TARGET : JEE (Main + Advanced) 2015

Course : VIJETA & VIJAY (ADP & ADR) Date : 28-04-2015

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

NO. 07

TEST INFORMATION

DATE : 29.04.2015 **PART TEST-03 (PT-03)**

Syllabus : Straight Line, Circle, Solution of Triangle, Matrices & Determinant

REVISION DPP OF

VECTORS AND THREE DIMENSIONAL GEOMETRY

Total Marks : 140 **Max. Time : 110 min.**

Single choice Objective (–1 negative marking) Q. 1 to 17	**(3 marks 2.5 min.)**	**[51, 42.5]**
Multiple choice objective (–1 negative marking) Q. 18 to 37	**(4 marks, 3 min.)**	**[80,60]**
Comprehension (–1 negative marking) Q.38 to Q.40	**(3 marks 2.5 min.)**	**[9, 7.5]**

1. If the points with position vectors $-\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\lambda\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar, then the value of λ is

(A) –1 (B) 0

(C) 1 (D) 2

2. If $\vec{a},\vec{b}$ and $\vec{c}$ are three non-coplanar uni-modular vectors, each inclined with other at an angle 30°, then volume of tetrahedron whose edges are $\vec{a},\vec{b}$ and $\vec{c}$ is

(A) $\dfrac{3\sqrt{3}-5}{4}$ (B) $\dfrac{3\sqrt{3}+5}{12}$

(C) $\dfrac{\sqrt{3\sqrt{3}-5}}{12}$ (D) $\dfrac{3\sqrt{3}-5}{24}$

3. If lines $\dfrac{x-1}{2}=\dfrac{y+1}{3}=\dfrac{z-1}{4}$ and $\dfrac{x-3}{1}=\dfrac{y-k}{2}=\dfrac{z}{1}$ intersect, then the value of k is

(A) $\dfrac{3}{2}$ (B) $\dfrac{9}{2}$

(C) $-\dfrac{2}{9}$ (D) $-\dfrac{3}{2}$

4. If the distance between point P and Q is d and the projections of PQ on the coordinate planes are d_1, d_2, d_3 respectively, then $d_1^2 + d_2^2 + d_3^2$ =

(A) d^2 (B) $2d^2$

(C) $3d^2$ (D) $4d^2$

5. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{c} = \hat{j}-\hat{k}$, $\vec{a}.\vec{b} = 3$ and $\vec{a}\times\vec{b} = \vec{c}$, then $\vec{b}$ is equal to

(A) $\frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right)$ (B) $\frac{1}{3}\left(5\hat{i}-2\hat{j}-2\hat{k}\right)$

(C) $5\hat{i}+3\hat{j}+2\hat{k}$ (D) $3\hat{i}+\hat{j}-\hat{k}$

6. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar non-zero vectors, then

$(\vec{a}.\vec{a})\vec{b} \times \vec{c} + (\vec{a}.\vec{b})\vec{c} \times \vec{a} + (\vec{a}.\vec{c})\vec{a}\times\vec{b} =$

(A) $[\vec{a}\ \vec{b}\ \vec{c}]\vec{a}$ (B) $[\vec{a}\ \vec{c}\ \vec{b}]\vec{a}$

(C) $[\vec{a}\ \vec{b}\ \vec{c}]\vec{b}$ (D) $[\vec{a}\ \vec{c}\ \vec{b}]\vec{c}$

7. Let L_1, L_2, L_3 be three distinct lines in a plane π. (Lines are not parallel) Another line L is equally inclined with these three lines

S_1 : L is perpendicular to the plane π.

S_2 : If a non-zero $\vec{v}$ is equally inclined to 3 non-zero coplanar vectors $\vec{v}_1$, $\vec{v}_2$ & $\vec{v}_3$, then it is perpendicular to the plane containing them.

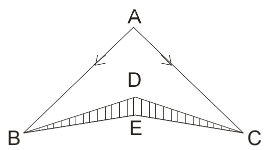
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(C) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

8. In given figure, $\overline{AB} = 3\hat{i} - \hat{j}$, $\overline{AC} = 2\hat{i} + 3\hat{j}$ & $\overline{DE} = 4\hat{i} - 2\hat{j}$. Then the area of the shaded region is



(A) 5 (B) 6

(C) 7 (D) 8

9. Four points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ are coplanar such that

$(\sin\alpha)\vec{a} + (2\sin2\beta)\vec{b} + (3\sin3\gamma)\vec{c} - \vec{d} = 0$. Then, the least value of the expression

$\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma$ is

(A) $\frac{1}{14}$ (B) 14

(C) $\sqrt{6}$ (D) $\frac{1}{\sqrt{16}}$

10. If a, b, c, x, y, z are real numbers and $a^2 + b^2 + c^2 = 9$, $x^2 + y^2 + z^2 = 16$ and $ax + by + cz = 12$, then $\frac{(a^3+b^3+c^3)^{1/3}}{(x^3+y^3+z^3)^{1/3}}$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$

(C) $\frac{3}{4}$ (D) $\frac{2}{3}$

11. If $\vec{a},\vec{b}$ are two unit vectors and $\vec{c}$ is such that $\vec{c} = \vec{a}\times\vec{c}+\vec{b}$, then the maximum value of $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ is

