* Simplex Method :

Note : ① If problem is of Maximisation type then make All $C_j - Z_j \leq 0$

② If probem is of Minimisation type then make All $C_j - Z_j \geq 0$

Example ① solve the following LPP using simplex method

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{subject to} \quad 3x_1 + 2x_2 \leq 18$$

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Solution: first we convert given LPP into standard form

we introduce the slack variables S_1, S_2, S_3

∴ The standard form of LPP is

$$\text{Maximize } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3$$

$$\text{Subject to} \quad 3x_1 + 2x_2 + S_1 + 0S_2 + 0S_3 = 18$$

$$x_1 + 0x_2 + 0S_1 + S_2 + 0S_3 = 4$$

$$0x_1 + x_2 + 0S_1 + 0S_2 + S_3 = 6$$

$$\text{All } x_1, x_2, S_1, S_2, S_3 \geq 0$$

* Initial iteration :

C_B	C_j	3	2	0	0	0	Solution	Ratio
	Basic variable	x_1	x_2	s_1	s_2	s_3		
0	s_1	3	2	1	0	0	18	$\frac{18}{3} = 6$
0	s_2	1	0	0	1	0	4	$\frac{4}{1} = 4$ ← min key row
0	s_3	0	1	0	0	1	6	-
	$Z_j = \sum C_B B_j$	0	0	0	0	0		
	$C_j - Z_j$	3	2	0	0	0		

↑ Max. (key column)

here, 1 is pivot element

* First iteration : (s_2 - outgoing , x_1 - incoming)

	C_B	C_j	3	2	0	0	0	Solution	Ratio
		Basic variable	x_1	x_2	s_1	s_2	s_3		
$R_1 - 3R_2$	0	s_1	0	2	1	-3	0	6	$\frac{6}{2} = 3$ ← Min (key row)
	3	x_1	1	0	0	1	0	4	-
	0	s_3	0	1	0	0	1	6	$\frac{6}{1} = 6$
		Z_j	3	0	0	3	0		
		$C_j - Z_j$	0	2	0	-3	0		

↑ Max (key column

∴ 2 is pivot element.

* Second Iteration :

(s_1 - outgoing, x_2 - incoming)

	C_j	3	2	0	0	0		
C_B	Basic Variable	x_1	x_2	S_1	S_2	S_3	solution	Ratio
2	x_2	0	1	$\frac{1}{2}$	$-\frac{3}{2}$	0	3	
3	x_1	1	0	0	1	0	4	
$R_3 - \frac{1}{2}R_1$ 0	S_3	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	1	3	
	z_j	3	2	1	0	0		
	$c_j - z_j$	0	0	-1	0	0		

here we can observe that

All $z_j - z_j \leq 0$

therefore,

$x_1 = 4$, $x_2 = 3$ is solution

and $Z = 3x_1 + 2x_2$

$= 3(4) + 2(3)$

$= 18$

$\therefore$ $\boxed{x_1 = 4, \; x_2 = 3 \text{ and } Z_{max} = 18}$

is Required solution of LPP

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Ex 2: solve the following L.P.P using simplex method.

Maximise $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \le 2$

$x_1 + 4x_3 \le 4$

$x_1, x_2, x_3 \geq 0$

Solution: first we convert given LPP into standard form

∴ we introduce the two slack variable s_1, s_2

∴ standard form of LPP is

Maximize $Z = 6x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2$

subject to $2x_1 - x_2 + 2x_3 + s_1 + 0s_2 = 2$

$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

* Initial iteration:

C_B	C_j	6	-2	3	0	0		
	Basic variable	x_1	x_2	x_3	s_1	s_2	Solution	Ratio
0	s_1	2	-1	2	1	0	2	$\frac{2}{2} = 1$ ← Min
0	s_2	1	0	4	0	1	4	$\frac{4}{1} = 4$
	z_j	0	0	0	0	0		
	$C_j - z_j$	6	-2	3	0	0		

↑ Maxi (key column)

∴ 2 is pivot element

* first Iteration: (s_1 - outgoing , x_1 - incoming)

C_B	C_j	6	-2	3	0	0	Solution	Ratio
	Basic variable	x_1	x_2	x_3	s_1	s_2		
$\frac{R_1}{2}$ 6	x_1	1	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	1	$\frac{1}{-\frac{1}{2}} = -2$
$R_2 - \frac{1}{2}R_1$ 0	s_2	0	$\frac{1}{2}$	3	$-\frac{1}{2}$	1	3	$\frac{3}{\frac{1}{2}} = 6$ ← Min (key row)
	Z_j	6	-3	6	3	0		
	$C_j - Z_j$	0	1	-3	-3	0		

↑ Max (key column)

∴ $\frac{1}{2}$ is pivot element

* second iteration:

(S_2 - outgoing , x_2 - incoming)

C_B	C_j	6	-2	3	0	0	Solution	Ratio.
	Basic variable	x_1	x_2	x_3	s_1	s_2		
$R_1 - R_2$ 6	x_1	1	0	4	0	1	4	
$R_2 \times 2$ -2	x_2	0	1	6	-1	2	6	
	Z_j	6	-2	12	2	10		
	$C_j - Z_j$	0	0	-9	-2	-10		

here, All $C_j - Z_j \leq 0$

∴ solution is $x_1 = 4$, $x_2 = 6$, $x_3 = 0$

$$\therefore Z = 6x_1 - 2x_2 + 3x_3$$
$$= 6(4) - 2(6) + 3(0)$$
$$= 12$$

∴ optimal solution is $x_1 = 4$, $x_2 = 6$, $x_3 = 0$, $Z_{max} = 12$