Ex ③ Solve the following LPP using simplex method

Minimise $z = x_1 - 3x_2 + x_3$

Subject
$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 + 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

* Duality : The phenomenon occuring in lineal programming that given a problem there exists another closely related problem with the same set of data and with the same solution is called the duality.

* Working Rule to write duel from primal

1> convert the inequality of All constraints to either '$\geq$' or '$\leq$' (by using (-1))

2> if primal is of minimisation type then convert it to maximisation and vice versa.

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Ex ① write the dual of following LPP.

Maximise $z = 2x_1 - x_2 + 4x_3$

subject to $x_1 + 2x_2 - x_3 \le 5$

$2x_1 - x_2 + x_3 \le 6$

$x_1 + x_2 + 3x_3 \le 10$

$4x_1 + x_3 \le 12$

$x_1, x_2, x_3 \ge 0$

Solution: Given LPP can be written as

Maximise $z = 2x_1 - x_2 + 4x_3$

subject to $x_1 + 2x_2 - x_3 \le 5$

$2x_1 - x_2 + x_3 \le 6$

$x_1 + x_2 + 3x_3 \le 10$

$4x_1 + 0x_2 + x_3 \le 12$

$x_1, x_2, x_3 \ge 0$

Therefore the dual of given LPP is

Minimise $z = 5y_1 + 6y_2 + 10y_3 + 12y_4$

subject to $y_1 + 2y_2 + y_3 + 4y_4 \ge 2$

$2y_1 - y_2 + y_3 + 0y_4 \ge -1$

$-y_1 + y_2 + 3y_3 + y_4 \ge 4$

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Ex② obtain the dual of the following LPP

Minimise $z = 3x_1 - 2x_2 + x_3$

subject to
$$2x_1 - 3x_2 + x_3 \le 5$$
$$4x_1 - 2x_2 \ge 9$$
$$-8x_1 + 4x_2 + 3x_3 = 8$$
$$x_1, x_2 \ge 0, \quad x_3 \text{ unrestricted.}$$

Solution: given LPP can be written as

Minimise $z = 3x_1 - 2x_2 + x_3$

subject to
$$-2x_1 + 3x_2 - x_3 \ge -5$$
$$4x_1 - 2x_2 + 0x_3 \ge 9$$
$$-8x_1 + 4x_2 + 3x_3 \ge 8$$
$$8x_1 - 4x_2 - 3x_3 \ge -8$$
$$x_1, x_2 \ge 0, \quad x_3 \text{ unrestricted}$$

since, x_3 is unrestricted

$\therefore$ we put $x_3 = x_3' - x_3''$, $x_3' \ge 0$, $x_3'' \ge 0$

$\therefore$ minimise $z = 3x_1 - 2x_2 + x_3' - x_3''$

subject to
$$-2x_1 + 3x_2 - x_3' + x_3'' \ge -5$$
$$4x_1 - 2x_2 + 0x_3' - 0x_3'' \ge 9$$
$$-8x_1 + 4x_2 + 3x_3' - 3x_3'' \ge 8$$
$$8x_1 - 4x_2 - 3x_3' + 3x_3'' \ge -8$$

with All $x_1, x_2, x_3', x_3'' \ge 0$

$\therefore$ The Dual of given LPP is

Maximise $z = -5y_1 + 9y_2 + 8y_3' - 8y_3''$

subject to
$$-2y_1 + 4y_2 - 8y_3' + 8y_3'' \le 3$$
$$3y_1 - 2y_2 + 4y_3' - 4y_3'' \le -2$$
$$-y_1 + 0y_2 + 3y_3' - 3y_3'' \le 1$$
$$y_1 + 0y_2 - 3y_3' + 3y_3'' \le -1$$

All $y_1, y_2, y_3', y'' \ge 0$

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It can be written as (by replacing $y_3' - y_3''$ by y_3)

Maximise $w = -5y_1 + 9y_2 + 8y_3$

subject to $-2y_1 + 4y_2 - 8y_3 \leq 3$

$3y_1 - 2y_2 + 4y_3 \leq -2$

$-y_1 + 3y_3 = 1$

$y_1, y_2 \geq 0$, y_3 unrestricted.

Ex. 3 obtain the dual of following LPP

Maximise $z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

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* Dual simplex method to solve the LPP :

Ex ① Use the dual simplex method to solve the LPP.

minimize $z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

Solution: The dual of given LPP is

Maximize $z = -6x_1 - x_2$

Subject to $-2x_1 - x_2 \leq -3$

$-x_1 + x_2 \leq 0$

∴ The standard form of above LPP is

Maximise $z = -6x_1 - x_2 + 0s_1 + 0s_2$

Subject to $-2x_1 - x_2 + s_1 + 0s_2 = -3$

$-x_1 + x_2 + 0s_1 + s_2 = 0$

$x_1, x_2, s_1, s_2 \geq 0$

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Initial iteration:

