TRIGONOMETRIC EQUATION

LEVEL-I

1. If $\sin(\pi\cos\theta) = \cos(\pi \sin\theta)$, then $\sin 2\theta$ may take value
   (A) 3/4 (B) –3/4
   (C) 1/4 (D) None of these

2. General solution to the equation $\tan^2\theta + \cos 2\theta - 1 = 0$ will be given by
   (A) $\theta = n\pi$ (B) $\theta = 2n\pi + \pi/4$
   (C) $\theta = n\pi + \pi/4$ (D) $\theta = 2n\pi - \pi/4$

3. If $\sin \alpha = p$ then the equation whose solution is $\tan \frac{\alpha}{2}$ is
   (A) $px^2 + 2xp - 1 = 0$ (B) $px^2 + 2x - p = 0$
   (B) $x^2 + 2x - p = 0$ (D) None of these

4. If $\tan(\cot x) = \cot(\tan x)$ then $\sin 2x$ is equal to
   (A) $\frac{2}{(2n+1)\pi}$ (B) $\frac{4}{(2n+1)\pi}$
   (C) $\frac{2}{n(n+1)\pi}$ (D) $\frac{4}{n(n+1)\pi}$

*5. If $\sin^{-1}x + \tan^{-1}x = \frac{\pi}{2}$, then $2x^2 + 1 =$
   (A) $\sqrt{5}$ (B) $\frac{\sqrt{5}-1}{2}$
   (C) 2 (D) none of these

6. Solution set of the equation $\sin^2 x + \cos^2 3x = 1$ is given by
   (A) $\left\{\frac{n\pi}{4},\ n \in I\right\}$ (B) $\left\{\frac{n\pi}{2},\ n \in I\right\}$
   (C) $\{n\pi,\ n \in I\}$ (D) none of these

*7. The difference between the roots in the first quadrant ($0 \le x \le \pi/2$) of the equation $4\cos x(2 - 3\sin^2 x) + (\cos 2x + 1) = 0$ is
   (A) $\pi/6$ (B) $\pi/4$
   (C) $\pi/3$ (D) $\pi/2$

8. The value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right)$ is equal to
   (A) $\frac{7}{17}$ (B) $\pi - \frac{7}{17}$
   (C) $-\frac{7}{17}$ (D) none of these

*9. The set of values of a for which $x^2 - ax - \sin^{-1}(\sin 3) > 0$ for all $x \in R$ is

(A) R (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) ϕ (D) none of these

10. If $\sin^{-1}(\sin x) = \pi - x$, then x belongs to

(A) $(-\infty, \infty)$ (B) $[0, \pi]$

(C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (D) $[\pi, 2\pi]$

*11. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$ then $x^2 + y^2 + z^2 - xy - yz - zx$ equals to

(A) 0 (B) 1

(C) 2 (D) 3

12. The number of real solutions of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

(A) zero (B) one (C) two (D) infinite

*13. If $x \geq 1$, then $2\tan^{-1} x + \sin^{-1}\frac{2x}{1+x^2}$ is equal to

(A) $4\tan^{-1} x$ (B) π

(C) 0 (D) None of these

14. If $\sin x + \sin y + \sin z = 3$, $x, y, z \in [0, 2\pi]$, then

(A) $x^2 + y^2 + z^2 - xy - yz - zx = 0$ (B) $x^3 + y^3 + z^3 = \pi^3/3$

(C) $x^3 + y^3 + z^3 = 0$ (D) $x + y + z = 0$

15. If $\cos\theta_1 + 2\cos\theta_2 + 3\cos\theta_3 = 6$ then $\tan\theta_1 + \tan\theta_2 + \tan\theta_3$ equals to

(A) 1/2 (B) 6

(C) 0 (D) 3

*16. The equation $e^{\sin x} + e^{-\sin x} = 2\sin x$ will have

(A) no solution (B) one solution

(C) two solution (D) none of these

17. If $1 + \tan\theta = \sqrt{2}$ then $\cos\theta - \sin\theta$ equals to ($\theta \neq (2n+1)\pi/2$)

(A) $2\sin\theta$ (B) $\sqrt{2}\sin\theta$

(C) $\sqrt{2}\cos\theta$ (D) $2\cos\theta$

18. Value of $\cos(2\cos^{-1}(4/5))$ equals to

(A) 6/25 (B) 7/25

(C) 4/25 (D) 8/25

19. If $4\cos^{-1} x + \sin^{-1} x = \pi$ then x equals to

(A) 1/2 (B) $1/\sqrt{2}$

(C) 1 (D) $\sqrt{3}/2$

*20. Number of solution to the equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

(A) one (B) two
(C) four (D) none of these.

