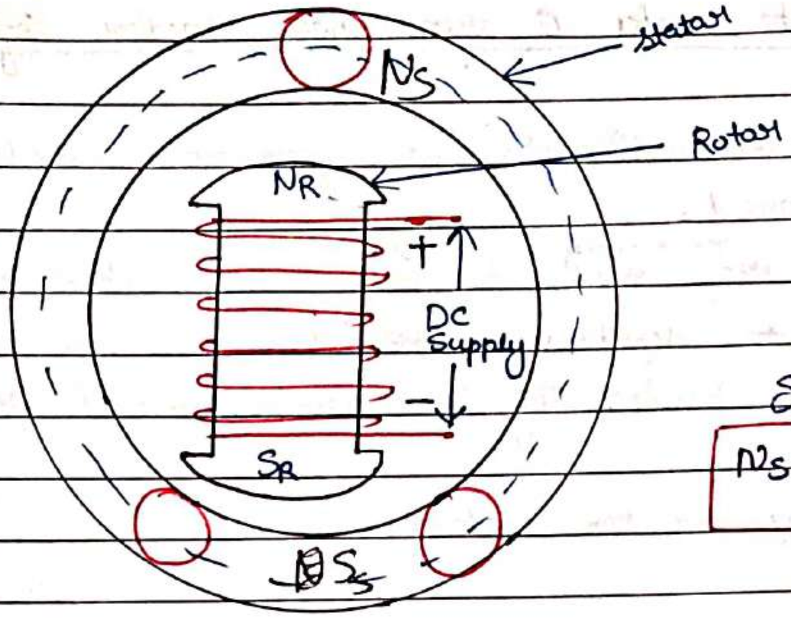
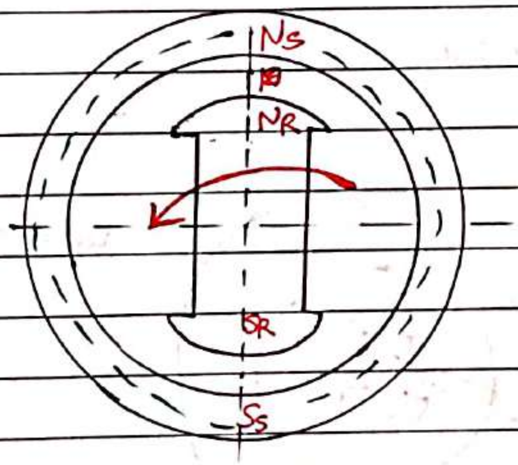
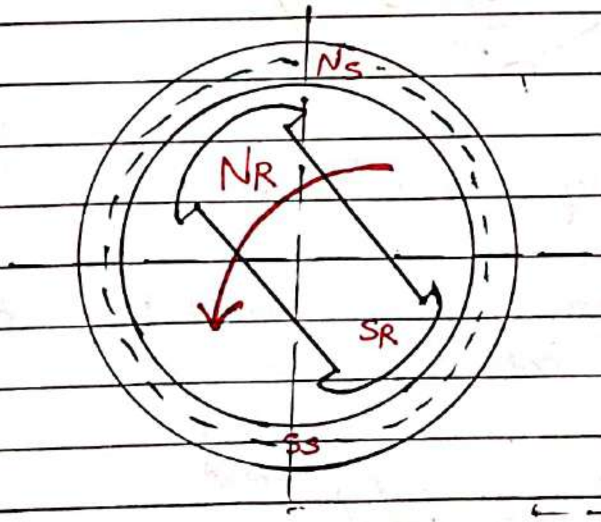
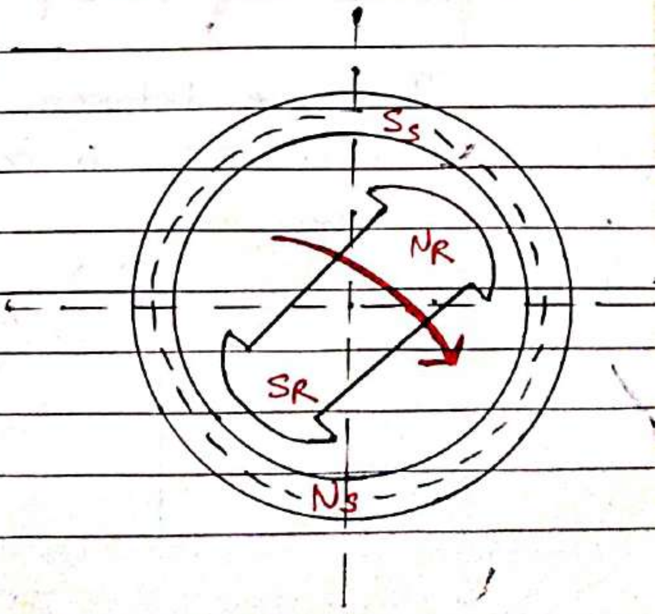


Synchronous speed

$$\boxed{N_S = \frac{120 f_S}{P}} \text{ rpm}$$







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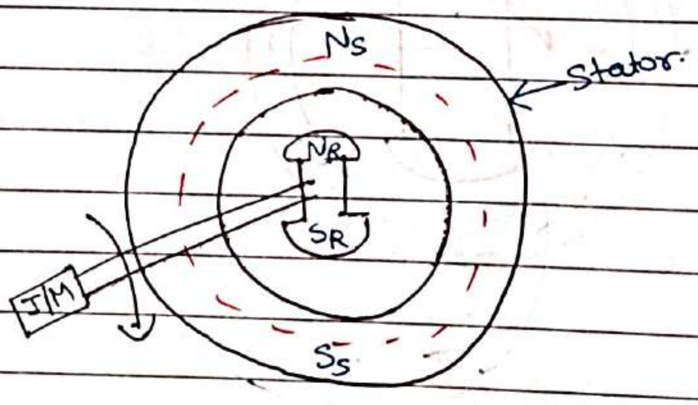
How to make a synchronous motor self starting?

From above discussion a synchronous motor is not self started.

