①

NETWORK THEOREMS

Different electric ckts (according to their properties) are:

1. **ckt** :- A ckt is a conducting path through which an electric current either flows or is intended to flow.

2. **Parameters** :- The various elements of an electric ckt. are called its parameters like resistance, inductance & capacitance. These parameters may be lumped or distributed.

3. **Linear ckt** :- A linear ckt. is one whose parameters are constants i.e. they do not change with voltage or current (It comprises independent sources, linear dependent source & linear passive elements like R, L, C, transformers.)

   (voltage & current relationship of a linear element is st. line)

4. **Non-linear ckt** :- A non linear ckt. is one whose parameters change with voltage or current. (See below)

   (Not st. line)

5. **Bilateral ckt** :- A bilateral ckt. is one whose ~~parameters~~ properties or characteristics are the same in either dirn. The usual transmission line is bilateral because it can be made to perform its fun: equally well in either dirn.

   (ch- means voltage-current relationship)
   (Resistor is an example of bilateral ckt. element)

6. **Unilateral ckt** :- It is that ckt. whose properties or characteristics change with the dirn of its operation. A diode (vacuum or semiconductor) rectifier is a unilateral ckt. because it cannot perform rectification in both dirn.

7. **Electric Network** : A combination of various electric elements, connected in any manner whatsoever, is called an electric network.

8. **Passive Network** :- is one which contains no source of e.m.f. in it. (Passive elements → Resistor, inductor, capacitor & transformer)

9. **Active Network** :- is one which contains one or more than one sources of emf. (Active elements → generators, amplifiers, & oscillators.)

10. **Node** :- is a junn in a ckt, where two or more ckt. elements are connected together.

11. **Branch** :- is that part of a network which lies between two junns.

It should be noted that, unless stated otherwise, an electric network would be assumed passive.

A network is said to be completely solved or analyzed when all voltages and all currents in its different elements are determined.

DC PCP

Non linear ckt : (i) When the current through a resistor is so large that the power dissipation changes its temp., the value of the resistance changes. (ii) An iron-core inductor is also non-linear, the permeability of iron depending on the current through the coil. (iii) The capacitance of a p-n junn is an example of a non linear capacitance, the value of c depending on the ~~current th~~ voltage applied across the junn. (iv) Many vacuum and semiconductor devices like triodes, transistors are non linear.

There are two general approaches to network analysis:

① ~~Direct Method~~ Direct Method :- Here, the network is left in its original form ~~which~~ while ~~~~ determining its different voltages and currents. Such methods are usually restricted to fairly simple ckts. and include Kirchhoff's laws, Loop analysis, Nodal analysis, Superposition theorem, Compensation theorem, Reciprocity theorem etc.

② Network Reduction Method :- Here, the original network is converted into a much simpler equivalent ckt. for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are: delta / Star and Star/delta conversions, Thevenin's theorem and Norton's theorem etc.

Kirchhoff's laws: These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff's laws, two in numbers, are particularly usefull (a) in determining the equivalent resistance of a complicated network of conductors and (b) for calculating the currents flowing in the various conductors.

(I) Kirchhoff's Current law (KCL) or Kirchhoff's Point Law.

It states as follows:

In any electrical network, the algebraic sum of the currents meeting at a point or jun. must be zero. It simply means that total current leaving a jun. equal to the total current entering that jun.

In other words $\Sigma I = 0$ .. at a junction

In this case incoming currents = outgoing currents.

Assuming the incoming currents to be a +ve & the outgoing currents -ve, we have $I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0 \Rightarrow I_1 + I_4 - I_2 - I_3 - I_5 = 0$

$\Rightarrow I_1 + I_4 = I_2 + I_3 + I_5$

$\boxed{I.C = O.C}$

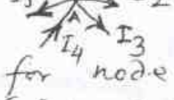


fig-a

Similarly, in fig-(b) for node-A

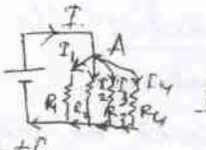


fig-b

$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \Rightarrow I = I_1 + I_2 + I_3 + I_4$

$\boxed{\text{incoming current} = \text{outgoing current}}$

(II) Kirchhoff's Mesh Law or Voltage law (KVL): It states as follows: the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emf's in that path is zero.

In other words $\Sigma IR + \Sigma e.m.f = 0$ .. round a mesh

The basis of this law: If we start from a particular junction and go round the mesh till we come back to the starting pt. then we must be at the same potential with which we started.

V_s — voltage source; current source

Determination of sign:

(a) Sign of Battery emf. A rise in voltage should be given a +ve sign and a fall in voltage a −ve sign.

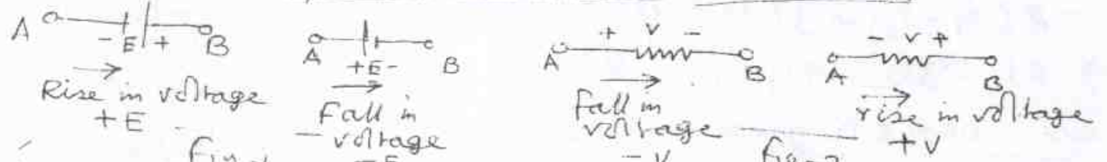


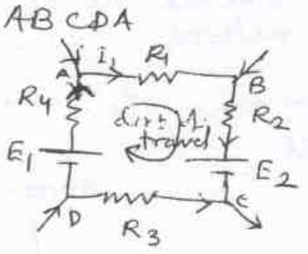
Fig-1 Fig-2

It is important to note that the sign of the battery emf is independent of the direction of current through the branch (Fig-1). It depends only on the polarity.

