Resonance
Educating for better tomorrow

TARGET : JEE (Advanced) 2015

Course : VIJETA & VIJAY (ADP & ADR) **Date : 21-04-2015**

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS
DPP NO. 05

TEST INFORMATION

DATE : 22.04.2015 **PART TEST-02 (PT-02)**

Syllabus : Application of Derivatives, Sequence & Series, Binomial Theorem

REVISION DPP OF STRAIGHT LINE AND CIRCLE

Total Marks : 147 **Max. Time : 113.5 min.**

Single choice Objective (–1 negative marking) Q. 1 to 11	**(3 marks 2.5 min.)**	**[33, 27.5]**
Multiple choice objective (–1 negative marking) Q. 12 to 38	**(4 marks, 3 min.)**	**[108, 81]**
Comprehension (–1 negative marking) Q.39 to 40	**(3 marks 2.5 min.)**	**[6, 5]**

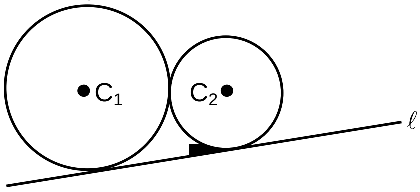
1. From a point 'P' on the line 2x + y + 4 = 0, which is nearest to the circle $x^2 + y^2 - 12y + 35 = 0$, tangents are drawn to given circle. The area of quadrilateral formed by these pair of tangents and pair of radii, is
(A) 8 (B) $\sqrt{110}$ (C) $\sqrt{19}$ (D) 19

2. The lines 5x + 12y – 10 = 0 and 5x – 12y – 40 = 0 touch a circle of radius 3 units. If the centre of circle lies in the first quadrant, then the coordinates of centre is
(A) (5, 3) (B) (5, 1) (C) (5, 2) (D) (5, 6)

3. Let A ≡ (– 2, 0) and B ≡ (2, 0), then the number of integral values of a, a ∈ [–10, 10] for which line segment AB subtends an acute angle at point C(a, a + 1) is
(A) 15 (B) 17 (C) 19 (D) 21

4. If the circles $x^2 + y^2 + (3 + \sin\beta)x + 2\cos\alpha \cdot y = 0$ and $x^2 + y^2 + 2\cos\alpha \cdot x + 2cy = 0$ touch each other, then the maximum value of 'c' is
(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

5. Two circles C_1 and C_2 of radii $\frac{3}{2}$ and $\frac{1}{2}$ respectively touch each other externally and 'ℓ' is their common tangent as shown in figure.



Then the perimeter of shaded region is :
(A) $\frac{5\pi}{6} + \sqrt{3}$ (B) $\frac{2\pi}{3} + \sqrt{3}$ (C) $\pi - \sqrt{3}$ (D) $\pi + \sqrt{3}$

6. Vertices of a variable triangle are (3, 4), (5cosθ, 5sinθ) and (5sinθ, – 5cosθ). Then locus of its orthocenter is
(A) $(x + y - 1)^2 + (x - y - 7)^2 = 100$ (B) $(x + y - 7)^2 + (x - y - 1)^2 = 100$
(C) $(x + y - 7)^2 + (x + y - 1)^2 = 100$ (D) $(x + y - 7)^2 + (x - y + 1)^2 = 100$

7. If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be the images of point P(x, y) about lines $L_1 = ax + by + c = 0$ and $L_2 = bx - ay + c' = 0$ respectively then the line joining points P_1 and P_2 always passes through

(A) Point of intersection of $L_1 = 0$ and $L_2 = 0$ (B) Point $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

(C) Point $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ (D) Information provided is incomplete

8. The base of a triangle passes through a fixed point (f, g) and its sides are bisected at right angles by the lines $y^2 - 8xy - 9x^2 = 0$. The locus of vertex of triangle is
(A) straight line (B) circle (C) parabola (D) ellipse

9. If $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$ where a, b, c > 0, then the family of lines $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$ always passes through the fixed point
(A) (1, 1) (B) (1, –1) (C) (–1, 2) (D) (–1, 1)

10. A line of fixed length 2 units moves so that its one end is on the positive x-axis and other end on that part of the line x + y = 0 which lies in the second quadrant. The locus of the mid-point of the line is given by
(A) $x^2 + 5y^2 + 4xy - 1 = 0$ (B) $x^2 + 5y^2 + 4xy + 1 = 0$
(C) $x^2 + 5y^2 - 4xy - 1 = 0$ (D) $4x^2 + 5y^2 + 4xy + 1 = 0$

11. A point P moves such that it is at a constant distance c from the origin. If Q is the image of P in the line mirror y = x and R is the image of Q in the line mirror y = –x then locus of R is
(A) $y^2 = x^2$ (B) $x^2 + y^2 = 2c^2$ (C) $xy = c^2$ (D) $x^2 + y^2 = c^2$

12. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is

(A) 1 (B) 2 (C) 3 (D) $\frac{1}{3}$

13. If $4a^2 + b^2 + 2c^2 + 4ab - 6ac - 3bc = 0$, then the family of lines ax + by + c = 0 may be concurrent at point(s)

(A) $\left(-1, -\frac{1}{2}\right)$ (B) (–1, –1) (C) (–2, –1) (D) (–1, 2)

14. Three vertices of a parallelogram are (1, 1), (2, 4) and (3, 5), then the fourth vertex of the parallelogram can be :
(A) (4, 8) (B) (5, 8) (C) (0, 0) (D) (2, 2)

15. The value(s) of t for which the lines 2x + 3y = 5, $t^2x + ty - 6 = 0$ and 3x – 2y – 1 = 0 are concurrent, can be
(A) 2 (B) –3 (C) –2 (D) 3

16. One diagonal of a square is the portion of the line $\sqrt{3}x + y = 2\sqrt{3}$ intercepted by the axes. Then an extremity of the other diagonal is :
(A) $(1+\sqrt{3}, \sqrt{3}-1)$ (B) $(1+\sqrt{3}, \sqrt{3}+1)$ (C) $(1-\sqrt{3}, \sqrt{3}-1)$ (D) $(1-\sqrt{3}, \sqrt{3}+1)$

17. The equation of the sides of the triangle having (3, –1) as a vertex and x – 4y + 10 = 0 and 6x + 10y = 59 as angle bisector and as median respectively drawn from different vertices, are :
(A) 6x + 5y – 13 = 0 (B) 2x + 9y – 65 = 0 (C) 18x + 13y – 41 = 0 (D) 6x – 7y – 25 = 0

18. C_1 and C_2 are two circles of radii a and b (a < b) touching both the coordinate axes and have their centres in the first quadrant. Then which of the following is true?

(A) If C_1, C_2 touch each other then $\frac{b}{a} = 3 + 2\sqrt{2}$

(B) If C_1, C_2 are orthogonal then $\frac{b}{a} = 2 + \sqrt{3}$

(C) If C_1, C_2 intersect in such a way that their common chord has maximum length, then $\frac{b}{a} = 3$.

(D) If C_2 passes through the centre of C_1, then $\frac{b}{a} = 2 + \sqrt{2}$.

19. If $g : R \to N \cup \{0\}$ and g(x) = n, where 'x' represents the area of triangle joining the two fixed points P(5, 0), Q(8, 4) and a variable point R such that $\angle PRQ = \frac{\pi}{2}$ and 'n' represents the number of such triangles, then
(A) g(5) = 4 (B) g(7) = 0 (C) g(6.25) = 2 (D) g (6.25) = 1

20. The line y = x is tangent at (0, 0) to a circle of radius 1. The centre of circle may be :
(A) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

21. If $(x-2)^2 + (y-2)^2 = 1$, then which of the following is true ?
(A) maximum value of x + y is $4 + \sqrt{2}$ (B) maximum value of x – y is $\sqrt{2}$
(C) maximum value of xy is $\frac{9+4\sqrt{2}}{2}$ (D) minimum value of x + y is $4 - \sqrt{2}$

22. The slope of median drawn from the vertex 'A' of triangle ABC is –2. Coordinates of vertices B and C are (–1,3) & (3, 5) respectively. If the area of triangle is 5 square units, then the distance of vertex A from the origin is
(A) 6 (B) 4 (C) $2\sqrt{2}$ (D) $3\sqrt{2}$

23. A(1, 2) and B(7, 10) are two fixed points. If P(x, y) is a point such that $\angle APB = 60°$ and the area of triangle APB is maximum, then
(A) point P lies on the line 3x + 4y = 36
(B) point P is on the circle passing through given points and having radius 10
(C) point P is on the circle passing through given points and having radius $\frac{10}{\sqrt{3}}$
(D) area of $\Delta PAB = \frac{75}{\sqrt{3}}$ sq. units

24. z_1, z_2, z_3 are three non collinear complex numbers such that $z = \frac{\ell z_1 + m z_2 + n z_3}{\ell + m + n}$ lies inside the triangle formed by z_1, z_2, z_3. If ℓ, m, n are roots of equation $x^3 + 3x^2 + px + q = 0$, then which of the following is **INCORRECT** ?
(A) p > 0, q > 0 (B) p < 0, q < 0 (C) p > 0, q < 0 (D) p < 0, q > 0

25. Equation of incircle of equilateral triangle ABC where B ≡ (2, 0), C ≡ (4, 0), is
(A) $x^2 + y^2 - 6x + \frac{2y}{\sqrt{3}} + 9 = 0$ (B) $x^2 + y^2 + 6x - \frac{2y}{\sqrt{3}} + 9 = 0$
(C) $x^2 + y^2 + 6x + \frac{2y}{\sqrt{3}} + 9 = 0$ (D) $x^2 + y^2 - 6x - \frac{2y}{\sqrt{3}} + 9 = 0$

