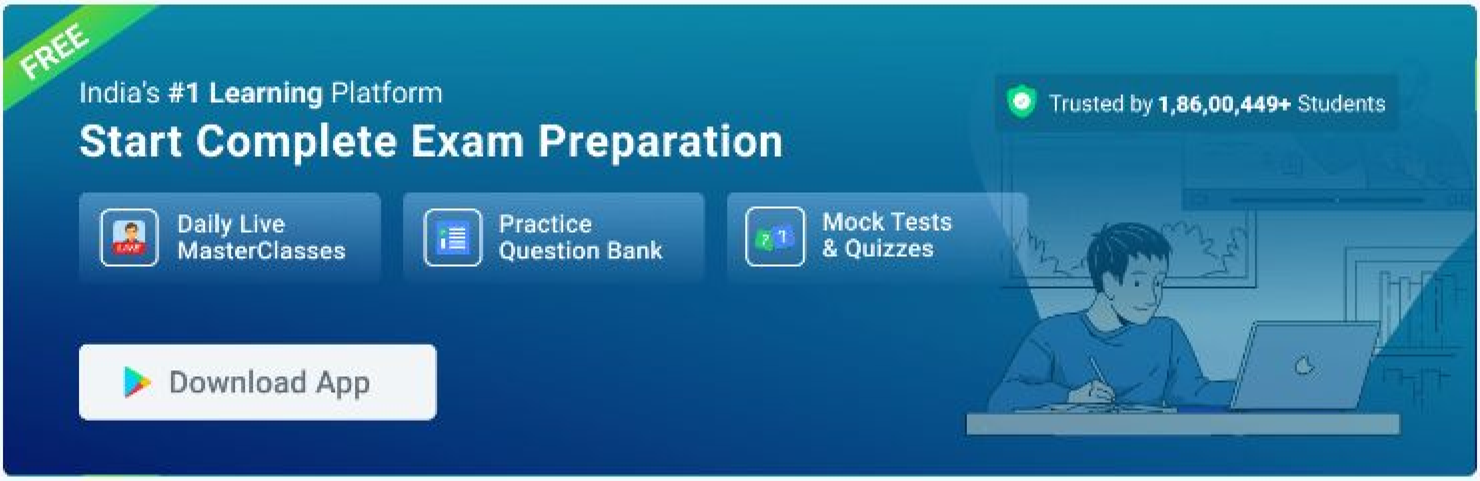
Mcq on solving linear differential equation 5eea6a1439140 f30f369f114

Engineering Mathematics (Lovely Professional University)

Solving Linear Differential Equation Questions

Latest Solving Linear Differential Equation MCQ Objective Questions



Question 1:

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Solution of the differential equation cos x dy = y (sin x - y) dx, $0 < x < \frac{\pi}{2}$ is

1. sec x = (tan x + c)y
2. y sec x = tan x + c
3. y tan x = sec x + c
4. tan x = (sec x + c).y

Answer (Detailed Solution Below)

Option 1 : sec x = (tan x + c)y

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Solving Linear Differential Equation Question 1 Detailed Solution

Concept:

Equation of the form $\frac{dy}{dx} + Py = Q$ solve by following the steps

1. find I.F = $e^{\int P dx}$
2. The solution will be y I.F = ∫ Q I.F dx + C

Formula used:

1. sin θ/coa θ = tan θ

2.1/cos θ = sec θ

3. $e^{\ln x} = x$

4. $\int \sec^2 x = \tan x$

Calculation:

cos x dy = y (sin x - y) dx

⇒ cos x dy = y $\frac{\sin x}{\cos x}$ - y^2 dx

⇒ $\frac{dy}{dx}$ = y tan x - y^2 sec x

⇒ $\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} tanx = -secx$

Now, let y = $\frac{1}{t}$

therefore $\frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx}$

Putting these values we get

$-\frac{dt}{dx} - t\ tanx = -secx$

$\frac{dt}{dx} + t\ tanx = secx$

Now,

I.F = $e^{\int tan\,x\,dx} = e^{log\,sec\,x} = sec\ x$

The solution of the equation will be

⇒ t (I.F) = ∫(I.F) sec x dx + c

⇒ t (sec x) = ∫(I.F) sec x dx + c

⇒ t sec x = ∫ sec^2 x + c

⇒ sec x = (tan x + c)y

∴ The solution of an equation is sec x = (tan x + c)y.

