FUNCTION

LEVEL–I

1. Let $f(x) = \ln(2x - x^2) + \sin\frac{\pi x}{2}$, then

(A) Graph of f is symmetrical about the line x = 1
(B) Graph of f is symmetrical about the line x = 2
(C) maximum value of f is 1
(D) minimum value of f does not exist .

2. The domain of definition of $f(x) = \sec^{-1}(\cos^2 x)$ is

(A) $m\pi$, $m \in I$ (B) $\pi/2$
(C) $\pi/4$ (D) none of these.

3. The period of $f(x) = \frac{1}{2}[\cos(\sin x) + \cos(\cos x)]$ is

(A) π (B) $\pi/2$
(C) $\pi/4$ (D) 2π

4. Domain of $f(x) = \log_{\left[x+\frac{1}{2}\right]}(x^2 - x - 2)$ is , where [.] denotes the greatest integer function.

(A) $\left[\frac{3}{2}, \infty\right)$ (B) $(2, \infty)$
(C) $\left[\frac{3}{2}, 2\right)$ (D) none of these

5. $f(x) = \begin{cases} |x|, & x \neq 0 \\ |[K]|, & x = 0 \end{cases}$ if f(x) is having minimum value –10 then k =

(A) 2 (B) –10
(C) 9 (D) not possible

6 Domain of $\cos^{-1}[2x^2 - 3]$ where [.] denotes greatest integer function, is

(A) $\left[1, \sqrt{\frac{5}{2}}\right]$ (B) $\left[-\sqrt{\frac{5}{2}}, -1\right]$
(C) $\left[-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right]$ (D) None of these.

7. Which of the following function(s) from $f : A \to A$ are invertible, where A = [-1,1]:

(A) f(x) = x/2 (B) g(x) = sin (πx/2)
(C) h(x) = |x| (D) $k(x) = x^2$

8 Solution of $0 < |x-3| \leq 5$ is

(A) [-2,8] (B) [-2,3) U (3,8] (C) [-2,3) (D) none of these

9. Solution of $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$ is

(A) (-6,-5] U [3, 7) U (4, 7) (B) [3,7]
(C) (-6,-5] (D) [3,4) (4,7]

10. f f(x) is a function that is odd and even simultaneously, then f(3) – f(2) is equal to
(A) 1 (B) -1 (C) 0 (D) none of these

11 If f(x) and g(x) be two given function with all real numbers as their domain, then h(x) = (f(x) + f(-x)) (g(x) – g(-x)). is
(A) always an odd function
(B) an odd function when both the f and g are odd
(C) an odd function when f is even and g is odd
(D) none of these

12 If f(x) = sin{x}, f: R $\to$R , then f is
(A) periodic (B) one-one
(C) many-one (D)none of these

13 If f(x) = $\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$ then the range of f(x) is
(A) $[-\pi/2,\pi/2]$ (B) $[0,\pi/2]$
(C) $[0,\pi/2)$ (D) $[-\pi/2,0)$

14. If the period of $\frac{\sin(nx)}{\tan(x/n)}$, where $n\in I$, is 6π, then
(A) n = 4 (B) n = -3
(C) n = 3 (D) none of these

15. If f(x) = {x} + sin ax (where { } denotes the fractional part function) is periodic, then
(A) 'a' is a rational multiple of π (B) 'a' is a natural number
(C) 'a' is any real number (D) 'a' is any positive real number

16. If f(x) = sin $\sqrt{[a]}$ x, (where [.] denotes the greatest integer function), has π as it's fundamental period, then
(A) a = 1 (B) a $\in$[1,2)
(C) a = 9 (D) a $\in$ [4,5)

17. Range of the function f(x) = $\frac{1}{\sqrt{|x|-x^2}}$ is . . . . .

18 The function f (x) = $\begin{cases} \{x\}, & x \geq 0 \\ \{-x\}, & x < 0 \end{cases}$ is ({.} : fractional part}
(A) even (B) odd
(C) neither (D) none of these

19 Period of |sin 2x| + |cos 8x| is:
(A) $\pi/2$ (B) $\pi/8$
(C) $\pi/16$ (D) None of these.

20 The domain of f(x) = $\sqrt{\log_{1/4}\left(\frac{5x-x^2}{4}\right)}$ + $^{10}C_x$ is
(A) (0, 1]U [4, 5) (B) (0, 5)
(C) {1, 4} (D) None of these

21 The expression $\left|a+\frac{1}{a}\right|$ is equal or …….…….. than …….…… for ……..values of a.

22 The absolute value of an expression is always ……

23 | x + y| = | x | + | y | holds good if and only if x and y are ……..

