CHAPTER

Three Dimensional Geometry 26

1. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is **[2002]**

(a) $x-y+z=1$ (b) $x+y+z=5$

(c) $x+2y-z=1$ (d) $2x-y+z=5$

2. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x+y=3$ are **[2002]**

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$

(c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

3. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is **[2003]**

(a) 39 (b) 26

(c) $11\frac{4}{13}$ (d) 13.

4. The two lines $x = ay + b$, $z = cy + d$ and $x = a¢y + b¢$, $z = c¢y + d¢$ will be perpendicular, if and only if **[2003]**

(a) $aa¢ + cc¢ + 1 = 0$

(b) $aa¢ + bb¢ + cc¢ + 1 = 0$

(c) $aa¢ + bb¢ + cc¢ = 0$

(d) $(a + a¢)(b + b¢) + (c + c¢) = 0$.

5. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ **[2003]**

and $\frac{x-1}{k}=\frac{y-4}{1}=\frac{z-5}{1}$ are coplanar if

(a) k = 3 or –2 (b) k = 0 or –1

(c) k = 1 or –1 (d) k = 0 or –3.

6. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **[2003]**

(a) 4 (b) 1

(c) 2 (d) 3

7. Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' from the origin then **[2003]**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$.

8. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{3}{2}$

9. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by **[2004]**

(a) $(2a, 3a, 3a), (2a, a, a)$

(b) $(3a, 2a, 3a), (a, a, a)$

(c) $(3a, 2a, 3a), (a, a, 2a)$

(d) $(3a, 3a, 3a), (a, a, a)$

10. If the straight lines **[2004]**

$x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$

and $x = \frac{t}{2}, y = 1 + t, z = 2 - t$, with parameters s and t respectively, are co-planar, then l equals.

(a) 0 (b) –1

(c) $-\frac{1}{2}$ (d) –2

11. The intersection of the spheres

$x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and

$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x - y - z = 1$ (b) $x - 2y - z = 1$

(c) $x - y - 2z = 1$ (d) $x - y - z = 1$

12. A line makes the same angle q, with each of the x and z axis. If the angle b, which it makes with y-axis, is such that $\sin^2 \beta = 3\sin^2 \theta$, then $\cos^2$q equals **[2004]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

13. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

14. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **[2005]**

(a) 0° (b) 90°

(c) 45° (d) 30°

15. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals **[2005]**

(a) – 1 (b) 1

(c) – 2 (d) 2

16. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{3}{10}$ (d) $\frac{10}{3}$

17. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is **[2005]**

(a) $(-1, 2)$ (b) $(-1, -2)$

(c) $(1, -2)$ (d) $\left(1, -\frac{1}{2}\right)$

18. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is **[2005]**

(a) the Geometric Mean of a and b
(b) the Arithmetic Mean of a and b
(c) equal to zero
(d) the Harmonic Mean of a and b

19. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius **[2005]**

(a) 3 (b) 1

(c) 2 (d) $\sqrt{2}$

20. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if **[2006]**

(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$

(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$

21. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is **[2006]**

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) $(15, 11, 4)$

(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) None of these

22. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

23. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) $(4, 3, 5)$ (b) $(4, 3, -3)$

(c) $(4, 9, -3)$ (d) $(4, -3, 3)$.

24. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$.

25. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of a and b? **[2008]**

(a) a = 2, b = 2 (b) a = 1, b = 2

(c) a = 2, b = 1 (d) a = 1, b = 1

26. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then **[2008]**

(a) $a = 2, b = 8$ (b) $a = 4, b = 6$

(c) $a = 6, b = 4$ (d) $a = 8, b = 2$

27. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**

(a) –5 (b) 5

(c) 2 (d) –2

28. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \text{a}z + \text{b} = 0$. Then (a, b) equals **[2009]**

(a) (–6, 7) (b) (5, –15)

(c) (–5, 5) (d) (6, –17)

29. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement -2: The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.

