MODULE 1

INTRODUCTION

Ever since computers were invented, we have wondered whether they might be made to learn. If we could understand how to program them to learn-to improve automatically with experience-the impact would be dramatic.

Imagine computers learning from medical records which treatments are most effective for new diseases

Houses learning from experience to optimize energy costs based on the particular usage patterns of their occupants.

Personal software assistants learning the evolving interests of their users in order to highlight especially relevant stories from the online morning newspaper

A successful understanding of how to make computers learn would open up many new uses of computers and new levels of competence and customization

Some successful applications of machine learning

Learning to recognize spoken words

Learning to drive an autonomous vehicle

Learning to classify new astronomical structures

Learning to play world-class backgammon

Why is Machine Learning Important?

- Some tasks cannot be defined well, except by examples (e.g., recognizing people).
- Relationships and correlations can be hidden within large amounts of data. Machine Learning/Data Mining may be able to find these relationships.
- Human designers often produce machines that do not work as well as desired in the environments in which they are used.
- The amount of knowledge available about certain tasks might be too large for explicit encoding by humans (e.g., medical diagnostic).
- Environments change over time.
- New knowledge about tasks is constantly being discovered by humans. It may be difficult to continuously re-design systems “by hand”.

WELL-POSED LEARNING PROBLEMS

Definition: A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.

To have a well-defined learning problem, three features needs to be identified:

1. The class of tasks
2. The measure of performance to be improved
3. The source of experience

Examples

1. ***Checkers game:*** A computer program that learns to play ***checkers*** might improve its performance as measured by its ability to win at the class of tasks involving playing checkers games, through experience obtained by playing games against itself.

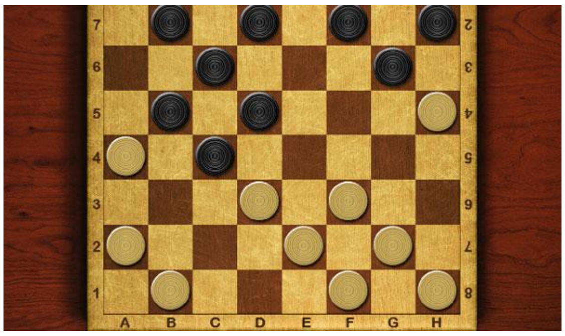


Fig: Checker game board

***A checkers learning problem*:**

Task T: playing checkers

Performance measure P: percent of games won against opponents

Training experience E: playing practice games against itself

2. ***A handwriting recognition learning problem:***

Task T: recognizing and classifying handwritten words within images

Performance measure P: percent of words correctly classified

Training experience E: a database of handwritten words with given classifications

3. ***A robot driving learning problem*:**

Task T: driving on public four-lane highways using vision sensors

Performance measure P: average distance travelled before an error (as judged by human overseer)

Training experience E: a sequence of images and steering commands recorded while observing a human driver

DESIGNING A LEARNING SYSTEM

The basic design issues and approaches to machine learning are illustrated by designing a program to learn to play checkers, with the goal of entering it in the world checkers tournament

1. Choosing the Training Experience
2. Choosing the Target Function
3. Choosing a Representation for the Target Function
4. Choosing a Function Approximation Algorithm
    1. Estimating training values
    2. Adjusting the weights
5. The Final Design

1. Choosing the Training Experience

The first design choice is to choose the type of training experience from which the system will learn.

The type of training experience available can have a significant impact on success or failure of the learner.

<u>There are three attributes which impact on success or failure of the learner</u>

1. Whether the training experience provides ***direct or indirect feedback*** regarding the choices made by the performance system.

    <u>For example, in checkers game</u>:
    In learning to play checkers, the system might learn from ***direct training examples***
    consisting of ***individual checkers board states*** and ***the correct move for each***.

    Indirect training examples consisting of the ***move sequences*** and ***final outcomes*** of various games played. The information about the correctness of specific moves early in the game must be inferred indirectly from the fact that the game was eventually won or lost.

    Here the learner faces an additional problem of ***credit assignment***, or determining the degree to which each move in the sequence deserves credit or blame for the final outcome. Credit assignment can be a particularly difficult problem because the game can be lost even when early moves are optimal, if these are followed later by poor moves.
    Hence, learning from direct training feedback is typically easier than learning from indirect feedback.

2. The degree to which the ***learner controls the sequence of training examples***

   For example, in checkers game:
   The learner might depends on the ***teacher*** to select informative board states and to provide the correct move for each.

   Alternatively, the learner might itself propose board states that it finds particularly confusing and ask the teacher for the correct move.

   The learner may have complete control over both the board states and (indirect) training classifications, as it does when it learns by playing against itself with ***no teacher present***.

3. How well it represents the ***distribution of examples*** over which the final system performance P must be measured

   For example, in checkers game:
   In checkers learning scenario, the performance metric P is the percent of games the system wins in the world tournament.

   If its training experience E consists only of games played against itself, there is a danger that this training experience might not be fully representative of the distribution of situations over which it will later be tested.
   It is necessary to learn from a distribution of examples that is different from those on which the final system will be evaluated.

2. Choosing the Target Function

The next design choice is to determine exactly what type of knowledge will be learned and how this will be used by the performance program.

Let's consider a checkers-playing program that can generate the legal moves from any board state.
The program needs only to learn how to choose the best move from among these legal moves. We must learn to choose among the legal moves, the most obvious choice for the type of information to be learned is a program, or function, that chooses the best move for any given board state.

1. Let ***ChooseMove*** be the target function and the notation is

$$\textit{ChooseMove} : B \rightarrow M$$

   which indicate that this function accepts as input any board from the set of legal board states B and produces as output some move from the set of legal moves M.

ChooseMove is a choice for the target function in checkers example, but this function will turn out to be very difficult to learn given the kind of indirect training experience available to our system

2. An alternative target function is an ***evaluation function*** that assigns a ***numerical score*** to any given board state
   Let the target function V and the notation

$$V : B \rightarrow R$$

   which denote that V maps any legal board state from the set B to some real value.
   Intend for this target function V to assign higher scores to better board states. If the system can successfully learn such a target function V, then it can easily use it to select the best move from any current board position.

Let us define the target value V(b) for an arbitrary board state b in B, as follows:

If b is a final board state that is won, then V(b) = 100
If b is a final board state that is lost, then V(b) = -100
If b is a final board state that is drawn, then V(b) = 0
If b is a not a final state in the game, then V(b) = V(b'),

Where b' is the best final board state that can be achieved starting from b and playing optimally until the end of the game

3. Choosing a Representation for the Target Function

Let's choose a simple representation - for any given board state, the function ***c*** will be calculated as a linear combination of the following board features:

x_1: the number of black pieces on the board
x_2: the number of red pieces on the board
x_3: the number of black kings on the board
x_4: the number of red kings on the board
x_5: the number of black pieces threatened by red (i.e., which can be captured on red's next turn)
x_6: the number of red pieces threatened by black

Thus, learning program will represent as a linear function of the form

$$\hat{V}(b) = w_0 + w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6$$

Where,

w_0 through w_6 are numerical coefficients, or weights, to be chosen by the learning algorithm.

Learned values for the weights w1 through w6 will determine the relative importance of the various board features in determining the value of the board

The weight w_0 will provide an additive constant to the board value

4. Choosing a Function Approximation Algorithm

In order to learn the target function **f** we require a set of training examples, each describing a specific board state b and the training value $V_{train}(b)$ for b.

Each training example is an ordered pair of the form (b, $V_{train}(b)$).

