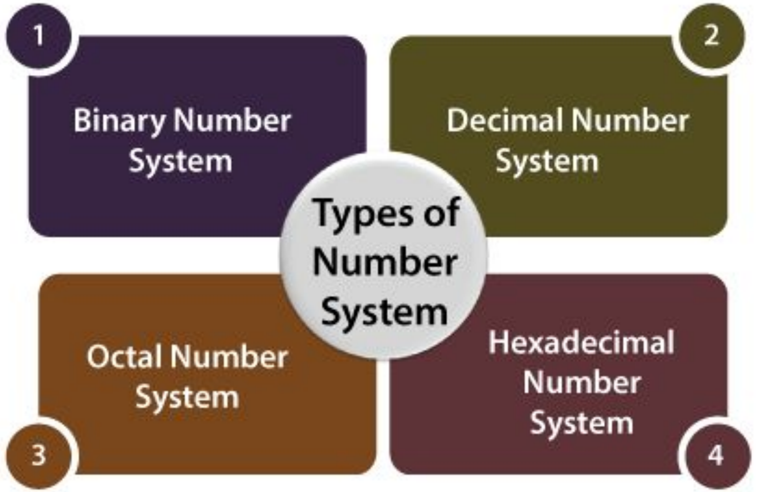
Module 1:
Fundamentals of Logic Design

Introduction to Number systems

- A number system is defined as the representation of numbers by using digits or other symbols in a consistent manner.
- The value of any digit in a number can be determined by a digit, its position in the number, and the base of the number system.
- In simple terms, for representing the information, we use the number system in the digital system.

Types of Number systems

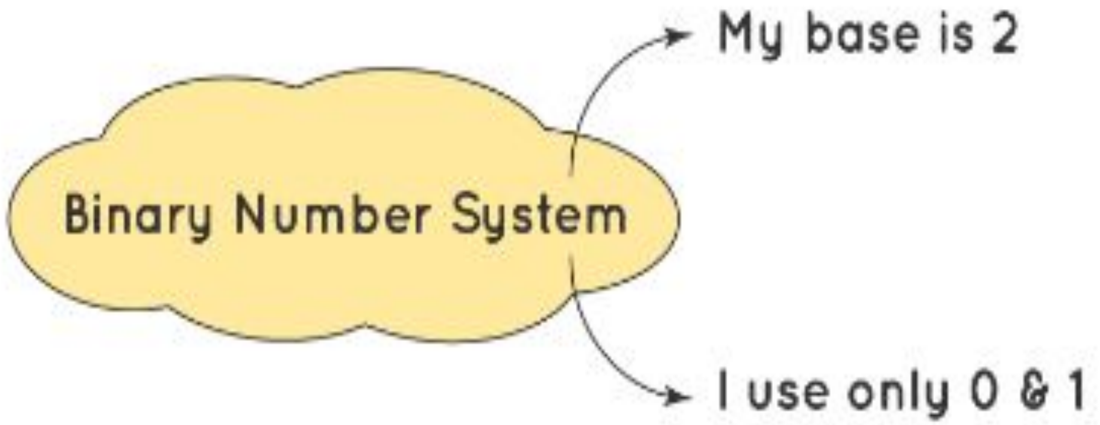
In the digital computer, there are various types of number systems used for representing information.



Binary number system

- The binary number system uses only two digits: 0 and 1. The numbers in this system have a base of 2.
- Digits 0 and 1 are called bits and 8 bits together make a byte.
- The data in computers is stored in terms of bits and bytes.
- For example: 10001_2, 111101_2, 1010101_2

