



Double integral area and length unit 3rd IIM

B.tech CSE (Lovely Professional University)

Unit – 3 Double Integral, Area & Length

1. $\int_0^3 \int_1^2 xy(x+y) dx dy$ is equal to
(a) 24 (b) 16
(c) 42 (d) 21
2. $\int_0^a \int_0^b (x^2 + y^2) dx dy$ is equal to
(a) $\frac{a^2 b^2 (a^2 + b^2)}{3}$ (b) $\frac{(a^2 + b^2)^2}{3}$
(c) $\frac{(a+b)(a^2 + b^2)}{3}$ (d) $\frac{ab(a^2 + b^2)}{3}$
3. $\int_0^1 \int_y^{\sqrt{y}} (x^2 + y^2) dy dx$ is equal to
(a) 2/35 (b) 3/35
(c) 7/35 (d) 1/35
4. $\int_0^1 \int_0^{x^2} e^{y/x} dx dy$ is equal to
(a) 1 (b) 1/2
(c) 0 (d) 2
5. $\int_0^1 \int_0^2 (x+y) dx dy$ is equal to
(a) 0 (b) 2
(c) 3 (d) 1
6. $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y dx dy$ is equal to
(a) $a/15$ (b) $3a/15$
(c) $a^3/15$ (d) $a^2/15$
7. $\int_0^2 \int_0^{\sqrt{2x-x^2}} x dx dy$ is equal to
(a) π (b) $\pi/3$
(c) $\pi/2$ (d) $\pi/4$
8. $\int_0^1 \int_0^{\sqrt{1-y^2}} 4y dy dx$ is equal to
(a) 1/3 (b) 2/3
(c) 7/3 (d) 4/3
9. $\int_1^2 \int_0^{3y} y dy dx$ is equal to
(a) 7 (b) 2
(c) 1 (d) 5
10. $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2} dy dx$ is equal to
(a) $\pi a^2/3$ (b) $\pi a^2/6$
(c) $\pi a^2/4$ (d) $\pi a^2/5$
11. $\int_0^1 \int_y^{\sqrt{y}} (x^2 + y^2) dy dx$ is equal to
(a) 2/17 (b) 3/13
(c) 3/35 (d) 2/35
12. $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \sin(x+y) dy dx$ is equal to
(a) 1 (b) 0
(c) 2 (d) 3

13. $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y) dy dx$ is equal to
 (a) 2 (b) 0
 (c) -2 (d) 1
14. Evaluate $\iint (x^2 + y^2) dy dx$. Over the region in the positive quadrant for which $x + y \leq 1$.
 (a) 1/3 (b) 1/4
 (c) 1/6 (d) 1/5
15. Evaluate $\iint x^2 y^2 dx dy$, over the region $x^2 + y^2 \leq 1$.
 (a) $\pi/24$ (b) $\pi/12$
 (c) $\pi/13$ (d) $\pi/3$
16. Evaluate $\iint \frac{xy}{\sqrt{1-y^2}} dx dy$, over the positive quadrant of the circle $x^2 + y^2 = 1$.
 (a) 1/4 (b) 1/5
 (c) 1/3 (d) 1/6
17. Evaluate $\iint xy dx dy$, where the region of integration is the positive quadrant of the circle $x^2 + y^2 = a^2$
 (a) $a^2/4$ (b) $a^3/5$
 (c) $a^4/8$ (d) $a^4/4$
18. Evaluate $\iint_R (x^2 + y^2) dx dy$ over the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 (a) $\pi ab(a^2 + b^2)/2$ (b) $\pi ab(a^2 + b^2)/4$
 (c) $\pi ab(a^2 + b^2)/3$ (d) $\pi ab(a^2 + b^2)/5$
19. Evaluate $\iint xy(x+y) dx dy$ over the area between the parabola $y = x^2$ and line $y = x$.
 (a) 2/19 (b) 4/39
 (c) 3/56 (d) 7/22
20. Evaluate $\iint y dx dy$, where the region of integration is the area bounded by parabolas $y^2 = 4ax$ and $x^2 = 4ay$
 (a) $\frac{48}{5} a^3$ (b) $\frac{48}{5} a^2$
 (c) $\frac{48}{5} a^4$ (d) $\frac{48}{5} a^7$
21. $\int_0^{\pi/2} \int_0^{a \cos \theta} r \sin \theta d\theta dr$
 (a) $a^2/6$ (b) $a^2/4$
 (c) $a^2/5$ (d) $a^2/7$
22. $\int_0^{\pi} \int_0^{a \cos \theta} r d\theta dr$
 (a) πa^2 (b) $\pi a^2/4$
 (c) $\pi a^2/2$ (d) $\pi a^2/3$
23. $\int_0^{\pi} \int_0^{a(1+\cos \theta)} r^2 \cos \theta d\theta dr$
 (a) $\pi a^3/2$ (b) $5\pi a^3/4$
 (c) $5\pi a^3/8$ (d) $7\pi a^3/2$
24. $\int_0^{\pi/2} \int_0^{2a \cos \theta} r^2 \sin \theta \cos \theta d\theta dr$
 (a) $8a/15$ (b) $8a^3/15$
 (c) $8a^2/17$ (d) $8a^3/17$

25. Evaluate $\iint r^2 d\theta dr$, over the area of the circle

$$r = a \cos \theta.$$

(a) $2a^3/17$

(b) $5a^2/7$

(c) $4a^3/9$

(d) $4a^2/7$

26. Integrate $r^2 \cos \theta$ over the area of the cardioid

$$r = a(1 + \cos \theta) \text{ above the initial line.}$$

(a) $3\pi a^4/8$

(b) $7\pi a^4/8$

(c) $3\pi a^2/8$

(d) $7\pi a^2/8$

27. Evaluate $\iint \frac{rd\theta dr}{\sqrt{a^2 + r^2}}$, over one loop of the

$$\text{lemniscates } r^2 = a^2 \cos 2\theta$$

(a) $\frac{(4-\pi)a^2}{3}$

(b) $\frac{(4-\pi)a}{3}$

(c) $\frac{(4-\pi)a^2}{2}$

(d) $\frac{(4-\pi)a}{2}$

28. $\int_0^1 \int_0^x \frac{x^3 dx dy}{\sqrt{x^2 + y^2}}$

(a) $\log(1+\sqrt{2})/2$

(b) $\log(1+\sqrt{2})/3$

(c) $\log(1+\sqrt{2})/4$

(d) $\log(1+\sqrt{2})/5$

29. $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2 + y^2} dx dy$

(a) $\pi a/10$

(b) $\pi a^5/20$

(c) $\pi a^2/10$

(d) $\pi a^3/20$

30. $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy$

(a) $\frac{3(\pi-8)}{8}$

(b) $\frac{3(\pi-8)}{2}$

(c) $\frac{3(\pi-8)}{4}$

(d) $\frac{3(\pi-8)}{7}$

31. $\int_0^2 \int_x^{\sqrt{2x-x^2}} \frac{xdx dy}{\sqrt{x^2 + y^2}}$

(a) $1/3$

(b) $2/3$

(c) $4/3$

(d) $5/3$

32. $\int_0^a \int_0^x f(x, y) dx dy$

(a) $\int_0^a \int_y^a f(x, y) dy dx$

(b) $\int_a^0 \int_x^a f(x, y) dy dx$

(c) $\int_a^0 \int_x^0 f(x, y) dy dx$

(d) $\int_0^a \int_a^y f(x, y) dy dx$

33. $\int_0^1 \int_x^{2x} f(x, y) dx dy$

(a) $\int_1^0 \int_x^{x/2} f(x, y) dy dx + \int_1^2 \int_{1/x/2}^1 f(x, y) dy dx$

(b) $\int_0^1 \int_{y/2}^y f(x, y) dy dx + \int_1^2 \int_{1/y}^{2/y} f(x, y) dy dx$

(c) $\int_0^1 \int_{2y}^y f(x, y) dy dx$

(d) $\int_0^1 \int_{y/2}^y f(x, y) dy dx + \int_1^2 \int_{1/y/2}^1 f(x, y) dy dx$

34. $\int_0^4 \int_x^{2\sqrt{x}} f(x, y) dx dy$

(a) $\int_0^4 \int_a^{y^2/4} f(x, y) dy dx$

(b) $\int_4^0 \int_a^{y^2/4} f(x, y) dy dx$

(c) $\int_0^4 \int_{y^2/4}^a f(x, y) dy dx$

(d) $\int_4^0 \int_a^{y^2/4} f(x, y) dy dx$

35. $\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y) dx dy$

(a) $\int_0^a \int_0^{\sqrt{ax}} f(x, y) dy dx + \int_0^{3a} \int_0^{3a-x} f(x, y) dy dx$

(b) $\int_0^a \int_0^{2\sqrt{ax}} f(x, y) dy dx + \int_0^{3a} \int_0^{3a-x} f(9x, y) dy dx$

(c) $\int_0^a \int_0^{\sqrt{ax}} f(x, y) dy dx + \int_0^a \int_0^{3a-x} f(x, y) dy dx$