(b) Sign of IR drop (fig-2): If we go through a resistor in the same dirn as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence this voltage fall should be taken −ve. However, if we go in a dirn opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a +ve sign.

It is clear that the sign of voltage drop across a resistor depends on the dirn of current through that resistor but is independent of the polarity of any other source of emf in the circuit under consideration.

Consider the closed path ABCDA

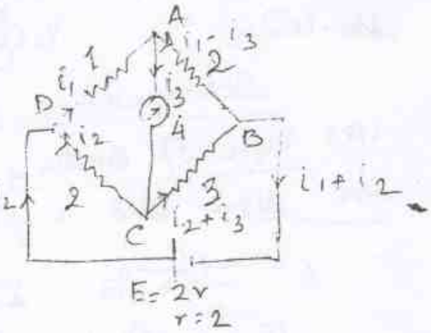


I_1R_1 is −ve (fall in potential)
I_2R_2 " (" ")
I_3R_3 +ve (rise ")
I_4R_4 −ve (fall ")
E_2 −ve (" ")
E_1 is +ve (rise ")

Using KVL, we get $-I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 - E_2 + E_1 = 0$

$\Rightarrow I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$.

Ex: Determine the currents in the unbalanced bridge ckt. Also determine the p.d. across BD and the resistance from B to D.



Soln:- Applying KVL to ckt DACD.

$-i_1 - 4i_3 + 2i_2 = 0 \Rightarrow i_1 - 2i_2 + 4i_3 = 0$ -- (1)

ckt. ABCA gives

$-2(i_1 - i_3) + 3(i_2 + i_3) + 4i_3 = 0$

$\Rightarrow 2i_1 - 3i_2 - 9i_3 = 0$ -- (2)

ckt DABED gives: $-i_1 - 2(i_1 - i_3) - 2(i_1 + i_2) + 2 = 0$

$\Rightarrow 5i_1 + 2i_2 - 2i_3 = 2$ -- (3)

by Cramer's rule
By solving, Current in DA $= i_1 = 30/91$ A

" " DC $= i_2 = 17/91$ "

" " AC $= i_3 = 1/91$ "

" " AB $= i_1 - i_3 = 29/91$ A

" " CB $= i_2 + i_3 = 18/91$ A

" " external ckt. $= i_1 + i_2 = \frac{47}{91}$ A

Internal voltage drop in cell $= 2(i_1 + i_2) = \frac{94}{91}$ V

∴ Pd across pts. D & B $= 2 - \frac{94}{91} = \frac{88}{91}$ V

Equivalent res of the bridge betn D & B

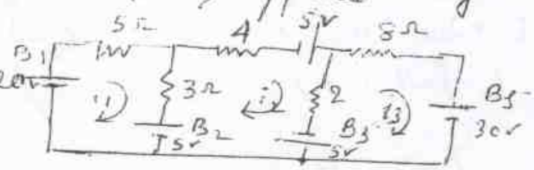
$= \dfrac{\text{Pd bet}^n \text{ pts. B \& D}}{\text{current through BD}} = \dfrac{88/91}{47/91} = \frac{88}{47} = 1.87\,\Omega$

Maxwell's Loop current Method:- (mesh)

If b be the no. of branches & j be the no. of junctions in a given network, Then the no. of independent eqns $b-(j-1)$

Ex: Determine the current supplied by each battery in the ckt.

Soln.



For loop 1, $20 - 5i_1 - 3(i_1 - i_2) - 5 = 0 \Rightarrow 8i_1 - 3i_2 = 15$ - (1)

For loop 2, $-4i_2 + 5 - 2(i_2 - i_3) + 5 + 5 - 3(i_2 - i_1) = 0 \Rightarrow 3i_1 - 9i_2 + 2i_3 = -15$ - (ii)

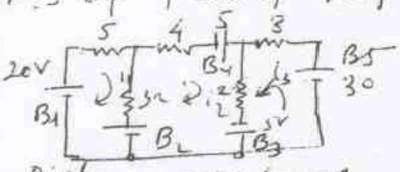
For loop-3, $-8i_3 - 30 - 5 - 2(i_3 - i_2) = 0 \Rightarrow 2i_2 - 10i_3 = 35$ - (iii)

Solving by Cramer's rule, $i_1 = 765/299$ A

$i_2 = 545/299$ A

$i_3 = -1875/598$ A

Since i_3 turns out to be −ve, actual dirn of flow of loop currents are as shown below



