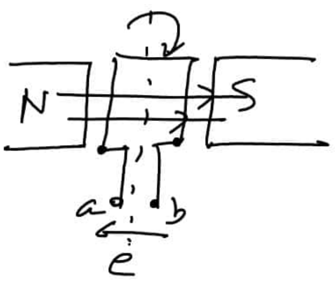
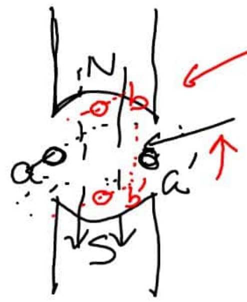
Lecture 11

28/12/2020

Single Phase System

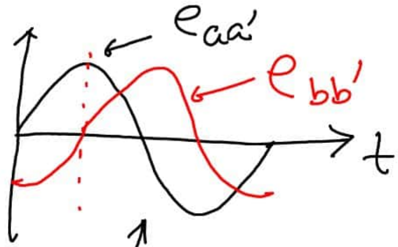


$\Rightarrow$



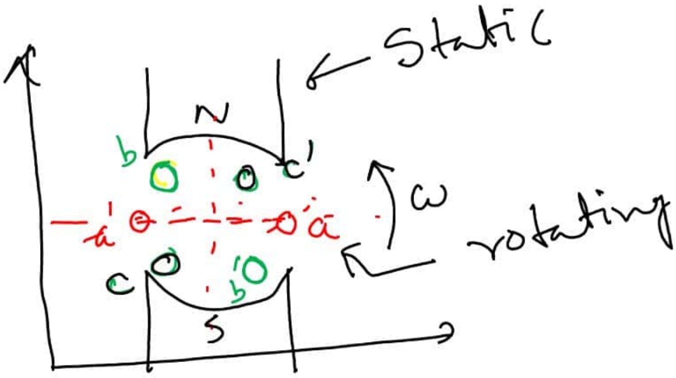
Coil axes are seperated by 90° (in space)

e.m.f between aa' $\Rightarrow$ of the Coil.



Voltages are phase separate by 90°.

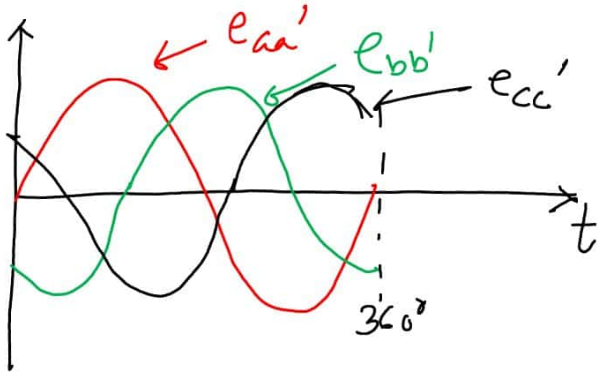
Two phase system



$aa' \rightarrow$

$bb' \rightarrow \angle 120^\circ$ of aa'

$cc' \rightarrow \angle -120^\circ$ of aa'



$\Rightarrow$ $e_{aa'}$, $e_{bb'}$ & $e_{cc'}$ these voltage are Phase seperated by 120°.

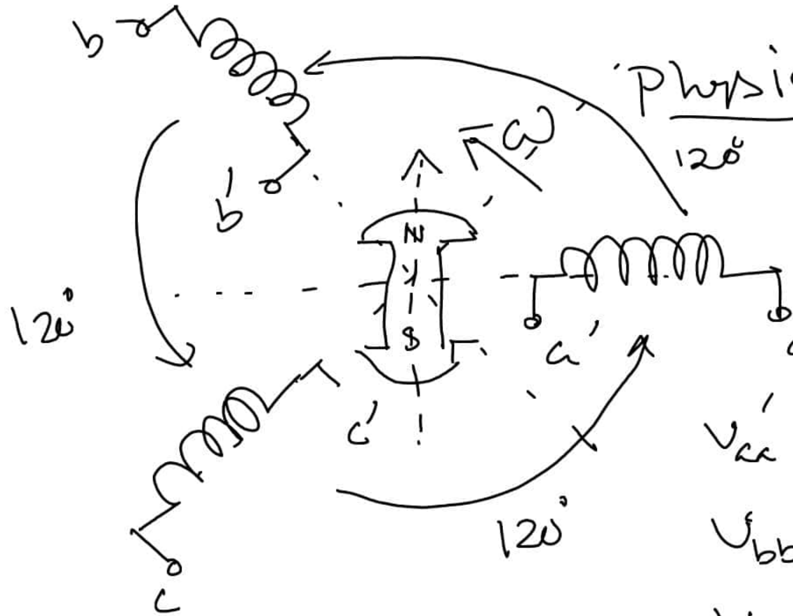
<u>Synchronous Generatar</u>

↳ This is used for electricity generation in thermal power plant, hydel power plants.

Magnetic field line generation $\Rightarrow$ is on the rotor

3 phase coils are on the stator.

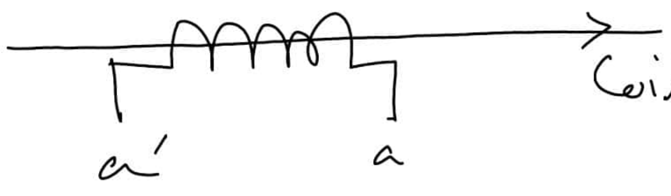
Physical spacing of 120°



$$V_{aa'} = V_m \sin\omega t$$

$$V_{bb'} = V_m \sin(\omega t - 120^\circ)$$

$$V_{cc'} = V_m \sin(\omega t - 240^\circ)$$
$$= V_m \sin(\omega t + 120^\circ)$$



Coil axis.

3ϕ System. $\Rightarrow$ Balanced voltage generation.

$v_{aa'} = V_m \sin\omega t$

$v_{bb'} = V_m \sin(\omega t - 120^\circ)$

$v_{cc'} = V_m \sin(\omega t + 120^\circ)$

* Equal in magnitude (Peak or RMS)
* Phase separated by 120°.

$$v_{aa'} + v_{bb'} + v_{cc'} = V_m\left[\sin\omega t + \sin(\omega t - 120^\circ) + \sin(\omega t + 120^\circ)\right]$$

