CHAPTER 6

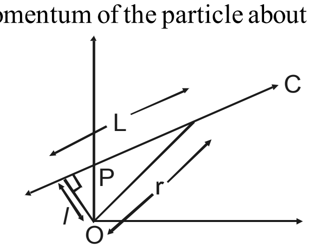
System of Particles and Rotational Motion

1. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? **[2002]**
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$.

2. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is **[2002]**
(a) v (b) $v/3$
(c) $v/2$ (d) zero

3. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[2002]**
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$.

4. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P? **[2002]**



(a) mvL (b) mvl
(c) mvr (d) zero.

5. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is **[2003]**
(a) $I_Y = 32 I_X$ (b) $I_Y = 16 I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64 I_X$

6. A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is **[2003]**
(a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $\frac{L}{2}$

7. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then **[2003]**
(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$
(b) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$
(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$
(d) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$

8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ? **[2004]**
(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that **[2004]**
(a) $I_A < I_B$ (b) $I_A > I_B$
(c) $I_A = I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$
where d_A and d_B are their densities.

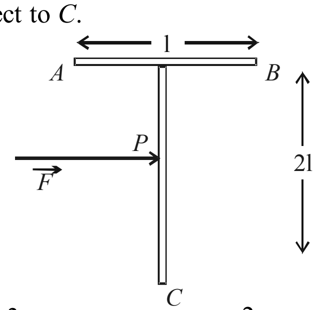
10. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[2005]**

(a) does not shift
(b) depends on height of breaking
(c) body B
(d) body C

11. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[2005]**

(a) $\frac{2}{5}Mr^2$ (b) $\frac{1}{4}Mr$
(c) $\frac{1}{2}Mr^2$ (d) Mr^2

12. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[2005]**



(a) $\frac{3}{2}\ell$ (b) $\frac{2}{3}\ell$
(c) ℓ (d) $\frac{4}{3}\ell$

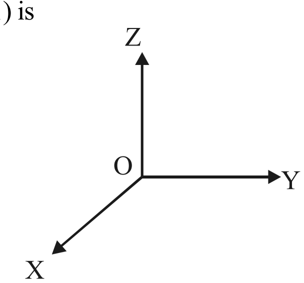
13. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle is moved, so as to keep the centre of mass at the same position? **[2006]**

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

14. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is **[2006]**

(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$
(c) $3m\ell^2$ (d) $m\ell^2$

15. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is **[2006]**



(a) $F(\hat{i}-\hat{j})$ (b) $-F(\hat{i}+\hat{j})$
(c) $F(\hat{i}+\hat{j})$ (d) $-F(\hat{i}-\hat{j})$

16. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$ **[2006]**

(a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$
(c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

17. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α / R form the centre of the bigger disc. The value of α is **[2007]**

(a) 1/4 (b) 1/3
(c) 1/2 (d) 1/6

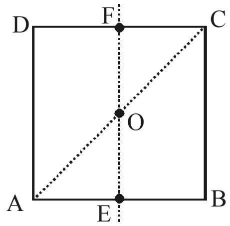
18. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[2007]**

(a) $\frac{g\sin\theta}{1-MR^2/I}$ (b) $\frac{g\sin\theta}{1+I/MR^2}$
(c) $\frac{g\sin\theta}{1+MR^2/I}$ (d) $\frac{g\sin\theta}{1-I/MR^2}$

19. Angular momentum of the particle rotating with a central force is constant due to **[2007]**

(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

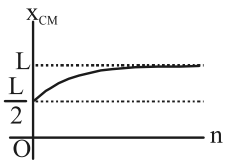
20. For the given uniform square lamina *ABCD*, whose centre is *O*, **[2007]**

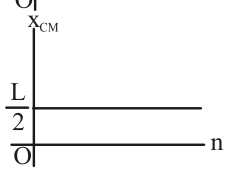


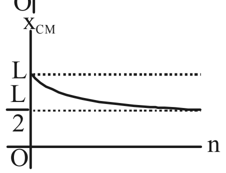
(a) $I_{AC} = \sqrt{2}\, I_{EF}$ (b) $\sqrt{2} I_{AC} = I_{EF}$