24 The solution of | x–3| = x is ……..

25 $\log_b a$ is meaningful only if a is …….. and b is ……..… or ……

26 If $\log_{a^k} N = y \log_a N$ then y = ……

27 The expression $ax^2 + bx + c > 0 \quad \forall x \in R$ implies that a is …….. and ……..……..

28 The domain of $f(x) = \sqrt{\frac{2-x}{x+1}}$ is

(A) (-1, 2) (B) R – (-1, 2]
(C) R – [-1, 2) (D) (-1, 2]

29 The range of $y = \sqrt{\log_3(\cos(\sin x))}$ contain(s)

(A) one element (B) infinitely many elements
(C) the function is undefined (D) none of these

30 The domain and range of $f(x) = \frac{1}{2-\cos 3x}$ are respectively

(A) R - $(2n+1)\frac{\pi}{3}$, R (B) R, R – [1/3, 1]
(C) R, [1/3, 1] (D) none of these

31 The equation x > [x] holds true for, where [-] denotes GIF

(A) all integral values of x (B) all $x \in R$
(C) all positive integers (D) R – I

32 The function and its inverse

(A) are symmetric about y = x line
(B) meet each other along the line y = x
(C) are symmetric about y + x = 0 line
(D) never intersect each other .

33 Let f (-x) = f (x). Then f′ (x) must be

(A) an even function (B) an odd function
(C) a periodic function (D) neither odd nor even

34. If $f(x) = \begin{cases} x \text{ when } x \text{ is rational} \\ 1-x \text{ when } x \text{ is irrational} \end{cases}$, then fof (x) is given as

(A) 1 (B) x
(C) 1 + x (D) None of these

35. If x -{x} = 2 then x belongs to……

36. Domain of the function f (x) = $\sqrt{\log_3(\cos(\sin x))}$ is ……

37. If f (x) = cos [π]x + cos [πx], where[.] stands for greatest integer function, then f (π/2) equals to……..

38 Solution set of inequation cos x $\geq$ -1/2 is

(A) $\left[2n\pi - \frac{2\pi}{3},\ 2n\pi + \frac{2\pi}{3}\right]$ (B) $\left(2n\pi - \frac{2\pi}{3},\ 2n\pi + \frac{2\pi}{3}\right)$

(C) $\left[n\pi - \frac{2\pi}{3},\ n\pi + \frac{2\pi}{3}\right]$ (D) none of these

39 Solution set of inequation tan x > $-\sqrt{3}$ is

(A) $n\pi - \frac{2\pi}{3} < x < \frac{\pi}{2}$ (B) $n\pi - \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$

(C) $2n\pi - \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{3}$ (D) none of these

40 Range of f(x) = $\sin^{-1}\sqrt{x^2 + x + 1}$ is

(A) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{3}, \frac{\pi}{4}\right]$

(C) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (D) none of these

41 Let f(x) = sinx + cos$\left(\sqrt{4 - a^2}\right)$x . Then the integral values of 'a' for which f(x) is a periodic function are given by

(A) {2, – 2} (B) [– 2, 2)

(C) (–2, 2) (D) none of these

42. The function f(x) = $(1 - x)^{1/3}$ is

(A) one– one & onto (B) many– one & onto

(C) one– one & into (D) many– one & into

43 Let f: R $\to$ R be any function. Define g : R $\to$ R by g(x) = |f(x)| for all x, then g is

(A) onto if f is onto (B) one– one if f is one– one

(C) continuous if f is continuous (D) differentiable if f is differentiable

44. The domain of definition of f(x) = $\sec^{-1}(\cos^2 x)$ is

(A) mπ , m $\in$ I (B) π/2

(C) π/4 (D) none of these.

45. Which of the following functions is /are periodic

(A) Sgn (e^{-x}) (B) sinx + |sinx|

(C) min (sinx, |x|) (D) $\left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$

Where [x] denotes the greatest integer function

46 The function defined as f : [0, π] $\to$ [–1, 1], f (x) = cos x is

(A) one-one onto (B) many-one onto

(C) one-one into (D) many-one into

47 Find the period of the function f (x) = cos [π^2]x + cos [$-\pi^2$]x

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

48 $y = \log_{|x|} |x|$, then find the domain

(A) R (B) R – {–1, 1}

(C) R – {0} (D) R – {0, –1, 1}

49. The range of the function $f(x) = \frac{x^2}{x^4+1}$ is

(A) $\left(0, \frac{1}{2}\right)$ (B) $\left(0, \frac{1}{2}\right]$

(C) $(0, \infty)$ (D) (0, 2]

50. $[\sin x] = [\cos x]$ for all $x \neq \frac{k\pi}{2}$, k is an integer

(A) true (B) false

51. If f (x) is an invertible function then $(f \circ f^{-1})(x) = x$ for all $x \in R$

(A) true (B) false

52. The range of the function $\ln (x^2-2x+6)$ is

(A) $(\ell n6, \infty)$ (B) $[\ln 5, \infty)$

(C) $(0, \infty)$ (D) R (set of real numbers)

53. Domain of $\log_{1/2}\log_4\log_3[(x-4)^2]$ is, [.] denotes the integer function .

(A) $(-\infty, 2] \cup [6, \infty)$ (B) $(-\infty, 2] \cup [6, 8)$

(C) (2, 6) (D) [2, 6]

