June 2023 – PG Examination

USN 1RV22SIT01

22MAT11CT

a

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU, Belagavi)

I Semester- Master of Technology

Common to MDC / MSE / MIT

LINEAR ALGEBRA & PROBABILITY THEORY

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer FIVE full questions selecting one from each unit.
2. Each unit consisting of two questions of 20 marks each.

UNIT-1

1	a	Show that the set $V = \{p\, sint + qe^t /\ p, q \in \mathbb{R}\}$, over the field $\mathbb{R}$ is a vector space under usual addition and scalar multiplication.	05
	b	Determine whether $\{(1,1,1,1),(1,2,3,2),(2,5,6,4),(2,6\,8,5)\}$ form a basis of R^4. If not, find the dimension of the subspace they span.	05
	c	Obtain the basis and dimension of the four fundamental subspaces of $\begin{bmatrix} 1 & -2 & 1 & 0 & 2 \\ 1 & -1 & 4 & 1 & 3 \\ -1 & 3 & 2 & 1 & -1 \\ 2 & -3 & 5 & 1 & 5 \end{bmatrix}$.	10
		OR	
2	a	Give the row and column picture of the following system of equations $2x + y = 3$ and $x + y = 2$.	05
	b	Find a basis for the subspace of M_{22} consisting of all 2×2 matirces A such that $A = A^T$. What is its dimension?	05
	c	Let $T: R^3 \to R^3$ be the linear mapping defined by $T(x,y,z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and dimension of i) Range space of T → Ra. ii) Kernel of T — Null Verify Rank and Nullity theorem.	10

UNIT-2

3	a	Compute the shortest distance between the vector $y = \begin{pmatrix} -9 \\ 1 \\ 6 \end{pmatrix}$ and the subspace W spanned by $\{u_1, u_2\}$ where $u_1 = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$, $u_2 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ using the concept of orthogonal projection of y onto W.	05
	b	Orthonormalize the vectors $\{(1,0,1),(-1,2,1),(2,1,1)\}$ using Gram-Schmidt process.	05
	c	Find the least square solution for $AX = b$ if $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$ and $b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix}$.	10
		OR	
4	a	Obtain the line of best fit for the equation $y = \beta_0 + \beta_1 x$ by least-squares for the given data points $(2,1),(5,2),(7,3),(8,3)$.	05

	b	Find the n^{th}-order Fourier approximation to the function $f(t) = t$ on the interval $[0, 2\pi]$.	05
	c	The columns of matrix A represent Linearly independent vectors in R^5. Using Schmidt process obtain the QR factorization of the matrix. $$A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}.$$	10

UNIT-3

5	a	Orthogonally diagonalize the matrix $A = \begin{bmatrix} 7 & -4 & 4 \\ -4 & 5 & 0 \\ 4 & 0 & 9 \end{bmatrix}$.	10
	b	Classify the quadratic forms: $Q(x) = 7x_1^2 + x_2^2 + 7x_3^2 - 8x_1x_2 - 4x_1x_3 - 8x_2x_3$. Hence find the maximum value subject to the constraint $x^Tx = 1$, a unit vector u where this maximum is attained, a unit vector v where the maximum occurs subject to $x^Tx = 1$ and $x^Tu = 0$.	10
		OR	
6	a	Measurements in three characteristics are made on two individuals in a random sample from a population. The observation matrix is given as $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$. Obtain the Singular Value Decomposition.	10
	b	Convert the matrix of observations $\begin{bmatrix} 19 & 22 & 6 & 3 & 2 & 20 \\ 12 & 6 & 9 & 15 & 13 & 5 \end{bmatrix}$ to mean-deviation form, and construct the sample covariance matrix and hence find the principal components of the data.	10

UNIT-4

7 a The joint probability distribution of two random variables X and Y is given by the following data.

