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PHYSICS
TARGET IIT JEE

ELECTROMAGNETIC INDUCTION

THEORY AND EXERCISE BOOKLET

CONTENTS

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ELECTROMAGNETIC INDUCTION

In previous lesson we studied about magnetic field produced by a moving charge and the force on it when placed in a magnetic field. In this lesson we will see how the current and emf are induced in a circuit when the magnetic flux through the circuit changes with time.

In 1820, the connection between electricity and magnetism was demonstrated by Faraday and independently by Joseph Henry. They showed that an electric current could be induced in a circuit by a changing magnetic field.

The result of these experiments led to a very basic and important law of electromagnetism known as Faraday's law of induction.

In this lesson we discuss the ideas of flux and the basic law of producing induced emf. Then we shall discuss inductor, which plays on important role in electrical circuits and its effect known as self induction and mutual induction. Finally, we examine the characteristics of circuits containing inductors, resistors and capacitors in various combinations.

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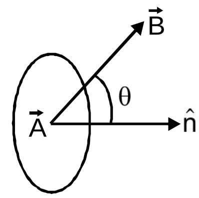
IIT-JEE Syllabus* : *Faraday's Law, Lenz's Law; Motional Emf; Self and mutual inductance. RC and LR circuits with d.c. sources.

1. MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field $\vec{B}$ in that region. The magnetic flux through the area $\vec{A}$ is given by



$\phi = \vec{B}.\vec{A}$ ...(i)

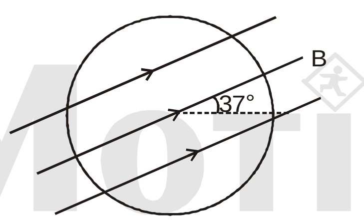
= BA cos θ

where θ is the angle which the vector B makes with the normal to the surface. If $\vec{B}$ is perpendicular to $\vec{A}$, then the flux through the closed area $\vec{A}$ is zero. SI unit of magnetic flux is weber (Wb).

Notes :

- Area vector is $\perp$ to the surface
- For open surface choose one direction as the area vector direction and stick to it for the whole problem.
- For closed surfaces outward normal is taken as area vector direction
- Flux is basically count of number of lines crossing a surface
- $\oint \vec{B}.\vec{ds} = 0$ because magnetic field lines exists in closed loop.

Ex.1 Find flux passing through Area ?



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Sol. Since $\vec{A}$ is $\perp$ to $\vec{B}$

$\phi = \vec{B}.\vec{A} = 0$

2. FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

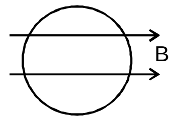
$$\varepsilon = -\frac{d\phi}{dt} \qquad \text{...(ii)}$$

where $\phi = \int \vec{B}.d\vec{A}$ the flux of magnetic field through the area.

The emf so produced drives an electric current through the loop. If the resistance of the loop is R, then the current

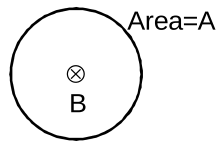
$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt} \qquad \text{...(iii)}$$

Ex.2 ***A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.***



Sol. ϕ = 0 (always) since area is perpendicular to magnetic field.
$\therefore$ emf = 0

Ex.3 ***Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.***



Sol. ϕ = BA (always) = const.
$\therefore$ emf = 0

Ex.4 ***Show that if the flux of magnetic induction through a coil changes from ϕ_1 to ϕ_2, then the charge q that flows through the circuit of total resistance R is given by q = $\frac{\phi_2 - \phi_1}{R}$, where R is the resistance of the coil.***

Sol. Let ϕ be the instantaneous flux. Then $\frac{d\phi}{dt}$ is the instantaneous rate of change of flux which is equal to the magnitude of the instantaneous emf. so the current in the circuit $|i| = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$, since the current is the rate of flow of charge, that is, $i = \frac{dq}{dt}$

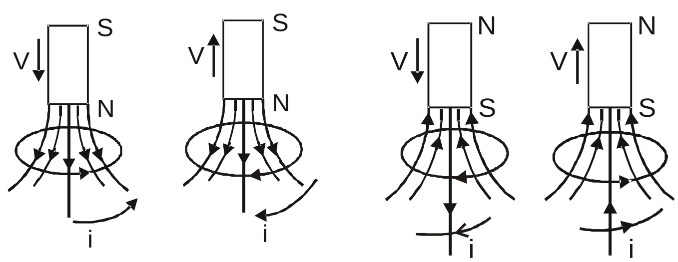
$$q = \int i\,dt \quad \text{or} \quad q = \int_{t=0}^{t=t}\left(\frac{1}{R}.\frac{d\phi}{dt}\right)dt$$

where τ is the time during which change takes place. but at t = 0, $\phi = \phi_1$, and at t = t, $\phi = \phi_2$

$$\therefore \quad \mathbf{q = \frac{1}{R}\int_{\phi=\phi_1}^{\phi=\phi_2} d\phi = \frac{\phi_2 - \phi_1}{R}}$$

3. **LENZ'S LAW**

The effect of the induced emf is such as to oppose the change in flux that produces it.



(a) **(c)** **(b)** **(d)**

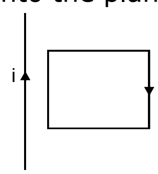
In figure (a & b) as the magnet approaches the loop, the flux through the loop increases. The induced current sets up an induced magnetic field B_{ind} whose flux opposes this change. The direction of B_{ind} is opposite to that of external field B_{ext} due to the magnet.

In figure (c & d) the flux through the loop decreases as the magnet moves away from the loop, the flux due to the induced magnetic field tries to maintain the flux through the loop. The direction of B_{ind} is same as that of B_{ext} due to magnet.

Lenz's law is closely related to the law of conservation of energy and is actually a consequence of this general law of nature. As the north pole of the magnet moves towards the loop an induced current is produced. This opposes the motion of N-pole of the bar magnet. Thus, in order to move the magnet toward the loop with a constant velocity an external force is to be applied. The work done by this external force gets transformed into electric energy, which induces current in the loop.

There is another alternative way to find the direction of current inside the loop which is described below.

Figure shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, then there will be an emf induced inside the loop. Due to this induced emf, an electric current is induced. To determine the direction of current inside the loop we put an arrow as shown. The right hand thumb rule shows that the normal to the loop is going into the plane. Again the same rule shows that the magnetic field at the site of the loop is also going into the plane of the diagram.

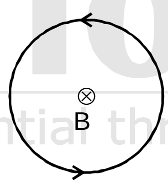


Thus $\vec{B}$ and $d\vec{A}$ are in the same direction. Therefore $\int \vec{B}.d\vec{A}$ is positive if i increases, the magnitude of flux ϕ increases. Since magnetic flux ϕ is positive and its magnitude increases, $\frac{d\phi}{dt}$ is positive. Thus ε is negative and hence the current is negative. Thus the current induced is opposite, to that of arrow.

Brain Teaser

Two identical coaxial circular loops carry equal currents circulating in the same direction. What will happen to the current in each loop if the loops approach each other ?

Ex.5 Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



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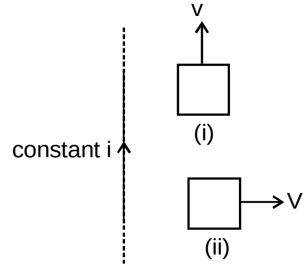
Sol. Inward flux is increasing with time. To oppose it outward magnetic field should be induced. Hence current will flow in anticlockwise.

Ex.6 Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



Sol. In loop (i) no emf will be induced because there is no flux change.

In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

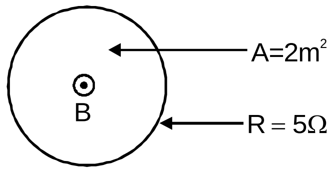
4. CALCULATION OF INDUCED EMF

As we know that magnetic flux (ϕ) linked with a closed conducting loop = BA cos θ where B is the strength of the magnetic field, A is the magnitude of the area vector and θ is the angle between magnetic field vector and area vector.

Hence flux will be affected by change in any of them, which is discussed in the next page.

4.1 By changing the magnetic field

Ex.7 ***Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10 T/s. Find out current in magnitude and direction***



Sol. ϕ = B.A

emf = A.$\frac{dB}{dt}$ = 2 × 10 = 20 v

$\therefore$ i = 20/5 = 4amp. From lenz's law direction of current will be anticlockwise.

Ex.8 ***The magnetic flux (ϕ_2) in a closed circuit of resistance 20 Ω varies with time (t) according to the equation $\phi = 7t^2 - 4t$ where ϕ is in weber and t is in seconds. The magnitude of the induced current at t = 0.25 s is***

(A) 25 mA (B) 0.025 mA (C) 47 mA (D) 175 mA

Sol. $\phi = 7t^2 - 4t$

$\Rightarrow$ Induced emf : $|e| = \frac{d\phi}{dt} = 14t - 4$

$\Rightarrow$ Induced current :

$$i = \frac{|e|}{R} = \frac{|14t-4|}{20} = \frac{|14 \times 0.25 - 4|}{20} \text{ (at t = 0.25 s)}$$

$$= \frac{0.5}{20} = 2.5 \times 10^{-2} \text{ A}$$

$\therefore$ (A)

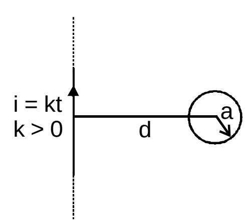
Ex.9 ***Consider a long infinite wire carrying a time varying current i = kt (k > 0). A circular loop of radius a and resistance R is placed with its centre at a distance d from the wire (a < < d). Find out the induced current in the loop?***

Sol. Since current in the wire is continuously increasing therefore we conclude that magnetic field due to this wire in the region is also increasing.