For instance, the following training example describes a board state b in which black has won the game (note $x_2 = 0$ indicates that red has no remaining pieces) and for which the target function value $V_{train}(b)$ is therefore +100.

$$((x_1=3, x_2=0, x_3=1, x_4=0, x_5=0, x_6=0), +100)$$

Function Approximation Procedure

1. Derive training examples from the indirect training experience available to the learner
2. Adjusts the weights w_i to best fit these training examples

1. Estimating training values

A simple approach for estimating training values for intermediate board states is to assign the training value of $V_{train}(b)$ for any intermediate board state b to be V(Successor(b))

Where ,

V is the learner's current approximation to V

Successor(b) denotes the next board state following b for which it is again the program's turn to move

Rule for estimating training values

$$\mathbf{V_{train}(b) \leftarrow V\ (Successor(b))}$$

2. Adjusting the weights

Specify the learning algorithm for choosing the weights w_i to best fit the set of training examples $\{(b, V_{train}(b))\}$

A first step is to define what we mean by the bestfit to the training data.

One common approach is to define the best hypothesis, or set of weights, as that which minimizes the squared error E between the training values and the values predicted by the hypothesis.

$$E \equiv \sum_{\langle b, V_{train}(b)\rangle \in \text{ training examples}} (V_{train}(b) - \hat{V}(b))^2$$

Several algorithms are known for finding weights of a linear function that minimize E. One such algorithm is called the ***least mean squares, or LMS training rule***. For each observed training example it adjusts the weights a small amount in the direction that reduces the error on this training example

LMS weight update rule :- For each training example $(b, V_{train}(b))$

Use the current weights to calculate V (b)

For each weight w_i, update it as

$$w_i \leftarrow w_i + \eta\,(V_{train}(b) - V(b))\,x_i$$

Here η is a small constant (e.g., 0.1) that moderates the size of the weight update.

Working of weight update rule

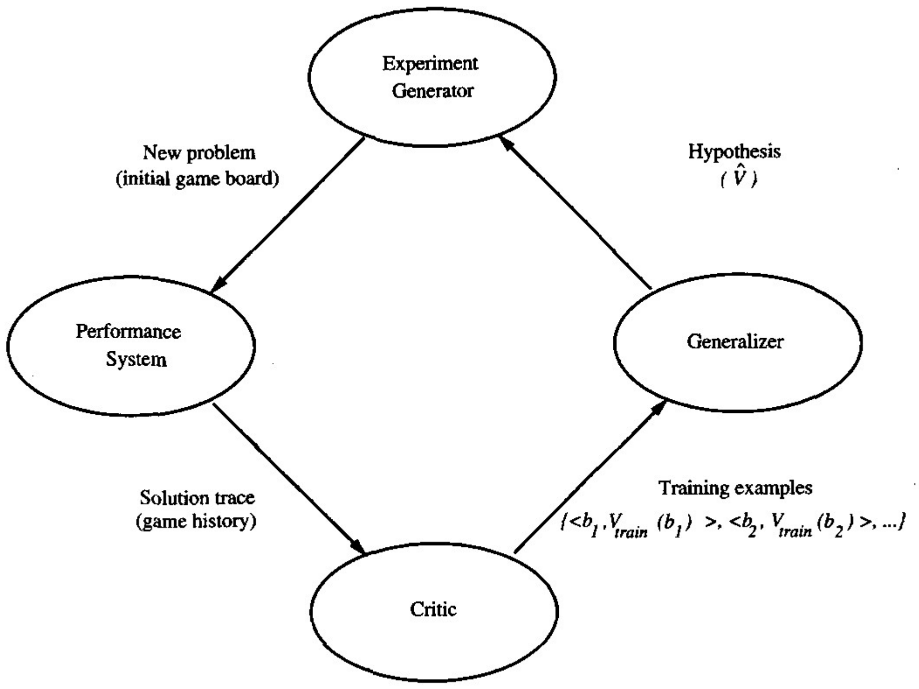
When the error $(V_{train}(b) - V(b))$ is zero, no weights are changed.

When $(V_{train}(b) - V(b))$ is positive (i.e., when V(b) is too low), then each weight is increased in proportion to the value of its corresponding feature. This will raise the value of V(b), reducing the error.

If the value of some feature x_i is zero, then its weight is not altered regardless of the error, so that the only weights updated are those whose features actually occur on the training example board.

5. *The Final Design*

The final design of checkers learning system can be described by four distinct program modules that represent the central components in many learning systems


Experiment Generator
New problem (initial game board)
Hypothesis ($\hat{V}$)
Performance System
Generalizer
Solution trace (game history)
Training examples
$\{<b_1, V_{train}(b_1)>, <b_2, V_{train}(b_2)>, ...\}$
Critic

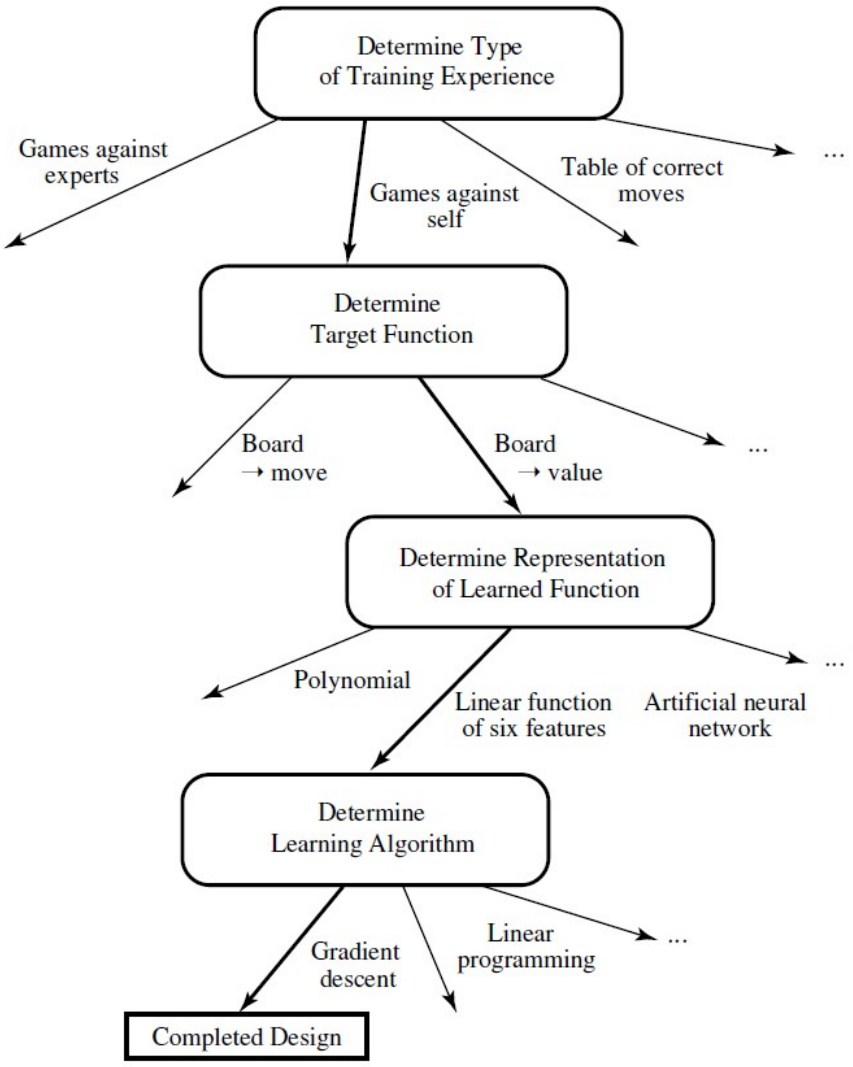
1. **The Performance System** is the module that must solve the given performance task by using the learned target function(s). It takes an instance of a new problem (new game) as input and produces a trace of its solution (game history) as output.

