CHAPTER 13

Oscillations

1. In a simple harmonic oscillator, at the mean position **[2002]**
   (a) kinetic energy is minimum, potential energy is maximum
   (b) both kinetic and potential energies are maximum
   (c) kinetic energy is maximum, potential energy is minimum
   (d) both kinetic and potential energies are minimum
2. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be **[2002]**
   (a) $T\sqrt{n}$ (b) $T/\sqrt{n}$
   (c) nT (d) T
3. A child swinging on a swing in sitting position, stands up, then the time period if the swing will **[2002]**
   (a) increase
   (b) decrease
   (c) remains same
   (d) increases if the child is long and decreases if the child is short
4. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is **[2003]**
   (a) $\frac{3}{5}$ (b) $\frac{25}{9}$
   (c) $\frac{16}{9}$ (d) $\frac{5}{3}$
5. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is **[2003]**
   (a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_2}{k_1}$
   (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1}{k_2}$
6. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[2003]**
   (a) 11% (b) 21%
   (c) 42% (d) 10%
7. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[2003]**
   (a) -4 (b) 4
   (c) $4\sqrt{2}$ (d) 8
8. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true ? **[2003]**
   (a) K.E. is maximum when $x = 0$
   (b) T.E is zero when $x = 0$
   (c) K.E is maximum when x is maximum
   (d) P.E is maximum when $x = 0$
9. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. Which relationship between t and t_0 is true? **[2004]**
   (a) $t = 2t_0$ (b) $t = t_0/2$
   (c) $t = t_0$ (d) $t = 4t_0$

10. A particle at the end of a spring executes S.H.M with a period t_1. while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T then **[2004]**

(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

11. The total energy of a particle, executing simple harmonic motion is **[2004]**

(a) independent of x (b) $\propto x^2$

(c) $\propto x$ (d) $\propto x^{1/2}$

where x is the displacement from the mean position.

12. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t(\omega \neq \omega_0)$ is applied to the oscillator. The displacement of the oscillator will be proportional to **[2004]**

(a) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$ (b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\dfrac{m}{\omega_0^2 - \omega^2}$ (d) $\dfrac{m}{(\omega_0^2 + \omega^2)}$

13. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force while the energy is maximum for a frequency ω_2 of the force; then **[2004]**

(a) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(b) $\omega_1 > \omega_2$

(c) $\omega_1 = \omega_2$

(d) $\omega_1 < \omega_2$

14. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \dfrac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{-\pi}{6}$

(c) $\dfrac{\pi}{6}$ (d) $\dfrac{-\pi}{3}$

15. The function $\sin^2(\omega t)$ represents **[2005]**

(a) a periodic, but not simple harmonic motion with a period $\dfrac{\pi}{\omega}$

(b) a periodic, but not simple harmonic motion with a period $\dfrac{2\pi}{\omega}$

(c) a simple harmonic motion with a period $\dfrac{\pi}{\omega}$

(d) a simple harmonic motion with a period $\dfrac{2\pi}{\omega}$

16. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[2005]**

(a) first decrease and then increase to the original value

(b) first increase and then decrease to the original value

(c) increase towards a saturation value

(d) remain unchanged

17. If a simple harmonic motion is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[2005]**

(a) $\dfrac{2\pi}{\sqrt{\alpha}}$ (b) $\dfrac{2\pi}{\alpha}$

(c) $2\pi\sqrt{\alpha}$ (d) $2\pi\alpha$

18. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is **[2006]**

(a) 0.01 s (b) 10 s

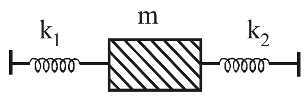
(c) 0.1 s (d) 100 s

19. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[2006]**

(a) $\dfrac{1}{6}$s (b) $\dfrac{1}{4}$s

(c) $\dfrac{1}{3}$s (d) $\dfrac{1}{12}$s

20. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[2007]**



(a) $2f$ (b) $f/2$
(c) $f/4$ (d) $4f$

21. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is **[2007]**

(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$
(c) $\frac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

22. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre.The time at which the maximum speed first occurs is **[2007]**

