CHAPTER 7

Permutations and Combinations

1. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**

(a) 216 (b) 375
(c) 400 (d) 720

2. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**

(a) 125 (b) 105
(c) 375 (d) 625

3. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are **[2002]**

(a) 312 (b) 3125
(c) 120 (d) 216

4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**

(a) 3000 (b) 3050
(c) 3600 (d) 3250

5. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2\times{}^nC_r$ equals **[2003]**

(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$
(c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$.

6. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**

(a) 346 (b) 140
(c) 196 (d) 280

7. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**

(a) $6!\times 5!$ (b) 6×5
(c) 30 (d) 5×4

8. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**

(a) 480 (b) 240
(c) 360 (d) 120

9. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**

(a) ${}^8C_3$ (b) 21
(c) 3^8 (d) 5

10. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**

(a) 601 (b) 600
(c) 603 (d) 602

11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**

(a) 5040 (b) 6210
(c) 385 (d) 1110

12. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size.
Thus $A \cup B \cup C = S$,
$A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**

(a) $\dfrac{12!}{(4!)^3}$ (b) $\dfrac{12!}{(4!)^4}$
(c) $\dfrac{12!}{3!(4!)^3}$ (d) $\dfrac{12!}{3!(4!)^4}$

13. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? **[2008]**

(a) $8.\,{}^6C_4.\,{}^7C_4$ (b) $6.7.\,{}^8C_4$

(c) $6.\,8.\,{}^7C_4.$ (d) $7.\,{}^6C_4.\,{}^8C_4$

14. From 6 different novels and 3 different dictionaries,4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**

(a) at least 500 but less than 750

(b) at least 750 but less than 1000

(c) at least 1000

(d) less than 500

15. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**

(a) 36 (b) 66

(c) 108 (d) 3

16. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

17. **Statement - 1 :** For each natural number n, $(n+1)^7-1$ is divisible by 7.

Statement - 2 : For each natural number $n, n^7 - n$ is divisible by 7. **[2011 RS]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

18. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then : **[2011RS]**

(a) $N \le 100$

(b) $100 < N \le 140$

(c) $140 < N \le 190$

(d) $N > 190$

19. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to: **[2014]**

(a) X (b) Y

(c) N (d) $Y - X$

(c) 8 (d) 64

20. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[2013, 2015]**

(a) 275 (b) 510

(c) 219 (d) 256

21. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[2015]**

(a) 120 (b) 72

(c) 216 (d) 192

22. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[2016]**

(a) 52^{nd} (b) 58^{th}

(c) 46^{th} (d) 59^{th}

23. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[2017]**

(a) 484 (b) 485

(c) 468 (d) 469

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(c)	(d)	(b)	(c)	(c)	(a)	(c)	(b)	(a)	(c)	(a)	(d)
14	15	16	17	18	19	20	21	22	23			
(c)	(c)	(a)	(a)	(a)	(b)	(c)	(d)	(b)	(b)			

SOLUTIONS

1. **(d)** Required number of numbers
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

2. **(c)** Required number of numbers
$= 3 \times 5 \times 5 \times 5 = 375$

3. **(d)** We know that a number is divisible by 3 only when the sum of the digits is divisible by 3. The given digits are 0, 1, 2, 3, 4, 5.
Here the possible number of combinations of 5 digits out of 6 are ${}^5C_4 = 5$, which are as follows–
$1+2+3+4+5 = 15 = 3 \times 5$
$0+2+3+4+5 = 14$ (not divisible by 3)
$0+1+3+4+5 = 13$ (not divisible by 3)
$0+1+2+4+5 = 12 = 3 \times 4$
$0+1+2+3+5 = 11$ (not divisible by 3)
$0+1+2+3+4 = 10$ (not divisible by 3)
Thus the number should contain the digits 1, 2, 3, 4, 5 or the digits 0, 1, 2, 4, 5.
Taking 1, 2, 3, 4, 5, the 5 digit numbers are $= 5! = 120$
Taking 0, 1, 2, 4, 5, the 5 digit numbers are $= 5! - 4! = 96$
$\therefore$ Total number of numbers $= 120 + 96 = 216$

4. **(b)** Required sum
$$= (2+4+6+...+100) + (5+10+15+...+100) - (10+20+...+100)$$
$$= 2550 + 1050 - 530 = 3050.$$

5. **(c)** ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$
$$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$$
$$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$$

6. **(c)** As for given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining question
$= {}^5C_4 \times {}^8C_6 = 140$ choices.

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions
$= {}^5C_5 \times {}^8C_5 = 56$ choices.

Therefore, total number of choices $= 140 + 56 = 196$.

7. **(a)** No. of ways in which 6 men can be arranged at a round table $= (6 - 1)! = 5!$
Now women can be arranged in ${}^6P_5$ $= 6!$ Ways.
Total Number of ways $= 6! \times 5!$

8. **(c)** Total number of arrangements of letters in the word GARDEN = 6 ! = 720 there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E.
So, vowels in alphabetical order in $\frac{1}{2} \times 720 = 360$

9. **(b)** We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$
$\therefore$ The required number of ways
$$= {}^{8-1}C_{3-1} = {}^7C_2 = \frac{7!}{2!5!} = \frac{7 \times 6}{2 \times 1} = 21$$

10. **(a)** Alphabetical order is
A, C, H, I, N, S
No. of words starting with A – 5!
No. of words starting with C – 5!

