CHAPTER 19

Determinants

1. If $a > 0$ and discriminant of $ax^2+2bx+c$ is –ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ *is* equal to **[2002]**

(a) +ve
(b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve
(d) 0

2. If the system of linear equations **[2003]**

$x+2ay+az=0$; $x+3by+bz=0$;

$x+4cy+cz=0$ has a non - zero solution, then a, b, c.

(a) satisfy $a+2b+3c=0$ (b) are in A.P
(c) are in G.P (d) are in H.P.

3. If $1, \omega, \omega^2$ are the cube roots of unity, then

$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to **[2003]**

(a) ω^2 (b) 0
(c) 1 (d) ω

4. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is **[2004]**

(a) $A^2 = I$
(b) $A = (-1)\, I$, where I is a unit matrix
(c) A^{-1} does not exist
(d) A is a zero matrix

5. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$. and $B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$.

If B is the inverse of matrix A, then α is **[2004]**

(a) 5 (b) –1
(c) 2 (d) –2

6. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots.$ are in G.P., then the value of the determinant **[2004]**

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is

(a) –2 (b) 1
(c) 2 (d) 0

7. The system of equations

$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$

has infinite solutions, if α is **[2005]**

(a) – 2 (b) either – 2 or 1
(c) not – 2 (d) 1

8. If $a^2+b^2+c^2 = -2$ and **[2005]**

$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f (x) is a polynomial of degree

(a) 1 (b) 0
(c) 3 (d) 2

9. If $a_1, a_2, a_3, ..., a_n, ...$ are in G. P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to **[2005]**

(a) 1 (b) 0
(c) 4 (d) 2

10. If $A^2 - A + I = 0$, then the inverse of A is **[2005]**

(a) $A + I$ (b) A
(c) $A - I$ (d) $I - A$

11. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is **[2007]**

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

12. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals **[2007]**

(a) 1/5 (b) 5
(c) 5^2 (d) 1

13. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2 = I$. **[2008]**

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

14. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[2008]**

(a) 2 (b) –1
(c) 0 (d) 1

15. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**

(a) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non integers
(c) If det $A = \pm 1$, then A^{-1} exists but all its entries are integers
(d) If det $A = \pm 1$, then A^{-1} need not exists

16. Let A be a 2×2 matrix

Statement -1 : adj (adj A) = A

Statement -2 : |adj A| = |A| **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

17. Let a, b, c be such that $b(a + c) \neq 0$ if **[2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is :

(a) any even integer

(b) any odd integer
(c) any integer
(d) zero

18. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $\mathrm{Tr}(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement - 1 : $\mathrm{Tr}(A) = 0$.

Statement -2 : $|A| = 1$. **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

19. Consider the system of linear equations; **[2010]**

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has
(a) exactly 3 solutions
(b) a unique solution
(c) no solution
(d) infinite number of solutions

20. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**

(a) 2 (b) 1
(c) zero (d) 3

21. If the trivial solution is the only solution of the system of equations **[2011RS]**

$x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

then the set of all values of k is :

(a) $R - \{2, -3\}$ (b) $R - \{2\}$
(c) $R - \{-3\}$ (d) $\{2, -3\}$

22 **Statement - 1:**
Determinant of a skew-symmetric matrix of order 3 is zero.

Statement - 2 :
For any matrix A, $\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$.
Where det (B) denotes the determinant of matrix B. Then : **[2011RS]**

(a) Both statements are true
(b) Both statements are false
(c) Statement-1 is false and statement-2 is true
(d) Statement-1 is true and statement-2 is false

23. Consider the following relation R on the set of real square matrices of order 3. **[2011RS]**

$R = \{(A, B) | A = P^{-1}BP$ for some invertible matrix $P\}$

Statement-1 : R is equivalence relation.

Statement-2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

(a) Statement-1 is true, statement-2 is true and statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(c) Statement-1 is true, stement-2 is false.
(d) Statement-1 is false, statement-2 is true.

24. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[2012]**

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

25. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : **[2012]**

(a) -2 (b) 1
(c) 0 (d) -1

26. The number of values of k, for which the system of equations :

$$(k+1)x+8y=4k$$
$$kx+(k+3)y=3k-1$$

has no solution, is **[2013]**

(a) infinite (b) 1
(c) 2 (d) 3

27. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A|=4$, then α is equal to : **[2013]**

(a) 4 (b) 11
(c) 5 (d) 0

28. If $\alpha,\beta\neq 0$, and $f(n)=\alpha^n+\beta^n$ and

$$\begin{vmatrix}3 & 1+f(1) & 1+f(2)\\ 1+f(1) & 1+f(2) & 1+f(3)\\ 1+f(2) & 1+f(3) & 1+f(4)\end{vmatrix}$$

$=K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$,

then K is equal to: **[2014]**

(a) 1 (b) –1
(c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

29. If A is an 3×3 non-singular matrix such that $AA'=A'A$ and $B=A^{-1}A'$, then BB' equals: **[2014]**

(a) B^{-1} (b) $(B^{-1})'$
(c) $I+B$ (d) I

30. The set of all values of λ for which the system of linear equations : **[2015]**

$2x_1-2x_2+x_3=\lambda x_1$
$2x_1-3x_2+2x_3=\lambda x_2$
$-x_1+2x_2=\lambda x_3$

has a non-trivial solution,

(a) contains two elements.
(b) contains more than two elements
(c) is an empty set.
(d) is a singleton

31. The system of linear equations **[2016]**

$x+\lambda y-z=0$
$\lambda x-y-z=0$
$x+y-\lambda z=0$

has a non-trivial solution for:

(a) exactly two values of λ.
(b) exactly three values of λ.
(c) infinitely many values of λ.
(d) exactly one value of λ.

32. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[2017]**

(a) $\left(2,\frac{1}{2}\right)$ (b) $\left(2,-\frac{1}{2}\right)$
(c) $\left(1,\frac{3}{4}\right)$ (d) $\left(1,-\frac{3}{4}\right)$

33. If S is the set of distinct values of 'b' for which the following system of linear equations **[2017]**

$x+y+z=1$
$x+ay+z=1$
$ax+by+z=0$

has no solution, then S is :

(a) a singleton
(b) an empty set
(c) an infinite set
(d) a finite set containing two or more elements

34. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3k$, then k is equal to : **[2017]**

(a) 1 (b) –z
(c) z (d) –1

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(d)	(b)	(a)	(a)	(d)	(a)	(d)	(b)	(d)	(d)	(a)	(d)	(d)	(c)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(a)	(b)	(b)	(c)	(a)	(a)	(d)	(b)	(d)	(c)	(b)	(b)	(a)	(d)	(a)
31	**32**	**33**	**34**											
(b)	(a)	(a)	(b)											

SOLUTIONS

1. **(c)** We have $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

By $R_3 \to R_3 - (xR_1 + R_2)$;

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

$= (ax^2+2bx+c)(b^2-ac) = (+)(-) = -$ve.

2. **(d)** For homogeneous system of equations to have non zero solution, $\Delta = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad C_2 \to C_2 - 2C_3 \sqrt{b^2-4ac}$$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \to R_3 - R_2, R_2 \to R_2 - R_1$$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

3. **(b)** $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$$= 1\left(\omega^{3n} - 1\right) - \omega^n\left(\omega^{2n} - \omega^{2n}\right) + \omega^{2n}\left(\omega^n - \omega^{4n}\right)$$

$$= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n}$$

$$= 1 - 1 + 1 - 1 = 0 \left[\because \omega^{3n} = 1\right]$$

4. **(a)** $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further

$$(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$

Also $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

5. **(a)** Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Also since, $B = A^{-1} \Rightarrow AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$$

6. **(d)** Let r be the common ratio, then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

$= 0 \quad \left[\text{Apply } c_2 \to c_2 - \frac{1}{2}c_1 - \frac{1}{2}c_3\right]$

7. (a) $\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1;$

$x + y + z\alpha = \alpha - 1$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$

$= \alpha\,(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$

For infinite solutions, $\Delta = 0$

$\Rightarrow \quad (\alpha - 1)[\alpha^2 + \alpha - 1 - 1] = 0$

$\Rightarrow \quad (\alpha - 1)[\alpha^2 + \alpha - 2] = 0$

$\Rightarrow (\alpha - 1)\,[\alpha^2 + 2\alpha - \alpha - 2] = 0$

$\Rightarrow (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)] = 0$

$(\alpha - 1) = 0, \ \alpha + 2 = 0$

$\Rightarrow \ \alpha = -2, 1;$

But $\alpha \neq 1$.