Magnetic field B due to wire $= \frac{\mu_0 i}{2\pi d}$ going into and perpendicular to the plane of the paper

Flux through the circular loop,

$$\phi = \frac{\mu_0 i}{2\pi d} \times \pi a^2$$



$$\phi = \frac{\mu_0 a^2 kt}{2d}$$

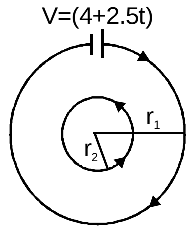
Induced e.m.f. in the loop

$$\varepsilon = -\frac{d\phi}{dt} = \frac{-\mu_0 a^2 k}{2d}$$

Induced current in the loop $i = \frac{|\varepsilon|}{R} = \frac{\mu_0 a^2 k}{2dR}$

Direction of induced current in the loop is anticlokwise.

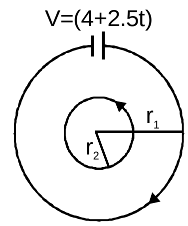
Ex.10 ***Two concentric coplanar circular loops have diameters 20 cm and 2 m and resistance of unit length of the wire = 10^{-4} Ω/m. A time -dependent voltage V =(4 + 2.5 t) volts is applied to the larger as shown. The current in the smaller loop is***



Sol. $r_1 = 1.0$ m, $r_2 = 10^{-1}$ m

Resistance of outer loop $= 2\pi \times 10^{-4}\ \Omega$

Resistance of inner loop $= 0.2\pi \times 10^{-4}\ \Omega$



Current in outer loop $= \dfrac{V}{R} = \dfrac{(4+2.5t)}{2\pi \times 10^{-4}}$ A

or $i_0 = \left[\left(\dfrac{2}{\pi}\right) \times 10^4 + \left(\dfrac{1.25}{\pi}\right) \times 10^4 \times t\right]$ A

Magnetic field produced at the common centre (see figure)

$$B = \frac{\mu_0 i}{2r_1}$$

or $$B = \frac{4\pi}{2} \times 10^{-7} \times \frac{[(2+1.25t)\times 10^4]}{\pi} = 2 \times 10^{-3}(2+1.25t)\ \text{T}$$

Hence, flux linked with the inner loop,

$\phi = BA = 2 \times 10^{-3}\ (2 + 1.25\ t) \times \pi(0.1)^2 = 2\pi \times 10^{-5}\ (2 + 1.25t)$ Wb

Hence, the e.m.f. induced in smaller loop =

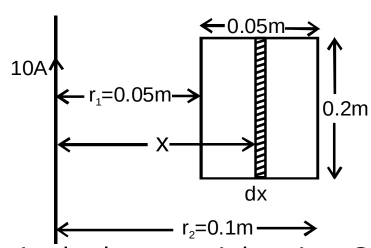
$$\varepsilon = -\frac{d\phi}{dt} = -2\pi \times 10^{-5} \times 1.25 = -2.5\pi \times 10^{-5}\ \text{V}$$

The negative sign indicates that the induced e.m.f. (or current) is opposite to applied e.m.f. (or current)

Hence, the current induced in the inner (smaller) loop is

$$i = \frac{|\varepsilon|}{R} = \frac{2.5\pi \times 10^{-5}\ \text{V}}{(0.2\pi \times 10^{-4})\Omega} = 1.25\ \text{A}$$

Ex.11 ***A rectangular wire frame of length 0.2 m, is located at a distance of 5 × 10^{-2} m from a long straight wire carrying a current of 10 A as shown in the figure. The width of the frame = 0.05 m. The wire is in the plane of the rectangle. Find the magnetic flux through the rectangular circuit. If the current decays uniformly to 0 in 0.2 s, find the emf induced in the circuit.***



Sol. A current, *i* = 10 A is flowing in the long straight wire. Consider a small rectangular strip (in the rectangular wire frame) of width dx at a distance x from the straight wire.

The magnetic flux at the location of the strip,

$$B_x = \frac{\mu_0 i}{2\pi x}$$

The flux linked with the infinitesimally small rectangular strip

$$= B_x \times \text{Area of the strip} = d\phi_x = \frac{\mu_0 i}{2\pi x}\, l\, dx$$

where l is the length of the rectangular wire circuit

$$= 2 \times 10^{-1}\,\text{m}$$

or $d\phi_x = (\mu_0\, i l/2\pi)\,(dx/x)$

Hence, the total magnetic flux linked with the rectangular frame

$$= \int d\phi_x = \phi = \frac{\mu_0 i l}{2\pi}\left[\log_e x\right]_{r_1}^{r_2}$$

or $\phi = \dfrac{\mu_0 i l}{2\pi}\left[\log_e r_2 - \log_e r_1\right] = \dfrac{\mu_0 i l}{2\pi}\log_e\left(\dfrac{r_2}{r_1}\right)$

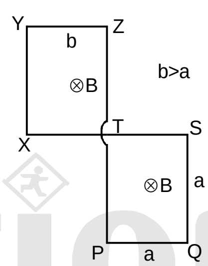
Substituting values, we get

$$\phi = 2 \times 10^{-7} \times 10 \times 2 \times 10^{-1} \times \log_e 2$$
$$= 2.772 \times 10^{-7}\ \text{Wb}$$

Induced e.m.f. $|\varepsilon| = \dfrac{d\phi}{dt} = \dfrac{\mu_o l \log_e\left(\dfrac{r_2}{r_1}\right)}{2\pi}\dfrac{di}{dt}$

$$= (2 \times 10^{-7} \times 2 \times 10^{-1} \log_e 2)\frac{10}{0.2}$$
$$= 1.386 \times 10^{-6}\ \text{V} = 1.386\ \mu\text{V}$$

Ex.12 Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as B = βt, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.



Sol. Induced emf in part PQST = βa^2 (in anticlockwise direction, from Lenz's Law) Similarly induced emf in part TXYZ = βb^2 (in anticlockwise direction, from Lenz's Law)

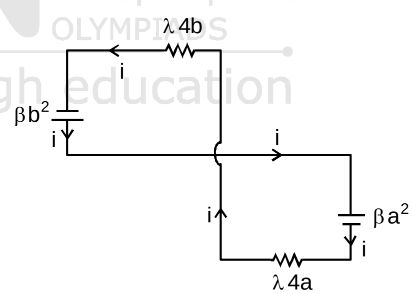
Total resistance of the part PQST = $\lambda 4a$

Total resistance of the part PQST = l4b.

The equivalent circuit it is shown in the diagram.

writing KVL along the current flow

$\beta b^2 - \beta a^2 - \lambda\, 4ai - \lambda\, 4bi = 0$



$$i = \frac{\beta}{4\lambda}(b - a)$$

Brain Teaser :

A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than or lesser than the acceleration due to gravity ?

4.2 BY CHANGING THE AREA

Solved Examples :

Ex.13 A space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ***ω about an axis passing through O, and perpendicular to the plane of the paper in the clockwise direction. The effective resistance of the loop is R.***

(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced emf and the time of rotation for two periods of rotation.

Sol. (a) As in time t, the arc swept by the loop in the field, i.e., region II.

$$A = \frac{1}{2}r(r\theta) = \frac{1}{2}r^2\omega t$$

So the flux linked with the rotating loop at time t,

$$\phi = BA = \frac{1}{2}B\omega r^2 t \quad [\theta = \omega t]$$

and hence the induced emf in the loop,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{1}{2}B\omega r^2 = \text{constant.}$$

And as the resistance of the loop is R, the induced current in it,

$$i = \frac{\varepsilon}{R} = -\frac{B\omega r^2}{2R}$$

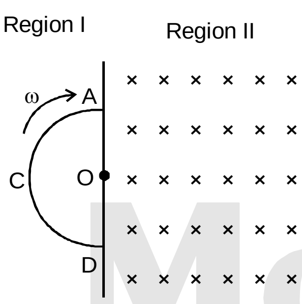


Fig (a)

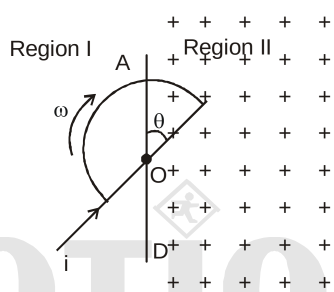


Fig (b)

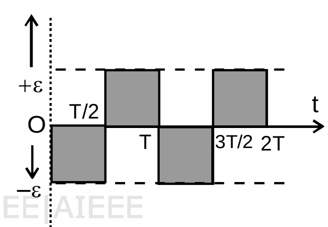


Fig.(c)

(b) When the loop is entering the region II, i.e., the field figure (b), the inward flux linked with it will increase, so in accordance with Lenz's law an anticlockwise current will be induced in it.

(c) Taking induced emf to the negative when flux linked with the loop is increasing and positive when decreasing, the emf versus time graph will be, as shown in figure (c)

Ex.14 Two parallel, long, straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now they start moving out with a constant velocity (v). (a) Will the induced emf be time dependent? (b) Will the current be time dependent?