2. **The Critic** takes as input the history or trace of the game and produces as output a set of training examples of the target function

3. **The Generalizer** takes as input the training examples and produces an output hypothesis that is its estimate of the target function. It generalizes from the specific training examples, hypothesizing a general function that covers these examples and other cases beyond the training examples.

4. **The Experiment Generator** takes as input the current hypothesis and outputs a new problem (i.e., initial board state) for the Performance System to explore. Its role is to pick new practice problems that will maximize the learning rate of the overall system.

The sequence of design choices made for the checkers program is summarized in below figure


Determine Type of Training Experience
Games against experts
Games against self
Table of correct moves
...
Determine Target Function
Board → move
Board → value
...
Determine Representation of Learned Function
Polynomial
Linear function of six features
Artificial neural network
...
Determine Learning Algorithm
Gradient descent
Linear programming
...
Completed Design

PERSPECTIVES AND ISSUES IN MACHINE LEARNING

Issues in Machine Learning

The field of machine learning, and much of this book, is concerned with answering questions such as the following

> What algorithms exist for learning general target functions from specific training examples? In what settings will particular algorithms converge to the desired function, given sufficient training data? Which algorithms perform best for which types of problems and representations?
>
> How much training data is sufficient? What general bounds can be found to relate the confidence in learned hypotheses to the amount of training experience and the character of the learner's hypothesis space?

When and how can prior knowledge held by the learner guide the process of generalizing from examples? Can prior knowledge be helpful even when it is only approximately correct?

What is the best strategy for choosing a useful next training experience, and how does the choice of this strategy alter the complexity of the learning problem?

What is the best way to reduce the learning task to one or more function approximation problems? Put another way, what specific functions should the system attempt to learn? Can this process itself be automated?

How can the learner automatically alter its representation to improve its ability to represent and learn the target function?

CONCEPT LEARNING

Learning involves acquiring general concepts from specific training examples. Example: People continually learn general concepts or categories such as "bird," "car," "situations in which I should study more in order to pass the exam," etc.

Each such concept can be viewed as describing some subset of objects or events defined over a larger set

Alternatively, each concept can be thought of as a Boolean-valued function defined over this larger set. (Example: A function defined over all animals, whose value is true for birds and false for other animals).

Definition: Concept learning - Inferring a Boolean-valued function from training examples of its input and output

A CONCEPT LEARNING TASK

Consider the example task of learning the target concept "Days on which ***Aldo*** enjoys his favorite water sport"

Example	**Sky**	**AirTemp**	**Humidity**	**Wind**	**Water**	**Forecast**	**EnjoySport**
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Table: Positive and negative training examples for the target concept ***EnjoySport.***

The task is to learn to predict the value of ***EnjoySport*** for an arbitrary day, based on the values of its other attributes?

What hypothesis representation is provided to the learner?

Let's consider a simple representation in which each hypothesis consists of a conjunction of constraints on the instance attributes.

Let each hypothesis be a vector of six constraints, specifying the values of the six attributes *Sky, AirTemp, Humidity, Wind, Water*, and *Forecast*.

For each attribute, the hypothesis will either

Indicate by a "?' that any value is acceptable for this attribute,

Specify a single required value (e.g., Warm) for the attribute, or

Indicate by a "Φ" that no value is acceptable

If some instance ***x*** satisfies all the constraints of hypothesis ***h***, then ***h*** classifies ***x*** as a positive example (***h(x) = 1***).

The hypothesis that ***PERSON*** enjoys his favorite sport only on cold days with high humidity is represented by the expression

(?, Cold, High, ?, ?, ?)

The most general hypothesis-that every day is a positive example-is represented by

(?, ?, ?, ?, ?, ?)

The most specific possible hypothesis-that no day is a positive example-is represented by

(Φ, Φ, Φ, Φ, Φ,
Φ)

Notation

The set of items over which the concept is defined is called the ***set of instances***, which is denoted by X.

Example: X is the set of all possible days, each represented by the attributes: Sky, AirTemp, Humidity, Wind, Water, and Forecast

The concept or function to be learned is called the ***target concept***, which is denoted by c. c can be any Boolean valued function defined over the instances X

$$c: X\rightarrow \{O, 1\}$$

Example: The target concept corresponds to the value of the attribute ***EnjoySport*** (i.e., c(x) = 1 if ***EnjoySport*** = Yes, and c(x) = 0 if ***EnjoySport*** = No).

Instances for which c(x) = 1 are called ***positive examples***, or members of the target concept.

Instances for which c(x) = 0 are called ***negative examples***, or non-members of the target concept.

The ordered pair (x, c(x)) to describe the training example consisting of the instance x and its target ***concept value c(x).***

D to denote the set of available training examples

The symbol ***H*** to denote the set of all possible hypotheses that the learner may consider regarding the identity of the target concept. Each hypothesis ***h*** in ***H*** represents a Boolean-valued function defined over ***X***

$$h: X \rightarrow \{O, 1\}$$

The goal of the learner is to find a hypothesis h such that h(x) = c(x) for all x in X.

Given:

Instances X: Possible days, each described by the attributes

Sky (with possible values Sunny, Cloudy, and Rainy),
AirTemp (with values Warm and Cold),
Humidity (with values Normal and High),
Wind (with values Strong and Weak),
Water (with values Warm and Cool),
Forecast (with values Same and Change).

Hypotheses *H*: Each hypothesis is described by a conjunction of constraints on the attributes *Sky, AirTemp, Humidity, Wind, Water*, and *Forecast*. The constraints may be "?" (any value is acceptable), "Φ" (no value is acceptable), or a specific value.

Target concept *c*: ***EnjoySport*** : X → {0, l}
Training examples *D*: Positive and negative examples of the target function

Determine:

A hypothesis h in H such that *h(x) = c(x)* for all *x* in X.

Table: The ***EnjoySport*** concept learning task.

Inductive learning hypothesis

- Our assumption is that the best hypothesis regarding unseen instances is the hypothesis that best fits the observed training data. This is the fundamental assumption of inductive learning.
- **The inductive learning hypothesis.** Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.

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Concept learning as search

Concept learning can be viewed as the task of **searching through a large space of hypotheses** implicitly defined by the hypothesis representation. **The goal of this search is to find the hypothesis that best fits the training examples.**

Consider, for example, the instances X and hypotheses H in the *EnjoySport* learning task. Given that the attribute *Sky* has three possible values, and that *AirTemp, Humidity, Wind, Water,* and *Forecast* each have two possible values, the instance space X contains exactly 3.2.2.2.2.2 = 96 distinct instances. A similar calculation shows that there are 5.4.4.4.4.4 = 5120 syntactically distinct hypotheses within H (including ? and Φ for each). Most practical learning tasks involve much larger, sometimes infinite, hypothesis spaces.

General-to-Specific Ordering of Hypotheses

Many algorithms for concept learning organize the search through the hypothesis space by relying on a very useful structure that exists for any concept learning problem: a general-to-specific ordering of hypotheses. To illustrate the general-to-specific ordering, consider the two hypotheses

h1 = (Sunny, ?, ?, Strong, ?, ?)

h2 = (Sunny, ?, ?, ?, ?, ?)

Now consider the sets of instances that are classified positive by hl and by h2. Because h2 imposes fewer constraints on the instance, it classifies more instances as positive. In fact, any instance classified positive by hl will also be classified positive by h2. Therefore, we say that **h2 is more general than hl.**

This intuitive "more general than" relationship between hypotheses can be defined more precisely as follows.

Definition: Let h_j and h_k ***be*** boolean-valued functions defined over X. Then ***hj*** is **more general-than-or-equal-to** h_k (written $h_j >=_g h_k$) if and only if

Type equation here. $\forall(x \in X)[h�(x) = 1) \rightarrow (h�(x) = 1)$

We will also find it useful to consider cases where one hypothesis is strictly more general than the other. Therefore, we will say that h_j is (strictly) more-general –than h_k (written $h_j >_g h_k$) if and only if $(h_j >=_g h_k)$ ^ $(h_k >=_g h_i)$. Finally, we will sometimes find the inverse useful and will say that h_j is more specific than h_k when h_k is more-general-than h_j.