$\therefore \alpha = -2$

8. (d) Applying, $C_1 \to C_1 + C_2 + C_3$ we get

$$f(x) = \begin{vmatrix} 1 + (a^2 + b^2 + c^2 + 2)x & (1 + b^2)x & (1 + c^2)x \\ 1 + (a^2 + b^2 + c^2 + 2)x & 1 + b^2 x & (1 + c^2 x) \\ 1 + (a^2 + b^2 + c^2 + 2)x & (1 + b^2)x & 1 + c^2 x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1 + b^2)x & (1 + c^2)x \\ 1 & 1 + b^2 x & (1 + c^2 x) \\ 1 & (1 + b^2)x & 1 + c^2 x \end{vmatrix}$$

[As given that $a^2 + b^2 + c^2 = -2$]

$\therefore a^2 + b^2 + c^2 + 2 = 0$

Applying $R_1 \to R_1 - R_2, \ R_2 \to R_2 - R_3$

$$\therefore f(x) = \begin{vmatrix} 0 & x - 1 & 0 \\ 0 & 1 - x & x - 1 \\ 1 & (1 + b^2)x & 1 + c^2 x \end{vmatrix}$$

$f(x) = (x - 1)^2$

Hence degree $= 2$.

9. (b) $\because a_1, a_2, a_3, ...$ are in G.P.

$\therefore$ Using $a_n = ar^{n-1}$, we get the given determinant, as

$$\begin{vmatrix} \log ar^{n-1} & \log ar^{n} & \log ar^{n+1} \\ \log ar^{n+2} & \log ar^{n+3} & \log ar^{n+4} \\ \log ar^{n+5} & \log ar^{n+6} & \log ar^{n+7} \end{vmatrix}$$

Operating $C_3 - C_2$ and $C_2 - C_1$ and

using $\log m - \log n = \log \frac{m}{n}$ we get

$$= \begin{vmatrix} \log ar^{n-1} & \log r & \log r \\ \log ar^{n+2} & \log r & \log r \\ \log ar^{n+5} & \log r & \log r \end{vmatrix}$$

$= 0$ (two columns being identical)

10. (d) Given $A^2 - A + I = 0$

$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$

(Multiplying A^{-1} on both sides)

$\Rightarrow A - 1 + A^{-1} = 0$ or $A^{-1} = I - A$.

11. (d) Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + x & 1 \\ 1 & 1 & 1 + y \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$ and $R \to R_3 - R_1$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y

12. (a) Given $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$

$$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$\therefore |A^2| = 25\ (25\alpha^2)$

$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

13. (d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = I$

$\Rightarrow a^2 + bc = 1$ and $ab + bd = 0$
$ac + cd = 0$ and $bc + d^2 = 1$

From these four relations,

$$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$$

and $b(a+d) = 0 = c(a+d) \Rightarrow a = -d$

We can take $a = 1, b = 0, c = 0, d = -1$ as one possible set of values, then

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Clearly $A \neq I$ and $A \neq -I$ and $\det A = -1$

$\therefore$ Statement 1 is true.

Also if $A \neq I$ then $tr(A) = 0$

$\therefore$ Statement 2 is false.

14. (d) The given equations are

$-x + cy + bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

$\because x, y, z$ are not all zero

$\therefore$ The above system should not have unique (zero) solution

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow -1(1 - a^2) - c(-c - ab) + b(ac + b) = 0$

$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

15. (c) $\because$ All entries of square matrix A are integers, therefore all cofactors should also be integers.

If $\det A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

16. (a) We know that $|adj\ (adj\ A)| = |A|^{n-2} A$.

$= |A|^0 A = A$

Also $|adj\ A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\therefore$ Both the statements are true but statement-2 is not a correct explanation for statement-1 .

17. (b) $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} +$

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2} a & (-1)^{n+1} b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2} a \\ b+1 & b-1 & (-1)^{n+1} b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$$

(Taking transpose of second determinant)

$C_1 \Leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2} a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2} c & c+1 & c-1 \end{vmatrix} = 0$$

$C_2 \Leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow \left[1 + (-1)^{n+2}\right] \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$C_2 - C_1,\ C_3 - C_1$

$$\Rightarrow \left[1+(-1)^{n+2}\right]\begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0 \; R_1+R_3$$

$$\Rightarrow \left[1+(-1)^{n+2}\right]\begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $[1+(-1)^{n+2}](a+c)(2b+1+2b-1)=0$

$\Rightarrow$ $4b(a+c)[1+(-1)^{n+2}]=0$

$\Rightarrow$ $1+(-1)^{n+2}=0$ as $b(a+c) \neq 0$

$\Rightarrow$ n should be an odd integer.