Sol. (a) Yes, φ (instantaneous flux) = B (a + 2vt)^2

$$\therefore \qquad \varepsilon = \frac{d\varphi}{dt} = 4Bv(a + 2vt)$$

(b) No,

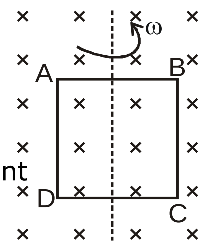
(instantaneous current) $i = \dfrac{\varepsilon}{R}$

Now $R = 4(a + 2vt)r$ where r = resistance per unit length

$\therefore \quad i = \dfrac{4Bv(a+2vt)}{4r(a+2vt)} = \dfrac{Bv}{r}$ (a constant)

The current will be time independent

Ex.15 ***Find the direction of induced of current in the wire AB. When rotated anticlockwise through angle θ, if it is placed initially as shown in the figure***

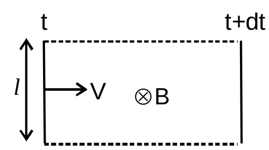


Sol.

Range of Angle	Rotated	Direction of induced current
0 – 90		A to B
90 – 180		A to B
180 – 270		B to A
270 – 360		B to A

4.3 Motional Emf

We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length l moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to lv and hence it will cut B l v lines per unit time.



Hence emf induced between the ends of the rod = Bvl

Also emf = $\dfrac{d\phi}{dt}$. Here ϕ denotes flux passing through the area, swept by the rod. The rod sweeps an area equal to l vdt in time interval dt. Flux through this area = Bl vdt. Thus $\dfrac{d\phi}{dt} = \dfrac{Bl\,vdt}{dt} = Bvl$

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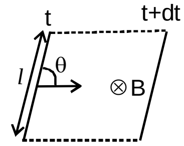
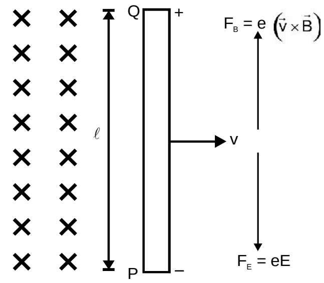
If the rod is moving as shown in the following figure, it will sweep area per unit time = v l sin θ



and hence it will cut B v l sin θ lines per unit time. Thus emf = $Bvl\sin\theta$.

4.3.1 Mechanism of The induced EMF a cross the ends of a moving rod :

Figure shows a conducting rod of length l moving with a constant velocity v in a uniform magnetic field. The length of the rod is perpendicular to magnetic field, and velocity is perpendicular to both the magnetic field and the length of the rod. An electron inside the conductor experiences a magnetic force $\vec{F}_B = -e(\vec{v} \times \vec{B})$ directed downward along the rod. As a result electrons migrate towards the lower end and leave unbalanced positive charges at the top. This redistribution of charges sets up an electric field E directed downward. This electric field exerts a force on free electrons in the upward direction. As redistribution continues electric field grows in magnitude until a situation, when

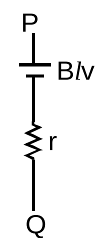


$$|q\vec{v} \times \vec{B}| = |q\vec{E}|$$

After this, there is no resultant force on the free electrons and the potential difference across the conductor is

$$\int d\varepsilon = -\vec{E}.d\vec{l} = \int (\vec{v} \times \vec{B}).d\vec{l} \qquad ...(4)$$

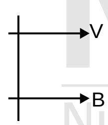
Thus it is the magnetic force on the moving free electrons that maintains the potential difference. So e.m.f. developed across the ends of the rod moving perpendicular to magnetic field with velocity perpendicular to the rod,



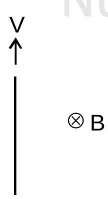
$$\varepsilon = vBl \qquad ...(5)$$

As this emf is produced due to the motion of the conductor, it is called motional emf.
In the problems related to motional e.m.f. we can replace the rod by a battery of e.m.f. vBl.
The moving rod can be represented (or equivalent) as electrical circuit as shown in figure.

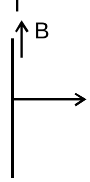
Ex.16 Find the value of emf induced in the rod for the following cases. The figures are self explanatory.

(a) 

Ans. emf = 0

(b) 

Ans. emf = 0

(c) 

Ans. emf = 0

Figure shows a closed coil ABCA moving in a uniform magnetic field B with a velocity v. The flux passing through the coil is a constant and therefore the induced emf is zero.

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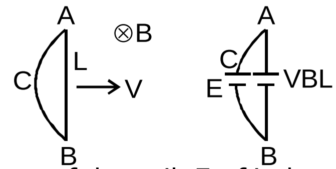


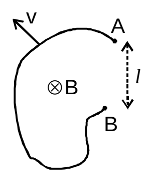
Now consider rod AB, which is a part of the coil. Emf induced in the rod = B L v. Now suppose the emf induced in part ACB is E, as shown in figure.
Since the emf in the coil is zero, Emf (in ACB) + Emf (in BA) = 0

or $\quad - E + vBL = 0$

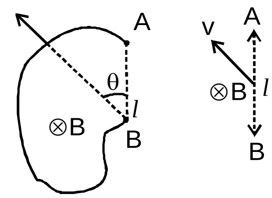
or $\quad E = vBL$

Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is BLv (here the two emf's are in parallel)

Ex.17 Figure shows an irregular shaped with AB moving with velocity v, as shown. Find the emf induced in the wire.

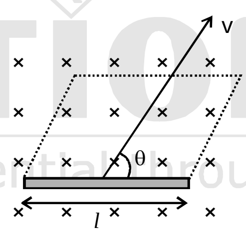


Sol. The same emf will be induced in the straight imaginary wire joining A and B, which is $Bvl \sin\theta$



Ex.18 A 0.4 meter long straight conductor moves in a magnetic field of magnetic induction 0.9 Wb/m² with a velocity of 7 m/sec. Calculate the emf induced in the conductor under the condition when it is maximum.

Sol. If a rod of length l is moved with velocity $\vec{v}$ and at angle θ to the length of the rod in a field $\vec{B}$ which is perpendicular to the plane of the motion, the flux linked with the area generated by the motion of rod in time t,



$\phi = Bl(v \sin\theta)t$ so, $|\varepsilon| = \dfrac{d\phi}{dt} = Bvl \sin\theta$

This will be maximum when $\sin\theta$ = max = 1, i.e., the rod is moving perpendicular to its length and then $(\varepsilon)_{max} = Bvl$

$E_{max} = 0.9 \times 7 \times 0.4 = 2.52$ V

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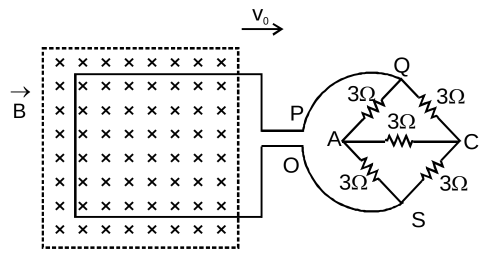
Ex.19 A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 Wb/m² as shown in figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 3 ohm. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? Find the direction of current in the loop ?



Sol. As the network AQCS is a balanced Wheatstone bridge, no current will flow through AC and hence the effective resistance of the network between QS,

$$R_{QS} = \frac{6 \times 6}{6 + 6} = 3 \text{ ohm}$$

and as the resistance of the square metal wire loop is 1 ohm, the total resistance of the circuit,

$$R = 3 + 1 = 4 \text{ ohm}$$

Now if the loop moves with speed v_0, the emf induced in the loop,

$$\varepsilon = Bv_0 l$$

So the current in the circuit, $i = \frac{\varepsilon}{R} = \frac{Bv_0 l}{R}$

Substituting the given data,

In accordance with Lenz's law, the induced current in the loop will be in clockwise direction.

Ex.20 A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

Sol. $E = B\,l\,V = \frac{\mu_0 i l v}{2\pi x}$

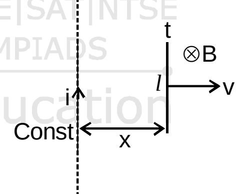
OR

Emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is l v dt. The magnetic field lines cut in dt time

$= B\,l\,vdt = \frac{\mu_0 i l\,vdt}{2\pi x}$

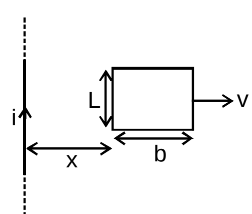
$\therefore$ The rate with which magnetic field lines are cut

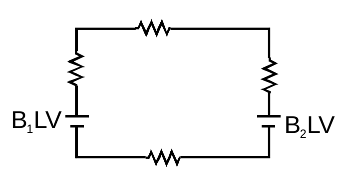
$= \frac{\mu_0 i l v}{2\pi x}$



Ex.21 A rectangular loop, as shown in the figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.

$E = B_1 L V - B_2 L v$

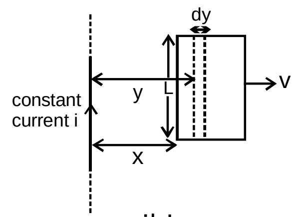




$$= \frac{\mu_0 i}{2\pi x} Lv - \frac{\mu_0 i}{2\pi(x+b)} Lv = \frac{\mu_0 i L b v}{2\pi x(x+b)}$$

Aliter : Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be $d\phi$.

$$\therefore \; d\phi = \frac{\mu_0 i}{2\pi y} L \; dy$$



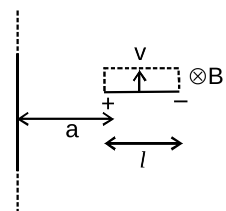
$$\therefore \; \phi = \frac{\mu_0 iL}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 iL}{2\pi}(\ln(x+b) - \ln x)$$

$$\text{Now } \frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right] = \frac{\mu_0 iL}{2\pi}\left[\frac{(-b)}{x(x+b)}\right]v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$$

$$\therefore \text{ induced emf} = \frac{\mu_0 ibLv}{2\pi x(x+b)}$$

Ex.22 A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance 'a' from the wire.