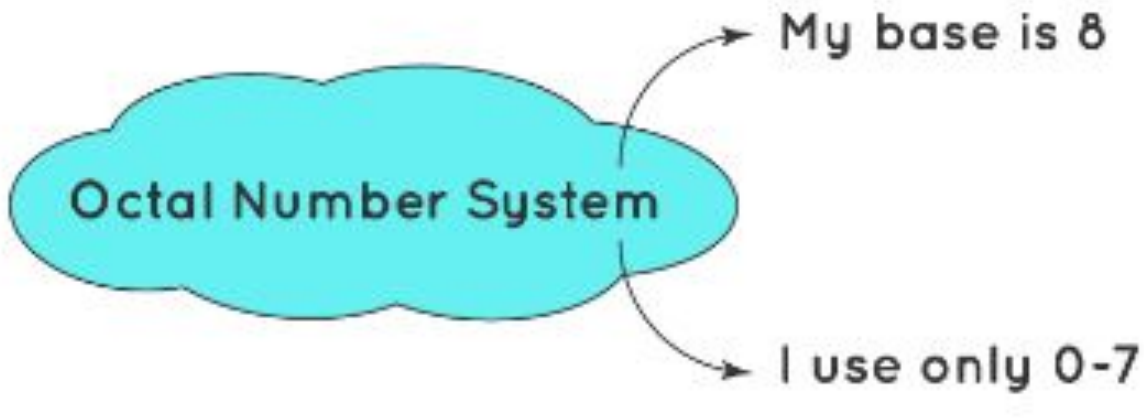
Binary number system



Octal Number System

- The octal number system uses eight digits: 0,1,2,3,4,5,6 and 7 with the base of 8.
- Digits like 8 and 9 are not included in the octal number system.
- Just as the binary, the octal number system is used in minicomputers but with digits from 0 to 7.
- For example: $35_8, 23_8, 141_8$

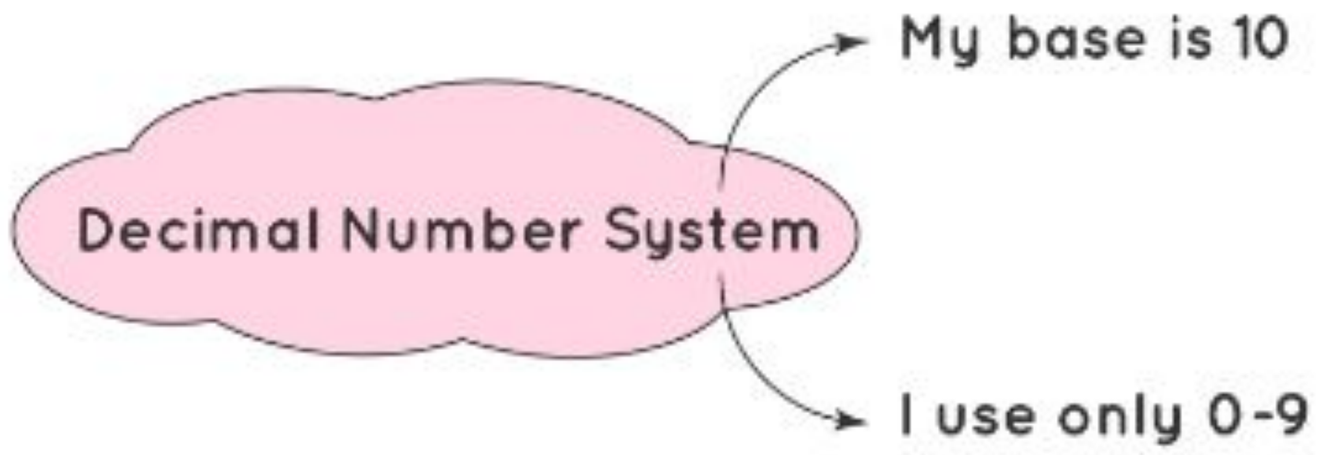
Octal Number System



Decimal Number System

- The decimal number system uses ten digits: 0,1,2,3,4,5,6,7,8 and 9 with the base number as 10.
- The decimal number system is the system that we generally use to represent numbers in real life.
- If any number is represented without a base, it means that its base is 10.
- For example: 723_{10}, 32_{10}, 4257_{10}