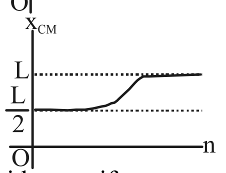
(c) $I_{AD} = 3I_{EF}$ (d) $I_{AC} = I_{EF}$

21. A thin rod of length '*L*' is lying along the *x*-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with *x* as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against '*n*', which of the following graphs best approximates the dependence of x_{CM} on *n*? **[2008]**

(a) 

(b) 

(c) 

(d) 

22. Consider a uniform square plate of side '*a*' and mass '*M*'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[2008]**

(a) $\frac{5}{6}Ma^2$ (b) $\frac{1}{12}Ma^2$

(c) $\frac{7}{12}Ma^2$ (d) $\frac{2}{3}Ma^2$

23. A thin uniform rod of length *l* and mass *m* is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[2009]**

(a) $\frac{1}{6}\frac{l\omega}{g}$ (b) $\frac{1}{2}\frac{l^2\omega^2}{g}$

(c) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{3}\frac{l^2\omega^2}{g}$

24. A mass *m* hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass *m* and radius *R*. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass *m*, if the string does not slip on the pulley, is: **[2011]**

(a) g (b) $\frac{2}{3}g$

(c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

25. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc. **[2011]**

(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

26. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where *t* is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of rotations made by the pulley before its direction of motion is reversed, is: **[2011]**

(a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9 (d) less than 3

27. A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the loop is zero. What will be the velocity of the centre of the loop when it ceases to slip ? **[2013]**

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$

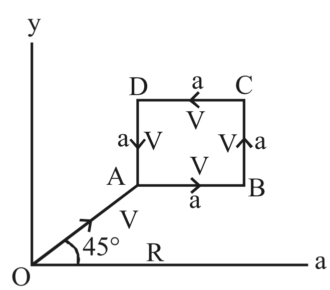
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

28. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: **[2014]**
(a) angular momentum is conserved.
(b) angular momentum changes in magnitude but not in direction.
(c) angular momentum changes in direction but not in magnitude.
(d) angular momentum changes both in direction and magnitude.

29. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[2015]**
(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$
(c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$

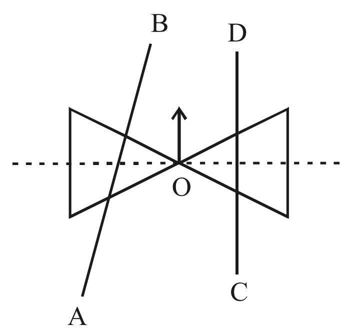
30. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is: **[2015]**
(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$
(c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

31. A particle of mass *m* is moving along the side of a square of side '*a*', with a uniform speed *v* in the *x-y* plane as shown in the figure : **[2016]**



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?
(a) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.
(b) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.
(c) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.
(d) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

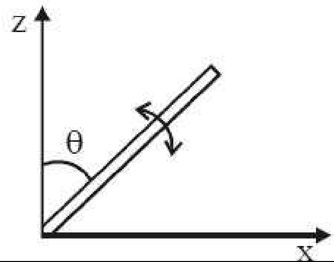
32. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to : **[2016]**



(a) go straight.
(b) turn left and right alternately.
(c) turn left.
(d) turn right.

33. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum? **[2017]**
(a) 1 (b) $\frac{3}{\sqrt{2}}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

34. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[2017]**



(a) $\frac{3g}{2\ell}\cos\theta$ (b) $\frac{2g}{3\ell}\cos\theta$

(c) $\frac{3g}{2\ell}\sin\theta$ (d) $\frac{2g}{2\ell}\sin\theta$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(a)	(d)	(d)	(a)	(d)	(b)	(a)	(a)	(c)	(d)	(c)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(b)	(b)	(d)	(d)	(a)	(d)	(c)	(b)	(c)	(a)	(c)	(c)	(d)	(a)
31	32	33	34											
(a)	(c)	(c)	(c)											

SOLUTIONS

1. (c) When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$\therefore\ I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

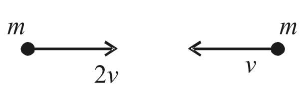
Here $I_1 = \frac{1}{2}MR^2$

and $I_2 = \frac{1}{2}MR^2 + 2mR^2$

$$\therefore \omega_2 = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2mR^2} \times \omega_1 = \frac{M}{M+4m}\omega_1$$

