TARGET : JEE (Advanced) 2015

Course : VIJETA & VIJAY (ADP & ADR) **Date : 12-05-2015**

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP NO. 11

TEST INFORMATION

DATE : 13.05.2015 **PART TEST (PT-05)**

Syllabus : Differential Equation, Complex Number, Permutation & Combination, Probability

DATE : 17.05.2015 **MAJOR TEST (MT)**

Syllabus : Full Syllabus

REVISION DPP OF
CONIC SECTION, TANGENTS & NORMALS

Total Marks : 153 **Max. Time : 113.5 min.**

Single choice Objective (–1 negative marking) Q. 1 to 5	**(3 marks 2.5 min.)**	**[15, 12.5]**
Multiple choice objective (–1 negative marking) Q. 6 to 30	**(4 marks, 3 min.)**	**[100, 75]**
Comprehension (MCQ)(–1 negative marking) Q.31 to 32	**(4 marks 3 min.)**	**[8, 6]**
Comprehension (SCQ) (–1 negative marking) Q.33 to 34	**(3 marks 2.5 min.)**	**[6, 5]**
Single digit type Questions (no negative marking) Q. 35 to 40	**(4 marks 2.5 min.)**	**[24, 15]**

1. Variable ellipses are drawn with $x = -4$ as a directrix and origin as corresponding foci. The locus of extremities of minor axes of these ellipses is

(A) $y^2 = 4x$ (B) $y^2 = 2x$

(C) $y^2 = x$ (D) $x^2 = 4y$

2. An endless inextensible string of length 15m passes around two pins, A & B which are 5m apart. This string is always kept tight and a small ring, R, of negligible dimensions, inserted in this string is made to move in a path keeping all segments RA, AB, RB tight (as mentioned earlier). The ring traces a path, given by conic C, then

(A) Conic C is an ellipse with eccentricity 1/2

(B) Conic C is an hyperbola with eccentricity 2

(C) Conic C is an ellipse with eccentricity 2/3

(D) Conic C is a hyperbola with eccentricity 3/2

3. Let PQ and RS be 2 perpendicular focal chords of a rectangular hyperbola, which are not parallel to its axes, then

(A) PQ = RS (B) $PQ^2 + (RS)^2 = (\text{latus rectum})^2$

(C) PQ + RS = (latus rectum) (D) None of these

4. If a variable line has intercepts of e_1 and e_2 on the co-ordinate axes, where $\frac{e_1}{2}$ & $\frac{e_2}{2}$ are the eccentricities of a hyperbola and its conjugate, then the line always touches a fixed circle $x^2 + y^2 = r^2$, then 'r' is

(A) 1 (B) 2 (C) 3 (D) 4

5. P is a point on the circle $x^2 + y^2 = 1$. Line OP, where O is origin, and x = 1 meet at Q. L_1 is a line parallel to x-axis drawn from Q. A line is drawn parallel to x axis from P meeting x = 1 at R. OR meets L_1 at S. Then locus of S is

(A) Circle (B) Parabola

(C) Ellipse (D) Hyperbola

6. Tangent to the curve $y = x^3 - 3x^2 + 2x + 1$ at $P(\alpha, \beta)$ does not meet the curve at any point other than P. Then identify the correct statement(s)

(A) There is only one such tangent

(B) There are two such tangents

(C) $\alpha + \beta = 2$

(D) Equation of normal at P can be y = x

7. Consider the curve $x^n + y^n = a^n (a > 0)$. Tangent at any arbitrary point $P(x_1, y_1)$ of the curve meets x-axis at A and y-axis at B.$(x_1y_1 \neq 0)$, then

(A) OA + OB = constant $\Rightarrow n = \frac{1}{2}$ (O is origin)

(B) AB = constant $\Rightarrow n = \frac{2}{3}$

(C) Mid-point of AB remain same $\forall (x_1, y_1) \Rightarrow n = 1$

(D) Slope of AB $= -\frac{x_1}{y_1} \Rightarrow n = 2$

8. The curve $y^2 = x^3 + 1$ touches a circle whose centre is (4, 0). Then abscissa of point of contact of these curves can be

(A) –1 (B) –2 (C) 4/3 (D) 1/3

9. Let $P(\alpha, 0)$ & $Q(0, \beta)$ be two-points on x-axis and y-axis respectively. Tangents from P touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $M_1 (x_1, y_1)$ & $M_2 (x_2, y_2)$, similarly tangents from Q to this hyperbola touches it at $M_3 (x_3, y_3)$ and $M_4 (x_4, y_4)$, then (given $\alpha, \beta \neq 0$)

(A) $x_1 = x_2$ & $y_1 + y_2 = 0$ (B) $x_1 + x_2 = 0$, $y_1 = y_2$

(C) $x_3 + x_4 = 0$, $y_3 = y_4$ (D) $x_3 + x_4$, $y_3 + y_4 = 0$

10. Circles are drawn with OA & OB as diameters, where A & B are points of parabola $y^2 = 4x$. These circles meet at P (other than O). m_1 and m_2 are slope of tangents at A & B respectively and m is slope of chord AB, then (given $m_1 + m_2 \neq 0$, A, B are points other than origin and 'O' is origin)

(A) A, P, B are collinear points
(B) m is harmonic mean of m_1 and m_2
(C) m is arithmetic mean of m_1 and m_2
(D) OP is perpendicular to AB

11. Tangents are drawn to hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$. ('b' being parameter) from A(0, 4). The locus of point of contact of these tangent is a conic C, then

(A) Eccentricity of conic C is 1
(B) (0, 3) is focus of C
(C) Eccentricity of conic C is 1/2
(D) (0, 5) is focus of C

12. Major and minor axis of an ellipse are 8 and 6 respectively. Initially it touches positive x and y axis and line joining the two focii is parallel to x-axis. It then rotates in anti-clockwise sense, always touching both the positive co-ordinate axes, and the rotation stops when the line joining their focii is vertical for the first time. C is centre of ellipse and O is origin, then

(A) Locus of C is the complete portion of $x^2 + y^2 = 25$ lying in I^{st} quadrant
(B) Locus of C is part of the circle $x^2 + y^2 = 100$
(C) Total distance covered by C is $5 \tan^{-1}\left(\frac{7}{24}\right)$
(D) Initial and final positions of C lies on the curve xy = 12

13. From centre O, of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, two perpendicular rays are drawn meeting the ellipse at P & Q, N is the foot of perpendicular from O to PQ, then

(A) $\frac{1}{OP^2} + \frac{1}{OQ^2} = \frac{25}{144}$
(B) $\frac{1}{OP^2} - \frac{1}{OQ^2} = \frac{25}{144}$
(C) ON = $\frac{12}{5}$
(D) ON = $\frac{6}{5}$

14. Two distinct tangents are drawn to parabola $y^2 = 4x$ from P(h, k) then (given $h \neq 0$).

(A) If slopes of both the tangents are positive then hk > 0
(B) if h < 0, slopes of the two tangents are of different signs
(C) If product of slopes of tangents is negative and hk > 0, then sum of slopes is positive
(D) If product of slopes of tangents is negative and hk > 0, the sum of slopes is negative

15. Tangents are drawn to the curve y = $\frac{3x+1}{x-2}$. These tangents meet x = 2 and y = 3 at P & Q respectively if point R is (2, 3) then