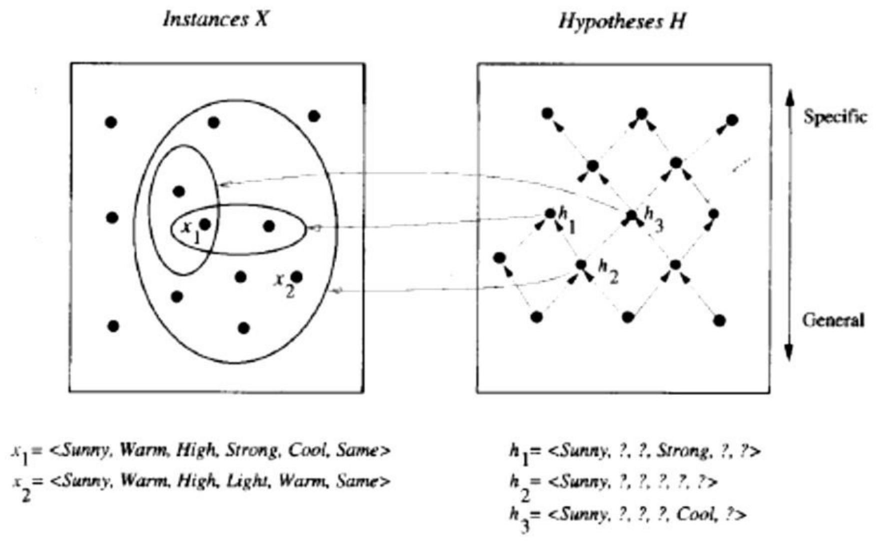

Instances X
Hypotheses H
Specific
General
x_1
x_2
h_1
h_2
h_3
x_1= <Sunny, Warm, High, Strong, Cool, Same>
x_2= <Sunny, Warm, High, Light, Warm, Same>
h_1= <Sunny, ?, ?, Strong, ?, ?>
h_2= <Sunny, ?, ?, ?, ?, ?>
h_3= <Sunny, ?, ?, ?, Cool, ?>

FIGURE 2.2
The hypothesis space search performed by FIND-S. The search begins (h_0) with the most specific hypothesis in H, then considers increasingly general hypotheses (h_1 through h_4) as mandated by the training examples. In the instance space diagram, positive training examples are denoted by "+," negative by "−," and instances that have not been presented as training examples are denoted by a solid circle.

The *pg* relation is important because it provides a useful structure over the hypothesis space H for ***any*** concept learning problem. The following sections present concept learning algorithms that take advantage of this partial order to efficiently organize the search for hypotheses that fit the training data.

Find-S: Finding A Maximally Specific Hypothesis

How can we use the ***more-general-than*** partial ordering to organize the search for a hypothesis consistent with the observed training examples? One way is to begin with the most specific possible hypothesis in H, then generalize this hypothesis each time it fails to cover an observed positive training example. FIND-S algorithm is used for this purpose.

```
FIND-S Algorithm.
1. Initialize $h$ to the most specific hypothesis in $H$
2. For each positive training instance $x$
       • For each attribute constraint $a_i$ in $h$
             If the constraint $a_i$ is satisfied by $x$
             Then do nothing
             Else replace $a_i$ in $h$ by the next more general constraint that is satisfied by $x$
3. Output hypothesis $h$
```

To illustrate this algorithm, assume the learner is given the sequence of training examples from Table 2.1 for the *EnjoySport* task.

Example	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*EnjoySport*
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

TABLE 2.1
Positive and negative training examples for the target concept *EnjoySport*.

The first step of FIND-S is to initialize h to the most specific hypothesis in H.

$$h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$$

Upon observing the **first training example** from Table 2.1, which happens to be a positive example, it becomes clear that our hypothesis is too specific. In particular, none of the "Φ" constraints in h are satisfied by this example, so each is replaced by the next more general constraint that fits the example; namely, the attribute values for this training example.

$$h \leftarrow \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle$$

This h is still very specific; it asserts that all instances are negative except for the single positive training example we have observed.

Next, the **second training example** (also positive in this case) forces the algorithm to further generalize h, this time substituting a "?' in place of any attribute value in h that is not satisfied by the new example. The refined hypothesis is

$$h \leftarrow \langle Sunny, Warm, ?, Strong, Warm, Same \rangle$$

Upon encountering the **third training example**-in this case a negative example-the algorithm makes no change to h. In fact, the FIND-S algorithm simply ignores every negative example.

The **fourth** (positive) example leads to a further generalization of h

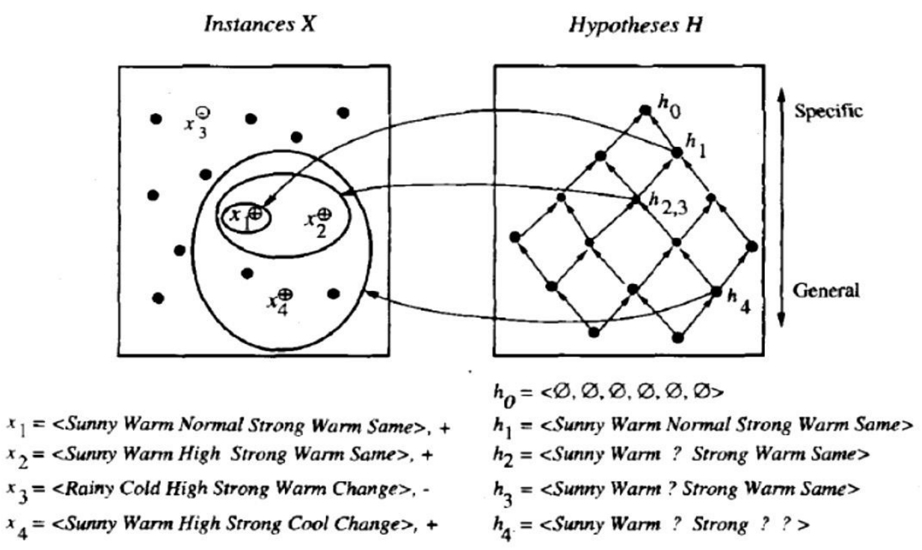
$$h \leftarrow \langle Sunny, Warm, ?, Strong, ?, ? \rangle$$

The FIND-S algorithm illustrates one way in which the more-general-than partial ordering can be used to organize the search for an acceptable hypothesis. The search moves from hypothesis to hypothesis, searching from the most specific to progressively more general hypotheses along one chain of the partial ordering.

Figure 2.2 illustrates this search in terms of the instance and hypothesis spaces.

Key Property

The key property of the Find-S algorithm is that for hypothesis spaces described by conjunctions of attribute constraints (such as H for the EnjoySport task). Find-S is guaranteed to output the most specific hypothesis within H that is consistent with the positive training examples.


Instances X
Hypotheses H
x_3
x_1
x_2
x_4
h_0
h_1
$h_{2,3}$
h_4
Specific
General
$h_0 = <\varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing>$
x_1 = <Sunny Warm Normal Strong Warm Same>, +
x_2 = <Sunny Warm High Strong Warm Same>, +
x_3 = <Rainy Cold High Strong Warm Change>, -
x_4 = <Sunny Warm High Strong Cool Change>, +
h_1 = <Sunny Warm Normal Strong Warm Same>
h_2 = <Sunny Warm ? Strong Warm Same>
h_3 = <Sunny Warm ? Strong Warm Same>
h_4 = <Sunny Warm ? Strong ? ? >

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The hypothesis space search performed by FIND-S. The search begins (h_0) with the most specific hypothesis in H, then considers increasingly general hypotheses (h_1 through h_4) as mandated by the training examples. In the instance space diagram, positive training examples are denoted by "+," negative by "−," and instances that have not been presented as training examples are denoted by a solid circle.

