Resonance Educating for better tomorrow	**PHYSICS**
TARGET : JEE (Advanced) 2015	**DPP** DAILY PRACTICE PROBLEMS
Course: VIJETA & VIJAY (ADP & ADR) **Date : 17-04-2015**	**NO. 04**

TEST INFORMATION

TEST : CUMULATIVE TEST (CT)-1 (6 hours) **Test Date : 19-04-2015**

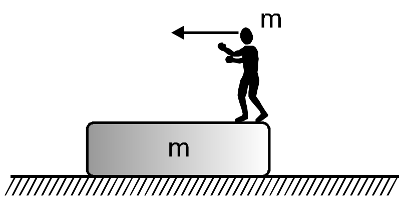
Syllabus : Geometrical Optics, Electrostatics, Gravitation, Kinematics, Newton's laws of motion, Friction, Current electricity, Capacitor, Magnetic field and force.

This DPP is to be discussed (22-04-2014)
CT-2 to be discussed (22-04-2014)

DPP No. # 04

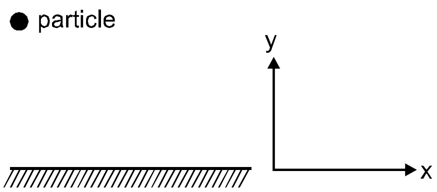
Total Total Marks : 151	**Max. Time : 116½ min.**	
Single choice Objective (–1 negative marking) Q. 1 to 15	(3 marks 2½ min.)	[45, 37½]
Multiple choice objective (–1 negative marking) Q. 16 to 23	(4 marks, 3 min.)	[32, 24]
Single Digit Subjective Questions (no negative marking) Q.24 to Q.30	(4 marks 2½ min.)	[28, 17½]
Double Digits Subjective Questions (no negative marking) Q. 31 to 32	(4 marks 2½ min.)	[8, 5]
Three Digits Subjective Questions (no negative marking) Q. 33	(4 marks 2½ min.)	[4, 2½]
Comprehension (–1 negative marking) Q.34 to 42	(3 marks 2½ min.)	[24, 20]
Match Listing (–1 negative marking) Q.43 to Q.44	(3 marks, 3 min.)	[6, 6]
Match the Following (no negative marking) (4 × 5) Q. 45	(8 marks 10 min.)	[8, 10]

1. A block of mass m and length ℓ is kept at rest on a rough horizontal ground of friction coefficient μ_k. A man of mass m is standing at the right end. Now the man starts walking towards left and reaches the left end within time 't'. During this time, the displacement of the block is : (Assume the pressing force between the block and the ground remains constant and its value is same as it was initially. Also as sume that the block slides during the entire time (t)) :



(A) $\frac{\ell - \mu_k g t^2}{2}$ (B) $\frac{\ell + \mu_k g t^2}{2}$ (C) $\frac{\mu_k g t^2 - \ell}{2}$ (D) $\frac{\ell}{2}$ (E) $\ell - \mu_k g t^2$

2. A particle moving with velocity $(2\hat{i} - 3\hat{j})$ m/s collides with a surface at rest in xz–plane as shown in figure and moves with velocity $(2\hat{i} + 2\hat{j})$ m/s after collision. Then coefficient of restitution is :



(A) $\frac{2}{3}$ (B) 1 (C) $\sqrt{\frac{8}{13}}$ (D) $\frac{4}{5}$

(5) None of these

3. A ball is suspended from the top of a cart by a light string of length 1.0 m. The cart and the ball are initially moving to the right at constant speed V, as shown in figure I. The cart comes to rest after colliding and sticking to a fixed bumper, as in figure II. The suspended ball swings through a maximum angle 60°. The initial speed V is (take g = 10 m/s^2) (neglect friction)

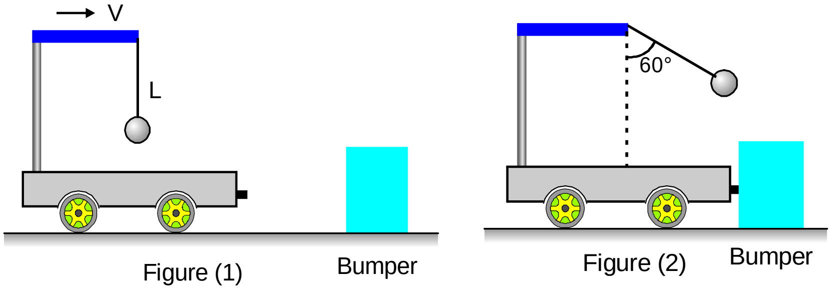


Figure (1)   Figure (2)

(A) $\sqrt{10}$ m/s   (B) $2\sqrt{5}$ m/s   (C) $5\sqrt{2}$ m/s   (D) 4 m/s

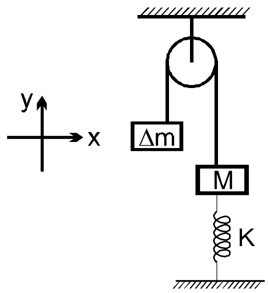
4. Two blocks each of mass m are joined together using an ideal spring of force constant K and natural length ℓ_0. The blocks are touching each other when the system is released from rest on a rough horizontal surface. Both the blocks come to rest simultaneously when the extension in the spring is $\frac{\ell_0}{4}$. The coefficient of friction between each block and the surface (assuming it to be same between any of the blocks and the surface) is :

(A) $\frac{K\ell_0}{40mg}$ (B) $\frac{K\ell_0}{8mg}$ (C) $\frac{3K\ell_0}{8mg}$ (D) $\frac{17}{20}\frac{K\ell_0}{mg}$

5. Two spherical bodies of masses m and 5m and radii R and 2R respectively, are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, the distance covered by smaller sphere just before collision is

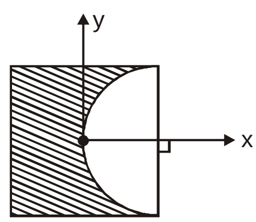
(A) $\frac{15R}{2}$ (B) $\frac{13R}{2}$ (C) 10R   (D) $\frac{17R}{2}$

6. Consider the system shown in figure. Pulley, string and spring are ideal and $\Delta m << M$. Initially spring is in it's natural length and both the blocks are at rest. (Assume that initially Δm was situated at origin). Maximum y coordinate of Δm in subsequent motion is xmg/k then value of x is .



