Resonance Educating for better tomorrow	**PHYSICS DPP** DAILY PRACTICE PROBLEMS
TARGET : JEE (ADVANCED) 2015	**NO. 11**
Course : VIJETA & VIJAY (ADP & ADR) **Date : 12-05-2015**	

TEST INFORMATION

DATE : 13.05.2015 **PART TEST(PT) - 05 (3 HOURS)**

Syllabus : String waves and sound waves, KTG, Heat and thermodynamics

DATE : 17.05.2015 **JEE PREPARATORY TEST (JPT) - 2 & 3**

Syllabus : Full syllabus

This DPP is to be discussed (15-05-2015)

PT-5 to be discussed (15-05-2015)

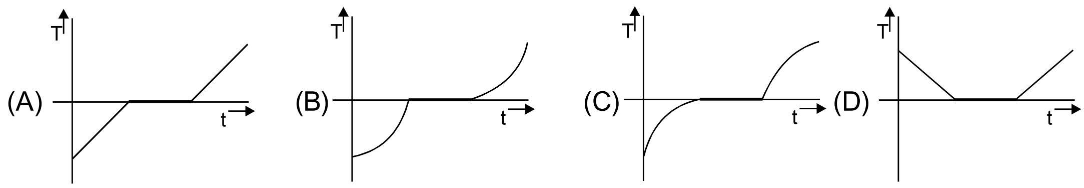
DPP No. # 11

Total Total Marks : 150	**Max. Time : 118 min.**	
Single choice Objective (–1 negative marking) Q. 1 to 14	**(3 marks 2½ min.)**	**[42, 35]**
Multiple choice objective (–1 negative marking) Q. 15 to 21	**(4 marks, 3 min.)**	**[28, 21]**
Single Digit Subjective Questions (no negative marking) Q.22 to Q.25	**(4 marks 2½ min.)**	**[16, 10]**
Double Digits Subjective Questions (no negative marking) Q.26 to Q.29	**(4 marks 2½ min.)**	**[16, 10]**
Comprehension (–1 negative marking) Q.30 to 41	**(3 marks 2½ min.)**	**[36, 30]**
Match Listing (–1 negative marking) Q.42 to Q.45	**(3 marks, 3 min.)**	**[12, 12]**

1. Let the wavelength at which the the spectral emissive power of a black body (at a temperature T) is maximum, be denoted by λ_{max} .As the temperature of the body is increased by 1 K , λ_{max} decreases by 1 percent .The temperature T of the black body is

(A) 100K (B) 200K (C) 400K (D) 288K

2. If specific heat capacity of a substance in solid and liquid state is proportional to temperature of the substance, then if heat is supplied to the solid initially at – 20°C (having melting point 0°C) at constant rate. Then the temperature dependence of solid with time will be best represented by :



3. A resistor has initial resistance 'R_0' at 0ºC. Now, it is connected to an ideal battery of constant emf = 'v'. If the temperature co–efficient of resistance is α, then after how much time, will its temperature be 'TºC'. Mass of the wire is m, specific heat capacity of the wire is S. (Assume the resistance veries linearly with temperature .Also neglect heat loss to the surrounding)

(A) $\frac{msR_0T}{v^2}$

(B) $\frac{m_0SR_0}{v^2}(T/2)$

(C) $\frac{mSR_0}{v^2}(T + \frac{\alpha T^2}{2})$

(D) $\frac{mSR_0}{v^2}T(1 + \alpha T)$

4. A sphere and a cube of same material and same total surface area are placed in same evacuated space after they are heated to same temperature. The ratio of their initial rate of cooling in space is :

(A) 1 (B) $\sqrt{\pi}$ (C) $\sqrt{\frac{\pi}{6}}$ (D) $\frac{1}{\sqrt{6}}$

5. A copper calorimeter of mass m_1 = 1 kg, contained with water of mass m_2 = 1 kg, their common temperature t = 10°C. Now a piece of ice of mass m_3 = 2 kg and temperature is –11°C dropped into the calorimeter. Neglecting any heat loss, the final temperature of system is. [specific heat of copper = 0.1 Kcal/ kg°C, specific heat of water = 1 Kcal/kg°C, specific heat of ice = 0.5 Kcal/kg°C, latent heat of fusion of ice = 78.7 Kcal/kg]

(A) 0°C (B) 4°C (C) – 4°C (D) – 2°C

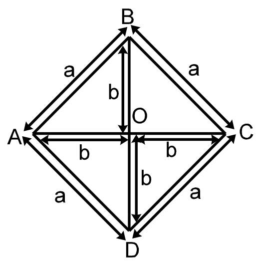
6. An insulated chamber at a height *h* above the earth's surface and maintained at 30ºC has a clock fitted with an uncompensated pendulum. The maker of the clock for the chamber mistakenly designs it to maintain correct time at 20ºC at that height. It is found that if the chamber were brought to earth's surface the clock in it would click correct time at 30°C. The coefficient of linear expansion of the material of pendulum is (earth's radius is *R*):

(A) $\frac{\Delta l}{10l}$ (B) $\frac{5R}{h}$ (C) $\frac{h}{5R}$ (D) $\frac{h}{20R}$

7. A force of constant magnitude F acts on a particle moving in a plane such that it is perpendicular to the velocity $\vec{v}$ ($|\vec{v}| = v$) of the body, and the force is always directed towards a fixed point. Then the angle turned by the velocity vector of the particle as it covers a distance S is :(take mass of the particle as m)

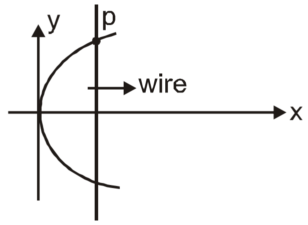
(A) $\frac{FS}{2mv^2}$ (B) $\frac{2FS}{mv^2}$ (C) $\frac{FS^2}{mv}$ (D) $\frac{FS}{mv^2}$

8. All the rods have same conductance 'K' and same area of cross section 'A'. If ends A and C are maintained at temperature $2T_0$ and T_0 respectively then which of the following is/are correct:



(A) Rate of heat flow through ABC, AOC and ADC is same
(B) Rate of heat flow through BO and OD is not same
(C) Total Rate of heat flow from A to C is $\frac{3KAT_0}{2a}$
(D) Temperature at junctions B, O and D are same