X \ Y	−4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

i) Determine the marginal PMF's of the random variable of X and Y.

ii) Compute the covariance matrix and correlation matrix for the above data. 10

b Let $X \sim N(\mu, \Sigma)$, $\mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}, \Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following

i) $P[X_1 > 7]$

ii) $P[-3X_1 + 3X_3 > 80]$

iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$ 10

OR

8	a	The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = K(2x + y)$, where x and y are integers such that $x = \{0, 1, 2\}$ and $y = \{0, 1, 2, 3\}$. Find i) K ii) $P(X = 1, Y = 2)$ iii) $P(X \geq 1, Y \leq 2)$ iv) $P(X = 1/Y = 3)$. Show that X and Y are dependent.	10
	b	The joint density function of two continuous random variables X and Y is $f(x,y) = \begin{cases} c(x^2 + y^2), & 0 < x < 1, \quad 0 < y < 1 \\ 0, & otherwise \end{cases}$ Find i) The value of c ii) $P\left(\frac{1}{4} < x < \frac{3}{2}\right)$ iii) $P\left(x < \frac{1}{2},\ y > \frac{1}{2}\right)$ iv) $P\left(y < \frac{1}{2}\right)$	10

UNIT-5

9	a	Show that $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ is regular stochastic matrix and compute the associated unique fixed probability vector.	10
	b	A habitual gambler is a member of two clubs A and B. He visits either of the clubs every day for playing cards. He never visits club A on two consecutive days. But if he visits club B on a particular day then the next day he is likely to visit club B or club A. i) Write the transition matrix of the Markov chain. ii) Verify whether it is irreduci~~ble?~~ → Regular iii) If the person had visited club B on Monday, what is the probability that he visits club A on Thursday. iv) Find the stationary distribution of the Markov process. → Unique probability distn.	10
		OR	
10	a	Assume that a computer system is in one of the states-busy, idle, or undergoing repair denoted by states- $0, 1, 2$. Observing the state at a specified time on each day it is found that the system approximately behaves like a Markov chain with transition probability matrix, $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$. Prove that the chain is irreducible and determine the steady state probabilities.	10
	b	Show that the random process $x(t) = A\cos(\omega_0 t + \theta)$ is wide sense stationary if it is assumed that A and ω_0 are constants and θ is uniformly distributed random variable in $(0,2\pi)$.	10

June-2024 PG Examinations

USN 1RV23SIT11

MMA203T

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU, Belagavi)

I Semester Master of Technology (Common to MDC, MSE, MIT)

LINEAR ALGEBRA AND PROBABILITY THEORY

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit.

UNIT-1

1	a	Give the row and column picture for the system of equations $x-2y=1$ and $2x+y=7$, Draw neat diagrams.	05
	b	Show that the transformation $T: R^3 \to R^2$ defined by $T(x,y,z)=(x+y+z,\ 2x-3y+4z)$ is a linear transformation.	05
	c	Compute the basis and dimension of the four fundamental subspaces of the matrix. $A=\begin{bmatrix}2 & -1 & 1 & 2\\ -8 & 4 & -6 & -4\\ 4 & -2 & 3 & 2\end{bmatrix}$	10
		OR	
2	a	Show that the set $V=\{p\,e^t+q\sin t \mid p,q\in\mathbb{R}\}$, over the field $\mathbb{R}$ is a vector space under usual addition and scalar multiplication.	05
	b	Determine the basis and dimension of the subspace spanned by $(1,-3,2,0)$, $(2,1,0,-1)$, $(4,8,-4,-3)$ and $(1,10,-6,-2)$ in $\mathbb{R}^4$.	05
	c	Given $T(e_1)=(0,1,0,2)$, $T(e_2)=(0,1,1,0)$, $T(e_3)=(0,1,-1,4)$. Obtain the linear transformation $T: R^3\to R^4$. Also find the range space, null space of the transformation and hence verify the rank-nullity theorem for the obtained transformation.	10