Discharge of current of $B_1 = 765/299$ A

Charging " $B_2 = i_1 - i_2 = 220/299$ A

Discharge " $B_3 = i_2 + i_3 = \frac{2965}{598}$ A

" " $B_4 = i_2 = 545/299$ A

Dirch. $B_5 = 1875/598$ A

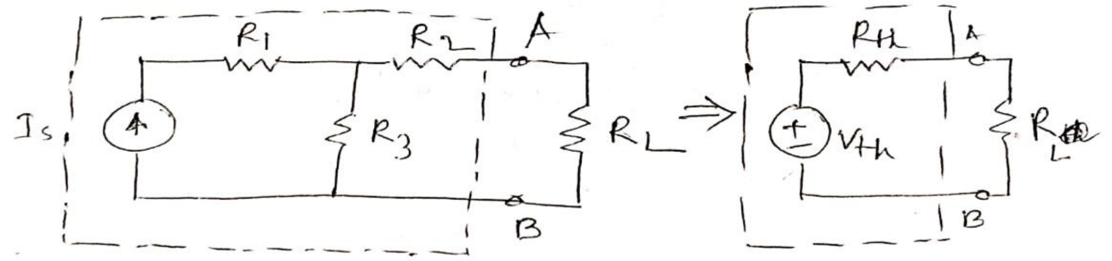
1. State Thevenin's theorem and draw the equivalent circuit .

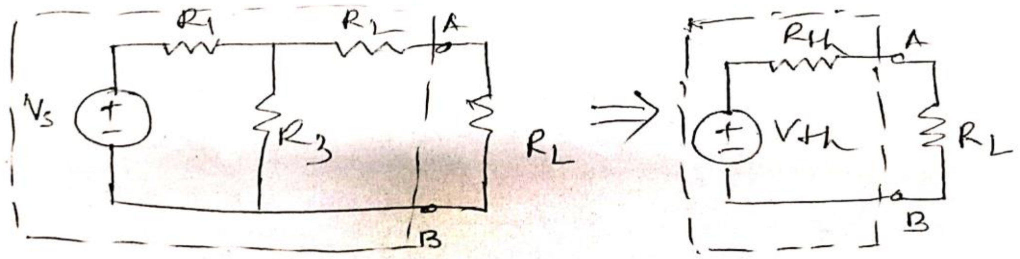
ckt .

Thevenin' Theorem:— The current flowing through a load resistance connected across any two terminals A & B of a linear, active, bilateral network is given by $V_{th}/(R_{th}+R_L)$

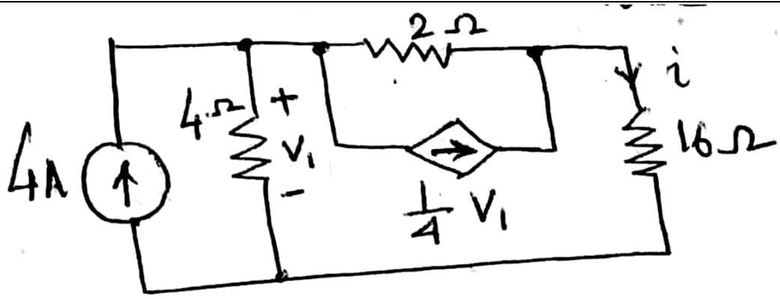
Where V_{th} = the o/c voltage (i.e. the voltage across the two terminals when R_L is removed.

R_{th} = internal resistance of the network as viewed back into the open-circuited network from the terminals with all sources in have been replaced by their internal resistances.

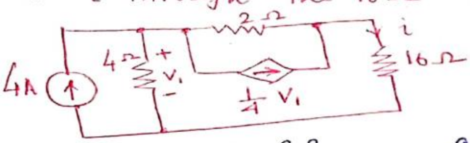




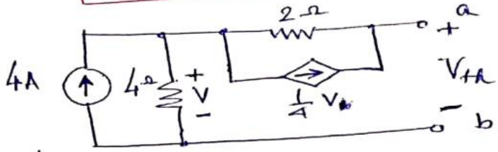
Determine the current i through the 16 ohm resistor by using Thevenin's theorem.



Q.8 Determine the current i through the $16\,\Omega$ resistor by the Thevenin Theorem.



Solution: For finding V_{th} :



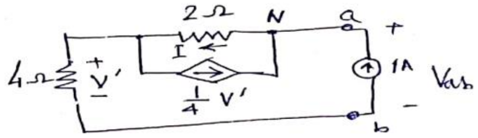
4A current passes through $4\,\Omega$ only.

$\therefore V = 4 \times 4 = 16$ volts.

$\therefore$ drop across $2\,\Omega = 2 \times \frac{1}{4} \times 16$
$= 8$ volts. [with a (+)ve]

$\therefore \boxed{V_{th} = 16 + 8 = 24 \text{ volts.}}$

For finding R_{th}, we connect 1 A source across a & b and 4A source is opened.



$V' = 4 \times 1 = 4$ Volts.

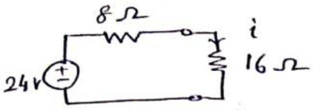
Applying KCL at node-N : $I = \frac{1}{4}V' + 1$
$= 1 + 1$
$\therefore I = 2$ A

$\therefore$ drop across $2\,\Omega = 2 \times 2 = 4$ volts.

$\therefore V_{ab} = 4 + 4 = 8$ volts.