It is ~~very~~ necessary ~~easy~~ to ~~run~~ rotate the rotor at a speed very near to synchronous speed (N_S)

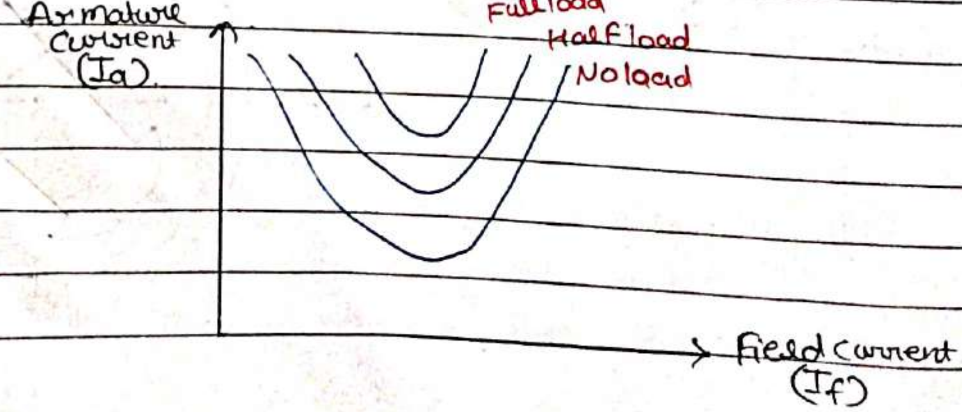
This is possible by the various methods in practice.

* By using induction motor
* By using DC motors
* By using Damper winding.



V curve of a synchronous motor:

The curve between field current (I_f) and armature current (I_a) is called V-curve.



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Applications of Synchronous motor :

(i) Synchronous motors are mainly used in constant speed applications.

(ii) Synchronous motors are used to improve the power factor and voltage regulation of transmission line.

* Cement mills
* Rolling mills
* Textile mills
* Paper mills
* Blowers

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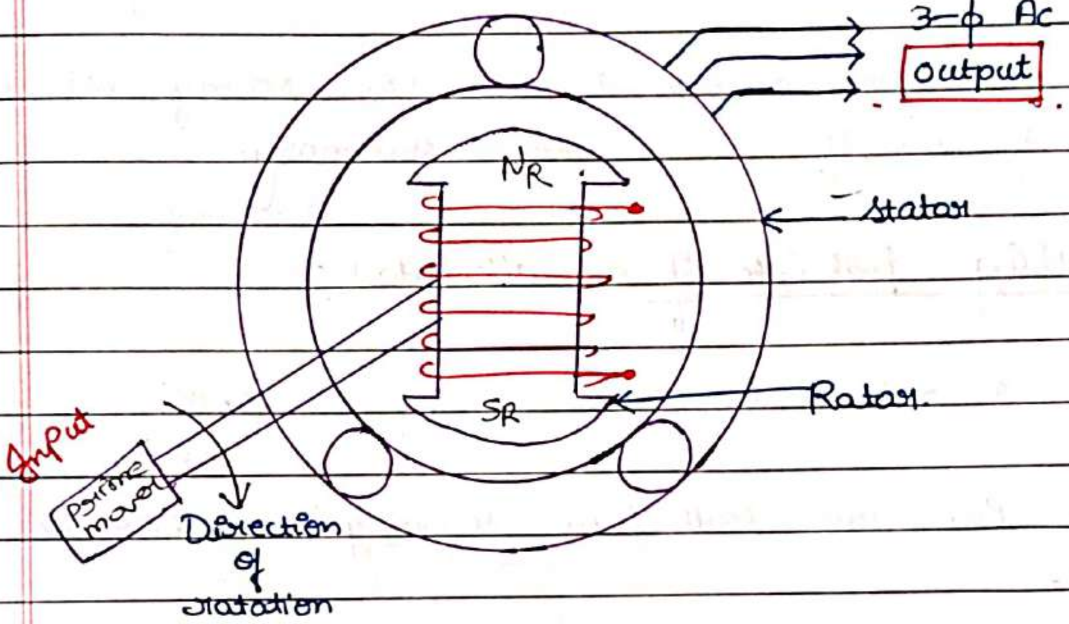
Three Phase Synchronous AC generator :
OR
Alternator.



Mechanical energy = Electrical energy

Field winding :

- To produce magnetic flux
- Rotating in nature

Armature winding :

- in which working emf is induced.
- Stationary in nature.

Advantages of rotating magnetic field

(i) It is always better to protect high voltage winding from centrifugal forces caused due to rotation .. so high voltage armature is generally keep stationary

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(ii) The problem of sparking at the slip rings can be avoided by keeping field rotating and armature stationary

(iii) Rotating field make overall construction very simple

(iv) The ventilation arrangment for high voltage side can be improved if it is kept stationary

Working principle of an alternator :

1. Let a rotor is excited by a DC supply.

2. North Poles and south poles (stationary) produced in the rotor.

3. Rotor is rotated by some prime mover then rotor cause will move means that north pole and south pole will rotate.

4. As the rotor rotates the flux wave form sweeps the coil sides.

5. By faraday's law a voltage is induced in the stator coil. Direction of induced emf can be found by Fleming's Right hand rule and frequency of induced emf is given by

$$f_s = \left(\frac{N_s P}{120}\right) Hz.$$

6. The stator and rotor magnetic field must be stationary wrt to each other so the stator should have a same

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number of poles as rotor.

Expression for frequency of an alternator :

$$\text{frequency} = \left[\frac{\text{cycle}}{\text{sec}}\right]$$

$$\text{frequency} = \left(\frac{\text{cycle}}{\text{revolution}}\right) \times \left(\frac{\text{revolution}}{\text{sec}}\right)$$

