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PHYSICS
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SOUND WAVES
THEORY AND EXERCISE BOOKLET

CONTENTS

Syllabus :

Weve motion (plane wave only), logitudinal and transverse waves, Superposition of waves; progressive and stationary waves; Vibration of strings and air columns. Resonance; Beats; Speed of sound in gases Doppler effect (in sound). Vibration of strings and air columns. Resonance; Beats; Speed of sound in gases; Doppler effect (in sound).

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1. SOUND WAVES

Sound is type of longitudnal wave. In general majority of longitudinal waves are termed as sound waves. Sound is produced by a vibrating source, like when a gong of a bell is struck with a hammer, sound is produced. The vibrations produced by gong are propagated through air, Through air these vibrations reach to the ear and ear drum is set into vibrations and these vibrations are communicated to human brain. By touching the gong of bell by hand, we can feel the vibrations.

2. PROPAGATION OF SOUND WAVES

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wavs, sound needs a medium having properties of inetia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of presure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.

normal atmospheric pressure
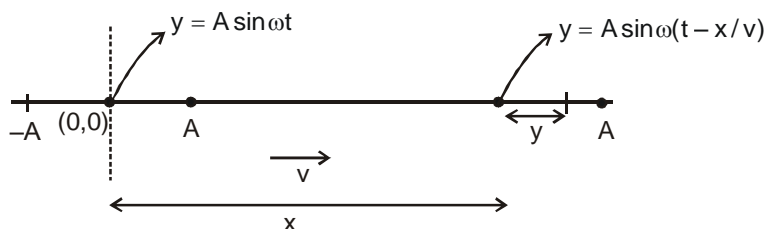
Undisturbed tuning fork
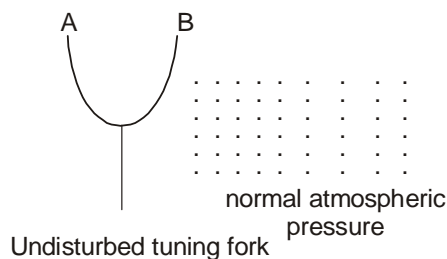
When Prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound

normal atmospheric pressure

Undisturbed tuning fork
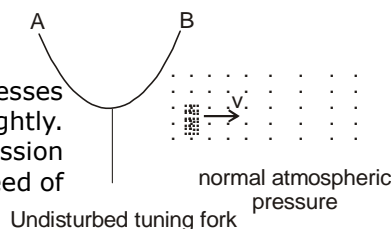
After producing the compression pulse, the prong B reverses its motion and moves inward. This drages away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.

rarefaction pulse

compression pulse
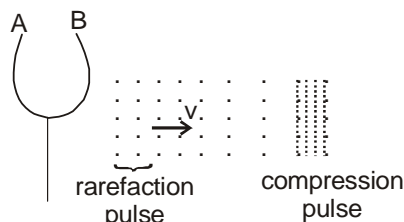
A longitudinal wave in a fluid is described either in tems of the longitudal displacements suffered by the particles of the medium.

$y = A\sin\omega t$

$y = A\sin\omega(t - x/v)$

$-A$ (0,0) A

y A

v

x

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Consider a wave going in the x-direction in a fluid. Suppose that at a time t, the particle at the undistrubed position x suffers a displacement y in the x-direction.

$$y = A \sin \omega \left(t - \frac{x}{v} \right) \qquad ...(i)$$

Position of any particle from origin at any time = x + y

   x = Distance of the mean position of the particle from the origin.

   y = Displacement of the particle from its mean position.

General Equation :

 $(0,0) \Rightarrow \quad y = A \sin (\omega t + \phi)$

   $(0,x) \Rightarrow y = A \sin [(t - x/v) + \phi]$

Displacement wave $y = A \sin (\omega t - kx + \phi)$

- If we fix $x = x_0$ then we are dealing with the particle whose mean position at distance x_0 from origin & this particle is performin SHM of amp. A with time period T phase difference $= - kx + \phi$

3.   **COMPRESSION WAVES**

When a longitudinal wave propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically. The region where compression occurs, the pressure is more than the normal pressure of the medium. Thus we can also describe longitudinal waves in a gaseous medium as pressure waves and these are also termed as compression waves in which the pressure at different point of medium also varies periodically with their displacements. Let us discuss the propagation of excess pressure in a medium in longitudinal wave analytically.
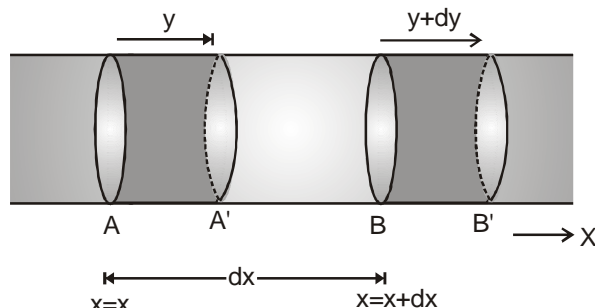
Consider a longitudinal wave propagating in positive x-direction as shown in figure. Figure shows a segmentAB of the medium of width dx. In this medium let a longitudinal wave is propagating whose equation is given as

$$y = A \sin (\omega t - kx) \qquad ...(1)$$

Where y is the displacement of medium particle situated at a distance x from the origin, along the direction of propagation of wave. In figure shown AB is the medium segment whose a medium particle is at position $x = x$ and B is at $x = x +dx$ at an instant. If after some time t medium particle at A reaches to a point A' which is displaced by y and the medium particle at b reaches to point B′ which is at a displacmenet y + dy from B. Here dy is given by equation (3.116) as

$$dy = - Ak \cos (\omega t - kx) \, dx$$

Here due to displacement of section AB to A′B′ the change in volume of it's section is given as

$$dV = - S \, dy \qquad [S \rightarrow \text{Area of cross-section}]$$
$$= SA \, k \cos (\omega t - kx) \, dx$$

The volume of section AB is $V = S \, dx$

Thus volume strain in section AB is

$$\frac{dV}{V} = \frac{-SAk \cos(\omega t - kx)dx}{S dx} \qquad \text{or} \qquad \frac{dV}{V} = -Ax \cos(\omega t - kx)$$

If B is the bulk modulus of the medium, then the excess pressure in the section AB can be given as

$$\Delta P = -B \left(\frac{dV}{V} \right) \qquad ...(2)$$

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$$\Delta P = BAk \cos(\omega t - kx)$$

or $\qquad \Delta P = \Delta P_0 \cos(\omega t - kx) \qquad\qquad\qquad ...(3)$

Here ΔP_0 is the pressure amplitude at a medium particle at position x from origin and ΔP is the excess pressure at that point. Equation shown that excess varies periodically at every point of the medium with pressure amplitude ΔP_0, which is given as

$$\Delta P_0 = BAk = \frac{2\pi}{\lambda} AB \qquad\qquad\qquad ...(4)$$

Equation shown is also termed as the equation of pressure wave in gaseous medium. We can also see that the pressure wave differs in phase is $\pi/2$ from the displacement wave and pressure maxima occurs where the displacement is zero and displacement maxima occur where the pressure is at its normal level. Remembers that pressure maxima implies that the presure at a point is pressure amplitude times more or less then the normal pressure level of the medium.

Ex.1 *A sound wave of wavelenth 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 2.0×10^{-3} N/m², find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m²*

Sol. The pressure amplitude is

$$p_0 = \frac{2.0 \times 10^{-3} N/m^2}{2} = 10^{-3} N/m^2$$

The displacement amplitudes s_0 is given by

$$p_0 = B k s_0$$

or, $\qquad s_0 = \frac{p_0}{Bk} = \frac{p_0 \lambda}{2\pi B} = \frac{10^{-3} N/m^2 \times (40 \times 10^{-2} m)}{2 \times \pi \times 14 \times 10^4 N/m^2} = \frac{100}{7\pi} Å = 6.6 Å$

Density Wave

In this section we will find the relation between pressure wave and density wave.

According to definition of bulk modulus (B),

$$B = \left(-\frac{dp}{dV/V} \right)$$

Further, $\qquad\qquad$ Volume $= \dfrac{mass}{density}$

or $\qquad\qquad V = \dfrac{m}{\rho} \qquad$ or $\qquad dV = -\dfrac{m}{\rho^2}.d\rho = -\dfrac{V}{\rho}.d\rho$

or $\qquad\qquad \dfrac{dV}{V} = -\dfrac{d\rho}{\rho}$

Substituting in Eq. (i), we get

$$d\rho = \frac{\rho(dP)}{B} = \frac{dp}{v^2} \qquad\qquad \left(\frac{\rho}{B} = \frac{1}{v^2} \right)$$

Or this can be written as,

$$\Delta\rho = \frac{\rho}{B}.\Delta P = \frac{1}{v^2}\Delta P$$

So, this relation relates the pressure equation with the density equation. For example, if

$$\Delta P = (\Delta P)_m \sin(kx - \omega t)$$

then $\qquad\qquad \Delta\rho = (\Delta\rho)_m \sin(kx - \omega t)$

where, $\qquad\qquad (\Delta\rho)_m = \dfrac{\rho}{B}.(\Delta P)_m = \dfrac{(\Delta P)_m}{v^2}$

Thus, density equation is in phase with the pressure equation and this is 90° out of phase with the displacement equation.

