CHAPTER 9

Sequences and Series

1. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P. then x equals **[2002]**
(a) $\log_3 4$ (b) $1-\log_3 4$
(c) $1-\log_4 3$ (d) $\log_4 3$

2. The value of $2^{1/4}.\ 4^{1/8}.\ 8^{1/16}\ \ldots\ \infty$ is **[2002]**
(a) 1 (b) 2
(c) 3/2 (d) 4

3. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**
(a) 256 (b) 512
(c) 1024 (d) none of these

4. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**
(a) 5 (b) 3/5
(c) 8/5 (d) 1/5

5. $1^3-2^3+3^3-4^3+\ldots+9^3=$ **[2002]**
(a) 425 (b) –425
(c) 475 (d) –475

6. The sum of the series **[2003]**
$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$.......... up to ∞ is equal to
(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$
(c) $\log_e 2-1$ (d) $\log_e 2$

7. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c},\frac{b}{a}$ and $\frac{c}{b}$ are in **[2003]**
(a) Arithmetic - Geometric Progression
(b) Arithmetic Progression
(c) Geometric Progression
(d) Harmonic Progression.

8. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to **[2004]**
(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$
(c) $n-1$ (d) $\frac{1}{2}n$

9. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m,n,\ m\neq n,\ T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals **[2004]**
(a) $\frac{1}{m}+\frac{1}{n}$ (b) 1
(c) $\frac{1}{mn}$ (d) 0

10. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is **[2004]**
(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

11. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+.....$ is **[2004]**

(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

12. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[2004]**

(a) $x^2-18x-16=0$

(b) $x^2-18x+16=0$

(c) $x^2+18x-16=0$

(d) $x^2+18x+16=0$

13. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2-m(4r-1)+4r^2-2=0$

(b) $m^2-m(4r+1)+4r^2+2=0$

(c) $m^2-m(4r+1)+4r^2-2=0$

(d) $m^2-m(4r-1)+4r^2+2=0$

14. If $x=\sum_{n=0}^{\infty}a^n$, $y=\sum_{n=0}^{\infty}b^n$, $z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in **[2005]**

(a) G.P.
(b) A.P.
(c) Arithmetic - Geometric Progression
(d) H.P.

15. The sum of the series **[2005]**

$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....ad inf. is

(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$

(c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

16. Let a_1, a_2, a_3........... be terms on A.P. If $\frac{a_1+a_2+..........a_p}{a_1+a_2+..........+a_q}=\frac{p^2}{q^2}$, $p\neq q$, then $\frac{a_6}{a_{21}}$ equals **[2006]**

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$

(c) $\frac{2}{7}$ (d) $\frac{11}{41}$

17. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2+a_2a_3+.........+a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1-a_n)$ (b) $(n-1)(a_1-a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

18. The sum of series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$

(c) e^{-2} (d) e^{-1}

19. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}\left(\sqrt{5}-1\right)$

(c) $\frac{1}{2}\left(1-\sqrt{5}\right)$ (d) $\frac{1}{2}\sqrt{5}$.

20. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12
(c) 12 (d) 4

21. The sum to infinite term of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$ is **[2009]**

(a) 3 (b) 4
(c) 6 (d) 2

22. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes
(c) 135 minutes (d) 24 minutes

23. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months
(c) 21 months (d) 18 months

24. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is **[2011]**

(a) $\alpha - \beta$ (b) $\frac{\alpha - \beta}{100}$
(c) $\beta - \alpha$ (d) $\frac{\alpha - \beta}{200}$

25. **Statement-1:** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400)$ is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

26. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is : **[2012]**

(a) – 150
(b) 150 times its 50^{th} term
(c) 150
(d) Zero

27. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[2013]**

(a) $\frac{7}{81}(179 - 10^{-20})$
(b) $\frac{7}{9}(99 - 10^{-20})$
(c) $\frac{7}{81}(179 + 10^{-20})$
(d) $\frac{7}{9}(99 + 10^{-20})$

28. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[2013]**

(a) $x = y = z$ (b) $2x = 3y = 6z$
(c) $6x = 3y = 2z$ (d) $6x = 4y = 3z$

29. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[2014]**

(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$
(c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

