CHAPTER 14

waves

1. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is **[2002]**
(a) 20 (b) 80
(c) 40 (d) 120

2. Tube *A* has both ends open while tube *B* has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube *A* and *B* is **[2002]**
(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1.

3. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is **[2002]**
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps

4. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is **[2002]**
(a) $y = a\sin(\omega t + kx)$
(b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$
(d) $y = -a\sin(\omega t - kx)$

5. When temperature increases, the frequency of a tuning fork **[2002]**
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material

6. The displacement y of a wave travelling in the x -direction is given by

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{metres}$$

where x is expressed in metres and t in seconds. The speed of the wave - motion, in ms^{-1}, is **[2003]**
(a) 300 (b) 600
(c) 1200 (d) 200

7. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[2003]**
(a) 50 Hz (b) 100 Hz
(c) 200Hz (d) 25Hz

8. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[2003]**
(a) $(256+2)$ Hz (b) $(256-2)$ Hz
(c) $(256-5)$ Hz (d) $(256+5)$ Hz

9. The displacement y of a particle in a medium can be expressed as,

$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)m$ where t is in second and x in meter. The speed of the wave is **[2004]**
(a) 20 m/s (b) 5 m/s
(c) 2000 m/s (d) 5π m/s

10. When two tuning forks are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? **[2005]**

(a) 202 Hz (b) 200 Hz
(c) 204 Hz (d) 196 Hz

11. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ? **[2005]**

(a) 0.5% (b) zero
(c) 20% (d) 5 %

12. A whistle producing sound waves of frequencies 9500 HZ and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 HZ, the maximum value of v upto which he can hear whistle is **[2006]**

(a) $15\sqrt{2}\ ms^{-1}$ (b) $\frac{15}{\sqrt{2}}\ ms^{-1}$
(c) $15\ ms^{-1}$ (d) $30\ ms^{-1}$

13. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[2006]**

(a) 105 Hz (b) 1.05 Hz
(c) 1050 Hz (d) 10.5 Hz

14. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of **[2007]**

(a) 100 (b) 1000
(c) 10000 (d) 10

15. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then **[2008]**

(a) $18 > x$ (b) $x > 54$
(c) $54 > x > 36$ (d) $36 > x > 18$

16. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0s, respectively, then α and β in appropriate units are **[2008]**

(a) $\alpha = 25.00\pi, \beta = \pi$
(b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$
(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$
(d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

17. Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu, (\nu + 1)$. They superpose to give beats. The number of beats produced per second will be : **[2009]**

(a) 3 (b) 2
(c) 1 (d) 4

18. A motor cycle starts from rest and accelerates along a straight path at $2 m/s^2$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}) **[2009]**

(a) 98 m (b) 147 m
(c) 196 m (d) 49 m

19. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is **[2010]**

(a) 4.0 N (b) 12.5 N
(c) 0.5 N (d) 6.25 N

20. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$.

This represents a: **[2011]**

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving in $+x$ direction speed $\sqrt{\frac{a}{b}}$

21. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is **[2011 RS]**

(a) A wave travelling along + x direction

(b) A wave travelling along – x direction

(c) A standing wave having nodes at $x = \frac{n\lambda}{2}, n = 0,1,2....$

(d) A standing wave having nodes at $x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2};\ n = 0,1,2....$

22. **Statement - 1 :** Two longitudinal waves given by equations : $y_1(x,t) = 2a\sin(\omega t - kx)$ and $y_2(x,t) = a\sin(2\omega t - 2kx)$ will have equal intensity. **[2011 RS]**

Statement - 2 : Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

(a) Statement-1 is true, statement-2 is false.

(b) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statement-1

(c) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1

(d) Statement-1 is false, statement-2 is true.

23. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now : **[2012]**

(a) f (b) $f/2$

(c) $3f/4$ (d) $2f$

24. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? **[2013]**

(a) 188.5 Hz (b) 178.2 Hz

(c) 200.5 Hz (d) 770 Hz

25. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[2014]**

(a) 12 (b) 8

(c) 6 (d) 4

26. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to : **[2015]**

(a) 18% (b) 24%

(c) 6% (d) 12%

27. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : **[2016]**

(a) 2f (b) f

(c) $\frac{f}{2}$ (d) $\frac{3f}{4}$

28. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is : **[2016]**

(take g = 10 ms^{-2})

(a) $2\sqrt{2}$s (b) $\sqrt{2}$ s

(c) $2\pi\sqrt{2}$ s (d) 2 s

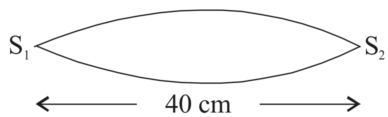
Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(b)	(b)	(b)	(a)	(a)	(c)	(b)	(d)	(c)	(c)	(a)	(a)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28		
(a)	(b)	(a)	(d)	(a)	(d)	(a)	(a)	(b)	(c)	(d)	(b)	(a)		

SOLUTIONS

1. **(b)** This will happen for fundamental mode of vibration as shown in the figure.

Here, $\frac{\lambda}{2} = 40 \ \therefore \lambda = 80\,\text{cm}$

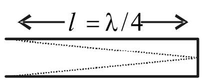


S_1 and S_2 are rigid support

2. **(c)** The fundamental frequency for closed organ pipe is given by

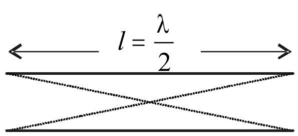
$$\upsilon_c = \frac{v}{4\ell} \qquad \left[\because \ell = \frac{\lambda}{4}\right]$$

Where ℓ = length of the tube and v is the velocity of sound in air.



The fundamental frequency for open organ pipe is given by

$$\upsilon_0 = \frac{v}{2\ell} \qquad \left[\because \ell = \frac{\lambda}{2}\right]$$



$$\therefore \frac{\upsilon_0}{\upsilon_c} = \frac{v}{2\ell} \times \frac{4\ell}{v} = \frac{2}{1}$$

3. **(b)** A tuning fork produces 4 beats/sec with another tuning fork of frequency 288 cps. From this information we can conclude that the frequency of unknown fork is 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

NOTE Had the frequency of unknown fork been 284 cps, then on placing wax its frequency would have decreased thereby increasing the gap between its frequency and the frequency of known fork. This would produce high beat frequency.

4. **(b)** To form a node there should be superposition of this wave with the reflected wave. The reflected wave should travel in opposite direction with a phase change of π. The equation of the reflected wave will be

$y = a \sin(\omega t + kx + \pi)$

$\Rightarrow y = -a \sin(\omega t + kx)$

5. **(b)** The frequency of a tuning fork is given by the expression

$$f = \frac{m^2 k}{4\sqrt{3}\,\pi \ell^2}\sqrt{\frac{Y}{\rho}}$$

As temperature increases, ℓ increases and therefore f decreases.

6. **(a)** $y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$

But $y = A\sin(\omega t - kx + \phi)$

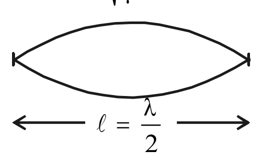
On comparing we get $\omega = 600$; $k = 2$

Also velocity of wave is given by

$$v = \frac{\omega}{k} = \frac{600}{2} = 300 \text{ ms}^{-1}$$

7. **(a)** For a string vibrating between two rigid support, the fundamental frequency is given by

$$n = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$



$$= \frac{1}{2 \times 1}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = 50\,\text{Hz}$$

As the string is vibrating in resonance to a.c of frequency n, therefore both the frequencies are same.

8. **(c)** A tuning fork of frequency 256 Hz makes 5 beats/second with the vibrating string of a piano. Therefore, the frequency of the vibrating string of piano is (256 ± 5) Hz. i.e., either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. Now since the beat frequency decreases, we can conclude that the frequency of piano string is 251Hz

9. **(b)** From equation given,

$\omega = 100$ and $k = 20$

$$v = \frac{\omega}{k} = \frac{100}{20} = 5\text{m/s}$$

10. **(d)** Frequency of fork 1 = 200 Hz = n_0

No. of beats heard when fork 2 is sounded with fork 1

$= \Delta n = 4$

Now we know that if on loading (attaching tape) an unknown fork, the beat frequency increases (from 4 to 6 in this case) then the frequency of the unknown fork 2 is given by,

$n = n_0 - \Delta n = 200 - 4 = 196$ Hz

11. **(c)** $$n' = n\left[\frac{v + v_0}{v}\right] = n\left[\frac{v + \frac{v}{5}}{v}\right] = n\left[\frac{6}{5}\right]$$

$$\frac{n'}{n} = \frac{6}{5}; \frac{n' - n}{n} = \frac{6-5}{5} \times 100 = 20\%$$

