U-2

Differential Eqn

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Diffination : an eqn which involves derivative of one dependant variable variable w.r.t one independent var is called a diff. eqn

eq $\frac{dy}{dx} + y = 0$

Order and Degree of Diff Eqn

$\left(\frac{dy}{dx}\right)^1 + y = 1$ — order = 1, degree = 1

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx}$ — ord = 2, degree = 1

$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + y = 0$ — ord = 3, degree = 2

Types of Diff Eqn

(1) $\frac{dy}{dx} + y = 0$ | This is linear Homogeneous diff Eqn with const. coff.

(2) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin x$ | This is linear non-homogeneous diff Eqn with const. coff.

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(3) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ | Non-linear, homogeneous diff eqn with const coff

(4) $\frac{d^3y}{dx^3} \times \frac{d^2y}{dx^2} + y = \sin x$ | Non lin, non-homo. diff eqn with const coff

$\frac{dy}{dx} - 1 = 0$

gen ~~Sol~~ — particular Sol

$y = x + c$ — $y = x$

Sol. of linear homo diff eqn

(1) $\frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} \cdots a_n \frac{d^{n-1}y}{dx^{n-1}} = 0$

Sit $\frac{d}{dx} = D$

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(1) If all the roots are real and unequal

$$y = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x} + \dots + C_n e^{\alpha_n x}$$

(2) $\alpha_1 = \alpha_2$ and all other roots are distinct

$$y = (C_1 + C_2 x) e^{\alpha_1 x} + C_3 e^{\alpha_3 x} + C_4 e^{\alpha_4 x} \dots C_n e^{\alpha_n x}$$

(3) $\alpha_1 = \alpha_2 = \alpha_3$

$$y = (C_1 + C_2 x + C_3 x^2) e^{\alpha_1 x}$$

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(4) $\alpha_1 = \alpha_2 = \dots = \alpha_n$

$$y = (c_1 + c_2 x + c_3 x^2 + \dots + c_n x^{n-1}) e^{\alpha_1 x}$$

(1) 2, 5

$$y = c_1 e^{2x} + c_2 e^{5x}$$

(2) 2, 5, 3

$$y = c_1 e^{2x} + c_2 e^{5x} + c_3 e^{3x}$$

(3) 2, 2, 5, 5

$$y = (c_1 + c_2 x) e^{2x} + (c_3 + c_4 x) e^{5x}$$

(4) 3 3 3 2 2 5

$$y = (c_1 + c_2 x + c_3 x^2) e^{3x} + (c_4 + c_5 x) e^{2x} + c_6 e^{5x}$$

(5) 3 3, 5 5, 7 7

$$y = (c_1 + c_2 x) e^{3x} + (c_3 + c_4 x) e^{5x} + (c_5 + c_6 x) e^{7x}$$

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(4) $\alpha \pm i\beta$ (when complex)

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

$2 + 3i$

$$y = e^{2x}(C_1 \cos 3x + C_2 \sin 3x)$$

$\alpha = 0$, $\pm i\beta$ (when purely complex)

$$y = C_1 \cos \beta x + C_2 \sin \beta x$$

$\pm 2i$

$$y = C_1 \cos 2x + C_2 \sin 2x$$

$\alpha + i\beta$, $\alpha \pm i\beta$ (when complex repeat 2 time)

$$y = e^{\alpha x}\left[(C_1 + C_2 x)\cos \beta x + (C_3 + C_4 x)\sin \beta x\right]$$

$\alpha \pm i\beta$, $\alpha \pm i\beta$, $\alpha \pm i\beta$ (when " " 3 time

$$y = e^{\alpha x}\left[(C_1 + C_2 + C_3 x^2)\cos \beta x + (C_4 + C_5 + C_6 x^2)\sin \beta x\right]$$

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$\alpha \pm i\beta$, $\gamma \pm i\delta$

$$y = e^{\alpha x}\left[C_1 \cos\beta x + C_2 \sin\beta x\right] + e^{\gamma x}\left[C_3 \cos\delta x + C_4 \sin\delta x\right]$$

$a_1 \pm ib$, $a_2 \pm ib_2$, $a_3 \pm b_3$

$$y = e^{a_1 x}\left[C_1 \cos b_1 x + C_2 \sin \beta x\right]$$

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^3$
4. $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
5. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
6. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
7. $a^2 - b^2 = (a-b)(a+b)$

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Factorize Date: / /

#
$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{-5} = i\sqrt{5}$$

$$\sqrt{-4} = i\sqrt{4} = i2$$

(1) $\frac{dy}{dx} - 2y = 0$

in Symbolic form : $(D - 2)y = 0$

In Axulariy Eq^n (AE) : (tak coff of $y = 0$)

$$D - 2 = 0$$

$$D = 2$$

$$\boxed{y = C_1 e^{2x}}$$

(2) $\frac{dy}{dx} + 5y = 0$

SF $(D + 5)y = 0$

AE : $D + 5 = 0$

$$D = -5$$

$$\boxed{y = C_1 e^{-5x}}$$

Eq^n of 2nd order

(3) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

SF = $(D^2 + 3D + 2)y = 0$ $1 \times 2 = 2$

AE = $D^2 + 3D + 2 = 0$

$$D^2 + 2D + D + 2 = 0$$

$$D(D+2) + 1(D+2) = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2, -1$$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-1x}}$$

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(4) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$

$SF = (D^2 + 7D + 12)y = 0$ $\qquad 1 \times 12$

$AE = D^2 + 7D + 12 = 0$

$D^2 + 4D + 3D + 12 = 0$

$D(D+4) + 3(D+4) = 0$

$(D+4)(D+3) = 0$

$D = -4, -3$

$y = c_1 e^{-4x} + c_2 e^{-3x}$

(5) $\frac{d^2y}{dx^2} + 9\frac{dy}{dx} + 20y = 0$

$1 \times 20 = 0$

$SF = (D^2 + 9D + 20)y = 0$ $\qquad 4 \times 5 = 20$

$AE = D^2 + 9D + 20 = 0$

$D^2 + 4D + 5D + 20 = 0$

$D(D+4) + 5(D+4) = 0$

$(D+4)(D+5$

$D = -4, -5$

$y = c_1 e^{-4x} + c_2 e^{-5x}$

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(6) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 0$