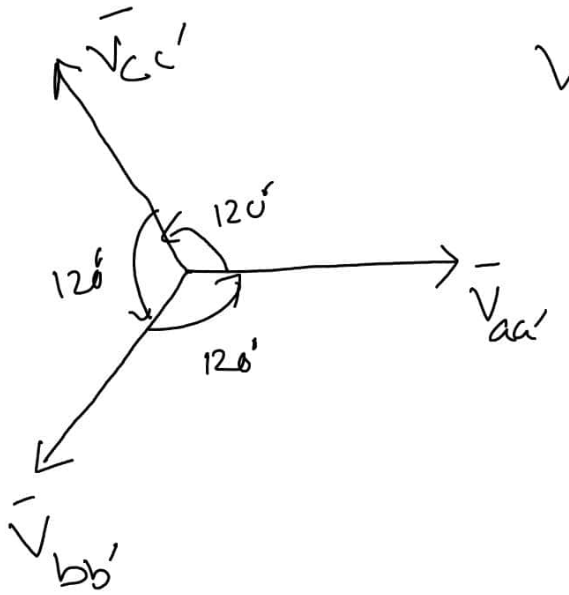
$$= 0$$

$\bar{V}_{aa'} = V\angle 0^\circ$

$V = \frac{V_m}{\sqrt{2}} \Rightarrow$ RMS value

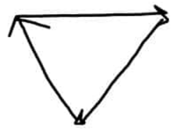
$\bar{V}_{bb'} = V\angle -120^\circ$

$\bar{V}_{cc'} = V\angle 120^\circ$



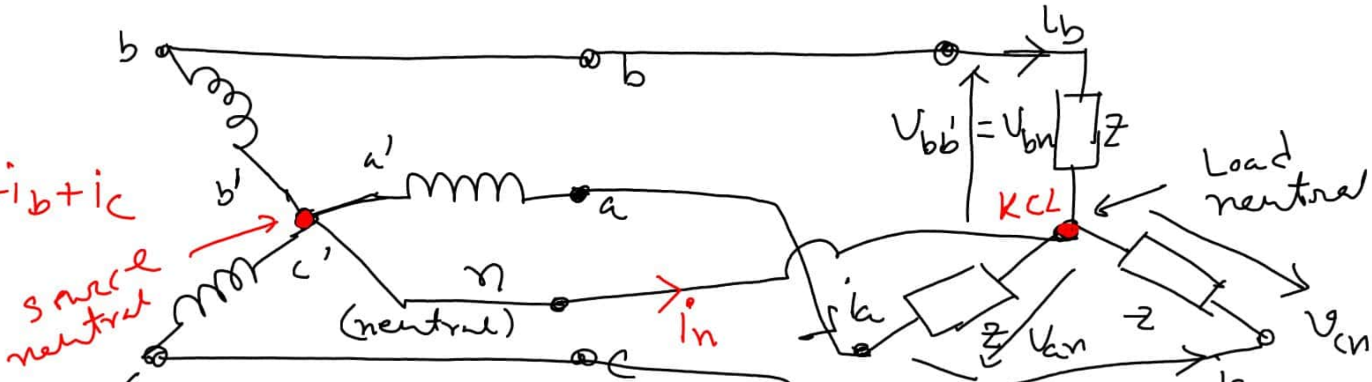
$$\bar{V}_{aa'} + \bar{V}_{bb'} + \bar{V}_{cc'} = 0$$

$$\Downarrow$$



Equilateral triangle.

$i_n = i_a + i_b + i_c$
$= 0$



Star Configuration

Star Connected Load

$$v_{an} = v_{aa'} = V_m \sin \omega t \qquad [Z \text{ impedance}]$$

$$v_{bn} = v_{bb'} = V_m \sin(\omega t - 120^\circ) \qquad ''$$

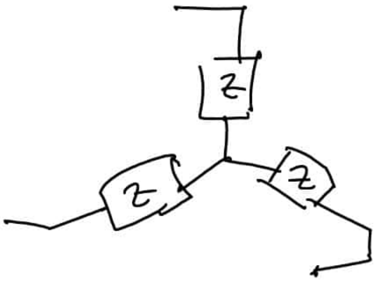
$$v_{cn} = v_{cc'} = V_m \sin(\omega t + 120^\circ) \qquad ''$$

$$\underline{Z = R + j\omega L} \qquad (\text{Let say})$$

$$i_a = \frac{V_m}{|Z|} \sin(\omega t - \phi) \qquad \phi = \tan^{-1} \frac{\omega L}{R}$$

$$i_b = \frac{V_m}{|Z|} \sin(\omega t - 120^\circ - \phi)$$

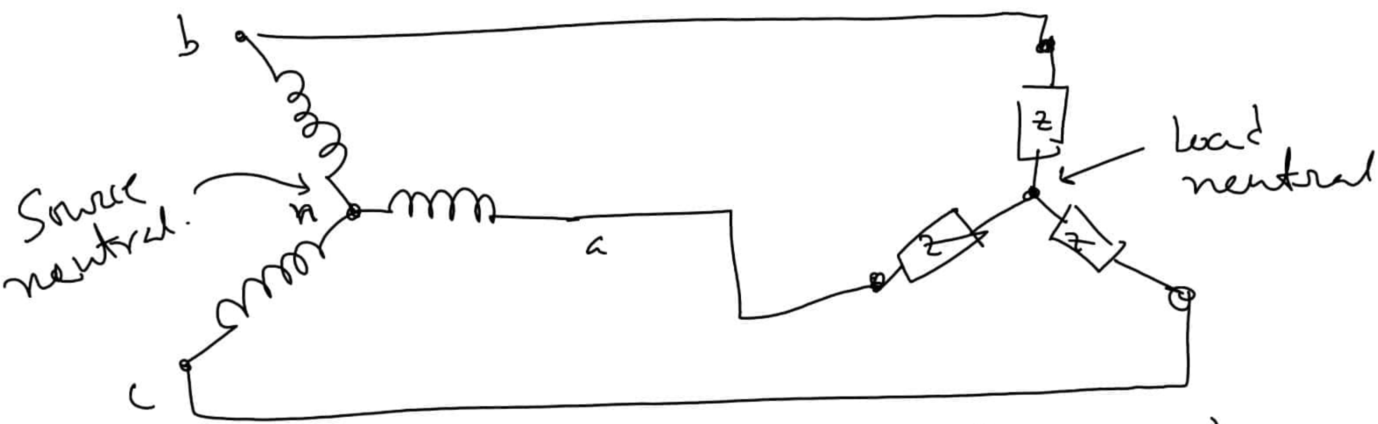
$$i_c = \frac{V_m}{|Z|} \sin(\omega t + 120^\circ - \phi)$$



$\Rightarrow$ Z equal in all branches
(So balanced loading)

$\underbrace{i_a, i_b, i_c}_{3\phi \text{ currents.}}$ are also balanced $\Rightarrow$
- $\rightarrow$ equal magnitude
- $\rightarrow$ $120°$ phase seperation

So, $i_a + i_b + i_c = 0$



3ϕ – 3Wire (3ϕ-3W)