(a) 0.25 s (b) 0.5 s
(c) 0.75 s (d) 0.125 s

23. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$,then **[2007]**

(a) $A = x_0\omega^2,\ \delta = 3\pi/4$
(b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2,\ \delta = \pi/4$
(d) $A = x_0\omega^2,\ \delta = -\pi/4$

24. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time? **[2009]**

(a) aT/x (b) $aT + 2\pi v$
(c) aT/v (d) $a^2T^2 + 4\pi^2v^2$

25. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is: **[2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

26. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: **[2011]**

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$
(c) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ (d) $\frac{M}{M+m}$

27. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T' **[2011 RS]**

(a) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (b) $2\pi\sqrt{\frac{\ell \rho}{dg}}$
(c) $2\pi\sqrt{\frac{\ell d}{(\rho - d)g}}$ (d) $2\pi\sqrt{\frac{\ell \rho}{(\rho - d)g}}$

28. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum in second is (assuming damping is small) **[2012]**

(a) $\frac{0.693}{b}$ (b) b
(c) $\frac{1}{b}$ (d) $\frac{2}{b}$

29. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
If two springs S_1 and S_2 of force constants k_1 and k_2 respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.
Statement 1 : If stretched by the same amount work done on S_1
Statement 2 : $k_1 < k_2$ **[2012]**

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

30. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[2013]**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

31. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[2013]**

(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

32. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then: **[2014]**

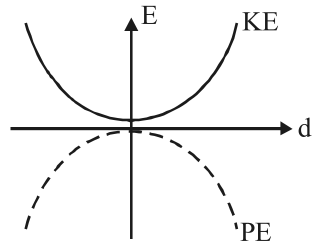
(a) amplitude of motion is 3a
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

33. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :
(g = *gravitational acceleration*) **[2015]**

(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

34. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*) **[2015]**

(a)


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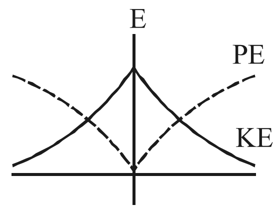

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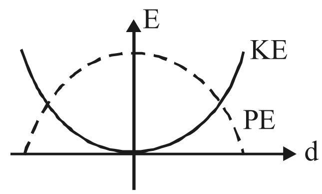
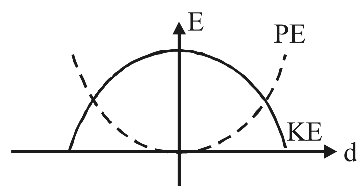


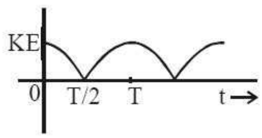
(d)


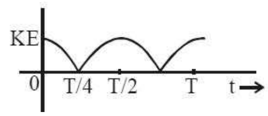
35. A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[2016]**

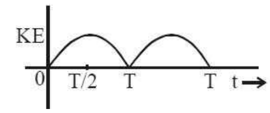
(a) $A\sqrt{3}$ (b) $\frac{7A}{3}$

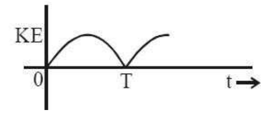
(c) $\frac{A}{3}\sqrt{41}$ (d) 3A

36. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like: **[2017]**

(a)


(b)


(c)


(d)


Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(b)	(c)	(c)	(d)	(c)	(a)	(a)	(b)	(a)	(b)	(c)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(a)	(a)	(a)	(a)	(b)	(b)	(a)	(a)	(a)	(c)	(a)	(d)	(b)	(c)
31	32	33	34	35	36									
(c)	(d)	(c)	(d)	(b)	(b)									

SOLUTIONS

1. **(c)** The kinetic energy (K. E.) and potential energy (U) of a simple harmonic oscillator is given by,

$$\text{K.E} = \frac{1}{2}k(A^2 - x^2); \qquad U = \frac{1}{2}kx^2$$

Where A = amplitude and $k = m\omega^2$

x = displacement from the mean position

At the mean position $x = 0$

$$\therefore \ \text{K.E.} = \frac{1}{2}kA^2 = \text{Maximum and } U = 0$$

2. **(b)** Let the spring constant of the original spring be k. Then its time period $T = 2\pi\sqrt{\frac{m}{k}}$

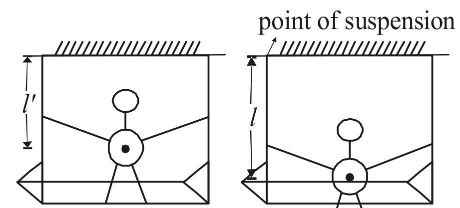
where m is the mass of oscillating body. When the spring is cut into n equal parts, the spring constant of one part becomes nk. Therefore the new time period,

$$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$$

3. **(b)** The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where ℓ = distance between the point of suspension and the centre of mass of the child.