54. The graph of $y = x + \frac{1}{x}$ is symmetrical

(A) about x – axis (B) about y - axis

(C) in opposite quadrants (D) None of these

55. Period of the function $|\cos 2x|$ is

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

56. The domain of $f(x) = \sin^{-1}(|x-1|-2)$ is

(A) $[-2, 0] \cup [2, 4]$ (B) $(-2, 0) \cup (2, 4)$

(C) $[-2, 0] \cup [1, 3]$ (D) $(-2, 0) \cup (1, 3)$

57. If $f(x) = x^2$, $g(x) = \sqrt{x}$, then what is g o f (x) is

(A) |x| (B) x

(C) –x (D) –|x|

58. Minimum of $2^{[(x^2-3)^3+27]}$ is

(A) 2^{27} (B) 1

(C) 2 (D) 2^{-27}

59. The function defined as $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$ is

(A) one-one onto (B) many-one onto

(C) one-one into (D) many-one into

60. The range of the function f (x) = $\frac{x-3}{|x-3|}$ is

(A) {–1, 1} (B) R
(C) R – {3} (D) R – {–1}

61. The solution set of log {x} = 0 is

(A) {ϕ} (B) [1, –1]
(C) (0, –1) (D) [0, 1]

62. The domain of the function f (x) = $\frac{1}{\sqrt{[x]-|x|}}$ is

(A) [0, ∞) (B) R
(C) (–∞, 0] (D) {Φ}

63. If f (x) = $\frac{1}{1-x}$, then f [f {f (x)}] is

(A) x – 1 (B) 1 – x
(C) x (D) –x

64. The value of x for $\log_{1/3}\left(x+\frac{2}{x}\right) < -1$ lies in

(A) (0, 1) $\cup$ (1, ∞) (B) (0, 1) $\cup$ (2, ∞)
(C) (0, 1) $\cup$ [2, ∞) (D) (0, 1] $\cup$ [2, ∞)

65. The range of the function f (x) = 11 – 3 sin x is

(A) [6, 14] (B) [8, 14]
(C) [8, 12] (D) [8, 11]

66. The period of the function f (x) = {x} + sin $\frac{\pi}{3}$x + tan 2x

(A) 1 (B) 2
(C) 3 (D) not periodic

67. The domain of the function f (x) = $\frac{\sin^{-1}x}{[x]}$ is

(A) [–1, 0) $\cup$ {1} (B) (–∞, 0) $\cup$ {1}
(C) (–1, 0) $\cup$ {1} (D) not defined

68. If f (x) = [x] and g (x) = |x|, then g o f $\left(\frac{5}{3}\right)$ – f o g $\left(\frac{5}{3}\right)$ is

(A) 0 (B) –1
(C) 1 (D) none of these

69. Which of the following is not periodic?

(A) f (x) = cos x (B) f (x) = |cos x|
(C) f (x) = cos x^2 (D) f (x) = $\cos^2$ x

70. The solution set of log [x] = 0 is

(A) [1, 2) (B) [1, 2]
(C) (1, 2] (D) (1, 2)

71. The domain of the function f (x) = $\frac{[x]+2}{[x]-2}$ is

(A) R (B) R – {2}
(C) R – [2, 3) (D) not defined

72. Domain of function f(x) = $\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$.

(A) $(-3, -2) - \left\{-\frac{5}{2}\right\}$ (B) $[0, 1] - \left\{\frac{1}{2}\right\}$
(C) $[-2, 1) - \{0\}$ (D) none of these

73. If f(x) = $\frac{2x+1}{2x^3+3x^2+x}$, interval when f(x) ≥ 0

(A) R (B) $R - [-1, 0]$
(C) R^+ (D) none of these

74. Domain of function f(x) = $\sqrt{1-2x}+3\sin^{-1}\left(\frac{3x-1}{2}\right)$

(A) $\left[-1, \frac{1}{2}\right)$ (B) $\left[-\frac{1}{3}, \frac{1}{2}\right]$
(C) $\left[-\frac{1}{3}, 1\right]$ (D) $\left[\frac{1}{2}, 1\right]$

75. Which of the following function is non-periodic

(A) f(x) = {x} (B) f(x) = cot(x + 7)
(C) f(x) = $1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$ (D) f(x) = x + sinx

76. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(A) gof(−2) = −2 (B) gof(4) = 4
(C) gof(3) = 6 (D) gof(2) = 4

77. The domain of f(x) = $\sqrt{\log(2x-x^2)}$ is, x =

(A) 1 (B) 2 (B) 3 (D) none of these

78. The range of f(x) = $\frac{x-3}{3-x}$, $x \neq 3$ is

(A) R (B) $R - \{-1\}$ (C) $R - \{1\}$ (D) none of these

79. The range of f(x) = $\frac{x}{1+|x|}$ is

(A) $R - \{-1, 1\}$ (B) R (C) $R - \{1\}$ (D) none of these

80. Let f(x) = $\frac{x-1}{x+1}$, $x \neq -1$ then $f^{-1}(x)$ is

(A) $\frac{1+x}{1-x}$ (B) $\frac{1-x}{1+x}$ (C) $\frac{1}{1-x}$ (D) none of these

81. If f(x) = 1 + αx, $\alpha \neq 0$ is the inverse of itself then the value of α is

(A) −1 (B) 1 (D) 2 (D) none of these

82. The value of $n \in I$ for which the function $f(x) = \dfrac{\sin nx}{\sin\left(\dfrac{x}{n}\right)}$ has 4π as its period is equal to