(A) Area of triangle PQR is 7 square units

(B) Area of triangle PQR is 14 square units

(C) Locus of circumcentre of triangle PQR is (y – 3)(x –2) = 1

(D) Locus of circumcentre of triangle PQR is (y – 3)(x –2) = 7

16. y = x is tangent to an ellipse whose foci are (1, 0) and (3, 0) then

(A) Major axis of ellipse is = $\sqrt{6}$

(B) Major axis of ellipse is = $\sqrt{10}$

(C) $\left(\frac{3}{4},\frac{3}{4}\right)$ is the point of contact of this ellipse and this tangent

(D) $\left(\frac{1}{2},\frac{1}{2}\right)$ is the point of contact of this ellipse and this tangent

17. Two perpendicular tangents to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}$ = 1 are such that the auxiliary circle intercepts chord of length ℓ_1 & ℓ_2 on these tangents, then

(A) These tangents intersect on the circle $x^2 + y^2 = 49$

(B) These tangents intersect on the circle $x^2 + y^2 = 25$

(C) $\ell_1^2 + \ell_2^2 = 28$

(D) $\ell_1^2 + \ell_2^2 = 25$

18. Focus & vertex of a parabola are A(5, 2) and B(8, 6) respectively. P & Q are two points on the parabola such that the tangents meet at T(11, 10). Then

(A) P & Q are mirror images of each other in the line 4x – 3y = 14

(B) Area of quadrilateral formed by tangent & normal at P & Q is 400 sq. units

(C) Area of quadrilateral formed by tangents & normal at P & Q is 200 sq. units

(D) P & Q are extremities of latus rectum of this parabola

19. Chord joining A(θ_1) & B(θ_2) is reflected by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1, at B. if AB is a focal chord and the reflected ray meets the ellipse again at C(θ_3), then (given $\theta_1, \theta_2 \neq \frac{n\pi}{2}$, n $\in$ Z)

(where e equal to eccentricity of ellipse)

(A) $e = \left|\frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}\right|$

(B) $e = \left|\frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right|$

(C) $\tan\frac{\theta_1}{2}\tan\frac{\theta_3}{2} = \cot^2\left(\frac{\theta_2}{2}\right)$

(D) $\tan\frac{\theta_1}{2}\tan\frac{\theta_3}{2} = \tan^2\left(\frac{\theta_2}{2}\right)$

20. Let set S consists of all the points (x, y) satisfying $16x^2 + 25y^2 \le 400$. For points in S let maximum and minimum value of $\frac{y-4}{x-9}$ be M and m respectively, then

(A) M = 1 (B) M = $\frac{65}{7}$ (C) m = 1 (D) m = $\frac{7}{65}$

21. Consider the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where x, y are real variables and a, b, c , f, g, h are real constants. Let $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, and curve S be the locus of point of intersection of perpendicular tangents of the above curve.

(A) If $\Delta \neq 0$ and $h^2 = ab$, then S is a straight line

(B) If $\Delta \neq 0$, h = 0, a = b $\neq$ 0 then S is a circle of radius $\sqrt{2\left(g^2+f^2-c\right)}$

(C) If $\Delta = 0$, a + b = 0, then S is a point only

(D) IF $\Delta = 0$, a + b = 0 then S is a pair of straight lines.

22. The ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ has a double contact with a circle at the extremity of latus rectum. The point of contact lying in first and fourth quadrant.

(A) Centre of circle is (0, 0) (B) Centre of circle is $\left(\frac{1}{4},0\right)$

(C) Radius of circle is $\frac{3\sqrt{5}}{4}$ (D) Radius of circle is $\frac{3\sqrt{5}}{2}$

23. Normal at point $P(x_1, y_1)$, not lying on x-axis, to the hyperbola $x^2 - y^2 = a^2$ meets x-axis at A and y-axis at B. If O is origin then

(A) Circumcentre of triangle OAB is P.

(B) Slope of OP + slope of AB = 0

(C) Slope of OP = slope of AB

(D) Locus of centroid of triangle OAB is a rectangular hyperbola

24. Tangents at $A(a\cos\theta_1, b\sin\theta_1)$ & $B(a\cos\theta_2, b\sin\theta_2)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ are perpendicular and their point of intersection is $T(x_1, y_1)$. Normal at A & B meet at point N(h, k). Then

(A) $(a^2 + b^2)\cos^2\left(\frac{\theta_1-\theta_2}{2}\right) = a^2\cos^2\left(\frac{\theta_1+\theta_2}{2}\right) + b^2\sin^2\left(\frac{\theta_1+\theta_2}{2}\right)$

(B) Origin, N and T are vertices of a right angle triangle

(C) $\cos^2\left(\frac{\theta_1-\theta_2}{2}\right) = \frac{a^2+b^2}{(a+b)^2}$

(D) Origin, N and T are collinear points

25. A & B two points on the curve $xy = a^2$. Let N be the mid-point of AB. The line through A and B meets. x-axis at P and y-axis at Q, then

(A) N bisects PQ
(B) ON is perpendicular to AB (where O is origin)
(C) AP = BQ
(D) AQ = BP

26. Two parabolas have same focus and same axis, but in opposite directions. Let P & Q be the point of intersection of these two parabolas, then

(A) PQ is latus rectum of at least one of these parabolas.
(B) PQ is a double ordinate of these parabolas
(C) These two parabolas meet orthogonally
(D) Tangent at P(or Q) to these parabola are equally inclined to both the axis

27. Let $A(x_1, y_1)$, $x_1 \neq 0$, be a point of the curve $y^2 = x^3$. Tangent at A meets the curve again at $B(x_2, y_2)$. M and N are foot of perpendicular drawn to x-axis from point A & B respectively. T is the point where tangent at A meets x-axis, then

(A) $y_1y_2 > 0$
(B) $y_1y_2 < 0$
(C) Area if triangle AMT = 8(Area of triangle BNT)
(D) Area of triangle AMT = 64(Area of triangle BNT)

28. Let, S, be a conic whose centre is M(p, q). Locus of middle points of chords of this conic, which passes through a fixed point $N(\alpha, \beta)$ is

(A) Another conic which has a centre
(B) Another conic with same focus
(C) Another conic with centre as $\left(\frac{\alpha+p}{2}, \frac{\beta+q}{2}\right)$
(D) Another conic with centre as $\left(\frac{\alpha-p}{2}, \frac{\beta-q}{2}\right)$

29. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+11)} = 1$, where f(x) is a strictly decreasing positive function, then

(A) the set of values of k for which the major axis of the ellipse is x-axis is (–3, 2)
(B) the set of values of k for which the major axis of the ellipse is y-axis is $(-\infty, 2)$
(C) the set of values of k for which the major axis of the ellipse is y-axis is $(-\infty, -3) \cup (2, \infty)$
(D) the set of values of k for which the major axis of the ellipse is x-axis is $(-3, \infty)$

30. Two concentric ellipses are such that the foci of one lie on the other and the length of their major-axes are equal. If e_1 & e_2 be their eccentricities, then

(A) the quadrilateral formed by joining their foci is a parallelogram
(B) the angle between their axes is given by $\cos\theta = \sqrt{\frac{1}{e_1^2} + \frac{1}{e_2^2} - \frac{1}{e_1^2 e_2^2}}$
(C) their axes are perpendicular if $e_1 = \sqrt{1-e_2^2}$
(D) None of these

Comprehension (Q. 31 to 32)

Consider the circle, S, with equation $x^2 + y^2 + 2gx + 2fy + c = 0$. This circle meets the parabola $y^2 = 4ax$ at $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$. Also let x-intercept of the circle, S, be X_L.

