TEST INFORMATION

DATE : 10.05.2015 **JEE PREPARTORY TEST (JPT)**

Syllabus : Full Syllabus

REVISION DPP OF PERMUTATION & COMBINATION AND PROBABILITY

Total Marks : 139		**Max. Time : 107.5 min.**	
Single choice Objective (–1 negative marking) Q. 1 to 18	**(3 marks 2.5 min.)**	**[54, 45]**
Multiple choice objective (–1 negative marking) Q. 19 to 33	**(4 marks, 3 min.)**	**[60, 45]**
Comprehension (–1 negative marking) Q.34 to 36	**(3 marks 2.5 min.)**	**[9, 7.5]**
Single digit type Questions (no negative marking) Q. 37,38	**(4 marks 2.5 min.)**	**[8, 5]**
Double digit type Questions (no negative marking) Q. 39	**(4 marks 2.5 min.)**	**[4, 2.5]**
Three digit type Questions (no negative marking) Q. 40	**(4 marks 2.5 min.)**	**[4, 2.5]**

1. A bag contains some white and some black balls, all combinations being equally likely. Total balls are 10. If three are drawn and found black then find probability that the bag contains 1 white and 9 black balls :

(A) $\frac{14}{55}$ (B) $\frac{13}{55}$ (C) $\frac{9}{100}$ (D) $\frac{1}{10}$

2. Each of 10 passengers board any of the three buses randomly which has no passenger initially. The probability that each bus has got at least one passenger, is :

(A) $\frac{{}^{10}P_3.3^7}{3^{10}}$ (B) $1-\frac{{}^{10}C_3.3^7}{3^{10}}$ (C) $1-\frac{2^{10}}{3^{10}}$ (D) $\frac{3^{10}-3.2^{10}+3}{3^{10}}$

3. The number of arrangements of the word "IDIOTS" such that vowels are at the places which from three consecutive terms of an A.P. is :

(A) 36 (B) 72 (C) 24 (D) 108

4. Let set A = {1, 2, 3, …….., 22}. Set B is a subset of A and B has exactly 11 elements. The sum of elements of all possible subsets of B is :

(A) $252\ {}^{21}C_{11}$ (B) $230\ {}^{21}C_{10}$ (C) $253\ {}^{21}C_9$ (D) $253\ {}^{21}C_{10}$

5. From a pack of 52 playing cards, half of the cards are randomly removed without looking at them. From the remaining cards, 3 cards are drawn randomly. The probability that all are king, is :

(A) $\frac{1}{25.17.13}$ (B) $\frac{1}{25.15.13}$ (C) $\frac{1}{52.17.13}$ (D) $\frac{1}{13.51.17}$

6. A fair coin is tossed repeatedly until two consecutive heads is obtained. The probability that two consecutive heads occur on the seventh and eight flips is equal to :

(A) $\frac{11}{256}$ (B) $\frac{15}{256}$ (C) $\frac{13}{256}$ (D) $\frac{17}{256}$

7. An insurance company believes that people can be divided into two classes, those who are accident prone and those who are not. Their statistics show that an accident prone person will not have an accident in a year period with probability 0.4 whereas this probability is 0.2 for the other kind. Given that 30% of people are accident prone, the probability that a new policy holder will have an accident within a year of purchasing a policy is :

(A) 0.74 (B) 0.28 (C) 0.34 (D) 0.66

8. Two cards are drawn one by one without replacement from a deck of 52 cards. The probability that the second card is higher in rank than the first card, is (Ranks in increasing order can be taken from Ace to King)

(A) $\frac{1}{17}$ (B) $\frac{8}{17}$ (C) $\frac{16}{17}$ (D) $\frac{9}{17}$

9. Number of ways in which 3 squares of unit length can be chosen on a 8 × 8 chessboard, so that all squares are in the same diagonal line, is :

(A) 360 (B) 392 (C) 112 (D) 224

10. Number of ways in which the letters of word ABBCABBC can be arranged such that the word ABBC does not appear in any word, is :

