CHAPTER 22

Integrals

1. $\int_0^{10\pi} |\sin x|\,dx$ is **[2002]**
(a) 20 (b) 8
(c) 10 (d) 18

2. $I_n = \int_0^{\pi/4} \tan^n x\,dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals **[2002]**
(a) $\frac{1}{2}$ (b) 1
(c) ∞ (d) zero

3. $\int_0^2 [x^2]dx$ is **[2002]**
(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
(c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

4. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is **[2002]**
(a) $\frac{\pi^2}{4}$ (b) π^2
(c) zero (d) $\frac{\pi}{2}$

5. If $f(a+b-x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to **[2003]**
(a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$
(b) $\frac{a+b}{2}\int_a^b f(b-x)dx$
(c) $\frac{a+b}{2}\int_a^b f(x)dx$
(d) $\frac{b-a}{2}\int_a^b f(x)dx$.

6. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\,g(x)dx$, is **[2003]**
(a) $e+\frac{e^2}{2}+\frac{5}{2}$ (b) $e-\frac{e^2}{2}-\frac{5}{2}$
(c) $e+\frac{e^2}{2}-\frac{3}{2}$ (d) $e-\frac{e^2}{2}-\frac{3}{2}$.

7. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is **[2003]**
(a) $\frac{1}{n+1}+\frac{1}{n+2}$ (b) $\frac{1}{n+1}$
(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1}-\frac{1}{n+2}$.

8. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is **[2004]**
(a) $e+1$ (b) $e-1$
(c) $1-e$ (d) e

9. The value of $\int_{-2}^{3} |1-x^2|dx$ is **[2004]**
(a) $\frac{1}{3}$ (b) $\frac{14}{3}$
(c) $\frac{7}{3}$ (d) $\frac{28}{3}$

10. The value of $I = \int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is **[2004]**
(a) 3 (b) 1
(c) 2 (d) 0

11. If $\int_0^{\pi} x f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is **[2004]**

(a) 2π (b) π

(c) $\frac{\pi}{4}$ (d) 0

12. If $f(x)=\frac{e^x}{1+e^x}$, $I_1=\int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2=\int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is **[2004]**

(a) 1 (b) –3

(c) –1 (d) 2

13. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha), +C,$ then value of (A, B) is **[2004]**

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

14. $\int \frac{dx}{\cos x - \sin x}$ is equal to **[2004]**

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

15. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to **[2005]**

(a) $\frac{\log x}{(\log x)^2+1}+C$ (b) $\frac{x}{x^2+1}+C$

(c) $\frac{xe^x}{1+x^2}+C$ (d) $\frac{x}{(\log x)^2+1}+C$

16. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then **[2005]**

(a) $I_2 > I_1$ (b) $I_1 > I_2$

(c) $I_3 = I_4$ (d) $I_3 > I_4$

17. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, $a > 0$, is **[2005]**

(a) $a\pi$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{a}$ (d) 2π

18. The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is **[2006]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 2 (d) 1

19. $\int_0^{\pi} xf(\sin x)dx$ is equal to **[2006]**

(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$

20. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}[(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to **[2006]**

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}-1$

21. The value of $\int_1^a [x] f'(x)dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is **[2006]**

(a) $af(a) - \{f(1) + f(2) + ...f([a])\}$

(b) $[a]f(a) - \{f(1) + f(2) + ...f([a])\}$

(c) $[a]f([a]) - \{f(1) + f(2) + ...f(a)\}$

(d) $af([a]) - \{f(1) + f(2) + ...f(a)\}$

22. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals **[2007]**

(a) $\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

(b) $\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

23. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_l^x \frac{\log t}{1+t}dt$, Then $F(e)$ equals **[2007]**