(A) 1
(B) 2
(C) 3
(D) 4

7. In the figure shown a semicircular area is removed from a uniform square plate of side 'ℓ' and mass 'm' (before removing). The x-coordinate of centre of mass of remaining portion is (The origin is at the centre of square)



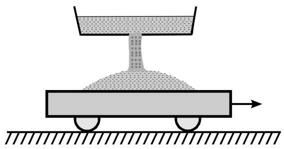
(A) $-\frac{\pi(\pi-2)\ell}{2(8-\pi)}$ (B) $\frac{\pi(\pi-2)\ell}{2(8-\pi)}$

(C) $-\frac{\pi(\pi-2)\ell}{8-\pi}$ (D) $-\frac{\ell\left(\pi-\frac{4}{3}\right)}{2(8-\pi)}$

8. Power of the only force acting on a particle of mass m = 1 kg moving in straight line depends on its velocity as P = v^2 where v is in m/s and P is in watt. If initial velocity of the particle is 1 m/s, then the displacement of the particle in ℓn2 second will be :

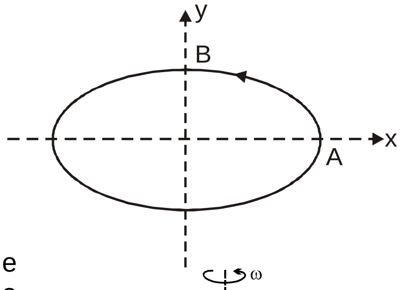
(A) (ℓn2 – 1)m   (B) (ℓn2)^2 m   (C) 1 m   (D) 2 m

9. Sand is falling on a flat car being pulled with constant speed. The rate of mass falling on the cart is constant. Then the horizontal component of force exerted by the falling sand on the cart (sand particles sticks to the cart)



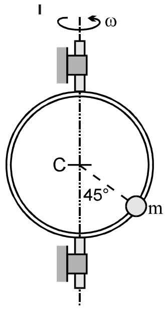
(A) increases
(B) decreases
(C) remains constant
(D) increases and then decreases

10. A particle is moving along an elliptical path with constant speed. As it moves from A to B, magnitude of its acceleration :



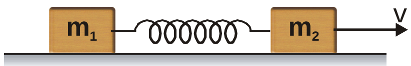
(A) continuously increases
(B) continuously decreases
(C) Remains constant
(D) first increases and then decreases

11. A small bead of mass m = 1 kg is free to move on a circular hoop. The circular hoop has centre at C and radius r = 1 m and it rotates about a fixed vertical axis. The coefficient of friction between bead and hoop is μ = 0.5. The maximum angular speed of the hoop for which the bead does not have relative motion with respect to hoop, at the position shown in figure is : (Take g = 10 m/s^2)



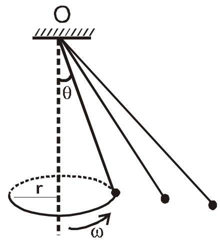
(A) $\left(5\sqrt{2}\right)^{1/2}$
(B) $\left(10\sqrt{2}\right)^{1/2}$
(C) $\left(15\sqrt{2}\right)^{1/2}$
(D) $\left(30\sqrt{2}\right)^{1/2}$

12. Two blocks of mass m_1 and m_2 ($m_1 < m_2$) are connected with an ideal spring on a smooth horizontal surface as shown in figure. At t = 0 m_1 is at rest and m_2 is given a velocity v towards right. At this moment, spring is in its natural length. Then choose the correct alternative :



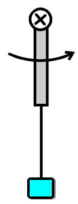
(A) Block of mass m_2 will be finally at rest after some time.
(B) Block of mass m_2 will never come to rest.
(C) Both the blocks will be finally at rest.
(D) None of these

13. Three point masses are attached by light inextensible strings of various lengths to a point O on the ceiling. All of the masses swing round in horizontal circles of various radii with the same angular frequency ω (one such circle is drawn in the shown figure.) Then pick up the correct statement.



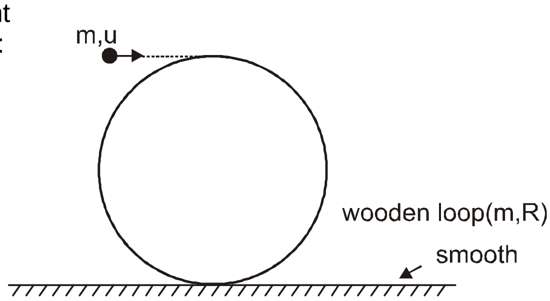
(A) The vertical depth of each mass below point of suspension from ceiling is different.
(B) The radius of horizontal circular path of each mass is same.
(C) All masses revolve in the same horizontal plane.
(D) All the particles must have same mass.

14. One end of a light rod of length 1 m is attached with a string of length 1m. Other end of the rod is attached at point O such that rod can move in a vertical circle. Other end of the string is attached with a block of mass 2kg. The minimum velocity that must be given to the block in horizontal direction so that it can complete the vertical circle is (g = 10 m/s^2).



(A) $4\sqrt{5}$
(B) $5\sqrt{5}$
(C) 10
(D) $3\sqrt{5}$

15. Particle sticks to wooden loop, If particle reach at the lowest position for first time after time T. Then displacement of centre of mass of system in this time interval will be :



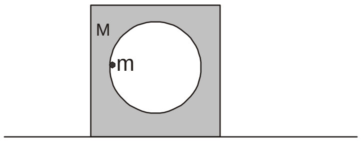
(A) $\sqrt{\left(\frac{uT}{2}\right)^2 + R^2}$

(B) $\sqrt{(uT)^2 + R^2}$

(C) $\frac{1}{2}\sqrt{(uT)^2 + R^2}$

(D) None of these

16. The figure shows a block of mass M=2m having a spherical smooth cavity of radius R placed on a smooth horizontal surface .There is a small ball of mass m moving at an instant vertically downward with a velocity v with respect to the block .At this instant :



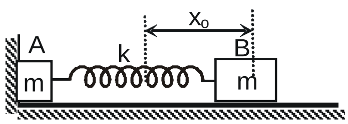
(A) The normal reaction on the ball by the block is $\frac{mv^2}{R}$

(B) The normal reaction on the ball by the block is $\frac{2}{3}\frac{mv^2}{R}$

(C) The acceleration of the block with respect to the ground is $\frac{v^2}{3R}$

(D) The acceleration of the block with respect to the ground is $\frac{v^2}{2R}$

17. Two identical blocks A and B of mass m each are connected to each other by spring of spring constant k. Block B is initially shifted to a small distance x_0 to the left and then released. Choose the correct statements for this problem, after the spring attains it's natural lenth.



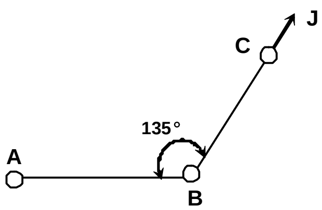
(A) Velocity of centre of mass of the system is $\frac{1}{2}\sqrt{\frac{k}{m}}\ x_0$

(B) Maximum elongation in spring during the subsequent motion is $\frac{x_0}{\sqrt{2}}$

(C) Maximum elongation in spring during the subsequent motion is x_0

(D) Maximum speed of block A during subsequent motion be $\sqrt{\frac{K}{m}}x_0$

18. Three identical particles A, B and C lie on a smooth horizontal table. Light inextensible strings which are just taut connect AB and BC and $\angle ABC$ is 135° . An impulse J is imparted to the particle C in the direction BC. Mass of each particle is m. Choose the correct options.



(A) Speed of A just after the impulse imparted is $\frac{\sqrt{2}J}{7m}$

(B) Speed of B just after the impulse imparted is $\frac{\sqrt{10}J}{7m}$

(C) Speed of C just after the impulse imparted is $\frac{3J}{7m}$

(D) Speed of A just after the impulse imparted is $\frac{2J}{7m}$

19. A particle is attached to an end of a rigid rod. The rod is hinged at the other end and rotates in a horizontal plane about the hinge. It’s angular speed is increasing at constant rate. The mass of the particle is 'm'. The force exerted by the rod on the particle is $\vec{F}$, then choose the correct alternative(s):

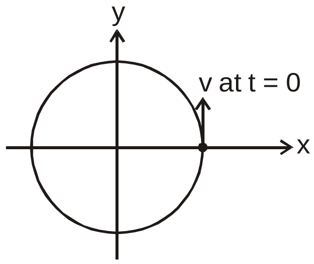
(A) $F \geq mg$

(B) F is constant

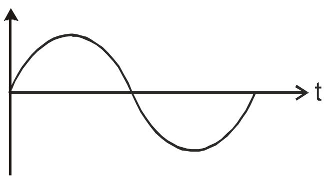
(C) The angle between $\vec{F}$ and horizontal plane decreases.

