CHAPTER 11

Conic Sections

1. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **[2002]**

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these

2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[2002]**

(a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$

(c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$

3. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[2002]**

(a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$

(c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$

4. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is **[2002]**

(a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$

(c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$

5. Two common tangents to the circle $x^2+y^2=2a^2$ and parabola $y^2=8ax$ are **[2002]**

(a) $x = \pm(y+2a)$

(b) $y = \pm(x+2a)$

(c) $x = \pm(y+a)$

(d) $y = \pm(x+a)$

6. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then **[2003]**

(a) $r > 2$ (b) $2 < r < 8$

(c) $r < 2$ (d) $r = 2$.

7. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is **[2003]**

(a) $x^2+y^2-2x+2y=62$

(b) $x^2+y^2+2x-2y=62$

(c) $x^2+y^2+2x-2y=47$

(d) $x^2+y^2-2x+2y=47$.

8. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then **[2003]**

(a) $t_2 = t_1+\frac{2}{t_1}$ (b) $t_2 = -t_1-\frac{2}{t_1}$

(c) $t_2 = -t_1+\frac{2}{t_1}$ (d) $t_2 = t_1-\frac{2}{t_1}$

9. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is **[2003]**

(a) 9 (b) 1

(c) 5 (d) 7

10. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[2004]**

(a) $2ax-2by-(a^2+b^2+4)=0$

(b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$

(d) $2ax+2by+(a^2+b^2+4)=0$

11. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis. The locus of the other end of the diameter through A is **[2004]**

(a) $(y-q)^2=4px$ (b) $(x-q)^2=4py$

(c) $(y-p)^2=4qx$ (d) $(x-p)^2=4qy$

12. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is **[2004]**

(a) $x^2+y^2+2x-2y-23=0$

(b) $x^2+y^2-2x-2y-23=0$

(c) $x^2+y^2+2x+2y-23=0$

(d) $x^2+y^2-2x+2y-23=0$

13. Intercept on the line $y = x$ by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is **[2004]**

(a) $x^2+y^2+x-y=0$

(b) $x^2+y^2-x+y=0$

(c) $x^2+y^2+x+y=0$

(d) $x^2+y^2-x-y=0$

14. If $a \neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then **[2004]**

(a) $d^2+(3b-2c)^2=0$

(b) $d^2+(3b+2c)^2=0$

(c) $d^2+(2b-3c)^2=0$

(d) $d^2+(2b+3c)^2=0$

15. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x=4$, then the equation of the ellipse is: **[2004]**

(a) $4x^2+3y^2=1$

(b) $3x^2+4y^2=12$

(c) $4x^2+3y^2=12$

(d) $3x^2+4y^2=1$

16. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for **[2005]**

(a) exactly one value of a

(b) no value of a

(c) infinitely many values of a

(d) exactly two values of a

17. A circle touches the x- axis and also touches the circle with centre at $(0,3)$ and radius 2. The locus of the centre of the circle is **[2005]**

(a) an ellipse (b) a circle

(c) a hyperbola (d) a parabola

18. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**

(a) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$

(b) $2ax+2by-(a^2-b^2+p^2)=0$

(c) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

(d) $2ax+2by-(a^2+b^2+p^2)=0$

19. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **[2005]**

(a) $3a^2-10ab+3b^2=0$

(b) $3a^2-2ab+3b^2=0$

(c) $3a^2+10ab+3b^2=0$

(d) $3a^2+2ab+3b^2=0$

20. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is **[2005]**

(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x + 2 = 0$

(c) $x^2 + 4y + 2 = 0$ (d) $x^2 - 4y + 2 = 0$

21. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is **[2005]**

(a) an ellipse (b) a circle

(c) a parabola (d) a hyperbola

22. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is **[2005]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

23. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is **[2006]**

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$

(b) $x^2 + y^2 + 2x - 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 62 = 0$

(d) $x^2 + y^2 - 2x + 2y - 47 = 0$

24. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is **[2006]**

(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$

25. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is **[2006]**

(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$

(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$

26. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is **[2006]**

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$

(c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

27. Consider a family of circles which are passing through the point (– 1, 1) and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **[2007]**

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

28. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies = ? **[2007]**

(a) abscissae of vertices

(b) abscissae of foci

(c) eccentricity (d) directrix.

29. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[2007]**

(a) $(2, 4)$ (b) $(-2, 0)$

(c) $(-1, 1)$ (d) $(0, 2)$

30. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[2008]**

(a) $(3, -4)$ (b) $(-3, 4)$

(c) $(-3, -4)$ (d) $(3, 4)$

31. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is **[2008]**

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

32. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at **[2008]**

(a) $(0, 2)$ (b) $(1, 0)$
(c) $(0, 1)$ (d) $(2, 0)$

33. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and (1, 1) for: **[2009]**

(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p

34. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[2009]**

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

35. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[2010]**

(a) $-35 < m < 15$ (b) $15 < m < 65$
(c) $35 < m < 85$ (d) $-85 < m < -35$

36. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is [2010]

(a) $2x + 1 = 0$ (b) $x = -1$
(c) $2x - 1 = 0$ (d) $x = 1$

37. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ ($c > 0$) touch each other if **[2011]**

(a) $|a| = c$ (b) $a = 2c$
(c) $|a| = 2c$ (d) $2|a| = c$

38. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is **[2011]**

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$
(c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

39. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is **[2011]**

(a) $5x^2 + 3y^2 - 48 = 0$ (b) $3x^2 + 5y^2 - 15 = 0$
(c) $5x^2 + 3y^2 - 32 = 0$ (d) $3x^2 + 5y^2 - 32 = 0$

40. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is **[2011 RS]**

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 - x - y = 0$
(c) $x^2 + y^2 + 2x + 2y - 7 = 0$
(d) $x^2 + y^2 + x + y - 2 = 0$

41. The equation of the hyperbola whose foci are (– 2, 0) and (2, 0) and eccentricity is 2 is given by : **[2011RS]**

(a) $x^2 - 3y^2 = 3$ (b) $3x^2 - y^2 = 3$
(c) $-x^2 + 3y^2 = 3$ (d) $-3x^2 + y^2 = 3$

42. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is: **[2012]**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$
(c) $\frac{6}{5}$ (d) $\frac{5}{3}$

43. **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$ **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

44. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[2012]**

(a) $4x^2+y^2=4$ (b) $x^2+4y^2=8$

(c) $4x^2+y^2=8$ (d) $x^2+4y^2=16$

45. The chord PQ of the parabola $y^2=x$, where one end P of the chord is at point (4, – 2), is perpendicular to the axis of the parabola. Then the slope of the normal at Q is **[2012]**

(a) -4 (b) $-\frac{1}{4}$

(c) 4 (d) $\frac{1}{4}$

46. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[2013]**

(a) (–5, 2) (b) (2, –5)

(c) (5, –2) (d) (–2, 5)

47. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is **[2013]**

(a) $x^2+y^2-6y-7=0$

(b) $x^2+y^2-6y+7=0$

(c) $x^2+y^2-6y-5=0$

(d) $x^2+y^2-6y+5=0$

48. **Given :** A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y=x+\sqrt{5}$.