$$\text{frequency} = \frac{P}{2} \times \frac{N}{60}$$

$$\boxed{\text{frequency} = \frac{PN}{120}}\ \text{Hz}$$

UNIT - 5

Electrical Installations

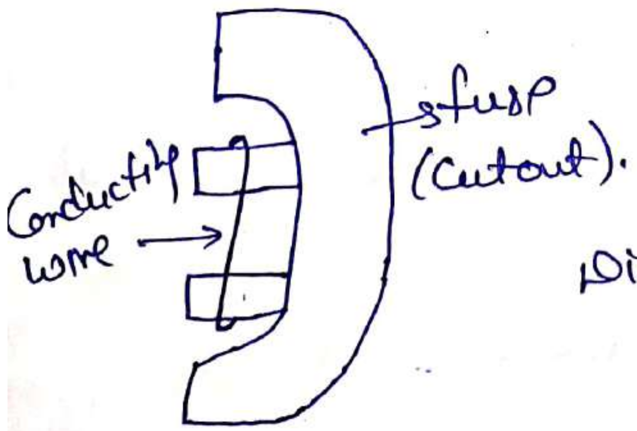
Syllabus :-

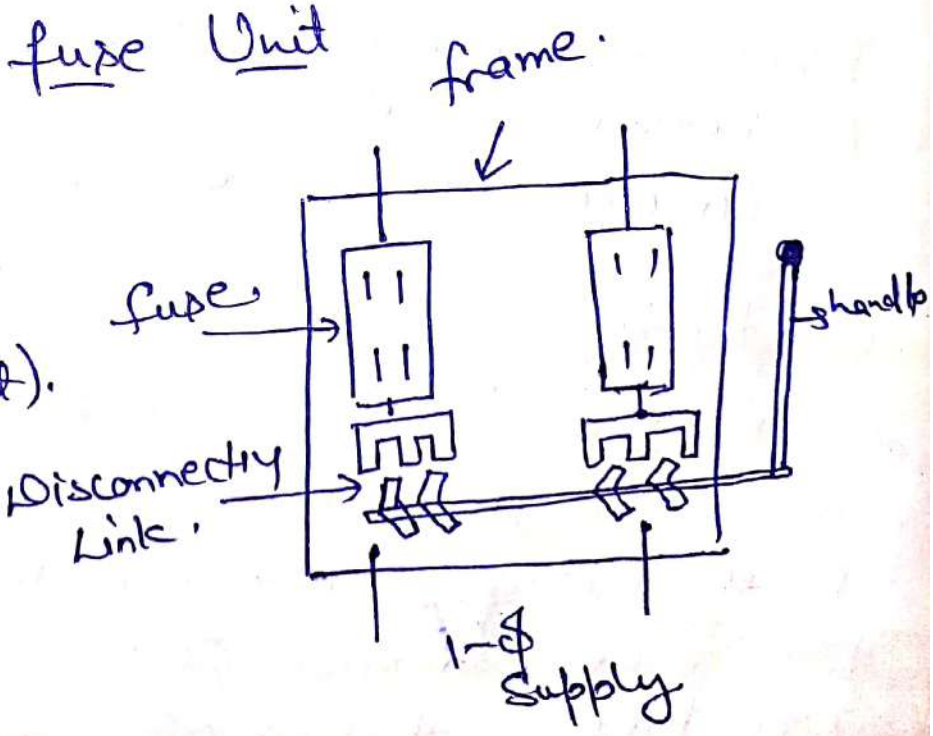
Components of LT Switchgear :-

1.) SFU (Switch fuse Unit)
2.) MCB (~~Minal~~ Miniature Circuit Breaker)
3.) ELCB(Earth Leakage Circuit Breaker)
4.) MCCB (Moulded Case Circuit Breaker)

Types of Wires of Cables
Importance of Earthing.
Types of Batteries.
Important Characteristics of Battery.
Elementary Calculation of Energy consumption
Battery Backup

SFU :- Switch fuse Unit





SFU is to be provided immediately after the meter board

It has one switch unit and one fuse unit. When we operate the Breaker, the Contacts will get close through switch and then the supply will pass through the fuse unit to the output

Whereas in SFU there is no seperate switch and fuse unit. There is only fuse unit which act itself as a switch. When we operate the fuse unit will close the input and output of the breaker.

S-FU has been used to trip the circuit, particularly for high Current Capacity tripping

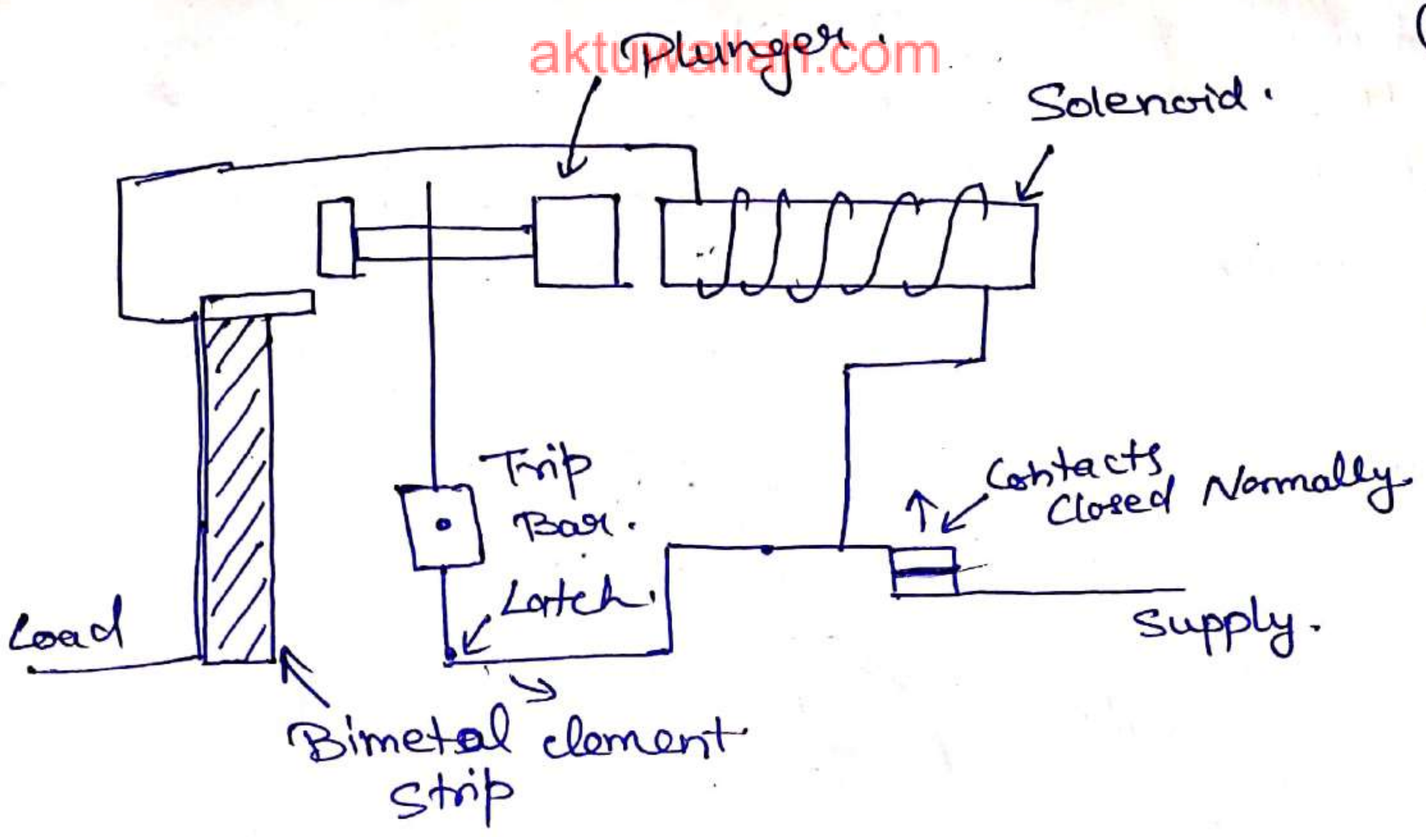
MCB :- Miniature Circuit Breaker

MCB are mechanically operated switch cum. Electro-mechanically operated automatic circuit protection devices. They are used to interrupt a Circuit during overload and short Circuits.