(b) Statement -1 is true, Statement -2 is false.

(c) Statement -1 is false, Statement -2 is true .

(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

30. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle q with the positive z-axis, then q equals **[2010]**

(a) 45° (b) 60°

(c) 75° (d) 30°

31. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point $(13, 32)$. The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is **[2010]**

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$

(c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

32. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then l equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

33. **Statement-1:** The point $A(1, 0, 7)$) is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

34. The distance of the point (1, – 5, 9) from the plane $x-y+z=5$ measured along a straight $x=y=z$ is **[2011RS]**

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$

(c) $3\sqrt{10}$ (d) $3\sqrt{5}$

35. The length of the perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is : **[2011RS]**

(a) $\sqrt{29}$ (b) $\sqrt{33}$

(c) $\sqrt{53}$ (d) $\sqrt{66}$

36. A equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is : **[2012]**

(a) $x-2y+2z-3=0$ (b) $x-2y+2z+1=0$

(c) $x-2y+2z-1=0$ (d) $x-2y+2z+5=0$

37. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to: **[2012]**

(a) –1 (b) $\frac{2}{9}$

(c) $\frac{9}{2}$ (d) 0

38. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

39. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have **[2013]**

(a) any value
(b) exactly one value
(c) exactly two values
(d) exactly three values

40. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line: **[2014]**

(a) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(b) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(c) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(d) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

41. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2$ is **[2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

42. The equation of the plane containing the line $2x-5y+z=3$; $x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is : **[2015]**

(a) $x+3y+6z=7$ (b) $2x+6y+12z=-13$

(c) $2x+6y+12z=13$ (d) $x+3y+6z=-7$

43. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=16$, is **[2015]**

(a) $3\sqrt{21}$ (b) 13

(c) $2\sqrt{14}$ (d) 8

44. The distance of the point (1, –5, 9) from the plane $x-y+z=5$ measured along the line $x=y=z$ is : **[2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$

(c) $3\sqrt{10}$ (d) $10\sqrt{3}$

45. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to : **[2016]**

(a) 5 (b) 2

(c) 26 (d) 18

46. If the image of the point P(1, –2, 3) in the plane, $2x+3y-4z+22=0$ measured parallel to line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to :

(a) $6\sqrt{5}$ (b) $3\sqrt{5}$

(c) $2\sqrt{42}$ (d) $\sqrt{42}$

47. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is : **[2017]**

(a) $\frac{10}{\sqrt{74}}$ (b) $\frac{20}{\sqrt{74}}$

(c) $\frac{10}{\sqrt{83}}$ (d) $\frac{5}{\sqrt{83}}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(d)	(a)	(d)	(d)	(a)	(c)	(b)	(d)	(a)	(c)	(a)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(c)	(a)	(b)	(a)	(d)	(b)	(c)	(c)	(d)	(c)	(a)	(a)	(a)	(b)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(c)	(d)	(a)	(a)	(c)	(a)	(c)	(c)	(c)	(c)	(c)	(a)	(b)	(d)	(b)
46	47													
(c)	(c)													

SOLUTIONS

1. (a) As the point (3, 2, 0) lies on the given line

$$\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$$

$\therefore$ There can be infinite many planes passing through this line. But here out of the four options only first option is satisfied by the coordinates of both the points (3, 2, 0) and (4, 7, 4)

$\therefore$ $x-y+z=1$ is the required plane.

2. (b) Equation of plane through (1, 0, 0) is

$a(x-1)+by+cz=0$ ...(i)

(i) passes through (0, 1, 0).

