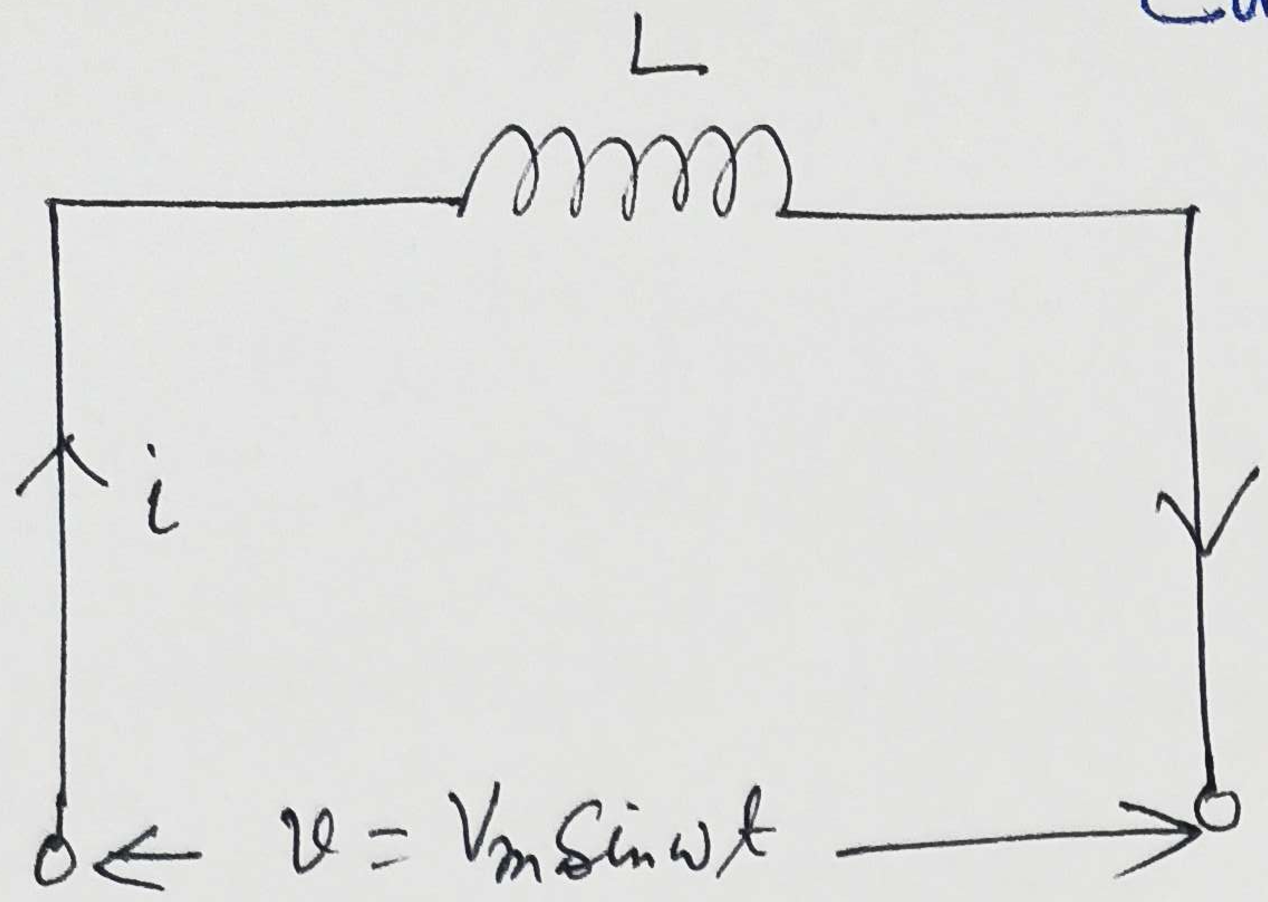
Current through a pure inductor.



$$e = -L\frac{di}{dt}$$

Applied voltage is equal and opposite to the self induced emf.

Hence, $v = -e$

$$= L\frac{di}{dt}.$$

$$v = V_m \sin\omega t$$

$$L\frac{di}{dt} = V_m \sin\omega t$$

$$di = \frac{V_m}{L}\sin\omega t\, dt$$

Integrating both sides of this equation.

$$i = \int \frac{V_m}{L}\sin\omega t \cdot dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t)$$

$$I_m = \frac{V_m}{\omega L}$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

The quantity ωL is called the inductive reactance and is denoted by X_L.

