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CHAPTER 5
Complex Numbers and Quadratic Equations

1. z and w are two non zero complex numbers such that $|z|=|w|$ and Arg z + Arg w = π then z equals **[2002]**
(a) $\bar{\omega}$ (b) $-\bar{\omega}$
(c) ω (d) $-\omega$

2. If $|z-4|<|z-2|$, its solution is given by **[2002]**
(a) $\text{Re}(z)>0$ (b) $\text{Re}(z)<0$
(c) $\text{Re}(z)>3$ (d) $\text{Re}(z)>2$

3. The locus of the centre of a circle which touches the circle $|z-z_1|=a$ and $|z-z_2|=b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

4. If $\alpha \neq \beta$ but $\alpha^2=5\alpha-3$ and $\beta^2=5\beta-3$ then the equation having α/β and β/α as its roots is **[2002]**
(a) $3x^2-19x+3=0$ (b) $3x^2+19x-3=0$
(c) $3x^2-19x-3=0$ (d) $x^2-5x+3=0$.

5. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then **[2002]**
(a) $a+b+4=0$ (b) $a+b-4=0$
(c) $a-b-4=0$ (d) $a-b+4=0$

6. Product of real roots of the equation $t^2x^2+|x|+9=0$ **[2002]**
(a) is always positive
(b) is always negative
(c) does not exist
(d) none of these

7. If p and q are the roots of the equation $x^2+px+q=0$, then **[2002]**
(a) $p=1, q=-2$ (b) $p=0, q=1$
(c) $p=-2, q=0$ (d) $p=-2, q=1$

8. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $Arg(z)-Arg(\omega)=\dfrac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**
(a) -1 (b) 1
(c) $-i$ (d) i

9. Let Z_1 and Z_2 be two roots of the equation $Z^2+aZ+b=0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then **[2003]**
(a) $a^2=4b$ (b) $a^2=b$
(c) $a^2=2b$ (d) $a^2=3b$

10. If $\left(\dfrac{1+i}{1-i}\right)^x=1$ then **[2003]**
(a) $x=2n+1$, where n is any positive integer
(b) $x=4n$, where n is any positive integer
(c) $x=2n$, where n is any positive integer
(d) $x=4n+1$, where n is any positive integer.

11. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other is **[2003]**
(a) $-\dfrac{1}{3}$ (b) $\dfrac{2}{3}$
(c) $-\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

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12. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[2003]**

(a) 3 (b) 2
(c) 4 (d) 1

13. Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals **[2004]**

(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

14. If $z = x - i y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to **[2004]**

(a) –2 (b) –1
(c) 2 (d) 1

15. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**

(a) an ellipse
(b) the imaginary axis
(c) a circle
(d) the real axis

16. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its root are **[2004]**

(a) $-1, 2$ (b) $-1, 1$
(c) $0, -1$ (d) $0, 1$

17. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots , then the value of 'q' is **[2004]**

(a) 4 (b) 12
(c) 3 (d) $\frac{49}{4}$

18. If the cube roots of unity are **1**, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**

(a) $-1, -1+2\omega, -1-2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1-2\omega, 1-2\omega^2$
(d) $-1, 1+2\omega, 1+2\omega^2$

19. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to **[2005]**

(a) $\frac{\pi}{2}$ (b) $-\pi$
(c) 0 (d) $\frac{-\pi}{2}$

20. If $\omega = \dfrac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**

(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

21. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then **[2005]**

(a) $a = b + c$ (b) $c = a + b$
(c) $b = c$ (d) $b = a + c$

22. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[2005]**

(a) $(5, 6]$ (b) $(6, \infty)$
(c) $(-\infty, 4)$ (d) $[4, 5]$

23. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is **[2006]**

(a) i (b) 1
(c) -1 (d) $-i$

24. If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots\ldots\ldots + \left(z^6+\frac{1}{z^6}\right)^2$ is **[2006]**

(a) 18 (b) 54
(c) 6 (d) 12

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25. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2 + q - p$ is **[2006]**

(a) 2 (b) 3

(c) 0 (d) 1

26. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval **[2006]**

(a) $-2 < m < 0$ (b) $m > 3$

(c) $-1 < m < 3$ (d) $1 < m < 4$

27. If $|z + 4| \leq 3$, then the maximum value of $|z+1|$ is **[2007]**

(a) 6 (b) 0

(c) 4 (d) 10

28. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(a) $(3, \infty)$ (b) $(-\infty, -3)$

(c) $(-3, 3)$ (d) $(-3, \infty)$.

