CHAPTER 27

Probability

1. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is **[2002]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

2. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[2002]**

(a) 8/3 (b) 3/8

(c) 4/5 (d) 5/4

3. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is **[2003]**

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

4. The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{7}{20}$ (d) $\frac{3}{20}$

5. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.1

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the $P(E \cup F)$ is **[2004]**

(a) 0.50 (b) 0.77

(c) 0.35 (d) 0.87

6. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$

(c) $\frac{128}{256}$ (d) $\frac{37}{256}$

7. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$

(c) $\frac{8}{9}$ (d) $\frac{7}{9}$

8. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals **[2005]**

(a) $\frac{2}{e^2}$ (b) 0

(c) $1-\frac{3}{e^2}$ (d) $\frac{3}{e^2}$

9. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$

(c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

10. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14.

11. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(a) 8/729 (b) 8/243
(c) 1/729 (d) 8/9.

12. It is given that the events A and B are such that

$P(A)=\frac{1}{4}, P(A|B)=\frac{1}{2}$ and $P(B|A)=\frac{2}{3}$.

Then $P(B)$ is **[2008]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

13. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[2009]**

(a) $\frac{1}{\log_{10}4+\log_{10}3}$ (b) $\frac{9}{\log_{10}4-\log_{10}3}$
(c) $\frac{4}{\log_{10}4-\log_{10}3}$ (d) $\frac{1}{\log_{10}4-\log_{10}3}$

14. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals: **[2009]**

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$
(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

15. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[2010]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$
(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

16. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[2011]**

(a) $\left(\frac{3}{4},\frac{11}{12}\right]$ (b) $\left[0,\frac{1}{2}\right]$
(c) $\left(\frac{11}{12},1\right]$ (d) $\left(\frac{1}{2},\frac{3}{4}\right]$

17. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is **[2011]**

(a) $P(C|D) \geq P(C)$
(b) $P(C|D) < P(C)$
(c) $P(C|D)=\frac{P(D)}{P(C)}$
(d) $P(C|D)=P(C)$

18. Let A, B, C, be pairwise independent events with P (C) > 0 and $P(A\cap B\cap C)$=0. Then $P\left(A^c \cap B^c / C\right)$. **[2011RS]**

(a) $\mathrm{P}(B^c)-\mathrm{P}(B)$ (b) $P(A^c)+P(B^c)$
(c) $P(A^c)-P(B^c)$ (d) $P(A^c)-P(B)$

19. Three numbers are chosen at random without replacement from $\{1,2,3,..8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[2012]**

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$
(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

20. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[2013]**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$

(c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

21. Let A and B be two events such that $P\left(\overline{A \cup B}\right)=\frac{1}{6}$, $P\left(\overline{A \cap B}\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[2014]**

(a) independent but not equally likely.
(b) independent and equally likely.
(c) mutually exclusive and independent.
(d) equally likely but not independent.

22. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : **[2014]**

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$

(c) 6 (d) 4

23. If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is : **[2014]**

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$

(c) $\frac{12}{55}$ (d) $\frac{14}{55}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(d)	(b)	(c)	(b)	(a)	(b)	(c)	(d)	(d)	(b)	(b)	(d)	(d)	(a)
16	17	18	19	20	21	22	23							
(b)	(a)	(d)	(b)	(c)	(a)	(b)	(b)							

SOLUTIONS

1. **(a)** $P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}$ and $P(E_3)=\frac{1}{4}$;

$$P(E_1 U E_2 U E_3)=1-P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$$

$$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$$

2. **(d)** The event follows binomial distribution with $n=5, p=3/6=1/2$.
$q=1-p=1/2.$; $\therefore$ Variance $=npq=5/4$.

3. **(b)** $\left.\begin{array}{l} np=4 \\ npq=2 \end{array}\right\} \Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$$P(X=1)={}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7$$

$$=8.\frac{1}{2^8}=\frac{1}{2^5}=\frac{1}{32}$$

4. **(c)** A and B will contradict each other if one speaks truth and other false . So , the required

$$\text{Probability }=\frac{4}{5}\left(1-\frac{3}{4}\right)+\left(1-\frac{4}{5}\right)\frac{3}{4}$$

$$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{7}{20}$$

5. **(b)** $P(E)=P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

$$=0.23+0.12+0.20+0.07=0.62$$

$$P(F)=P(1 \text{ or } 2 \text{ or } 3)$$

$$=0.15+0.23+0.12=0.50$$

$$P(E\cap F)=P(2 \text{ or } 3)$$

$$=0.23+0.12=0.35$$

$$\therefore P(EUF)=P(E)+P(F)-P(E\cap F)$$

$$=0.62+0.50-0.35=0.77$$

6. **(a)** mean $=np=4$ and variance $=npq=2$

$$\therefore p=q=\frac{1}{2} \text{ and } n=8$$

$\therefore P(2 \text{ success}) = {}^8C_2 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2$

$= \frac{28}{2^8} = \frac{28}{256}$

7. (b) For a particular house being selected

Probability $= \frac{1}{3}$

P (all the persons apply for the same house)

$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) 3 = \frac{1}{9}$.