$-a+b=0 \Rightarrow b=a$; Also,

$$\cos 45^\circ=\frac{a+a}{\sqrt{2(2a^2+c^2)}}\Rightarrow 2a=\sqrt{2a^2+c^2}$$

$$\Rightarrow 2a^2=c^2 \qquad \Rightarrow c=\sqrt{2}a.$$

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

3. (d) Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere – radius

$$=\left|\frac{-2\times12+4\times1+3\times3-327}{\sqrt{144+9+16}}\right|-\sqrt{4+1+9+155}$$

$=26-13=13$

4. (a) $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c};\ \frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}.$

For perpendicularity of lines

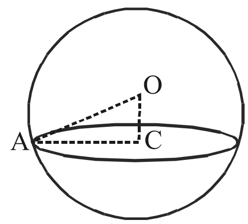
$aa'+1+cc'=0$

5. (d) $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0$$

$$\begin{vmatrix}1&-1&-1\\1&1&-k\\k&2&1\end{vmatrix}=0\Rightarrow\begin{vmatrix}0&0&-1\\2&1+k&-k\\k+2&1&1\end{vmatrix}=0$$

$k^2+3k=0\Rightarrow k(k+3)=0 \ or\ k=0\, or -3$

6. (d)



centre of sphere = (-1, 1, 2)

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC=d=\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right|=\frac{12}{3}=4.$$

$$AC^2=AO^2-OC^2=5^2-4^2=9$$

$\Rightarrow AC=3$

7. (a) Eq. of planes be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ &

$\frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

8. (c) The planes are $2x+y+2z-8=0$. ...(1)

and $4x+2y+4z+5=0$

or $2x+y+2z+\frac{5}{2}=0$ ...(2)

$\therefore$ Distance between (1) and (2)

$$=\left|\frac{\frac{5}{2}+8}{\sqrt{2^2+1^2+2^2}}\right|=\left|\frac{21}{2\sqrt{9}}\right|=\frac{7}{2}$$

9. (b) Let a point on the line $x=y+a=z$ is $(\lambda,\lambda-a,\lambda)$ and a point on the line

$x+a=2y=2z$ is $\left(\mu-a,\frac{\mu}{2},\frac{\mu}{2}\right)$, then direction ratio of the line joining these points are $\lambda-\mu+a,\ \lambda-a-\frac{\mu}{2},\ \lambda-\frac{\mu}{2}$

If it rspresents the required line, then

$$\frac{\lambda-\mu+a}{2}=\frac{\lambda-a-\frac{\mu}{2}}{1}=\frac{\lambda-\frac{\mu}{2}}{2}$$

on solving we get $\lambda=3a, \mu=2a$

$\therefore$ The required points of intersection are

$(3a, 3a\text{-}a, 3a)$ and $\left(2a-a, \frac{2a}{2}, \frac{2a}{2}\right)$

or $(3a, 2a, 3a)$ and (a, a, a)

10. (d) The given lines are

$$x-1=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s \qquad \text{..........(1)}$$

and $2x=y-1=\frac{z-2}{-1}=t$

.........(2)

The lines are coplanar, if

$$\begin{vmatrix} 0-(-1) & -1-3 & -2-(-1) \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=0$$

$$c_2\to c_2+c_3;\ \begin{vmatrix} 1 & -5 & -1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix}=0$$

$$\Rightarrow 5(-1-\frac{\lambda}{2})=0\Rightarrow\lambda=-2$$

11. (a) The equations of spheres are

$S_1: x^2+y^2+z^2+7x-2y-z-13=0$ *and*

$S_2: x^2+y^2+z^2-3x+3y+4z-8=0$

Their plane of intersection is

$S_1-S_2=0\Rightarrow 10x-5y-5z-5=0$

$\Rightarrow 2x-y-z=1$

12. (c) The direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore \cos^2\theta+\cos^2\beta+\cos^2\theta=1$

$\Rightarrow 2\cos^2\theta=\sin^2\beta=3\sin^2\theta$ (given)

$\Rightarrow 2\cos^2\theta=3-3\cos^2\theta$

$\therefore \cos^2\theta=\frac{3}{5}$

13. (a) If θ is the angle between line and plane then $\left(\frac{\pi}{2}-\theta\right)$ is the angle between line and normal to plane given by