$\therefore \boxed{R_{th} = 8\,\Omega}$

$\therefore$ Thevenin equivalent ckt :

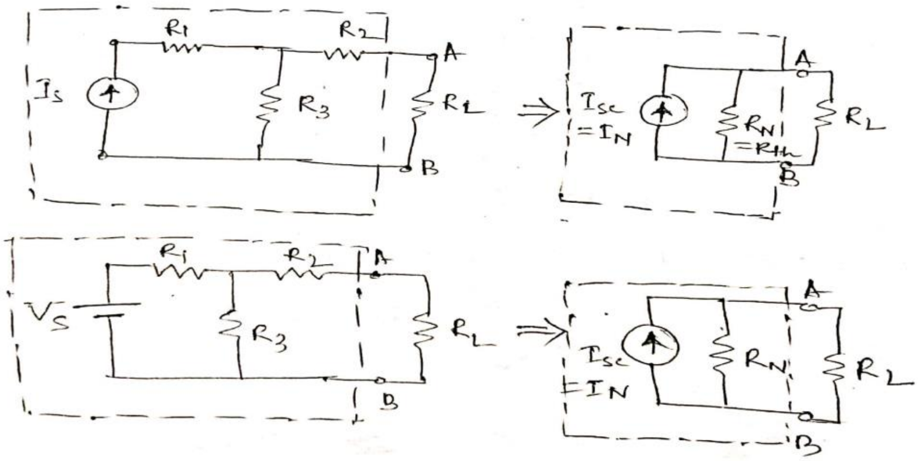


$\therefore$ Current through $16\,\Omega = i = \frac{24}{8+16} = \frac{24}{24} = 1$ A

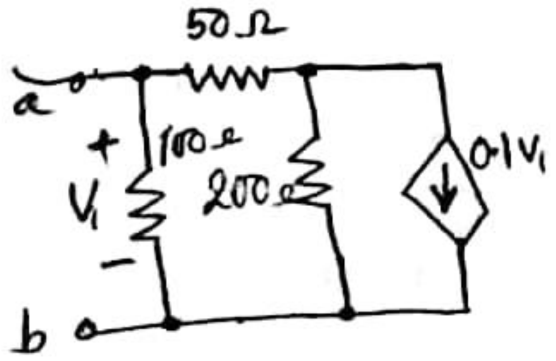
$\therefore \boxed{i = 1A}$ Ans

2. State Norton's theorem and draw the equivalent circuit .

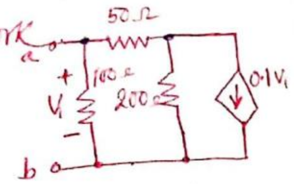
Norton's Theorem:— Any two-terminals active network when viewed from its o/p terminals is equivalent to a constant current source and a parallel resistance. The constant current is equal to the current which would flow in a short ckt. placed across the terminals and parallel resistance is the resistance of the network when viewed from these o/p terminals after all sources have been replaced by their internal resistances.



Find the Norton equivalent of the network.

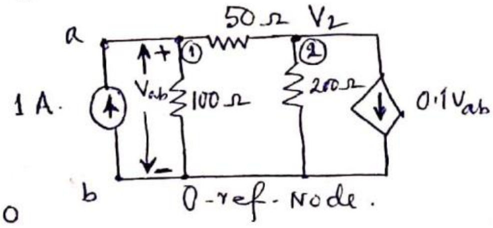


10. Find the Norton equivalent of the network



Solution: As there is no independent source in the circuit, so $I_{sc} = 0$

For finding R_N, we connect 1 A current source to the terminals a & b.



Applying KCL to Node-1:

$$1 - \frac{V_{ab}}{100} - \frac{V_{ab}-V_2}{50} = 0$$

$$\Rightarrow \frac{100 - V_{ab} - 2V_{ab} + 2V_2}{100} = 0$$

$$\Rightarrow 100 - 3V_{ab} + 2V_2 = 0$$

$$\Rightarrow 3V_{ab} - 2V_2 = 100 \quad \text{------ (1)}$$

Applying KCL to Node-2: $\frac{V_{ab}-V_2}{50} - \frac{V_2}{200} - 0.1V_{ab} = 0$

$$\Rightarrow \frac{4V_{ab} - 4V_2 - V_2 - 20V_{ab}}{200} = 0$$

$$\Rightarrow -16V_{ab} - 5V_2 = 0$$

$$\Rightarrow 16V_{ab} + 5V_2 = 0 \quad \text{--- (2)}$$

For solving, (1) × 5 $\Rightarrow 15V_{ab} - 10V_2 = 500$

(2) × 2 $\Rightarrow 32V_{ab} + 10V_2 = 0$

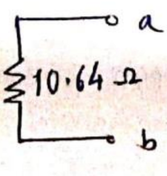
(Add) $47V_{ab} = 500$

$$\Rightarrow V_{ab} = \frac{500}{47}$$

$$\therefore \boxed{V_{ab} = 10.64 \text{ volts}}$$

$\therefore R_N = 10.64\ \Omega$

∴ Norton equivalent ckt.



