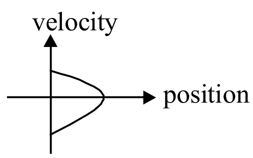
JEE MAIN SOLVED PAPER–2018

PHYSICS

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

(1) 2.5% (2) 3.5%
(3) 4.5% (4) 6%

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1)


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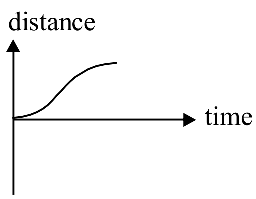

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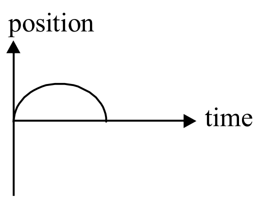

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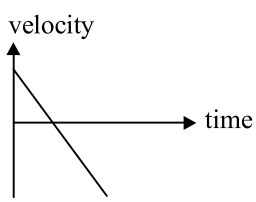
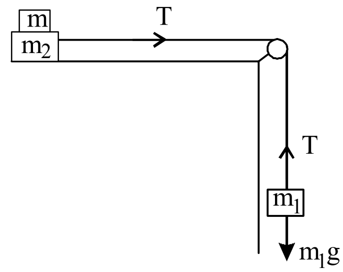


3. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 18.3 kg (2) 27.3 kg
(3) 43.3 kg (4) 10.3 kg

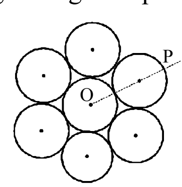
4. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(1) $-\frac{k}{4a^2}$ (2) $\frac{k}{2a^2}$
(3) zero (4) $-\frac{3}{2}\frac{k}{a^2}$

5. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(1) $\frac{v_0}{4}$ (2) $\sqrt{2}v_0$
(3) $\frac{v_0}{2}$ (4) $\frac{v_0}{\sqrt{2}}$

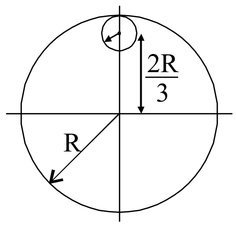
6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$

(3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

7. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is :



(1) $4\,MR^2$ (2) $\frac{40}{9}\,MR^2$

(3) $10\,MR^2$ (4) $\frac{37}{9}MR^2$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then:

(1) $T \propto R^{3/2}$ for any n. (2) $T \propto R^{n/2+1}$

(3) $T \propto R^{(n+1)/2}$ (4) $T \propto R^{n/2}$

9. A solid sphere of radius *r* made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass *m* is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is :

(1) $\frac{Ka}{mg}$ (2) $\frac{Ka}{3mg}$

(3) $\frac{mg}{3Ka}$ (4) $\frac{mg}{Ka}$

10. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) 2.7 kJ

(2) (a) 195 K (b) –2.7 kJ

(3) (a) 189 K (b) –2.7 kJ

(4) (a) 195 K (b) 2.7 kJ

11. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

(1) $2.35\times10^3\ N/m^2$ (2) $4.70\times10^3\ N/m^2$

(3) $2.35\times10^2\ N/m^2$ (4) $4.70\times10^2\ N/m^2$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 6.4 N/m (2) 7.1 N/m

(3) 2.2 N/m (4) 5.5 N/m

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 5 kHz (2) 2.5 kHz

(3) 10 kHz (4) 7.5 kHz

14. Three concentric metal shells A, B and C of respective radii *a*, *b* and *c* (*a* < *b* < *c*) have surface charge densities +σ, –σ and +σ respectively. The potential of shell B is:

(1) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (2) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(3) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (4) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $k = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:

(1) 1.2 n C (2) 0.3 n C
(3) 2.4 n C (4) 0.9 n C

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively:

(1) 50W, 10A (2) $\frac{1000}{\sqrt{2}}$ W, 10A
(3) $\frac{50}{\sqrt{2}}$ W, 0 (4) 50W, 0

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of $10\,\Omega$. The internal resistances of the two batteries are $1\,\Omega$ and $2\,\Omega$ respectively. The voltage across the load lies between:

(1) 11.6 V and 11.7 V (2) 11.5 V and 11.6 V
(3) 11.4 V and 11.5 V (4) 11.7 V and 11.8 V

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is :

(1) $r_e > r_p = r_\alpha$ (2) $r_e < r_p = r_\alpha$
(3) $r_e < r_p < r_\alpha$ (4) $r_e < r_\alpha < r_p$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

(1) 2 (2) $\sqrt{3}$
(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

20. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by:

(1) $\frac{\omega_0 L}{R}$ (2) $\frac{\omega_0 R}{L}$
(3) $\frac{R}{(\omega_0 C)}$ (4) $\frac{CR}{\omega_0}$

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z - ct)\right]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is nonmagnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$ (2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
(3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$ (4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another indentical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 0° (2) 30°
(3) 45° (4) 60°

23. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.)

(1) 25 μm (2) 50 μm
(3) 75 μm (4) 100 μm

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + \dfrac{B}{\lambda_n^2}$ (2) $\Lambda_n \approx A + B\lambda_n$
(3) $\Lambda_n^2 \approx A + B\lambda_n^2$ (4) $\Lambda_n^2 \approx \lambda$

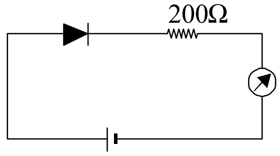
25. If the series limit frequency of the Lyman series is v_1, then the series limit frequency of the P-fund series is :

(1) $25\ \nu_L$ (2) $16\ \nu_L$
(3) $\nu_L/16$ (4) $\nu_L/25$

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively:

(1) $(\cdot 89, \cdot 28)$ (2) $(\cdot 28, \cdot 89)$
(3) (0, 0) (4) (0, 1)

27. The reading of the ammeter for a silicon diode in the given circuit is :



(1) 0 (2) 15 mA
(3) 11.5 mA (4) 13.5 mA

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^3 (2) 2×10^4
(3) 2×10^5 (4) 2×10^6

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1 Ω (2) 1.5 Ω
(3) 2 Ω (4) 2.5 Ω

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 990 Ω (2) 505 Ω
(3) 550 Ω (4) 910 Ω

CHEMISTRY

31. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is :

(1) $C_3H_6O_3$ (2) C_2H_4O
(3) $C_3H_4O_2$ (4) $C_2H_4O_3$

32. Which type of 'defect' has the presence of cations in the interstitial sites?

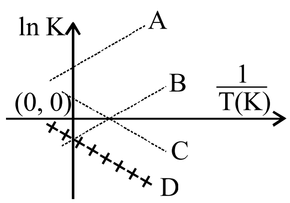
(1) Schottky defect
(2) Vacancy defect
(3) Frenkel defect
(4) Metal deficiency defect

33. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) He_2^{2+} (2) He_2^{+}

(3) H_2^{-} (4) H_2^{2-}

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



(1) A and B (2) B and C
(3) C and D (4) A and D

35. The combustion of benzene (l) gives CO_2 (*g*) and H_2O (*l*). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be : (R= 8.314 JK^{-1} mol^{-1})

(1) 4152.6 (2) –452.46
(3) 3260 (4) –3267.6

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_6]Cl_3$
(2) $[Co(H_2O)_5Cl]Cl_2.H_2O$
(3) $[Co(H_2O)_4Cl_2]Cl.\ 2H_2O$
(4) $[Co(H_2O)_3Cl_3].3H_2O$

37. An aqueous solution contains 0.10 MH_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

(1) 5×10^{-8} (2) 3×10^{-20}
(3) 6×10^{-21} (4) 5×10^{-19}

38. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+} ?