(D) The angle between $\vec{F}$ and the rod decreases.

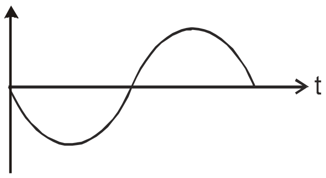
20. A particle is moving in a uniform circular motion on a horizontal surface. Particle's position and velocity at time t = 0 are shown in the figure in the coordinate system. Which of the indicated variable on the vertical axis is/are correctly matched by the graph(s) shown alongside for particle's motion ?



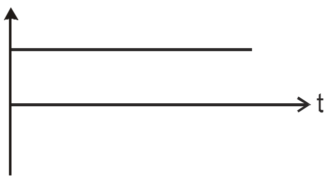
(A) x component of velocity



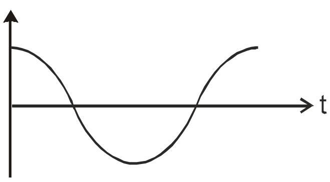
(B) y component of force keeping particle moving in a circle



(C) Angular velocity of the particle



(D) x coordinate of the particle



21. The linear momentum of a particle is given by $\vec{P} = (a \sin t\,\hat{i} - a \cos t\,\hat{j})$ kg–m/s. A force $\vec{F}$ is acting on the particle. Select correct alternative/s :

(A) Linear momentum $\vec{P}$ of particle is always parallel to $\vec{F}$

(B) Linear momentum $\vec{P}$ of particle is always perpendicular to $\vec{F}$

(C) Linear momentum $\vec{P}$ is always constant

(D) Magnitude of linear momentum is constant with respect to time.

22. A circular road of radius r is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible. Choose the correct alternatives :

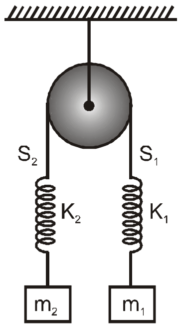
(A) The car can make a turn without skidding.

(B) If the car turns at a speed less than 40 km/hr, it will slip down

(C) If the car turns at the constant speed of 40 km/hr, the force by the road on the car is equal to $\sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$

(D) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$

23. Consider the condition shown in the figure. Pulley is massless and frictionless, springs are massless. Both the blocks are released with the springs in their natural lengths. Choose the correct options.



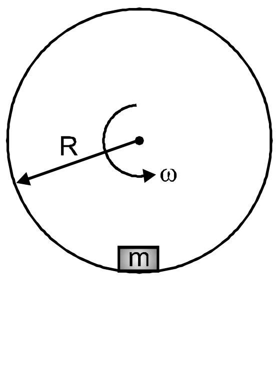
(A) Maximum elongation in the spring S_1 is $\frac{4m_1m_2g}{K_1(m_1+m_2)}$

(B) Maximum elongation in the spring S_1 is $\frac{4m_1m_2g}{K_2(m_1+m_2)}$

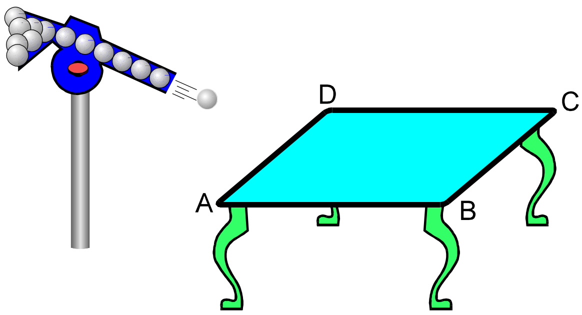
(C) If $m_1 = m_2$ both the blocks will come to instantaneous rest simultaneously.

(D) If $K_1 = K_2$ both the blocks will come to instantaneous rest simultaneously.

24 A cylinder of radius R is rotating about its horizontal axis with constant $\omega = \sqrt{\frac{5g}{R}}$. A block of mass m is kept on the inner surface of the cylinder. Block is moving in vertical circular motion without slipping. co–efficient of friction between block and surface of cylinder is μ. If minimum value of μ for complete vertical circular motion of block is $\frac{2\sqrt{6}}{3x}$ then find 'x'.

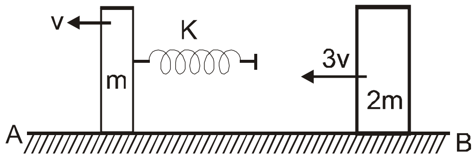


25. A gun which fires small balls each of mass 20 gm is firing 20 balls per second on the smooth horizontal table surface ABCD. If the collision is perfectly elastic and balls are striking at the centre of table with a speed 5 m/sec at an angle of 60º with the vertical just before collision, then force exerted by one of the leg on ground is (in N) (assume total weight of the table is 0.2 kg and g = 10 m/s^2) :



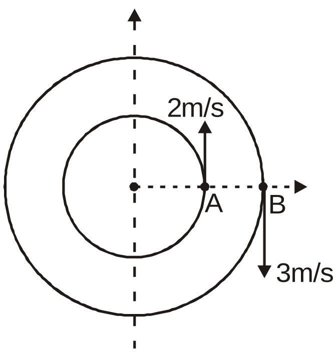
26. A rocket of total mass 1000kg initially is launched from ground. The gases are ejected at the rate 20kg/s with velocity 1000 m/s relative to rocket vertically downwards. The initial acceleration of the rocket is a (in m/s^2). Find $\frac{a}{g}$. (Take g = 10m/s^2)

27. AB is a long frictionless horizontal surface. One end of an ideal spring of spring constant K is attached to a block of mass m, which is being moved left with constant velocity v, and the another end is free. Another block of mass 2m is given a velocity 3v towards the spring. Magnitude of work done by external agent in moving m with constant velocity v in long time is β times mv^2. Find the value of β

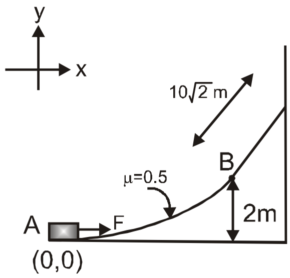


28. In a region, potential energy varies with x as U(x) = 30 – (x – 5)^2 Joule, where x is in meters. A particle of mass 0.5 kg is projected from x = 11 m towards origin with a velocity 'u'. u is the minimum velocity, so that the particle can reach the origin. (x = 0). Find the value of $\frac{u}{2}$ in meter/second. (Take $\sqrt{44}$ = 6.5)

29. Two particles A and B are revolving with constant angular velocity on two concentric circles of radius 1m and 2m respectively as shown in figure. The positions of the particles at t = 0 are shown in figure. If m_A = 2kg, m_B = 1kg and $\vec{P}_A$ and $\vec{P}_B$ are linear momentum of the particles then what is the maximum value of $\left|\vec{P}_A + \vec{P}_B\right|$ in kg-m/sec in subsequent motion of the two particles.



