CHAPTER 17

Inverse Trigonometric Functions

1. $\cot^{-1}(\sqrt{\cos\alpha})-\tan^{-1}(\sqrt{\cos\alpha})=x$, then $\sin x=$ **[2002]**

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

2. The domain of $\sin^{-1}[\log_3(x/3)]$ is **[2002]**

(a) $[1,9]$ (b) $[-1,9]$

(c) $[-9,1]$ (d) $[-9,-1]$

3. The trigonometric equation $\sin^{-1}x=2\sin^{-1}a$ has a solution for **[2003]**

(a) $|a|\le\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}<|a|<\frac{1}{\sqrt{2}}$

(c) all real values of a (d) $|a|<\frac{1}{2}$

4. The domain of the function

$f(x)=\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) $[1,2]$ (b) $[2,3)$ **[2004]**

(c) $[1,2]$ (d) $[2,3]$

5. If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to **[2005]**

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

6. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then the values of x is **[2007]**

(a) 4 (b) 5

(c) 1 (d) 3.

7. The largest interval lying in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ for which the function,

$f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$, is defined, is **[2007]**

(a) $\left[-\frac{\pi}{4},\frac{\pi}{2}\right)$ (b) $\left[0,\frac{\pi}{2}\right)$

(c) $[0,\pi]$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

8. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$

(c) $\frac{4}{17}$ (d) $\frac{5}{17}$

9. Let $\tan^{-1}y=\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where or $|x|<\frac{1}{\sqrt{3}}$. Then a value of y is : **[2015]**

(a) $\frac{3x-x^3}{1+3x^2}$ (b) $\frac{3x+x^3}{1+3x^2}$

(c) $\frac{3x-x^3}{1-3x^2}$ (d) $\frac{3x+x^3}{1-3x^2}$

10. If for $x\in\left(0,\frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then g(x) equals : **[2017]**

(a) $\frac{3}{1+9x^3}$ (b) $\frac{9}{1+9x^3}$

(c) $\frac{3x\sqrt{x}}{1-9x^3}$ (d) $\frac{3x}{1-9x^3}$

Answer Key

1	2	3	4	5	6	7	8	9	10					
(a)	(a)	(a)	(b)	(c)	(d)	(b)	(a)	(c)	(b)					

SOLUTIONS

1. **(a)** $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow \tan^{-1}\dfrac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$

$\Rightarrow \tan^{-1}\dfrac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x$

$\Rightarrow \tan x = \dfrac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$ OR $\cot x = \dfrac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$

[Considering a Δ with perpendicular

$= (1-\cos\alpha)$ and base $= 2\sqrt{\cos\alpha}$]

$\Rightarrow \sin x = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \dfrac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1}$

or $\sin x = \tan^2\dfrac{\alpha}{2}$

2. **(a)** $f(x) = \sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$ exists

if $-1 \le \log_3\left(\frac{x}{3}\right) \le 1 \Leftrightarrow 3^{-1} \le \frac{x}{3} \le 3^1$

$\Leftrightarrow 1 \le x \le 9$ or $x \in [1, 9]$

3. **(a)** $\sin^{-1}x = 2\sin^{-1}a$

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$; $\therefore -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4}$ or $\frac{-1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\therefore |a| \le \frac{1}{\sqrt{2}}$

4. **(b)** $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined

if (i) $-1 \le x-3 \le 1 \Rightarrow 2 \le x \le 4$

and (ii) $9 - x^2 > 0 \Rightarrow -3 < x < 3$

Taking common solution of (i) and (ii),

we get $2 \le x < 3$ $\therefore$ Domain $= [2, 3)$

5. **(c)** $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$

$\cos^{-1}\left(\dfrac{xy + \sqrt{4-y^2-4x^2+x^2y^2}}{2}\right) = \alpha$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2$

$= 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha\,\cdot$

6. **(d)** $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

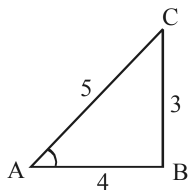
$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \text{cosec}^{-1}\left(\frac{5}{4}\right)$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$[\because \sin^{-1}x + \cos^{-1}x = \pi/2]$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$ ....(i)

Let $\cos^{-1}\frac{4}{5} = A \Rightarrow \cos A = \frac{4}{5}$



$\Rightarrow A = \cos^{-1}(4/5)$

$\Rightarrow \sin A = \dfrac{3}{5}$

$\Rightarrow A = \sin^{-1}\dfrac{3}{5}$

$\therefore \quad \cos^{-1}(4/5) = \sin^{-1}(3/5)$

$\therefore$ equation (i) become,

$\sin^{-1}\dfrac{x}{5} = \sin^{-1}\dfrac{3}{5} \Rightarrow \dfrac{x}{5} = \dfrac{3}{5} \Rightarrow x =$ 3

7. (b) $f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$

$f(x)$ is defined if $-1 \le \left(\dfrac{x}{2} - 1\right) \le 1$ and $\cos x > 0$

or $\quad 0 \le \dfrac{x}{2} \le 2$ and $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

or $\quad 0 \le x \le 4$ and $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

$\therefore \quad x \in \left[0, \dfrac{\pi}{2}\right)$

8. (a) $\cot\left(\operatorname{cosec}^{-1}\dfrac{5}{3} + \tan^{-1}\dfrac{2}{3}\right) =$

$\cot\left[\tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{2}{3}\right]$

$= \cot\left[\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]$

$= \cot\left[\tan^{-1}\dfrac{17}{6}\right]$

$= \cot\left(\cot^{-1}\dfrac{6}{17}\right) = \dfrac{6}{17}$

9. (c) $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left[\dfrac{2x}{1-x^2}\right]$

$= \tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x$

$\tan^{-1}y = \tan^{-1}\left[\dfrac{3x - x^3}{1-3x^2}\right]$

$\Rightarrow y = \dfrac{3x - x^3}{1-3x^2}$

10. (b) Let $F(x) = \tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right)$ where $x \in \left(0, \dfrac{1}{4}\right)$.

$= \tan^{-1}\left(\dfrac{2.(3x^{3/2})}{1-(3x^{3/2})^2}\right) = 2\tan^{-1}(3x^{3/2})$

As $3x^{3/2} \in \left(0, \dfrac{3}{8}\right)$

$\left[\because 0 < x < \dfrac{1}{4} \Rightarrow 0 < x^{3/2} < \dfrac{1}{8} \Rightarrow 0 < 3x^{3/2} < \dfrac{3}{8}\right]$

So $\dfrac{dF(x)}{dx} = 2 \times \dfrac{1}{1+9x^3} \times 3 \times \dfrac{3}{2} \times x^{1/2}$

$= \dfrac{9}{1+9x^3}\sqrt{x}$

On comparing

$\therefore g(x) = \dfrac{9}{1+9x^3}$