Applied category theory, 18.S097 at MIT.

Pawan Sasanka Ammanamanchi

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I. Chapter 1

1.1 Lecture 1

Category theory is a fundamental part of mathematics, it has branched out into a variety of subjects like computer science and physics. Applied category theory is a relatively new field.

1.1.1 Generative/ Cascade effects

A set of different objects while might be observed to not have any common interactions, but when looked at differently might interact with each. Eg: contagion.

Definition I..1 *A set is a bag of dots.*

$$A = \{a, b, c\} \quad where, a, b, c \in A$$

Similarly, there are different sets of numbers :

$$\begin{aligned} &\mathbb{N}, \quad the\, set\, of\, natural\, numbers \\ &\mathbb{Z}, \quad the\, set\, of\, integers \\ &\mathbb{R}, \quad the\, set\, of\, real\, numbers \\ &\mathbb{B}, \quad the\, set\, of\, booleans \end{aligned}$$

Definition I..2 *Product Sets: Suppose A, B are sets then,*

$$A * B = \{(a, b) \mid a \in A, b \in B\}$$

In category theory, we think of objects in terms of the roles they play.

Definition I..3 *A relation, R, on sets A and B is defined by*

$$R \subset A * B$$

Every function is a relation. Properties like order, equivalence and tolerance are relations as well.

Definition I..4 *A function, f, from A to B, denoted $f : A \to B$ is a relation on A and B. $R \subset A * B$, satisfying*

- *For all $a \in A$, there exists an element $b \in B$ such that $(a, b) \in R$.*

- *For all a, b1, b2, if $(a, b1) \in R$ and $(a, b2) \in R$, then $b1 = b2$.*

Definition of injective (no two x'es are mapped to the same y) and surjective (for every y there exists an x in the mapping) functions.

We can order partitions, Say we have two partitions P1 and P2, then we say $P1 \leq P2$ if there is a function $P1 \to P2$ making the diagram commute.

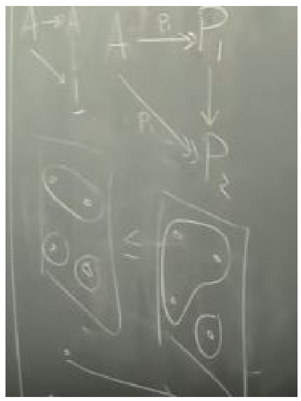

Figure 1: When is a partition lesser than another partititon

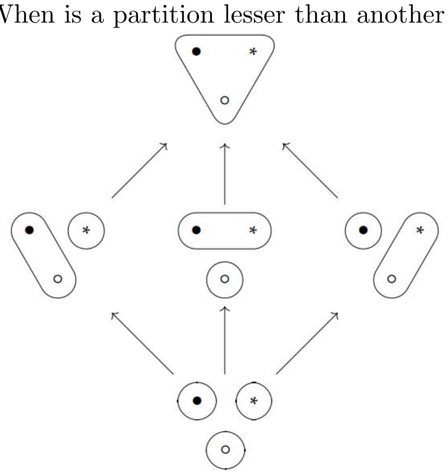

Figure 2: Can be thought of as a lattice theory structure/ poset

A *pre-order* is

1. a set S
2. a relation $"\leq" \subset S * S$

and also satisfying two properties i.e its reflexive and transitive.
Order creates join.

1.2 Lecture 2

1.3 Additional Chapter Notes