(A) 360 (B) 361 (C) 358 (D) 392

11. Let A = $\{x_1, x_2, \ldots, x_8\}$ and B = $\{y_1, y_2, y_3, y_4\}$. The total number of functions $f : A \to B$ that are onto and there are exactly three elements x in A such that $f(x) = y_1$, is :

(A) 11088 (B) 10920 (C) 13608 (D) None of these

12. A man has a T.V. having only 4 channels all of them quite boring. He changes channels after every one minute. The number of ways he can come back to the original channel for the first time after 5 minutes is

(A) 4 (B) 24 (C) 64 (D) 27

13. Find the number of positive integers not exceeding 100 which are divisible by 2 or 3 but not by 4.

(A) 40 (B) 58 (C) 42 (D) 43

14. The number of different rational numbers of the type p/q where p, q are co prime positive integers such that pq = 20! are

(A) 64 (B) 128 (C) 256 (D) 512

15. Maximum number of points of intersection of 5 parabolas is

(A) 12 (B) 6 (C) 40 (D) 50

16. If an unbiased coin is tossed 10 times then the probability that no two consecutive heads occurs is

(A) $\frac{9}{64}$ (B) $1 - \frac{1}{2^{10}}$ (C) $\left(\frac{1}{2}\right)^{10}$ (D) $\frac{1}{2}$

17. Number of 6 digit numbers which can be formed if the sum of their digits has to be 51, is

(A) 56 (B) 50 (C) 36 (D) 30

18. A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. If internal arrangement inside the car does not matter then the number of ways in which they can travel, is

(A) 91 (B) 182 (C) 126 (D) 3920

19. Four people sit round a circular table and each person will roll a normal six sided die once. The probability that no two people sitting next to each other will roll the same number is $\frac{N}{1296}$ then N is divisible by :

(A) 3 (B) 7 (C) 9 (D) 7^2

20. A committee of 10 members is to chosen from among 9 democrats and 7 republicans so that atleast two members of each party serve on the committee. Number of possible ways it can be done, is

(A) $28({}^{13}C_3)$ (B) $7(4\ {}^{13}C_3 - 1)$ (C) ${}^{16}C_{10} - 7$ (D) 8008

21. If w is imaginary cube root of unity and a, b are integers such that $|aw + b| = 1$ then

(A) $(a - b)^2 = 0$ or 1 (B) ab = 0 or 1 (C) $a^2 + b^2 = 0$ or 1 (D) $a^2 + b^2 = 1$ or 2

22. The number of ordered quadruples (a_1, a_2, a_3, a_4) of positive odd integers that satisfy $a_1 + a_2 + a_3 + a_4 = 32$ is equal to :

(A) ${}^{31}C_3$ (B) ${}^{17}C_3$ (C) 4495 (D) $\frac{9}{2}({}^{17}C_3) - {}^{17}C_4$

23. If polynomial of the form $x^3 + ax^2 + bx + c$ is divisible by $x^2 + 2$, then
(A) b = 1
(B) b = 2
(C) 2a = c
(D) $n\{(a, b, c) : a, b, c \in N;\ a, b, c \le 3\} = 1$

24. A player throws an ordinary die with faces numbered 1 to 6. Whenever he throws 1, he has a further throw. If P(n) is the probability of getting a total score of n then

(A) $P(5) = \frac{1}{5}\left(1 - \frac{1}{6^4}\right)$

(B) $P(5) = 5\left(1 - \frac{1}{6^4}\right)$

(C) $P(8) = \frac{1}{180}\left(1 - \frac{1}{6^5}\right)$

(D) $P(8) = \frac{1}{180}\left(1 - \frac{1}{6^4}\right)$

25. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then

(A) $P\left(\frac{B}{A \cup B^c}\right) = 0.25$

(B) P(A/B) = 0.5

(C) $P(A/B^c) = 5/6$

(D) P(neither A nor B) = 0.2

26. There is a group of 6 persons. They play a game in which each has to select a number from 1 to 4. Let A_n is event that n persons have selection of same number, then