Sol. Consider a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

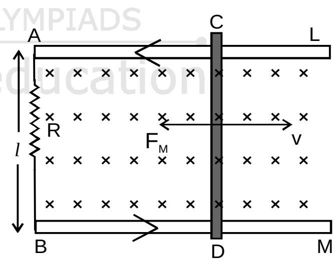


$$d_\epsilon = \frac{\mu_0 i}{2\pi x} dx.v$$

$$\therefore \; \epsilon = \int_a^{a+l}\frac{\mu_0 iv dx}{2\pi x} = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{l+a}{a}\right)$$

Ex.23 Two parallel wires AL and BM placed at a distance l are connected by a resistor R and placed in a magnetic field B which is perpendicular to the plane containing the wires. Another wire CD now connects the two wires perpendicularly and made to slide with velocity v. Calculate the work done per second needed to slide the wire CD. Neglect the resistance of all the wires.

Sol. When a rod of length l moves in a magnetic field with velocity v as shown in figure, an emf $\varepsilon = Bvl$ will be induced in it. Due to this induced emf, a current $i = \frac{\varepsilon}{R} = \frac{Bvl}{R}$ will flow in the circuit as shown in figure. Due to this induced current, the wire will experience a magnetic force



$$F_M = Bil = \frac{B^2 l^2 v}{R}$$

which will oppose its motion, So to maintain the motion of the wire CD, a force $F = F_M$ must be applied in the direction of motion.

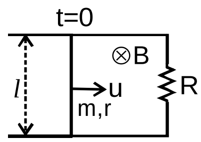
The work done per second, i.e., power needed to slide the wire is given by

$$P = \frac{dW}{dt} = Fv = F_M\, v = \frac{B^2 v^2 l^2}{R}$$

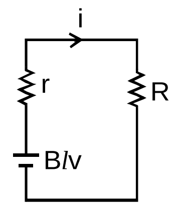
Note :

- *The power delivered by the external agent is converted into joule heating in a by using the circuit(as shwon above). It means magnetic field helps in converting the mechanical energy into joule heating.*

Ex.24 A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u Find its velocity as a function of time.



Sol. Let at an instant the velocity of the rod be v. The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \dfrac{Bl\,v}{R+r}$

At time t

Magnetic force acting on the rod is $F = i\,l\,B$, opposite to the motion of the rod.

$$i\,l\,B = -\,m\,\frac{dv}{dt} \qquad \text{...(1)}$$

$$i = \frac{Bl\,v}{R+r} \qquad \text{...(2)}$$

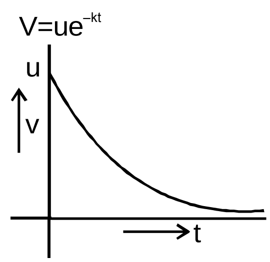
Now solving these two equation

$$\frac{B^2l^2v}{R+r} = -m.\frac{dv}{dt} \quad \Rightarrow \quad -\frac{B^2l^2}{(R+r)m}.dt = \frac{dv}{v}$$

let $\Rightarrow\ -\ K.\ dt = \dfrac{dV}{V}$

$$\int_u^v \frac{dv}{v} = \int_0^t -K.dt$$

$$\ln\left(\frac{v}{u}\right) = -Kt \quad \Rightarrow \quad V = ue^{-Kt}$$



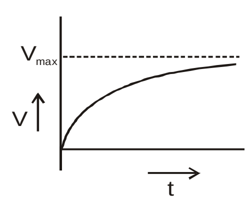
Ex.25 In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Sol. $m\dfrac{dv}{dt} = F - i\,l\,B$ ..(1)

$i = \dfrac{Bl\,v}{R+r}$ ...(2)

$$m\frac{dv}{dt} = F - \frac{B^2l^2v}{R+r}$$

let $K = \dfrac{B^2l^2}{R+r} \quad \Rightarrow \displaystyle\int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$



$$\Rightarrow \quad -\frac{1}{K}\ln(F-KV)\int_0^v \frac{t}{m}$$

$$\ln\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m}$$

$F - KV = F\,e^{-kt/m}$

$$V = \frac{F}{K}\left(1 - e^{-kt/m}\right)$$

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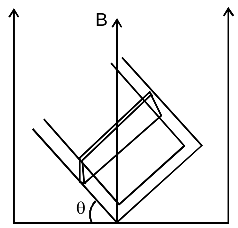
Ex.26 A square wire of length l, mass m and resistance R slides without friction down the parallel conducting wires of negligible resistance as shown in figure.
The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and rails form a closed rectangular loop. The plane of the rails makes an angle θ with horizontal and a uniform vertical field of magnetic induction B exists throughout the region. Show that the wire acquires a steady state velocity of magnitude $v = \dfrac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$



Sol. Force down the plane = mg sin θ
At any instant if the velocity is v the induced e.m.f
$= l$ B cosθ × v

Current in the loop × $\dfrac{Bl\,v\cos\theta}{R}$

Force on the conductor in the horizontal direction

$$= Bl \times \frac{Bl\,v\cos\theta}{R}$$

The component of force parallel to the incline

$$= \frac{B^2 l^2\, v\cos\theta}{R} \times \cos\theta$$

If v is constant, $\dfrac{B^2 l^2 \cos^2\theta}{R} \times v = mg\sin\theta$

$$\therefore \quad \mathbf{v = \frac{mRg\sin\theta}{B^2 l^2 \cos^2\theta}}$$

Ex.27 A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod at the instant its velocity is v.

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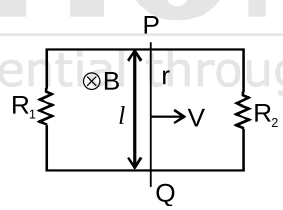
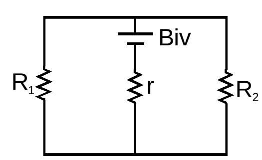
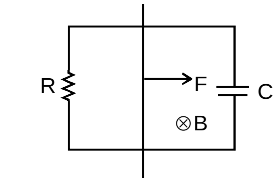


Sol. The equivalant circuit of above figure



$$i = \frac{Bl\,v}{r + \dfrac{R_1 R_2}{R_1 + R_2}}$$

Ex.28 In the above question if one resistance is replaced by a capacitor of capacitance C as shown. Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.

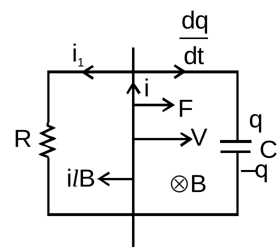


Sol. At any time t, let the velocity of the rod be v.

Applying Newtons law :

$F - i\,lB = ma$ ...(1)

Also $B\,l\,v = i_1 R = \dfrac{q}{c}$



Applying KCL,

$i = i_1 + \dfrac{dq}{dt} = \dfrac{Bl\,v}{R} + \dfrac{d}{dt}(Bl\,vC)$ or $i = \dfrac{Bl\,v}{R} + Bl\,Ca$

Putting the value of i in eq. (1),

$$F - \frac{B^2l^2v}{R} = (m + B^2l^2C)a = (m + B^2l^2C)\frac{dv}{dt}$$

$$(m + B^2l^2C)\frac{dv}{F - \frac{B^2l^2v}{R}} = dt$$

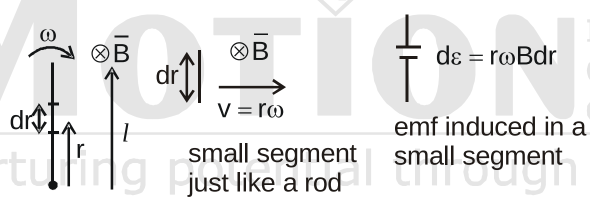
Integrating both sides, and solving we get

$$v = \frac{FR}{B^2l^2}\left(1 - e^{\frac{tB^2l^2}{R(m+CB^2l^2)}}\right)$$

4.4 Induced e.m.f due to rotation

4.4.1 Rotation of the rod

Consider a conducting rod of length l rotating in a uniform magnetic field.

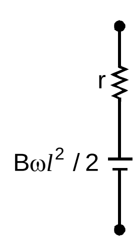


Emf induced in a small segment of length dr, of the rod = v B dr = r ω B dr

∴ emf induced in the rod

$$= \omega B \int_0^{\ell} r\,dr = \frac{1}{2}B\omega l^2$$

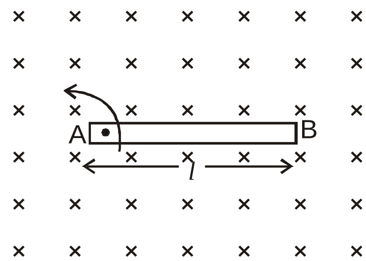
equivalent of this rod is as following



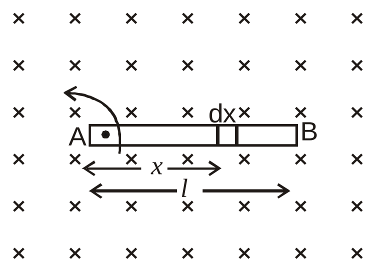
$$\text{or } \varepsilon = \frac{d\phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}l^2\omega dt}{dt} = \frac{1}{2}B\omega l^2$$

Ex.29 Find out the potential difference between A & B :



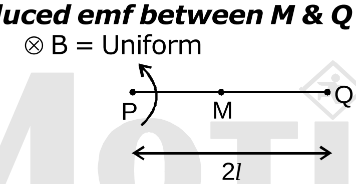
Sol.



$$\int dE = \int_0^{\ell} B\omega x dx$$

$$V_A - V_B = \frac{B\omega l^2}{2}$$

Ex.30 A rod PQ of length 2l is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V.

