TEST INFORMATION

DATE : 06.05.2015 **PART TEST (PT-04)**

Syllabus : Vectors & Three Dimensional Geometry, Definite Integration & Its Application, Indefinite Integration

REVISION DPP OF DIFFERENTIAL EQUATION AND COMPLEX NUMBER

Total Marks : 142	**Max. Time : 110 min.**	
Single choice Objective (–1 negative marking) Q. 1 to 12	**(3 marks 2.5 min.)**	**[36, 30]**
Multiple choice objective (–1 negative marking) Q. 13 to 32	**(4 marks, 3 min.)**	**[80, 60]**
Comprehension (–1 negative marking) Q. 33 to Q.38	**(3 marks 2.5 min.)**	**[18, 15]**
Single digit type (no negative marking) Q. 39 to 40	**(4 marks 2.5 min.)**	**[8, 5]**

1. If z_1 & z_2 are two complex numbers satisfying $|z - 4| = \text{Re}(z)$ and having greatest and least argument respectively, then area of triangle formed by origin, z_1 & z_2 is
(A) 10 sq. units (B) 12 sq. units (C) 16 sq. units (D) 20 sq. units

2. The number of ordered pairs (a, b) of real numbers such that $(a + ib)^{2015} = a - ib$ is
(A) 2015 (B) 2014 (C) 2016 (D) 2017

3. If the complex number $z \neq 0, 1$, then the area of the quadrilateral with vertices z, $\bar{z}$, $\frac{1}{z}$, $\frac{1}{\bar{z}}$ is
(A) $\frac{1}{4}|z^2 - \bar{z}^2|$ (B) $\frac{|z|^4 + 1}{4|z|^2}$ (C) $\frac{1}{4}\cdot\frac{\left||z|^4 - 1\right|}{|z|^2}$ (D) $\frac{1}{4}\cdot\left|1 - \frac{1}{|z|^4}\right|\cdot|z^2 - \bar{z}^2|$

4. If $\omega = e^{i\frac{2\pi}{3}}$ then last digit of the value of $(1 + \omega)(1 + \omega^2) \ldots\ldots (1 + \omega^{1988})$ is
(A) 2 (B) 4 (C) 6 (D) 8

5. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| = 1$, then minimum value of $|z_1 + 1| + |z_2 + 1| + |z_1z_2 + 1|$ is
(A) 1 (B) 2 (C) 4 (D) 3

6. The solution of differential equation $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$ is
(A) $\sin(x^2y^2) - 1 = 0$ (B) $\cos\left(\frac{\pi}{2} + x^2y^2\right) + x = 0$
(C) $\sin(x^2y^2) = e^{x-1}$ (D) $\sin(x^2y^2) = e^{2(x-1)}$

7. If $\frac{dy}{dx}\left(\frac{1 + \cos x}{y}\right) = -\sin x$ and $f\left(\frac{\pi}{2}\right) = -1$, then f(0) is
(A) 2 (B) 1 (C) 3 (D) 4

8. The solution of differential equation $yy' = x\left(\frac{y^2}{x^2} + \frac{f(y^2/x^2)}{f'(y^2/x^2)}\right)$ is
(A) $f(y^2/x^2) = cx^2$ (B) $x^2 f(y^2/x^2) = c^2y^2$ (C) $x^2 f(y^2/x^2) = c$ (D) $f(y^2/x^2) = \frac{cy}{x}$

9. A tangent to a curve intersects the y-axis at a point P. A line perpendicular to this tangent and passing through P also passes through (1, 0). The differential equation of the curve is
(A) $yy' - x(y')^2 = 1$ (B) $yy' + x = 1$ (C) $xy'' + (y')^2 = 0$ (D) $yy' + (y'')^2 = 0$

10. A differentiable function f(x) satisfies the equation $(x + 1) f'(x) - 2(x^2 + x) f(x) = \frac{e^{x^2}}{x+1}$. If f(0) = 5, then f(x) =

(A) $\left(\frac{3x+5}{x+1}\right)e^{x^2}$ (B) $\left(\frac{6x+5}{x+1}\right)e^{x^2}$ (C) $\left(\frac{6x+5}{(x+1)^2}\right)e^{x^2}$ (D) $\left(\frac{5-6x}{x+1}\right)e^{x^2}$