31. Identify the correct identity (identities)

(A) $y_1 + y_2 + y_3 + y_4 = 0$
(B) $x_1 + x_2 + x_3 + x_4 = -(8a + 4g)$
(C) $y_1y_2y_3y_4 = a^2c$
(D) $y_1y_2y_3y_4 = 16a^2c$

32. If A, B, C are co-normal points and $X_L = 2\sqrt{9^2 + f^2 - c}$, then

(A) $x_4 = 0$
(B) $x_1x_2x_3 = 0$
(C) Circle, touches parabola
(D) $(y_1 + y_2)(y_2 + y_3)(y_3 + y_1) = 0$

Comprehension : (Q.33 to 34)

Let P, Q, R be three distinct points on the circle $x^2 + y^2 = 25$. L, M, N are points on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. PL, QM, NR are perpendicular to x-axis, with each segment not intersecting the x-axis. Further none of these points lie on coordinate axes and P,Q,R have been so chosen that area of triangle PQR is maximum.

33. Area of triangle LMN is (in square units)

(A) $45\sqrt{3}$
(B) $\frac{75\sqrt{3}}{4}$
(C) $25\sqrt{3}$
(D) $15\sqrt{3}$

34. Normals to the ellipse at L, M and N are

(A) Concurrent at a point.
(B) such that they all pass through origin
(C) sides of an equilateral triangle with non-zero area.
(D) such that two of them are necessarily perpendicular.

35. AB is focal chord of a parabola. Let D and C be foot of perpendicular from A & B on it's directrix respectively. If CD= 6 and area of trapezium ABCD is 24 square units, then find length of chord AB.

36. A circle is drawn whose centre is on x-axis and it touches y-axis. If no part of the circle lies ouside the parabola $y^2 = 8x$, then maximum possible radius of the circle is

37. Let P be a point on the curve $y = \ell n\left(1+\sqrt{1-x^2}\right) - \ell nx - \sqrt{1-x^2}$, Tangent at point P meet y-axis at Q, then find the length of segment PQ.

38. Normal at the point P to the parabola $y^2 = 4x$, intersects the circle with SP as diameter at Q also. If PQ = 2 units (given that point S is focus of the given parabola), then find the abscissa of point P.

39. Parabola, P_1 has focus at S(2, 2) and y-axis is it's directrix. Parabola, P_2 is confocal with P_1 and it's directrix is x-axis. Let $Q(x_1, y_1)$ and $R(x_2, y_2)$ be real points of intersection of parabolas P_1 and P_2. If the ratio $\frac{RS}{QS} = a + b\sqrt{b}$ find (a + b) (given $x_2 > x_1$ and a, b $\in$ N)

40. From point P $\left(-\frac{5}{4}, 2\right)$ variable straight lines are drawn to meet the curve y = 2 $\sqrt{x}$ at A & B. Q is a point on this line such that PA.PB = $(PQ)^2$, then locus of point Q is the line ax + y = b, where (a + b) is equal to

ANSWER KEY

DPP # 10

REVISION DPP OF PERMUTATION & COMBINATION AND PROBABILITY

1.	(A)	**2.**	(D)	**3.**	(D)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(A)
8.	(B)	**9.**	(B)	**10.**	(B)	**11.**	(D)	**12.**	(B)	**13.**	(C)	**14.**	(C)
15.	(C)	**16.**	(A)	**17.**	(A)	**18.**	(C)	**19.**	(A,B,C,)	**20.**	(B,C)		
21.	(A,B,D)	**22.**	(B,D)	**23.**	(B,C,D)	**24.**	(A,C)	**25.**	(A,B,C)	**26.** (B,C)	**27.**	(A,B,C)	
28.	(A,C)	**29.**	(A,B,C,D)			**30.**	(B,C,D)	**31.**	(A,B,C,D)	**32.**	(A,B)		
33.	(C,D)	**34.**	(D)	**35.**	(A)	**36.**	(B)	**37.**	4	**38.**	5	**39.**	16
40.	175												

ANSWER KEY

DPP # 11

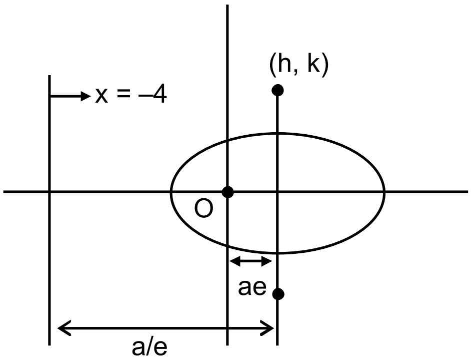
REVISION DPP OF CONIC SECTION, TANGENTS & NORMALS

1.	(A)	**2.**	(A)	**3.**	(A)	**4.**	(B)	**5.**	(D)	**6.**	(A,C,D)
7.	(A,B,C,D)			**8.**	(A,C)	**9.**	(A,C)	**10.**	(A,B,D)	**11.**	(A,B)
12.	(C,D)	**13.**	(A,C)	**14.**	(A,B,C)	**15.**	(B,D)	**16.**	(B,C)	**17.**	(B,C)
18.	(A,C,D)	**19.**	(A,C)	**20.**	(A,D)	**21.**	(A,C)	**22.**	(B,C)	**23.**	(A,B,D)
24.	(A,D)	**25.**	(A,C,D)	**26.**	(B,C)	**27.**	(B,D)	**28.**	(A,C)	**29.**	(A,C)
30.	(A,B,C)	**31.**	(A,B,D)	**32.**	(A,B,C,D)			**33.**	(D)	**34.**	(A)
35.	8	**36.**	4	**37.**	1	**38.**	3	**39.**	5	**40.**	5

! ! BEST OF LUCK FOR JEE-ADVANCED 2015 ! !

MATHEMATICS

1.



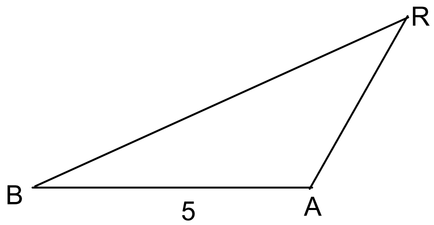
Now $\quad k = b \quad$ and $h = ae$

Also $\quad \frac{a}{e} - ae = 4 \quad \Rightarrow \quad a(1 - e^2) = 4e \quad \Rightarrow \quad a^2(1 - e^2) = 4ae$

$\Rightarrow \quad b^2 = 4h$

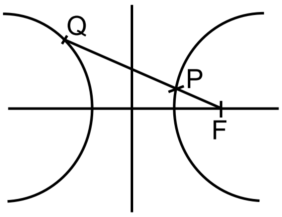
$\Rightarrow \quad k^2 = 4h \quad \Rightarrow \quad \boxed{y^2 = 4x}$

2.



Since length of string is constant, RA + RB = 10, hence locus of R, i.e. conic C is an ellipse with eccentricity $\frac{5}{10} = \frac{1}{2}$.

3.