26. Equation of circle touching the circle $x^2 + y^2 - 15x + 5y = 0$ at (1, 2) and having radius $\sqrt{\frac{5}{2}}$ is
(A) $5x^2 + 5y^2 - 23x + 11y + 20 = 0$ (B) $5x^2 + 5y^2 - 23x - 11y + 20 = 0$
(C) $5x^2 + 5y^2 + 3x - 29y + 30 = 0$ (D) $5x^2 + 5y^2 + 3x + 29y + 30 = 0$

27. The equation of circle which is touched by line y = x, has its centre on the x-axis and cuts off a chord of length 2 units along the line $\sqrt{3}y - x = 0$ is
(A) $x^2 + y^2 - 4x + 2 = 0$ (B) $x^2 + y^2 - 4x + 6 = 0$ (C) $x^2 + y^2 - 6x + 2 = 0$ (D) $x^2 + y^2 + 4x + 2 = 0$

28. Let C be a circle with two diameters intersecting at an angle of 30°. A circle S having radius unity, touches both the diameters and also the circle C, then the radius of circle 'C' can be
(A) $1 + \sqrt{6} + \sqrt{2}$ (B) $1 + \sqrt{6} - \sqrt{2}$ (C) $\sqrt{6} + \sqrt{2} - 1$ (D) $\sqrt{6} - \sqrt{2} - 1$

29. If from (α, β), two tangents are drawn to circle $x^2 + y^2 = 4$ so that slopes of tangents are in the ratio 1 : 2 and $f(x) = \alpha^2 x^2 + 12x - \frac{\beta^2}{4}$, then
(A) $f(x) > 0 \ \forall x \in R$ (B) Locus of (α^2, β^2) is a hyperbola
(C) least positive integral value of α is 1 (D) eccentricity of locus of (α^2, β^2) is $\sqrt{2}$

30. Equation of the chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$ which passes through origin such that the origin divides it in the ratio 4 : 1 is
(A) y = 0 (B) 24x + 7y = 0 (C) 7x + 24y = 0 (D) 7x – 24y = 0

31. If the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is inscribed in a triangle whose two sides are coordinate axes and one side has negative slope cutting intercepts a and b on x and y axis respectively, then
(A) $\frac{1}{a} + \frac{1}{b} - 1 = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$ (B) $\frac{1}{a} + \frac{1}{b} - 1 = -\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$
(C) $\frac{1}{a} + \frac{1}{b} > 1$ (D) $\frac{1}{a} + \frac{1}{b} < 1$

32. The lines x + y = 1, $(m-1)x + (m^2-7)y - 5 = 0$ and $(m-2)x + (2m-5)y = 0$
(A) are concurrent for m = 3 (B) form a triangle for m = 2
(C) are concurrent for no value of m (D) are parallel for m = 3

33. If m_1, m_2 are roots of equation $x^2 - ax - a - 1 = 0$, then the area of the triangle formed by the three straight lines $y = m_1x$, $y = m_2x$ and $y = a$ $(a \neq -1)$ is

(A) $\frac{a^2(a+2)}{2(a+1)}$ if $a > -1$ (B) $\frac{-a^2(a+2)}{2(a+1)}$ if $a < -1$

(C) $\frac{-a^2(a+2)}{2(a+1)}$ if $-2 < a < -1$ (D) $\frac{a^2(a+2)}{2(a+1)}$ if $a < -2$

34. If one of the lines in $2x^2 + axy + 3y^2 = 0$ coincide with one of those given by $4x^2 + bxy - 6y^2 = 0$ and the other lines represented by them be perpendicular then which of the following may be true ?
(A) a + b = 7 (B) a + b = –6 (C) a – b = –3 (D) a – b = 3

35. The diagonals of a rhombus ABCD intersect in (1, 2) and its two sides are parallel to the lines x – y + 2 = 0 and 7x – y + 3 = 0. If the vertex A is (0, k) then the value of k is/are
(A) 2/5 (B) 3/5 (C) 5/2 (D) 0

36. The point $(\alpha, \alpha + 1)$ lies inside the triangle ABC whose vertices are A(0, 3), B(–2, 0) and C(6, 1) if
(A) $\alpha = -1$ (B) $\alpha = \frac{-1}{2}$ (C) $\alpha = \frac{1}{2}$ (D) $\frac{-6}{7} < \alpha < \frac{3}{2}$

37. The straight line ax + by + c = 0 where $abc \neq 0$ will pass through the first quadrant if :
(A) ac > 0 and bc > 0 (B) c > 0 and bc < 0
(C) bc > 0 and/or ac > 0 (D) ac < 0 and/or bc < 0

38. The equation of a circle in which the chord joining the points (1, 2) and (2, –1) subtends an angle of $\pi/4$ at any point on the circumference is
(A) $x^2 + y^2 = 5$ (B) $x^2 + y^2 - 6x - 2y + 5 = 0$
(C) $x^2 + y^2 + 6x + 2y - 15 = 0$ (D) $x^2 + y^2 + 7x - 2y + 14 = 0$

Comprehension (For Q. No. 39 to 40)
Let C : $x^2 + y^2 - 4x - 6y - 3 = 0$ is a circle and S is a family of circles passing through two fixed points A(3, 7) and B(6, 5).