30. Work done by force F to move block of mass 2kg from A to C very slowly is (76 λ)J. Force F is always acting tangential to path. Equation of path AB is $x^2 = 8y$ and BC is straight line which is tangent on curve AB at point B (μ between block and path ABC is 0.5). Then value of 'λ' is **[g = 10 m/s²]** :



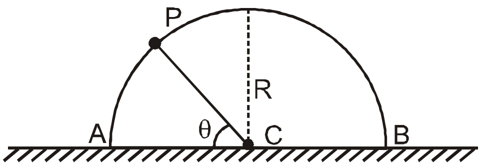
31. A ball of mass 'm' is suspended from a point with a massless string of length 'ℓ' in form of a pendulum. This ball is given a horizontal velocity $\sqrt{4g\ell}$ at bottom most point. When string makes an angle 60° from lower vertical, $\frac{a_c}{a_t}$ = p. Write the value of p^2. (g = 10 m/s²)

32. Two blocks of masses m_1 = 10 kg and m_2 = 20 kg are connected by a spring of stiffness k = 200 N/m. The coefficient of friction between the blocks and the fixed horizontal surface is μ = 0.1. Find the minimum constant horizontal force F (in Newton) to be applied to m_1 in order to slide the mass m_2. (Take g = 10 m/s²)

33. A particle of mass m = 1 kg is lying at rest on x-axis, experiences a net force given by law F = x (3x – 2) Newton, where x is the x-coordinate of the particle in meters. The magnitude of minimum velocity in negative x-direction to be imparted to the particle placed at x = 4 meters such that it reaches the origin is $\sqrt{\frac{P}{27}}$ m/s. Find the value of P.

COMPREHENSION

A hemispherical bowl of uniform mass distribution of mass 'M' is at rest on frictionless horizontal ground surface. There is a small insect of mass 'm' on bowl at point 'A' of the bowl at rest. Now the insect moves with constant speed 'v' relative to bowl in vertical plane. Assume that insect does not slip on the bowl.



34. When the insect reaches point 'P' of the hemisphere the displacement of hemisphere w.r.t. ground is :

(A) Zero

(B) $\frac{MR}{(M+m)}(1-\cos\theta)$ horizontally towards left

(C) $\frac{mR}{(M+m)}(1-\cos\theta)$ horizontally towards left

(D) $\frac{mR(1+\cos\theta)}{(M+m)}(1+\cos\theta)$

35. When the insect is at point 'P' of the hemispherical bowl the acceleration of bowl is :

(A) Zero

(B) $\left(\frac{m}{M+m}\right)\frac{v^2}{R}\cos\theta$ horizontally towards left

(C) $\left(\frac{M}{M+m}\right)\frac{v^2}{R}\sin\theta$ horizontally towards right

(D) $\left(\frac{m}{M+m}\right)\frac{v^2}{R}\cos\theta$ horizontally towards right

36. When insect at point 'P' on the bowl the displacement of centre of mass of the system (bowl + insect) is :

(A) $\frac{mR\cos\theta}{(M+m)}$ vertically upwards

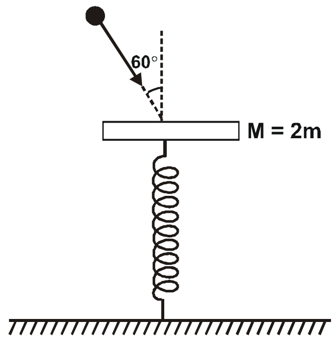
(B) Zero

(C) $\frac{mR\sin\theta}{(M+m)}$ vertically upwards

(D) $\frac{mR\sin\theta}{(M+m)}$ vertically upwards

COMPREHENSION

A particle of mass m collides elastically with the pan of mass (M = 2m) of a spring balance, as shown in figure. Pan is in equillibrium before collision. Spring constant is k and speed of the particle before collision is v_0. Answer the following three questions regarding this collision.



37. Maximum compression in the spring after the collision is

(A) $\sqrt{\frac{2m}{k}}\frac{v_0}{3}$ (B) $\sqrt{\frac{2m}{3k}}\,v_0$ (C) $\sqrt{\frac{m}{3k}}\,v_0$ (D) $\sqrt{\frac{m}{k}}\,v_0$

38. Maximum height attained by the particle from the point of collision after collision is

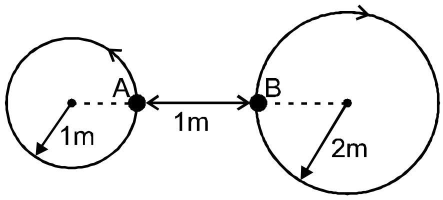
(A) $\frac{v_0^2}{16g}$ (B) $\frac{v_0^2}{8g}$ (C) $\frac{v_0^2}{36g}$ (D) $\frac{v_0^2}{72g}$

39. Minimum kinetic energy of the particle after collision is

(A) $\frac{mv_0^2}{8}$ (B) $\frac{3mv_0^2}{8}$ (C) $\frac{3mv_0^2}{4}$ (D) $\frac{mv_0^2}{2}$

COMPREHENSION

Two particles are moving in different circles in same plane with different angular velocities as shown in figure. At t = 0, initial positions of particles A and B are shown by dots on the respective circles. Initial distance between particles is 1m. Particle A move anticlockwise in the first circle whereas B moves clockwise in the second circle. Angle described (rotated) by A and B in time 't' are $\theta_A = \left(\frac{\pi}{2}t\right)$ and $\theta_B = (\pi t)$ respectively. Here θ is in radian and t is in second. Radius of each circle is shown in diagram.



40. Find the magnitude of acceleration of A at t = 1 sec

(A) $\frac{\pi^2}{3}$ m/s (B) $\frac{\pi^2}{7}$ (C) $\frac{\pi^2}{4}$ (D) None of these

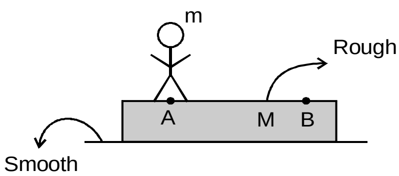
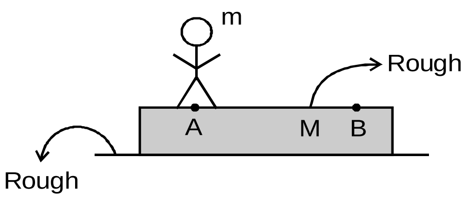
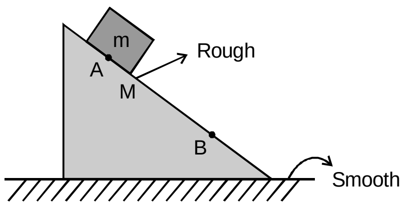
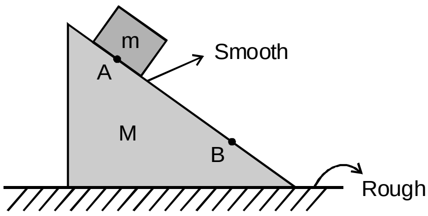
41. At time t = 1 sec, the magnitude of acceleration of A with respect to B is

(A) $\frac{\pi^2}{4}\sqrt{65}\,\frac{m}{sec^2}$ (B) $\frac{\pi^2}{2}\sqrt{7}\,\frac{m}{sec^2}$ (C) $\frac{\pi^2}{3}\sqrt{15}\,\frac{m}{sec^2}$ (D) $\frac{\pi^2}{4}\sqrt{7}\,\frac{m}{sec^2}$