(A) $P(A_5) = \frac{18}{4^6}$

(B) $P(A_5) = \frac{18}{4^5}$

(C) $P(A_6) = \frac{1}{4^5}$

(D) $P(A_5 / A_6) = \frac{1}{2}$

27. For any two events A and B, $P(A \cap B)$ is
(A) Not less than P(A) + P(B) – 1
(B) Not greater than P(A) + P(B)
(C) Equal to $P(A) + P(B) - P(A \cup B)$
(D) Equal to $P(A) + P(B) + P(A \cup B)$

28. One die has three faces marked 1, two faces marked 2 and one face marked 3. Another has one face marked 1, two faces marked 2 and three faces marked 3 then
(A) The most probable throw with two dice is 4
(B) The probability of most probable throw is 1/4
(C) The probability of most probable throw is 7/18
(D) None of these

29. For the 3 events A, B and C, P (at least one occurring) = $\frac{3}{4}$, P (at least two occurring) = $\frac{1}{2}$ and P (exactly two occurring) = $\frac{2}{5}$. Which of the following relations is / are **CORRECT**?

(A) P(ABC) = $\frac{1}{10}$

(B) P(AB) + P(BC) + P(CA) = $\frac{7}{10}$

(C) P(A) + P(B) + P(C) = $\frac{27}{20}$

(D) $P(A\bar{B}\bar{C}) + P(\bar{A}\bar{B}C) + P(\bar{A}B\bar{C}) = \frac{1}{4}$

30. Each of 2010 boxes in a line contains one red marble, and for $1 \le k \le 2010$, the box at the k^{th} position also contains k white marbles. A child begins at the first box and successively draws a single marble at random from each box in order. He stops when he first draws a red marble. Let P(n) be the probability that he stops after drawing exactly n marbles. The possible value(s) of n for which $P(n) < \frac{1}{2010}$, is

(A) 44 (B) 45 (C) 46 (D) 47

31. Let all letters of word 'MATHEMATICS' are arranged in all possible order. Three events A, B and C are defined as :

A : Both M are together B : Both T are together C : Both A are together

Which of the following hold(s) good ?

(A) $P(A) = P(B) = \frac{2}{11}$ (B) $P(A \cap B) = P(B \cap C) = P(C \cap A) = \frac{2}{55}$

(A) $P(A \cap B \cap C) = \frac{4}{495}$ (D) $P((A \cap \bar{B}) \mid \bar{C}) = \frac{58}{405}$

32. Two class rooms A and B have capacity of 25 and n – 25 seats respectively. A_n denotes the number of possible seating arrangements of persons in room A when n persons are to be seated in these rooms, starting from room A which is to be filled up to its capacity. If $A_n - A_{n-1} = 25!\ ({}^{49}C_{25})$ then n is divisible by

(A) 5 (B) 2 (C) 4 (D) 6

33. The number of sides of a polygon in which the number of diagonals is at least 10 more than the number of sides can be :

(A) 6 (B) 7 (C) 8 (D) 9

Comprehension # 1 (For Q. 34 to 36)

Let 1, $\alpha_1, \alpha_2, \ldots\ldots, \alpha_k$ are divisors of number $N = 2^{n-1}(2^n - 1)$ where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \ldots\ldots < \alpha_k$

34. The value of k is :

(A) $n.2^n$ (B) 2^n (C) 2n (D) None of these

35. The value of $1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \ldots\ldots + \frac{1}{\alpha_k}$ is :

(A) 2 (B) 3 (C) k (D) k + 1

36. Number of ways to express N as a product of two co-prime factors is

(A) 1 (B) 2 (C) 4 (D) 8

37. A game uses a deck of n different cards with $n \geq 6$. If the number of possible sets of 6 cards that can be drawn from the deck is 6 times the number of possible sets of 3 cards that can be drawn then the sum of the digits of n is

38. If A be any event in sample space the maximum value of $3\sqrt{P(A)} + 4\sqrt{P(\bar{A})}$ is

39. Let $f(x) = ax^4 + bx^2 + 3x + 7$ such that $f(-4) = 2286$ and $f(4) = N$. Find number of ways in which the number N can be resolved as a product of two divisors which are relatively prime.

