Determinants

LEVEL–I

1. Let f (x) = x(x – 1), then $\Delta = \begin{vmatrix} f(0) & f(1) & f(2) \\ f(1) & f(2) & f(3) \\ f(2) & f(3) & f(4) \end{vmatrix}$ is equal to

(A) –2! (B) –3! – 2! (C) 0 (D) none of these

2. If f (x) = $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then f (100) is equal to

(A) 0 (B) 1 (C) 100 (D) –100

3. The determinant $\Delta(x) = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ ($abc \neq 0$) is divisible by

(A) 1 + x (B) $(1 + x)^2$
(C) x^2 (D) none of these

4. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p^2-qr & q^2-pr & r^2-pq \end{vmatrix}$ is

(A) pqr (B) p + q + r
(C) p + q + r – pqr (D) 0

5. If a, b, c > 0 and x, y, z $\in$ R, then the determinant $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$ is equal to

(A) $a^x + b^y + c^z$ (B) $a^{-x} b^{-y} c^{-z}$
(C) $a^{2x} b^{2y} c^{2z}$ (D) 0

6. Given a system of equations in x, y, z: x + y + z = 6; x + 2y + 3z = 10 and x + 2y + az = b. If this system has infinite number of solutions, then

(A) a = 3, b = 10 (B) a = 3, b $\neq$ 10
(C) a $\neq$ 3, b = 10 (D) a $\neq$ 3, b $\neq$ 10

7. If each element of a determinant of 3rd order with value A is multiplied by 3, then the value of the newly formed determinant is

(A) 3A (B) 9A (C) 27A (D) none of these

8. If the value of 3rd order determinant is 11, then the value of the determinant formed by the cofactors will be

(A) 11 (B) 121 (C) 1331 (D) 14641

9. If $a^{-1} + b^{-1} + c^{-1} = 0$ such that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$, then the value of λ is

(A) 0 (B) abc (C) –abc (D) none of these

10. If a, b, c are real numbers, then $\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix}$ is

(A) 0 (B) 6
(C) 9 (D) None of these

11. Let D be the determinant of order 3×3 with the entry I^{i+k} in I^{th} row and k^{th} column ($I = \sqrt{-1}$. Then value of D is

(A) imaginary (B) Zero
(C) real and positive (D) real and negative

12. The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is

(A) $a^3+b^3+c^3-3abc$ (B) $a^2+b^2+c^2-bc-ca-ab$
(C) $a^2b^2+b^2c^2+c^2a^2$ (D) None of these

13. Let $\Delta = \begin{vmatrix} x & l & m & 1 \\ \alpha & x & n & 1 \\ \alpha & \beta & \gamma & 1 \\ \alpha & \beta & \gamma & 1 \end{vmatrix}$. Then, the roots of the equation are

(A) α, β, γ (B) l, m ,n
(C) $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ (D) l+m, m+n, n+l

14. Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$; a>0 , b>0, c >0. Then,

(A) $\Delta \neq 0$ (B) a+b+c = 0
(C) $\Delta > 0$ (D) $\Delta \in R$

15. The value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$ is

(A) $3\sqrt{3}\ i$ (B) $-3\sqrt{3}\ i$ (C) $-\sqrt{3}\ i$ (D) $\sqrt{3}\ i$

16. If a, b, c are negative different real numbers, then $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(A) < 0 (B) ≤ 0 (C) > 0 (D) ≥ 0

17. The equation x + 2y + 3z = 1, x – y + 4z = 0, 2x + y + 7z = 1 have
(A) one solution only (B) two solutions only (C) no solution (D) infinitely may solution

18. The value of λ and μ for which the system of equation $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have unique solution are

(A) $\lambda = 3, \mu \in R$ (B) $\lambda = 3, \mu = 10$ (C) $\lambda \neq 3, \mu = 10$ (D) $\lambda \neq 3, \mu \neq 10$