Let the parameteric equation of chord be

$x = a\sqrt{2} + r\cos\theta \quad \Rightarrow \quad y = r\sin\theta$

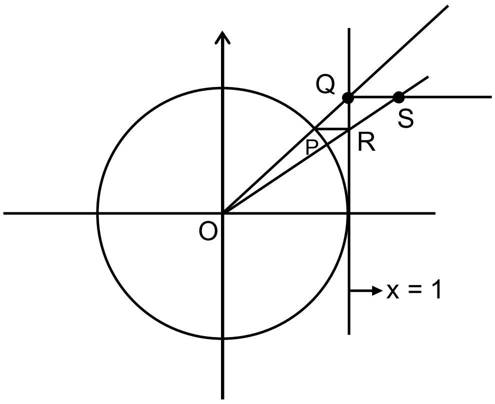
Solving it with $x^2 - y^2 = a^2$ $\quad$ We get $r^2(\cos 2\theta) + (2\sqrt{2}\, a\cos\theta)r + a^2 = 0$

$(PQ)^2 = (r_1 + r_2)^2 - 4r_1r_2 = \frac{8a^2\cos^2\theta - 4a^2\cos 2\theta}{\cos^2 2\theta} = 4a^2\sec^2 2\theta$

Hence $(RS)^2 = 4a^2\sec^2\left\{2\left(\frac{\pi}{2} + \theta\right)\right\} = 4a^2\sec^2 2\theta$

4. Line is $xe_2 + ye_1 - e_1e_2 = 0$ $\quad$ ...(i) $\quad$ also $4\left(e_1^2 + e_2^2\right) = e_1^2e_2^2$ $\quad$ ...(ii)

It is tangent to circle if $r = \frac{e_1e_2}{\sqrt{e_1^2 + e_2^2}} = 2$

5. 

Let point P be ($\cos\theta$, $\sin\theta$), so equation of OP is y = ($\tan\theta$)x, hence point Q is (1, $\tan\theta$). Equation of L_1 is y = $\tan\theta$. Now equation of line PR is y = $\sin\theta$, hence point R is (1, $\sin\theta$). Therefore equation of OR is y = ($\sin\theta$)x. Point of intersection of OR and L_1 is S($\sec\theta$, $\tan\theta$). Hence locus of S is $x^2 - y^2 = 1$, a hyperbola

6. If the tangent at P does not meet the curve at any other point then the equation
$x^3 - 3x^2 + 2x + 1 = mx + c$ (where y = mx + c in the equation tangent at P) has 3 coincident roots α.
Hence $x^3 - 3x^2 + (2 - m)x + (1 - c) \equiv (x - \alpha)^3$
$\Rightarrow$ $3\alpha = 3$ $\Rightarrow$ $\alpha = 1$ $\Rightarrow$ $2 - m = 3$ & $1 - c = -1$
$\Rightarrow$ $m = -1$ & $c = 2$ hence point P is (1, 1)
and corresponding tangent is x + y = 2 & normal is y = x
So (A) (C) (D)

Aliter: Since $y = x^3 - 3x^2 + 2x + 1 = f(x)$ & y = mx + c touch each other at P(α, β)
$\Rightarrow$ $f'(\alpha) = m$, since the equation is a cubic equation these two curves when equated will give 3 roots and as 2 are real, third root too has to be real & as the given condition states 3rd root can be α only.
Hence $f''(\alpha) = 0$ $\Rightarrow$ $\alpha = 1$ m = –1 and c = 2 $\Rightarrow$ (A)(C)(D)

7. Tangent at P is $xx_1^{n-1} + yy_1^{n-1} = a^n$
$\Rightarrow$ A is ($a^n x_1^{1-n}$, 0) & B is (0, $a^n y_1^{1-n}$)

OA + OB = $a^n(x_1^{1-n} + y_1^{1-n})$ = constant $\Rightarrow$ 1 – n = n $\Rightarrow$ $n = \frac{1}{2}$.

AB = $a^n \sqrt{x_1^{2-2n} + y_1^{2-2n}}$ = constant $\Rightarrow$ 2 – 2n = n $\Rightarrow$ n = 2/3

Mid-point of AB is $\left(\frac{a^n x_1^{1-n}}{2}, \frac{a^n y_1^{1-n}}{2}\right)$ remain same $\Rightarrow$ 1 – n = 0 $\Rightarrow$ n = 1

Slope of AB = $-\left(\frac{y_1}{x_1}\right)^{1-n} = -\frac{x_1}{y_1}$ $\Rightarrow$ 1 – n = –1 $\Rightarrow$ n = 2

8. For $y^2 = x^3 + 1$, $\frac{dy}{dx} = \frac{3x^2}{2y}$ (if $y \neq 0$) $\Rightarrow$ $\left.\frac{dy}{dx}\right|_{\text{for circle}} = \frac{4 - x}{y}$ (if $y \neq 0$)

If they touch each other for $y \neq 0$

$\frac{3x^2}{2y} = \frac{4 - x}{y}$ $\Rightarrow$ $3x^2 + 2x - 8 = 0$ $\begin{cases} 4/3 \\ -2 \text{ (rejected)} \end{cases}$

(Since x = –2 $\Rightarrow$ $y^2 = -7$ for first curve)
If y = 0, x = –1 & tangents to both the curves are parallel to y-axis

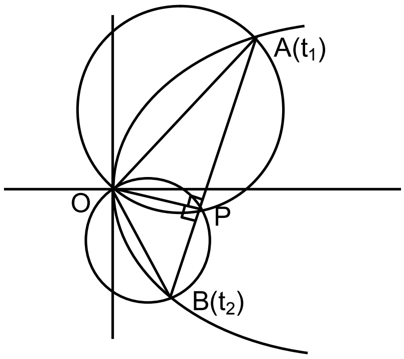
9. Tangent at (p, q) to the hyperbola is $\frac{xp}{a^2} - \frac{yq}{b^2} = 1$

If they pass through (α, 0) $\Rightarrow$ $p = \frac{a^2}{\alpha}$ $\Rightarrow$ $x_1 = x_2$ $\Rightarrow$ $y_1 + y_2 = 0$

If they pass through (0, β)

$q = -\frac{b^2}{\beta}$ $\Rightarrow$ $y_3 = y_4$ $\Rightarrow$ $x_3 + x_4 = 0$

10.

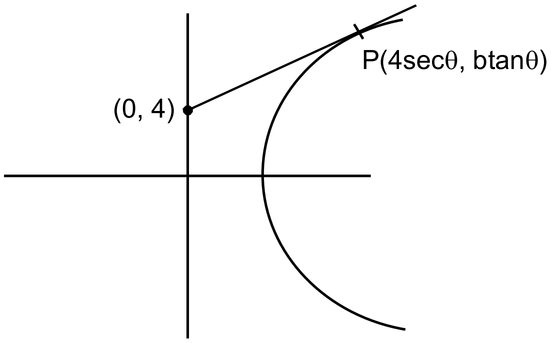


Since OA & OB are diameters of circles $\angle OPA = \angle OPB = 90°$
Hence A, P, B are collinear

Now $m = \dfrac{2}{t_1 + t_2} = \dfrac{2}{\dfrac{1}{m_1} + \dfrac{1}{m_2}} = \dfrac{2m_1m_2}{m_1 + m_2}$ $\left(m_1 = \dfrac{1}{t_1} \quad \& \quad m_2 = \dfrac{1}{t_2}\right)$

Hence (A), (B), (D)

11.



Tangent at P is $\dfrac{x\sec\theta}{4} - \dfrac{y\tan\theta}{b} = 1$.

It passes through (0, 4) Hence $b = -4\tan\theta$ ...(1)

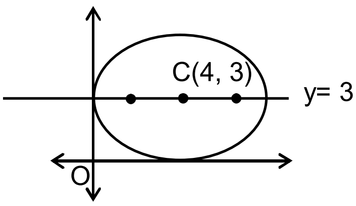
Now $h = 4\sec\theta$ and $k = b\tan\theta = -4\tan^2\theta$ (from (1))

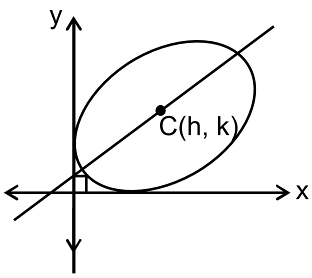
$\Rightarrow \quad K = -4(\sec^2\theta - 1) \quad \Rightarrow \quad k = -4\left(\dfrac{h^2}{16} - 1\right)$.