8. (c) According to Poission distribution, prob. of getting k successes is

$P(x = k) = e^{-\lambda} \frac{\lambda^k}{k!}$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right) = 1 - \frac{3}{e^2}$.

9. (d) $P(X = r) = \frac{e^{-m} m^r}{r!}$

P (at most 1 phone call)

$= P(X \leq 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = \frac{6}{e^5}$

10. (d) Given : Probability of aeroplane I, scoring a target correctly i.e., $P(I) = 0.3$ probability of scoring a target correctly by aeroplane II, i.e. $P(II) = 0.2$

$\therefore P(\overline{I}) = 1 - 0.3 = 0.7$

$\therefore$ The required probability

$= P(\overline{I} \cap II) = P(\overline{I}).P(II) = 0.7 \times 0.2 = 0.14$

11. (b) A pair of fair dice is thrown, the sample space $S = (1, 1), (1, 2) (1, 3) .... = 36$

Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3,6)

$\therefore$ Possibility of getting score 9 in a single throw

$= \frac{4}{36} = \frac{1}{9}$

$\therefore$ Probability of getting score 9 exactly twice

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 . \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$

$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

12. (b) $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A) P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

13. (d) We have

$P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow 1 - P(x = 0) \geq \frac{9}{10}$

$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow 1 - \frac{9}{10} \geq \left(\frac{3}{4}\right)^n$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq \left(\frac{1}{10}\right)$

Taking log to the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \geq \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow n \geq -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)}$

$= \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

14. (d) Let $A \equiv$ Sum of the digits is 8

$B \equiv$ Product of the digits is 0

Then A = {08, 17, 26, 35, 44}

B = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40,}

$A \cap B = \{08\}$

$\therefore$ P (A/B) $= \frac{P(A \cap B)}{P(B)} = \frac{1/50}{14/50} = \frac{1}{14}$

15. (a) $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

Probability =

$\frac{3 \times 4 \times 2}{{}^9C_3} = \frac{24 \times 3!}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$

16. **(b)** p (at least one failure) $\geq \frac{31}{32}$

$\Rightarrow 1 - p \text{ (no failure)} \geq \frac{31}{32}$

$\Rightarrow 1 - p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \frac{1}{32}$

$\Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

Hence p lies in the interval $\left[0, \frac{1}{2}\right]$.

17. **(a)** In this case

$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$$

Where, $0 \leq P(D) \leq 1$, hence

$$P\left(\frac{C}{D}\right) \geq P(C)$$

18. **(d)** $P(A^c \cap B^c / C) =$

$$\frac{P\left(\left(A^c \cap B^c\right) \cap C\right)}{P(C)} = \frac{P\left((A \cup B)^c \cap C\right)}{P(C)}$$

$$= \frac{P((1 - A \cup B) \cap C)}{P(C)}$$

$$= \frac{P((1 - A - B + A \cap B) \cap C)}{P(C)}$$

$$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)}$$

$= 1 - P(A) - P(B)$

$= P(A^c) - P(B)$

19. **(b)** Given sample space = {1,2,3,.....,8}

Let Event

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3.

This is the case of conditional probability

We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2}$$

$$= \frac{2}{10} = \frac{1}{5}$$

20. **(c)** $p = p$ (correct answer), $q = p$ (wrong answer)

$\Rightarrow \pi = \frac{1}{3}, q = \frac{2}{3}, n = 5$

By using Binomial distribution

Required probability

$$= {}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5$$

$$= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$$

21. **(a)** Given,

$$P(\overline{A \cup B}) = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$$

$$\left(\because P(A \cap B) = \frac{1}{4}\right)$$

$$\Rightarrow P(B) = \frac{1}{3}$$

$\because P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$= P(A \cap B)$

So A & B are independent.

22. **(b)** We can apply binomial probability distribution

We have n = 10

p = Probability of drawing a green ball = $\frac{15}{25} = \frac{3}{5}$

Also q = $1 - \frac{3}{5} = \frac{2}{5}$

Variance = npq

$= 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

23. **(b)** Let A $\equiv$ {0, 1, 2, 3, 4, ..., 10}

n (S) = ${}^{11}C_2 = 55$ where 'S' denotes sample space

Let E be the given event

$\therefore$ E $\equiv$ {(0,4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)}

$\Rightarrow$ n (E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{55}$$