(A) 2 (B) $\frac{1}{2}$

(C) 1 (D) $\frac{3}{2}$

12. If $[\vec{a}\,\vec{b}\,\vec{c}]$ = 2, then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})\times(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\times(\vec{b}\times\vec{d})$ =

(A) $-5\vec{d}$ (B) $-3\vec{d}$

(C) $-4\vec{d}$ (D) $3\vec{d}$

13. A variable plane moves so that the sum of reciprocals of its intercepts on the three coordinate axes is constant λ. It passes through a fixed point whose coordinate are

(A) $(\lambda, \lambda, \lambda)$ (B) $\left(\frac{-1}{\lambda},\frac{-1}{\lambda},\frac{-1}{\lambda}\right)$

(C) $\left(\frac{1}{\lambda},\frac{1}{\lambda},\frac{1}{\lambda}\right)$ (D) $(-\lambda,-\lambda,-\lambda)$

14. The line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + 1 = z$ and $x - 2y + 1 = 0$ & $y - z = 0$. The coordinates of each of the points of intersection are

(A) (2, 1, 2), (1, 1, 1) (B) (3, 2, 3), (1, 1, 1)

(C) (3, 2, 3), (1, 1, 2) (D) (2, 3, 3), (2, 1, 1)

15. If $\vec{r}\,.\vec{a} = \vec{r}\,.\vec{b} = \vec{r}\,.\vec{c} = \frac{1}{2}$ for some non-zero vector $\vec{r}$, then the area of the triangle whose vertices are A($\vec{a}$), B($\vec{b}$), C($\vec{c}$) is (Origin does not lie in the plane of ΔABC)

(A) $|[\vec{a}\ \ \vec{b}\ \ \vec{c}]|$ (B) $|\vec{r}|$

(C) $|[\vec{a}\ \ \vec{b}\ \ \vec{c}]\vec{r}|$ (D) None of these

16. Let $x - y\sin\alpha - z\sin\beta = 0$

$x\sin\alpha - y + z\sin\gamma = 0$

& $x\sin\beta + y\sin\gamma - z = 0$ be three planes such that $\alpha + \beta + \gamma = \frac{\pi}{2}$ ($\alpha, \beta, \gamma \neq 0$) then the planes

(A) intersect in a point

(B) intersect in a line

(C) are parallel to each other

(D) are mutually perpendicular and intersect in a point

17. L_1 and L_2 are two lines whose vector equations are

$L_1 = \vec{r}_1 = \lambda[(\cos\theta + \sqrt{3})\hat{i} + (\sqrt{2}\sin\theta)\hat{j} + (\cos\theta - \sqrt{3})\hat{k}]$

& $L_2 = \vec{r}_2 = \mu(a\hat{i} + b\hat{j} + c\hat{k})$,

where λ and μ are scalars. If 'α' is the acute angle between L_1 and L_2, which is independent of 'θ', then α =

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{5\pi}{12}$

18. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ can be

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

19. $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 1$, If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then

(A) $|\vec{c}| = 2\sqrt{3}$ (B) $|\vec{c}| = 4\sqrt{3}$

(C) $\vec{b} \wedge \vec{c} = \frac{2\pi}{3}$ (D) $\vec{b} \wedge \vec{c} = \frac{5\pi}{6}$

20. The lines $x = y = z$ and $x = \frac{y}{2} = \frac{z}{3}$ and a third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units, (1, 1, 1) being one of the vertices of the triangle. Then the point of intersection of the third line with the second is

(A) (1, 2, 3) (B) (2, 4, 6)

(C) $\left(\frac{4}{3}, \frac{8}{3}, 4\right)$ (D) (–2, –4, –6)

21. Let O (O being the origin) be an interior point of ΔABC such that $\overrightarrow{OA} + 2\,\overrightarrow{OB} + 3\,\overrightarrow{OC} = 0$. If Δ, Δ_1, Δ_2 and Δ_3 are areas of ΔABC, ΔOAB, ΔOBC & ΔOCA respectively, then

(A) $\Delta = 3\Delta_1$ (B) $\Delta_1 = 3\Delta_2$

(C) $2\Delta_1 = 3\Delta_3$ (D) $\Delta = 3\Delta_3$

22. A unit vector $\hat{k}$ is rotated by 135° in such a way that the plane made by it bisects the angle between $\hat{i}$ & $\hat{j}$. The vector in the new position is

(A) $-\frac{\hat{i}}{2}+\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$ (B) $\frac{\hat{i}}{2}+\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

(C) $-\frac{\hat{i}}{2}-\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i}}{2}-\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

23. If $\hat{a}$ and $\hat{b}$ are unit vectors, then the vector $\vec{v} = (\hat{a} \times \hat{b}) \times (\hat{a} + \hat{b})$ is collinear with

(A) $\hat{a}+\hat{b}$ (B) $\hat{b}-\hat{a}$

(C) $\hat{a}-\hat{b}$ (D) $\hat{a}+2\hat{b}$

24. $[\vec{a} \times \vec{b} \;\; \vec{c} \times \vec{d} \;\; \vec{e} \times \vec{f}\,] =$