Power in purely inductive circuit

$$P = v \times i$$

$$= V_m \sin\omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

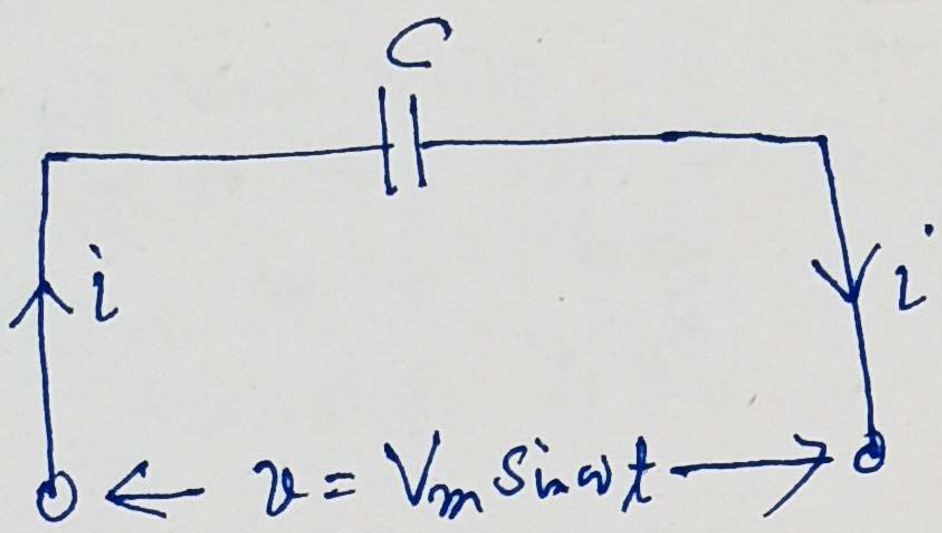
$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= -\frac{1}{2} V_m I_m \sin 2\omega t$$

Average power for the complete cycle

$$P = -\frac{1}{2} V_m I_m \text{ Average of } (\sin 2\omega t) = 0$$

A.C circuits containing capacitance only



$i = C \times$ Rate of change of Potential difference

$= C \frac{dv}{dt}$

$v = V_{max} \sin \omega t$

$i = \frac{d}{dt}(V_m \sin \omega t)$

$= C \omega V_m \cos \omega t$

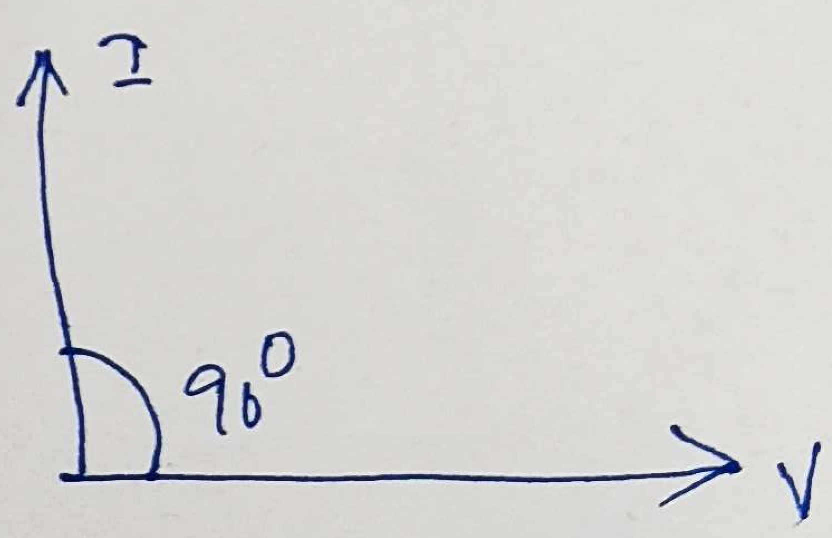
$= C \omega V_m (\sin \omega t + \frac{\pi}{2})$

$= \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2)$

The Max^m current $I_m = \frac{V_m}{1/\omega C}$

$\therefore i = I_m \sin(\omega t + \frac{\pi}{2})$

The term $\frac{1}{\omega C}$ is called the capacitive reactance and is denoted by X_C.



purely capacitive circuit

Power in purely capacitive circuit

$P = v \times i$

$= V_m \sin \omega t \times I_m \sin(\omega t + \frac{\pi}{2})$

$= V_m \sin \omega t \; I_m \cos \omega t$

$= \frac{1}{2} V_m I_m \sin 2\omega t$

Average power for one complete cycle

$= \frac{1}{2} V_m I_m \times$ Average of $(\sin 2\omega t) = 0$

P1 A non inductive load takes 20A at 200 volts. Calculate the inductance of the reactor to be connected in series in order to the same current be supplied from 230 volts, 50 Hz A.C supply. Also determine the power factor of the circuit.

Sln:- $R = \dfrac{V_{r.m.s}}{I_{r.m.s}} = \dfrac{200}{20} = 10\Omega$

$I_{r.m.s} = \dfrac{V_{r.m.s}}{Z}$; $Z = \dfrac{230}{20} = \dfrac{23}{2}\Omega$.

$X_L = \sqrt{Z^2 - R^2} \rightarrow L = 0.018\,H$.

$P.f = \cos\phi = \dfrac{R}{Z} = 0.8945$ (lagging).

