Q) Find the eigen values & eigen vectors of adj A.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

(i) adj A → 6, 2, 3

(ii) adj (adj A) → $\frac{6\times2\times3}{6}$ = 6, 18, 12

10/2/23

Caley Hamilton theorem - Every square matrix satisfies its characteristic equation.

(expected ki manually kiya hai)

ex. ① Verify Caley-Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and find A^{-1}, A^4

→

characteristic eqⁿ of A is,

$\lambda^3 - 0\lambda^2 + (2-24+2)\lambda - |A| = 0$

$\lambda^3 - 20\lambda + 8 = 0$

$1(-12-12)$
$-1(-4-8)$
$+(-4+6)$

By CHT,

$A^3 - 20A + 8I = 0$

$A = [\ \ ]$

$A^2 = [\ \ ][\ \ ]$

$A^3 = A^2 \cdot A$

$A^3 - 20A + 8I$

$[\ \ ] - 20[\ \ ] + 8[\ \ ]$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ CHT is verified.

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(i) $A^3 - 20A + 8I = 0$

$A^{-1}(A^3 - 20A + 8I) = 0$

$A^2 - 20I + 8A^{-1} = 0$

$8A^{-1} = -A^2 + 20I$

$= \begin{bmatrix} 24 & 8 & 12 \\ -10 & -2 & -6 \\ -2 & -2 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{8}\begin{bmatrix} 24 & 8 & 12 \\ -10 & -2 & -6 \\ -2 & -2 & -2 \end{bmatrix}$

(ii) $A^3 - 20A + 8I = 0 \times A$

$A^4 - 20A^2 + 8A = 0$

$A^4 = 20A^2 - 8A$

$= \begin{bmatrix} -88 & -168 & -264 \\ 192 & 416 & 144 \\ 56 & 72 & 472 \end{bmatrix}$

(Q) Find $A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$

where $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$.

Humesha Q me mentioned nahi hoga to decide karna

Characteristic eq^n of A is $|A - \lambda I| = 0$.

$\Rightarrow \lambda^3 - 4\lambda^2 + (-10 - 4 - 6)\lambda - 35I = 0$

$\lambda^3 - 4\lambda^2 - 20\lambda - 35I = 0 \rightarrow$ ch. eq^n — (1)

Let $f(A) = A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$

$(2-6)$
$3(4-3)$
$+7(8-2)$
$-4-3+42$

$$A^4 + A$$

$$A^3 - 4A^2 - 20A - 35I \;\big)\; A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$$

$$\cancel{A^7} - \cancel{4A^6} - \cancel{20A^5} - 35A^4$$

$$- \quad + \quad + \quad +$$

$$A^4 - 4A^3 - 20A^2 - 33A + I$$

$$\cancel{A^4} - \cancel{4A^3} - \cancel{20A^2} - 35A$$

$$+$$

$$2A + I$$

$$f(A) = 0 \times (A^4 + A) + 2A + I \quad (\text{from } 1)$$

$$= 2A + I.$$

$$f(A) = \begin{bmatrix} 2 & 6 & 14 \\ 8 & 4 & 6 \\ 2 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 14 \\ 8 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix}$$

13/2/23

Diagonalization of a matrix

Def: Two matrices A and B are called similar to each other if and only if there exists a non-singular matrix m such that $m^{-1} A m = B$ or $A = m B m^{-1}$

$\underbrace{\hphantom{xxxxxx}}_{\text{same}}$

Note. - If A and B are similar and B and C are similar then A and C are also similar.

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★ Defⁿ – A matrix 'A' is said to be diagonalizable if 'A' is similar to a diagonal matrix i.e there exist a matrix 'M' such that $M^{-1}AM = D$ where 'D' is a diagonal matrix.

Note – If A and B are diagonalizable and have same eigen values then they are similar.

★ Theorem – Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of a $n \times n$ matrix 'A' and $x_1, x_2, \ldots, x_n$ are corresponding n linearly independent eigen vectors then for

$$M = (x_1, x_2, \ldots, x_n);$$

$$M^{-1}AM = D, \quad \text{where } D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

★ The matrix M is called modal matrix or diagonalizing matrix and matrix D is called spectral matrix or diagonal matrix of A.

★ NOTE

1) A $n \times n$ matrix 'A' is diagonalizable if and only if A has ~~one~~ linearly independent eigen vectors.
2) If a $n \times n$ matrix 'A' has n distinct type in values then it is always ~~dig~~ diagonalizable.

→ To get spectral matrix eigen values are enough first we should know that if matrix is diagonalizable.

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(a) - Is the given matrix diagonalizable?
- Find one 'M'. (modal) (Tabhi hi eigen vectors nikalenge).
Find spectral matrix (eigen vectors nahi lagte ise).

(Q) $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

$\frac{2 \pm \sqrt{4+44}}{2}$

$\frac{2 \pm \sqrt{48}}{2} = 2 \pm 4\sqrt{3}$

$1 \pm \sqrt{12}$

$\lambda^3 - (1+4-3)\lambda^2 + (4-\cancel{2}-3)\lambda = 0$

$1(-12+12)$
$6(0) - 4(0)$

$\lambda^3 - 2\lambda^2 + \lambda = 0$

$\lambda(\lambda^2 - 2\lambda + 1) = 0$

$\lambda(\lambda-1)(\lambda-1)$

$\lambda = 0; 1, 1$

For 3X3 we need 3 linearly independent eigen vectors → diagona.

For $\lambda = 1$,

$(A - \lambda I)X = 0$

$(A - I)X = 0.$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 3 & 2 \\ 0 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_2 + 2x_3 = 0$

$x_3 = \frac{-3x_2}{2}$

(independent)

Kitne eigen values aayenge wo nikalna yaad rakhna Isk

conclusion - There are 2 linearly independent eigen vectors.

→ A is diagonalizable.

Spectral matrix / Diagonal matrix of A is,

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$

For $\lambda = 0$,

$$(A - 0I)x = 0$$

$$\begin{bmatrix} 1 & 6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 6x_2 - 4x_3 = 0, \quad 4x_2 + 2x_3 = 0$$

$$x_3 = -2x_2, \quad x_1 = -2x_2$$

$$\cancel{X}_3 = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$$

$\therefore$ Modal matrix or diagonalizing matrix for A is

$$M = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 0 & -3 & -2 \end{bmatrix}$$

D me jis order me eigen values ~~data hai usi~~ order me eigen vectors M me dalne hai.

(Q) $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\lambda^3 - 5\lambda^2 + (4+2+2)\lambda - 4 = 0$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$2(2)$
$-3(0)$
$+4$

$$\lambda = 2, 2, 1 \rightarrow \text{Since } \Delta\text{gular matrix.}$$

$\lambda = 2$,

$$(A - 2I)X = 0$$

$$\begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$3x_2 + 4x_3 = 0$$
$$x_3 = 0$$
$$x_2 = 0$$

Conclusion.

There is only 'one' linearly independent ~~value~~ eigen vector.

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Cond^n:- A is not diagonalizable.

Def^n - (1) Algebraic multiplicity of an eigen value: No. of times the eigen value is repeated.

(2) Geometric multiplicity: No of linearly independent eigen vectors possible for the eigen values.

Note / Result - A matrix A is diagonalizable if and only if for every eigen values of A the algebraic multiplicity is equal to geometric multiplicity.

λ	AM	GM
2	2	1
1	1	1

A is not diagonalizable. (At $\lambda = 2$, AM $\neq$ GM).