Construction :-

1.) External Casing ⇒ It holds all the internal Components firms and protects them from dust It is made of insulating materials such as plastic or ceramics etc.

2.) Contacts :- A pair of Contact can be found inside MCB. One of them is fixed and



one of them is moveable,

3.) Knob ⇒ MCB can be turned on and off using this knob

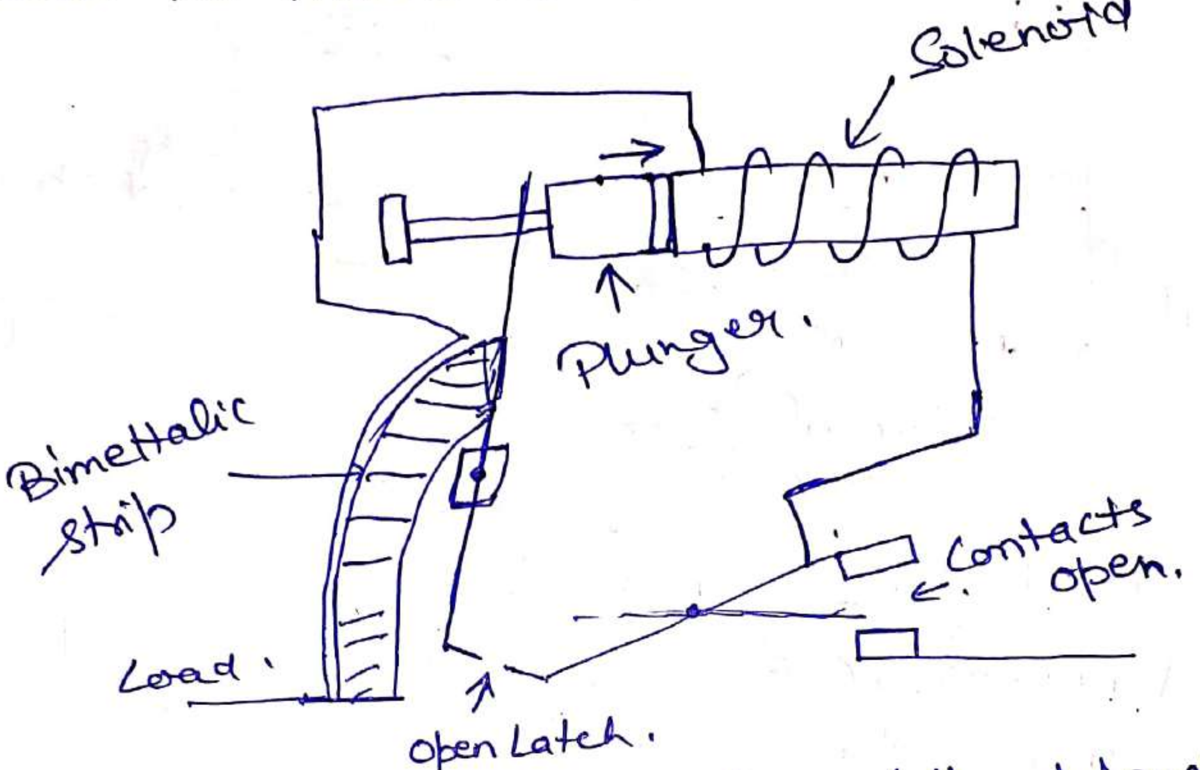
4.) Latch :- A Latch arrangement is made inside MCB to holds the contact under spring tension at ON position.

5.) Bimetallic Strip :- Bimetallic strip offers delayed overload protection by sensing prolonged flow of current greater than its rated current.

6.) Solenoid :- Solenoid offers instantaneous protection against short circuit by releasing the mechanical latch.

Working principle :- In case of overload A current more than the rated current driven through the MCB. As the current

flows through the Bimetallic strip. it gets heated up and deflects by bending and releases the mechanical latch.



Deflection time of Bimetallic strip depends on the amount of current flowing through the strip. Higher the current faster will be the deflection of bimetallic ~~str~~ strip.

During short circuit, a transient current flowing through the solenoid forces the plunger towards the latch. This action instantaneously release the mechanical latch and opens the contact immidiately.

During short circuit and over current, MCB can isolate the equipment from the supply.

Application :- It is used in the protection

of light, refrigerator, AC and Submersible etc.

Advantages of MCB over fuse

1.) It can act faster than fuse during short circuit
2.) MCB can offer better overload protection than fuse.
3.) MCB can be reset after the clearence of fault. But fuse need to be rewired or replaced.
4.) Knob makes operation of MCB much easier than fuse.

MCCB :- Moulded Case Circuit Breaker

It is a electromechanical device, which protect a circuit from overcurrent and short circuit.

They provide overcurrent and short circuit protection for circuit ranging from 63 Amp upto 3000 Amp.

Their primary function are to provide a means to manually open a circuit and automatically open a circuit under over load and short circuit condition. The overcurrent, in an electrical circuit, may result from short circuit, overload or faulty design.

MCCB is an alternative to a fuse since it does not require replacement once an overload is detected. Unlike fuse, an MCCB can be easily reset after a fault and offers improved operational safety and convenience without incurring operating cost.

Charateristics of MCCB

1.) The range of rated current is upto 1000 Amp.

2.) Trip current may be adjusted.