(d) $\int_0^{2a} \int_0^{\sqrt{ax}} f(x, y) dy dx + \int_0^a \int_0^{3a-x} f(x, y) dy dx$

36. $\int_0^a \int_{x^2/a}^{2a-x} f(x, y) dx dy$

(a) $\int_0^{2a} \int_0^a f(x, y) dy dx + \int_0^{2a-y} \int_0^a f(x, y) dy dx$

(b) $\int_0^{2a} \int_0^{\sqrt{ax}} f(x, y) dy dx + \int_0^a \int_0^{2a-y} f(x, y) dy dx$

(c) $\int_0^a \int_0^{\sqrt{ax}} f(x, y) dy dx + \int_0^{2a} \int_0^{2a-y} f(x, y) dy dx$

(d) $\int_0^a \int_0^{2\sqrt{ax}} f(x, y) dy dx + \int_0^{2a} \int_0^{2a-y} f(x, y) dy dx$

37. $\int_0^a \int_x^{a^2/x} f(x, y) dx dy$

(a) $\int_0^a \int_0^y + \int_0^{\infty} \int_0^{a^2/y} f(x, y) dy dx$

(b) $\int_0^{2a} \int_0^y + \int_0^{\infty} \int_0^{a^2/y} f(x, y) dy dx$

(c) $\int_0^{2a} \int_0^y + \int_0^{a^2/y} \int_0^a f(x, y) dy dx$

(d) $\int_0^{2a} \int_0^y + \int_0^{a^2/y} \int_0^{a^2} f(x, y) dy dx$

38. $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+2a} f(x, y) dx dy$

Evaluate by changing order of integration (Q.39-Q.45)

39. $\int_0^\infty \int_0^x x e^{-x^2/y} dx dy$

(a) $1/2$

(b) 0

(c) 1

(d) $1/3$

40. $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x dx dy}{\sqrt{x^2+y^2}}$

(a) $\frac{1}{2}\sqrt{2}$

(b) $1 - \frac{1}{2}\sqrt{2}$

(c) $-\frac{1}{2}\sqrt{2}$

(d) $2 - \frac{1}{2}\sqrt{2}$

41. $\int_0^1 \int_y^1 x^2 \cos(x^2 - xy) dy dx$
 (a) $(1 - \cos 1)/3$ (b) $1 - \cos 1$
 (c) $-\cos 1/2$ (d) $(1 - \cos 1)/2$
42. $\int_0^1 \int_{\sqrt{x}}^1 e^{x/y} dx dy$
 (a) 0 (b) 1
 (c) $\frac{1}{2}$ (d) -1
43. $\int_0^1 \int_y^1 e^{x^2} dy dx$
 (a) $\frac{1}{3}(e - 1)$ (b) $\frac{1}{2}(e - 1)$
 (c) $\frac{1}{4}(e - 1)$ (d) $\frac{1}{5}(e - 1)$
44. When the region of integration is the circle is $x^2 + y^2 = 2ay$. Then $\iint \sqrt{4ay - x^2} dx dy$ is equal to
 (a) $\frac{1}{2}(3\pi + 8)a^3$ (b) $\frac{1}{3}(3\pi + 8)a^2$
 (c) $\frac{1}{3}(3\pi + 8)a^3$ (d) $\frac{1}{2}(3\pi + 8)a^2$
45. The value of $\iint x^{l-1} y^{-l} e^{x+y} dx dy$ extended to all position values, subject to $x + y < h$, is
 (a) $\frac{\pi(e^h - 1)}{\sin l\pi}$ (b) $\frac{\pi(e^h - 1)}{\cos l\pi}$
 (c) $\frac{e^h - 1}{\sin l\pi}$ (d) $\frac{e^h - 1}{\cos l\pi}$
46. Evaluate $\iint \sqrt{\frac{1 - (x^2/a^2) - (y^2/b^2)}{1 + (x^2/a^2) + (y^2/b^2)}} dx dy$, over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (a) $\frac{ab\pi}{2} \left(\frac{\pi}{2} - 1 \right)$ (b) $\frac{ab\pi}{4} (\pi - 1)$
 (c) $\frac{ab\pi}{3} \left(\frac{\pi}{2} - 1 \right)$ (d) $\frac{ab\pi}{4} \left(\frac{\pi}{2} - 1 \right)$
47. The area bounded by the x -axis, ordinates and the curves: $y = \log_e x; x = a, x = b (b > a > 1)$ is
48. The area bounded by the x -axis, ordinates and the curves: $y = c \cosh(x/c); x = 0, x = a$ [Catenary]
49. The area bounded by the x -axis, ordinates and the curves: $xy = c^2, x = a, x = b (a > b)$ [Hyperbola]
50. The area bounded by the x -axis, ordinates and the curves: $y = \tan x; x = -\frac{\pi}{3}, x = \frac{\pi}{3}$
 (a) $2 \log 1$ (b) $\log 1$
 (c) $3 \log 2$ (d) $2 \log 2$
51. The whole area of the following curves: is $x^2 + y^2 = a^2$ is
 (a) $2\pi a^2$ (b) πa^2
 (c) πa (d) πa^3
52. The whole area of the following curves: $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ [Hypo-cycloid]
 (a) $3\pi ab/8$ (b) $3\pi ab/5$
 (c) $3\pi ab/7$ (d) $3\pi ab/11$

53. The whole area of the curve: is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

- (a) $\pi a^2 b^2$ (b) $\pi(a+b)$
(c) πab (d) $\pi(a^2 + b^2)$

54. The whole area of the curve: is $a^2 y^2 = x^3(2a-x)$

- (a) πa (b) $2\pi a$
(c) πa^2 (d) πa^3

55. The whole area of the curve: is

$$a^2 y^2 = x^2(a^2 - x^2)$$

- (a) $a^2/2$ (b) $a^3/3$
(c) $4a^3/2$ (d) $4a^2/3$

56. The whole area of the curve: is $a^2 x^2 = y^3(2a-y)$

- (a) πa (b) πa^3
(c) πa^2 (d) πa^5

57. The area of the loop of the curve: is

$$ay^2 = x^2(a-x)$$

- (a) $8a^2/15$ (b) $8a/15$
(c) $8a^3/15$ (d) $8a^5/15$

58. The area of the loop of the curve:

$$y^2(a+x) = x^2(a-x) \text{ [Strophoid] is}$$

- (a) $2a^2(1-\pi/4)$ (b) $2a^2(1-\pi/2)$
(c) $2a^2(1-\pi/3)$ (d) $2a^2(1-\pi/6)$

59. The area of the loop of the curve: $a^3 y^2 = x^4(b+x)$ is

- (a) $32b^{7/2}/105a^2$ (b) $32b^{7/2}/105a^{3/2}$
(c) $35b^{7/2}/105a^{5/2}$ (d) $32b^{7/2}/105a^{3/2}$

60. The area of the loop of the curve:

$$a^4 y^2 = x^4(a^2 - x^2) \text{ is}$$

- (a) $\pi a^2/3$ (b) $\pi a^2/4$
(c) $\pi a^2/8$ (d) $\pi a^2/2$

61. The area of the loop of the curve: $3ay^2 = x(x-a)^2$ is

- (a) $4a^2/15\sqrt{3}$ (b) $a^3/15\sqrt{3}$
(c) $7a^2/15\sqrt{3}$ (d) $8a^2/15\sqrt{3}$

62. The area bounded by the curves and their asymptotes: $x^2 y^2 = a^2(y^2 - x^2)$

- (a) $4a^2$ (b) a^3
(c) $7a^2$ (d) a^5

63. The area bounded by the curves and their asymptotes: $y^2(2a-x) = x^3$

- (a) $2\pi a$ (b) $3\pi a^2$
(c) $2\pi a^2$ (d) $4\pi a$

64. The area bounded by the curves and their asymptotes: $x^2(x^2 + y^2) = a^2(y^2 - x^2)$

- (a) $a^2\left(\frac{\pi}{2} + 1\right)$ (b) $3a^2(\pi + 1)$
(c) $2a^2\left(\frac{\pi}{2} + 1\right)$ (d) $3a^2\left(\frac{\pi}{2} + 1\right)$