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3.1 Velocity and Acceleration of particle :

General equation of wave is given by

$$y = A \sin (\omega t - kx)$$

$$v_\rho = \frac{\partial y}{\partial t} = A \omega \cos (\omega t - kx) \qquad ...(1)$$

$$a_\rho = \frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin (\omega t - kx) \qquad ...(2)$$

$$\frac{\partial y}{\partial x} = -Ak \cos(\omega t - kx) \qquad ...(3)$$

Here $\frac{\partial y}{\partial x}$ = slope of (y, x) curve Now again differentiate eq. – 3

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \qquad ...(4)$$

from eq. (2) & (4)

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

4. VELOCITY OF SOUND/LONGITUDINAL WAVES IN SOLIDS

Consider a section AB of medium as shown in figure(a) of cross-sectional area S. Let A and B be two cross section as shown. Let in this medium sound propagation is from left to right. If wave source is at origin O and when it oscillates, the oscillations at that point propagate along the rod.
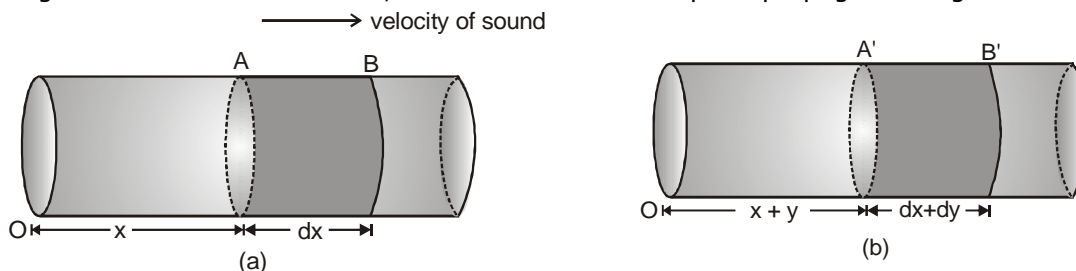
Here we say an elastic wave has propagated along the rod with a velocity determined by the physical properties of the medium. Due to oscillations say a force F is developed at every point of medium which produces a stress in rod and is the cause of strain or propagation of disturbance along the rod. This stress at any cross-sectional area can be given as

$$\text{Stress } S_1 = \frac{F}{S} \qquad ...(1)$$

If we consider the section AB of medium at a general instant of time t. The end A is at a distance x from O and B is at a distance x + dx from O. Let in time dt due to oscillations, medium particles at a are displaced along the length of medium by y and those at B by y + dy. The resulting position of section and A′ and B′ shown in figure (b), Here we can say that the section AB is deformed (elongated) by a length dy. Thus strain produced in it is

Strain in section AB $\qquad E = \frac{dy}{dx} \qquad ...(2)$

If Young's modulus of the material of medium is Y, we have

$$\text{Young's Modulus } Y = \frac{\text{Stess}}{\text{Strain}} = \frac{S_1}{E}$$

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From equation (1) and (2), we have $Y = \dfrac{F/S}{dy/dx}$

or $\qquad\qquad F = YS\dfrac{dy}{dx}$ ...(3)

If net force acting of secting AB is dF then it is given as

$\qquad\qquad dF = dma$ ...(4)

Where *dm* is the mass of section AB and a be its acceleration, which can be given as for a medium of density ρ.

$\qquad\qquad dm = \rho S dx \qquad$ and $\qquad a = \dfrac{d^2 y}{dt^2}$

From equation (4), we have $\ dF = (\rho S dx)\,\dfrac{d^2 y}{dx^2}$

or $\qquad\qquad \dfrac{dF}{dx} = \rho S \dfrac{d^2 y}{dt^2}$ ...(5)

From equation (3) on differentiating w.r. to x, we can write

$\qquad\qquad \dfrac{dF}{dx} = YS \dfrac{d^2 y}{dt^2}$ ...(6)

From equation (5) and (6) we get

$\qquad\qquad \dfrac{d^2 y}{dx^2} = \left(\dfrac{Y}{\rho}\right)\dfrac{d^2 y}{dx^2}$ ....(7)

Equation (7) is the differential form of wave equation, comparing it with previous equation we get the wave velocity in the medium can be given as

$\qquad\qquad v = \sqrt{\dfrac{Y}{\rho}}$

Similar to the case of a solid in fluid, instead of Young's Modulus we use Bulk modulus of the medium hence the velocity of longitudinal waves in a fluid medium is given as

$\qquad\qquad v = \sqrt{\dfrac{B}{\rho}}$

Where B is the Bulk modulus of medium.

For a gaseous medium bulk modulus is defined as

$\qquad\qquad B = \dfrac{dp}{(-dV/V)} \quad$ or $\qquad B = -V\dfrac{dP}{dV}$

4.1 Newton's Formula for velocity of Sound in Gases

Newton assumed that during sound propagation temperature of medium remains constant hence the stated that propagation of sound in a gasesous medium is an isothermal phenomenon, thus Boyal's law can be applied in the process. So for a section of medium we use

$\qquad\qquad PV = $ constant

Differentiating we get

$\qquad\qquad PdV + V\,dP = 0$

or $\qquad\qquad -V\dfrac{dP}{dV} = P$

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or bulk modulus of medium can be given as

B = P (Pressure of medium)

Newton found that during isothermal propagation of sound in a gaseous medium, bulk modulus of medium is equal to the pressure of the medium, hence sound velocity in a gaseous medium can be given as

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad \qquad ...(1)$$

From gas law we have
$$\frac{P}{\rho} = \frac{RT}{M} \qquad \qquad ...(2)$$

From (1) & (2) we have
$$v = \sqrt{\frac{RT}{M}} \qquad \qquad ...(3)$$

From the expression in equation (1) if we find the sound veloicty in air at normal temperature and atompsheric pressure we have

Normal atmospheric pressure is $\qquad$ P = 1.01 × 10⁵ Pa

Density of air at NTP is $\qquad\qquad$ ρ = 1.293 kg/m³

Now from equation (1) $\qquad\qquad$ $v = \sqrt{\frac{P}{\rho}} \Rightarrow v = \sqrt{\frac{1.01 \times 10^5}{1.293}}$ = 279.45 m/s

But the experimental value of veloicty of sound determined from various experiments gives the velocity of sound at NTP, 332 m/s. Therefore there is a difference of about 52 m/s between the theoretical and experimental values. This large difference can not be attributed to the experimental errors. Newton was unable to explain error in his formula. This correction was explained by a French Scientist Laplace.

4.2 Laplace Correction

Laplace explained that when sound waves propagated in a gaseous medium. There is compression and rarefaction in the particles of medium. Where there is compression, particles come near to each other and are heated up, where there is rarefaction, medium expends and there is fall of temperature. Therefore, the temperature of medium at every point does not remain constant so the process of sound propagation is not isothermal. The total quantity of heat of the system as a whole remains constant. medium does not gain or loose any heat to the surrounding. Thus in a gaseous medium sound propagation is an adiabatic process. For adiabatic process the relation in pressure and volume of a section of medium can be given as

$$PV^\gamma = \text{constant} \qquad\qquad ...(1)$$

Here $\gamma = \dfrac{C_p}{C_v}$, ratio of specific heats of the medium.

Differentiating equation (1) we get,

$$dPV^\gamma + \gamma V^{\gamma-1} dVP = 0$$

or $\qquad\qquad$ $dP + \gamma \dfrac{PdV}{dV} = 0$

or $\qquad\qquad$ $- V \dfrac{dP}{dV} = \gamma P$

Bulk modulus of medium $\qquad$ B = γ P

Thus Laplace found that during adiabatic propagation of sound, the Bulk modulus of gaseous medium is equal to the product of ratio of specific heats and the pressure of medium. Thus velocity of sound propagation can be given as

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

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From gas law $v = \sqrt{\dfrac{\gamma RT}{M}}$

From above equation we find sound velocity in air at NTP, we have

Normal atmospheric pressure $P = 1.01 \times 10^5$ Pa

Density of air at NTP P = 1.293 kg/m^3

Ratio of specific heat of air $\gamma = \dfrac{C_p}{C_v} = 1.42$

$\Rightarrow v = \sqrt{\dfrac{\gamma \rho}{\rho}} = \sqrt{\dfrac{1.42 \times 1.01 \times 10^5}{1.293}} = 333.04$ m/s

This value is in agreement with exerimental value.

Now at any temperature t°C velocity of sound $v_t = \sqrt{\dfrac{\gamma R(273 + t°)}{M}} = \sqrt{\dfrac{\gamma R 273}{M}}\left(1 + \dfrac{t}{273}\right)^{1/2}$

$$v_t = v_0\left(1 + \dfrac{t}{546}\right)$$

4.3   Effect of Temperature on Velocity of Sound

We have velocity of sound propagation in a gasous medium as

$$v = \sqrt{\dfrac{\gamma RT}{M}}$$

For a given gaseous medium γ, R and M remains constant, thus velocity of sound is directly proportional to square root of absolute temperature of the medium. Thus

$$v \propto \sqrt{T}$$

If at two different temperatures T_1 and T_2, sound velocities in medium are v_1 and v_2 then from above equation we have

$$\dfrac{v_1}{v_2} = \sqrt{\dfrac{T_1}{T_2}}$$

4.4   Effect of Pressure on Velocity of Sound

We know form gas law

$$\dfrac{P}{\rho} = \dfrac{RT}{M}$$

If temperature of a medium remains constant then on changing pressure, density of medium proportionally

changes so that the ratio $\dfrac{P}{\rho}$ remains constant.

Hence if in a medium,   T = constant

Then, $\dfrac{P}{\rho}$ = constant

Thus velocity of sound, $v = \sqrt{\dfrac{\gamma P}{\rho}}$ = constant

Therefore, the velocity of sound in air or in a gas is independent of change in pressure.