3.) Thermal Magnetic operation

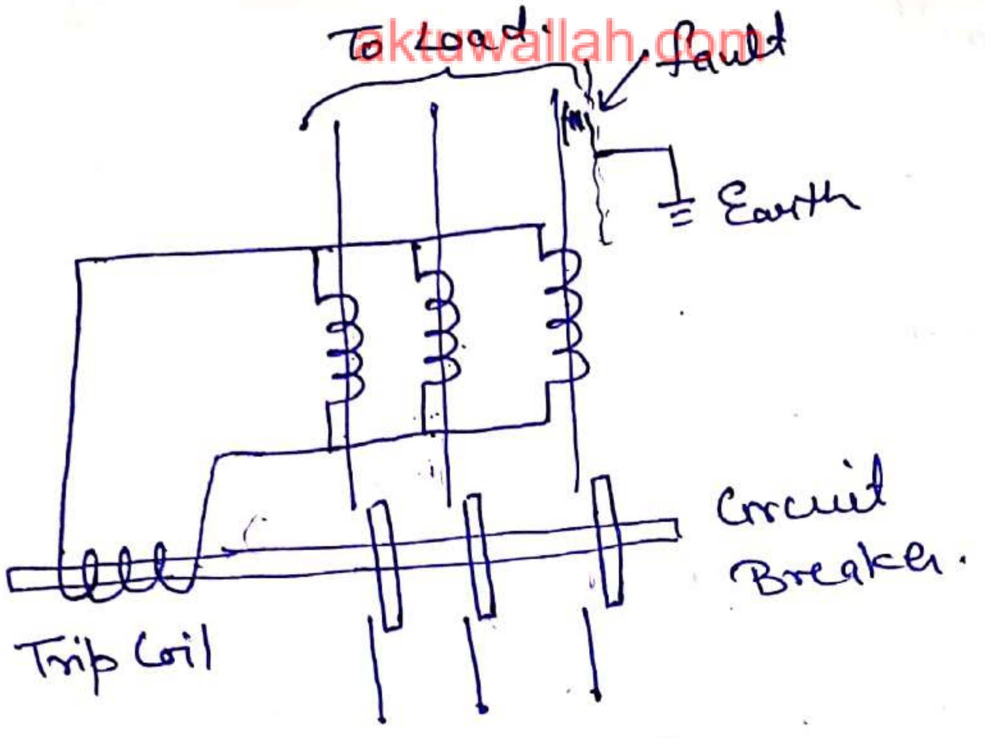
ELCB :- Earth Leakage Circuit Breaker

It is a device that provide protection against earth leakage. These are of two types

1.) Current operated Earth Leakage circuit Breaker :- It is also known as RCCB (Residual Current Circuit Breaker.

:> It is used when the product of the operating current in amp and the Earth loop impedance in ohm does not exceed 40V. where such a circuit Breaker is used, the consumer earthing terminal is connected to suitable earth electrode

A current operated earth leakage is applied to a 3-Φ 3 wire circuit as shown in



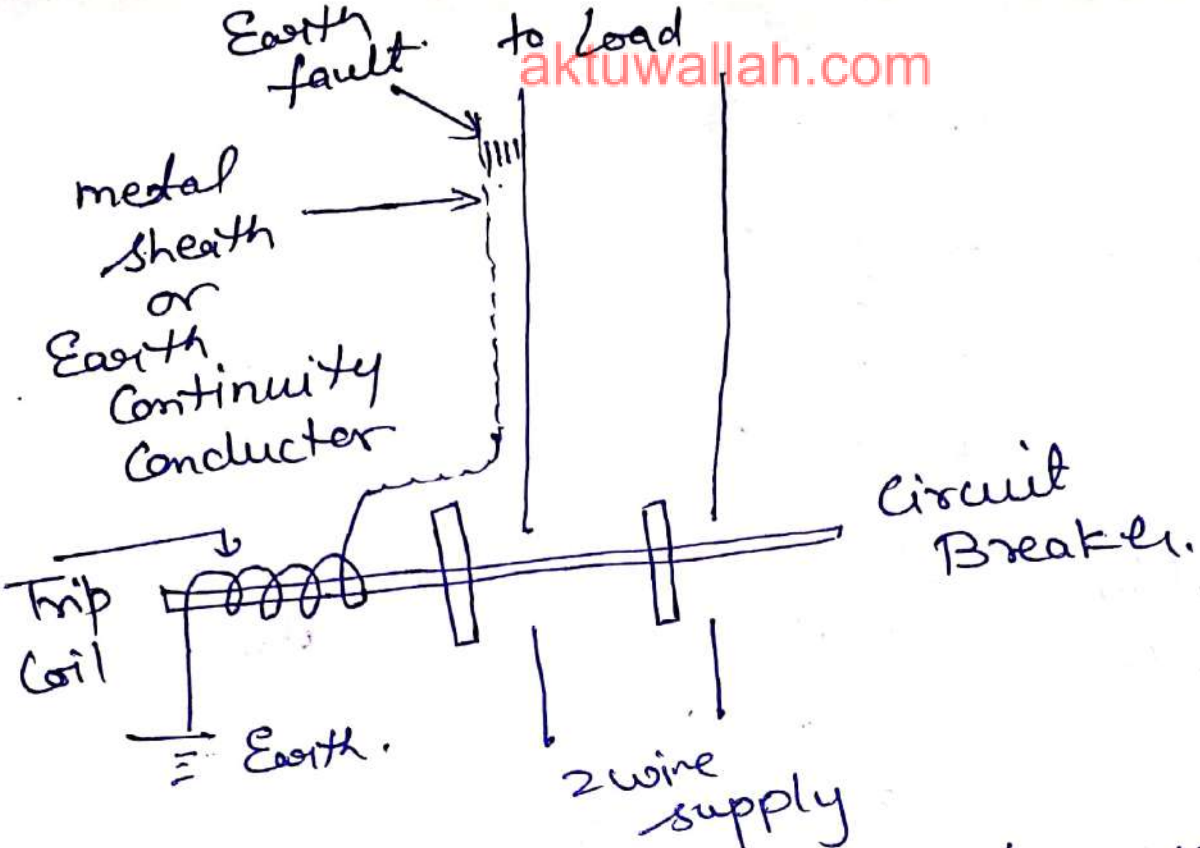
3-ϕ Supply.

Note

Trip Coil is working as a Relay.
Relay is a device that is used to sense the fault and give signal to Circuit Breaker and Circuit Breaker Break the Circuit

fig. In Normal Condition where there is no earth leakage the algebraic sum of the Currents in the three coils of the current transformers is zero. and no current flows through the trip coil. In case of any earth leakage, the current are unbalanced and the trip coil is energised and thus the Circuit Breaker is tripped.

2.) Voltage operated Earth leakage Circuit Breaker :- It is suitable for use when the Earth Loop impedence exceeds the values applicable to fuses or excess current Circuit



Breaker or to Current operated earth leakage Circuit Breaker. Such the earth leakage trip in a 2 wire circuit as shown in fig. When the voltage between the earth Continuity Conductor (ECC) and the earth electrode rises to a sufficient value, the trip coil will carry the required current to trip the Circuit Breaker. With such a Circuit Breaker the earthing leads between the trip coil and the earth electrode must be insulated; In Addition, the earth electrode must be placed outside the resistance area of any other parallel earth which may exist.