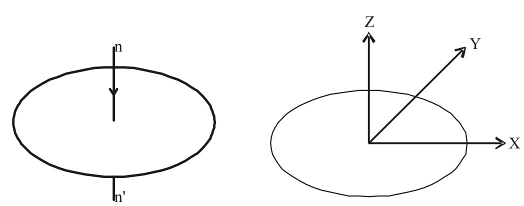
2. (c) The velocity of centre of mass of two particle system is given by

$$v_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$



$$= \frac{m(2v) + m(-v)}{m+m} = \frac{v}{2}$$

3. (a) M. I of a circular wire about an axis nn' passing through the centre of the circle and perpendicular to the plane of the circle $= MR^2$



As shown in the figure, X-axis and Y-axis lie in the plane of the ring . Then by perpendicular axis theorem

$I_X + I_Y = I_Z$

$\Rightarrow 2\,I_X = MR^2$ $[\because I_X = I_Y$ (by symmetry) and $I_Z = MR^2]$

$\therefore I_X = \frac{1}{2}MR^2$

4. (d) Angular momentum (L)

= (linear momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)

$= mv \times r$ [Here $r = 0$ because the line of

$= mv \times 0$ action of momentum passes

$= 0$ through the axis of rotation]

5. **(d)** We know that density $(d) = \frac{mass(M)}{volume(V)}$

$\therefore \quad M = d \times V = d \times (\pi R^2 \times t)$.

The moment of inertia of a disc is given by

$I = \frac{1}{2}MR^2$

$\therefore \quad I = \frac{1}{2}(d \times \pi R^2 \times t)R^2 = \frac{\pi d}{2} t \times R^4$

$\therefore \quad \frac{I_X}{I_Y} = \frac{t_X R_X^4}{t_Y R_Y^4} \quad = \frac{t \times R^4}{\frac{t}{4} \times (4R)^4} = \frac{1}{64}$

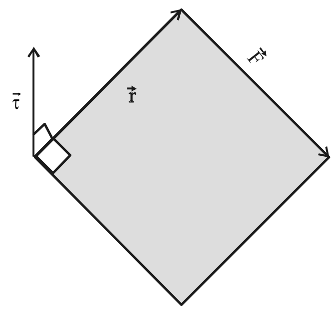
6. **(a)** $K.E. \frac{1}{2} I\omega^2$, but $L = I\omega \Rightarrow I = \frac{L}{\omega}$

$\therefore \quad K.E. = \frac{1}{2}\frac{L}{\omega} \times \omega^2 = \frac{1}{2}L\omega$

$\therefore \quad \frac{K.E}{K.E'} = \frac{L \times \omega}{L' \times \omega'} \Rightarrow \frac{K.E}{\frac{K.E}{2}} = \frac{L \times \omega}{L' \times 2\omega}$

$\therefore \quad L' = \frac{L}{4}$

7. **(d)** We know that $\vec{\tau} = \vec{r} \times \vec{F}$



The angle between $\vec{\tau}$ and $\vec{r}$ is 90° and the angle between $\vec{\tau}$ and $\vec{F}$ is also 90°. We also know that the dot product of two vectors which have an angle of 90° between them is zero. Therefore (d) is the correct option.

8. **(b)** Angular momentum will remain the same since external torque is zero.

9. **(a)** The moment of inertia of solid sphere A about its diameter $I_A = \frac{2}{5}MR^2$.

The moment of inertia of a hollow sphere B about its diameter $I_B = \frac{2}{3}MR^2$.

$\therefore I_A < I_B$

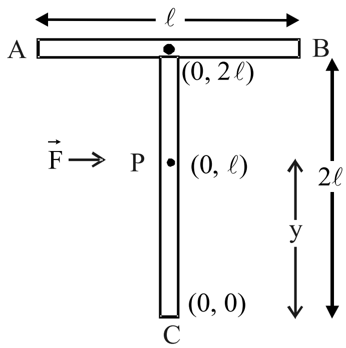
10. **(a)** Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

11. **(c)** The disc may be assumed as combination of two semi circular parts.