UNIT-2

3	a	Let S be the subspace of R^4 spanned by the vectors $v_1=(1,1,1,1)$, $v_2=(1,-1,2,2)$, $v_3=(1,2,-3,-4)$. Apply Gram-Schmidt orthogonalization process to find an orthogonal basis of S and hence find the projection of $v=(1,2,-3,\ 4)$ onto S.	10
	b	Find the least square solution for $AX=B$ if $A=\begin{bmatrix}1&1&0&0\\1&1&0&0\\1&0&1&0\\1&0&1&0\\1&0&0&1\\1&0&0&1\end{bmatrix}$ and $B=\begin{bmatrix}-3\\-1\\0\\2\\5\\1\end{bmatrix}$	10
		OR	
4	a	Find the QR factorization of the matrix A by applying Gram-Schmidt process, where $A=\begin{bmatrix}1&2&2\\-1&1&2\\-1&0&1\\1&1&2\end{bmatrix}$.	10

	b	Obtain the line of best fit for the equation $y = \beta_0 + \beta_1 x$ by least-squares for the given data points $(1,14), (2,27), (3,40), (4,55), (5,68)$.	10

8

UNIT-3

5 a Determine an orthogonal matrix P such that $D = P^{-1}AP$ is a diagonal matrix, where matrix $= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$. **10**

7-

b Classify the quadratic forms $Q(x) = 5x_1^2 + 6x_2^2 + 7x_3^2 + 48x_1x_2 - 4x_2x_3$.
Determine the following:

i) Classify the quadratic form $Q(x)$
ii) the maximum value of $Q(x)$ subject to the constraint $x^T x = 1$
iii) a unit vector u where the maximum of (ii) is attained.
iv) a unit vector v where the maximum occurs subject to the constraints $x^T x = 1$ and $x^T u = 0$. **10**

5

OR

6 a Find singular value decomposition for the matrix $A = \begin{bmatrix} 1 & -1 \\ -3 & 3 \\ -1 & 1 \end{bmatrix}$ **10**

b The following table lists the weights and heights of five boys

Boy	1	2	3	4	5
Weight (lb)	120	125	125	135	145
Height (m)	61	60	64	68	72

Compute the sample covariance matrix. Also find the principal component of the data and the percentage of information captured by the first and second principal components. **10**

UNIT-4

7 a The joint distribution of two discrete random variables X and Y is given by the following table.

$X \backslash Y$	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

7-

Find

i) The marginal distribution of X and Y,
ii) Covariance and Correlation matrix of (X, Y) **10**

b Let $X \sim N(\mu, \Sigma)$, $\mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following:

4

i) $P[X_1 < 7]$
ii) $P[-3X_1 + 3X_3 > 80]$
iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$ **10**

OR

8 a If the joint density function of two continuous random variable X and Y is $f(x,y) = \begin{cases} k(x^2+y^2), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & otherwise \end{cases}$

Find

i) the value of the constant k

ii) marginal density function of x and y

iii) $P\left(x < \frac{1}{2},\ y > \frac{1}{2}\right)$

iv) $P\left(\frac{1}{4} < x < \frac{3}{4}\right)$. **10**

b The joint distribution of two random variables X and Y is given by the following table:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

i) Find $P(X+Y > 11)$

ii) Determine the individual (marginal) probability distributions of X and Y and verify that X and Y are independent.

iii) Compute $P(Y|X=4)$, and $P(X|Y=3)$ **10**

UNIT-5

9 a Compute the unique fixed probability vector of the stochastic matrix

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}.$$ Also show that A is regular. **10**

7

b A company executive changes his car every year. If he has a car of make A he changes over to a car of make B. If he has a car of make B he changes over to a car of make C. If he has a car of make C, he is just as likely to change over to a car of make C, B or A. If he had a car of make C in the year 2008, find the probability that he will have a car of

7

i) make A in 2010

ii) make C in 2010

iii) make B in 2011

iv) make C in 2011. **10**

OR

10 a The transition probability matrix of a Markov Chain $\{X_n\}$, having 3 states 1,2,3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$, find:

i) $P(X_2 = 3)$

ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ **10**

b Show that the mean and variance of a random process $\{X(t)\}$ given by the probability law $\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ is identical. **10**

RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

33/60

Date	**18.03.2024**	**Time**	**9.30 – 11.30**
TEST	**I**	**Maximum Marks**	**50+10**
Course Title	**LINEAR ALGEBRA & PROBABILITY THEORY**	**Course Code**	**MMA203T**
Semester		**Programs**	