(A) ± 2 (B) 2 (C) ± 1 (D) none of these

LEVEL−II

1. Which of the following is correct?
(A) sin1 > sin2 (B) sin1 < sin2
(C) sin2 > sin3 (D) sin2 < sin3.

2. The range of the function $\sin^2 x - 5\sin x - 6$ is
(A) [– 10, 0] (B) [– 1, 1]
(C) [0, π] (D) [– 49/4, 0]

3. If $f(x) = (1 - x^n)^{1/n}$, $0 < x < 1$, n being an odd positive integer and $h(x) = f(f(x))$, then $h'\left(\frac{1}{2}\right)$ is equal to
(A) 2^n (B) 2
(C) $n.\ 2^{n-1}$ (D) none of these

4. If $f : I \to I$ be defined by $f(x) = [x + 1]$, where [.] denotes the greatest integer function, then $f^{-1}(x)$ is equal to
(A) x – 1 (B) [x + 1]
(C) $\frac{1}{[x-1]}$ (D) $\frac{1}{x+1}$

5. Which pair of functions is identical?
(A) $\sin^{-1}(\sin x)$, $\sin(\sin^{-1}x)$ (B) $\ln e^x$, $e^{\ln x}$
(C) $\ln x^2$, $2\ln x$ (D) none of these.

6. If g is the inverse function of f and $f'(x) = \sin x$, then $g'(x)$ is equal to
(A) sin(g(x)) (B) cosec(g(x))
(C) tan(g(x)) (D) none of these.

7. Value(s) of x for which tangent drawn to the curve $f(x) = |1 - 2e^{-|x|}|$ would be lying entirely below the curve, is given by
(A) $x \in (\ln 2, \infty)$ (B) $x \in (-\ln 2, 0)$
(C) $x \in (-\infty, -\ln 2)$ (D) $x \in (0, \ln 2)$

8. Solution set of $[\sin^{-1}x] > [\cos^{-1}x]$, where [.] denotes greatest integer function
(A) [sin1, 1] (B) $\left[\frac{1}{\sqrt{2}}, 1\right]$
(C) (cos1, sin1) (D) None of these

9. If P(x) be a polynomial satisfying the identity $P(x^2) + 2x^2 + 10x = 2x\,P(x+1) + 3$, then P(x) is
(A) 2x +3 (B) 3 x- 4
(C) 3x + 2 (D) 2 x –3

10. Let $f(x) = \begin{cases} x^3 - 1, & x < 2 \\ x^2 + 3, & x \geq 2 \end{cases}$. Then
(A) $f^{-1}(x) = \begin{cases} (x+1)^{1/3}, & x < 2 \\ (x-3)^{1/2}, & x \geq 2 \end{cases}$ (B) $f^{-1}(x) = \begin{cases} (x+1)^{1/3}, & x < 7 \\ (x-3)^{1/2}, & x \geq 7 \end{cases}$
(C) $f^{-1}(x) = \begin{cases} (x+1)^{1/3}, & x < 1 \\ (x-3)^{1/2}, & x \geq 7 \end{cases}$ (D) $f^{-1}(x)$ does not exist

11. Which of the following is/are true, (you may use f(x) = $\frac{\ln(\ln x)}{\ln x}$)

(A) $(\ln 2.1)^{\ln 2.2} > (\ln 2.2)^{\ln 2.1}$ (B) $(\ln 4)^{\ln 5} < (\ln 5)^{\ln 4}$
(C) $(\ln 30)^{\ln 31} > (\ln 31)^{\ln 30}$ (D) $(\ln 28)^{\ln 30} < (\ln 30)^{\ln 28}$

12. sin ax + cosax and |sinx| + |cosx| are periodic functions of same fundamental period if a equals

(A) 0 (B) 1
(C) 2 (D) 4

13. If {x} denotes the fractional part of x, then $\left\{\frac{4^{2n}}{15}\right\}$, n∈N, is

(A) $\frac{1}{15}$ (B) $\frac{14}{15}$
(C) $\frac{7}{8}$ (D) None of these

14. If f (x) = minimum {sin x, cos x} ∀ x ∈ R. then range of g (x) = [f (x)] is, [ ] denotes the greatest integer function

(A) {–1, 0, 1} (B) {0, 1}
(C) {–1, 0} (D) none of these

15. If $f(x-1/x) = x^2 + 1/x^2$, $x \neq 0$, then f(x) is

(A) is an even function (B) always greater or equal to2 ∀ x ∈R
(C) onto if f : R→ [3,∞) (D) none of these

16 .If $f(x) = \begin{cases} x^2 & \text{for } x \geq 0 \\ x & \text{for } x < 0 \end{cases}$, then fof(x) is given by

(A) x^2 for $x \geq 0$, x for $x < 0$ (B) x^4 for $x \geq 0$, x^2 for $x < 0$
(C) x^4 for $x \geq 0$, $-x^2$ for $x < 0$ (D) x^4 for $x \geq 0$, x for $x < 0$

17. The range of the function $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] is the greatest integer function, is