*21. The solutions of the equation $(\cos^2 x - 4\sin x + 6)(1-\sin x) = \cos^2 x$ are

(A) $2n\pi$, $n \in I$ (B) $(4n-1)\pi/2$, $n \in I$
(C) $(4n+1)\frac{\pi}{2}$, $n \in I$ (D) None of these

22. If $\tan^2\frac{x}{2} = \sec x - \cos x$, then

(A) $x = 2n\pi$, $n \in I$ (B) $x = (2n+1)\pi$, $n \in I$
(C) $x = (2n+1)\frac{\pi}{2}$, $n \in I$ (D) None of these

23. The inequality $\log_2 x < \sin^{-1}(\sin 5)$ holds if

(A) $x \in (0, 2^{5-2\pi})$ (B) $x \in (2^{5-2\pi}, \infty)$
(C) $x \in (2^{2\pi-5}, \infty)$ (D) None of these

24. The value(s) of y for which the equation $4\sin x + 3\cos x = y^2 - 6y + 14$ has a real solution, is (are)

(A) 3 (B) 5
(C) –3 (D) None of these.

25. The most general values of x for which $\sin x + \cos x = \min.\{y^2 - 10y + 26, y^2 - 6y + 12\}$, $\forall\ y \in R$, are given by

(A) $[2n\pi, (2n+1)\pi]$ (B) $[n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}, (n+1)\pi]$
(C) $[2n\pi, (4n+1)\frac{\pi}{2}]$ (D) None of these

26. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then $x^3 + y^3 + z^3$ is equal to

(A) –3 (B) 3
(C) 0 (D) None of these

*27. The solution(s) x, of the equation $\sqrt{3}\cos x - \sin x = (\cos^{10}y + \sec^{10}y)$, is (are)

(A) $\pi/6$ (B) $-\pi/6$
(C) $-\pi/3$ (D) $\pi/3$

28. If $\cot x \cot y = k$ and $x + y = \pi/3$, then $\tan x$, $\tan y$ satisfy the equation

(A) $kt^2 - \sqrt{3}(k-1)t + 1 = 0$ (B) $kt^2 + \sqrt{3}(k-1)t + 1 = 0$
(C) $kt^2 - \sqrt{3}(k+1)t + 1 = 0$ (D) $kt^2 + \sqrt{3}(k+1)t + 1 = 0$

29. If $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)(1+\tan^2 2y)(3+\sin 3z) = 4$, then

(A) x may be a multiple of π (B) x can not be an even multiple of π
(C) z can be a multiple of π (D) y can be a multiple of $\pi/2$.

30. $\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$. Then θ is equal to
(A) $\pi/12$ (B) $5\pi/12$
(C) $-3\pi/12$ (D) $-7\pi/12$

31. If $-1 < x < 0$ then $\tan^{-1}x$ equals

(A) $\pi - \cos^{-1}\left(\sqrt{1-x^2}\right)$ (B) $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(C) $-\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ (D) $\text{cosec}^{-1} x$

32. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by
(A) $\left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (B) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (C) $\left(\frac{\pi}{10}, -\frac{3\pi}{10}\right)$ (D) none of these.

33. The number of roots of the equation $x + 2\tan x = \pi/2$ in the interval $[0, 2\pi]$ is
(A) 1 (B) 2 (C) 3 (D) infinite

34. The general solution of the equation $\sin x + \cos x = 1$, for $n = 0, \pm 1, \pm 2, \ldots\ldots$ is
(A) $x = 2n\pi$ (B) $x = 2n\pi + \frac{1}{2}\pi$
(C) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$ (D) none of these

35. The solution set of $(2\cos x - 1)(3 + 2\cos x) = 0$ in the interval $0 \le x \le 2\pi$ is
(A) $\left\{\frac{\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$
(C) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \cos^{-1}(-3/2)\right\}$ (D) none of these.

36. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is
(A) 0 (B) 1
(C) 2 (D) 3

37. The general solution of the equation $\tan^2\theta + 2\sqrt{3}\tan\theta = 1$ is given by

(A) $\theta = \frac{\pi}{2}$ (B) $\theta = \left(n + \frac{1}{2}\right)\pi$

(C) $\theta = (6n+1)\frac{\pi}{12}$ (D) $\frac{n\pi}{12}$

38. The general solution of sinx - 3sin2x + sin3x = cosx - 3cos2x + cos3x is

(A) $n\pi + \frac{\pi}{8}$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}$

(C) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (D) $2n\pi + \cos^{-1}(3/2)$

*39. The value of $\tan[\cos^{-1} 4/5 + \tan^{-1} 2/3]$ or $\tan[\sin^{-1}(3/5) + \cot^{-1} 3/2]$ is

(A) 6/17 (B) 7/16

(C) 17/6 (D) none of these.

40. The principal value of $\sin^{-1}(\sin\frac{2\pi}{3})$ is

(A) $-2\pi/3$ (B) $2\pi/3$

(C) $4\pi/3$ (D) None of these

*41. If $1 + |\sin x| + \sin^2 x + |\sin^3 x| + ... \infty = 4 + 2\sqrt{3}$, then

(A) $x = \frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $x = \frac{2\pi}{3}$ (D) $x = \frac{5\pi}{6}$

42. The number of ordered pair (x, y), where x and y satisfy $x + y = 2\pi/3$ and $\cos x + \cos y = 3/2$ is

(A)0 (B)1

(C)2 (D) infinity

43. The number of solutions of $\cos^2\theta + \sin\theta + 1 = 0$, is $(\theta \in [0, 2\pi])$

(A) 0 (B)1

(C) 2 (D) infinity

44. If $\sin^{-1}x > \cos^{-1}x$, then

(A) $x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$ (B) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$

(C) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (D) $x \in \left(-\frac{1}{\sqrt{2}}, 0\right)$

45. The set of all values of x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$ contains

(A)$[0, \pi/6]$ (B)$[0, \pi/3]$

(C)$[2\pi/3, \pi]$ (D) $[0, \pi/6] \cup \{\pi/2\} \cup [5\pi/6, \pi]$

*46. If the expression $\frac{\sin(x/2)+\cos(x/2)+i\tan x}{1+2i\sin(x/2)}$ is real If x belong to the set

(A) {$n\pi$: $n\in I$}
(B) {$2n\pi$: $n\in I$}
(C) {$n\pi+\pi/4$: $n\in I$}
(D) {$2n\pi+\pi/4$: $n\in I$}

47. sinx, sin2x, sin3x are in A.P. if (for $n \in I$)

(A) $x = \frac{n\pi}{2}$
(B) $x = n\pi$
(C) $x = 2n\pi$
(D) $x = \frac{n\pi}{3}$

*48. sinx cosx cos 2x = k has a solution, if k belong to the interval

(A)[0, 1]
(B)[-1,0]
(C) [$-\pi/2$, $\pi/2$]
(D) [-1/4, ¼]

LEVEL-II

1. The values of x in $[0, 2\pi]$ which satisfy the equation $2^{1+|\sin x|+|\sin 2x|+|\sin 3x|+\ldots} = 2$ are
   (A) 0 (B) π
   (C) 2π (D) $3\pi/2$

2. The values of θ in the interval $(-\pi/2, \pi/2)$ satisfying the equation $\left(\sqrt{3}\right)^{\sec^2\theta} = \tan^4\theta + 2\tan^2\theta$ is
   (A) $\pi/4$ (B) $-\pi/4$
   (C) π (D) none of these

