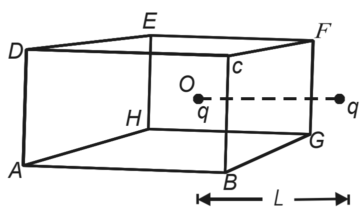
CHAPTER 15

Electric Charges and Fields

1. A charged particle q is placed at the centre O of cube of length L ($A\ B\ C\ D\ E\ F\ G\ H$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is **[2002]**



(a) $q/4\pi\epsilon_0 L$ (b) zero

(c) $q/2\pi\epsilon_0 L$ (d) $q/3\pi\epsilon_0 L$

2. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is **[2002]**

(a) $Q/2$ (b) $-Q/2$

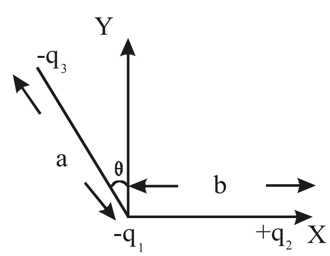
(c) $Q/4$ (d) $-Q/4$

3. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[2003]**

(a) $(\phi_2 - \phi_1)\varepsilon_o$ (b) $(\phi_1 + \phi_2)/\varepsilon_o$

(c) $(\phi_2 - \phi_1)/\varepsilon_o$ (d) $(\phi_1 + \phi_2)\varepsilon_o$

4. Three charges $-q_1$, $+q_2$ and $-q_3$ are place as shown in the figure. The x - component of the force on $-q_1$ is proportional to **[2003]**



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

5. Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that B but **uncharged** is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[2004]**

(a) $F/8$ (b) $3F/4$

(c) $F/4$ (d) $3F/8$

6. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is **[2004]**

(a) $-\frac{Q}{2}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{4}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

7. A charged oil drop is suspended in a uniform field of 3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass of the charge $= 9.9\times10^{-15}$ kg and $g = 10$ m/s^2) **[2004]**

(a) 1.6×10^{-18} C (b) 3.2×10^{-18} C

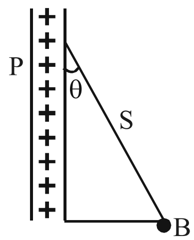
(c) 3.3×10^{-18} C (d) 4.8×10^{-18} C

8. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is **[2005]**

(a) $\frac{L}{4}$ (b) $2L$

(c) $4L$ (d) $8L$

9. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to **[2005]**



(a) $\cot\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\sin\theta$

10. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[2006]**

(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

11. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **[2006]**

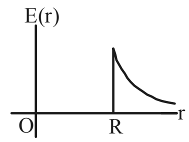
(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

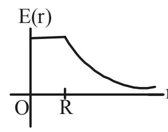
12. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio **[2007]**

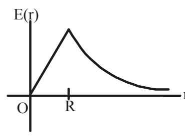
$\dfrac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

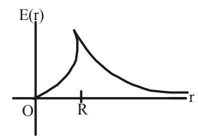
(a) g_M / g_E (b) 1
(c) 0 (d) g_E / g_M

13. A thin spherical shell of radus R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[2008]**

(a) 

(b) 

(c) 

(d) 

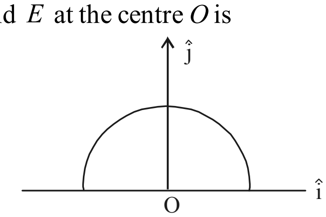
14. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[2009]**

(a) -1 (b) 1
(c) $-\dfrac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

15. Let $\rho(r) = \dfrac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'P' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is : **[2009]**

(a) $\dfrac{Q}{4\pi \epsilon_0 r_1^2}$ (b) $\dfrac{Q r_1^2}{4\pi \epsilon_0 R^4}$
(c) $\dfrac{Q r_1^2}{3\pi \epsilon_0 R^4}$ (d) 0

16. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is **[2010]**



(a) $\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{j}$ (b) $-\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{j}$
(c) $-\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{j}$ (d) $\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{j}$

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$

for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by **[2010]**

(a) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

18. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **[2011]**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$

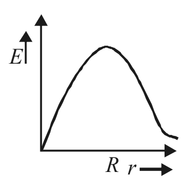
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

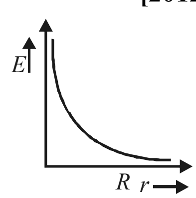
19. The potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre and a, b are constants. Then the charge density inside the ball is: **[2011]**