42. At time t = 2 second, the angular velocity of the particle A with respect to the particle B is

(A) 5π rad/sec (B) $\frac{3\pi}{2}$ rad/sec (C) $\frac{2\pi}{3}$ rad/sec (D) $\frac{5\pi}{6}$ rad/sec

43. List-I shows some arrangements in which motion of masses are described and list-II defines motion of centre of mass of the system (m + M).
Match appropriate possible options in list-II

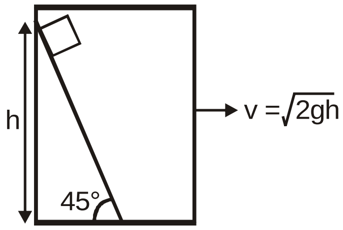
List-I	List-II
(P)  person moves with constant acceleration (towards right)	(1) Acceleration of centre of mass may be zero.
(Q)  At t = 0, a person at point A is moving with constant velocity (towards right) and the plank is at rest , then for his motion between point A and B.	(2) Centre of mass must move with constan velocity
(R) 	(3) Centre of mass must remain at rest
(S) 	(4) Centre of mass must have a component of acceleration in the downward direction

Codes :

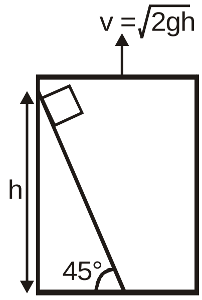
	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

44. Figure shows four situations in which a small block of mass 'm' is released from rest (with respect to smooth fixed wedge) as shown in figure. Column-II shows work done by normal reaction with respect to an observer who is stationary with respect to ground till block reaches at the bottom of inclined wedge, match the appropriate column (Assume that there is infinite friction between block and floor of cabin) :

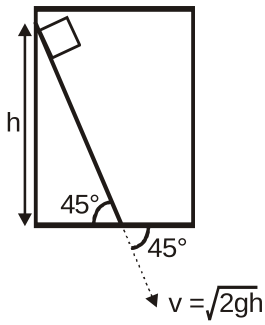
Column–I | **Column–II**

(P) 

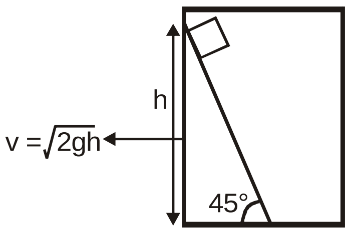
(1) Positive

(Q) 

(2) Negative

(R) 

(3) equal to mgh in magnitude

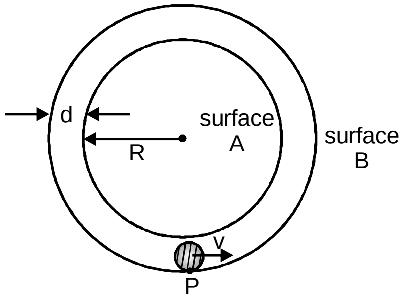
(S) 

(4) equal to zero

Codes :

	P	Q	R	S
(A)	1	4	2	3
(B)	1	1	4	2
(C)	4	2	1	3
(D)	4	1	2	3

45. A small spherical ball of mass m is projected from lowest point (point P) in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. Speed of ball at lowest point is v. N_A and N_B represent magnitudes of the normal reaction force on the ball exerted by the spheres A and B respectively. Match the value of v given in column–I with corresponding results in column–II.



Column–I	**Column–II**
(A) $v = \sqrt{gR}$	(p) maximum value of $N_A = 0$
(B) $v = \sqrt{2gR}$	(q) minimum value of $N_B = 0$
(C) $v = \sqrt{3gR}$	(r) maximum value of $N_B = 6$ mg
(D) $v = \sqrt{5gR}$	(s) maximum value of $N_B = 4$ mg
	(t) maximum value of $N_B = 2$ mg

ANSWER KEY OF DPP NO. # 03

1.	(C)	**2.**	(C)	3.	(B)	**4.**	(C)	**5.**	(D)	**6.**	(B)	**7.**	(B)
8.	(A)	**9.**	(A)	**10.**	(C)	**11.**	(D)	**12.**	(B)	**13.**	(D)	**14.**	(A)
15.	(4)	**16.**	(A,B,D)	**17.**	(A,B,C,D)			**18.**	(B,D)	**19.**	(A,B,C,D)	**20.**	(A,D)
21.	**9**	**22.**	2	**23.**	**5**	**24.**	**2**	**25.**	0	**26.**	2	**27.**	3
28.	52	**29.**	49	**30.**	300	**31.**	400	**32.**	(D)	**33.**	(D)	**34.**	(C)
35.	(B)	**36.**	(D)	**37.**	(B)	**38.**	(C)	**39.**	(B)	**40.**	(B)	**41.**	(B)
42.	(A)	**43.**	(B)	**44.**	(A)	**45.**	(D)						

PHYSICS

1. $(F_{net})_{ext} = \mu_k(2m)g = (m_{total})a_{cm}$

$a_{cm} = \mu_k g$

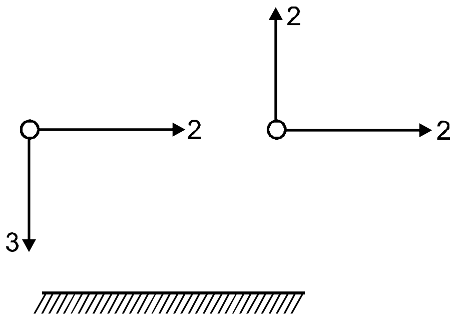
$S_{cm} = 0 + \frac{1}{2}(\mu_k g)t^2$

$S_{cm} = \frac{m_1S_1 + m_2S_2}{m_1 + m_2}$

$-\frac{1}{2}(\mu_k g)t^2 = \frac{(m)(x) + (m)(x - \ell)}{m + m}$

$x = \frac{\ell - (\mu_k g)t^2}{2}$.

2.



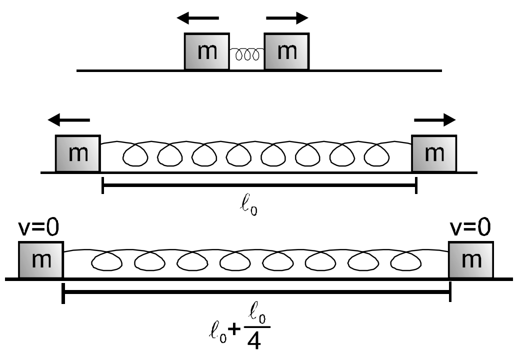
$e = \frac{v_{sep}}{v_{opp}} = \frac{2}{3}$.

3. As string does no work on the ball, energy conservation can be applied.

$$\frac{1}{2}mV^2 = mg\,(L - L\cos\theta) \Rightarrow \quad V = \sqrt{2gL\,(1 - \cos\theta)}$$

on putting values $V = \sqrt{10}$ m/s

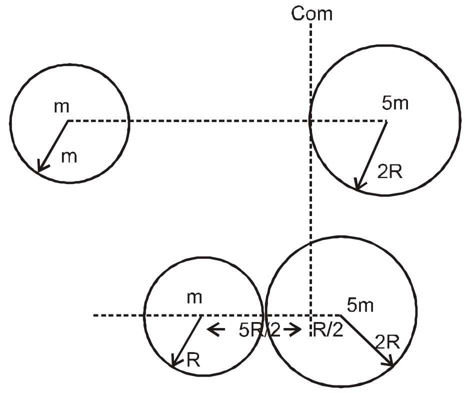
4.