(A) $[\vec{a}\;\vec{b}\;\vec{d}\,][\vec{c}\;\vec{e}\;\vec{f}\,] - [\vec{a}\;\vec{b}\;\vec{c}\,][\vec{d}\;\vec{e}\;\vec{f}\,]$ (B) $[\vec{a}\;\vec{b}\;\vec{e}\,][\vec{f}\;\vec{c}\;\vec{d}\,] - [\vec{a}\;\vec{b}\;\vec{f}\,][\vec{e}\;\vec{c}\;\vec{d}\,]$

(C) $[\vec{c}\;\vec{d}\;\vec{a}\,][\vec{b}\;\vec{e}\;\vec{f}\,] - [\vec{c}\;\vec{d}\;\vec{b}\,][\vec{a}\;\vec{e}\;\vec{f}\,]$ (D) $[\vec{a}\;\vec{c}\;\vec{e}\,][\vec{b}\;\vec{d}\;\vec{f}\,]$

25. $a_1, a_2, a_3 \in R - \{0\}$ and $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0\ \forall\ x \in R$, then which of the following is/are true ?

(A) $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular

(B) $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$ are parallel

(C) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then (a_1, a_2, a_3) can be $(1, -1, -2)$

(D) If $2a_1 + 3a_2 + 6a_3 = 26$ then $\left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right| = 2\sqrt{6}$ units

26. If $\vec{p}, \vec{q}, \vec{r}$ are three non-zero non-collinear vectors satisfying $\vec{p} \times \vec{q} = \vec{r}$ & $\vec{q} \times \vec{r} = \vec{p}$ then which the following is always true

(A) $|\vec{q}\,| = 1$ (B) $|\,\vec{p}\,| = |\vec{r}\,|$

(C) $|\vec{r}\,| = 1$ (D) $\vec{r} \times \vec{p} = [\vec{p}\;\;\vec{q}\;\;\vec{r}\,]\vec{q}$

27. A rod of length 2 units in such that its one end is (1, 0, –1) and the other end touches the plane $x - 2y + 2z + 4 = 0$. Then

(A) The rod sweeps a figure with volume π cubic units

(B) The area of the region which the rod traces on the plane is 2π.

(C) The length of projection of the rod on the plane is $\sqrt{3}$ units

(D) The centre of the region which the rod traces on the plane is $\left(\frac{2}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

28. The position vector of the vertices A, B & C of a tetrahedron ABCD are (1, 1, 1), (1, 0, 0) & (3, 0, 0) respectively. The altitude from the vertex D to the opposite face ABC meets the median through A of ΔABC at point E. If AD = 4 units and volume of tetrahedron = $\frac{2\sqrt{2}}{3}$, then the correct statement(s) among the following is/are :

(A) The altitude from vertex D = 2 units

(B) There is only one possible position for point E

(C) There are two possible positions for point E

(D) vector $\hat{j}-\hat{k}$ is normal to the plane ABC

29. The equation of the plane which is equally inclined to the lines

$L_1 \equiv \frac{x-1}{2} = \frac{y}{-2} = \frac{z+2}{-1}$ & $L_2 \equiv \frac{x+3}{8} = \frac{y-4}{1} = \frac{z}{-4}$ and passing through origin is/are

(A) 14x – 5y – 7z = 0 (B) 2x + 7y – z = 0

(C) 3x – 4y – z = 0 (D) x + 2y – 5z = 0

30. Let $\vec{u}$ be a vector in the x-y plane with slope $\sqrt{3}$. Further $|\vec{u}|$, $|\vec{u}-\hat{i}|$, $|\vec{u}-2\hat{i}|$ are in G.P. , $\hat{i}$ being the unit vector along positve x-axis, then $|\vec{u}|$ is equal to

(A) $\sqrt{3-2\sqrt{2}}$ (B) $\sqrt{3+2\sqrt{2}}$

(C) $\tan\frac{9\pi}{8}$ (D) $\cot\frac{3\pi}{8}$

31. Let OABC is a regular tetrahedron and $\hat{p},\hat{q},\hat{r}$ are unit vectors along bisectors of angle between $\overrightarrow{OA},\overrightarrow{OB}$; $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OC},\overrightarrow{OA}$ respectively. If $\hat{a},\hat{b}$ and $\hat{c}$ are unit vectors along $\overrightarrow{OA},\overrightarrow{OB}$ & $\overrightarrow{OC}$ respectively, then

(A) $\frac{[\hat{a}\ \hat{b}\ \hat{c}]}{[\hat{p}\ \hat{q}\ \hat{r}]} = \frac{3\sqrt{3}}{2}$ (B) $\frac{[\hat{p}+\hat{q}\ \ \hat{q}+\hat{r}\ \ \hat{r}+\hat{p}]}{[\hat{a}+\hat{b}\ \ \hat{b}+\hat{c}\ \ \hat{c}+\hat{a}]} = \frac{3\sqrt{3}}{2}$