40. Find the number of different four digit numbers which can be made out of one 1, two 2's, three 3's and four 4's.

ANSWER KEY
DPP # 9

REVISION DPP OF DIFFERENTIAL EQUATION AND COMPLEX NUMBER

1.	(C)	**2.**	(D)	**3.**	(D)	**4.**	(B)	**5.**	(B)	**6.**	(C)	**7.**	(A)
8.	(A)	**9.**	(A)	**10.**	(B)	**11.**	(D)	**12.**	(C)	**13.**	(B,C)	**14.**	(B,D)
15.	(A,B,C)	**16.**	(A,C,D)	**17.**	(A,C,D)	**18.**	(A,C,D)	**19.**	(B,C,D)	**20.**	(A, C)	**21.**	(B,C,D)
22.	(A,C,D)	**23.**	(A,B,C)	**24.**	(A, D)	**25.**	(B, C, D)	**26.**	(A, B, C, D)			**27.**	(C, D)
28.	(A, B, C)	**29.**	(A, C, D)	**30.**	(A,B,C)	**31.**	(A, B, D)	**32.**	(B, C)	**33.**	(C)	**34.**	(B)
35.	(C)	**36.**	(B)	**37.**	(A)	**38.**	(B)	**39.**	8	**40.**	2		

MATHEMATICS

1. Let A = three black balls are drawn E_i = Bag contains i white and 10 – i black balls

$$P(E_1/A) = \frac{P(A\,|\,E_1)P(E_1)}{\sum_{i=0}^{10} P(A\,|\,E_i)P(E_i)} = \frac{\frac{1}{11}\times\frac{{}^9C_3}{{}^{10}C_3}}{\frac{1}{11}\left(\frac{{}^{10}C_3 + {}^9C_3 + ...+{}^3C_3}{{}^{10}C_3}\right)} = \frac{{}^9C_3}{{}^{11}C_4} = \frac{14}{55}$$

2. Total ways = 3^{10} Favorable ways = $3^{10} - {}^3C_1 \times 1 - {}^3C_2\,(2^{10} - 2)$

3. Vowels I, I, O are at place (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (1, 3, 5), or (2, 4, 6)

$\Rightarrow \quad 6 \times \frac{3!}{2!} \times 3! = 108$

4. Sum = $1({}^{21}C_{10}) + 2({}^{21}C_{10}) + ... + 22({}^{21}C_{10})$

5.
$$P(A|E) = \frac{P(A\cap E)}{P(E)} = \frac{\sum_{i=0}^{4} P(E_i)\,P(A\,|\,E_i)}{\sum_{i=0}^{4} P(E_i)} = \frac{\frac{{}^{48}C_{26}}{{}^{52}C_{26}}\times\frac{{}^4C_3}{{}^{26}C_3} + \frac{{}^4C_1\;{}^{48}C_{25}}{{}^{52}C_{26}}\times\frac{{}^3C_3}{{}^{26}C_3} + 0 + 0 + 0}{\frac{{}^4C_0\,{}^{48}C_{26} + {}^4C_1\,{}^{48}C_{25} + {}^4C_2\,{}^{48}C_{24} + {}^4C_3\,{}^{48}C_{23} + {}^4C_4\,{}^{48}C_{22}}{{}^{52}C_{26}}}$$

$$= \frac{4\left({}^{48}C_{26} + {}^{48}C_{25}\right)}{{}^{26}C_3({}^{52}C_{26})} = \frac{4(49!)3!\,23!\,26!\,26!}{26!\,23!\,26!\,52!} = \frac{4(3!)}{52\times51\times50} = \frac{1}{13.17.25}$$

6. × × × × × T H H

First five are (no consecutive heads) 5T or 4T, 1H or 3T, 2H or 2T, 3H

i.e. T T T T T T H H

or H T T T T T H H or H T H T T T H H or H T H T H T H H

$\Rightarrow$ Required probability = $\frac{1 + {}^5C_1 + {}^4C_2 + {}^3C_3}{2^8} = \frac{13}{256}$