Question 2:

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The solution of the differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is :

1. $y(1 + x^2) = c + \tan^{-1}x$
2. $\frac{y}{1+x^2} = c + \tan^{-1}x$
3. $y \log(1 + x^2) = c + \tan^{-1}x$
4. $y(1 + x^2) = c + \sin^{-1}x$

Answer (Detailed Solution Below)

Option 1 : $y(1 + x^2) = c + \tan^{-1}x$

Solving Linear Differential Equation Question 2 Detailed Solution

<u>Concept:</u>

<u>1)</u> The solution of the linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is given by

$y \times I.F = \int Q(x)(I.F)dx + C$

Where P and Q are the functions of 'x' and I.F = $e^{\int P(x)dx}$

2) $\int \frac{1}{(1+x^2)} dx$ = $\tan^{-1}x + C$

<u>Calculation:</u>

Given $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$

This is a differential equation in the form $\frac{dy}{dx} + P(x)y = Q(x)$

Here P(x) = $\frac{2x}{1+x^2}$ and Q(x) = $\frac{1}{(1+x^2)^2}$

Integrating factor (I.F) = $e^{\int \frac{2x}{1+x^2} dx}$

$\therefore$ I.F = $e^{\int \frac{2x}{1+x^2} dx}$ = $e^{\log|1+x^2|}$ = $1 + x^2$

The solution of the differential equation is given by:

$y \times I.F = \int (I.F)Q(x)dx$

$\Rightarrow y \cdot (1 + x^2) = \int \frac{1+x^2}{(1+x^2)^2} dx$

$\Rightarrow y \cdot (1 + x^2) = \int \frac{1}{(1+x^2)} dx$

$\therefore$ **$y \cdot (1 + x^2) = \tan^{-1} x + c$**

The correct answer is option 1.

Question 3:

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The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a dissipative force is described by $\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 16x = 6te^{-8t} + 4t^2e^{-2t}$ The general form of the particular solution, in terms of constants A, B etc., is

1. $t(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}$
2. $(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}$
3. $t(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$
4. $(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$

Answer (Detailed Solution Below)

Option 3 : $t(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$

Solving Linear Differential Equation Question 3 Detailed Solution

Concept:

The forced harmonic oscillator in the presence of a dissipative force is in the form

y" + py' + q = s

where p, q and s are constants.

Calculation:

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 16x = 6te^{-8t} + 4t^2e^{-2t}$$

The roots of the characteristics equation

$q + (k^2 + kp) = 0$

The characteristics equation is

$k^2 + 10k + 16 = 0$

The roots of the equation are

$k_1 = -8$ and $k_2 = -2$

The general solution is

$x(t) = C_1\, e^{-8t} + C_2\, e^{-2t}$

The particular equation can be solved by using the variation of the parameter method

$y_1(t)\, \frac{d}{dt} C_1(t) + y_2(t)\, \frac{d}{dt} C_2(t) = 0$

$\frac{d}{dt} C_1(t) y_1(t) + \frac{d}{dt} C_2(t) \frac{d}{dt} y_2(t) = f(t)$

$f(t) = 6te^{-8t} + 4t^2e^{-2t}$

$e^{-2t}\, \frac{d}{dt} + e^{-8t}\, \frac{d}{dt} = 0$

$\frac{d}{dt} C_1(t) y_1(t) + \frac{d}{dt} C_2(t) \frac{d}{dt} y_2(t) = 6te^{-8t} + 4t^2e^{-2t}$

$-2\, e^{-2t}\, \frac{d}{dt}\, C_2(t) - 8\, e^{-8t}\, \frac{d}{dt}\, C_1(t) = 6te^{-8t} + 4t^2e^{-2t}$

$\frac{d}{dt} C_1(t) = \frac{-2t^2e^{6t}}{3} - t\, e^{5t}$

$\frac{d}{dt} C_2(t) = \frac{2t^2}{3} + t\, e^{-t}$

$C_1(t) = C_3 + \int \frac{-2t^2e^{6t}}{3} - t\, e^{5t}\, dt$

$C_2(t) = C_4 + \int \frac{2t^2}{3} + t\, e^{-t}\, dt$

The final answer is

$x(t) = \frac{2t^3e^{-2t}}{9} - \frac{t^2e^{-2t}}{9} + \frac{te^{-2t}}{27} - \frac{e^{-2t}}{162}$

The complete solution is

$B' = t[At^2 + Bt + C]e^{-2t}$

Thus the Particular Integral is

$t(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$

The correct answer is option (3).

Question 4:

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What is the solution of the differential equation

$\frac{dy}{dx} + \frac{y}{x} = x^3$

Where c is an arbitrary constant.