(a) 1 (b) 2

(c) 1/2 (d) 0,

24. The solution for x of the equation

$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is **[2007]**

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) 2 (d) None of these

25. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true? **[2008]**

(a) $I > \frac{2}{3}$ and $J > 2$ (b) $I < \frac{2}{3}$ and $J < 2$

(c) $I < \frac{2}{3}$ and $J > 2$ (d) $I > \frac{2}{3}$ and $J < 2$

26. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is **[2008]**

(a) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

(b) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

(c) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

(d) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

27. $\int_0^{\pi} [\cot x]dx$, where [.] denotes the greatest integer function, is equal to : **[2009]**

(a) 1 (b) –1

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

28. Let $p(x)$ be a function defined on **R** such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals **[2010]**

(a) 21 (b) 41

(c) 42 (d) $\sqrt{41}$

29. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is **[2011]**

(a) $\frac{\pi}{8}\log 2$ (b) $\frac{\pi}{2}\log 2$

(c) $\log 2$ (d) $\pi \log 2$

30. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x\left[x^2\right]dx$ is :. **[2011 RS]**

(a) 0 (b) $\frac{3}{2}$

(c) $\frac{3}{4}$ (d) $\frac{5}{4}$

31. If the $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to : **[2012]**

(a) – 1 (b) – 2

(c) 1 (d) 2

32. If $g(x)=\int_0^x \cos 4t\,dt$, then $g(x+\pi)$ equals **[2012]**

(a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(p)$

(c) $g(x)-g(p)$ (d) $g(x).g(p)$

33. If $\int f(x)dx=\psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[2013]**

(a) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$

(b) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(c) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

34. **Statement-1 :** The value of the integral

$\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$

Statement-2 : $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx.$ **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

35. The intercepts on x-axis made by tangents to thecurve, $y=\int_0^x |t|\,dt,\ x\in R,$ which are parallel to the line $y=2x$, are equal to : **[2013]**

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

36. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[2014]**

(a) $(x+1)e^{x+\frac{1}{x}}+c$ (b) $-xe^{x+\frac{1}{x}}+c$

(c) $(x-1)e^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

37. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals: **[2014]**

(a) $4\sqrt{3}-4$ (b) $4\sqrt{3}-4-\frac{\pi}{3}$

(c) $\pi-4$ (d) $\frac{2\pi}{3}-4-4\sqrt{3}$

38. The integral $\int\frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[2015]**

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

39. The integral $\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to : **[2015]**

(a) 1 (b) 6

(c) 2 (d) 4

40. The integral $\int\frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[2016]**

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\dfrac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(d) $\dfrac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.

41. The integral $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : **[2017]**

(a) –1 (b) –2

(c) 2 (d) 4

42. Let $I_n = \int \tan^n x\, dx, (n>1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : **[2017]**

(a) $\left(-\frac{1}{5}, 0\right)$ (b) $\left(-\frac{1}{5}, 1\right)$

(c) $\left(\frac{1}{5}, 0\right)$ (d) $\left(\frac{1}{5}, -1\right)$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(d)	(b)	(c)	(d)	(d)	(b)	(d)	(c)	(b)	(d)	(b)	(a)	(d)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(b)	(b)	(b)	(d)	(c)	(b)	(c)	(c)	(d)	(b)	(c)	(c)	(a)	(d)	(c)
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**			
(d)	(b, c)	(c)	(d)	(a)	(d)	(b)	(b)	(a)	(d)	(c)	(c)			

SOLUTIONS

1. **(a)** $I = \displaystyle\int_0^{10\pi} |\sin x|\, dx = 10\int_0^{\pi} |\sin x|\, dx$

$= 10\displaystyle\int_0^{\pi} \sin x\, dx$

[$\because$ $|\sin x|$ is periodic with period π and $\sin x > 0$ if $0 < x < \pi$]

$I = 20\displaystyle\int_0^{\pi/2} \sin x\, dx = 20[-\cos x]_0^{\pi/2} = 20$

2. **(b)** $I_n + I_{n+2} = \displaystyle\int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx$

$= \displaystyle\int_0^{\pi/4} \tan^n x \sec^2 x\, dx = \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\pi/4}$