However, there are several **questions still left unanswered**, such as:

- ***Has the learner converged to the correct target concept?*** Although FIND-S will find a hypothesis consistent with the training data, it has no way to determine whether it has found the only hypothesis in H consistent with the data (i.e., the correct target concept), or whether there are many other consistent hypotheses as well.

- ***Why prefer the most specific hypothesis?*** In case there are multiple hypotheses consistent with the training examples, FIND-S will find the most specific. It is unclear whether we should prefer this hypothesis over, say, the most general, or some other hypothesis of intermediate generality.
- ***Are the training examples consistent?*** In most practical learning problems there is some chance that the training examples will contain at least some errors or noise. Such inconsistent sets of training examples can severely mislead FIND-S, given the fact that it ignores negative examples.
- ***What if there are several maximally specific consistent hypotheses?*** There can be several maximally specific hypotheses consistent with the data. Find S finds only one.

Version Space and Motivation to Candidate Elimination algorithm

Candidate Elimination algorithm (CEA), addresses limitations of FIND-S. It finds all describable hypotheses that are consistent with the observed training examples. In order to define this algorithm precisely, we begin with a few basic definitions.

Definition: A hypothesis h is **consistent** with a set of training examples D if and only if $h(x) = c(x)$ for each example $\langle x, c(x)\rangle$ in D.

$$Consistent(h, D) \equiv (\forall \langle x, c(x)\rangle \in D)\ h(x) = c(x)$$

Notice the key difference between this definition of ***consistent*** and our earlier definition of ***satisfies***. An example x is said to satisfy hypothesis h when *h(x) = 1*, regardless of whether x is a positive or negative example of the target concept. However, whether such an example is consistent with h depends on the target concept, and in particular, whether *h(x) = c(x).*

This subset of all hypotheses is called the **version space** with respect to the hypothesis space H and the training examples D, because it contains all plausible versions of the target concept.

Definition: The **version space**, denoted $VS_{H,D}$, with respect to hypothesis space H and training examples D, is the subset of hypotheses from H consistent with the training examples in D.

$$VS_{H,D} \equiv \{h \in H | Consistent(h, D)\}$$

The List-Then-Eliminate algorithm

One obvious way to represent the version space is simply to list all of its members. This leads to a simple learning algorithm, which we might call the List-Then-Eliminate algorithm.

The LIST-THEN-ELIMINATE Algorithm

```
1. $VersionSpace \leftarrow$ a list containing every hypothesis in $H$
2. For each training example, $\langle x, c(x) \rangle$
        remove from $VersionSpace$ any hypothesis $h$ for which $h(x) \neq c(x)$
3. Output the list of hypotheses in $VersionSpace$
```

The List-Then-Eliminate algorithm first initializes the version space to contain all hypotheses in H, and then eliminates any hypothesis found inconsistent with any training example. The version space of candidate hypotheses thus shrinks as more examples are observed, until ideally just one hypothesis remains that are consistent with all the observed examples.

It is intuitively plausible that we can represent the version space in terms of its most specific and most general members.

Definition: The **general boundary** G, with respect to hypothesis space H and training data D, is the set of maximally general members of H consistent with D.

$$G \equiv \{g \in H | Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$$

Definition: The **specific boundary** S, with respect to hypothesis space H and training data D, is the set of minimally general (i.e., maximally specific) members of H consistent with D.

$$S \equiv \{s \in H | Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$$

As long as the sets G and S are well defined, they completely specify the version space. In particular, we can show that the version space is precisely the set of hypotheses contained in G, plus those contained in S, plus those that lie between G and S in the partially ordered hypothesis space. (This is stated precisely in Theorem 2.1. Refer text book for more details)

Candidate Elimination algorithm

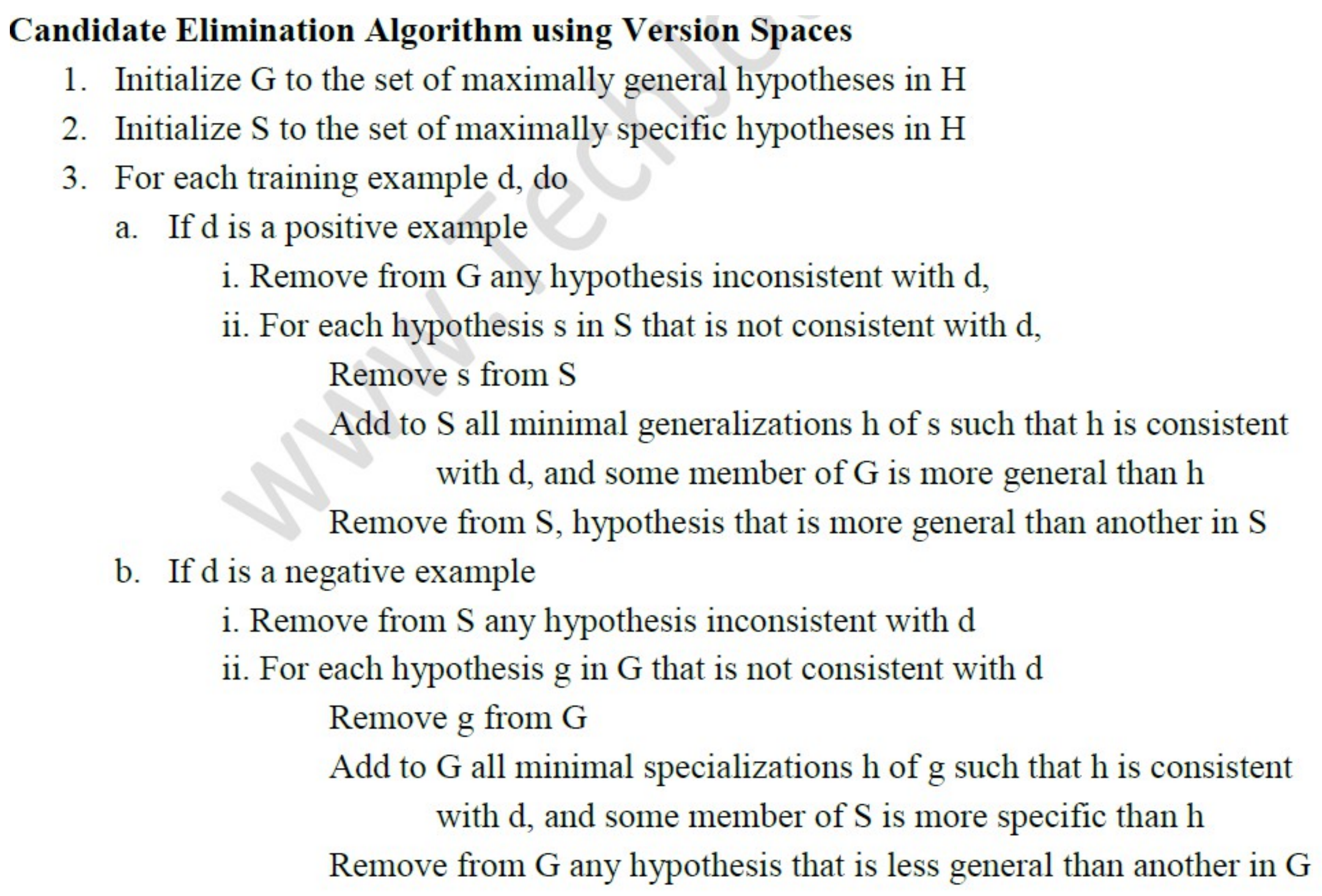
It computes the version space containing all hypotheses from H that are consistent with an observed sequence of training examples. It begins by initializing the version space to the set of all hypotheses in H; that is, by initializing the G boundary set to contain the most general hypothesis in H

$$G_0 \leftarrow \{\langle ?, ?, ?, ?, ?, ? \rangle\}$$

and initializing the S boundary set to contain the most specific (least general) hypothesis

$$S_0 \leftarrow \{\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle\}$$

These two boundary sets delimit the entire hypothesis space, because every other hypothesis in H is both more general than So and more specific than Go. As each training example is considered, the S and G boundary sets are generalized and specialized, respectively, to eliminate from the version space any hypotheses found inconsistent with the new training example. After all examples have been processed, the computed version space contains all the hypotheses consistent with these examples and only these hypotheses. This algorithm is summarized in given below.