39. The chords in which the circle C cuts the member of the family S are concurrent at point
(A) (2, 3) (B) $\left(2, \frac{23}{3}\right)$ (C) $\left(3, \frac{23}{2}\right)$ (D) (3, 2)

40. Equation of member of the family S that bisects the circumference of circle C is
(A) $x^2 + y^2 - 5x - 1 = 0$ (B) $x^2 + y^2 - 5x + 6y - 1 = 0$
(C) $x^2 + y^2 - 5x - 6y - 1 = 0$ (D) $x^2 + y^2 + 5x - 6y - 1 = 0$

DPP # 4

REVISION DPP OF SEQUENCE & SERIES AND BINOMIAL THEOREM

1.	(B)	**2.**	(A)	**3.**	(B)	**4.**	(C)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(C)	**9.**	(C)	**10.**	(D)	**11.**	(D)	**12.**	(D)	**13.**	(B)	**14.**	(B,D)
15.	(A,B,C)	**16.**	(A,B,D)	**17.**	(A,C)	**18.**	(A,B,C)	**19.**	(B,C)	**20.**	(A,D)	**21.**	(C,D)
22.	(A,B,D)	**23.**	(B,C)	**24.**	(A,C)	**25.**	(A,C)	**26.**	(A,D)	**27.**	(A,B,D)	**28.**	(B,D)
29.	(A,B,D)	**30.**	(A,B,C)	**31.**	(A,B,C,D)			**32.**	(A,C,D)	**33.**	(A,C)	**34.**	(A,C)
35.	(A)	**36.**	(B)	**37.**	(C)	**38.**	2	**39.**	4	**40.**			1

MATHEMATICS

1_. Nearest point will be foot of perpendicular drawn from centre.

$$\frac{x-0}{2} = \frac{y-6}{1} = \frac{-(0+6+4)}{4+1} \Rightarrow (x, y) = (-4, 4).$$

2_. Let coordinates of centre is (h, k)

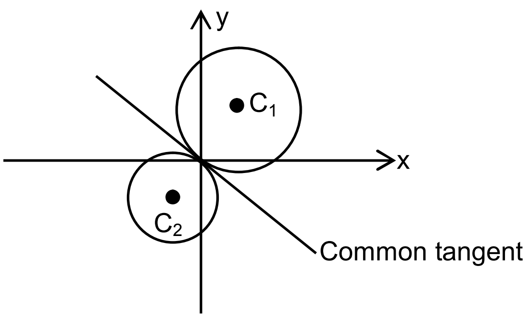
$$\left|\frac{5h+12k-10}{13}\right| = 3 \quad \text{and} \quad \left|\frac{5h-12k-40}{13}\right| = 3$$

$\Rightarrow$ $5h + 12k - 10 = 39$ & $-(5h - 12k - 40) = 39$ $\Rightarrow$ $(h, k) = (5, 2)$

3_. Point $C(a, a + 1)$ must lie outside the circle $x^2 + y^2 = 4$.

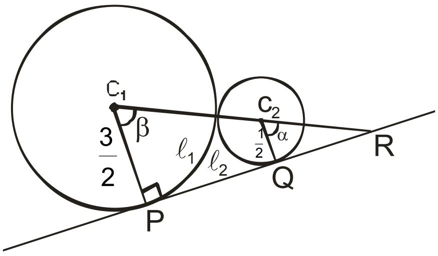
$\Rightarrow$ $S_1 > 0$ $\Rightarrow$ $a^2 + (a + 1)^2 - 4 > 0$ $\Rightarrow$ $a < \frac{-1-\sqrt{7}}{2}$ or $a > \frac{-1+\sqrt{7}}{2}$

4_.



$$\left.\frac{dy}{dx}\right|_{(0,0)} = -\frac{(3+\sin\beta)}{2\cos\alpha} = -\frac{2\cos\alpha}{2c} \Rightarrow c = \frac{2\cos^2\alpha}{3+\sin\beta} \qquad \therefore \quad c_{max} = 1.$$

5_.



$$PQ = \sqrt{C_1C_2^2-(r_1-r_2)^2} = \sqrt{4-1} = \sqrt{3}$$

$$\frac{RC_1}{RC_2} = \frac{3/2}{1/2} \quad \Rightarrow \quad RC_1 = 3RC_2 \quad \Rightarrow \quad RC_2 = 1$$

$$\therefore \quad \cos\alpha = \frac{1/2}{1} \quad \Rightarrow \quad \alpha = \frac{\pi}{3} = \beta$$

Required perimeter $= \ell_1 + \ell_2 + \sqrt{3} = \frac{3}{2}.\frac{\pi}{3}+\frac{1}{2}.\frac{2\pi}{3}+\sqrt{3} = \frac{5\pi}{6}+\sqrt{3}$