LEVEL−II

1. The value of $\begin{vmatrix} i^{m} & i^{m+1} & i^{m+2} \\ i^{m+5} & i^{m+4} & i^{m+5} \\ i^{m+6} & i^{m+7} & i^{m+8} \end{vmatrix}$, where $i = \sqrt{-1}$ is

(A) 1 if m is multiple of 4 (B) 0 for all real m
(C) –i if m is a multiple of 3 (D) none of these

2. If the equations $a(y + z) = x$, $b(z + x) = y$ and $c(x + y) = z$, where $a \neq -1$, $b \neq -1$, $c \neq -1$ admit non-trivial solution, then $(1 + a)^{-1} + (1 + b)^{-1} + (1 + c)^{-1}$ is
(A) 2 (B) 1 (C) 1/2 (D) none of these

3. The number of values of t for which the system of equations $(a - t)x + by + c = 0$, $bx + (c - t)y + az = 0$, $cx + ay + (b - t)z = 0$ has non-trivial solution is
(A) 1 (B) 2 (C) 3 (D) 4

4. If α, β are non real numbers satisfying $x^3 - 1 = 0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to
(A) 0 (B) λ^3 (C) $\lambda^3 + 1$ (D) none of these

5. The system of equations $ax + 4y + z = 0$, $bx + 3y + z = 0$, $cx + 2y + z = 0$ has non trivial solutions if a, b, c are in
(A) A.P (B) G.P (C) H.P (D) none of these

6. The maximum value of $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\cos 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$ is
(A) 3 (B) 4 (C) 5 (D) 6

7. There are three points (a, x), (b, y) and (c, z) such that the straight lines joining any two of them are not equally inclined to the coordinate axes where $a, b, c, x, y, z \in R$.
If $\begin{vmatrix} x+a & y+b & z+c \\ y+b & z+c & x+a \\ z+c & x+a & y+b \end{vmatrix} = 0$ and $a + c = -b$, then x, $-\frac{y}{2}$, z are in
(A) A. P. (B) G.P.
(C) H.P. (D) none of these

8. **If x, y, z are the integers in A.P, lying between 1 and 9 and x51, y41 and z31 are three digits numbers, then the value of** $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ **is**
(A) x + y + z **(B) x –y + z**
(C) 0 **(D) None of these**

9. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), and (a_1,b_1), (a_2,b_2) (a_3,b_3) are

(A) Congruent (B) Similar
(C) Of equal area (D) Of equal altitude

10. Let $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$. Then $\sum_{a=1}^{n} \Delta a$ is equal to

(A) 0 (B) (a-1) $\sum n^2$
(C) (a-1)n $\sum$n (D) None of these

11. The determinant $\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\theta \\ -\sin\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

(A) α (B) β
(C) α and β (D) None of these

12. Let $\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$. $\forall$ a, b, c $\in$ R. Then,

(A) $\Delta = 0$ (B) $\Delta < 0$
(C) $\Delta > 0$ (D) None of these

13. If A +B +C = π , then the value of $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is

(A) sinA sinB sinC (B) sinA sinB+ sinC sinA +sinB sinC
(C) 0 (D) sinA cosBsinC+sinA sinB cosC+cosA sinBsinC

14. Let $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$. Then

(A) $\Delta_1 = 3(\Delta_2)^2$ (B) $\left(\frac{d}{dx}\right)\Delta_1 = 3\Delta_2$
(C) $\left(\frac{d}{dx}\right)\Delta_1 = 3(\Delta_2)^2$ (D) $\Delta_1 = 3(\Delta_2)^{3/2}$

15. Let $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$. Then

(A) x, y, z are in A.P (B) x,y,z are in G.P
(C) x, y, z are in H.P (D) xy, yz , zx are in A.P

16. Let f(x) = $\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where 'p' is a constant. Then $\frac{d^3}{dx^3}[f(x)]$ at x =0 is