29. The conjugate of a complex number is $\dfrac{1}{i-1}$ then that complex number is **[2008]**

(a) $\dfrac{-1}{i-1}$ (b) $\dfrac{1}{i+1}$

(c) $\dfrac{-1}{i+1}$ (d) $\dfrac{1}{i-1}$

30. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2009]**

(a) 1 (b) 4

(c) 3 (d) 2

31. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is : **[2009]**

(a) less than $4ab$

(b) greater than $-4ab$

(c) less than $-4ab$

(d) greater than $4ab$

32. If $\left|z - \dfrac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to : **[2009]**

(a) $\sqrt{5}+1$ (b) 2

(c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

33. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals **[2010]**

(a) 1 (b) 2

(c) ∞ (d) 0

34. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[2010]**

(a) -1 (b) 1

(c) 2 (d) -2

35. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\operatorname{Re} z = 1$, then it is necessary that : **[2011]**

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$

(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

36. If $\omega (\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals **[2011]**

(a) $(1, 1)$ (b) $(1, 0)$

(c) $(-1, 1)$ (d) $(0, 1)$

37. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**

(a) 6, 1 (b) 4, 3

(c) $-6, -1$ (d) $-4, -3$

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38. Let for $a \neq a_1 \neq 0$,

$f(x) = ax^2 + bx + c,\ g(x) = a_1x^2 + b_1x + c_1$

and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is : **[2011 RS]**

(a) 3 (b) 9
(c) 6 (d) 18

39. If $z \neq 1$ and $\dfrac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**

(a) either on the real axis or on a circle passing through the origin.
(b) on a circle with centre at the origin
(c) either on the real axis or on a circle not passing through the origin.
(d) on the imaginary axis.

40. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then $a : b : c$ is **[2013]**

(a) $1:2:3$ (b) $3:2:1$
(c) $1:3:2$ (d) $3:1:2$

41. If z is a complex number of unit modulus and argument θ, then arg $\left(\dfrac{1+z}{1+\bar{z}}\right)$ equals: **[2013]**

(a) $-\theta$ (b) $\dfrac{\pi}{2} - \theta$
(c) θ (d) $\pi - \theta$

42. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \dfrac{1}{2}\right|$: **[2014]**

(a) is strictly greater than $\dfrac{5}{2}$
(b) is strictly greater than $\dfrac{3}{2}$ but less than $\dfrac{5}{2}$
(c) is equal to $\dfrac{5}{2}$
(d) lie in the interval $(1, 2)$

43. If $a \in R$ and the equation

$-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$

(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**

(a) $(-2,-1)$
(b) $(-\infty,-2) \cup (2,\infty)$
(c) $(-1,0) \cup (0,1)$
(d) $(1,2)$

44. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[2015]**

(a) circle of radius 2.
(b) circle of radius $\sqrt{2}$.
(c) straight line parallel to x-axis
(d) straight line parallel to y-axis.

45. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to : **[2015]**

(a) 3 (b) -3
(c) 6 (d) -6

46. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is : **[2016]**

(a) 6 (b) 5
(c) 3 (d) -4

47. A value of θ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **[2016]**

(a) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$
(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

48. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to : **[2017]**

(a) 11 (b) 12
(c) 9 (d) 10

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Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(b)	(a)	(a)	(a)	(a)	(a)	(d)	(b)	(b)	(c)	(c)	(a)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(d)	(c)	(c)	(c)	(b)	(c)	(d)	(d)	(b)	(c)	(a)	(c)	(c)	(d)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(b)	(a)	(a)	(b)	(c)	(a)	(a)	(d)	(a)	(a)	(c)	(d)	(c)	(a)	(a)
46	47	48												
(c)	(b)	(a)												

SOLUTIONS

1. **(b)** Let $|z| = |\omega| = r$

$\therefore z = re^{i\theta},\ \omega = re^{i\phi}$ where $\theta + \phi = \pi$.

$\therefore z = re^{i(\pi-\phi)} = re^{i\pi} . e^{-i\phi} = -re^{-i\phi} = -\bar{\omega}$.

$[\because \bar{\omega} = re^{-i\phi}]$

2. **(c)** Given $|z-4| < |z-2|$ Let $z = x + iy$

$\Rightarrow |(x-4)+iy)| < |(x-2)+iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \mathrm{Re}(z) > 3$

3. **(b)** Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ and $|z_0 - z_2| = r + b$. Eliminating r, we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.