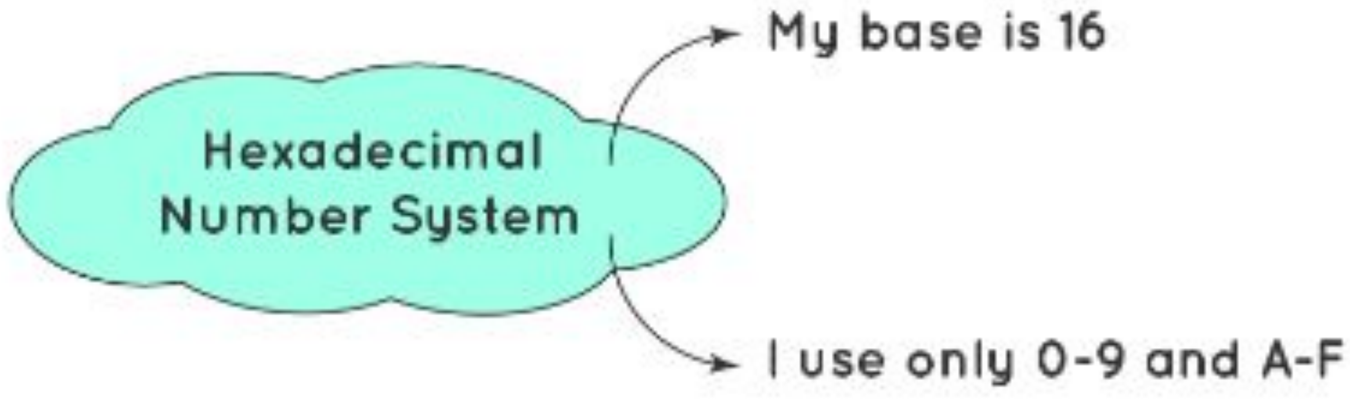
Decimal Number System



Hexadecimal Number System

- The hexadecimal number system uses sixteen digits/alphabets: 0,1,2,3,4,5,6,7,8,9 and A,B,C,D,E,F with the base number as 16.
- Here, A-F of the hexadecimal system means the numbers 10-15 of the decimal number system respectively.
- This system is used in computers to reduce the large-sized strings of the binary system.
- For example, $7B3_{16}$, $6F_{16}$,

Hexadecimal Number System



Conversion of Binary to Decimal Number System

If the number is integer - Multiply each digit of the given number by 2 from right to left, with the exponents of the base. The exponents should start with 0 and increase by 1 every time as we move from right to left i.e., 2^0, 2^1, 2^2 and so on from right to left and add them.

Conversion of Binary to Decimal Number System

Example 1:

$100111 = (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= (1 \times 32) + (0 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$

$= 32 + 0 + 0 + 4 + 2 + 1$

$= 39$

Thus, $100111_2 = 39_{10}$.

Conversion of Binary to Decimal Number System

Q.1: Convert the binary number 1001 to a decimal number.

Solution: Given, binary number = 1001_2

Hence, using the binary to decimal conversion formula, we have:

$1001_2 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 8 + 0 + 0 + 1$

$= (9)_{10}$

Q.2: Convert 1101001_2 into an equivalent decimal number.

Solution: Using binary to decimal conversion method, we get;

$(1101001)_2 = (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 64 + 32 + 0 + 8 + 0 + 0 + 1$

$= (105)_{10}$

Conversion of Binary to Decimal Number System

If the number is fractional - for the fractional binary numbers to the right of the binary point, the weight of each digit becomes more negative giving: $2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}$, and so on as shown.

Example 2: Convert binary number 0.001(base 2) into decimal form

0.001(base 2) $= 0 \times 2^{0} + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 0 + 0 + 0 + 0.125$

$= 0.125$

Conversion of Binary to Decimal Number System

Example 3:

$(101.01)_2$

$= 1 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0} + 0 \times 2^{-1} + 1 \times 2^{-2}$

$= 4 + 0 + 1 + 0 + 0.25$

$= (5.25)_{10}$

Conversion of Binary to Decimal Number System

Example 4: $(11001011.01101)_2 = (?)_{10}$.

Soln. :

$$(11001011)_2 = 1\times 2^7 + 1\times 2^6 + 0\times 2^5 + 0\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0$$

$$= 128 + 64 + 8 + 2 + 1$$

$$= (203)_{10}$$

$$(0.01101)_2 = 0\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3} + 0\times 2^{-4} + 1\times 2^{-5}$$

$$= 0.25 + 0.125 + 0.03125$$

$$= (0.40625)_{10}$$

$$\therefore (11001011.01101)_2 = (203.40625)_{10}$$

Conversion from decimal to binary number

Decimal numbers can be converted to binary by repeated division of the number by 2 while recording the remainder.

The remainders are to be read from bottom to top to obtain the binary equivalent.

