(3) Method of Undetermined Coefficients :-

~~We have The method of~~ ~~Method of undetermined~~

This method is used to find the particular integral of the differential equation

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}\frac{dy}{dx} + a_n y = b(x), \quad \text{--- (1)}$$

where $b(x)$ may be some special form like e^{ax}, $\sin ax$ or $\cos a$.

where p is constant

Case I: If $b(x) = pe^{ax}$, In this case we choose

$$y_p(x) = C e^{ax}$$, where C is constant. and

determine C by substituting in (1) ;

Case II :- If $b(x) = p\sin ax$ or $p\cos ax$, In this case we choose $y_p(x) = C_1 \cos ax + C_2 \sin ax$,

and determine C_1 and C_2 by substituting in (1)

Case III :- If $b(x) = px^m$, In this case we choose

$$y_p(x) = C_0 x^m + C_1 x^{m-1} + \cdots + C_{m-1} x + C_m$$ and

determine $C_0, C_1, C_2, C_{m-1}, C_n$ by substituting in (1).

Find Complementary function by using method of undetermined coefficients

Ex ① $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = -8e^{x}$ — ①

Here $b(x) = -8e^{x}$ (pe^{x})

Let $y_p(x) = ce^{x}$ — ② $\Rightarrow y_p'(x) = ce^{x}$

$\Rightarrow y_p''(x) = ce^{x}$ — ③

From equation ① we have

$$\frac{d^2y_p}{dx^2} - 2\frac{dy_p}{dx} - 3y_p = -8e^{x}$$

$$ce^{x} - 2ce^{x} - 3ce^{x} = -8e^{x}$$

$$ce^{x} - 5ce^{x} = -8e^{x}$$

$$\Rightarrow -4c\cancel{e^{x}} = -8\cancel{e^{x}} \Rightarrow c = \frac{8}{4}$$

$$\Rightarrow c = 2$$

Using $c = 2$ in ②, we have,

$$\boxed{y_p(x) = 2e^{x}}$$

Ex ② $\frac{d^2y}{dx^2}+2y = \cos 3x$ ——— ①

Solⁿ:

Here, $b(x) = \cos 3x$,

Let $y_p(x) = C_1 \cos 3x + C_2 \sin 3x$ ——— ②

$\Rightarrow y_p'(x) = -3C_1 \sin 3x + 3C_2 \cos 3x$

$\Rightarrow y_p''(x) = -9C_1 \cos 3x - 9C_2 \sin 3x$

From equation ① we have

$$\frac{d^2y_p}{dx^2} + 2y_p = \cos 3x$$

~~$-9C_1 \cos 3x - 9C_2 \sin 3x = \cos 3x + 0.\sin 3x$~~

$-9C_1 = 1$, $-9C_2 = 0$

$\Rightarrow C_1 = -\frac{1}{9}$, $C_2 = 0$

$-9C_1 \cos 3x - 9C_2 \sin 3x + 2(C_1 \cos 3x + C_2 \sin 3x) = \cos 3x$

$-9C_1 \cos 3x - 9C_2 \sin 3x + 2C_1 \cos 3x + 2C_2 \sin 3x = \cos 3x$

$-7C_1 \cos 3x - 7C_2 \sin 3x = \cos 3x + 0.\sin 3x$

$-7C_1 = 1$, $-7C_2 = 0$ $\Rightarrow$ $C_1 = -\frac{1}{7}$ & $C_2 = 0$

Hence $\boxed{y_p(x) = -\frac{1}{7}\cos 3x}$

Ex ③ $\dfrac{d^2y}{dx^2} + y = 32x^3$ — ①

Here $b(x) = 32x^3$.

Let $y_p(x) = a_0x^3 + a_1x^2 + a_2x + a_3$ — ②

$\Rightarrow y_p'(x) = 3a_0x^2 + 2a_1x + a_2$

$\Rightarrow y_p''(x) = 6a_0x + 2a_1$

From equation ①, we have

$$\frac{d^2y_p}{dx^2} + y_p = 32x^3$$

$$(6a_0x + 2a_1) + (a_0x^3 + a_1x^2 + a_2x + a_3) = 32x^3$$

$$a_0x^3 + a_1x^2 + (6a_0 + a_2)x + (2a_1 + a_3) = 32x^3$$

$\Rightarrow$ Comparing the coefficients of various powers of x, we get

$a_0 = 32$, $a_1 = 0$, $6a_0 + a_2 = 0 \Rightarrow a_2 = -6a_0$

$\Rightarrow a_2 = -6 \times 32 \Rightarrow a_2 = -192$, $2a_1 + a_3 = 0$

$\Rightarrow a_3 = -2a \Rightarrow a_3 = -2 \times 0 \Rightarrow a_3 = 0$

As, $a_0 = 32$, $a_1 = 0$, $a_2 = -192$, $a_3 = 0$

Therefor after using the value of a_0, a_1, a_2 and a_3 in equation ② ~~$y_p(x) =$~~ we have

$$\boxed{y_p(x) = 32x^3 - 192x^2}$$