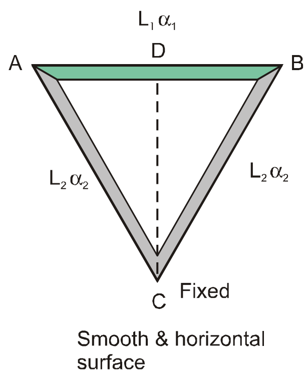
9. A wire is bent in a parabolic shape followed by equation $x = 4y^2$. Consider origin as vertex of parabola. A wire parallel to y axis moves with constant speed 4 m/s along x-axis in the plane of bent wire. Then the acceleration of touching point of straight wire and parabolic wire is (when straight wire has x coordinate = 4 m) :



(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 2 (D) 4

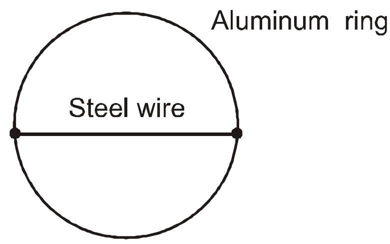
10. Equal volumes of water and alcohol when put in similar calorimeters take 100 sec and 74 sec. respectively to cool from 50ºC to 40ºC. The thermal capacity (in cal/°C) of each calorimeter is numerically equal to the volume (in cm^3) of either liquid. The specific gravity of alcohol is 0.8. If the specific heat capacity of water is 1 cal/g°C, the specific heat capacity of alcohol will be : (Assume Newton's law of cooling to be valid).
(A) 0.6 cal/gºC (B) 0.8 cal/gºC (C) 1.6 cal/gºC (D) 1.8 cal/gºC

11. Figure shows isosceles triangle frame ABC of two different material shown in figure. Thermal expansion cofficient of the rod ADB is α_1 and for rod ACB is α_2. End C is fixed and whole system is placed on smooth horizontal surface and D is midpoint of rod AB and CD is perpendicular to the AB. If temperature of the system is increase such as it is found that distance CD remain fixed then.



(A) $\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$ (B) $\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_1}{\alpha_2}}$ (C) $\frac{\ell_1}{\ell_2} = \sqrt{\frac{\alpha_1}{\alpha_2}}$ (D) $\frac{\ell_1}{\ell_2} = \sqrt{\frac{\alpha_2}{\alpha_1}}$

12. A steel wire is rigidly fixed along diameter of aluminium ring of radius R as shown. Linear expansion coefficient of steel is half of linear expansion coefficient for aluminium, then the thermal stress developed in steel wire is: (α_{Al} is linear expansion coefficient for aluminium and Young's modulus for steel is Y)



(1) more than R $\alpha_{Al}\,\Delta\theta Y$ (2) less then R $\alpha_{Al}\,\Delta\theta Y$
(3) Equal to R $\alpha_{Al}\,\Delta\theta Y$ (4) equal to 2R $\alpha_{Al}\,\Delta\theta Y$

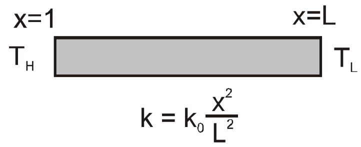
13. The temperature of an spherical isolated black body falls from T_1 to T_2 in time 't'. Then time t is :

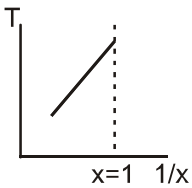
(A) $t \propto \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$ (B) $t \propto \left(\frac{1}{T_2^2} - \frac{1}{T_1^2}\right)$ (C) $t \propto \left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$ (D) $t \propto \left(\frac{1}{T_2^4} - \frac{1}{T_1^4}\right)$

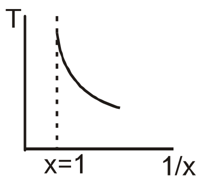
14. Two containers having boiling water and ice are connected through a conducting metal rod. The whole ice melts in time T. Now the rod is cut into two equal parts and both parts are connected in parallel between the containers. The time required to melt the same amount of ice will be -

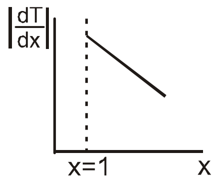
(A) T (B) $\frac{T}{2}$ (C) $\frac{T}{4}$ (D) 2T

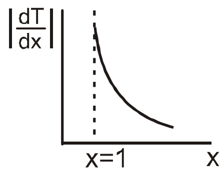
15. A rod of uniform cross–section but non–uniform thermal conductivity which vary as $k = k_0 \frac{x^2}{L^2}$ $(1 \le x \le L)$ (as shown in figure) is kept between fixed temperature difference for a long time. Select the correct option(s) :



(A)


(B)


(C)


(D)


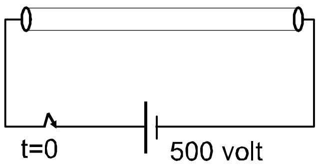
16. A vessel is partly filled with a liquid. Co-efficients of voltmetric expansion of material of the vessel and liquid are γ_V & γ_L respectively. If the system is heated then volume unoccupied by the liquid will necessarily :

(A) increase if $\gamma_V > \gamma_L$
(B) decrease if $\gamma_V < \gamma_L$
(C) remain unchanged if $\gamma_V = \gamma_L$
(D) increase if $\gamma_V = \gamma_L$

17. The ends of a rod of uniform thermal conductivity are maintained at different (constant) temperatures. After the steady state is achieved :

(A) heat flows in the rod from high temperature to low temperature even if the rod has nonuniform cross sectional area.
(B) temperature gradient along length is same even if the rod has non uniform cross sectional area.
(C) heat current is same even if the rod has non-uniform cross sectional area.
(D) if the rod has uniform cross sectional area the temperature is same at all points of the rod.

18. A straight nicrome wire is initially at room temperature 20ºC. It is connected to an ideal battery of 500 volt. Just after switching on, the current detected is 5 amp. Due to heating effect its temperature increases, and is also loosing heat to the environment according to newton's cooling law as $\frac{dQ_{loss}}{dt}$ = 45(T – 20ºC)J/sec. At steady state, the current detected is 4.5 amp.