Conversion from decimal to binary number

Example 1: convert decimal number 43 to binary.

		Remainder	
2	43		
2	21	1	MSB
2	10	1	
2	5	0	
2	2	1	
2	1	0	
	0	1	LSB

The remainders are to be read from bottom to top to obtain the binary equivalent. $43_{10} = 101011_2$

Conversion from decimal to binary number

Example 2: convert the given decimal number 13 into a binary number.

Step 1: Divide the given number 13 repeatedly by 2 until you get '0' as the quotient

$13 \div 2 = 6$ (Remainder 1)

$6 \div 2 = 3$ (Remainder 0)

$3 \div 2 = 1$ (Remainder 1)

$1 \div 2 = 0$ (Remainder 1)

Step 2: Write the remainders in the reverse order 1 1 0 1

$$\therefore 13_{10} = 1101_2$$

(Decimal) (Binary)

Conversion from decimal to binary number

Example 3: convert $(10.625)_{10}$ to binary.

2	10	
2	5	0
2	2	1
2	1	0
	0	1

0.625

$0.625 \times 2 = 1.250$ ----- 1

$0.250 \times 2 = 0.500$ ----- 0

$0.500 \times 2 = 1.0$ -------- 1

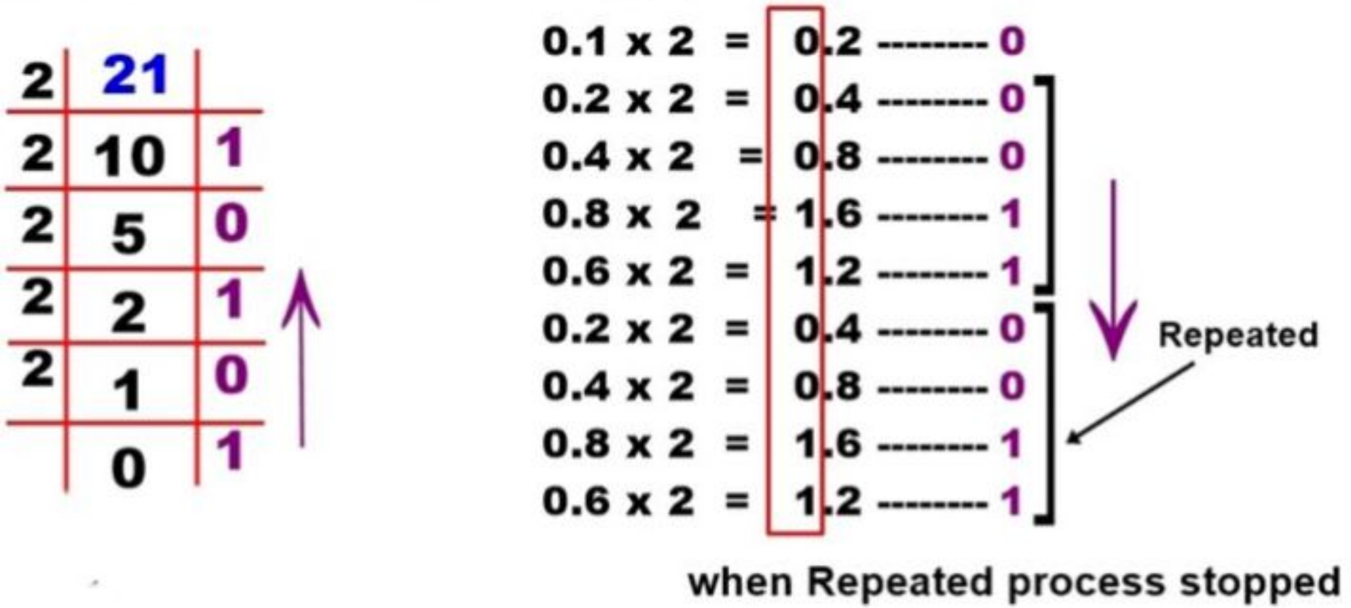
0×2 end process

$(10.625)_{10} = (1010.101)_2$

Practice: Find the binary fraction equivalent of the decimal fraction: 0.8125_{10}