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{(\hat{i}+2\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+\sqrt{\lambda}\hat{k})}{3\sqrt{4+1+\lambda}}$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3}\Rightarrow 4\lambda=5+\lambda$$

$$\Rightarrow\lambda=\frac{5}{3}.$$

14. (b) The given lines are $2x=3y=-z$

or $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ [Dividing by 6]

and $6x=-y=-4z$

or $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$$\cos\theta=\frac{3.2+2.(-12)+(-6).(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$=\frac{6-24+18}{\sqrt{49}\sqrt{157}}=0\Rightarrow\theta=90^\circ$$

15. (c) Plane $2ax-3ay+4az+6=0$ passes through the mid point of the centre of spheres

$x^2+y^2+z^2+6x-8y-2z=13$ and

$x^2+y^2+z^2-10x+4y-2z=8$

respectively centre of spheres are (– 3, 4, 1) and (5, – 2, 1). Mid point of centres is (1, 1, 1).

Satisfying this in the equation of plane, we get

$2a-3a+4a+6=0$

$\Rightarrow a=-2.$

16. (b) The given line is

$\vec{r} = 2\vec{i} - 2\vec{j} + 3\vec{k} + \lambda(\vec{i} - \vec{j} + 4\vec{k})$

and the plane is $\vec{r}\cdot(\vec{i} + 5\vec{j} + \vec{k}) = 5$

or $x + 5y + z = 5$

Required distance

$$= \left|\frac{2 - 10 - 2 + 3 - 5}{\sqrt{1 + 25 + 1}}\right| = \frac{10}{3\sqrt{3}}$$

17. (c) a, b, c are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$$

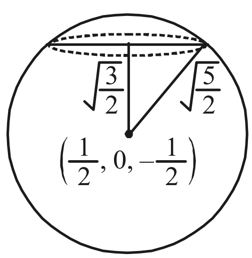
$\therefore \frac{x}{a} + \frac{y}{a} + \frac{1}{c} = 0$ passes through $(1, -2)$

18. (a) Vector $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

$\therefore c$ is G.M. of a and b.

19. (b)



Perpendicular distance of centre

$\left(\frac{1}{2}, 0, -\frac{1}{2}\right)$

from $x + 2y - 2 = 4$ is given by

$$\frac{\left|\frac{1}{2} + \frac{1}{2} - 4\right|}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

$$\text{radius of sphere} = \sqrt{\frac{1}{4} + \frac{1}{4} + 2} = \sqrt{\frac{5}{2}}$$

$$\therefore \text{radius of circle} = \sqrt{\frac{5}{2} - \frac{3}{2}} = 1.$$

20. (a) Equation of lines $\frac{x - b}{a} = \frac{y}{1} = \frac{z - d}{c}$

$\frac{x - b'}{a'} = \frac{y}{1} = \frac{z - d'}{c'}$

Line are perpendicular

$\Rightarrow aa' + 1 + cc' = 0$

21. (d) If (α, β, γ) be the image, then mid point of (α, β, γ) and $(-1, 3, 4)$ must lie on $x - 2y = 0$

$$\therefore \frac{\alpha - 1}{2} - 2\left(\frac{\beta + 3}{2}\right) = 0$$

$\therefore \alpha - 1 - 2\beta - 6 = 0 \Rightarrow \alpha - 2\beta = 7 \ldots (1)$

Also line joining (α, β, γ) and $(-1, 3, 4)$ should be parallel to the normal of the plane $x - 2y = 0$

$$\therefore \frac{\alpha + 1}{1} = \frac{\beta - 3}{-2} = \frac{\gamma - 4}{0} = \lambda$$

$\Rightarrow \alpha = \lambda - 1, \beta = -2\lambda + 3, \gamma = 4 \ldots (2)$

From (1) and (2)

$$\alpha = \frac{9}{5},\ \beta = -\frac{13}{5},\ \gamma = 4$$

None of the option matches.