$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2| = a - b$, which represents a hyperbola.

4. **(a)** We have $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;

$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$ or $x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$

Thus, the equation having $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ as its roots is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $3x^2 - 19x + 3 = 0$

5. **(a)** Let α, β and γ, δ be the roots of the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ respectively.

$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0\ (\because a \neq b)$

6. **(a)** Product of real roots $= \frac{9}{t^2} > 0,\ \forall\ t \in R$

$\therefore$ Product of real roots is always positive.

7. **(a)** $p + q = -p$ and $pq = q \Rightarrow q(p-1) = 0$

$\Rightarrow q = 0$ or $p = 1$.

If $q = 0$, then $p = 0$. i.e. $p = q$

$\therefore p = 1$ and $q = -2$.

8. **(a)** $|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1$

$\mathrm{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$

$= -\arg(z) + \arg\omega = -\frac{\pi}{2}$

$\therefore \bar{z}\omega = -1$

ALTERNATE SOLUTION

Let $z = r_1 e^{i\theta}$ and $w = r_2 e^{i\phi}$, $\therefore \bar{z} = r_1 e^{-i\theta}$

Now $|z\omega| = 1 \Rightarrow \left|r_1 r_2 e^{i(\theta+\phi)}\right| = 1 \Rightarrow r_1 r_2 = 1$

Also $\arg(z) - \arg(\omega) = \frac{\pi}{2} \Rightarrow \theta - \phi = \frac{\pi}{2}$

Now $\bar{z}\omega = r_1 e^{-i\theta} . r_2 e^{i\phi}$

$$= r_1 r_2 e^{-i(\theta-\phi)} = e^{-\frac{i\pi}{2}} = -1$$

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9. (d) $Z^2 + aZ + b = 0$;

$Z_1 + Z_2 = -a \;\&\; Z_1 Z_2 = b$

$0, Z_1, Z_2$ form an equilateral Δ

$\therefore\; 0^2 + Z_1^2 + Z_2^2 = 0.Z_1 + Z_1.Z_2 + Z_2.0$

(for an equilateral triangle,

$Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$)

$\Rightarrow Z_1^2 + Z_2^2 = Z_1 Z_2$

$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1Z_2$

$\therefore\; a^2 = 3b$

10. (b) $\left(\frac{1+i}{1-i}\right)^x = 1 \;\Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$

$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1;\; \therefore x = 4n;\;\; n \in I^+$

11. (b) Let the roots of given equation be α and 2α then

$$\alpha + 2\alpha = 3\alpha = \frac{1-3a}{a^2-5a+3}$$

and $\alpha.2\alpha = 2\alpha^2 = \dfrac{2}{a^2-5a+3}$

$$\Rightarrow \alpha = \frac{1-3a}{3\left(a^2-5a+3\right)}$$

$$\therefore\; 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right] = \frac{2}{a^2-5a+3}$$

$\dfrac{(1-3a)^2}{(a^2-5a+3)} = 9$ or $9a^2 - 6a + 1$

$= 9a^2 - 45a + 27$

or $39a = 26$ or $a = \dfrac{2}{3}$

12. (c) $x^2 - 3|x| + 2 = 0 \Rightarrow |x|^2 - 3|x| + 2 = 0$

$(|x|-2)(|x|-1) = 0$

$|x| = 1, 2$ or $x = \pm 1, \pm 2$

$\therefore$ No.of solution $= 4$

13. (c) $\arg zw = \pi \Rightarrow \arg z + \arg w = \pi$...(1)

$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$

$\therefore\; z = iw \Rightarrow \arg z = \dfrac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \dfrac{\pi}{2} + \pi - \arg z$ (from (1))

$\therefore \arg z = \dfrac{3\pi}{4}$

14. (a) $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p+iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2 \Rightarrow \dfrac{x}{p} = p^2 - 3q^2$

$y = q^3 - 3p^2q \Rightarrow \dfrac{y}{q} = q^2 - 3p^2$

$\therefore \dfrac{x}{p} + \dfrac{y}{q} = -2p^2 - 2q^2$

$\therefore \left(\dfrac{x}{p} + \dfrac{y}{q}\right) \Big/ (p^2+q^2) = -2$

15. (b) $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2-1|^2 = (z\bar{z}+1)^2$

$\Rightarrow (z^2-1)(\bar{z}^2-1) = (z\bar{z}+1)^2$

$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$

$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0 \Rightarrow (z+\bar{z})^2 = 0 \Rightarrow z = -\bar{z}$