11. If ω is a complex number such that $|\omega| = r \neq 1$, and $z = \omega + \frac{1}{\omega}$, then locus of z is a conic. The distance between the focii of conic is

(A) 2 (B) $2\left(\sqrt{2}-1\right)$ (C) 3 (D) 4

12. Two regular polygon are inscribed in the same circle. The first polygon has 1982 sides and the second has 2973 sides. If the polygons have any common vertices, then total number of such common vertices is

(A) 989 (B) 1 (C) 991 (D) 992

13. For $|z - 1| = 1$, $\arg\left(\tan\left(\frac{\arg(z-1)}{2}\right) - \frac{2i}{z}\right) =$

(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) $-\frac{3\pi}{2}$

14. Let z_1 and z_2 are two complex numbers such that $(1 - i)z_1 = 2z_2$ and $\arg(z_1z_2) = \frac{\pi}{2}$, then $\arg(z_2)$ can be equal to

(A) $3\pi/8$ (B) $\pi/8$ (C) $5\pi/8$ (D) $-7\pi/8$

15. If z = x + iy such that |z| = 4, then possible values of $|\,\mathrm{Re}(z) - \mathrm{Im}(z)|$ is/are

(A) 1 (B) 4 (C) $\frac{11}{2}$ (D) 6

16. The argument of a root of the equation $z^6 + z^3 + 1 = 0$ can be

(A) 320° (B) 120° (C) 160° (D) 280°

17. Let z_1 and z_2 are two non-zero complex numbers such that $\left|\frac{z_1}{z_2}\right| = 2$ and $\arg(z_1z_2) = \frac{3\pi}{2}$. If z represents the centroid of triangle formed by complex numbers $\frac{\bar{z}_1}{z_2}$, ω and ω^2 (where ω is imaginary cube root of unity), then arg(z) lies in

(A) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{2\pi}{3}, \pi\right)$ (C) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{7\pi}{12}, \frac{5\pi}{6}\right)$

18. If z_1 and z_2 two distinct complex numbers and z is a complex which lies on the line joining z_1 & z_2, then

(A) $\frac{z-z_1}{z_2-z_1}$ is a real number

(B) $\arg\left(\frac{z-z_1}{z_2-z_1}\right) = 0$

(C) there exist a real number t for which $z = (1 - t)z_1 + tz_2$

(D) $\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$

19. Two curves C_1 and C_2 are represented by $\arg\left(\frac{z+i}{z+1}\right) = \pm\frac{\pi}{4}$ and $\left|\frac{z+i}{z+1}\right| = 1$ respectively, then which of the following is/are true?

(A) Both C_1 and C_2 pass through (1, 1)

(B) Area bounded by curve C_1 is $\left(\frac{3\pi}{2}+1\right)$ square units

(C) Both curves are orthogonal

(D) Curve C_2 bisects the area bounded by curve C_1

20. If $\left|\frac{6z - i}{2 + 3iz}\right| \le 1$, then which of the following can't be possible?

(A) $|z| = \frac{1}{2}$ (B) $|z| = \frac{1}{3}$ (C) $|z| = 1$ (D) $|z| = \frac{1}{4}$

21. Let $\omega = e^{i\frac{2\pi}{2n+1}}$, $n \ge 1$ and $z = \frac{1}{2} + \omega + \omega^2 + ... + \omega^n$, then

(A) $\text{Re}(z^{2k}) = 0, k \in N$ (B) $\text{Im}(z^{2k}) = 0, k \in N$

(C) $(2z + 1)^{2n+1} + (2z - 1)^{2n+1} = 0$ (D) $\text{Re}(z^{2k+1}) = 0, k \in N$

22. A,B,C are the points representing the complex numbers z_1, z_2, z_3 respectively on the complex plane and the circumcentre of the triangle ABC lies at the origin. If the altitude of the triangle through the vertex A meets the circumcircle again at P, then point P represents the complex number

(A) $-z_2z_3/z_1$ (B) $-\bar{z}_1z_2z_3$ (C) $-\frac{\bar{z}_1z_2}{\bar{z}_3}$ (D) $-\frac{\bar{z}_1z_3}{\bar{z}_2}$

23. Solution of the equation $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ is

(A) $y = \frac{c}{1 + \cos x}$ (B) $y = \frac{c}{1 - \cos x}$ (C) $x = 2\sin^{-1}\sqrt{\frac{c}{y}}$ (D) $y = c\cos^2\frac{x}{2}$