(A) steady state temperature of the wire is 70 ºC
(B) steady state temperature of the wire is 75.5ºC
(C) temperature co–efficient of resistance of the wire is nearly 2.2×10^{-3} /ºC
(D) temperature co–efficient of resistance of the wire is nearly 1.57×10^{-3} /ºC

19. Water at 50 ºC is filled in a cubical container of side 1 m. The thickness of the walls of the container is 1 mm. The container is surrounded by large amount of ice at 0 ºC. The temperature of the water becomes 25 ºC in 10 ℓn 2 seconds. Choose the correct options. Find the thermal conductivity of the material of the container and the ice melted in that time.
[Given, specific heat of water = 1 cal/gm degree ;
latent heat of fusion of ice = 80 cal/gm; density of water = 1 gm/cm^3;
heat capacity of the container $\cong$ 0]
(A) thermal conductivity of the material is 70 J/m ºC
(B) thermal conductivity of the material is 60 J/m ºC
(C) Mass of the ice melted is 312.5 kg
(D) Mass of the ice melted is 252 kg

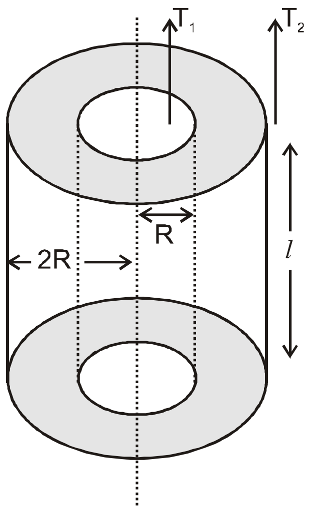
20. When the temperature of a copper coin is raised by 80 ºC, its diameter increases by 0.2%,
(A) percentage rise in the area of a face is 0.4%
(B) percentage rise in the thickness is 0.4%
(C) percentage rise in the volume is 0.6%
(D) coefficient of linear expansion of copper is 0.25×10^{-4} / ºC.

21. The emissive power of a black body at T = 300 K is 100 Watt/m^2. Consider a body B of area A = 10 m^2 coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5. Its temperature is 300 K. Then which of the following is correct :
(A) The emissive power of B is 20 W/m^2
(B) The emissive power of B is 200 W/m^2
(C) The power emitted by B is 200 Watts
(D) The emissivity of B is = 0.2

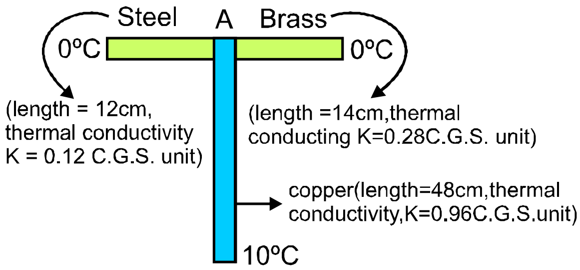
22. A solid cube of side a, density d and specific heat 's' is at temperature 400 K. It is placed in an ambient temperature of 200 K. Take : a = 0.9 m, d = 4.8×10^3 Kg/m^3, s = 2.0×10^3 J/kg/K, Stefan's constant $\sigma = 6 \times 10^{-8}$ W/K^4-m^2. Consider the cube to be a black body. If the time for the temperature of the cube to drop by 5 K is 1000 X seconds, find X in nearest integer.

23. A sphere P(emissivity=1) of radius 2R and and another sphere Q(emissivity =1/2) of radius R are placed in vacuum at some distance.There are no other objects. The temperature of the sphere Q is maintained at 200K by the means of a heater. A fraction 1/32 of the power emitted by the sphere Q falls on the sphere P. If the equilibrium temperature of the sphere P is 10 T Kelvin, find the value of T.

24. Inner surface of a cylindrical shell of length ℓ and of material of thermal conductivity k is kept at constant temperature T_1 and outer surface of the cylinder is kept at constant temperature T_2 such that $(T_1 > T_2)$ as shown in figure. Heat flows from inner surface to outer surface radially outward. Inner and outer radii of the shell are R and 2R respectively. Due to lack of space this cylinder has to be replaced by a smaller cylinder of length $\frac{\ell}{2}$ inner and outer radii $\frac{R}{4}$ and R respectively and thermal conductivity of material nk. If rate of radially outward heat flow remains same for same temperatures of inner and outer surface i.e. T_1 and T_2, then find the value of n.



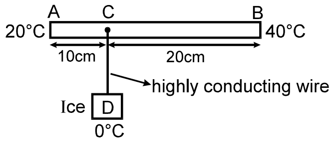
25. Three rods of copper, brass and steel are joined together to form T shape as shown in figure. The cross–sectional area of each rod is 4cm^2. The end of copper rod is maintained at 10ºC and the ends of brass and steel rods at 0ºC. Assume there is no loss of heat to surrounding. What is temperature of junction point A in ºC.



26. A steel wire is rigidly fixed at both ends. Its length, mass and cross sectional area are 1m, 0.1kg and 10^{-6} m^2 respectively. Then the temperature of the wire is lowered by 20 ºC. If the transverse waves are setup by plucking the wire at 0.25m from one end and assuming that wire vibrates with minimum number of loops possible for such a case. Find the frequency of vibration (in Hz). [coefficient of linear expansion of steel =1.21×10^{-5}/ºC and Young's modulus = 2×10^{11} N/m^2]

27. A body cools in 7 minutes from 60 ºC to 40 ºC. What will be its temperature (in °C) after the next 7 minutes. The temperature of surroundings is 10 ºC.

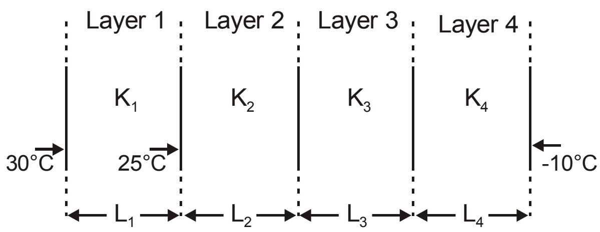
28. In the figure shown AB is a rod of length 30 cm and area of cross-section 1.0 cm^2 and thermal conductivity 336 S. I. units. The ends A & B are maintained at temperatures 20º C and 40 ºC respectively. A point C of this rod is connected to a box D, containing ice at 0º C, through a highly conducting wire of negligible heat capacity. Find the initial rate (in mg/s) at which ice melts in the box. [Assume latent heat of fusion for ice L_f = 80 cal/gm]



29. A scale ($\alpha = 10^{-3}$/ºC) gives correct reading at 0ºC. It is used at a different temperature where the scale measured length of 1.015 m line as 1m. Find the temperature, in ºC, at which this scale is used while taking the measurement.