4.5   Effect of Humidity on Velocity of Sound

The density of water vapour at NTP is 0.8 kg/m^3 whereas the density of dry air at NTP is 1.293 kg/^{3}. Therefore water vapour has a density less than the density of dry air. As atmospheric pressure remains approximately same, the velocity of sound is more in moist air then the velocity of sound in dry air.

$$v_{moist\ air} > v_{dry\ air}   \text{(from the previous equation)}$$

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4.6   Effect of Wind on Velocity of Sound

If wind is blowing in the direction of propagation of sound, it will increase the velocity of sound, On the other hand if wave propagation is opposite to the direction of propagation of wind, wave velocity is decreased. If wind blows at speed v_ω then sound velocity in the medium can be given as

$$\vec{V} = \vec{V_s} + \vec{V_w}$$

Where $\vec{V_s}$ is the velocity of sound in still air.

5.   APPEARANCE OF SOUND TO HUMAN EAR

The appearance of sound to a human ear is characterised by three parameters (a) pitch (b) loudness and (c) quality.

(a)   Pitch and Frequency

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice and a female voice. We say that a buffalo voice is of low pitch, a male voice has higher pitch and a female voice has still higher pitch. This sensation primarly depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa.

(b)   Loudness and Intensity

The loudness that we sense is related to the intensity of sound though it is not directly proportional to it. Our perception of loudness is beter correlated with the sound level measured in decibels (abbreviated as dB) and defined as follows.

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2 The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level $\beta = 0$.

(c)   Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.
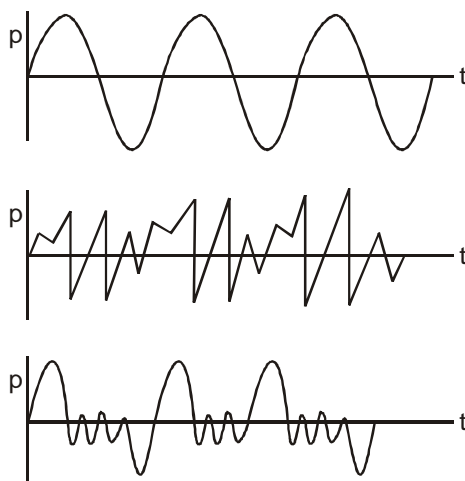
We differenatiate between the sound from a tabla and that from a mridang by saying that they have different quality.

(d) **Energy in sound Waves**

$$P_{avg} = 2\pi^2 f^2 A^2 \, \mu v$$

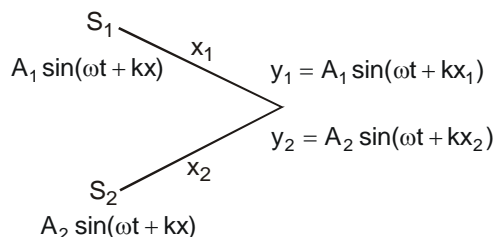$$\text{Intensity} = 2\pi^2 A^2 f^2 \, \rho \, v$$

$$\Delta P = \Delta P_0 \cos(\omega t - kx)$$

$$\Delta P_0 = \beta^3 Ak \quad \Rightarrow \quad A = \frac{\Delta P_0}{\beta K}$$

$$\text{Intensity} = \frac{\omega^2 A^2 \rho v}{2} = \frac{\omega^2 \Delta P_0^2 \rho v}{2\beta^2 k^2} = \frac{\omega^2 \Delta P_0^2 \rho v^3}{2\beta^2 \omega^2} = \frac{\Delta P_0^2 \rho v^2 . v}{2\beta^2}$$

$$= \frac{\Delta P_0^2 \rho \beta . v}{2\beta^2 \rho} \qquad \left(v = \sqrt{\frac{\beta}{\rho}} \right)$$

$$I = \frac{\Delta P_0^2 \, V}{2.\beta}$$

6. ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES

Interference implies super position of waves. Whenever two or more than two waves superimpose each other at some position then the resutant displacement of the particle is given by the vector sum of the individual displacements.

Let the two waves coming from sources S_1 & S_2 be

$$y_1 = A_1 \sin (\omega t + kx_1)$$
$$y_2 = A_2 \sin (\omega t + kx_2) \text{ respectively.}$$

Due to superposition

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_1 \sin (\omega t + kx_1) + A_2 \sin (\omega t + kx_2)$$

Phase difference between y_1 & $y_2 = k(x_2 - x_1)$

i.e., $\Delta\phi = k(x_2 - x_1)$

As $\quad \Delta\phi = \dfrac{2\pi}{\lambda}\Delta x \quad$ (where Δx = path difference & $\Delta\phi$ = phase difference)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$\Rightarrow \quad A_{net}^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi$

$\therefore \quad I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi \quad$ (as $I \propto A^2$)

When the two displacements are in phase, then the resultant amplitude will be sum of the two amplitude & I_{net} will be maximum, this is known of constructive interference.

For I_{net} to be maximum

$$\cos\phi = 1 \quad \Rightarrow \quad \phi = 2n\pi \text{ where } n = \{0,1,2,3,4,5............\}$$

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi \quad \Rightarrow \qquad \Delta x = n\lambda$$

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For constructive interference

$$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $I_1 = I_2 = I$

$I_{net} = 4 I$

$A_{net} = A_1 + A_2$

When superposing waves are in opposite phase, the resultant amplitude is the difference of two amplitudes & I_{net} is minimum; this is known as destructive interference.

For I_{net} to be minimum,

$\cos \Delta\phi = -1$

$\Delta\phi = (2n + 1) \pi$ where n = {0,1,2,3,4,5...........}

$\frac{2\pi}{\lambda}\Delta x = (2n + 1) \pi \Rightarrow$ $\Delta x = (2n+1)\frac{\lambda}{2}$

For destructive interference

$$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If $I_1 = I_2$

$I_{net} = 0$

$A_{net} = A_1 - A_2$

Generally,

$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

If $I_1 = I_2 = I$

$I_{net} = 2I + 2I\cos\phi$

$I_{net} = 2I(1 + \cos \phi) = 4I\cos^2 \frac{\Delta\phi}{2}$

Ratio of I_{max} & I_{min} = $\dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

7.   **LONGITUDINAL STANDING WAVES**

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering waves are given by

$p_1 = p_0 \sin (\omega t - kx)$ and $p_2 = p_0 \sin (\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p = p_1 + p_2 = 2p_0\cos (kx + \frac{\phi}{2}) \sin (\omega t + \frac{\phi}{2})$$

$\Rightarrow$ $p = p_0' \sin (\omega t + \frac{\phi}{2})$ ...(1)

This is equation of SHM* in which the amplitude p_0' depends on position as

$p_0' = 2p_0\cos (kx + \frac{\phi}{2})$ ...(2)

Points where pressure remains permanently at its average value, i.e., pressure amplitude is zero is called a pressure node, and the condition for a pressure node would be given by

$p_0' = 0$

i.e. $\cos(kx + \frac{\phi}{2}) = 0$

i.e. $kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2}$,   n = 0,,1,2.......

Similarly points where pressure amplitude is maximum is called a pressure antinode and condition for a pressure antinode would be given by

$$p_0' = \pm 2p_0$$

i.e. $\cos(kx + \frac{\phi}{2}) = \pm 1$

or $(kx + \frac{\phi}{2}) = n\pi,$ $n = 0, 1, 2, ...$

Note :

● Note that a pressure node in a standing wave would correspond to a displacement antinode; and a pressure anti-node would correspond to a displacement node.

• (when we label eqn. (1) as SHM, what we mean that excess pressure at any point varies simple harmonically. if the sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to (1) would be

$$s = s_0' \cos(\omega t + \frac{\phi}{2}) \text{ where } s_0' = 2s_0 \sin(kx + \frac{\phi}{2})$$

This can be easily observed to be an equation of SHM. It represents the medium particles moving simple harmonically about their mean position at x.

8.   REFLECTION OF SOUND WAVES

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) as analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion i.e. an abrupt phase change of p. This is consistent with the requirement of displacement amplitude of remains zero at the rigid end, since a medium particle at the rigid end can not vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, a displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure waves from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogies to reflection of string wave from a free end. This point corresponds to a displacment maxima, so that the incident & reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence thereflected presure wave would be out of phase by π with respect to the incident wave. i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

9.   WAVES IN A VIBRATING AIR COLUMN

Hollow pipes have long used for making musical sounds. A hollow pipe we call organ pipe. To understand how these work, first we examine the behaviour of air in a hollow pipe that is open at both ends. If we blow air across one end, the disturbance due to the moving air at that end propagates along the pipe to the far end. When it reaches far end, a part of the wave is reflected, similar in the case when a wave is reflected along a string whose end point is free to move. Since the air particles are free to move at the open end, the end point is an antinode. if one end of the pipe is closed off, the the air is not free to move any further in that direction and closed end becomes a node. now the resonant behaviour of pipe is completely changed. Similar in the case of string, here also all harmonic frequencies are possible and resonance may take place it the frequency of external source matches with any of the one harmonic freuquency of pipe. Let us discuss in detail.
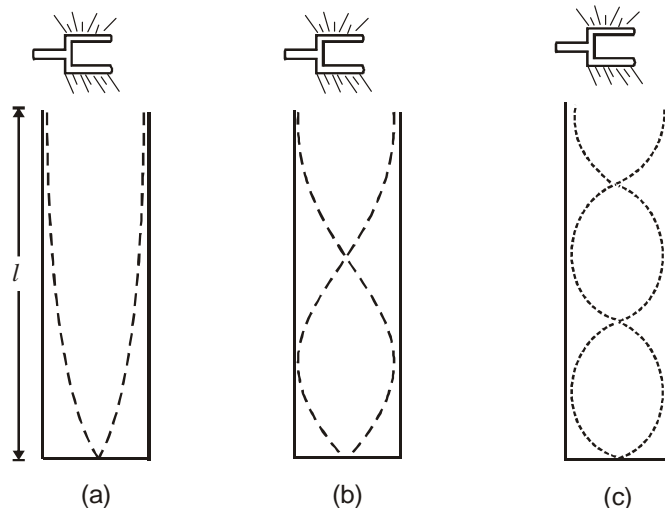
9.1   Vibration of Air in a Closed Organ Pipe

When a tuning fork is placed near the open end of a pipe. The air in the pipe oscillates with the same frequency as that of tuning fork. Here the open end should be an antinode and closed and should be a node for perfect reflection of waves from either end or for formation of stationary waves. Since one end is a node and other is an antinode, the lowest frequency (largest wavelength) vibration has no other nodes or antinodes between ends as shown in figure(a). This is the fundamental (minimum) frequency at which stationary waves can be formed in a closed organ pipe. Thus if the wavelength is λ then we can see from figure (a), which shows the displacement wave of longitudinal waves in the closed organ pipe.