7. $P(A\,|\,B_1) = 0.6$, $P(A\,|\,B_2) = 0.8$

$P(B_1) = 0.3$, $P(B_2) = 0.7$

$$P(A) = \sum_{i=1}^{2} P(B_i)\;P(A\,|\,B_i) = \frac{3}{10}\times\frac{6}{10} + \frac{7}{10}\times\frac{8}{10} = 0.74$$

8. P(cards are higher or lower in rank) = $\frac{{}^{13}C_2\;{}^4C_1\;{}^4C_1}{{}^{52}C_2} = \frac{16}{17}$ $\Rightarrow$ P(same) = $\frac{1}{17}$

As P(H) + P(L) + P(same) = 1 $\Rightarrow$ P(H) = P(L) = $\frac{8}{17}$

9. Total ways = $2({}^8C_3) + 4({}^7C_3 + {}^6C_3 + {}^5C_3 + {}^4C_3 + {}^3C_3) = 392$

10. Total words formed = $\frac{8!}{4!\,2!\,2!}$ = 420

Let ABBC = ×

Number of ways in which × ABBC can be arranged = $\frac{5!}{2!}$ = 60 but this includes ×ABBC and ABBC×.

But the word ABBCABBC is counted twice in 60 hence it should be 59 so required number of ways = 420 – 59 = 361

11. No. of functions = $^8C_3 \times (3^5 - {}^3C_1 2^5 + {}^3C_2\, 1^5) = 8400$

12. He has 3, 2, 2, 2, 1 ways respectively at the end of 1 minute, 2 min, 3min, 4 min and 5 min
so $3 \times 2 \times 2 \times 2 \times 1 = 24$ ways

13. $n(2 \cup 3) - n(3 \cap 4) - n(2 \cap 4) + n(2 \cap 3 \cap 4) = 67 - 8 - 25 + 8 = 42$

14. No. of ways = 2(No. of ways to express 20! as product of two co-prime factors)= $2(2^{n-1}) = 2^n = 2^8 = 256$

15. $^5C_2 \times 4$

16. $p_n = \frac{p_{n-1}}{2} + \frac{p_{n-2}}{4}$, $n \geq 4$

	T	H

$= p_{n-2} \times \frac{1}{4}$

		T

$= p_{n-1} \times \frac{1}{2}$

As $p_2 = \frac{3}{4}$ and $p_3 = \frac{5}{8}$ $\therefore$ By above formula, $p_4 = \frac{8}{16}$

similarly $p_5 = \frac{13}{32}$, $p_6 = \frac{21}{64}$, $p_7 = \frac{34}{128}$, $p_8 = \frac{55}{256}$, $p_9 = \frac{89}{512}$, $p_{10} = \frac{144}{1024}$

17. Digits to be used are ≥ 6
999996 $\Rightarrow$ 6 ; 999987 $\Rightarrow$ 30 ; 999888 $\Rightarrow$ 20 $\therefore$ total = 56

18. Required number = $\frac{8!}{5! \times 3!} + \frac{8! \times 2!}{4! \times 4! \times 2!} = 126$

19. Required probability = P(A & C throw same number) + P(A & C throw different number)

$$= \frac{6 \times 5 \times 1 \times 5}{6^4} + \frac{6 \times 4 \times 5 \times 4}{6^4} = \frac{150 + 480}{1296} = \frac{630}{1296}$$

20. Required number of ways = $^{16}C_{10} - {}^9C_9\, {}^7C_1 = {}^{16}C_6 - 7$

21. $|a\omega + b|^2 = 1 \Rightarrow a^2 - ab + b^2 = 1 \Rightarrow (a - b)^2 + ab = 1$

($a^2 - ab + b^2 = 1 \Rightarrow ab$ cannot be negative integer)

When $(a - b)^2 = 0$ then $ab = 1 \Rightarrow (a, b) = (1, 1), (-1, -1)$

When $(a - b)^2 = 1$ then $ab = 0 \Rightarrow (a, b) = (0, 1), (1, 0), (0, -1), (-1, 0)$