(C) $\frac{[\hat{p}\times\hat{q}\ \ \hat{q}\times\hat{r}\ \ \hat{r}\times\hat{p}]}{[\hat{a}\times\hat{b}\ \ \hat{b}\times\hat{c}\ \ \hat{c}\times\hat{a}]} = \frac{4}{27}$ (D) $\frac{[\hat{a}\ \hat{b}\ \hat{c}]}{[\hat{p}+\hat{q}\ \ \hat{q}+\hat{r}\ \ \hat{r}+\hat{p}]} = \frac{3\sqrt{3}}{4}$

32. Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k},\ \vec{b} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$, magnitude of whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is

(A) $4\hat{i}-\hat{j}+4\hat{k}$ (B) $2\hat{i}+\hat{j}+2\hat{k}$

(C) $3\hat{i}+\hat{j}-3\hat{k}$ (D) $3\hat{i}-\hat{j}+3\hat{k}$

33. OA, OB, OC are the sides of a rectangular parallelopiped whose diagonals are OO', AA', BB' and CC'. D is the centre of the rectangle AC'O'B' and D' is the centre of rectangle O' B' CA'. If the sides OA, OB, OC are in the ratio 1 : 2 : 3 then the $\angle$DOD' is equal to

(A) $\cos^{-1}\frac{24}{\sqrt{697}}$ (B) $\cos^{-1}\frac{11}{\sqrt{697}}$

(C) $\sin^{-1}\frac{11}{\sqrt{697}}$ (D) $\tan^{-1}\frac{11}{24}$

34. Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then $\vec{c}$ is

(A) $\frac{1}{\sqrt{6}}\left(\hat{i}-2\hat{j}+\hat{k}\right)$ (B) $\frac{1}{\sqrt{6}}\left(\hat{i}+2\hat{j}+\hat{k}\right)$

(C) $\frac{1}{\sqrt{6}}\left(2\hat{i}-\hat{j}+\hat{k}\right)$ (D) $\frac{1}{\sqrt{6}}\left(\hat{j}-2\hat{i}-\hat{k}\right)$

35. If $\vec{a},\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $\left[\vec{a}\times\left(\vec{b}+\vec{c}\right)\ \ \vec{b}\times\left(\vec{c}-2\vec{a}\right)\ \ \vec{c}\times\left(\vec{a}+3\vec{b}\right)\right]$ is equal to

(A) $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$ (B) $7\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

(C) $-5\ \left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]$ (D) $7\left[\vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\right]$

36. Let a, b, c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then

(A) $\frac{a^2+b^2}{2}>c^2$ (B) $\frac{1}{a}+\frac{1}{b}>\frac{2}{c}$

(C) a + b < 2c (D) a + b > 2c

37. If θ is the angle between the vectors $\vec{p}$ = $a\hat{i}+b\hat{j}+c\hat{k}$ and $\vec{q}$ = $b\hat{i}+c\hat{j}+a\hat{k}$, where a, b, c, $\in$ R, then all possible values of θ lies in

(A) $\left[0,\frac{5\pi}{6}\right]$ (B) $\left[\frac{5\pi}{6},\pi\right]$

(C) $\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$ (D) $\left[0,\frac{2\pi}{3}\right]$

Comprehension (Q. 38 to Q.40)

Consider two lines :

$L_1 : \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $L_2 : \frac{x-2}{3} = \frac{y-3}{1} = \frac{z-1}{2}$ then

38. If π denotes the plane x + by + cz + d = 0 parallel to the lines L_1, L_2 and which is equidistant from both L_1 and L_2 , then

(A) $1 + b^2 = c^2 + d^2$ (B) $d = \sqrt{bc}$

(C) b = cd (D) 2b + c + d = 0

39. Shortest distance between the two lines L_1 and L_2 is

(A) $\frac{2\sqrt{3}}{5}$ (B) $\frac{4\sqrt{3}}{5}$

(C) $\frac{6\sqrt{3}}{5}$ (D) $\frac{8\sqrt{3}}{5}$

40. Number of straight lines that can be drawn through the point (1, 4, –1) to intersect the lines L_1 and L_2 is

(A) 0 (B) 1

(C) 2 (D) infinite

DPP # 6

REVISION DPP OF SOLUTION OF TRIANGLE AND MATRICES & DETERMINANT

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(D)	**5.**	(C)	**6.**	(C)	**7.**	(D)
8.	(D)	**9.**	(A)	**10.**	(C)	**11.**	(A)	**12.**	(C,D)	**13.**	(A,D)	**14.**	(A,B)
15.	(A,B,C)	**16.**	(C,D)	**17.**	(A,D)	**18.**	(C,D)	**19.**	(A,B,C,D)	**20.**	(A,C)		
21.	(A,B,C)	**22.**	(B,D)	**23.**	(A,D)	**24.**	(A,B,D)	**25.**	(A,B,C)	**26.**	(A,B,D)		
27.	(B,C,D)	**28.**	(A,C,D)	**29.**	(B,C,D)	**30.**	(B,C)	**31.**	(A,C,D)	**32.**	(A,C,D)	**33.**	(C)
34.	(B)	**35.**	(D)	**36.**	(D)	**37.**	(A)	**38.**	(B)	**39.**	3		
40.	$(A \to P, Q); (B \to S); (C \to P, R); (D \to R)$												