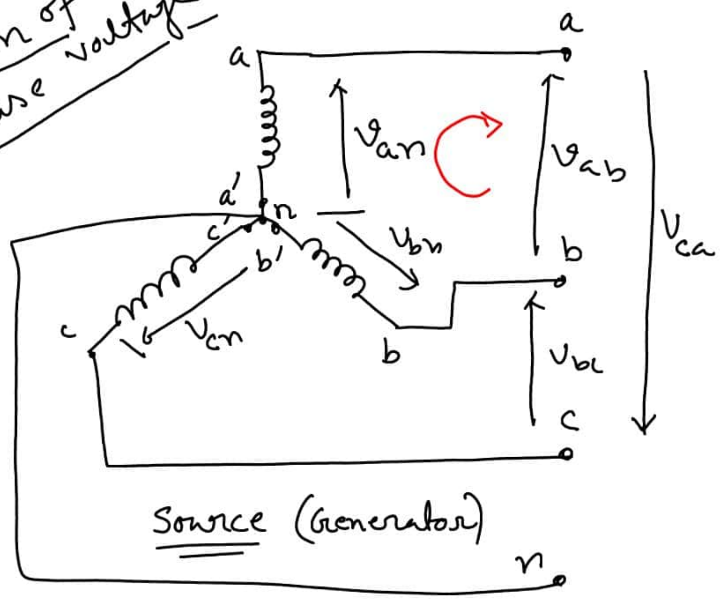
Balanced System

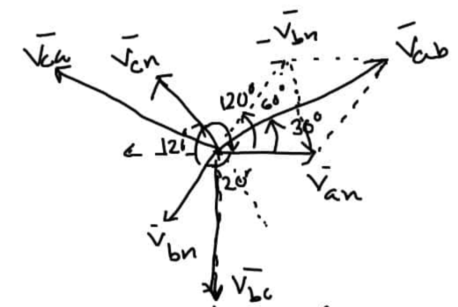
Lecture-12

29/12/2020

Phasor diagram of Line and phase voltage

Star (⅄) Connected Source





$|\bar{V}_{ab}| = |\bar{V}_{an}| \cos 30^\circ + |-\bar{V}_{bn}| \cos 30^\circ$

$= \sqrt{3}\, V_P$

$v_{an}, v_{bn}, v_{cn} \Rightarrow$ Phase voltages.

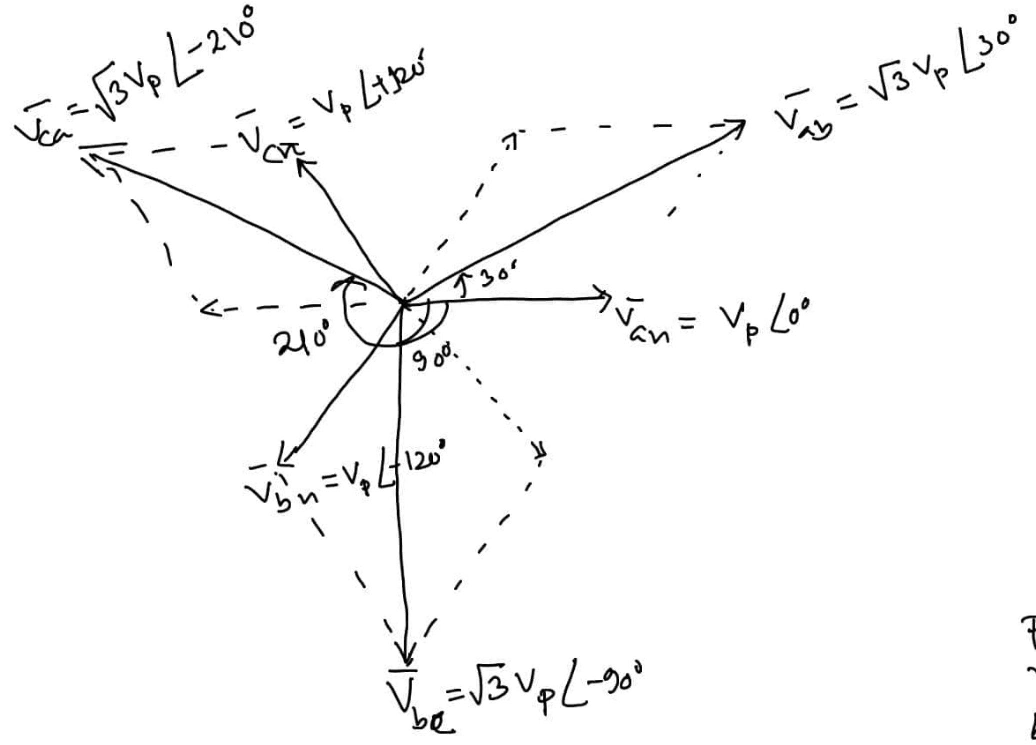
$v_{ab}, v_{bc}, v_{ca} \Rightarrow$ Line voltages.

Phase Voltage $\bar{V}_{an} = V_P\angle 0^\circ$; $\bar{V}_{bn} = V_P\angle -120^\circ$; $\bar{V}_{cn} = V_P\angle +120^\circ$

$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn} = \sqrt{3} V_P \angle 30^\circ$ [referenced w.r.t. $\bar{V}_{an}$]

$$\bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn} = \sqrt{3}V_p \angle -90^\circ$$

$$\bar{V}_{ca} = \bar{V}_{cn} - \bar{V}_{an} = \sqrt{3}V_p \angle -210^\circ$$



Phase voltage magnitude.

In the star connected system line voltage $|V_{LL}| = \sqrt{3}\,|V_p|$

Line voltages are also equal in magnitude & Phase separated by 120° $\Rightarrow$ A balanced system.

$$v_{an} = V_m \sin\omega t$$

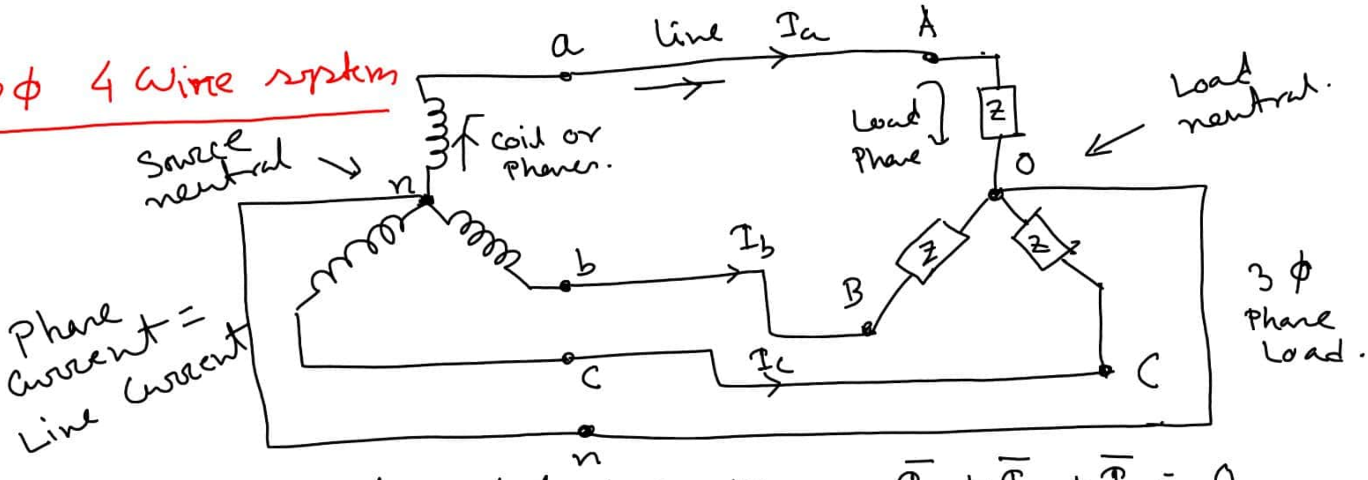
$$v_{ab} = \sqrt{3}\, V_m \sin(\omega t + 30^\circ)$$

phase lead of 30° w.r.t. v_{an}.