24. The portion of tangent to a curve intercepted between lines y = x & y = –x is bisected by the point of tangency. If the curve passes through the point (2, 1) then

(A) eccentricity of the curve is $\sqrt{2}$

(B) eccentricity of the curve is $\frac{1}{\sqrt{2}}$

(C) sum of focal distance of any point on the curve is $2\sqrt{3}$

(D) difference of focal distance of any point on the curve is $2\sqrt{3}$

25. Solution of the differential equation $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$ is

(A) $x^2 - y^2 + c(x - y) = 0$ (B) $x^2 + y^2 + c(x + y) = 0$

(C) a straight line if it passes through (1, –1) (D) a circle if it passes through (1, 1)

26. The integral curve of the equation $(1 - x^2)\frac{dy}{dx} + xy = x$ is

(A) a conic whose centre is (0, 1) (B) a conic, length of whose one axis is 2

(C) an ellipse if $|x| < 1$ (D) a hyperbola if $|x| > 1$

27. Let z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| = |z_2|$, then $\frac{z_1}{z_2}$ may be

(A) $1 + \omega$ (B) $1 + \omega^2$ (C) ω (D) ω^2

(where ω is an imaginary cube root of unity)

28. If a differentiable function satisfies $(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3)$ $\forall$ x, y $\in$ R, and f(1) = 2, then

(A) f(x) must be a polynomial function (B) area bounded by f(x) with x-axis is $\frac{1}{6}$

(C) f(3) = 12 (D) f(3) = 13

29. z_1 and z_2 are two complex numbers satisfying $i|z_1|^2z_2 - |z_2|^2z_1 = z_1 - iz_2$. Then which of the following is/are correct?

(A) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$ (B) $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$ (C) $z_1\bar{z}_2 + \bar{z}_1z_2 = 0$ (D) $|z_1|\,|z_2| = 1$ or $|z_1| = |z_2|$

30. If α ($\alpha \ne 1$) is the fifth root of unity, then

(A) $|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4| = 0$ (B) $|1 + \alpha + \alpha^2 + \alpha^3| = 1$

(C) $|1 + \alpha + \alpha^2| = 2\cos\frac{\pi}{5}$ (D) $|1 + \alpha| = 2\cos\frac{\pi}{10}$

31. If $z_1 = 5 + 12i$ and $|z_2| = 4$, then

(A) maximum $(|z_1 + iz_2|) = 17$

(B) minimum $(|z_1 + (1 + i)z_2|) = 13 - 4\sqrt{2}$

(C) minimum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{4}$

(D) maximum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{3}$

32. If $|z| = 1$ and $\omega = \dfrac{(1-z)^2}{1-z^2}$ where $z \neq 1$ then the locus of ω is represented by

(A) $|z - 2 - 4i| = |z - 2 + 4i|$

(B) $|z - 3 + 4i| = |z + 3 + 4i|$

(C) $|z - 2| = |z + 2|$

(D) $||z - i| - |z + i|| = 2$

Comprehension # 1 (Q.no. 33 to 35)

Suppose f(x) and g(x) are differentiable functions such that $x\, g(f(x)).\, f'(g(x)).g'(x) = f(g(x)).g'(f(x))\, f'(x)$ $\forall\, x \in R$ and f(x) & g(x) are positive for all $x \in R$. Also $\int_0^x f(g(t))\,dt = \dfrac{1}{2}\,(1 - e^{-2x})\ \forall\, x \in R$, $g(f(0)) = 1$ and $h(x) = \dfrac{g(f(x))}{f(g(x))}\ \forall\, x \in R$.

33. The graph of y = h(x) is symmetric with respect to the line:

(A) x = –1 (B) x = 0 (C) x = 1 (D) x = 2

34. The value of f(g(0)) + g(f(0)) is equal to:

(A) 1 (B) 2 (C) 3 (D) 4

35. The largest possible value of h(x) $\forall\, x \in R$ is

(A) 1 (B) $e^{1/3}$ (C) e (D) e^2

Comprehension # 2 (Q. No. 36 to 38)

Let A, B, C be three sets of complex numbers as defined below.

