CHAPTER 8

Binomial Theorem

1. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are **[2002]**
(a) equal
(b) equal with opposite signs
(c) reciprocals of each other
(d) none of these

2. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is **[2002]**
(a) 1594 (b) 792
(c) 924 (d) 2924

3. The positive integer just greater than $(1+0.0001)^{10000}$ is **[2002]**
(a) 4 (b) 5
(c) 2 (d) 3

4. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals **[2002]**
(a) $3r$ (b) $3r+1$
(c) $2r$ (d) $2r+1$

5. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is **[2003]**
(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

6. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is **[2003]**
(a) 35 (b) 32
(c) 33 (d) 34

7. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of is the same if α equals **[2004]**
(a) $\frac{3}{5}$ (b) $\frac{10}{3}$
(c) $\frac{-3}{10}$ (d) $\frac{-5}{3}$

8. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is **[2004]**
(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

9. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[2005]**
(a) ${}^{55}C_4$ (b) ${}^{55}C_3$
(c) ${}^{56}C_3$ (d) ${}^{56}C_4$

10. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation **[2005]**
(a) $a-b=1$ (b) $a+b=1$
(c) $\frac{a}{b}=1$ (d) $ab=1$

11. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as **[2005]**
(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

12. For natural numbers m, n if $(1-y)^m(1+y)^n$ $= 1+a_1y+a_2y^2+\ldots\ldots$ and $a_1=a_2=10$, then (m,n) is **[2006]**

(a) (20,45) (b) (35,20)
(c) (45,35) (d) (35,45)

13. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals **[2007]**

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$.

14. The sum of the series **[2007]**

$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$

(c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

15. **Statement -1 :** $\sum_{r=0}^{n}(r+1)\ {}^nC_r = (n+2)2^{n-1}$.

Statement-2: $\sum_{r=0}^{n}(r+1)\ {}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$. **[2008]**

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

16. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is: **[2009]**

(a) 2 (b) 7
(c) 8 (d) 0

17. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$. **[2010]**

Statement -1 : $S_3 = 55 \times 2^9$.
Statement - 2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation or Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

18. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is **[2011]**

(a) –132 (b) –144
(c) 132 (d) 144

19. If n is a positive integer , then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is : **[2012]**

(a) an irrational number
(b) an odd positive integer
(c) an even positive integer
(d) a rational number other than positive integers

20. The term independent of x in expansion of

$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is **[2013]**

(a) 4 (b) 120
(c) 210 (d) 310

21. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[2013]**

(a) 7 (b) 5
(c) 10 (d) 8

22. The sum of coefficients of integral power of x in the binomial expansion $\left(1-2\sqrt{x}\right)^{50}$ is : **[2015]**

(a) $\frac{1}{2}\left(3^{50}-1\right)$ (b) $\frac{1}{2}\left(2^{50}+1\right)$

(c) $\frac{1}{2}\left(3^{50}+1\right)$ (d) $\frac{1}{2}\left(3^{50}\right)$

23. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[2016]**

(a) 243 (b) 729
(c) 64 (d) 2187

24. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is : **[2017]**

(a) $2^{20} - 2^{10}$ (b) $2^{21} - 2^{11}$
(c) $2^{21} - 2^{10}$ (d) $2^{20} - 2^9$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(d)	(c)	(d)	(c)	(c)	(b)	(d)	(d)	(c)	(d)	(b)	(d)	(b)
16	17	18	19	20	21	22	23	24						
(a)	(b)	(b)	(a)	(c)	(b)	(c)	(b)	(a)						

SOLUTIONS

1. **(a)** We have $t_{p+1} = {}^{p+q}C_p x^p$ and $t_{q+1} = {}^{p+q}C_q x^q$ ${}^{p+q}C_p = {}^{p+q}C_q$. [Remember ${}^nC_r = {}^nC_{n-r}$]

2. **(c)** We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;
the greatest coeff = coeff of middle term.
So middle term
$= t_7$; $t_7 = t_{6+1}$