30. If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to: **[2014]**

(a) 100 (b) 110
(c) $\frac{121}{10}$ (d) $\frac{441}{100}$

31. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[2014]**

(a) $2 - \sqrt{3}$ (b) $2 + \sqrt{3}$
(c) $\sqrt{2} + \sqrt{3}$ (d) $3 + \sqrt{2}$

32. The sum of first 9 terms of the series.

$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$ **[2015]**

(a) 142 (b) 192
(c) 71 (d) 96

33. If m is the A.M. of two distinct real numbers l and n(l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$

(c) $4\,l^2mn$ (d) $4\,lm^2n$

34. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **[2016]**

(a) 1 (b) $\frac{7}{4}$

(c) $\frac{8}{5}$ (d) $\frac{4}{3}$

35. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots,$$

is $\frac{16}{5}$m, then m is equal to : **[2016]**

(a) 100 (b) 99

(c) 102 (d) 101

36. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **[2017]**

(a) a, b and c are in G.P.

(b) b, c and a are in G.P.

(c) b, c and a are in A.P.

(d) a, b and c are in A.P.

37. Let a, b, c ∈ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and $f(x + y) = f(x) + f(y) + xy$, ∀ x, y ∈ R, then $\sum_{n=1}^{10} f(n)$ is equal to : **[2017]**

(a) 255 (b) 330

(c) 165 (d) 190

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(b)	(b)	(a)	(a)	(d)	(d)	(d)	(b)	(b)	(b)	(c)	(d)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(d)	(d)	(b)	(b)	(a)	(a)	(c)	(b)	(b)	(d)	(c)	(a)	(b)	(a)
31	32	33	34	35	36	37								
(b)	(d)	(d)	(d)	(d)	(c)	(b)								

SOLUTIONS

1. **(b)** 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x - 1)$ are in A.P.

$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3[3(4\,.\,3^x - 1)]$

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3.$

Put $3^x = t$

$\Rightarrow \frac{3}{t} + 2 = 12t - 3$ or $12t^2 - 5t - 3 = 0;$

Hence $t = -\frac{1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$ (as $3^x \neq -ve$)

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$

$\Rightarrow x = 1 - \log_3 4$

2. **(b)** The product is $P = 2^{1/4}.2^{2/8}.2^{3/16}$.........

$= 2^{1/4+2/8+3/16+\ldots\ldots\infty}$

Now let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \ldots\ldots\infty$ ......(1)

$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \ldots\ldots\infty$ ......(2)

Subtracting (2) from (1)

$\Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots\infty$

or $\frac{1}{2}S = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S = 1$

$\therefore P = 2^S = 2$

3. **(b)** $ar^4 = 2$

$a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$
$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

4. **(b)** Let a = first term of G.P. and r = common ratio of G.P.; Then G.P. is a, ar, ar^2

Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$

$\Rightarrow a = 20(1-r)$... (i)

Also $a^2 + a^2r^2 + a^2r^4 + ...$ to $\infty = 100$

$\Rightarrow \frac{a^2}{1-r^2} = 100$

$\Rightarrow a^2 = 100(1-r)(1+r)$... (ii)

From (i), $a^2 = 400(1-r)^2$;

From (ii), we get $100(1-r)(1+r) = 400(1-r)^2$

$\Rightarrow 1+r = 4-4r \Rightarrow 5r = 3 \Rightarrow r = 3/5$.

5. **(a)** $1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3$

$= 1^3 + 2^3 + 3^3 + ... + 9^3 - 2(2^3 + 4^3 + 6^3 + 8^3)$

$= \left[\frac{9 \times 10}{2}\right]^2 - 2.2^3\left[1^3 + 2^3 + 3^3 + 4^3\right]$

$= (45)^2 - 16.\left[\frac{4 \times 5}{2}\right]^2 = 2025 - 1600 = 425$

6. **(a)** $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4}$...............$\infty$

$|T_n| = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S = T_1 - T_2 + T_3 - T_4 + T_5$............$\infty$

$= \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right)$.....

$= 1 - 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}\text{...............}\infty\right]$

$= 1 - 2[-\log(1+1) + 1] = 2\log 2 - 1 = \log\left(\frac{4}{e}\right)$.