<u>Types of wires and Cables</u> :- The wires employed for internal wiring of building may be devided into different groups

1.) Conductor used.

2.) No. of Cores used.
3.) Voltage Grading.
4.) Types of Insulation used.

Conductor Used :- According to Conductor material used in cables, these may be devided into two classes known as copper conductor Cables and Aluminium Conductor cables.

No. of Cores used :- According to No. of Cores, the Cable consist of, the cables may be divided into classes known as.

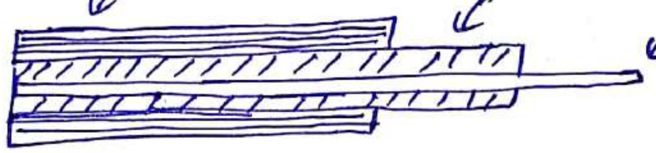
Single Core Cables
Twin Core cables
Three Core Cables
Two Core with ECC (Earth Continuity Conductor)
etc.

Voltage Grading :- According to Voltage Grading the cables may be devided into two classes
1.) 250/440 volt Cables
2.) 650/1100 volt Cables

Types of Insulation Used ⇒ These insulating cables are of following types
1.) Vulcanized Indian Rubber (VIR)

2.) Tough Rubber sheathed (TRS) or Cab tyre sheathed (CTS) cables

3.) Lead Sheathed Cables
4.) Polyvinyl Chloride (PVC) cables
5) Weatherproof Cables.
6.) flexible Cord and Cables
7.) Multi- strand Cables.
8.) XLPE Cables.



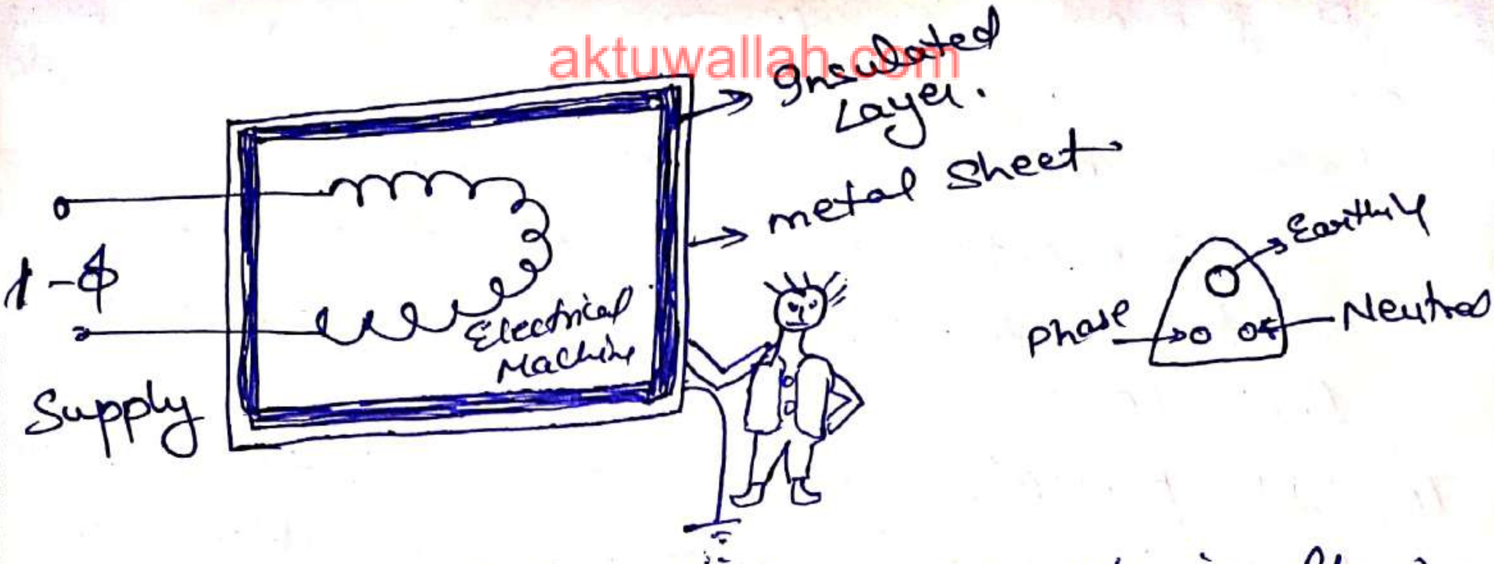
Importance of Earthing ⇒

Earthing ÷ Earthing or Grounding is the process of transferring the immidiate discharge of electricity directly to the earth plate, by means of low resistance electrical cables of wires.

The earthing is done by connecting the Non- current carrying part of the equipment or neutral of supply system to the ground.

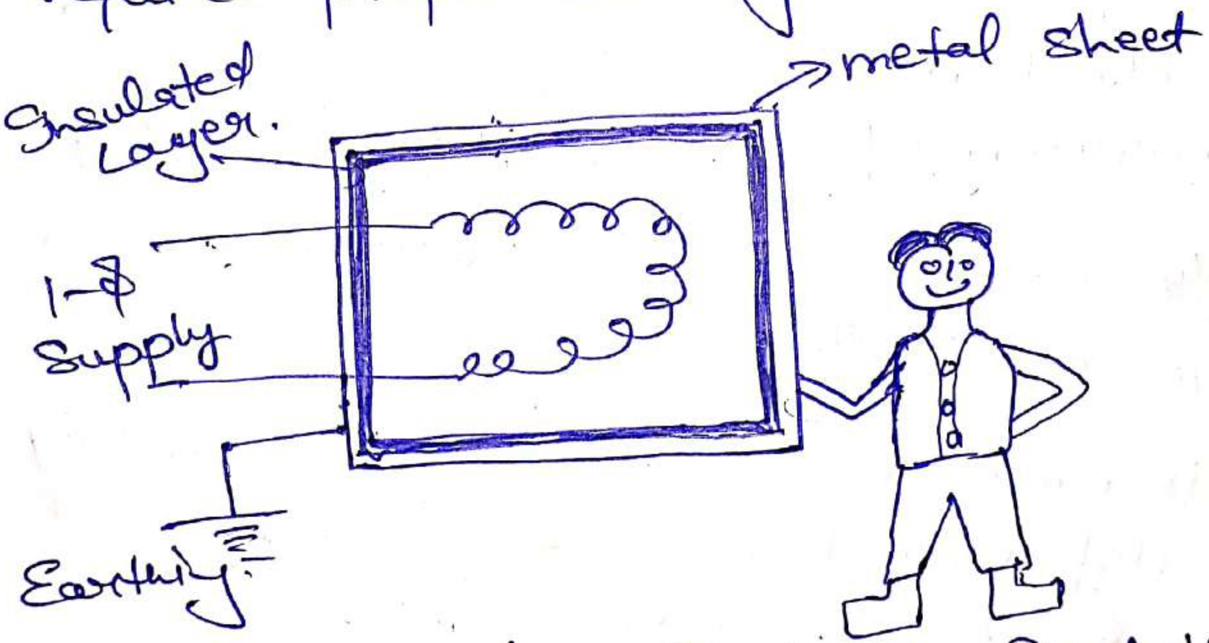
Necessity of Earthing ÷

1.) It keeps people safe by preventing electric shocks.