Sl. No	PART – A Instructions: Answer all questions.	M	CO	BT
1	For the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ the basis of null space of A is ______.	1	2	1
2	If A is a (9 X 11) matrix with a three dimensional null space, the rank of A is ______.	1	3	2
3	The standard basis of $P_3 = \{c_0 + c_1 t + c_2 t^2 + c_3 t^3 \mid c_i \in R\}$ is ______.	1	1	1
4	If $(1, -2, k)$ is a linear combination of $(3,0,-2)$ and $(2,-1,5)$, then $k =$ ______	2	2	2
5	$T: R^3 \rightarrow R^1$ is defined as $T(x,y,z) = x + y + z$, then Rank of $T =$ ______.	1	3	1
6	A basis for the column space of the matrix B where $B = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is ______.	1	3	1
7	Let $T: R^2 \rightarrow R^3$ be a linear transformation with $T(x_1, x_2) = (2x_1 - x_2, -3x_1 + x_2, 2x_1 - 3x_2)$, then $T(1,2)=$ ______.	1	2	1
8	Orthonormalize the basis {(1,-2), (1,-1)}.	2	3	2
	PART - B			
1a	Show that the set $V = \{a + b\sqrt{5} \,/\, a, b, c \in Q\}$, over the field $\mathbb{Q}$ is a vector space under usual addition and scalar multiplication.	6	1	1
1b	Give the row and column picture for the system of equations $2x + y = 3$ and $x - 2y = -1$, Draw neat diagrams.	4	1	1
2	Compute the basis and dimension of the four fundamental subspaces of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$	10	4	4

-5

-5

-3

-8

P.T.C.

Academic year 2023-2024 (Odd Semester 2023)

3a	Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace U of R^4 spanned by $v_1 = (5,1,-3,1)$ and $v_2 = (9,7,-5,5)$.	5	3	4	-3-
3b	Prove that the set $W = \{(x,y,z) \mid x - 3y + 4z = 0\}$ of the vector space R^3 is a subspace of R^3.	5	1	2	-3-
4	Find the Linear transformation $T: R^3 \to R^2$ such that $T(1,2,1) = (4,1)$, $T(2,1,0) = (4,3)$, $T(-1,1,2) = (-1,-1)$. Also find the bases for the range space and null space of the linear transformation.	10	3	4	-4
5a	Show that the vectors (1, 1, 2, 4), (2, -1, -5, 2), (1, -1, -4,0) and (2, 1,1,6) are linearly dependent in R^4 and extract a linearly independent subset. Also find the dimension and a basis of the subspace spanned by them.	6	2	3	-8
5b	Find the shortest distance between the vector $y = (5,-9,5)$ and the subspace W spanned by $\{u_1, u_2\}$ where $u_1 = (-3,-5,1)$, $u_2 = (-3,2,1)$ using the concept of orthogonal projection of y onto W.	4	2	3	-0-

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test Max Marks	15	10	15	10	10	5	10	25		

RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

$\frac{38}{50}$

Date	11.05.2024	Time	9.30 – 11.30	
TEST	III	Maximum Marks	50+10	
Course Title	LINEAR ALGEBRA & PROBABILITY THEORY		Course Code	MMA203T
Semester	I	Programs	MDC, MSE, MIT	