Work energy theorem,

$$-\mu mg\left(\ell_0 + \frac{\ell_0}{4}\right) = \frac{1}{2}K\left(\frac{\ell_0}{4}\right)^2 - \frac{1}{2}K\ell_0^{\,2} \qquad \mu = \frac{3K\ell_0}{8mg} \qquad \textbf{Ans.}$$

5.



Distance covered by the smaller sphere = $10R - \frac{5R}{2} = \frac{15R}{2}$

6. As $\Delta m << M$

So, we can assume that motion of mass M will not be influenced by Δm. Now, when total force on mass M is zero, let the compression in the spring is x.

by energy conservation

$$\Rightarrow kx = mg \Rightarrow x = \frac{mg}{k}$$

Now, maximum downwards displacement of M

$$2x = 2\frac{Mg}{k}$$

As block Δm is connected to mass M so its maximum upward displacement = $\frac{2Mg}{k}$ **Ans.**

7. $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

m_1 = mass of square plate

= m

x_1 = c. m. of square plate = 0

m_2 = mass of removed part

$$= -\frac{m}{\ell^2}\left(\frac{\pi\frac{\ell^2}{4}}{2}\right) = -\frac{\pi}{8}m$$

x_2 = c.m. of removed part

$$= \frac{\ell}{2} - \frac{4}{3\pi}\left(\frac{\ell}{2}\right) = \frac{\ell}{2}\left(1 - \frac{4}{3\pi}\right)$$

$$\therefore \quad x_{cm} = \frac{-\frac{\pi m}{8}.\frac{\ell}{2}\left(1-\frac{4}{3\pi}\right)}{m - \frac{\pi}{8}m} \qquad x_{cm} = -\frac{\ell\left(\pi - \frac{4}{3}\right)}{2\,(8-\pi)}$$

8. $P = Fv$

$v^2 = Fv$

$F = v$

$Ma = v$

$$1 \times v\frac{dv}{dx} = v$$

$$\int_1^v dv = \int_0^x dx$$

$v - 1 = x$

$v = x + 1$

$$\frac{dx}{dt} = x + 1$$

$$\int_0^x \frac{dx}{x+1} = \int_0^t dt$$

$\ell n(x + 1) - \ell n(0 + 1) = t$

$x + 1 = e^t$

$x = e^t - 1$

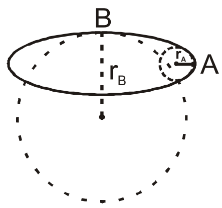
$x = e^t - 1$

at $t = \ell n2$

$x = 2 - 1 = 1$ m.

9. The horizontal component of velocity of sand just before falling on the cart is $v_s = 0$.
The horizontal speed of cart = v_C (constant).
The rate of mass falling on cart = μ (constant).
Horizontal force exerted by falling sand on cart = $\mu v_{rel} = \mu (v_c - v_s) = \mu v_c$
$\because$ μ and v_c are constant, the horizontal force is constant.

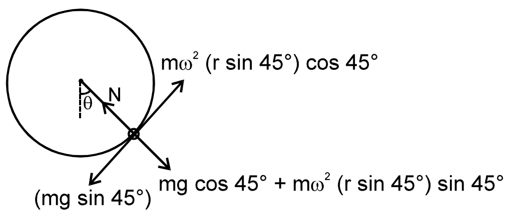
10. $$a_t = \frac{dv}{dt} = 0$$



$$a_c = \frac{v^2}{R}$$

From A to B radius of curvature increases
So, acceleration decreases.

11. The maximum angular speed of the hoop corresponds to the situation when the bead is just about to slide upwards.
The free body diagram of the bead is



For the bead not to slide upwards.

$m\omega^2$ (r sin 45°) cos 45° – mg sin 45° < μN .... (1)

where N = mg cos 45° + $m\omega^2$ (r sin 45°) sin 45° .... (2)

From 1 and 2 we get.

$$\omega = \sqrt{30\sqrt{2}} \text{ rad / s.}$$

12. If velocity of m_2 is zero then by momentum conservation

$m_1 v' = m_2 v$

$$v' = \frac{m_2 v}{m_1}$$

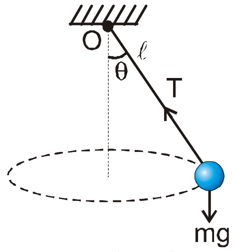
Now kinetic energy of m_1

$$= \frac{1}{2} m^1 v'^2 = \frac{1}{2} m_1 \left(\frac{m_2}{m_1}\right)^2 v^2 = \frac{1}{2}\left(\frac{m_2}{m_1}\right) m_2 v^2 = \left(\frac{m_2}{m_1}\right)\frac{1}{2} m_2 v^2 = \frac{m_2}{m_1} \times \text{initial Kinetic energy}$$

Kinetic energy of m_1 > initial mechanical energy of system

Hence proved

13. For conical pendulum of length ℓ, mass m moving along horizontal circle as shown



$T\cos\theta = mg$ .... (1)

$T\sin\theta = m\omega^2\ell\sin\theta$ .... (2)

From equation (1) and equation (2), $\ell\cos\theta = \dfrac{g}{\omega^2}$

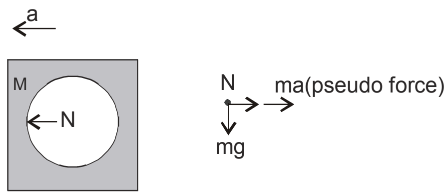
$\ell\cos\theta$ is the vertical distance of bob below O point of suspension. Hence if ω of all three pendulums are same, they shall revolve in same horizontal plane.

Alternate :

If we remember that time period T of conical pendulum is

$T = 2\pi\sqrt{\dfrac{L}{g}}$ where L is the vertical depth of bob below point of suspension. If T is same for three pendulums even L shall be also same. Hence all three particles shall revolve in same horizontal plane.

16. Let the normal force between the block and the ball be N.

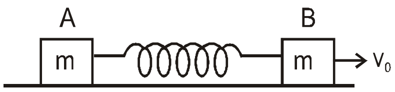


For the block, from Newton's IInd law , we have N = Ma = 2ma

For ball (with respect to the block), from Newton's IInd law , we have $N + ma = \dfrac{mv^2}{R}$

Solve the two equations.

17.