MATHEMATICS

1. $A\left(-\hat{j}-\hat{k}\right)$ joined to $B\left(4\hat{i}+5\hat{j}+\lambda\hat{k}\right)$, $C\left(3\hat{i}+9\hat{j}+4\hat{k}\right)$, $D\left(-4\hat{i}+4\hat{j}+4\hat{k}\right)$

$$\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}\right] = \begin{vmatrix} 4 & 6 & \lambda+1 \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0 \qquad \Rightarrow \qquad \lambda = 1$$

2.
$$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2 = \begin{vmatrix} 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 1 \end{vmatrix} = \frac{3\sqrt{3}-5}{4}$$

$$\text{Volume} = \frac{1}{6}\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \frac{\sqrt{3\sqrt{3}-5}}{12}$$

3.
$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \qquad \Rightarrow \qquad k = \frac{9}{2}$$

4. Let $\overrightarrow{PQ} = x\hat{i}+y\hat{j}+z\hat{k}$ $\qquad \therefore \qquad d^2 = x^2 + y^2 + z^2$

Now, projection of $\overrightarrow{PQ}$ on xy-plane is d_1 $\qquad \therefore \qquad d^2 = d_1^2 + z^2$

similarly $\qquad d^2 = d_2^2 + x^2$

$\qquad d^2 = d_3^2 + y^2$ $\qquad \therefore \qquad d_1^2 + d_2^2 + d_3^2 = 2d^2$

5. $\vec{a}\times\vec{b} = \vec{c}$

$\Rightarrow \qquad \vec{a}\times\left(\vec{a}\times\vec{b}\right) = \vec{a}\times\vec{c} \qquad \Rightarrow \qquad 3\vec{a}-3\vec{b} = -2\hat{i}+\hat{j}+\hat{k} \qquad \Rightarrow \qquad \vec{b} = \frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right)$

6. $\vec{a}\ \vec{b}\ \vec{c}$ non coplanar

$\Rightarrow \qquad \vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ are also non-coplanar

$\Rightarrow \qquad \vec{a} = \lambda\vec{b}\times\vec{c} + \mu\vec{c}\times\vec{a} + \nu\vec{a}\times\vec{b} \qquad \Rightarrow \qquad \vec{a}.\vec{a} = \lambda[\vec{a}\ \vec{b}\ \vec{c}]$

similarly μ & ν $\qquad \therefore \qquad \vec{a} = \frac{\left(\vec{a}\ .\ \vec{a}\right)\left(\vec{b}\times\vec{c}\right)}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]} + \frac{\left(\vec{a}\ .\ \vec{b}\right)\left(\vec{c}\times\vec{a}\right)}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]} + \frac{\left(\vec{a}\ .\ \vec{c}\right)\left(\vec{a}\times\vec{b}\right)}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$

7. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be unit vectors along L_1, L_2, L_3 & L respectively

$\Rightarrow \qquad \vec{a}.\vec{d} = \vec{b}.\vec{d} = \vec{c}.\vec{d} \qquad \Rightarrow \qquad (\vec{a}-\vec{b}).\vec{d} = 0$

$(\vec{b}-\vec{c}).\vec{d} = 0$ & $(\vec{c}-\vec{a}).\vec{d} = 0 \Rightarrow$ is perpendicular to plane π

8. Required area = $\frac{1}{2}\left|\overline{BE}\times\overline{DE}+\overline{EC}\times\overline{DE}\right|$

$= \frac{1}{2}\left|\overline{BC}\times\overline{DE}\right|$

$= \frac{1}{2}\left|\left(-\hat{i}+4\hat{j}\right)\times\left(4\hat{i}-2\hat{j}\right)\right| = 7$

9. $\sin\alpha + 2\sin2\beta + 3\sin3\gamma = 1$ ....(1)

also $|\sin\alpha + 2\sin2\beta + 3\sin3\gamma| \le \sqrt{1+4+9}\,\sqrt{\sin^2\alpha+\sin^2 2\beta+\sin^2 3\gamma}$ as $|\vec{p}\,.\,\vec{q}| \le |\vec{p}\,||\,\vec{q}|$

$\therefore$ $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma \ge \frac{1}{14}$

10. Let $\vec{r}_1 = a\hat{i}+b\hat{j}+c\hat{k}$ & $\vec{r}_2 = x\hat{i}+y\hat{j}+z\hat{k}$

$\because$ $\vec{r}_1.\vec{r}_2 = |\vec{r}_1|\,|\vec{r}_2|$ $\therefore$ $\vec{r}_1 \,||\, \vec{r}_2$ $\Rightarrow$ $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}$

11. $\vec{c} = \vec{a}\times\vec{c}+\vec{b}$

$\Rightarrow$ $\left|\vec{c}-\vec{b}\right| = \left|\vec{a}\times\vec{c}\right|$ $\Rightarrow$ $c^2 + 1 - 2\vec{b}.\vec{c} = c^2\sin^2\theta$, where $\theta = \vec{a}\,\hat{}\,\vec{c}$

$\Rightarrow$ $2\,\vec{b}.\vec{c} = c^2\cos^2\theta + 1$ $\Rightarrow$ $2\vec{b}.\left(\vec{a}\times\vec{c}+\vec{b}\right) = c^2\cos^2\theta + 1$