Statement-2 : If the line, $y=mx+\frac{\sqrt{5}}{m}$ $(m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0$. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

49. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is **[2014]**

(a) $\left(x^2+y^2\right)^2=6x^2+2y^2$

(b) $\left(x^2+y^2\right)^2=6x^2-2y^2$

(c) $\left(x^2-y^2\right)^2=6x^2+2y^2$

(d) $\left(x^2-y^2\right)^2=6x^2-2y^2$

50. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

51. The slope of the line touching both the parabolas $y^2=4x$ and $x^2=-32y$ is **[2014]**

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

52. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : **[2015]**

(a) $y^2=2x$ (b) $x^2=2y$

(c) $x^2=y$ (d) $y^2=x$

53. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is : **[2015]**

(a) 3 (b) 4

(c) 1 (d) 2

54. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is : **[2015]**

(a) $\frac{27}{2}$ (b) 27

(c) $\frac{27}{4}$ (d) 18

55. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k \in \mathbf{R}$, is a : **[2015]**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis

56. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis, lie on: **[2016]**

(a) a hyperbola

(b) a parabola

(c) a circle

(d) an ellipse which is not a circle

57. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[2016]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{4}{3}$ (d) $\frac{4}{\sqrt{3}}$

58. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is: **[2016]**

(a) 5 (b) 10

(c) $5\sqrt{2}$ (d) $5\sqrt{3}$

59. Let P be the point on the parabola, $y^2=8x$ which is at a minimum distance from the centre C of the circle, $x^2+(y+6)^2=1$. Then the equation of the circle, passing through C and having its centre at P is: **[2016]**

(a) $x^2+y^2-\frac{x}{4}+2y-24=0$

(b) $x^2+y^2-4x+9y+18=0$

(c) $x^2+y^2-4x+8y+12=0$

(d) $x^2+y^2-x+4y-12=0$

60. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : **[2017]**

(a) $\left(-\sqrt{2},-\sqrt{3}\right)$ (b) $\left(3\sqrt{2},2\sqrt{3}\right)$

(c) $\left(2\sqrt{2},3\sqrt{3}\right)$ (d) $\left(\sqrt{3},\sqrt{2}\right)$

61. The radius of a circle, having minimum area, which touches the curve $y=4-x^2$ and the lines, $y=|x|$ is : **[2017]**

(a) $4\left(\sqrt{2}+1\right)$ (b) $2\left(\sqrt{2}+1\right)$

(c) $2\left(\sqrt{2}-1\right)$ (d) $4\left(\sqrt{2}-1\right)$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(a)	(b)	(c)	(b)	(b)	(d)	(b)	(d)	(b)	(d)	(d)	(d)	(d)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(d)	(d)	(d)	(a)	(d)	(a)	(d)	(d)	(a)	(a)	(d)	(b)	(b)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(a)	(b)	(a)	(a)	(a)	(b)	(a)	(a)	(d)	(b)	(b)	(a)	(b)	(d)	(a)
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
(c)	(a)	(b)	(a)	(b)	(c)	(b)	(a)	(b)	(a)	(b)	(a)	(d)	(c)	(c)
61														
None)														

SOLUTIONS

1. **(c)** Equation of circle $x^2+y^2=1=(1)^2$
$\Rightarrow x^2+y^2=(y-mx)^2$
$\Rightarrow x^2=m^2x^2-2\,mxy;$
$\Rightarrow x^2\,(1-m^2)+2mxy=0.$ Which represents the pair of lines between which the angle is 45°.

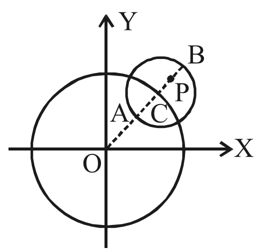
$$\tan 45 = \pm\frac{2\sqrt{m^2-0}}{1-m^2}=\frac{\pm 2m}{1-m^2};$$

$\Rightarrow 1-m^2=\pm\,2m \Rightarrow m^2\pm 2m-1=0$

$$\Rightarrow m=\frac{-2\pm\sqrt{4+4}}{2}$$

$$=\frac{-2\pm 2\sqrt{2}}{2}=-1\pm\sqrt{2}.$$

2. **(a)** For any point P (x, y) in the given circle,



we should have
$OA \le OP \le OB$

$$\Rightarrow (5-3)\le\sqrt{x^2+y^2}\le 5+3$$

$$\Rightarrow 4\le x^2+y^2\le 64$$

3. **(b)** Let the required circle be
$x^2+y^2+2gx+2fy+c=0$
Since it passes through (0, 0) and (1, 0)

$$\Rightarrow c=0 \text{ and } g=-\frac{1}{2}$$

Points (0, 0) and (1, 0) lie inside the circle $x^2+y^2=9$, so two circles touch internally
$\Rightarrow c_1c_2=r_1-r_2$

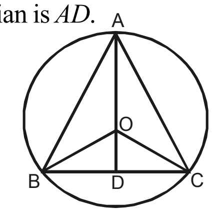
$$\therefore \sqrt{g^2+f^2}=3-\sqrt{g^2+f^2}\Rightarrow\sqrt{g^2+f^2}=\frac{3}{2}$$

$$\Rightarrow f^2=\frac{9}{4}-\frac{1}{4}=2 \qquad \therefore f=\pm\sqrt{2}.$$

Hence, the centres of required circle are

$$\left(\frac{1}{2},\sqrt{2}\right)\text{ or }\left(\frac{1}{2},-\sqrt{2}\right)$$

4. **(c)** Let ABC be an equilateral triangle, whose median is AD.