(1) 5×10^{-9} M (2) 2×10^{-9} M
(3) 1.1×10^{-9} M (4) 1.0×10^{-10} M

39. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

(1) 2 (2) 3
(3) 1 (4) 0

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(1) 6.4 hours (2) 0.8 hours
(3) 3.2 hours (4) 1.6 hours

41. The recommended concentration of fluoride ion in drinking water is up to 1ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.\ Ca(OH)_2]$ to :

(1) $[CaF_2]$
(2) $[3(CaF_2).Ca(OH)_2]$
(3) $[3Ca_3(PO_4)_2.CaF_2]$
(4) $[3\{(Ca(OH)_2\}.CaF_2]$

42. Which of the following compounds contain(s) no covalent bond(s)?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, B_2H_6, PH_3 (2) KCl, H_2SO_4
(3) KCl (4) KCl, B_2H_6

43. Which of the following are Lewis acids?

(1) PH_3 and BCl_3 (2) $AlCl_3$ and $SiCl_4$
(3) PH_3 and $SiCl_4$ (4) BCl_3 and $AlCl_3$

44. Total number of lone pair of electrons in I_3^- ion is :

(1) 3 (2) 6
(3) 9 (4) 12

45. Which of the following salts is the most basic in aqueous solution?

(1) $Al(CN)_3$ (2) CH_3COOK
(3) $FeCl_3$ (4) $Pb(CH_3COO)_2$

46. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are respectively:

(1) $(H_2O + O_2)$ and H_2O
(2) $(H_2O + O_2)$ and $(H_2O + OH^-)$
(3) H_2O and $(H_2O + O_2)$
(4) H_2O and $(H_2O + OH^-)$

47. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O)_2(NH_3)]$ respectively are :

(1) +3, +4, and +6 (2) +3, +2, and +4
(3) +3, 0, and +6 (4) +3, 0, and + 4

48. The compound that **does not** produce nitrogen gas by the thermal decomposition is :

(1) $Ba(N_3)_2$ (2) $(NH_4)_2Cr_2O_7$
(3) NH_4NO_2 (4) $(NH_4)_2SO_4$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

(1) Zn (2) Ca
(3) Al (4) Fe

50. Consider the following reaction and statements:

$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.
(II) Two isomers are produced if the reactant complex ion is a *trans* -isomer
(III) Only one isomer is produced if the reactant complex ion is a *trans* -isomer
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

The correct statements are:

(1) (I) and (II) (2) (I) and (III)
(3) (III) and (IV) (4) (II) and (IV)

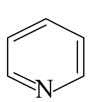
51. Glucose on prolonged heating with HI gives :

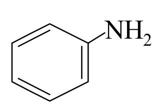
(1) *n*-Hexane (2) 1- Hexene
(3) Hexanoic acid (4) 6-iodohexanal

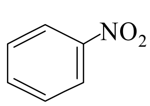
52. The *trans*-alkenes are formed by the reduction of alkynes with:

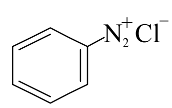
(1) H_2-Pd/C, $BaSO_4$ (2) $NaBH_4$
(3) Na/liq. NH_3 (4) Sn - HCl

53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

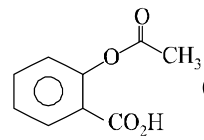
(1) 

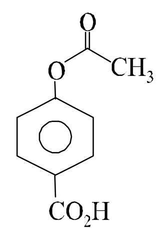
(2) 

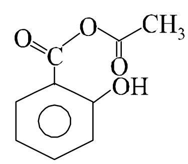
(3) 

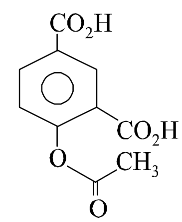
(4) 

54. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

(1) 

(2) 

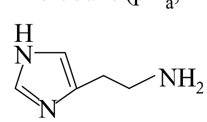
(3) 

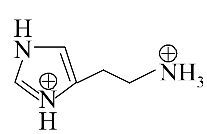
(4) 

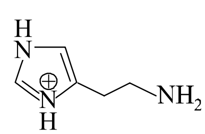
55. An alkali is titrated against an acid with methyl orange as indicator, which of the following in a correct combination?

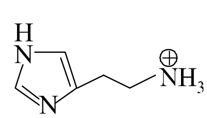
	Base	Acid	End point
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to Pinkish red
(4)	Strong	Strong	Pink to colourless

56. The predominant form of histamine present in human blood is (pK_a, Histidine – 6.0)

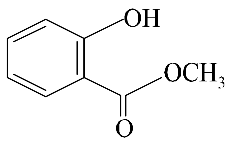
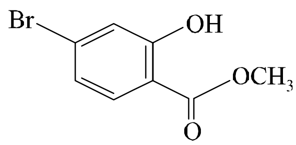
(1) 

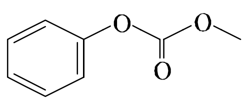
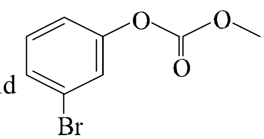
(2) 

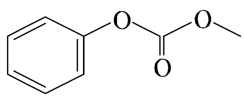
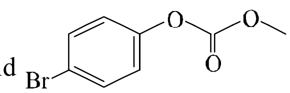
(3) 

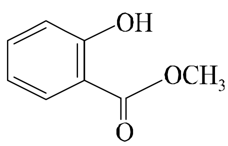
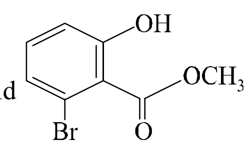
(4) 

57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

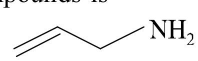
(1)  and 

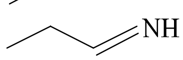
(2)  and 

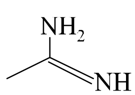
(3)  and 

(4)  and 

58. The increasing order of basicity of the following compounds is

(a) 

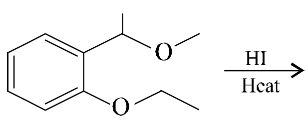
(b) 

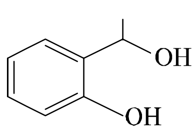
(c) 

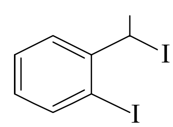
(d) 

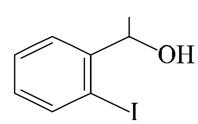
(1) (a) < (b) < (c) < (d)
(2) (b) < (a) < (c) < (d)
(3) (b) < (a) < (d) < (c)
(4) (d) < (b) < (a) < (c)

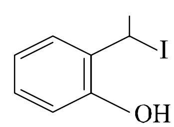
59. The major product formed in the following reaction is :



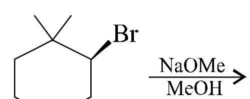
(1) 

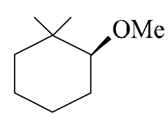
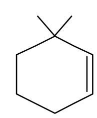
(2) 

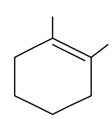
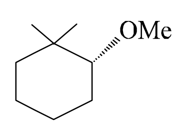
(3) 

(4) 

60. The major product of the following reaction is :



(1) 
(2) 

(3) 
(4) 

MATHEMATICS

61. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to :

(1) $\frac{-1}{3(1+\tan^3 x)} + C$ (2) $\frac{1}{1+\cot^3 x} + C$

(3) $\frac{-1}{1+\cot^3 x} + C$ (4) $\frac{1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

62. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$

(3) $36\sqrt{5}$ (4) $45\sqrt{5}$

63. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is :

(1) 2 (2) 3

(3) $\frac{4}{3}$ (4) $\frac{1}{2}$

64. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b} - 24$, then $|\vec{u}|^2$ is equal to :

(1) 315 (2) 256

(3) 84 (4) 336

65. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) 0 (2) 1

(3) 2 (4) -1

66. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is :

(1) $\frac{1}{2}(\sqrt{3}+1)$ (2) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(3) $\frac{1}{2}(\sqrt{2}-1)$ (4) $\frac{1}{2}(\sqrt{3}-1)$

67. The sum of the co-efficients of all odd degree terms in the expansion of

$(x+\sqrt{x^3-1})^5 + (x-\sqrt{x^3-1})^5, (x > 1)$ is :

(1) 0 (2) 1

(3) 2 (4) -1

68. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that

$\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If

$a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to :

(1) 68 (2) 34

(3) 33 (4) 66

69. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is :

(1) 4 (2) 2

(3) 3 (4) 9

70. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) 50 (2) $100\sqrt{3}$

(3) $50\sqrt{2}$ (4) 100

71. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\};$

$B = \{(a, b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$.

Then :

(1) $A \subset B$

(2) $A \cap B = \phi$ (an empty set)

(3) neither $A \subset B$ nor $B \subset A$

(4) $B \subset A$

72. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) less than 500

(2) at least 500 but less than 750

(3) at least 750 but less than 1000

(4) at least 1000

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is :

(1) -3 (2) $-2\sqrt{2}$

(3) $2\sqrt{2}$ (4) 3

74. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 15.

(2) is equal to 120.

(3) does not exist (in R).

(4) is equal to 0.

75. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is :

(1) $\frac{\pi}{2}$ (2) 4π

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

76. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$

(3) $\frac{3}{4}$ (4) $\frac{3}{10}$

77. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

78. If sum of all the solutions of the equation

$$8\cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) - 1 \quad \text{in}$$

$[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$

(3) $\frac{20}{9}$ (4) $\frac{2}{3}$

79. A straight the through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$

(3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

80. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2 \cdot 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$

If $B - 2A = 100\lambda$, then λ is equal to :

(1) 248 (2) 464

(3) 496 (4) 232

81. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{7}{2}$ (2) 4

(3) $\frac{9}{2}$ (4) 6

82. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$

(3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

83. Let S = $\{t \in R : f(x) = |x - \pi| (e^{|x|} - 1) \sin |x|$ is not differentiable at t}. Then the set S is equal to :

(1) {0} (2) {π}

(3) {0, π} (4) φ (an empty set)

84. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$,

then the ordered pair (A, B) is equal to :

(1) (– 4, 3) (2) (– 4, 5)

(3) (4, 5) (4) (– 4, – 5)

85. The Boolean expression

$\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) p (2) q

(3) ~q (4) ~p

86. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 10 (2) – 30

(3) 30 (4) – 10

87. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S :

(1) contains exactly one element.

(2) contains exactly two elements.

(3) contains exactly four elements.

(4) is an empty set.

88. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 185 (2) 85

(3) 95 (4) 195

89. Let $y - y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{-8}{9\sqrt{3}}\pi^2$ (2) $-\frac{8}{9}\pi^2$

(3) $-\frac{4}{9}\pi^2$ (4) $\frac{4}{9\sqrt{3}}\pi^2$

90. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Hints and Solutions

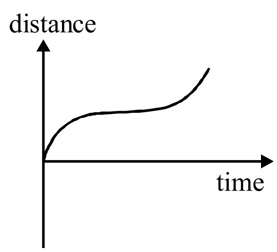
PHYSICS

1. **(3)** $\text{Density}(d) = \frac{\text{Mass}(M)}{\text{Volume}(V)} = \frac{M}{L^3}$

$\therefore$ Error in density, $\frac{\Delta d}{d} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

$= 1.5\% + 3(1\%) = 4.5\%$

2. **(2)** Graphs in option (3) position-time and option (1) velocity-position are corresponding to velocity-time graph option (4) and its distance-time graph is as given below. Hence distance-time graph option (2) is incorrect.



3 **(2)** Given : $m_1 = 5\text{kg}; m_2 = 10\text{kg}; \mu = 0.15$

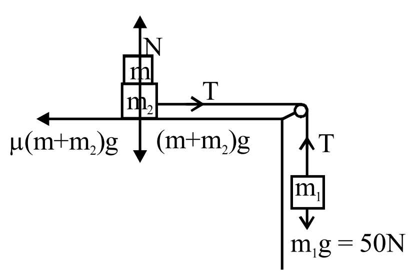
FBD for m_1, $m_1 g - T = m_1 a$

$\Rightarrow 50 - T = 5 \times a$

and, $T - 0.15(m + 10)g = (10 + m)a$

For rest $a = 0$

or, $50 = 0.15(m + 10)\,10$



$\Rightarrow \quad 5 = \frac{3}{20}(m + 10)$

$\frac{100}{3} = m + 10 \quad \therefore \; m = 23.3\text{kg}$; close to option (2)

4. **(3)** $F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$

Since particle is moving in circular path

$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$

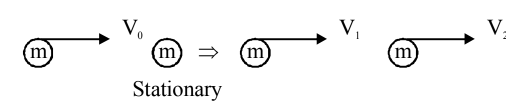
$\therefore \; K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$

Total energy = P.E. + K.E.

$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero}$

$(\because \text{P.E.} = -\frac{K}{2r^2} \text{ given})$

5. **(2)**



$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$

$\Rightarrow \; v_1^2 + v_2^2 = \frac{3}{2}v_0^2$ ....(i)

From momentum conservation

$mv_0 = m(v_1 + v_2)$ ....(ii)

Squarring both sides,

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow \; v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

$2v_1v_2 = -\frac{v_0^2}{2}$

$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$

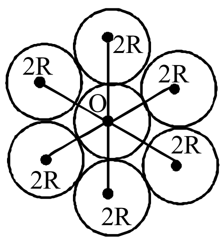
Solving we get relative velocity between the two particles

$v_1 - v_2 = \sqrt{2}v_0$

6. **(4)** Using parallel axes theorem, moment of inertia about 'O'

$I_o = I_{cm} + md^2$

$$= \frac{7MR^2}{2} + 6(M \times (2R)^2) = \frac{55MR^2}{2}$$

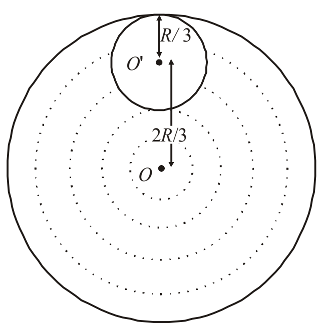


Again, moment of inertia about point P,

$I_p = I_o + md^2$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

7. **(1)** Let σ be the mass per unit area.



The total mass of the disc

$= \sigma \times \pi R^2 = 9M$

The mass of the circular disc cut

$$= \sigma \times \pi\left(\frac{R}{3}\right)^2 = \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it.

Moment of inertia (I_1) of the complete disc

$= \frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

$M.I.$ of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

$\therefore$ $M.I.$ (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

$\therefore$ The total $M.I.$ of the system about an axis passing through O and perpendicular to the plane of the disc is

$I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

8. **(3)** $m\omega^2 R = \text{Force} \propto \frac{1}{R^n}$

$\left(\text{Force} = \frac{mv^2}{R}\right)$

$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}}$ $\qquad \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

Time period $T = \frac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

9. **(3)** Bulk modulus, $K = \dfrac{\text{volumetric stress}}{\text{volumetric strain}}$

$$K = \frac{mg}{a\left(\dfrac{dV}{V}\right)}$$

$$\Rightarrow \frac{dV}{V} = \frac{mg}{Ka} \quad \text{....(i)}$$

volume of sphere , $V = \frac{4}{3}\pi R^3$

Fractional change in volume $\dfrac{dV}{V} = \dfrac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\dfrac{3dr}{r} = \dfrac{mg}{Ka}$

$\therefore \dfrac{dr}{r} = \dfrac{mg}{3Ka}$ (fractional decrement in radius)

10. **(3)** In an adiabatic process

$TV^{\gamma-1}$ = Constant

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \dfrac{5}{3}$

$(300)V^{2/3} = T_2(2V)^{2/3}$

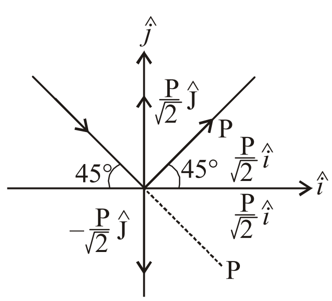
$$\Rightarrow T_2 = \frac{300}{(2)^{2/3}}$$

T_2 = 189 K (final temperature)

Change in internal energy $\Delta U = n\dfrac{f}{2}R\ \Delta T$

$$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7\,\text{kJ}$$

11. **(1)** Change in momentum



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule}$$

$$\Rightarrow I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{F}{A} = \frac{\sqrt{2}P}{A}n \quad (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}}$$

$= 2.35 \times 10^3 \text{N/m}^2$

12. **(2)** As we know, frequency in SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

where m = mass of one atom

Mass of one atom of silver,

$$= \frac{108}{\left(6.02 \times 10^{23}\right)} \times 10^{-3}\,\text{kg}$$

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

Solving we get, spring constant, K = 7.1N/m

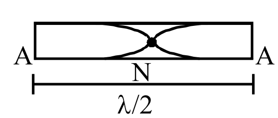
13. **(1)** In solids, Velocity of wave

$$V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$v = 5.85 \times 10^3$ m/sec

Since rod is clamped at middle fundamental wave shape is as follow

$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$



$\lambda = 1.2\text{m}$ ($\because$ L = 60 cm = 0.6m (given)

Using $v = f\lambda$

$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{5.85 \times 10^3}{1.2}$

$= 4.88 \times 10^3 \text{ Hz} \simeq 5 \text{ KHz}$

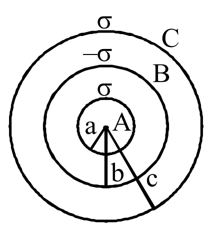
14. (2) Potential outside the shell,

$V_{outside} = \frac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{inside} = \frac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi \in_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

15. (1) Charge on Capacitor, $Q_i = CV$
After inserting dielectric of dielectric constant $= K$ $Q_f = (kC)V$
Induced charges on dielectric

$$Q_{ind} = Q_f - Q_i = KCV - CV$$

$$= (K-1)CV = \left(\frac{5}{3} - 1\right) \times 90\text{pF} \times 2\text{V} = 1.2\text{nc}$$

16. (2) As we know, average power $P_{avg} = V_{rms} I_{rms} \cos\theta$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45°$$

