CHAPTER 16

Electrostatic Potential and Capacitance

1. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is **[2002]**

(a) 0.1 V (b) 8V

(c) 2V (d) 0.5V.

2. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to **[2002]**

(a) CV (b) $\frac{1}{2}nCV^2$

(c) CV^2 (d) $\frac{1}{2n}CV^2$

3. Capacitance (in F) of a spherical conductor with radius 1 m is **[2002]**

(a) 1.1×10^{-10} (b) 10^{-6}

(c) 9×10^{-9} (d) 10^{-3}

4. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[2003]**

(a) decreases

(b) remains unchanged

(c) becomes infinite

(d) increases

5. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P, a distance $\frac{R}{2}$ from the centre of the shell is **[2003]**

(a) $\frac{2Q}{4\pi\varepsilon_o R}$ (b) $\frac{2Q}{4\pi\varepsilon_o R}-\frac{2q}{4\pi\varepsilon_o R}$

(c) $\frac{2Q}{4\pi\varepsilon_o R}+\frac{q}{4\pi\varepsilon_o R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_o R}$

6. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is **[2003]**

(a) 16×10^{-32} joule

(b) 3.1×10^{-26} joule

(c) 4×10^{-10} joule

(d) 32×10^{-32} joule

7. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is **[2005]**

(a) $\frac{q}{2\pi\in_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(b) $\frac{qR}{4\pi\in_0 d^2}$

(c) $\frac{q}{4\pi\in_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(d) zero

8. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C' then the resultant capacitance is **[2005]**

(a) $(n+1)C$ (b) $(n-1)C$

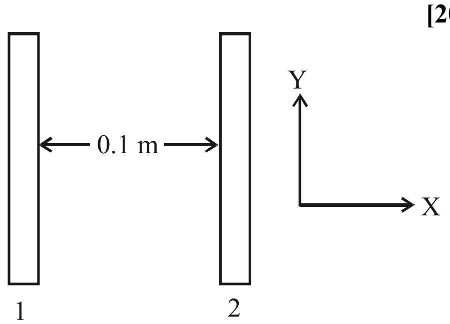
(c) nC (d) C

9. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is **[2005]**

(a) $\frac{mC\Delta T}{s}$ (b) $\sqrt{\frac{2mC\Delta T}{s}}$

(c) $\sqrt{\frac{2ms\Delta T}{C}}$ (d) $\frac{ms\Delta T}{C}$

10. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg) **[2006]**



(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s

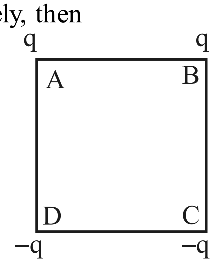
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s

11. An electric charge $10^{-3}\,\mu$ C is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be **[2007]**

(a) 4.5 volts (b) 9 volts

(c) Zero (d) 2 volt

12. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **[2007]**



(a) $\vec{E}$ changes, V remains unchanged

(b) $\vec{E}$ remains unchanged, V changes

(c) both $\vec{E}$ and V change

(d) $\vec{E}$ and V remain unchanged

13. The potential at a point x (measured in μ m) due to some charges situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volt. The electric field E at $x = 4\ \mu$ m is given by **[2007]**

(a) (10/9) volt/μ m and in the +ve x direction

(b) (5/3) volt/μ m and in the –ve x direction

(c) (5/3) volt/μ m and in the +ve x direction

(d) (10/9) volt/μ m and in the –ve x direction

14. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[2007]**

(a) zero (b) $\frac{1}{2}(K-1)\,CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

15. A parallel plate capacitor with air between the plates has capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **[2008]**

(a) 1.8 pF (b) 45 pF

(c) 40.5 pF (d) 20.25 pF

16. Two points P and Q are maintained at the potentials of 10 V and – 4 V, respectively. The work done in moving 100 electrons from P to Q is: **[2009]**