Candidate Elimination Algorithm using Version Spaces

```
1. Initialize G to the set of maximally general hypotheses in H
2. Initialize S to the set of maximally specific hypotheses in H
3. For each training example d, do
   a. If d is a positive example
        i. Remove from G any hypothesis inconsistent with d,
        ii. For each hypothesis s in S that is not consistent with d,
              Remove s from S
              Add to S all minimal generalizations h of s such that h is consistent
                    with d, and some member of G is more general than h
              Remove from S, hypothesis that is more general than another in S
   b. If d is a negative example
        i. Remove from S any hypothesis inconsistent with d
        ii. For each hypothesis g in G that is not consistent with d
              Remove g from G
              Add to G all minimal specializations h of g such that h is consistent
                    with d, and some member of S is more specific than h
              Remove from G any hypothesis that is less general than another in G
```

An Illustrative Example

The Figure given below traces the algorithm. As described above, the boundary sets are first initialized to G0 and S0, the most general and most specific hypotheses in H, respectively.

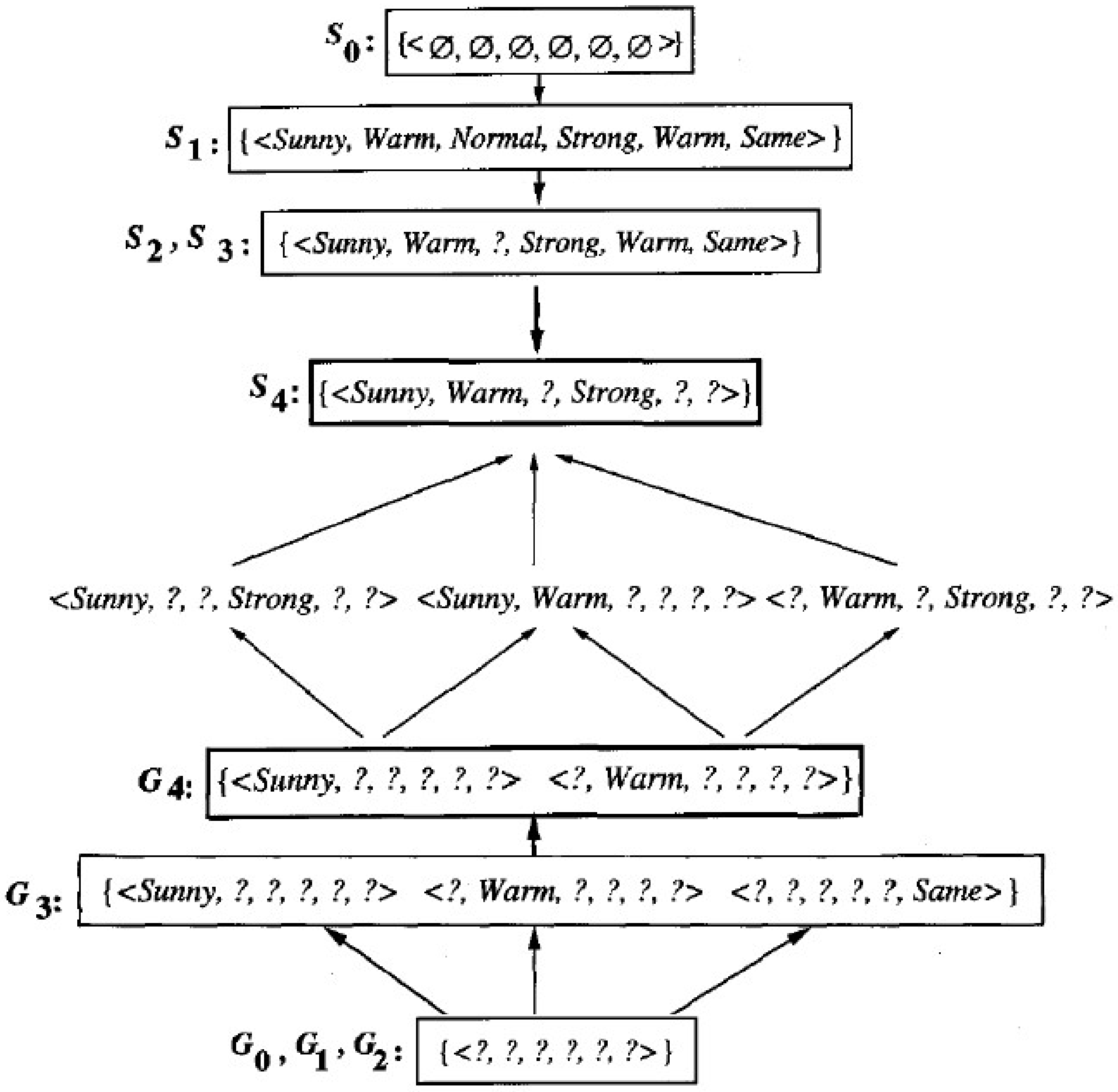
First training Sample: When the first training example is presented (a positive example in this case), the algorithm checks the S boundary and finds that it is overly specific-it fails to cover the positive example. The boundary is therefore revised by moving it to the least more general hypothesis that covers this new example. This revised boundary is shown as S1 in Figure 2.4. No update of the G boundary is needed in response to this training example because G0 correctly covers this example.

Second Training Sample: When the second training example (also positive) is observed, it has a similar effect of generalizing S further to S2, leaving G again unchanged (i.e., G2 = G1= G0). Notice the processing of these first two positive examples is very similar to the processing performed by the Find-S algorithm.

Third Sample: Negative training examples play the complimentary role of forcing the G boundary to become increasingly specific. Consider the third training example (negative sample), This negative example reveals that the G boundary of the version space is overly general; that is, the hypothesis in G incorrectly predicts that this new example is a positive example. The hypothesis in the G boundary must therefore be specialized until it correctly classifies this new negative example. There are several alternative minimally more specific hypotheses. All of these become members of the new G3 boundary set.

Given that there are six attributes that could be specified to specialize G2, why are there only three new hypotheses in G3? For example, the hypothesis *h = (?, ?, Normal, ?, ?, ?)* is a minimal specialization of G2 that correctly labels the new example as a negative example, but it is not included in G3. The reason this hypothesis is excluded is that it is inconsistent with the previously encountered positive examples. The algorithm determines this simply by noting that h is not more general than the current specific boundary, S2. In fact, the S boundary of the version space forms a summary of the previously encountered positive examples that can be used to determine whether any given hypothesis is consistent with these examples. Any hypothesis more general than S will, by definition, cover any example that S covers and thus will cover any past positive example. In a dual fashion, the G boundary summarizes the information from previously encountered negative examples. Any hypothesis more specific than G is assured to be consistent with past negative examples. This is true because any such hypothesis, by definition, cannot cover examples that G does not cover.