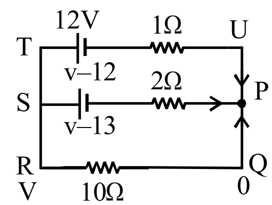
$(\because \theta = 45°)$

$$P_{avg} = \frac{1000}{\sqrt{2}} \text{ watt}$$

Wattless current $I = I_{rms} \sin\theta$

$$= \frac{I_0}{\sqrt{2}} \sin\theta = \frac{20}{\sqrt{2}} \sin 45° = 10\text{A}$$

17. (2)



Using Kirchhoff's law at P we get

$$\frac{V-12}{1} + \frac{V-13}{2} + \frac{V-0}{10} = 0$$

[Let potential at P, Q, U = 0 and at R = V

$$\Rightarrow \frac{V}{1} + \frac{V}{2} + \frac{V}{10} = \frac{12}{1} + \frac{13}{2} + \frac{0}{10}$$

$$\Rightarrow \frac{10+5+1}{10} V = \frac{24+13}{2}$$

$$\Rightarrow V\left(\frac{16}{10}\right) = \frac{37}{2}$$

$$\Rightarrow V = \frac{37 \times 10}{16 \times 2} = \frac{370}{32} = 11.56 \text{ volt}$$

18. (2) As we know, radius of circular path in magnetic field

$$r = \frac{\sqrt{2Km}}{qB}$$

For electron, $r_e = \frac{\sqrt{2Km_e}}{eB}$ ....(i)

For proton, $r_p = \frac{\sqrt{2Km_p}}{eB}$ ....(ii)

For α particle,

$$r_\alpha = \frac{\sqrt{2Km_a}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \text{ ...(iii)}$$

$\therefore \quad r_e < r_p = r_\alpha \qquad (\because m_e < m_p)$

19. **(3)** Magnetic field at the centre of loop,

$$B_1 = \frac{\mu_0 I}{2R}$$

Dipole moment of circular loop is m = IA

$m_1 = I.A = I.\pi R^2$ {R = Radius of the loop}

If moment is doubled (keeping current constant)

R becomes $\sqrt{2}R$

$$m_2 = I.\pi\left(\sqrt{2}R\right)^2 = 2.I\pi R^2 = 2m_1$$

$$B_2 = \frac{\mu_0 I}{2\left(\sqrt{2}R\right)}$$

$$\therefore \frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}} = \sqrt{2}$$

20. **(1)** Quality factor $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

21. **(3)** Velocity of EM wave is given by $v = \frac{1}{\sqrt{\mu \in}}$

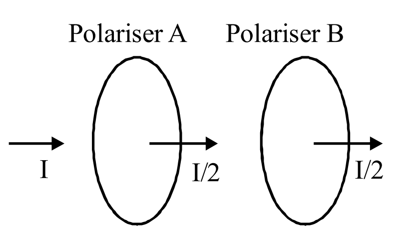
Velocity in air $= \frac{\omega}{k} = C$

Velocity in medium $= \frac{C}{2}$

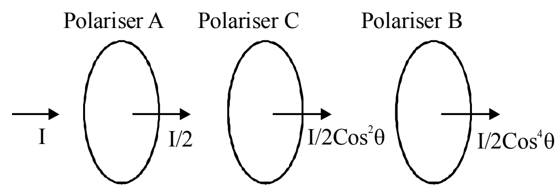
Here, $\mu_1 = \mu_2 = 1$ as medium is non-magnetic

$$\therefore \frac{\frac{1}{\sqrt{\in_{r_1}}}}{\frac{1}{\sqrt{\in_{r_2}}}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \quad \Rightarrow \quad \frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$$

22. **(3)** Axis of transmission of A & B are parallel.



After introducing polariser C between A and B,



$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ or, } \theta = 45°$$

23. **(1)** Angular width of central maxima $= \frac{2\lambda}{d}$

or, $\lambda = \frac{d}{2}$; Fringe width, $\beta = \frac{\lambda \times D}{d'}$

$$10^{-2} = \frac{d}{2} \times \frac{50\times10^{-2}}{d'} = \frac{10^{-6}\times50\times10^{-2}}{2\times d'}$$

Therefore, slit separation distance, d' = 25μm

24. **(1)** Wavelength of emitted photon from n^{th} state to the ground state,

$$\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

Since n is very large, using binomial theorem

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

As we know,

$$\lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

25. **(4)** $h\nu_L = E_\infty - E_1$ ...(i)

$h\nu_f = E_\infty - E_5$ ...(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

$$\text{Eqn (i) / (ii)} \Rightarrow \frac{h\nu_L}{h\nu_f} = \frac{E_1}{E_5}$$

$$\Rightarrow \frac{\nu_L}{\nu_f} = \frac{25}{1} \Rightarrow \nu_f = \frac{\nu_L}{25}$$

26. **(1)** For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13}v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

27. **(3)** Clearly from fig. given in question, Silicon diode is in forward bias.

$\therefore$ Potential barrier across diode

$\Delta V = 0.7$ volts

Current,

$$I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200} = \frac{2.3}{200} = 11.5\text{mA}$$

28. **(3)** If n = no. of channels

10% of 10 GHz = n × 5 KHz

or, $\frac{10}{100} \times 10 \times 10^9 = n \times 5 \times 10^3$

$\Rightarrow n = 2 \times 10^5$

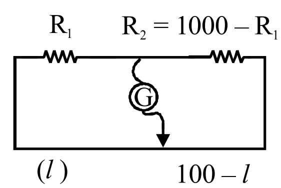
29. **(2)** Using formula, internal resistance,

$$r = \left(\frac{l_1 - l_2}{l_2}\right)s$$

$$= \left(\frac{52 - 40}{40}\right) \times 5 = 1.5\Omega$$

30. **(3)** $R_1 + R_2 = 1000$

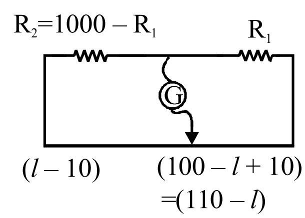
$\Rightarrow R_2 = 1000 - R_1$



On balancing condition

$R_1(100 - l) = (1000 - R_1)l$ ...(i)

On Interchanging resistance balance point shifts left by 10 cm



On balancing condition

$(1000 - R_1)(110 - l) = R_1(l - 10)$

or, $R_1(l - 10) = (1000 - R_1)(110 - l)$ ...(ii)

Dividing eqn (i) by (ii)

$$\frac{100 - l}{l - 10} = \frac{l}{110 - l}$$

$\Rightarrow (100 - l)(110 - l) = l(l - 10)$

$\Rightarrow 11000 - 100l - 110l + l^2 = l^2 - 10l$

$\Rightarrow 11000 = 200l$

or, $l = 55$

Putting the value of 'l' in eqn (i)

$R_1(100-55) = (1000 - R_1)\,55$

$\Rightarrow R_1(45) = (1000 - R_1)\,55$

$\Rightarrow R_1(9) = (1000 - R_1)\,11$

$\Rightarrow 20\,R_1 = 11000$

$\therefore R_1 = 550\,K\Omega$

CHEMISTRY

31. (4)

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12}=0.5$	1
H	1	$\frac{1}{1}=1$	2

So, $X = 1, Y = 2$

Equation for combustion of C_XH_Y

$$C_XH_Y + \left(X+\frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required $= 2\left(X+\frac{Y}{4}\right)$

As mentioned,

$$2\left(X+\frac{Y}{4}\right) = 2Z$$

$$\Rightarrow \left(1+\frac{2}{4}\right) = Z \quad \Rightarrow \quad Z = 1.5$$

$\therefore$ molecule can be written as

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow C_2H_4O_3$

32. (3) In Frenkel defect some of ion (usually cation due to their small size) missing from their normal position and occupies position in interstitial position.

33. (4) Electronic configuration Bond order

He_2^+	$(Z=3)$	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^1}$	$\frac{2-1}{2}=0.5$
H_2^-	$(Z=3)$	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^1}$	$\frac{2-1}{2}=0.5$
H_2^{2-}	$(Z=4)$	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^2}$	$\frac{2-2}{2}=0$
He_2^{2+}	$(Z=2)$	σ_{1s^2}	$\frac{2-0}{2}=1$

Molecule having zero bond order will not be a viable molecule.

34. (1) From thermodynamics

$$\ln K = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

for exothermic reaction,

$\Delta H = -ve$

$$\text{slope} = \frac{-\Delta H^\circ}{R} = +ve$$

So from graph, line should be A & B.