Given $AD=3a$.
In ΔABD, $AB^2=AD^2+BD^2$;
$\Rightarrow x^2=9a^2+(x^2/4)$ where $AB=BC=AC=x$.

$$\frac{3}{4}x^2=9a^2\Rightarrow x^2=12a^2.$$

In ΔOBD, $OB^2=OD^2+BD^2$

$$\Rightarrow r^2=(3a-r)^2+\frac{x^2}{4}$$

$\Rightarrow r^2=9a^2-6ar+r^2+3a^2;\Rightarrow 6ar=12a^2$
$\Rightarrow r=2a$
So equation of circle is $x^2+y^2=4a^2$

5. **(b)** Any tangent to the parabola $y^2=8ax$ is

$$y=mx+\frac{2a}{m} \qquad \text{...(i)}$$

If (i) is a tangent to the circle, $x^2+y^2=2a^2$

$$\text{then, }\sqrt{2}a=\pm\frac{2a}{m\sqrt{m^2+1}}$$

$\Rightarrow m^2(1+m^2)=2\Rightarrow(m^2+2)(m^2-1)=0$
$\Rightarrow m=\pm 1.$
So from (i), $y=\pm(x+2a)$.

6. **(b)** $|r_1-r_2|<C_1C_2$ for intersection

$\Rightarrow r-3<5\Rightarrow r<8$ ...(1)

and $r_1 + r_2 > C_1C_2$, $r + 3 > 5 \Rightarrow r > 2$ ...(2)

From (1) and (2), $2 < r < 8$.

7. **(d)** $\pi r^2 = 154 \Rightarrow r = 7$

For centre on solving equation

$2x - 3y = 5$ & $3x - 4y = 7$

we get $x = 1, y = -1$

$\therefore$ centre $= (1, -1)$

Equation of circle, $(x-1)^2 + (y+1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

8. **(b)** Equation of the normal to a parabola $y^2 = 4bx$ at point $\left(bt_1^2, 2bt_1\right)$ is

$y = -t_1 x + 2bt_1 + bt_1^3$

As given, it also passes through $\left(bt_2^2, 2bt_2\right)$ then

$2bt_2 = -t_1\, bt_2^2 + 2\,bt_1 + bt_1^3$

$$2t_2 - 2t_1 = -t_1\left(t_2^2 - t_1^2\right)$$

$$= -t_1(t_2 + t_1)(t_2 - t_1)$$

$$\Rightarrow 2 = -t_1(t_2 + t_1) \Rightarrow t_2 + t_1 = -\frac{2}{t_1}$$

$$\Rightarrow t_2 = -t_1 - \frac{2}{t_1}$$

9. **(d)** $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, \quad e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$

$\therefore$ Foci $= (\pm 3, 0)$

$\therefore$ foci of ellipse $=$ foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

$\therefore$ $e = \frac{3}{4}$

Then $b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 16\left(1 - \frac{9}{16}\right) = 7$$

10. **(b)** Let the variable circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ......(1)

It passes through (a, b)

$\therefore a^2 + b^2 + 2ga + 2fb + c = 0$ ......(2)

(1) cuts $x^2 + y^2 = 4$ orthogonally

$\therefore 2(g \times 0 + f \times 0) = c - 4 \Rightarrow c = 4$

$\therefore$ from (2) $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$a^2 + b^2 - 2ax - 2by + 4 = 0$

or $2ax + 2by = a^2 + b^2 + 4$

11. **(d)** Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(1)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0$ ....(2)

Circle (1) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (2)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0$ ....(3)

Let the other end of diameter through (p, q) be (h, k), then

$\frac{h+p}{2} = -g$ and $\frac{k+q}{2} = -f$

Put in (3)

$$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$$

$\Rightarrow h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k)

is $x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow (x - p)^2 = 4qy$

12. **(d)** Two diameters are along

$2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

solving we get centre $(1, -1)$

circumference $= 2\pi r = 10\pi$

$\therefore r = 5$.

Required circle is, $(x-1)^2+(y+1)^2=5^2$

$\Rightarrow x^2+y^2-2x+2y-23=0$

13. (d) Solving $y=x$ and the circle

$x^2+y^2-2x=0$, we get

$x=0, y=0$ and $x=1, y=1$

$\therefore$ Extremities of diameter of the required circle are (0, 0) and (1, 1). Hence, the equation of circle is

$(x-0)(x-1)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

14. (d) Solving equations of parabolas

$y^2=4ax$ and $x^2=4ay$

we get (0, 0) and ($4a, 4a$)

Substituting in the given equation of line

$2bx+3cy+4d=0$,

we get $d=0$ and $2b+3c=0$

$\Rightarrow d^2+(2b+3c)^2=0$

15. (b) $e=\frac{1}{2}$. Directrix, $x=\frac{a}{e}=4$

$\therefore a=4\times\frac{1}{2}=2$

$\therefore b=2\sqrt{1-\frac{1}{4}}=\sqrt{3}$

Equation of ellipse is

$\frac{x^2}{4}+\frac{y^2}{3}=1 \Rightarrow 3x^2+4y^2=12$

16. (b) $s_1=x^2+y^2+2ax+cy+a=0$

$s_2=x^2+y^2-3ax+dy-1=0$

Equation of common chord of circles

s_1 and s_2 is given by $s_1-s_2=0$

$\Rightarrow 5ax+(c-d)y+a+1=0$

Given that $5x+by-a=0$ passes through P and Q

$\therefore$ The two equations should represent the same line

$\Rightarrow \frac{a}{1}=\frac{c-d}{b}=\frac{a+1}{-a} \Rightarrow a+1=-a^2$

$a^2+a+1=0$

No real value of a.

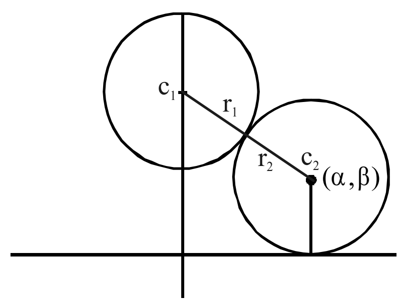
17. (d) Equation of circle with centre (0, 3) and radius 2 is $x^2+(y-3)^2=4$

Let locus of the variable circle is (α, β)

$\because$ It touches x - axis.

$\therefore$ It's equation is

$(x-\alpha)^2+(y+\beta)^2=\beta^2$



Circle touch externally $\Rightarrow c_1c_2=r_1+r_2$

$\therefore \sqrt{\alpha^2+(\beta-3)^2}=2+\beta$

$\alpha^2+(\beta-3)^2=\beta^2+4+4\beta$

$\Rightarrow \alpha^2=10(\beta-1/2)$

$\therefore$ Locus is $x^2=10\left(y-\frac{1}{2}\right)$

Which is parabola.