7. **(d)** $ax^2 + bx + c = 0, \ \alpha + \beta = \frac{-b}{a}, \ \alpha\beta = \frac{c}{a}$

As for given condition, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} - \frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c} \quad \Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

8. **(d)** $S_n = \frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + ... + \frac{1}{^nC_n}$

$t_n = \frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + ... + \frac{n}{^nC_n}$

$t_n = \frac{n}{^nC_n} + \frac{n-1}{^nC_{n-1}} + \frac{n-2}{^nC_{n-2}} + ... + \frac{0}{^nC_0}$

Add,

$2t_n = (n)\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} + ... \frac{1}{^nC_n}\right] = nS_n$

$\therefore \frac{t_n}{S_n} = \frac{n}{2}$

9. **(d)** $T_m = a + (m-1)d = \frac{1}{n}$ .....(1)

$T_n = a + (n-1)d = \frac{1}{m}$ .....(2)

$(1) - (2) \Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (1) $a = \frac{1}{mn} \Rightarrow a - d = 0$

10. **(b)** If n is odd, the required sum is

$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2.(n-1)^2 + n^2$

$= \frac{(n-1)(n-1+1)^2}{2} + n^2$

[$\because$ $(n-1)$ is even

$\therefore$ using given formula for the sum of (n–1) terms.]

$$= \left(\frac{n-1}{2}+1\right)n^2 = \frac{n^2(n+1)}{2}$$

11. (b) We know that $e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......$

and $e^{-1} = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.......$

$$\therefore e+e^{-1} = 2\left[1+\frac{1}{2!}+\frac{1}{4!}+....\right]$$

$$\therefore \frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+...... = \frac{e+e^{-1}}{2}-1$$

$$= \frac{e^2+1-2e}{2e} = \frac{(e-1)^2}{2e}$$

12. (b) Let two numbers be a and b then $\frac{a+b}{2} = 9$

and $\sqrt{ab} = 4$

$\therefore$ Equation with roots a and b is

$$x^2-(a+b)x+ab = 0$$

$$\Rightarrow x^2-18x+16 = 0$$

13. (c) Given ${}^mC_{r-1},\ {}^mC_r,\ {}^mC_{r+1}$ are in A.P.

$$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$$

$$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r}+\frac{{}^mC_{r+1}}{{}^mC_r}$$

$$= \frac{r}{m-r+1}+\frac{m-r}{r+1}$$

$$\Rightarrow m^2-m(4r+1)+4r^2-2 = 0.$$

14. (d) $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad a = 1-\frac{1}{x}$

$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \qquad b = 1-\frac{1}{y}$

$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \qquad c = 1-\frac{1}{z}$

a, b, c are in A.P. OR $2b = a+c$

$$2\left(1-\frac{1}{y}\right) = 1-\frac{1}{x}+1-\frac{1}{y}$$

$$\frac{2}{y} = \frac{1}{x}+\frac{1}{z} \Rightarrow x, y, z \text{ are in H.P.}$$

15. (d) $\frac{e^x+e^{-x}}{2} = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}.............$

Putting $x = \frac{1}{2}$ we get

$$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+......$$

$$\infty = \frac{e^{\frac{1}{2}}+e^{\frac{-1}{2}}}{2} = \frac{\sqrt{e}+\frac{1}{\sqrt{e}}}{2} = \frac{e+1}{2\sqrt{e}}$$

16. (d) $\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d} = \frac{p}{q}$$

$$\frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11, q = 41 \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

17. (d) $\frac{1}{a_2}-\frac{1}{a_1} = \frac{1}{a_3}-\frac{1}{a_2} = ... = \frac{1}{a_n}-\frac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \frac{a_1-a_2}{d}$, $a_2a_3 = \frac{a_2-a_3}{d}$,

$$..........., a_{n-1}a_n = \frac{a_{n-1}-a_n}{d}$$

$$\therefore \quad a_1a_2+a_2a_3+.........+a_{n-1}a_n$$

$$= \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + ... + \frac{a_{n-1} - a_n}{d}$$

$$= \frac{1}{d}[a_1 - a_2 + a_2 - a_3 + ... + a_{n-1} - a_n]$$

$$= \frac{a_1 - a_n}{d}$$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\Rightarrow \frac{a_1 - a_n}{a_1 a_n} = (n-1)d$$

$$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1 a_n$$

Which is the required result.