P2 The voltage applied to a circuit: $v = 100 \sin(\omega t + 30°)$ and the current flowing in the circuit is: $i = 15 \sin(\omega t + 60°)$ Determine the impedance, resistance, reactance, power and power factor.

Soln:- $V_{r.m.s} = \dfrac{100}{\sqrt{2}}V$, $I_{r.m.s} = \dfrac{15}{\sqrt{2}}A$

$Z = \dfrac{V_{r.m.s}}{I_{r.m.s}} = 6.666\,\Omega$

Phase angle, $\phi = 30°$ (leading).

Reactance, $X_C = Z\sin\phi = 3.333\,\Omega$

$R = 5.773\,\Omega$, $P.f = \cos\phi = \cos 30° = 0.866$ (leading)

Power, $P = VI\cos\phi = 70.7 \times 10.6 \times 0.866 = 649\,W$.

P3 The current in a circuit is given by $(4.5 + J12)$, when the applied voltage is $(100 + J150V)$. Determine : (i) Complex expression for the impedance ii) Power and iii) P.f.

Soln:- $I = 12.816 \angle 69.44° A$,

$V = 180.27 \angle 56.30° V$

$$Z = \frac{V}{I} = \frac{180.27\angle 56.30^\circ}{12.816\angle 69.44^\circ} = 14.05\angle -13.14^\circ\,\Omega$$

$= (13.68 - J3.19)\,\Omega$ capacitive.

(iii) $P = VI\cos\phi = 180.27 \times 12.816 \times \cos(-13.14^\circ)$

$= 2.250\,W$

p.f = Power factor $= \cos(-13.14^\circ) = 0.9738$

P4 A coil takes 2.5A, when connected across 200V, 50HZ main. The power consumed by the coil is found to be 400W. Find the inductance and power factor of the coil.

Soln:- Power, $P = I^2R = (2.5)^2 \times R = 400\,W$

$R = \frac{400}{6.25} = 64\,\Omega$

$$Z = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{200}{2.5} = 80\,\Omega$$

$X_L = 48\,\Omega$, $L = 0.153\,H$

$\cos\phi = \frac{R}{Z} = 0.8$ (lagging).

Power in A.C. circuit

$P = v \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$

$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

It remains constant with — average value

Average power in one cycle

Reactive power $= VI\sin\phi$

$P = V_{rms} I_{rms} \cos\phi$ → Active power

Apparent Power $= V_{rms} \times I_{rms}$

Instantaneous values of voltage & current -

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \phi)$

$\phi \rightarrow$ phase angle by which current lags the voltage.

V_m, I_m are the peak values of voltage & current respectively.

ω = angular frequency in rad/sec

f = supply frequency in Hz.

$\omega = 2\pi f$.

$X \rightarrow$ reactance in Ω.

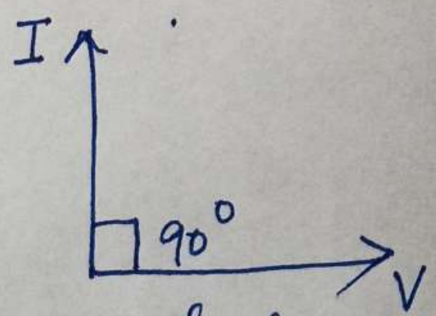
$Z \rightarrow$ Impedance in Ω

$X_L \rightarrow$ Inductive reactive in Ω

$X_C \rightarrow$ Capacitive reactance in Ω

$Z = R + jX$ in Complex form.

$|Z| = \sqrt{R^2 + X^2}$, $\theta = \tan^{-1} \frac{X}{R}$.



For purely ~~inductive~~ capacitive circuit

Problem:-

1. Find the current that will flow through a coil of negligible resistance and inductance of 60 mH, when connected to 230V, 50 HZ single phase supply. What will be the current if the frequency is (a) decreased to 20 HZ (b) increased to 60 HZ.

$$I = \frac{V}{X_L} = 12.2\,A = \frac{230}{18.86}\ \text{amps}$$

→ All values are r.m.s

a) $I = 30.49\,A$

b) $I = 10.16\,A$

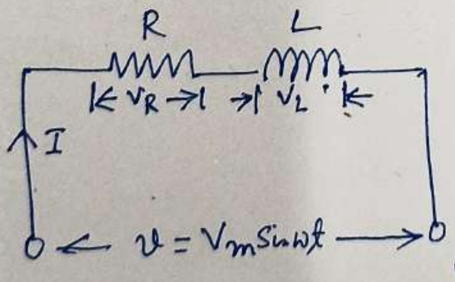
2. A capacitor of 100 uf is connected across a 200V, 50HZ single phase supply. Calculate (i) the reactance of the capacitor ii) rms value of current iii) maximum current.