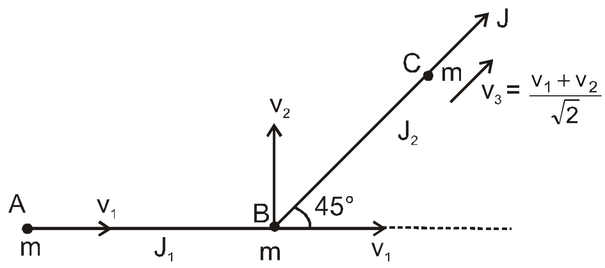
$1/2\, mv_0^2 = 1/2\, kx_0^2$

$v_{cm} = v_0/2 = \dfrac{1}{2}\sqrt{\dfrac{k}{m}}x_0$, $\quad 1/2\, kx^2_{max} = \dfrac{1}{2}\left(\dfrac{m}{2}\right)v_0^2$

$1/2\, kx^2_{max} = \dfrac{1}{2}\left(\dfrac{1}{2}kx_0^2\right) \quad \Rightarrow x_{max} = \dfrac{x_0}{\sqrt{2}}$

$(V_A)_{max} = (V_B)_{max} = v_0 = \sqrt{\dfrac{k}{m}}x_0$

18. $\frac{J_2}{\sqrt{2}} = 2mv_1 \quad \frac{J_2}{\sqrt{2}} = mv_2 \quad \Rightarrow \quad v_2 = 2\,v_1$



$J - J_2 = \frac{m}{\sqrt{2}}(v_1 + v_2)$

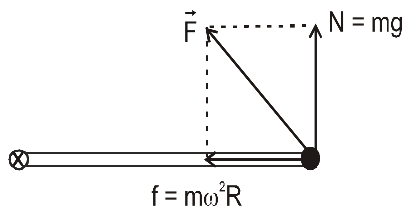
$J - 2\sqrt{2}\,mv_1 = \frac{3mv_1}{\sqrt{2}} \quad \Rightarrow J = \frac{7mv_1}{\sqrt{2}} \quad \Rightarrow v_1 = \frac{\sqrt{2}J}{7m}$

$v_2 = \frac{2\sqrt{2}J}{7m}$

$v_A = v_1 = \frac{\sqrt{2}J}{7m} \qquad v_B = \frac{\sqrt{10}J}{7m}$

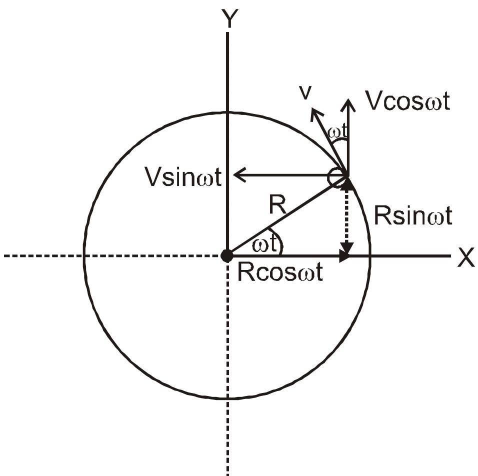
$v_c = \frac{v_1 + v_2}{\sqrt{2}} = \frac{3J}{7m}.$

19. $F = \sqrt{f^2 + (mg)^2}$



Now when the angular speed of the rod is increasing at const. rate the resultant force will be more inclined towards $\vec{f}$.
Hence the angle between $\vec{F}$ and horizontal plane decreases so as with the rod.

20.



So X component of velocity $V_x = -V\sin\omega t$
y component of force $F_y = -mv^2/R\sin\omega t = -m\omega^2 R\sin\omega t$
Angular velocity of particle ω = constant.
X–coordinate of the particle $x = R\cos\omega t$. So B, C, D are correctly matched

21. $\vec{F} = \frac{\overrightarrow{dp}}{dt}$ = acost $\hat{i}$ + asint $\hat{j}$

$\vec{F}.\vec{P}$ = 0

magnitude of momentum :

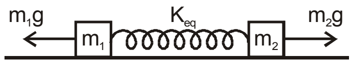
= $\sqrt{a^2\cos^2 t + a^2 \sin^2 t}$ = a

22. When speed of car is 40 km/hr, car can make a turn without skidding. If speed is less than 40 km/hr than tendency of slipping is downward so it will slip down. If speed is greater than 40 km/hr than tendency of slipping upward so it will slip up.

If the car's turn at correct speed 40 km/hr

$$N \sin\theta = \frac{mv^2}{r}, \qquad N = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)}$$ **Ans.**

23. Above situation can be represented as



Now at maximum elongation $v_{2/1} = 0$

Say at any moment elongation of spring is x

$a_{2/1} = 2g - k_{eq}x\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

$vdv = \left[2g - k_{eq}x\left(\frac{m_1+m_2}{m_1 m_2}\right)\right]dx$

$\Rightarrow \qquad x_{max} = \frac{4m_1m_2g}{K_{eq}(m_1+m_2)}$

Ans. $k_1x_1 = k_2x_2 = k_{eq}x$

24 N = mg cos θ + mω²R = mg cos θ + m $\left(\frac{5g}{R}\right)$R $\Rightarrow$ N = 5 mg + mg cosθ > 0 so block does not leave circular motion

$f_r = ma_T$ For limiting case $\mu N = ma_T$ $\Rightarrow$ μ (mg (5 + cos θ)) = mg sin θ $\Rightarrow \mu = \frac{\sin\theta}{5+\cos\theta}$

$$\frac{d\mu}{d\theta} = \frac{(5+\cos\theta)(\cos\theta) - \sin\theta(-\sin\theta)}{(5+\cos\theta)^2} = 0 \Rightarrow \cos\theta = -\frac{1}{5}$$

$$\mu_{max.} = \frac{\sqrt{1-\frac{1}{25}}}{5-\frac{1}{5}} = \frac{2\sqrt{6}}{24} \Rightarrow x = 8$$

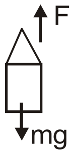
25. Force on table due to collision of balls :

$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2\text{ N}$$

Net force on one leg $= \frac{1}{4}$ (2 + 0.2 × 10) = 1 N

26. Thrust force

$F = \frac{dm}{dt} \cdot u_{rel} = 20 \times 1000$ N

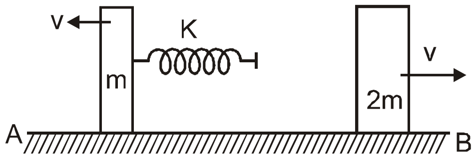


$F_{net} = F - mg$
$= 20000 - 10000$
$= 10000$ N

$\therefore \quad a = 10$ m/s^2

$\therefore \quad \frac{a}{g} = 1$

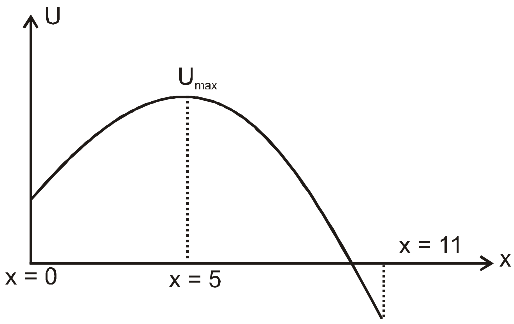
27. Situation after long time



Work done $= \Delta K = \frac{1}{2}(2m)(v)^2 - \frac{1}{2}(2m)(3v)^2 =$

$= mv^2 - 9mv^2 = -8mv^2$.

28. Draw U v/s x graph. There is a maxima of potential energy between x = 11 to x = 0. So to bring the particle from x = 11 to x = 0, the particle has to cross the maxima (x = 5) and to just cross the point x = 5, velocity at x = 5 should be 0^+



$\Rightarrow$ Applying energy conservation between x = 11 to x = 5.

$k_i + U_i = k_f + U_f$

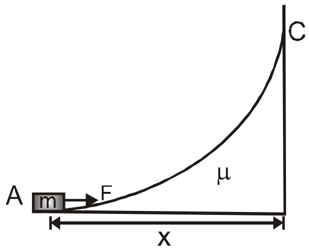
$\frac{1}{2}(0.5)\,u^2 + (30 - (11 - 5)^2) = 0^+ + (30 - (5 - 5)^2)$

u = 12 m/sec $\quad\Rightarrow\quad \frac{u}{2} = 6$ m/sec

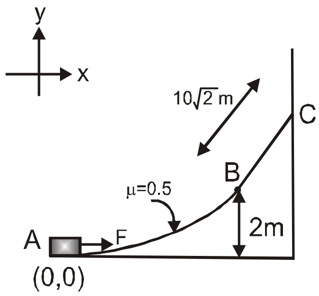
29. Since angular velocities of the particles are different, after some time, two particles may move parallel. In such case $\left|\vec{P}_A + \vec{P}_B\right|$ is maximum.