$= \dfrac{1-0}{n+1} = \dfrac{1}{n+1}$

$\therefore\ I_n + I_{n+2} = \dfrac{1}{n+1} \Rightarrow \lim\limits_{n\to\infty} n\,[I_n + I_{n+2}]$

$= \lim\limits_{n\to\infty} n.\dfrac{1}{n+1} = \lim\limits_{n\to\infty} \dfrac{n}{n+1}$

$= \lim\limits_{n\to\infty} \dfrac{n}{n\left(1+\frac{1}{n}\right)} = 1$

3. **(d)** $\displaystyle\int_0^2 \left[x^2\right] dx = \int_0^1 \left[x^2\right] dx + \int_1^{\sqrt{2}} \left[x^2\right] dx + \int_{\sqrt{2}}^{\sqrt{3}} \left[x^2\right] + \int_{\sqrt{3}}^{2} \left[x^2\right] dx$

$= \displaystyle\int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$

$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$

$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$= 5 - \sqrt{3} - \sqrt{2}$

4. (b) $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$

$= \int_{-\pi}^{\pi} \frac{2x\, dx}{1+\cos^2 x} + 2\int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$= 0 + 4\int_0^{\pi} \frac{x \sin x\, dx}{1+\cos^2 x}$; $\left[\because \int_{-a}^{a} f(x)\, dx = 0\right.$

if f(x) is odd

$\left. = 2\int_0^{a} f(x)\, dx \text{ if } f(x) \text{ is even.}\right]$

$I = 4\int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$

$I = 4\int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$

$\Rightarrow I = 4\pi\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x} - 4\int \frac{x \sin x\, dx}{1+\cos^2 x}$

$\Rightarrow 2I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$

put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\therefore I = -2\pi \int_1^{-1} \frac{1}{1+t^2} dt = 2\pi \int_{-1}^{1} \frac{1}{1+t^2} dt$

$= 2\pi\left[\tan^{-1} t\right]_{-1}^{1}$

$= 2\pi\left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$

$= 2\pi\left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] = 2\pi . \frac{\pi}{2} = \pi^2$

5. (c) $I = \int_a^b x f(x) dx = \int_a^b (a+b-x) f(a+b-x) dx$

$= (a+b)\int_a^b f(a+b-x) dx - \int_a^b x f(a+b-x) dx$

$= (a+b)\int_a^b f(x)\, dx - \int_a^b x f(x) dx$

[$\because$ given that $f(a+b-x) = f(x)$]

$2I = (a+b)\int_a^b f(x) dx$

$\Rightarrow \quad I = \frac{(a+b)}{2}\int_a^b f(x) dx$

6. (d) Given $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating

$\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$

$f(0) = 1 \Rightarrow f(x) = e^x$

$\therefore \int_0^1 f(x) g(x) dx = \int_0^1 e^x (x^2 - e^x) dx$

$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$

$= \left[x^2 e^x\right]_0^1 - 2\left[x e^x - e^x\right]_0^1 - \frac{1}{2}\left[e^{2x}\right]_0^1$

$= e - \left[\frac{e^2}{2} - \frac{1}{2}\right] - 2\left[e - e + 1\right] = e - \frac{e^2}{2} - \frac{3}{2}$

7. (d) $I = \int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-1+x)^n dx$

$= \int_0^1 (1-x) x^n dx = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$

$= \frac{1}{n+1} - \frac{1}{n+2}$

8. (b) $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ [Using definite integrals as limit of sum]

$= \int_0^1 e^x dx = e - 1$

9. (d) $\int_{-2}^{3} |1 - x^2|\, dx = \int_{-2}^{3} |x^2 - 1|\, dx$

Now $|x^2 - 1| = \begin{cases} x^2 - 1 & \text{if} \quad x \le -1 \\ 1 - x^2 & \text{if} \quad -1 \le x \le 1 \\ x^2 - 1 & \text{if} \quad x \ge 1 \end{cases}$