3. State and prove Maximum Power transfer theorem.

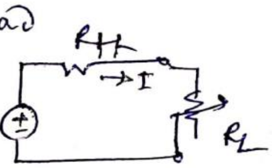
Max^m power Transfer Theorem :-

A resistive load will consume max^m power from a network when the load resistance is equal to the resistance of the network as viewed from the o/p terminals, with all energy sources replaced by their internal resistances (i.e called R_{th})

So Cond^n of Max^m power transfer, $R_L = R_{th}$

and Max^m power, $P_{max} = \frac{V_{th}^2}{4R_{th}}$.

Proof: Power consumed by load



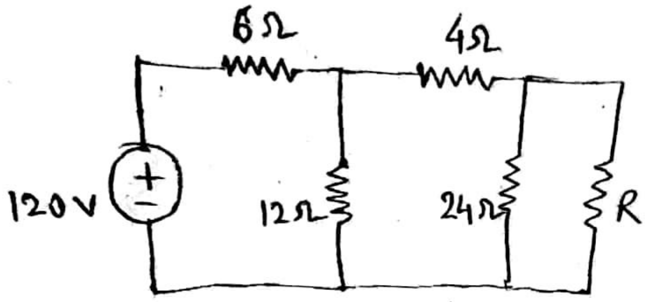
$$P = I^2 R_L$$

$$= \frac{V_{th}^2 R_L}{(R_L + R_{th})^2}$$

for P to max^m, $\frac{dP}{dR_L} = 0$

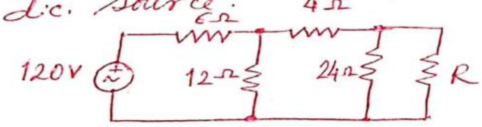
$$\Rightarrow \boxed{R_L = R_{th}}$$

Determine the maximum power dissipated by the resistor R from the dc source.

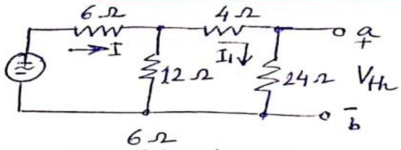


25 Determine the Maximum power dissipated by the resistor R from the d.c. source.

CEM-12

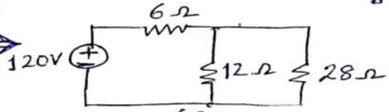


Solution: For finding V_{th},



To find the total current (I) supplied by 120 V source,

⇒



⇒

$\left[= \frac{12 \times 28}{40}\right]$

⇒

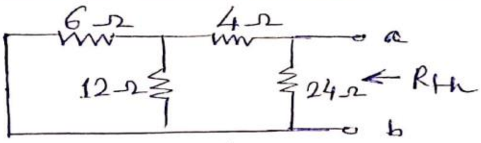
$\therefore I = \frac{120}{14.4}$ Amp.

$\therefore$ Current through 24 Ω, $I_1 = \frac{120}{14.4} \times \frac{12}{40} = \frac{10}{4} = 2.5$ A

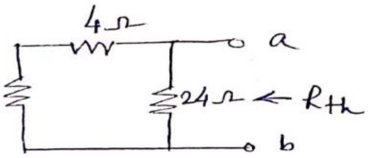
$\therefore V_{th}$ = drop across 24 Ω
$= 2.5 \times 24$
$= 60$ volt.

$\boxed{V_{th} = 60 \text{ volt}}$

For finding R_{th},

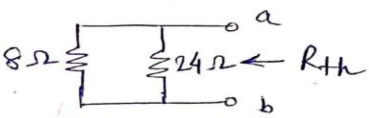


⇒



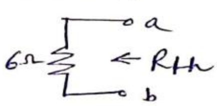
$\left[\frac{6 \times 12}{18} = \right] 4\ \Omega$

⇒

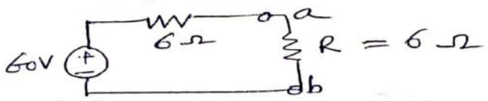


$\therefore R_{th} = \frac{8 \times 24}{32}$

$\boxed{R_{th} = 6\ \Omega}$



$\therefore$ Thevenin equivalent ckt.



For Maximum power dissipation, $R = R_{th} = 6\ \Omega$

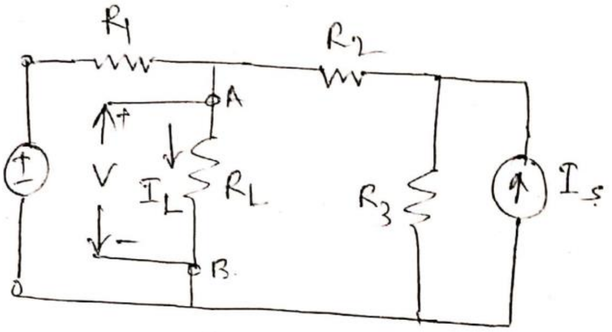
$\therefore$ Maximum power dissipated $= \frac{V_{th}^2}{4R_{th}}$

$= \frac{60 \times 60}{4 \times 6}$

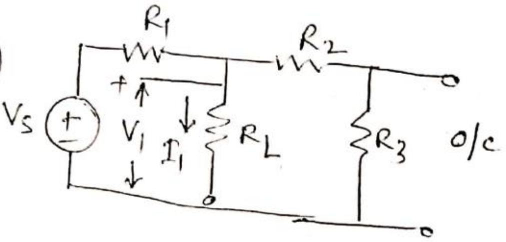
$\therefore \boxed{P_{max} = 150\ W}$

4. State Superposition theorem.

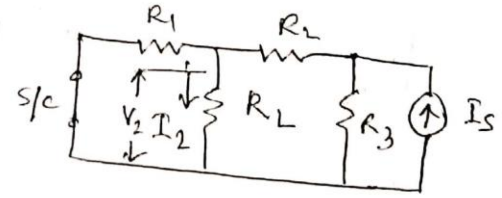
Superposition Theorem:— In any linear resistive network containing several sources, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltage or current caused by the separate sources acting alone, with all other sources replaced by their internal resistances.