As shown in the figure, $\ell' < \ell$

$\therefore \ T' < T$ i.e., the period decreases.



Case (ii) child standing   Case (i) child sitting

4. **(c)** $T = 2\pi\sqrt{\frac{M}{k}}$

$$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$$

$$\therefore \ 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}}$$

$$M + m = \frac{25}{9} \times M$$

$$1 + \frac{m}{M} = \frac{25}{9} \ \Rightarrow \ \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$$

5. **(c)** Maximum velocity during SHM $= A\omega$

But $k = m\omega^2$

$$\therefore \ \omega = \sqrt{\frac{k}{m}}$$

$\therefore$ Maximum velocity $= A\sqrt{\dfrac{k}{m}}$

Here the maximum velocity is same and m is also same

$\therefore\ A_1\sqrt{k_1} = A_2\sqrt{k_2} \qquad \therefore\ \dfrac{A_1}{A_2} = \sqrt{\dfrac{k_2}{k_1}}$

6. (d) $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ and $T' = 2\pi\sqrt{\dfrac{1.21\ell}{g}}$

$(\because\ \ell' = \ell + 21\%\text{ of }\ell)$

$\%\text{ increase} = \dfrac{T'-T}{T}\times 100$

$= \dfrac{\sqrt{1.21\ell}-\sqrt{\ell}}{\sqrt{\ell}}\times 100 = \left(\sqrt{1.21}-\sqrt{1}\right)\times 100$

$= (1.1-1)\times 100 = 10\%$

7. (c) $x = 4(\cos\pi t + \sin\pi t)$

$= \sqrt{2}\times 4\left(\dfrac{\sin\pi t}{\sqrt{2}} + \dfrac{\cos\pi t}{\sqrt{2}}\right)$

$= 4\sqrt{2}(\sin\pi t\cos 45° + \cos\pi t\sin 45°)$

$x = 4\sqrt{2}\sin(\pi t + 45°)$

on comparing it with $x = A\sin(\omega t + \phi)$

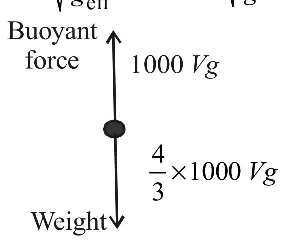
we get $A = 4\sqrt{2}$

8. (a) $\text{K.E.} = \dfrac{1}{2}m\omega^2(a^2 - x^2)$

When $x = 0$, K.E is maximum and is equal to

$\dfrac{1}{2}m\omega^2 a^2$.

9. (a) $t = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}}$; $t_0 = 2\pi\sqrt{\dfrac{\ell}{g}}$



Net force $= \left(\dfrac{4}{3}-1\right)\times 1000\,Vg = \dfrac{1000}{3}Vg$

$g_{eff} = \dfrac{1000\,Vg}{3\times\dfrac{4}{3}\times 1000V} = \dfrac{g}{4}$

$\therefore\ t = 2\pi\sqrt{\dfrac{\ell}{g/4}}$

$t = 2t_0$

10. (b) For first spring, $t_1 = 2\pi\sqrt{\dfrac{m}{k_1}}$,

For second spring, $t_2 = 2\pi\sqrt{\dfrac{m}{k_2}}$

when springs are in series then,

$k_{eff} = \dfrac{k_1k_2}{k_l + k_2}$

$\therefore\ T = 2\pi\sqrt{\dfrac{m(k_l + k_2)}{k_1k_2}}$

$\therefore\ T = 2\pi\sqrt{\dfrac{m}{k_2} + \dfrac{m}{k_1}} = 2\pi\sqrt{\dfrac{t_2^2}{(2\pi)^2} + \dfrac{t_1^2}{(2\pi)^2}}$

$\Rightarrow T^2 = t_1^2 + t_2^2$

where x is the displacement from the mean position

11. (a) At any instant the total energy is

$\dfrac{1}{2}kA_0^2 = \text{constant}$, where $A_0 = \text{amplitude}$

hence total energy is independent of x.