$$V_{LL} = \sqrt{3}\, V_P$$

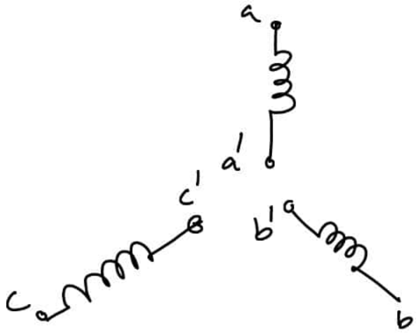
RMS RMS

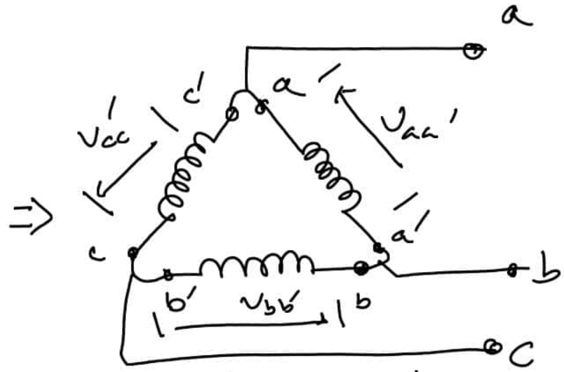
3ϕ 4 Wire system



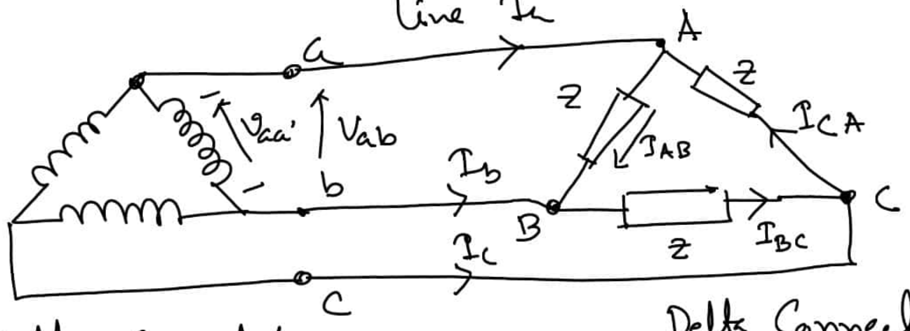
under balanced load Condn. $\quad \bar{I}_a + \bar{I}_b + \bar{I}_c = 0$

Delta Connected System





Delta Connected System



Delta Connected Source

Delta Connected load

$$\bar{I}_{AB} = \frac{\bar{V}_{ab}}{Z}$$

$$\bar{I}_{BC} = \frac{\bar{V}_{bc}}{Z}$$

$$\bar{I}_{CA} = \frac{\bar{V}_{ca}}{Z}$$

Phase Currents in Load.

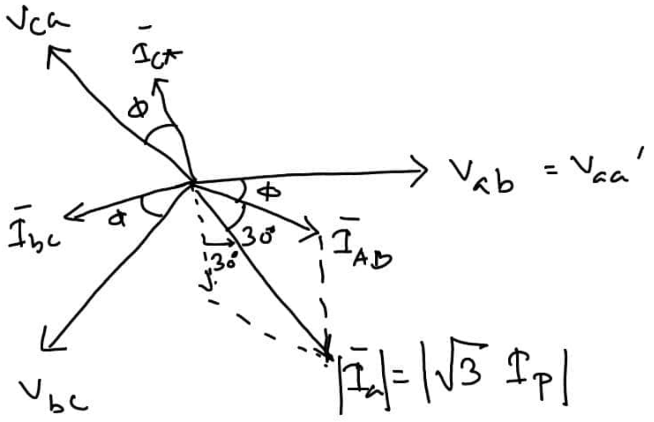
Coil Voltage = Phase voltage = Line voltage

Delta Connected system => Phase voltage = Line voltage.

Line currents.
$$\begin{cases} \bar{I}_a = \bar{I}_{AB} - \bar{I}_{CA} \\ \bar{I}_b = \bar{I}_{BC} - \bar{I}_{AB} \\ \bar{I}_c = \bar{I}_{CA} - \bar{I}_{BC} \end{cases}$$

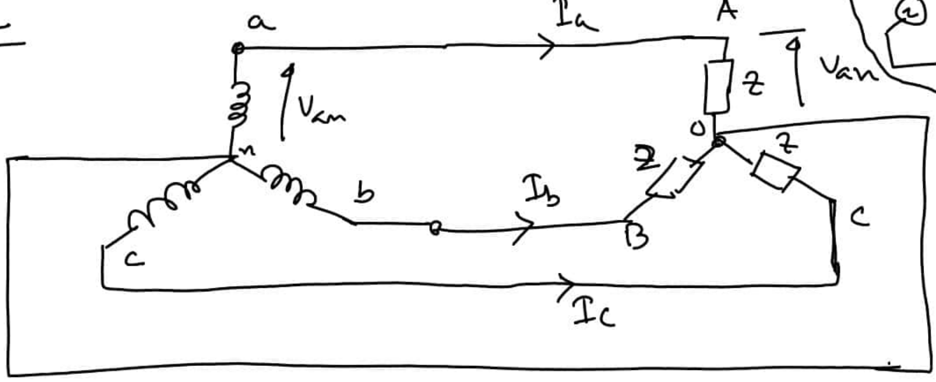
$|\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}| = I_P$

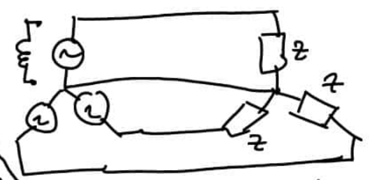
Phase Current magnitude in load.



Line current = $\sqrt{3}$ x Phase current => Delta Connected System

Power





Source & load connection can be represented by above figure.

Power in Phase A → $p_a = v_{an} \cdot i_a$

Instantaneous Power in the 3 phases —

$$p = p_a + p_b + p_c$$
$$= v_{an} i_a + v_{bn} i_b + v_{cn} i_c$$
$$= 3 \underbrace{V_{ph}}_{\text{RMS}} \underbrace{I_{ph}}_{\text{RMS}} \cos\phi$$

$v_{an} = V_m \sin \omega t$

$i_a = I_m \sin(\omega t - \phi)$

$I_m = \frac{V_m}{|Z|}$; $\phi = \angle Z$

Average power $P = \frac{1}{2\pi}\int_0^{2\pi} p \, d(\omega t) = 3 V_{ph} I_{ph} \cos\phi$