$A = \{z : |z + 1| \le 2 + \text{Re}(z)\}$, $B = \{z : |z - 1| \ge 1\}$ and $C = \left\{z : \left|\dfrac{z-1}{z+1}\right| \ge 1\right\}$

36. The number of point(s) having integral coordinates in the region $A \cap B \cap C$ is

(A) 4 (B) 5 (C) 6 (D) 10

37. The area of region(in sq. units) bounded by $A \cap B \cap C$ is

(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) $4\sqrt{3}$ (D) 2

38. The real part of the complex number in the region $A \cap B \cap C$ having maximum amplitude, may be

(A) –1 (B) $\dfrac{-3}{2}$ (C) $-\dfrac{1}{2}i$ (D) – 2

39. Find the number of complex numbers z satisfying $|z| = 1$ and $\left|\dfrac{z}{\bar{z}} + \dfrac{\bar{z}}{z}\right| = 1$.

40. If $z_1, z_2, z_3 \in C$ satisfying $|z_1| = |z_2| = |z_3| = 1$, $z_1 + z_2 + z_3 = 1$ and $z_1 z_2 z_3 = 1$. Also $\text{Im}(z_1) < \text{Im}(z_2) < \text{Im}(z_3)$. Then find the value of $[|z_1 + z_2^2 + z_3^3|]$, where [.] denotes the greatest integer function.

ANSWER KEY

DPP # 8

REVISION DPP OF

DEFINITE INTEGRATION & ITS APPLICATION AND INDEFINITE INTEGRATION

1.	(B)	**2.**	(C)	**3.**	(B)	**4.**	(C)	**5.**	(B)	**6.**	(C)	**7.**	(A)
8.	(A)	**9.**	(C)	**10.**	(B)	**11.**	(D)	**12.**	(C)	**13.**	(D)	**14.**	(A)
15.	(B,C,D)	**16.**	(A,B,C)	**17.**	(B,D)	**18.**	(A,C,D)	**19.**	(A,B,C,D)	**20.**	(A,C)	**21.**	(A,B)
22.	(B,D)	**23.**	(A,C)	**24.**	(B,D)	**25.**	(B,D)	**26.**	(A,B)	**27.**	(A,B,C,D)		
28.	(A,B,D)	**29.**	(A,B,C,D)			**30.**	(A,D)	**31.**	(B,C,D)	**32.**	(B)	**33.**	(C)
34.	(C)	**35.**	(B)	**36.**	(C)	**37.**	3	**38.**	1	**39.**	5	**40.**	67

MATHEMATICS

1. $|z-4| = \text{Re}(z) \;\Rightarrow\; y^2 = 8(x-2)$

Tangent is $y = m(x-2) + \dfrac{2}{m} \Rightarrow 0 = -2m + \dfrac{2}{m} \Rightarrow m = \pm 1 \;\Rightarrow\; z_1 = 4 + 4i \;\&\; z_2 = 4 - 4i$

2. Let $z = a + ib$, $z^{2015} = \bar{z} \;\Rightarrow\; |z|^{2015} = |z|$

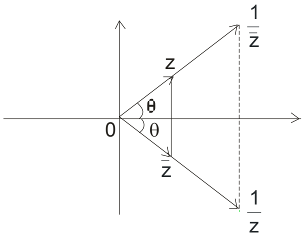
$\Rightarrow\; |z|\,(|z|^{2014} - 1) = 0 \;\Rightarrow\; |z| = 0$ or $|z| = 1$

when $|z| = 0,\ z = 0$

when $|z| = 1,\ z^{2015} = \bar{z} \;\Rightarrow\; z^{2016} = z\bar{z} = 1 \;\Rightarrow\;$ 2016 roots

$\therefore$ equation $z^{2015} =$ has total 2017 roots.

3. Required area = $\left|\dfrac{1}{2}.\left|\dfrac{1}{\bar{z}}\right|.\left|\dfrac{1}{z}\right|\sin 2\theta - \dfrac{1}{2}|z||\bar{z}|\sin 2\theta\right|$



$= \dfrac{1}{2}|\sin 2\theta|.\left|\dfrac{1}{|z|^2} - |z|^2\right| = \dfrac{1}{2}.\left|\dfrac{z^2 - \bar{z}^2}{2i|z|^2}\right|.\left|\dfrac{1}{|z|^2} - |z|^2\right|$ $\quad\left\{\begin{array}{c} \because\ z - \bar{z} = 2ir\sin\theta \\ z + \bar{z} = 2r\cos\theta \\ \Rightarrow\ z^2 - \bar{z}^2 = 2ir^2\sin 2\theta \end{array}\right\}$