i) $X_C = \frac{1}{\omega C} = 31.8\,\Omega$

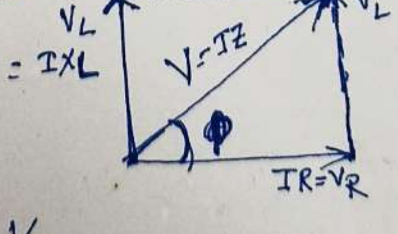
ii) $I_{rms} = \frac{V}{X_C} = 6.29\,A$ → R.M.S value of applied voltage

iii) $I_{max} = I\sqrt{2} = 8.87\,A$

Series R-L circuit



$v = V_m \sin\omega t$
$i = I_m \sin(\omega t - \varphi)$

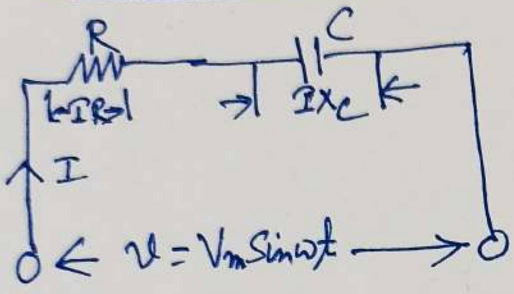


$$V = \left(V_R^2 + V_L^2\right)^{1/2}$$
$$= I\left(R^2 + X_L^2\right)^{1/2}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\tan\varphi = \frac{IX_L}{IR} = \frac{X_L}{R} \rightarrow \varphi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{\omega L}{R}$$

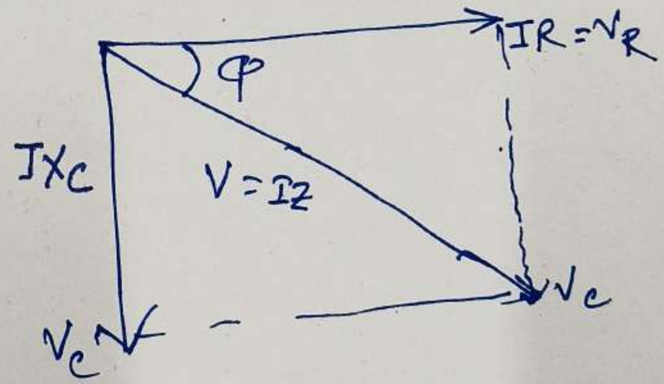
FOR series RC circuit



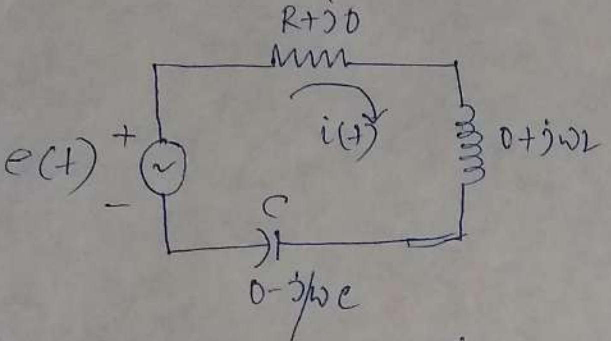
$v = V_m \sin\omega t$

$i = I_m \sin(\omega t + \varphi)$

$$\varphi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{1}{\omega CR}$$



Series R-L-C circuits



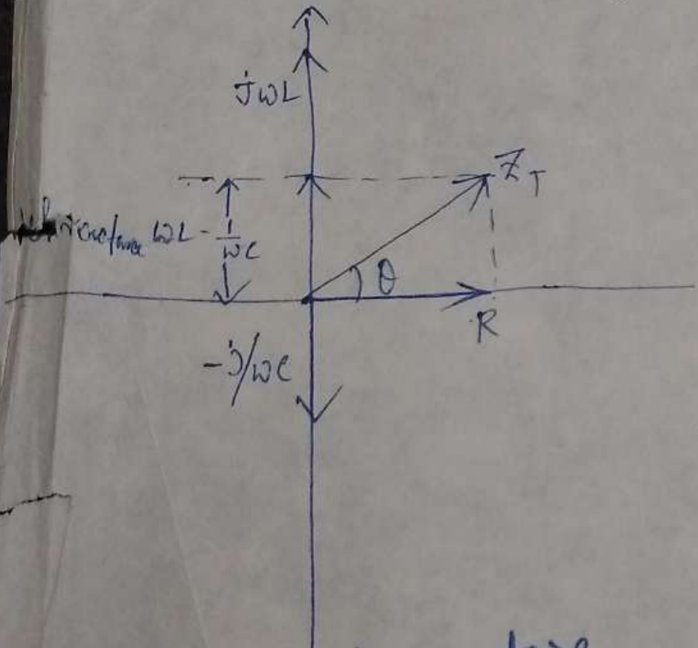
In this case, the total impedance is

$$Z_T = (R+j0) + (0+j\omega L) + (0 - j/\omega C).$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right) \Omega$$