$\therefore$ Integral is

$$\int_{-2}^{-1}(x^2-1)dx+\int_{-1}^{1}(1-x^2)dx+\int_{1}^{3}(x^2-1)dx$$

$$=\left[\frac{x^3}{3}-x\right]_{-2}^{-1}+\left[x-\frac{x^3}{3}\right]_{-1}^{1}+\left[\frac{x^3}{3}-x\right]_{1}^{3}$$

$$=\left(-\frac{1}{3}+1\right)-\left(-\frac{8}{3}+2\right)+\left(\frac{27}{3}-3\right)-\left(\frac{1}{3}-1\right)$$

$$=\frac{2}{3}+\frac{2}{3}+\frac{4}{3}+6+\frac{2}{3}=\frac{28}{3}$$

10. **(c)** $I=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$

We know $[(\sin x+\cos x)^2=1+\sin 2x]$, so

$$I=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{(\sin x+\cos x)}dx$$

$$=\int_0^{\frac{\pi}{2}}(\sin x+\cos x)dx$$

$$\left[\because \sin x+\cos x>0 \ if\ 0<x<\frac{\pi}{2}\right]$$

or $I=[-\cos x+\sin x]_0^{\frac{\pi}{2}}=2$

11. **(b)** Let $I=\int_0^{\pi}xf(\sin x)dx$

$$=\int_0^{\pi}(\pi-x)f(\sin x)dx$$

$$\therefore 2I=\pi\int_2^{\pi}f(\sin x)dx \ =\pi.2\int_0^{\frac{\pi}{2}}f(\sin x)dx$$

$$\therefore I=\pi\int_0^{\frac{\pi}{2}}f(\sin x)dx\Rightarrow A=\pi$$

12. **(d)** $f(x)=\frac{e^x}{1+e^x}\Rightarrow f(-x)=\frac{e^{-x}}{1+e^{-x}}$

$$=\frac{1}{e^x+1}$$

$\therefore f(x)+f(-x)=1\ \forall\ x$

Now $I_1=\int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$

$$=\int_{f(-a)}^{f(a)}(1-x)g\{x(1-x)\}dx$$

$$\left[\text{using}\int_a^b f(x)\,dx\ a=\int_a^b f(a+b-x)\,dx\right]$$

$$=I_2-I_1\Rightarrow 2I_1=I_2$$

13. **(b)** $\int\frac{\sin x}{\sin(x-\alpha)}dx=\int\frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$$=\int\frac{\sin(x-\alpha)\cos\alpha+\cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$$

$$=\int\{\cos\alpha+\sin\alpha\cot(x-\alpha)\}dx$$

$$=(\cos\alpha)x+(\sin\alpha)\log\sin(x-\alpha)+C$$

$\therefore A=\cos\alpha,\ B=\sin\alpha$

14. **(a)** $\int\frac{dx}{\cos x-\sin x}=\int\frac{dx}{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}$

$$=\frac{1}{\sqrt{2}}\int\sec\left(x+\frac{\pi}{4}\right)dx$$

$$=\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{\pi}{8}\right)\right|+C$$

$$\left[\because\int\sec x\,dx=\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|\right]$$

$$=\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$$

15. **(d)** $\int\frac{(\log x-1)^2}{(1+(\log x)^2)^2}dx$

$$=\int\frac{1+(\log x)^2-2\log x}{\left[1+(\log x)^2\right]^2}dx$$

$$=\int\left[\frac{1}{(1+(\log x)^2)}-\frac{2\log x}{(1+(\log x)^2)^2}\right]dx$$

$$= \int \left[\frac{e^t}{1+t^2} - \frac{2t\,e^t}{(1+t^2)^2}\right]dt \quad \text{put } \log x = t$$

$$\Rightarrow \quad dx = e^t\,dt$$

$$= \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]dt$$

$$\left[\text{Which is of the form } \int e^x \left(f(x) + f'(x)\right)dx\right]$$

$$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\log x)^2} + c$$

16. (b) $I_1 = \int_0^1 2^{x^2}dx,\ I_2 = \int_0^1 2^{x^3}dx,$

$$I_3 = \int_0^1 2^{x^2}dx,\ I_4 = \int_0^1 2^{x^3}dx \ \forall 0 < x < 1,\ x^2 > x^3$$

$$\Rightarrow \int_0^1 2^{x^2}dx > \int_0^1 2^{x^3}dx \ \Rightarrow \ I_1 > I_2$$