18. **(d)** We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$

Put $x = -1$

$$\therefore \quad e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} ....\infty$$

$$\therefore \quad e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} ....\infty$$

19. **(b)** Let the series a, ar, ar^2, ... are in geometric progression.

given, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \quad \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2}$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$$

$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$ [$\because$ terms of G.P. are positive $\therefore$ r should be positive]

20. **(b)** As per question,

$a + ar = 12$ ...(1)

$ar^2 + ar^3 = 48$ ...(2)

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$$

($\because$ terms are = + ve and –ve alternately)

$\Rightarrow a = -12$

21. **(a)** We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty \quad ....(1)$$

Multiplying both sides by $\frac{1}{3}$ we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty \quad ....(2)$$

Subtracting eqn. (2) from eqn. (1) we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$$

22. **(a)** Till 10^{th} minute number of counted notes = 1500

$$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$$

$n^2 - 149n + 3000 = 0$

$\Rightarrow n = 125, 24$

But $n = 125$ is not possible

$\therefore$ total time = 24 + 10 = 34 minutes.

23. **(c)** Let required number of months = n

$\therefore 200 \times 3 + (240 + 280 + 320 + ... + (n-3)^{th}$ term)

= 11040

$$\Rightarrow \frac{n-3}{2}[2 \times 240 + (n-4) \times 40]$$

$= 11040 - 600$

$\Rightarrow (n-3)[240 + 20n - 80] = 10440$

$\Rightarrow (n-3)(20n + 160) = 10440$

$\Rightarrow (n-3)(n+8) = 522$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21) = 0$

$\therefore n = 21$

24. **(b)** Let A.P. be $a, a+d, a+2d, ...$

$$a_2 + a_4 + ... + a_{200} = \alpha$$

$\Rightarrow \quad \frac{100}{2}\left[2(a+d)+(100-1)d\right]=\alpha$ ....(i)

and $a_1+a_3+a_5+.........+a_{199}=\beta$

$\Rightarrow \quad \frac{100}{2}$ [2a + (100 – 1) d]= β ....(ii)

On solving (i) and (ii), we get

$$d=\frac{\alpha-\beta}{100}$$

25. **(b)** nth term of the given series

$$=T_n=(n-1)^2+(n-1)n+n^2$$

$$=\frac{\left((n-1)^3-n^3\right)}{(n-1)-n}=n^3-(n-1)^3$$

$$\Rightarrow S_n=\sum_{k=1}^{n}\left[k^3-(k-1)^3\right]\Rightarrow 8000=n^3$$

$\Rightarrow \quad n=20$ which is a natural number.

Now, put $n=1,2,3,....20$

$T_1=1^3-0^3$

$T_2=2^3-1^3$

⋮

$T_{20}=20^3-19^3$

Now, $T_1+T_2+---+T_{20}=S_{20}$

$\Rightarrow \quad S_{20}=20^3-0^3=8000$

Hence, both the given statement is true.

26. **(d)** Let 100th term of an AP is $a+(100-1)\,d$ $=a+99d$ where 'a' is the first term of $A.P$ and 'd' is the common difference of $A.P.$

Similarly, 50th term $=a+(50-1)\,d$

$=a+49d$

Now, According to the question

$100\,(a+99d)=50\,(a+49d)$

$\Rightarrow \quad 2a+198\,d=a+49d \;\Rightarrow\; a+149\,d=0$

This is the 150th term of an $A.P.$

Hence, $T_{150}=a+149\,d=0$

27. **(c)** Given sequence can be written as

$$\frac{7}{10}+\frac{77}{100}+\frac{777}{10^3}+......+\text{ up to 20 terms}$$

$$=7\left[\frac{1}{10}+\frac{11}{100}+\frac{111}{10^3}+.....+\text{up to 20 terms}\right]$$

Multiply and divide by 9

$$=\frac{7}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+......\text{up to 20 terms}\right]$$

$$=\frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+......\text{up to 20 terms}\right]$$

$$=\frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right]$$

$$=\frac{7}{9}\left[\frac{179}{9}+\frac{1}{9}\left(\frac{1}{10}\right)^{20}\right]$$

$$=\frac{7}{81}\,[179+(10)^{-20}]$$

28. **(a)** Since, x, y, z are in A.P.