COMPREHENSION-1

Figure shows in cross section a wall consisting of four layers with thermal conductivities K_1 = 0.06 W/mK; K_3 = 0.04 W/mK and K_4 = 0.10 W/mK. The layer thicknesses are L_1 = 1.5 cm ; L_3 = 2.8 cm and L_4 = 3.5 cm. The temperature of interfaces is as shown in figure. Energy transfer through the wall is in steady state.



30. The temperature of the interface between layers 3 and 4 is :
(A) – 1°C (B) – 3°C (C) 2°C (D) 0°C

31. The temperature of the interface between layers 2 and 3 is :
(A) 11°C (B) 8°C (C) 7.2°C (D) 5.4°C

32. If layer thickness L_2 is 1.4 cm, then its thermal conductivity K_2 will have value (in W/mK) :
(A) 2×10^{-2} (B) 2×10^{-3} (C) 4×10^{-2} (D) 4×10^{-3}

COMPREHENSION-2

Temperature inside the thin hollow sphere (which is placed in vacuum) of surface area 1m^2 is maintained at 100°C with the help of an electric heater of constant power. The outer surface of sphere losses 120 W power in radiation and absorbs 50 watt radiation incident on it at steady state. It is also observed in other experiment with same sphere that it emits 50 watt power when temperature of outer surface is 40°. (Assume temperature of the surrounding is constant (20°C) and Newtons law of cooling is valid, thermal conductivity of material 7×10^{-4} Watt/m °C)

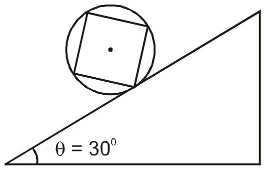
33. Electric power of heater is :
(A) 50 watt (B) 60 watt (C) 20 watt (D) 70 watt

34. Temperature of outer surface of sphere is
(A) 100° (B) 80° (C) 48° (D) 60°

35. Thickness of hollow sphere is :
(A) 0.6 mm (B) 0.52 mm (C) 0.7 mm (D) none of these

COMPREHENSION-3

Four identical uniform rods of mass M = 6kg each are welded at their ends to form a square and then welded to a uniform ring having mass m = 4kg & radius R = 1 m. The system is allowed to roll down on the rough and fixed incline of inclination θ = 30^0. (Assume no sliding anywhere)



36. The moment of inertia of system about the axis of ring will be -

(A) 20 kg m^2 (B) 40 kg m^2 (C) 10 kg m^2 (D) 60 kg m^2.

37. The acceleration of centre of mass of system is -

(A) $\frac{g}{2}$ (B) $\frac{g}{4}$ (C) $\frac{7g}{24}$ (D) $\frac{g}{8}$

38. The minimum value of coefficient of friction to prevent slipping is -

(A) $\frac{5}{7}$ (B) $\frac{5}{12\sqrt{3}}$ (C) $\frac{5\sqrt{3}}{7}$ (D) $\frac{7}{5\sqrt{3}}$

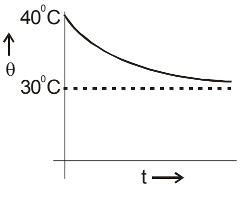
COMPREHENSION-4

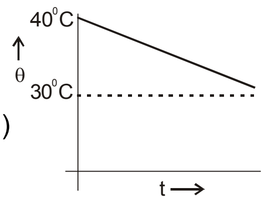
A body cools in a surrounding of constant temperature 30 ºC. Its heat capacity is 2J/ºC. Initial temperature of the body is 40ºC . Assume Newton's law of cooling is valid. The body cools to 38ºC in 10 minutes.

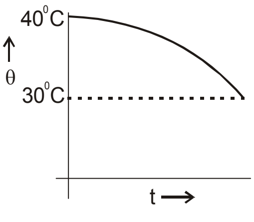
39. In further 10 minutes it will cool from 38ºC to _____ :

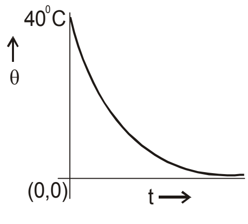
(A) 36ºC (B) 36.4ºC (C) 37ºC (D) 37.5ºC

40. The temperature of the body in ºC denoted by θ the variation of θ versus time t is best denoted as

(A)


(B)


(C)


(D)


41. When the body temperature has reached 38 ºC, it is heated again so that it reaches to 40ºC in 10 minutes .The total heat required from a heater by the body is:

(A) 3.6J (B) 0.364J (C) 8 J (D) 4 J

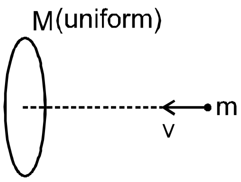
42. **Match the following :**

	Column–I		Column–II
(P)	In a negative β^- decay reaction $_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + \beta^{-1} + \overline{\upsilon}$	(1)	Total Energy is conserved
(Q)	When a heavy object is projected from the surface of earth	(2)	Total linear momentum of the system is conserved
(R)	When an electron jumps from higher energy level to ground state.	(3)	Total angular momentum is conserved
(S)	When a rocket moves due to thrust of ejected gases	(4)	Total mass is conserved

	P	Q	R	S
(A)	4	2	3	1
(B)	4	2	1	3
(C)	2	1	3	4
(D)	3	1	4	2

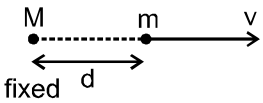
43. Column-I has some statements about the system shown in column-II, Match approprtate column.

Column-I **Column-II**

(P) 

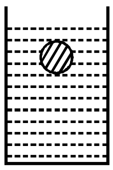
Fixed ring

[Neglect other gravitational forces, system consists of ring and point mass] — (1) kinetic energy of system continiously increases..

(Q) 

$v > \sqrt{\frac{2GM}{d}}$

[Neglect other gravitational force, system consists of two point massage M and m.] — (2) Potential energy of system continiously increases.