Sl. No	PART – A Instructions: Answer all questions.	M	CO	BT	
1	The joint distribution of two random variables X and Y is given by the following table <table><tr><td>X \ Y</td><td>0</td><td>1</td></tr><tr><td>0</td><td>0.1</td><td>0.2</td></tr><tr><td>1</td><td>0.4</td><td>0.2</td></tr><tr><td>2</td><td>0.1</td><td>0</td></tr></table>Find $P(X+Y \geq 1)$ and $P(XY > 1)$.	2	1	1	2
2	If the Joint distribution of two random variables X and Y is given by $P_{ij} = k(i+j),\ i = 1,2,3,4;\ j = 1,2,3,$ *then* $k =$ ____.	2	2	1	1
3	The joint distribution of two random variables X and Y is given by the following table <table><tr><td>X \ Y</td><td>2</td><td>3</td><td>4</td></tr><tr><td>1</td><td>0.06</td><td>0.15</td><td>0.09</td></tr><tr><td>2</td><td>0.14</td><td>0.35</td><td>0.21</td></tr></table>Find $P(X = 2 / Y = 3)$	2	1	1	2
4	If Eigen values of A are 1, 2, 4 then $trace(A^2) =$ ____ & $\lvert (A^{-1})^T \rvert =$ ____.	2	2	2	1
5	Compute X^TAX for the matrix $A = \begin{bmatrix} 3 & -2 \\ -2 & 7 \end{bmatrix}$ where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.	2	2	1	1
	PART - B				
1	The joint distribution of two random variables X and Y is given by the following table where X is scaled temperature and Y is difference in pressure (scaled) of a reactor. <table><tr><td>X \ Y</td><td>1</td><td>2</td></tr><tr><td>0</td><td>0.3</td><td>0.1</td></tr><tr><td>1</td><td>0.2</td><td>0.1</td></tr><tr><td>2</td><td>0.1</td><td>0.2</td></tr></table>Determine (i) Covariance and Correlation matrix of (X, Y)	10	2	2	8
2	Let $X \sim N(\mu, \Sigma),\ \mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix},\ \Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following i) $P[X_1 < 7]$ ii) $P[-3X_1 + 3X_3 > 80]$ iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$	10	2	2	8

3	Measurements in three characteristics are made on two individuals in a random sample from a population. The observation matrix is given as $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$. Using Singular Value Decomposition find the first singular value.	10	4	4
4	The joint distribution of two random variables X and Y is given by the following table: (see below)	10	3	3
5	The following table lists the weights and heights of five boys (see below)	10	4	4

4

Q4. The joint distribution of two random variables X and Y is given by the following table:

X \ Y	0	1
0	0.1	0.2
1	0.4	0.2
2	0.1	0

(a) Find $P(X+Y>1)$

(b) Determine the individual (marginal) probability distributions of X and Y and verify that X and Y are not independent.

(c) Compute $P(Y|X=2)$, $P(X|Y=1)$

8

Q5. The following table lists the weights and heights of five boys

Boy	1	2	3	4	5
Weight ([illegible])	120	125	125	135	145
Height (m)	61	60	64	68	72

Compute the sample covariance matrix. Also find the principal component of the data and the percentage of information captured by the first and second principal components.

8

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test Max Marks	5	20	5	20	-	25	5	20		

RV Educational Institutions
RV College of Engineering

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi

Approved by AICTE, New Delhi

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Academic year 2022-2023 (Odd Sem)

DEPARTMENT OF MATHEMATICS

CIE - II		**Maximum Marks**	**10+50**
Course Code	**21MAT11CT**	**Date: 26-04-2023**	
Sem	**I Semester**	**Time: 9:30 AM-11:30 AM**	
Linear Algebra and Probability (MDC, MIT, MSE)			

Sl. No.	Questions *Instructions to candidates:* Answer all questions from part A. Part A questions should be answered in the first two pages of the answer book only.	M	BT	CO
	PART-A			
1.	For X in R^3, Let $Q(x) = 5x_1^2 + 3x_2^2 + 2x_3^2 - x_1x_2 + 8x_2x_3$. Write this quadratic form as $X^T AX$.	2	2	1
2.	Classify the quadratic form $Q(x) = -5x_1^2 + 4x_1x_2 - 2x_2^2$.	2	2	1
3.	If the characteristic roots of the matrix A is -1, 5,4, then the characteristic roots of the matrix A^2+3A is __________.	2	2	2
4.	The set $\{u_1, u_2\}$, where $u_1 = (2,-5,1), u_2 = (4,-4,2)$ forms a basis for a subspace W. Using Gram Schmidt process, it can be produced to $\{v_1, v_2\}$ an orthogonal basis for W. Letting $v_1 = u_1, v_2$ can be obtained as $v_2 = (x, y, z)$ where $x =$ _______.	2	1	2
5.	Find a least square solution of the inconsistent system $Ax = b$ for $A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$	2	1	3
	PART-B			
1	The columns of matrix A represents Linearly independent vectors in R^4 Using Gram Schmidt process obtain the QR factorization of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.	10	2	1