22. (b) Let the angle of line makes with the positive direction of z-axis is α direction cosines of line with the +ve directions of x-axis, y-axis, and z-axis is l, m, n respectively.

$\therefore\ l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}, n = \cos\alpha$

as we know that, $l^2 + m^2 + n^2 = 1$

$$\therefore \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\alpha = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\alpha = 1$$

$$\Rightarrow \cos^2\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$.

23. (c) We know that equation of sphere is $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

where centre is $(-u, -v, -w)$

given $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$

$\therefore$ centre $\equiv (3, 6, 1)$

Coordinates of one end of diameter of the sphere are (2, 3, 5). Let the coordinates of the other end of diameter are (α, β, γ)

$\therefore \dfrac{\alpha+2}{2}=3, \dfrac{\beta+3}{2}=6, \dfrac{\gamma+5}{2}=1$

$\Rightarrow \alpha=4, \beta=9$ and $\gamma=-3$

$\therefore$ Coordinate of other end of diameter are $(4, 9, -3)$

24. (c) Let the direction cosines of line L be l, m, n, then

$2l+3m+n=0$ ....(i)

and $l+3m+2n=0$ ....(ii)

on solving equation (i) and (ii), we get

$\dfrac{l}{6-3}=\dfrac{m}{1-4}=\dfrac{n}{6-3} \Rightarrow \dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}$

Now $\dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\because l^2+m^2+n^2=1$

$\therefore \dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{1}{\sqrt{27}}$

$\Rightarrow l=\dfrac{3}{\sqrt{27}}=\dfrac{1}{\sqrt{3}}, m=-\dfrac{1}{\sqrt{3}}, n=\dfrac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

$\therefore l=\cos\alpha \Rightarrow \cos\alpha=\dfrac{1}{\sqrt{3}}$

25. (d) $\because \vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$

$\therefore \quad \vec{a}=\vec{b}+\lambda\vec{c}$

$\Rightarrow \alpha\hat{i}+2\hat{j}+\beta\hat{k}=\hat{i}+\hat{j}+\lambda(\hat{j}+\hat{k})$

$\Rightarrow \alpha=1, 2=1+\lambda, \beta=\lambda \Rightarrow \alpha=1, \beta=1$

ALTERNATE SOLUTION

$\because \vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$.

$\therefore \vec{a}=\lambda(\hat{b}+\hat{c})$

$\Rightarrow \alpha\hat{i}+2\hat{j}+\beta\hat{k}=\dfrac{\lambda(\hat{i}+2\hat{j}+\hat{k})}{\sqrt{2}}$

$\Rightarrow \alpha=\dfrac{\lambda}{\sqrt{2}}, \lambda=\sqrt{2}, \beta=\dfrac{\lambda}{\sqrt{2}}$

$\Rightarrow \alpha=\beta=1$

26. (c) Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is $\dfrac{x-5}{-2}=\dfrac{y-1}{b-1}=\dfrac{z-a}{1-a}=\lambda$

$\therefore$ Any point on this line is a $[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$

It crosses yz plane where $-2\lambda+5=0$

$\lambda=\dfrac{5}{2}$

$\therefore \left(0, (b-1)\dfrac{5}{2}+1, (1-a)\dfrac{5}{2}+a\right)=\left(0, \dfrac{17}{2}, \dfrac{-13}{2}\right)$

$\Rightarrow (b-1)\dfrac{5}{2}+1=\dfrac{17}{2}$ and $(1-a)\dfrac{5}{2}+a=-\dfrac{13}{2}$

$\Rightarrow b=4$ and $a=6$

27. (a) The two lines intersect if shortest distance between them is zero i.e.