65. The area included between the curves: $x^2 + y^2 = 8$ and $y^2 = 2x$ is
 (a) $(2\pi + \sqrt{3})/2$ (b) $(2\pi + 9\sqrt{3})/3$
 (c) $(5\pi + 9\sqrt{3})/2$ (d) $(4\pi + 9\sqrt{3})/3$
66. The area included between the curves: $y^2 = 4ax$ and $x^2 = 4ay$
 (a) $4a^2/3$ (b) $7a^2/3$
 (c) $16a^2/3$ (d) $5a^2/3$
67. The area included between the curves: $y^2 = 2ax - x^2$ and $y^2 = ax$
 (a) $2a^2 \left(\pi - \frac{2}{3} \right)$ (b) $a^2 \left(\frac{\pi}{4} - \frac{2}{3} \right)$
 (c) $2a^2 \left(\frac{\pi}{4} - \frac{2}{3} \right)$ (d) $3a^2 \left(\frac{\pi}{2} - \frac{2}{3} \right)$
68. The area included between the curves: $y^2 = 4a(x+a)$ and $y^2 = 4b(b-x)$
 (a) $8(a+b)\sqrt{ab}/2$ (b) $8(a+b)\sqrt{ab}/5$
 (c) $8(a+b)\sqrt{ab}/3$ (d) $8(a+b)\sqrt{a+b}/3$
69. The area included between the following curves: $x^2 = 4ay$ and $y(x^2 + 4a^2) = 8a^3$
 (a) $a^2(3\pi - 2)/2$ (b) $2a^2(3\pi - 2)/3$
 (c) $2a(3\pi - 2)/3$ (d) $2a^2(3\pi - 2)/5$
70. The area included between the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ and its base is
 (a) $3\pi a^2$ (b) $2\pi a$
 (c) $2\pi a^2$ (d) πa^2
71. The area enclosed by the curves by double integral: parabola $y = 4x - x^2$ and the line $y = x$
 (a) $9/2$ (b) $5/2$
 (c) $7/2$ (d) $11/2$
72. The area enclosed by the curves by double integral: parabola $y = x^2$ and the line $y = x + 2$
 (a) $5/2$ (b) $7/2$
 (c) $9/2$ (d) $11/2$
73. The area enclosed by the curves by double integral: parabola $x^2 = 4y$ and the line $x = 4y - 2$
 (a) $7/24$ (b) $21/24$
 (c) $11/24$ (d) $31/24$
74. The area lying between the curves $y^2 = x^3$ and $y = x$ by double integrations is
 (a) $1/3$ (b) $1/7$
 (c) $1/10$ (d) $1/11$
75. The area between the curve and radii vectors: $r = ae^{m\theta}$ $\theta = \alpha, \theta = \beta$
76. The area between the curve and radii vectors: $\frac{1}{r} = 1 + \cos \theta$; $\theta = 0, \theta = \alpha$
77. The area of the curves: $r = a(1 - \cos \theta)$ [cardioid]
78. The area of the curves: $r = a + b \cos \theta$; $a > b$ [limaçon]
79. The area of the curve: $r^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$
80. The area common to the following curves:
 (i) $r = a, r = a(1 + \cos \theta)$
 (ii) $r = a(1 + \cos \theta), r = a(1 - \cos \theta)$

81. The area situated outside the circle $r = 2a \cos \theta$ and inside the cardioids $r = a(1 + \cos \theta)$.
 (a) $2\pi a$ (b) πa^2
 (c) $\pi a^2/2$ (d) $3\pi a^2/2$
82. By double integration the area lying inside the circle $r = a \sin \theta$ and outside the cardioid $r = a(1 - \cos \theta)$
 (a) $a(1 - \pi/4)$ (b) $a^2(1 - \pi/4)$
 (c) $a^5(1 - \pi/4)$ (d) $a^6(1 - \pi/4)$
83. The area enclosed by the curves:
 $x = a \cos t + b \sin t + c$; $y = a' \cos t + b' \sin t + c'$
 (a) $\pi(ab' - a'b)$ (b) $\pi(ab - a'b')$
 (c) $\pi(a'b - ab')$ (d) $\pi(a'b' - ab)$
84. The area enclosed by the curves:
 $x = a(3 \sin t - \sin^3 t)$; $y = a \cos^3 t$
 (a) $7\pi a^2/3$ (b) $8\pi a^2/5$
 (c) $8\pi a^2/15$ (d) $15\pi a^2/8$
85. The area enclosed by the curves:
 $x = \frac{1-t^3}{1+t^2}$; $y = \frac{2t}{1+t^2}$
 (a) 0 (b) 2π
 (c) π (d) 1
86. The area enclosed by the curves:
 $x = a \frac{1-t^2}{1+t^2}$; $y = \frac{2at}{1+t^2}$
 (a) πa (b) πa^2
 (c) $2\pi a$ (d) $\pi a^2/2$
87. The area of the loop of the curves:
 $x = a \sin 2t$; $y = a \sin t$
 (a) $2a^2/3$ (b) $7a^2/3$
 (c) $4a^2/3$ (d) $5a^2/2$
88. The area of the loop of the curves:
 $x = \frac{a \sin 3t}{\sin t}$; $y = \frac{a \sin 3t}{\cos t}$
 (a) $3a^2/2$ (b) $3\sqrt{3}a^2$
 (c) $3\sqrt{3}a^2/3$ (d) $3\sqrt{3}a^2/2$
89. The area of the loop of the curves:
 $x = \frac{3at}{1+t^3}$; $y = \frac{3at^2}{1+t^3}$
 (a) $a^2/2$ (b) $3a^2$
 (c) $3a^2/2$ (d) $7a^2/2$
90. The length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$
 (a) $\log_e 2$ (b) $\log_e (2 - \sqrt{3})$
 (c) $\log_e \sqrt{3}$ (d) $\log_e (2 + \sqrt{3})$
91. The length of the arc of the parabola $y^2 = 4ax$ cut off by the latus rectum is.
92. Length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant.
 (a) $\log(\sqrt{5} + 2)$ (b) $\sqrt{5} + \log(\sqrt{5} + 2)$
 (c) $5 + \log(\sqrt{5} + 2)$ (d) $5 + \log(\sqrt{5} + 2)$

93. The length of the arc of the curve $x = e^\theta \sin \theta, y = e^\theta \cos \theta$ from $\theta = 0$ to $\theta = \pi/2$.
 (a) $\sqrt{2}(e^\pi - 1)$ (b) $\sqrt{2}(e^{\pi/2} - 2)$
 (c) $\sqrt{2}(e^{\pi/2} - 1)$ (d) $(e^{\pi/2} - 1)$
94. The area of a loop of the curve $x^4 = a^2(x^2 - y^2)$ is
 (a) $2a^2/3$ (b) $2a^2/5$
 (c) $2a^2/7$ (d) $2a^2/9$
95. The area enclosed by the curve $xy^2 = 4(2 - x)$ and Y -axis
 (a) π (b) 2π
 (c) 3π (d) 4π
96. The area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$.
97. The area of a loop of the curve $a^4 y^2 = x^4(a^2 - x^2)$.
 (a) $\pi a^2/3$ (b) $\pi a^2/4$
 (c) $\pi a^2/7$ (d) $\pi a^2/8$
98. The area of the infinite region between the curve $y^2(2a - x) = x^3$ and its asymptote.
 (a) $3a^2\pi$ (b) $3a\pi$
 (c) $3a\pi^2$ (d) $3a^2\pi^2$
99. The area enclosed by the curves $x^2 = 4ay$ and $x^2 + 4a^2 = 8a^3/y$.
 (a) $\frac{2}{3}(3\pi - 2)a$ (b) $\frac{1}{3}(3\pi - 2)a^2$
 (c) $\frac{1}{3}(3\pi - 2)a$ (d) $\frac{2}{3}(3\pi - 2)a^2$
100. The area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$.
 (a) $17a^2/2$ (b) $16a^2/3$
 (c) $5a^2/2$ (d) $7a^2/2$
101. The area enclosed by the curve given by the equations $x = a \cos^3 \theta, y = b \sin^3 \theta$
 (a) $\frac{3}{4}\pi ab$ (b) $\frac{3}{8}\pi^2 ab$
 (c) $\frac{3}{7}\pi^2 a^2 b^2$ (d) $\frac{3}{8}\pi ab$
102. The area of the smaller portion enclosed by the curves $x^2 + y^2 = 9, y^2 = 8x$
103. The area of the loop of the curve $y^2 x + (x + a)^2(x + 2a) = 0$
 (a) $a^2(4 - \pi)$ (b) $\frac{1}{3}a^2(4 - \pi)$
 (c) $\frac{1}{2}a^2(4 - \pi)$ (d) $\frac{1}{4}a^2(4 - \pi)$
104. The whole area of the curve $x^2(x^2 + y^2) = a^2(x^2 - y^2)$
 (a) $a^2(\pi - 1)$ (b) $a^2(\pi - 2)$
 (c) $a^2(\pi - 3)$ (d) $a^2(\pi - 4)$
105. The area of a loop of the curve $r = a \sin 2\theta$
 (a) $\pi a^2/3$ (b) $\pi a^2/4$
 (c) $\pi a^2/6$ (d) $\pi a^2/8$
106. The area of a loop of the curve $r = a \sin 3\theta$
 (a) $\pi a^2/4$ (b) $\pi a^2/6$
 (c) $\pi a^2/8$ (d) $\pi a^2/12$
107. The area of the cardioids $r = a(1 + \cos \theta)$
 (a) $3\pi a^2/2$ (b) πa^2
 (c) $\pi a^2/3$ (d) $3\pi a^2/5$
108. The area outside the circle $r = 2a \cos \theta$ and inside the cardioid $r = a(1 + \cos \theta)$
 (a) πa^2 (b) $\pi a^2/2$
 (c) $\pi a^2/3$ (d) $\pi^2 a^2/3$
109. The area of a loop of the curve $x^4 + y^2 = 2a^2 xy$.
 (a) $\pi a^2/2$ (b) $\pi a^2/3$
 (c) $\pi a^2/4$ (d) $\pi a^2/5$

110. The area bounded by the curve

$$(x^2 + y^2)^2 = a^2 x^2 + b^2 y^2$$

- (a) $\frac{\pi^2(a^2 + b^2)}{3}$ (b) $\frac{\pi^2(a^2 + b^2)}{2}$
 (c) $\frac{\pi(a^2 + b^2)}{2}$ (d) $\frac{\pi(a^2 + b^2)}{3}$

111. The area enclosed by the curve

$$x = a \cos^3 t, y = b \sin^3 t$$

- (a) $3\pi ab/5$ (b) $3\pi ab/8$
 (c) $3\pi a^2 b^2/5$ (d) $3\pi a^2 b^2/8$

112. The area enclosed by the curve

$$x = a \cos t + b \sin t + c,$$

$$y = a' \cos t + b' \sin t + c'.$$

113. The length of the cardioids $r = a(1 - \cos \theta)$, lying

outside the circle $r = a \cos \theta$.