Let I be the moment of inertia of the uniform semicircular disc

$\Rightarrow 2I = \frac{2Mr^2}{2} \Rightarrow I = \frac{Mr^2}{2}$

12. **(d)**

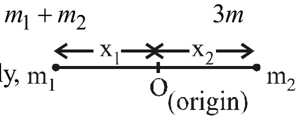


To have linear motion, the force $\vec{F}$ has to be applied at centre of mass.

i.e. the point 'P' has to be at the centre of mass

$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$

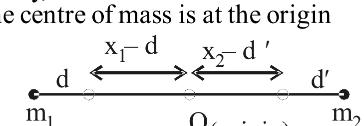
13. **(c)** Initially,



$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2} \Rightarrow m_1 x_1 = m_2 x_2 \; ...(1)$

Finally,

The centre of mass is at the origin

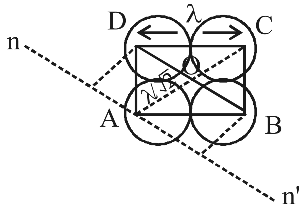


$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$

$\Rightarrow 0 = m_1 d - m_1 x_1 + m_2 x_2 - m_2 d'$

$\Rightarrow d' = \frac{m_1}{m_2} d$ [From (1).]

14. (c)



$I_{nn'}$ = $M.I$ due to the point mass at B + $M.I$ due to the point mass at D + $M.I$ due to the point mass at C.

$$I_{nn'} = 2 \times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$

$$= m\ell^2 + 2m\ell^2 = 3m\ell^2$$

15. (c) Torque $\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$

$$= F[-\hat{i} \times \hat{k} + \hat{j} \times \hat{k}]$$

$$= F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j})$$

$$\left[\text{Since } \hat{k} \times \hat{i} = \hat{j} \text{ and } \hat{j} \times \hat{k} = \hat{i}\right]$$

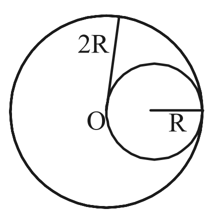
16. (d) Applying conservation of angular momentum $I'\omega' = I\omega$

$$(mR^2 + 2MR^2)\omega' = mR^2\omega$$

$$\Rightarrow \omega' = \omega\left[\frac{m}{m + 2M}\right]$$

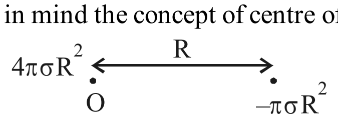
17. (b) Let the mass per unit area be σ. Then the mass of the complete disc

$$= \sigma[\pi(2R)^2] = 4\pi\sigma R^2$$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{c.m} = \frac{(4\pi\sigma R^2) \times 0 + (-\pi\sigma R^2)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore \quad x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore \quad x_{c.m} = -\frac{R}{3} \Rightarrow \alpha = \frac{1}{3}$$

18. (b) This is a standard formula and should be memorized.

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

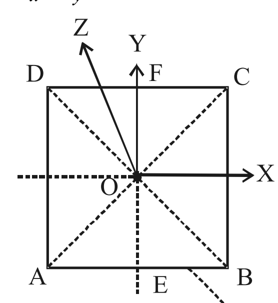
19. (d) We know that $\vec{\tau_c} = \frac{d\vec{L_c}}{dt}$

where $\vec{\tau_c}$ torque about the center of mass of the body and $\vec{L_c}$ = Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therefore torque about center of mass is zero.

When $\vec{\tau_c} = 0$ then $\vec{L_c}$ = constt.

20. (d) By the theorem of perpendicular axes,
$I_z = I_x + I_y$ or, $I_z = 2I_y$

($\because I_x = I_y$ by symmetry of the figure)



$$\therefore \quad I_{EF} = \frac{I_z}{2} \qquad \text{...(i)}$$

Again, by the same theorem $I_z = I_{AC} + I_{BD} = 2I_{AC}$
($\therefore I_{AC} = I_{BD}$ by symmetry of the figure)

$$\therefore \quad I_{AC} = \frac{I_z}{2} \qquad \text{...(ii)}$$

From (i) and (ii), we get, $I_{EF} = I_{AC}$.

21. (a)
- When $n = 0$, $x = k$ where k is a constant. This means that the linear mass density is constant. In this case the centre of mass

will be at the midelle of the rod ie at $L/2$. Therefore (c) is ruled out

- n is positive and as its value increases, the rate of increase of linear mass density with increase in x increases. This shows that the centre of mass will shift towards that end of the rod where $n = L$ as the value of n increases. Therefore graph (b) is ruled out.

