CHAPTER

Applications of Integrals 23

1. If $y=f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is **[2002]**
   (a) 3/2 (b) 1
   (c) 5/4 (d) –3/4

2. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln |x||$ is **[2002]**
   (a) 4sq. units (b) 6 sq. units
   (c) 10 sq. units (d) none of these

3. The area of the region bounded by the curves $y = |x-1|$ *and* $y = 3-|x|$ is **[2003]**
   (a) 6 sq. units (b) 2 sq. units
   (c) 3 sq. units (d) 4 sq. units.

4. The area of the region bounded by the curves $y = |x-2|, x = 1, x = 3$ and the x-axis is **[2004]**
   (a) 4 (b) 2
   (c) 3 (d) 1

5. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is **[2005]**
   (a) 1 (b) 2
   (c) 3 (d) 4

6. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is **[2005]**
   (a) $1:2:1$ (b) $1:2:3$
   (c) $2:1:2$ (d) $1:1:1$

7. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y = f(x)$, x - axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ **[2005]**
   (a) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (b) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$
   (c) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (d) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

8. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[2007]**
   (a) 1/6 (b) 1/3
   (c) 2/3 (d) 1

9. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[2008]**
   (a) $\frac{5}{3}$ (b) $\frac{1}{3}$
   (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

10. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point (2, 3) and the x-axis is: **[2009]**
    (a) 6 (b) 9
    (c) 12 (d) 3

11. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[2010]**
    (a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$
    (c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

12. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is **[2011]**

(a) 1 square unit (b) $\frac{3}{2}$ square units

(c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit

13. The area bounded by the curves **[2011 RS]**

$y^2 = 4x$ and $x^2 = 4y$ is:

(a) $\frac{32}{3}$ sq units (b) $\frac{16}{3}$ sq units

(c) $\frac{8}{3}$ sq. units (d) 0 sq. units

14. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : **[2012]**

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$

(c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

15. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : **[2013]**

(a) 9 (b) 36

(c) 18 (d) $\frac{27}{4}$

16. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is: **[2014]**

(a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$

(c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$

17. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is **[2015]**

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$

(c) $\frac{7}{32}$ (d) $\frac{5}{64}$

18. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is : **[2016]**

(a) $\pi - \frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

(c) $\pi - \frac{4}{3}$ (d) $\pi - \frac{8}{3}$

19. The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is : **[2017]**

(a) $\frac{5}{2}$ (b) $\frac{59}{12}$

(c) $\frac{3}{2}$ (d) $\frac{7}{3}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(a)	(d)	(d)	(a)	(d)	(d)	(a)	(d)	(b)	(d)	(b)	(b)	(c)	(a)
16	17	18	19											
(c)	(b)	(d)	(a)											

SOLUTIONS

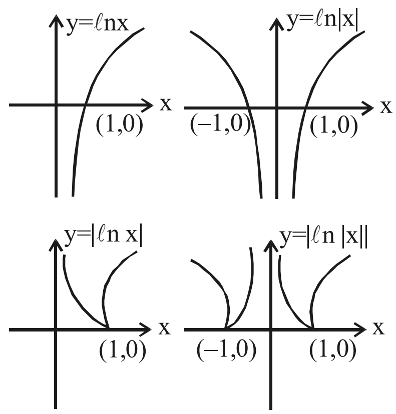
1. **(d)** We have $\int_0^2 f(x)dx = \frac{3}{4}$; Now,

$$\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$$

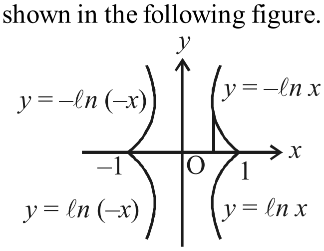
$$=[x\ f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$$

$$=0 - \frac{3}{4}\ (\because f(2)=0) = -\frac{3}{4}.$$

2. **(a)** First we draw each curve as separate graph



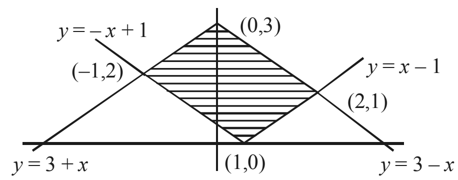
Note: Graph of $y=|f(x)|$ can be obtained from the graph of the curve $y=f(x)$ by drawing the mirror image of the portion of the graph below x-axis, with respect to x-axis. Clearly the bounded area is as shown in the following figure.