$\Rightarrow$ $-2\left[\vec{a}\,\vec{b}\,\vec{c}\right] + 2 = c^2\cos^2\theta + 1$ $\Rightarrow$ $2\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 1 - c^2\cos^2\theta \le 1$ $\Rightarrow$ $\left[\vec{a}\,\vec{b}\,\vec{c}\right] \le 1/2$

12. Let $\vec{d} = \alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$

Now $\vec{d}.\vec{b}\times\vec{c} = 2\alpha$

$\vec{d}.\vec{c}\times\vec{a} = 2\beta$

$\vec{d}.\vec{a}\times\vec{b} = 2\gamma$ $\therefore$ $\left[\vec{d}\;\vec{b}\;\vec{c}\right]\vec{a}+\left[\vec{d}\;\vec{c}\,\vec{a}\right]\vec{b}+\left[\vec{d}\;\vec{a}\;\vec{b}\right]\vec{c} = 2\vec{d}$

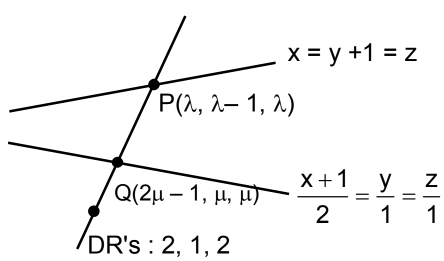
Now, $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{b}\times\vec{d}\right)$

$= \left[\vec{a}\;\vec{b}\;\vec{d}\right]\vec{c}-2\vec{d}+\left[\vec{b}\;\vec{c}\;\vec{d}\right]\vec{a}-2\vec{d}+\left[\vec{c}\;\vec{a}\;\vec{d}\right]\vec{b}-2\vec{d} = -4\vec{d}$

13. Let equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$

Given that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \lambda$ $\because$ fixed point is $\left(\frac{1}{\lambda},\frac{1}{\lambda},\frac{1}{\lambda}\right)$

14.



$\frac{2\mu-\lambda-1}{2}=\frac{\mu-\lambda+1}{1}=\frac{\mu-\lambda}{2}$ $\Rightarrow$ $\lambda = 3$ & $\mu = 1$

15. $\vec{r} = x(\vec{a}\times\vec{b}) + y(\vec{b}\times\vec{c}) + z(\vec{c}\times\vec{a})$

dot with $\vec{a}$, $\vec{b}$ & $\vec{c}$ $\Rightarrow$ $x = \frac{\vec{r}.\vec{c}}{\left[\vec{a}\;\;\vec{b}\;\;\vec{c}\right]}$ and so on

$\Rightarrow \quad \vec r\,[\vec a\ \ \vec b\ \ \vec c\,] = \dfrac{1}{2}\,(\vec a\times\vec b + \vec b\times\vec c + \vec c\times\vec a\,) \Rightarrow \quad$ Ar ΔABC = $|[\vec a\ \ \vec b\ \ \vec c\,]\,\vec r\,|$

16. Line of intersect of plane (1) and (2) is $\dfrac{x}{\cos\gamma} = \dfrac{y}{\cos\beta} = \dfrac{z}{\cos\alpha}$
which passes through origin and is perpendicular to the normal of the third plane

17. $\cos\alpha = \dfrac{\vec r_1\cdot\vec r_2}{|\vec r_1||\vec r_2|} = \dfrac{(a+c)\cos\theta + b\sqrt{2}\sin\theta + \sqrt{3}\,(a-c)}{\sqrt{a^2+b^2+c^2}\,\sqrt{8}}$
for 'α' to be independent of θ, a + c = 0 & b = 0

18. $\vec r_1 = \hat i\times(\hat i+\hat j) = \hat k$

$\vec r_2 = (\hat i-\hat j)\times(\hat i+\hat k) = -\hat i-\hat j+\hat k$

Now $\vec a = \lambda(\vec r_1\times\vec r_2) = \lambda(\hat i-\hat j)$

19. $\vec c = (2\vec a\times\vec b\,) - 3\vec b$

$\Rightarrow \quad \vec b\,.\,\vec c = -3|\vec b\,|^2 \qquad ...(1)$

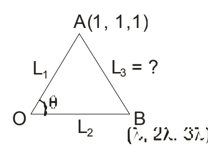
also, $|\vec c\,|^2 = 4|\vec a\times\vec b\,|^2 + 9|\vec b\,|^2 - \underbrace{6\vec b.(2\vec a\times\vec b)}_{\text{zero}} \quad\Rightarrow\quad |\vec c\,|^2 = 4[|\vec a\,|^2\,|\vec b\,|^2 - (\vec a\,.\,\vec b\,)^2] + 9|\vec b\,|^2$

$\Rightarrow \quad |\vec c\,|^2 = 48 \quad\Rightarrow\quad |\vec c\,| = 4\sqrt{3} \qquad ...(2)$

$\therefore \quad \cos\theta = \dfrac{\vec b.\vec c}{|\vec b||\vec c|} = \dfrac{-3|\vec b\,|^2}{|\vec b||\vec c|} = \dfrac{-\sqrt{3}}{2}$