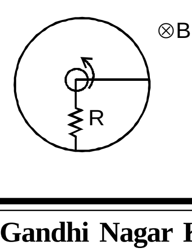


Sol. $E_{MQ} + E_{PM} = E_{PQ}$ corner $\rightarrow \frac{Bwl^2}{2} = 100$

$$E_{MQ} + \frac{B\omega\left(\frac{l}{2}\right)^2}{2} = \frac{B\omega l^2}{2}$$

$$E_{MQ} = \frac{3}{8}B\omega l^2 = \frac{3}{4} \times 100\,V = 75\,V$$

Ex.31 A rod of length L and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.



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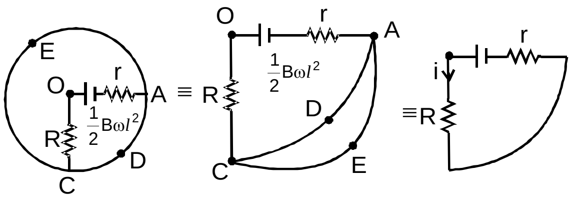
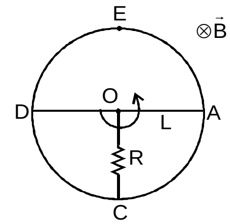
Sol.

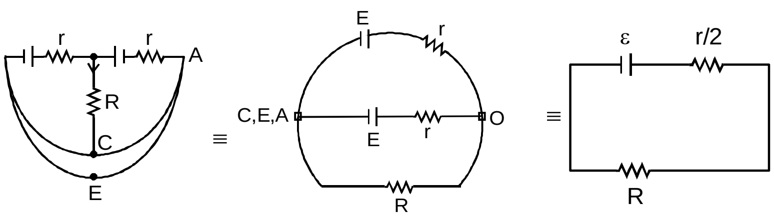


current $i = \dfrac{\frac{1}{2}B\omega l^2}{R + r}$

Ex.32 Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring.

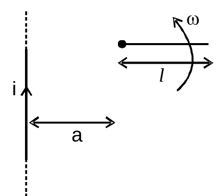


Sol.



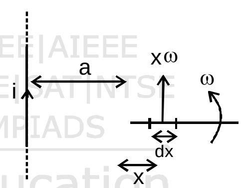
$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Ex.33 A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant as shown in the figure.



Consider a small segment of rod of length dx, at a distance x from one end of the rod.

Emf induced in the segment



$$dE = \frac{\mu_0 i}{2\pi(x + a)}(x\omega)dx$$

$$\therefore\ E = \int_0^l \frac{\mu_0 i}{2\pi(x + a)}(x\omega)dx = \frac{\mu_0 i\,\omega}{2\pi}\left[l - a.\ln\left(\frac{l + a}{a}\right)\right]$$

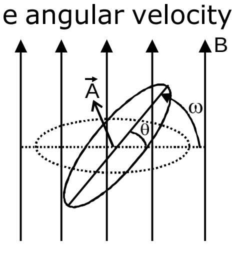
4.5 By Changing The Angle

Let us consider the case when the magnitude of the magnetic field strength and the area of the coil remains constant. When the coil is rotated relative to the direction of the field, an induced current is produced which lasts as long as the coil is rotating.

We have, ϕ = BA cos θ [where B is the magnetic field strength, A is the magnitude of the area vector & θ is the angle between them] If the angular velocity with which the coil is rotating is ω, then $\theta = \omega t$

Induced e.m.f. in the coil

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega \sin \omega t$$



Induced current in the coil

$$= i = \frac{|\varepsilon|}{R} = \frac{B\omega A}{R}\sin \omega t$$

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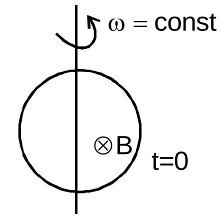
Ex.34 ***A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.***



Sol. At any time t, ϕ = BA cos θ = BA cos ωt

Now induced emf in the loop

$$e = \frac{-d\phi}{dt} = BA\,\omega \sin \omega t$$

If there are N turns

emf = BAωN sin ωt

BA ωN is the amplitude of the emf $e = e_m \sin \omega t$

$$i = \frac{e}{R} = \frac{e_m}{R} \sin \omega t = i_m \sin \omega t$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is the principle which is always used in generator.

5. INDUCED ELECTRIC FIELD DUE TO A TIME VARYING MAGNETIC FIELD

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till t = 0 and then changes with time. An induced current starts in the loop at t = 0.

The free electrons were at rest till t = 0 (we are not interested in the random motion of the electrons.) The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field. So we conclude that an electric field appears at time t = 0.

This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is nonelectrostatic and nonconservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field of q$\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is E. $d\vec{l}$.The emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

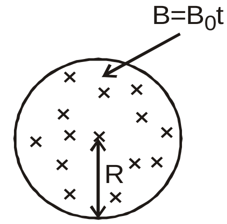
Using Faraday's law of induction,

$$\varepsilon = -\frac{d\phi}{dt}$$

or, $$\oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt} \quad ...(6)$$

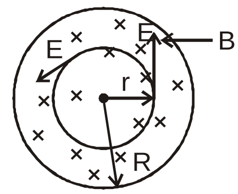
The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

Ex.35 ***What will be the electric field at a distance r from axis of changing cylindrical magentic field B, which is parallel to the axis of cylinder ?***



Sol. **(i)** When r < R

let at a distance r electric field is E

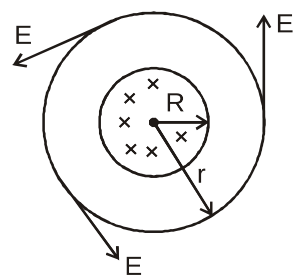


$$\varepsilon = \left|\oint \vec{E}.d\vec{l}\right| = \left|-\frac{d\phi}{dt}\right|$$

$$E.\ 2\pi r = \left|-\frac{d[B.(\pi r^2)]}{dt}\right|$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{B_0 r}{2}$$

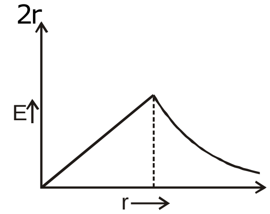
(ii) When r > R



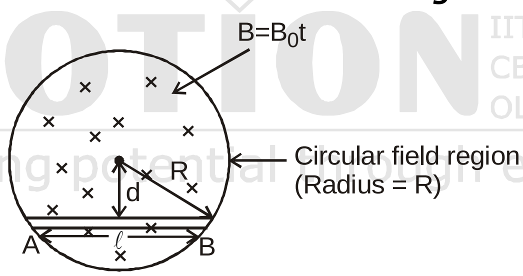
$$\left|\oint \vec{E}.d\vec{\ell}\right| = \left|\frac{-d\phi}{dt}\right|$$

$$E.2\pi r = \frac{d(B\pi R^2)}{dt}$$

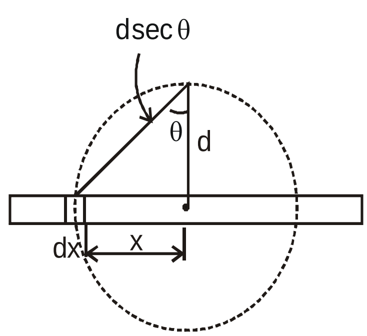
$$\Rightarrow \qquad E = \frac{R^2}{2r}.\frac{dB}{dt} = \frac{B_0 R^2}{2r}$$

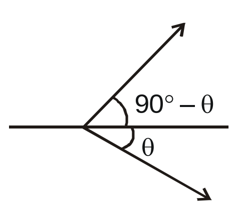


Ex.36 Find the e.m.f induced in the rod as shown in the figure.



Sol.





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$$E = \frac{r}{2}\frac{dB}{dt} = \frac{d\sec\theta}{2}B_0$$

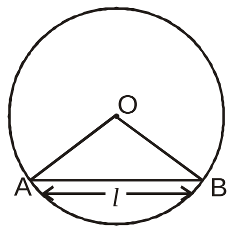
$$dE = E.dx\cos\theta \qquad = \frac{d\sec\theta B_0 dx\cos\theta}{2}$$

$$\Rightarrow \int_0^E dE = \frac{B_0 d}{2}\int_{-\ell/2}^{\ell/2} dx \Rightarrow \quad E = \frac{B_0 \ell d}{2}$$

Alternate

Induced emf in OA & OB is zero (because $\vec{B}$ & $d\vec{l}$ are perpendicular)

Total induced emf in OAB is in AB



$$\text{Area} = \frac{1}{2}ld$$

$$\phi = \frac{B_0 l dt}{2}$$

$$\frac{d\phi}{dt} = \frac{B_0 l d}{2}$$

Ex.37 ***A thin, nonconducting ring of mass m, carrying a charge q, can rotate freely about its axis. At the instant t = 0 the ring was at rest and no magnetic field was present. Then suddenly a magnetic field B was set perpendicular to the plane. Find the angular velocity acquired by the ring.***

Sol. Due to the sudden change of flux, an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$\varepsilon(\text{induced emf}) = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}.dA$$

Also $\varepsilon = \oint \vec{E}.d\vec{l}$ where E is the induced electric field.

$$\therefore \quad \oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{A} \Rightarrow E.2\pi r = -\frac{d}{dt}(B\pi r^2)$$

$$\Rightarrow \quad E = -\frac{r}{2}\frac{dB}{dt}$$

Force experienced by the ring $= q\,|\vec{E}|$

Torque experienced by the ring $\tau = (qE)r = \frac{qr^2}{2}\frac{dB}{dt}$

$\therefore$ Angular impulse experienced by the ring

$$= \int \tau\, dt = \frac{qr^2}{2}\int \frac{dB}{dt}dt = qr^2\frac{B}{2}$$

Also angular impulse acquired $= I\omega$ where I is moment of inertia of the ring about its axis = mr^2

$\therefore$ $mr^2\,\omega = qr^2\, B/2$

$\Rightarrow$ Angular velocity acquired by the ring $\omega = qB/2m$

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6. SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change. Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = L i$ where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}$$

The instantaneous emf is given as

$$\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$$

S.I unit of inductance is wb/amp or Henry (H)

L - self inductance is +ve quantity.