35. (4)

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$$

$= -3263.9 + (-3.71)$

$= -3267.6\ kJ\ mol^{-1}$

36. (4) Number of particles (i)

(1)	$[Co(H_2O)_6]Cl_3$	1
(2)	$[Co(H_2O)_5Cl]Cl_2.H_2O$	3
(3)	$[Co(H_2O)_4Cl_2]Cl.2H_2O$	2
(4)	$[Co(H_2O)_3Cl_3].3H_2O$	4

$\Delta T_f \propto i$; where $\Delta T_f = (T_f - T_f')$

Remember, the greater the no. of particles, the lower will be the freezing point. Compound (d) will have the highest freezing point due to least no of particles.

37. **(2)** In presence of external H^+,

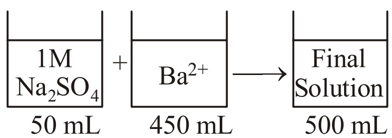
$$H_2S \rightleftharpoons 2H^+ + S^{2-},\ K_{a_1} . K_{a_2} = K_{eq}$$

$$\therefore \quad \frac{\left[H^+\right]^2\left[S^{2-}\right]}{\left[H_2S\right]} = 1\times10^{-7}\times1.2\times10^{-13}$$

$$\frac{\left[0.2\right]^2\left[S^{2-}\right]}{\left[0.1\right]} = 1.2\times10^{-20}$$

$$\left[S^{2-}\right] = 3\times10^{-20}$$

38. **(3)**



Concentration of SO_4^{2-} in $BaSO_4$ solution

$M_1V_1 = M_2V_2$

$1 \times 50 = M_2 \times 500$

$$M_2 = \frac{1}{10}$$

For just precipitation

Ionic product = K_{sp}

$[Ba^{2+}]\ [SO_4{}^{2-}] = K_{sp}(BaSO_4)$

$$[Ba^{2+}] \times \frac{1}{10} = 10^{-10}$$

$[Ba^{2+}] = 10^{-9}$ M in 500 mL solution

Thus $[Ba^{2+}]$ in original solution (450 mL)

$\Rightarrow\ M_1 \times 450 = 10^{-9} \times 500$

[where M_1 = molarity of original solution]

$$M_1 = \frac{500}{450}\times10^{-9} = 1.11\times10^{-9}\ \text{M}$$

39. **(1)** $CH_3CHO \longrightarrow CH_4 + CO$

Generally $r \propto (a-x)m$

$r_1 = 1$ torr s^{-1}, when 5% reacted

$r_2 = 0.5$ torr s^{-1}, when 33 % reacted

$(a - x_1) = 0.95$(unreacted)

$(a - x_2) = 0.67$(unreacted)

$$\frac{r_1}{r_2} = \left[\frac{(a-x_1)}{(a-x_2)}\right]^m ; \frac{1}{0.5} = \left(\frac{0.95}{0.67}\right)^m$$

$$2 = (1.41)^m \Rightarrow 2 = (\sqrt{2})^m$$

$$\Rightarrow m = 2$$

40. **(3)** $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

27.66 g of B_2H_6 (1 mole) requires 3 moles of oxygen (O_2) for complete burning.

Required O_2 (3 moles) is obtained by electrolysis of 6 moles of H_2O

On electrolysis : $6H_2O \longrightarrow 6H_2 + 3O_2$

Number of Faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

$$t = \frac{12\times96500}{100}\text{second}$$

$$= \frac{12\times96500}{100\times3600}\text{hour}$$

= 3.2 hours

41. **(3)** $$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} + \underset{\substack{\text{(drinking}\\\text{water}\\\text{upto 1ppm)}}}{2F^-} \longrightarrow \underset{\substack{\text{Fluorapatite}\\\text{(Harder teeth}\\\text{enamel)}}}{[3Ca_3(PO_4)_2.CaF_2]} + 2OH^-$$

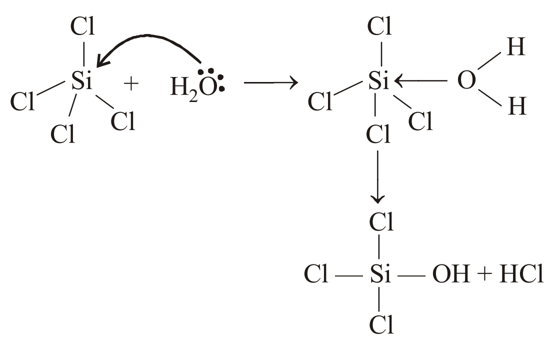
42. **(3)** KCl is an ionic compound while other (PH_3, O_2, B_2H_6, H_2SO_4) are covalent compounds.

43. **(2, 4)** BCl_3 and $AlCl_3$, both have vacant *p*-orbital and incomplete octet thus they behave as Lewis acids.

$SiCl_4$ can accept lone pair of electron in *d*-orbital of silicon hence it can act as Lewis acid.

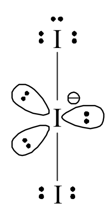
Although the most suitable answer is (c). However, both options (c) and (a) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



i.e., option (a) $AlCl_3$ and $SiCl_4$ is also correct.

44. **(3)**



$\therefore$ Total number of lone pair of electrons is 9.

45. **(2)** CH_3COOK is a salt of weak acid (CH_3COOH) and strong base (KOH).

$FeCl_3$ is a salt of weak base [$Fe(OH)_3$] and strong acid (HCl).

Pb $(CH_3COO)_2$ is a salt of weak base $Pb(OH)_2$ and weak acid (CH_3COOH)

$Al(CN)_3$ is a salt of weak base [Al $(OH)_3$] and weak acid (HCN).

46. **(3)** $[Fe(CN)_6]^{4-} + \frac{1}{2}H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

$[Fe(CN)_6]^{3-} + \frac{1}{2}H_2O_2 + OH^- \longrightarrow [Fe(CN)_6]^{4-} + H_2O + \frac{1}{2}O_2$

47. **(3)** $[Cr(H_2O)_6]Cl_3$

$\Rightarrow x + 6 \times 0 + (-1) \times 3 = 0$

$\Rightarrow x = +3$

$[Cr(C_6H_6)_2]$

$x + 2 \times 0 = 0 ; x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2 \times 1 + x + 2 \times (-1) + 2 \times (-2) + (-2) + 0 = 0$

$x = +6$

48. **(4)**

(1) $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

(2) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$

(3) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(4) $(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

NH_3 is evolved in reaction (c).

49. **(3)**

$$Al + 3H_2O \xrightarrow{NaOH} \underset{(X)}{Al(OH)_3} \downarrow + 3/2H_2(g)$$

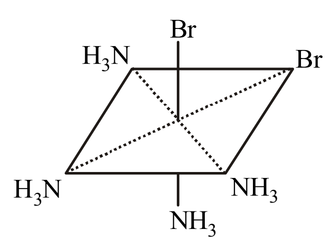
White gelatinous ppt.

$$Al(OH)_3 \xrightarrow{\text{excess of NaOH}} \underset{\text{Soluble}}{Na[Al(OH)_4]}$$

$$\underset{(X)}{2Al(OH)_3} \xrightarrow{\Delta} Al_2O_3 + 3H_2O$$

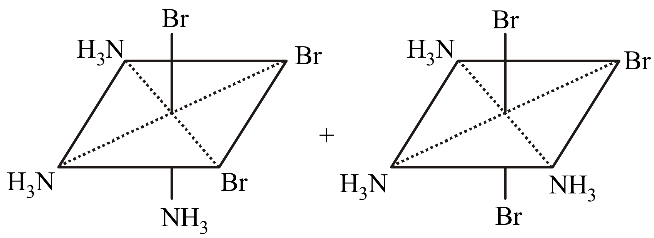
Al_2O_3 is used as adsorbent in chromatography. Thus, metal 'M' is Al.