- (a) $4a/\sqrt{2}$ (b) $4a/\sqrt{3}$
 (c) $5a/\sqrt{2}$ (d) $5a/\sqrt{3}$

114. The length of the curve defined by the equations

$$x \cos \theta = a \cos(\tan \theta - \theta),$$

$$y \cos \theta = a \sin(\tan \theta - \theta),$$

between the points for which $\theta = 0$ and

$$\theta = \alpha < \frac{1}{2}\pi.$$

- (a) $\frac{1}{2} a \tan^2 \alpha$ (b) $\frac{1}{3} a \tan^2 \alpha$
 (c) $\frac{1}{4} a \tan^2 \alpha$ (d) $\frac{1}{5} a \tan^2 \alpha$

115. The arc length of the curve $y = f(x)$ lying

between two points for which $x = a$ and

$x = b (b > a)$ is given by:

- (a) $\int_a^b y dx$ (b) $\pi \int_a^b y^2 dx$
 (c) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ (d) $\int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$

116. The length of the arc of the parabola $x^2 = 4ay$ from the vertex to one extremity of the latus rectum is given by:

- (a) $\int_0^{2a} \sqrt{1 + \frac{x^2}{4a^2}} dx$ (b) $\int_0^{2a} \sqrt{1 + \frac{4a^2}{x^2}} dx$
 (c) $\int_0^a \sqrt{\frac{1+y}{a}} dx$ (d) $\int_0^a \sqrt{1 + \frac{x^2}{4a^2}} dx$

117. The length of the arc of the curve $y = \log \sec x$ between $x = 0$ and $x = \pi/6$ is equal to:

- (a) $\log 3$ (b) $2 \log 3$
 (c) $\frac{1}{2} \log 3$ (d) None of these

118. Length of the arc of the curve

$x = e^\theta \sin \theta, y = e^\theta \cos \theta$ from $\theta = 0$ to $\theta = \pi/2$ is:

- (a) $e^{\pi/2}$ (b) $\sqrt{2}(e^{\pi/2} - 1)$
 (c) $\sqrt{2}(e^{\pi/2} + 1)$ (d) $\frac{e^{\pi/2}}{\sqrt{2}}$

119. The length of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$.

- (a) $\log(2 + \sqrt{1})$ (b) $\log(2 + \sqrt{2})$
 (c) $\log(2 + \sqrt{3})$ (d) $\log(2 + \sqrt{5})$

120. The length of the arc of the curve

$$y = \log \tanh(x/2)$$

from $x = 1$ to $x = 2$

- (a) $\log[(e^2 + 1)/e]$ (b) $\log[(e^2 + 1)/e^2]$
 (c) $\log[(e^2 + 1)/3e]$ (d) $\log[(e^2 + 1)/2e]$

121. The length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum.

- (a) $2a \left[\sqrt{2} + \log(1 + \sqrt{2}) \right]$
 (b) $2a \log(1 + \sqrt{2})$
 (c) $2a \left[1 - \log(1 + \sqrt{2}) \right]$
 (d) $2a \left[\sqrt{2} - \log(1 + \sqrt{2}) \right]$

122. The length of the arc of the catenary

$$y = c \cosh(x/c)$$

- (a) $c \sinh(x/c)$ (b) $c \sinh(c/x)$
 (c) $c \cosh(x/c)$ (d) $c \cosh(c/x)$

123. A curve is given by the equations

$$x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta); \text{ find the length of the arc from } \theta = 0 \text{ to } \theta = \alpha$$

- (a) $\frac{1}{2} a \alpha$ (b) $a \alpha$
 (c) $\frac{1}{2} a^2 \alpha^2$ (d) $\frac{1}{2} a \alpha^2$

124. The length of an arc of the curve $r = ae^{\theta \cot \alpha}$ taking $s = 0$ when $\theta = 0$

125. The length of the curve $r = a \cos^3(\theta/3)$

- (a) $\frac{3}{2} a \pi$ (b) $\frac{3}{2} a^2 \pi^2$
 (c) $\frac{3}{5} a \pi$ (d) $\frac{3}{5} a^2 \pi^2$

126. The length of the loop of the curve $r = a(\theta^2 - 1)$

- (a) $8a/5$ (b) $8a/3$
 (c) $7a/5$ (d) $7a/3$

127. The length of the arc of the curve

$$x = t^2 \cos t, y = t^2 \sin t$$

128. The length of the arc of the curve $y = x(2 - x)$ as

x varies from 0 to 2

129. The length of the arc of the parabola $l/r = 1 + \cos \theta$

130. The length of the curve $x = e^\theta \sin \theta, y = e^\theta \cos \theta$ from $\theta = 0$ to $\theta = \pi/2$

- (a) $\sqrt{2}(e^\pi - 1)$ (b) $(e^{\pi/2} - 1)$
 (c) $\sqrt{2}(e^{\pi/2} - 1)$ (d) $\sqrt{2}(e^{\pi/3} - 1)$

131. Evaluate double integrals: $\int_1^2 \int_0^{3y} y \, dy \, dx$

- (a) 5 (b) 6
 (c) 7 (d) 8

132. Evaluate double integrals: $\int_0^2 \int_0^x \frac{1}{x^2 + y^2} \, dx \, dy$

- (a) $\log 2$ (b) $\frac{1}{3} \log 2$
 (c) $\frac{1}{4} \log 2$ (d) $\frac{1}{5} \log 2$

133. Evaluate double integrals: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2} \, dx \, dy$

- (a) $\log(1 + \sqrt{2})$ (b) $\log \sqrt{2}$
 (c) $\log(1 + \sqrt{3})$ (d) $\log \sqrt{3}$

134. Evaluate double integrals: $\int_0^1 \int_0^{x^2} e^{y/x} \, dx \, dy$

- (a) $\frac{1}{2}$ (b) 2
 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

135. Evaluate double integrals: $\int_0^2 \int_0^{\sqrt{2x-x^2}} x \, dx \, dy$

- (a) π (b) 2π
 (c) $\pi/2$ (d) $\pi/4$

136. Evaluate double integrals: $\int_0^1 \int_0^{\sqrt{y}} (x^2 + y^2) dy dx$

(a) $\frac{3}{13}$

(b) $\frac{3}{25}$

(c) $\frac{3}{35}$

(d) $\frac{3}{34}$

137. Evaluate double integrals: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} dx dy$

(a) $\frac{\pi}{4} \log \sqrt{2}$

(b) $\frac{\pi}{2} \log(1 + \sqrt{2})$

(c) $\frac{\pi}{4} \log(1 + \sqrt{2})$

(d) $\frac{\pi}{2} \log \sqrt{2}$

138. Evaluate double integrals: $\int_0^{\pi/2} \int_0^{\pi/2} \cos(x+y) dy dx$

(a) -1

(b) 0

(c) 1

(d) -2

139. Evaluate double integrals:

$\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2} dy dx$

(a) $\frac{\pi a^2}{3}$

(b) $\frac{\pi a^3}{6}$

(c) $\frac{\pi a^2}{5}$

(d) $\frac{\pi a^3}{3}$

140. Evaluate double integrals: $\int_0^{\pi a \sin \theta} \int_0^{\pi a \sin \theta} r d\theta dr$

(a) $\frac{\pi a^2}{3}$

(b) $\frac{\pi a^3}{4}$

(c) $\frac{\pi a^2}{4}$

(d) $\frac{\pi a^3}{6}$

141. Evaluate double integrals:

$\int_0^{\pi/2} \int_0^a r^n \sin^n \theta \cos \theta d\theta dr, \text{ for } n+1 > 0$