$= \dfrac{1}{4}\left|z^2 - \bar{z}^2\right|\left|\dfrac{1}{|z|^4} - 1\right|$

4. $(1+\omega)(1+\omega^2) \ldots\ldots (1+\omega^{1988}) = \{(1+\omega)(1+\omega^2)(1+\omega^3)\}^{662}.(1+\omega^{1987})(1+\omega^{1988}) = 2^{662} = 4^{331}$

5. $|z_1 + 1| + |z_2 + 1| + |z_1z_2 + 1| \geq |z_1 + 1| + |(z_2 + 1) - (z_1z_2 + 1)|$

$\geq |z_1 + 1| + |z_2(1 - z_1)|$

$\geq |1 + z_1| + |1 - z_1| \geq 2$

6. Put $x^2y^2 = t \;\Rightarrow\; 2xy^2 + 2x^2y.\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow\; \tan t = \dfrac{dt}{dx} \;\Rightarrow\; \int dx = \int \cot t\, dt \;\Rightarrow\; x = \ell n|\sin t| + c$

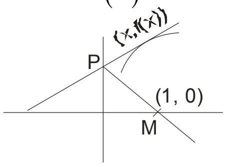
7. $\dfrac{dy}{y} + \dfrac{\sin x}{1 + \cos x}dx = 0$

$\Rightarrow\; \int\dfrac{dy}{y} + \int \tan\dfrac{x}{2}dx = c \;\Rightarrow\; \ell n|y| + \dfrac{\ell n\left|\sec\dfrac{x}{2}\right|}{\dfrac{1}{2}} = \ell n c \;\Rightarrow\; |y|.\sec^2\dfrac{x}{2} = c$

8. $\dfrac{y}{x}\dfrac{dy}{dx} = \dfrac{y^2}{x^2} + \dfrac{f\left(y^2/x^2\right)}{f'\left(y^2/x^2\right)} \;\Rightarrow\; \dfrac{y}{x} = v \;\Rightarrow\; \dfrac{2vf'\left(v^2\right)dv}{f\left(v^2\right)} = 2\dfrac{dx}{x} \;\Rightarrow\; f(v^2) = cx^2$

9. Equation of tangent $Y - f(x) = f'(x)(X - x)$ $\Rightarrow$ $P(0, f(x) - xf'(x))$

$m_{PM} = \dfrac{-1}{f'(x)} = \dfrac{f(x) - xf'(x)}{-1}$ $\Rightarrow$ $yy' - x(y')^2 = 1$



10. $f'(x) - \dfrac{2x(x+1)}{(x+1)} f(x) = \dfrac{e^{x^2}}{(x+1)^2}$ I.F. $= e^{-x^2}$ $\Rightarrow$ $f(x)\, e^{-x^2} = \displaystyle\int \frac{dx}{(x+1)^2}$

11. Let $\omega = re^{i\theta}$ and $z = x + iy$

$\therefore\ x + iy = re^{i\theta} + \dfrac{e^{-i\theta}}{r}$ $\Rightarrow x = \left(r + \dfrac{1}{r}\right)\cos\theta$ & $y = \left(r - \dfrac{1}{r}\right)\sin\theta$ $\Rightarrow \dfrac{x^2}{\left(r+\frac{1}{r}\right)^2} + \dfrac{y^2}{\left(r-\frac{1}{r}\right)^2} = 1$

Distance between focii = 2ae = $2\sqrt{\left(r+\dfrac{1}{r}\right)^2 - \left(r - \dfrac{1}{r}\right)^2} = 4$

12. The number of common vertices is given by the number of common roots of $z^{1982} - 1 = 0$ and $z^{2973} - 1 = 0$, which is equal to HCF (1982, 2973) = 991.