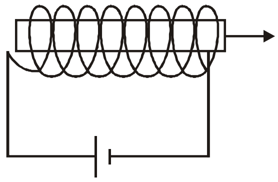
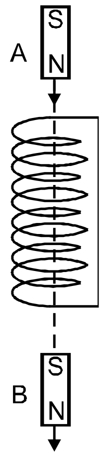
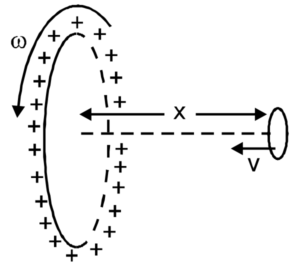
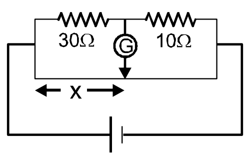
(R)  $\rho_{ball} > \rho_{liquid}$

‘liquid’ is viscous and filled in a very long tube. — (3) Total mechanical energy of system remains conserved

(S) Q •--------------• q
both the charges are free to move and release from rest — (4) Total mechanical energy of system continiously decreases.

	P	Q	R	S
(A)	4	1	2	3
(B)	1	3	2	4
(C)	3	2	4	1
(D)	1	2	3	4

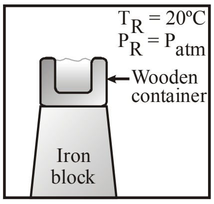
44. In column–II, some situations are given and in column–I, some results are given. Match the column according to the correct results.

Column–I	**Column–II**
(P) Current will increase	(1)  An inductor coil having some resistance is connected with a battery for a long time. Now the iron rod is suddenly pulled out. Magnetic energy stored in the inductor is E. During the small time intervel in which the rod is comming out.
(Q) Current will decrease	(2) 
(R) E will increase	A short circuited solenoid, having some resistance is fixed with its axis vertical. A powerful bar magnet is released from position A. Here 'E' is mechanical + electromagnetic energy of the magnet–solenoid system. During the motion of the magnet from A to B.
(S) E will decrease	(3)  A highly charged ring rotating with a constant angular velocity. A small loop, made of a resistance wire, whose radius is very much smaller than that of the ring, is moving along the axis with a constant velocity. E is total thermal energy produced in the loop. While the loop is moving from $x \to \infty$ to $x = 0$
	(4)  In the meter bridge circuit, torque produced in the galvanometer coil is 'E' and current through the galvanometer is i. As the jockey is moved from x = 50 to x = 70 then :

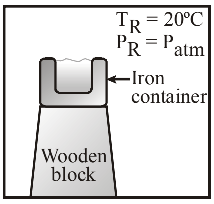
	P	Q	R	S
(A)	1	3	4	2
(B)	1	4	3	2
(C)	2	3	4	1
(D)	3	4	1	2

45. Containers of different materials are filled with water at room temperature 25ºC and kept in a closed room (at temperature T_R and pressure P_R) in different conditions shown in column-I, Match each situation given in column I with the phenomenon given in column-II . Assume air and wood are bad conductors of heat.

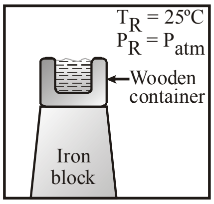
Column-I **Column-II**

(P) 

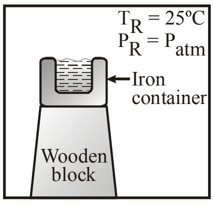
(1) Heat exchange occurs between water and surrounding by conduction

(Q) 

(2)Heat exchange occurs between water and surrounding by convection

(R) 

(3) Water emits radiations to surroundings.

(S) 

(4) Water absorbs radiations from surroundings.

	P	Q	R	S
(A)	4	1	2	3
(B)	1	3	2	4
(C)	2	1	3	4
(D)	1	2	3	4

ANSWER KEY OF DPP No. # 10

1.	(C)	**2.**	(D)	**3.**	(C)	**4**	(D)	**5.**	(A)	**6.**	(A)	**7.**	(C)
8.	(D)	**9.**	(D)	**10.**	(D)	**11.**	(C)	**12.**	(B)	**13.**	(B)	**14.**	(B,C)
15.	(A,C,D)	**16.**	(B,C,D)	**17.**	(A,B,C)	**18.**	(A,B)	**19.**	**5**	**20.**	**5**	**21.**	**3**
22.	**7**	**23.**	**4**	**24.**	**2**	**25.**	11	**26.**	**14**	**27.**	10	**28.**	(A)
29.	(C)	**30.**	(A)	**31.**	(C)	**32.**	(A)	**33.**	(C)	**34.**	(C)	**35.**	(B)
36.	(A)	**37.**	(A)	**38.**	(D)	**39.**	(C)	**40.**	(A)	**41.**	(C)	**42.**	(A)
43.	(B)	**44.**	(C)	**45.**	(D)								

Solution of DPP # 11

TARGET : JEE (ADVANCED) 2015
COURSE : VIJAY & VIJETA (ADR & ADP)

PHYSICS

1. $\lambda_m T$ = const.
$\ell n\, \lambda_m + \ell n T = C$

$\frac{d\lambda_m}{\lambda_m} + \frac{dT}{T} = 0$ $\quad\quad \therefore \frac{d\lambda_m}{\lambda_m} = -\frac{dT}{T}$

Now $\frac{d\lambda_m}{\lambda_m} = -1\ \% = -\frac{1}{100}$ (–ve sign indicates decrease)

dT = 1 (given)
$\therefore$ T = 100 K.

2. As dQ = msdT

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

From question : $S \propto T$
or $S = K_1T$. (K_1 being propertionality constant)

Also, $\frac{d\theta}{dt}$ = constant = K_2 (say) $\quad \Rightarrow ms\frac{dT}{dt} = K_2 \quad \Rightarrow m(K_1T)\frac{dT}{dt} = K_2$

$$\Rightarrow \left(m\frac{K_1}{K_2}\right)\frac{T^2}{2} = t \Rightarrow \quad T \propto \sqrt{t}$$

3. Rate of heat produced

$$\frac{dQ}{dt} = \frac{v^2}{R} = \frac{v^2}{R_0(1+\alpha(T-0))} = \frac{v^2}{R_0(1+\alpha T)} \quad \text{and} \quad \frac{dQ}{dt} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad ms\frac{dT}{dt} = \frac{v^2}{R_0(1+\alpha T)}$$

$$\int_{T=0}^{T=T}(1+\alpha T)dT = \frac{v^2}{R_0 ms}\int_{t=0}^{t=t}dt$$

$$T + \frac{\alpha T^2}{2} = \frac{v^2}{R_0 ms}t, \quad\quad t = \frac{R_0 ms}{v^2}\left(T + \frac{\alpha T^2}{2}\right).$$