$\Rightarrow z$ is purely imaginary

ALTERNATE SOLUTION 1

Let $z = r(\cos\theta + i\sin\theta)$

Then $|z^2 - 1| = |r^2(\cos 2\theta + i\sin 2\theta) - 1|$

$= \sqrt{r^4 - 2r^2\cos 2\theta + 1}$ and

$|z^2-1|^2 = (|z|^2+1)^2$

$\Rightarrow r^4 - 2r^2\cos 2\theta + 1 = r^4 + 2r^2 + 1$

$\Rightarrow 2\cos^2\theta = 0 \Rightarrow \cos\theta = \pm\dfrac{\pi}{2}$

$\therefore z$ lies on imaginary axis.

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ALTERNATE SOLUTION 2

We know that, if $|z_1+z_2|=|z_1|+|z_2|$ then origin, z_1 and z_2 are collinear

$\Rightarrow \arg(z_1)=\arg(z_2)$

As per question $|z^2+(-1)|=|z^2|+|-1|$

$\Rightarrow \arg(z^2)=\arg(-1)$

$\Rightarrow 2\arg(z)=\pi \Rightarrow \arg(z)=\dfrac{\pi}{2}$

$\Rightarrow z$ lies on imaginary axis.

16. (c) Let the second root be α.

Then $\alpha+(1-p)=-p \Rightarrow \alpha=-1$

Also $\alpha.(1-p)=1-p$

$\Rightarrow (\alpha-1)(1-p)=0 \Rightarrow p=1[\because \alpha=-1]$

$\therefore$ Roots are $\alpha=-1$ and $p-1=0$

17. (d) 4 is a root of $x^2+px+12=0$

$\Rightarrow 16+4p+12=0 \Rightarrow p=-7$

Now, the equation $x^2+px+q=0$ has equal roots.

$\therefore p^2-4q=0 \Rightarrow q=\dfrac{p^2}{4}=\dfrac{49}{4}$

18. (c) $(x-1)^3+8=0 \Rightarrow (x-1)=(-2)\,(1)^{1/3}$

$\Rightarrow x-1=-2$ or -2ω or $-2\omega^2$

or $x=-1$ or $1-2\omega$ or $1-2\omega^2$.

19. (c) $|z_1+z_2|=|z_1|+|z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1-\arg z_2=0$.

20. (c) As given $w=\dfrac{z}{z-\frac{1}{3}i}$

$\Rightarrow |w|=\dfrac{|z|}{|z-\frac{1}{3}i|}=1$

$\Rightarrow |z|=\left|z-\frac{1}{3}i\right|$

$\Rightarrow$ distance of z from origin and point $\left(0,\frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence z lies on a straight line.

21. (b) $\tan\left(\frac{P}{2}\right), \tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0$

$\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)=-\dfrac{b}{a}$

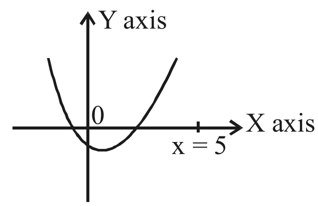
$\tan\left(\frac{P}{2}\right)\cdot\tan\left(\frac{Q}{2}\right)=\dfrac{c}{a}$

$$\frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}=\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=1$$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}}=1 \Rightarrow -\frac{b}{a}=\frac{a}{a}-\frac{c}{a}$$

$\Rightarrow -b=a-c$ or $c=a+b$.

22. (c) both roots are less than 5



then (i) Discriminant ≥ 0

(ii) $p(5)>0$

(iii) $\dfrac{\text{Sum of roots}}{2}<5$

Hence (i) $4k^2-4(k^2+k-5)\geq 0$

$4k^2-4k^2-4k+20\geq 0$

$4k\leq 20 \Rightarrow k\leq 5$

(ii) $\Rightarrow f(5)>0\,;\,25-10k+k^2+k-5>0$

or $k^2-9k+20>0$

or $k(k-4)-5(k-4)>0$

or $(k-5)(k-4)>0$

$\Rightarrow k\in(-\infty,4)\cup(-\infty,5)$

(iii) $\Rightarrow \dfrac{\text{Sum of roots}}{2}=-\dfrac{b}{2a}=\dfrac{2k}{2}<5$

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The intersection of (i), (ii) & (iii) gives

$k \in (-\infty, 4)$.