When Insulation failure current is flowing from metal sheet to earth via human body If no-earthing present. Human body feel Shock or blast in the body.

To protect the human body from current we required proper earthing



If proper Earthing present, due to Insulation failure, total Current passed through Earth via earthing cable. No Current pass through human body. because Current follows short Circuit path.

2.) It prevents demage to electrical appliances and devices by preventing excessive current from running through the circuit

3.) The prevents the risk of fire that Could otherwise be caused by current leakage

4.) It provide the easiest path to the flow of short circuit current even after the failure of the insulation.

Types of Batteries ⇒.

Battery ⇒ A battery is a source of electrical energy, which is provided by one or more electrochemical cells of the battery after conversion of stored chemical energy

Types of Battery :- There are two type of Battery.

1.) Primary Battery ⇒ A Primary battery is a disposable kind of Battery. Once used, it cannot be recharged.

Primary Battery are of five types.

a.) Alkaline Battery
b.) Zinc-Carbon Battery
c.) Mercury Battery
d.) Zinc-Chloride Battery
e.) Silver Oxide Battery

a) Alkaline Battery :- It is non-rechargeable high energy density, batteries that have a long life span. It has 1.5V output
It consists of a Zinc anode and manga-nese dioxide cathode in an alkaline electrolyte (potassium hydroxide).
It works with high efficiency even with contineous use, due to Low internal resistancy
Application :- Remote Control, Clocks, radios etc
b) Zinc Carbon Battery :- Zinc Carbon Battery are also known as dry cell (as the nature of electrolyte used in these cells is dry), which comes in a combination of a carbon rod (cathode) surrounded by a mixture of carbon powder and manganese dioxide.
This whole combination is packed in a Zinc Container acting as an anode. This electrolyte is a mixture of ammonium chloride and zinc chloride.
The typical voltage value is a little less than 1.5V. These battery are dureable and have long life
Applications :- flash light, Projects, Toys etc.
c) Mercury Battery :- It Contain murcuric oxide with maganese dioxide. They are deep discharge battery. and Voltage

level does not fall below 1.35V until 5% energy level is reached.

These battery are less popular because of low output voltage furthermore, mercury is toxic and can cause hazards for humans.

Applications :- Calculators, electronic devices etc

d.) Zinc-Chloride Battery :- This cell is also referred to as a heavy duty type battery. It is modified Zinc- carbon battery. It has little ~~chang~~ chance of liquid leakage because the cell consumes water along with the chemically active materials. This cell is usually dry at the end of its useful life

Applications :- Torch, Gas Geyser, Radio etc

e.) Silver-Oxide Battery :-> This Battery consists of a zinc anode, silver oxide cathode, and potassium or sodium hydroxide electrolyte. It is typically available as 1.5V, miniature button form

Applications :- Wrist watch, Laser Light Hearing machies etc.

2.) Secondary Battery :- These are rechargeable Batteries. Once used, it can be recharged again.

There are two types

a.) Lead - Acid Battery.
b.) Lithium - ion Battery

a.) Lead Acid Battery :- These batteries are rechargeable kind of batteries. These large, heavy weight batteries find the major applications in automobiles as these fulfill the high current requirements of the heavy motors at start

The Composition of Lead acid battery changes in charged and discharged state.

A Combination of Pb (negative) and PbO_2 (Positive) as electrode and H_2SO_4 as electrolyte in charged form and $PbSO_4$ and water in discharged form.

$$\underset{\text{Positive Plate}}{PbO_2} + 2H_2SO_4 + \underset{\text{Negative Plate}}{Pb} \rightleftharpoons \underset{\text{Positive Plate}}{PbSO_4} + 2H_2O + \underset{\text{Negative Plate}}{PbSO_4} + \text{Electrical Energy}$$

Applications :- Automobiles, used as power backup supply (Inverter), UPS etc.

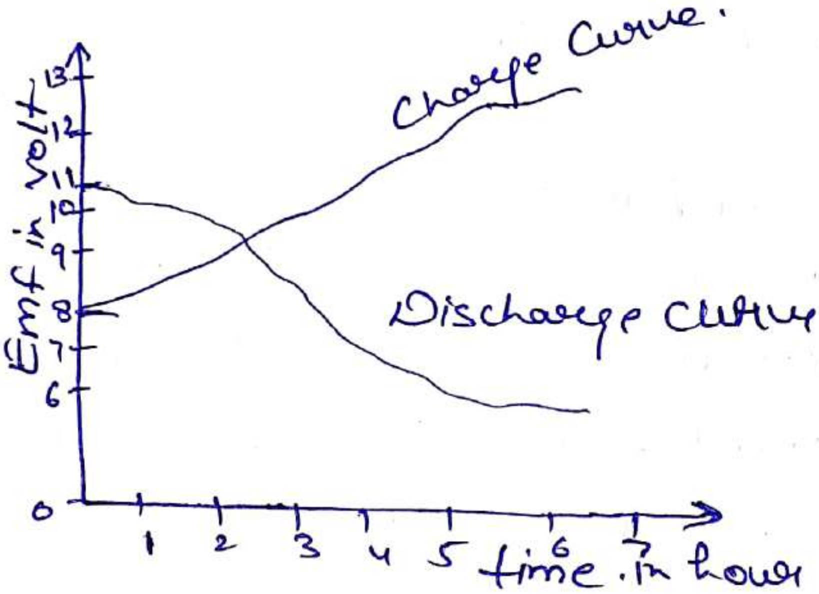
b.) Lithium - ion Battery :- These batteries are recharge batteries, where lithium in its pure ion compound form is used.

Depending on the design and chemical Compounds used, lithium batteries can produce voltage from 1.5V to 3.7V

The most common type of lithium battery used in consumer applications uses mangnese dioxide as cathode and metallic lithium as anode.