(A) $\left\{\frac{\pi}{2}, \pi\right\}$ (B) $\left\{0, \frac{\pi}{2}\right\}$
(C) $\{\pi\}$ (D) $\left(0, \frac{\pi}{2}\right)$

18. If |x| + [x] = 2x (where [.] denotes the greatest integer function), then number of solutions of the equation in [-1,1) is/are

(A) one only (B) infinitely many
(C) two only (D) none of these

19. If $f(x) = \cos\left|x\right| + \left[\left|\frac{\sin x}{2}\right|\right]$, (where [.] denotes the greatest integer function), then

(A) f(x) is periodic (B) f(x) is odd
(C) f(x) is even (D) f(x) is non-periodic

20. Let f : (2,4) $\rightarrow$(1,3) where f(x) = x – [x/2] (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is
(A) not defined (B) x – 1
(C) x + 1 (D) none of these

21. The fundamental period of cos (cos 2x) + cos (sin3x) is
(A) π (B) 2π
(C) $\pi/4$ (D) $\pi/2$

22. Let f : R $\rightarrow$ R, where $f(x) = 2^{|x|} - 2^{-x}$, then
(A) Range of f(x) is all non-negative R (B) f(x) is many-one
(C) f(x) is into (D) f(x) is non-periodic8

23. If $f(x) = \sqrt{4 - x^2} + \frac{1}{\sqrt{|\sin x| - \sin x}}$, then the domain of f(x) is
(A) [-2,0] (B) (0,2]
(C) [-2,2} (D) [-2,0)

24. Number of real roots of $3^x + 4^x + 5^x - 6^x = 0$ is/are
(A) two (B) more then two
(C) one (D) equation will not have any real root

25. Range of function [| sinx| + | cosx|] , where [.] denotes the greatest integer function is . . . .

26. f: { x, y, z } $\rightarrow$ { a, b, c} be a one one function. It is known that only one of the following statements is true
(i) $f(x) \neq b$ (ii) f(y) = b (iii) $f(z) \neq a$ then $f^{-1}(a)$ =
(A) x (B) y
(C) z (D) none of these

27. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is
(A) even (B) odd
(C) neither even nor odd (D) none of these

28. Let f : [-10,10] $\rightarrow$ R, where $f(x) = \sin x + [x^2/a]$ be an odd function. Then set of values of parameter 'a' is/are:
(A) (-10,10) ~ {0} (B) (0,10)
(C) [100,∞) (D) (100,∞)

29. If fog = |sin x| and gof = $\sin^2\sqrt{x}$ then f(x) and g(x) are:
(A) $f(x) = \sqrt{\sin x}$, $g(x) = x^2$ (B) f(x) = |x|, g(x) = sin x
(C) $f(x) = \sqrt{x}$, $g(x) = \sin^2 x$ (D) $f(x) = \sin\sqrt{x}$, $g(x) = x^2$

30. If $f(x) + 2f(1- x) = x^2 + 2 \ \forall \ x \in R$, then f(x) is given as
(A) $\frac{(x-2)^2}{3}$ (B) $x^2 - 2$
(C) 1 (D) None of these

31. Let f (x) be a function whose domain is [-5, 7]. Let g (x) = |2x + 5|, then the domain of fog (x) is

(A) [-5, 1] (B) [-4, 0]
(C) [-6, 1] (D) none of these

32. Let f:[-π/3, 2π/3] → [0,4] be a function defined as f(x) = $\sqrt{3}$ sin x – cos x + 2. Then $f^{-1}(x)$ is given by

(A) $\sin^{-1}\left(\frac{x-2}{2}\right)-\frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{x-2}{2}\right)+\frac{\pi}{6}$

(C) $\frac{2\pi}{3}-\cos^{-1}\left(\frac{x-2}{2}\right)$ (D) None of these.

33. The function f : R → R, $f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ is

(A) one-one and onto (B) one-one and into
(C) many-one and onto (D) many-one and into

34. The function $f(x)=\begin{cases} x|x|, & x\le -1 \\ [1+x]+[1-x], & -1<x<1 \\ -x|x|, & x\ge 1 \end{cases}$ is (where [-] denotes GIF)

(A) even (B) odd
(C) neither even nor odd (D) symmetric with y-axis

35. Let $f(x)=\begin{cases} \sin x+\cos x \text{ for } & 0\le x<\pi/2 \\ b \quad \text{for} & x=\pi/2 \\ \tan^2 x+\operatorname{cosec} x \text{ for } & \pi/2<x<\pi \end{cases}$, Then its odd extension is

(A) $-\tan^2 x - \operatorname{cosec} x$, $-\pi < x < -\pi/2$
-b for $x = -\pi/2$
$-\sin x + \cos x$ for $-\pi/2 < x < 0$

(B) $-\tan^2 x + \operatorname{cosec} x$, $-\pi < x < -\pi/2$
-b for $x = -\pi/2$
$\sin x - \cos x$ for $-\pi/2 < x < 0$

(C) $-\tan^2 x + \operatorname{cosec} x$, $-\pi < x < -\pi/2$
b for $x = -\pi/2$
$\sin x - \cos x$, $-\pi/2 < x < 0$