50. **(2)**



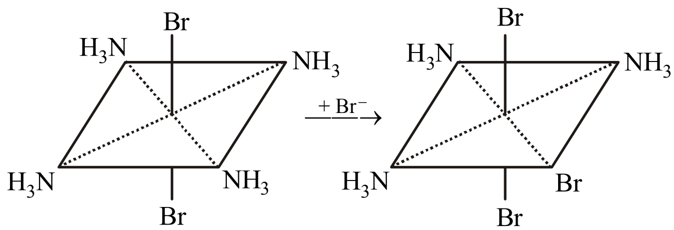
cis-isomer

$+Br^-$



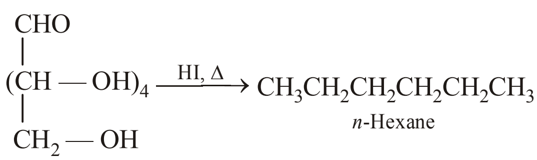
fac– *mer–*

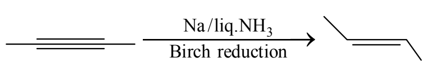
(2 isomers)



trans *mer* (1 isomer)

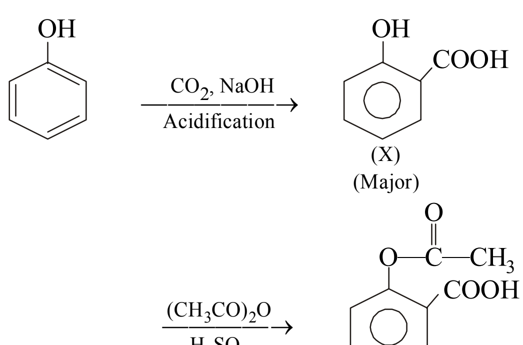
51. (1)



52. (3) 

53. (1) Kjeldahl's method is not applicable for compounds containing nitrogen in nitro and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions.

54. (1)



(X)
(Major)

Acetyl salicylic acid
(Aspirin)

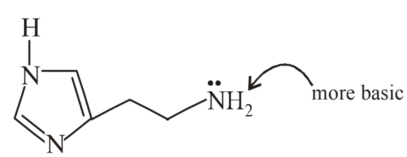
55. (3) pH range for methyl orange is

$\xleftarrow[\text{Pinkish red}]{} 3.9 - 4.5 \xrightarrow[\text{Yellow}]{}$

Generally, weak bases have pH greater than 7. When methyl orange is added to a weak base solution, solution becomes yellow. This solution is then titrated by a strong acid and at the end point pH will be less than 3.1.

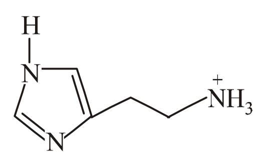
$\therefore$ Solution becomes pinkish red.

56. (4) Structure of histamine

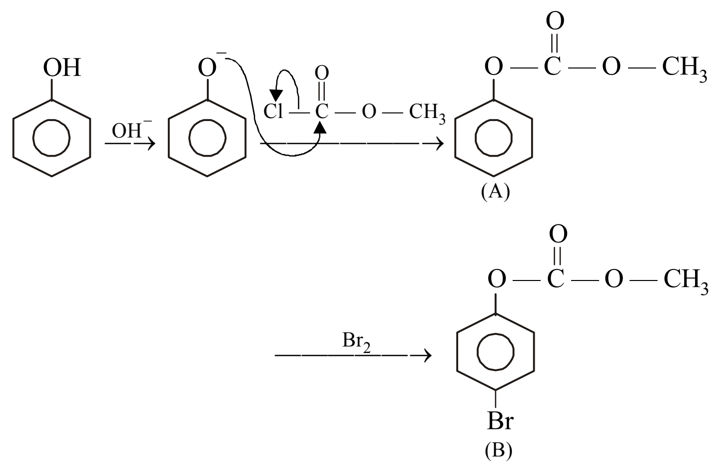


Blood is slightly basic in nature (7.35 pH). At this pH, terminal NH_2 will get protonated due to more basic nature.

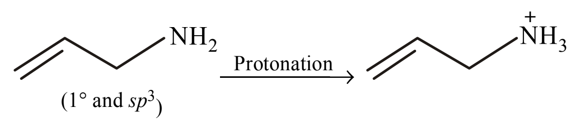
$\therefore$ Predominant structure of histamine is



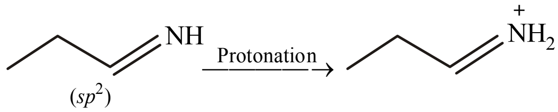
57. (3)



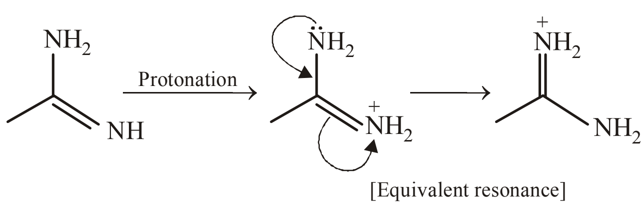
58. (3) (a)



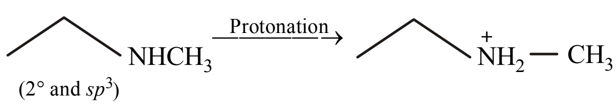
(b)



(c)

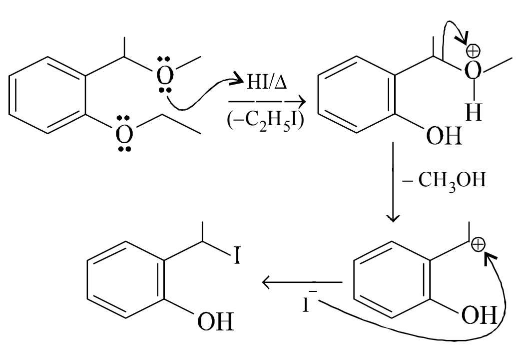


(d)

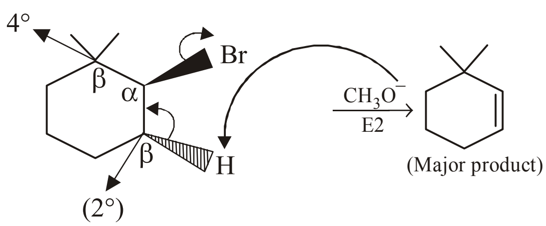


Hence, correct order of basicity will be : b < a < d < c.

59. (4)



60. **(2)** CH_3O^- is a strong base and strong nucleophile, so favourable condition is $S_N2/E2$.
The given alkyl halide is 2° and β carbons are 4° and 2°, so sufficiently hindered, thus E2 dominates over S_N2.
Also, polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated by E2.



MATHEMATICS

61. **(1)** Let I

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{[(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)]^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$$

$$= \int \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2} dx$$

Now, put $(1 + \tan^3 x) = t$

$\Rightarrow$ $3 \tan^2 x \sec^2 x \, dx = dt$

$\therefore$ $I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t} + C = \frac{-1}{3(1 + \tan^3 x)} + C$

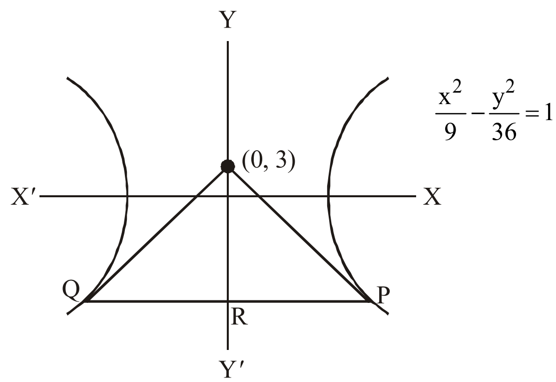
62. **(4)** Here equation of hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

Now, PQ is the chord of contant

$\therefore$ Equation of PQ is : $\frac{x(0)}{9} - \frac{y(3)}{36} = 1$

$\Rightarrow$ $y = -12$



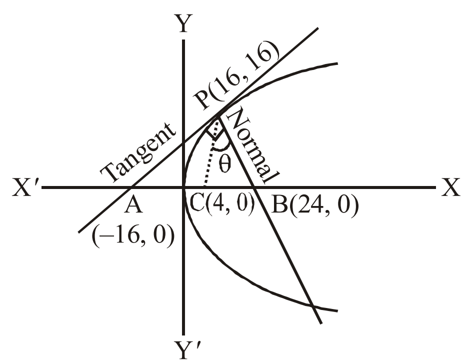
$\therefore$ Area of $\Delta PQT = \frac{1}{2} \times TR \times PQ$

$\because$ $P \equiv (3\sqrt{5}, -12)$ $\therefore$ $TR = 3 + 12 = 15$,

$\therefore$ Area of

$$\Delta PQT = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5} \text{ sq. units}$$

63. **(1)** Equation of tnagent at P(16, 16) is given as:
$x - 2y + 16 = 0$



Slope of PC $(m_1) = \frac{4}{3}$

Slope of PB $(m_2) = -2$

Hence, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right| = \left|\dfrac{\frac{4}{3}+2}{1-\frac{4}{3}\cdot 2}\right|$

$\Rightarrow$ $\tan\theta = 2$

64. **(4)** $\because \vec{u}, \vec{a}$ & $\vec{b}$ are coplanar

$\therefore \vec{u} = \lambda(\vec{a}\times\vec{b})\times\vec{a} \quad = \lambda\{\vec{a}^2.\vec{b} - (\vec{a}.\vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i} + 8\hat{j} + 16\hat{k}\} = \lambda'\{-\hat{i} + 2\hat{j} + 4\hat{k}\}.$