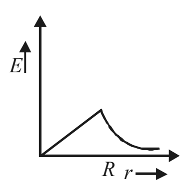
(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$

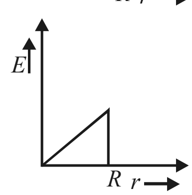
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

20. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre, The graph which would correspond to the above will be: **[2012]**

(a) 

(b) 

(c) 

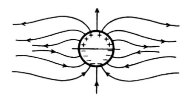
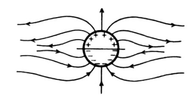
(d) 

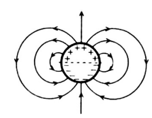
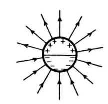
21. Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y <<a) along the y-axis, the net force acting on the particle is proportional to **[2013]**

(a) y (b) –y

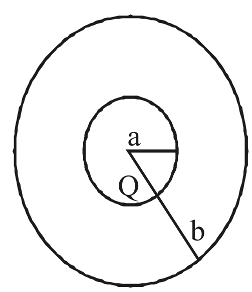
(c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

22. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge - σ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) **[2015]**

(a)  (b) 

(c)  (d) 

23. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[2016]**



(a) $\frac{2Q}{\pi\left(a^2-b^2\right)}$ (b) $\frac{2Q}{\pi a^2}$

(c) $\frac{Q}{2\pi a^2}$ (d) $\frac{Q}{2\pi\left(b^2-a^2\right)}$

24. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{i}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T_2} = -\vec{T_1}$. The angle θ is : **[2017]**

(a) 60° (b) 90°

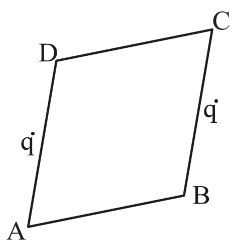
(c) 30° (d) 45°

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(a)	(b)	(d)	(b)	(c)	(b)	(c)	(c)	(c)	(b)	(a)	(d)	(b)
16	17	18	19	20	21	22	23	24						
(c)	(a)	(d)	(c)	(c)	(a)	(c)	(c)	(a)						

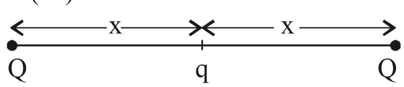
SOLUTIONS

1. **(b)** Both the charges are identical and placed symmetrically about $ABCD$. The flux crossing $ABCD$ due to each charge is $\frac{1}{6}\left[\frac{q}{\epsilon_0}\right]$ but in opposite directions. Therefore the resultant is zero.



2. **(d)** For equilibrium of charge Q

$$k\frac{Q \times Q}{(2x)^2} + k\frac{Qq}{x^2} = 0$$



$$\Rightarrow \quad q = -\frac{Q}{4}$$

3. **(a)** The flux entering an enclosed surface is taken as negative and the flux leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface $= \phi_2 - \phi_1$

$\therefore$ the charge enclosed in the surface by Gauss's law is $q = \epsilon_0 (\phi_2 - \phi_1)$

4. **(b)** Force on charge q_1 due to q_2 is

$$F_{12} = k\frac{q_1 q_2}{b^2}$$

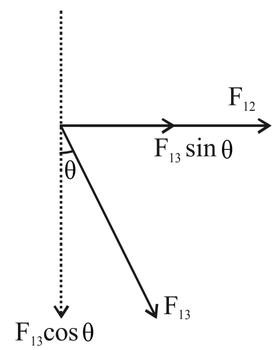
Force on charge q_1 due to q_3 is

$$F_{13} = k\frac{q_1 q_3}{a^2}$$

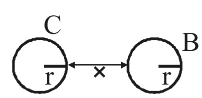
The X - component of the force (F_x) on q_1 *is* $F_{12} + F_{13}\sin\theta$

$$\therefore \quad F_x = k\frac{q_1 q_2}{b^2} + k\frac{q_1 q_2}{a^2}\sin\theta$$

$$\therefore \quad F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$



5. **(d)**



$$F \propto \frac{Q_B Q_C}{x^2}$$

x is distance between the spheres. After first operation charge on B is halved i.e $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$.

Now it is touched to C, charge then equally distributes them selves to make potential same, hence charge on C becomes $\left(Q + \frac{Q}{2}\right)\frac{1}{2} = \frac{3Q}{4}$.