Instantaneous power $p = v_{an} i_a + v_{bn} i_b + v_{cn} i_c$

$$= V_m I_m \sin(\omega t) \sin(\omega t - \phi)$$
$$+ V_m I_m \sin(\omega t - 120^\circ) \sin(\omega t - 120^\circ - \phi)$$
$$+ V_m I_m \sin(\omega t + 120^\circ) \sin(\omega t + 120^\circ - \phi)$$

$$V_m I_m \sin(\omega t) \sin(\omega t - \phi) = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$V_m I_m \sin(\omega t - 120^\circ) \sin(\omega t - 120^\circ - \phi) = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - 240^\circ - \phi)\right]$$

$$V_m I_m \sin(\omega t + 120^\circ) \sin(\omega t + 120^\circ - \phi) = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + 240^\circ - \phi)\right]$$

$$\therefore p = \frac{3}{2} V_m I_m \cos\phi - \underbrace{\frac{V_m I_m}{2}\left[\cos(2\omega t - \phi) + \cos(2\omega t - \phi - 120^\circ) + \cos(2\omega t - \phi + 120^\circ)\right]}_{=0}$$

$$\Rightarrow p = \frac{3}{2} V_m I_m \cos\phi = 3 \times V_{ph} \times I_{ph} \cos\phi$$ [Instantaneous power is constant]

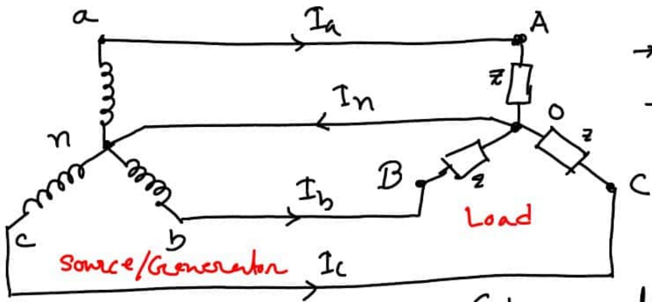
V_{ph}, I_{ph} } RMS values.

Scanned with CamScanner

Lecture - 13

01/01/2021

1.



3 Phase - 4 wire. (Y connected)

→ Balanced load

→ Balanced Source

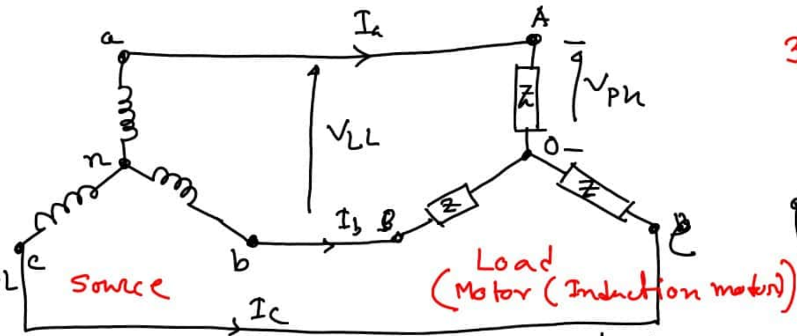
$I_a + I_b + I_c = I_n = 0$

2.

$\sqrt{3}\, V_{ph} = V_{LL}$

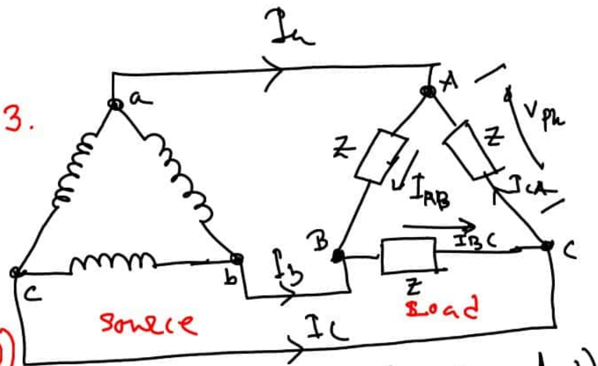
$|I_a| = |I_b| = |I_c|$

$= I_{ph} = I_L$



3 Phase - 3 Wire (Y Connected)

3.



3 Phase - 3 Wire (Δ Connected)

* Balanced load (for this course)

$I_a + I_b + I_c = 0$
apply KCL

$V_{ph} = V_{LL}$

Phase current → $|I_{AB}| = |I_{BC}| = |I_{CA}| = I_{ph}$

Line current $|I_a| = |I_b| = |I_c| = \sqrt{3}\, I_{ph}$
$= I_L$

Active Power (Load consumes this to perform work)

active power = Average value of instantaneous power

$$P = 3 V_{ph} I_{ph} \cos\phi$$

Star Connected Load

$$P = 3 V_{ph} I_{ph} \cos\phi$$

$$= \sqrt{3} V_{LL} I_L \cos\phi$$

$$\left[V_{ph} = \frac{V_{LL}}{\sqrt{3}} \right]$$

$$I_{ph} = I_L$$

Delta Connected Load

$$P = 3 V_{ph} I_{ph} \cos\phi$$

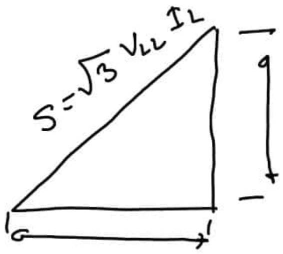
$$= \sqrt{3} V_{LL} I_L \cos\phi$$

$$\left[\begin{array}{l} V_{ph} = V_{LL} \\ I_{ph} = \frac{I_L}{\sqrt{3}} \end{array} \right]$$

Apparent Power

$$S = 3 V_{ph} I_{ph} \quad \text{(unit in VA)}$$

$$= \sqrt{3} V_{LL} I_L$$



$\sqrt{3}\,V_{LL} I_L \sin\phi = 3V_{ph} I_{ph} \sin\phi \Rightarrow$ Reactive Power. (Q)

Unit VAR

$P=\sqrt{3}\,V_{LL} I_L \cos\phi$

Complex Power

$\bar{Z} = \dfrac{\bar{V}_{ph}}{\bar{I}_{ph}}$ or, $\bar{I}_{ph} = \dfrac{\bar{V}_{ph}}{\bar{Z}} = \left|\dfrac{V_{ph}}{Z}\right| \angle Z$

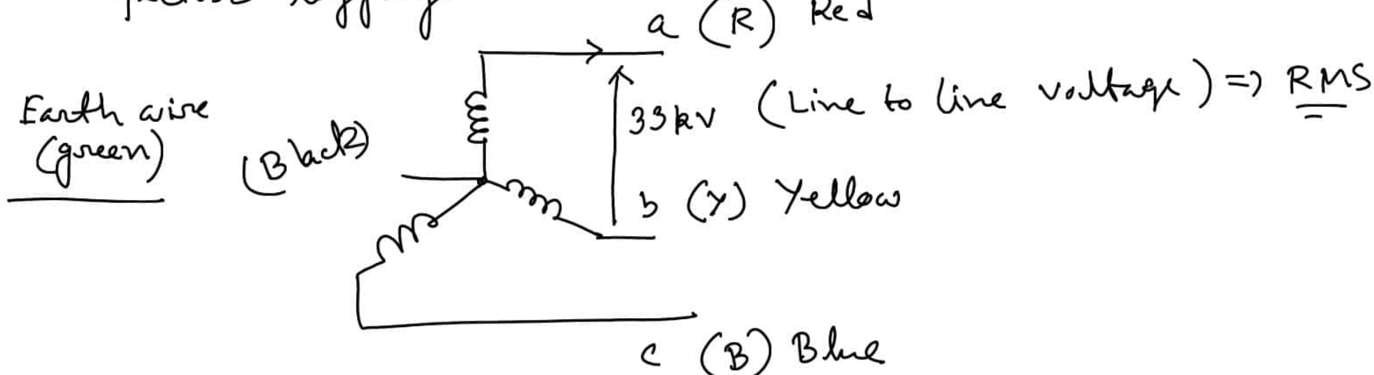
Individual Phases $\Rightarrow \mathrm{Re}\left[3\times \bar{V}_{ph} \times I_{ph}^{*}\right]$ = Active power P (Watt)

$\hookrightarrow \mathrm{Im}\left[3\times \bar{V}_{ph} \times I_{ph}^{*}\right]$ = Reactive power Q (VAR)

$\hookrightarrow \bar{V}_{ph} \times I_{ph}^{*}$ = Apparent power S (VA)

$S = P + jQ$.