$\Rightarrow \quad 4K - 16 = -h^2 \quad \Rightarrow \quad x^2 = -4(y - 4) \Rightarrow$ (A) & (B)

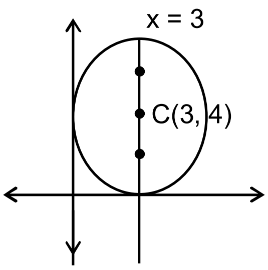
12.

Initially





Finally

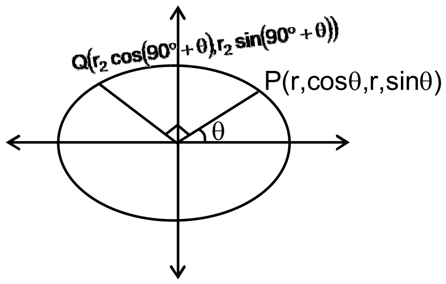


Consider the ellipse in some intermediate position with C being (h, k).

Now $h^2 + k^2 = a^2 + b^2 = 16 + 9 = 25$.
Hence C moves in a circle of radius 5 units whose centre is O. Initially major axis is along y = 3 hence C is (4, 3) and finally major axis is along x = 3 hence C is (3, 4)

Distance curved by C in this motion in $5\left(\tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right) = 5\tan^{-1}\left(\frac{7}{24}\right)$

13. Let $OP = r_1$ & $OQ = r_2$



Now P & Q lie on the ellipse hence

$$r_1^2\left(\frac{\cos^2\theta}{16} + \frac{\sin^2\theta}{9}\right) = 1 \quad \Rightarrow \quad \frac{\cos^2\theta}{16} + \frac{\sin^2\theta}{9} = \frac{1}{r_1^2} \quad \text{....(1)}$$

$$r_2^2\left(\frac{\sin^2\theta}{16} + \frac{\cos^2\theta}{9}\right) = 1 \quad \Rightarrow \quad \frac{\sin^2\theta}{16} + \frac{\cos^2\theta}{9} = \frac{1}{r_2^2} \quad \text{.....(2)}$$

Now (1) + (2) $\Rightarrow \frac{1}{16} + \frac{1}{9} = \frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{25}{144}$

Let equation of chord PQ be $x\cos\alpha + y\sin\alpha = p$, homogenizing the equation of ellipse with this chord gives

$$\frac{x^2}{16} + \frac{y^2}{9} - \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2 = 0$$

As OP & OQ are perpendicular
coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad \left(\frac{1}{16} - \frac{\cos^2\alpha}{p^2}\right) + \left(\frac{1}{9} - \frac{\sin^2\alpha}{p^2}\right) = 0 \quad \Rightarrow \quad \frac{1}{16} + \frac{1}{9} = \frac{1}{p^2} \Rightarrow p^2 = \frac{144}{25} \Rightarrow p = 12/5$$

14. Let equation of tangent be $y = mx + \frac{1}{m}$, as this tangent passes through (h, k), we get $k = mh + \frac{1}{m}$

$$\Rightarrow \quad hm^2 - km + 1 = 0 \; < \begin{matrix} m_1 \\ m_2 \end{matrix} \qquad m_1 + m_2 = \frac{k}{h}$$

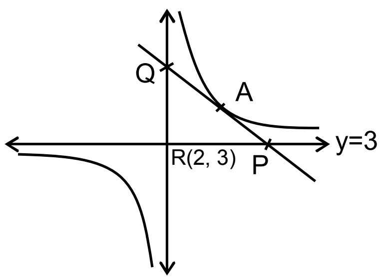
$$m_1 m_2 = \frac{1}{h}$$

If $m_1, m_2 > 0 \quad \Rightarrow \quad \frac{k}{h} > 0$ & $\frac{1}{h} > 0 \quad \Rightarrow \quad k > 0$ & $h > 0 \quad \Rightarrow \quad hk > 0$

If $h < 0 \quad \Rightarrow \quad m_1m_2 < 0$

If $m_1m_2 < 0 \quad \Rightarrow \quad h < 0$ & if $hk > 0 \quad \Rightarrow \quad k < 0 \quad \Rightarrow \quad m_1 + m_2 = \frac{k}{h} > 0$

15.



$y = \dfrac{3(x-2)+7}{x-2} \quad \Rightarrow \quad (x-2)(y-3) = 7$

The given curve a rectangular hyperbola, now shifting origin at (2, 3) the curve transforms to xy = 7. We know that in a rectangular hyperbola portion between axes is bisected by point of tangency and area of triangle PQR is $2c^2$ (for $xy = c^2$). Hence, here area is 14 & circumcentre of PQR is mid-point of PQ which lies on the given curve.

16. Product of perpendicular from two foci on any tangent = $b^2 = \dfrac{3}{\sqrt{2}}.\dfrac{1}{\sqrt{2}} = \dfrac{3}{2} \quad \Rightarrow \quad b = \sqrt{\dfrac{3}{2}}$

Now ae = 1 $\quad \Rightarrow \quad a^2 = b^2 + a^2e^2 \quad \Rightarrow \quad a = \sqrt{\dfrac{5}{2}}$

We know that tangent and normal bisect the angle between focal distances of a point.

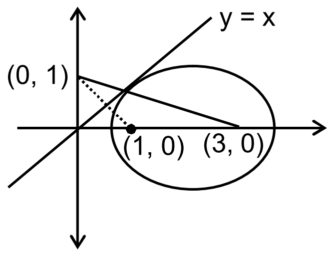


Image of (1, 0) in y = x is (0, 1), line joining (0, 1) & (3, 0) is x + 3y = 3. Point of contact of y = x & ellipse is the point of intersection of y = x and x + 3y = 3, i.e. $\left(\dfrac{3}{4}, \dfrac{3}{4}\right)$

17. As these tangents are perpendicular they meet on director circle of the ellipse, hence locus of point of intersection of these tangents is $x^2 + y^2 = 16 + 9 = 25$.

Let equation of a tangent be $y = mx + \sqrt{16m^2+9}$

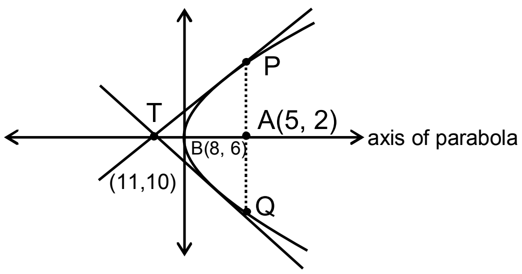
$\ell_1 = 2\sqrt{16-p^2}$, where p is perpendicular distance of the tangent from origin, here $p^2 = \dfrac{16m^2+9}{1+m^2}$

So $\ell_1^2 = 4\left(16 - \dfrac{16m^2+9}{1+m^2}\right) = \dfrac{28}{1+m^2}$

Similarly $\ell_2^2 = \dfrac{28m^2}{1+m^2}$ (replacing m by $-\dfrac{1}{m}$)

Hence $\ell_1^2 + \ell_2^2 = \dfrac{28(1+m^2)}{1+m^2} = 28$

18.



Note that B is mid-point of AT, hence tangents at the extremities of latus rectum meet at T.

