EXERCISE – V   JEE PROBLEMS

1. Evaluate $\int \sin^{-1}\left(\dfrac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx.$ **[JEE 2001]**

2. For any natural number m, evaluate

$\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx,$ where x > 0

[JEE 2002 (Mains), 4]

3. $\int \dfrac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}} dx$ is equal to   **[JEE 2006, 3]**

(A) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^2} + C$ (B) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^3} + C$

(C) $\dfrac{\sqrt{2x^4-2x^2+1}}{x} + C$ (D) $\dfrac{\sqrt{2x^4-2x^2+1}}{2x^2} + C$

4.(a) Let f(x) = $\dfrac{x}{(1+x^n)^{1/n}}$ for n ≥ 2 and   **[JEE 2007]**

g(x) = $\underbrace{(f \circ f \circ \ldots \circ f)}_{\text{f occurs n times}}$(x). Then $\int x^{n-2}g(x)dx$ equals

(A) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$ (B) $\dfrac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$

(C) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$ (D) $\dfrac{1}{(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

(b) Let F(x) be an indefinite integral of $\sin^2 x$.
Statement–1 : The function F(x) satisfies
F(x + π) = F(x) for all real x.
because
Statement–2 : $\sin^2(x + π) = \sin^2 x$ for all real x.
(A) Statement (1) is true and statement (2) is true
and statement (2) is correct explanation for (1)
(B) Statement (1) is true and statement (2) is true
and statement (2) is NOT correct explanation for (1)
(C) Statement (1) is true but (2) is false
(D) Statement (1) is false but (2) is true **[JEE 2007]**

5. Let I = $\int \dfrac{e^x}{e^{4x}+e^{2x}+1}dx$, J = $\int \dfrac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$

Then, for an arbitrary constant C, the value of J – I
equals   **[JEE 2008]**

(A) $\dfrac{1}{2}\ell n\left(\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$ (B) $\dfrac{1}{2}\ell n\left(\dfrac{e^{2x}+e^x+1}{e^{2x}-e^x+1}\right)+C$

(C) $\dfrac{1}{2}\ell n\left(\dfrac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right)+C$ (D) $\dfrac{1}{2}\ell n\left(\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

6. The integral $\int \dfrac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals
(for some arbitrary constant K)   **[JEE 2012]**

(A) $-\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right\}+K$

(B) $\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right\}+K$

(C) $-\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right\}+K$

(D) $\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right\}+K$