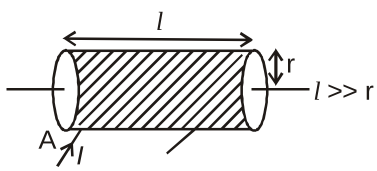
L depends on : (1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current. L is a scalar quantity.

Brain Teaser

If a circuit has large self-inductance, what inference can you draw about the circuit.

6.1 Self Inductance of solenoid



Let the volume of the solenoid be V, the number of turns per unit length be n. Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B = \mu_0 nI$. The magnetic flux through one turn of solenoid $\phi = \mu_0 n I A$.

The total magnetic flux through the solenoid $= N\phi$

$$= N \mu_0 n I A$$

$$= \mu_0 n^2 I A l$$

$\therefore$ $L = \mu_0 n^2 l A = \mu_0 n^2 V$

$\phi = \mu_0 n i \pi r^2 (n l)$

$$L = \frac{\phi}{i} = \mu_0 n^2 \pi r^2 l$$

Inductance per unit volume $= \mu_0 n^2$

Ex.38 The current in a coil of self-inductance L = 2H is increasing according to the law i = 2 sin t^2. Find the amount of energy spent during the period when the current changes from 0 to 2 ampere.

Sol. Let the current be 2 amp at $t = \tau$

Then $2 = 2\sin\tau^2 \Rightarrow \tau = \sqrt{\frac{\pi}{2}}$

When the instantaneous current is i, the self induced emf is $L\frac{di}{dt}$. If the amount of charge that is displaced in time dt is dq, then the elementary work done

$$= L.\left(\frac{di}{dt}\right)dq = L.\frac{di}{dt}idt = Lidi$$

$$W = \int_0^{\tau} Lidi = \int_0^{\tau} L(2\sin t^2)d(2\sin t^2)$$

$$W = \int_0^{\tau} 8L\sin t^2 \cos t^2 (tdt) = 4L\int_0^{\tau} \sin 2t^2 (tdt)$$

Let $\theta = 2t^2$

Differentiating $d\theta = 4t\ dt$

$$\therefore W = 4L\int \frac{\sin\theta d\theta}{4}$$

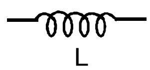
$$= L\ (-\cos\theta) \quad = -L\cos 2t^2$$

$$W = -L\left[\cos 2t^2\right]_0^{\sqrt{\pi/2}}$$

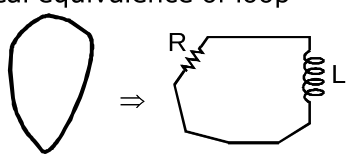
$$= 2L = 2 \times 2 = 4 \text{ joule}$$

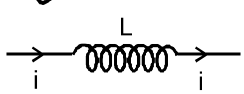
7. INDUCTOR

It is represent by

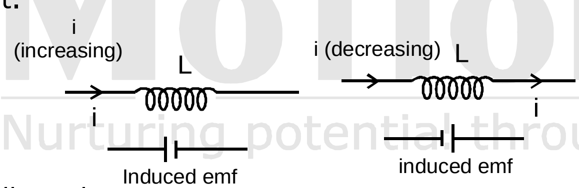


electrical equivalence of loop

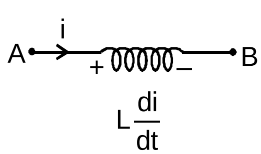




If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current. If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.



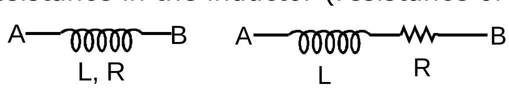
Over all result



$$V_A - L\frac{di}{dt} = V_B$$

Note

➢ If there is a resistance in the inductor (resistance of the coil of inductor) then :



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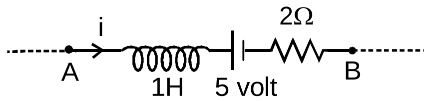
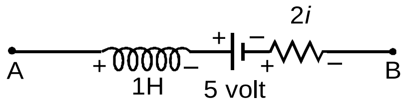
Ex.39 A B is a part of circuit. Find the potential difference $v_A - v_B$ if



(i) current i = 2A and is constant
(ii) current i = 2A and is increasing at the rate of 1 amp/sec.
(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Sol.



$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2\,i = V_B$$

(i) Put i = 2, $\frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$ $\quad\therefore\quad V_A - V_B = 9$ volt

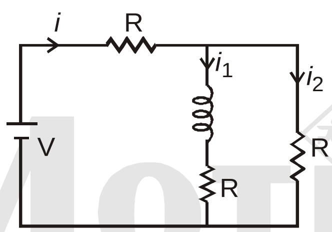
(ii) Put i = 2, $\frac{di}{dt} = 1$;

$V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\ V_0$

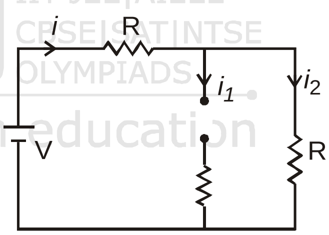
(iii) Put i = 2, $\frac{di}{dt} = -1$

$V_A + 1 - 5 - 2 \times 2 = V_B$
$V_A = 8$ volt

Ex.40 Find current i, i_1 and i_2 in the following circuit.

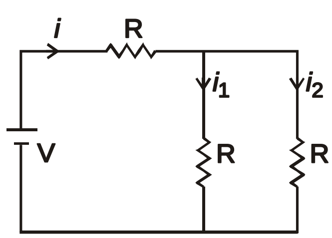


Sol. **at t = 0**



$$i = i_2 = \frac{V}{2R} \text{ and } i_1 = 0$$

at **t = ∞**



$$\Rightarrow\ \mathbf{i_1 = i_2 = \frac{i}{2} = \frac{V}{2R}}$$

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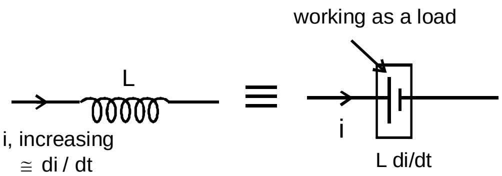
7.1 Energy stored in an inductor :

If current in an inductor at an instant is i and is increasing at the rate di/dt, the induced emf will oppose the current. Its behaviour is shown in the figure.



Power consumed by the inductor = $i\,L\,\frac{di}{dt}$

Energy consumed in dt time = $i\,L\,\frac{di}{dt}\,dt$

$\therefore$ total energy consumed as the current increases from 0 to I = $\int_0^i ILdi = \frac{1}{2}Li^2$

$$= \frac{1}{2}L\,i^2 \Rightarrow U = \frac{1}{2}L\,i^2$$

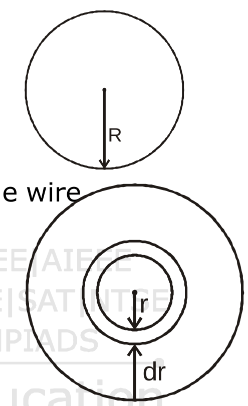
Note :

➤ This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r}dV$

Ex.41 Find out the energy per unit length ratio inside the solid long wire having current density J.



Sol. Take a ring of radius r and thickness dr as an element inside the wire

$$\frac{dE}{dv} = \frac{B^2}{2\mu_0}$$

using $B = \frac{\mu_0 jr}{2}$

$$\frac{dE}{dv} = \frac{\mu_0^2 J^2 r^2}{4(2\mu_0)} \Rightarrow \int dE = \int \frac{\mu_0 j^2 r^2}{8} 2\pi r dr \ell \Rightarrow \frac{E}{\ell} = \frac{\pi\mu_0 j^2 R^4}{16}$$

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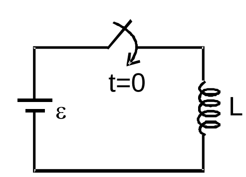
Ex.42 A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open. It is closed at t = 0. Find the current
as a function of time.

