CHAPTER 6

Linear Inequality

1. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is **[2002]**
   (a) less than 1 (b) equal to 1
   (c) greater than 1 (d) any real no.

2. If x is real, the maximum value of $\dfrac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**
   (a) $\dfrac{1}{4}$ (b) 41
   (c) 1 (d) $\dfrac{17}{7}$

3. **Statement-1 :** For every natural number $n \geq 2$,
$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+........+\frac{1}{\sqrt{n}} > \sqrt{n}.$$
Statement-2 : For every natural number $n \geq 2$,
$\sqrt{n(n+1)} < n+1.$ **[2008]**
   (a) Statement -1 is false, Statement-2 is true
   (b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
   (c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
   (d) Statement -1 is true, Statement-2 is false

Answer Key

1	2	3										
(a)	(b)	(b)										

SOLUTIONS

1. **(a)** $\because (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$
$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) > 0$
$\Rightarrow 2 > 2(ab+bc+ca) \Rightarrow ab+bc+ca < 1$

2. **(b)** $y = \dfrac{3x^2+9x+17}{3x^2+9x+7}$
$3x^2(y-1)+9x(y-1)+7y-17=0$
$D \geq 0 \ \because$ x is real
$81(y-1)^2 - 4\times 3(y-1)(7y-17) \geq 0$
$\Rightarrow (y-1)(y-41) \leq 0 \Rightarrow 1 \leq y \leq 41$

$\therefore$ Max value of y is 41

ALTERNATE SOLUTION

Given $f(x) = \dfrac{3x^2+9x+17}{3x^2+9x+7}$

$\Rightarrow f(x) = 1 + \dfrac{10}{3x^2+9x+7}$

Clearly $f(x)$ is maximum when $g(x) = 3x^2 + 9x + 7$ is min.

Here $g(x) = 3\left(x^2+3x+\dfrac{9}{4}\right)+7-\dfrac{27}{4}$

$$= 3\left(x + \frac{3}{2}\right)^2 + \frac{1}{4}$$

which is minimum when $x = \frac{-3}{2}$

$$\therefore\ f_{\max} = 1 + \frac{10}{3 \times \frac{9}{4} - 9 \times \frac{3}{2} + 7} =$$

$$1 + \frac{10 \times 4}{27 - 54 + 28} = 41$$

3. **(b)** Statement 2 is $\sqrt{n(n+1)} < n + 1, n \geq 2$

$\Rightarrow \sqrt{n} < \sqrt{n+1}, n \geq 2$ which is true

$\Rightarrow \sqrt{2} < \sqrt{3} < \sqrt{4} < \sqrt{5} < \text{--------} \sqrt{n}$

Now $\sqrt{2} < \sqrt{n} \Rightarrow \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}$

$\sqrt{3} < \sqrt{n} \Rightarrow \frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}$;

$\sqrt{n} \leq \sqrt{n} \Rightarrow \frac{1}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$

Also $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}$ $\therefore$ Adding all, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots\ldots\ldots + \frac{1}{n} > \frac{n}{\sqrt{n}} = \sqrt{n}$$

Hence both the statements are correct and statement 2 is a correct explanation of statement-1.