18. (d) Let the centre be (α, β)

$\because$ It cuts the circle $x^2+y^2=p^2$ orthogonally

$\therefore$ Using $2g_1g_2+2f_1f_2=c_1+c_2$, we get

$2(-\alpha)\times 0+2(-\beta)\times 0=c_1-p^2$

$c_1 = p^2$

Let equation of circle is

$x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

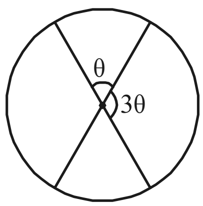
It passes through

$(a,b) \Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

$\therefore$ Locus of (α, β) is

$\therefore 2ax + 2by - (a^2 + b^2 + p^2) = 0$.

19. (d)



As per question area of one sector = 3 area of another sector

$\Rightarrow$ angle at centre by one sector

$= 3\times$ angle at centre by another sector

Let one angle be θ then other $= 3\theta$

Clearly $\theta + 3\theta = 180 \Rightarrow \theta = 45^\circ$

$\therefore$ Angle between the diameters represented by combined equation

$ax^2 + 2(a+b)xy + by^2 = 0$ is 45°

$\therefore$ Using $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$

we get $\tan 45^\circ = \dfrac{2\sqrt{(a+b)^2 - ab}}{a+b}$

$\Rightarrow 1 = \dfrac{2\sqrt{a^2 + b^2 + ab}}{a+b}$

$\Rightarrow (a+b)^2 = 4(a^2 + b^2 + ab)$

$\Rightarrow a^2 + b^2 + 2ab = 4a^2 + 4b^2 + 4ab$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0$

20. (a) $P = (1, 0)$ $Q = (h, k)$ Such that $K^2 = 8h$

Let (α, β) be the midpoint of PQ

$\alpha = \dfrac{h+1}{2}$, $\quad \beta = \dfrac{k+0}{2}$

$2\alpha - 1 = h \qquad 2\beta = k.$

$(2\beta)^2 = 8(2\alpha - 1) \Rightarrow \beta^2 = 4\alpha - 2$

$\Rightarrow y^2 - 4x + 2 = 0$.

21. (d) Tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola

$\Rightarrow \quad m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

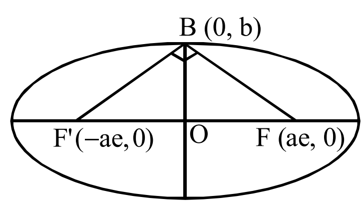
$\therefore \quad a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

22. (a) $\because \angle FBF' = 90^\circ \Rightarrow FB^2 + F'B^2 = FF'^2$

$\therefore \left(\sqrt{a^2e^2 + b^2}\right)^2 + \left(\sqrt{a^2e^2 + b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2 \Rightarrow e^2 = \dfrac{b^2}{a^2}$



Also $e^2 = 1 - b^2/a^2 = 1 - e^2$

$\Rightarrow 2e^2 = 1, \; e = \dfrac{1}{\sqrt{2}}$.

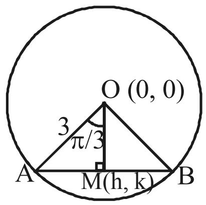
23. (d) Point of intersection of $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ is $(1, -1)$ which is the centre of the circle and radius = 7

$\therefore$ Equation is $(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow \; x^2 + y^2 - 2x + 2y - 47 = 0$

24. **(d)** Let $M(h, k)$ be the mid point of chord AB

where $\angle AOB = \dfrac{2\pi}{3}$



$\therefore\ \angle AOM = \dfrac{\pi}{3}$. Also $OM = 3\cos\dfrac{\pi}{3} = \dfrac{3}{2}$

$\Rightarrow \sqrt{h^2+k^2} = \dfrac{3}{2} \Rightarrow h^2+k^2 = \dfrac{9}{4}$

$\therefore$ Locus of (h, k) is $x^2+y^2 = \dfrac{9}{4}$

25. **(a)** Given parabola is $y = \dfrac{a^3x^2}{3} + \dfrac{a^2x}{2} - 2a$

$\Rightarrow y = \dfrac{a^3}{3}\left(x^3 + \dfrac{3}{2a}x + \dfrac{9}{16a^2}\right) - \dfrac{3a}{16} - 2a$

$\Rightarrow y + \dfrac{35a}{16} = \dfrac{a^3}{3}\left(x + \dfrac{3}{4a}\right)^2$

$\therefore$ Vertex of parabola is $\left(\dfrac{-3}{4a}, \dfrac{-35a}{16}\right)$

To find locus of this vertex,

$x = \dfrac{-3}{4a}$ and $y = \dfrac{-35a}{16}$

$\Rightarrow a = \dfrac{-3}{4x}$ and $a = -\dfrac{16y}{35}$

$\Rightarrow \dfrac{-3}{4x} = \dfrac{-16y}{35} \Rightarrow 64xy = 105$

$\Rightarrow xy = \dfrac{105}{64}$ which is the required locus.

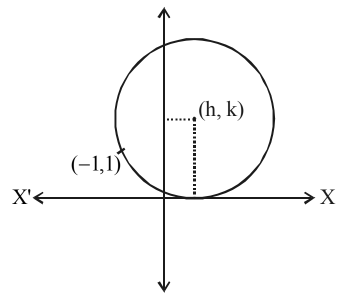
26. **(a)** $2ae = 6 \Rightarrow ae = 3$; $2b = 8 \Rightarrow b = 4$

$b^2 = a^2(1-e^2)$; $16 = a^2 - a^2e^2$

$\Rightarrow a^2 = 16 + 9 = 25 \Rightarrow a = 5$

$\therefore\ e = \dfrac{3}{a} = \dfrac{3}{5}$

27. **(d)** Equation of circle whose centre is (h, k)
i.e $(x-h)^2 + (y-k)^2 = k^2$



(radius of circle $= k$ because circle is tangent to x-axis)

Equation of circle passing through $(-1, +1)$

$\therefore\ (-1-h)^2 + (1-k)^2 = k^2$

$\Rightarrow\ 1 + h^2 + 2h + 1 + k^2 - 2k = k^2$

$\Rightarrow\ h^2 + 2h - 2k + 2 = 0$

$D \geq 0$

$\therefore\ (2)^2 - 4 \times 1.(-2k+2) \geq 0$

$\Rightarrow\ 4 - 4(-2k+2) \geq 0 \Rightarrow 1 + 2k - 2 \geq 0$

$\Rightarrow\ k \geq \dfrac{1}{2}$

28. **(b)** Given, equation of hyperbola is

$\dfrac{x^2}{\cos^2\alpha} - \dfrac{y^2}{\sin^2\alpha} = 1$

We know that the equation of hyperbola is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ Here, $a^2 = \cos^2\alpha$ and