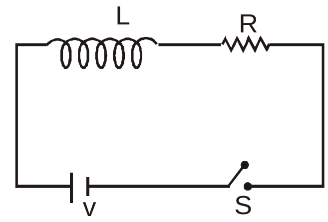


Sol. $\varepsilon = L\frac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i Ldi$

$\varepsilon t = Li \Rightarrow i = \frac{\varepsilon t}{L}$

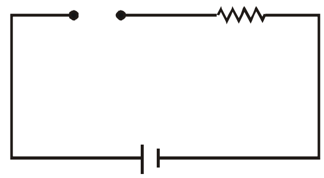
8. L.R. CIRCUIT

As the switch S is closed in given figure, current in circuit wants to rise upto $\frac{v}{R}$ in no time but inductor opposes it $\left(\frac{di}{dt} \rightarrow \frac{V}{L}\right)$



hence at time t = 0 inductor will behave as an open circuit

at t = 0

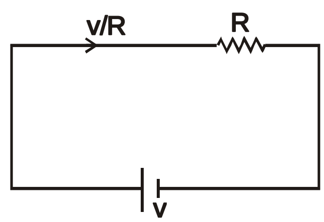


As the time passes, i in the circuit rises and $\frac{di}{dt}$ decreases. At any instant t.

$$\frac{Ldi}{dt} + iR = V$$

current reaches the value $\frac{v}{R}$ at time t = ∞ or we can say, inductor will behave as a simple wire.

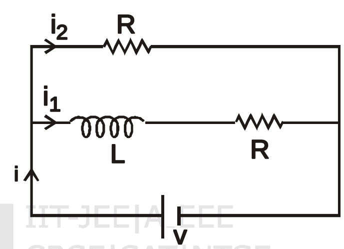
at t = ∞



Ex.43 Find value of current i, i_1 and i_2 in given figure at

(a) time t = 0

(b) time t = ∞

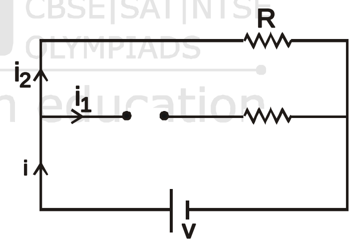


Sol. **(a) At time t = 0 inductor behaves as open circuit**

$i = v/R$

$i_1 = 0$

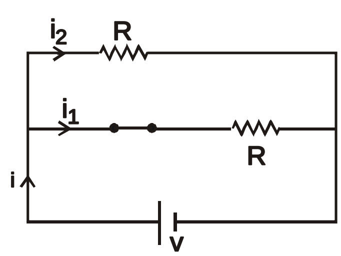
$i_2 = i = v/R$



(b) At time t = ∞. Inductor will behaves as simple wire

$$i = \frac{v}{(R/2)} = \frac{2v}{R}$$

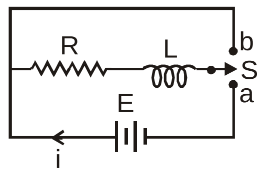
$$i_1 = i_2 = \frac{v}{R}$$



9. GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT

9.1 Growth of Current

Consider a circuit containing a resistance R, an inductance L, a two way key and a battery of e.m.f E connected in series as shown in figure. When the switch S is connected to a, the current in the circuit grows from zero value. The inductor opposes the growth of the current. This is due to the fact that when the current grows through inductor, a back e.m.f. is developed which opposes the growth of current in the circuit. So the rate of growth of current is reduced. During the growth of current in the circuit, let i be the current in the circuit at any instant t. Using Kirchhoff's voltage law in the circuit, we obtain



$E - L\dfrac{di}{dt} = Ri$ or $E - Ri = L\dfrac{di}{dt}$

or $\dfrac{di}{E - Ri} = \dfrac{dt}{L}$

Multiplying by – R on both the sides, we get

$$\frac{-R\,di}{E - Ri} = \frac{-R\,dt}{L}$$

Integrating the above equation, we have

$\log_e(E - Ri) = -\dfrac{R}{L}t + A$ ...(1)

where A is integration constant. The value of this constant can be obtained by applying the condition that current i is zero just at start i.e., at t = 0. Hence

$\log_e E = 0 + A$

or $A = \log_e E$ ...(2)

Substituting the value of A from equation (2) in equation (1), we get

$$\log_e(E - Ri) = -\frac{R}{L}t + \log_e E$$

or $\log_e\left(\dfrac{E - Ri}{E}\right) = -\dfrac{R}{L}t$

or $\left(\dfrac{E - Ri}{E}\right) = \exp\left(-\dfrac{R}{L}t\right)$

or $1 - \dfrac{Ri}{E} = \exp\left(-\dfrac{R}{L}t\right)$

or $\dfrac{Ri}{E} = \left\{1 - \exp\left(-\dfrac{R}{L}t\right)\right\}$

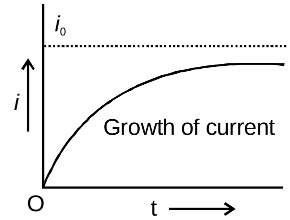
$\therefore$ $i = \dfrac{E}{R}\left\{1 - \exp\left(-\dfrac{R}{L}t\right)\right\}$

The maximum current in the circuit $i_0 = E/R$. So

$i = i_0\left\{1 - \exp\left(-\dfrac{R}{L}t\right)\right\}$ ...(3)

Equation (3) gives the current in the circuit at any instant t. It is obvious from equation (3) that $i = i_0$, when

$$\exp\left(-\frac{R}{L}t\right) = 0 \text{ i.e., at } t = \infty$$



Hence the current never attains the value i_0 but it approaches it asymptotically. A graph between current and time is shown in figure.

- We observe the following points

(i) When t = (L/R) then

$$i = i_0\left\{1 - \exp\left(-\frac{R}{L}\times\frac{L}{R}\right)\right\} = i_0\{1 - \exp(-1)\} = i_0\left(1 - \frac{1}{e}\right) = 0.63\, i_0$$

Thus after an interval of (L/R) second, the current reaches to a value which is 63% of the maximum current. The value of (L/R) is known as time constant of the circuit and is represented by τ. Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 63% of its final value. In terms of τ,

$$i = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

(ii) The rate of growth of current (di/dt) is given by

$$\frac{di}{dt} = \frac{d}{dt}\left[i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}\right]$$

$$\Rightarrow \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\exp\left(-\frac{R}{L}t\right) \qquad ...(4)$$

From equation (3), $\exp\left(-\frac{R}{L}t\right) = \frac{i_0 - i}{i_0}$

$$\therefore \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\left(\frac{i_0 - i}{i_0}\right) = \frac{R}{L}(i_0 - i) \qquad ...(5)$$

This shows that the rate of growth of the current decreases as i tends to i_0. For any other value of current, it depends upon the value of R/L. Thus greater is the value of time constant, smaller will be the rate of growth of current.

Note :

- Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.
- After one time constant, current in the circuit=63% of the final current (verify yourself)
- More time constant in the circuit implies slower rate of change of current.
- If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$$L_1 i_1 = L_2 i_2$$

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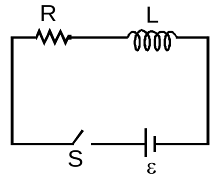
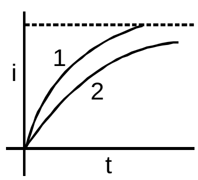
Ex.44 At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser $\left(\frac{L}{\eta}\right)$ then its initial value, find out instant current just after the operation.



Sol. Using above result (note 4)

$L_1 i_1 = L_2 i_2 \quad \Rightarrow \quad i_2 = \frac{\eta\varepsilon}{R}$

Ex.45 Which of the two curves shown has less time constant.



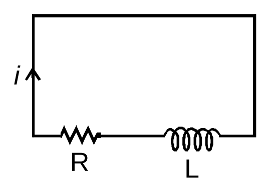
Sol. curve 1

9.2 Decay of Current

Let the circuit be disconnected from battery and switch S is thrown to point b in the figure. The current now begins to fall. In the absence of inductance, the current would have fallen from maximum i_0 to zero almost instantaneously. But due to the presence of inductance, which opposes the decay of current, the rate of decay of current is reduced.

suppose during the decay of current, i be the value of current at any instant t. Using Kirchhoff's voltage law in the circuit, we get

$$-L\frac{di}{dt} = Ri \qquad \text{or} \qquad \frac{di}{dt} = -\frac{R}{L}i$$



Integrating this expression, we get

$$\log_e i = -\frac{R}{L}t + B$$

where B is constant of integration. The value of B can be obtained by applying the condition that when t = 0, $i = i_0$

$\therefore \quad \log_e i_0 = B$

Substituting the value of B, we get

$$\log_e i = -\frac{R}{L}t + \log_e i_0$$

or $$\log_e \frac{i}{i_0} = -\frac{R}{L}t$$

or $$(i/i_0) = \exp\left(-\frac{R}{L}t\right) \qquad \ldots(6)$$

or $$i = i_0 \exp\left(-\frac{R}{L}t\right) = i_0 \exp\left(-\frac{t}{\tau}\right)$$

where τ = L/R = inductive time constant of the circuit.

It is obvious from equation that the current in the circuit decays exponentially as shown in figure.

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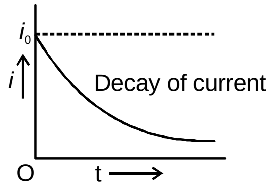
- We observe the following points

(i) After t = L/R, the current in the circuit is given by



$$i = i_0 \exp\left(-\frac{R}{L} \times \frac{L}{R}\right) = i_0 \exp(-1)$$

$$= (i_0 / e) = i_0/2.718 = 0.37\, i_0$$

So after a time (L/R) second, the current reduces to 37% of the maximum current i_0. (L/R) is known as time constant τ. This is defined as the time during which the current decays to 37% of the maximum current during decay.