(A) p
(B) p+p^2
(C) p+p^3
(D) independent of 'p'

17. Let Δ = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$, then Δ lies in the interval

(A) [2, 3] (B) [3, 4] (C) [2, 4] (D) (2, 4)

18. If α, β, γ are roots of $x^3 + ax^2 + b = 0$, then the value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) – a^3 (B) a^3 – 3b (C) a^3 (D) a^2 – 3b

19. Given $a_i^2 + b_i^2 + c_i^2 = 1$, (i = 1, 2, 3) and $a_ia_j + b_ib_j + c_ic_j = 0$ ($i \neq j$, i, j = 1, 2, 3), then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is

(A) 0 (B) 1/2 (C) 1 (D) 2

20. If Δ (x) = $\begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)\,dx$ is equal to

(A) 1/4 (B) 1/2 (C) 0 (D) –1/2

21. If A + B + C = π, then the value of determinant $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ is equal to

(A) 0
(B) 1
(C) -1
(D) None of these

LEVEL–III

1. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$ = (y-z)(z – x) (x – y) $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then

(A) n =2 (B) n = -2
(C) n = -1 (D) n = 1

2. Let m be a positive integer and $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin(m^2) \end{vmatrix}$.

Then the value of $\sum_{r=0}^{m} \Delta_r$ is given by

(A) 0 (B) m^2-1
(C) 2^m (D) $2^m \sin^2(2^m)$

3. If Δ (x) = $\begin{vmatrix} x & 1+x^2 & x^3 \\ \log(1+x^2) & e^x & \sin x \\ \cos x & \tan x & \sin^2 x \end{vmatrix}$ then

(A) Δ (x) is divisible by x (B) Δ (x) = 0
(C) Δ' (x) = 0 (D) None of these

4. If $f_r(x)$, $g_r(x)$, $h_r(x)$, (r=1,2,3) are polynomials in x such that $f_r(A) = g_r(A) = h_r(A)$, r = 1,2,3 and F(x) = $\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$, then F ′ (x) at x = a is

(A) 0 (B) 1
(C) $\sum f_r(x)+ \sum g_r(x)+ \sum h_r(x)$ (D) None of these

5. Let f(x) = $\begin{vmatrix} \sec x & \cos x & \sec^2 x+\operatorname{cosec} x\cot x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$. Then $\int_0^{\pi/2} f(x)dx$ is equal to

(A) $\left[\frac{8}{15}-\frac{\pi}{4}\right]$ (B) $\left[\frac{8}{15}+\frac{\pi}{4}\right]$

(C) $-\left[\frac{8}{15}+\frac{\pi}{4}\right]$ (D) None of these

6. Let $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$. Then $\sum_{r=1}^{n} D_r$ is equal to

(A) $\alpha+\beta+\gamma$ (B) $\alpha\beta\gamma$
(C) $2^n\, 3^n\, 5^n$ (D) 0

7. If maximum and minimum values of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$

are α and β, then

(A) α $+\beta^{99} = 4$

(B) $\alpha^3 - \beta^{17} = 26$

(C) $(\alpha^{2n} - \beta^{2n})$ is always an even integer for $n \in N$

(D) a triangle can be constructed having it's sides as α, β and $\alpha - \beta$.

8. The parameter on which the value of the determinant

$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is

(A) a (B) p

(C) d (D) x

L−I

1.	B	2.	A
3.	C	4.	D
5.	D	6.	
7.	C	8.	
9.	B	10.	A
11.	B	12.	D
13.	A	14.	D
15.	A	16.	C
17.		18.	C

L−II

1.	D	2.	
3.		4.	A
5.		6.	A
7.	A	8.	D
9.	C	10.	A
11.	A	12.	C
13.	C	14.	B
15.	B	16.	D
17.	C	18.	C
19.	A	20.	D

L−III

1.	C	2.	A
3.		4.	A
5.	C	6.	D
7.	B	8.	B