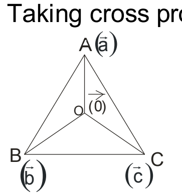
20. $\cos\theta = \dfrac{6}{\sqrt{42}} \quad\Rightarrow\quad \sin\theta = \dfrac{\sqrt{6}}{\sqrt{42}}$

Area ΔOAB = $\dfrac{1}{2}$ (OA)(OB) $\sin\theta$



$= \dfrac{1}{2}\,(\sqrt{3}\,)\,|\lambda|\,(\sqrt{14}\,)\,\dfrac{\sqrt{6}}{\sqrt{42}} = \sqrt{6} \qquad\Rightarrow\qquad \lambda = \pm 2$

21. $\vec a + 2\vec b + 3\vec c = 0$
Taking cross product with $\vec a$ and $\vec b$,



$\vec a\times\vec b = \dfrac{3}{2}(\vec c\times\vec a\,) = 3(\vec b\times\vec c\,)$ Now $\Delta = \dfrac{1}{2}\left|\vec a\times\vec b+\vec b\times\vec c+\vec c\times\vec a\right| = \dfrac{1}{2}.2\left|\vec a\times\vec b\right| = \dfrac{1}{2}.3\left|\vec c\times\vec a\right| = \dfrac{1}{2}.6\left|\vec b\times\vec c\right|$

22. Let $\hat r$ be the new vector $\quad\Rightarrow\quad \hat r = \lambda\hat k + \mu(\hat i+\hat j\,)$

$\hat r\,.\,\hat k = -\dfrac{1}{\sqrt{2}} \quad \& \quad |\hat r\,| = 1 \qquad \lambda = -\dfrac{1}{\sqrt{2}}\ \&\ \mu = \pm\dfrac{1}{2}$

23. Apply VTP to get = $(1 + \hat a\,.\,\hat b\,)\,(\hat b - \hat a\,)$

24. Use $\vec a\,.(\vec b\times\vec c\,) = \vec b\,.(\vec c\times\vec a\,) = \vec c\,.(\vec a\times\vec b\,)$

25. $(a_1 + a_2) + \sin^2x(a_3 - 2a_2) = 0 \quad \Rightarrow \quad a_1 + a_2 = 0$

& $\quad a_3 - 2a_2 = 0 \quad \Rightarrow \quad \frac{a_1}{-1} = \frac{a_2}{1} = \frac{a_3}{2} = \lambda$

26. $\vec{q} \times \vec{r} = \vec{p}$

$(\vec{q} \times \vec{r}) \times \vec{q} = \vec{p} \times \vec{q} = \vec{r}$

$\Rightarrow \quad |\vec{q}| = 1$ & $\vec{r}\,.\,\vec{q} = 0$ & $\because \vec{q} \times \vec{r} = \vec{p}$

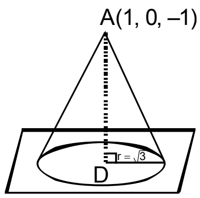
$\Rightarrow \quad |\vec{p}| = |\vec{r}|$

27. The rod sweeps a cone
AD = 1 unit

slant height $\ell = 2$ units $\Rightarrow r = \sqrt{3} \quad \Rightarrow \quad$ volume = $\frac{1}{3}\pi r^2 h = \pi$ cubic units

also, area of circle = $\pi(\sqrt{3})^2 = 3\pi$

& centre is foot of perpendicular of A in plane = $\left(\frac{2}{3}, \frac{2}{3}, \frac{-5}{3}\right)$



28. Volume = $\frac{2\sqrt{2}}{3}$

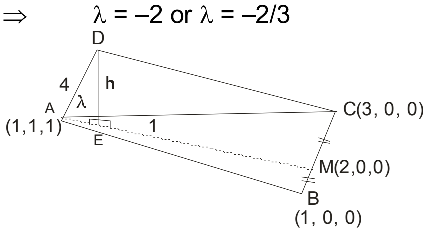
$$\Rightarrow \quad \frac{1}{3}\cdot\frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{matrix}\right\| \times h = \frac{2\sqrt{2}}{3}$$

$\Rightarrow \quad h\,|\hat{j} - \hat{k}| = 2\sqrt{2} \quad \Rightarrow \quad h = 2$

for E , let AE : EM = $\lambda : 1$

$\Rightarrow \quad E \equiv \left(\frac{2\lambda+1}{\lambda+1}, \frac{1}{\lambda+1}, \frac{1}{\lambda+1}\right)$ & $(AE)^2 + (ED)^2 = (AD)^2$

$\Rightarrow \quad \lambda = -2$ or $\lambda = -2/3$



29. The plane is perpendicular to the angle bisectors of the line, which are $\frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} \pm \frac{8\hat{i} + \hat{j} - 4\hat{k}}{9}$

30. $\vec{u}\,.\,\hat{i} = |\vec{u}|\cos 60° = \frac{|\vec{u}|}{2} \quad \because \quad$ slope = $\sqrt{3}$

also $|\vec{u} - \hat{i}|^2 = |\vec{u}||\vec{u} - 2\hat{i}| \quad \Rightarrow \quad u^2 + 1 - u = u.\sqrt{u^2 + 4 - 2u} \quad \Rightarrow \quad |\vec{u}| = \sqrt{2} - 1$