23. **(d)** $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$

$= i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11}-i\sin\frac{2k\pi}{11}\right)$

$= i\sum_{k=1}^{10} e^{-\frac{2k\pi}{11}i} = i\left\{\sum_{k=0}^{10} e^{-\frac{2k\pi}{11}i}-1\right\}$

$= i\left[1+e^{-\frac{2\pi}{11}i}+e^{-\frac{4\pi}{11}i}+....11 \text{ terms}\right]-i$

$= i\left[\frac{1-\left(e^{-\frac{2\pi}{11}}\right)^{11}}{1-e^{-\frac{2\pi}{11}i}}\right]-i \;= i\left[\frac{1-e^{-2\pi i}}{1-e^{-\frac{2\pi}{11}i}}\right]-i$

$= i\times 0-i \qquad [\because e^{-2\pi i}=1]$

$= -i$

24. **(d)** $z^2+z+1=0 \;\Rightarrow\; z=\omega$ or ω^2

So, $z+\frac{1}{z}=\omega+\omega^2=-1$

$z^2+\frac{1}{z^2}=\omega^2+\omega=-1,$

$z^3+\frac{1}{z^3}=\omega^3+\omega^3=2$

$z^4+\frac{1}{z^4}=-1,\; z^5+\frac{1}{z^5}=-1$

and $z^6+\frac{1}{z^6}=2$

$\therefore$ The given sum $= 1+1+4+1+1+4$
$= 12$

25. **(b)** $x^2+px+q=0$

Sum of roots $= \tan 30°+\tan 15° = -p$

Product of roots $= \tan 30° . \tan 15° = q$

$\tan 45° = \frac{\tan 30°+\tan 15°}{1-\tan 30°.\tan 15°} = \frac{-p}{1-q} = 1$

$\Rightarrow -p = 1-q \;\Rightarrow\; q-p=1$

$\therefore\; 2+q-p=3$

26. **(c)** Equation $x^2-2mx+m^2-1=0$

$(x-m)^2-1=0$ or

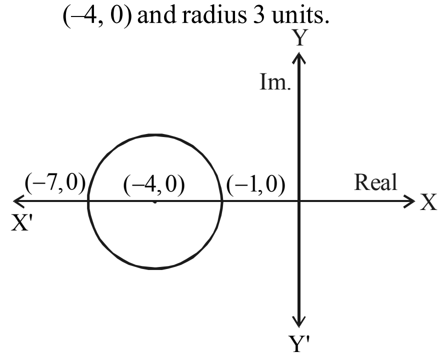
$(x-m+1)(x-m-1)=0$

$x=m-1, m+1$

$m-1>-2$ and $m+1<4$

$\Rightarrow\; m>-1$ and $m<3$ or, $-1<m<3$

27. **(a)** z lies on or inside the circle with centre (–4, 0) and radius 3 units.



From the Argand diagram maximum value of $|z+1|$ is 6

ALTERNATE SOLUTION $|z+1| = |z+4-3|$

$\leq |z+4|+|-3| \leq |3|+|-3|$

$\Rightarrow |z+1|\leq 6 \;\Rightarrow\; |z+1|_{max}=6$

28. **(c)** Let α and β are roots of the equation $x^2+ax+1=0$

$\alpha+\beta=-a$ and $\alpha\beta=1$

given $|\alpha-\beta|<\sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2-4\alpha\beta}<\sqrt{5}$

$\left(\because\; (\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta\right)$

$\Rightarrow \sqrt{a^2-4}<\sqrt{5} \;\Rightarrow\; a^2-4<5$

$\Rightarrow a^2-9<0 \Rightarrow a^2<9 \Rightarrow -3<a<3$

$\Rightarrow a\in(-3,3)$

29. **(c)** $\left(\frac{1}{i-1}\right)=\frac{1}{-i-1}=\frac{-1}{i+1}$

30. **(d)** Let the roots of equation $x^2-6x+a=0$ be α and 4β and that of the equation $x^2-cx+6=0$ be α and 3β .Then

$\alpha+4\beta=6; \qquad 4\alpha\beta=a$

and $\alpha+3\beta=c; \qquad 3\alpha\beta=6$

$\Rightarrow \quad a=8$

$\therefore$ The equation becomes $x^2-6x+8=0$

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$\Rightarrow \quad (x-2)(x-4)=0$

$\Rightarrow$ roots are 2 and 4

$\Rightarrow \quad \alpha=2, \beta=1 \quad \therefore$ Common root is 2.