18. **(b)** Let $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d $\neq 0$

$$A^2=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\Rightarrow A^2=\begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$\Rightarrow a^2+bc=1, bc+d^2=1$

$ab+bd=ac+cd=0$

$c \neq 0$ and $b \neq 0 \Rightarrow a+d=0$

$|A| = ad-bc=-a^2-bc=-1$

19. **(c)** $D=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix}=0$

$$D_1=\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

20. (a) $\Delta = 0$

$$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix}=0$$

$\Rightarrow 4(4-2)-k(k-2)+2(2k-8)=0$

$\Rightarrow 8-k^2+2k+4k-16=0$

$k^2-6k+8=0$

$\Rightarrow (k-4)(k-2)=0 \Rightarrow k=4,2$

21. (a)

$x-ky+z=0$

$kx+3y-kz=0$

$3x+y-z=0$

The given system of equations will have non trivial solution, if

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix}=0$$

$\Rightarrow$ $1(-3+k)+k(-k+3k)+1(k-9)=0$

$\Rightarrow$ $k-3+2k^2+k-9=0$

$\Rightarrow$ $k^2+k-6=0$

$\Rightarrow$ $k=-3, k=2$

So the equation will have only trivial solution,

when $k \in R-\{2,-3\}$

22. (d) **Statement-1 :** Determinant of skew symmetric matrix of odd order is zero.

Statement-2 : det $\left(A^T\right)$ = det (A).

det (– A) = – (– 1)^n det (A).

where A is a $n \times n$ order matrix.

23. **(b)** **For reflexive**

$(A, A) \in R$

$A=P^{-1}AP$ is true,

For P = I, which is an invertible matrix.

$\therefore$ R is reflexive.

For symmetry

As $(A, B) \in$ R for matrix P

$A=P^{-1}BP$

$\Rightarrow$ $PAP^{-1}=B$

$\Rightarrow$ $B=PAP^{-1}$

$\Rightarrow$ $B=\left(P^{-1}\right)^{-1} A\,(P^{-1})$

$\therefore$ (B, A) $\in$ R for matrix P^{-1}

$\therefore$ R is symmetric.

For transitivity

$A=P^{-1}BP$

and $B=P^{-1}CP$

$\Rightarrow$ $A=P^{-1}\left(P^{-1}CP\right)P$

$\Rightarrow \quad A=\left(P^{-1}\right)^2 CP^2$

$\Rightarrow \quad A=\left(P^2\right)^{-1} C\left(P^2\right)$

$\therefore$ $(A, C) \in R$ for matrix P^2
$\therefore$ R is transitive.
So R is equivalence

24. (d) Let $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$

Then, $Au_1+Au_2=\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}$

$\Rightarrow \quad A(u_1+u_2)=\begin{pmatrix}1\\1\\0\end{pmatrix}$ ...(1)

Also, $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$

$\Rightarrow \quad |A|=1(1)-0(2)+0(4-3)=1$

We know,

$A^{-1}=\frac{1}{|A|}adjA$

$\Rightarrow A^{-1}=adj(A) \quad (\because |A|=1)$

Now, from equation (1), we have

$u_1+u_2=A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix}$

$=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

25. (c) Given $P^3=Q^3$ ...(1)

and $P^2Q=Q^2P$ ...(2)

Subtracting (1) and (2), we get

$P^3-P^2Q=Q^3-Q^2P$

$\Rightarrow \quad P^2(P-Q)+Q^2(P-Q)=0$

$\Rightarrow \quad (P^2+Q^2)(P-Q)=0$

If $|P^2+Q^2|\neq 0$ then P^2+Q^2 is invertible.

$\Rightarrow \quad P-Q=0 \Rightarrow P=Q$

Which gives a contradiction $(\because P\neq Q)$

Hence $|P^2+Q^2|=0$

26. (b) From the given system, we have

$\frac{k+1}{k}=\frac{8}{k+3}\neq\frac{4k}{3k-1}$

($\because$ System has no solution)

$\Rightarrow \quad k^2+4k+3=8k$

$\Rightarrow \quad k=1, 3$

If $k=1$ then $\frac{8}{1+3}\neq\frac{4.1}{2}$ which is false

and if $k=3$ then $\frac{8}{6}\neq\frac{4.3}{9-1}$ which is true,

therefore $k=3$

Hence for only one value of k. System has no solution.

