

Topic-wise Solved Papers Mathematics

CHAPTER

1

Sets

- If A , B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
[2009]
(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$
- Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is: [2012]
- If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$; then S : [2016]
(a) contains exactly two elements.
(b) contains more than two elements.
(c) is an empty set.
(d) contains exactly one element.

Answer Key

1	2	3											
(b)	(b)	(a)											

SOLUTIONS

- (b) Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cup B$
 $\Leftrightarrow x \in A \cup C$ ($\because A \cup B = A \cup C$)
 $\Leftrightarrow x \in C \quad \therefore B = C$
 Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cap B$
 $\Leftrightarrow x \in A \cap C$ ($\because A \cap B = A \cap C$)
 $\Leftrightarrow x \in C \quad \therefore B = C$
- (b) Let $X = \{1, 2, 3, 4, 5\}$
 Total no. of elements = 5
 Each element has 3 options. Either set Y or set Z or none. ($\because Y \cap Z = \phi$)
 So, number of ordered pairs = 3^5
- (a) $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ (1)
 $f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ (2)

$$\text{Adding (1) and (2)} \Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x}$$

Subtracting (1) from (2)

$$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x$$

On adding the above equations

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$$

$$x^2 = 2 \quad \text{or} \quad x = \sqrt{2}, -\sqrt{2}$$