Academic year 2022-2023 (Odd Sem)

2.	The electric currents and voltages in a tuned radio is given by the symmetric matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$. Factorize the matrix A in the form PDP^{-1}.	10	3	3
3.	A middle-aged man was stretched on a rack to lengths $L = 5, 6$ and 7 feet under applied forces of $F = 1, 2$ and 3 tons. Assuming Hooke's law $L = a + bF$, find his normal length 'a' by least squares.	10	3	3
4a	The following table lists the weights and heights of five boys: Boy: #1, #2, #3, #4, #5 Weight (lb): 120, 125, 125, 125, 145 Height (in.): 61, 60, 64, 68, 72 Find the covariance matrix for the data.	6	2	2
4b	Find the maximum value of the quadratic form, given $Q(x) = 3x_1^2 + 3x_2^2 + 4x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$ subject to the constraint $X^TX = 1$, and find a unit vector at which this maximum is attained.	4	2	2
5	The matrix of observation of a certain process is given by $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. Obtain the Singular Value Decomposition of the matrix A.	10	3	4

Table in question 4a:

Boy	#1	#2	#3	#4	#5
Weight (lb)	120	125	125	125	145
Height (in.)	61	60	64	68	72

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Quiz	Max Marks	4	4	2	-	4	6	-	-	-	
	Test		10	10	30	-	-	20	30	-	-	-

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Academic year 2022-2023 (Odd Sem)

DEPARTMENT OF MATHEMATICS

CIE - I		Maximum Marks	10+50
Course Code	21MAT11CT	Date: 27-03-2023	
Sem	I Semester	Time: 9:30 AM-11:30 AM	
Linear Algebra and Probability (MDC, MIT, MSE)			

Sl. No.	Questions *Instructions to candidates:* Answer all questions from part A. Part A questions should be answered in the first two pages of the answer book only.	M	BT	CO
	PART-A			
1.	For the equations $x + 2y = 2;\ x - y = 2$, the column picture is represented as ______.	2	2	1
2.	Given V(R) is vector space of all real valued functions defined on R, then determine the subset S = {family of all functions such that $f(0) = 1$} is a subspace.	1	2	1
3.	Given A is 7 X 9 matrix with rank of A is 5 then dim{N(A^T)} is ______.	1	2	2
4.	The value of K such that the polynomials $2 + kx + 4x^2, 1 - 3x + 2x^2, 2 - 4x - x^2$ are linearly dependent is ______.	2	1	2
5.	Let $T: R^2 \to R^2$ be the transformation defined by $T(x, y) = (x, 0)$. The null space of T is ______.	2	1	2
6.	Find the third column, so that the matrix $\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & - \\ 1/\sqrt{3} & 2/\sqrt{14} & - \\ 1/\sqrt{3} & -3/\sqrt{14} & - \end{bmatrix}$ is orthogonal.	2	2	3
	PART-B			
1a.	Show that the set $V = \{ a + b\sqrt{2} + c\sqrt{3} \ / \ a, b, c \in \mathbb{Q} \}$, over the field $\mathbb{Q}$ is a vector space under usual addition and scalar multiplication.	5	2	1

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1b.	Let W be a set of all vectors of the form $\begin{bmatrix} 3b \\ 2b-c \\ 2a+3b \\ a-2c \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$.	5	2	1
2.	Given $T(3,2,1) = (4,7,-2,12)$, $T(-1,2,3) = (-4,9,14,4)$, $T(2,-1,3) = (5,0,8,1)$. Obtain the linear transformation $T: R^3 \to R^4$. . Also find the range space, null space of the transformation and hence verify the rank-nullity theorem for the obtained transformation.	10	3	3
3.	Find the basis and dimension of the four fundamental subspaces of the matrix $A = \begin{pmatrix} 2 & -1 & 1 & 2 \\ -8 & 4 & -6 & -4 \\ 4 & -2 & 3 & 2 \end{pmatrix}$	10	3	3
4a	Determine the dimension and basis of the subspace spanned by $(1,-3,2,0)$, $(2,1,0,-1)$, $(4,8,-4,-3)$, $(1,10,-6,-2)$ in $\mathbb{R}^4$.	6	2	2
4b	Compute the orthogonal projection of $y = \begin{pmatrix} 5 \\ -9 \\ 5 \end{pmatrix}$ onto span of $\{u_1, u_2\}$ where $u_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}$ $u_2 = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$. Also find the distance from y to the plane in $\mathbb{R}^3$ spanned by u_1 and u_2.	4	2	2
5	Find an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$	10	3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
Marks Distribution	Quiz	Max Marks	3	5	2	-	4	6	-	-	-	
	Test		10	10	20	10	-	20	30	-	-	-