Area of quadrilateral formed by tangents an normal at the extremities of latus rectum = $\dfrac{1}{2}$ (latus rectum)^2

$= \dfrac{1}{2} \times (20)^2 = 200$

19. Equation of chord AB is $\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1-\theta_2}{2}\right)$

It passes through (ae, 0) or (–ae, 0)

Hence e = $\left|\frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}\right|$

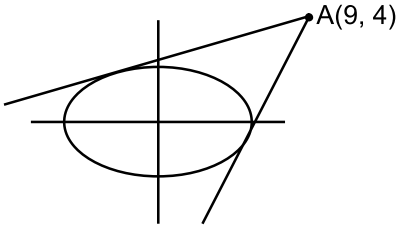
WLOG let it passes through (ae, 0) then BC passes through (–ae, 0)

Hence e = $\frac{1+\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}}{1-\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}}$

$\Rightarrow \quad \tan\frac{\theta_1}{2}\,\tan\frac{\theta_2}{2} = \frac{e-1}{e+1}$ & similarly $\tan\frac{\theta_2}{2}\,\tan\frac{\theta_3}{2} = \frac{-e-1}{-e+1} = \frac{e+1}{e-1}$

So, $\left(\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)\left(\tan\frac{\theta_2}{2}\tan\frac{\theta_3}{2}\right) = 1$

20.



$\frac{y-4}{x-9}$ is the slope of line joining A(9, 4) & (x, y)

For maximum & minimum value of this expression we have to determine the slope of tangents to the ellipse $\frac{x^2}{25}+\frac{y^2}{16} = 1$ from (9, 4)

Hence $y = Kx \pm \sqrt{16k^2+9}$ It passes through (9, 4)

Hence $(4 - 9k)^2 = 16K^2 + 9 \quad \Rightarrow \quad 65K^2 - 72K + 7 = 0$

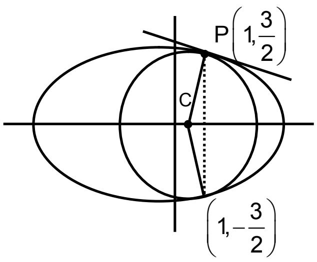
Hence K = 1 or $\frac{7}{65} \quad \Rightarrow \quad M = 1$ & $m = \frac{7}{65}$

21. If $\Delta \neq 0$, $h^2 = ab \Rightarrow$ curve is a parabola, hence S is a straight line

If $\Delta \neq 0$, h = 0, $a = b \neq 0 \Rightarrow$ curve is a circle & S is a circle of radius $\sqrt{2\left(g^2+f^2-c\right)}$ (provided a = b = 1)

If $\Delta = 0$, $a + b = 0 \Rightarrow$ curve is a pair of perpendicular straight lines for which S is a point which is the point of intersection of the two lines.

22.



By symmetry centre of circle lies on x-axis

Normal at P is $\frac{4x}{1}-\frac{3y}{3/2} = 1 \quad \Rightarrow \quad$ point C is $\left(\frac{1}{4},0\right)$

Radius $= \sqrt{\left(1-\frac{1}{4}\right)^2+\left(\frac{3}{2}\right)^2} = \sqrt{\frac{9}{16}+\frac{9}{4}} = \frac{3\sqrt{5}}{4}$

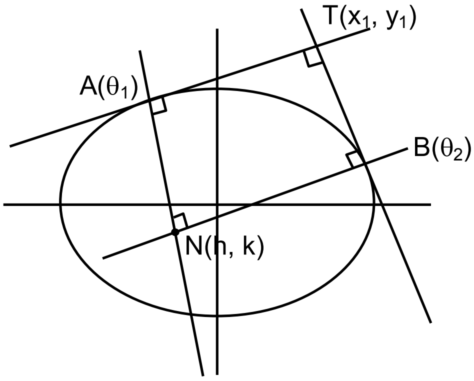
23. Equation of normal at P is $\frac{x}{x_1}+\frac{y}{y_1} = 2$ $\Rightarrow$ A($2x_1$, 0), B(0, $2y_1$)

Hence P is mid-point of AB, i.e. circumcentre of ΔOAB

$m_{AB} = -\frac{y_1}{x_1}$, $m_{OP} = \frac{y_1}{x_1}$

Let (h, k) be centroid of the triangle OAB $\therefore$ 3h = 2asecθ_1 3k = 2atanθ $\Rightarrow$ $x^2 - y^2 = \frac{4a^2}{9}$

24.



$$x_1 = \frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_1}{2}\right)} \quad ..... (1), \qquad y_1 = \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_1}{2}\right)} \quad ....(2)$$

As tangents are perpendicular $x_1^2 + y_1^2 = (a^2 + b^2)$

Hence $(a^2 + b^2)\cos^2\left(\frac{\theta_1-\theta_1}{2}\right) = a^2\cos^2\left(\frac{\theta_1+\theta_2}{2}\right) + b^2\sin^2\left(\frac{\theta_1+\theta_2}{2}\right)$

Now it is clear that ATBN is a rectangle, hence diagonals bisect each other, therefore.

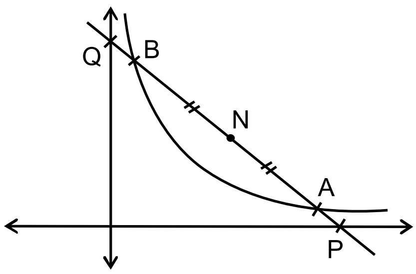
$h + x_1 = a(\cos\theta_1 + \cos\theta_2)$ & $k + y_1 = b(\sin\theta_1 + \sin\theta_2)$

$\Rightarrow \quad \frac{k+y_1}{h+x_1} = \frac{b}{a}\tan\frac{(\theta_1+\theta_2)}{2} = \frac{b}{a}\cdot\frac{ay_1}{bx_1}$ (from (1) & (2))

$\Rightarrow \quad \frac{k+y_1}{h+x_1} = \frac{y_1}{x_1} \quad \Rightarrow \quad kx_1 = hy_1 \quad \Rightarrow \quad \frac{y_1}{x_1} = \frac{k}{h}$

So origin (O), T, N are collinear

25.



Equation of AB is $x + t_1t_2y = a(t_1 + t_2)$

Hence point P is $(a(t_1 + t_2), 0)$ and Q is $\left(0, a\left(\frac{1}{t_1}+\frac{1}{t_2}\right)\right)$

N is $\left(\frac{a(t_1+t_2)}{2}, \frac{a}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)\right)$

Hence N bisects AB as well as PQ

$m_{ON} = \frac{a(t_1+t_2)}{2t_1t_2}\cdot\frac{2}{a(t_1+t_2)} = \frac{1}{t_1t_2}$

Now AN = BN and PN = QN $\Rightarrow$ AP + AN = BQ + BN $\Rightarrow$ AP = BQ

Further AP + AB = BQ + AB $\Rightarrow$ BP = AQ

26. Let the parabola be $y^2 = 4ax$ and $y^2 = -4b(x - a - b)$ (where $ab > 0$)
Let the point of intersection of parabola be (x_1, y_1)
then slope of tangents are say m_1 & m_2

$m_1 = \frac{2a}{y_1}$ & $m_2 = -\frac{2b}{y_1}$

Also, $4ax_1 = -4b(x_1 - a - b) \quad \Rightarrow \quad x_1 = b$ & $y_1 = \pm 2\sqrt{ab}$

Hence $y_1^2 = 4ab$ & PQ is perpendicular to axis, i.e. it is a double ordinate. further $m_1m_2 = -\frac{4ab}{y_1^2} = -1$.