⇒ (1)

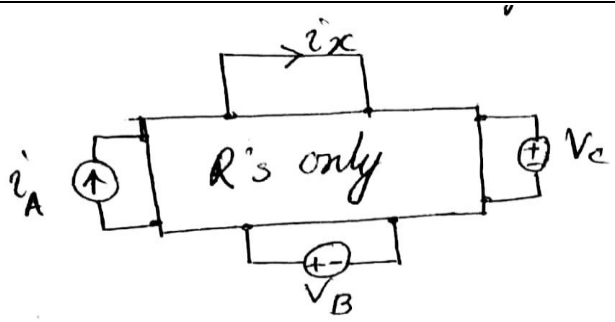


⇒ (2)



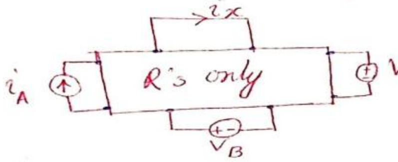
$$\left.\begin{aligned} \therefore\ I_L &= I_1 + I_2 \\ \&\ V_L &= V_1 + V_2 \end{aligned}\right\}$$

With source i_a and v_b on in the circuit and v_c=0 and i_x=20 amp. With i_a and v_c on and v_b=0, i_x=5 amp. ; and finally with all the three sources on, i_x=12 amp. Find i_x if the only source operating is (a) i_a (b) v_b (c) v_c (d) find i_x if i_a and v_c are doubled in amplitude and v_b is reversed.



9. With Sources i_A and V_B on in the Circuit and $V_c = 0$, $i_x = 20A$. With i_A and V_c on and $V_B = 0$, $i_x = -5A$; and finally, with all Three Sources on, $i_x = 12A$. Find i_x if the only Source operating is (a) i_A; (b) V_B; (c) V_c. (d) Find i_x if i_A and V_c are doubled in amplitude and V_B is reversed.

CEM-3



Solution: Let the value of i_x when only i_A is operating be i_1
Similarly " " " i_x " " V_B " " " i_2
and " " " i_x " " V_c " " " i_3

From Superposition theorem and according to the question,

$$i_1 + i_2 = 20 \quad \text{---} (1)$$
$$i_1 + i_3 = -5 \quad \text{---} (2)$$
$$i_1 + i_2 + i_3 = 12 \quad \text{---} (3)$$

from (1) & (3), $\Rightarrow$ $i_3 = 12 - (i_1 + i_2)$
$= 12 - 20 = -8$

$\therefore \boxed{i_3 = -8A}$ Am .... (c)

from (2) & (3), $\Rightarrow$ $i_2 = 12 - (i_1 + i_3)$
$= 12 - (-5) = 17$

$\therefore \boxed{i_2 = 17A}$ Am .... (b)

from (1), $i_1 = 20 - i_2$
$= 20 - 17 = 3$

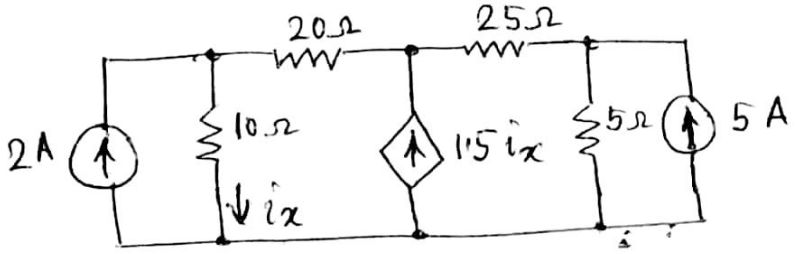
$\therefore \boxed{i_1 = 3A}$ Am .... (a)

(d). If i_A and V_c are doubled in amplitude and V_B is reversed

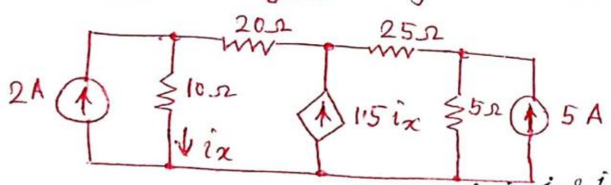
Then $i_x = 2i_1 - i_2 + 2i_3$
$= 2 \times 3 - 17 + 2(-8)$
$= 6 - 17 - 16$
$= 6 - 33 = -27$

$\therefore \boxed{i_x = -27A}$ Am .... (d)

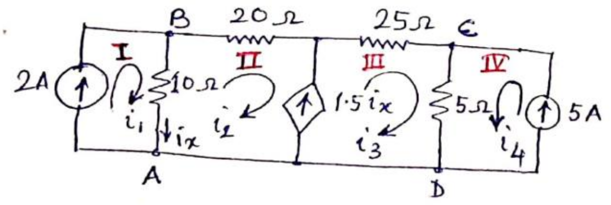
5. Use Mesh analysis to find the power generated by each of the five sources.