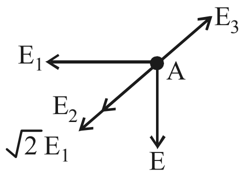
$$\therefore F_{new} \propto \frac{Q_C' Q_B'}{x^2} = \frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2} = \frac{3}{8}\frac{Q^2}{x^2}$$

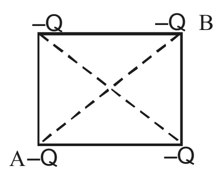
or $\quad F_{new} = \frac{3}{8}F$

6. **(b)** Net field at A should be zero

$$\sqrt{2}\, E_1 + E_2 = E_3$$

$$\therefore \quad \frac{kQ \times \sqrt{2}}{a^2} + \frac{kQ}{(\sqrt{2}\,a)^2} = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$$





$$\Rightarrow \frac{Q\sqrt{2}}{1}+\frac{Q}{2}=2q \Rightarrow q=\frac{Q}{4}(2\sqrt{2}+1)$$

7. (c) At equilibrium, electric force on drop balances weight of drop.

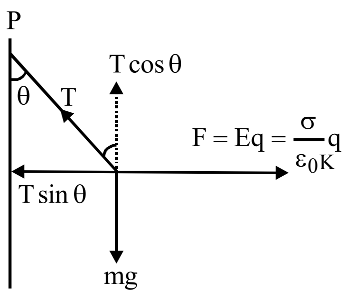
$$qE = mg \Rightarrow q = \frac{mg}{E}$$

$$q = \frac{9.9\times10^{-15}\times10}{3\times10^{4}} = 3.3\times10^{-18}\text{C}$$

8. (b) $\dfrac{-K2q}{(x-L)^2}+\dfrac{K8q}{x^2}=0 \Rightarrow \dfrac{1}{(x-L)^2}=\dfrac{4}{x^2}$

or $\dfrac{1}{x-L}=\dfrac{2}{x} \Rightarrow x = 2x-2L$ or $x = 2L$

9. (c)



$T\sin\theta = \dfrac{\sigma}{\varepsilon_0 K}.q$ .... (i)

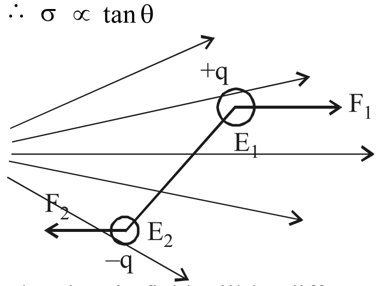
$T\cos\theta = mg$ .... (ii)

Dividing (i) by (ii),

$$\tan\theta = \frac{\sigma q}{\varepsilon_0 K.mg}$$

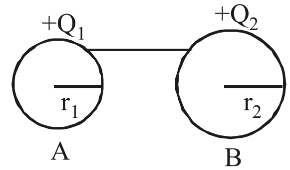
$\therefore\ \sigma \propto \tan\theta$

10. (c)



The electric field will be different at the location of the two charges. Therefore the two forces will be unequal. This will result in a force as well as torque.

11. (c)



After connection, $V_1 = V_2$

$$\Rightarrow k\frac{Q_1}{r_1}=k\frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{r_1}=\frac{Q_2}{r_2}$$

The ratio of electric fields

$$\frac{E_1}{E_2}=\frac{k\frac{Q_1}{r_1^2}}{k\frac{Q_2}{r_2^2}} \Rightarrow \frac{E_1}{E_2}=\frac{Q_1}{r_1^2}\times\frac{r_2^2}{Q_2}$$

$$\Rightarrow \frac{E_1}{E_2}=\frac{r_1\times r_2^2}{r_1^2\times r_2} \Rightarrow \frac{E_1}{E_2}=\frac{r_2}{r_1}=\frac{2}{1}$$

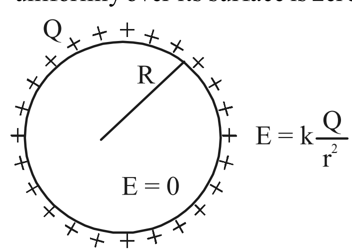
Since the distance between the spheres is large as compared to their diameters, the induced effects may be ignored.

12. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.

So, electronic charge on earth
= electronic charge on moon

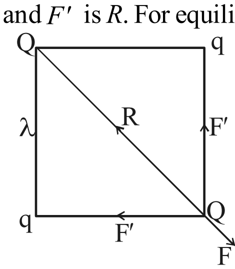
$\therefore$ Required ratio = 1.