27. (b) $|P|=1(12-12)-\alpha(4-6)+3(4-6)=2\alpha-6$

Now, adj A = P $\Rightarrow |\text{adj A}|=|P|$

$\Rightarrow |A|^2=|P|$

$\Rightarrow |P|=16$

$\Rightarrow \quad 2a-6=16$

$\Rightarrow \quad a=11$

28. (a) Consider

$$\begin{vmatrix}3&1+f(1)&1+f(2)\\1+f(1)&1+f(2)&1+f(3)\\1+f(2)&1+f(3)&1+f(4)\end{vmatrix}$$

$$=\begin{vmatrix}1+1+1&1+\alpha+\beta&1+\alpha^2+\beta^2\\1+\alpha+\beta&1+\alpha^2+\beta^2&1+\alpha^3+\beta^3\\1+\alpha^2+\beta^2&1+\alpha^3+\beta^3&1+\alpha^4+\beta^4\end{vmatrix}$$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}\times\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}$$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}^2$$

$=[(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

So, $\boxed{K=1}$

29. **(d)** $BB' = B(A^{-1}A')' = B(A')'(A^{-1})'$

$= BA(A^{-1})'$

$= (A^{-1}A')(A(A^{-1})')$

$= A^{-1}A.A'.(A^{-1})'$ $\{\text{as } AA' = A'A\}$

$= I(A^{-1}A)' = I.I = I^2 = I$

30. **(a)** $\left.\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}\right\}$

$\Rightarrow$ $(2-\lambda)x_1 - 2x_2 + x_3 = 0$
$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$
$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution,
$\Delta = 0$

i.e. $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda)-4]+2[-2\lambda+2]+1[4-(3+\lambda)]=0$

$\Rightarrow \lambda^3+\lambda^2-5\lambda+3=0$

$\Rightarrow \lambda = 1, 1, 3$

Hence λ has 2 values.

31. **(b)** For trivial solution,

$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow -\lambda(\lambda+1)(\lambda-1) = 0$

$\Rightarrow \lambda = 0, +1, -1$

32. **(a)** We have

$\frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\| = 28$

$\Rightarrow 5k^2 + 13k - 46 = 0$

or $5k^2 + 13k + 66 = 0$

Now, $5k^2 + 13k - 46 = 0$

$\Rightarrow$ $k = \dfrac{-13 \pm \sqrt{1089}}{10}$ $\therefore$ $k = \dfrac{-23}{5}; k = 2$

since k is an integer, $\therefore$ k = 2

Also $5k^2 + 13k + 66 = 0$

$\Rightarrow$ $k = \dfrac{-13 \pm \sqrt{-1151}}{10}$

So no real solution exist

For orthocentre

BH $\perp$ AC

$\therefore$ $\left(\dfrac{\beta-2}{\alpha-5}\right)\left(\dfrac{8}{-4}\right) = -1$

$\Rightarrow$ $\alpha - 2\beta = 1$ ...(1)

Also CH $\perp$ AB

$\therefore$ $\left(\dfrac{\beta-2}{\alpha+2}\right)\left(\dfrac{8}{3}\right) = -1$

$\Rightarrow$ $3\alpha + 8\beta = 1$ ... (2)

Solving (1) and (2), we get

$\alpha = 2, \beta = \dfrac{1}{2}$

orthocentre is $\left(21\dfrac{1}{2}\right)$

33. **(a)** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$

$\Rightarrow 1[a-b] - 1[1-a] + 1[b-a^2] = 0$

$\Rightarrow (a-1)^2 = 0$

$\Rightarrow a = 1$

For a = 1, First two equations are identical

ie. $x + y + z = 1$

To have no solution with $x + by + z = 0$

$b = 1$

So $b = \{1\} \Rightarrow$ It is singleton set.

34. **(b)** Given $2\omega + 1 = z$;

$z = \sqrt{3}i$

$\Rightarrow \omega = \dfrac{\sqrt{3}i - 1}{2}$

$\Rightarrow \omega$ is complex cube root of unity

Applying $R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$= 3(-1-\omega-\omega) = -3(1+2\omega) = -3z$

$\Rightarrow k = -z$