6. Use mesh analysis to find i_x in the Circuit.



Solution: Assume mesh currents i_1, i_2, i_3 & i_4 in respective meshes



Here
$$i_1 = 2\text{ A}$$
$$i_4 = 5\text{A}$$
$$i_1 - i_2 = i_x$$
$$1.5\, i_x = i_3 - i_2$$
Constrained eqns.

or, $1.5(i_1 - i_2) = i_3 - i_2$

or, $1.5 \times 2 - 1.5 i_2 - i_3 + i_2 = 0$

or, $0.5 i_2 + i_3 = 3$ ...(1)

Applying KVL to loop ABCDA

$$-20 i_2 - 25(i_3) - 5(i_3 + i_4) - 10(i_2 - i_1) = 0$$

$$\Rightarrow 20 i_2 + 25 i_3 + 5 i_3 + 5 \times 5 + 10 i_2 - 10 \times 2 = 0$$

$$\Rightarrow 30 i_2 + 30 i_3 + 5 = 0$$

$$\Rightarrow 6 i_2 + 6 i_3 = -1 \quad \text{.....(2)}$$

8

for solving,

$(1) \times 6 \Rightarrow 3 i_2 + 6 i_3 = 18$

$(2) \times 1 \Rightarrow 6 i_2 + 6 i_3 = -1$

$\quad - \quad - \quad +$

(sub.) $-3 i_2 = 19$

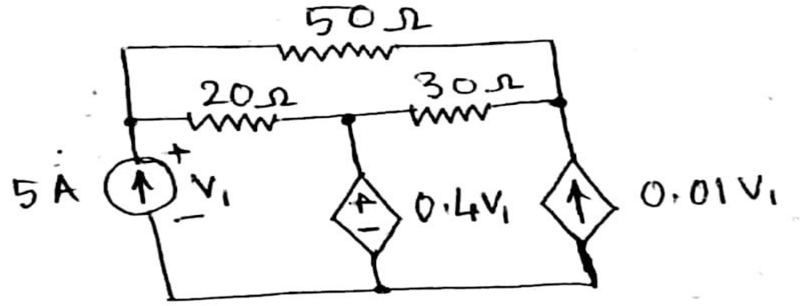
$$\Rightarrow i_2 = -\frac{19}{3} = -6.33$$

$$\therefore \boxed{i_2 = -6.33\text{ A}}$$

$$\therefore i_x = i_1 - i_2$$
$$= 2 - (-6.33) = 8.33$$

$\boxed{i_x = 8.33\text{ A}}$ Ans.

6. Use Nodal analysis to determine V_1 and power being supplied by the dependent current source in the circuit.



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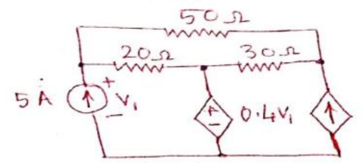
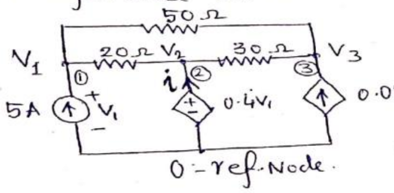
3. Use nodal analysis to determine V_1 and power being supplied by the dependent current source in the circuit.



Solution: Defining nodal voltages V_1, V_2 & V_3 with respect to the reference node '0'



Here $V_2 = 0.4V_1$; [Constrained equation]

Applying KCL at node-1 : $5 - \frac{V_1 - V_2}{20} - \frac{V_1 - V_3}{50} = 0$

or, $50 - \frac{V_1 - V_2}{2} - \frac{V_1 - V_3}{5} = 0$

or, $\frac{500 - 5V_1 + 5V_2 - 2V_1 + 2V_3}{10} = 0$

or, $7V_1 - 5V_2 - 2V_3 = 500$

or, $7V_1 - 2V_1 - 2V_3 = 500$

or, $5V_1 - 2V_3 = 500$ ---- (1)

Applying KCL at node-3 : $\frac{V_1 - V_3}{50} + \frac{V_2 - V_3}{30} + 0.01V_1 = 0$

or, $\frac{3V_1 - 3V_3 + 5V_2 - 5V_3 + 1.5V_1}{150} = 0$

or, $4.5V_1 + 5V_2 - 8V_3 = 0$

or, $4.5V_1 + 2V_1 - 8V_3 = 0$

or, $6.5V_1 - 8V_3 = 0$ --- (2)

for Solving, ① × 4 $\Rightarrow$ $20V_1 - 8V_3 = 2000$

② × 1 $\Rightarrow$ $6.5V_1 - 8V_3 = 0$

 $-$ $+$ $-$

(Sub.) $13.5V_1 = 2000$

or, $V_1 = \frac{2000}{13.5} = 148.15$

$\therefore \boxed{V_1 = 148.15 \text{ Volts.}}$ Ans.

from ② ; $V_3 = \frac{6.5V_1}{8}$

$= \frac{6.5 \times 148.15}{8}$

$\therefore V_3 = 120.37$ Volts.

