CHAPTER 13

Mathematical Reasoning

DIRECTIONS: Given below question contains two statements: Statement-1(Assertion) and Statement-2(Reason). This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

1. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number". **[2008]**

Statement-1 : r is equivalent to either q or p

Statement-2 : r is equivalent to $\sim(p \leftrightarrow \sim q)$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

2. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to **[2008]**

(a) $p \rightarrow (p \rightarrow q)$ (b) $p \rightarrow (p \vee q)$

(c) $p \rightarrow (p \wedge q)$ (d) $p \rightarrow (p \leftrightarrow q)$

DIRECTIONS: Given below question contains two statements: Statement-1(Assertion) and Statement 2(Reason). This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

3. **Statement-1 :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement-2 : $\sim(p \leftrightarrow \sim q)$ is a tantology **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

4. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x \in$ S such that $x > 0$.

Which of the following statements is the negation of the statement P ? **[2010]**

(a) There is no rational number $x \in$ S such than $x \leq 0$.

(b) Every rational number $x \in$ S satisfies $x \leq 0$.

(c) $x \in$ S and $x \leq 0 \Rightarrow x$ is not rational.

(d) There is a rational number $x \in$ S such that $x \leq 0$.

5. Consider the following statements **[2011]**

***P* :** Suman is brilliant

***Q* :** Suman is rich

***R* :** Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$

(b) $\sim Q \leftrightarrow \sim P \wedge R$

(c) $\sim(P \wedge \sim R) \leftrightarrow Q$

(d) $\sim P \wedge (Q \leftrightarrow \sim R)$

6. The only statement among the following that is a tautology is **[2011RS]**

(a) $A \wedge (A \vee B)$

(b) $A \vee (A \wedge B)$

(c) $[A \wedge (A \to B)] \to B$

(d) $B \to [A \wedge (A \to B)]$

7. The negation of the statement "If I become a teacher, then I will open a school", is : **[2012]**

(a) I will become a teacher and I will not open a school.

(b) Either I will not become a teacher or I will not open a school.

(c) Neither I will become a teacher nor I will open a school.

(d) I will not become a teacher or I will open a school.

8. Consider

Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

9. The statement $\sim(p \leftrightarrow \sim q)$ is: **[2014]**

(a) a tautology

(b) a fallacy

(c) equavalent to $p \leftrightarrow q$

(d) equivalent to $\sim p \leftrightarrow q$

10. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[2015]**

(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$

(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

11. The Boolean Expression

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to: **[2016]**

(a) $p \vee q$ (b) $p \vee \sim q$

(c) $\sim p \wedge q$ (d) $p \wedge q$

12. The following statement **[2017]**

$(p \to q) \to [(\sim p \to q) \to q]$ is :

(a) a fallacy

(b) a tautology

(c) equivalent to $\sim p \to q$

(d) equivalent to $p \to \sim q$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12			
None	(b)	(b)	(b)	(a)	(c)	(a)	(b)	(c)	(b)	(a)	(b)			

SOLUTIONS

1. **(None)**

p : x is an irrational number

q : y is a transcendental number

r : x is a rational number iff y is a transcendental number.

clearly $r : \sim p \leftrightarrow q$

Let us use truth table to check the equivalence of 'r' and 'q or p'; 'r' and $\sim(p \leftrightarrow \sim q)$

p	q	~p	~q	1 ~p ↔q	2 q or p	p↔~q	3 ~(p↔~q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

From columns (1), (2) and (3), we observe, none of the these statements are equivalent to each other.

∴ Statement 1as well as statement 2 both are false.

∴ None of the options is correct.

2. **(b)** Let us make the truth table for the given statements, as follows :

p	q	p∨q	q→p	p →(q→p)	p →(p∨q)
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe

$p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

3. (b) The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns for $\sim(p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are identical, therefore $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim(p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

$\therefore$ Statement-1 is true but statement-2 is false.

4. (b) P : there is a rational number $x \in S$ such that $x > 0$

$\sim$P : Every rational number $x \in S$ satisfies $x \le 0$

5. (a) Suman is brilliant and dishonest if and only if Suman is rich is expressed as

$Q \leftrightarrow (P \wedge \sim R)$

Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$

6. (c)

A	B	A ∨ B	A ∧ B	A ∧ (A ∨ B)	A ∨ (A ∧ B)	A → B	A ∧ (A → B)	[A ∧ (A → B) → B]	[B → [A ∧ (A → B)]
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

$\therefore$ It is tautology.

7. (a) Let p : I become a teacher.

q : I will open a school

Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

8. (b) **Statement-2 :** $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true.

So statement 2 is true

Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$= p \wedge \sim q \wedge \sim p \wedge q$

$= p \wedge \sim p \wedge \sim q \wedge q$

$= f \wedge f = f$

So statement-1 is true

9. (c)

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
F	F	T	F	T
F	T	F	T	F
T	F	T	T	F
T	T	F	F	T

Clearly equivalent to $p \leftrightarrow q$

10. (b) $\sim[\sim s \vee (\sim r \wedge s)]$

$= s \wedge \sim(\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= (s \wedge r) \vee 0$

$= s \wedge r$

11. (a) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

40. (1) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge T\} \vee (\sim p \wedge q)$

$\Rightarrow (p \vee q) \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q)$

$\Rightarrow T \wedge (p \vee q)$

$\Rightarrow p \vee q$

12. (b) We have

p	q	~ p	p → q	~ p → q	(~ p → q) → q	(p → q) → ((~p → q) → q)
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

$\therefore$ It is tautology.