***Fourth training example**:* This further generalizes the S boundary of the version space. It also results in removing one member of the G boundary, because this member fails to cover the new positive example. This last action results from the first step under the condition "If d is a positive example" in the algorithm. To understand the rationale for this step, it is useful to consider why the offending hypothesis must be removed from G. Notice it cannot be specialized, because specializing it would not make it cover the new example. It also cannot be generalized, because by the definition of G, any more general hypothesis will cover at least one negative training example. Therefore, the hypothesis must be dropped from the G boundary, thereby removing an entire branch of the partial ordering from the version space of hypotheses remaining under consideration.


S_0: {<∅, ∅, ∅, ∅, ∅, ∅>}
S_1: {<Sunny, Warm, Normal, Strong, Warm, Same>}
S_2, S_3: {<Sunny, Warm, ?, Strong, Warm, Same>}
S_4: {<Sunny, Warm, ?, Strong, ?, ?>}
<Sunny, ?, ?, Strong, ?, ?> <Sunny, Warm, ?, ?, ?, ?> <?, Warm, ?, Strong, ?, ?>
G_4: {<Sunny, ?, ?, ?, ?, ?> <?, Warm, ?, ?, ?, ?>}
G_3: {<Sunny, ?, ?, ?, ?, ?> <?, Warm, ?, ?, ?, ?> <?, ?, ?, ?, ?, Same>}
G_0, G_1, G_2: {<?, ?, ?, ?, ?, ?>}

Training examples:

1. <Sunny, Warm, Normal, Strong, Warm, Same>, Enjoy Sport = Yes
2. <Sunny, Warm, High, Strong, Warm, Same>, Enjoy Sport = Yes
3. <Rainy, Cold, High, Strong, Warm, Change>, EnjoySport=No
4. <Sunny, Warm, High, Strong, Cool, Change>, EnjoySport = Yes

After processing these four examples, the boundary sets S4 and G4 delimit the version space of all hypotheses consistent with the set of incrementally observed training examples. The entire version space, including those hypotheses bounded by S4 and G4. This learned version space is independent of the sequence in which the training examples are presented (because in the end it contains all hypotheses consistent with the set of examples). As further training data is encountered, the S and G boundaries will move monotonically closer to each other, delimiting a smaller and smaller version space of candidate hypotheses.

2.6 Inductive Bias.

The CEA will converge toward the true target concept provided it is given accurate training examples and provided its initial hypothesis space contains the target concept.

- What if the target concept is not contained in the hypothesis space?
- Can we avoid this difficulty by using a hypothesis space that includes every possible hypothesis?
- How does the size of this hypothesis space influence the ability of the algorithm to generalize to unobserved instances?
- How does the size of the hypothesis space influence the number of training examples that must be observed?

These are fundamental questions for inductive inference in general. Here we examine them in the context of the CEA. The conclusions we draw from this analysis will apply to any concept learning system that outputs any hypothesis consistent with the training data.

2.6.1 A Biased Hypothesis Space

Suppose we wish to assure that the hypothesis space contains the unknown target concept. The obvious solution is to enrich the hypothesis space to include every possible hypothesis. To illustrate, consider *EnjoySport* example in which we restricted the hypothesis space to include only conjunctions of attribute values. Because of this restriction, the hypothesis space is unable to represent even simple disjunctive target concepts such as "*Sky = Sunny or Sky = Cloudy.*"

In fact, given the following three training examples of this disjunctive hypothesis, our algorithm would find that there are zero hypotheses in the version space.

Example	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*EnjoySport*
1	Sunny	Warm	Normal	Strong	Cool	Change	Yes
2	Cloudy	Warm	Normal	Strong	Cool	Change	Yes
3	Rainy	Warm	Normal	Strong	Cool	Change	No

To see why there are no hypotheses consistent with these three examples, note that the most specific hypothesis consistent with the first two examples and representable in the given hypothesis space H

$S_2 : \langle ?, Warm, Normal, Strong, Cool, Change \rangle$ is

This hypothesis, although it is the maximally specific hypothesis from H that is consistent with the first two examples, is already overly general: **it incorrectly covers the third (negative) training example.**

The problem is that we have biased the learner to consider only conjunctive hypotheses. In this case we require a more expressive hypothesis space.

2.6.2 An Unbiased Learner

The obvious solution to the problem of assuring that the target concept is in the hypothesis space H is to provide a hypothesis space capable of representing ***every teachable concept***; that is, it is capable of representing every possible subset of the instances X. (In general, the set of all subsets of a set X is called the ***power-set*** of X).

In the *EnjoySport* learning task, for example, the size of the instance space X of days described by the six available attributes is 96. In general, the number of distinct subsets that can be defined over a set X containing |x| elements is 2|X|. Thus, there are 296, or approximately distinct target concepts that could be defined over this instance space and that our learner might be called upon to learn. Our conjunctive hypothesis space is able to represent only 973 of these-a very biased hypothesis space indeed!

Let us reformulate the *Enjoysport* learning task in an unbiased way by defining a new hypothesis space H' that can represent every subset of instances; that is, let H' correspond to the power set of X. One way to define such an H' is to allow arbitrary disjunctions, conjunctions, and negations of our earlier hypotheses.

For instance, the target concept "Sky = Sunny or Sky = Cloudy" could then be described as

$$\langle Sunny, ?, ?, ?, ?, ?\rangle \vee \langle Cloudy, ?, ?, ?, ?, ?\rangle$$

However, while this hypothesis space eliminates any problems of expressibility, it unfortunately raises a new, equally difficult problem: our concept learning algorithm is now completely unable to generalize beyond the observed examples! To see why, suppose we present three positive examples (xl, x2, x3) and two negative examples (x4, x5) to the learner. At this point, the S boundary of the version space will be

$$S : \{(x_1 \vee x_2 \vee x_3)\}$$

That of G will be

$$G : \{\neg(x_4 \vee x_5)\}$$

Here in order to converge to a single, final target concept, we will have to present every single instance in X as a training example!

2.6.3 The Futility of Bias-Free Learning

The fundamental property of inductive inference: *a learner that makes no a priori assumptions regarding the identity of the target concept has no rational basis for classifying any unseen instances*. In fact, the only reason that the CEA was able to generalize beyond the observed training examples in our original formulation of the *EnjoySport* task is that it was biased by the implicit assumption that the target concept could be represented by a conjunction of attribute values. In cases where this assumption is correct (and the training examples are error-free), its classification of new instances will also be correct. If this assumption is incorrect, however, it is certain that the CEA will mis-classify at least some instances from X.

Let us define this notion of inductive bias more precisely. Consider the general setting in which an arbitrary learning algorithm L is provided an arbitrary set of training data Dc = {⟨x, c(x) ⟩} of some arbitrary target concept c. After training, L is asked to classify a new instance xi. Let L(xi, Dc) denote the classification (e.g., positive or negative) that L assigns to xi after learning from the training data Dc. We can describe this inductive inference step performed by L as follows

$$(D_c \wedge x_i) \succ L(x_i, D_c)$$

where the notation y z indicates that z is inductively inferred from y. For example, if we take L to be the CEA, Dc, to be the training data from Table 2.1, and xi to be the first instance from Table 2.6, then the inductive inference performed in this case concludes that L(xi, Dc) = (EnjoySport = yes).

Instance	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*EnjoySport*
A	Sunny	Warm	Normal	Strong	Cool	Change	?
B	Rainy	Cold	Normal	Light	Warm	Same	?
C	Sunny	Warm	Normal	Light	Warm	Same	?
D	Sunny	Cold	Normal	Strong	Warm	Same	?

TABLE 2.6
New instances to be classified.