31. **(b)** Given that roots of the equation $bx^2+cx+a=0$ are imaginary

$\therefore \quad c^2-4ab<0$ ....(i)

Let $y=3b^2x^2+6\,bc\,x+2c^2$

$\Rightarrow \quad 3b^2x^2+6\,bc\,x+2c^2-y=0$

As x is real, $D\geq 0$

$\Rightarrow \quad 36\,b^2c^2-12\,b^2(2c^2-y)\geq 0$

$\Rightarrow \quad 12\,b^2\,(3\,c^2-2\,c^2+y)\geq 0$

$\Rightarrow \quad c^2+y\geq 0 \quad \Rightarrow \quad y\geq -c^2$

But from eqn. (i), $c^2<4ab$ or $-c^2>-4ab$

$\therefore$ we get $y\geq -c^2>-4ab$

$\Rightarrow \quad y>-4\,ab$

32. **(a)** Given that $\left|z-\dfrac{4}{z}\right|=2$

N o w

$$|z| = \left|z-\frac{4}{z}+\frac{4}{-z}\right| \leq \left|z-\frac{4}{z}\right|+\frac{4}{|z|}$$

$$\Rightarrow |z| \leq 2+\frac{4}{|z|}$$

$$\Rightarrow |z|^2-2|z|-4\leq 0$$

$$\Rightarrow \left(|z|-\frac{2+\sqrt{20}}{2}\right)\left(|z|-\frac{2-\sqrt{20}}{2}\right)\leq 0$$

$$\Rightarrow \left(|z|-(1+\sqrt{5})\right)\left(|z|-(1-\sqrt{5})\right)\leq 0$$

$$\Rightarrow \left(-\sqrt{5}+1\right)\leq |z|\leq\left(\sqrt{5}+1\right)$$

$\Rightarrow \quad |z|_{max}=\sqrt{5}+1$ Hence T is an equivalence relation.

33. **(a)** Let $z=x+iy$

$|z-1|=|z+1|(x-1)^2+y^2=(x+1)^2+y^2$

$\Rightarrow$ Re $z=0 \quad \Rightarrow \quad x=0$

$|z-1|=|z-i|(x-1)^2+y^2=x^2+(y-1)^2$

$\Rightarrow \quad x=y$

$|z+1|=|z-i|(x+1)^2+y^2=x^2+(y-1)^2$

Only (0, 0) will satisfy all conditions.

$\Rightarrow$ Number of complex number $z=1$

34. **(b)** $x^2-x+1=0 \Rightarrow x=\dfrac{1\pm\sqrt{1-4}}{2}$

$$x=\frac{1\pm\sqrt{3}\,i}{2}$$

$$\alpha=\frac{1}{2}+i\frac{\sqrt{3}}{2}=-\omega^2$$

$$\beta=\frac{1}{2}-\frac{i\sqrt{3}}{2}=-\omega$$

$$\alpha=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3},\quad \beta=\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$$

$$\alpha^{2009}+\beta^{2009}=(-\omega^2)^{2009}+(-\omega)^{2009}$$

$$=-\omega^2-\omega=1$$

35. **(c)** Since both the roots lie in the line $Re\,z=1$ i.e., $x=1$, hence real part of both the roots are 1.

Let both roots be $1+i\alpha$ and $1-i\alpha$

Product of the roots, $1+\alpha^2=\beta$

$\because \quad \alpha^2+1\geq 1$

$\therefore \beta\geq 1 \Rightarrow \because \beta\in(1,\infty)$

36. **(a)** $(1+\omega)^7=A+B\omega$

$(-\omega^2)^7=A+B\omega$

$-\omega^2=A+B\omega$

$1+\omega=A+B\omega$

$\Rightarrow A=1, B=1.$

37. **(a)** Let the correct equation be

$ax^2+bx+c=0$

Now Sachin's equation

$ax^2+bx+c'=0$

Roots found by Sachin's are 4 and 3

Rahul's equation, $ax^2+b'x+c=0$

Roots found by Rahul's are 3 and 2

$-\dfrac{b}{a}=7$ ....(i)

$\dfrac{c}{a}=6$ ...(ii)

From (i) and (ii), roots of the correct equation $x^2-7x+6=0$ are 6 and 1.