$\left|\vec{P}_A + \vec{P}_B\right|_{max} = (2 \times 2 + 1 \times 3)$ kg m/s = 7 kg m/s

30. Slope of line BC = $\frac{dy}{dx} = \frac{2x}{8} = \frac{2\times4}{8} = 1 \quad \Rightarrow \quad \theta = 45^{o}$



If the mass m is taken from A to C slowly work done by friction will always be equal to the $W_f = -\mu mgx$



Now, by $\quad W_{net} = \Delta KE = 0$

$W_F - mg(10 + 2) - \mu mg(10 + 4) = 0$

$\Rightarrow \quad W_F = 380 = 76 \times 5 \quad \Rightarrow \quad \lambda = 5.$

31. $a_t = g \sin 60^{o} = \frac{\sqrt{3}g}{2}$

$a_c = \frac{v^2}{R}$

$\frac{1}{2} mv^2 - mgR \cos 60 = \frac{1}{2} m (4gR) - mgR.$

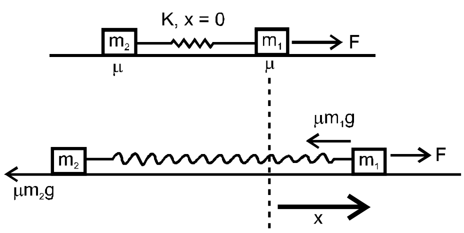
$\frac{1}{2} mv^2 = \frac{3}{2} mgR$

$v^2 = 3gR .$

$a_c = \frac{3gR}{R} = 3g, \quad a_t = \frac{\sqrt{3}g}{2}, a_c = 3g$

$P = \frac{a_c}{a_t} = \frac{2\times3g}{\sqrt{3}g} = 2\sqrt{3}$ **Ans.**

32.

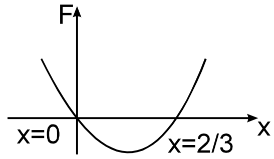


WF + WSp + Wfric = ΔK

$\Rightarrow \quad Fx - \frac{1}{2} Kx^2 - \mu m_1 g x = 0 \quad$ and $\quad Kx = \mu m_2 g$

$\Rightarrow \quad F - \frac{1}{2} \mu m_2 g - \mu m_1 g = 0 \quad \Rightarrow \quad F = \mu m_1 g + \frac{\mu m_2 g}{2} = 0.1 \times 10 \times 10 + \frac{0.1\times20\times10}{2} = 20\text{ N}$

33. The particle is at equilibrium at x = 0 and $x = \frac{2}{3}$.



The minimum speed imparted to the particle should be such that it just reaches $x = \frac{2}{3}$ from there on it shall automatically reach x = 0

$$\frac{1}{2}mv^2 = -\int_4^{2/3} F\,dx = -\int_4^{2/3} x(3x-2)dx = \frac{1300}{27} \quad \text{or} \quad v = \sqrt{\frac{2600}{27}}\ \text{m/s}$$

34. $MX = m(R - R\cos\theta - X] \Rightarrow \quad X = \frac{mR(1-\cos\theta)}{(M+m)}$

35. Momentum of system in horizontal will remain conserved, so

$$MV = m(v\sin\theta - V)$$

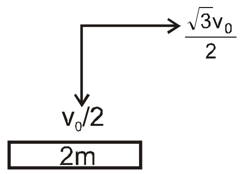
$$\Rightarrow \quad V = \frac{mv}{M+m}\sin\theta$$

$$\frac{dV}{dt} = \frac{mv}{M+m}\cos\theta\left(\frac{d\theta}{dt}\right)$$

$$\left(\frac{dV}{dt}\right) = \left(\frac{m}{M+m}\right)\frac{v^2}{R}\cos\theta$$

37 to 39.

JBC



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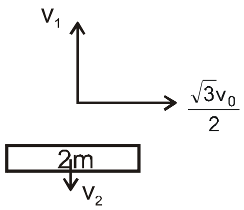


$v_1 + v_2 = V_0/2$

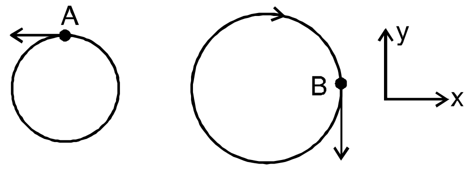
$2mv_2 - mv_1 = m\frac{v_0}{2} \quad \Rightarrow \quad v_1 = \frac{v_0}{6} \qquad v_2 = \frac{v_0}{3}$

Maximum compression $= \dfrac{v_2}{\sqrt{\dfrac{K}{2m}}} = \sqrt{\dfrac{2m}{k}}\,\dfrac{v_0}{3}$

Maximum height $= \dfrac{v_1^2}{2g} = \dfrac{v_0^2}{72g}$

minimum kinetic energy $= \dfrac{1}{2}m\left(\dfrac{\sqrt{3}v_0}{2}\right)^2 = \dfrac{3mv_0^2}{8}$

40. At time t = 1 sec positions of A and B are



acceleration of A $\vec{a}_A = \omega_1^2 r_1(-\hat{j})$ = $\left(\frac{\pi}{2}\right)^2 (1)(-\hat{j})$

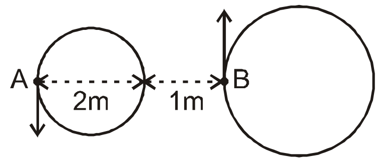
41. At time t = 1 sec

$\vec{a}_B = \omega_2^2 r_2 (-\hat{i})$ = $2\pi^2(-\hat{i})$

$\vec{a}_A - \vec{a}_B = \frac{\pi^2}{4}(-\hat{j}) + 2\pi^2(\hat{i})$

a_{rel} = $\pi^2\left[\frac{1}{16}+4\right]^{1/2}$ = $\frac{\pi^2}{4}\sqrt{65}$ m/sec^2

42. At time t = 2 sec, position of A and B are



$v_A = \omega_1 r_1 = \frac{\pi}{2}(1) = \frac{\pi}{2}$ m/sec.
$v_B = \omega_2 r_2 = 2\pi$ m/sec.
distanceAB = 3m

$\omega = \frac{v_A + v_B}{AB}$ = $\frac{\pi/2 + 2\pi}{3}$ = $\frac{5\pi}{6}$ rad/sec.

43. (P) Since external force in horizontal direction is zero there for COM remains at rest.
(Q) If the block remains at rest then centre of mass moves with constant velocity.
(R) If m does not slips on M then COM remains at rest otherwise COM is accelerated when m moves from point A to B.
(S) The COM is accelerated vertically downwards by the gravity force.

45. Ball only loose contact with surface B when v is in range $\sqrt{2Rg} < v < \sqrt{5Rg}$ so for A,B,D maximum value of N_A is zero for option C ball lose contact with surface B at some point.
maximum value of N_B is lowest point and given

$$N = mg + \frac{mv^2}{R}.$$