Definition: Consider a concept learning algorithm L for the set of instances X. Let c be an arbitrary concept defined over X, and let Dc = {⟨x, c(x) ⟩} be an arbitrary set of training examples of c. Let L(xi, Dc) denote the classification assigned to the instance xi by L after training on the data Dc. The **inductive bias** of L is any minimal set of assertions B such that for any target concept c and corresponding training examples Dc

$$(\forall x_i \in X)[(B \wedge D_c \wedge x_i) \vdash L(x_i, D_c)]$$

Inductive bias of CEA: The target concept c is contained in the given hypothesis space H.

The figure given below summarizes the situation schematically.

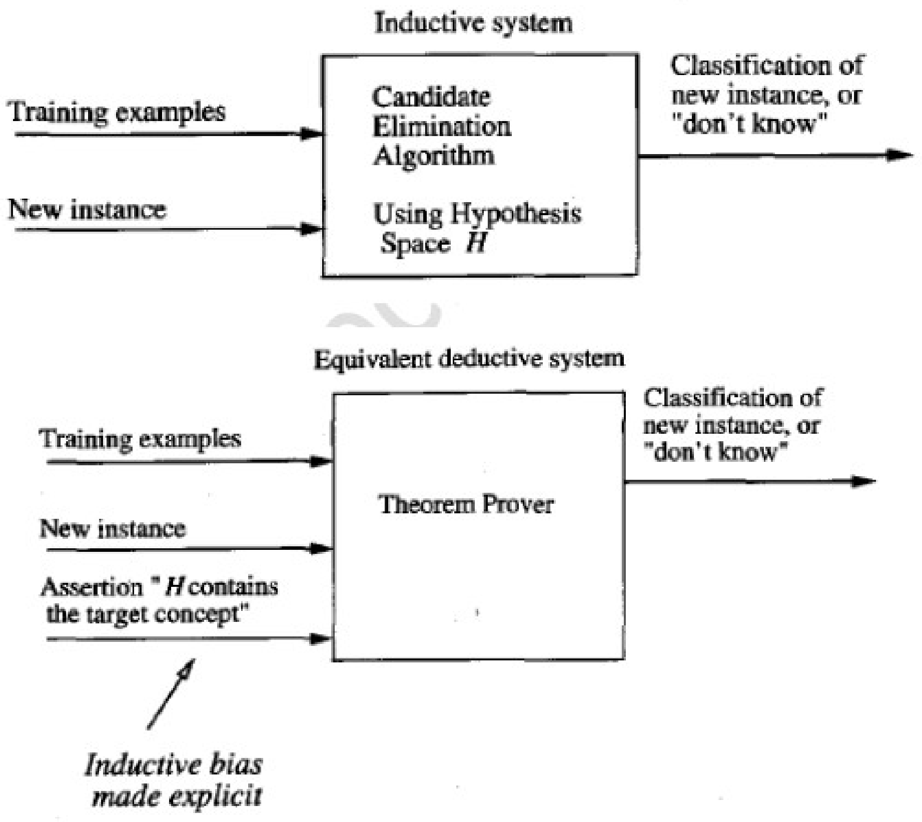

Inductive system
Training examples
New instance
Candidate Elimination Algorithm
Using Hypothesis Space H
Classification of new instance, or "don't know"
Equivalent deductive system
Training examples
New instance
Assertion "H contains the target concept"
Theorem Prover
Classification of new instance, or "don't know"
Inductive bias made explicit

FIGURE 2.8
Modeling inductive systems by equivalent deductive systems. The input-output behavior of the CANDIDATE-ELIMINATION algorithm using a hypothesis space H is identical to that of a deductive theorem prover utilizing the assertion "H contains the target concept." This assertion is therefore called the *inductive bias* of the CANDIDATE-ELIMINATION algorithm. Characterizing inductive systems by their inductive bias allows modeling them by their equivalent deductive systems. This provides a way to compare inductive systems according to their policies for generalizing beyond the observed training data.

One advantage of viewing inductive inference systems in terms of their inductive bias is that it provides a nonprocedural means of characterizing their policy for generalizing beyond the observed data. A second advantage is that it allows comparison of different learners according to the strength of the inductive bias they employ. Consider, for example, the following three learning algorithms, which are listed from weakest to strongest bias.

- Rote-Learner: Learning corresponds simply to storing each observed training example in memory. Subsequent instances are classified by looking them up in memory. If the instance is found in memory, the stored classification is returned. Otherwise, the system refuses to classify the new instance.
- CEA: New instances are classified only in the case where all members of the current version space agree on the classification. Otherwise, the system refuses to classify the new instance.
- FIND-S: This algorithm, described earlier, finds the most specific hypothesis consistent with the training examples. It then uses this hypothesis to classify all subsequent instances.

The Rote-Learner has no inductive bias. The classifications it provides for new instances follow deductively from the observed training examples, with no additional assumptions required. The CEA has a stronger inductive bias: that the target concept can be represented in its hypothesis space. Because it has a stronger bias, it will classify some instances that the Rote-Learner will not. Of course, the correctness of such classifications will depend completely on the correctness of this inductive bias. The FIND-S algorithm has an even stronger inductive bias. In addition to the assumption that the target concept can be described in its hypothesis space, it has an additional inductive bias assumption: that all instances are negative instances unless the opposite is entailed by its other knowledge.

Summary

Machine learning addresses the question of how to build computer programs that improve their performance at some task through experience. Major points of this topic include:

• Machine learning algorithms have proven to be of great practical value in a variety of application domains. They are especially useful in (a) data mining problems where large databases may contain valuable implicit regularities that can be discovered automatically (b) poorly understood domains where humans might not have the knowledge needed to develop effective and (c) domains where the program must dynamically adapt to changing conditions

• Machine learning draws on ideas from a diverse set of disciplines, including artificial intelligence, probability and statistics, computational complexity, information theory, psychology and neurobiology, control theory, and philosophy.

• A well-defined learning problem requires a well-specified task, performance metric, and source of training experience.

• Designing a machine learning approach involves a number of design choices, including choosing the type of training experience, the target function to be learned, a representation for this target function, and an algorithm for learning the target function from training examples.

• Learning involves search: searching through a space of possible hypotheses to find the hypothesis that best fits the available training examples and other prior constraints or knowledge.

The main points in the Concept Learning include:

• Concept learning can be cast as a problem of searching through a large predefined space of potential hypotheses.

• The general-to-specific partial ordering of hypotheses, which can be defined for any concept learning problem, provides a useful structure for organizing the search through the hypothesis space.

• The Find-S algorithm utilizes this general-to-specific ordering, performing a specific-to-general search through the hypothesis space along one branch of the partial ordering, to find the most specific hypothesis consistent with the training examples.

- The CEA utilizes this general-to-specific ordering to compute the version space (the set of all hypotheses consistent with the training data) by incrementally computing the sets of maximally specific (S) and maximally general (G) hypotheses.
- The version space of alternative hypotheses can be examined to determine whether the learner has converged to the target concept, to determine when the training data are inconsistent, to generate informative queries to further refine the version space, and to determine which unseen instances can be unambiguously classified based on the partially learned concept.
- Version spaces and the CEA provide a useful conceptual framework for studying concept learning. However, this learning algorithm is not robust to noisy data or to situations in which the unknown target concept is not expressible in the provided hypothesis space.
- Inductive learning algorithms are able to classify unseen examples only because of their implicit inductive bias for selecting one consistent hypothesis over another. The bias associated with the CEA is that the target concept can be found in the provided hypothesis space ($c \in H$). The output hypotheses and classifications of subsequent instances follow deductively from this assumption together with the observed training data.
- If the hypothesis space is enriched to the point where there is a hypothesis corresponding to every possible subset of instances (the power set of the instances), this will remove any inductive bias from the CEA. Unfortunately, this also removes the ability to classify any instance beyond the observed training examples. An unbiased learner cannot make inductive leaps to classify unseen exampl