$b^2 = \sin^2\alpha$

We know that, $b^2 = a^2(e^2-1)$

$\Rightarrow\ \sin^2\alpha = \cos^2\alpha(e^2-1)$

$\Rightarrow\ \sin^2\alpha + \cos^2\alpha = \cos^2\alpha.e^2$

$\Rightarrow\ e^2 = 1 + \tan^2\alpha = \sec^2\alpha$

$\Rightarrow\ e = \sec\alpha$

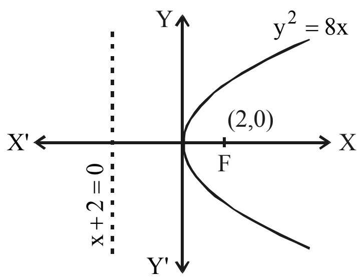
$\therefore\ \ ae = \cos\alpha \,.\, \dfrac{1}{\cos\alpha} = 1$

Co-ordinates of foci are $(\pm ae, 0)$

i.e. $(\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

29. **(b)** Parabola $y^2 = 8x$



We know that the locus of point of intersection of two perpendicular tangents to a parabola is its directrix.

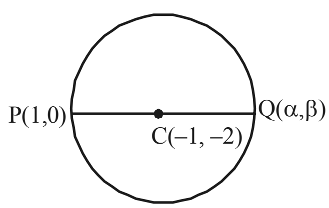
Point must be on the directrix of parabola

$\because$ equation of directrix $x + 2 = 0$

$\Rightarrow x = -2$

Hence the point is $(-2, 0)$

30. **(c)** The given circle is $x^2 + y^2 + 2x + 4y - 3 = 0$



Centre $(-1, -2)$

Let $Q(\alpha, \beta)$ be the point diametrically opposite to the point $P(1, 0)$,

then $\dfrac{1+\alpha}{2} = -1$ and $\dfrac{0+\beta}{2} = -2$

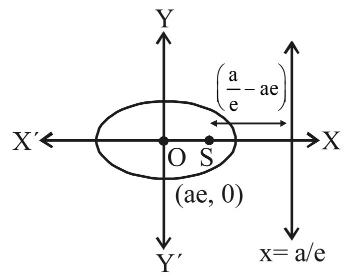
$\Rightarrow \alpha = -3, \beta = -4$

So, Q is $(-3, -4)$

31. **(a)** Perpendicular distance of directrix from focus

$= \dfrac{a}{e} - ae = 4$

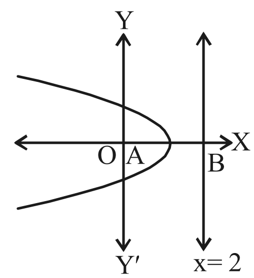
$\Rightarrow a\left(2 - \dfrac{1}{2}\right) = 4$



$\Rightarrow a = \dfrac{8}{3}$

$\therefore$ Semi major axis $= 8/3$

32. **(b)** Vertex of a parabola is the mid point of focus and the point



where directrix meets the axis of the parabola.

Here focus is $O(0, 0)$ and directrix meets the axis at $B(2, 0)$

$\therefore$ Vertex of the parabola is $(1, 0)$

33. **(a)** The given circles are

$S_1 \equiv x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ ....(1)

$S_2 \equiv x^2 + y^2 + 2x + 2y - p^2 = 0$ ....(2)

$\therefore$ Equation of common chord PQ is

$S_1 - S_2 = 0$

$\Rightarrow L \equiv x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow$ Equation of circle passing through P and Q is

$S_1 + \lambda L = 0$

$\Rightarrow (x^2 + y^2 + 3x + 7y + 2p - 5)$
$+ \lambda (x + 5y + p^2 + 2p - 5) = 0$

As it passes through (1, 1), therefore

$(7 + 2p) + \lambda (2p + p^2 + 1) = 0$

$\Rightarrow \lambda = -\dfrac{2p+7}{(p+1)^2}$

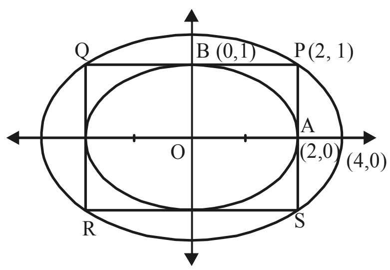
which does not exist for p = – 1

34. **(a)** The given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1$

So $A=(2,0)$ and $B=(0,1)$

If $PQRS$ is the rectangle in which it is inscribed, then $P=(2,1)$.

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the ellipse circumscribing the rectangle $PQRS$.



Then it passed through $P\ (2,1)$

$\therefore\ \frac{4}{a^2}+\frac{1}{b^2}=1$ ....(A)

Also, given that, it passes through (4, 0)

$\therefore\ \frac{16}{a^2}+0=1 \Rightarrow a^2=16$

$\Rightarrow b^2=4/3$ [substituting $a^2=16$ in eq^n (A)]

$\therefore$ The required ellipse is $\frac{x^2}{16}+\frac{y^2}{4/3}=1$

or $x^2+12y^2=16$

35. **(a)** Circle $x^2+y^2-4x-8y-5=0$

Centre = (2, 4), Radius = $\sqrt{4+16+5}=5$

If circle is intersecting line $3x-4y=m$, at two distinct points.

$\Rightarrow$ length of perpendicular from centre to the line < radius

$\Rightarrow \frac{|6-16-m|}{5}<5 \quad \Rightarrow |10+m|<25$

$\Rightarrow -25<m+10<25 \Rightarrow -35<m<15$

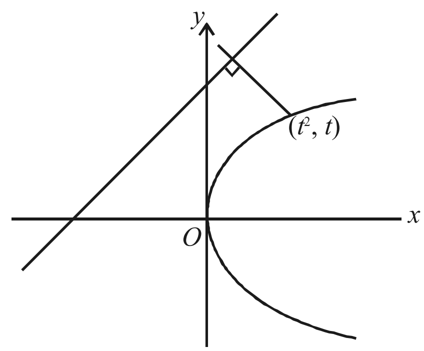
36. **(b)** The locus of perpendicular tangents is directrix i.e., $x=-a; x=-1$

37. **(a)** If the two circles touch each other, then they must touch each other internally.