(a) $\frac{a^n}{(n+1)}$

(b) $\frac{a^{n+1}}{n+1}$

(c) $\frac{a^{n+1}}{(n+1)^2}$

(d) $\frac{a^{n-1}}{(n+1)^2}$

142. Evaluate double integrals: $\iint \frac{rd\theta dr}{\sqrt{a^2+r^2}}$ over one

loop of $r^2 = a^2 \cos 2\theta$

(a) $\left(2 - \frac{\pi}{2}\right)a$

(b) $\left(2 - \frac{\pi}{2}\right)a^2$

(c) $\left(2 - \frac{\pi}{2}\right)a^3$

(d) $\left(2 - \frac{\pi}{2}\right)a^4$

143. Evaluate double integrals: $\iint r^2 d\theta dr$ over the area of the circle $r = a \cos \theta$

(a) $\pi a/2$

(b) $\pi a^2/2$

(c) $\pi a^3/5$

(d) $\pi a^3/9$

144. Evaluate $\iint x^2 y^2 dx dy$ over the region $x^2 + y^2 \leq 1$.

(a) $\pi/12$

(b) $\pi/6$

(c) $\pi/15$

(d) $\pi/24$

145. Evaluate $\iint_A (x^2 + y^2) dx dy$ over the region bounded by $x=0, y=0, x+y=1$

(a) $1/2$

(b) $1/3$

(c) $1/4$

(d) $1/6$

146. Evaluate $\iint_A \frac{xy}{\sqrt{1-y^2}} dx dy$, where the region of integration is the positive quadrant of the circle $x^2 + y^2 = 1$

(a) $1/2$

(b) $1/3$

(c) $1/5$

(d) $1/6$

147. Evaluate $\iint xy dx dy$ over the region in the positive quadrant for which $x+y \leq 1$

(a) $7/24$

(b) $9/24$

(c) $1/24$

(d) $5/24$

148. By double integration the area of the region bounded by $y = 4x - x^2$ and $y = x$

(a) $7/2$

(b) $9/2$

(c) $11/2$

(d) $5/2$

149. The area of the region bounded by quadrant of $x^2 + y^2 = a^2$ and $x + y = a$

- (a) $\frac{1}{4}(\pi^2 - 6)$ (b) $\frac{1}{4}(\pi - 2)a^2$
 (c) $\frac{1}{4}(\pi - 2)a^3$ (d) $\frac{1}{4}(\pi - 2)a^5$

150. The double integration the area of the region bounded by $y^2 = x$ and $y = x$

- (a) $\frac{1}{7}$ (b) $\frac{1}{8}$
 (c) $\frac{1}{9}$ (d) $\frac{1}{10}$

151. The area of the loop of the curve $r = a\theta \cos \theta$ between $\theta = 0$ and $\theta = \pi/2$ by double integration

- (a) $\frac{\pi a^2}{16}(\pi^2 - 6)$ (b) $\frac{\pi a^2}{26}(\pi^2 - 6)$
 (c) $\frac{\pi a^2}{76}(\pi^2 - 6)$ (d) $\frac{\pi a^2}{96}(\pi^2 - 6)$

152. The area of the curve $r^2 = a^2 \cos 2\theta$ by double integration.

- (a) a^2 (b) a^3
 (c) a^5 (d) a^7

153. By double integration that the area lying inside the cardioids $r = a(1 + \cos \theta)$ and outside the circle $r = a$ is

- (a) $\frac{1}{4}a(\pi + 8)$ (b) $\frac{1}{4}a^2(\pi + 4)$
 (c) $\frac{1}{4}a^2(\pi + 8)$ (d) $\frac{1}{4}a^3(\pi + 8)$

154. The volume bounded by the paraboloid $x^2 + y^2 = z$ and the plane $z = 4$

- (a) 2π (b) 4π
 (c) 6π (d) 8π

155. The volume bounded by the paraboloid $4x^2 + y^2 = 4z$ and the plane $z = 2$

- (a) 2π (b) 4π
 (c) 6π (d) 8π

156. The volume bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

- (a) $\frac{abc}{2}$ (b) $\frac{abc}{3}$
 (c) $\frac{abc}{5}$ (d) $\frac{abc}{6}$

157. The area of the surface $z^2 = 2xy$ included between planes $x = 0, x = a, y = 0, y = b$

Change the order of integration in the following double integral (Q. 158 to Q. 169)

158. $\int_0^a \int_0^x f(x, y) dx dy$

159. $\int_a^b \int_a^x f(x, y) dx dy$

160. $\int_0^a \int_{mx}^{lx} V(x, y) dx dy$

161. $\int_0^a \int_x^{a^2/x} f(x, y) dx dy$

162. $\int_0^b \int_0^{\sqrt{a^2-x^2}} f(x, y) dx dy$

163. $\int_0^3 \int_1^{\sqrt{4-y}} (x + y) dy dx$

164. $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V dx dy$

165. $\int_0^a \int_{\frac{1}{2}\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} V dx dy$

166. $\int_0^{a/2} \int_{x^2/a}^{x-x^2/a} V dx dy$

167. $\int_0^{a/2} \int_0^{\sqrt{a^2-x^2}} f(x, y) dx dy$

168. $\int_0^1 \int_x^{2-x} f(x, y) dx dy$

(c) $\frac{\pi a^3}{9}$

(d) $\frac{\pi a^5}{10}$

169. $\int_0^{\pi/3} \int_0^{2a \cos \theta} f(r, \theta) r dr d\theta$

173. By using the transformation $x + y = u$, $y = uv$; find

the value $\int_0^1 \int_0^{1-x} e^{y/(x+y)} dx dy = \frac{1}{2}(e-1)$

(a) $(e-1)$ (b) $\frac{1}{3}(e-1)$

(c) $\frac{1}{4}(e-1)$ (d) $\frac{1}{2}(e-1)$

174. Find the area enclosed by the parabolas $y^2 = 4ax$, $y^2 = 4bx$, $x^2 = 4cy$, $x^2 = 4dy$.

175. Transform the integral $\int_0^1 \int_x^{1/x} V dx dy$, by the substitutions $u = 1+x$ and $v = xy$

176. When the region of integration R is the triangle bounded by $y = 0$, $y = x$ and $x = 1$ find

$\iint_R (4x^2 - y^2) dx dy$

(a) $\frac{1}{3}\left(\frac{\pi}{3} + \sqrt{2}\right)$ (b) $\frac{1}{3}\left(\frac{\pi}{2} + \sqrt{2}\right)$

(c) $\frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$ (d) $\frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{3}\right)$

177. Evaluate $\iint (x+y)^2 dx dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(a) $\frac{\pi ab(a^2 b^2)}{2}$ (b) $\frac{\pi ab(a^2 + b^2)}{2}$

(c) $\frac{\pi ab(a^2 + b^2)}{4}$ (d) $\frac{\pi ab(a^2 b^2)}{4}$

178. Evaluate $\iint_R dx dy$, where R is the positive

quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(a) $\frac{1}{7} \pi ab$ (b) $\frac{1}{3} \pi ab$

(a) $9 \frac{61}{103}$

(b) $9 \frac{61}{105}$

(c) $8 \frac{61}{102}$

(d) $8 \frac{61}{105}$

171. $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x dx dy}{\sqrt{x^2 + y^2}}$

(a) $1 - \sqrt{2}$

(b) $1 - \sqrt{5}$

(c) $1 - \frac{\sqrt{2}}{3}$

(d) $1 - \frac{\sqrt{2}}{2}$

172. Change the following integrals into polar coordinates and find the value

(i) $\int_0^a \int_y^a \frac{1}{x^2 + y^2} dy dx = \frac{\pi a}{4}$;

(a) $\frac{\pi a}{2}$

(b) $\frac{\pi a}{3}$

(c) $\frac{\pi a}{4}$

(d) $\frac{\pi a}{5}$

(ii) $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy = \frac{3\pi}{8} - 1$;

(a) $\frac{3\pi}{8} - 1$

(b) $\frac{\pi}{8} - 1$

(c) $\frac{3\pi}{8} - 2$

(d) $\frac{\pi}{8} - 2$

(iii) $\int_0^a \int_0^{\sqrt{a^2-x^2}} y \sqrt{x^2 + y^2} dx dy = \frac{\pi a^5}{20}$