* Calculate the line current of $\curlywedge$ connected 3ϕ alternator delivering 5 MW at 33 kV and working at 0.8 power factor lagging.



$$|V_{ab}| = |V_{bc}| = |V_{ca}| = 33\text{ kV (RMS)}$$

Line voltage $V_{LL} = 33 \times 10^3$ V

Active Power $P = 5\text{ MW} = 5 \times 10^6$ W

power factor $\cos\phi = 0.8$ (lagging)

$$\sqrt{3}\, V_{LL}\, I_L \cos\phi = P \quad \Rightarrow \quad I_L = \underline{\underline{109.4\text{ A}}}.$$

2. 3ϕ Load $\Rightarrow$ 3 similar inductive coil ($R = 50\Omega$; $L = 0.3H$)

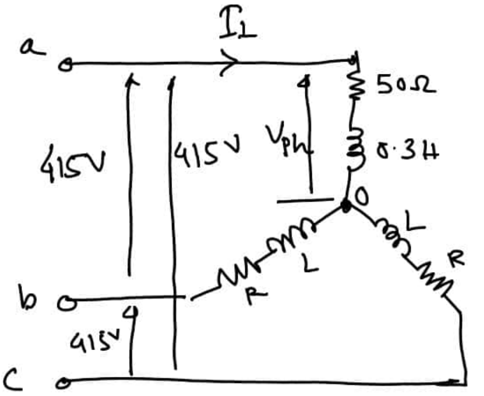
The supply in 3ϕ, 415V & 50Hz.

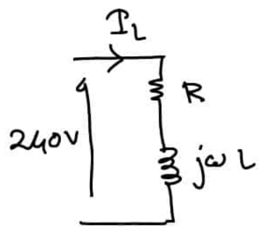
a) The line current
b) power factor
c) The total power

} i) star connected
ii) delta connected.

* Supply is 3ϕ, 415V, 50Hz
  ↳ Balanced supply
  ↳ $V_{LL} = 415V$.

$$Z = R + j2\pi f L$$
$$= 50 + j94.2$$





$$\bar{I}_L = \bar{I}_{ph}$$

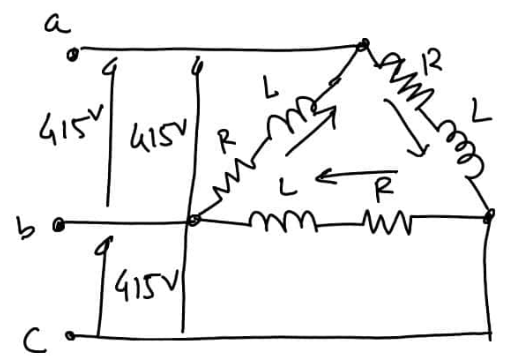
$$V_{ph} = \frac{V_{LL}}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

$$\bar{I}_L = \frac{\bar{V}_{ph}}{R + j\omega L} = \frac{240\angle 0^\circ}{50 + j94.2}$$

Balanced load.

$P = 3\, V_{ph}\, I_{ph} \cos\phi = \;?$ $\quad$ $\cos\phi = \angle Z$

b)



$Z = R + j\omega L$

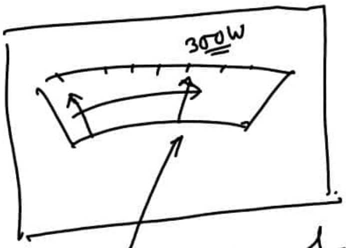
$|V_{LL}| = |V_{ph}| = 415$

$|I_{ph}| = \dfrac{415}{|R + j\omega L|}$

Check the power consumed by the two loads.

Power Measurement

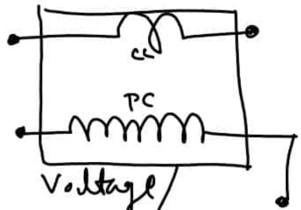
Wattmeter



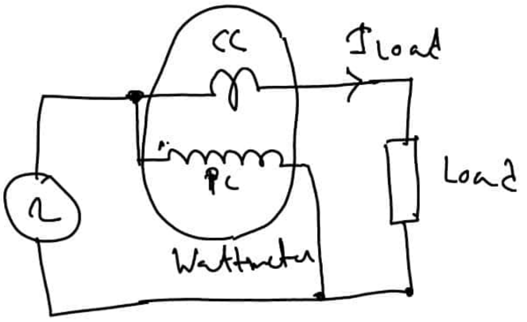
Average value of power

(very low impedance) → almost zero voltage drop in it.

Current Coil



Voltage/pressure/potential coil
(very high impedance/Resistance)
almost zero current drawn by it



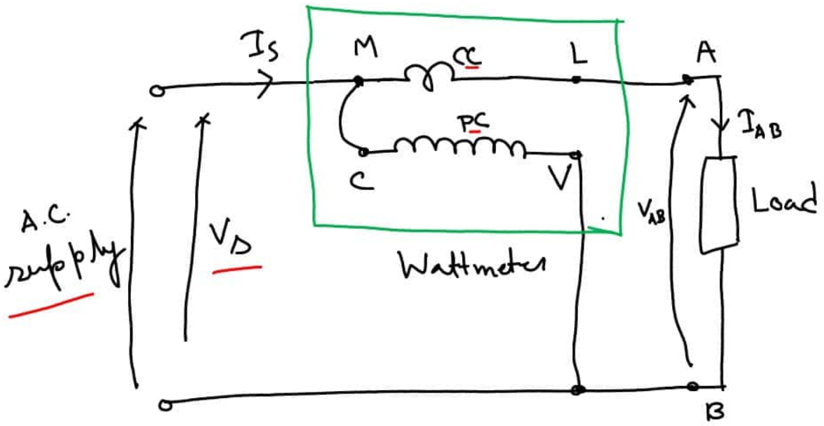
$P = V \; I \; \cos\phi$

RMS RMS p.f.