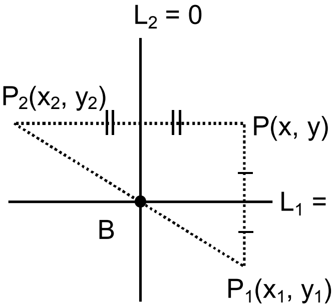
6. $OA = OB = OC$ $\Rightarrow$ $(0, 0)$ is circumcentre

as $OG : GH = 1 : 2 \Rightarrow$ $H = 3G$

$\Rightarrow$ $x = 3 + 5\cos\theta + 5\sin\theta$ and $y = 4 + 5\sin\theta - 5\cos\theta$

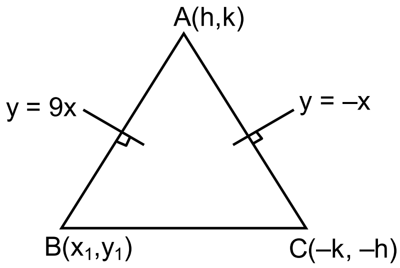
$\Rightarrow$ $x + y = 7 + 10\sin\theta$ and $x - y = -1 + 10\cos\theta$ $\Rightarrow$ $(x + y - 7)^2 + (x - y + 1)^2 = 10^2$

7.



$\because$ B is circumcentre of triangle PP_1P_2

8.



$x_1 = \dfrac{9k - 40h}{41}$, $y_1 = \dfrac{9h + 40k}{41}$

BC का समीकरण

$y + h = \dfrac{50h + 40k}{50k - 40h}(x + k)$

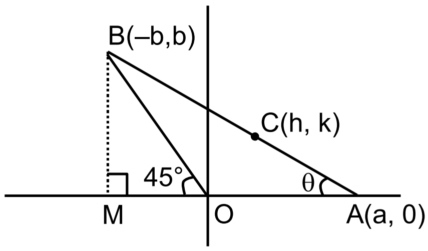
(f, g) lies on BC $\Rightarrow$ $g + h = \dfrac{5h + 4k}{5k - 4h}(f + k)$

$\Rightarrow$ locus of (h, k) is $4x^2 + 4y^2 + x(5f + 4g) + y(4f - 5g) = 0$ है।

9. $a = (\sqrt{b} + \sqrt{c})^2 \Rightarrow$ $(\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0$ $\Rightarrow$ $\sqrt{a} - \sqrt{b} - \sqrt{c} = 0$

$\Rightarrow$ $(\sqrt{b} + \sqrt{c})x + \sqrt{b}\ y + \sqrt{c} = 0$ $\Rightarrow$ $\sqrt{b}(x + y) + \sqrt{c}(x + 1) = 0$

10.



$BM = 2\sin\theta$ $\Rightarrow$ $MO = 2\sin\theta$

$MA = 2\cos\theta$

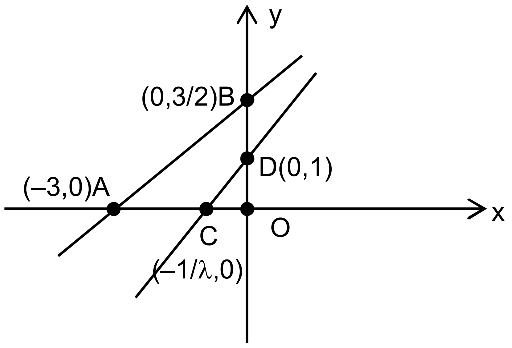
$\therefore$ $A(2\cos\theta - 2\sin\theta, 0)$ तथा $B(-2\sin\theta, 2\sin\theta)$

$\therefore$ $2h = 2\cos\theta - 4\sin\theta$ तथा $2k = 2\sin\theta$

As $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow$ $k^2 + (h + 2k)^2 = 1$ $\Rightarrow$ $h^2 + 5k^2 + 4hk = 1$

11. P lies on circle $x^2 + y^2 = c^2$. As curve is symmetrical about y = x and y = –x. So locus of Q and R will remain same.

12_*.



Case – I If $\frac{-1}{\lambda} \neq -3$ i.e., $\lambda \neq \frac{1}{3}$, then

$OA \cdot OC = OB \cdot OD \Rightarrow \frac{3}{\lambda} = \frac{3}{2} \Rightarrow \lambda = 2.$

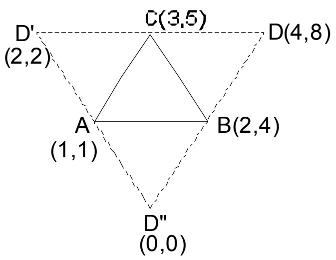
Case-II If $\lambda = \frac{1}{3}$, then a unique circle will always pass through these point.