(a) 9.60×10^{-17} J (b) -2.24×10^{-16} J

(c) 2.24×10^{-16} J (d) -9.60×10^{-17} J

17. Two positive charges of magnitude 'q' are placed, at the ends of a side (side 1) of a square of side '$2a$'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[2011 RS]**

(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

18. This questions has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero. **[2012]**

Statement -1 When a charge q is taken from the centre to the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement -2 The electric field at a distance r $(r<R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of statement 1.
(b) Statement 1 is true Statement 2 is false.
(c) Statement 1 is false Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1

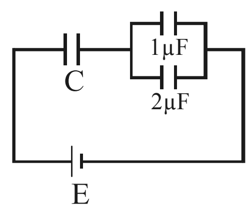
19. Assume that an electric field $\vec{E}=30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is: **[2014]**

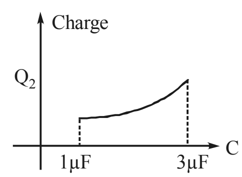
(a) 120 J/C (b) –120 J/C
(c) –80 J/C (d) 80 J/C

20. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[2014]**

(a) $6\times10^{-7}\ C/m^2$ (b) $3\times10^{-7}\ C/m^2$
(c) $3\times10^{4}\ C/m^2$ (d) $6\times10^{4}\ C/m^2$

21. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by: (*figures are drawn schematically and are not to scale*) **[2015]**



(a)


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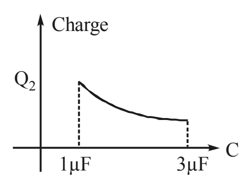

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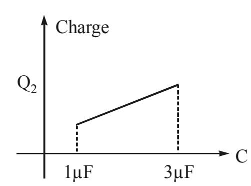
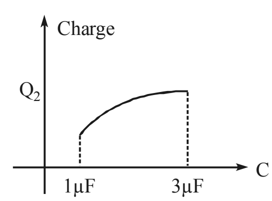


(d)


22. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[2015]**

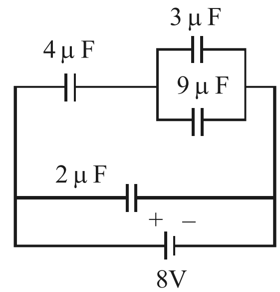
(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(b) $2R < R_4$

(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

23. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μ F and 9 μ F capacitors), at a point distance 30 m from it, would equal : **[2016]**



(a) 420 N/C (b) 480 N/C

(c) 240 N/C (d) 360 N/C

24. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **[2017]**

(a) 24 (b) 32

(c) 2 (d) 16

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(a)	(c)	(a)	(a)	(a)	(c)
16	17	18	19	20	21	22	23	24						
(c)	(d)	(c)	(c)	(a)	(d)	(a)	(a)	(b)						

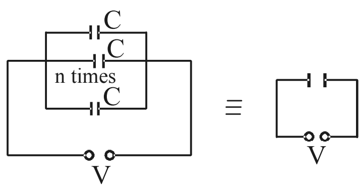
SOLUTIONS

1. **(a)** We know that $\frac{W_{AB}}{q} = V_B - V_A$

$$\therefore\ V_B - V_A = \frac{2\,\text{J}}{20\,\text{C}} = 0.1\,\text{J/C} = 0.1\,\text{V}$$

2. **(b)** The equivalent capacitance of n identical capacitors of capacitance C is equal to nC. Energy stored in this capacitor

$$E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$$



Alternatively

Each capacitor has a potential difference of V between the plates.