$\phi = \angle_{V}^{I}$

Lecture-14

04/01/2021



M => Mains
L => Load
C => Common
V => Voltage

CC => Current coil
PC => Pressure coil/voltage coil

Wattmeter Connection

$V_s \simeq V_{AB}$ (negligible drop in 'CC')

Wattmeter Reading → Power (W) $P = |V_{AB}|\,|I_a| \times$ (Cosine of angle betn voltage across 'PC' and current through 'cc')

$|I_a|$: I_{AB} — RMS value of current through 'cc'

$|V_{AB}|$: RMS value of voltage across 'PC'

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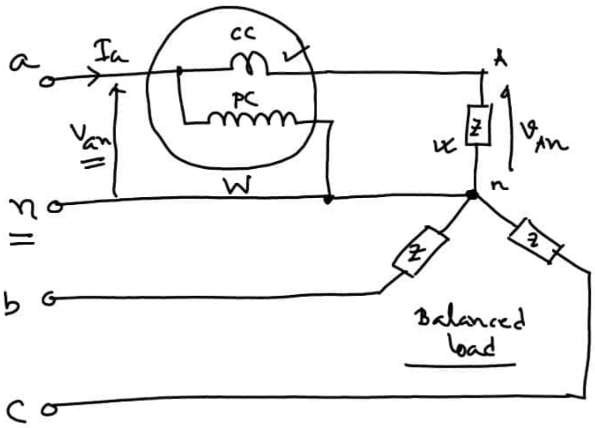
Measurement of three phase power

(Balanced system → 3 phase 4 wire
→ 3 phase 3 wire)

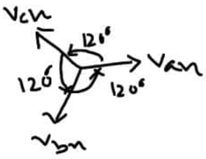
One Wattmeter Method :-



Total power = 3 × power in one phase

* Neutral point is available for connection (3ϕ 4 wire system)

Two Wattmeter Method



⇒ Phasor relationship of supply Phase voltage.

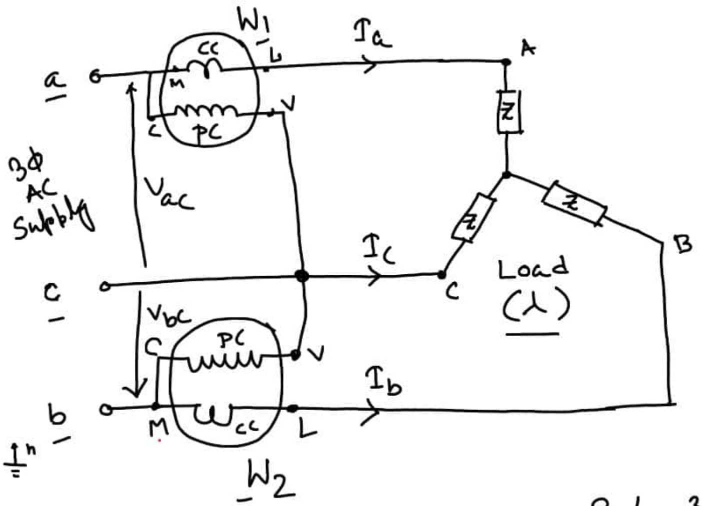
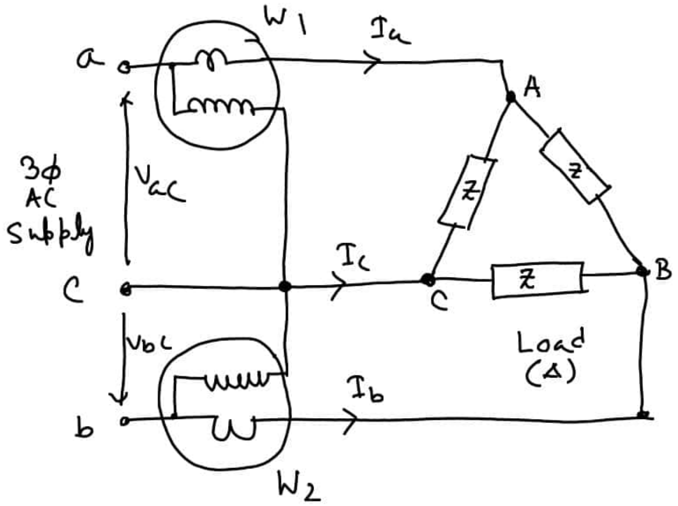


Fig. L



3φ - 3 Wire system (Balanced)

* Neutral point is not available.

Wattmeter W_1 →

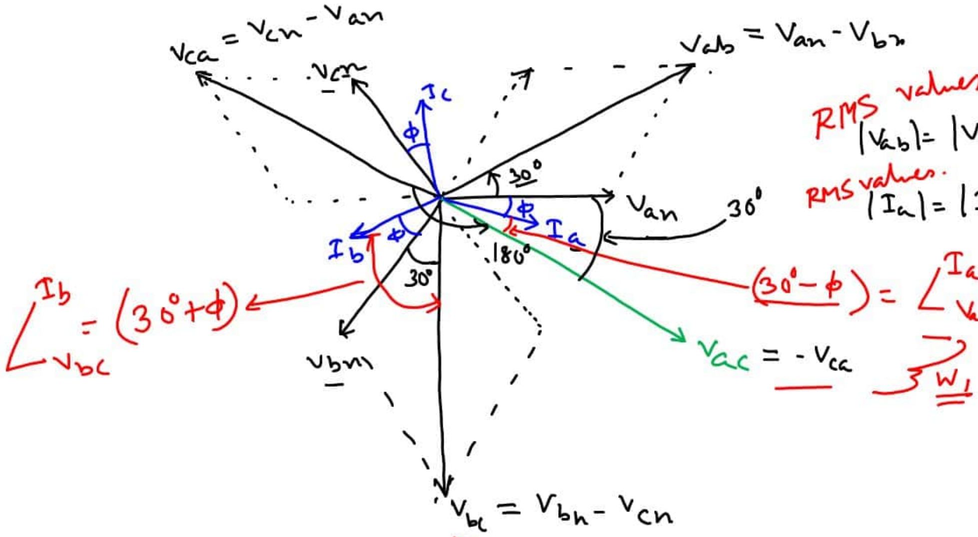
CC → I_a & PC = V_{ac} ⇒ $W_1 = |V_{ac}|\,|I_a| \cos\left(\angle_{V_{ac}}^{I_a}\right)$

Wattmeter W_2 →

CC → I_b & PC = V_{bc} ⇒ $W_2 = |V_{bc}|\,|I_b| \cos\left(\angle_{V_{bc}}^{I_b}\right)$

Lagging load

Current I_a, I_b & I_c are lagging respective phase voltages by angle ϕ.



RMS values $|V_{ab}| = |V_{bc}| = |V_{ca}| = V_L$ ✓

RMS values. $|I_a| = |I_b| = |I_c| = I_L$ ✓

Reading of $W_1 \rightarrow$

$$W_1 = |V_{ac}||I_a| \cos(30^\circ - \phi)$$

Reading of $W_2 \rightarrow$

$$W_2 = |V_{bc}||I_b| \cos(30^\circ + \phi)$$

* Summation of $W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi = P$ (3ϕ Active Power)

* $W_1 - W_2 = V_L I_L \cos(30^\circ - \phi) - V_L I_L \cos(30^\circ + \phi)$

$$= V_L I_L \sin\phi = \frac{Q}{\sqrt{3}} = \frac{\text{Reactive Power}}{\sqrt{3}}$$

$$\therefore \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \tan\phi \quad \Big| \quad \therefore \phi = \tan^{-1}\left(\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)$$

↳ p.f. angle.