$\dfrac{(\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}=0$

$\Rightarrow (\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2=0$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$,

$\vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$\Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$

$\Rightarrow 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow -2k^2-5k+25=0 \Rightarrow k=-5$ or $\dfrac{5}{2}$

$\because k$ is an integer, therefore $k=-5$

28. (a) $\because$ The line $\dfrac{x-2}{3}=\dfrac{y-1}{-5}=\dfrac{z+2}{2}$ lie in the plane

$x+3y-\alpha z+\beta=0$

$\therefore Pt\,(2, 1, -2)$ lies on the plane

i.e. $2+3+2\alpha+\beta=0$

or $2\alpha+\beta+5=0$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore 3\times1-5\times3+2\times(-\alpha)=0$

$\Rightarrow \alpha=-6$

From equation (i) then, $\beta=7$

$\therefore (\alpha, \beta)=(-6, 7)$

29. (a) A(3, 1, 6); B = (1, 3, 4)

Mid-point of AB = (2, 2, 5) lies on the plane.

and d.r's of AB = (2, –2, 2)

d.r's of normal to plane = (1, –1, 1).

Direction ratio of AB and normal to the plane are proportional therefore,

AB is perpendicular to the plane

$\therefore$ A is image of B

Statement-2 is correct but it is not correct explanation.

30. **(b)** Direction cosines of the line :

$\ell = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$,

$n = \cos\theta$

where θ is the angle, which line makes with positive z-axis.

Now $\ell^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2}$ (θ being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

31. **(c)** Slope of line L $= -\frac{b}{5}$

Slope of line K $= -\frac{3}{c}$

Line L is parallel to line *k*.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

(13, 32) is a point on L.

$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$

Equation of K :

$y - 4x = 3 \Rightarrow 4x - y + 3 = 0$

Distance between L and K

$= \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

32. **(d)** If θ be the angle between the given line and plane, then

$$\sin\theta = \frac{1\times1 + 2\times2 + \lambda\times3}{\sqrt{1^2+2^2+\lambda^2}.\sqrt{1^2+2^2+3^2}}$$

$$= \frac{5+3\lambda}{\sqrt{14}.\sqrt{5+\lambda^2}}$$

$$\Rightarrow \cos\theta = \sqrt{1 - \frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\therefore \theta = \cos^{-1}\sqrt{1 - \frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

But it is given that $\theta = \cos^{-1}\sqrt{\frac{5}{14}}$

$$\therefore \sqrt{1 - \frac{(5+3\lambda)^2}{14(5+\lambda^2}} = \sqrt{\frac{5}{14}}$$

$\Rightarrow \lambda = \frac{2}{3}$

33. **(a)** The direction ratio of the line segment joining points A(1, 0, 7) and B(1, 6, 3) is 0, 6, –4.

The direction ratio of the given line is 1, 2, 3.

Clearly $1 \times 0 + 2 \times 6 + 3 \times (-4) = 0$

So, the given line is perpendicular to line AB.

Also , the mid point of A and B is (1, 3, 5) which lies on the given line.

So, the image of B in the given line is A, because the given line is the perpendicular bisector of line segment joining points A and B.

34. **(a)** Equation of line through $P(1,-5,9)$ and parallel to the plane $x = y = z$ is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda(say)$$

Q= $(x = 1+\lambda, y = -5+\lambda, z = 9+\lambda)$

Given plane x – y + z =5

$\therefore$ $1 + \lambda + 5 - \lambda + 9 + \lambda = 5$

$\Rightarrow$ $\lambda = -10$

$\therefore$ Q = (– 9 , – 15, – 1)

$\therefore$ PQ $= \sqrt{(1+9)^2 + (15-5)^2 + (9+1)^2}$

$= \sqrt{300} = 10\sqrt{3}$

35. **(c)** Let feet of perpendicular is

$(2\alpha, 3\alpha + 2, 4\alpha + 3)$

$\Rightarrow$ Direction ratio of the $\perp$ line is

$2\alpha - 3, 3\alpha + 3, 4\alpha - 8.$ and

Direction ratio of the line 2, 3, 4 are

$\Rightarrow$ $2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$

$\Rightarrow$ $29\alpha - 29 = 0$

$\Rightarrow$ $\alpha = 1$

$\Rightarrow$ Feet of $\perp$ is (2, 5, 7)