12. (b) Equation of displacement is given by

$x = A\sin(\omega t + \phi)$

where $A = \dfrac{F_0}{m\sqrt{(\omega_0{}^2 - \omega^2)^2}}$

$= \dfrac{F_0}{m(\omega_0{}^2 - \omega^2)}$

Here damping effect is considered to be zero

$\therefore\ x \propto \dfrac{1}{m(\omega_0{}^2 - \omega^2)}$

13. (c) Since energy $\propto$ (Amplitude)^2, the maximum for both of them occurs at the same frequency

$\therefore\ \omega_1 = \omega_2$

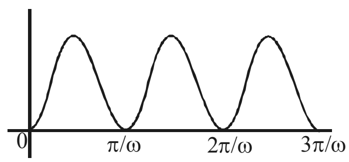
14. (b) $v_1 = \dfrac{dy_1}{dt} = 0.1\times 100\pi\cos\left(100\pi t + \dfrac{\pi}{3}\right)$

$v_2 = \dfrac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \dfrac{\pi}{2}\right)$

$\therefore$ Phase diff. $= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6}$

$= -\frac{\pi}{6}$

15. (c) Clearly $\sin^2 \omega t$ is a periodic function as sin ωt is periodic with period $\frac{\pi}{\omega}$



For SHM $\frac{d^2 y}{dt^2} \propto -y$

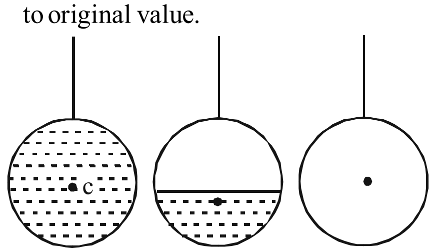
$\frac{dy}{dt} = 2\omega \sin \omega t \cos \omega t = \omega \sin 2\omega t$

$\frac{d^2 y}{dt^2} = 2\omega^2 \cos 2\omega t$ which is not proportional to $-y$. Hence, it is not in SHM.

16. (b) Centre of mass of combination of liquid and hollow portion (at position ℓ), first goes down (to $\ell + \Delta\ell$) and when total water is drained out, centre of mass regain its original position (to ℓ),

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore$ 'T' first increases and then decreases to original value.



17. (a) $\frac{d^2 x}{dt^2} = -\alpha x = -\omega^2 x$

$\Rightarrow \omega = \sqrt{\alpha}$ or $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$

18. (a) Maximum velocity,

$v_{max} = a\omega$

$v_{max} = a \times \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01\ s$

19. (a) K.E. of a body undergoing SHM is given by,

$K.E. = \frac{1}{2} m a^2 \omega^2 \cos^2 \omega t$

$T.E. = \frac{1}{2} m a^2 \omega^2$

Given K.E. = 0.75 T.E.

$\Rightarrow 0.75 = \cos^2 \omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6}\ s$

20. (a) The two springs are in parallel.

$\therefore$ Effective spring constant,

$k = k_1 + k_2$

Now, frequency of oscillation is given by

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

or, $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$ ....(i)

When both k_1 and k_2 are made four times their original values, the new frequency is given by

$f' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4(k_1 + 4k_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}\right)$

21. (b) The kinetic energy of a particle executing S.H.M. is given by

$K = \frac{1}{2} m a^2 \omega^2 \sin^2 \omega t$

where, m = mass of particle
a = amplitude
ω = angular frequency
t = time

Now, average K.E. $= \langle K \rangle = \langle \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t \rangle$

$= \frac{1}{2} m\omega^2 a^2 \langle \sin^2 \omega t \rangle$

$= \frac{1}{2} m\omega^2 a^2 \left(\frac{1}{2}\right) \quad \left(\because \langle \sin^2 \theta \rangle = \frac{1}{2}\right)$

$= \frac{1}{4} m\omega^2 a^2 = \frac{1}{4} m a^2 (2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$

or, $\langle K \rangle = \pi^2 m a^2 \nu^2$

22. **(b)** Here, $x = 2 \times 10^{-2} \cos \pi t$

Speed is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2}\, \pi \sin \pi t$$