Required area $= 4\int_0^1 (-\ell nx)dx$

$= -4[x\ \ell n\ x - x]_0^1 = 4$ sq. units

3. **(d)**



$$A = \int_{-1}^{0} \{(3+x)-(-x+1)\}\,dx +$$

$$\int_0^1 \{(3-x)-(-x+1)\}\,dx + \int_1^2 \{(3-x)-(x-1)\}\,dx$$

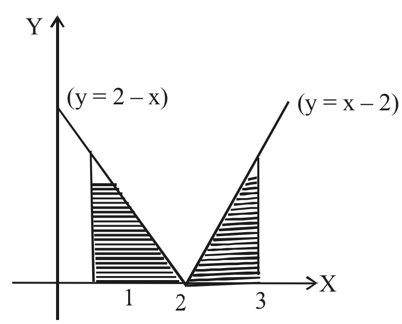
$$= \int_{-1}^{0} (2+2x)dx + \int_0^1 2dx + \int_1^2 (4-2x)dx$$

$$= \left[2x - x^2\right]_{-1}^{0} + [2x]_0^1 + \left[4x - x^2\right]_1^2$$

$$= 0-(-2+1)+(2-0)+(8-4)-(4-1)$$

$$= 1+2+4-3 = 4\ sq.\ units$$

4. **(d)** The required area is shown by shaded region

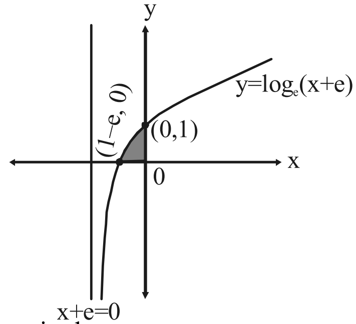


Required Area

$$A = \int_1^3 |x-2|dx = 2\int_2^3 (x-2)dx$$

$$= 2\left[\frac{x^2}{2} - 2x\right]_2^3 = 1$$

5. **(a)** The graph of the curve $y = \log_e(x+e)$ is as shown in the fig.



Required area

$$A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)\,dx$$

put $x + e = t \Rightarrow dx = dt$ also At $x = 1 - e$, $t = 1$

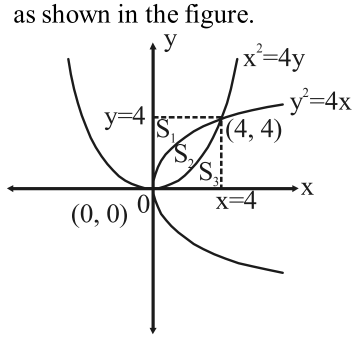
At $x = 0$, $t = e$

$$\therefore A = \int_1^e \log_e t\,dt = [t \log_e t - t]_1^e$$

$e - e - 0 + 1 = 1$

Hence the required area is 1 square unit.

6. **(d)** Intersection points of $x^2 = 4y$ and $y^2 = 4x$ are (0, 0) and (4, 4). The graph is as shown in the figure.



By symmetry, we observe

$$S_1 = S_3 = \int_0^4 y\,dx$$

$$= \int_0^4 \frac{x^2}{4}\,dx = \left[\frac{x^3}{12}\right]_0^4 = \frac{16}{3}\ sq.\ units$$

Also $S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4$

$$= \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3}\ \text{sq. units}$$

$\therefore S_1 : S_2 : S_3 = 1:1:1$

7. **(d)** Given that

$$\int_{\pi/4}^{\beta} f(x)dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$$

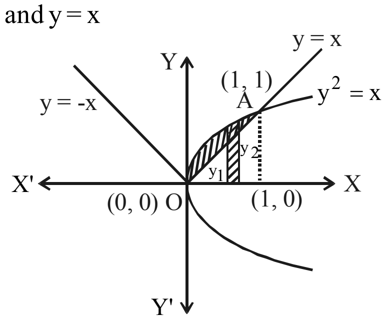
Differentiating w. r . t β

$$f(\beta) = \beta\ \cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$$

$$f\left(\frac{\pi}{2}\right) = \left(1 - \frac{\pi}{4}\right)\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$$

8. **(a)** The area enclosed between the curves $y^2 = x$ and $y = |x|$

From the figure, area lies between $y^2 = x$ and $y = x$



$$\therefore \text{Required area} = \int_0^1 (y_2 - y_1)dx$$

$$= \int_0^1 (\sqrt{x} - x)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$$\therefore \text{Required area} = \frac{2}{3}\left[x^{3/2}\right]_0^1 - \frac{1}{2}\left[x^2\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