(D) None of these

36. Period of the function f (x) = cos (cos x) + cos (sin x) is……

37. Let f : (-∞, 1] → (-∞, 1] such that f (x) = x (2 –x). then f^{-1} (x) is

(A) $1+\sqrt{1-x}$ (B) $1-\sqrt{1-x}$
(C) $\sqrt{1-x}$ (D) none of these

38. Number of solutions of the equation cos x = |x|, x ∈ [-π/2, π/2] is

(A) 1 (B) 2
(C) 3 (D) 4

39. The number solutions of equation tan x = x in interval $\left[0, \frac{3\pi}{2}\right]$

(A) 1 (B) 2
(C) 3 (D) 4

40. Let f be a function satisfying f(x +y) =f(x).f(y) for all x,y $\in$ R. If f(1) =3 then $\sum_{r=1}^{n} f(r)$ is equal to

(A) $\frac{3}{2}(3^n - 1)$ (B) $\frac{3}{2}n(n+1)$

(C) $3^{n+1} - 3$ (D) None of these

41. Let f: R $\to$R be a function such that f(x) = $x^3 + x^2 + 3x + \sin x$. Then

(A) f is one– one and into (B) f is one– one and onto

(C) f is many– one and into (D) f is many– one and onto

42. Let f(x) = $\sec^{-1}[1 + \cos^2 x]$ where [.] denotes the greatest integer function. Then

(A) the domain of f is R (B) the domain of f is [1,2]

(C) The range of f is [1,2] (D) the range of f is [$\sec^{-1}1$, $\sec^{-1}2$]

43. Range of the function f(x) = $\sqrt{a-x} + \sqrt{x-b}$, where a > b > 0

(A) $(-\infty, \sqrt{a-b}]$ (B) $[\sqrt{a-b}, \sqrt{2(a-b)}]$

(C) $[\sqrt{a-b}, \infty)$ (D) none of these

44. If $|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2$, then f(x) is necessarily non– negative in

(A) [– 2, 2] (B) $(-\infty, -2) \cup (2, \infty)$

(C) [– $\sqrt{6}$, $\sqrt{6}$] (D) none of these

45. The period of f(x) = $\frac{1}{2}[\cos(\sin x) + \cos(\cos x)]$ is

(A) π (B) $\pi/2$

(C) $\pi/4$ (D) 2π

46. Total number of roots of the equation $3^{\cos x} = |\sin x|$, belonging to [- 2π, 2π], are;

(A) 6 (B) 8

(C) 10 (D) 12

47. If f(x) = $[x^2] - [x]^2$, where[.] denotes the greatest integer function, and x $\in$ [0,2], then the set of values of f(x) is

(A) {-1, 0} (B) {-1,0,1}

(C) {0} (D) {0,1,2}

48. Range of f (x) = 2 cos $\sqrt{\frac{\pi^2}{9} - x^2}$ is

(A) [–1, 2] (B) [1, 0]

(C) (0, 1) (D) [1, 2]

49. If $[x]^2 - 5[x] + 6 = 0$, then x belongs to

(A) [2, 4) (B) [2, 4) – {3}

(C) {3} (D) {2}

50. Range of y = $\cos^{-1} \frac{2}{2 + \sin x}$ is

(A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[0, \cos^{-1}\frac{2}{3}\right]$

(C) [0, $\cos^{-1}$ 2] (D) $\left[\cos^{-1}\frac{2}{3}, \pi\right]$

51. Number of solution of $\sin x + \cos x = 2$ are
(A) 1 (B) 2
(C) 0 (D) infinite

52. The period of the function $f(x) = 2 + (-1)^{[x]}$ is
(A) 1 (B) 0
(C) 2 (D) 0.5

53. The number of solutions of $|\ln |x|| = \sqrt{5 - x^2}$ is
(A) 1 (B) 2
(C) 3 (D) 4

54. The function $f(x) = \dfrac{x^2 + 4x + 7}{x^2 + x + 1}$, where $f : R \to R$ is
(A) one-one into (B) many-one into
(C) one-one onto (D) many-one onto

55. Total number of solutions of $2^{|\cos x|} = 3|\sin x|$, belonging to the interval $[-10\pi, 10\pi]$ are;
(A) 20 (B) 40 (C) 80 (D) none of these

56. If $f: [1,\infty) \to [2,\infty)$ is given by $f(x) = x + \dfrac{1}{x}$ then $f^{-1}(x)$ equals
(A) $\dfrac{x + \sqrt{x^2 - 4}}{2}$ (B) $\dfrac{x - \sqrt{x^2 - 4}}{2}$ (C) $\dfrac{x + \sqrt{x^2 + 4}}{2}$ (D) none of these

57. The solution of the inequality $\log_{1/2}\sin^{-1}x > \log_{1/2}\cos^{-1}x$ is
(A) $x \in \left[0, \dfrac{1}{\sqrt{2}}\right)$ (B) $x \in \left(\dfrac{1}{\sqrt{2}}, 1\right]$
(C) $x \in \left(0, \dfrac{1}{\sqrt{2}}\right)$ (D) None of these

