Tutorial

(1) $D^2 - 9D + 20 = 7e^{4x}$

$$P.I = \frac{1}{D^2 - 9D + 20} 7e^{4x}$$

$$= \frac{1}{(D-4)(D-5)} 7e^{4x}$$

Here $a = 4$, $f(D) = (D-4)(D-5)$

$f(a) = f(4) = 0$,

Now $\phi(D) = D - 5$, $\phi(4) = (4-5) = -1$

$$P.I = \frac{7}{(D-4)(D-5)} e^{4x} = \frac{x}{\phi(a)} e^{4x} = 7\frac{x}{-1} e^{4x}$$

$$\boxed{P.I = -7x e^{4x} = y_p(x)}$$

(2) $(D^2 - 9D + 20) = -11e^{5x}$

$$P.I = y_p(x) = \frac{1}{(D-4)(D-5)} 11e^{5x} = 11\frac{1}{(D-4)(D-5)} e^{5x}$$

Here, $a = 5$, $f(D) = (D-4)(D-5)$, $f(5) = 0$,

Now, $\phi(D) = (D-4)$, $\phi(a) = \phi(5) = 5 - 4 = 1$

$$P.I = \frac{11}{(D-4)(D-5)} e^{5x} = \frac{11x}{\phi(a)} e^{5x} = \frac{11}{1} x e^{5x}$$

$$\boxed{P.I = 11x e^{5x}}$$

$(D-2)(D+3)$

$= D^2 + 3D - 2D - 6$

$(D-1)(D-3)$

$= D^2 - 3D - D + 3$

$= D^2 - 4D + 3$

$(D-4)(D-5)$

$D^2 - 9D + 20$

Tutorial

Q① $D^2 - 9D + 20 = 7e^{4x}$

$$P.I = \frac{1}{D^2 - 9D + 20} 7e^{4x}$$

$$= \frac{1}{(D-4)(D-5)} 7e^{4x}$$

Here $a = 4$, $f(D) = (D-4)(D-5)$

$f(a) = f(4) = 0$,

Now $\phi(D) = D - 5$, $\phi(4) = (4-5) = -1$

$$P.I = \frac{7}{(D-4)(D-5)} e^{4x} = \frac{x}{\phi(a)} e^{4x} = 7 \frac{x}{-1} e^{4x}$$

$$\boxed{P.I = -7x e^{4x} = y_p(x)}$$

Q② $(D^2 - 9D + 20) = -11 e^{5x}$

$$P.I. = y_p(x) = \frac{1}{(D-4)(D-5)} 11 e^{5x} = 11 \frac{1}{(D-4)(D-5)} e^{5x}$$

Here, $a = 5$, $f(D) = (D-4)(D-5)$, $f(5) = 0$,

Now, $\phi(D) = (D-4)$, $\phi(a) = \phi(5) = 5 - 4 = 1$

$$P.I = \frac{11}{(D-4)(D-5)} e^{5x} = \frac{11x}{\phi(a)} e^{5x} = \frac{11}{1} x e^{5x}$$

$$\boxed{P.I = x e^{5x}}$$

$(D-2)(D-3)$

$= D^2 - 5D -$

$(D-2)(D-3)$

$= D^2 - 2D =$

$(D-4)(D-5)$

$D^2 - 4D - 5D + 20$

$D^2 - 9D + 20$

D^2+2D+1 D^2+2D+1 $D^2-7D+10$

Q: $(D^2-7D+10)y = e^{2x} + e^{5x}$

$$P.I = \frac{1}{(D^2-7D+10)}(e^{2x} + e^{5x})$$

$$= \underbrace{\frac{1}{D^2-7D+10} e^{2x}}_{I} + \underbrace{\frac{1}{D^2-7D+10} e^{5x}}_{II} \quad \text{---(1)}$$

Let $I = \frac{1}{D^2-7D+10} e^{2x} = \frac{1}{(D-2)(D-5)} e^{2x}$

Here $f(D) = (D-2)(D-5)$

$f(2) = f(2) = 0$, So $\phi(D) = D-5$, $\phi(2) = \phi(2) = -3$

Now $I = \frac{1}{(D^2-7D+10)} e^{2x} = \frac{x}{\phi(2)} e^{2x} = \frac{x}{\phi(2)} e^{2x}$

$$I = \frac{x}{-3} e^{2x} \rightarrow \boxed{I = -\frac{x}{3} e^{2x}} \quad \text{---(2)}$$

Next,

$$II = \frac{1}{D^2-7D+10} e^{5x} = \frac{1}{(D-2)(D-5)} e^{5x}$$

Here, $a=5$, $f(D) = (D-2)(D-5)$

$f(5) = 0$, $\phi(D) = D-2$

and $\phi(a) = \phi(5) = 5-2 = 3$

Now

$$II = \frac{1}{D^2-7D+10} e^{5x} = \frac{x}{\phi(a)} \cdot e^{5x} = \frac{x}{3} e^{5x}$$

$$II = \frac{x}{3} e^{5x} \quad \text{— (3)}$$

Using the value of (2) and (3) in (1) we get

$$P.I = y_p(x) = \frac{1}{D^2-7D+10} e^{2x} + \frac{1}{D^2-7D+10} e^{5x}$$

$$\boxed{y_p(x) = \frac{x}{-3} e^{2x} + \frac{x}{3} e^{5x}}$$