CHAPTER 15

Probability

1. A and B are events such that $P(A \cup B)=3/4$, $P(A \cap B)=1/4$, $P(\overline{A})=2/3$ then $P(\overline{A} \cap B)$ is **[2002]**

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4

2. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$ The set of possible values of x are in the interval. **[2003]**

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$
(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

3. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

4. Let A and B be two events such that $P(\overline{A \cup B})=\frac{1}{6}$, $P(A \cap B)=\frac{1}{4}$ and $P(\overline{A})=\frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are **[2005]**

(a) equally likely and mutually exclusive
(b) equally likely but not independent
(c) independent but not equally likely
(d) mutually exclusive and independent

5. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[2008]**

(a) $\frac{3}{5}$ (b) 0
(c) 1 (d) $\frac{2}{5}$

6. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, ...20\}$. **[2010]**

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $(\pm1, \pm2, \pm3, \pm4, \pm5)$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1
(b) Statement -1 is true, Statment -2 is false
(c) Statement -1 is false, Statment -2 is true.
(d) Statement -1 is true, Statement -2 is true ; Statement -2 is a correct explanation for Statement -1.

7. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is : **[2012]**
   (a) 880 (b) 629
   (c) 630 (d) 879

8. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$.
   Then the probability that at least one of the events occurs, is : **[2017]**
   (a) $\frac{3}{16}$ (b) $\frac{7}{32}$
   (c) $\frac{7}{16}$ (d) $\frac{7}{64}$

Answer Key

1	2	3	4	5	6	7	8							
(a)	(b)	(a)	(c)	(c)	(b)	(d)	(c)							

SOLUTIONS

1. **(a)** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$

$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3}$;

Now, $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$.

2. **(b)** $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$,

$P(C) = \frac{1-2x}{2}$

$\because$ For any event $E, 0 \le P(E) \le 1$

$\Rightarrow 0 \le \frac{3x+1}{3} \le 1, \quad 0 \le \frac{1-x}{4} \le 1$

and $0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow -1 \le 3x \le 2, -3 \le x \le 1$ and $-1 \le 2x \le 1$

$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3} \le -3 \le x \le 1$, and

$-\frac{1}{2} \le x \le \frac{1}{2}$

Also for mutually exclusive events A, B, C,

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$

$\therefore 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$

$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$

Considering all inequations, we get

$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$

$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

3. **(a)** Let 5 horses are H_1, H_2, H_3, H_4 and H_5. Selected pair of horses will be one of the 10 pairs (i.e.; ${}^5C_2$): $H_1 H_2$, $H_1 H_3$, $H_1 H_4$, $H_1 H_5$, $H_2 H_3$, $H_2 H_4$, $H_2 H_5$, $H_3 H_4$, $H_3 H_5$ and $H_4 H_5$.
Any horse can win the race in 4 ways.
For example : Horses H_2 win the race in 4 ways $H_1 H_2$, $H_2 H_3$, $H_2 H_4$ and $H_2 H_5$.

Hence required probability $= \frac{4}{10} = \frac{2}{5}$

4. **(c)** $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and

$P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = \frac{5}{6},\ P(A) = \frac{3}{4}$

Also

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$

$\Rightarrow P(A)\ P(B) = \frac{3}{4} - \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

Hence A and B are independent but not equally likely.

5. **(c)** $A \equiv$ number is greater than 3

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

$B \equiv$ number is less than 5

$\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$A \cap B \equiv$ number is greater than 3 but less than 5.

$\Rightarrow P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = 1$$

6. **(b)** $n(S) = {}^{20}C_4$

Statement-1:

common difference is 1; total number of cases = 17

common difference is 2; total number of cases = 14

common difference is 3; total number of cases = 11

common difference is 4; total number of cases = 8

common difference is 5; total number of cases = 5

common difference is 6; total number of cases = 2

$$\text{Prob.} = \frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}$$

Statement -2 is false, because common difference can be 6 also.

7. **(d)** Number of white balls = 10

Number of green balls = 9

and Number of black balls = 7

$\therefore$ Required probability

$= (10+1)(9+1)(7+1) - 1$

$= 11.10.8 - 1 = 879$

[$\because$ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is

$(p+1)(q+1)(r+1) - 1$]

$$= \frac{64}{127}$$

8. **(c)** P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ...(1)

P (Exactly one of B or C occurs)

$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ...(2)

P (Exactly one of C or A occurs)

$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ...(3)

Adding (1), (2) and (3), we get

$2\Sigma P(A) - 2\Sigma P(A \cap B) = \frac{3}{4}$

$\therefore \Sigma P(A) - \Sigma P(A \cap B) = \frac{3}{8}$

Now, $P(A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A \cup B \cup C)$

$= \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$