13. (a) The electric field inside a thin spherical shell of radius R has charge Q spread uniformly over its surface is zero.



Outside the shell the electric field is $E = k\dfrac{Q}{r^2}$. These characteristics are represented by graph (a).

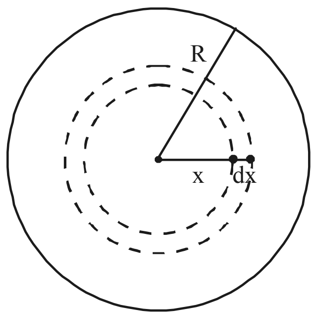
14. (d) Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. The resultant of F' and F' is R. For equilibrium



$\vec{R}+\vec{F}=0 \qquad \sqrt{2}\,F'=-F$

$$\sqrt{2}\times k\frac{Qq}{\ell^2}=-k\frac{Q^2}{(\sqrt{2}\,\ell)^2} \Rightarrow \frac{Q}{q}=-2\sqrt{2}$$

15. (b)



Let us consider a spherical shell of thickness dx and radius x. The volume of this spherical shell $= 4\pi x^2 dx$. The charge enclosed within shell

$$=\left[\frac{Qx}{\pi R^4}\right][4\pi x^2 dx]=\frac{4Q}{R^4}x^3 dx$$

The charge enclosed in a sphere of radius r_1 is

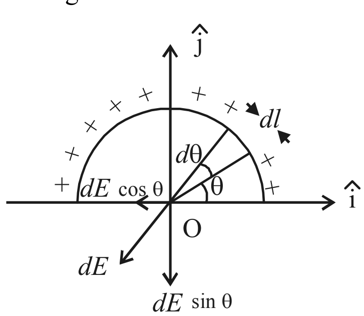
$$=\frac{4Q}{R^4}\int_0^{r_1}x^3 dx=\frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1}=\frac{Q}{R^4}r_1^4$$

$\therefore$ The electric field at point P inside the sphere at a distance r_1 from the centre of the sphere is

$$E=\frac{1}{4\pi\in_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2}$$

$$=\frac{1}{4\pi\in_0}\frac{Q}{R^4}r_1^2$$

16. (c) Let us consider a differential element dl. charge on this element.



$$dq=\left(\frac{q}{\pi r}\right)dl$$

$$=\frac{q}{\pi r}(rd\theta) \qquad (\because dl=rd\theta)$$

$$=\left(\frac{q}{\pi}\right)d\theta$$

Electric field at O due to dq is

$$dE=\frac{1}{4\pi\in_0}.\frac{dq}{r^2}=\frac{1}{4\pi\in_0}.\frac{q}{\pi r^2}d\theta$$

The component $dE\cos\theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant of the component $dE\sin\theta$ only.

$$\therefore\ E=\int dE\sin\theta=\int_0^{\pi}\frac{q}{4\pi^2 r^2\in_0}\sin\theta d\theta$$

$$=\frac{q}{4\pi^2 r^2\in_0}[-\cos\theta]_0^{\pi}$$

$$=\frac{q}{4\pi^2 r^2\in_0}(+1+1)=\frac{q}{2\pi^2 r^2\in_0}$$

The direction of E is towards negative y-axis.

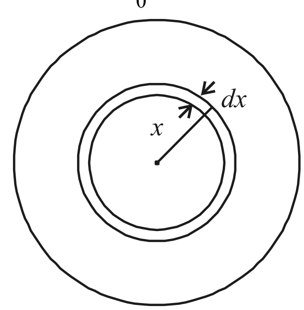
$$\therefore\ \vec{E}=-\frac{q}{2\pi^2 r^2\in_0}\hat{j}$$

17. (a) Let us consider a spherical shell of radius x and thickness dx.
Charge on this shell

$$dq=\rho.4\pi x^2 dx=\rho_0\left(\frac{5}{4}-\frac{x}{R}\right).4\pi x^2 dx$$

$\therefore$ Total charge in the spherical region from centre to r ($r<R$) is

$$q=\int dq=4\pi\rho_0\int_0^r\left(\frac{5}{4}-\frac{x}{R}\right)x^2 dx$$



$$=4\pi\rho_0\left[\frac{5}{4}.\frac{r^3}{3}-\frac{1}{R}.\frac{r^4}{4}\right]=\pi\rho_0 r^3\left(\frac{5}{3}-\frac{r}{R}\right)$$