22. $(2k_1 + 1) + (2k_2 + 1) + (2k_3 + 1) + (2k_4 + 1) = 32$

$\Rightarrow k_1 + k_2 + k_3 + k_4 = 14 \Rightarrow {}^{14+4-1}C_{4-1} = {}^{17}C_3 = 680$

23. $x^3 + ax^2 + bx + c = (x^2 + 2)(x + a) + (b - 2)x + (c - 2a) \Rightarrow b = 2$ & $c = 2a$

24. P(score of 5) = P(5) + P(14) + P(113) + P(1112) $= \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \frac{1}{6^4}$

P(score of 8) = P(116) + P(1115) + P(11114) + P(111113) + P(1111112)
$= \frac{1}{6^3} + \frac{1}{6^4} + \frac{1}{6^5} + \frac{1}{6^6} + \frac{1}{6^7}$

25. $P\left(\frac{B}{A \cup B^c}\right) = \frac{0.2}{0.8} = \frac{1}{4}$

$P(A/B) = \frac{0.2}{0.4} = \frac{1}{2} \Rightarrow P(A/B^c) = \frac{0.5}{0.6} = 5/6$

26. P(5 persons has same selection) = ${}^6C_5 \times {}^4C_1 \times \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^1$

P(6 persons has same selection) = ${}^4C_1 \times \left(\frac{1}{4}\right)^6 = \frac{1}{4^5}$ Also $P\left(\frac{A_5}{A_6}\right) = 0$

27. $P(A \cup B) \le 1 \Rightarrow P(A) + P(B) - P(A \cap B) \le 1 \Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

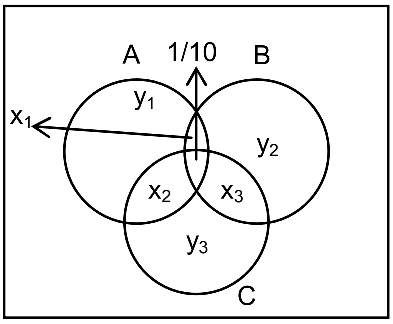
28.

x	P(x)
2	$\frac{3}{36}$
3	$\frac{8}{36}$
4	$\frac{14}{36}$
5	$\frac{8}{36}$
6	$\frac{3}{36}$

29. $P(A \cap B \cap C) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

$x_1 + x_2 + x_3 = \frac{2}{5}$

$y_1 + y_2 + y_3 = \frac{3}{4} - \frac{2}{5} - \frac{1}{10} = \frac{1}{4}$



$\therefore P(A) + P(B) + P(C) = P(A \cup B \cup C) + P(AB) + P(BC) + P(CA) - P(ABC)$

$$= \frac{3}{4} + \frac{7}{10} - \frac{1}{10} = \frac{27}{20}$$

30.

1R	1R	1R	1R	1R	1R	$\rightarrow$	red marbles in the box.
B_1	B_2	B_3........B_k		B_{2009}	B_{2010}	$\rightarrow$	boxes
1W	2W	3W	kW	200W	2010W	$\rightarrow$	white marbles in the box.

Now P(n) = probability that child stops after drawing exactly n marbles.

i.e. at the n^{th} position red marble must be drawn.

$\therefore \quad P(n) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)......\left(\frac{n-2}{n-1}\right)\left(\frac{n-1}{n}\right)\underbrace{\left(\frac{1}{n+1}\right)}_{\text{red}} = \frac{1}{n(n+1)}$

$\therefore \quad \frac{1}{n(n+1)} < \frac{1}{2010} \Rightarrow \frac{2}{n(n+1)} < \frac{1}{1005} \Rightarrow \frac{n(n+1)}{2} > 1005$

$\Rightarrow \quad n \ge 45 \Rightarrow$ B, C, D

31. $P(A) = \dfrac{\dfrac{10!}{2!\,2!}}{\dfrac{11!}{2!\,2!\,2!}} = \dfrac{2}{11}$ = P(B) = P(C)

$P(A \cap B) = \dfrac{\dfrac{9!}{2!}}{\dfrac{11!}{2!\,2!\,2!}} = \dfrac{2}{55}$ = $P(A \cap C) = P(B \cap C)$