(ii) The rate of decay of current in given by

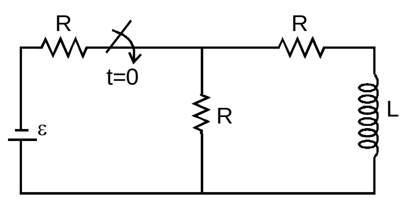
$$\frac{di}{dt} = \frac{d}{dt}\left\{i_0 \exp\left(-\frac{R}{L}t\right)\right\}$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{R}{L} i_0 \exp\left(-\frac{R}{L}t\right) = -\frac{R}{L} i \qquad ...(7)$$

or $\quad -\dfrac{di}{dt} = \dfrac{R}{L} i$

This equation shows that when L is small, the rate of decay of current will be large i.e., the current will decay out more rapidly.

Ex.46 In the following circuit the switch is closed at t = 0. Initially there is no current in inductor. Find out current the inductor coil as a function of time.

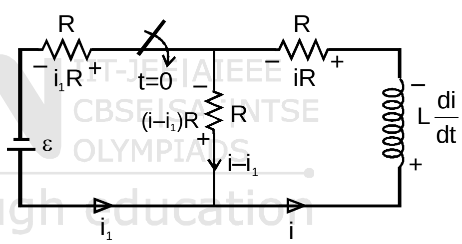


Sol. At any time t

$$-\varepsilon + i_1 R - (i - i_1) R = 0$$

$$-\varepsilon + 2i_1 R - i R = 0$$

$$i_1 = \frac{iR + \varepsilon}{2R}$$



Now, $-\varepsilon + i_1 R + iR + L.\dfrac{di}{dt} = 0$

$$-\varepsilon + \left(\frac{iR + \varepsilon}{2}\right) + iR + i.\frac{di}{dt} = 0 \Rightarrow -\frac{\varepsilon}{2} + \frac{3IR}{2} = -L\frac{di}{dt}$$

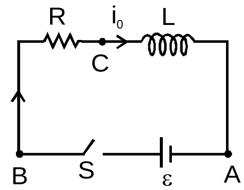
$$\left(\frac{-\varepsilon + 3iR}{2}\right) dt = -L.\, di \qquad \Rightarrow -\frac{-di}{dt} = \frac{di}{-\varepsilon + 3iR}$$

$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon + 3iR} \quad \Rightarrow -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) = \frac{3Rt}{2L}$$

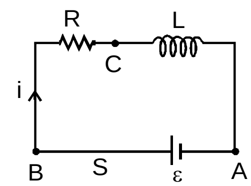
$$i = +\frac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

Ex.47 Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Sol. Let an instant t current in the circuit is i which is increasing at the rate di/dt.
Writing KVL along the circuit, we have

$$\varepsilon - L\frac{di}{dt} - iR = 0 \Rightarrow L\frac{di}{dt} = \varepsilon - iR$$



$$\Rightarrow \int_{i_0}^{i}\frac{di}{\varepsilon - iR} = \int_{0}^{t}\frac{dt}{L} \quad \Rightarrow \quad \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L}$$

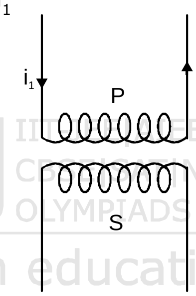
$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0R)e^{-Rt/L} \qquad \Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

10. MUTUAL INDUCTANCE

Consider two coils P and S placed close to each other as shown in the figure. When the current passing through a coil increases or decreases, the magnetic flux linked with the other coil also changes and an induced e.m.f. is developed in it. This phenomenon is known as mutual induction. This coil in which current is passed is known as primary and the other in which e.m.f. is developed is called as secondary.

Let the current through the primary coil at any instant be i_1. Then the magnetic flux ϕ_2 in the secondary at any time will be proportional to i_1 i.e., $\phi_2 \propto i_1$

Therefore the induced e.m.f. in secondary when i_1 changes is given by



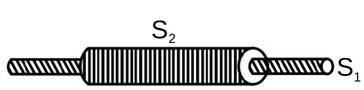
$$\varepsilon = -\frac{d\phi_2}{dt} \text{ i.e., } \varepsilon \propto -\frac{di_1}{dt}$$

$$\therefore \qquad \varepsilon = -M\frac{di_1}{dt} = -\frac{dMi_1}{dt} \Rightarrow \phi_2 = M\,i_1$$

where M is the constant of proportionality and is known as mutual inductance of two coils. It is defined as the e.m.f. induced in the secondary coil by unit rate of change of current in the primary coil. The unit of mutual inductance is henry (H).

10.1 Mutual Inductance of a Pair of Solenoids one Suurounding the other coil

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.



To calculate mutual inductance M between them, let us assume a current i_1 through the inner solenoid S_1

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0\left(\frac{N_1}{l_1}\right)i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B_2} through the surrounding coil is, therefore,

$$\phi_{B_2} = B(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\pi R_1^2$$

Now, $$M = \frac{N_2\phi_{B_2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 \Rightarrow \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

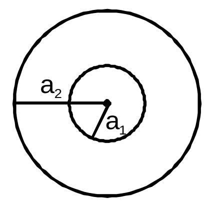
Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior.

Brain Teaser

What is the meaning of the statement "The coefficient of mutual inductance for a pair of coils is large" ?

Note :
- $M \le \sqrt{L_1 L_2}$
- For two coils in series if mutual inductance is considered then $L_{eq} = L_1 + L_2 \pm 2M$

Ex.48 Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Sol. Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2}\pi a_1^2$

or $\quad M\, i = \frac{\mu_0 i}{2a_2}\pi a_1^2 \quad$ or $\quad M = \frac{\mu_0 \pi a_1^2}{2a_2}$

Ex.49 Solve the above question, if the planes of coil are perpendicular.

Sol. Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0

Ex.50 Solve the above problem if the planes of coils make θ angle with each other.

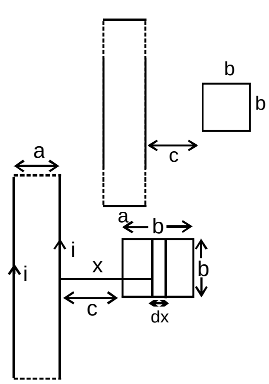
Sol. If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

Thus flux = $\vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2} . \pi a_1^2 \cos\theta$

or $\quad M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

Ex.51 Find the mutual inductance between two rectangular loops, shown in figure.



Sol. Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\, dx$$

$$\Rightarrow \phi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx$$

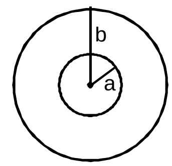
$$= \frac{\mu_0 ib}{2\pi}\left[\ln\frac{c+b}{c} - \ln\frac{a+b+c}{a+c}\right]$$

Ex.52 ***Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop***

(a) Find the mutual inductance of the two coils

(b) Find the emf induced in the larger coil

(c) If the resistance of the larger loop is R find the current in it as a function of time



Sol. (a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil.

Magnetic field at the centre $= \frac{\mu_0 i}{2b}$.

flux through the smaller coil $= \frac{\mu_0 i}{2b}\pi a^2$

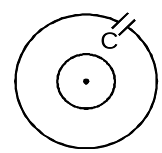
$\therefore$ $M = \frac{\mu_0}{2b}\pi a^2$

(ii) |emf induced in larger coil| $= M\left[\left(\frac{di}{dt}\right) \text{in smaller coil}\right]$

$$= \frac{\mu_0}{2b}\pi a^2 \ (2) = \frac{\mu_0 \pi a^2}{b}$$

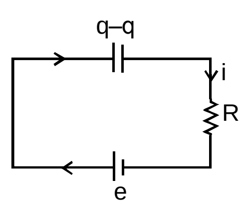
(iii) current in the larger coil $= \frac{\mu_0 \pi a^2}{bR}$

Ex.53 ***If the current in the inner loop changes according to i = 2t² then, find the current in the capacitor as a function of time.***



Sol. $M = \frac{\mu_0}{2b}\pi a^2$

|emf induced in larger coil| $= M\left[\left(\frac{di}{dt}\right) \text{in smaller coil}\right]$



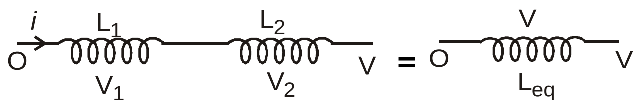
$e = \frac{\mu_0}{2b}\pi a^2 \ (4t) = \frac{2\mu_0 \pi a^2 t}{b}$

Applying KVL : -

$+e - \frac{q}{c} - iR = 0 \qquad \Rightarrow \quad \frac{2\mu_0\pi a^2 t}{b} - \frac{q}{c} - iR = 0$

differentiate wrt time : $\frac{2\mu_0\pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$ on solving it $i = \frac{2\mu_0\pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$

11. SERIES COMBINATION OF INDUCTORS

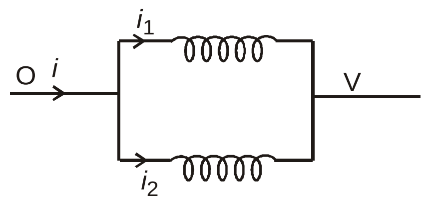


$\therefore \quad V = V_1 + V_2$

$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

$L_{eq} = L_1 + L_2 + \ldots\ldots\ldots\ldots\ldots$

Parallel Combination of inductor



$i = i_1 + i_2 \qquad \Rightarrow \qquad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$$\frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots\ldots$$

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