CHAPTER 9

Mechanical Properties of Fluids

1. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is **[2002]**

(a) 10 (b) 20
(c) 25.5 (d) 5

2. Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is **[2004]**

(a) inversely proportional to both radius 'R' and velocity 'v'
(b) directly proportional to both radius 'R' and velocity 'v'
(c) directly proportional to 'R' but inversely proportional to 'v'
(d) inversely proportional to 'R' but directly proportional to velocity 'v'

3. If two soap bubbles of different radii are connected by a tube **[2004]**

(a) air flows from the smaller bubble to the bigger
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(d) there is no flow of air.

4. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be **[2005]**

(a) 10 cm (b) 8 cm
(c) 20 cm (d) 4 cm

5. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid **[2006]**

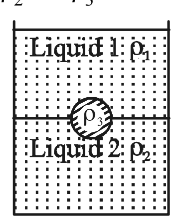
(a) 0.4 m/s (b) 0.133 m/s
(c) 0.1 m/s (d) 0.2 m/s

6. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_1 $(\rho_2 < \rho_1)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ $(k > 0)$. The terminal speed of the ball is **[2008]**

(a) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\dfrac{Vg\rho_1}{k}$

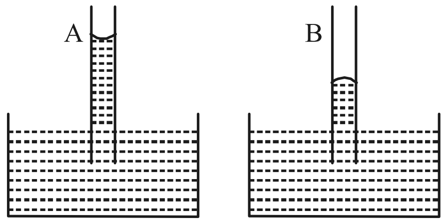
(c) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (d) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$

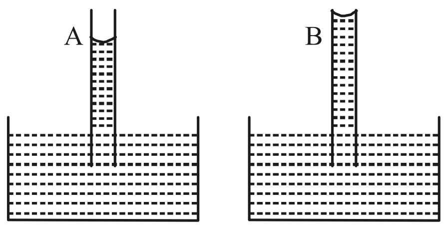
7. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and, ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3? **[2008]**

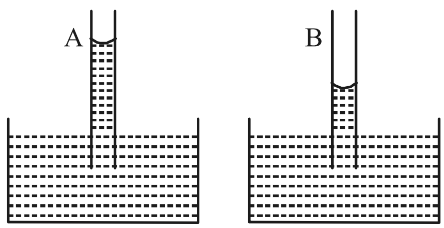


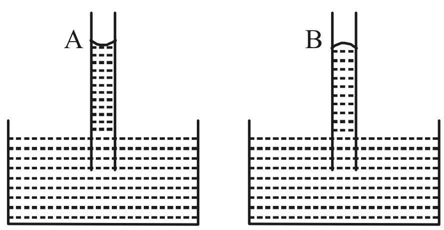
(a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

8. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[2008]**

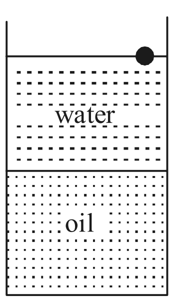
(a)


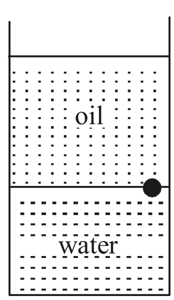
(b)


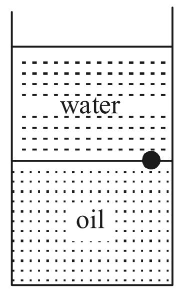
(c)


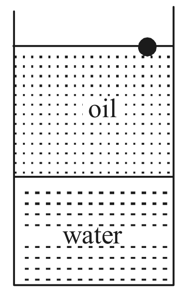
(d)


9. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[2010]**

(a)


(b)


(c)


(d)


10. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) **[2011]**

(a) 0.2πmJ (b) 2πmJ
(c) 0.4πmJ (d) 4πmJ

11. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to: **[2011]**

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

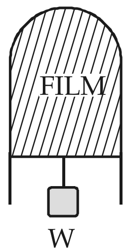
12. Two mercury drops (each of radius 'r') merge to form bigger drop. The surface energy of the bigger drop, if T is the surface tension, is : **[2011 RS]**

(a) $4\pi r^2 T$ (b) $2\pi r^2 T$
(c) $2^{8/3}\pi r^2 T$ (d) $2^{5/3}\pi r^2 T$

13. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in water (Coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s), then, its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be, nearly **[2011 RS]**

