Continuous Random Variable- Continuous variable is a type of variable that can take on any value within a given range. Unlike discrete variables, which consist of distinct, separate values, continuous variables can represent an infinite number of possible values, including fractional and decimal values. Continuous variables often represent measurements or quantities.

Example of continuous variables are:

- Height: Height is a continuous variable because it can take on any value within a range (e.g., 150.5 cm, 162.3 cm, 175.9 cm).
- Weight: Weight is continuous because it can be measured with precision and can take on any value within a range (e.g., 55.3 kg, 68.7 kg, 72.1 kg).
- Time: Time can be measured with precision, and it can take on any value (e.g., 10:30:15.5 AM, 10:45:30.75 AM).
- Analysts denote a continuous random variable as X and its possible values as x, just like the discrete version. However, unlike discrete random variables, the chances of X taking on a specific value for continuous data is zero. In other words: $P(X = x) = 0$, where x is any specific value.
- Instead, probabilities greater than zero only exist for ranges of values, such as $P(a \leq X \leq b)$, where a and b are the lower and upper bounds of the range.

A probability density function (PDF) describes the probability distribution of a continuous random variable. These functions use a curve displaying probability densities, which are ranges of one unit.

Continuous random variables must satisfy the following:

- Probabilities for all ranges of X are greater than or equal to zero: $P(a \leq X \leq b) \geq 0$.
- The total area under the curve equals one: $P(-\infty \leq X \leq +\infty) = 1$.

Continuous Random Variable

Consider a generalized experiment rather than taking some particular experiment. Suppose that in your experiment, the outcome of this experiment can take values in some interval (a, b). That means that each and every single point in the interval can be taken up as the outcome values when you do the experiment. Hence, you do not have discrete values in this set of possible values but rather an interval.

Thus, X= {x: x belongs to (a, b)}

Example of a Continuous Random Variable

Some examples of Continuous Random Variable are:

- The height of an adult male or female.
- The weight of an object.
- The time is taken to complete a task.
- The temperature of a room.

• The speed of a vehicle on a highway.

Random Variable

Random variable is a fundamental concept in statistics that bridges the gap between theoretical probability and real-world data. random variable in statistics is a function that assigns a real value to an outcome in the sample space of a random experiment. For example: if you roll a die, you can assign a number to each possible outcome.

There are two basic types of random variables,

• Discrete Random Variables (which take on specific values)

• Continuous Random Variables (assume any value within a given range)

What is a Random Variable?

Random Variable Probability is a mathematical concept that assigns numerical values to outcomes of a sample space. They can describe the outcomes of objective randomness (like tossing a coin) or subjective randomness(results of a cricket game).

There are two types of Random Variables- Discrete and Continuous.

A random variable is considered a discrete random variable when it takes specific, or distinct values within an interval. Conversely, if it takes a continuous range of values, then it is classified as a continuous random variable.

Random Variable Definition

Random variable in statistics is a variable whose possible values are numerical outcomes of a random phenomenon. It is a function that assigns a real number to each outcome in the sample space of a random experiment. We define a random variable as a function that maps from the sample space of an experiment to the real numbers. Mathematically, Random Variable is expressed as,

$X: S \rightarrow R$

where,

• X is Random Variable (It is usually denoted using capital letter)

• S is Sample Space

• R is Set of Real Numbers

Random Variable Example

Example 1

If two unbiased coins are tossed then find the random variable associated with that event.

Solution:

Suppose Two (unbiased) coins are tossed

X = number of heads. [X is a random variable or function]

Here, the sample space S = {HH, HT, TH, TT}

Example 2

Suppose a random variable X takes m different values i.e. sample space

X = {x1, x2, x3………xm} with probabilities

P(X = xi) = pi

where $1 \leq i \leq m$

The probabilities must satisfy the following conditions :

- $0 \leq pi \leq 1$; where $1 \leq i \leq m$
- p1 + p2 + p3 + ……. + pm = 1 Or we can say $0 \leq pi \leq 1$ and Σpi = 1

Hence possible values for random variable X are 0, 1, 2.