(a) $\frac{\pi a^5}{20}$

(b) $\frac{\pi a^4}{10}$

(c) $\frac{1}{3}\pi a^2 b^2$

(d) $\frac{1}{2}\pi a^2 b^2$

185. Value of $\int_0^a \int_0^b (x^2 + y^2) dx dy$ is

(a) $ab(a^2 + b^2)$

(b) $\frac{1}{3}ab(a^2 + b^2)$

(c) $\frac{1}{2}ab(a^2 + b^2)$

(d) $3ab(a^2 + b^2)$

179. Evaluate $\iint_R y dx dy$, where R is the region

bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$

(a) $48/3$

(b) $48/7$

(c) $48/5$

(d) $48/11$

180. Evaluate $\iint_R y dx dy$ over the part R of the plane

bounded by the line $y = x$ and the parabola

$y = 4x - x^2$

(a) $54/8$

(b) $58/8$

(c) $58/4$

(d) $58/2$

181. Evaluate $\iint_R \frac{xy}{x^2 + y^2} dx dy$, where R is the region

of integration bounded by $y = x$, $y = 2x$, $x = 2$

(a) $\log(5/3)$

(b) $\log(5/2)$

(c) $\log(5/4)$

(d) $\log(5/6)$

182. Change the order of integration in

$I = \int_0^1 \int_y^1 x^2 \cos(x^2 - xy) dy dx$ and hence evaluate it

(a) $-\cos 1$

(b) $\frac{1}{2} - \cos 1$

(c) $\frac{1}{2}(1 - \cos 1)$

(d) $\frac{1}{2}$

183. Value of $\int_1^2 \int_0^{1/2} y dy dx$ is

(a) $7/6$

(b) $1/6$

(c) $2/3$

(d) $7/3$

184. Value of $\int_1^2 \int_0^{3y} y dy dx$ is

(a) 3

(b) 5

(c) 7

(d) 9

186. Value of $\int_0^1 \int_y^{\sqrt{y}} (x^2 + y^2) dy dx$ is

(a) $\frac{1}{35}$

(b) $\frac{2}{35}$

(c) $\frac{3}{35}$

(d) $\frac{4}{35}$

187. Value of $\int_0^{\pi/2} \int_0^{a \cos \theta} r \sin \theta dr d\theta$ is

(a) $\frac{1}{6}a^2$

(b) $\frac{1}{3}a^2$

(c) $\frac{1}{2}a^2$

(d) $\frac{5}{6}a^2$

188. Area lying between the parabola $y = 4x - x^2$ and the line $y = k$ is

(a) $\frac{1}{2}$ unit

(b) $\frac{3}{2}$ unit

(c) $\frac{5}{2}$ unit

(d) $\frac{9}{2}$ unit

189. Value of $\int_1^a \int_1^b \frac{dx dy}{xy}$ is:

(a) $\log(ab)$

(b) $\log(a/b)$

(c) $(\log a) \cdot (\log b)$

(d) $\frac{(\log a)}{(\log b)}$

190. Value of $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y) dy dx$ is:

(a) 0

(b) 2

(c) -2

(d) 1

191. The double integral $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$ is equal to :

- (a) 0 (b) 1
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

192. If $\int_0^1 \int_{2y}^2 e^x dx dy = k(e^4 - 1)$, then k equals _____

(JAM MA 2020)

193. The area bounded by the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$ and the straight lines $y = x$ and $y = 0$ is.

- (a) $3\left(\frac{\pi}{2} + \frac{1}{4}\right)$ (b) $3\left(\frac{\pi}{4} + \frac{1}{2}\right)$
(c) $2\left(\frac{\pi}{4} + \frac{1}{3}\right)$ (d) $2\left(\frac{\pi}{3} + \frac{1}{4}\right)$

(JAM MA 2020)

194. The value of the integral

$$\int_{y=0}^1 \int_{x=0}^{1-y^2} y \sin(\pi(1-x)^2) dx dy \text{ is}$$

- (a) $\frac{1}{2\pi}$ (b) 2π
(c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$ (JAM MA 2019)

195. The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

- (a) $\frac{\pi}{2}(17^{3/2} - 1)$ (b) $\pi(17^{3/2} - 1)$
(c) $\frac{\pi}{6}(17^{3/2} - 1)$ (d) $\frac{\pi}{3}(17^{3/2} - 1)$

(JAM MA 2019)

196. The value of the integral $\int_{-1}^1 \int_{-1}^1 |x + y| dx dy$ (round off to 2 decimal places) is _____ (JAM MA 2019)

197. The value of the integral $\int_0^1 \int_x^1 y^4 e^{xy^2} dy dx$ is _____ (correct up to two decimal places). (JAM MA 2018)

198. The area of the parametrized surface

$$S = \left\{ ((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u) \right.$$

$$\left. \in \mathbb{R}^3 \mid 0 \leq u \leq \frac{\pi}{2}, 0 \leq v \leq \frac{\pi}{2} \right\}$$

is _____ (correct up to two decimal places).

(JAM MA 2018)

199. $\int_0^1 \int_x^1 \sin(y^2) dy dx =$

- (a) $\frac{1 + \cos 1}{2}$ (b) $1 - \cos 1$
(c) $1 + \cos 1$ (d) $\frac{1 - \cos 1}{2}$

(JAM MA 2017)

200. The area of the surface $z = \frac{xy}{3}$ intercepted by the

cylinder $x^2 + y^2 \leq 16$ lies in the interval

- (a) $(20\pi, 22\pi]$ (b) $(22\pi, 24\pi]$
(c) $(24\pi, 26\pi]$ (d) $(26\pi, 28\pi]$

(JAM MA 2017)

201. Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq 1$. Then the value of $\iint_R |xy| dx dy$ is

_____ (JAM MA 2016)

202. The value of the double integral $\int_0^\pi \int_0^x \frac{\sin y}{\pi - y} dy dx$

is _____ (JAM MA 2016)

203. The area of the planar region bounded by the curves $x = 6y^2 - 2$ and $x = 2y^2$ is

- (a) $\frac{\sqrt{2}}{3}$ (b) $\frac{2\sqrt{2}}{3}$
(c) $\frac{4\sqrt{2}}{3}$ (d) $\sqrt{2}$ (JAM MA 2015)

204. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{y}{\sin y}, & y \neq 0 \\ 1, & y = 0 \end{cases} \quad \text{Then the integral}$$

$\frac{1}{\pi^2} \int_{x=0}^1 \int_{y=\sin^{-1}x}^{\frac{\pi}{2}} f(x, y) dy dx$ correct upto three decimal places, is _____ (JAM MA 2015)

205. The value of the integral

$$\iint_D \sqrt{x^2 + y^2} dx dy, D = \{(x, y) \in \mathbb{R}^2 : x \leq x^2 + y^2 \leq 2x\}$$

is

- (a) 0 (b) $\frac{7}{9}$
(c) $\frac{14}{9}$ (d) $\frac{28}{9}$ (JAM MA 2013)

206. Change the order of integration in the double

$$\text{integral } \int_{-1}^2 \left(\int_{-x}^{2-x^2} f(x, y) dy \right) dx \quad (\text{JAM MA 2011})$$

207. Evaluate $\int_{1/4}^1 \int_{\sqrt{x-x^2}}^{\sqrt{x}} \frac{x^2 - y^2}{x^2} dy dx$ by changing the order of integration (JAM MA 2013)

208. Evaluate $\int_{x=0}^4 \int_{y=\sqrt{4-x}}^2 e^{y^3} dy dx$ (JAM MA 2012)

209. Find the area of the portion of the surface

$z = x^2 - y^2$ in \mathbb{R}^3 which lies inside the solid

cylinder $x^2 + y^2 \leq 1$ (JAM MA 2012)

210. Find the area of the surface of the solid bounded

by the cone $z = 3 - \sqrt{x^2 + y^2}$ and the paraboloid

$z = 1 + x^2 + y^2$ (JAM MA 2011)

211. The value of $\iint_G \frac{\ln(x^2 + y^2)}{x^2 + y^2} dx dy$, where

$G = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq e^2\}$, is

- (a) π (b) 2π
(c) 3π (d) 4π (JAM MA 2010)

212. Change the order of integration in the integral

$$\int_0^1 \int_{x-1}^{\sqrt{1-x^2}} f(x, y) dy dx \quad (\text{JAM MA 2010})$$

213. Let $F : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function and

$a > 0$. Then the integral $\int_0^a \left[\int_0^x F(y) dy \right] dx$ equals

- (a) $\int_0^a yF(y) dy$
(b) $\int_0^a (a-y)F(y) dy$
(c) $\int_0^a (y-a)F(y) dy$
(d) $\int_a^0 yF(y) dy$ (JAM MA 2009)

214. Evaluate $\iint_R \cos\left(\max\{x^3, y^{3/2}\}\right) dx dy$, where

$R = [0, 1] \times [0, 1]$ (JAM MA 2009)

215. Find the surface area of the portion of the cone

$z^2 = x^2 + y^2$ that is inside the cylinder $z^2 = 2y$

(JAM MA 2008)

216. Let $A(t)$ denote the area bounded by the curve $y = e^{-|x|}$, the x -axis and the straight lines $x = -t$ and $x = t$. Then $\lim_{t \rightarrow \infty} A(t)$ is equal to

- (a) 2 (b) 1
(c) $1/2$ (d) 0 (JAM MA 2007)

217. Compute the double integral $\iint_D (x+2y) dx dy$, where D is the region in the xy -plane bounded by the straight lines $y = x+3$, $y = x-3$, $y = -2x+4$ and $y = -2x-2$. (JAM MA 2007)

218. Evaluate $\int_0^{\pi/2} \left[\int_{x/2}^x \frac{\sin x}{x} dx \right] dy + \int_{\pi/2}^x \left[\int_y^{\pi} \frac{\sin x}{x} dx \right] dy$. (JAM MA 2007)

219. Evaluate $\iint_R x e^{y^2} dx dy$, where R is the region bounded by the lines $x = 0$, $y = 1$ and the parabola $y = x^2$. (JAM MA 2006)