So energy stored in each capacitor

$$= \frac{1}{2}CV^2.$$

∴ Energy stored in n capacitor

$$= \left[\frac{1}{2}CV^2\right] \times n$$

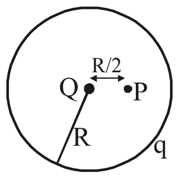
3. **(a)** For an isolated sphere, the capacitance is given by

$$C = 4\pi \in_0 r = \frac{1}{9\times 10^9} \times 1 = 1.1\times 10^{-10}\,\text{F}$$

4. **(b)** The capacitance of a parallel plate capacitor in which a metal plate of thickness t is inserted is given by

$$C = \frac{\varepsilon_o A}{d - t}.\quad \text{Here } t \to 0\ \therefore C = \frac{\varepsilon_o A}{d}$$

5. (c) Electric potential due to charge Q placed at the centre of the spherical shell at point P is



$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2} = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$$

Electric potential due to charge q on the surface of the spherical shell at any point inside the shell is

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

$\therefore$ The net electric potential at point P is

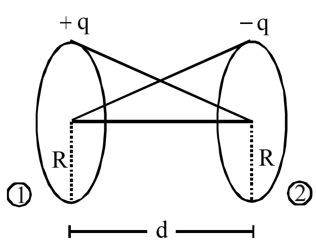
$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R} + \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

6. (d) The work done is stored as the potential energy. The potential energy stored in a capacitor is given by

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\times\frac{\left(8\times10^{-18}\right)^2}{100\times10^{-6}}$$

$= 32\times10^{-32}$ J

7. (a)



At (1) using, potential

$(V_1) = V_{\text{self}} + V_{\text{due to (2)}}$

$$\Rightarrow \; V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

At (2) using potential

$(V_2) = V_{\text{self}} + V_{\text{due to (1)}}$

$$\Rightarrow \; V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R} + \frac{q}{\sqrt{R^2+d^2}}\right]$$

$\Delta V = V_1 - V_2$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} + \frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

8. (b) As n plates are joined, it means $(n-1)$ combination joined in parallel.

$\therefore$ resultant capacitance $= (n-1)\,C$

9. (c) Applying conservation of energy,

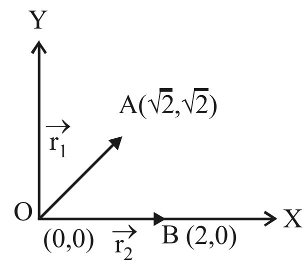
$$\frac{1}{2}CV^2 = m.s\,\Delta t;\;\; \text{V} = \sqrt{\frac{2m.s.\Delta t}{C}}$$

10. (a) $eV = \frac{1}{2}mv^2$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.1\times10^{-31}}}$$

$= 2.65\times10^6$ m/s

11. (c)



The distance of point $A\left(\sqrt{2},\sqrt{2}\right)$ from the origin,

$OA = |\vec{r_1}| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2$ units.

The distance of point $B(2, 0)$ from the origin,

$OB = |\vec{r_2}| = \sqrt{(2)^2 + (0)^2} = 2$ units.

Now, potential at A, $V_A = \frac{1}{4\pi\in_0}.\frac{Q}{(OA)}$

Potential at B, $V_B = \frac{1}{4\pi\in_0}.\frac{Q}{(OB)}$

$\therefore$ Potential difference between the points A and B is given by

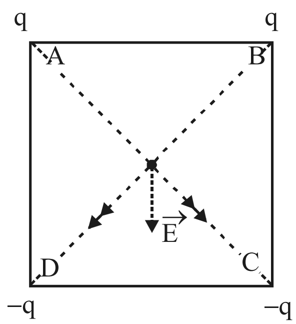
$$V_A - V_B = \frac{1}{4\pi\in_0}.\frac{Q}{OA} - \frac{1}{4\pi\in_0}.\frac{Q}{OB}$$

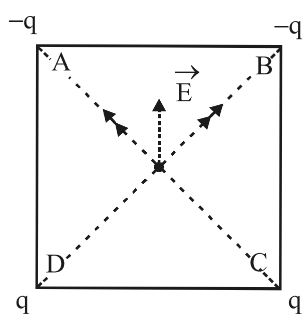
$$= \frac{Q}{4\pi\in_0}\left(\frac{1}{OA} - \frac{1}{OB}\right) = \frac{Q}{4\pi\in_0}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{Q}{4\pi\in_0}\times 0 = 0.$$

12. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude, but opposite directions.