17. (b) Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$ ....(1)

$$= \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}}dx$$

$$\left[\text{Using } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$= \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx \quad ....(2)$$

Adding equations (1) and (2) we get

$$2I = \int_{-\pi}^{\pi} \cos^2 x\left(\frac{1+a^x}{1+a^x}\right)dx = \int_{-\pi}^{\pi} \cos^2 x\,dx$$

$$= 2\int_0^{\pi} \cos^2 x\,dx$$

$$= 2\times 2\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = 4\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = 2\int_0^{\frac{\pi}{2}} \left(1-\cos^2 x\right)dx$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} dx - 2\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

$$\Rightarrow I + I = 2\left(\frac{\pi}{2}\right) = \pi \ \Rightarrow \mathrm{I} = \frac{\pi}{2}$$

18. (b) $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ ...(1)

$$I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx \quad ...(2)$$

$$[\text{ using } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\ ]$$

Adding equation (1) and (2)

$$2I = \int_3^6 dx = 3 \ \Rightarrow \ I = \frac{3}{2}$$

19. (d) $I = \int_0^{\pi} xf(\sin x)dx = \int_0^{\pi} (\pi - x)f(\sin x)dx$

$$= \pi\int_0^{\pi} f(\sin x)dx - I$$

$$\Rightarrow 2I = \pi\int_0^{\pi} f(\sin x)dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = \pi\int_0^{\pi/2} f(\sin x)dx$$

$$= \pi\int_0^{\pi/2} f(\cos x)dx$$

20. (c) $I = \int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$

Put $x + \pi = t$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [t^3 + \cos^2 t]dt = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt$$

[using the property of even and odd function]

$$= \int_0^{\frac{\pi}{2}} (1+\cos 2t)dt = \frac{\pi}{2} + 0$$

21. (b) Let $a = k + h$ where k is an integer such that $[a] = k$ and $0 \le h < 1$

$$\therefore \int_1^a [x] f'(x) dx = \int_1^2 1 f'(x) dx + \int_2^3 2 f'(x) dx +$$

$$\ldots \int_{k-1}^{k} (k-1) dx + \int_{k}^{k+h} k f'(x) dx$$

$= \{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\} + \ldots\ldots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\}$

$= -f(1) - f(2) - f(3) \ldots\ldots - f(k) + kf(k+h)$

$= [a] f(a) - \{f(1) + f(2) + f(3) + \ldots f([a])\}$

22. (c) $I = \int \frac{dx}{\cos x + \sqrt{3} \sin x}$

$$\Rightarrow \quad I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$$

$$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]}$$

$$= \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$$

$$\Rightarrow I = \frac{1}{2}.\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right) dx$$

But we know that

$$\int \operatorname{cosec} x\, dx = \log|(\tan x/2)| + C$$

$$\therefore \quad I = \frac{1}{2}.\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

23. (c) Given $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where

$$f(x) = \int_1^x \frac{\log t}{1+t} dt$$

$$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt \quad \ldots.(A)$$

Now for solving, $I = \int_1^{1/e} \frac{\log t}{1+t} dt$

$\therefore$ Put $\frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$

and limit for $t = 1 \Rightarrow z = 1$ and for $t = 1/e \Rightarrow z = e$

$$\therefore \quad I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1 + \frac{1}{z}}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e \frac{(\log 1 - \log z).z}{z+1}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e -\frac{\log z}{(z+1)}\left(-\frac{dz}{z}\right) \qquad [\because \log 1 = 0]$$

$$= \int_1^e \frac{\log z}{z(z+1)} dz$$

$$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$$

[By property $\int_a^b f(t) dt = \int_a^b f(x) dx$]

Equation (A) becomes

$$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{t.\log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$$