4. For sphere :

$$\sigma T^4 S = m_1 C.\left(\frac{-d\theta}{dt}\right)_{sphere}$$

For cube :

$$\sigma T^4.\ S = m_2.C\left(\frac{-d\theta}{dt}\right)_{cube}$$

$$\therefore \frac{\left(\frac{-d\theta}{dt}\right)_{sphere}}{\left(-\frac{d\theta}{dt}\right)_{cube}} = \frac{m_2}{m_1} = \frac{V_2}{V_1} \quad [S = 6a^2 = (4\pi r^2)] = \sqrt{\frac{\pi}{6}}$$

5. Loss in heat from calorimeter + water as temperture changes from 10°C to 0°C

$= m_1C_1 10 + m_2C_2 10 \quad = 1 \times 1 \times 10 + 1 \times 0.1 \times 10 = 11$ kcal

Gain in heat of ice as its temperature changes from –11°C to 0°C

$= m_3C_3 \times 11 = 2 \times 0.5 \times 11 = 11$ kcal

Hence ice and water will coexist at 0°C without any phase change.

6. Clock is designed to indicate correct time at 20°C at height 'h'. It will indicate correct time at 30°C on the ground if in this case the time period is same as the earlier.

$$\therefore 2\pi\sqrt{\frac{L}{g_h}} = 2\pi\sqrt{\frac{L'}{g_s}}$$

here $L' = L(1 + \alpha\, 10)$, $\quad g_s = \frac{GM}{R^2}$ and $g_h = \frac{GM}{(R+h)^2} \Rightarrow \frac{L}{g_n} = \frac{L'}{g_s}$

$\Rightarrow L(R + h)^2 = L(1 + \alpha\, 10)R^2 \qquad \Rightarrow \qquad 1 + \frac{h}{R} = (1 + \alpha\, 10)^{1/2}$

$\Rightarrow 1 + \frac{h}{R} = 1 + 5\alpha$ (by binomial expansion) $\qquad \Rightarrow \qquad \alpha = \frac{h}{5R}$

7. Since $\vec{F} \perp \vec{V}$, the particle will move along a circle.

$\therefore \qquad F = \frac{mv^2}{R} \qquad \& \ \theta = \frac{S}{R} \qquad \Rightarrow \qquad \theta = \frac{FS}{mv^2}$

8. By symmetry

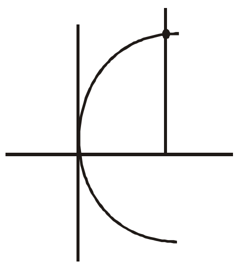
$I_{AB} = I_{BC}$ & $I_{AD} = I_{DC}$

$\therefore$ No current in BO and OD

$\therefore$ $T_B = T_O = T_D$

9. $x = 4y^2$

$\frac{dx}{dt} = 8y\frac{dy}{dt}$



$V_x = 8y\,V_y$

$V_x = 4$

$a_x = 0$

$0 = a_x = 8[y.a_y + V_y^2]$

$-y\,a_y = V_y^2$

$|a_y| = \frac{v_y^2}{y}$

$|a_y| = \frac{v_x^2}{64\,y^3} = \frac{16}{64 \times y^3}$

at $y = 1 \Rightarrow \quad |a_y| = \frac{1}{4}$

10. Applying Newton's Law on water calorimeter :

$$(m_1s_1 + m_2s_2)\,\frac{dT}{dt} = kA\,(T - T_0)$$

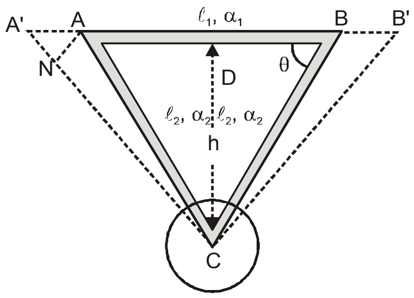
$$[(\rho v1) + v)]\left(\frac{50-40}{100}\right) = kA\,(45 - T_0)$$

$$[(0.8\rho\,)vS + v)\left[\frac{50-40}{74}\right] = kA\,(45 - T_0)$$

using $\rho = 1$ gm/cm^3, by solving

$S = 0.6$ cal/gm°C

11.



According to condition of the problem, height of the isosceles triangle ABC is unchanged. The dotted lines show configuration after a temperature rise. Increase in length of rod AB,

$\Delta\ell_1 = \ell_1\alpha_1\Delta T$

Thus $AA' = 1/2\ \ell_1\alpha_1\Delta T$

We draw a normal from A to A'C (the final length of AC). Increase in length of AC is A'N

$A'N = \ell_2\alpha_2\Delta T$

Considering increase in angle θ to be very small.

$$A'N \simeq AA' \cos\theta$$

Where $\cos\theta = \dfrac{\ell_1}{2\ell_2}$

Thus, we have $\ell_2\alpha_2\Delta T = \left(\dfrac{1}{2}\ell_1\alpha_1\Delta T\right)\left(\dfrac{\ell_1}{2\ell_2}\right)$

Hence $\dfrac{\ell_1}{\ell_2} = 2\sqrt{\dfrac{\alpha_2}{\alpha_1}}$

12. Stress = $2R\,[\alpha_{Al} - \alpha_{st}]\,\Delta\theta Y$ If aluminium ring is alowed to expand freely

so stress < $2R\,(\alpha_{Al})\,\Delta\theta Y$

13. Power radiated $P = 4\pi r^2\sigma T^4 = -\left(ms\dfrac{dT}{dt}\right)$

$$-\frac{dT}{T^4} = \frac{4\pi r^2\sigma dt}{m} = c\,dt$$

$$-\int_{T_1}^{T_2}\frac{dT}{T^4} = ct \qquad \Rightarrow \qquad t = K\left[\frac{1}{T_2^{\,3}} - \frac{1}{T_1^{\,3}}\right].$$

14. $\Delta Q = \frac{kA(100-0)}{L} . T$ ....(i)

In second case :

$\Delta Q' = 2. \frac{kA(100-0)}{L/2} . T'$

since $\Delta Q = \Delta Q'$

$\therefore$ $T = 4T'$ $T' = \frac{T}{4}$

15. $i = -\frac{k_0 x^2}{L^2} A \frac{dT}{dx}$ $\Rightarrow$ $\int_{T_H}^{T} dT = -\frac{iL^2}{k_0 A} \int_1^x \frac{dx}{x^2}$

$T - T_H = \frac{iL^2}{k_0 A} \left[\frac{1}{x}\right]_1^x$

17. Heat obviously flows from higher temperature to lower temperature in steady state. $\Rightarrow$ A is true.

Temperature gradient $\propto \frac{1}{\text{cross sec tion area}}$ in steady state. $\Rightarrow$ B is false.