Applications :- Mobile phones, cameras, Laptop etc.

Charge and Discharge curve of Secondary Battery :-



Important characteristics of Battery :-

1) Amp / Amp-hour :- It is also known as Amp. This is the rate at which electrons flow in a wire. This unit are coulomb per second.

One Amp-hour = Current of one Amp flowing for one hour.

A Unit quantity of electricity used as a measure of the amount of electrical charge that may be obtained from a storage battery before its require recharging b.) Capacity :- The Capacity of a battery is expressed as the total quantity of electricity involved in the electro chemical reaction and is defined in terms of Coulombs or Amp-hour (Ah) or the total No. of Ah or watt hour that can be withdrawn from a fully charged cell or battery, under specified Conditions of discharge, is termed as the capacity of the battery.

c.) Power :- The power generated by a battery can be calculated as

watt = $W = VI$ → current; ↳ voltage

d.) Power density

$$\text{Power density} = \text{Energy}(E)/\text{time}(t)/\text{mass (kg)}$$

$$= \text{Energy}(qV)/\text{time}(t)/\text{mass (kg)}$$

$$= \text{Power}/\text{mass}$$

Its unit is watt/kg

Power density is defined as the power per unit mass.

e.) Energy Density :- The energy density is determined by the voltage of the battery and the amount of charge that can be stored $\boxed{E = qV}$

f) Cycle life for rechargeable batteries the duration of satisfactory performance, measured in years or in the number of charge/discharge cycle. In practice, end of life is usually considered to be reached when the cell or battery delivers approx. 80% of the rated amp-hour capacity.

g) Efficiency :- for rechargeable batteries efficiency are of three types.

1.) Voltage efficiency :- It is described as the ratio of average voltage during discharge to average voltage during charge under specified condition

2.) Watt-hour efficiency :- It is known as the ratio of watt hour delivered on discharge to the watt hour needed to restore it to its original state of a battery under specified condition of charge and discharge

3.) Amp-Hour efficiency :- It is the ratio of the output of a battery, measured in Amp-hour, to the Input required to restore the initial state of charge under specified conditions.

Elementary Calculation of Energy Consumption and Savings :-

1.) Energy Consumption per Day :- To Calculate the daily energy consumption, the difference between the meter readings is devided by the Number of days in the period covered.

Mathematically

$$\text{Energy Consumption (day)} = \frac{\text{Meter Reading (end)} - \text{Meter Reading (start)}}{\text{Days in period}}$$

2.) Energy Consumption per Year :- The daily electricity consumption can be multiplied with 365 days

$$\text{Electricity Consumption (year)} = \text{Energy Consumption (day)} \times 365$$

3.) Energy Saving :- The electricity Consumption in the previous period, extrapolated to 1year, can be calculated by dividing the electricity consumption in the previous period by the No. of days in the period and multiplying with 365 days

The difference between the yearly electricity consumption in the previous period and in the monitoring period is the Annual electricity saving acheived.

$$\boxed{\text{Energy saving} = \text{Energy Consumption (Previous Year)} - \text{Energy Consumption (Monitoring year)}.}$$

Battery Backup :- Time taken to discharge a battery is known as battery backup

Battery Backup time can be computed using this formula.

$$\boxed{\underset{\text{Time in hour}}{\text{Battery Back up}} = \frac{Ah \times \text{Voltage}}{\text{Load}}}$$

when $Ah \rightarrow$ Ampere hour of the battery

$V \rightarrow$ Voltage of the battery

(20) $\text{Battery Back-up} = \frac{V_{battery} \times Ah_{battery}}{Load}$

Two bulb 15 watt each

One fan 100 watt each

Total Load = 2 bulb + 1 fan

$= 15 \times 2 + 100 \times 1$

$= 130$ watt

$V_{battery} = 12V$

Ah rating of battery = 150 Ah.

$\text{Battery Backup} = \frac{12 \times 15\cancel{0}}{13\cancel{0}}$

$= \underline{13.84 \text{ hours}}$ Ans

current period.

(19)

Date	Reading
1 Dec 2016	12300
28 feb 2017	13200

No. of day

90 days 900 unit (kwh)

$\text{Electricity Consumption (day)} = \frac{\text{Total unit}}{\text{No. of days}} = \frac{\overset{10}{\cancel{900}}}{\cancel{90}}$

$= 10$ unit (kwh)

$\boxed{1 \text{ unit} = 1 \text{kwh}}$

Energy Consumption (year) = Energy Consumption (day) × 365 ②

$= 10 \times 365$

$= 3650$ kwh.

Previous period

Total unit Consumption in previous bill = 3960 kwh.

10 Nov 2015 to 5 Nov 2016

No. of days = 360 days

$$\text{Energy Consumption (day)} = \frac{\text{No. of unit}}{\text{No. of days}} = \frac{3960}{360}$$

$= 11$ kwh.

Energy Consumption (year) = Energy Consumption (day) × 365

$= 11 \times 365$

$= \cancel{40}\ \cancel{4105}$ kwh.
4015

Electricity saving = Energy Consumption (previous year) − Energy Consumption (Current year)

$= 4005 - 3650$

$\boxed{\text{Electrical Saving} = \cancel{455\ kwh}}$ 365 unit kwh

(17) Ah rating of battery = 100 Ah

Load = 100 watt

V_{supply} = 12V

$$\text{Battery Backup} = \frac{V_{battery} \times \text{Ah rating battery}}{\text{Load}}$$

$$= \frac{12 \times \cancel{100}}{\cancel{10}\,\cancel{100}}$$

= 12 hour Ans

(16) 5 CFL 20 watt each.

Total Load of CFL = 5×20 = 100 watt.

3 fan of 60 watt each.

Total Load of fan = 60 × 3 = 180 watt

Total Load = Total Load of fan + Total Load of CFL.

= 100 + 180 = 280 watt

Unit of Energy is kWh

Total Energy Consumption (day) = 280 × 3

= 840 wh

= 0.840 kWh.

(15) Current → 4A
time → 12h
Ah rating = 4×12 = 48 Ah.
$V_{battery}$ = 1.2V
} discharge

Watt hour → VIh = 1.2×4×12 = 57.6 watthour.

Current = 3A
time — 20h
Ah rating = 3×20 = 60 Ah.
$V_{battery}$ = 1.44 V
} → charging

Watt hour
↓ Power (VI)
↓ time

watthour = VIh
= 1.44×3×20
= 86.4 watt hour

Watt hour or kilo watthour is the unit of Energy
Energy = P×t = V×I×t