Also, the potential will be same in both cases as it is a scalar quantity.





13. (a) Here, $V(x) = \dfrac{20}{x^2 - 4}$ volt

We know that $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}\left(\dfrac{20}{x^2-4}\right)$

or, $E = +\dfrac{40x}{(x^2-4)^2}$

At $x = 4\,\mu\text{m}$,

$E = +\dfrac{40 \times 4}{(4^2-4)^2} = +\dfrac{160}{144} = +\dfrac{10}{9}$ volt / μm.

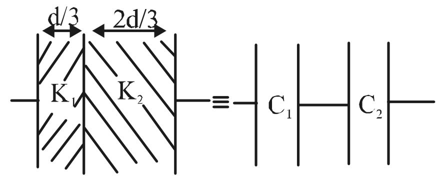
Positive sign indicates that $\vec{E}$ is in +ve x-direction.

14. (a) The potential energy of a charged capacitor is given by $U = \dfrac{Q^2}{2C}$.

If a dielectric slab is inserted between the plates, the energy is given by $\dfrac{Q^2}{2KC}$, where K is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

15. (c)



The given capacitance is equal to two capacitances connected in series where

$$C_1 = \frac{k_1 \epsilon_0 A}{d/3} = \frac{3k_1 \epsilon_0 A}{d}$$

$$= \frac{3 \times 3 \epsilon_0 A}{d} = \frac{9 \epsilon_0 A}{d}$$

and

$$C_1 = \frac{k_2 \epsilon_0 A}{2d/3} = \frac{3k_2 \epsilon_0 A}{2d}$$

$$= \frac{3 \times 6 \epsilon_0 A}{2d} = \frac{9 \epsilon_0 A}{d}$$

The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d}{9\epsilon_0 A} + \frac{d}{9\epsilon_0 A} = \frac{2d}{9\epsilon_0 A}$$

$$\therefore C_{eq} = \frac{9}{2}\frac{\epsilon_0 A}{d} = \frac{9}{2} \times 9\ pF = 40.5\,pF$$

16. (c) $\dfrac{W_{PQ}}{q} = (V_Q - V_P)$

$\Rightarrow W_{PQ} = q(V_Q - V_P)$

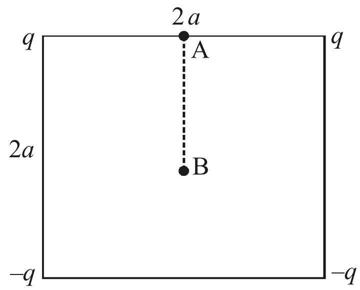
$= (-100 \times 1.6 \times 10^{-19})(-4 - 10)$

$= +2.24 \times 10^{-16}$J

17. (d) Potential at point A,

$$V_A = \frac{2kq}{a} - \frac{2kq}{a\sqrt{5}}$$

$\therefore$ (potential due to each $q = \dfrac{kq}{a}$ and potential due to each $-q = \dfrac{-kq}{a\sqrt{5}}$)



Potential at point. B,

$$V_B = 0$$

($\because$ Point B is equidistant from all the four charges)

$\therefore$ Using work energy theorem,

$$(W_{AB})_{electric} = Q(V_A - V_B)$$

$$= \frac{2kqQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2Qq}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

18. (c) The potential energy at the centre of the sphere

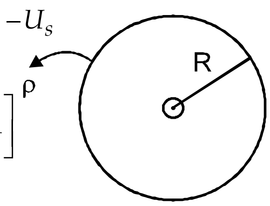
$$U_c = \frac{3}{2}\frac{KQq}{R}$$

The potential energy at the surface of the sphere

$$U_s = \frac{KQq}{R}$$

Now change in the energy

$$\Delta U = U_c - U_s$$



$$= \frac{KQq}{R}\left[\frac{3}{2} - 1\right]$$

$$= \frac{KQq}{2R}$$

Where $Q = \rho.V = \rho.\frac{4}{3}\pi R^3$

$$\Delta U = \frac{2K}{3}\frac{\pi R^3 \rho q}{R}$$

$$\Delta U = \frac{2}{3} \times \frac{1}{4\pi \in_0}\frac{\pi R^3 \rho q}{R}$$

$$\Delta U = \frac{R^2 \rho q}{6 \in_0}$$

19. (c) Potential difference between any two points in an electric field is given by,

$$dV = -\vec{E}.\vec{dx}$$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80J/C$$