Thermal current through each cross section area is same. $\Rightarrow$ C is true.

Temperature decreases along the length of the rod from higher temperature end to lower temperature end. $\Rightarrow$ D is false.

18. For steady state

$\left(\frac{dQ}{dt}\right)_{in} = \left(\frac{dQ}{dt}\right)_{out}$

$(V)\,(i_{55}) = 45(T-20)$

$(500)\,(4.5) = 45(T-20)$

$T_{55} = 70°C.$

Resistance at 20ºC is $R = \frac{v}{i} = \frac{500}{5}$

$R_{20} = 100\ \Omega$

Resistance at 70ºC is $R = \frac{v}{i} = \frac{500}{4.5} \simeq 111\ \Omega$

$R_f = R_0(1 + \alpha\Delta T)$

$111 = 100(1 + \alpha(50))$

$\alpha = \frac{0.11}{50} \simeq 2.2 \times 10^{-3}$ /ºC.

19. Let at any instant temperature of water be T, then heat current

$i = \frac{kA}{x} . (T - 0)$ —— (1)

where $A = 6\ a^2 = 6\ m^2$; x = thickness = 1 mm = 10^{-3} m

Rate of heat lost from water, $\frac{dQ}{dt} = +\, m\, s \frac{dT}{dt}$ —— (2)

So, we get from (1) & (2), $-\, m\, s \frac{dT}{dt} = \frac{kAT}{x}$ $\Rightarrow$ $-\int_{50°}^{25°} \frac{dT}{T} = \frac{kA}{mSx} \int_0^{10\ell n2} dt$

$\Rightarrow$ $\ell n\,(2) = \frac{kA}{mSx} . 10\,(\ell n\,2)$ So, $\frac{kA\,(10)}{mSx} = 1$

Putting values $\Rightarrow$ $k = \frac{mSx}{10A} = \frac{(10^3 \text{ kg})(4.2 \times 10^3 \text{ J/kg°C})\, 10^{-3} \text{ m}}{10 \times (6 \text{ m}^3)}$ = k = 70 J/m ºC

$\Rightarrow$ Total heat transferred will be = total heat

$Q = \int dQ = \int \frac{kA}{x} T\, dt$ Lost by water.

$Q = m S \Delta T = 10^3 \times 4200 \times 25$ J $= (10^6 \text{ gm})(1 \text{ cal})(25) = m_{ice} L$

Giving $m_{ice} = \left(\frac{10^6 \times 25}{80}\right)$ gm $= \frac{25000}{80}$ kg = 312.5 kg

mass of ice melted = 312.5 kg

20. $\frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta \ell}{A}\right) \times 100$

$\Rightarrow$ % increase in Area = 2 × 0.2 = 0.4

$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6$ %

Since $\Delta \ell = \ell \alpha \Delta T$

$\frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta T \times 100 = 0.2$

$\Rightarrow$ $\alpha = 0.25 \times 10^{-4}$ / ºC

21. Since , e = a = 0.2 (Since, a = (1 – r – t) = 0.2 for the body B)
E = (100) (0.2) = 20 W/m^2
Power emitted = e.A = 20 × 10 = 200 Watt

22. $\sigma(T^4 - T_S^4) . (6a^2)\, t = (d . a^3)\, s.\Delta T$

$\Rightarrow$ $t = \frac{d a s \Delta T}{6.\sigma(T^4 - T_s^4)} = \frac{4.8 \times 10^3 \times 0.9 \times 2.0 \times 10^3 \times 5}{6 \times 6 \times 10^{-8} \times (400^4 - 200^4)}$ = 5000 s.

X = 5

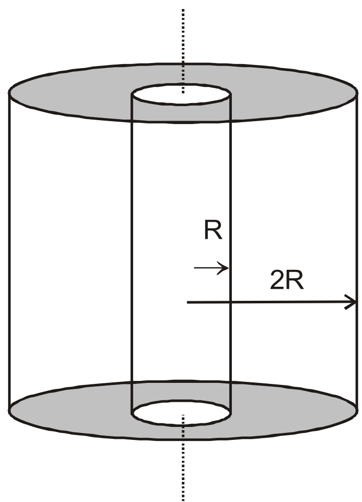
23. In equilibrium, power released= power absorbed

or $4\pi(2R)^2(1)\sigma T^4 = \frac{1}{32} 4\pi(R)^2(\frac{1}{2})\sigma 200^4$

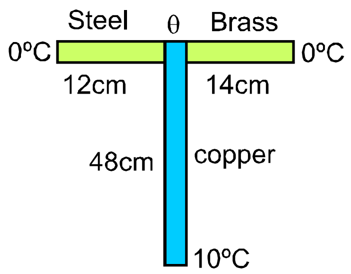
24. $H = -K.\ 2\pi r l \frac{dT}{dr}$

$\int_{R_1}^{R_2} \frac{H\, dr}{2\pi r l} = -K \int_{T_1}^{T_2} dT$

$H = \frac{2\pi l k(T_1 - T_2)}{l n \frac{R_2}{R_1}}$ $H_i = H_f$ $\therefore$ **Ans.** n = 4



25. $i = i_1 + i_2$



$$\frac{0.96 \times 4 \times (10-\theta)}{48} = \frac{0.28 \times 4(\theta - 0)}{14} + \frac{0.12 \times 4 \times (\theta - 0)}{12}$$

$0.02(10 - \theta) = 0.02\theta + 0.01\theta$

$\theta = 4°C.$

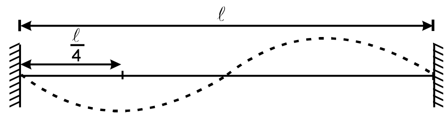
26. The mechanical strain $= \frac{\Delta \ell}{\ell} = \alpha \, \Delta T = 1.21 \times 10^{-5} \times 20 = 2.42 \times 10^{-5}$

The tension in wire $= T = Y \frac{\Delta \ell}{\ell} A = 2 \times 10^{11} \times 2.42 \times 10^{-5} \times 10^{-6} = 48.4$ N

$\therefore$ speed of wave in wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{0.1}} = 22 \text{ m/s}$$

Since the wire is plucked at $\frac{\ell}{4}$ from one end



The wire shall oscillate in 1st overtone (for minimum number of loops)

$\lambda = \ell = 1$m

Now $V = f\lambda$ or $f = \frac{V}{\lambda} = 22$ Hz.

