

Practice Questions: Independence and Conditional Independence

Concept Recap

- Independent Events: Two events A and B are independent if:

$$P(A \cap B) = P(A) * P(B)$$

- Conditionally Independent Events: Two events A and B are conditionally independent given C if:

$$P(A \cap B | C) = P(A | C) * P(B | C)$$

Practice Questions with Answers

Q1. Check for Independence

Let $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.15$.

Solution: $0.3 * 0.5 = 0.15 \rightarrow$ Yes, independent.

Q2. Conditional Independence Check

$P(A|C) = 0.6$, $P(B|C) = 0.3$, $P(A \cap B|C) = 0.18$

$$P(A \cap B|C) = P(A|C) \cdot P(B|C) = 0.6 \cdot 0.3 = 0.18$$

✅ **Answer:** Yes, A and B are conditionally independent given C.

\rightarrow Yes, conditionally independent.

Q3. Find Union Probability (Independent Events)

$P(A) = 0.4$, $P(B) = 0.7 \rightarrow$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B) = 0.4 * 0.7 = 0.28$$

$$P(A \cup B) = 0.4 + 0.7 - 0.28 = 0.82$$

Q4. Independence Violation Check

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.3 \rightarrow$

$0.5 * 0.4 = 0.2 \neq 0.3 \rightarrow$ Not independent

Q5. Conditional Independence Application

If A = 'Free' in email, B = 'suspicious link', and C = 'email is spam', then given C, A and B are independent \rightarrow simplifies Naive Bayes.

Given that an email is spam, the presence of the word "free" does not affect the probability of a suspicious link and vice versa.

✅ **Interpretation:** In spam emails, the features behave independently, simplifying modeling in Naive Bayes classifiers.

Practice Problems (Try Yourself)

Q6. Prove or Disprove Conditional Independence

$$P(A|C) = 0.4, P(B|C) = 0.2, P(A \cap B|C) = 0.1$$

Q7. Compute Union and Intersection

$$P(A) = 0.6, P(B) = 0.5, A \perp B \rightarrow \text{Find } P(A \cap B) \text{ and } P(A \cup B)$$

Q8. Real-World ML Scenario

A: symptom, B: test result, C: disease.

Given C, A and B are independent. What does this imply for diagnostics?