13_. $4a^2 - 2(3c - 2b)a + (b^2 + 2c^2 - 3bc) = 0$

$$\Rightarrow \quad a = \frac{2(3c-2b) \pm \sqrt{4(3c-2b)^2 - 4.4.(b^2+2c^2-3bc)}}{2.4}$$

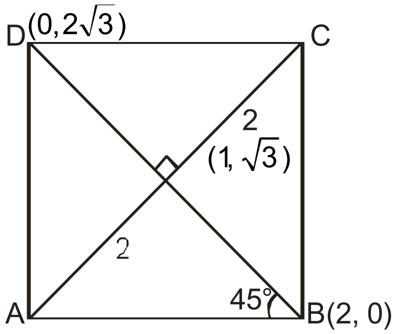
$\Rightarrow \quad 2a + b - 2c = 0 \quad \& \quad 2a + b - c = 0$

14_.



15_. $\begin{vmatrix} t^2 & t & 6 \\ 2 & 3 & 5 \\ 3 & -2 & 1 \end{vmatrix} = 0 \Rightarrow \quad 13t^2 + 13t - 78 = 0 \quad \Rightarrow \quad t^2 + t - 6 = 0 \quad \Rightarrow \quad t = -3, 2$

16_.



$\left(1 \pm 2\cos\frac{\pi}{6}, \sqrt{3} \pm 2\sin\frac{\pi}{6}\right)$

17_.

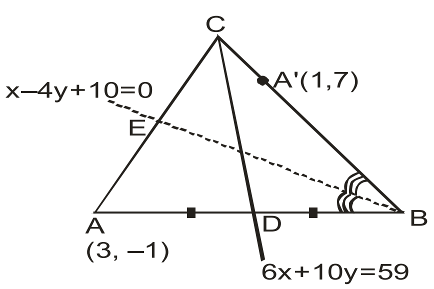


Image of A(3, –1) about line BE is

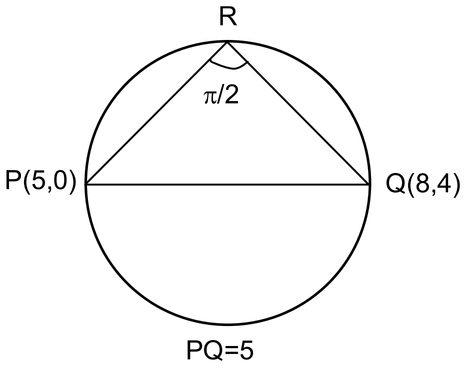
$\frac{x-3}{1} = \frac{y+1}{-4} = -2(1) \quad \Rightarrow \quad (x, y) \equiv (1, 7)$ lies on side BC.

Let vertex B is $(4\alpha - 10, \alpha)$.

Mid point of AB lies on $6x + 10y = 59$ $\quad \therefore \quad \alpha = 5 \quad \Rightarrow \quad B(10, 5)$

18_. Equation of C_1 : $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Equation of C_2 : $x^2 + y^2 - 2bx - 2by + b^2 = 0$

19_.



Maximum area of $\Delta PQR = \frac{1}{2} \times 5 \times \frac{5}{2} = 6.25$ sq. units

20_. Let equation of circle is $x^2 + y^2 + \lambda(x - y) = 0$. Radius = $\sqrt{\frac{\lambda^2}{4} + \frac{\lambda^2}{4}} = 1 \quad \Rightarrow \quad \lambda = \pm\sqrt{2}$

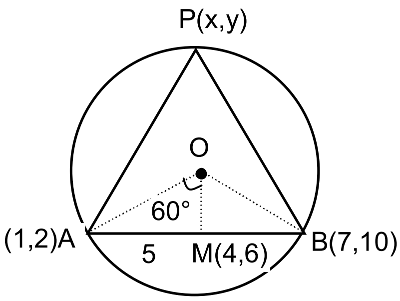
21_. Let $x - 2 = \cos\theta$ & $y - 2 = \sin\theta$

22_*. Equation of median through A is $y - 4 = -2(x - 1) \quad \Rightarrow \quad y = -2x + 6$

Let coordinates of point A is $(\alpha, 6 - 2\alpha)$

Now, $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \alpha & -1 & 3 \\ 6-2\alpha & 3 & 5 \end{vmatrix} = 5 \quad \Rightarrow \quad \alpha = 0, 2$

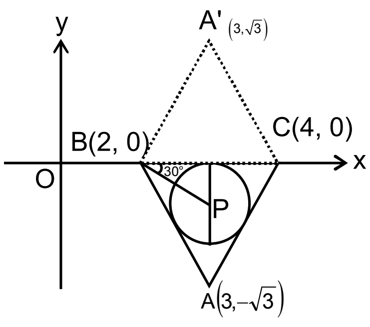
23_*.



$OA = 5\text{cosec } 60° = \frac{10}{\sqrt{3}}$ $\qquad OM = 5\cot 60° = \frac{5}{\sqrt{3}}$

24^*. If z lies inside triangle then ℓ, m, n are all of same sign

25^*.