$\therefore$ Electric field at r,

$$E=\frac{1}{4\pi\in_0}.\frac{q}{r^2}$$

$$=\frac{1}{4\pi\in_0}.\frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3}-\frac{r}{R}\right)=\frac{\rho_0 r}{4\in_0}\left(\frac{5}{3}-\frac{r}{R}\right)$$

18. **(d)** At any instant

$T\cos\theta = mg$ ....(i)

$T\sin\theta = F_e$ ....(ii)

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{F_e}{mg}$

$\Rightarrow F_e = mg\tan\theta$

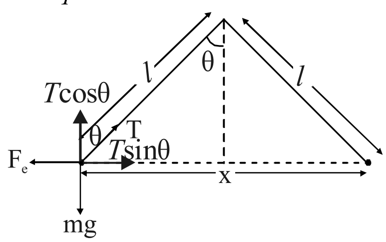
$\Rightarrow \dfrac{kq^2}{x^2} = mg\tan\theta$

$\Rightarrow q^2 \propto x^2\tan\theta$

$\sin\theta = \dfrac{x}{2l}$

For small θ, $\sin\theta \approx \tan\theta$

$\therefore q^2 \propto x^3$



$\Rightarrow q\dfrac{dq}{dt} \propto x^2\dfrac{dx}{dt}$

$\therefore \dfrac{dq}{dt} = \text{const.}$

$\therefore q \propto x^2.v$

$\Rightarrow x^{\frac{3}{2}} \alpha x^2.v$

$\Rightarrow v \propto x^{-1/2}$ $[\because q^2 \propto x^3]$

19. **(c)** Electric field

$E = -\dfrac{d\phi}{dr} = -2ar$ ....(i)

By Gauss's theorem

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ ....(ii)

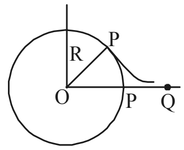
From (i) and (ii),

$Q = -8\pi\varepsilon_0 ar^3$

$\Rightarrow dq = -24\pi\varepsilon_0 ar^2\,dr$

Charge density, $\rho = \dfrac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

20. **(c)** $\vec{E}$ inside the charged sphere



$\vec{E}_{in} = 0$ …(i)

$\vec{E}$ on the surface of the charged sphere

$\vec{E}_s = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^2}$ i.e., $\vec{E}_s \propto \dfrac{1}{R^2}\hat{n}$ …(ii)

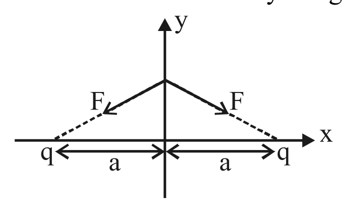
$\vec{E}$ on any point away from the uniformly charged sphere is given

$\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}\hat{n}$

$\vec{E} \propto \dfrac{1}{r^2}\hat{n}$ …(iii)

$\because R$ is the radius of the sphere, which is constant, thus $\vec{E}$ is maximum and constant at the surface of the sphere. But decreases on moving away from the uniformly charged sphere.

21. **(a)**



$\Rightarrow$ F sin θ ← → F sin θ $\Rightarrow$ F_{net}

2Fcos θ

$= 2F\cos\theta$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3}$$

So, $F \propto y$

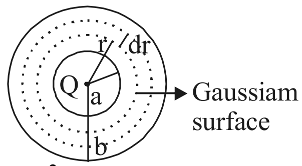
22. **(c)** Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

23. **(c)** Applying Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$$

$$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi ar^2 - 2\pi Aa^2}{\epsilon_0}$$

$$\rho = \frac{dr}{dv}$$



$Q = \rho 4\pi r^2$

$$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A\,[r^2 - a^2]$$

$$E = \frac{1}{4\pi \epsilon_0}\left[\frac{Q - 2\pi A a^2}{r^2} + 2\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$$\therefore \ A = \frac{Q}{2\pi a^2}$$

24. **(a)** T = PE sinθ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\ \hat{i} + p\sin\theta\ \hat{j}$

$\vec{E}_1 = E\vec{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}\,E_1\hat{j}$

$\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}\,E_1\hat{j}$

$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

From eqns. (*i*) and (*ii*)

$pE\sin\theta = \sqrt{3}\,pE\cos\theta$

$\tan\theta = \sqrt{3} \quad \therefore \quad \theta = 60°$