13. $z - 1 = e^{i\theta}$ $\Rightarrow$ $z = 2\cos(\theta/2)\, e^{i(\theta/2)}$ $\Rightarrow$ $\tan\left(\arg\dfrac{(z-1)}{2}\right) - \left(\dfrac{2i}{z}\right) = \tan\left(\dfrac{\theta}{2}\right) - \dfrac{i}{\cos\frac{\theta}{2}} e^{-i\theta/2} = -i$

14. $\theta_1 - \pi/4 = \theta_2 + 2m\pi$ and $\theta_1 + \theta_2 = 2n\pi + \pi/2$

15. $|x - y| = 4|\cos\theta - \sin\theta| = 4\sqrt{1 - \sin 2\theta} = \left[0, 4\sqrt{2}\right]$ (putting $z = 4e^{i\theta}$)

16. $z^3 = t$ $\Rightarrow$ $t = \omega$ or ω^2

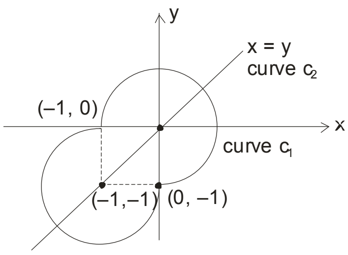
$z = e^{(i)\left[\left(2m\pi + \frac{2\pi}{3}\right)/3\right]}$ or $e^{i\left[\left(2m\pi + \frac{4\pi}{3}\right)/3\right]}$ $\Rightarrow$ $\theta = \dfrac{2m\pi}{3} + \dfrac{2\pi}{9}$ or $\dfrac{2m\pi}{3} + \dfrac{4\pi}{9}$

17. $\dfrac{\bar{z}_1}{z_2} = \dfrac{r_1 e^{-i\theta_1}}{r_2 e^{i\theta_2}} = 2i$ $\therefore$ $z = \dfrac{2i + \omega + \omega^2}{3} = \dfrac{2i - 1}{3}$

18. Equation of line passing through z_1 & z_2 is

$z = z_1 + t(z_2 - z_1)$; $t \in R$ $\Rightarrow$ $\dfrac{z - z_1}{z_2 - z_1} = t$ = purely real number

19.



20. $|6z - i| \le |2 + 3iz|$ $\Rightarrow$ $|6z - i|^2 \le |2 + 3iz|^2$ $\Rightarrow$ $|z| \le \dfrac{1}{3}$

21. We have $\omega^{2n+1} = 1$ & $1 + \omega + \omega^2 + \ldots + \omega^{2n} = 0$

$\Rightarrow$ $1 + \omega + \omega^2 + \ldots + \omega^n + \omega^n(\omega + \omega^2 \ldots\ \omega^n) = 0$ $\Rightarrow$ $1 + z - \dfrac{1}{2} + \omega^n\left(z - \dfrac{1}{2}\right) = 0$

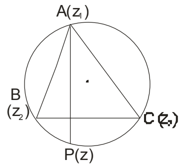
$\Rightarrow$ $(2z + 1) = -\omega^n.(2z - 1)$ $\Rightarrow$ $(2z + 1)^{2n+1} = -\omega^{n(2n+1)}(2z - 1)^{2n+1}$

$\Rightarrow$ $(2z + 1)^{2n+1} + (2z - 1)^{2n+1} = 0$ Ans. (B)

further $z = \dfrac{1}{2} \cdot \dfrac{\omega^n - 1}{\omega^n + 1}$ $\Rightarrow$ $\bar{z} = \dfrac{1}{2}\,\dfrac{\frac{1}{\omega^n} - 1}{\frac{1}{\omega^n} + 1}$ $\Rightarrow$ $\bar{z} = -z$ $\Rightarrow$ $(\bar{z})^{2k} = z^{2k}$ & $(\bar{z})^{2k+1} = -z^{2k+1}$

22. $|z_1| = |z_2| = |z_3| = |z|$ $\Rightarrow$ $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = z\bar{z}$

$\because$ AP $\perp^r$ BC $\therefore$ $\dfrac{z - z_1}{\bar{z} - \bar{z}_1} + \dfrac{z_2 - z_3}{\bar{z}_2 - \bar{z}_3} = 0$ $\Rightarrow$ $\dfrac{z - z_1}{\dfrac{z_1\bar{z}_1}{z} - \bar{z}_1} + \dfrac{z_2 - z_3}{\dfrac{z_3\bar{z}_3}{z_2} - \bar{z}_3} = 0$