$P(A \cap B \cap C) = \dfrac{8!}{\dfrac{11!}{2!\,2!\,2!}} = \dfrac{4}{495}$ $\Rightarrow$ $P\ ((A \cap \bar{B}) \,|\, \bar{C}) = \dfrac{\dfrac{2}{11} - \dfrac{2}{55} - \dfrac{2}{55} + \dfrac{4}{495}}{1 - \dfrac{2}{11}} = \dfrac{58}{405}$

32. $A_n - A_{n-1} = 25!\ (^{49}C_{25})$

$\Rightarrow$ $^nC_{25}(25!) - {}^{n-1}C_{25}(25!) = 25!\ (^{49}C_{25})$ $\Rightarrow$ $^nC_{25} - {}^{n-1}C_{25} = {}^{49}C_{25}$

$\Rightarrow$ $^{n-1}C_{24} = {}^{49}C_{24}$ $\Rightarrow$ $n - 1 = 49$ $\Rightarrow$ $n = 50$

33. $^nC_2 - n = n + k,\ k \geq 10$

$\Rightarrow$ $\dfrac{n(n-1)}{2} = 2n + k$ $\Rightarrow$ $n^2 - n = 4n + 2k$ $\Rightarrow$ $n^2 - 5n = 2k$

$\Rightarrow$ $\left(n - \dfrac{5}{2}\right)^2 = 2k + \dfrac{25}{4} \geq \dfrac{105}{4}$ $\Rightarrow$ $n - \dfrac{5}{2} \geq \dfrac{\sqrt{105}}{2}$ $\Rightarrow$ $n \geq \dfrac{5 + \sqrt{105}}{2}$ $\Rightarrow$ $n \geq 7$

34. Number of divisors = $(n - 1 + 1)(1 + 1) = 2n$ $\Rightarrow k = 2n - 1$

35. Divisors of N are $1, 2, 2^2, \ldots\ldots, 2^{n-1}, 2^n - 1, 2(2^n - 1), 2^2(2^n - 1), 2^{n-1}(2^n - 1)$

$\therefore$ $1 + \dfrac{1}{\alpha_1} + \dfrac{1}{\alpha_2} + \ldots\ldots + \dfrac{1}{\alpha_k} = \left(1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \ldots\ldots + \dfrac{1}{2^{n-1}}\right) + \dfrac{1}{2^n - 1}\left(1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \ldots\ldots + \dfrac{1}{2^{n-1}}\right)$

$= 2\left[1 - \left(\dfrac{1}{2}\right)^n\right]\left(1 + \dfrac{1}{2^n - 1}\right) = 2\left(\dfrac{2^n - 1}{2^n}\right)\left(\dfrac{2^n}{2^n - 1}\right) = 2$

36. Number of ways = $2^{p-1} = 2^{2-1} = 2^1 = 2$

37. $^nC_6 = 6\ {}^nC_3$ $\Rightarrow$ $(n - 3)(n - 4)(n - 5) = 10 \times 9 \times 8$ $\Rightarrow$ $n = 13$

38. Let $P(A) = \sin^2\theta$ $\Rightarrow$ Given expression = $3\sin\theta + 4\cos\theta$ whose maximum value is 5.
where $0 \leq \theta \leq 90°$

39. $f(x) - f(-x) = 6x$ $\Rightarrow$ $f(4) - f(-4) = 24$

$\Rightarrow$ N = 2310 = 2.3.5.7.11

Hence number of divisors = $2^{n-1} = 2^{5-1} = 16$

40.

	Category	**Selection**	**Arrangement**
1.	All 4 alike	1	=1
2.	3 alike + 1 different	$2 \times {}^3C_1 = 6$	$6 \times \dfrac{4!}{3!} = 24$
3.	2 alike + 2 different	$^3C_1 \times {}^3C_2 = 9$	$9 \times \dfrac{4!}{2!} = 108$
4.	2 alike + 2 other alike	$^3C_2 = 3$	$3 \times \dfrac{4!}{2!2!} = 18$
5.	All 4 different	1	4! = 24
			Total = 175