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$$l = \frac{\lambda}{4} \qquad \ldots (1)$$

or $\quad \ell = 4\,l$

(a)   (b)   (c)

Thus fundamental frequency of oscillations of closed organ pipe of length l be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{4l} \qquad \ldots (2)$$

Similarly first overtone of closed pipe vibrations is shown in figure (b) here wavelength λ' and pipe length l are related as

$$l = \frac{3\lambda'}{4} \qquad \ldots (3)$$

or $\qquad \lambda' = \frac{4l}{3}$

Thus frequency of first overtone oscillatinos of a closed organ pipe of length l can be given as

$$n_2 = \frac{v}{\lambda'} = \frac{3v}{4l} \qquad \ldots (4)$$

$$= 3n_1$$

This is three times the fundamental frequency thus after fundamental only third harmonic frequency exist for a closed organ pipe at which resonance can take place or stationary waves can be formed in it.

Similarly next overtone, second overtone is shown in figure(c). Here the wavelength λ'' and pipe length l are related as

$$l = \frac{5\lambda''}{4}$$

or $\qquad \lambda'' = \frac{4l}{5}$

Thus the frequency of second overtone oscillation of a closed organ pipe of length l can be given as

$$n_3 = \frac{v}{\lambda''} = \frac{5v}{4l} = 5n_1$$

This is fifth harmonic frequency of fundamental oscillations.

In general f = $\dfrac{(2n-1)v}{4\ell}$

Here frequency of oscillation is called $(2n-1)^{th}$ harmonic and $(n-1)^{th}$ overtone

From above analysis it is clear that the resonant frequecies of the closed organ pipe are only odd harnomics of the fundamental frequency.

## 9.2   Vibration of Air in Open Organ Pipe

Figure shown the resonant oscillations of an open organ pipe. The least frequency at which an open organ pipe resonates is the one with longest wavelength when at both the open ends of pipe antinodes are formed and there is one node is between as shown in figure (a). In this situation the wavelengths of sound in air λ is related to length of organ pipe as

$$l = \frac{\lambda}{2}$$

or $\lambda = 2\,l$ ...(1)

Thus the fundamental frequency of organ pipe can be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{2l}$$

(a)	(b)	(c)
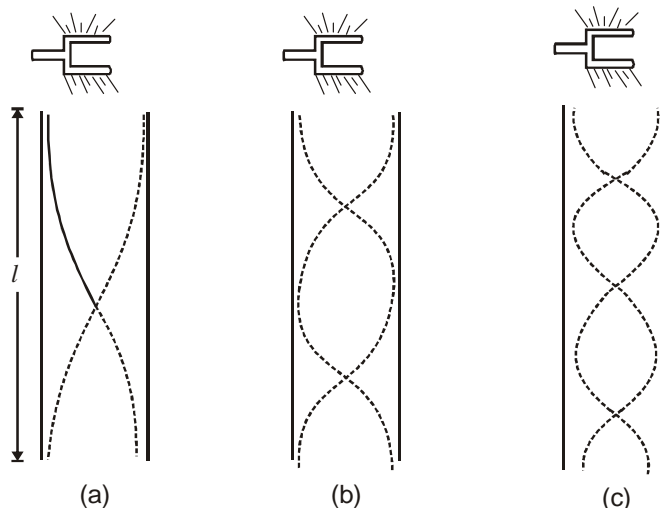
Similarly next higher frequency at which the open organ pipe resonate is shown in figure (b) which we call first overtone. Here the wavelength λ' is related to the length of pipe as

$$l = \lambda'$$ ....(2)

Thus here resonant frequency for first overtone is given as

$$n_2 = \frac{v}{\lambda'} = \frac{v}{l}$$ ...(3)

Which is second harmonic of fundamental frequency. Similarly as shown in figure (c), in second overtone oscillations, the wavelength λ'' of sound is releated to the length of pipe as

$$l = \frac{3\lambda''}{2}$$ ...(4)

or $\lambda'' = \frac{2l}{3}$ ...(5)

Thus be frequency of second overtone oscillations of an open organ pipe can given as

$$n_3 = \frac{v}{\lambda''} = \frac{3v}{2l}$$ ...(6)

$$= 3n_1$$ ...(7)

Which is third harmonic of fundamental frequency.

In General f = $\dfrac{nv}{2\ell}$

we can say frequency of oscillation is called n^{th} harmonic and $(n - 1)^{th}$ overtone

The above analysis shown that resonant frequencies for formation of stationary waves includes all the possible harmonic frequencies for an open organ pipe.

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9.3 End correction

As mentioned earlier the displacment antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

e = 0.6 r
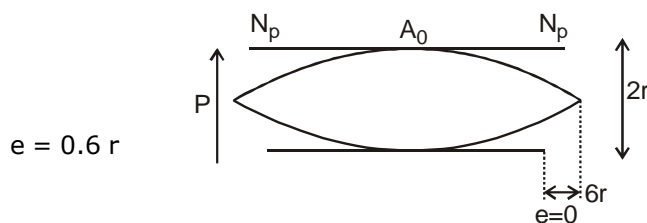
where r = radius of the organ pipe.
with end correction, the fundamental frequency of a closed pipe (f_e) and an open argon pipe (f_0) will be given by

$$f_e = \frac{v}{4(\ell + 0.6r)} \quad \text{and} \quad f_0 = \frac{v}{2(\ell + 1.2r)}$$

9.4 Resonance Tube

This an apparatus used to determine velocity of sound in air experimentally and also to compare frequncies of two tuning forks.

shown figure the setup of a resonance the experiment. There is a long tube T in which initially water is filled upto the top and the eaer level can be change by moving a reservoir R up and down.

A tunning fork of known frequency n_0 is struck gently on a rubber pad and brought near the open and tube T due to which oscillations are transferred to the air coloumn in the tube above water level. Now we gradually decrease the water level in the tube. This air column behaves like a closed organ pipe and the water level as closed end of pipe. As soon as water level reaches a position where there is a node of corresponding stationary wave, in air column, resonance takes place and maximum sound intensity is detected. Let at this position length of air column be I_1. If water level is further decreased, again maximum sound intensity is observed when water leel is at another node i.e. at a length l_2 as shown in figure. Here if we find two successive resonance lengths l_1 and l_2, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2}$$

or $\quad \lambda = 2(l_2 - l_1)$

Thus sound velocity in air can be given as

$$v = n_0\lambda = 2n_0(l_2 - l_1)$$

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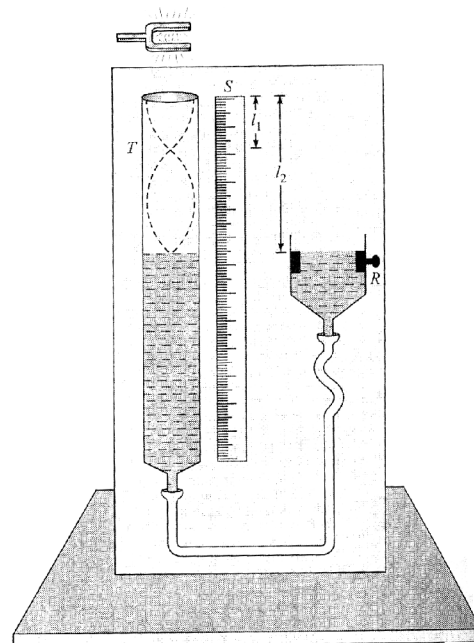
Ex.2 *A tube of certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.*

Sol. The displacement curves of longitudinal waves in a tube open at both ends is shown in figure (a) and (b).

(a)   (b)
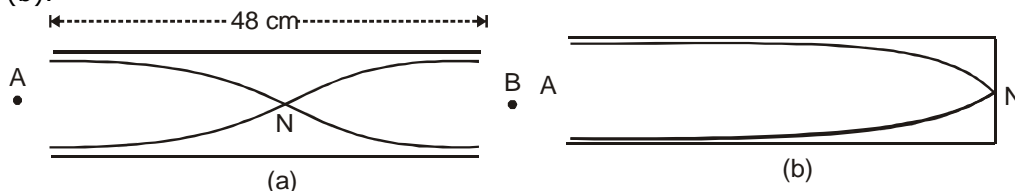
Let r be the radius of the tube. We know the antinodes occur slightly outside the tube at a distance 0.6 r from the tube end.

