CHAPTER 4

# Principle of Mathematical Induction

1. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+......}}}$ having n radical signs then by methods of mathematical induction which is true **[2002]**

(a) $a_n > 7 \ \forall \ n \geq 1$ (b) $a_n < 7 \ \forall \ n \geq 1$

(c) $a_n < 4 \ \forall \ n \geq 1$ (d) $a_n < 3 \ \forall \ n \geq 1$

2. Let $S(K) = 1+3+5...+(2K-1) = 3+K^2$. Then which of the following is true **[2004]**

(a) Principle of mathematical induction can be used to prove the formula

(b) $S(K) \Rightarrow S(K+1)$

(c) $S(K) \nRightarrow S(K+1)$

(d) $S(1)$ is correct

**Answer Key**

| 1 | 2 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (b) | (b) | | | | | | | | | | | | | |

## SOLUTIONS

1. **(b)** $a_1 = \sqrt{7} < 7$. Let $a_m < 7$

Then $a_{m+1} = \sqrt{7+a_m} \Rightarrow a^2_{m+1} = 7 + a_m$

$< 7 + 7 < 14$.

$\Rightarrow a_{m+1} < \sqrt{14} < 7$; So by the principle of mathematical induction $a_n < 7 \ \forall \ n$.

$\therefore$ r = 0,8,16,24,........256 , total 33 values.

2. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3+K^2$

$S(1): 1 = 3+1$, which is not true

$\because$ $S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(K)$ is true, i.e.

$1+3+5....+(2K-1) = 3+K^2$

$\Rightarrow 1+3+5....+(2K-1)+2K+1$

$= 3+K^2+2K+1 = 3+(K+1)^2$

$\therefore S(K) \Rightarrow S(K+1)$