*3. $\tan^{-1}\left(\frac{\sin 1 - 1}{\cos 1}\right)$ equals
   (A) 0 (B) $1 - \frac{\pi}{2}$
   (C) $\frac{\pi}{2} - 1$ (D) $\frac{1}{2} - \frac{\pi}{4}$

*4. The value of x that satisfies the equation $\tan^2 x = \tan^{-1}(\tan 3)$ is
   (A) $\pi/3$ (B) $-\pi/3$
   (C) $\sqrt{\tan^{-1} 3}$ (D) none of these

5. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$, then the number of ordered pairs (x, y) is
   (A) 0 (B) 1
   (C) 2 (D) none of these

6. The number of real solutions of $\cos^{-1}x + \cos^{-1} 2x = -\pi$ is
   (A) 0 (B) 1
   (C) 2 (D) infinitely many

7. $\sin x + \cos x = y^2 - y + a$ has no value of x for any y if 'a' belongs to
   (A) $(0, \sqrt{3})$ (B) $(-\sqrt{3}, 0)$
   (C) $(-\infty, -\sqrt{3})$ (D) $(\sqrt{3}, \infty)$

8. The values of k, for which the system of equations $\cos x \cos 2y = (k^2 - 4)^2 + 1$ and $\sin x \sin 2y = k + 2$ holds, is (are) given by
   (A) $k = \pm 2$ (B) $k = -2$
   (C) $k = 2$ (D) none of these

9. The value of $\tan[\sin^{-1}(\cos(\sin^{-1}x))]\tan[\cos^{-1}(\sin(\cos^{-1}x))]$, $(x \in (0, 1))$ is equal to
   (A) 0 (B) 1
   (C) –1 (D) none of these.

10. The value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$, for $0 < A < \pi/4$, is

(A) $\tan^{-1} 2$ (B) $\tan^{-1}(\cot A)$
(C) $4\tan^{-1}(1)$ (D) $2\tan^{-1}(2)$

*11. The value of a for which the equation $4\operatorname{cosec}^2(\pi (a + x)) + a^2 - 4a = 0$ has a real solution, is

(A) a = 1 (B) a = 2
(C) a = 10 (D) None of these

*12. $\cos\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \cos\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$ is equal to

(A) $\pm\sqrt{\frac{a+b}{b}}$ (B) $\sqrt{\frac{b}{a+b}}$
(C) $\sqrt{\frac{a+b}{b}}$ (D) None of these

13. If $2\sin^{-1}x = \cos^{-1}(1 - 2x^2)$, then

(A) $-1 \le x \le 1$ (B) $-1 \le x \le 0$
(C) $x = 1/\sqrt{2}$ (D) $0 \le x \le 1$

14. If $\sqrt{1-\sin A} = \sin\frac{A}{2} - \cos\frac{A}{2}$, then $\frac{A}{2} - \frac{\pi}{4}$ could lie in quadrant

(A) first (B) second
(C) third (D) fourth

15. General solution to the equation $\tan^2\theta + \cos 2\theta - 1 = 0$ will be given by

(A) $\theta = n\pi$ (B) $\theta = 2n\pi + \pi/4$
(C) $\theta = n\pi + \pi/4$ (D) $\theta = 2n\pi - \pi/4$

16. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$, $x \in [0, \pi]$, then

(A) $x = \pi/4$ (B) $y = 0$ (C) $y = 1$ (D) $x = 3\pi/4$.

17. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

(A) 1 (B) $2 - \frac{1}{\sqrt{2}}$ (C) $2^{-1/\sqrt{2}}$ (D) $2^{1-\frac{1}{\sqrt{2}}}$

*18. The number of solutions of the equation

$\tan^{-1}\frac{1}{2x+1} + \tan^{-1}\frac{1}{4x+1} = \tan^{-1}\frac{2}{x^2}$ is

(A) 1 (B)2
(C)3 (D) 4

*19. The value of $\tan^{-1}\left(\frac{a_1x-y}{a_1y+x}\right)+\tan^{-1}\frac{a_2-a_1}{1+a_1a_2}+\tan^{-1}\frac{a_3-a_2}{1+a_2a_3}+\ldots+\tan^{-1}\frac{a_n-a_{n-1}}{1+a_na_{n-1}}+\tan^{-1}\frac{1}{a_n}$ is