31. $\hat{p} = \dfrac{\hat{a}+\hat{b}}{2\cos\frac{\pi}{6}} = \dfrac{\hat{a}+\hat{b}}{\sqrt{3}}$

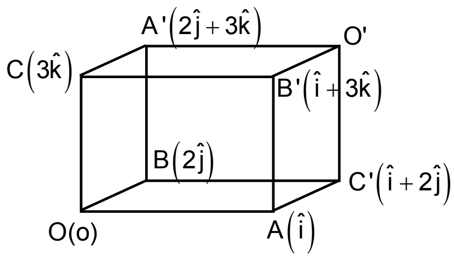
Similarly $\hat{q} = \dfrac{\hat{b}+\hat{c}}{\sqrt{3}}$ & $\hat{r} = \dfrac{\hat{c}+\hat{a}}{\sqrt{3}}$

Now $[\hat{p}\ \ \hat{q}\ \ \hat{r}] = \dfrac{1}{3\sqrt{3}}\left[\hat{a}+\hat{b}\quad \hat{b}+\hat{c}\quad \hat{c}+\hat{a}\right] = \dfrac{2}{3\sqrt{3}}\left[\hat{a}\ \ \hat{b}\ \ \hat{c}\right]$

32. Let required vector is $\vec{r} = x\vec{a} + y\vec{b}$

Now $\vec{r}.\hat{c} = \pm\dfrac{1}{\sqrt{3}}$ $\Rightarrow$ $2x - y = \pm 1$

33.



p.v. of point D = $\overrightarrow{OD} = \dfrac{2\hat{i}+2\hat{j}+3\hat{k}}{2}$

p.v. of point D' = $\overrightarrow{OD} = \dfrac{\hat{i}+2\hat{j}+6\hat{k}}{2}$

now $\cos\theta$ = $\dfrac{\overrightarrow{OD}.\overrightarrow{OD}}{|\overrightarrow{OD}||\overrightarrow{OD}|} = \dfrac{24}{\sqrt{697}}$

34. $\vec{a}\times(\vec{a}\times\vec{b}) = (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}$

$= -(\hat{i}+\hat{j}-\hat{k}) - 3(\hat{i}-\hat{j}+\hat{k}) = -4\hat{i}+2\hat{j}-2\hat{k} = -2(2\hat{i}-\hat{j}+\hat{k})$

Required unit vector = $\pm\dfrac{(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{6}}$

35. $\left[\vec{a}\times\vec{b}-\vec{c}\times\vec{a}\quad \vec{b}\times\vec{c}+2\vec{a}\times\vec{b}\quad \vec{c}\times\vec{a}-3\vec{b}\times\vec{c}\right]$

$= \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & -3 & 1 \end{vmatrix}\left[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\right] = 7\left[\vec{c}\times\vec{a}\quad \vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\right]$

36. $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$ $\Rightarrow$ $c^2 = ab$ $\Rightarrow$ a, c, b are in G.P.

37. $\cos\theta$ = $\dfrac{\vec{p}.\vec{q}}{|\vec{p}||\vec{p}|} = \dfrac{ab+bc+ca}{a^2+b^2+c^2}$

$\because$ we know that $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$ and $(a + b + c)^2 \geq 0$

$\therefore$ $-\dfrac{1}{2} \leq \dfrac{ab+bc+ca}{a^2+b^2+c^2} \leq 1 \Rightarrow -\dfrac{1}{2} \leq \cos\theta \leq 1 \Rightarrow \theta \in \left[0, \dfrac{2\pi}{3}\right]$

38. Normal of plane $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix} = \hat{i} + 7\hat{j} - 5\hat{k}$

Let equation of plane x + 7y – 5z + d = 0

Now $\left|\dfrac{1+14-15+d}{\sqrt{75}}\right| = \left|\dfrac{2+21-5+d}{\sqrt{75}}\right|$

$\Rightarrow$ $|d| = |18 + d|$

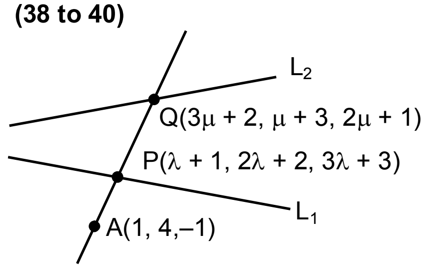
$\Rightarrow$ $d = -9$

$\therefore$ equation of plane is $x + 7y - 5z - 9 = 0$.

39. Shortest distance = $\left|\left(-\hat{i} - \hat{j} + 2\hat{k}\right).\dfrac{\left(\hat{i} + 7\hat{j} - 5\hat{k}\right)}{\sqrt{75}}\right|$

$= \dfrac{18}{5\sqrt{3}} = \dfrac{6\sqrt{3}}{5}$

40. **(38 to 40)**



Now $\overrightarrow{AP} \parallel \overrightarrow{AQ}$

$\therefore$ $\dfrac{\lambda}{3\mu+1} = \dfrac{2\lambda-2}{\mu-1} = \dfrac{3\lambda+4}{2\mu+2}$ $\Rightarrow$ $\lambda = 1, -\dfrac{1}{2}$

but $\lambda = 1$ is not possible