$\phi \Rightarrow$ Can vary from $0°$ to $90°$

i) When $\phi = 0°$ then $\cos\phi = 1$. (u.p.f.) unity power factor

$\therefore W_1 = V_L I_L \cos(30° - \phi)\Big|_{\phi = 0°} = \frac{\sqrt{3}}{2} V_L I_L$

$W_2 = V_L I_L \cos(30° + \phi)\Big|_{\phi = 0°} = \frac{\sqrt{3}}{2} V_L I_L$

$\Big\}$ Hence $W_1 = W_2$.

ii) When $0° \leq \phi \leq 60°$ i.e. $\underbrace{1.0}_{\cos\phi \text{ when } \phi = 0°} \geq \cos\phi \geq \underbrace{0.5}_{\cos\phi \text{ when } \phi = 60°} \Rightarrow$ W_1 & W_2 both are positive

and $W_1 > W_2$.

At $\phi = 60° \Rightarrow$ $W_1 = V_L I_L \cos(30° - 60°) = \frac{\sqrt{3}}{2} V_L I_L$

$W_2 = V_L I_L \cos(30° + 60°) = 0$

$\Big\}$ So W_2 will read zero.

$W = W_1 + W_2$

iii) When $60° < \phi \leq 90°$ i.e. $\underbrace{0.5}_{\cos\phi \text{ when } \phi = 60°} > \cos\phi \geq \underbrace{0}_{\cos\phi \text{ when } \phi = 90°} \Rightarrow W_1$ is positive but W_2 is negative.

* when W_2 reads negative then either 'PC' or 'CC' needs to reversed and measurement of W_2 should be taken.

Then $W = W_1 + (-W_2) = W_1 - W_2$ (Calculation of Total power 'W' when W_2 is negative)

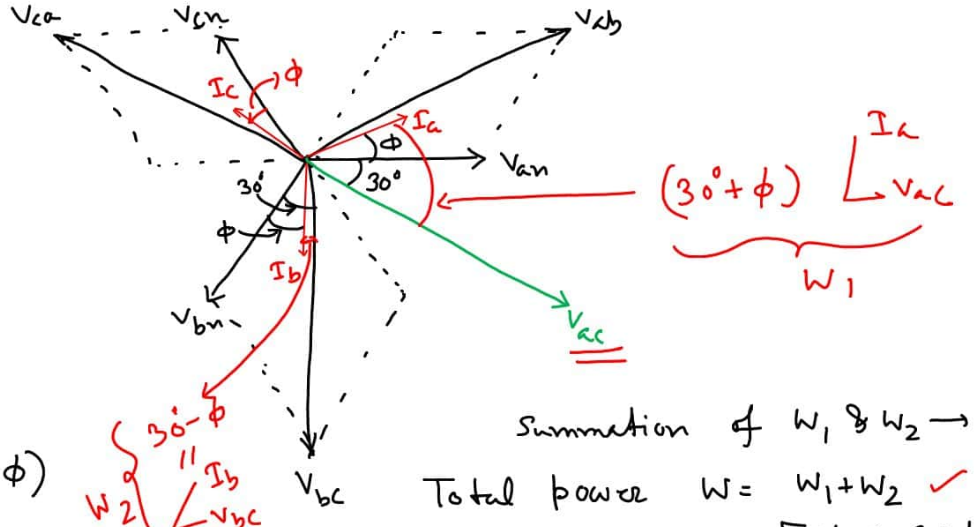
iv) At $\phi = 90°$; $W_1 = 0.5\, V_L I_L$

$W_2 = -0.5\, V_L I_L$

$\Big\}$ total power $W = W_1 + W_2 = 0$.

Leading load

Capacitive load => Current is leading voltage



Reading of W_1 →

$W_1 = V_L I_L \cos(30° + \phi)$

Reading of W_2 →

$W_2 = V_L I_L \cos(30° - \phi)$

Summation of W_1 & W_2 →

Total power $W = W_1 + W_2$

$= \sqrt{3} V_L I_L \cos\phi$

Calculation of $W_1 - W_2 = -V_L I_L \sin\phi$

$\Rightarrow W_2 - W_1 = V_L I_L \sin\phi$ ← $\frac{Q}{\sqrt{3}}$

* For leading power factor the readings of the Wattmeter are interchanged compared to the readings for lagging power factor.

	$\phi = 0^\circ$	$\phi = 30^\circ$	$\phi = 60^\circ$	$\phi = 90^\circ$
$\cos(30^\circ+\phi)$	$\sqrt{3}/2$	$\frac{1}{2}$	0	$-\frac{1}{2}$
$\cos(30^\circ-\phi)$	$\sqrt{3}/2$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

$\Rightarrow \cos(30^\circ-\phi) \geqslant \cos(30^\circ+\phi)$
$[0^\circ \le \phi \le 90^\circ]$

$$* \tan\phi = \sqrt{3} \times \frac{\text{Greater Wattmeter Reading} - \text{Smaller Wattmeter reading}}{\text{Sum of Wattmeter reading}}.$$

Preferred Way:

1. Connect the wattmeter with phase convention a, b, c (or R, Y, B) as shown in the connection diagram.
   (Phase convention wise → V_{an} is leading V_{bn} by 120° & V_{bn} is leading V_{cn} by 120°.)

2. Check the readings of wattmeters. If the readings are equal then its u.p.f.
   If readings are unequal then mark 'W_1' to the wattmeter which is showing higher value. The other one as W_2.

3. Reverse connection of W_2 if it is showing deflection in opposite direction. (Remember sign of W_2).

4. Now calculate $W = W_1 + W_2$ = Active power P

and p.f. $\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\ \frac{W_1 - W_2}{W_1 + W_2}\right)\right]$

↳ leading or lagging from phase Convention & marking of W_1 & W_2.

lagging load ↓ Wattmeter 'cc' connected to Phase a is W_1 | leading load ↓ Wattmeter 'cc' connected to Phase b is W_1

Reactive power $Q = \sqrt{3}\ (W_1 - W_2)$.

* 5.2 kW & 1.7 kW (motor load / lagging load). $V_L = 400V$.

↑ obtained after reversing the Wattmeter connection.

a) Total power = ? ; b) p.f. = ? ; I_L = ?

a) $W = W_1 + W_2 = 5.2 + (-1.7) = 3.5$ kW.

b) p.f. = $\cos\left(\tan^{-1}\left(\sqrt{3}\ \frac{W_1 - W_2}{W_1 + W_2}\right)\right) = 0.281$ lagging.

c) $I_L = \frac{P}{\sqrt{3} V_L \cos\phi} = \frac{3.5 \times 10^3}{\sqrt{3} \times 400 \times 0.281} = 17.98$ A.