CHAPTER 7

Gravitation

1. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is **[2002]**
(a) $mgR/2$ (b) $2mgR$
(c) mgR (d) $mgR/4$.

2. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will **[2002]**
(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit in the same velocity
(c) become stationary in its orbit
(d) move towards the earth

3. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is **[2002]**
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$.

4. The escape velocity of a body depends upon mass as **[2002]**
(a) m^0 (b) m^1
(c) m^2 (d) m^3

5. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[2003]**
(a) 10 hours (b) 80 hours
(c) 40 hours (d) 20 hours

6. Two spherical bodies of mass M and $5M$ & radii R & $2R$ respectively are released in free space with initial separation between their centres equal to $12\ R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is **[2003]**
(a) $2.5R$ (b) $4.5R$
(c) $7.5R$ (d) $1.5R$

7. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45°with the vertical, the escape velocity will be **[2003]**
(a) $11\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km/s

8. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[2004]**
(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$
(c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

9. The time period of an earth satellite in circular orbit is independent of **[2004]**
(a) both the mass and radius of the orbit
(b) radius of its orbit
(c) the mass of the satellite
(d) neither the mass of the satellite nor the radius of its orbit.

10. If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is **[2004]**
(a) $\frac{1}{4}mgR$ (b) $\frac{1}{2}mgR$
(c) $2\,mgR$ (d) mgR

11. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to **[2004]**
(a) R^n (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) $R^{\left(\frac{n+1}{2}\right)}$ (d) $R^{\left(\frac{n-2}{2}\right)}$

12. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct? **[2005]**

(a) $d=\frac{3h}{2}$ (b) $d=\frac{h}{2}$

(c) $d=h$ (d) $d=2h$

13. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere **[2005]**

(you may take $G = 6.67\times 10^{-11}\ Nm^2/kg^2$)

(a) 3.33×10^{-10} J (b) 13.34×10^{-10} J

(c) 6.67×10^{-10} J (d) 6.67×10^{-9} J

14. Average density of the earth **[2005]**

(a) is a complex function of g

(b) does not depend on g

(c) is inversely proportional to g

(d) is directly proportional to g

15. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[2008]**

(a) 1.1 km s^{-1} (b) 11 km s^{-1}

(c) 110 km s^{-1} (d) 0.11 km s^{-1}

16. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2008]**

Statement-1 : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides 4 π GM. and

Statement-2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

17. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[2009]**

(a) $\frac{R}{\sqrt{2}}$ (b) $R/2$

(c) $\sqrt{2}R$ (d) 2R

18. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is: **[2011]**

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$

(c) $-\frac{9Gm}{r}$ (d) zero

19. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[2011 RS]**

(a) $\sqrt{\frac{Gm}{4R}}$ (b) $\sqrt{\frac{Gm}{3R}}$

(c) $\sqrt{\frac{Gm}{2R}}$ (d) $\sqrt{\frac{Gm}{R}}$

20. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be **[2012]**

(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules

(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

21. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[2013]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$

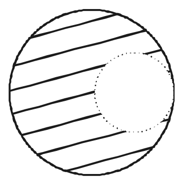
(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

22. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: **[2014]**

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

23. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : **[2015]**
(*G = gravitational constant*)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$

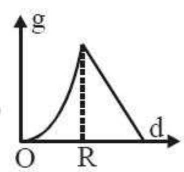
(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

24. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[2016]**

(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}\left(\sqrt{2}-1\right)$

(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

25. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): **[2017]**

(a)


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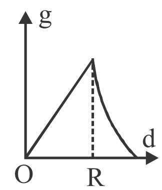

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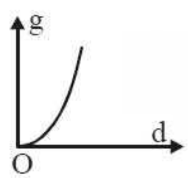
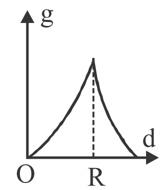


(d)


Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(d)	(a)	(c)	(c)	(c)	(d)	(c)	(b)	(c)	(d)	(c)	(d)	(c)
16	17	18	19	20	21	22	23	24	25					
(b)	(d)	(c)	(a)	(d)	(a)	(d)	(d)	(b)	(b)					

SOLUTIONS

1. **(c)** $K.E = \frac{1}{2}mv_e^2$ where v_e = escape velocity

$= \sqrt{2gR}$

$\therefore K.E = \frac{1}{2}m \times 2gR = mgR$

2. **(c)** Due to inertia of motion it will move tangentially to the original orbit in the same velocity.