58. Total number of roots of the equation $7^{|x|}(|5 - |x||) = 1$, are;
(A) 6 (B) 8
(C) 4 (D) 12

59. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$ is
(A) $[3/4, \infty)$ (B) $(3/4, \infty)$
(C) $(7, \infty)$ (D) $[7, \infty)$

60. Let reflection of function $f(x) = (4 - (x - 7)^3)^{1/5}$ about a line $y = x$ is $g(x)$ then
(A) $g(x) = 7 - (4 - x^3)^{1/5}$ (B) $g(x) = x$
(C) $g(x) = -x^2 + 1$ (D) $g(x) = 7 + (4 - x^5)^{1/3}$

61. The period of the function $f(x) = \sin^4 x + \cos^4 x$
(A) π (B) $\pi/4$
(C) $\pi/2$ (D) 2π

62. The function $f: R \to R$ given by $f(x) = x^3 + ax^2 + bx + c$ is one−one if
(A) $a < b$ (B) $a^2 < 3b$
(C) $a^2 > 3b^2$ (D) $a^2 = c^2$

63. Let $f(x) = \dfrac{x - 2}{x - 3}$ is an invertible function then domain $f^{-1}(x)$ is
(A) R^+ (B) $R - \{3\}$

(C) R − {1} (D) none of these

64. Let g(x) = 1 + x − [x] and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$. Then for all x, fog(x) is equal to

(A) x (B) 1 (C) f(x) (D) g(x)

LEVEL−III

1. If the derivative of f(x) w.r. t. x is $\frac{\frac{1}{2}-\sin^2 x}{f(x)}$, then f(x) is a periodic function with period

(A) π (B) 2π
(C) $\pi/2$ (D) none of these.

2. If $\tan^{-1}(x+h) = \tan^{-1}(x) + (h\ \text{siny})(\text{siny}) - (h\ \text{siny})^2.\frac{\sin 2y}{2} + (h\ \text{siny})^3.\frac{\sin 3y}{3} + \ldots.$,

where $x \in (0, 1)$, $y \in (\pi/4, \pi/2)$, then
(A) $y = \tan^{-1}x$ (B) $y = \sin^{-1}x$
(C) $y = \cot^{-1}x$ (D) $y = \cos^{-1}x$

3. The domain of the function $f(x) = \frac{x^{1/2}}{\sqrt{\sin(\ln x) - \cos(\ln x)}}$ is

(A) $\bigcup_{n\in I}\left(e^{2n\pi},\ e^{\left(3n+\frac{1}{2}\right)\pi}\right)$ (B) $\bigcup_{n\in I}\left(e^{\left(2n+\frac{1}{4}\right)\pi},\ e^{\left(2n+\frac{5}{4}\right)\pi}\right)$
(C) $\bigcup_{n\in I}\left(e^{\left(2n+\frac{1}{4}\right)\pi},\ e^{\left(3n-\frac{3}{4}\right)\pi}\right)$ (D) $\bigcup_{n\in I}\left(e^{\left(2n-\frac{3}{4}\right)\pi},\ e^{\left(3n+\frac{3}{4}\right)\pi}\right)$

4. If $f(x) = \log_{[x-1][}\frac{|x|}{x}$, where [.] denotes greatest integer function, then
(A) domain of f = (2, ∞) (B) range of f = {0, 1}
(C) domain of f = [3, ∞) (D) range of f = {0}

5. Let $f(x) = \sin x + ax + b$. Then $f(x) = 0$ has
(A) only one real root which is positive if $a > 1, b < 0$
(B) only one real root which is negative if $a > 1, b > 0$
(C) only one real root which is negative if $a < -1, b < 0$
(D) none of these.

6. If $f(x) = [x^2] - [x]^2$, where [.] denotes the greatest integer function, and $x \in [0, n]$, $n \in N$, then the number of elements in the range of f(x) is
(A) $2n + 1$ (B) $4n - 3$
(C) $3n - 3$ (D) $2n - 1$

7. Total number of solutions of $x^2 - 2x - [x] = 0$ is equal to
(A) 2 (B) 4
(C) 6 (D) none of these

8. If $f(x) = \frac{1}{[|\sin x| + |\cos x|]}$ (where [.] denotes the greatest integer function), then
(A) f(x) is an even function (B) f(x) is an odd function
(C) range of f(x) contains only one element (D) none of these

9. Let f and g be functions from the interval $[0, \infty)$ to the interval $[0, \infty)$, f being an increasing function and g being a non-increasing function. If $f\{g(0)\} = 0$ then
(A) $f\{g(x)\} \geq f\{g(0)\}$ (B) $g\{f(x)\} \leq g\{f(0)\}$
(C) $f\{g(2)\} = 0$ (D) None of these

10. If P(x) be a polynomial satisfying the identity $P(x^2) + 2x^2 + 10x = 2x\,P(x+1) + 3$, then P(x) is
(A) 2x +3 (B) 3 x- 4
(C) 3x + 2 (D) 2 x –3

11. If $k\sin^2 x + \frac{1}{k}\operatorname{cosec}^2 x = 2$, $x \in (0, \pi/2)$, then $\cos^2 x + 5 \sin x \cos x + 6 \sin^2 x$ is equal to
(A) $\frac{k^2 + 5k + 6}{k^2}$ (B) $\frac{k^2 - 5k + 6}{k^2}$
(C) 6 (D) none of these