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$SF = (D^2 + 7D + 10) = 0$

$D^2 + 5D + 2D + 10 = 0$

$D(D+5) + 2(D+5) = 0$

(7) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

$1 \times 9 = 9$

AE: $D^2 + 6D + 9 = 0$

$D^2 + 3D + 3D + 9 = 0$

$D(D+3) + 3(D+3) = 0$

$(D+3)(D+3) = 0$

$D = -3, -3$

$y = (C_1 + C_2)e^{-3x}$

(8) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$1 \times 1 = 1$

$D^2 - 2D + 1 = 0$

$D^2 - D - D + 1 = 0$

$D(D-1) - 1(D-1) = 0$

$(D-1)(D-1)$

$D = 1, 1$

$y = (C_1 + C_2)e^{1x}$

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(9) $(D^2 - 4)y = 0$

$D^2 - 4 = 0$

$D^2 - 2^2 = 0$

$(D-2)(D+2) = 0$

$D = 2, -2$

$y = C_1 e^{2x} + C_2 e^{-2x}$

(10) $(D^2 - 5)y = 0$

AE $D^2 - 5 = 0$

$D^2 - \sqrt{5}^2 = 0$

$(D - \sqrt{5})(D + \sqrt{5}) = 0$

$y = C_1 e^{\sqrt{5}x} + C_2 e^{-\sqrt{5}x}$

(4) $D^2 - a^2 = 0$

$(D+a)(D-a) = 0$

$D = -a, a$

$y = C_1 e^{-ax} + C_2 e^{ax}$

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(12) $D^2 - w = 0$

$D^2 - \sqrt{w}^2 = 0$

$(D + \sqrt{w})(D - \sqrt{w}) = 0$

$D = -\sqrt{w}, \sqrt{w}$

$y = c_1 e^{-\sqrt{w}x} + c_2 e^{\sqrt{w}x}$

(13) $D^2 - 16 = 0$

$D^2 - 4^2 = 0$

$(D+4)(D-4)$

$D = -4, 4$

$y = c_1 e^{-4x} + c_2 e^{4x}$

(14) $D^2 + D + 1 = 0$

$a = 1, b = 1, c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1}$

$= \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1}{2} \pm \frac{i\sqrt{3}}{2}$

$y = e^{\frac{-1}{2}x}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right)$

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(15) $2D^2 + D + 2 = 0$

$a = 2, \; b = 1, \; c = 2$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{4} = \frac{-1 \pm \sqrt{-15}}{4}$$

$$= \frac{-1}{4} \pm \frac{i\sqrt{15}}{4}$$

$$y = e^{-\frac{1}{4}x}\left(C_1 \cos\frac{\sqrt{15}}{4}x + C_2 \sin\frac{\sqrt{15}}{4}x\right)$$

(16) $D^2 + D + 5 = 0$

$a = 1, \; b = 1, \; c = 5$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$=$

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(17) $D^2+4=0$

$D^2=-4$

$D=\pm\sqrt{-4}$

$D=\pm i\sqrt{4}$

$D=\pm i2$

$y = C_1 \cos 2x + C_2 \sin 2x$

(18) $D^2+1=0$

$D^2=-1$

$D=\pm\sqrt{-1}$

$D=\pm i\sqrt{1}$

$y = C_1 \cos\sqrt{1}x + C_2 \sin\sqrt{1}x$

(19) $D^2+9=0$

$D^2=-9$

$D=\pm\sqrt{-9}$

$D=\pm i\sqrt{9}$

$y = C_1 \cos\sqrt{9}x + C_2 \sin\sqrt{9}x$

(20) $(D^2+16)=0$

$D^2=-16 \Rightarrow$

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$\frac{d}{dx} e^{mx} = m e^{mx}$

$\frac{d}{dx}(\text{const}) = 0$

$\frac{d}{dx} e^{2x} = 2e^{2x}$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(e^{-x}) = -e^{-x}$

$\frac{d}{dx} x^2 = 2x$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} x^3 = 3x^2$

$\frac{d}{dx} \sin 2x = 2 \cos 2x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \cos 2x = -2 \sin 2x$

$$d(u.v) = u \frac{d}{dx}(v) + v \frac{d}{dx}(u)$$

Question

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(Q) Solve the following diff eq^n

① $\frac{dy}{dx} - 5y = 0$; $y(0) = 1$

In SF : $(D-5)y = 0$

In AE : $D - 5 = 5$

$D = 5$

$y = C_1 e^{5x}$

when $x = 0$, $y = 1$

$1 = C_1 e^{0x}$

$1 = C_1$

$y = e^{5x}$

Alt method

$\frac{dy}{dx} - 5y = 0$, $y(0) = 1$

what is the sol of given problem

(a) $y = 2e^{5x}$ (b) $y = 3e^{-5x}$

(c) $y = 4e^{5x}$ (d) $y = e^{5x}$

Then we check each option with $x = 0$ and y must $= 1$

② Solve the following diff eq^n :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0 ; y(0) = 0 ; y'(0) = 5$$

In AE : $D^2 + 3D + 2 = 0$

$D^2 + 2D + D + 2 = 0$

$D(D+2) + 1(D+2) = 0$

$(D+1)(D+2) = 0$

$D = -1, -2$

$y = C_1 e^{-1x} + C_2 e^{-2x}$ — ①

~~At~~ Sat $x = 0$, $y = 0$

$0 = C_1 + C_2$

$C_1 = -C_2$ — ②

$y'(x) = -C_1 e^{-x} - 2C_2 e^{-2x}$

when $x = 0$ $y' = 5$

$5 = -C_1 - 2C_2$

$5 = -(-C_2) - 2C_2$

$5 = C_2 - 2C_2$

$5 = -C_2$

$C_2 = -5$

from ②

$C_1 = -(-5) = 5$

$y = 5e^{-x} - 5e^{-2x}$

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Solu:

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$

(b) $4y'' - 8y' + 3y = 0$

(c) $4y'' + 4y' + y = 0$

(d) $y'' + 6y' + 9y = 0$

(e) $y'' - 4y' - 5y = 0$

(f) $y'' + 2y' + 2y = 0$

(g) $y'' + 4y' + 13y = 0$

(a) $D^2 - D - 6 = 0$

$\cancel{D^2 - 3D + D - 6 = 0}$

$D^2 - 3D + 2D - 6 = 0$

$D(D-3) + 2(D-3) = 0$

$(D+2)(D-3) = 0$

$D = -2, 3$

$y = C_1 e^{-2x} + C_2 e^{3x}$

(b) $4D^2 - 8D + 3 = 0$

$4D^2 - 6D - 2D + 3 = 0$

$2D(2D-3) - 1(2D-3) = 0$

$(2D-1)(2D-3) = 0$

$D = \frac{1}{2}, \frac{3}{2}$

$y = C_1 e^{x/2} + C_2 e^{3x/2}$

(c) $4D^2 + 4D + 1 = 0$

$4D^2 + 2D + 2D + 1 = 0$

$2D(2D+1) + 1(2D+1) = 0$

$(2D+1)(2D+1) = 0$

$2D = -1$

$D = -\frac{1}{2}$

$y = (C_1 + C_2 x) e^{-\frac{1}{2}x}$

(d) $D^2 + 6D + 9 = 0$

$D^2 + 3D + 3D + 9 = 0$

$D(D+3) + 3(D+3) = 0$

$(D+3)(D+3) = 0$

$D = -3, -3$

$y = (C_1 + C_2 x) e^{-3x}$

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(e) $D^2 - 4D - 5 = 0$

$D^2 - 5D + D - 5 = 0$

$D(D-5) + 1(D-5) = 0$

$(D+1)(D-5) = 0$

$D = -1, 5$

(f) $D^2 + 2D + 2 = 0$

$a = 1, b = 2, c = 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2 \times 1}$

$= \frac{-2 \pm \sqrt{-4}}{2}$

$= \frac{-2}{2} \pm \frac{i.2}{2}$

$= -1 \pm i1$

$y = e^{-1x}(C_1 \cos x + C_2 \sin x)$

(g) $D^2 + 4D + 13 = 0$

$a = 1, b = 4, c = 13$

$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 13}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{-36}}{2}$

$= \frac{-4}{2} \pm \frac{i6}{2}$

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Q $(D^2+9)y=0$

$D^2+9=0$

$D^2=-9$

$D=\pm\sqrt{-9}$

$D=\pm i3$

$y = C_1 \cos 3x + C_2 \sin 3x$

Q $D^3+3D^2+3D+1=0$

in last no can write as cube of any number

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(D+1)^3=0$

$D=-1,-1,-1$

$y=(C_1+C_2x+C_3x^2)e^{-x}$

Solve the foll. diff eqn

$$\frac{d^2y}{dx^2} + \frac{12dy}{dx} + 20y = 0$$

$\Rightarrow D^2 + 12D + 20 = 0$

$D^2 + 10D + 2D + 20 = 0$

$D(D + 10) + 2(D+10) = 0$

$(D+10)(D+2)$

$D = -10, -2$

$y = c_1 e^{-10x} + c_2 e^{-2x}$

$1 \times 20 = 20$

$D^2 + 10D + 25 = 0$

$D^2 + 5D + 5D + 25 = 0$

$D(D+5) + 5(D+5) = 0$

$D = -5, -5$

$y = (c_1 x + c_2 x)e^{-5x}$

$D^2 + 1 = 0$

$D = \pm\sqrt{-1}$

$D = \pm i$

$y = [c_1 \cos 1x + c_2 \sin 1x]$

$D^2 + 4 = 0$

$D = \pm\sqrt{-4}$

$D = \pm i2$

$y = [c_1 \cos 2x + c_2 \sin 2x]$

$D^5 - D^3 = 0$

$D^3(D^2 - 1) = 0$

$D^3(D-1)(D+1) = 0$

$D = 0, 0, 0, -1, +1$

$y = (c_1 + c_2 x + c_3 x)e^0 + c_4 e^{-1x} + c_5 e^{1x}$

$D^3 - 6D^2 + 11D - 6 = 0$ $\xrightarrow{\text{factors}}$ $(\pm 1, \pm 2, \pm 3, \pm 6)$

Let $D = 1$

$1^3 - 6(1)^2 + 11 \times 1 - 6 = 0$

Satisfy

$0 = 0$

$\Rightarrow$ Coff of Polyn:

+1	1	-6	11	-6
			-5	6
	1	-5	6	0

$(D-1)(D^2 - 5D + 6) = 0$

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$(D-1)(D^2-3D-2D+6)=0$

$(D-1)(D(D-3)-2(D-3))=0$

$(D-1)(D-2)(D-3)=0$

$D=1,2,3$

$y=c_1e^{1x}+c_2e^{2x}+c_3e^{3x}$

$D^3+6D^2+11D+6=0$ | $\pm1, \pm2, \pm3, \pm6$

$-1+6-11+6=0$

$12-12=0$

-1

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$D^3+3D^2+3D+(1)^3=0$ → when cube of 9

$(D+1)^3=0$

$(D+1)(D+1)(D+1)=$

$D=-1,-1,-1$

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$$D^4 - 5D^2 + 4 = 0$$
$$D^4 - 4D^2 - D^2 + 4 = 0$$
$$D^2(D^2 - 4) - 1(D^2 - 4) = 0$$
$$(D^2 - 1)(D^2 - 4)$$

$$D = 1, -1, -2, 2$$

$$D^4 + 5D^2 + 4 = 0$$
$$D^4 + 4D^2 - D^2 + 4 = 0$$
$$D^2(D^2 + 4) - 1(D^2 + 4) = 0$$
$$(D^2 - 1)(D^2 + 4) = 0$$
$$D^2 = -1 \quad D^2 = -4$$
$$D = \pm i \quad D = \pm i2$$

$$y = [C_1 \cos x + C_2 \sin x] + [C_3 \cos 2x + C_4 \sin 2x]$$

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① $(D^2+1)y=0$

② $y''' - y'' + 100y' - 100y = 0$

③ $\frac{d^4y}{dx^4} - \frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^3} - 11\frac{dy}{dx} - 4y = 0$

④ $(D^4 + K^4)y = 0$

⑤ $(D^4 - K^4)y = 0$

⑥ $(D^2 + 6D + 4)y = 0$

⑦ $(D^2+1)^3 (D^2 + D + 1)^2 y = 0$

⑧ $y_2 - y_1 - 2y = \sinh 2x$

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① $D^2 + 1 = 0$

$(D+1)(D+1) = 0$

$D = -1, -1$

② $D^3 - D^2 + 100D - 100 = 0$

Put $D = +1$ $(\pm 1, \pm 5, \pm 10, \pm 50, \pm 100)$

$1^3 - 1^2 + 100 - 100 = 0$

$0 = 0$

1	1	-1	+100	-100
		1	0	100
	1	0	100	0

$(D-1)(D^2 + 0D + 100) = 0$

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$$(D-1)(D^2+100)=0$$
$$(D-1)(D+10)(D+10)=0$$
$$D=1,\ -10,\ -10$$

$$y = c_1 e^{-1x} + c_2 e^{-10x} + c_3 e^{-10x}$$

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(3) $D^4 - D^3 - 9D^2 - 11D - 4 = 0$