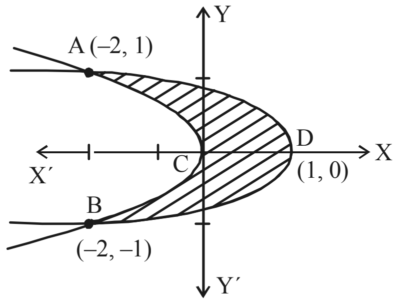
9. **(d)** $x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

[Left handed parabola with vertex at (0, 0)]

$x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

[Left handed parabola with vertex at (1, 0)]

Solving the two equations we get the points of intersection as (–2, 1), (–2, –1)



The required area is ACBDA, given by

$$= \left|\int_{-1}^{1}(1-3y^2-2y^2)dy\right| = \left|\left[y-\frac{5y^3}{3}\right]_{-1}^{1}\right|$$

$$= \left|\left(1-\frac{5}{3}\right)-\left(-1+\frac{5}{3}\right)\right| = 2\times\frac{2}{3}=\frac{4}{3} \text{ sq. units.}$$

10. (b) The given parabola is $(y-2)^2 = x-1$

Vertex (1, 2) and it meets x –axis at (5, 0)

Also it gives $y^2 - 4y - x + 5 = 0$

So, that equation of tangent to the parabola at (2, 3) is

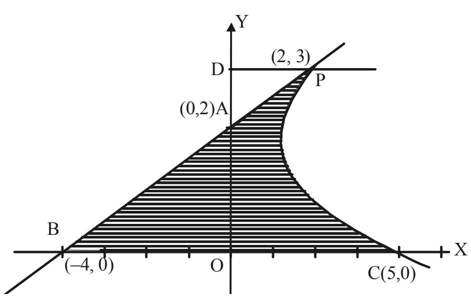
$$y.3-2(y+3)-\frac{1}{2}(x+2)+5=0$$

or $\quad x-2y+4=0$

which meets x-axis at (– 4, 0).

In the figure shaded area is the required area.

Let us draw PD perpendicular to y – axis.



Then required area = Ar ΔBOA + Ar (OCPD) – Ar (ΔAPD)

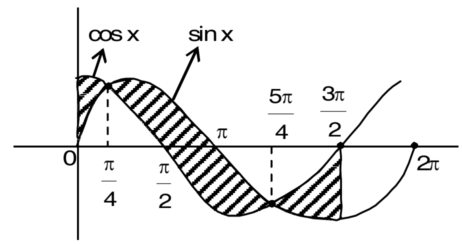
$$= \frac{1}{2}\times 4\times 2+\int_0^3 x\,dy-\frac{1}{2}\times 2\times 1$$

$$= 3+\int_0^3 (y-2)^2+1\,dy$$

$$= 3+\left[\frac{(y-2)^3}{3}+y\right]_0^3$$

$$= 3+\left[\frac{1}{3}+3+\frac{8}{3}\right] = 3+6=9 \text{ Sq. units}$$

11. (d)



Area above x-axis = Area below x-axis

∴ Required area

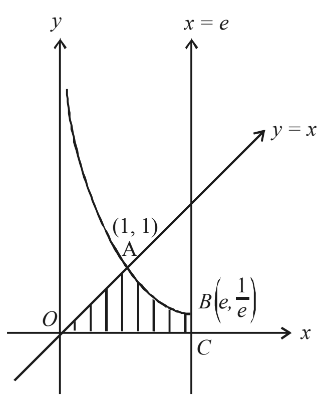
$$= 2\left[\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx+\int_{\frac{\pi}{2}}^{\pi}\sin x\,dx\right]$$

$$= 4\sqrt{2}-2$$

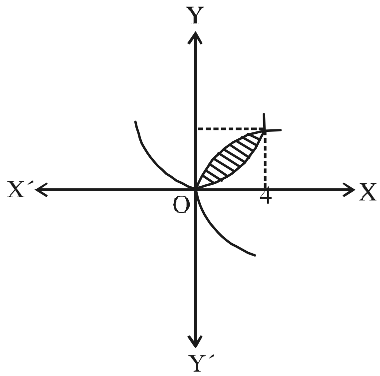
12. (b) Area of required region $AOBC$

$$= \int_0^1 x\,dx+\int_1^e \frac{1}{x}dx$$

$$= \frac{1}{2}+1=\frac{3}{2} \text{ sq. units}$$



13. **(b)**

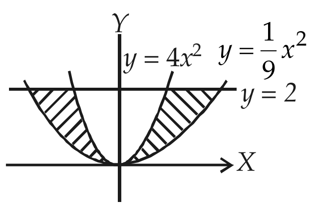


$$\text{Area} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left[2\left(\frac{x^{3/2}}{3/2}\right) - \frac{x^3}{12}\right]_0^4 = \frac{4}{3}\times 8 - \frac{64}{12}$$

$$= \frac{32}{3} - 16 = \frac{16}{3} \text{ sq. units}$$

14. **(c)** Given curves $x^2 = \frac{y}{4}$ and $x^2 = 9y$ are the parabolas whose equations can be written as $y = 4x^2$ and $y = \frac{1}{9}x^2$.