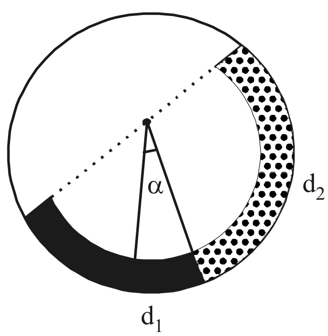
(a) 6.25×10^{-4} cm s^{-1}
(b) 6.45×10^{-4} cm s^{-1}
(c) 1.5×10^{-5} cm s^{-1}
(d) 1.6×10^{-5} cm s^{-1}

14. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight is negligible. The surface tension of the liquid film is **[2012]**



(a) $0.0125\ Nm^{-1}$ (b) $0.1\ Nm^{-1}$
(c) $0.05\ Nm^{-1}$ (d) $0.025\ Nm^{-1}$

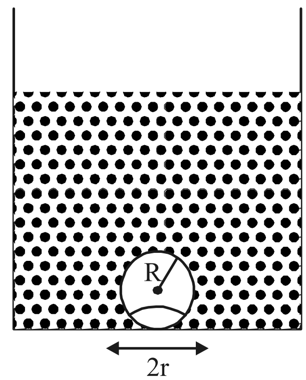
15. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[2014]**



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

16. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r \ll R$ and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) **[2014]**



(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

17. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?

(Atmospheric pressure = 76 cm of Hg) **[2014]**

(a) 16 cm (b) 22 cm
(c) 38 cm (d) 6 cm

18. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **[2017]**

(a) 81 (b) $\frac{1}{81}$
(c) 9 (d) $\frac{1}{9}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(a)	(c)	(c)	(a)	(d)	(c)	(b)	(c)	(c)	(c)	(a)	(d)	(c)
16	17	18												
(a)	(a)	(c)												

SOLUTIONS

1. **(b)** The velocity of efflux is given

$v = \sqrt{2gh}$

Where h is the height of the free surface of liquid from the hole

$\therefore\ v = \sqrt{2 \times 10 \times 20} = 20\,\text{m/s}$

2. **(b)** From Stoke's law,

viscous force $F = 6\pi\eta rv$

hence F is directly proportional to radius & velocity.

3. **(a)** Let pressure outside be P_0.

$$\therefore P_1 \text{ (in smaller bubble)} = P_0 + \frac{2T}{r}$$

$$P_2 \text{ (in bigger bubble)} = P_0 + \frac{2T}{R} \quad (R > r)$$

$\therefore P_1 > P_2$

hence air moves from smaller bubble to bigger bubble.

4. **(c)** Water fills the tube entirely in gravityless condition i.e., 20 cm.

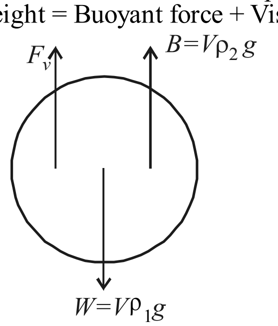
5. **(c)** Terminal velocity, $v_T = \dfrac{2r^2(d_1 - d_2)g}{9\eta}$

$$\frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$$

$\therefore v_{T_2} = 0.1$ m/s

6. **(a)** The condition for terminal speed (v_t) is

Weight = Buoyant force + Viscous force



$$\therefore\ V\rho_1 g = V\rho_2 g + kv_t^2$$

$$\therefore v_t = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$$

7. **(d)** From the figure it is clear that liquid 1 floats on liquid 2. The lighter liquid floats over heavier liquid. Therefore we can conclude that $\rho_1 < \rho_2$

Also $\rho_3 < \rho_2$ otherwise the ball would have sink to the bottom of the jar.

Also $\rho_3 > \rho_1$ otherwise the ball would have floated in liquid 1. From the above discussion we conclude that $\rho_1 < \rho_3 < \rho_2$.

8. **(c)** In case of water, the meniscus shape is concave upwards. Also according to ascent formula $h = \dfrac{2T\cos\theta}{r\rho g}$

The surface tension (T) of soap solution is less than water. Therefore rise of soap solution in the capillary tube is less as compared to water. As in the case of water, the meniscus shape of soap solution is also concave upwards.

9. **(b)** Oil will float on water so, (2) or (4) is the correct option. But density of ball is more than that of oil,, hence it will sink in oil.