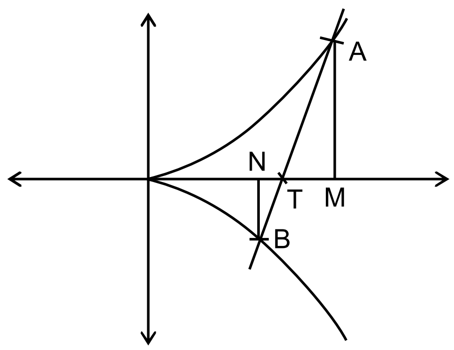
27. Let point A be (t_1^2, t_1^3), hence equation of tangent is $y - t_1^3 = \frac{3t_1}{2}(x - t_1^2)$

If point B is $(t_2^2\ t_2^3)$, then $t_2^3 - t_1^3 = \frac{3t_1}{2}(t_2^2 - t_1^2)$.

$\Rightarrow \quad t_1^2 + 2t_1t_2 + 2t_2^2 = 3t_1t_2 + 3t_1^2$

$\Rightarrow \quad 2t_2^2 - t_1t_2 - t_1^2 = 0 \quad \Rightarrow \quad (t_2 - t_1)(2t_2 + t_1) = 0 \quad \Rightarrow \quad t_2 = -\frac{t_1}{2}$

So point B is $\left(\frac{t_1^2}{4}, \frac{-t_1^3}{8}\right)$



M is $(t_1^2, 0)$, N is $\left(\frac{t_1^2}{4}, 0\right)$ and T is $\left(\frac{t_1^2}{3}, 0\right)$

Triangle AMT and BNT are similar triangle

Hence $\frac{\Delta(AMT)}{\Delta(BNT)} = \left(\frac{AM}{BN}\right)^2 = 8^2 = 64$

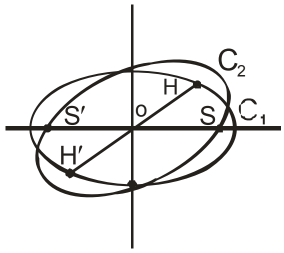
28. Since the above conic has a centre it must be a hyperbola or an ellipse
Let origin be shifted to M(p,q) and axis be so rotated that it coincides with the principle axis of conic S, hence its equation is $Ax^2 + By^2 = 1$, and new-co-ordinates of N be (α', β')
Equation of chord whose mid-point is (h, k) is $T = S_1$, i.e.
$Axh + Byk = Ah^2 + Bk^2$
it passes through (α', β')
Hence $A(x^2 - x\alpha') + B(y^2 - y\beta') = 0$

$\Rightarrow \quad A\left(x - \frac{\alpha'}{2}\right)^2 + B\left(y - \frac{\beta'}{2}\right)^2 = \frac{A(\alpha')^2}{4} + \frac{B(\beta')^2}{4}$

Hence locus is a similar conic whose centre is $\left(\frac{\alpha'}{2}, \frac{\beta'}{2}\right)$

i.e. mid-point of MN.

29. For major axis to be x-axis,
$f(k^2 + 2k + 5) > f(k + 11)$
$\Rightarrow \quad k^2 + 2k + 5 < k + 11 \quad \Rightarrow \quad k \in (-3, 2)$

30. 

HH′ & SS′ have same mid-point $\Rightarrow$ HSH′S′ is a parallelogram

let one ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ...(i) $\therefore$ H lies on it

also H $\equiv$ ($ae_2\cos\theta$, $ae_2\sin\theta$)

putting in equation (i)

$$\cos^2\theta = \frac{1}{e_1^2}+\frac{1}{e_2^2}-\frac{1}{e_1^2e_2^2}$$

$\therefore$ (A), (B) & (C) are correct. (C) follows from (B)

(31 to 32)

Considering a point (at^2, $2at$) and substitute it in equation of circle, S, we get

$a^2t^4 + 2a(2a + g)t^2 + 4aft + c = 0$ $\Big\langle$ t_1, t_2, t_3, t_4

$t_1 + t_2 + t_3 + t_4 = 0$

$\Sigma t_1t_2 = \frac{2a(2a+g)}{a^2}$

$\Sigma y_i = 2a\Sigma t_i = 0$

$\Sigma x_i = a\Sigma t_i^2 = a\{(\Sigma t_i)^2 - 2\Sigma t_1t_2\} = -4(2a + g)$

$\Pi t = \frac{c}{a^2} \Rightarrow \frac{\Pi y_i}{16a^4} = \frac{c}{a^2} \Rightarrow \Pi y_i = 16a^2c$

If A, B, C are co-normal point $t_1 + t_2 + t_3 = 0$ and as $X_L = 2$ (radius of S), centre lies on x-axis

$\Rightarrow$ f = 0 and $t_4 = 0$ (as $\Sigma t_i = 0$)

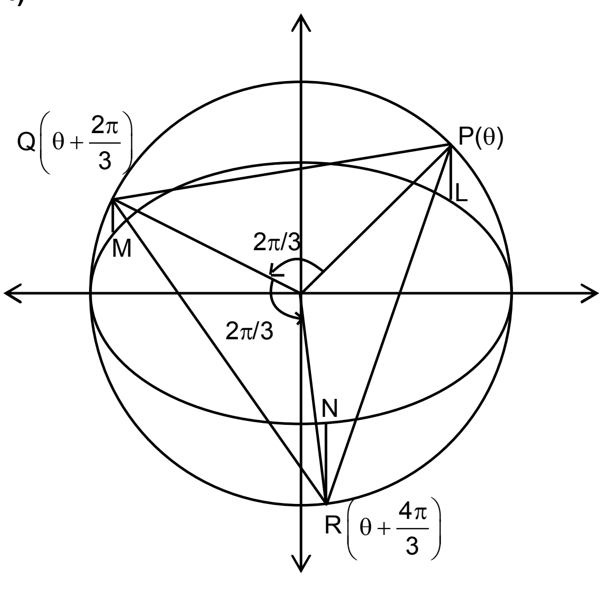
Hence t = 0 is a repeated root of circle and parabola

$\Rightarrow$ one of A, B, C is origin apart from D being origin, i.e. O coincides with one of the points amongst A,B,C

$\Rightarrow$ one of t_1, t_2, t_3 is zero $\Rightarrow$ $t_1 + t_2 = 0$ or $t_2 + t_3 = 0$ or $t_3 + t_1 = 0$

and circle has double contact with parabola at origin.

(33 to 34)



Note that PQR must be an equilateral triangle hence if P is ($5\cos\theta$, $5\sin\theta$), Q & R would be $\left(5\cos\left(\theta+\frac{2\pi}{3}\right), 5\sin\left(\theta+\frac{2\pi}{3}\right)\right)$ & $\left(5\cos\left(\theta+\frac{4\pi}{3}\right), 5\sin\left(\theta+\frac{4\pi}{3}\right)\right)$.