20. (a) Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

$$E = \frac{\sigma}{K\varepsilon_0}$$

Then, charge density

$$\sigma = K\varepsilon_0 E$$
$$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$
$$\approx 6 \times 10^{-7}\ \text{C/m}^2$$

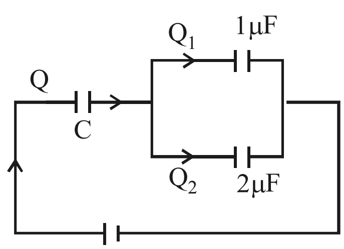
$= \frac{\sigma.s}{v}$ (Let surface area of plates = s)

$$= \frac{\sigma.s}{\frac{\sigma}{\lambda}\ln\left(1 + \frac{\lambda d}{K_0}\right)}$$

$= s\lambda.\frac{d}{d}\frac{1}{\ln\left(1+\frac{\lambda d}{K_0}\right)}$ ($\because$ in vacuum $\varepsilon_0 = 1$)

$c = \frac{\lambda d}{\ln\left(1+\frac{\lambda d}{K_0}\right)}\cdot C_0$ $\left(\text{here, } C_0 = \frac{s}{d}\right)$

21. (d)

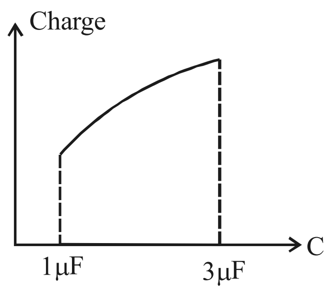


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$$Q = E\left(\frac{C \times 3}{C+3}\right)$$

$$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$$

Therefore graph d correctly dipicts.



22. **(a)** We know, $V_0 = \frac{Kq}{R} = $ Vsurface

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For $r < R$]

At the centre of sphere r = 0. Here

$V = \frac{3}{2}V_0$

Now, $\frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$

$R_2 = \frac{R}{\sqrt{2}}$

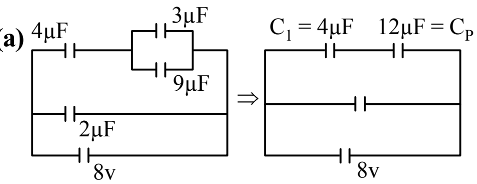
$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$

$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$

$R_4 = 4R$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

23. **(a)**



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 =$

$24\mu c$

The voltage across C_P is $V_P = \frac{4}{4+12}\times 8 = 2v$

$\therefore$ Voltage across $9\mu F$ is also 2V

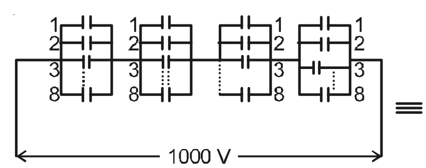
$\therefore$ Charge on $9\mu F$ capacitor $= 9\times 2 = 18\mu C$

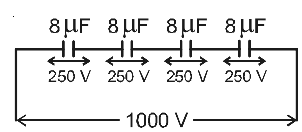
$\therefore$ Total charge on 4 μF and 9μF = 42μc

$$\therefore E = \frac{KQ}{r^2} = 9\times 10^9 \times \frac{42\times 10^{-6}}{30\times 30}$$

$$= 420\, NC^{-1}$$

24. **(b)** To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i.e., 32 such capacitors are required.





$$\frac{1}{C_{eq}} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} \quad \therefore\ C_{eq} = 2\ \mu F$$