X = {0, 1, 2} where m = 3

- P(X = 0) = (Probability that number of heads is 0) = P(TT) = 1/2×1/2 = 1/4
- P(X = 1) = (Probability that number of heads is 1) = P(HT | TH) = 1/2×1/2 + 1/2×1/2 = 1/2
- P(X = 2) = (Probability that number of heads is 2) = P(HH) = 1/2×1/2 = 1/4

Here, you can observe that, ($0 \leq$ p1, p2, p3 $\leq 1/2$)

p1 + p2 + p3 = 1/4 + 2/4 + 1/4 = 1

For example,

Suppose a dice is thrown (X = outcome of the dice). Here, the sample space S = {1, 2, 3, 4, 5, 6}.

The output of the function will be:

- P(X=1) = 1/6
- P(X=2) = 1/6
- P(X=3) = 1/6
- P(X=4) = 1/6
- P(X=5) = 1/6
- P(X=6) = 1/6

Continuous Random Variable

Continuous Random Variable takes on an infinite number of values. The probability function associated with it is said to be PDF (Probability Density Function).

PDF (Probability Density Function)

If X is a continuous random variable. P (x < X < x + dx) = f(x)dx then,

• $0 \leq f(x) \leq 1$; for all x

• $\int f(x)\,dx = 1$ over all values of x

Then P (X) is said to be a PDF of the distribution.

Continuous Random Variables Example

Find the value of $P(1 < X < 2)$

Such that,

• $f(x) = kx3; 0 \leq x \leq 3 = 0$

Otherwise f(x) is a density function.

Solution:

If a function f is said to be a density function, then the sum of all probabilities is equal to 1.

Since it is a continuous random variable Integral value is 1 overall sample space s.

$\int f(x)\,dx = 1$

$\int kx3\,dx = 1$

$K[x4]/4 = 1$

Given interval, $0 \leq x \leq 3 = 0$

$K[34 – 04]/4 = 1$

$K(81/4) = 1$

$K = 4/81$

Thus,

$P(1 < X < 2) = k\times[X4]/4$

$P = 4/81\times[16-1]/4$

$P = 15/81$

Random Variable Formulas

There are two main random variable formulas,

• Mean of Random Variable

• Variance of Random Variable

Let’s learn about the same in detail,

Mean of Random Variable

For any random variable X where P is its respective probability we define its mean as, Mean(μ) = Σ X.P

where,

- X is the random variable that consist of all possible values.
- P is the probability of respective variables

Variance of Random Variable

The variance of a random variable tells us how the random variable is spread about the mean value of the random variable. Variance of Random Variable is calculated using the formula,

Var(x) = σ2 = E(X2) – {E(X)}2

where,

- E(X2) = ΣX2P
- E(X) = ΣXP

Random Variable Functions

For any random variable X if it assume the values x1, x2,…xn where the probability corresponding to each random variable is P(x1), P(x2),…P(xn), then the expected value of the variable is,

Expectation of X, E(x) = Σ x.P(x)

Now for any new random variable Y in which the random variable X is its input, i.e. Y = f(X), then the cumulative distribution function of Y is,

Fy(Y) = P(g(X) ≤ y)

Random Variable Example with Solutions

Here are some of the solved examples on Random variable. Learn random variables by practicing these solved examples.

1. Find the mean value for the continuous random variable, f(x) = x2, 1 ≤ x ≤ 3

Solution:

Given,

f(x) = x2

1 ≤ x ≤ 3

E(x) = ∫3 1 x.f(x)dx

E(x) = ∫31 x.x2.dx

E(x) = ∫31 x3.dx

$E(x) = [x4/4]31$

$E(x) = 1/4\{34– 14\} = 1/4\{81 – 1\}$

$E(x) = 1/4\{80\} = 20$

2. Find the mean value for the continuous random variable, $f(x) = ex, 1 \leq x \leq 3$

Solution:

Given,

$f(x) = ex\ 1 \leq x \leq 3$

$E(x) = \int 3$

$1\ x.f(x)dx$

$E(x) = \int 31\ x.ex.dx$

$E(x) = [x.ex – ex]31$

$E(x) = [ex(x – 1)]31$

$E(x) = e3(2) – e(0)$

3. Find the mean value for the continuous random variable, $f(x) = x2, 1 \leq x \leq 3$

Solution :

f(x) = x^2, $1 \leq x \leq 3$

Mean = ∫(x^3)dx / ∫(x^2)dx from 1 to 3

$= [(1/4)x^4]_1^3 / [(1/3)x^3]_1^3$

= (81/4 – 1/4) / (27/3 – 1/3)

= 20 / 26/3

= 20 * 3/26 = 60/26 ≈ 2.31

4. Find the mean value for the continuous random variable, $f(x) = 2x + 1, 0 \leq x \leq 4$

Solution :

$f(x) = 2x + 1, 0 \leq x \leq 4$

Mean = ∫(x(2x+1))dx / ∫(2x+1)dx from 0 to 4

$= [(2/3)x^3 + (1/2)x^2]_0^4 / [x^2 + x]_0^4$

= (170/3) / 20

= 17/6 ≈ 2.83

5. Find the mean value for the continuous random variable, $f(x) = x3, -1 \leq x \leq 2$

Solution :

$f(x) = x3, -1 \leq x \leq 2$

Mean = ∫(x^4)dx / ∫(x^3)dx from -1 to 2

$= [(1/5)x^5]_{-1}^{2} / [(1/4)x^4]_{-1}^{2}$

= (32/5 + 1/5) / (16/4 + 1/4)

= 33/5 / 17/4

= 334 / (517) ≈ 1.55

6. Find the mean value for the continuous random variable, $f(x) = \sqrt{x}, 1 \leq x \leq 9$

Solution :

$f(x) = \sqrt{x}, 1 \leq x \leq 9$

Mean = ∫(x√x)dx / ∫(√x)dx from 1 to 9

$= [(2/5)x^{(5/2)}]_{1}^{9} / [(2/3)x^{(3/2)}]_{1}^{9}$

= (486 – 2/5) / (18 – 2/3)

≈ 5.2

7. Find the mean value for the continuous random variable, $f(x) = 3x2 – 2x, 0 \leq x \leq 3$

Solution :

$f(x) = 3x^2 – 2x, 0 \leq x \leq 3$

Mean = ∫(x(3x^2-2x))dx / ∫(3x^2-2x)dx from 0 to 3

$= [(3/4)x^4 – (2/3)x^3]_{0}^{3} / [x^3 – x^2]_{0}^{3}$

= (81/4 – 18) / (27 – 9)

= 27/4 / 18 = = 27/72 = 3/8 ≈ 0.375

8. Find the mean value for the continuous random variable, $f(x) = \sin(x), 0 \leq x \leq \pi$

Solution :

$f(x) = \sin(x), 0 \leq x \leq \pi$

Mean = ∫(x sin(x))dx / ∫(sin(x))dx from 0 to π

$= [-x \cos(x) + \sin(x)]_{0}^{pi} / [-\cos(x)]_{0}^{pi}$

= (π + 1) / 2

≈ 2.07

9. Find the mean value for the continuous random variable, $f(x) = ex, 0 \leq x \leq 2$

Solution :

$f(x) = e^x, 0 \leq x \leq 2$

Mean = $\int(x\, e^x)dx / \int(e^x)dx$ from 0 to 2

$= [x\, e^x - e^x]_0^{\,2} / [e^x]_0^{\,2}$

$= (2e^2 - e^2 + 1) / (e^2 - 1)$

≈ 1.54

10. Find the mean value for the continuous random variable, $f(x) = \ln(x), 1 \leq x \leq e$

Solution :

$f(x) = \ln(x), 1 \leq x \leq e$

Mean = $\int(x \ln(x))dx / \int(\ln(x))dx$ from 1 to e

$= [(1/4)x^2(2\ln(x)-1)]_1^{\,e} / [x \ln(x) - x]_1^{\,e}$

$= (e^2/2 - 1/4) / (e - 1)$

≈ 1.95