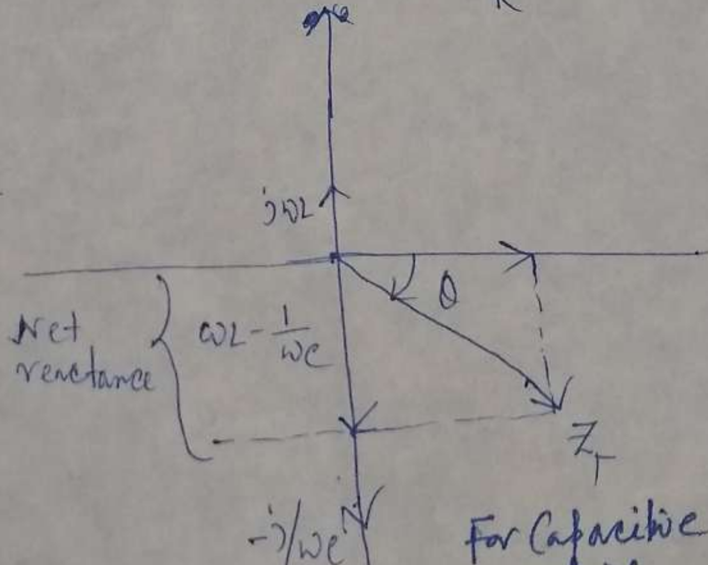
In terms of the magnitudes of the reactances, we have

$$Z_T = R + j(|X_L| - |X_C|) \Omega$$

$$|Z_T| = \sqrt{R^2 + (|X_L| - |X_C|)^2} \, \Omega \;, \quad \theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$
$$= \tan^{-1} \frac{|X_L| - |X_C|}{R}.$$



For inductive nature



For Capacitive nature

Resonance in series R-L-C circuit - 2

For resonance, $X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$\boxed{Z = R}$

Current becomes maximum at resonance condition.

Then at resonance, the frequency is called resonant frequency and determined from the relation,

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}.$$

We know, V_L = voltage across inductor $= IX_L$.

At resonance,

$$V_L = I \cdot 2\pi f_r L$$

$$= I \cdot 2\pi \cdot \frac{1}{2\pi\sqrt{LC}} \cdot L$$

$$= I\sqrt{L/C}.$$

$$= (V/R) \cdot \sqrt{L/C} = V \cdot \sqrt{\frac{L}{CR^2}}.$$

Voltage across a capacitor,

$$V_C = IX_C = \frac{I}{2\pi f_r C} \text{ at resonance}$$

$$= \frac{V}{R}\left[\frac{1}{\sqrt{C/L}}\right] = V \cdot \sqrt{\frac{L}{CR^2}}.$$

Again at resonance, voltage drop across the inductor and capacitor is equal & opposite.

We see that at resonance, voltage magnification occurs during resonance.

voltage magnification at resonance

$$= \frac{\text{Voltage across L or C}}{\text{Supply voltage}}$$

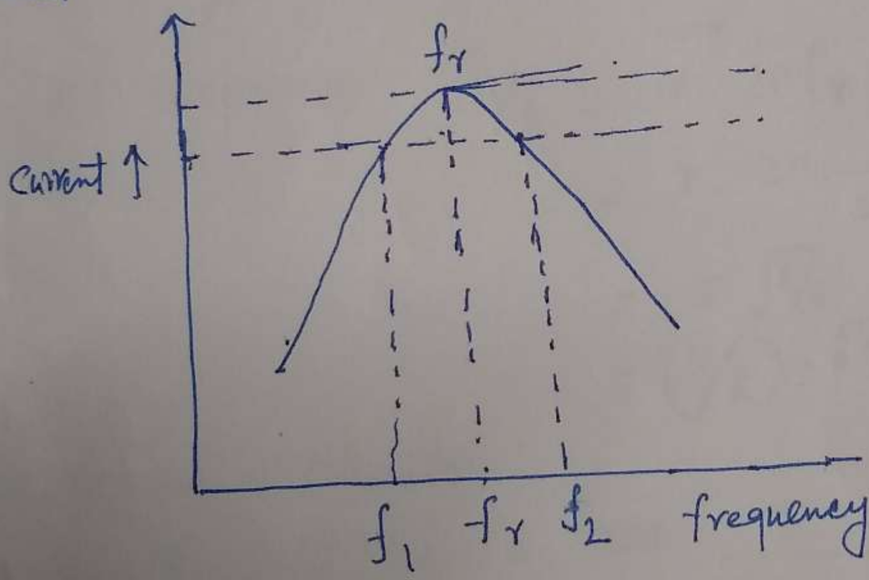
$$= \frac{V\sqrt{\frac{L}{CR^2}}}{V} = \sqrt{\frac{L}{CR^2}}$$

$$= \frac{\sqrt{L}}{R} \cdot \frac{1}{\sqrt{C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{\sqrt{L}}{R} \cdot 2\pi f_r \cdot \sqrt{L} = \frac{\omega_r L}{L}.$$

= This is called the Q factor

Q he circuit



f_1 & f_2 → half power frequency

$$I = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}.$$

at resonance,

$I_m = \frac{V}{R}$ at resonance.

