Tutorial 1
SCS 2212- Automata Theory

1. Define the following languages:
   a) The language of all strings consisting of n 0's followed by n 1's, for some n>=0
   b) The set of strings of 0's and 1's with an equal number of each.
   c) The set of binary numbers whose value is prime

2. Let L1 = {a, ab} and L2 = { λ, a, c, ac} be two finite languages over the alphabet Σ= {a, b, c}
   Define the following languages:
   a) L1 U L2
   b) LI ∩ L2
   c) L1.L2
   d) L2L1
   e) $L1^2$

3. Let Σ= {0,1}. Define the following languages
   a) Σ^0
   b) Σ^1
   c) Σ^2
   d) Σ^3
   e) Σ^*
   f) Σ^+

4. Let X = {a, b, c} and Y = {abb, ba}. Find the following :
   a) XY
   b) X^0
   c) X^1
   d) X^2
   e) X^3

5. Consider the following production rules. Note: the start symbol of the following grammar is <sentence>.

   I. <sentence> → <noun-phrase> <verb-phrase>
   II. <noun-phrase> → <proper-noun> | <determiner> <common-noun>
   III. <proper-noun> → John | Jill
   IV. <common-noun> → car | hamburger
   V. <determiner> → a | the
   VI. <verb-phrase>→ <verb> <adverb> | <verb>
   VII. <verb> → drives | eats
   VIII. <adverb> → slowly | frequently

   a) Determine the set of non-terminals in the grammar by looking at the production rules
   b) Determine the set of terminals in the grammar by looking at the production rules
   c) Can the statement "Jill drives frequently" be derived from this grammar? Justify
   d) Can the statement "Jill eats a hamburger slowly" be derived from this grammar? Justify

6. Consider the following grammar

G = (N, T, P, S)
N = {S, A, B}
T = {a, b, c}
S = Initial state

P : $S \to aSa$
$S \to aAa$
$A \to bB$
$B \to bB$
$B \to c$ is

a) is type 3
b) is type 2 but not type 3
c) is type 1 but not type 2
d) is type 0 but not type 1

7. Consider the following grammar

G = (N, T, P, S)
N = {S, A, B, C, D, E}
T = {a, b, c}
P : $S \to aAB$
$AB \to CD$
$CD \to CE$
$C \to aC$
$C \to b$
$bE \to bc$

a) is type 3
b) is type 2 but not type 3
c) is type 1 but not type 2
d) is type 0 but not type 1

8.
Consider the following grammar

G = (N, T, P, S)
N = {S, A, B, C}
T = {a, b, c}
P : $S \to aS$
$A \to bB$
$B \to cC$
$C \to a$

a) is type 3
b) is type 2 but not type 3
c) is type 1 but not type 2
d) is type 0 but not type 1

9. Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$ by using induction.

10. Let L = {ab, aa, baa}. Which of the following strings are in L*:

    abaabaaabaa,
    aaaabaaaa,
    baaaaabaaaab,
    baaaaabaa?

    Write all the strings that belong to L^4?

11. Let Σ = {a, b} and L = {aa, bb}. Use set notation to describe L'.

12. Let L be any language on a non-empty alphabet. Show that L and L' cannot both be finite.

13. Find grammars for Σ = {a, b} that generate the sets of
    (a) all strings with exactly one a.
    (b) all strings with at least one a.
    (c) all strings with no more than three a's.
    (d) all strings with at least three a's. In each case, give convincing arguments that the grammar you give does indeed generate the indicated language.

14. Give a simple description of the language generated by the grammar with productions

    $S \longrightarrow aA$
    $A \longrightarrow bS$
    $S \longrightarrow \varepsilon$

15. What language does the grammar with these productions generate?

    $S \longrightarrow Aa$
    $A \longrightarrow B$
    $B \longrightarrow Aa$

16. Are the two grammars with respective productions

    $S \longrightarrow aSb \mid ab \mid \lambda$
    And
    $S \longrightarrow aAb \mid ab$
    $A \longrightarrow aAb \mid \lambda$

    equivalent? Assume that S is the start symbol in both cases.

17. Show that the grammar

    $S \longrightarrow aSb \mid bSa \mid SS \mid a$
    and
    $S \longrightarrow aSb \mid bSa \mid a$

    are not equivalent.