38. **(d)** $p(x)=0$

$\Rightarrow \quad f(x)=g(x)$

$\Rightarrow \quad ax^2+bx+c=a_1x^2+b_1x+c_1$

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$\Rightarrow \quad (a-a_1)x^2+(b-b_1)x+(c-c_1)=0.$

It has only one solution, $x=-1$

$\Rightarrow \quad b-b_1=a-a_1+c-c_1 \quad$ ...(1)

vertex $=(-1,0)$

$\Rightarrow \quad \dfrac{b-b_1}{2(a-a_1)}=-1$

$\Rightarrow \quad b-b_1=2(a-a_1) \quad$ ...(2)

Now p $(-2)=2$

$\Rightarrow \quad f(-2)-g(-2)=2$

$\Rightarrow \quad 4a-2b+c-4a_1+2b_1-c_1=2$

$\Rightarrow \quad 4(a-a_1)-2(b-b_1)+(c-c_1)=2$ ...(3)

From equations, (1), (2) and (3)

$a-a_1=c-c_1=\dfrac{1}{2}(b-b_1)=2$

Now, $p(2)=f(2)-g(2)$

$=4(a-a_1)+2(b-b_1)+(c-c_1)$

$=8+8+2=18$

39. **(a)** Since we know $z=\overline{z}$ if z is real.

Therefore, $\dfrac{z^2}{z-1}=\dfrac{\overline{z}^2}{\overline{z}-1}$

$\Rightarrow z\overline{z}z-z^2=z.\overline{z}.\overline{z}-\overline{z}^2$

$\Rightarrow |z|^2.z-z^2=|z|^2.\overline{z}-\overline{z}^2$

$\Rightarrow |z|^2(z-\overline{z})-(z-\overline{z})(z+\overline{z})=0$

$\Rightarrow (z-\overline{z})\left(|z|^2-(z+\overline{z})\right)=0$

Either $z-\overline{z}=0$ or $|z|^2-(z+\overline{z})=0$

Either $z=\overline{z}\Rightarrow$ real axis

or $|z|^2=z+\overline{z}\Rightarrow z\overline{z}-z-\overline{z}=0$

represents a circle passing through origin.

40. **(a)** Given equations are

$x^2+2x+3=0 \quad$ ...(i)

$ax^2+bx+c=0 \quad$ ...(ii)

Roots of equation (i) are imaginary roots. According to the question (ii) will also have both roots same as (i). Thus

$\dfrac{a}{1}=\dfrac{b}{2}=\dfrac{c}{3}=\lambda$ (say)

$\Rightarrow \quad a=\lambda, b=2\lambda, c=3\lambda$

Hence, required ratio is 1 : 2 : 3

41. **(c)** Given $|z|=1$, $\arg z=\theta$

As we know, $\overline{z}=\dfrac{1}{z}$

$\therefore \arg\left(\dfrac{1+z}{1+\overline{z}}\right)=\arg\left(\dfrac{1+z}{1+\dfrac{1}{z}}\right)=\arg(z)=\theta.$

42. **(d)** We know minimum value of $|Z_1+Z_2|$ is $||Z_1|-|Z_2||$ Thus minimum value of $\left|Z+\dfrac{1}{2}\right|$ is

$\left||Z|-\dfrac{1}{2}\right| \le \left|Z+\dfrac{1}{2}\right| \le |Z|+\dfrac{1}{2}$

Since, $|Z|\ge 2$ therefore

$2-\dfrac{1}{2}<\left|Z+\dfrac{1}{2}\right|<2+\dfrac{1}{2}$

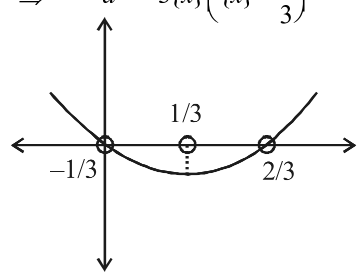
$\Rightarrow \quad \dfrac{3}{2}<\left|Z+\dfrac{1}{2}\right|<\dfrac{5}{2}$

43. **(c)** Consider $-3(x-[x])^2+2[x-[x]]+a^2=0$

$\Rightarrow \quad 3\{x\}^2-2\{x\}-a^2=0 \quad (\because x-[x]=\{x\})$

$\Rightarrow \quad 3\left(\{x\}^2-\dfrac{2}{3}\{x\}\right)=a^2, a\neq 0$

$\Rightarrow \quad a^2=3\{x\}\left(\{x\}-\dfrac{2}{3}\right)$



Now, $\{x\}\in(0,1)$ and $\dfrac{-2}{3}\le a^2<1$

(by graph)