Also, $\vec{u}.\vec{b} = 24 \quad \Rightarrow \lambda' = 4$

$\therefore$ $\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow$ $|\vec{u}|^2 = 336$

65. **(2)** α, β are roots of $x^2 - x + 1 = 0$

$\therefore$ $\alpha = -\omega$ and $\beta = -\omega^2$

where ω is cube root of unity

$\therefore$ $\alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega)^{107}$

$= -[\omega^2 + \omega] = -[-1] = 1$

66. **(4)** Here, $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow$ $(3x - \pi)(6x - \pi) = 0$

$\Rightarrow$ $\alpha = \dfrac{\pi}{6}, \beta = \dfrac{\pi}{3}$

Also, $gof(x) = \cos x$

$\therefore$ Req. area $= \displaystyle\int_{\pi/6}^{\pi/3} \cos x\,dx \quad = \dfrac{\sqrt{3}-1}{2}$

67. **(3)** Since we know that,

$(x + a)^5 + (x - a)^5$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3 \cdot a^2 + {}^5C_4 x \cdot a^4]$

$\therefore$ $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3(x^3 - 1) + {}^5C_4 x(x^3 - 1)^2]$

$\Rightarrow$ $2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

$\therefore$ Sum of coefficients of odd degree terms = 2.

68. **(2)** $\because \displaystyle\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow \dfrac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow$ $a_1 + 24d = 32$ ...(1)

Now, $a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(2)

From eq. (1) & (2) we get; $d = 1$ and $a_1 = 8$

Also, $\displaystyle\sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}[8 + (r-1)1]^2 = 140\text{ m}$

$\Rightarrow$ $\displaystyle\sum_{r=1}^{17}(r+7)^2 = 140\text{ m}$

$\Rightarrow$ $\displaystyle\sum_{r=1}^{17}(r^2 + 14r + 49) = 140\text{ m}$

$\Rightarrow$ $\left(\dfrac{17\times18\times35}{6}\right) + 14\left(\dfrac{17\times18}{2}\right) + (49\times17) = 140$

$\Rightarrow$ $m = 34$

69. **(2)** Given $\displaystyle\sum_{i=1}^{9}(x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$ ...(i)

Also, $\displaystyle\sum_{i=1}^{9}(x_i - 5)^2 = 45$

$\Rightarrow$ $\displaystyle\sum_{i=1}^{9} x_i{}^2 - 10\sum_{i=1}^{9} x_i + 9(25) = 45$ ...(ii)

From (i) and (ii) we get,

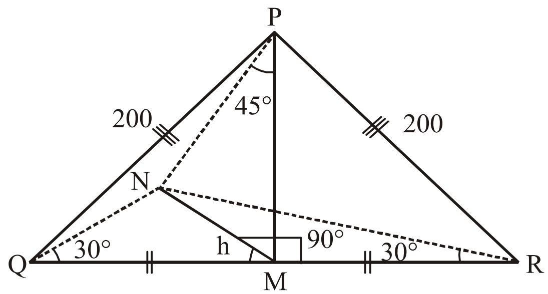
$\displaystyle\sum_{i=1}^{9} x_i^2 = 360$

Since, variance $= \dfrac{\sum x_i{}^2}{9} - \left(\dfrac{\sum x_i}{9}\right)^2$

$= \dfrac{360}{9} - \left(\dfrac{54}{9}\right)^2 = 40 - 36 = 4$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}} = 2$

70. (4)



Let height of tower MN = h

In ΔQMN we have

$$\tan 30° = \frac{MN}{QM}$$

$\therefore$ $QM = \sqrt{3}h = MR$ ...(1)

Now in ΔMNP

$MN = PM$ ...(2)

In ΔPMQ we have :

$$MP = \sqrt{(200)^2 - (\sqrt{3}\,h)^2}$$

$\therefore$ From (2), we get :

$$\sqrt{(200)^2 - (\sqrt{3}\,h)^2} = h \Rightarrow h = 100\text{m}$$

71. (1) $A = \{(a,b) \in R \times R : |a-5| < 1, |b-5| < 1\}$

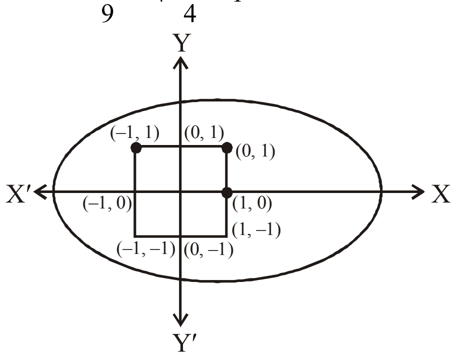
Let $a - 5, = x, b - 5 = y$

Set A contains all points inside $|x| < 1, |y| < 1$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(B-5)^2 \le 36\}$

Set B contains all points inside or on

$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$$



$\therefore$ $(\pm 1, \pm 1)$ lies inside the ellipse.

Hence, $A \subset B$.

72. (4) $\therefore$ Required number of ways = ${}^6C_4 \times {}^3C_1 \times 4!$

$= 15 \times 3 \times 24 = 1080$

73. (3) Here, $h(x) = \dfrac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \dfrac{2}{x - \frac{1}{x}}$

When $x - \frac{1}{x} < 0$

$$\therefore \quad x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \le -2\sqrt{2}$$

Hence, $-2\sqrt{2}$ will be local maximum value of h(x).

When $x - \frac{1}{x} > 0$

$$\therefore \quad x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \ge 2\sqrt{2}$$

Hence, $2\sqrt{2}$ will be local minimum value of h(x).

74. (2) Since, $\lim\limits_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

$$= \lim_{x \to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots + \left\{\frac{15}{x}\right\}\right)$$

$$\because\ 0 \le \left\{\frac{r}{x}\right\} < 1 \quad \Rightarrow \quad 0 \le x\left\{\frac{r}{x}\right\} < x$$

$$\therefore \lim_{x \to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) = \frac{15 \times 16}{2} = 120$$

75. (3) Let, $I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ ...(i)

Using, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we get :

$$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^{-x}}dx \quad ...(ii)$$

Adding (i) and (ii), we get;

$$2I=\int_{-\pi/2}^{\pi/2}\sin^2 x\,dx \quad \Rightarrow$$

$$2I=2\cdot\int_0^{x/2}\sin^2 x\,dx$$

$$\Rightarrow \quad 2I=2\times\frac{\pi}{4}\Rightarrow I=\frac{\pi}{4}$$

76. (1) Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.

Now, in the given bag there are 4 red and 6 black balls.

$$\therefore \quad P(R_1)=\frac{4}{10} \text{ and } P(B_1)=\frac{6}{10}$$

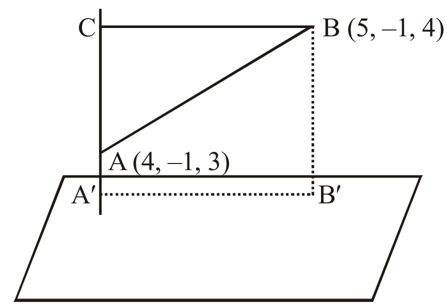
And, $P\left(\frac{R_2}{R_1}\right)=\frac{6}{12}$ and $P\left(\frac{R_2}{B_1}\right)=\frac{4}{12}$

Now, required probability

$$=P(R_1)\times P\left(\frac{R_2}{R_1}\right)+P(B_1)\times P\left(\frac{R_2}{B_1}\right)$$

$$=\left(\frac{4}{10}\times\frac{6}{12}\right)+\left(\frac{6}{10}\times\frac{4}{12}\right)=\frac{2}{5}$$

77. (3)



$$AC=\overrightarrow{AB}.\widehat{AC} \quad =\left(\hat{i}+\hat{k}\right).\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}=\frac{2}{\sqrt{3}}$$