C_B	C_j / Basic variable	-6 / x_1	-1 / x_2	0 / s_1	0 / s_2	solution	Ratio
0	s_1	-2	-1	1	0	-3	$\frac{-3}{-1} = 3$ ← Min (key Row)
0	s_2	-1	1	0	1	0	$\frac{0}{1} = 0$
	z_j	0	0	0	0		
	$c_j - z_j$	-6	-1	0	0		

↑ Max (key column) under x_2 , -1 is pivot element.

first Iteration:

(s_1 - outgoing , x_2 - incoming)

C_B	C_j / Basic variable	-6 / x_1	-1 / x_2	0 / s_1	0 / s_2	solution	Ratio
-1	x_2	2	1	-1	0	3	$\frac{3}{2} = 1.5$
0	s_2	-3	0	1	1	-3	$\frac{-3}{-3} = 1$ ← Min key row
	z_j	-2	-1	1	0		
	$c_j - z_j$	-4	0	-1	0		

↑ Max (key column) under x_1

$\therefore$ -3 is pivot element.

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* Second iteration :

(s_2 - outgoing , x_2 - incoming)

	C_B	C_j	-6	-1	0	0	solution	Ratio
		Basic variable	x_1	x_2	s_1	s_2		
$R_1+\frac{2}{3}R_2$	-6	x_2	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	1	
$\frac{R_2}{-3}$	-1	x_1	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	
		z_j	-1	-6	$\frac{7}{3}$	$\frac{-11}{3}$		
		$c_j - z_j$	-5	5	$-\frac{7}{3}$	$\frac{11}{3}$		

$\therefore$ $x_1 = 1, \ x_2 = 1$ is solution of given LPP

and $z = 6x_1 + x_2$

$= 6(1) + 1$

$= 7$

$\therefore$ optimal solution is

$x_1 = 1, \ x_2 = 1, \ Z_{min} = 7$

HW.

Ex. 2. Use simplex method to solve the following LPP.

minimize $Z = x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

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* Big - M method : (Charne's method)

Note! In this method we use Artificial Variable denoted as A_i $(i=1,2,\ldots)$

EX ① Use Big-M method to solve the following LPP.

Minimise $Z = 2x_1 + 3x_2$

subject to $x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$

Solution: we introduce the surplus variable s_1, s_2 are artifficial variable A_1, A_2

∴ The standard form of given LPP is

Minimise $Z = 2x_1 + 3x_2 + 0s_1 + 0s_2$

s.t $x_1 + x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 5$

$x_1 + 2x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 6$

All $x_1, x_2, s_1, s_2 \geq 0$

Initial iteration:

C_B	C_j	2	3	0	0	M	M		
	Basic variable	x_1	x_2	s_1	s_2	A_1	A_2	Solution	Ratio
M	A_1	1	1	-1	0	1	0	5	$\frac{5}{1} = 5$
M	A_2	1	2	0	-1	0	1	6	$\frac{6}{2} = 3$ ← min (key row)
	Z_j	2M	3M	-M	-M	M	M		
	$C_j - Z_j$	2-2M	3-3M	M	M	0	0		

↑ Max. (key column)

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first iteration:

(A_2 - outgoing , x_2 - incoming)

	C_B	C_j / Basic variable	2	3	0	0	M	M	soln	Ratio	
			x_1	x_2	S_1	S_2	A_1	A_2			
$R_1 - \frac{1}{2}R_2$	M	A_1	$\frac{1}{2}$	0	-1	$\frac{1}{2}$	1	$-\frac{1}{2}$	2	4	← Min (key row)
$\frac{R_2}{2}$	3	x_2	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	3	6	
		Z_j	$\frac{M}{2}+\frac{3}{2}$	3	$-M$	$\frac{M}{2}-\frac{3}{2}$	M	$-\frac{M}{2}+\frac{1}{2}$			
		$C_j - Z_j$	$\frac{1}{2}-\frac{M}{2}$	0	M	$\frac{3}{2}-\frac{M}{2}$	0	$\frac{3M}{2}-\frac{1}{2}$			

↑ Max (key column)

$\therefore$ $\frac{1}{2}$ is pivot element

* second iteration: (A_1 - outgoing , x_1 - incoming)

	C_B	C_j / Basic variable	2	3	0	0	M	M	Solution	Ratio.
			x_1	x_2	S_1	S_2	A_1	A_2		
$2R_1$	2	x_1	1	0	-2	1	2	-1	4	
$R_2 - R_1$	3	x_2	0	1	1	-1	-1	1	1	
		Z_j	2	3	-1	-1	1	1		
		$C_j - Z_j$	0	0	1	1	$M-1$	$M-1$		

$\therefore$ $x_1 = 4$, $x_2 = 1$ is feasible solution

Now $Z = 2x_1 + 3x_2$

$= 2(4) + 3(1)$

$= 8 + 3$

$= 11$

$\therefore$ optimal solution are

$x_1 = 4$, $x_2 = 1$, $Z_{min} = 11$

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(H.W)

Ex(2) Use Big-M method to solve the following LPP.

Minimise $z = 2x_1 + x_2$

subject to
$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(Hint: use two Artificial variable A_1, A_2 for first & second constraint.)

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