Power at resonance, $I_m^2 R = \frac{V^2}{R^2} \cdot R$

$$= \frac{V^2}{R} \cdot \text{Watt.}$$

At ~~resonance~~ half power ~~frequency~~, frequencies — ~~current $= \frac{I_m}{\sqrt{2}}$~~.

Current, becomes $\frac{1}{\sqrt{2}}$ of current at resonance condition.

So power at those frequencies become,

$$= \left(\frac{V}{\sqrt{2} R}\right)^2 \cdot R$$

$$= \frac{V^2}{2R^2} \cdot R = \frac{V^2}{2R} \text{ Watt}$$

$\Delta f = f_1 \sim f_2$ is the bandwidth.

It can be shown,

$$\Delta f = f_1 \sim f_2 = \frac{R}{2\pi L}$$

$$\text{Bandwidth} = \Delta f = \frac{f_r R}{2\pi f_r L}$$

Again Q factor, $Q = \frac{2\pi f_r L}{R}$.

$$\therefore \boxed{\Delta f = \frac{f_r}{Q}}$$

Bandwidth, $= \frac{\text{Resonant frequency}}{\text{Q factor.}}$

1. A Series R.L.C circuit is composed of a 15 μH, 10 Ω inductor and and a 100 PF Capacitor. Determine it's band width.

Solution: - $f_r = \frac{1}{2\pi\sqrt{LC}} = 4.11\times10^6$ HZ

$X_L = 2\pi f_r L = 387.2\ \Omega$

$Q = \frac{X_L}{R} = \frac{387.2}{10} = 38.72$

Band width, $BW = \frac{f_r}{Q} = \frac{4.11\times10^6}{38.72} = 106.15$ KHZ

2. A Series LC circuit is resonance at 150 KHZ and has a Q of 50. Find upper and lower cut off frequencies: -

$BW = \frac{f_r}{Q} = \frac{150}{50} = 3$ KHZ

upper half power frequency,

$f_{upp} = \frac{BW}{2} + f_r = \frac{3}{2} + 150 = 151.5$ KHZ

lower cut off freq, $f_{low} = f_r - \frac{BW}{2} = 150 - \frac{3}{2}$
$= 148.5$ KHZ

Q factor

Q factor of a series circuit indicates how many times the P.d across L or C is greater than the applied voltage at resonance. If Q factor is 20. and 240 V is the applied voltage across a series R.L.C circuit, then the voltage across the L or C at resonance is $240\times20 = 4800$ V

Deduction of relation,

$$\Delta f = \frac{R}{2\pi L}.$$

at half power frequency, i.e at f_1 or at f_2

$$\frac{V}{\sqrt{2}R} = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$\therefore\ 2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\therefore\ R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

or $$\omega L - \frac{1}{\omega C} = \pm R$$

$$\therefore\ \omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\Downarrow$$

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}.$$

for low values of R, the term, $\frac{R^2}{4L^2}$ can be neglected in comparison with the term $\frac{1}{LC}$.

then, $$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}}.$$

Again, $$\omega_r = \frac{1}{\sqrt{LC}}.$$

$$\therefore\ \omega = \pm\frac{R}{2L} + \omega_r$$ ω_r can be positive value only.

$$\omega_1 = \omega_r + \frac{R}{2L}.$$

$$\omega_2 = \omega_r - \frac{R}{2L}.$$

$$\therefore\ \omega_2 - \omega_1 = \frac{R}{L}$$ or, $$f_2 - f_1 = \frac{R}{2\pi L}.$$

Admittance and susceptance

The reciprocal of impedance is called admittance.

$$Y = \frac{1}{Z}.$$

The terms "impedance" and "admittance" are quite descriptive: Impedance is a measure of the extent to which a component impedes the flow of ac current through it, while admittance is a measure of how well it admits the flow of current. The greater the impedance, the smaller the admittance and vice versa.

Conductance $G = \frac{1}{R\angle 0^0} = \frac{1}{R}\angle 0^0$ Siemens. Conductance is one form of admittance.