Also, given $y = 2$.

Now, shaded portion shows the required area which is symmetric.

$$\therefore \quad \text{Area} = 2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right)dy$$

$$\text{Area} = 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy$$

$$= 2\left[\frac{3.y^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2}.\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^2$$

$$= 2\left[\frac{2}{3}\times 3.y^{\frac{3}{2}} - \frac{1}{2}\times\frac{2}{3}.y^{\frac{3}{2}}\right]_0^2$$

$$= 2\left[2y^{\frac{3}{2}} - \frac{1}{3}y^{\frac{3}{2}}\right] = 2\times\frac{5}{3}y^{\frac{3}{2}}\Big|_0^2$$

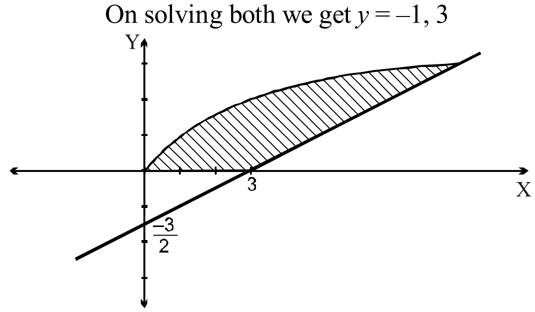
$$= 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

15. **(a)** Given curves are

$y = \sqrt{x}$ …(1)

and $2y - x + 3 = 0$ …(2)

On solving both we get $y = -1, 3$



$$\text{Required area} = \int_0^3 \left\{(2y+3) - y^2\right\} dy$$

$$= y^2 + 3y - \frac{y^3}{3}\Big|_0^3 = 9.$$

16. **(c)** Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.

Intersecting points are $x = 0, 1$

Area of shaded portion is the required area.

So, Required Area = Area of semi-circle + Area bounded by parabola

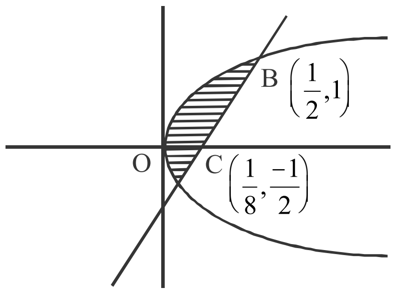
$$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}dx$$

$$= \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx \quad (\because \text{ radius of circle} = 1)$$

$$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1$$

$$= \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ Sq. unit}$$

17. **(b)** Required area

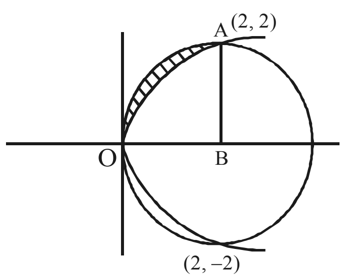


$$= \int_{-1/2}^{1} \frac{y+1}{4}dy - \int_{-1/2}^{1} \frac{y^2}{2}dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2}+y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$

$$= \frac{1}{4}\left[\frac{3}{2}+\frac{3}{8}\right]-\frac{9}{48}=\frac{15}{32}-\frac{9}{48}=\frac{27}{96}=\frac{9}{32}$$

18. **(d)**

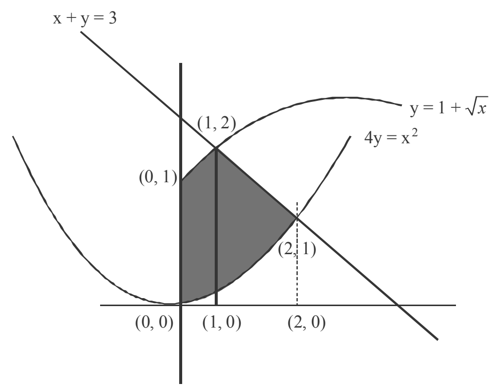


Points of intersection of the two curves are (0, 0), (2, 2) and (2, –2)

Area = Area (OAB) – area under parabola (0 to 2)

$$= \frac{\pi\times(2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x}\,dx \quad = \pi - \frac{8}{3}$$

19. **(a)**



Area of shaded region

$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= x\Big]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + 3x\Big]_1^2 - \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{5}{2} \text{ Sq.unit}$$