$\Rightarrow$ Length $\perp$ is $\sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

36. **(a)** Given equation of a plane is $x - 2y + 2z - 5 = 0$

So, Equation of parallel plane is given by $x - 2y + 2z + d = 0$

Now, it is given that distance from origin to the parallel plane is 1.

$$\therefore \left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right| = 1 \Rightarrow d = \pm 3$$

So equation of required plane

$x - 2y + 2z \pm 3 = 0$

37. **(c)** Given lines are $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$

and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$

Thus, $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are given as

$\vec{a}(1,-1,1)$, $\vec{b}(2,3,4)$, $\vec{c}(3,k,0)$; and $\vec{d}(1,2,1)$

These lines will intersect if lines are coplanar

i.e., $\vec{a}-\vec{c}, \vec{b}$ & $\vec{d}$ are coplanar

$\therefore \quad \left[\vec{a}-\vec{c}, \vec{b}, \vec{d}\right] = 0$

Now, $\vec{a}-\vec{c} = (3-1, k+1, 0-1)$

$= (2, k+1, -1)$

$$\Rightarrow \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(3-8) - k+1(2-4) - 1(4-3) = 0$

$\Rightarrow \quad 2(-5) - (k+1)(-2) - 1(1) = 0$

$\Rightarrow \quad -10 + 2k + 2 - 1 = 0 \Rightarrow k = \dfrac{9}{2}$

38. **(c)** $2x + y + 2z - 8 = 0$ ...(Plane 1)

$2x + y + 2z + \dfrac{5}{2} = 0$ ...(Plane 2)

Distance between Plane 1 and 2

$$= \left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{-21}{6}\right| = \frac{7}{2}$$

39. **(c)** Given lines will be coplanar

$$\text{If} \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & \end{vmatrix} = 0$$

$\Rightarrow \; -1(1+2k) - (1+k^2) + 1(2-k) = 0$

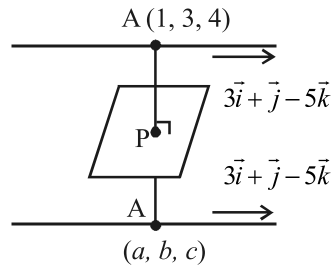
$\Rightarrow \; k = 0, -3$

40. **(c)** $\dfrac{a-1}{2} = \dfrac{b-3}{-1} = \dfrac{c-4}{1} = \lambda(\text{let})$

$\Rightarrow \quad a = 2\lambda + 1$

$b = 3 - \lambda$

$c = 4 + \lambda$



$$P = \left(\frac{a+1}{2}, \frac{b+3}{2}, \frac{c+4}{2}\right)$$

$$= \left(\lambda+1, \frac{6-\lambda}{2}, \frac{\lambda+8}{2}\right)$$

$\therefore \quad 2(\lambda+1) - \dfrac{6-\lambda}{2} + \dfrac{\lambda+8}{2} + 3 = 0$

$3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$a = -3, b = 5, c = 2$

Required line is $\dfrac{x+3}{3} = \dfrac{y-5}{1} = \dfrac{z-2}{-5}$

41. **(c)** Given, $l + m + n = 0$ and $l^2 = m^2 + n^2$

Now, $(-m-n)^2 = m^2 + n^2$

$\Rightarrow \quad mn = 0 \Rightarrow m = 0$ or $n = 0$

If $m = 0$ then $l = -n$

We know $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\dfrac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1) = \left(-\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

If $n = 0$ then $l = -m$

$l^2 + m^2 + n^2 = 1 \quad \Rightarrow 2m^2 = 1$

$\Rightarrow \quad m = \pm\dfrac{1}{\sqrt{2}}$

Let $m = \dfrac{1}{\sqrt{2}} \quad \Rightarrow l = -\dfrac{1}{\sqrt{2}}$ and $n = 0$