So, $\frac{|a|}{2}=c-\frac{|a|}{2} \Rightarrow |a|=c$

38. **(a)** Shortest distance between two curve occurred along the common normal, so – $2t=-1$

$\Rightarrow t=1/2$



So shortest distance between them is $\frac{3\sqrt{2}}{8}$

39. **(d)** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

It passes through (–3, 1) so $\frac{9}{a^2}+\frac{1}{b^2}=1$ ..(i)

Also, $b^2=a^2(1-2/5)$

$\Rightarrow 5b^2=3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2=\frac{32}{3}, b^2=\frac{32}{5}$

So, the equation of the ellipse is

$$3x^2+5y^2=32$$

40. **(b)** Circle whose diametric end points are (1,0) and (0,1) will be of smallest radius. Equation of this smallest circle is

$(x-1)(x-0)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

41. **(b)** $ae=2$

$e=2$

$\therefore\ a=1$

$b^2=a^2(e^2-1)$

$b^2=1(4-1)$

$b^2=3$

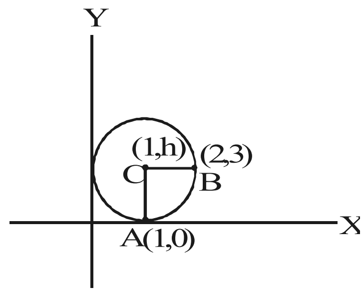
Equation of hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$

$3x^2 - y^2 = 3$

42. (a) Let centre of the circle be (1,h)

[$\because$ circle touches x-axis at (1,0)]



Let the circle passes through the point B (2,3)

$\therefore \quad CA = CB$ (radius)

$\Rightarrow \quad CA^2 = CB^2$

$\Rightarrow \quad (1-1)^2 + (h-0)^2 = (1-2)^2 + (h-3)^2$

$\Rightarrow \quad h^2 = 1 + h^2 + 9 - 6h$

$\Rightarrow \quad h = \frac{10}{6} = \frac{5}{3}$

43. (b) Given equation of ellipse is $2x^2 + y^2 = 4$

$\Rightarrow \frac{2x^2}{4} + \frac{y^2}{4} = 1 \Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$

Equation of tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ is

$y = mx \pm \sqrt{2m^2 + 4}$ ...(1)

($\because$ equation of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $y = mx + c$ where $c = \pm\sqrt{a^2 m^2 + b^2}$)

Now, Equation of tangent to the parabola $y^2 = 16\sqrt{3}x$ is $y = mx + \frac{4\sqrt{3}}{m}$ ...(2)

($\because$ equation of tangent to the parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$)

On comparing (1) and (2), we get

$\frac{4\sqrt{3}}{m} = \pm\sqrt{2m^2 + 4}$

Squaring on both the sides, we get

$16(3) = (2m^2 + 4)m^2$

$\Rightarrow \quad 48 = m^2(2m^2 + 4)$

$\Rightarrow \quad 2m^4 + 4m^2 - 48 = 0$

$\Rightarrow \quad m^4 + 2m^2 - 24 = 0$

$\Rightarrow \quad (m^2 + 6)(m^2 - 4) = 0$

$\Rightarrow \quad m^2 = 4 \ (\because m^2 \neq -6) \Rightarrow \quad m = \pm 2$

$\Rightarrow$ Equation of common tangents are $y = \pm 2x \pm 2\sqrt{3}$

Thus, statement-1 is true.

Statement-2 is obviously true.

44. (d) Equation of circle is $(x-1)^2 + y^2 = 1$

$\Rightarrow$ radius = 1 and diameter = 2

$\therefore$ Length of semi-minor axis is 2.

Equation of circle is $x^2 + (y-2)^2 = 4 = (2)^2$

$\Rightarrow$ radius = 2 and diameter = 4

$\therefore$ Length of semi major axis is 4

We know, equation of ellipse is given by

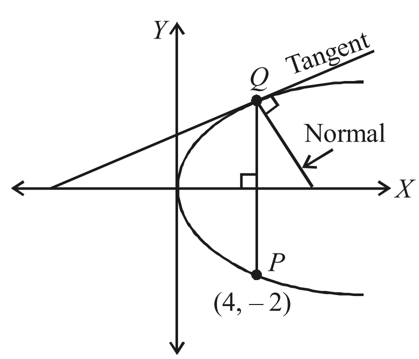
$$\frac{x^2}{(\text{Major axis})^2} + \frac{y^2}{(\text{Minor axis})^2} = 1$$

$\Rightarrow \quad \frac{x^2}{(4)^2} + \frac{y^2}{(2)^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

$\Rightarrow \quad x^2 + 4y^2 = 16$

45. (a) Point P is (4, –2) and $PQ \perp x$-axis

So, Q = (4, 2)



Equation of tangent at (4, 2) is

$yy_1 = \frac{1}{2}(x + x_1)$

$\Rightarrow 2y = \frac{1}{2}(x + 2) \Rightarrow 4y = x + 2$

$\Rightarrow y = \frac{x}{4} + \frac{1}{2}$

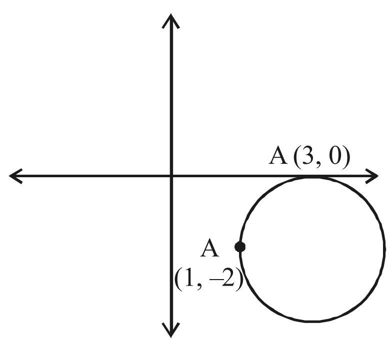
So, slope of tangent $= \frac{1}{4}$

$\therefore$ Slope of normal $= -4$

46. (c) Since circle touches x-axis at (3, 0)

$\therefore$ the equation of circle be

$(x-3)^2 + (y-0)^2 + \lambda y = 0$



As it passes through (1, –2)

$\therefore$ Put $x = 1, y = -2$

$\Rightarrow (1-3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow \lambda = 4$

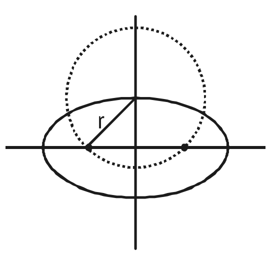
$\therefore$ equation of circle is

$(x-3)^2 + y^2 - 8 = 0$

Now, from the options (5, –2) satisfies equation of circle.