For the first time, the speed to be maximum,

$\sin \pi t = 1$ or, $\sin \pi t = \sin \dfrac{\pi}{2}$

$$\Rightarrow \quad \pi t = \frac{\pi}{2} \quad \text{or, } t = \frac{1}{2} = 0.5 \text{ sec.}$$

23. **(a)** Here,

$x = x_0 \cos(\omega t - \pi/4)$

$\therefore$ Velocity, $v = \dfrac{dx}{dt} = -x_0 \omega \sin\left(\omega t - \dfrac{\pi}{4}\right)$

Acceleration,

$$a = \frac{dv}{dt} = -x_0 \omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0 \omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= x_0 \omega^2 \cos\left(\omega t + \frac{3\pi}{4}\right) \quad \text{...(1)}$$

Acceleration, $a = A \cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0 \omega^2$ and $\delta = \dfrac{3\pi}{4}$.

24. **(a)** For an SHM, the acceleration $a = -\omega^2 x$ where ω^2 is a constant. Therefore, $\dfrac{a}{x}$ is a constant. The time period T is also constant. Therefore, $\dfrac{aT}{x}$ is a constant.

25. **(a)** Let, $x_1 = A \sin \omega t$ and $x_2 = A \sin(\omega t + \phi)$

$$x_2 - x_1 = 2A \cos\left(\omega t + \frac{\phi}{2}\right) \sin\frac{\phi}{2}$$

As the maximum separation between the particles is A,

$$\therefore\ 2A \sin\frac{\phi}{2} = A \qquad \Rightarrow \phi = \frac{\pi}{3}$$

26. **(c)** The net force becomes zero atthe mean point.

Therefore, linear momentum must be conserved.

$\therefore\ Mv_1 = (M+m)v_2$

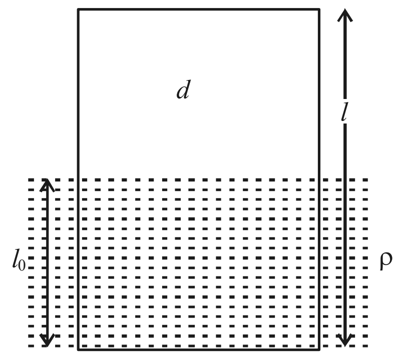
$$MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{m+M}}$$

$$\therefore \left(V = A\sqrt{\frac{k}{M}}\right)$$

$$\because\ A_1\sqrt{M} = A_2\sqrt{M+m}$$

$$\therefore\ \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

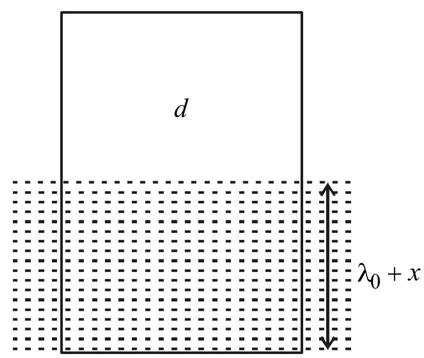
27. **(a)**



At equilibrium

$F_b = mg$

$\rho A \ell_0 g = dA\ell g$ ....(i)



Restoring force,

$F = mg - F_b'$

$F = mg - \rho A(\ell_0 + x)g$

$dA\ell a = dA\ell g - \rho A\ell_0 g - \rho g A x$

$$a = -\frac{\rho g}{d\ell}x$$

Therefore, wooden cube performs S.H.M.

$$\therefore\ \omega = \sqrt{\frac{\rho g}{d\ell}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{\ell d}{\rho g}}$$

28. **(d)** The equation of motion for the pendulum, suffering retardation

$F = -kx - bv$

$$\Rightarrow\ m\frac{d^2x}{dt^2} + kx + b\frac{dk}{dt} = 0$$

$$\Rightarrow\ \frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = 0$$

$$\Rightarrow\ \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \qquad \ldots(1)$$

Let $x = e^{\lambda t}$ is the solution of the equation (1)

$$\frac{dx}{dt} = \lambda e^{\lambda t} \Rightarrow \frac{d^2x}{dt^2} = \lambda^2 e^{\lambda t}$$