Since, x is not an integer

$\therefore \quad a\in(-1,1)-\{0\}$

$\Rightarrow a\in(-1,0)\cup(0,1)$

44. **(a)** $\left|\dfrac{z_1-2z_2}{2-z_1\overline{z}_2}\right|=1$

$\Rightarrow |z_1-2z_2|^2=|2-z_1\overline{z}_2|^2$

$\Rightarrow (z_1-2z_2)\overline{(z_1-2z_2)}=(2-z_1\overline{z}_2)\overline{(2-z_1\overline{z}_2)}$

$\Rightarrow (z_1-2z_2)(\overline{z}_1-2\overline{z}_2)=(2-z_1\overline{z}_2)(2-\overline{z}_1z_2)$

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$\Rightarrow \quad (z_1\bar{z}_1) - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2\bar{z}_2$

$= \quad 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$

$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2 |z_2|^2$

$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2 |z_2|^2 = 0$

$\left(|z_1|^2 - 4\right)\left(1 - |z_2|^2\right) = 0$

$\because \quad |z_2| \neq 1$

$\therefore \quad |z_1|^2 = 4$

$\Rightarrow \quad |z_1| = 2$

$\Rightarrow$ Point z_1 lies on circle of radius 2.

45. (a) $\alpha, \beta = \dfrac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$

$\alpha = 3 + \sqrt{11}$, $\beta = 3 - \sqrt{11}$

$\therefore a_n = \left(3+\sqrt{11}\right)^n - \left(3-\sqrt{11}\right)^n$

$\dfrac{a_{10} - 2a_8}{2a_9}$

$$= \frac{\left(3+\sqrt{11}\right)^{10} - \left(3-\sqrt{11}\right)^{10} - 2\left(3+\sqrt{11}\right)^8 + 2\left(3-\sqrt{11}\right)^8}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{\left(3+\sqrt{11}\right)^8\left[\left(3+\sqrt{11}\right)^2 - 2\right] + \left(3-\sqrt{11}\right)^8\left[2 - \left(3-\sqrt{11}\right)^2\right]}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{\left(3+\sqrt{11}\right)^8\left(9+11+6\sqrt{11}-2\right) + \left(3-\sqrt{11}\right)^8\left(2-9-11+6\sqrt{11}\right)}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{6\left(3+\sqrt{11}\right)^9 - 6\left(3-\sqrt{11}\right)^9}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]} = \frac{6}{2} = 3$$

46. (c) $(x^2 - 5x + 5)^{x^2+4x-60} = 1$

Case I

$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number

$\Rightarrow x = 1, 4$

Case II

$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number

$\Rightarrow x = 2, 3$

where 3 is rejected because for $x = 3$, $x^2 + 4x - 60$ is odd.

Case III

$x^2 - 5x + 5$ can be any real number and

$x^2 + 4x - 60 = 0$

$\Rightarrow x = -10, 6$

$\Rightarrow$ Sum of all values of $x = -10 + 6 + 2 + 1 + 4 = 3$

47. (b) Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$\Rightarrow \dfrac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \dfrac{1}{3}$

$\Rightarrow \sin\theta = \pm\dfrac{1}{\sqrt{3}}$

48. (a) We have

$$\sum_{r=1}^{n} (x+r-1)(x+r) = 10n$$

$$\sum_{r=1}^{n} (x^2 + xr + (r-1)x + r^2 - r = 10n$$

$$\Rightarrow \sum_{r=1}^{n} (x^2 + (2r-1)x + r(r-1) = 10n$$

$\Rightarrow nx^2 + \{1+3+5+....+(2n-1)\}x + \{1.2 + 2.3 +.... + (n-1)n\} = 10n$

$\Rightarrow nx^2 + n^2 x + \dfrac{(n-1)n(n+1)}{3} = 10n$

$\Rightarrow x^2 + nx + \dfrac{n^2-31}{3} = 0$

Let α and $\alpha + 1$ be its two solutions

($\because$ it has two consequtive integral solutions)

$\Rightarrow \alpha + (\alpha+1) = -n$

$\Rightarrow \alpha = \dfrac{-n-1}{2}$ ...(1)

Also $\alpha(\alpha+1) = \dfrac{n^2-31}{3}$ ...(2)

Putting value of (1) in (2), we get

$-\left(\dfrac{n+1}{2}\right)\left(\dfrac{1-n}{2}\right) = \dfrac{n^2-31}{3}$

$\Rightarrow n^2 = 121 \Rightarrow n = 11$