- The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$

$$Here \quad \frac{x}{L} \le 1$$

With increase in the value of n, the centre of mass shift towards the end $x = L$ such that first the shifting is at a higher rate with increase in the value of n and then the rate decreases with the value of n.

These characteristics are represented by graph (a).

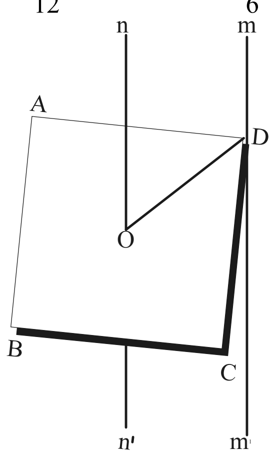
$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda\, dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n .xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{k\,x^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0, x_{CM} = \frac{L}{2}; n = 1,$

$x_{CM} = \frac{2L}{3}; n = 2, x_{CM} = \frac{3L}{4};....$

22. (d) $I_{nn'} = \frac{1}{12}M(a^2 + a^2) = \frac{Ma^2}{6}$



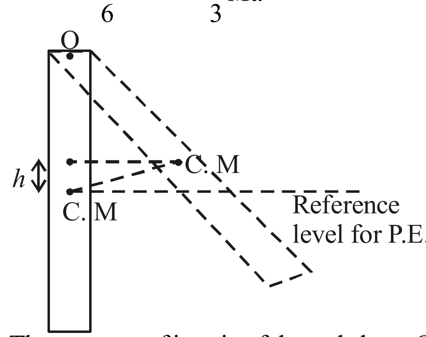
Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

According to parallel axis theorem

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$= \frac{Ma^2 + 3Ma^2}{6} = \frac{2}{3}Ma^2$$

23. (c)



The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$. The maximum angular speed of the rod is when the rod is instantaneously vertical. The energy of the rod in this condition is $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in its extreme portion, its angular velocity is zero momentarily. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

$$\therefore\ mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

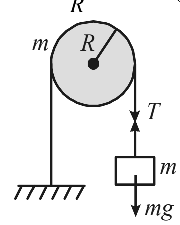
$$\Rightarrow\ h = \frac{\ell^2\omega^2}{6g}$$

24. (b) For translational motion,

$mg - T = ma$ .....(1)

For rotational motion,

$T.R = I\alpha = I\frac{\alpha}{R}$ ....(2)



Solving (1) & (2),

$$a = \frac{mg}{\left(m + \frac{I}{R^2}\right)} = \frac{mg}{m + \frac{mR^2}{2R^2}} = \frac{2mg}{3m} = \frac{2g}{3}$$

25. (c) As insect moves along a diameter, the effective mass and hence the M.I. first decreases then increases so from principle of conservation of angular momentum, angular speed, first increases then decreases.

26. (a) $F = 20t - 5t^2$

$$\therefore \alpha = \frac{FR}{I} = 4t - t^2$$

$$\Rightarrow \frac{d\omega}{dt} = 4t - t^2$$

$$\Rightarrow \int_0^{\omega} d\omega = \int_0^{t} \left(4t - t^2\right) dt$$

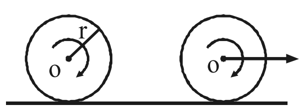
$$\Rightarrow \omega = 2t^2 - \frac{t^3}{3} \text{ (as } \omega = 0 \text{ at } t = 0, 6\text{s)}$$

$$\int_0^{\theta} d\theta = \int_0^{6} \left(2t^2 - \frac{t^3}{3}\right) dt$$

$$\Rightarrow \theta = 36 \text{ rad}$$

$$\Rightarrow n = \frac{36}{2\pi} < 6$$

27. (c)

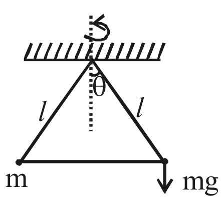


From conservation of angular momentum about any fix point on the surface,

$$mr^2\omega_0 = 2mr^2\omega$$

$$\Rightarrow \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2} \quad [\because v = r\omega]$$

