Mathematical Conceptualization of Starlings

Shashwat Banchhor , Himanshu Rewar

April 15, 2018

This document is a Mathematical Modelling of the flocking of Birds.Flocking behavior is the behavior exhibited when a group of birds, called a flock, are foraging or in flight. There are parallels with the shoaling behavior of fish, the swarming behavior of insects, and herd behavior of land animals.

1 INTRODUCTION

Flocking behavior is controlled by three simple rules:

Separation - Avoid crowding neighbors (short range repulsion)
Alignment - Steer towards average heading of neighbors
Cohesion - Steer towards average position of neighbors (long range attraction)

ASSUMPTIONS :-
1. The paper assumes flocking in a 2D world, the same can be extended to 3D by incorporating/involving the z-component of the properties.
2.The mass of each bird is assumed to be same,this can be made different by relating assuming mass proportional to a physical property of the symbol with which each bird is represented,such as area of the triangle etc. .
3. No other force would effect the motion of the birds.

2 Modelling of the birds

Each bird is represented by two concurrent lines (Similar to letter A without the sleeping line or water image of letter V). The mass of the bird can be considered as proportionate to area in the closed figure formed by bird. Each bird has 3 points in its representation $(x_1, y_1), (x_2, y_2), (x_3, y_3)$
Consider a bird in the flock. Let the initial coordinates of the bird be $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and the initial velocity be v_i and the final velocity be (v_{fx}, v_{fy}) then the new coordinates will be

$$x'_1 = x_2 - pcos\theta - bsin\theta$$

$$y'_1 = y_2 - psin\theta + bcos\theta$$

$$x'_2 = x_2 y'_2 = y_2$$

$$x'_3 = x_2 - pcos\theta + bsin\theta$$

$$y'_3 = y_2 - psin\theta - bcos\theta$$

Where $tan\theta = v_{fy}/v_{fx}$.

3 Modelling of the Rules

Consider the following factosr effective only in local of a bird.Where 'local of a bird' can be understood as a circle of a radius r with the bird at its centre. This bird at the centre is referred as CentreBird.

3.1 Seperation

A bird must try to a move away birds in the local to avoid the overcrowding of the birds.

Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the cordinates of nearset bird in the local is (x_n, y_n)

$$v_{final} = v_t + v_{add}$$

$$v_{add} = (1/1000) * (r/s - 1)cos\theta\hat{i} + (1/1000) * (r/s - 1)sin\theta\hat{j}$$

$$tan\theta = (x_t - x_n)/(y_t - y_n)$$

3.2 Alignment

A bird must move in the direction of the average velocity of the birds in the local.

Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the average velocity of the birds in the local is v_{avg}

$$v_{final} = (v_t + v_{avg})/10$$

3.3 Cohesion

A bird must try to move towards the COM of the local birds.

Let the initial cordinates of the CentreBird be (x_t, y_t) and its velocity be (v_t) and the cordinates of COM of the birds in the local is (x_{com}, y_{com})

$$v_{final} = v_t + v_{add}$$

$$v_{add} = (1/100) * (s/r)cos\theta\hat{i} + (1/100) * (s/r)sin\theta\hat{j}$$

$$tan\theta = (x_t - x_{com})/(y_t - y_{com})$$

4 Calciulating the Physical Properties

4.1 Average Energy spent by each Bird

Let E be the total Energy of the birds in the flock. $E = (1/2) * m(\sum_{n=1}^{/birdCount} {v_i}^2/birdCount)$ where v_i is the veloicity of i^{th} and birCount is the total number of birds in the environment .

4.2 Angular Momentum of a Bird

Consider a bird in the flock and let I be the Moment of Inertia and ω be the angular velocity of the bird at any given time then we have

$$\omega = \frac{d\theta}{dt}$$

$$= \frac{\delta\theta}{\delta t}$$

$$= \frac{\theta_{final} - \theta_{initial}}{\delta t}$$

$$tan\theta_{final} = \frac{v_{fy}}{v_{fx}} tan\theta_{initial} = \frac{v_{iy}}{v_{ix}}$$

$$AngularMomentum = I\omega$$

$$= I * (\theta final - \theta_{initial})/t$$

4.3 Force on each Bird

Consider a bird and let it's initial velocity bev_i and the final velocity be v_f and the angle between the two velocties be θ then the change on magnitude of the velocity of the bird is

$$\delta v = |v_f - v_i|$$

$$= \sqrt{v_i^2 + v_f^2 + 2v_i v_f cos\theta}$$

Now the force F on the bird is $m\frac{dv}{dt}$

$$= m\frac{\delta v}{\delta t}$$

$$= m\frac{\sqrt{v_i^2 + v_f^2 + 2v_i v_f cos\theta}}{\delta t}$$