$(l_2, m_2, n_2) = \left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0\right)$

$\therefore \quad \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

42. **(a)** Equation of the plane containing the lines $2x - 5y + z = 3$ and $x + y + 4z = 5$ is

$2x - 5y + z - 3 + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow (2+\lambda)x + (-5+\lambda)y + (1+4\lambda)z + (-3-5\lambda) = 0$ ...(i)

Since the plane (i) parallel to the given plane x

$+3y+6z=1$

$\therefore \frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\Rightarrow \lambda=-\frac{11}{2}$

Hence equation of the required plane is

$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$

$\Rightarrow (4-11)x+(-10-11)y+(2-44)z+(-6+55)=0$

$\Rightarrow -7x-21y-42z+49=0$

$\Rightarrow x+3y+6z-7=0$

$\Rightarrow x+3y+6z=7$

43. **(b)** General point on given line $\equiv P(3r+2, 4r-1, 12r+2)$

Point P must satisfy equation of plane

$(3r+2)-(4r-1)+(12r+2)=16$

$11r+5=16$

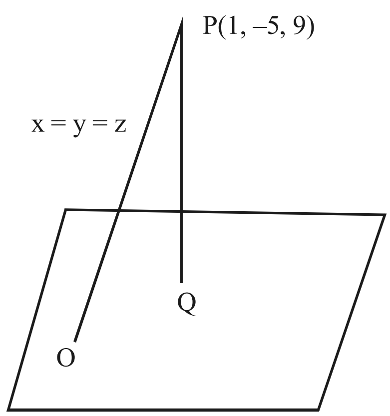
$r=1$

$P(3\times 1+2, 4\times 1-1, 12\times 1+2)=P(5, 3, 14)$

distance between P and (1, 0, 2)

$D=\sqrt{(5-1)^2+3^2+(14-2)^2}=13$

44. **(d)**



eq^n of PO : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$\Rightarrow x=\lambda+1;\ y=\lambda-5; z=\lambda+9.$

Putting these in eq^n of plane :-

$\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \lambda=-10$

$\Rightarrow$ O is $(-9,-15,-1)$

$\Rightarrow$ distance $OP=10\sqrt{3}$

45. **(b)** Line lies in the plane $\Rightarrow (3,-2,-4)$ lie in the plane

$\Rightarrow 3\ell-2m+4=9$ or $3\ell-2m=5$ ..... (1)

Also, $\ell, m, -1$ are dr's of line perpendicular to plane and $2, -1, 3$ are dr's of line lying in the plane

$\Rightarrow 2\ell-m-3=0$ or $2\ell-m=3$ .....(2)

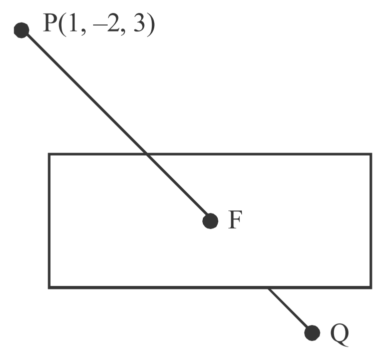
Solving (1) and (2) we get $\ell=1$ and $m=-1$

$\Rightarrow \ell^2+m^2=2.$

46. **(c)** Equation of line PQ is

$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$

Let F be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



Since F lies on the plane

$\therefore 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$2\lambda+2+12\lambda-6-20\lambda-12+22=0$

$\Rightarrow -6\lambda+6=0 \Rightarrow \lambda=1$

$\therefore$ F is (2, 2, 8)

$PQ=2\,PF=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

47. **(c)** Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

Normal vector

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}=5\hat{i}+7\hat{j}+3\hat{k}$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow 5x+7y+3z+5=0$

Distance of point $(1, 3, -7)$ from the plane is

$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$