Reactance is another special case of impedance, so the reciprocal of reactance is a special case of admittance. The reciprocal of reactance is ~~called~~ a special case of admittance. The reciprocal of reactance is called susceptance, denoted by B.

$$B = \frac{1}{X}$$

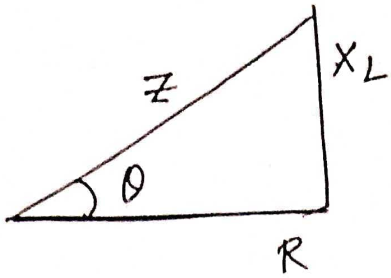
$$B_L = \frac{1}{X_L} = \frac{1}{\omega L\angle 90^0} = \frac{1}{\omega L}\angle -90^0 \text{ siemens}$$

$$= 0 - j\left(\frac{1}{\omega L}\right) \text{ siemens.}$$

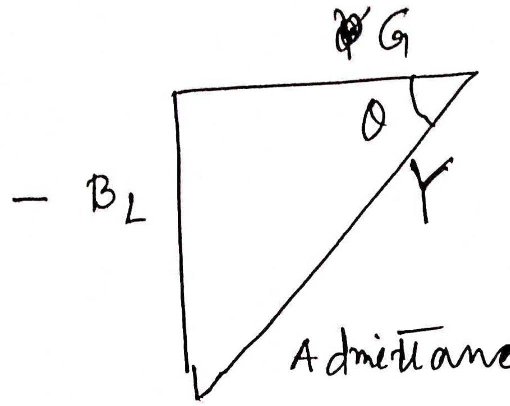
$$B_C = \frac{1}{X_C} = \frac{1}{(1/\omega C)\angle -90^0} = \omega C\angle 90^0 \text{ Siemens.}$$

Impedance traingle

For Inductive circuit -



$$Z = (R^2 + X_L^2)^{1/2}$$



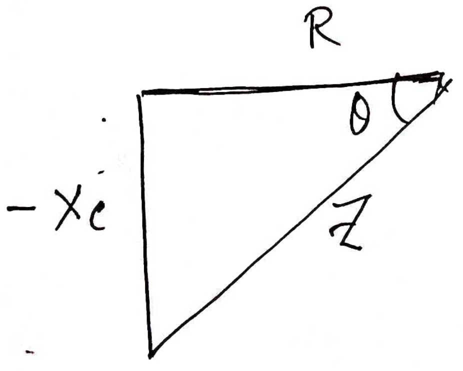
Admittance traingle

$$G = Y\cos\theta$$
$$= Y \cdot \frac{R}{Z} = \frac{1}{Z} \cdot \frac{R}{Z}$$
$$= \frac{R}{Z^2}$$

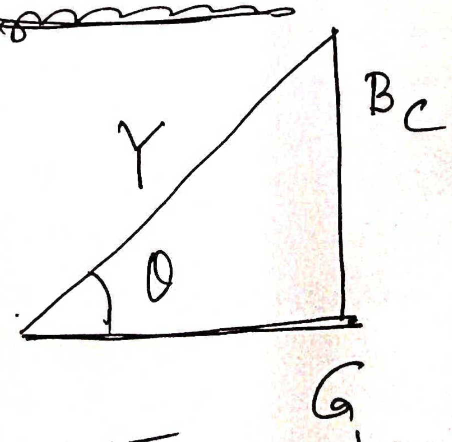
$$B_L = Y\sin\theta$$
$$= \frac{1}{Z} \cdot \frac{X_L}{Z}$$
$$= \frac{X_L}{Z^2}.$$

$$\tan\theta = -\frac{B_L}{G}.$$

For Capacitive circuit



Impedance traingle



Admittance traingle

$$Z = (R^2 + X_C^2)^{1/2}$$

$$G = Y\cos\theta$$
$$= \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2}$$

$$\tan\theta = \frac{B_C}{Z}.$$

$$B_C = Y\sin\theta$$
$$= \frac{1}{Z} \cdot \frac{X_C}{Z} = \frac{X_C}{Z^2}$$

P. A 10 Ω resistor, a 15.9 mH inductor and 159 μF capacitor are connected in a parallel to a 200V, 50 Hz source. Calculate the supply current and p.f.

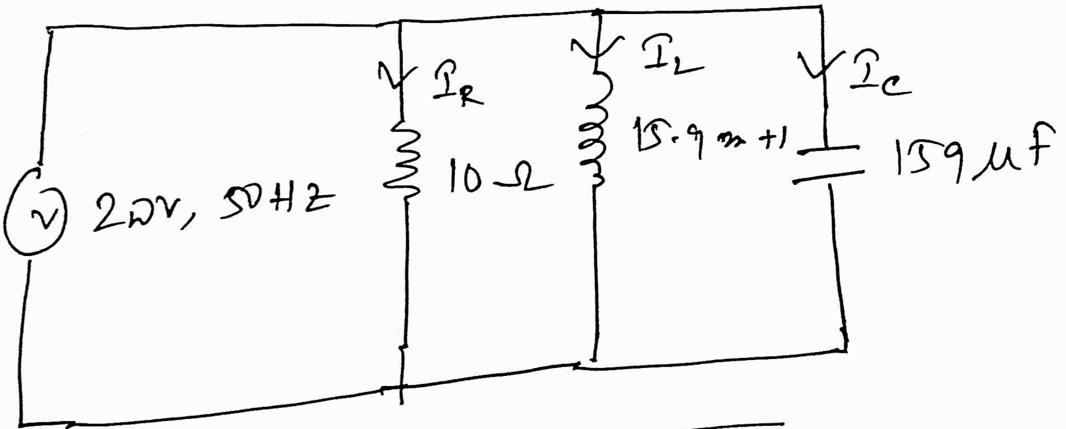
$$X_L = 2\pi f L = 5\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = 20\,\Omega$$