28. (c) Torque working on the bob of mass m is, $\tau = mg \times \ell \sin\theta$. (Direction parallel to plane of rotation of particle)



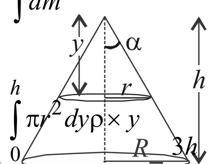
As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

$$= \frac{\frac{1}{2} + \frac{\sqrt{3}}{5} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{5} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{2}\left(1 + \frac{3}{5}\right)}{\frac{\sqrt{3}}{5}\left(1 - \frac{1}{5}\right)} = \frac{\frac{1}{2} \times \frac{8}{5}}{\frac{\sqrt{3} \times 4}{10}}$$

$$= \frac{\frac{8}{10}}{\frac{\sqrt{3} \times 4}{10}} = \frac{8}{\sqrt{3} \times 4} = \frac{2}{\sqrt{3}}$$

29. (d) $y_{cm} = \frac{\int y\,dm}{\int dm}$

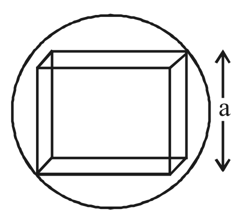


$$= \frac{\int_0^h \pi r^2 dy \rho \times y}{\frac{1}{3}\pi R^2 h \rho} = \frac{3h}{4}$$

30. (a) Here $a = \frac{2}{\sqrt{3}} R$

Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}} R\right)^3} = \frac{\sqrt{3}}{2}\pi.$$

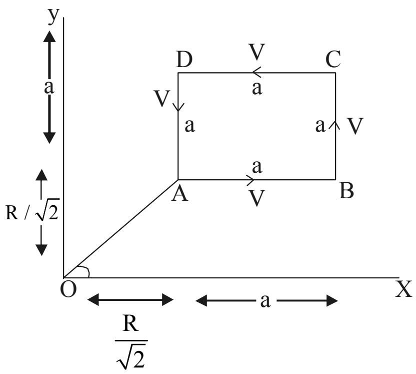


$$M' = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about the given axis, $I = \frac{M'a^2}{6}$

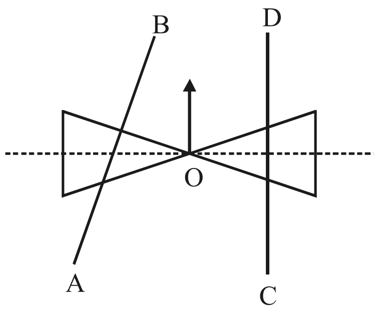
$$= \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}} R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

31. **(a)** We know that $|L| = mvr_{\perp}$



In none of the cases, the perpendicular distance $r_{\perp}$ is $\left(\frac{R}{\sqrt{2}} - a\right)$

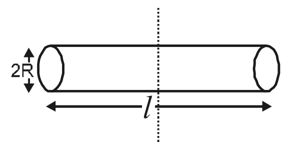
32. **(c)** As shown in the diagram, the normal reaction of AB on roller will shift towards O.
This will lead to tending of the system of cones to turn left.



33. **(c)** As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3}\right]$$



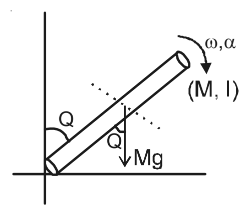
$$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3}\right] \Rightarrow \frac{dl}{dl} = \frac{m}{4}\left[\frac{-v}{\pi l^2} + \frac{2l}{3}\right] = 0$$

$$\frac{v}{\pi l^2} = \frac{2l}{3} \quad \Rightarrow v = \frac{2\pi l^3}{3}$$

$$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2} \text{ or, } \frac{l}{R} = \sqrt{\frac{3}{2}}$$

34. **(c)** Torque at angle θ

$$\tau = Mg\sin\theta . \frac{l}{2}$$



Also $\tau = I\alpha$

$$\therefore \quad I\alpha = Mg\sin\theta\frac{l}{2}$$

$$\frac{Ml^2}{3}.\alpha = Mg\sin\theta\frac{l}{2} \quad \left[\because I_{rod} = \frac{Ml^2}{3}\right]$$

$$\Rightarrow \frac{l\alpha}{3} = g\frac{\sin\theta}{2} \qquad \therefore \quad \alpha = \frac{3g\sin\theta}{2l}$$