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(+2)

8-8
-4-(+4)
4+4

4-8

-12+8

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Academic year 2022-2023 (Odd Sem)

DEPARTMENT OF MATHEMATICS

IMPROVEMENT TEST		**Maximum Marks**	**10+50**
Course Code	**21MAT11CT**	**Date: 29-04-2023**	
Sem	**I Semester**	**Time: 9:30 AM-11:30 AM**	
Linear Algebra and Probability (MDC, MIT, MSE)			

Sl. No.	Questions	M	BT	CO
	PART-A			
1.	The joint distribution of two random variables X and Y is given by the following table	2	2	1
2.	The value of the constant k if $f(x,y) = \begin{cases} kxy, & 0<x<2, 1<y<2 \\ 0 & otherwise \end{cases}$ is a joint probability density function.	2	2	1
3.	If the matrix $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ is regular stochastic matrix for M^n, then n is equal to.........	2	2	2
4.	Fixed probability vector for A = $\begin{pmatrix} 0.7 & 0.3 \\ 0.8 & 0.2 \end{pmatrix}$ is ____.	2	1	2
5.	The joint distribution of two random variables X and Y is given by the following table	2	1	2

Table for Part-A Question 1:

X \ Y	0	1
0	0.1	0.2
1	0.4	0.2
2	0.1	0

Find $P(X+Y \geq 1)$ and $P(XY > 1)$.

Table for Part-A Question 5:

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Find $P(X = 2 \ / \ Y = 3)$

PART-B

Sl. No.	Questions	M	BT	CO
1	The joint distribution of two random variables X and Y is given by the following table where X is scaled temperature and Y is difference in pressure (scaled) of a reactor.	10	2	1
2.	Let $X \sim N(\mu, \Sigma)$, $\mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following	10	3	3

Table for Part-B Question 1:

Y \ X	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine (*i*) Marginal distributions of X and Y, (*ii*) Covariance of (X, Y), (*iii*) Correlation of X and Y *and* (*iv*) What is the relationship between the variables X and Y.

Computations for Part-B Question 2:

i) $P[X_1 < 7]$

ii) $P[-3X_1 + 3X_3 > 80]$

iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$

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3 The joint distribution of two random variables X and Y is given by the following table where X is scaled temperature and Y is difference in pressure (scaled) of a reactor. — 10 | 2 | 2

X \ Y	1	2
0	0.3	0.1
1	0.2	0.1
2	0.1	0.2

Determine (i), (ii) Covariance and Correlation matrix of (X,Y), (iii) $P(X\,/\,Y=2)$.

4 Suppose that the error in the reaction temperature X, in C and pressure Y for a controlled laboratory experiment are modelled as continuous random variables having the joint density function. — 10 | 2 | 2

$$f(x,y) = \begin{cases} \frac{1}{96}xy, & 0 < x < 4, 1 < y < 5 \\ 0, & elsewhere \end{cases}$$

Determine (i) $P(1 < x < 2, 2 < y < 3)$.
(ii) $P(x > 3, y \le 2)$
(iii) $P(y \le x)$,
(iv) $P(x + y \le 3)$.

5 Show that the stochastic matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is regular and hence find the fixed probability vector. — 10 | 3 | 4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

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Marks Distribution	Quiz	Max Marks	4	6	-	-	4	6	-	-	-	
	Test		10	20	10	10	-	20	30	-	-	-

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Multiply same matrix with itself
A × A