1. $xy = x^5 + c$
2. $xy = x^4 + c$
3. $4xy = x^4 + c$
4. $5xy = x^5 + c$

Answer (Detailed Solution Below)

Option 4 : $5xy = x^5 + c$

Solving Linear Differential Equation Question 4 Detailed Solution

Concept:

Linear differential equation:

A differential equation of the form $\frac{dy}{dx} + Py = Q$

Where P and Q are the functions of x or constants.

The general solution of a linear differential equation of the form $\frac{dy}{dx} + Py = Q$ is given by:

$I.F \times y = \int (I.F) Q dx$

Where I.F is known as Integrating Factor and it is calculated as follows: $I.F = e^{\int p dx}$

Solution:

Given differential equation is

$\frac{dy}{dx} + \frac{y}{x} = x^3$

Comparing with the standard form we get $P = \frac{1}{x}$ and $Q = x^3$

The integrating factor is calculated as:

$I.F = e^{\int \frac{dx}{x}}$

$= e^{\ln x}$

$= x$

Therefore, the general solution is given by:

$I.F \times y = \int (I.F)Q dx$

$xy = \int x(x^3)dx$

$xy = \int x^4 dx$

$xy = \frac{x^5}{5} + K$

$5xy = x^5 + C$ where $C = 5K$

∴ The correct option is (4)

Question 5:

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The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

1. e^{-y}

2. e^{-x}

3. x

4. $\frac{1}{x}$

Answer (Detailed Solution Below)

Option 4 : $\frac{1}{x}$

Solving Linear Differential Equation Question 5 Detailed Solution

Given:

$x\frac{dy}{dx} - y = 2x^2$

Concept:

- The general form of a linear differential equation is : $\frac{dy}{dx} + P(x)y = Q(x)$
- Its general solution is y(IF) = ∫[(IF)Q(x)]dx + C where IF is the integrating factor = $e^{\int P(x)dx}$

Solution:

Given DE : $x\frac{dy}{dx} - y = 2x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$

On comparing with the general form of a linear differential equation P(x) = -1/x

IF = $e^{\int P(x)dx}$

Now, ∫P (x)dx = ∫ (-1/x)dx

= -lnx

= ln(1/x)

⇒ IF = $e^{\ln(1/x)}$ = 1/x

Top Solving Linear Differential Equation MCQ Objective Questions

Question 6

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The integrating factor of the differential equation $\frac{dy}{dx} + xy = x$ is

1. $e^{\frac{-x^2}{2}}$
2. $e^{\frac{x^2}{2}}$
3. e^{x}
4. None of these

Answer (Detailed Solution Below)

Option 2 : $e^{\frac{x^2}{2}}$

Solving Linear Differential Equation Question 6 Detailed Solution

Concept:

Integrating factor, (IF) for a differential equation, $\frac{dx}{dy} + Px = Q$, where P and Q are given continuous function of y.

IF = $e^{\int Pdy}$

Calculation:

Given differential equation

$$\frac{dy}{dx} + xy = x$$

Now, this differential equation is in the form

$$\frac{dy}{dx} + yP(x) = Q(x)$$

where, P(x) = x and Q(x) = x

Integrating Factor (I.F.) = $e^{\int P(x)dx}$

I.F. = $e^{\int x dx} = e^{\frac{x^2}{2}}$

Question 7

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Find the integral factor of $\frac{dy}{dx} + \frac{y}{x} = 3\sin x$

1. e^x
2. x
3. $e^{(1/x)}$
4. none of these

Answer (Detailed Solution Below)

Option 2 : x

Solving Linear Differential Equation Question 7 Detailed Solution

Concept:

In first order linear differential equation;

$\frac{dy}{dx} + Py = Q$, where P and Q are function of x

Integrating factor (IF) = $e^{\int P\,dx}$

y × (IF) = ∫ Q(IF) dx

$y \times (IF) = \int Q(IF)\,dx$

Calculation:

$\frac{dy}{dx} + \frac{y}{x} = 3\sin x$

IF = $e^{\int \frac{1}{x} dx}$

⇒ **IF = $e^{\ln x}$**

⇒ **IF = x**

Question 8

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What is the general solution of the differential equation $y dx - (x + 2y^2)\, dy = 0$?

1. $x = y^2 + cy$
2. $x = 2cy^2$
3. $x = 2y^2 + cy$
4. None of the above

Answer (Detailed Solution Below)

Option 3 : $x = 2y^2 + cy$

Solving Linear Differential Equation Question 8 Detailed Solution

Concept:

Solution of Linear Differential equation:

If the D.E. has a form of $\frac{dx}{dy} + Px = Q$ then, where P and Q are functions of y.