~~(4)~~ (7) $(D^2+1)^3 (D^2+D+1)^2 y = 0$

$D^2+1=0$

$D = \pm\sqrt{-1}$

$D = \pm i$

$D = \pm i, \pm i, \pm i$

$D^2 + D + 1 = 0$

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(e) $D^4 + K^4 = 0$

$D^4 = -K^4$

$D = \pm \sqrt[4]{-K^4}$

$D = \pm -K$

(1) $y'' - y' - 6y = 0$

(2) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$

(3) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

(4) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 5y = 0$

(5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

(7) $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 8y = 0$

(8) $\frac{d^2y}{dx^2} - y = 0$

(9) $\frac{d^3y}{dx^3} - y = 0$

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$D^2 + 6D + 4 = 0$

$a = 1,\ b = 6\ c = 4$

$D = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

$= \frac{-6 \pm \sqrt{36 - 16}}{2} = \frac{-6 \pm \sqrt{20}}{2}$

$1 \times 6 = 1$

$- + - = 6$

(1) $D^2 - D - 6 = 0$

$a = 1\ b = -1\ c = -6$

$D = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot -6}}{2 \times 1}$

$= \frac{1 \pm \sqrt{1 + 24}}{2}$

$= \frac{1 \pm 5}{2}$

$= \frac{1}{2} \pm \frac{5}{2}$ / $\frac{1}{2} + \frac{5}{2}$ | $\frac{1}{2} - \frac{5}{2}$

$\frac{6}{2}$ $\frac{-4}{2}$

$y = C_1 e^{6/2 x} + C_2 e^{-4/2 x}$

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(2) ~~4D²~~

$4D^2 + 4D + 1 = 0$

$\Rightarrow 4D^2 + 2D + 2D + 1 = 0$

~~$2D(D+1)$~~ ~~$4D^2$~~

$2D(2D+1) + 1(2D+1) = 0$

$(2D+1)(2D+1) = 0$

$D = -\frac{1}{2}, -\frac{1}{2}$

$y = \cancel{c_1 e^{-\frac{1}{2}x} + c_2 e^{-\frac{1}{2}x}}$
$(c_1 + c_2 x) e^{-\frac{1}{2}x}$

(3) $D^2 + 6D + 9 = 0$

$D^2 + 3D + 3D + 9 = 0$

$D(D+3) + 3(D+3) = 0$

$(D+3)(D+3) = 0$

$D = -3, -3$

$y = (c_1 + c_2 x) e^{-3x}$

(4) $D^2 + 4D - 5 = 0$

$a = 1,\ b = 4,\ c = -5$

$D = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -5}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{16 + 20}}{2} = \frac{-4 \pm \sqrt{36}}{2}$

$= \frac{-4 \pm 6}{2} = \frac{\cancel{-4}^{-2}}{\cancel{2}} \pm \frac{\cancel{6}^{3}}{\cancel{2}}$

$= (-2+3)(-2-3) = 1, -5$

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$y = c_1 e^{1x} + c_2 e^{-5x}$

(5) $D^2 + 2D + 2 = 0$

$a = 1,\ b = 2,\ c = 2 = 0$

$\cancel{D} = x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2 \times 1} = \dfrac{-2 \pm \sqrt{4 - 8}}{2}$

$= \dfrac{-2 \pm \sqrt{-4}}{2} = \dfrac{-2 \pm 2\sqrt{-1}}{2} = \dfrac{-\cancel{2}}{\cancel{2}} \pm \dfrac{\cancel{2}i}{\cancel{2}}$

$= -1 \pm i$

$y = e^{-1x}(c_1 \cos x + c_2 \sin x)$

(6) $D^2 + 4D + 13 = 0$

$a = 1,\ b = 4,\ c = 13$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 13}}{2 \times 1}$

$\begin{array}{r} 52 \\ -16 \\ \hline 36 \end{array}$

$= \dfrac{-4 \pm \sqrt{16 - 52}}{2} = \dfrac{-4 \pm \sqrt{-36}}{2}$

$= \dfrac{-4 \pm 6i}{2} = \dfrac{-4}{2} \pm \dfrac{6i}{2} = -2 \pm 3i$

$y = e^{-2x}(c_1 \cos 3x + c_2 \sin 3x)$

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(7) $D^3 + 6D^2 + 12D + 8 = 0$

$(D+2)^3 = 0$

$D = -2, -2, -2$

$y = (c_1 + c_2 x \; c_3 x^2) e^{-2x}$

(8) $D^2 - 1 = 0$

$D = 1, 1$

$y = (c_1 + c_2 x) e^x$

(9) $D^3 - 1^3 = 0$

$(D-1)(D^2 + D + 1) = 0$

$D^2 + D + 1 = 0$

$a = 1, \; b = 1, \; c = 1$

$$x = \frac{-1 \pm \sqrt{4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{4}}{2} \qquad = \frac{-1 \pm 2}{2}$$

$$= \frac{-1}{2} \pm \frac{2}{2} \qquad \Rightarrow \frac{-1}{2} \pm 1$$

$$= \frac{-1}{2} + 1 \;, \quad \frac{-1}{2} - 1$$

$$= \frac{-1+2}{2} \;, \quad \frac{-1-2}{2}$$

$$= \frac{1}{2}, \; -3/2$$

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(1) $$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

$$D^3 - 6D^2 + 11D - 6 = 0 \qquad (\pm1, \pm2, \pm3, \pm6)$$

Sit $D = 1$

$$1 - 6 + 11 - 6 = 0$$

$$0 = 0$$

1	1	-6	11	-6
		1	-5	6
	1	-5	6	0

$$(D-1)(D^2 - 5D + 6) = 0$$

$$(D-1)(D^2 - 3D - 2D + 6) = 0$$

$$(D-1)(D(D-3) - 2(D-3)) = 0$$

$$(D-1)(D-3)(D-2) = 0$$

$$D = 1, 3, 2$$

$$y = C_1 e^{1x} + C_2 e^{3x} + C_3 e^{2x}$$

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Results:

(1) ~~a^3+b~~ $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

(2) $(a-b)^3 = a^3-b^3-3a^2b+3ab^3$

Question 1: $D^3+3D^2+3D+1^3=0$ → is last cube of any number

$(D+1)^3 = 0$

$D = -1, -1, -1$

$y = (C_1 + C_2x + C_3x^2)e^{-x}$

Q2 $d^3 - 6d^2 + 12d - 8$

$(D-2)^3 = 0$

$D = 2, 2, 2$

$y = (C_1 + C_2x + C_3x^2)e^{2x}$

$D^4 - 5D^2 + 4 = 0$ $\left\{ D^2 = z \right.$

$z^2 - 5z + 4 = 0$

$z^2 - 4z - z + 4 = 0$

$z(z-4) - 1(z-4) = 0$

~~$z = 1, 4$~~ $(z-1)(z-4)$

$(D^2-1)(D^2-4)$

$D = -1, 1, 2, +2$

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$D^4 + 5D^2 + 4 = 0$ $\{ D^2 = 2$

$\cancel{z^2 + 5z + 4 = 0}$

Result

(1) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Q1 $D^3 + 1 = 0$

$(D+1)(D^2 - D + 1) = 0$

$a = 1,\ b = -1,\ c = 1$

$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1-4}}{2}$

$= \frac{1 \pm i\sqrt{3}}{2} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y = C_1 e^{-1x} + e^{\frac{1}{2}x}\left(C_2 \cos\frac{\sqrt{3}}{2}x + C_3 \sin\frac{\sqrt{3}}{2}x\right)$

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Results:

① $\frac{d}{dx}(\text{constant}) = 0$

② $\frac{d}{dx}(e^{ax}) = ae^{ax}$

③ $\frac{d(x)}{dx} = 1$

④ $\frac{d(x^2)}{dx} = 2x$

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⑤ $\frac{d}{dx}(4x^2) = 8x$

---

① $D - 5 = 0$; $y(0) = 1$

$D = 5$

Sol

$y = c_1 e^{5x}$ — ①

When $x = 0$ $y = 1$

$1 = c_1$

$y = e^{5x}$

---

② $D^2 - 3D + 2 = 0$, $y(0) = 0$, $y'(0) = 3$

$D^2 - 2D - D + 2 = 0$

$D(D-2) - 1(D-2) = 0$

$(D-1)(D-2) = 0$

$D = 1, 2$

$y = c_1 e^{x} + c_2 e^{2x}$ — ①

when $x = 0$, $y = 0$

$0 = c_1 + c_2$

$c_1 = -c_2$ — ②

$y' = c_1 e^{x} + 2c_2 e^{2x}$

when $x = 0$, $y' = 3$

$3 = c_1 + 2c_2$

$3 = -c_2 + 2c_2$

$3 = c_2$

$c_1 = -3$

from ①

$y = -3e^{x} + 3e^{2x}$

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1. $(D-2)y=0$, $y(0)=8$
2. $(D+5)y=0$, $y(0)=3$
3. $(D^2-6D+9)y=0$, $y(0)=1$, $y'(0)=5$
4. $(D^2-7D+12)y=0$, $y(0)=3$, $y'(0)=8$
5. $(D^2-9D+14)y=0$, $y(0)=0$, $y'(0)=4$

(1) $D^{-1}X = \frac{1}{D}X = \int X\,dx$ { X function of x could be exp, poly etc.

(2) $\frac{1}{D-a}X = e^{ax}\int e^{-ax}X\,dx$

(3) $\frac{1}{D+a}X = e^{-ax}\int e^{ax}X\,dx$

(4) $\frac{1}{D-a}\phi(x) = e^{ax}\int e^{-ax}\phi(x)\,dx$

Non - Homo diff eqn

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + a_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_n y = f(x)$$

In SF

$$(D^n + a_1 D^{n-1} + \dots + a_n)y = f(x)$$

$$G(D)y = f(x)$$

$$\boxed{y = \frac{1}{G(D)}f(x)} \longrightarrow 5 \text{ types to find particular integral}$$

This is called $\underbrace{\text{perticular}}_{\text{no const}}$ integral of diff eqn

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Type ① When $f(x)$ is expo fun $= e^{ax}$

Particular Integral $PI = \frac{1}{G(D)} e^{ax}$

Oulti' replace D by a to get the Sol^n

$= \frac{1}{G(a)} e^{ax}$ { Only when $G(a) \neq 0$

if $G(a) = 0$

then, $PI = x \frac{1}{G'(D)} e^{ax}$

replace $D \to a$

$= x \frac{1}{G'(a)} e^{ax}$ { if $G'(a) \neq 0$

if $G'(a) = 0$

Then, $PI = x^2 \frac{1}{G''(D)} e^{ax}$

replace $D \to a$ { if $G''(a) \neq 0$

$= x^2 \frac{1}{G''(a)} e^{ax}$

⋮

Keep repeating till

$G''(a) \neq 0$

⋮

Q. Find the PI of :

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(1) $\frac{1}{D^2+4} e^{3x}$

$PI = \frac{1}{D^2+4} e^{3x}$

Put $D \to 3$

$PI = \frac{1}{13} e^{3x}$

(2) $\frac{1}{D^2+3D+5} e^{2x}$

$PI = \frac{1}{4+6+5} e^{2x}$

$= \frac{1}{15} e^{2x}$

(3) $\frac{1}{D^2-3D+2} e^{2x}$

$PI = \frac{1}{4-6+2} e^{2x}$

$= \frac{1}{0} e^{2x}$

$= x \frac{1}{2\times2-3} e^{2x} = \frac{x}{1} e^{2x}$

Take derivative $\to 2x - 3$

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Q $\frac{1}{D^2-4D+4} e^{2x}$

$\frac{1}{2^2-4(2)+4} e^{2x}$

$= \frac{1}{4-8+4} e^{2x}$

$= x \frac{1}{0}$

Q $PI = \frac{1}{D^3+1} \cdot (1+e^x)^2$

$= \frac{1}{D^2+1} (1^2+(e^x)^2+2x1\times e^x$

$= \frac{1}{D^2+1} (1+e^{2x}+2e^x$

$= \frac{1}{D^2+1} (e^{0x}+e^{2x}+2e^x)$

$= \frac{1}{D^2+1} e^{0x} + \frac{1}{D^2+1} e^{2x} + \frac{2e^x}{D^2+1})$

$= \frac{1}{D^2+1} e^{0x} + \frac{1}{2}$

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Date: / /

(1) $PI = \frac{1}{D^3 - D^2 + 4D - D} e^x$

(2) $\frac{1}{D^3 - 6D^2 + 11D - 6} e^{2x}$

(3) $\frac{1}{D^2 + 1} (e^{mx} + e^{nx})$

(4) $\frac{1}{D^3 + 1} e^{x/2}$

(5) $\frac{1}{D^2 - D + 1} (48e^{2x} + 64e^{4x})$

---

(1) $PI = \frac{1}{1^3 - 1^2 + 4 - 1} e^x$

$= \frac{1}{1 + 1 + 4 - 1} e^x = \frac{1}{5} e^x$

(2) $\frac{1}{2^3 - 6(2)^2 + 11 \times 2 - 6} e^{2x} = \frac{1}{8 - 24 + 22 - 6} e^{2x}$