Conversion from decimal to binary number

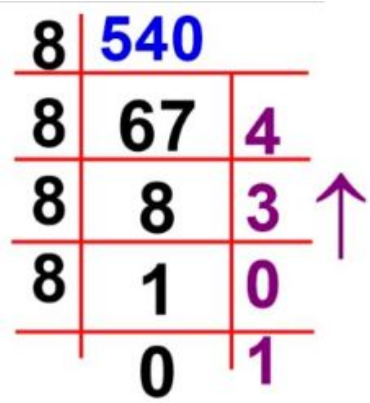
Example 4: $(21.1)_{10} = (?)_2$



$(21.1)_{10} = (10101.000110011)_2$

Conversion from decimal to octal number

Example 1:



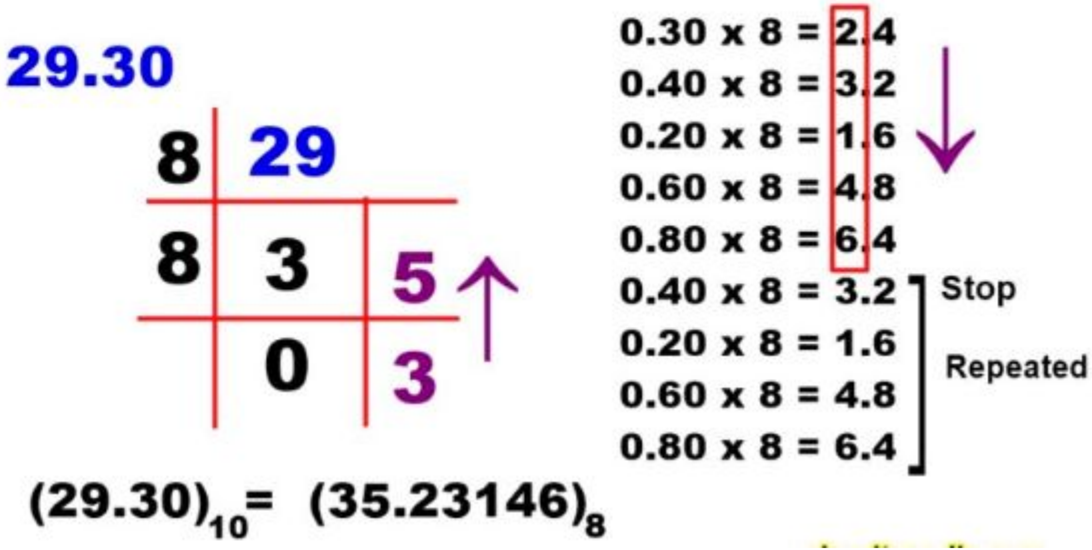
$(540)_{10} = (1034)_8$

Practice: Convert $(127)_{10}$ to Octal.

Convert 100_{10} to octal.

Conversion from decimal to octal number

Example 2:



Conversion from octal to decimal number

To obtain the decimal equivalent of an octal number, individual digits of octal number should be multiplied by powers of 8 starting with rightmost digit multiplied by 8^0, second last digit multiplied by 8^1, third last digit multiplied by 8^2 and so on up to the leftmost digit. For digits after the decimal point, the leftmost digit should be multiplied by 8^{-1} and the next digit by 8^{-2} and so on up to the rightmost digit.

Example:

$(127.34)_8$

$= 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 3 \times 8^{-1} + 4 \times 8^{-2}$

$= 64 + 16 + 7 + 0.375 + 0.0625$

$= (87.4375)_{10}$

Conversion from octal to binary number

1 2 . 5

001 010 101

$(12.5)_8 = (001010.101)_2$

7 2 1

111 010 001

$(721)_8 = (111010001)_2$

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Conversion from Decimal to Hexadecimal number

Repeatedly divide the decimal number by 16 until the quotient becomes less than 16 and record all the remainders. The remainders should be written bottom to upwards to get the hexadecimal equivalent of the decimal number.

Here,

- **10 = A**
- **11 = B**
- **12 = C**
- **13 = D**
- **14 = E**
- **15 = F**

Conversion from Decimal to Hexadecimal number

Example:

$(1973)_{10}$

16	1973	
16	123	**5**
	7	**11**

Here, 11 = B

$(1973)_{10} = (7B5)_{16}$

Binary Arithmetic