The distance between two antinodes is given by

$$\frac{\lambda}{2} = 48 + 2 \times 0.6r$$

We have

$$\lambda = \frac{v}{n} = \frac{32000}{320} = 100 \text{ cm}$$

or

$$50 = 48 + 1.2r$$

or

$$r = \frac{2}{1.2} = 1.67 \text{ cm}$$

Thus diameter of the tube is D = 2r = 3.33

When one end is closed, then

$$\frac{\lambda}{4} = 48 + 0.6r = 48 + 0.6 \times 1.67 = 49$$

or

$$\lambda = 4 \times 49 = 196 \text{ cm}$$

Now

$$n = \frac{v}{\lambda} = \frac{32000}{196} = 163.3 \text{ Hz}$$

9.5   Quink's Tube

This is an apparatus used to demonstrate the phenomenon of interference and also used to measure velocity of sound in air. This is made up of two U-tubes A and B as shown in figure. Here the tube B can slide in and out from the tube A. There are two openings P and Q in the tube A. At opening P, a tuning fork or a sound source of known frequency n_0 is placed and at the other opening a detector is placed to detect the resultant sound of interference occurred due to superposition of two sound waves coming from the tubes A and B.
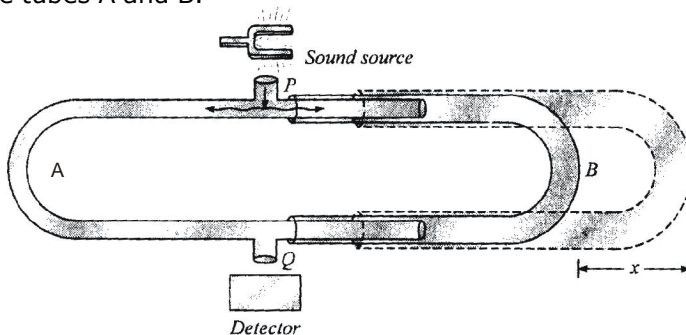
Initially tube B is adjusted so that detector detects a maximum. At this instant if length of paths covered by the two waves from P to Q from the side of A and side of B are l_1 are l_2 respectively then for constructive interference we must have

$$l_2 - l_2 = N\lambda \qquad \qquad ...(1)$$

If now tube B is further pulled out by a distance x so that next maximum is obtained and the length of path from the side of B is l_2' then we have

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$$l_2' = l_2 + 2x \qquad\qquad ...(2)$$

Where x is the displacement of the tube. For next constructive interference of sound at point Q, we have

$$l_2' - l_1 = (N+1)\lambda \qquad\qquad ..(3)$$

From equation (1), (2) and (3), we get

or $\qquad\qquad x = \dfrac{\lambda}{2} \qquad\qquad ...(4)$

Thus by experiment we get the wavelength of sound as for two sucessive points of constructive interference, the path difference must be λ. As the tube B is pulled out by x, this introduces a path difference 2x in the path of sound wave through tube B. If the frequency of the source is known, n_0, the velocity of sound in the air filled in tube can be given as

$$v = n_0\lambda = 2n_0x \qquad\qquad ...(5)$$

Ex.3 *In a large room a person recieves direct sound waves from a source 120 m away from him. he also receives waves from the same source which reach him, being reflected from the 25m high ceiling at a point halfway between them. For which wavelength will these two sound waves interfere constructivity?*

Sol. As shown in figure for reflection from the ceilling

Path SCP = SC + CP = 2SC $\qquad\qquad$ [As $\angle i = \angle r$, SC = CP]

or $\qquad$ Path SCP = $2\sqrt{60^2 + 25^2}$ = 130 cm

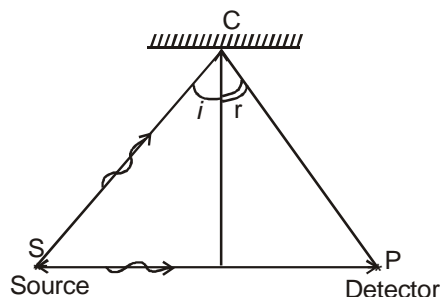So path differene between interfering waves along path SCP, and SP,

$\qquad\qquad \Delta x = 130 - 120 = 10$ m

Now for consructive interfernece at P.

$\qquad\qquad \Delta x = n\lambda$. i.e., $\quad 10 = n\lambda$

or $\qquad\qquad \lambda = \dfrac{10}{n}$ with $\;n = 1, 2, 3 ....$

i.e. $\qquad\qquad \lambda = 10$ m, 5m, (10/3) m and so on

Ex.4 *Figure shown a tube structure in which a sound signal is sent from one end and is received at the othe end. The semicircular part has a radius of 20.0 cm. The frequency of the sound source can be varied electronically between 1000 and 4000 Hz. Find the frequencies at which maxima of intensity are detected. The speed of sound in air = 340 m/s.*
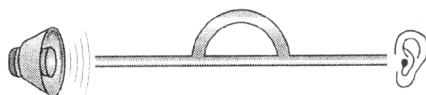
Sol. The sound wave reaches detector by two paths simultaneously be straight as well as semicircular track. The wave through the straight path travels a distance $l_1 = 2 \times 20$ cm and the wave through the curved part travles a distane $l_2 = \pi (20$ cm) = 62.8 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore,

$$\Delta l = l_2 - l_2 = 62.8 \text{ cm} - 40 \text{ cm} = 22.8 \text{ cm} = 0.228 \text{ m}$$

The wavelength of either wave is $\dfrac{v}{n} = \dfrac{340}{n}$. For constructive interfernece, $\Delta l = N\lambda$, where N is an integer.
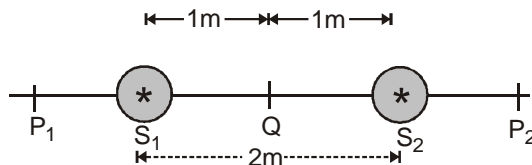
or, $\qquad\qquad 0.228 = N\left(\dfrac{340}{n}\right)$

or, $\qquad\qquad n = N\left(\dfrac{340}{0.228}\right) = N(1491.2)$ Hz = N(1490) Hz

Thus, the frequencies within the specific range which cause maxima of intensity are 1490 Hz and 2980 Hz.

Ex.5 *Two sources S_1 and S_2, separated by 2.0 m, vibrate according to equation $y_1 = 0.03 \sin \pi t$ and $y_2 = 0.02 \sin \pi t$ where y_1, y_2 and t are in M.K.S unit. They send out waves of velocity 1.5m/s. Calculate the amplitude of the resultant motion of the particle co-linear with S_1 and S_2 and located at a point (a) to the right of S_2 (b) to the left of S_2 and (C) in the middle of S_1 and S_2.*

Sol. The situation of shown in figure

The oscillations y_1 and y_2 have amplitudes $A_1 = 0.03$ m and $A_2 = 0.02$ respectively.

The frequency of both sources in n = $\dfrac{\omega}{2\pi}$ = $\dfrac{1}{2}$ = 0.5 Hz

Now wavelength of each wave $\lambda = \dfrac{v}{n} = \dfrac{1.5}{0.5}$ = 3.0 m

(a) The path difference for all points P_2 to the right of S_2 is

$\Delta = (S_1P_2 - S_2P_2) = S_1S_2 = 2m$

Phase difference $\phi = \dfrac{2\pi}{\lambda} \times$ Path difference

$= \dfrac{2\pi}{3} \times 2.0 = \dfrac{4\pi}{3}$

The resultant amplitude for this point is given by

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi} = \sqrt{(0.03)^2 + (0.02)^2 + 2 \times 0.03 \times 0.02 \times \cos(4\pi/3)}$$

Solving we get   R = 0.0265 m

(b) The path differnece for all point P, to the left of S_1
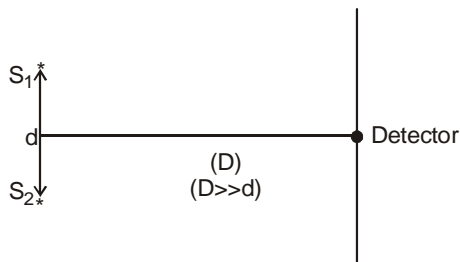
$\Delta = (S_2P - S_1P) = S_1S_2 = 2.0$ m

Hence the resultant amplitude for all points to the left of S_1 is also 0.0265 m

(c) For a point Q, between S_1 and S_2, the path difference is zero i.e., $\phi = 0$. Hence constructive interference take place at Q, thus amplitude at this point is maximum and given as

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2} = A_1 + A_2 = 0.03 + 0.02 = 0.05 \text{ m}$$

Ex.6 *Two point sources of sound are placed at a distance d and a detector moves on a straight line parallel to the line joining the sources as shown in figure. at a distane D away from sources. Initially Detector is situated on the line so that it is equidistant from both the sources. Find the displacement of detertor when it detects n^{th} maximum sound and also find its displacement when it detects n^{th} minimum sound.*

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Sol.   The situation is shown in figure

Let us consider the situation when detector move by a distance x as shown. Let at this position the path diference between the waves

from S_1 and S_2 to detector by Δ then we have

$\Delta = S_2D - S_1D \simeq S_2Q$ [Where S_1Q is perpendicular on line S_2D]

Here if θ is small angle as D >> d, we have

$$S_2Q = d \sin \theta \simeq d \tan\theta = d\frac{x}{D}$$

Thus at the position of detector, path deference is

$$\Delta = \frac{dx}{D} \qquad\qquad\qquad ....(1)$$

The expression for path difference in equation (1) is an important formula for such problems. Students are advised to keep this formula in mind for future use.