$= \frac{1}{0} e^{2x} \Rightarrow x \frac{1}{3D^2 - 12D + 11} e^{2x}$

$\Rightarrow x \frac{1}{3(2)^2 - 12(2) + 11} e^{2x} \Rightarrow \frac{1}{12 - 24 + 11}$

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Type 2 $\dfrac{1}{f(D^2)}$ (Sin ax or Cos ax)

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must be polynomial of even degree

Replace D^2 by $-(a)^2$

$= \dfrac{1}{f(-a^2)}$ Sin ax or cos ax

final if $f(-a^2) \neq 0$

If $f(-a^2)$ then I put on $\dfrac{1}{f(D^2)}$, few

Q1 PI $\dfrac{1}{D^2+9}$ Sin 2x

replace D^2 by $-(a)^2$

$PI = \dfrac{1}{-4+9}$ Sin 2x

$= \dfrac{1}{5}$ Sin 2x

Q

$\dfrac{1}{-9+16}$ Cos 3x

$\dfrac{1}{7}$ Cos 3x

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Q $$\frac{1}{D^2+9}\cos 3x$$

$$= \frac{1}{-9+9}\cos 3x = \frac{1}{0}\cos 3x$$

$$= x\frac{1}{2D}\cos 3x$$ $\left\{ \frac{1}{D} \text{ is integration} \right.$

$$= \frac{x}{2}\int \cos 3x = \frac{x}{2}\frac{\sin 3x}{3} = \frac{x}{6}\sin 3x$$

Q $$\frac{1}{D^2+a^2}\sin ax = \frac{1}{-a^2+a^2}\sin ax$$

$$= x\frac{1}{2D}\sin ax = \frac{x}{2}\int \sin ax$$

$$= \frac{-x}{2a}\cos ax = \frac{-x}{2a}\cos ax$$

(1) $$\frac{1}{D^3+3D^2+16}(1+e^{2x})^3$$

(2) $$\frac{1}{D^4+D^2+16}(\sin 3x)$$

(3) $$\frac{1}{D^4-5D^2+4}e^{x}$$

(4) $$\frac{1}{D^4+5D^2+4}\cos 2x$$

(5) $$\frac{1}{D^2+1}(e^{3x}+\sin 3x+1)$$

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Type ① $f(x) = e^{ax}$

$$PI = \frac{1}{G(D)}$$

$D \to a$

Type ② $f(x) = \sin ax$ or $\cos ax$

$$\frac{1}{f(D^2)} (\sin ax \text{ or } \cos ax)$$

$D^2 \to -(a^2)$

Q1 ⇒ $\frac{1}{D^2+5} \cos 6x$

$$\frac{1}{-36+5} \cos 6x$$

$$= \frac{1}{-31} \cos 6x$$

Q_2 ⇒ $\frac{1}{D^3+1} \cos 2x$

$$\frac{1}{D\times -4+1} \cos 2x$$

$$\frac{1}{1-4D} \cos 2x$$

Whu Mult/div

$(1+4D)$

$$= \frac{1+4D}{(1-4D)(1+4D)} \cos 2x$$

$$= \frac{1+4D}{1^2-(4D)^2} \cos 2x$$

$$= \frac{1+4D}{1-16D^2} \cos 2x$$

$$= \frac{1+4D}{1-16\times(-4)} \cos 2x = \frac{1+4D}{1+64} \cos 2x$$

Q_3 ⇒

$$= \frac{1}{65}\left(\cos 2x + 4D\cos 2x\right)$$

$$= \frac{1}{65}\left(\cos 2x + 4\times 2\sin 2x\right)$$

$$= \frac{1}{65}\left(\cos 2x - 8\sin 2x\right)$$

$Q_3 \Rightarrow \frac{1}{D^2+3D+7}\sin 3x \Rightarrow \frac{1}{-9+3D+7}\sin 3x$

$\Rightarrow \frac{1}{3D-2}\sin 3x \Rightarrow \frac{3D+2}{(3D-2)(3D+2)}\sin 3x$

$\Rightarrow \frac{3D+2}{9D^2-4}\sin 3x \Rightarrow \frac{3D+2}{-81-4}\sin 3x$

$\Rightarrow \frac{3D+2}{-85}\sin 3x \Rightarrow -\frac{1}{85}\,3D\sin 3x + 2\sin 3x$

$\Rightarrow -\frac{1}{85}\,3\times 3\cos 3x + 2\sin 3x$

$= -\frac{1}{85}\,9\cos 3x + 2\sin 3x$

Find the PI HW (1)

(1) $PI = \frac{1}{D^2+4} 6\cos x$

(2) $PI = \frac{1}{2D^2+D-1} 16\cos 2x$

(3) $PI = \frac{1}{D^3-D^2+4D-4} \sin 3x$ X

(4) $PI = \frac{1}{D^2+1} 6\sin x$

Type (3) Polynomial fun

$$PI = \frac{1}{f(D)} g(x)$$

When $g(x)$ is a polynomial function.

