Probability Density Estimation & Maximum Likelihood Estimation

Practice Problems with Solutions

1. Conceptual Questions

1. What is the difference between a probability mass function (PMF) and a probability density function (PDF)?

Answer: PMF is used for discrete random variables and gives the probability that a discrete random variable is exactly equal to some value. PDF is used for continuous random variables and represents the relative likelihood that the variable falls within a particular range.

2. What assumptions does Maximum Likelihood Estimation (MLE) make about data?

Answer: MLE assumes that the data are independent and identically distributed (i.i.d) and that the underlying distribution form is known.

2. Short Problems

3. Given a sample x = [2.3, 2.1, 2.4, 2.2] from a normal distribution with unknown mean and known variance = 0.01, compute the MLE for .

Answer: MLE for is the sample mean: (2.3 + 2.1 + 2.4 + 2.2) / 4 = 2.25

4. Let X, X, ..., X be i.i.d. samples from an exponential distribution with PDF: f(x;) = e^{-x}. Derive the MLE for .

Answer: MLE for = 1 / (sample mean) = n / x

3. Application Problem

5. You are given the following dataset: [1.2, 0.9, 1.5, 1.1, 1.3]. Assuming it comes from a Gaussian distribution with unknown and , estimate and using MLE.

Answer: Sample mean = (1.2 + 0.9 + 1.5 + 1.1 + 1.3) / 5 = 1.2

Sample variance = [(1.2-1.2) + (0.9-1.2) + (1.5-1.2) + (1.1-1.2) + (1.3-1.2)] / 5 = 0.04