27. Rate of cooling

$$\frac{\Delta T}{\Delta t} = K(T - T_0)$$

For cooling from 60°C to 40°C

$$\Rightarrow \frac{60-40}{7} = K\left(\frac{60+40}{2} - 10\right)$$

$$\Rightarrow K = \frac{20}{7 \times 40} = \frac{1}{14}$$

For cooling from 40°C to T

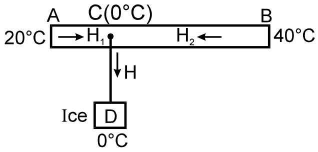
$$\frac{40-T}{7} = K\left(\frac{40+T}{2} - 10\right) \Rightarrow \frac{40-T}{7} = \frac{1}{14}\left(\frac{40+T-20}{2}\right)$$

$\Rightarrow 160 - 4T = 20 + T$

$\Rightarrow 140 = 5T$

$$\Rightarrow T = \frac{140}{5} = 28°C$$

$\Rightarrow$ T = 28°C

28. 

Thermal resistance of AC $\left(=\frac{L}{KA}\right) = \frac{0.1}{336 \times 1 \times 10^{-4}} = \frac{10^3}{336}$ = R (suppose)

thermal resistance of BC = $\frac{0.2}{336 \times 10^{-4}}$ = 2R

temperature of C = 0°C

$\therefore \quad H_1 = \frac{20}{R} \quad ; \; H_2 = \frac{40}{2R} = \frac{20}{R}$

$\therefore \quad H = H_1 + H_2 = \frac{40}{R} = \frac{40 \times 336}{10^3} = \frac{13440}{10^3}$ = 13.44 watt

Rate of melting of ice= $\frac{H}{L_f} = \frac{13.44/4.2}{80}$ g/s = 40 mg/s

29. $L = L_0 (1 - \alpha_s \Delta t)$

$1 = 1.015 - 1.015\, \alpha \Delta T$

$0.015 = 1.015 \times 10^{-3} \times \Delta T$

$\Delta T = \frac{15}{1.015} = 15.$

30. In steady state $\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 4}}$

$\Rightarrow \frac{0.06 \times A \times (30 - 25)}{1.5 \times 10^{-2}} = \frac{0.10 \times A \times \Delta T}{3.5 \times 10^{-2}} \qquad \Rightarrow DT = 7°C$

$T_3 = (-10 + 7)°C = -3°C$

31. $\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 3}}$

$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{0.04 \times A \times \Delta T}{2.8 \times 10^{-2}} \qquad \Rightarrow DT = 14°C$

$T_3 = (-3 + 14)°C = 11°C$

32. $\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 2}}$

$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{K_2 \times A \times 14}{1.4 \times 10^{-2}} \quad \Rightarrow K_2 = 0.02$ W/mK

33 to 35

For spherical surface, at steady state

$P_{Heater} + 50\text{ W} = 120\text{ W}$

$\Rightarrow \quad P_{Heater} = 70\text{ W}$

At steady state $P_{Heater} = \dfrac{(100 - t_{out})\,kA}{\ell} = 70 = k'(t_{out} - 20)$ ....(i)

From the given observation

$50 = k'(40 - 20)$ ...(ii)

from equation (ii) $k' = \dfrac{5}{2}$

from equation (i)

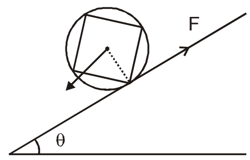
$t_{out} = 48°$

$\ell = 0.52$ mm

36 to 38 $I = \left[\dfrac{M(R\sqrt{2})^2}{12} + M\left(\dfrac{R}{\sqrt{2}}\right)^2\right] \times 4 + mR^2$

$= 20\text{ kgm}^2.$

$(4M + m)g\sin\theta - F = (4M + m)a.$



$F.R. = I\left(\dfrac{a}{R}\right)$

Solving

$a = \dfrac{7g}{24}$

$F = 20a \le \mu\,(4M + m)g\cos 30$

$\mu \ge \dfrac{5}{12\sqrt{3}}$

$\therefore \quad \mu_{min} = \dfrac{5}{12\sqrt{3}}$

39. We have $\theta - \theta_s = (\theta_0 - \theta_s)\,e^{-kt}$

where θ_0 = Initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40 to 38 in 10min, we have

$38 - 30 = (40 - 30)\,e^{-k\,10}$ .... (1)

Let after 10 min, The body temp. be θ

$\theta - 30 = (38 - 30)\,e^{-k\,10}$ .... (2)

$\dfrac{(1)}{(2)}$ gives $\dfrac{8}{\theta - 30} = \dfrac{10}{8}$, $\theta - 30 = 6.4 \quad \Rightarrow \theta = 36.4$ °C

40. Self Explanatory

41. During heating process from 38 to 40 in 10 min. The body will lose heat in the surrounding which will be exactly equal to the heat lost when it cooled from 40 to 38 in 10 min, which is equal to ms $\Delta\theta = 2 \times 2 = 4$ J.

During heating process heat required by the body = m s $\Delta\theta$ = 4 J.

$\therefore$ Total heat required = 8 J.

42. (P) Total energy, total linear momentum, total angular momentum is conserved but mass is converted into energy.

(Q) Total energy, total linear momentum, total angular momentum and total mass is conserved

(R) Total energy, total linear momentum, total angular momentum and total mass is conserved

(S) Total energy, total linear momentum, total angular momentum is conserved but mass is converted into energy.