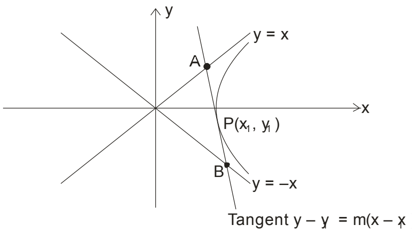


$\Rightarrow$ $-\dfrac{z}{\bar{z}_1} - \dfrac{z_2}{\bar{z}_3} = 0$ $\Rightarrow$ $z = -\dfrac{\bar{z}_1 z_2}{\bar{z}_3} = -\dfrac{z_2 z_3}{z_1}$

23. $\dfrac{dy}{dx} = \dfrac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2.1}$ $\Rightarrow$ $\dfrac{dy}{dx} = (-\cot x \pm \text{cosec}x)y$

$\Rightarrow$ $\dfrac{dy}{dx} = -\cot\dfrac{x}{2}.y$ or $\dfrac{dy}{dx} = \tan\dfrac{x}{2}.y$ $\Rightarrow$ $y = c\ \text{cosec}^2\dfrac{x}{2}$ or $y = c\ \sec^2\dfrac{x}{2}$

24.



$A\left(\dfrac{mx_1 - y_1}{m-1}, \dfrac{mx_1 - y_1}{m-1}\right)$ & $B\left(\dfrac{mx_1 - y_1}{m+1}, \dfrac{y_1 - mx_1}{m+1}\right)$ $\because$ P is mid point of AB

$\therefore$ $2x_1 = \dfrac{mx_1 - y_1}{m-1} + \dfrac{mx_1 - y_1}{m+1}$ $\Rightarrow$ $m = \dfrac{x_1}{y_1}$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{x}{y}$ $\Rightarrow$ $x^2 - y^2 = c$

25. Put $y = tx$ $\Rightarrow$ $t + x\dfrac{dt}{dx} = \dfrac{t^2 - 2t - 1}{t^2 + 2t - 1}$

$\Rightarrow$ $\dfrac{-t^2 - 2t + 1}{(t+1)(t^2+1)}dt = \dfrac{dx}{x}$ $\Rightarrow$ $\left(\dfrac{1}{t+1} - \dfrac{2t}{t^2+1}\right)dt = \dfrac{dx}{x}$ $\Rightarrow$ $x^2 + y^2 = c(x + y)$

26. $(1 - x^2)\dfrac{dy}{dx} = x(1 - y)$ $\Rightarrow$ $\dfrac{dy}{y-1} = \dfrac{x}{x^2 - 1}dx$

Integrating both sides

$2\displaystyle\int \dfrac{dy}{y-1} = \displaystyle\int \dfrac{2x}{x^2-1}dx$ $\Rightarrow$ $2\ell n\,|y - 1| = \ell n|x^2 - 1| + \ell nc$ $\Rightarrow$ $(y - 1)^2 = c|x^2 - 1|$

27. $|z_1 + z_2|^2 = |z_1|^2 = |z_2|^2$ $\Rightarrow$ $|z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$

divide by $z_2\bar{z}_2$

$\dfrac{z_1}{z_2} + \overline{\left(\dfrac{z_1}{z_2}\right)} + 1 = 0$ $(\because z_1\bar{z}_1 = z_2\bar{z}_2)$ $\Rightarrow$ $\dfrac{z_1}{z_2} = \omega$ or ω^2

28. Differentiate both sides w.r.t. y, then put y = 0

$2xf'(x) - 2f(x) = 2x^2$ $\Rightarrow$ $\dfrac{dy}{dx} - \dfrac{1}{x}.y = x$ $\Rightarrow$ $y = x^2 + x$

29. $iz_2(|z_1|^2 + 1) = z_1(1 + |z_2|^2)$ $\Rightarrow$ $\dfrac{z_1}{z_2}$ = pure imaginary

further $iz_1\bar{z}_1 z_2 - z_2\bar{z}_2 z_1 = z_1 - iz_2$ $\Rightarrow$ $\bar{z}_1 z_2(iz_1 + z_2) = -i(z_2 + iz_1)$ $(\because z_1\bar{z}_2 = -\bar{z}_1 z_2)$

$\Rightarrow$ $\bar{z}_1 z_2 = -i$ or $iz_1 = -z_2$

$\Rightarrow$ $|z_1 z_2| = 1$ or $|z_1| = |z_2|$

30. (A) Standard result (B) $|1 + \alpha + \alpha^2 + \alpha^3| = |-\alpha^4| = 1$