Poly → $g(x) = x^3+2x+5$, x^2+1, x^2+5x+2

Write $f(D)$ in ascending power of D
Apply binomial expansion for find the solution of the problem

→ $f(D) = D^2+D+1$

(1) $(1+D)^{-1} = 1-D+D^2-D^3+D^4 \dots \infty$

(2) $(1-D)^{-1} = 1+D^2+D^3+D^4 \dots \infty$

(3) $(1+D)^{-2} = 1-2D+3D^2+4D^3 \dots \infty$

(4) $(1-D)^{-2} = 1+2D+3D^2+4D^3 \dots \infty$

$D^2(x) = 0$

if degree of D > the degree of x then ans = 0

Q1) find

$$PI = \frac{1}{D+1}(x^2)$$

$$= \frac{1}{1+D}x^2 = (1+D)^{-1}x^2$$

$$= (1 - D + D^2)x^2 = (x^2 - Dx^2 + D^2x^2$$

$$= (x^2 - 2x + 2)$$

Q2) $$PI = \frac{1}{D^2+D}(x^2+2x+4) = \frac{1}{D+D^2}(x^2+2x+4$$

$$= \frac{1}{D(1+D)}(x^2+2x+4) \Rightarrow \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$$

$$= \frac{1}{D}(1-D+D^2)(x^2+2x+4)$$

$$= \frac{1}{D}((x^2+2x+4) - (x^2+2x+4)D + (x^2+2x+4)D^2$$

$$= \frac{1}{D}((x^2+\cancel{2x}+4) - (\cancel{2x}+2) + (2))$$

$$= \cancel{\frac{1}{D}(x^2+2x+4)} - 2 = \frac{1}{D}(x^2+4) = \int(x^2+4)dx$$

$$= \frac{x^3}{3} + 4x$$

3

HW①

①

②

Q3) $PI = \frac{1}{D^2+16} 64x^2$

$= \frac{1}{16+D^2} 64x^2 = \frac{64}{16+D^2} x^2$

$= 64 \frac{1}{16(1+\frac{D^2}{16})} x^2 = \frac{64}{16}\left(1+\frac{D^2}{16}\right)^{-1} x^2$

$= 4\left[1-\frac{D^2}{16}\right]x^2 = 4\left[x^2-\frac{D^2(x^2)}{16}\right]$

$= 4\left[x^2-\frac{2}{16}\right] = 4x^2-\frac{4}{8} = 4x^2-\frac{1}{2}$

HW②

① $\frac{1}{D^2+7D+5} x$

② $\frac{1}{D^4+D^2} 108x^2$

③ $\frac{1}{D^2+4D+4}(x^2+2x+5)$ X

$$\begin{array}{r} 16 \\ \times 9 \\ \hline 144 \end{array}$$

HW①

① $P.I = \frac{1}{D^2+4} 6\cos x$

$= \frac{1}{-1+4} 6\cos x = \frac{1}{3} 6\cos x = 2\cos x$

② $P.I = \frac{1}{2D^2+D-1} 16\cos 2x$

$= \frac{1}{2\times -4 + D - 1} 16\cos 2x = \frac{1}{-8+D-1} 16\cos 2x$

$= \frac{1}{D-9} 16\cos 2x = \frac{D+9}{(D-9)(D+9)} 16\cos 2x$

$= \frac{D+9}{D^2-81} 16\cos 2x = \frac{D+9}{-4-81} 16\cos 2x$

$= \frac{D+9}{-85} 16\cos 2x = \frac{1}{-85} D16\cos 2x + \cos 2x$

$= \frac{1}{85} -32\sin 2x + 144\cos 2x$

$= 144\cos 2x - 32\sin 2x - \frac{1}{85}$

(4) $PI = \frac{1}{D^2+1} 6\sin x$

$= \frac{1}{-1+1} 6\sin x = \frac{1}{0} 6\sin x$

$= \frac{1}{2D} 6\sin x = \frac{2D}{4D^2} 6\sin x$

$= \frac{\cancel{2}D}{-\cancel{4}_2} 6\sin x = -\frac{1}{2} 6\cos x$

HW (2) (1) $PI = \frac{1}{D^2+7D+5} x$

$= \frac{1}{5+7D+D^2} x = (5+7D-D^2)^{-1} x$

$= 0$

(2) $\frac{1}{D^4+D^2} 108x^2 = \frac{1}{D^2+D^4} 108x^2$

$= (D^2+D^4)^{-1} 108x^2$

(3) $\frac{1}{D^2+4D+4}(x^2+2x+5) = \frac{1}{4+4D+D^2}(x^2+2x+5)$

$= (4+4D+D^2)\cancel{(x^2+2x+5)}$

$D^2 + 2^2 + 2\cdot 2D$

$$= \frac{1}{4 + 4D + D^2}(x^2 + 2x + 5)$$

$$= \frac{1}{2^2 + D^2}(x^2 + 2x + 5)$$

Type (4) Product of two fun one of them is expo and anothe trig or poly

$$\frac{1}{f(D)}\left(e^{ax}\, V\right) = e^{ax}\left[\frac{1}{f(D+a)} V\right]$$

$$D \to D+a$$

① $(D^2 - 2D + 4)y = e^x \cos x$

$$PI = \frac{1}{D^2 - 2D + 4} e^x \cos x$$

$$= e^x \left[\frac{1}{D^2 - 2D + 4} \cos x \right]$$

$$= e^x \left[\frac{1}{(D+1)^2 - 2(D+1) + 4} \cos x \right]$$

$$= e^x \frac{1}{D^2 + 1 + \cancel{2D} - \cancel{2D} - 2 + 4} \cos x$$

$$= e^x \left(\frac{1}{D^2 + 3} \cos x \right)$$

$$= e^x \frac{1}{2} \cos x$$

② $$PI = \frac{1}{D^2 + 1} e^{2x} x$$

$$= e^{2x} \frac{1}{(D+2)^2 + 1} x = e^{2x} \frac{1}{D^2 + 4 + 4D + 1} x$$

$$= e^{2x} \frac{1}{D^2 + 4D + 5} x = e^{2x} \frac{1}{5 + 4D + D^2} x$$

$$= \frac{e^{2x}}{5} \frac{1}{1 + \frac{4D}{5} + \frac{D^2}{5}} x = \frac{e^{2x}}{5} \left(1 + \left(\frac{4D}{5} + \frac{D^2}{5} \right)^{-1} \right) x$$

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$$= \frac{1}{5} e^{2x} \left(1 - \left(\frac{4D}{5} + \frac{D^2}{5}\right)\right) x$$

$$= \frac{1}{5} e^{2x} \left(1 - \frac{4D}{5}\right) x$$

$$= \frac{1}{5} e^{2x} \left(x - \frac{4Dx}{5}\right) = \frac{1}{5} e^{2x} \left(x - \frac{4}{5}\right)$$

Type 5 $\boxed{e^{iax} = \cos ax + i \sin ax}$

$\cos ax$ = Real part of e^{iax}

$\sin ax$ = img part of e^{iax}

Type 5 $\frac{1}{f(D)} \left[g(x)\, z(x) \right]$ Product of 2 funˣ

when $g(x)$ is poly funˣ

$z(x)$ is trignomat funˣ ($\sin ax$, $\cos ax$)

we try to convert Type 5 to Type 4

Q.