10. **(c)** $W = 2T\Delta V$

$W = 2T4\pi[(5^2) - (3)^2] \times 10^{-4}$

$= 2 \times 0.03 \times 4\pi\,[25 - 9] \times 10^{-4}$ J

$= 0.4\pi \times 10^{-3}$ J

$= 0.4\pi$ mJ

11. **(c)** From Bernoulli's theorem,

$$P_0 + \frac{1}{2}\rho v_1^2 \rho gh = P_0 + \frac{1}{2}\rho v_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{0.16 + 2 \times 10 \times 0.2}$$

$= 2.03$ m/s

From equation of continuity

$A_2 v_2 = A_1 v_1$

$$\pi\frac{D_2^2}{4} \times v_2 = \pi\frac{D_1^2}{4} v_1$$

$$\Rightarrow D_2 = D_1\sqrt{\frac{v_1}{v_2}} = 3.55 \times 10^{-3}\text{ m}$$

12. **(c)** Sum of volumes of 2 smaller drops = Volume of the bigger drop

$$2.\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2^{1/3} r$$

Surface energy $= T.4\pi R^2$

$= T4\pi 2^{2/3} r^2 = T.2^{8/3}\pi r^2$.

13. (a) $V\rho g = 6\pi\eta rv + V\rho_\ell g$

$\Rightarrow \; Vg(\rho - \rho_\ell) = 6\pi\eta rv$

Also $\; Vg(\rho - \rho_i') = 6\pi\eta' rv'$

$\therefore \; v'\eta' = \dfrac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times v\eta$

$\Rightarrow \; v' = \dfrac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times \dfrac{v\eta}{\eta'}$

$= \dfrac{(7.8 - 1.2)}{(7.8 - 1)} \times \dfrac{10 \times 8.5 \times 10^{-4}}{13.2}$

$\therefore \; v' = 6.25 \times 10^{-4}$ cm/s

14. (d) The surface tension of the liquid film is given as $T = \dfrac{F}{2l}$, where F is the force, and l = length of the slider.

F = $2lT$

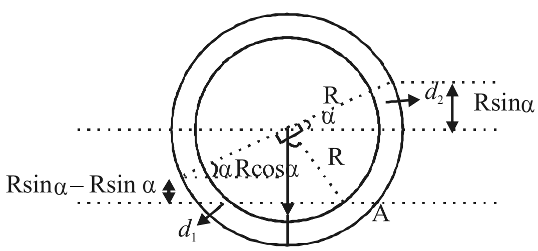
At equilibrium, $F = W$

$2Tl = mg$

$T = \dfrac{mg}{2l} = \dfrac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}} = \dfrac{1.5}{60}$

$= 0.025 \text{ Nm}^{-1}$

15. (c) Pressure at interface A must be same from both the sides to be in equilibrium.

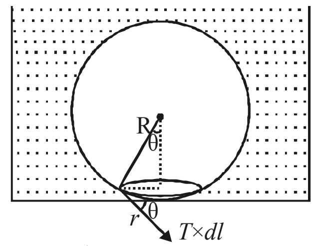


$\therefore \; (R\cos\alpha + R\sin\alpha)d_2 g$

$= (R\cos\alpha - R\sin\alpha)d_1 g$

$\Rightarrow \; \dfrac{d_1}{d_2} = \dfrac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \dfrac{1 + \tan\alpha}{1 - \tan\alpha}$

16. (a) When the bubble gets detached, Buoyant force = force due to surface tension



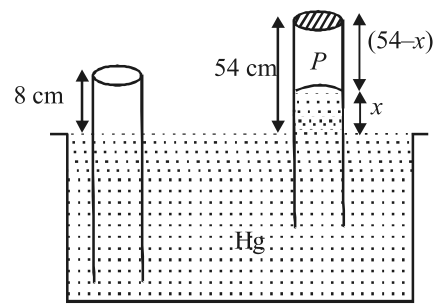
Force due to excess pressure = upthrust

Access pressure in air bubble = $\dfrac{2T}{R}$

$\dfrac{2T}{R}(\pi r^2) = \dfrac{4\pi R^3}{3T}\rho_w g$

$\Rightarrow r^2 = \dfrac{2R^4 \rho_w g}{3T} \Rightarrow r = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

17. (a)



Length of the air column above mercury in the tube is,

$P + x = P_0$

$\Rightarrow \; P = (76 - x)$

$\Rightarrow \; 8 \times A \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore \; x = 38$

Thus, length of air column

$= 54 - 38 = 16$ cm.

18. (c) As linear dimension increases by a factor of 9

$\therefore \; \dfrac{v_f}{v_i} = 9^3$

$\because$ Density remains same

So, mass $\propto$ Volume

$\dfrac{m_f}{m_i} = 9^3 \quad \Rightarrow \quad \dfrac{(Area)_f}{(Area)_i} = 9^2$

Stress $(\sigma) = \dfrac{force}{area} = \dfrac{(mass) \times g}{area}$

$\dfrac{\sigma_2}{\sigma_1} = \left(\dfrac{m_f}{m_i}\right)\left(\dfrac{A_i}{A_f}\right) = \dfrac{9^3}{9^2} = 9$