Let $\log t = x$ $\qquad \therefore \frac{1}{t} dt = dx$

[for limit $t = 1, x = 0$ and $t = e, x = \log e = 1$]

$$\therefore F(e) = \int_0^1 x\, dx \quad F(e) = \left[\frac{x^2}{2}\right]_0^1$$

$$\Rightarrow F(e) = \frac{1}{2}$$

24. (d) $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$

$$\therefore \left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$$

$$\left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x\right]$$

$$\Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1}x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1}x = \frac{\pi}{2}+\frac{\pi}{4}$$

$$\Rightarrow \sec^{-1}x = \frac{3\pi}{4} \Rightarrow x = \sec\frac{3\pi}{4}$$

$$\Rightarrow x = -\sqrt{2}$$

25. **(b)** We know that $\frac{\sin x}{x} < 1$, for $x \in (0, 1)$

$$\Rightarrow \frac{\sin x}{\sqrt{x}} < \sqrt{x} \text{ on } x \in (0,1)$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \sqrt{x}dx = \left[\frac{2x^{3/2}}{3}\right]_0^1$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \frac{2}{3} \Rightarrow I < \frac{2}{3}$$

Also $\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$ for $x \in (0, 1)$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 x^{-1/2}dx = \left[2\sqrt{x}\right]_0^1 = 2$$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}dx < 2 \Rightarrow J < 2$$

26. **(c)** Let $I = \sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ put $x-\frac{\pi}{4} = t$

$$\Rightarrow dx = dt \Rightarrow I = \sqrt{2}\int \frac{\sin\left(t+\frac{\pi}{4}\right)}{\sin t}dt$$

$$= \frac{\sqrt{2}}{\sqrt{2}}\int\left(\frac{\sin t+\cos t}{\sin t}\right)dt$$

$$\Rightarrow I = \int(1+\cot t)dt = t + \log|\sin t| + c_1$$

$$= x - \frac{\pi}{4} + \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c_1$$

$$= x + \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c \quad \left(\text{where } c = c_1 - \frac{\pi}{4}\right)$$

27. **(c)** Let $I = \int_0^\pi [\cot x]\, dx$ ....(1)

$$= \int_0^\pi [\cot(\pi - x)]\, dx = \int_0^\pi [-\cot x]\, dx \quad ....(2)$$

Adding two values of I in eq^n s (1) & (2), We get

$$2I = \int_0^\pi ([\cot x]+[-\cot x])dx$$

$$= \int_0^\pi (-1)dx$$

$[\because [x]+[-x] = -1,$ if $x \notin z$ and $[x]+[-x] = 0,$ if $x \in z]$

$$= [-x]_0^\pi = -\pi \Rightarrow I = -\frac{\pi}{2}$$

28. **(a)** $p'(x) = p'(1-x)$

$\Rightarrow p(x) = -p(1-x) + c$

at $x = 0$

$p(0) = -p(1) + c \Rightarrow 42 = c$

Now, $p(x) = -p(1-x) + 42$

$\Rightarrow p(x) + p(1-x) = 42$

$$\Rightarrow I = \int_0^1 p(x)dx \quad ...(i)$$

$$\Rightarrow I = \int_0^1 p(1-x)dx \quad ...(ii)$$

on adding (i) and (ii),

$$2I = \int_0^1 (42)dx \Rightarrow I = 21$$

29. **(d)** $I = \int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Put $x = \tan\theta$,

$$\therefore \frac{dx}{d\theta} = \sec^2\theta \Rightarrow dx = \sec^2\theta d\theta$$

$$\therefore I = 8\int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta\, d\theta$$

$$I = 8\int_0^{\pi/4} \log(1+\tan\theta)d\theta \quad ...(i)$$

$$= 8\int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$= 8\int_0^{\pi/4} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$= 8\int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right]d\theta$$