$\tan 30° = \frac{PQ}{1} \quad \Rightarrow \quad PQ = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad r = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad P\left(3\ ,\ -\dfrac{1}{\sqrt{3}}\right) \quad \Rightarrow \quad (x-3)^2 + \left(y \pm \dfrac{1}{\sqrt{3}}\right)^2 = \dfrac{1}{3}$

26^*. Equation of tangent at (1, 2) is $13x - 9y + 5 = 0$

Required equation is $(x-1)^2 + (y-2)^2 + \lambda(13x - 9y + 5) = 0$

$\Rightarrow \quad x^2 + y^2 - (2 - 13\lambda)x - (4 + 9\lambda)y + 5(1 + \lambda) = 0$

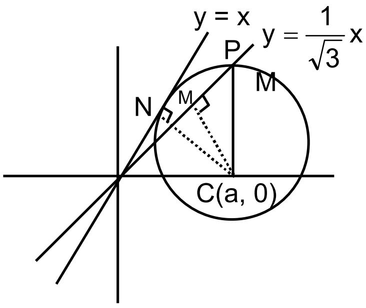
Radius = $\sqrt{\dfrac{(2-13\lambda)^2}{4} + \dfrac{(4+9\lambda)^2}{4} - 5(1+\lambda)} = \sqrt{\dfrac{5}{2}} \quad \Rightarrow \quad \lambda = \pm\, \dfrac{1}{5}$

equation of required circle is

$\Rightarrow \quad 5\{(x-1)^2 + (y-2)^2\} \pm 1(13x - 9y + 5) = 0 \quad \Rightarrow \quad 5x^2 + 5y^2 + 3x - 29y + 30 = 0$

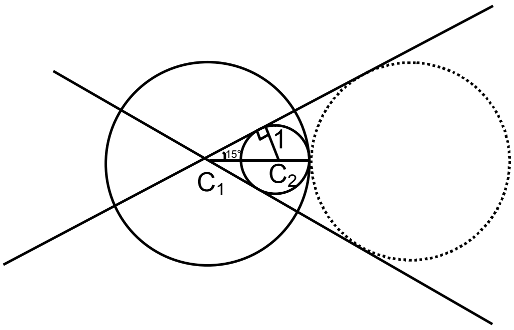
& $\quad 5x^2 + 5y^2 - 23x - 11y + 20 = 0$

27^*.



Radius = CN = $\dfrac{|a|}{\sqrt{2}} \quad \Rightarrow \quad CP^2 = CM^2 + MP^2 \quad \Rightarrow \quad \dfrac{a^2}{2} = \left(\dfrac{0 - \dfrac{a}{\sqrt{3}}}{\sqrt{1 + \dfrac{1}{3}}}\right)^2 + 1^2 \quad \Rightarrow \quad a = 2$

28^*.



$\text{cosec}15° = \dfrac{C_1C_2}{1} \quad \Rightarrow \quad r \pm 1 = \text{cosec}15° \quad \Rightarrow \quad r = \sqrt{6} + \sqrt{2} \pm 1$

29^*. Equation of tangent is $y = mx \pm 2\sqrt{1+m^2}$

$\Rightarrow \quad (\beta - m\alpha)^2 = 4 + 4m^2 \quad \Rightarrow \quad (\alpha^2 - 4)m^2 - 2\alpha\beta\, m + (\beta^2 - 4) = 0$

$\Rightarrow \quad m_1 + 2m_1 = \dfrac{2\alpha\beta}{\alpha^2 - 4} \quad$ and $\quad 2m_1^2 = \dfrac{\beta^2 - 4}{\alpha^2 - 4}$

$\Rightarrow \quad 2\left(\dfrac{2\alpha\beta}{3(\alpha^2 - 4)}\right)^2 = \dfrac{\beta^2 - 4}{\alpha^2 - 4} \quad \Rightarrow \quad \dfrac{8\alpha^2\beta^2}{9(\alpha^2 - 4)} = \beta^2 - 4$

$\Rightarrow \quad 8\alpha^2\beta^2 = 9(\alpha^2\beta^2 - 4\alpha^2 - 4\beta^2 + 16) \quad \Rightarrow \quad \alpha^2\beta^2 - 36(\alpha^2 + \beta^2) + 144 = 0$

Disc of f(x) is $144 + \alpha^2\beta^2 = 36(\alpha^2 + \beta^2) > 0$ so (A) is not true

Locus of (α^2, β^2) is $xy - 36x - 36y + 144 = 0$ which is a hyperbola

As $\beta^2 = \dfrac{36(\alpha^2 - 4)}{(\alpha^2 - 36)} \quad \Rightarrow \quad \dfrac{\alpha^2 - 4}{\alpha^2 - 36} > 0 \quad \Rightarrow \quad \alpha \in (-\infty, -6) \cup (-2, 2) \cup (6, \infty)$

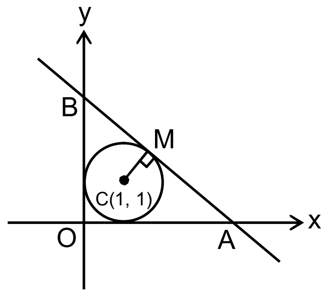
30^*. Let $y = mx$ be the chord.