$$I_R = V/R = \frac{200}{10} = 20\,A \quad \text{in phase with } V$$

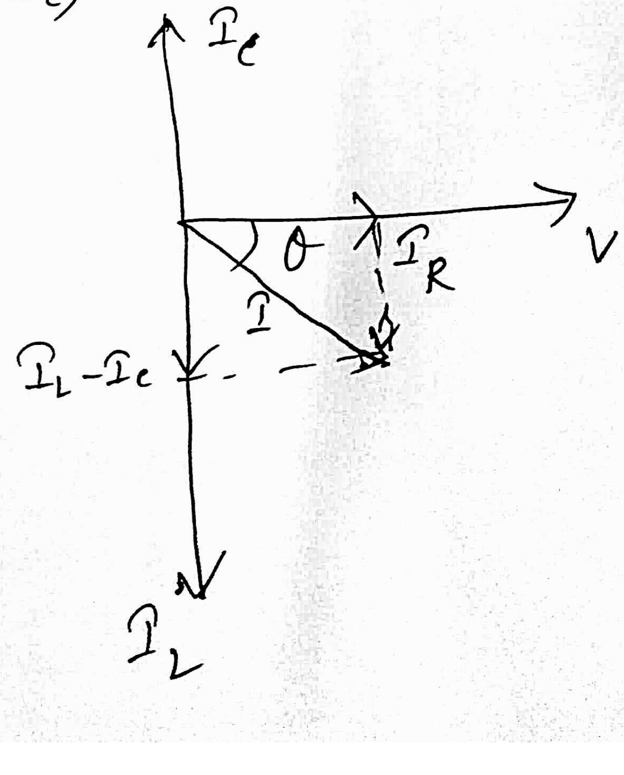
$$I_L = V/X_L = \frac{200}{5} = 40\,A \quad \text{lags } V \text{ by } 90^\circ$$

$$I_C = V/X_C = \frac{200}{20} = 10\,A \quad \text{leads } V \text{ by } 90^\circ$$

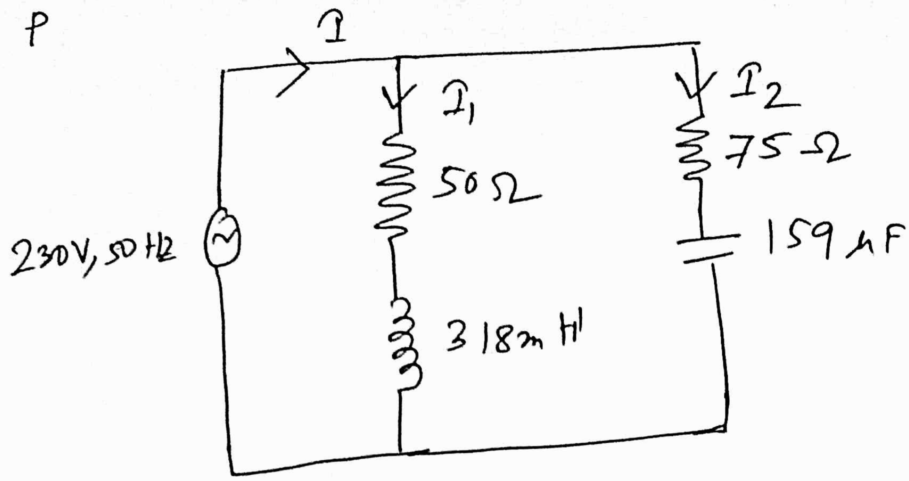


$$I = \sqrt{I_R^2 + (I_L - I_C)^2} = 36\,A$$

$$\cos\theta = \frac{I_R}{I} = \frac{20}{36} = 0.56 \text{ lag}$$



P



Find circuit current & P.f.

$V = 230\angle 0^\circ$ V

$X_L = 100\ \Omega$

$Z_1 = R + jX_L$

$= 112\angle 63.5^\circ\ \Omega$

$I_1 = \frac{V}{Z_1} = 0.91 - j1.83$ A

$X_C = 20\ \Omega$, $Z_2 = R_2 - jX_C$

$= 77.6\angle -15^\circ\ \Omega$

$I_2 = \frac{V}{Z_2} = 2.96\angle 15^\circ$ A

$= (2.86 + j0.766$ A$)$

Supply current, $I = I_1 + I_2 = 3.92\angle -15.7^\circ$ A

$\cos\phi = 0.963$ lag.