(A) 0 (B) 1

(C) $\tan^{-1}\frac{x}{y}$ (D) $\tan^{-1}\frac{y}{x}$

*20. If sinx + cosx = 1 + sinx cosx, then

(A) $\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ (B) $\sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

(C) $\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ (D) $\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

21. If $\alpha \le \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \le \beta \quad \forall\, x \in (0, 1]$ then

(A) $\alpha = 0$, $\beta = \pi/2$ (B) $\alpha = 0$, $\beta = \pi$

(C) $\alpha = \pi/2$, $\beta = \pi$ (D) $\alpha = \pi/2$, $\beta = \pi$

LEVEL-III

1. If all the solutions 'x' of $a^{\cos x} + a^{-\cos x} = 6$ $(a > 1)$ are real, then set of values of a is
   (A) $[3+2\sqrt{2}, \infty)$ (B) (6, 12)
   (C) $(1, 3 + 2\sqrt{2})$ (D) none of these.

2. The value of $\sin^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right\}$ is
   (A) 0 (B) $\pi/4$
   (C) $\pi/6$ (D) $\pi/2$

3. The number of integral values of p for which the equation cos (psinx) = sin(p cosx) has a solution in $[0, 2\pi]$ is
   (A) 1 (B) 2
   (C) 3 (D) none of these

4. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x equals to
   (A) -1 (B) 1
   (C) 0 (D) none of these

*5. The number of points inside or on the circle $x^2 + y^2 = 4$ satisfying $\tan^4 x + \cot^4 x + 1 = 3\sin^2 y$ is
   (A) one (B) two
   (C) four (D) infinite

*6. If $\cos\left[\pi\left\{\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{3}\right)\right\}\right] = 0$, then x is
   (A) $n\pi + \pi/4$, $n \in I$. (B) $n\pi - \pi/2$, $n \in I$.
   (C) $n\pi - \pi/4$, $n \in I$. (D) none of these

7. Indicate the relation which is true
   (A) $\tan|\tan^{-1}x| = |x|$ (B) $\cot|\cot^{-1}x| = x$ (C) $\tan^{-1}|\tan x| = |x|$
   (D) $\sin|\sin^{-1}x| = |x|$

8. The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. with common difference
   (A) $\pi/4$ (B) $\pi/8$ (C) $3\pi/8$ (C) $5\pi/8$

9. In a triangle ABC, the angle B is greater than angle A. If the values of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the value of C is
   (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$
   (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

10. If $A = 2\tan^{-1}(2\sqrt{2} - 1)$ and $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$, then
(A) A = B (B) A < B
(C) A > B (D) none of these

*11. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable, has real roots. Then the interval of p may be
(A) $(0, 2\pi)$ (B) $(-\pi, 0)$
(C) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (D) $(0, \pi)$

ANSWERS

LEVEL –I

1.	A	2.	A,C	3.	D	4.	B
5.	A	6.	A	7.	A	8.	C
9.	C	10.	C	11.	A	12.	C
13.	B	14.	A	15.	C	16.	A
17.		18.	B	19.	D	20.	A
21.	C			22.	A		
23.	A	24.	A				
25.	D	26.	A				
27.	B	28.	A				
29.	A,D	30.	A, B, C, D	31.	B	32.	A
33.	C	34.	C	35.	B	36.	C
37.	C	38.		39.	C	40.	D
41.	B, C	42.	A	43.	B	44.	C
45.	D	46.	B,C	47.	A, C	48.	D

LEVEL –II

1.	A, B, C	2.	A, B	3.	D	4.	D
5.	B	6.	A	7.	D	8.	B
9.	B	10.	C	11.	C	12.	C
13.	D	14.	A,B	15.		16.	A, C
17.	D	18.	B	19.	C	20.	A, D
21.							

LEVEL –III

1.	A	2.	A	3.	D	4.	A
5.	C	6.		7.	A, B, D	8.	A
9.	C	10.	C	11.	D		