Mathematical Conceptualization of Starlings

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This document is a Mathematical Modelling of the flocking of Birds.Flocking behavior is the behavior exhibited when a group of birds, called a flock, are foraging or in flight. There are parallels with the shoaling behavior of fish, the swarming behavior of insects, and herd behavior of land animals.

1 INTRODUCTION

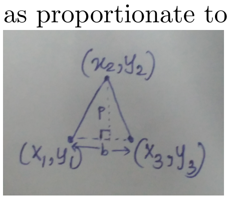
Flocking behavior is controlled by three simple rules:

Separation - Avoid crowding neighbors (short range repulsion)
Alignment - Steer towards average heading of neighbors
Cohesion - Steer towards average position of neighbors (long range attraction)

ASSUMPTIONS :-
1. The paper assumes flocking in a 2D world, the same can be extended to 3D by incorporating/involving the z-component of the properties.
2.The mass of each bird is assumed to be same,this can be made different by relating assuming mass proportional to a physical property of the symbol with which each bird is represented,such as area of the triangle etc. .
3. No other force would effect the motion of the birds.

2 Modelling of the birds

Each bird is represented by two concurrent lines (Similar to letter A without the sleeping line or water image of letter V). The mass of the bird can be considered as proportionate to area in the closed figure formed by bird.



Consider a bird in the flock. Let the initial coordinates of the bird be $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and the initial velocity be v_i and the final velocity be (v_{fx}, v_{fy}) then the new coordinates will be

$$x'_1 = x_2 - pcos\theta - bsin\theta$$

$$y'_1 = y_2 - psin\theta + bcos\theta$$

$$x'_2 = x_2 y'_2 = y_2$$

$$x'_3 = x_2 - pcos\theta + bsin\theta$$

$$y'_3 = y_2 - psin\theta - bcos\theta$$

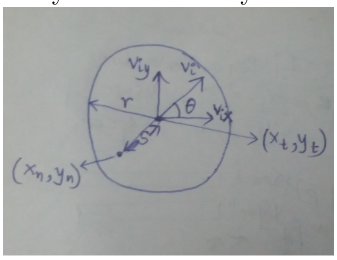
Where $tan\theta = v_{fy}/v_{fx}$.

3 Modelling of the Rules

Consider the following factosr effective only in local of a bird.Where 'local of a bird' can be understood as a circle of a radius r with the bird at its centre. This bird at the centre is referred as CentreBird.

3.1 Seperation

A bird must try to a move away birds in the local to avoid the overcrowding of the birds.



Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the cordinates of nearset bird in the local is (x_n, y_n)

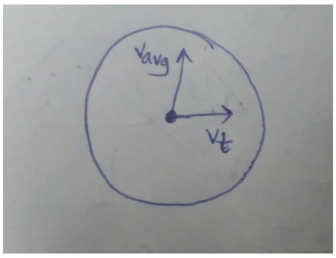
$$v_{final} = v_t + v_{add}$$

$$v_{add} = (1/1000) * (r/s - 1)cos\theta\hat{i} + (1/1000) * (r/s - 1)sin\theta\hat{j}$$

$$tan\theta = (x_t - x_n)/(y_t - y_n)$$

3.2 Alignment

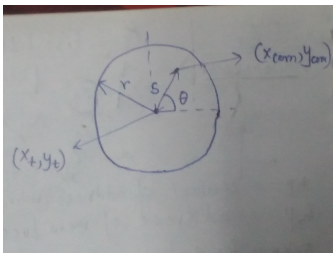
A bird must move in the direction of the average velocity of the birds in the local.



Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the average velocity of the birds in the local is v_{avg}

$$v_{final} = (v_t + v_{avg})/10$$

3.3 Cohesion



A bird must try to move towards the COM of the local birds.
Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the cordinates of COM of the birds in the local is (x_{com}, y_{com})

$$v_{final} = v_t + v_{add}$$

$$v_{add} = (1/100) * (s/r)cos\theta\hat{i} + (1/100) * (s/r)sin\theta\hat{j}$$

$$tan\theta = (x_t - x_{com})/(y_t - y_{com})$$

4 Calciulating the Physical Properties

4.1 Average Energy spent by each Bird

Let E be the total Energy of the birds in the flock. $E = (1/2)*m(\sum_{n=1}^{/birdCount} {v_i}^2/birdCount)$ where v_i is the veloicity of i^{th} and birCount is the total number of birds in the environment .

4.2 Angular Momentum of a Bird

Consider a bird in the flock and let I be the Moment of Inertia and ω be the angular velocity of the bird at any given time then we have

$$\omega = \frac{d\theta}{dt}$$

$$= \frac{\delta\theta}{\delta t}$$

$$= \frac{\theta_{final} - \theta_{initial}}{\delta t}$$

$$tan\theta_{final} = \frac{v_{fy}}{v_{fx}} tan\theta_{initial} = \frac{v_{iy}}{v_{ix}}$$

$$AngularMomentum = I\omega$$

$$= I * (\theta final - \theta_{initial})/t$$

4.3 Force on each Bird

Consider a bird and let it's initial velocity bev_i and the final velocity be v_f and the angle between the two velocties be θ then the change on magnitude of the velocity of the bird is

$$\delta v = |v_f - v_i|$$

$$= \sqrt{v_i^2 + v_f^2 + 2v_i v_f cos\theta}$$

Now the force F on the bird is $m\frac{dv}{dt}$

$$= m\frac{\delta v}{\delta t}$$

$$= m\frac{\sqrt{v_i^2 + v_f^2 + 2v_i v_f cos\theta}}{\delta t}$$