$\Rightarrow$ coeff of $t_7 = {}^{12}C_6 = \dfrac{12!}{6!6!} = 924.$

3. **(d)** $(1+0.0001)^{10000} = \left(1+\dfrac{1}{n}\right)^n, n = 10000$

$$= 1 + n.\frac{1}{n} + \frac{n(n-1)}{2!}\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\frac{1}{n^3} + ...$$

$$= 1 + 1 + \frac{1}{2!}\left(1-\frac{1}{n}\right) + \frac{1}{3!}\left(1-\frac{1}{n}\right) + \left(1-\frac{2}{n}\right) + ...$$

$$< 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{(9999)!}$$

$$= 1 + \frac{1}{1!} + \frac{1}{2!} + ...\infty = e < 3$$

4. **(c)** $t_{r+2} = {}^{2n}C_{r+1}\ x^{r+1}; t_{3r} = {}^{2n}C_{3r-1}\ x^{3r-1}$
Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;
$\Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$
$\Rightarrow 2n - r - 1 = 3r - 1 \Rightarrow 2n = 4r \Rightarrow n = 2r$

5. **(d)**

$$T_{r+1} = \frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$$

For first negative term,
$n - r + 1 < 0 \Rightarrow r > n + 1$

$\Rightarrow r > \dfrac{32}{5} \therefore r = 7.\left(\because n = \dfrac{27}{5}\right)$

Therefore, first negative term is T_8.

6. **(c)** $T_{r+1} = {}^{256}C_r(\sqrt{3})^{256-r}(\sqrt[8]{5})^r$

$= {}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\dfrac{256-r}{2}$ & $\dfrac{r}{8}$ both are +ve integer, which is so if r is an integral multiple of 8. As $0 \le r \le 256$

7. **(c)** The middle term in the expansion of
$(1+\alpha x)^4 = T_3 = {}^4C_2(\alpha x)^2 = 6\alpha^2 x^2$
The middle term in the expansion of
$(1-\alpha x)^6 = T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3 x^3$
According to the question

$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\dfrac{3}{10}$

8. **(b)** Coeff. of x^n in $(1+x)(1-x)^n$
= coeff of x^n in

$$(1+x)(1 - {}^nC_1 x + {}^nC_2 x^2 - .... + (-1)^{n\,n}C_n x^n)$$

$$= (-1)^{n\,n}C_n + (-1)^{n-1\,n}C_{n-1} = (-1)^n + (-1)^{n-1}.n$$

$= (-1)^n(1-n)$

ALTERNATE SOLUTION

Coeff of x^n in $(1+x)(1-x)^n$
= Coeff of x^n in
$(1-x)^n$ + Coeff of x^{n-1} in $(1-x)^n$

$= (-1)^n\ {}^nC_n + (-1)^{n-1}\ {}^nC_{n-1}$

$= (-1)^n 1 + (-1)^{n-1} n$

$=(-1)^n[1-n]$

9. **(d)** $^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$

$$={}^{50}C_4+\left[{}^{55}C_3+{}^{54}C_3+{}^{53}C_3+{}^{52}C_3+{}^{51}C_3+{}^{50}C_3\right]$$

We know $\left[{}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right]$

$=({}^{50}C_4+{}^{50}C_3)$

$+{}^{51}C_3+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

$=$

$({}^{51}C_4+{}^{51}C_3)+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

Proceeding in the same way, we get

$={}^{55}C_4+{}^{55}C_3={}^{56}C_4$.

10. **(d)** T_{r+1} in the expansion

$$\left[ax^2+\frac{1}{bx}\right]^{11}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$$

$={}^{11}C_r(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the Coefficient of x^7, we have

$22-3r=7\Rightarrow r=5$

$\therefore$ Coefficient of x^7

$={}^{11}C_5(a)^6(b)^{-5}$

...(1)

Again T_{r+1} in the expansion

$$\left[ax-\frac{1}{bx^2}\right]^{11}={}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$$

$={}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r}(x)^{11-r}$

For the Coefficient of x^{-7}, we have

Now $11-3r=-7\Rightarrow 3r=18\Rightarrow r=6$

$\therefore$ Coefficient of x^{-7}

$={}^{11}C_6a^5\times1\times(b)^{-6}$

$\therefore$ Coefficient of x^7 = Coefficient of x^{-7}

$\Rightarrow {}^{11}C_5(a)^6(b)^{-5}={}^{11}C_6a^5\times(b)^{-6}$

$\Rightarrow ab=1$.

11. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore\ \frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^3}{\left(1-x^{2}\right)^{\frac{1}{2}}}$$

$$=(1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{2!}x^2\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$=\left[1+\frac{x}{2}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right]=\frac{-3}{8}x^2$$

(as x^3 and higher powers of x can be neglected)

12. **(d)** $(1-y)^m(1+y)^n$

$=[1-{}^mC_1y+{}^mC_2y^2-......]$

$[1+{}^nC_1y+{}^nC_2y^2+.....]$

$$=1+(n-m)+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+.....$$

$\therefore\ a_1=n-m=10$

and $a_2=\dfrac{m^2+n^2-m-n-2mn}{2}=10$

So, $n-m=10$ and $(m-n)^2-(m+n)=20$

$\Rightarrow\ m+n=80$

$\therefore\ m=35, n=45$

13. **(b)** $T_{r+1}=(-1)^r.{}^nC_r(a)^{n-r}.(b)^r$ is an expansion of $(a-b)^n$

$\therefore$ 5th term $=t_5=t_{4+1}$

$=(-1)^4.{}^nC_4(a)^{n-4}.(b)^4={}^nC_4.a^{n-4}.b^4$

6th term $=t_6=t_{5+1}=(-1)^5\,{}^nC_5(a)^{n-5}(b)^5$

Given $t_5+t_6=0$

$\therefore\ {}^nC_4.a^{n-4}.b^4+(-{}^nC_5.a^{n-5}.b^5)=0$

$$\Rightarrow\ \frac{n!}{4!(n-4)!}.\frac{a^n}{a^4}.b^4-\frac{n!}{5!(n-5)!}.\frac{a^nb^5}{a^5}=0$$

$$\Rightarrow \frac{n!.a^n b^4}{4!(n-5)!.a^4}\left[\frac{1}{(n-4)}-\frac{b}{5.a}\right]=0$$

or, $\frac{1}{n-4}-\frac{b}{5a}=0 \Rightarrow \frac{a}{b}=\frac{n-4}{5}$

14. **(d)** We know that, $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x+{}^{20}C_2x^2+\ldots\ldots {}^{20}C_{10}x^{10}+\ldots.. {}^{20}C_{20}x^{20}$

Put $x=-1$, $(0)={}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+\ldots\ldots+{}^{20}C_{10}-{}^{20}C_{11}\ldots.+{}^{20}C_{20}$

$\Rightarrow\ 0=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+\ldots..-{}^{20}C_9]+{}^{20}C_{10}$

$\Rightarrow\ {}^{20}C_{10}=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+\ldots\ldots-{}^{20}C_9+{}^{20}C_{10}]$

$\Rightarrow\ {}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+\ldots.+{}^{20}C_{10}=\frac{1}{2}\,{}^{20}C_{10}$

15. **(b)** We have

$$\sum_{r=0}^{n}(r+1)\,{}^nC_rx^r=\sum_{r=0}^{n}r.{}^nC_rx^r+\sum_{r=0}^{n}{}^nC_rx^r$$

$$=\sum_{r=1}^{n}r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}x^r+(1+x)^n$$

$$=nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}x^{r-1}+(1+x)^n$$

$=nx(1+x)^{n-1}+(1+x)^n=\text{RHS}$

$\therefore$ Statement 2 is correct.

Putting $x=1$, we get

$$\sum_{r=0}^{n}(r+1)\,{}^nC_r=n\cdot 2^{n-1}+2^n=(n+2)\cdot 2^{n-1}.$$

$\therefore$ Statement 1 is also true and statement 2 is a correct explanation for statement 1.

16. **(a)** $(8)^{2n}-(62)^{2n+1}$

$=(64)^n-(62)^{2n+1}$

$=(63+1)^n-(63-1)^{2n+1}$

$=$

$$\left[{}^nC_0(63)^n+{}^nC_1(63)^{n-1}+{}^nC_2(63)^{n-2}+\ldots\ldots..+{}^nC_{n-1}(63)+{}^nC_n\right]$$

$$=\left[{}^{2n+1}C_0(63)^{2n+1}-{}^{2n+1}C_1(63)^{2n}+{}^{2n+1}C_2(63)^{2n-1}-\ldots\ldots..+(-1)^{2n+1}\,{}^{2n+1}C_{2+1}\right]$$

$=63\times$

$$\left[{}^nC_0(63)^{n-1}+{}^nC_1(63)^{n-2}+{}^nC_2(63)^{n-3}+\ldots\ldots.\right]+1-63\times$$

$$\left[{}^{2n+1}C_0(63)^{2n}-{}^{2n+1}C_1(63)^{2n-1}+\ldots\ldots.\right]+1$$

$=63\times$ some integral value $+2$

$=8^{2n}-(62)^{2n+1}$ when divided by 9 leaves 2 as the remainder.