3. **(d)** Energy required = (Potential energy of the Earth -mass system when mass is at distance 3*R*) – (Potential energy of the Earth -mass system when mass is at distance 2*R*)

$$= \frac{-GMm}{3R} - \left(\frac{-GMm}{2R}\right) = \frac{-GMm}{3R} + \frac{GMm}{2R}$$

$$= \frac{-2GMm + 3GMm}{6R} = \frac{GMm}{6R}$$

4. **(a)** Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$\Rightarrow v_e \propto m^0$

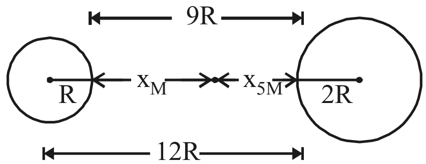
Where *M, R* are the mass and radius of the planet respectively. In this expression the mass of the body (*m*) is not present showing that the escape velocity is independent of the mass.

5. **(c)** According to Kepler's law of planetary motion $T^2 \propto R^3$

$$\therefore T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 5 \times \left[\frac{4R}{R}\right]^{3/2}$$

$$= 5 \times 2^3 = 40 \text{ hours}$$

6. (c) The gravitational force acting on both the masses is the same. We know that Force = mass × acceleration.



For same force, acceleration $\propto \dfrac{1}{\text{mass}}$

$$\therefore \frac{a_{5M}}{a_M} = \frac{M}{5M} = \frac{1}{5} \quad \text{.....(i)}$$

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass 5*M*

$$u = 0,\ S = x_{5M},\ t = t,\ a = a_{5M}$$

$$S = ut + \frac{1}{2}at^2$$

$$\therefore x_{5M} = \frac{1}{2}a_{5M}t^2 \quad \text{....(ii)}$$

For mass *M*

$$u = 0, s = x_M, t = t,\ a = a_M$$

$$\therefore s = ut + \frac{1}{2}at^2$$

$$\Rightarrow x_M = \frac{1}{2}a_M t^2 \quad \text{...(iii)}$$

Dividing (ii) by (iii)

$$\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5} \quad \text{[From (i)]}$$

$$\therefore 5x_{5M} = x_M \quad \text{....(iv)}$$

From the figure it is clear that

$$x_{5M} + x_M = 9R \quad \text{....(v)}$$

Where O is the point where the two spheres collide.

From (iv) and (v)

$$\frac{x_M}{5} + x_M = 9R$$

$$\therefore 6x_M = 45R$$

$$\therefore x_M = \frac{45}{6}R = 7.5R$$

7. (c) $v_e = \sqrt{2gR}$

The escape velocity is independent of the angle at which the body is projected.

8. (d) Gravitational force provides the necessary centripetal force.

$$\therefore \frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2} \text{ also } g = \frac{GM}{R^2}$$

$$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}\frac{n!}{r!(n-r)!}$$

$$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

9. (c) We have, $\dfrac{mv^2}{R+x} = \dfrac{GmM}{(R+x)^2}$

x = height of satellite from earth surface

m = mass of satellite

$$\Rightarrow v^2 = \frac{GM}{(R+x)} \text{ or } v = \sqrt{\frac{GM}{R+x}}$$

$$T = \frac{2\pi(R+x)}{v} = \frac{2\pi(R+x)}{\sqrt{\dfrac{GM}{R+x}}}$$

which is independent of mass of satellite

10. (b) At earth surface, $P.E.$ of system is $-\dfrac{GmM}{R}$

At a distance R from the earth's surface,

$P.E$ .of system is $-\dfrac{GmM}{2R}$

$$\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R};\ \Delta U = \frac{GmM}{2R}$$

$$\text{Now } \frac{GM}{R^2} = g;\ \therefore \frac{GM}{R} = gR$$

$$\therefore \Delta U = \frac{1}{2}mgR$$

11. (c) $F = KR^{-n} = MR\omega^2 \Rightarrow \omega^2 = KR^{-(n+1)}$

or $\omega = KR^{\frac{-(n+1)}{2}}$

$\frac{2\pi}{T} \propto R^{\frac{-(n+1)}{2}}$

$\therefore T \propto R^{\frac{+(n+1)}{2}}$

12. (d) Variation of g with altitude is,

$g_h = g\left[1 - \frac{2h}{R}\right]$;

variation of g with depth is,

$g_d = g\left[1 - \frac{d}{R}\right]$

Equating g_h and g_d, we get $d = 2h$

13. (c) Workdone,

$$W = \Delta U = U_f - U_i = 0 - \left[\frac{-GMm}{R}\right]$$

$$W = \frac{6.67 \times 10^{-11} \times 100}{0.1} \times \frac{10}{1000}$$

$= 6.67 \times 10^{-10}$ J

14. (d) $g = \frac{GM}{R^2} = \frac{G\rho \times V}{R^2} \Rightarrow g = \frac{G \times \rho \times \frac{4}{3}\pi R^3}{R^2}$

$g = \frac{4}{3}\rho\pi G.R$ where $\rho \rightarrow$ average density

$\rho = \left(\frac{3g}{4\pi GR}\right)$

$\Rightarrow \rho$ is directly proportional to g.