No. of words starting with H – 5!
No. of words starting with I – 5!
No. of words starting with N – 5!
SACHIN – 1
$\therefore$ sachin appears at serial no 601

11. (c) $^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

12. (a) Set S = {1, 2, 3, ... 12}

$A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$

$\therefore$ The number of ways to partition
$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$$= \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$$

13. (d) First let us arrange M, I, I, I, P, P

Which can be done in $\frac{7!}{4!2!}$ ways

√ M √ I √ I √ I √ I √ P √ P √

Now 4 S can be kept at any of the ticked places in $^8C_4$ ways so that no two S are adjacent.
Total required ways

$$= \frac{7!}{4!2!}\,{}^8C_4 = \frac{7!}{4!2!}\,{}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$$

14. (c) 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in $^6C_4 \times {}^3C_1$ ways

Then 4 novels with one dictionary in the middle can be arranged in 4! ways.
$\therefore$ Total ways of arrangement

$= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

15. (c) Total number of ways = $^3C_2 \times {}^9C_2$

$$= 3 \times \frac{9 \times 8}{2} = 3 \times 36 = 108$$

16. (a) The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box empty is same as the number of ways of selecting $(r - 1)$ places out of $(n - 1)$ different places, that is $^{n-1}C_{r-1}$.

Hence require number of ways

$= {}^{10-1}C_{4-1} = {}^9C_3$

17. (a) **Statement 2 :**

$P(n) : n^7 - n$ is divisible by 7

Put $n = 1, 1 - 1 = 0$ is divisible by 7, which is true
Let $n = k, P(k) : k^7 - k$ is divisible by 7, true
Put $n = k + 1$

$\therefore\quad P(k+1) : (k+1)^7 - (k+1)$ is div. by 7
$P(k + 1) : k^7 + {}^7C_1k^6 + {}^7C_2k^2 + ... + {}^7C_6k + 1 - k - 1$, is div. by 7.
$P(k + 1) : (k^7 - k) + ({}^7C_1k^6 + {}^7C_2k^5 + ... + {}^7C_6k)$ is div. by 7.
Since 7 is coprime with 1, 2, 3, 4, 5, 6.

So $^7C_1, {}^7C_2, ...{}^7C_6$ are all divisible by 7

$\therefore\quad P(k + 1)$ is divisible by 7
Hence $P(n) : n^7 - n$ is divisible by 7

Statement 1 : $n^7 - n$ is divisible by 7

$\Rightarrow\ (n+1)^7 - (n+1)$ is divisible by 7

$\Rightarrow\ (n+1)^7 - n^7 - 1 + (n^7 - n)$

is divisible by 7

$\Rightarrow\ (n+1)^7 - n^7 - 1$ is divisible by 7

Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1.

18. (a) Number of required triangles = $^{10}C_3 - {}^6C_3$

$$= \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100$$

19. (b) $4^n - 3n - 1 = (1 + 3)^n - 3n - 1$
$= [{}^nC_0 + {}^nC_1.3 + {}^nC_2.3^2 + ... + {}^nC_n3^n] - 3n - 1$
$= 9\ [{}^nC_2 + {}^nC_3.3 + ... + {}^nC_n.3^{n-2}]$
$\therefore\ 4^n - 3n - 1$ is a multiple of 9 for all n.
$\therefore\ X = \{x : x$ is a multiple of $9\}$
Also, $Y = \{9\,(n - 1) : n \in \mathbf{N}\}$
$= \{$All multiples of $9\}$
Clearly $X \subset Y$. $\therefore\ X \cup Y = Y$

20. (c) Given
$n(A) = 2, n(B) = 4, n(A \times B) = 8$

Required number of subsets =
$^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

21. **(d)** Four digits number can be arranged in $3 \times 4!$ ways.

Five digits number can be arranged in 5! ways.

Number of integers $= 3 \times 4! + 5! = 192$.

22. **(b)** ALLMS
No. of words starting with

A : $\underline{A}\,\_\,\_\,\_\,\_\;\dfrac{4!}{2!} = 12$

L : $\underline{L}\,\_\,\_\,\_\,\_\;4! = 24$

M : $\underline{M}\,\_\,\_\,\_\,\_\;\dfrac{4!}{2!} = 12$

S : $\underline{S}\,\underline{A}\,\_\,\_\,\_\,\_\;\dfrac{3!}{2!} = 3$

 : $\underline{S}\,\underline{L}\,\_\,\_\,\_\;3! = 6$

SMALL $\to 58^{th}$ word

23. **(b)** $X\begin{cases}\to 4 \text{ ladies}\\ \to 3 \text{ men}\end{cases}$ $\quad Y\begin{cases}\to 3 \text{ ladies}\\ \to 4 \text{ men}\end{cases}$

Possible cases for X are
(1) 3 ladies, 0 man
(2) 2 ladies, 1 man
(3) 1 lady, 2 men
(4) 0 ladies, 3 men
Possible cases for Y are
(1) 0 ladies, 3 men
(2) 1 lady, 2 men
(3) 2 ladies, 1 man
(4) 3 ladies, 0 man
No. of ways $= {}^4C_3 \,.\, {}^4C_3 + ({}^4C_2 \,.\, {}^3C_1)^2 + ({}^4C_1 \,.\, {}^3C_2)^2 + ({}^3C_3)^2$
$= 16 + 324 + 144 + 1 = 485$