(C) $|1 + \alpha + \alpha^2| = |-\alpha^3 - \alpha^4| = |1 + \alpha| = \left|1 + \cos\dfrac{2\pi}{5} + i\sin\dfrac{2\pi}{5}\right| = 2\cos\dfrac{\pi}{5}$

31. $|z_1 + i z_2| \le |z_1| + |z_2| = 17$ Also, $|z_1 + (i + 1)z_2| \ge |\,|z_1| - |(1 + i)z_2|\,| = 13 - 4\sqrt{2}$

Further, $\left|z_2 + \frac{4}{z_2}\right| \le |z_2| + \frac{4}{|z_2|} = 5$ & $\left|z_2 + \frac{4}{z_2}\right| \ge \left||z_2| - \frac{4}{|z_2|}\right| = 3$

32. $\omega = \frac{1-z}{1+z} = \frac{\bar{z}-1}{\bar{z}+1} = -\overline{\left(\frac{1-z}{1+z}\right)} = -\bar{\omega}$ or $\omega + \bar{\omega} = 0$ $\Rightarrow$ ω lies on y-axis

33. to 35. $\int_0^x f(g(t))dt = \frac{1}{2}(1 - e^{-2x})$

differentiating both sides w.r.t. x

$f(g(x)) = e^{-2x}$ $\Rightarrow$ $f'(g(x)).g'(x) = -2e^{-2x}$

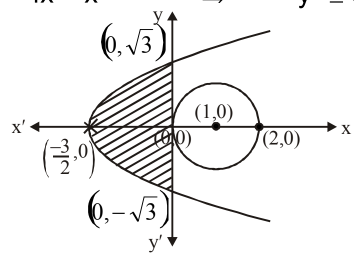
let $g(f(x)) = y$

$\therefore$ $x.y.(-2).e^{-2x} = e^{-2x}.\frac{dy}{dx}$ $\Rightarrow$ $\frac{dy}{dx} = -2yx$ $\Rightarrow$ $\frac{dy}{y} + 2x\,dx = 0$

$\Rightarrow$ $\ell n\, y + x^2 = c$ $\Rightarrow$ $y = e^{c-x^2}$ $\Rightarrow$ $g(f(x)) = e^{-x^2}$ $\therefore h(x) = \frac{e^{-x^2}}{e^{-2x}} = e^{2x-x^2}$

36. to 38. For A, $|z + 1| \le 2 + \text{Re}(z)$ $\Rightarrow$ $(x+1)^2 + y^2 \le 4 + 4x + x^2$ $\Rightarrow$ $y^2 \le 3 + 2x$

$\Rightarrow$ $y^2 \le 2\left(x + \frac{3}{2}\right)$ …(i)



For B, $|z - 1| \ge 1$ $\Rightarrow$ $(x-1)^2 + y^2 \ge 1$ ......(2)

For C, $|z - 1|^2 \ge |z + 1|^2$ $\Rightarrow$ $x \le 0$ .....(3)

(i) (–1,0), (–1, 1) , (–1,–1), (0,0), (0,1), (0,–1) but $z \ne -1$

$\therefore$ Total number of point(s) having integral coordinates in the region $A \cap B \cap C$ is 5.

(ii) Required area = $2\int_{-3/2}^{0} \sqrt{2\left(x + \frac{3}{2}\right)}\,dx = 2\sqrt{3}$

(iii) Clearly $z = \frac{-3}{2} + 0i$ is the complex number in the region $A \cap B \cap C$ having maximum amplitude. $\therefore$ $\text{Re}(z) = -3/2$

39. Let $z = e^{i\theta}$; $\theta \in [0, 2\pi)$

$\because$ $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ $\Rightarrow$ $|2\cos 2\theta| = 1$

$\Rightarrow$ $\cos 2\theta = \pm 1/2$ $\Rightarrow$ Total 8 solutions.

40. $z_1z_2 + z_2z_3 + z_3z_1$

$= z_1z_2z_3\,(\bar{z}_1 + \bar{z}_2 + \bar{z}_3)$

$\Rightarrow$ $z_1z_2 + z_2z_3 + z_3z_1 = 1$

$\therefore$ z_1, z_2, z_3 satisfy

$z^3 - z^2 + z - 1 = 0$

or $z_1 = -i$

$z_2 = 1$

$z_3 = i$ $\Rightarrow$ $|z_1 + z_2^2 + z_3^3| = |-i + 1 - i| = \sqrt{5}$