When detector was at point O, path difference was zero and it detects a maxima, now if detector detects n^{th} maximum then its path difference at a distance x from O can be given as

$$\Delta = n \lambda$$

or $\qquad \dfrac{dx}{D} = n \lambda \qquad$ or $\quad x = \dfrac{n\lambda D}{d}$

Similarly if detector detects n^{th} minima then the path difference between two waves at detector can be given as

$$\Delta = (2n + 1) \frac{\lambda}{2} \qquad \text{or} \qquad \frac{dx}{D} = (2n+1)\frac{\lambda}{2} \quad \text{or} \qquad x = \frac{(2n+1)\lambda D}{2d}$$

Ex.7   ***Two coherent narrow slits emitting of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ the sound is detected by moving a detector on the screen S at a distance D (>> λ) from the slit S_1 as shown in figure. Find the distance x such that the intensity at P is equal to the intensity at O.***
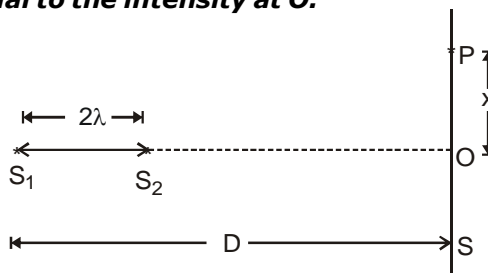
Sol.   When detector is at O, we can see that the path differnece in the two waves reaching O is d = 2λ thus at O detector receives a maximum sound. When it reaches P and again there is a maximum sound detected at P the path difference between two waves must $\Delta = \lambda$. Thus shown figure the path difference at P can be given as

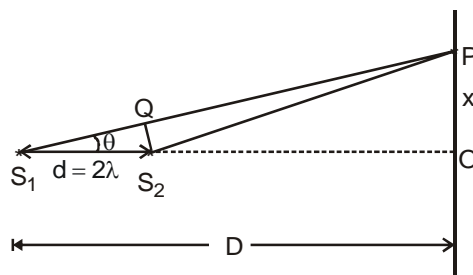$$\Delta = S_1P - S_2P \simeq S_1Q$$
$$= d \cos \theta = 2\lambda \cos \theta$$

And we have at point P, path difference $\Delta = \lambda$, Thus
$$\Delta = 2\lambda \cos \theta = \lambda$$

or, $\qquad\qquad \cos \theta = \dfrac{1}{2}$

or, $\qquad\qquad \theta = \dfrac{\pi}{3}$

Thus the value of x can be written as $x = D \tan \theta = D \tan \left(\dfrac{\pi}{3}\right) = \sqrt{3}\,D$

Ex.8 *Figure shown two coherent sources S_1 and S_2 which emit sound of wavelength λ in phase. The separation between the sources is 3λ. A ciruclar wire of large radius is placed in such a way that S_1S_2 lies in its plane and the middle point of S_1S_2 is at the centre of the wire. Find the angular position θ on the wire for which constructive interference takes place.*
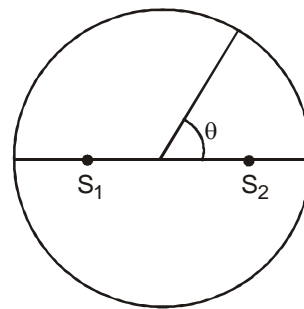
Sol.
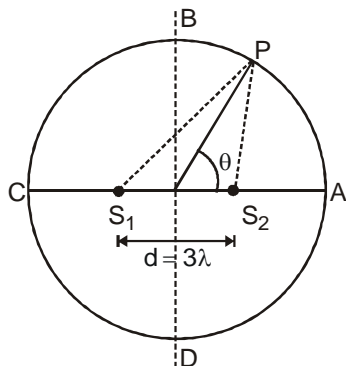
From previous question, we can say that for a point P on the circle shown in fiure shown. The path difference in the two waves at P is

$$\Delta = S_1P - S_2P = d \cos \theta \qquad = 3\lambda \cos \theta$$

We know for constructive interference at P. The path difference must be an integral multiple of wavelength λ. Thus for a maxima at P, we have

$$3\lambda \cos \theta = 0 ; \qquad 3\lambda \cos \theta = \lambda ; \qquad 3\lambda \cos \theta = 2\lambda ; \qquad 3\lambda \cos \theta = 3\lambda ;$$
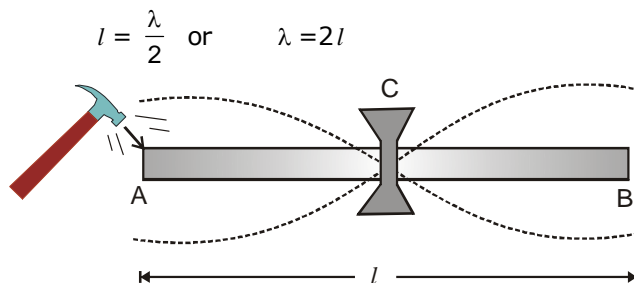
or, $\quad \theta = \dfrac{\pi}{2} \quad$ or $\quad \theta = \cos^{-1}\dfrac{1}{3} \quad$ or $\quad \theta = \cos^{-1}\dfrac{2}{3} \quad$ or $\quad \theta = 0$

There are four points A, B, C and D on circle at which $\theta = 0$ or $\dfrac{\pi}{2}$ and there are two points in each quadrant at $\theta = \cos^{-1}\dfrac{1}{3}$ and $\theta = \cos^{-1}\dfrac{2}{3}$ at which constructive interference takes place. Thus there are total twelve points on circle at which maxima occurs.

9.6   Vibrations of Clamped Rod

We have discussed the resonant vibrations of a string clamped at two ends. Now we discuss the ocillations of a rod clamped at a point on its length as shown in figure. Figure shows a rod AB clamped at its middle point. If we gently hit the rod at its one end, it begin to oscillate and in the natural oscillations the rod vibrates at its lowest frequency and maximum wavelength, which we call fundamental mode of oscillations. With maximum wavelength when transverse stationary waves setup in the rod, the free ends vibrates as antinodes and the clamped end a node as shown in figure. Here if λ be the wavelength of the wave, we have
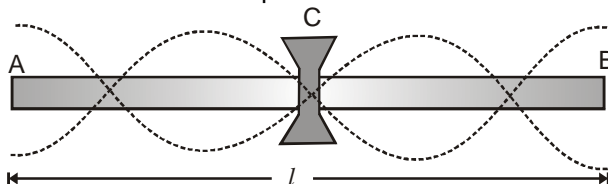
$$l = \dfrac{\lambda}{2} \quad \text{or} \qquad \lambda = 2l$$

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Thus the frequency of fundamental oscillations of a rod damped at mid point can be given as

$$n_0 = \frac{v}{\lambda} = \frac{l}{2l}\sqrt{\frac{Y}{\rho}} \qquad \qquad ...(1)$$

Where Y is the Young's modulus of the material of rod and ρ is the density of the material of rod.
Next higher frequency at which rod vibrates will be then one when wave length is decreased to a value so that one node is inserted between mid point and an end of rod as shown in figure

In this case if λ be the wavelength of the waves in rod, we have

$$l = \frac{3\lambda}{2}$$

or $$\lambda = \frac{2l}{3} \qquad \qquad ...(2)$$

Thus in this case the oscillation frequency of rod can be given as

$$n_1 = \frac{v}{\lambda} = \frac{3}{2l}\sqrt{\frac{Y}{\rho}} = 3n_0 \qquad \qquad ...(3)$$

This is called first overtone frequency of the damped rod or thid harmonic frequency. Similarly, the next higher freqnecy of oscillation i.e. second overtone of the oscillating rod can be shown in figure shown. Here is λ be the wavelength of the wave then it can be given as

$$l = \frac{5\lambda}{2} \quad \text{or} \quad \lambda = \frac{2l}{5} \qquad \qquad ...(4)$$
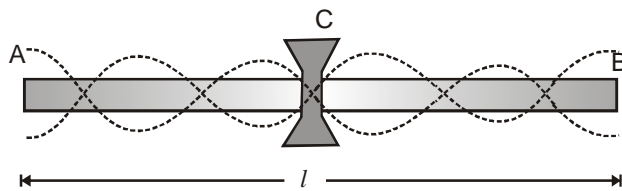
Thus the frequency of oscillation of rod can be given as

$$n_2 = \frac{v}{\lambda} = \frac{5}{2l}\sqrt{\frac{Y}{\rho}} = 5n_0 \qquad \qquad ..(5)$$

Thus the second overtone frequency is the fifth harmonic of the fundamental oscillation frequency of rod. We can also see from the above analysis that the resonant freqencies at which stationary waves are setup in a damped rod are only odd harmonics of fundamental frequency.
Thus when an external source of frequency matching with any of the harmonic of the damped rod then stationary waves are setup in the rod.

9.7   Natural Oscillation of Organ Pipes

When we initiate some oscillations in an organ pipe, which harmonics are excited in the pipe depends on how initial disturbance is produced in it. For example, if you gently blow across the top of an organ pipe it resonates softly at its fundamental frequency. But if you blow must harder you hear the higher pitch of an overtone because the faster airstream higher frequencies in the exciting disturbance. This sound effect can also be achieved by increasing the air pressure to an organ pipe.