220. Using the change of variables, evaluate $\iint_R xy dx dy$, where the region R is bounded by the curves $xy = 1$, $xy = 3$, $y = 3x$ and $y = 5x$ in the first quadrant. (JAM MA 2006)

221. The length of curve $y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 7$ from $x = 1$ to $x = 8$ equals
(a) $\frac{99}{8}$ (b) $\frac{117}{8}$
(c) $\frac{99}{4}$ (d) $\frac{117}{4}$ (JAM MS 2019)

222. The value (round off to 2 decimal places) of the

double integral $\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{1+y^3} dy dx$ equals _____

(JAM MS 2019)

223. Let

$$S = \left\{ (x, y) \in \mathbb{R}^2 : x, y \geq 0, \sqrt{4-(x-2)^2} \leq y \leq \sqrt{9-(x-3)^2} \right\}$$

Then the area of S equals _____ (JAM MS 2018)

224. Let $S = \left\{ (x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1 \right\}$. Then the area of S equals _____ (JAM MS 2018)

225. The value of $\int_0^{\pi/2} \left(\int_0^x e^{\sin y} \sin x dy \right) dx$ equals _____ (JAM MS 2018)

226. If

$$\int_{y=0}^1 \int_{x=y}^{2-\sqrt{1-(y-1)^2}} f(x, y) dx dy = \int_{x=0}^1 \int_{y=0}^{\alpha(x)} f(x, y) dy dx + \int_{x=1}^2 \int_{y=0}^{\beta(x)} f(x, y) dy dx$$

then $\alpha(x)$ and $\beta(x)$ are

- (a) $\alpha(x) = x, \beta(x) = 1 + \sqrt{1-(x-2)^2}$
(b) $\alpha(x) = x, \beta(x) = 1 - \sqrt{1-(x-2)^2}$
(c) $\alpha(x) = 1 + \sqrt{1-(x-2)^2}, \beta(x) = x$
(d) $\alpha(x) = 1 - \sqrt{1-(x-2)^2}, \beta(x) = x$

(JAM MS 2017)

227. The area bounded between two parabolas

$$y = x^2 + 4 \text{ and } y = -x^2 + 6$$

is _____ (JAM MS 2017)

228. A tangent is drawn on the curve

$$y = \frac{1}{3}\sqrt{x^3}, (x > 0) \text{ at the point } P\left(1, \frac{1}{3}\right) \text{ which}$$

meets the x -axis at Q . Then the length of the closed curve $OQPO$, where O is the origin

is _____ (JAM MS 2017)

229. Let $g : [0, 2] \rightarrow \mathbb{R}$ be defined by

$$g(x) = \int_0^x (x-t)e^t dt. \text{ The area between the curve}$$

$y = g''(x)$ and the x -axis over the interval $[0, 2]$ is

(a) $e^2 - 1$ (b) $2(e^2 - 1)$

(c) $4(e^2 - 1)$ (d) $8(e^2 - 1)$

(JAM MS 2016)

230. Consider a differentiable function f on $[0, 1]$ with

the derivative $f'(x) = 2\sqrt{2x}$. The arc length of

curve $y = f(x)$, $0 \leq x \leq 1$, is _____

(JAM MS 2016)

231. The value of the real number m in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2) dy dx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3 dr d\theta$$

is _____ (JAM MS 2016)

232. Suppose $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a continuous function.

Define $h : \mathbb{R} \rightarrow \mathbb{R}$ by $h(x) = \int_0^x \int_0^x f(u, v) du dv$.

Then $h'(1)$ is equal to

(a) $2f(1, 1)$

(b) $f(1, 0) + f(0, 1)$

(c) $\int_0^1 f(t, t) dt$

(d) $\int_0^1 (f(1, t) + f(t, 1)) dt$ (JAM MS 2015)

233. The length of the curve $y = \sqrt{4-x^2}$ from

$x = -\sqrt{2}$ to $x = \sqrt{2}$ is equal to _____ (JAM MS 2015)

234. The area of the region in the first quadrant

enclosed by the curves $y = 0$, $y = x$ and $y = \frac{2}{x} - 1$ is

equal to _____ (JAM MS 2015)

235. The area of the region bounded by $y = 8$ and

$y = |x^2 - 1|$, is

(a) $\frac{50}{3}$ (b) $\frac{100}{3}$

(c) $\frac{110}{3}$ (d) $\frac{52}{3}$ (JAM MS 2014)

236. The integral $\int_0^1 \int_{x^2}^{2x} f(x, y) dy dx$ is equal to

(a) $\int_0^1 \int_{y/2}^{\sqrt{y}} f(x, y) dy dx + \int_1^2 \int_{y/2}^1 f(x, y) dx dy$

(b) $\int_0^2 \int_y^{y/2} f(x, y) dx dy$

(c) $\int_0^1 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy + \int_1^2 \int_1^{2y} f(x, y) dx dy$

(d) $\int_0^2 \int_y^{2\sqrt{y}} f(x, y) dx dy$ (JAM MS 2014)

237. Let $f : [0, \infty) \rightarrow [0, \infty)$ be a twice differentiable and increasing function with $f(0) = 0$. Suppose that, for any $t \geq 0$, the length of the arc of the curve $y = f(x)$, $x \geq 0$ between $x = 0$ and $x = t$ is

$$\frac{2}{3} \left[(1+t)^{3/2-1} \right]. \text{ Then } f(4) \text{ is equal to}$$

- (a) $\frac{11}{3}$ (b) $\frac{13}{3}$
(c) $\frac{14}{3}$ (d) $\frac{16}{3}$ (JAM MS 2013)

238. Let $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. The change of order of integration in the integral gives I as

(a) $I = \int_0^1 \int_0^{\sqrt{y}} xy \, dx \, dy + \int_1^2 \int_0^{2-y} xy \, dx \, dy.$

(b) $I = \int_0^1 \int_0^{2-y} xy \, dx \, dy + \int_1^2 \int_0^{2-y} xy \, dx \, dy.$

(c) $I = \int_0^1 \int_0^{\sqrt{y}} xy \, dx \, dy + \int_0^1 \int_0^{2-y} xy \, dx \, dy.$

(d) $I = \int_0^1 \int_0^{2-y} xy \, dx \, dy + \int_1^2 \int_0^{2-y} xy \, dx \, dy.$

(JAM MS 2012)

239. Let D be the triangle bounded by the y -axis, the line $2y = \pi$ and the line $y = x$. Then the value of the integral $\iint_D \frac{\cos(y)}{y} \, dx \, dy$ is

- (a) $\frac{1}{2}$ (b) 1

- (c) $\frac{3}{2}$ (d) 2 (JAM MS 2011)

240. The value of $\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^y e^{-\frac{1}{2}(x^2+y^2)} \, dx \, dy$ equals

- (a) $\frac{\pi}{4}$ (b) $\frac{1}{2\pi}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (JAM MS 2010)

241. The value of $\iint_S e^{-(x+y)} \, dx \, dy$, where

$S = \{(x, y) : 0 < x < 1, y > 0, 1 < x + y < 2\}$, equals

- (a) 1 (b) 2
(c) $e^{-1} - e^{-2}$ (d) $e^2 - e$ (JAM MS 2010)

242. Find the area of the smaller of the two regions enclosed between $\frac{x^2}{9} + \frac{y^2}{2} = 1$ and $y^2 = x$

(JAM MS 2010)

243. Evaluate $\int_1^{\infty} \int_0^{y-1} e^{\frac{-y}{x+1}-x} \, dx \, dy$

(JAM MS 2010)

244. The area of the region bounded by $y = x^3$, $x + y - 2 = 0$ and $y = 0$ is

- (a) 0.25 (b) 0.5
(c) 0.75 (d) 1.0 (JAM MS 2009)

245. The area of the region enclosed by the curve $y = x^2$ and the straight line $x + y = 2$ is

- (a) 3 (b) $\frac{27}{2}$
(c) $\frac{9}{2}$ (d) 9 (JAM MS 2008)

246. By changing the order of integration, the integral

$$\int_0^1 \int_1^{e^x} f(x, y) dy dx \text{ can be expressed as}$$

(a) $\int_0^1 \int_1^{\ln y} f(x, y) dx dy$ (b) $\int_0^1 \int_0^{\ln y} f(x, y) dx dy$

(c) $\int_1^e \int_1^{e^y} f(x, y) dx dy$ (d) $\int_1^e \int_{\ln y}^1 f(x, y) dx dy$

(JAM MS 2007)

247. Evaluate the integral $\iint_R e^{(x^2+y^2)/2} dx dy$, where R is

the region bounded by the lines $y=0$ and $y=x$,
and the arcs of the circles $x^2 + y^2 = 1$ and
 $x^2 + y^2 = 2$

(JAM MS 2007)

248. Area enclosed by the curves $y^2 = x$ and
 $y^2 = 2x - 1$ lying in the first quadrant is

(a) $1/6$ (b) $1/4$
(c) $1/2$ (d) $1/3$

(JAM CA 2005)