$$= 8\int_0^{\pi/4} [\log 2 - \log(1+\tan\theta)]d\theta$$

$$I = 8.(\log 2)[x]_0^{\pi/4} - 8\int_0^{\pi/4} \log(1+\tan\theta)d\theta$$

$$I = 8.\frac{\pi}{4}.\log 2 - I \quad \text{[From equation (i)]}$$

$\Rightarrow\ 2I = 2\pi\log 2$

$\therefore\ I = \pi\log 2$

30. (c)

$$\int_0^{1.5} x\left[x^2\right]dx = \int_0^1 x[x^2]dx + \int_1^{\sqrt{2}} x\left[x^2\right]dx + \int_{\sqrt{2}}^{1.5} x\left[x^2\right]dx$$

$$= \int_0^1 x.0\,dx + \int_1^{\sqrt{2}} xdx + \int_{\sqrt{2}}^{1.5} 2xdx$$

$$= 0 + \left[\frac{x^2}{2}\right]_1^{\sqrt{2}} + \left[x^2\right]_{\sqrt{2}}^{1.5}$$

$$= \frac{1}{2}(2-1) + (2.25-2) \quad = \frac{1}{2} + 0.25$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

31. (d) $\displaystyle\int \frac{5\tan x}{\tan x - 2}dx = \int \frac{5\dfrac{\sin x}{\cos x}}{\dfrac{\sin x}{\cos x} - 2}dx$

$$= \int\left(\frac{5\sin x}{\cos x} \times \frac{\cos x}{\sin x - 2\cos x}\right)dx$$

$$= \int \frac{5\sin x\,dx}{\sin x - 2\cos x}$$

$$= \int\left(\frac{4\sin x + \sin x + 2\cos x - 2\cos x}{\sin x - 2\cos x}\right)dx$$

$$= \int \frac{(\sin x - 2\cos x) + (4\sin x + 2\cos x)}{\sin x - 2\cos x}dx$$

$$= \int \frac{(\sin x - 2\cos x) + 2(\cos x + 2\sin x)}{(\sin x - 2\cos x)}dx$$

$$= \int \frac{\sin x - 2\cos x}{\sin x - 2\cos x}dx + 2\int\left(\frac{\cos x + 2\sin x}{\sin x - 2\cos x}\right)dx$$

$$= \int dx + 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x}dx$$

$= I_1 + I_2$

where

$$I_1 = \int dx \text{ and } I_2 = 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x}dx$$

put $\sin x - 2\cos x = t$

$\Rightarrow\quad (\cos x + 2\sin x)\,dx = dt$

$$\therefore\ I_2 = 2\int \frac{dt}{t} = 2\ln t + C$$

$= 2\,ln\,(\sin x - 2\cos x) + C$

Hence,

$$I_1 + I_2 = \int dx + 2\ln(\sin x - 2\cos x) + c$$

$= x + 2ln\,|(\sin x - 2\cos x)| + k \Rightarrow a = 2$

32. (b, c) $\displaystyle g(x+\pi) = \int_0^{x+\pi} \cos 4t\,dt$

$$= \int_0^x \cos 4t\,dt + \int_x^{\pi+x} \cos 4t\,dt$$

$$= g(x) + \int_0^{\pi} \cos 4t\,dt$$

(from graph of cos 4t, it is clear that

$$\int_x^{\pi+x} \cos 4t\,dt = \int_0^{\pi} \cos 4t\,dt$$

$= g(x) + g(\pi) = g(x) - g(\pi)$

($\because$ from graph of cos 4t, $g(\pi) = 0$)

33. (c) Let $\int f(x)dx = \psi(x)$

Let $I = \int x^5 f(x^3)dx$

put $x^3 = t$

$\Rightarrow\quad 3x^2dx = dt$

$$I = \frac{1}{3}\int 3 \cdot x^2 \cdot x^3 \cdot f(x^3) \cdot dx$$

$$= \frac{1}{3}\int tf(t)dt \ = \frac{1}{3}\left[t\int f(t)dt - \int f(t)dt\right]$$

$$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$$

$$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$$

$$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$$

34. **(d)** Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\, dx}{1+\sqrt{\tan x}}$ ...(1)

Also, given

$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\, dx}{1+\sqrt{\tan x}}$ ...(2)

By adding (1) and (2), we get

$2I = \int_{\pi/6}^{\pi/3} dx$

$\Rightarrow \quad I = \frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right] = \frac{\pi}{12},$

statement-1 is false

$\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

It is fundamental property.