9.8   Kundt's Tube

This is an apparatus used to find velocity of sound in a gaseous medium or in different materials. It consists of a glass tube as shown in figure. one end of which a piston B is fitted which is attached to a wooden handle H and can be moved inside and outside the tube and fixed, the rod M of the required material is fixed at clamp C in which the velocity of sound is required, at one end of rod a disc A is fixed as shown.

a thin layer of lycopodium powder
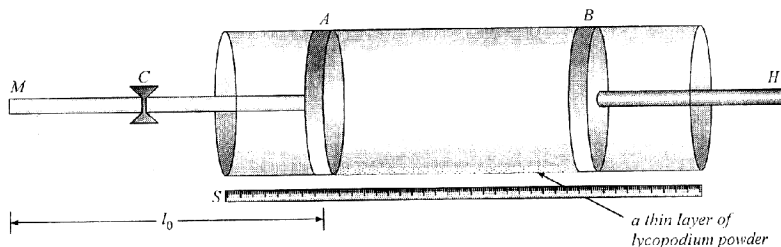
In tube air is filled at room temperature and a thin layer of lycopodium powder is put along the length of the tube. It is a very fine powder particles of which can be displaced by the air particles also. When rod M is gently rubbed with a resin cloth or hit gently, it starts oscillating in fundamental mode as shown in figure, frequency of which can be given as

$$n_{rod} = \frac{v}{\lambda} = \frac{1}{2l_0}\sqrt{\frac{Y}{\rho}} \qquad [\text{As } \lambda_0 = \frac{\lambda}{2}]$$
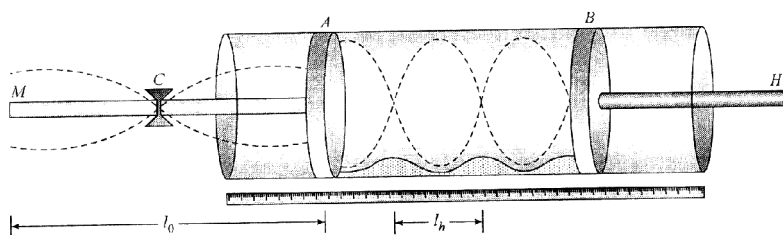
10.   BEATS

When two sources of sound that have almost the same frequency are sounded together, an interesting phenomenon occurs. A sound with a frequency average of the two is heard and the loudness of sound repeatedly grows and then decays, rather than being constant. Such a repeated variation in amplitude of sound are called 'beats'.

If the frequency of one of the source is changed, there is a corresponding change in the rate at which the amplitude varies. This rate is called beat fequency. As the frequencies come close together, the beat frequency becomes slower. A musician can tune a guitar to another source by listening for the beats while increasing or decreasing the tension in each string, eventually the beat frequency becomes very low so that effectively no beats are heard, and the two sources are then in tune.

We can also expalin the phenomenon of beat mathematically. Let us consider the two superposing waves have frequencies n_1 and n_2 then their respective equations of oscillation are

$$y_1 = A \sin 2\pi n_1 t \qquad\qquad\qquad ...(1)$$
and $\qquad\quad y_2 = A \sin 2\pi n_2 t \qquad\qquad\qquad ...(2)$

On superposition at a point, the displacement of the medium particle is given as

$$y = y_1 + y_2$$
$$y = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$$

$$y = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t\sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \qquad ...(3)$$

$$y = R\sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \qquad\qquad ...(4)$$

There equation (4) gives the displacement of medium particle where susperposition takes place, it shows that the particle executes SHM with frequency $\dfrac{n_1 + n_2}{2}$, average of the two superposing frequencies and with amplitude R which varies with time, given as

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$$R = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t \qquad\qquad ...(5)$$

Here R becomes maximum when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$$

or $\qquad 2\pi\left(\frac{n_1 - n_2}{2}\right)t = N\pi \qquad\qquad [N \in I]$

or $\qquad t = \dfrac{N}{n_1 - n_2}$

or at time $\qquad t = 0, \dfrac{1}{n_1 - n_2}, \dfrac{2}{n_1 - n_2}, ...$

At all the above time instants the sound of maximum loudness is heard, similarly we can find the time instans when the loudness of sound is minimum, it occurs when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = 0$$

or $\qquad 2\pi\left(\frac{n_1 - n_2}{2}\right)t = (2N + 1)\dfrac{\pi}{2} \qquad\qquad [N \in I]$

or $\qquad t = \dfrac{2N + 1}{2(n_1 + n_2)}$

or at time instants $\qquad t = \dfrac{1}{2(n_1 - n_2)}, \dfrac{3}{2(n_1 - n_2)}, ...$

Here we can see that these time instants are exactly lying in the middle of the instans when loudest sound is heard. Thus on superposition of the above two frequencies at a medium particle, the sound will be increasing, decreasing, again increasing and decreasing and so on. This effect is called beats. Here the time between two successive maximum or minimum sounds is called beat period, which is given as
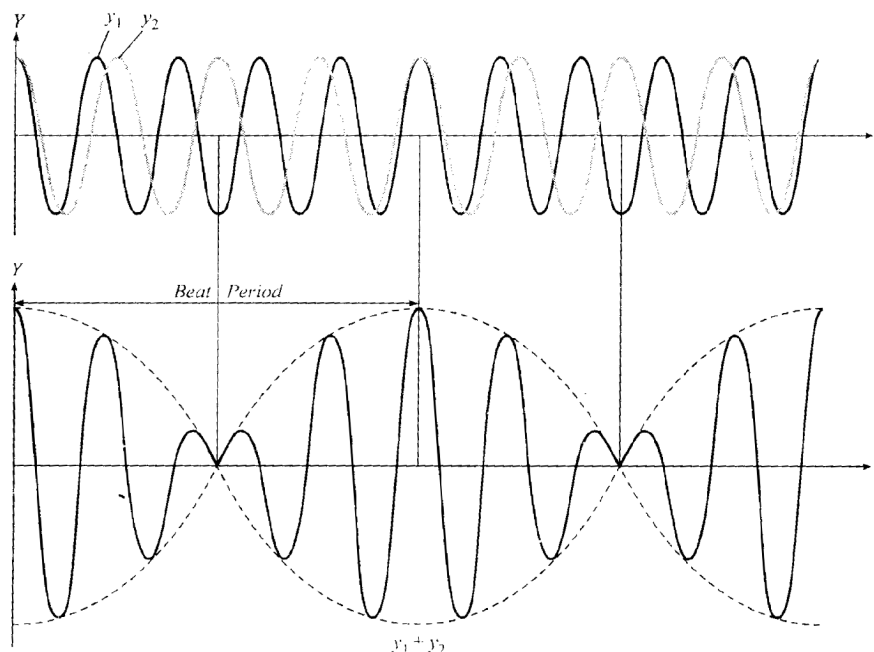
Beat Period T_B = time between two successive maxima = time between two successive minima

$$= \dfrac{1}{n_1 - n_2}$$

Thus beat frequency or number of beats heard per second can be given as

$$f_B = \dfrac{1}{T_B} = n_1 - n_2$$

The superposition of two waves of slightly different frequencies is graphically shown in figure. The resulting envelope of the wave formed after superposition is also shown in figure (b). Such a wave when propagates, produces "beat" effect at the medium particles.

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10.1 Echo

The repetition of sound produced due to reflection by a distant extended surface like a different, hill well, building etc. is called an echo. The effect of sound on human ear remains for approximately one tenth of a second. If the sound is reflected back in a time less then 1/10 of a second, no echo is heard. Hence human ears are not able to distinguish a beat frequency of 10 Hz or more than 10 Hz.
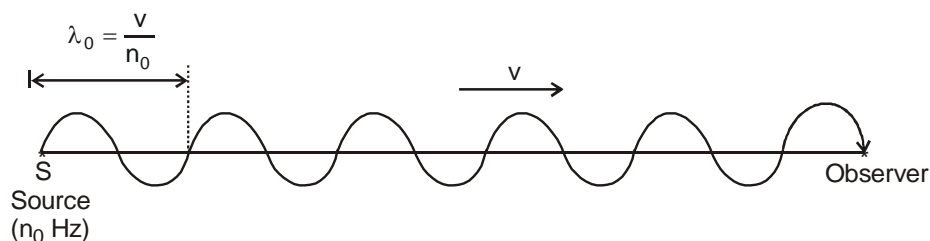
11. DOPPLER'S EFFECT

When a car at rest on a road sounds its high frequency horn and you are also standing on the road near by, you'll hear the sound of same frequency it is sounding but when the car approaches you with its horn sounding, the pich (frequency) of its sound seems to drop as the car passes. This phenomenon was first described by an Austrain Scientist Christien Doppler, is called the Doppler effect, He explained that when a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. Lets discuss the Doppler effect in detail for different cases.

11.1 Stationary Source and Stationary Observer

Figure shown a stationary sources of frequency n_0 which produces sound waves in air of wavelength λ_0 given as

$$\lambda_0 = \frac{v}{n_0}$$ [v = speed of sound in air]

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Although sound waves are longitudinal, here we represent sound weaves by the transverse displacement curve as shown in figure to understand the concept in a better way. As source produces waves, these waves travel towards, stationary observer O in the medium (air) with speed v and wavelength λ_0. As observer is at rest here it will observe the same wavelength λ_0 is approaching it with speed v so it will listen the frequency n given as

$$n = \frac{v}{\lambda_0} = n_0 \qquad \text{[same as that of source]} \qquad ...(1)$$

This is why when a stationary observer listends the sound from a stationary source of sound, it detects the same frequency sound which the source is producting. Thus no Doppler effect takes place if there is no relative motion between source and observer.