Substituting in the equation (1)

$$\lambda^2 e^{\lambda t} + \frac{b}{m}\lambda e^{\lambda t} + \frac{k}{m}e^{\lambda t} = 0$$

$$\lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{k}{m}}}{2} = \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

Solving the equation (1) for x, we have

$$x = e^{\frac{-b}{2m}t}$$

$$\omega = \sqrt{\omega_0{}^2 - \lambda^2} \text{ where } \omega_0 = \frac{k}{m},\ \lambda = \frac{+b}{2}$$

The average life $= \frac{1}{\lambda} = \frac{2}{b}$

29. **(b)** $\because\ w = \frac{1}{2}kx^2$

$$w_1 = \frac{1}{2}k_1x^2;\quad w_2 = \frac{1}{2}k_2x^2$$

Since $w_1 > w_2$ Thus $(k_1 > k_2)$

30. **(c)** $\because A = A_0 e^{-\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)

According to the questions, after 5 second,

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \ldots\text{(i)}$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots\text{(ii)}$$

From eq^{n}s (i) and (ii)

$A = 0.729\,A_0\ \therefore\ \alpha = 0.729$

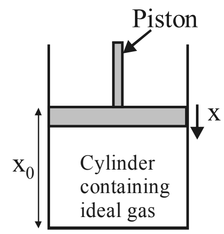
31. **(c)** $\frac{Mg}{A} = P_0$

$P_0 V_0{}^{\gamma} = PV^{\gamma}$

$Mg = P_0 A \quad \ldots(1)$

$P_0 A x_0{}^{\gamma} = PA(x_0 - x)^{\gamma}$

$$P = \frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}$$



Let piston is displaced by distance x

$$Mg - \left(\frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right)A = F_{\text{restoring}}$$

$$P_0 A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) = F_{\text{restoring}}$$

$[x_0 - x \approx x_0]$

$$F = -\frac{\gamma P_0 A x}{x_0}$$

$\therefore$ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

32. **(d)** In simple harmonic motion, starting from rest,

At $t = 0$, $x = A$

$x = A\cos\omega t \quad$ ..... (i)

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

From equation (i)

$A - a = A\cos\omega\tau \quad$ ......(ii)

$A - 3a = A\cos 2\omega\tau \quad$ ....(iii)

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$ ...(iv)

From equation (ii), (iii) and (iv)

$$\frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$\Rightarrow 2a^2 = aA$

$\Rightarrow A = 2a$

$\Rightarrow \quad \frac{a}{A} = \frac{1}{2}$

Now, $A - a = A\cos\omega\tau$

$\Rightarrow \quad \cos\omega\tau = \frac{A-a}{A}$

$\Rightarrow \quad \cos\omega\tau = \frac{1}{2}$ or $\frac{2\pi}{T}\tau = \frac{\pi}{3}$

$\Rightarrow \quad T = 6\tau$

33. (c) As we know, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$

$\frac{T_M}{T} = \sqrt{\frac{\ell+\Delta\ell}{\ell}}$ or $\left(\frac{T_M}{T}\right)^2 = \frac{\ell+\Delta\ell}{\ell}$

or, $\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{Ay}$

$\left[\because \Delta\ell = \frac{Mg\ell}{Ay}\right]$

$\therefore \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

34. (d) $K.E = \frac{1}{2}k(A^2 - d^2)$

and $P.E. = \frac{1}{2}kd^2$

At mean position $d = 0$. At extrement positions $d = A$

35. (b) We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3v = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get $\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$

$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9}$

$\therefore A' = \frac{7A}{3}$

36. (b) For a particle executing SHM

At mean position; $t = 0$, $\omega t = 0$, $y = 0$, $V = V_{max} = a\omega$

$\therefore \quad K.E. = KE_{max} = \frac{1}{2}m\omega^2 a^2$

At extreme position : $t = \frac{T}{4}$, $\omega t = \frac{\pi}{2}$, $y = A$, $V = V_{min} = 0$

$\therefore \quad K.E. = KE_{min} = 0$

Kinetic energy in *SHM*, $KE = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2}m\omega^2 a^2\cos^2\omega t$

Hence graph (2) correctly depicts kinetic energy time graph.