12. The number of distinct values of $f(x) = [x^3] - [x]^3$ for $\forall\ x \in [0, 2]$
(A) 4 (B) 5
(C) 7 (D) 8

13. If f(x) is an odd function also periodic function with period 2 then f(4) equal to
(A) 1 (B) 2 (C) 0 (D) none of these

14. Domain of f(x) satisfying $2^x + 2^{f(x)} = 2$ is
(A) $(\infty, -1)$ (B) $[0, 1]$ (C) $(-1, 1)$ (D) $(-\infty, 1)$

15. If $f : R \to R$, where $f(x) = ax + \cos x$ is an invertible function then
(A) $a \in (-2, -1] \cup [1, 2)$ (B) $a \in [-2, 2]$
(C) $a \in (-\infty, -1] \cup [1, \infty)$ (D) $a \in [-1, 1]$

16. Total number of solutions of $x^2 - 4 - [x] = 0$ (where $[\cdot]$ denotes G. I. F.) is
(A) 0 (B) 1 (C) 2 (D) 3

17. The fundamental period of $f(x) = [x] + [2x] + [3x] + \ldots. + [nx]$; where $x \in N$ and $[\cdot] \to$ G. I. F.; is
(A) 1 (B) n (C) 1/n (D) Non-periodic

18. If $f(x) = \begin{cases} 2 + x; & x \geq 0 \\ 2 - x; & x < 0 \end{cases}$; then f(f(x)) is given by
(A) $\begin{cases} 2 + x, & x \geq 0 \\ 2 - x, & x < 0 \end{cases}$ (B) $\begin{cases} 4 + x, & x \geq 0 \\ 4 - x, & x < 0 \end{cases}$
(C) $\begin{cases} 4 - x, & x \geq 0 \\ 4 + x, & x < 0 \end{cases}$ (D) $\begin{cases} 2 - x, & x \geq 0 \\ 2 + x, & x < 0 \end{cases}$

19. Period of $f(x) = x - [x + a] + b + a \sin(2\pi x)$; where $a, b \in R^+$ and $[\cdot]$ denotes G. I. F.; os
(A) π (B) 1 (C) a + b (D) Don't exist

20. $f : R \to (0, \pi/2]$ where $f(x) = \cot^{-1}(x^2 + x + a)$ complete set of values of 'a' such that $f(x)$ is onto is:
(A) $[3/4, \infty)$ (B) $[1/2, \infty)$ (C) $[1, \infty)$ (D) $[1/4, \infty)$

ANSWERS

LEVEL −I

1.	A, C, D	2.	A	3.	B	4.	B
5.	D	6.	D	7.	A, B	8.	A
9.	A	10.	C	11.	A	12.	A
13.	C	14.	C	15.	A	16.	D
17.	$[2, \infty)$	18.	A	19.	$\pi/2$	20.	C
21.	Greater, 2, all			22.	positive		
23.	both positive & both negative			24.	$x = \frac{3}{2}$		
25.	positive, positive & $\neq 1$			26.	$\frac{1}{k}$		
27.	positive, $D < 0$			28.	C		
29.	A	30.	C	31.	D	32.	A
33.	B	34.	B	35.	[2, 3]	36.	$x \in n\pi$
37.	cos4	38.	A	39.	B	40.	A
41.	A	42.	A	43.	C	44.	A
45.	C	46.	B	47.	B	48.	D
49.	B	50.	B	51.	B	52.	B
53.	A	54.	C	55.	C	56.	A
57.	A	58.	B	59.	A	60.	A
61.	A	62.	D	63.	C	64.	B
65.	B	66.	D	67.	A	68.	A
69.	C	70.	A	71.	C	72.	C
73.	B	74.	B	75.	D	76.	D
77.	A	78.	D	79.	D	80.	A
81.	A	82.	A				

LEVEL −II

1.	C	2.	A	3.	D	4.	A
5.	D	6.	B	7.	B, D	8.	A
9.	A	10.	B	11.	C	12.	D
13.	A	14.	A	15.	A	16.	D
17.	A	18.	C	19.	A, C	20.	C
21.	A	22.	A, C	23.	D	24.	C
25.	1	26.	B	27.	B	28.	D
29.	C	30.	A	31.	C	32.	B
33.	B	34.	A	35.	B	36.	$\frac{\pi}{2}$
37.	B	38.	B	39.	B	40.	A
41.	B	42.	A	43.	B	44.	A
45.	B	46.	B	47.	D	48.	D
49.	A	50.	B	51.	C		
52.	C	53.	D	54.	B	55.	B
56.	A	57.	C	58.	C	59.	D
60.	D	61.	C	62.	B	63.	C
64.	B						

LEVEL −III

1.	A	2.	C	3.	B	4.	C
5.	A, B, C	6.	D	7.	A	8.	A, C
9.	B	10.	A	11.	D	12.	C
13.	C	14.	D	15.	C	16.	C
17.	A	18.	B	19.	B	20.	D.