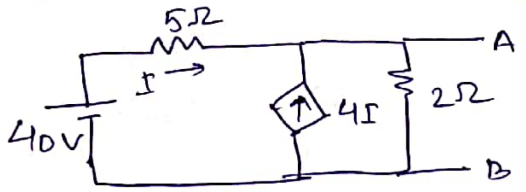
$\therefore$ Power supplied by the dependent current source $0.01V_1$

$P_{0.01V_1} = V_3 \times 0.01V_1$

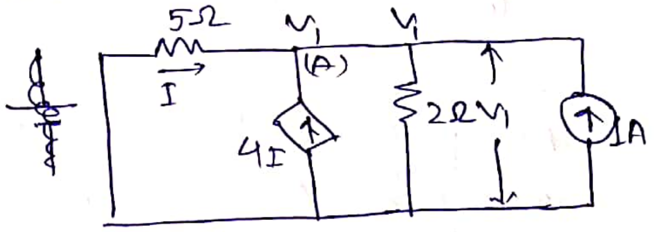
$= 120.37 \times 0.01 \times 148.15 = 178.33$

$\therefore \boxed{P_{0.01V_1} = 178.33 \text{ Watts.}}$ Ans.

Q Find the Norton's equivalent to the left of terminal AB of the circuit shown in Fig. 2.



Sol For the calculation of R_N we will the terminal AB with 1A current source and short circuit the voltage source.



Now, applying nodal analysis at node A.

$$\frac{V_1}{2} - I - 4I - 1 = 0 \quad — ①$$

$$V_1 = -5I \quad — ②$$

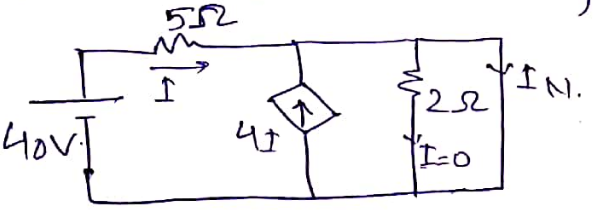
using equ. ② in ① we get.

$$-\frac{5I}{2} - 5I = 1 \Rightarrow I = -\frac{2}{15} \text{ A}$$

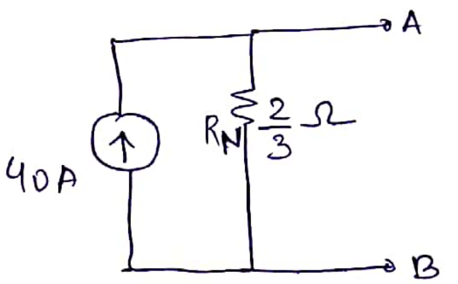
so, $V_1 = -5I = \frac{2}{3}$ V

$$\boxed{R_N = \frac{V_1}{1} = \frac{2}{3}\,\Omega}$$

For the calculation of I_N, we will short the terminal AB.



$\Rightarrow$ Norton's equivalent ckt

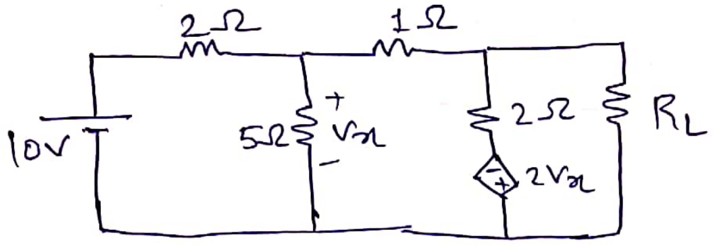


$$I_N = I + 4I = 5I$$

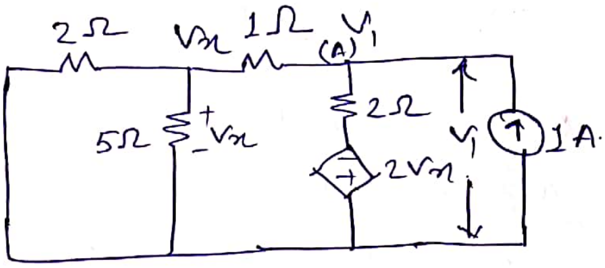
$$I = \frac{40}{5} = 8A$$

$$\Rightarrow \boxed{I_N = 5 \times 8 = 40\text{ A}}$$

Q Find R_{th} across load resistor R_L.



Sol Since this circuit consist of dependent voltage sources, we will apply a 1A current source across load resistance & short circuit the ideal voltage source.



Applying nodal analysis at node A, we get

$$\frac{V_1+2V_x}{2} + \frac{V_1-V_x}{1} - 1 = 0$$

$$V_1 + 2V_x + 2V_1 - 2V_x = 2$$

$$3V_1 = 2$$

$$V_1 = 2/3 \text{ V}$$

$$\boxed{R_{th} = \frac{V_1}{1} = \frac{2}{3}\ \Omega}$$

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