Now,

$$A'B'=BC=\sqrt{AB^2-AC^2}=\sqrt{2-\frac{4}{3}}=\sqrt{\frac{2}{3}}$$

$$\therefore \quad \text{Length of projection}=\sqrt{\frac{2}{3}}$$

78. (1) $\because \; 8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$$\Rightarrow \quad 8\cos x\left(\frac{3}{4}-\frac{1}{2}-\sin^2 x\right)=1$$

$$\Rightarrow \quad 8\cos x\left(\frac{1}{4}-(1-\cos^2 x)\right)=1$$

$$\Rightarrow \quad 8\cos x\left(\frac{1}{4}-1+\cos^2 x\right)=1$$

$$\Rightarrow \quad 8\cos x\left(\cos^2 x-\frac{3}{4}\right)=1$$

$$\Rightarrow \quad 8\left(\frac{4\cos^3 x-3\cos x}{4}\right)=1$$

$$\Rightarrow \quad 2(4\cos^3 x-3\cos x)=1$$

$$\Rightarrow \quad 2\cos 3x=1\Rightarrow \cos 3x=\frac{1}{2}$$

$$\therefore \quad 3x=2n\pi\pm\frac{\pi}{3}, n\in 1$$

$$\Rightarrow \quad x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$$

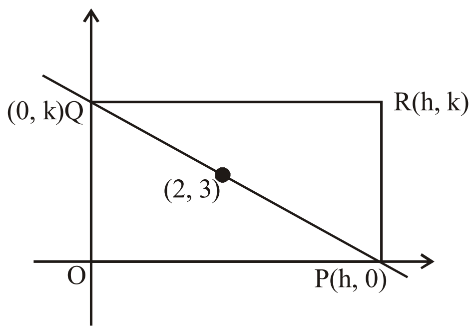
In $x \in [0, \pi] : x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}$, only

Sum of all the solutions of the equation

$$= \left(\frac{1}{9} + \frac{2}{3} + \frac{1}{9} + \frac{2}{3} - \frac{1}{9}\right)\pi = \frac{13}{9}\pi$$

79. **(2)** Equation of PQ is

$$\frac{x}{h} + \frac{y}{k} = 1 \qquad ...(1)$$



Since, (1) passes through the fixed point (2, 3) Then, $\frac{2}{h} + \frac{3}{k} = 1$

Then, the locus of R is $\frac{2}{x} + \frac{3}{y} = 1$ or $3x + 2y = xy$.

80. **(1)** Here, $B - 2A =$

$$\sum_{n=1}^{40} a_n - 2\sum_{n=1}^{20} a_n = \sum_{n=21}^{40} a_n - 2\sum_{n=1}^{20} a_n$$

$$B - 2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + ... + 40^2) - (1^2 + 2.2^2 + 3^2 + 2.4^2 ..... + 20^2)$$

$$= 20\,[22 + 2.24 + 26 + 2.28 + ... + 60]$$

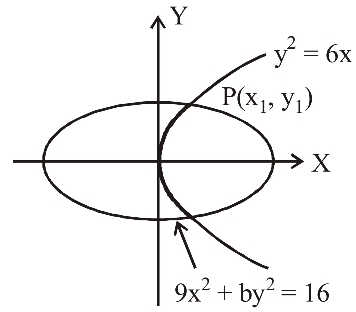
$$= 20\left[\underbrace{(22 + 24 + 26....60)}_{20 \text{ terms}} + \underbrace{(24 + 28 + ... + 60)}_{10 \text{ terms}}\right]$$

$$20\left[\frac{20}{2}(22 + 60) + \frac{10}{2}(24 + 60)\right]$$

$$= 10[20.82 + 10.84]$$

$$= 100[164 + 84] = 100.248$$

81. **(3)** Let curve intersect each other at point $P(x_1, y_1)$



Since, point of intersection is on both the curves, then

$$y_1^2 = 6x_1 \qquad ...(i)$$

and $9x_1^2 + by_1^2 = 16 \qquad ...(ii)$

Now, find the slope of tangent to both the curves at the point of intersection $P(x_1, y_1)$

For slope of curves:

curve (i):

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_1 = \frac{3}{y_1}$$

curve (ii):

and $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_2 = -\frac{9x_1}{by_1}$

Since, both the curves intersect each other at right angle then,

$$m_1 m_2 = -1 \Rightarrow \frac{27x_1}{by_1^2} = 1 \Rightarrow b = 27\frac{x_1}{y_1^2}$$

$\therefore$ from equation (i), $b = 27 \times \frac{1}{6} = \frac{9}{2}$

82. **(2)** Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore AB : BC = 2 : 1$

Now, $AB = \frac{2}{3}AC$

$\Rightarrow \quad AC = \frac{3}{2}AB = \frac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$= \frac{AC}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

83. **(4)** $f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$

Check differentiability of f(x) at $x = \pi$ and $x = 0$

at $x = \pi$:

$$\text{R.H.D.} = \lim_{h\to 0}\frac{|\pi+h-\pi|(e^{|x+h|}-1)\sin|\pi+h|-0}{h}$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{|\pi-h-\pi|(e^{|\pi-h|}-1)\sin|\pi-h|-0}{-h}$$

$= 0$

$\because$ RHD = LHD

Therefore, function is differentiable at $x = \pi$

at $x = 0$:

$$\text{R.H.D.} = \lim_{h\to 0}\frac{|h-\pi|(e^{|h|}-1)\sin|h|-0}{h} = 0$$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{|-h-\pi|(e^{|-h|}-1)\sin|-h|-0}{-h} = 0$$

$\therefore$ RHD = LHD

Therefore, function is differentiable.

at $x = 0$.

Since, the function f(x) is differentiable at all the points including π and 0.

i.e., f(x) is every where differentiable .

Therefore, there is no element in the set S.

$\Rightarrow S = \phi$ (an empty set)

84. **(2)** Here, $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put

$$x = 0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$$

$\Rightarrow A = -4$

$\Rightarrow$

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$$

Now take x common from both the sides

$$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = (B-\frac{4}{x})(1+\frac{4}{x})^2$$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$$

$\therefore$ ordered pair (A, B) is (–4, 5)

85. **(4)** $\sim(p\vee q)\vee(\sim p\wedge q)$

$(\sim p\wedge\sim q)\vee(\sim p\wedge q)$

$\Rightarrow \sim p\wedge(\sim q\vee q)$

$\Rightarrow \sim p\wedge t \equiv \sim p$

86. **(1)** For non zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow k = 11$

Now equations become

$x + 11y + 3z = 0$ ...(1)

$3x + 11y - 2z = 0$ ...(2)

$2x + 4y - 3z = 0$ ...(3)

Adding equations (1) & (3) we get

$3x + 15y = 0$

$\Rightarrow x = -5y$

Now put $x = -5y$ in equation (1), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

$$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$$

87. (2) **Case-I:** $x \in [0,9]$

$$2(3-\sqrt{x})+x-6\sqrt{x}+6=0$$

$$\Rightarrow x-8\sqrt{x}+12=0 \Rightarrow \sqrt{x}=4,2$$
$$\Rightarrow x=16,4$$

Since $x \in [0,9]$

$\therefore$ $x=4$

Case-II: $x \in [9,\infty]$

$$2(\sqrt{x}-3)+x-6\sqrt{x}+6=0$$

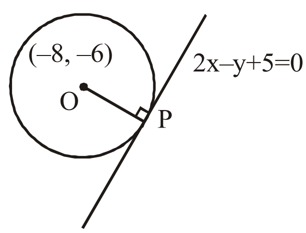
$$\Rightarrow x-4\sqrt{x}=0 \Rightarrow x=16,0$$

Since $x \in [9,\infty]$

$\therefore$ $x=16$

Hence, $x = 4$ & 16

88. (3) Equation of tangent at (1, 7) to $x^2 = y - 6$ is
$2x - y + 5 = 0.$



Now, perpendicular from centre O(–8, –6) to
$2x - y + 5 = 0$ should be equal to radius of the circle

$$\therefore \left|\frac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-C}$$

$$\Rightarrow \sqrt{5} = \sqrt{100-c}$$
$$\Rightarrow c = 95$$

89. (2) Consider the given differential equation the
$\sin x dy + y \cos x dx = 4x dx$

$\Rightarrow$ $d(y.\sin x) = 4x dx$

Integrate both sides

$\Rightarrow$ $y.\sin x = 2x^2 + C$ ...(1)

$$\Rightarrow \quad y(x) = \frac{2x^2}{\sin x} + c \text{ ...(2)}$$

$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$

$$\Rightarrow 0 = \frac{\pi^2}{2} + C$$

$$\Rightarrow C = -\frac{\pi^2}{2}$$

Now, put the value of C in (1)

Then, $y \sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$$\therefore y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$$

90. (1) Equation of plane passing through the line of intersection of first two planes is:

$$(2x-2y+3z-2)+\lambda(x-y+z+1)=0$$

or

$$x(\lambda+2)-y(2+\lambda)+z(\lambda+3)+(\lambda-2)=0 \quad \text{...(i)}$$

is having infinite number of solution with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$, then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \lambda = 5$$

Now put $\lambda = 5$ in (i), we get

$7x - 7y + 8z + 3 = 0$

Now perpendicular distance from (0, 0, 0) to the place containing L_1 and L_2 =

$$\frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$$