15. (c) $\frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$= \sqrt{\frac{10M_e}{M_e} \times \frac{R_e}{R_e/10}} = 10$

$\therefore (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110\ km/s$

16. (b) Gravitational flux through a closed surface is given by

$\int \vec{E_g} \cdot \vec{dS} = -4\pi GM$

where, M = mass enclosed in the closed surface

This relationship is valid when $|\vec{E}_g| \propto \frac{1}{r^2}$.

17. (d) We know that $\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

$\therefore \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2$

$\therefore \frac{R}{R+h} = \frac{1}{3}$

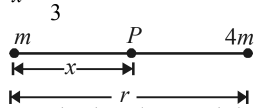
$\therefore \quad h = 2R$

18. (c) Let the gravitational field at P, distant x from mass m, be zero.

$\therefore \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$

$\Rightarrow \frac{1}{x} = \frac{2}{r-x} \quad \therefore r - x = 2x$

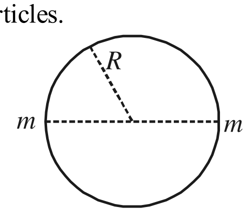
$x = \frac{r}{3}$



Gravitational potential at P,

$V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$

19. (a) Here, centripetal force will be given by the gravitational force between the two particles.



$\frac{Gm^2}{(2R)^2} = m\omega^2 R \Rightarrow \frac{Gm}{4R^3} = \omega^2$

$\Rightarrow \omega = \sqrt{\frac{Gm}{4R^3}}$

If the velocity of the two particles with respect to the centre of gravity is v then $v = \omega R$

$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$

20. (d) The work done to launch the spaceship

$$W = -\int_R^\infty \vec{F}.\vec{dr} = -\int_R^\infty \frac{GMm}{r^2} dr$$

$$W = +\frac{GMm}{R} \quad \ldots(i)$$

The force of attraction of the earth on the spaceship, when it was on the earth's surface

$$F = \frac{GMm}{R^2}$$

$$\Rightarrow \ mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2} \quad \ldots(ii)$$

The required energy for this work is given by

$= U - W = mgR$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 6.4 \times 10^{10}$ Joule

21. **(a)** As we know,

Gravitational potential energy $= \dfrac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

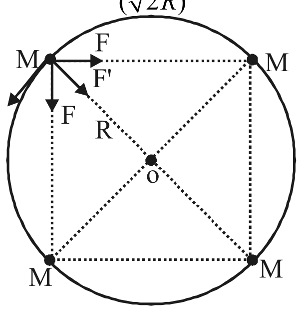
$E_i = E_f$

Therefore minimum required energy,

$$K = \frac{5GMm}{6R}$$

22. **(d)** $2F\cos 45° + F' = \dfrac{Mv^2}{R}$ (From figure)

Where $F = \dfrac{GM^2}{(\sqrt{2}R)^2}$ and $F' = \dfrac{GM^2}{4R^2}$



$$\Rightarrow \ \frac{2 \times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

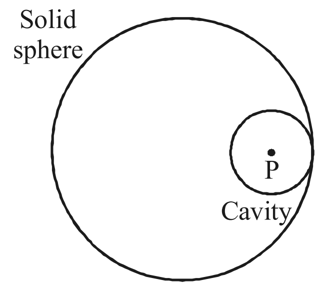
$$\Rightarrow \ \frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$$

$$\therefore \ v = \sqrt{\frac{Gm}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$$

23. **(d)** Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$= V_{sphere} - V_{cavity}$

$$= -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

24. **(b)** For h << R, the orbital velocity is $\sqrt{gR}$

Escape velocity $= \sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\,(\sqrt{2} - 1)$

25. **(b)** Variation of acceleration due to gravity, g with distance 'd' from centre of the earth

If $\quad d < R, g = \dfrac{Gm}{R^2}.d \quad$ *i.e.*, $g \propto d$ (straight line)

If $\quad d = R, g_s = \dfrac{Gm}{R^2}$

If $\quad d > R, g = \dfrac{Gm}{d^2} \quad$ *i.e.*, $g \propto \dfrac{1}{d^2}$