249. The value of $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$ is

(a) $\frac{e+2}{2}$ (b) $\frac{e-2}{2}$
(c) $\frac{e-1}{2}$ (d) $\frac{e+1}{2}$

(JAM CA 2005)

250. The value of $\int_0^1 \int_y^1 \frac{x}{(x^2 + y^2)} dx dy$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{5}$

251. The value of the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ is

(a) 0 (b) 1
(c) 2 (d) ∞

(JAM CA 06,09)

252. The entire area bounded by the curve

$$r^2 = a \cos 2\theta \text{ is}$$

(a) a (b) 2a
(c) πa (d) $2\pi a$

(JAM CA 2006)

253. The double integral $\int_1^2 \int_x^{2x} f(x, y) dy dx$ under the
transformation $x = u(1-v)$, $y = uv$ is
transformed into

(a) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) du dv$
(b) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) u du dv$
(c) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) v du dv$
(d) $\int_{2/3}^1 \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) u du dv$

(JAM CA 2006)

254. The area bounded by the curve $y = (x+1)^2$, its
tangent at $(1, 4)$ and the x-axis is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{4}$ (d) $\frac{4}{3}$

(JAM CA 2006)

255. Arc length of the curve $y = x^{3/2}$, $z = 0$ from
 $(0, 0, 0)$ to $(4, 8, 0)$ is 1

(a) $\frac{8}{27}(10^{3/2} + 1)$
(b) $\frac{8}{27}(10^{3/2} - 2)$

(c) $\frac{8}{27}(10^{3/2} - 1)$

(d) $\frac{8}{27}(10^{3/2} + 2)$

(JAM CA 2006)

256. If Ω denotes the region bounded by the x-axis and the lines $y = x$ and $x = 1$, then the value of the

integral $\int_{\Omega} \frac{\cos(2x)}{x} dx dy$ is

(a) $\frac{\sin 2}{2}$

(b) $\frac{\cos 2}{2}$

(c) $\cos 2$

(d) $\sin 2$

(JAM CA 2007)

257. Let D the region in the first quadrant lying between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The value of the integral $\int_D \sin(x^2 + y^2) dx dy$ is

(a) $\frac{\pi}{4}(\cos 1 - \cos 2)$

(b) $\frac{\pi}{4}(\cos 1 - \cos 4)$

(c) $\frac{\pi}{2}(\cos 1 - \cos 2)$

(d) $\frac{\pi}{2}(\cos 1 - \cos 4)$

(JAM CA 2007)

258. If the line $y = mx, 0 \leq x \leq 2$ is rotated about the line $y = -1$, then the area of the generated surface is

(a) $4\pi(1+m)\sqrt{1+m}$

(b) $4\pi(1+m^2)\sqrt{1+m}$

(c) $4\pi(1+\sqrt{m})\sqrt{1+m^2}$

(d) $4\pi(1+m)\sqrt{1+m^2}$

(JAM CA 2007)

259. Let f be an increasing, differentiable function. If the curve $y = f(x)$ passes through $(1, 1)$ and has length

$$L = \int_1^2 \sqrt{1 + \frac{1}{4x^2}} dx, 1 \leq x \leq 2$$

then the value is

(a) $y = \ln(\sqrt{x}) - 1$

(b) $y = 1 - \ln(\sqrt{x})$

(c) $y = \ln(1 + \sqrt{x})$

(d) $y = 1 + \ln(\sqrt{x})$

(JAM CA 2007)

260. Consider the double integral $\int_0^1 \int_x^{2+x} f(x, y) dy dx$.

After reversing the order of the integration, the integral becomes

(a)

$$\int_0^1 \int_0^{y-2} f(x, y) dx dy + \int_1^2 \int_0^1 f(x, y) dx dy + \int_2^3 \int_y^1 f(x, y) dx dy$$

(b)

$$\int_0^1 \int_0^y f(x, y) dx dy + \int_1^2 \int_0^1 f(x, y) dx dy + \int_2^3 \int_{y-2}^1 f(x, y) dx dy$$

(c)

$$\int_0^1 \int_0^1 f(x, y) dx dy + \int_1^2 \int_0^y f(x, y) dx dy + \int_2^3 \int_0^y f(x, y) dx dy$$

(d)

$$\int_0^1 \int_0^{y-2} f(x, y) dx dy + \int_1^2 \int_0^y f(x, y) dx dy + \int_2^3 \int_y^1 f(x, y) dx dy$$

(JAM CA 2008)

261. The double integral $\int_0^2 \int_x^{4-x} f(x, y) dy dx$ under

the transformation $u = x + y - 2x$, is transformation into

(a) $\int_0^4 \int_{u/2}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$

(b) $3 \int_0^4 \int_{u/2}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$

(c) $\frac{1}{3} \int_0^4 \int_{u/2}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$

(a) $\frac{3}{2}$

(b) $\frac{1}{2}$

(d) $\frac{1}{3} \int_0^4 \int_{-u/2}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$

(c) $\frac{1}{4}$

(d) 0

(JAM CA 2008)

(JAM CA 2009)

262. The area of the region bounded by the curves $x^2 = 2y$ and $y^2 = 2x$ is

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) 4 (JAM CA 2008)

266. The area bounded by the curves $y^2 = x$ and $x^2 = y$ is

(a) $1/3$

(b) $2/3$

(c) $4/3$

(d) $5/3$

(JAM CA 2009)

263. Let value of the integral $\int_0^3 \int_0^{\sqrt{3x}} \frac{dy dx}{\sqrt{x^2 + y^2}}$ is

(a) $3 \log(2 + \sqrt{3})$

(b) $3 \log(2 - \sqrt{3})$

(c) $3 \log 2$

(d) $\frac{3}{2} \log(2 + \sqrt{3})$

(JAM CA 2008)

267. The value of the integral $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y} dy dx$

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

(JAM CA 2009)

264. Changing the order of integration of

$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{1-x^2} f(x, y) dy dx$ gives

(a) $\int_0^1 \int_{\sqrt{1-y}}^{\sqrt{1+y}} f(x, y) dx dy + \int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx dy$

(b) $\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x, y) dx dy + \int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx dy$

(c) $\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x, y) dx dy + \int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx dy$

(d) $\int_0^1 \int_{\sqrt{1-y}}^{\sqrt{1+y}} f(x, y) dx dy - \int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx dy$

(JAM CA 2009)

268. The area of the region bounded by the curves $r = 1$ and $r^2 \cos 2\theta, 0 \leq \theta \leq \frac{\pi}{2}$, is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{8}$

(e) None of the above

(JAM CA 2010)

265. The value of $\iint_D [x+y] dx dy$, where $[x+y]$ is

the greatest integer less than or equal to $x+y$ and D is the region bounded by $x=0, y=0$ and $x+y=2$ is

269. Let $I = \int_0^2 \int_{\sqrt{4-y^2}}^{\sqrt{9-y^2}} 2xy dx dy + \int_2^3 \int_2^{\sqrt{9-y^2}} 2xy dx dy$.

Then using the transformation

$x = r \cos \theta, y = r \sin \theta$, integral I is equal to

(a) $\int_0^{\pi/2} \int_0^3 r^2 \sin 2\theta dr d\theta$

(b) $\int_0^{\pi/2} \int_0^2 r^3 \sin 2\theta dr d\theta$

(c) $\int_0^{\pi/2} \int_2^3 r^3 \sin 2\theta dr d\theta$

(d) $\int_0^{\pi/2} \int_2^3 r^2 \sin 2\theta dr d\theta$

(JAM CA 2010)

Dr. Ashutosh Sharma

Dr. Onkar Singh Bhati

Innovative Institute of Mathematics, Ground Floor Krishna Tower, Gopalpura Mod, Near Big Bazaar, Jaipur (Raj.)

Mob. : 7792988108, 8696149555

270. The area included between the curves

$$x^2 + y^2 = a^2 \text{ and } b^2 x^2 + a^2 y^2 = a^2 b^2 \quad (a > 0, b > 0),$$

is

(a) $\frac{\pi a}{2} |a - b|$

(b) $\pi |a^2 - 3ab + b^2|$

(c) $\pi a |a - b|$

(d) $\pi |a^2 - b^2|$ **(JAM CA 2011)**

271. Changing the order of integration of

$$\int_1^2 \int_0^x f(x, y) dy dx \text{ gives}$$

(a) $\int_0^1 \int_1^2 f(x, y) dx dy + \int_0^1 \int_0^1 f(x, y) dx dy$

(b) $\int_0^1 \int_1^2 f(x, y) dx dy + \int_1^2 \int_y^2 f(x, y) dx dy$

(c) $\int_0^1 \int_{y/2}^y f(x, y) dx dy + \int_1^2 \int_y^{2y} f(x, y) dx dy$

(d) $\int_0^1 \int_y^1 f(x, y) dx dy + \int_1^2 \int_1^y f(x, y) dx dy$

(JAM CA 2011)

272. The area bounded by the curves $x^2 = 4 - 2y$ and

$$x^2 = y + 4 \text{ is}$$

(a) 16

(b) 24

(c) 30

(d) 36 **(JAM CA 2011)**