Points of intersection of chord and circle are given by $(1 + m^2)x^2 - (3 + 4m)x - 4 = 0$

$\Rightarrow \quad x_1 + x_2 = \dfrac{3+4m}{1+m^2}$ and $x_1x_2 = \dfrac{-4}{1+m^2}$

As $\quad x_2 = -4x_1 \Rightarrow \quad 9 + 9m^2 = 9 + 16m^2 + 24m \quad \Rightarrow \quad 7m^2 + 24m = 0 \Rightarrow \quad m = 0, -\dfrac{24}{7}$

31.



Equation of AB is $\dfrac{x}{a}+\dfrac{y}{b} = 1 \quad CM = 1$

$$\Rightarrow \quad \left|\frac{\frac{1}{a}+\frac{1}{b}-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = 1 \quad \Rightarrow \quad -\left(\frac{1}{a}+\frac{1}{b}-1\right) = \sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

32^*. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ m-1 & m^2-7 & 5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = m^3 - 4m^2 + 5m - 6 = (m-3)(m^2 - m + 2) \Rightarrow \Delta = 0 \quad \Rightarrow \quad m = 3$

33*. Area $= \left|\dfrac{1}{2}\begin{vmatrix} \frac{a}{m_1} & a & 1 \\ \frac{a}{m_2} & a & 1 \\ 0 & 0 & 1 \end{vmatrix}\right| = \left|\dfrac{a^2(m_1-m_2)}{2m_1m_2}\right| = \left|\dfrac{a^2(a+2)}{2(a+1)}\right|$

34*. $3m^2 + am + 2 = 0$ and $6m^2 - bm - 4 = 0$

$\Rightarrow \quad m_1 + m_2 = \dfrac{-a}{3}, \ m_1m_2 = \dfrac{2}{3}, m_1 - \dfrac{1}{m_2} = \dfrac{b}{6}, \dfrac{-m_1}{m_2} = \dfrac{-2}{3}$

$\Rightarrow \quad (m_1m_2)\left(\dfrac{m_1}{m_2}\right) = \dfrac{4}{9} \quad \Rightarrow \quad m_1^2 = \dfrac{4}{9} \quad \Rightarrow \quad m_1 = \pm \dfrac{2}{3}$

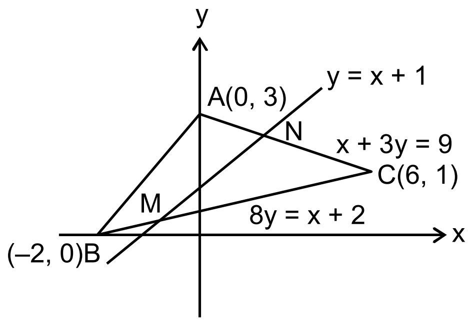
(i) $\quad m_1 = \dfrac{2}{3} \quad \Rightarrow \quad m_2 = 1 \quad \Rightarrow \quad a = -5, b = -2$

(ii) $\quad m_1 = -\dfrac{2}{3} \quad \Rightarrow \quad m_2 = -1 \quad \Rightarrow \quad a = 5, b = 2$

35*. As diagonals are bisectors of angle A so their equations are $\dfrac{x-y+k}{\sqrt{2}} = \pm\dfrac{7x-y+k}{5\sqrt{2}}$

As they pass through (1, 2) $\quad \Rightarrow \quad k = 0, 5/2$

36*.

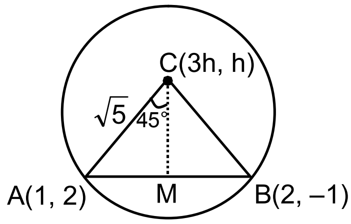


$N\left(\frac{3}{2},\frac{5}{2}\right)$ and $M\left(\frac{-6}{7},\frac{1}{7}\right)$ $\Rightarrow$ $\frac{-6}{7} < \alpha < \frac{3}{2}$

37*. Line meets x-axis at $A\left(-\frac{c}{a},0\right)$ & y-axis at $\left(0,\frac{-c}{b}\right)$

$\therefore$ $\frac{-c}{a} > 0$ and/or $\frac{-c}{b} > 0$

38*.



$AM = \frac{AB}{2} = \frac{\sqrt{10}}{2}$ $\Rightarrow$ $AC = \sqrt{5}$

CM equation of CM is x = 3y

let C(3h, h) $\Rightarrow$ $(3h - 1)^2 + (h - 2)^2 = 5$ $\Rightarrow$ h = 0, 1

Sol. **(39 & 40)**

Equation of line joining points A(3, 7) and B(6, 5) is $2x + 3y - 27 = 0$

Equation of family of circles S is $(x - 3)(x - 6) + (y - 5)(y - 7) + \lambda(2x + 3y - 27) = 0$

$\Rightarrow$ $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0$.

Equation of common chord $-5x - 6y + 56 + \lambda(2x + 3y - 27) = 0$.