47. (a) From the given equation of ellipse, we have

$a = 4, b = 3, e = \sqrt{1 - \frac{9}{16}}$



$\Rightarrow e = \frac{\sqrt{7}}{4}$

Now, radius of this circle $= a^2 = 16$

$\Rightarrow$ Focii $= (\pm\sqrt{7}, 0)$

Now equation of circle is $(x-0)^2 + (y-3)^2 = 16$

$x^2 + y^2 - 6y - 7 = 0$

48. (b) Let common tangent be

$y = mx + \frac{\sqrt{5}}{m}$

Since, perpendicular distance from centre of the circle to the common tangent is equal to radius of the circle, therefore

$$\frac{\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}} = \sqrt{\frac{5}{2}}$$

On squaring both the side, we get

$m^2(1+m^2) = 2$

$\Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2+2)(m^2-1) = 0$

$\Rightarrow m = \pm 1 \qquad (\because m \neq \pm\sqrt{2})$

$y = \pm\left(x + \sqrt{5}\right)$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement-2.

49. (a) Given equation of ellipse can be written as

$\frac{x^2}{6} + \frac{y^2}{2} = 1$

$\Rightarrow a^2 = 6, b^2 = 2$

Now, equation of any variable tangent is

$$y = mx \pm \sqrt{a^2m^2 + b^2} \qquad \text{...(i)}$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y = \frac{-x}{m} \qquad \text{...(ii)}$$

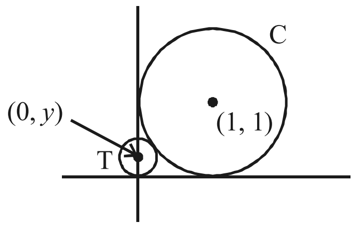
Eliminating m, we get

$$(x^4 + y^4 + 2x^2y^2) = a^2x^2 + b^2y^2$$

$$\Rightarrow \quad (x^2+y^2)^2 = a^2x^2 + b^2y^2$$

$$\Rightarrow \quad \boxed{(x^2+y^2)^2 = 6x^2+2y^2}$$

50. **(b)**



Equation of circle

$$C \equiv (x-1)^2 + (y-1)^2 = 1$$

Radius of T = $|y|$

T touches C externally therefore,

Distance between the centres = sum of their radii

$$\Rightarrow \quad \sqrt{(0-1)^2+(y-1)^2} = 1 + |y|$$

$$\Rightarrow \quad (0-1)^2 + (y-1)^2 = (1+|y|)^2$$

$$\Rightarrow \quad 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$$

$$2|y| = 1 - 2y$$

If $y > 0$ then $2y = 1 - 2y \Rightarrow y = \frac{1}{4}$

If $y < 0$ then $-2y = 1 - 2y \Rightarrow 0 = 1$ (not possible)

$$\therefore \quad y = \frac{1}{4}$$

51. **(c)** Given parabolas are

$y^2 = 4x$ ...(i)

$x^2 = -32y$ ...(ii)

Let m be slope of common tangent

Equation of tangent of parabola (1)

$$y = mx + \frac{1}{m} \quad ...(i)$$

Equation of tangent of parabola (2)

$$y = mx + 8m^2 \quad ...(ii)$$

(i) and (ii) are identical

$$\Rightarrow \quad \frac{1}{m} = 8m^2 \Rightarrow m^3 = \frac{1}{8} \Rightarrow \boxed{m = \frac{1}{2}}$$

ALTERNATIVE METHOD:

Let tangent to $y^2 = 4x$ be $y = mx + \frac{1}{m}$

Since this is also tangent to $x^2 = -32y$

$$\therefore \quad x^2 = -32\left(mx + \frac{1}{m}\right)$$

$$\Rightarrow \quad x^2 + 32mx + \frac{32}{m} = 0$$

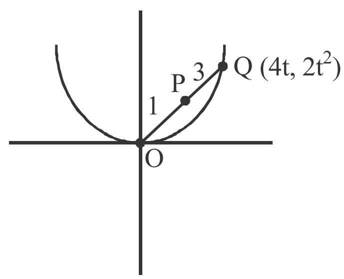
Now, D = 0

$$(32)^2 - 4\left(\frac{32}{m}\right) = 0$$

$$\Rightarrow m^3 = \frac{4}{32} \quad \Rightarrow m = \frac{1}{2}$$

52. **(b)** Let P(h, k) divides OQ in the ratio 1 : 3

Let any point Q on $x^2 = 8y$ is $(4t, 2t^2)$.



Then by section formula

$$\Rightarrow \quad k = \frac{t^2}{2} \text{ and } h = t$$

$$\Rightarrow \quad 2k = h^2$$

Required locus of P is $x^2 = 2y$

53. **(a)** $x^2 + y^2 - 4x - 6y - 12 = 0$ ...(i)

Centre, $C_1 = (2, 3)$

Radius, $r_1 = 5$ units

$x^2 + y^2 + 6x + 18y + 26 = 0$ ...(ii)

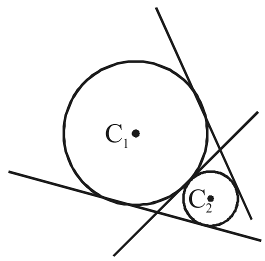
Centre, $C_2 = (-3, -9)$

Radius, $r_2 = 8$ units

$$C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2} = 13 \text{ units}$$

$$r_1 + r_2 = 5 + 8 = 13$$

$\therefore\ C_1C_2 = r_1 + r_2$



Therefore there are three common tangents.

54. **(b)** The end point of latus rectum of ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant is $\left(ae, \frac{b^2}{a}\right)$

and the tangent at this point intersects x-axis at

$\left(\frac{a}{e}, 0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

Then $a^2 = 9, b^2 = 5$

$\Rightarrow \quad e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

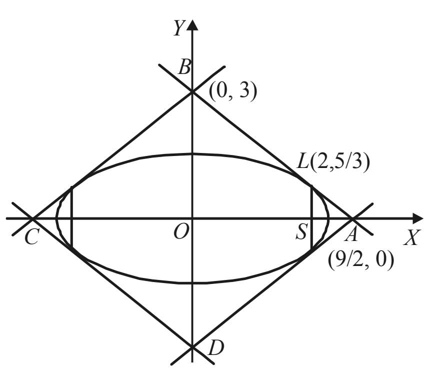
$\therefore$ end point of latus rectum in first quadrant is

L (2, 5/3)

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$

It meets x-axis at A (9/2, 0) and y-axis at B (0, 3)

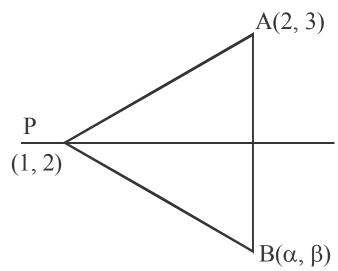
$\therefore \quad$ Area of ΔOAB $=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$



By symmetry area of quadrilateral

$= 4 \times$ (Area ΔOAB) $= 4\times\frac{27}{4}=27$ sq. units.