1. $PI = \frac{1}{f(D)} x^n \cos ax$

$= \cancel{\frac{1}{f(D)}}$

Real part of $\frac{1}{f(D)} x^n e^{iax}$

Real part of $e^{iax} \frac{1}{f(D+ia)} x^n$
(give increment of a to D)

2. $\frac{1}{f(D)} x^n \sin ax$

~~Real part~~ img part of $\frac{1}{f(D)} x^n e^{iax}$

img part of $e^{iax} \frac{1}{f(D+ia)} x^n$

Solution of ~~the~~ Non-Homogen diff. eqn.

SF $(D^n + a_1 D^{n-1} + \dots + a_n) y = X$

AE = $D^n + a_1 D^{n-1} + \dots a_n = 0$

Let $\alpha_1, \alpha_2 \dots \alpha_n$ be n roots

Soln

$y = C_1 y_1 + C_2 y_2 + \dots C_n y_n$

This sol is called complentry fun (CF)

$y = C_1 e^{x} + C_2 e^{x \alpha_1}$ $\alpha + i\beta$

$y = C_1 y_1 + C_2 y_2$

$$y = \frac{1}{D^n + aD^{n-1} + \dots + a_n} x$$

This is called (PI)

Sum of Complenty funn + PI

= Complete Solution

When Homogenous eqn

PI = 0

f. eqn.

(i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{5x}$

$D^2 + 3D + 2 = e^{5x}$

$D^2 + 2D + D + 2 = e^{5x}$

$D(D+2) + 1(D+2) = e^{5x}$

$(D+1)(D+2) = e^{5x}$

$D = -1, -2$

CF $y = C_1 e^{-x} C_2 e^{-2x}$

PI $= \frac{1}{D^2 + 3D + 2} e^{5x}$

$= \frac{1}{5^2 + 3 \times 5 + 2} e^{5x} = \frac{1}{42} e^{5x}$

CS $= CF + PI$

$= C_1 e^{-x} C_2 e^{-2x} + \frac{1}{42} e^{5x}$

$(D^2 - 3D + 2)y = e^{2x}$

AE $\Rightarrow D^2 - 3D + 2 = 0$

$D^2 - 2D - D + 2 = 0$

$D(D-2) - 1(D-2) = 0$

$(D-2)(D-1) = 0$

$D = 1, 2$

$CF = C_1 e^{x} + C_2 e^{2x}$

$$PI = \frac{1}{D^2 - 3D + 2} e^{2x}$$

$$= \frac{1}{4 - 6 + 2} e^{2x}$$

$$= \frac{1}{0} e^{2x}$$

$$= x \frac{1}{2D - 3} e^{2x}$$

$$= x \frac{1}{1} e^{2x}$$

$$= x e^{2x}$$

$CS = C_1 e^{x} + C_2 e^{2x} + x e^{2x}$

(3) $(D^2-4)y = e^{5x}$

AE : $D^2 - 4 = 0$

$D^2 - 2^2 = 0$

$(D-2)(D+2) = 0$

$CF = C_1 e^{2x} + C_2 e^{-2x}$

$PI = \frac{1}{D^2-4} e^{5x} = \frac{1}{25-4} e^{5x} = \frac{1}{21} e^{5x}$

$GS = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{21} e^{5x}$

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(4) $D^2 + 4 = 0$

$\cancel{(D+2)(D+2) = 0}$

$D = \pm\sqrt{-4}$

$D = \pm i2$

$y = C_1 \cos 2x + C_2 \sin 2x$

$PI = \frac{1}{D^2+4} \cos 3x$

$= \frac{1}{-9+4} \cos 3x = \frac{1}{-5} \cos 3x$

$GS = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{5} \cos 3x$

$= \cos 2x$

$D^2 + 7D + 12 = 0$

$D^2 + 4D + 3D + 12 = 0$

$D(D+4) + 3(D+4) = 0$

$(D+3)(D+4) = 0$

$D = -3, -4$

$CF \; y = C_1 e^{-3x} + C_2 e^{-4x}$

$PI = \dfrac{1}{D^2 + 7D + 12} \cos 2x$

$= \dfrac{1}{-4 + 7D + 12} \cos 2x$

$= \dfrac{1}{7D + 8} \cos 2x$

$= \dfrac{(7D-8)}{(7D+8)(7D-8)} \cos 2x$

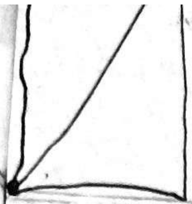
$= \dfrac{(7D-8)}{7D^2 - 8^2} \cos 2x$

$\dfrac{7D-8}{(7D)^2 - (8)^2} \cos 2x$

$= \dfrac{7D - 8}{49(-4) - 64} \cos 2x$

$= \dfrac{-1}{260} \; 7D \cos 2x - 8 \cos 2x$

$= \dfrac{-1}{260} \; 7 \cdot -2 \sin 2x - 8 \cos 2x = \dfrac{1}{260} \; 14 \sin 2x + 8 \cos 2x$



$$D^2 + 3D + 2 = x+5$$

$$D^2 + 3D + 2 = 0$$

$$D^2 + 2D + D + 2 = 0$$

$$D(D+2) + 1(D+2) = 0$$

$$D = -1, -2$$

$$CF = C_1 e^{-x} + C_2 e^{-2x}$$

$$PI = \frac{1}{D^2 + 3D + 2} \, x+5$$

$$= \frac{1}{2 + 3D + D^2} \, x+5 = \frac{1}{2\left(1 + \frac{3D}{2} + \frac{D^2}{2}\right)} (x+5)$$

$$= \frac{1}{2} \frac{1}{1 + \frac{3D}{2} + \frac{D^2}{2}} (x+5)$$

$$= \frac{1}{2} \left(1 + \left(\frac{3D}{2} + \frac{D^2}{2}\right)\right)^{-1} (x+5)$$

$$= \frac{1}{2} \left(1 - \left(\frac{3D}{2} + \frac{D^2}{2}\right)\right) (x+5)$$

$$= \frac{1}{2} \left(1 - \frac{3}{2} D\right) (x+5) = \frac{1}{2} \left((x+5) - \frac{3}{2} D (x+5)\right)$$

$$= \frac{1}{2} \left(x + 5 - \frac{3}{2}\right) = \quad \boxed{(1+D)^{-1} = 1 - D - D^2 \ldots}$$

$$= \frac{1}{2} \left(\frac{2x + 10 - 3}{2}\right) = \frac{1}{4} (2x+7)$$