Also area of $\Delta PQR = \frac{\sqrt{3}}{4}(10\sin 60°)^2 = \frac{\sqrt{3}}{4}.100\times\frac{3}{4}=\frac{75\sqrt{3}}{4}$

Now $\frac{\text{area of }\Delta PQR}{\text{area of }\Delta LMN}=\frac{5}{4}$ $\Rightarrow$ Area of $\Delta LMN = 15\sqrt{3}$

Now normals at L, M, N are

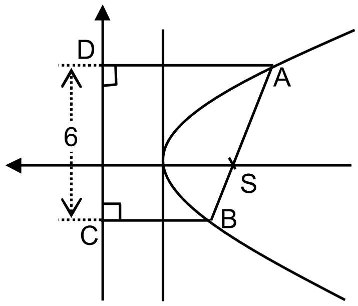
$(5\sin\theta)x - (4\cos\theta)y = \frac{9}{2}\sin 2\theta$

$5\sin\left(\theta+\frac{2\pi}{3}\right)x - 4\cos\left(\theta+\frac{2\pi}{3}\right)y = \frac{9}{2}\sin\left(2\theta+\frac{4\pi}{3}\right)$

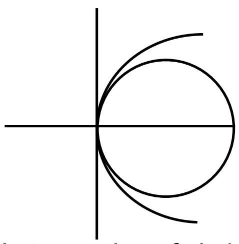
$5\sin\left(\theta+\frac{4\pi}{3}\right)x - 4\cos\left(\theta+\frac{4\pi}{3}\right)y = \frac{9}{2}\sin\left(2\theta+\frac{8\pi}{3}\right)$

Now $\frac{1}{2}\begin{vmatrix} 5\sin\theta & 4\cos\theta & 9\sin 2\theta \\ 5\sin\left(\theta+\frac{2\pi}{3}\right) & 4\cos\left(\theta+\frac{2\pi}{3}\right) & 9\sin\left(2\theta+\frac{4\pi}{3}\right) \\ 5\sin\left(\theta+\frac{4\pi}{3}\right) & 4\cos\left(\theta+\frac{4\pi}{3}\right) & 9\sin\left(2\theta+\frac{8\pi}{3}\right)\end{vmatrix} = 0$ (by $R_1 \to R_1 + R_2 + R_3$)

Hence the normals are concurrent

35. 

Let S be focus AS =AD & BS = BC
Area of trapezium
$= \frac{1}{2}\{AD + BC\}.6$
$= 3\,(AS + BS)$
$= 3AB$
hence AB = 8 units

36. 

Let equation of circle be $(x - r)^2 + y^2 = r^2$
Solving at with $y^2 = 8x_1$ we get
$x^2 - 2rx + r^2 + 8x = r^2$ $\Rightarrow$ $x = 0$ or $x = 2r - 8$
Now $2r - 8 \le 0$ $\Rightarrow$ $r \le 4$ Hence $r_{max} = 4$

Alter: Normal at $(2t^2, 4t)$ to $y^2 = 8x$, meets x-axis at $(4 + 2t^2, 0)$, So x-coordinate of centre should be such that $r \le 4 + 2t^2 \le 4$, hence $r_{max} = 4$

37. Tangent to a curve at (x_1, y_1) meets y-axis at $(0, y_1 - mx_1)$ where. $m = \left.\frac{dy}{dx}\right|_{(x_1,\, y_1)}$

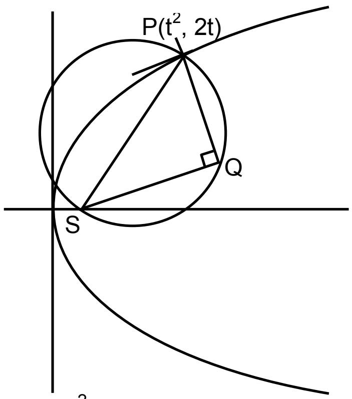
Hence $PQ = |x_1|\sqrt{1+m^2}$

Now $\frac{dy}{dx} = \frac{1}{1+\sqrt{1-x^2}}.\left(\frac{-x}{\sqrt{1-x^2}}\right) - \frac{1}{x} + \frac{x}{\sqrt{1-x^2}}$

$$= \frac{-x^2 - \sqrt{1-x^2}(1+\sqrt{1-x^2}) + x^2(1+\sqrt{1-x^2})}{(1+\sqrt{1-x^2}).x.\sqrt{1-x^2}} = \frac{-\cancel{x^2} - \sqrt{1-x^2} - 1 + \cancel{x^2} + x^2 + x^2\sqrt{1-x^2}}{x(1+\sqrt{1-x^2})\sqrt{1-x^2}}$$

$$= \frac{-(1-x^2)(1+\sqrt{1-x^2})}{x(1+\sqrt{1-x^2})\sqrt{1-x^2}} = \frac{-\sqrt{1-x^2}}{x}$$

Hence $PQ^2 = x_1^2\left(1+\frac{1-x_1^2}{x_1^2}\right) = 1 \quad \Rightarrow \quad PQ = 1$

38. 

Let P be $(t^2, 2t)$, then equation of normal is $y + tx = 2t + t^3$

Therefore $SQ = \left|\frac{t(t^2+1)}{\sqrt{1+t^2}}\right| = |t\sqrt{1+t^2}|$

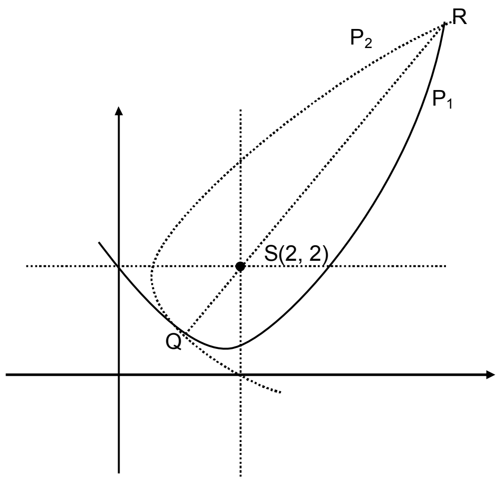
Now $SP = (1 + t^2)$

So $\quad PQ = \sqrt{SP^2 - SQ^2}$

$= \sqrt{(1+t^2)^2 - t^2(1+t^2)} = \sqrt{(1+t^2)(1+t^2-t^2)} = \sqrt{1+t^2}$

Now $PQ = 2 \quad \Rightarrow \quad t^2 = 3$ Hence abscissa of point P is 3.

39.



$$\left.\begin{array}{ll} P_1 & (y-2)^2 = 4(x-1) \\ P_2 & (x-2)^2 = 4(y-1) \end{array}\right\}$$ Subtracting them we get $(x - y)(x + y) = 0 \Rightarrow$ line QR is $y = x$

Hence x_1 & x_2 are roots of the equation

$(x-2)^2 = 4(x-1) \quad \Rightarrow \quad x^2 - 8x + 8 = 0 \left\langle \begin{matrix} 4-2\sqrt{2} = x_1 \\ 4+2\sqrt{2} = x_2 \end{matrix} \right. \quad$ (given $x_2 > x_1$)

So $\quad \dfrac{RS}{QS} = \dfrac{2\sqrt{2}+2}{2\sqrt{2}-2} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} = 3+2\sqrt{2}$

40. Let PA = r_1 and PB = r_2 where r_1 and r_2 are roots of equation $(2 + r\sin\theta)^2 = 4\left(-\dfrac{5}{4} + r\cos\theta\right)$,

$\Rightarrow \quad r^2\sin^2\theta + 4(\sin\theta - \cos\theta)r + 9 = 0$

Let Q be (h, k) and PQ = r $\quad \Rightarrow \quad h = -\dfrac{5}{4} + r\cos\theta,\ k = 2 + r\sin\theta$

then $r_1r_2 = \dfrac{9}{\sin^2\theta} = r^2$

$\Rightarrow \quad 9 = (k-2)^2 \quad \Rightarrow \quad k^2 - 4k - 5 = 0$

Hence k = 5 or k = – 1(rejected)

Hence locus is y = 5 $\quad \Rightarrow \quad$ a = 0, b = 5 $\quad \Rightarrow \quad \boxed{a+b=5}$