$\therefore \quad 2y=x+z$

Also, we have

$2\tan^{-1}y=\tan^{-1}x+\tan^{-1}(z)$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right)=\tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{x+z}{1-y^2}=\frac{x+z}{1-xz} \qquad (\because \; 2y=x+z)$$

$\Rightarrow y^2=xz \quad$ or $\quad x+z=0 \;\Rightarrow\; x=y=z$

29 **(b)** Let p, q, r are in AP

$\Rightarrow 2q=p+r$ ...(i)

Given $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}=4$$

We have $\alpha+\beta=-q/p$ and $\alpha\beta=\frac{r}{p}$

$$\Rightarrow \frac{-\frac{q}{p}}{\frac{r}{p}}=4\Rightarrow q=-4r \qquad ....(ii)$$

From (i), we have

$2(-4r)=p+r$

$p=-9r$

$q = -4r$
$r = r$

Now $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2 - 4pr}}{|p|}$$

$$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$$

30. **(a)** Let $10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9$

Multiplied by $\frac{11}{10}$ on both the sides

$$\frac{11}{10}x = 11.10^8 + 2.(11)^2.(10)^7 + ... + 9(11)^9 + 11^{10}$$

$$x\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2 \times (10)^7 + ... + 11^9 - 11^{10}$$

$$\Rightarrow -\frac{x}{10} = 10^9\left[\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right] - 11^{10}$$

$$\Rightarrow -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$$

$\Rightarrow x = 10^{11} = k.10^9$ Given

$\Rightarrow k = 100$

31. **(b)** Let a, ar, ar^2 are in G.P.

According to the question

$a, 2ar, ar^2$ are in A.P.

$\Rightarrow 2 \times 2ar = a + ar^2$

$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

Since $r > 1$

$\therefore r = 2 - \sqrt{3}$ is rejected

Hence, $r = 2 + \sqrt{3}$

32. **(d)** n^{th} term of series

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2} = \frac{1}{4}(n+1)^2$$

Sum of n term $= \Sigma \frac{1}{4}(n+1)^2$

$$= \frac{1}{4}\left[\Sigma n^2 + 2\Sigma n + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$$

Sum of 9 terms

$$= \frac{1}{4}\left[\frac{9 \times 10 \times 19}{6} + \frac{18 \times 10}{2} + 9\right]$$

$$= \frac{384}{4} = 96$$

33. **(d)** $m = \frac{l+n}{2}$ and common ratio of G.P.

$$= r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$$

$\therefore G_1 = l^{3/4}n^{1/4}, G_2 = l^{1/2}n^{1/2}, G_3 = l^{1/4}n^{3/4}$

$G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$

$= ln(l+n)^2$

$= ln \times 2m^2$

$= 4lm^2n$

34. **(d)** Let the GP be a, ar and ar^2 then $a = A + d$; $ar = A + 4d$; $ar^2 = A + 8d$

$$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$$

$$r = \frac{4}{3}$$

35. **(d)**

$$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 ... + \left(\frac{44}{5}\right)^2$$

$$S = \frac{16}{25}\left(2^2 + 3^2 + 4^2 + ... + 11^2\right)$$

$$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right)$$

$$= \frac{16}{25} \times 505 = \frac{16}{5} \times 101$$

$$\Rightarrow \frac{16}{5} m = \frac{16}{5} \times 101$$

$\Rightarrow m = 101.$

36. (c) We have

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15bc = 0$

$$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$$

it is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b = 5$

$$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$$

$$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$$

$\Rightarrow a + b = 2c$

$\Rightarrow b, c, a$ are in A.P.

37. (b) $f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3 \Rightarrow f(1) = 3$

Now $f(x + y) = f(x) + f(y) + xy$ ...(1)

Put $x = y = 1$ in eqn (1)

$f(2) = f(1) + f(1) + 1$

$= 2f(1) + 1$

$f(2) = 7$

$\Rightarrow f(3) = 12$

Now, $S_n = 3 + 7 + 12 + \dots\dots\dots t_n$ ...(1)

$S_n = 3 + 7 + \dots\dots\dots t_{n-1} + t_n$ ...(2)

Subtract (2) from (1)

$t_n = 3 + 4 + 5 + \dots.$ upto n terms

$$t_n = \frac{(n^2 + 5n)}{2}$$

$$S_n = \sum t_n = \sum \frac{(n^2 + 5n)}{2}$$

$S_n =$

$$\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right] = \frac{n(n+1)(n+8)}{6}$$

$$S_{10} = \frac{10 \times 11 \times 18}{6} = 330$$