55. **(a)** Intersection point of $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$ is (1, 2)



Since, P is the fixed point for given family of lines

So, PB = PA

$(\alpha - 1)^2 + (\beta - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

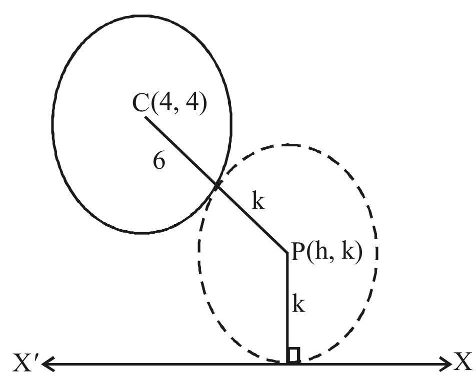
$(\alpha - 1)^2 + (\beta - 2)^2 = 1 + 1 = 2$

$(x - 1)^2 + (y - 2)^2 = (\sqrt{2})^2$

$(x - a)^2 + (y - b)^2 = r^2$

Therefore, given locus is a circle with centre (1, 2) and radius $\sqrt{2}$.

56. **(b)**



For the given circle,
centre : (4, 4)
radius = 6

$6+k=\sqrt{(h-4)^2+(k-4)^2}$

$(h - 4)^2 = 20k + 20$

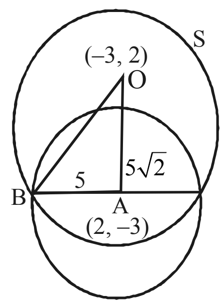
$\therefore$ locus of (h, k) is
$(x-4)^2 = 20(y+1)$,
which is a parabola.

57. **(a)** $\frac{2b^2}{a} = 8$ and $2b = \frac{1}{2}(2ae)$

$\Rightarrow 4b^2 = a^2e^2 \Rightarrow 4a^2(e^2-1) = a^2e^2$

$\Rightarrow 3e^2 = 4 \Rightarrow e = \frac{2}{\sqrt{3}}$

58. **(d)**



Centre of S : O (–3, 2) centre of given circle A(2, –3)

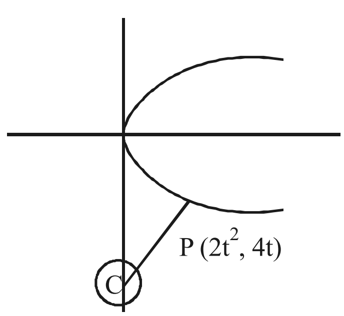
$\Rightarrow$ OA $= 5\sqrt{2}$

Also AB = 5 ($\because$ AB = r of the given circle)

$\Rightarrow$ Using pythagoras theorem in ΔOAB

$r = 5\sqrt{3}$

59. **(c)** Minimum distance $\Rightarrow$ perpendicular distance
Eq^n of normal at p111$(2t^2, 4t)$
$y = -tx + 4t + 2t^3$
It passes through C(0, –6)
$\Rightarrow t^3 + 2t + 3 = 0 \Rightarrow t = -1$



Centre of new circle = $P(2t^2, 4t)$
$= P(2, -4)$

Radius = PC $= \sqrt{(2-0)^2 + (-4+6)^2}$

$= 2\sqrt{2}$

$\therefore$ Equation of circle is :

$(x-2)^2 + (y+4) = \left(2\sqrt{2}\right)^2$

$\Rightarrow$ $x^2 + y^2 - 4x + 8y + 12 = 0$

60. **(c)** Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

foci is $(\pm 2, 0) \Rightarrow ae = 2 \Rightarrow a^2e^2 = 4$
Since $b^2 = a^2(e^2 - 1)$
$b^2 = a^2e^2 - a^2 \therefore a^2 + b^2 = 4$ ...(1)
Hyperbola passes through $(\sqrt{2}, \sqrt{3})$

$\therefore \frac{2}{a^2} - \frac{3}{b^2} = 1$ ...(2)

$\frac{2}{4-b^2} \frac{-3}{b^2} = 1$

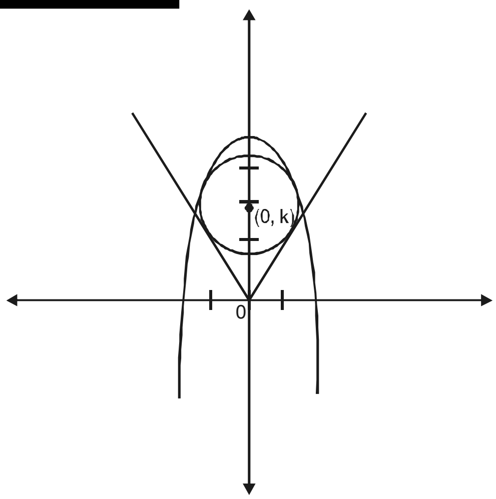
$\Rightarrow b^4 + b^2 - 12 = 0$
$\Rightarrow (b^2 - 3)(b^2 + 4) = 0$
$\Rightarrow b^2 = 3$
$b^2 = -4$ (Not possible)
For $b^2 = 3$

$\Rightarrow a^2 = 1 \therefore \frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent is $\frac{\sqrt{2}x}{1} - \frac{\sqrt{3}y}{3} = 1$

Clearly $(2\sqrt{2}, 3\sqrt{3})$ satisfies it.

61. **(None)**
(Let the equation of circle be
$x^2 + (y-k)^2 = r^2$
It touches $x - y = 0$



$\Rightarrow \quad \left|\frac{0-k}{\sqrt{2}}\right| = r$

$\Rightarrow \quad k = r\sqrt{2}$

$\therefore$ Equation of circle becomes

$$x^2 + (y-k)^2 = \frac{k^2}{2} \qquad \text{...(i)}$$

It touches $y = 4 - x^2$ as well

$\therefore$ Solving the two equations

$\Rightarrow \quad 4 - y + (y-k)^2 = \frac{k^2}{2}$

$\Rightarrow \quad 1y^2 - y(2k+1) + \frac{k^2}{2} + 4 = 0$

It will give equal roots $\therefore$ $D = 0$

$\Rightarrow \quad (2k+1)^2 = 4\left(\frac{k^2}{2} + 4\right)$

$\Rightarrow \quad 2k^2 + 4k - 15 = 0$

$\Rightarrow \quad k = \frac{-2+\sqrt{34}}{2}$

$\therefore \quad r = \frac{k}{\sqrt{2}} = \frac{-2+\sqrt{34}}{2\sqrt{2}}$

Which is not matching with any of the option given here.1