35. **(a)** Since, $y = \int_0^x |t|\,dt,\ x \in R$

therefore $\frac{dy}{dx} = |x|$

But from $y = 2x,\ \frac{dy}{dx} = 2$

$\Rightarrow \quad |x| = 2 \quad \Rightarrow \quad x = \pm 2$

Points $y = \int_0^{\pm 2} |t|\,dt = \pm 2$

$\therefore$ equation of tangent is
$y-2 = 2(x-2)$ or $y+2 = 2(x+2)$
$\Rightarrow$ x-intercept $= \pm 1$.

36. **(d)** Let $I = \int\left(1+x-\frac{1}{x}\right)e^{x+1/x}dx$

$= \int e^{x+1/x}dx + \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+1/x} - \int x\left(1-\frac{1}{x^2}\right)e^{x+1/x}dx$

$+\int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+\frac{1}{x}} - \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$+\int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= xe^{x+\frac{1}{x}} + C$

37. **(b)** Let $I = \int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$= \int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx$

$= \int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx + \int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$

$\left[\because \ \sin\frac{x}{2} = \frac{1}{2} \Rightarrow \frac{x}{2} = \frac{\pi}{6}\right.$

$\left.\Rightarrow x = \frac{\pi}{3}, \frac{x}{2} = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{3}\right]$

$= \left[x+4\cos\frac{x}{2}\right]_0^{\pi/3} + \left[-4\cos\frac{x}{2}-x\right]_{\pi/3}^{\pi}$

$= \frac{\pi}{3}+4\frac{\sqrt{3}}{2}-4+\left(0-\pi+4\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)$

$= 4\sqrt{3}-4-\frac{\pi}{3}$

38. **(b)** $I = \int\frac{dx}{x^2(x^4+1)^{3/4}} = \int\frac{dx}{x^3(1+x^{-4})^{3/4}}$

Let $x^{-4} = y$
$\Rightarrow -4x^{-3}\,dx = dy$

$\Rightarrow dx = \frac{-1}{4}x^3 dy$

$\therefore I = \frac{-1}{4}\int\frac{x^3 dy}{x^3(1+y)^{3/4}} = \frac{-1}{4}\int\frac{dy}{(1+y)^{3/4}}$

$$= \frac{-1}{4} \times 4(1+y)^{1/4} = -(1+x^{-4})^{1/4} + C$$

$$= -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

39. (a) $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx \quad \text{...(i)}$$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx \quad \text{...(ii)}$$

Adding (i) and (ii)

$$2I = \int_2^4 dx = [x]_2^4 = 2$$

$$I = 1$$

40. (d) $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Substitute $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right) dx = dt$$

$$\Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx = -dt$$

This gives,

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2} + C$$

$$= \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

41. (c) $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} \quad \text{...(i)}$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 - \cos x} \quad \text{...(ii)}$$

Using $\int_a^b f(x) dx = \int_a^b f(a+b-x)\, dx$

Adding (i) and (ii)

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{\sin^2 x} dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x\, dx$$

$$I = -(\cot x)_{\pi/4}^{3\pi/4} = -\left[\cot\frac{3\pi}{4} - \cot\frac{\pi}{4}\right] = 2$$

42. (c) $I_n = \int \tan^n x\, dx, n > 1$

Let $I = I_4 + I_6$

$$= \int (\tan^4 x + \tan^6 x) dx = \int \tan^4 x \sec^2 x\, dx$$

Let $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \int t^4 dt$$

$$= \frac{t^5}{5} + C$$

$$= \frac{1}{5}\tan^5 x + C$$ $\Rightarrow$ On comparing, we have

$$a = \frac{1}{5}, b = 0$$