11.2 Stationary Source and Moving Observer

Figure shown the case when a stationary sources of frequency n_0 produces sound waves which have wavelength in air given as

$$\lambda_0 = \frac{v}{n_0}$$
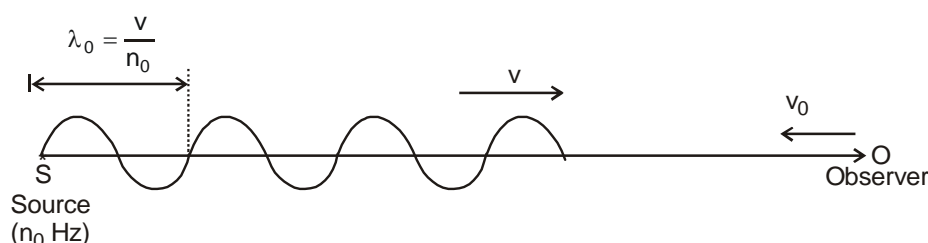
These waves travel toward moving observer with velocity v_0 towards, the source. When sound waves approach observer, it will receive the waves of wavelength λ_0 with speed $v + v_0$ (relative speed). Thus the frequency of sound heard by observer can be given as
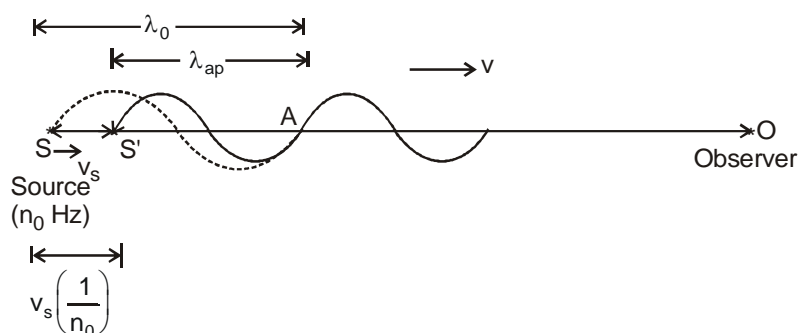
Apparent frequency $\quad n_{ap} = \dfrac{v + v_0}{\lambda_0}$

$$= \frac{v + v_0}{\left(\dfrac{v}{n_0}\right)} = n_0\left(\frac{v + v_0}{v}\right) \qquad ...(2)$$

Similarly we can say that if the observer is receding away from the source the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{v - v_0}{v}\right) \qquad ...(3)$$

11.3 Moving Source and Stationary Observer

Figure shows the situation when a moving source S of frequency n_0 produces sound waves in medium (air) and the waves travel toward observer with velocity v.

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Here if we carefully look at the initial situation when source starts moving with velocity v_s as well as it starts producing waves. The period of one oscillation is $\left(\dfrac{1}{n_0}\right)$ sec and in this duration source emits one wavelength λ_0 in the direction of propagation of waves with speed v, but in this duratin the source will also move forward by a distance $v_s \left(\dfrac{1}{n_0}\right)$. Thus the effective wavelength of emitted sound in air is slightly compressed by this distance as shown in figure. This is termed as apparent wavelength of sound in medium (air) by the moving source. This is given as

Apparent wavelength $\lambda_{ap} = \lambda_0 - v_s\left(\dfrac{1}{n_0}\right)$ 　　　　　　...(1)

$$= \frac{\lambda_0 n_0 - v_s}{n_0} = \frac{v - v_s}{n_0}$$

Now this wavelength will approach observer with speed v (O is at rest). Thus the frequency of sound heard by observer can be given as
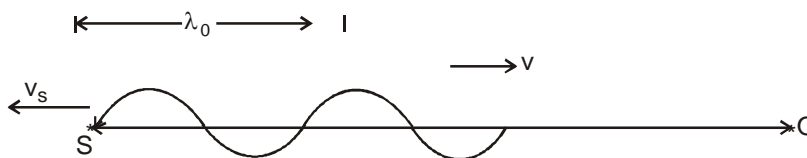
Apparent frequecy 　　　　$n_{ap} = \dfrac{v}{\lambda_{ap}}$

$$= \frac{v}{(v - v_s)/n_0} = n_0\left(\frac{v}{v - v_s}\right)$$ 　　　　　...(2)

Similarly if source is receding away from observer, the apparent wavelength emitted by source in air toward observer will be slightly expanded and the apparent frequency heard by the stationary observer can be given as

$$n_{ap} = n_0\left(\frac{v}{v + v_s}\right)$$ 　　　　　　...(3)

11.4　Moving Source and Moving Observer

Let us consider the situation when both source and observer are moving in same direction as shown in figure at speeds v_s and v_0 respectively.

In this case the apparent wavelength emitted by the source behind it is given as

$$\lambda_{ap} = \frac{v + v_s}{n_0}$$

Now this wavelength will approach the observer at relative speed $v + v_0$ thus the apparent frequency of sound heard by the observer is given as

$$n_{ap} = \frac{v + v_0}{\lambda_{ap}} = n_0\left(\frac{v + v_0}{v + v_s}\right)$$ 　　　　　...(1)

By looking at the expression of apparent frequency given by equation, we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to a moving source as

$$n_{ap} = n_0\left[\frac{v \pm v_0}{v \mp v_s}\right]$$ 　　　　　　...(2)

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Here + and − signs are chosen according to the direction of motion of source and observer. The sign covention related to the motion direction can be stated as :

(i) For both source and observer v_0 and v_s are taken in equation with −ve sign if they are moving in the direction of $\vec{v}$ i.e. the direction of propagation of sound from source to observer.

(ii) For both source and observer v_0 and v_s are taken in equation (2) with +ve sign if they are moving in the direction opposite to $\vec{v}$ i.e. opposite to the direction of propagation of sound from source to observer.

11.5   Doppler Effect in Reflected Sound

When a car is moving toward a stationary wall as shown in figure. If the car sounds a horn, wave travels towards the wall and is reflected from the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch. If we wish to measure the frequency of reflected sound then the problem must be handled in two steps.
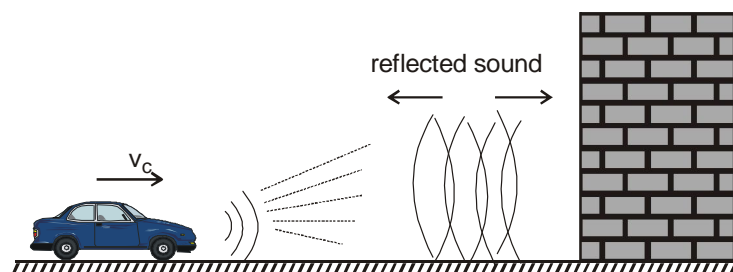

First we treat the stationary wall as stationary observer and car as a moving source of sound of frequency n_0. In this case the frequency received by the wall is given as

$$n_1 = n_0\left(\frac{v}{v - v_c}\right) \qquad \qquad ...(1)$$

Now wall reflects this frequency and behaves like a stationary source of sound of frequency n_1 and car (driver) behave like a moving observer with velocity v_c. Here the apparent frequency heard by the car driver can be given as

$$n_{ap} = n_1\left(\frac{v + v_c}{v}\right)$$

$$= n_0\left(\frac{v}{v - v_c}\right) \times \left(\frac{v + v_c}{v}\right) = n_0\left(\frac{v + v_c}{v - v_c}\right) \qquad (2)$$

Same problem can also be solved in a different manner by using method of sound images. In this procedure we assume the image of the sound source behind the reflector. In previous example we can explain this by situation shown in figure.
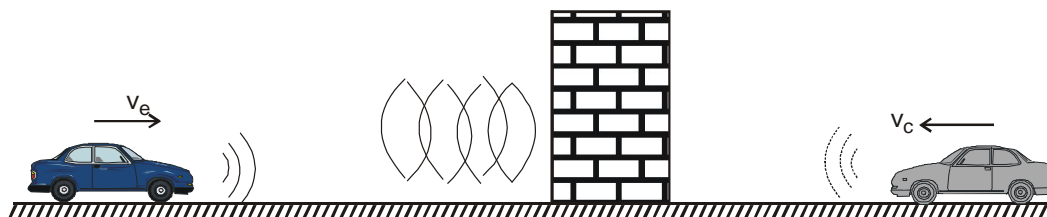
Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming toward it with velocity v_c. Now the frequency of sound heard by car driver can directly be given as

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$$n_{ap} = n_0 \left[\frac{v + v_c}{v - v_c} \right] \qquad ...(3)$$

This method of images for solving problems of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequenctly when the reflector of sound is moving.

11.6 Doppler's Effect for Accelerated Motion

For the case of a moving source and a moving observer, we known the apparent frequency observer can be given as

$$n_{ap} = n_0 \left[\frac{v \pm v_0}{v \mp v_s} \right] \qquad ...(4)$$

Here v is the velocity of sound and v_0 and v_s are the velocity of observer and source respectively.

When a source of observer has accelerated or ratarded motion then in equation (4) we use that value of v_0 at which observer receies the sound and for source, we use that value of v_s at which it has emitted the wave.

The alternative method of solving this case is by the traditional method of compressing or expending wavelength of sound by motion of source and using relative velocity of sound with respect to observer

11.7 Doppler's Effect when Source and Observer are not in Same Line of Motion

Consider the situation shown in figure. Two cars 1 and 2 are moving along perpendicular roads at speed v_1 and v_2. When car - 1 sound a horn of frequency n_0, it emits sound in all directions and say car - 2 is at the position, shown in figure. when it receives the sound. In such cases we use velocity components of the cars along the line joining the source and observer thus the apparent frequency of sound heard by car-2 can be given as

$$n_{ap} = n_0 \left[\frac{v + v_2 \cos \theta_2}{v - v_1 \cos \theta_1} \right] \qquad ...(6.266)$$

car-1

θ_1 $\rightarrow v_1$

v_2 ↑

Car-2

θ_2

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