The solution is given as, $x \times I.F. = \int I.F. \times Qdy + c$

where, I.F. is integrating factor which is given as,

$I.F. = e^{\int Pdy}$

Calculation:

Given: ydx − (x + 2y^2) dy = 0

$y\frac{dx}{dy} = x + 2y^2$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$

Differential equation is in form of, $\frac{dx}{dy} + Px = Q$

Integrating factor, $I.F. = e^{\int Pdy}$

$I.F. = e^{\int -\frac{1}{y}dy}$

$\Rightarrow I.F. = e^{-\ln y}$

$\Rightarrow I.F. = \frac{1}{y}$

Differential equation is given as,

$x \times \frac{1}{y} = \int \frac{1}{y} \times (2y)dy + c$

$\Rightarrow \frac{x}{y} = 2y + c$

⇒ x = 2y^2 + cy

Question 9

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Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = 4x^2$

1. $x^2 + c$
2. $x^3 + \frac{c}{x}$
3. $x^2 + \frac{c}{x}$
4. $x^3 + c$

Answer (Detailed Solution Below)

Option 2 : $x^3 + \frac{c}{x}$

Solving Linear Differential Equation Question 9 Detailed Solution

Concept:

In first order linear differential equation;

$\frac{dy}{dx} + Py = Q$, where P and Q are function of x

Integrating factor (IF) = $e^{\int P\,dx}$

y × (IF) = ∫ Q(IF) dx

Calculation:

Linear differential equation is of first order

$\frac{dy}{dx} + \frac{y}{x} = 4x^2$

Comparing with $\frac{dy}{dx} + Py = Q$

So, P = 1/x and Q = $4x^2$

IF = $e^{\int \frac{1}{x} dx}$

IF = $e^{\ln x}$

⇒ IF = x (∵ $e^{\ln x} = x$)

Now, y × (IF) = ∫ Q (IF) dx

⇒ y × x = ∫ $4x^2$ × x dx

⇒ yx = ∫ $4x^3$ dx

Integrating,

⇒ yx = x^4 + c (where c is integration constant)

⇒ y = $\backslash\text{boldsymbol}x^3 + \frac{c}{x}$

Question 10 View this Question Online >

Find y(e) for $x^2 \frac{dy}{dx} + 4xy = 4\frac{\ln x}{x^3}$, and y(1) = 1

1. $\frac{2}{e^3}$
2. $\frac{3}{e^4}$
3. $\frac{4}{e^2}$
4. $\frac{5}{e^5}$

Answer (Detailed Solution Below)

Option 2 : $\frac{3}{e^4}$

Question 11 View this Question Online >

If y(x) satisfies the differential equation

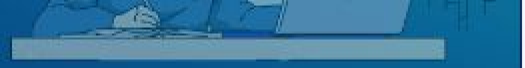
$$(\sin x)\frac{dy}{dx} + y\cos x = 1$$

subject to the condition y(π/2) = π/2, then y(π/6) is

1. 0
2. π/6
3. π/3
4. π/2

Answer (Detailed Solution Below)

Option 3 : π/3



Question 12 View this Question Online >

Solve $(2y + x) \frac{dy}{dx} = 1$

1. $x + y + 1 = ce^{-y}$

2. $x + 2y + 2 = ce^{y}$

3. $x + 2y + 1 = ce^{y}$

4. $x + 2y + 2 = ce^{-y}$

Answer (Detailed Solution Below)

Option 2 : $x + 2y + 2 = ce^{y}$

Question 13 View this Question Online >

Solve the differential equation $\frac{dy}{dx} + y \cos x = 3 \cos x$

1. $ye^{-\sin x} = 3e^{-\sin x} + c$

2. $ye^{-\sin x} = 3e^{\sin x} + c$

3. $ye^{\sin x} = -3e^{-\sin x} + c$

4. $ye^{\sin x} = 3e^{\sin x} + c$

5. None of these

Answer (Detailed Solution Below)

Option 4 : $ye^{\sin x} = 3e^{\sin x} + c$

Question 14 View this Question Online >

What is the solution of the differential equation

$\frac{dx}{dy} + \frac{x}{y} - y^2 = 0$?

Where c is an arbitrary constant.

1. $xy = x^4 + c$
2. $xy = y^4 + c$
3. $4xy = y^4 + c$
4. $3xy = y^3 + c$

Answer (Detailed Solution Below)

Option 3 : $4xy = y^4 + c$

Question 15

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The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is:

1. xe^{x}

2. $xe^{1/x}$

$-e^{x}$

3. $\frac{\quad}{x}$

4. $\frac{x}{e^x}$

5. $\frac{e^x}{x}$

Answer (Detailed Solution Below)

Option 5 : $\frac{e^x}{x}$