17. **(b)** $$S_2=\sum_{j=1}^{10}j\,{}^{10}C_j=\sum_{j=1}^{10}10\,{}^9C_{j-1}$$

$$=10\left[{}^9C_0+{}^9C_1+{}^9C_2+\ldots.+{}^9C_9\right]=10.2^9$$

18. **(b)** $(1-x-x^2+x^3)^6=[(1-x)-x^2(1-x)]^6$

$=(1-x)^6(1-x^2)^6$

$=(1-6x+15x^2-20x^3+15x^4-6x^5+x^6)\times(1-6x^2+15x^4-20x^6+15x^8-6x^{10}+x^{12})$

Coefficient of $x^7=(-6)(-20)+(-20)(15)+(-6)(-6)=-144$

19. **(a)** Consider $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$

$$=2\left[{}^{2n}C_1\left(\sqrt{3}\right)^{2n-1}+{}^{2n}C_3\left(\sqrt{3}\right)^{2n-3}+{}^{2n}C_5\left(\sqrt{3}\right)^{2n-5}+\ldots.\right]$$

(Using binomial expansion of $(a+b)^n$ and $(a-b)^n$)

$=$ which is an irrational number.

20. **(c)** Given expression can be written as

$$\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$=\left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}\right)^{10}$$

$=(x^{1/3}-x^{-1/2})^{10}$

General term $=T_{r+1}$

$={}^{10}C_r\,(x^{1/3})^{10-r}(-x^{-1/2})^r$

$$= {}^{10}C_r\, x^{\frac{10-r}{3}} \cdot (-1)^r \cdot x^{-\frac{r}{2}}$$

$$= {}^{10}C_r (-1)^r \cdot x^{\frac{10-r}{3}-\frac{r}{2}}$$

Term will be independent of x when

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$\Rightarrow r = 4$

So, required term $= T_5 = {}^{10}C_4 = 210$

21. **(b)** We know,

$T_n = {}^nC_3,\ T_{n+1} = {}^{n+1}C_3$

ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow {}^nC_2 = 10$

$\Rightarrow n = 5.$

22. **(c)** $(1-2\sqrt{x})^{50} = {}^{50}C_0 - {}^{50}C_1\, 2\sqrt{x} + {}^{50}C_2 (2\sqrt{x})^2$ ...(1)

$(1+2\sqrt{x})^{50} = {}^{50}C_0 + {}^{50}C_1\, 2\sqrt{x} - {}^{50}C_2 (2\sqrt{x})^2$

$+ \ldots + {}^{50}C_3 (2\sqrt{x})^3 - {}^{50}C_4 (2\sqrt{x})^4$ ...(2)

Adding equation (1) and (2)

$$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}$$

$$= 2\left[{}^{50}C_0 + {}^{50}C_2 2^2 x + {}^{50}C_4 2^3 x^2 + \ldots\right]$$

Putting x = 1, we get above as $\dfrac{3^{50}+1}{2}$

23. **(b)** Total number of terms $= {}^{n+2}C_2 = 28$

$(n+2)(n+1) = 56;\ x = 6$

24. **(a)** We have $({}^{21}C_1 + {}^{21}C_2 \ldots\ldots + {}^{21}C_{10})$

$- ({}^{10}C_1 + {}^{10}C_2 \ldots\ldots {}^{10}C_{10})$

$$= \frac{1}{2}[({}^{21}C_1 + \ldots. + {}^{21}C_{10}) + ({}^{21}C_{11} + \ldots. {}^{21}C_{20})]$$

$-(2^{10} - 1)$

$(\because {}^{10}C_1 + {}^{10}C_2 + \ldots. + {}^{10}C_{10} = 2^{10} - 1)$

$$= \frac{1}{2}[2^{21} - 2] - (2^{10} - 1)$$

$$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$$