12. **(c)** $$\nu' = \nu\left[\frac{v}{v - v_s}\right]$$

$$\Rightarrow 10000 = 9500\left[\frac{300}{300 - v}\right]$$

$$\Rightarrow 300 - v = 300 \times 0.95$$

$$\Rightarrow v = 300 - 285 = 15\ \text{ms}^{-1}$$

13. **(a)** Given $\frac{nv}{2\ell} = 315$ and $(n+1)\frac{v}{2\ell} = 420$

$$\Rightarrow \frac{n+1}{n} = \frac{420}{315} \Rightarrow n = 3$$

Hence $3 \times \frac{v}{2\ell} = 315 \Rightarrow \frac{v}{2\ell} = 105$ Hz

The lowest resonant frequency is when $n = 1$

Therefore lowest resonant frequency $= 105$ Hz.

14. **(a)** We have, $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$;

$$L_2 = 10\log\left(\frac{I_2}{I_0}\right)$$

$$\therefore L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$$

$$\text{or, } \Delta L = 10\log\left(\frac{I_1}{I_0} \times \frac{I_0}{I_2}\right)$$

$$\text{or, } \Delta L = 10\log\left(\frac{I_1}{I_2}\right)$$

$$\text{or, } 20 = 10\log\left(\frac{I_1}{I_2}\right) \text{ or, } 2 = \log\left(\frac{I_1}{I_2}\right)$$

$$\text{or, } \frac{I_1}{I_2} = 10^2 \text{ or, } I_2 = \frac{I_1}{100}.$$

$\Rightarrow$ Intensity decreases by a factor 100.

15. **(b)** For first resonant length

$$\nu = \frac{v}{4\ell_1} = \frac{v}{4 \times 18} \text{ (in winter)}$$

For second resonant length

$$\nu' = \frac{3v'}{4\ell_2} = \frac{3v'}{4x} \text{ (in summer)}$$

$$\therefore \frac{v}{4 \times 18} = \frac{3v'}{4 \times x}$$

$$\therefore x = 3 \times 18 \times \frac{v'}{v}$$

$$\therefore x = 54 \times \frac{v'}{v}\text{cm}$$

$v' > v$ because velocity of light is greater in summer as compared to winter $(v \propto \sqrt{T})$

$\therefore x > 54$cm

16 **(a)** $y(x, t) = 0.005 \cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave

$y(x,t) = a \cos(kx - \omega t)$ we get

$k = \alpha$ and $\omega = \beta$

But $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$\Rightarrow \frac{2\pi}{\lambda} = \alpha$ and $\frac{2\pi}{T} = \beta$

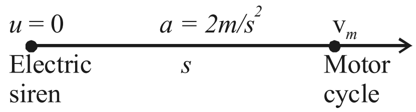
Given that $\lambda = 0.08$ m and $T = 2.0$s

$\therefore \alpha = \frac{2\pi}{0.08} = 25\pi$ and $\beta = \frac{2\pi}{2} = \pi$

17. (b) Maximum number of beats

$= (\nu + 1) - (\nu - 1) = 2$

18. (a)



$v_m^2 - u^2 = 2as$

$\therefore v_m^2 = 2 \times 2 \times s$

$\therefore v_m = 2\sqrt{s}$

According to Doppler's effect

$$\nu' = \nu\left[\frac{v - v_m}{v}\right]$$

$$0.94\nu = \nu\left[\frac{330 - 2\sqrt{s}}{330}\right]$$

$\Rightarrow s = 98.01\ m$

19. (d) $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

Comparing this equation with the standard wave equation

$y = a\sin(\omega t - kx)$

we get

$\omega = \frac{2\pi}{0.04}$

$\Rightarrow \nu = \frac{1}{0.04} = 25\ Hz$

$k = \frac{2\pi}{0.50} \Rightarrow \lambda = 0.5$ m

$\therefore$ velocity, $v = \nu\lambda = 25 \times 0.5$ m/s
$= 12.5$ m/s

Velocity on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

$\therefore T = v^2 \times \mu = (12.5)^2 \times 0.04 = 6.25$ N

20. (a) Given wave equation is $y(x,t)$

$= e^{\left(-ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$

$= e^{-[(\sqrt{a}x)^2 + (\sqrt{b}t)^2 + 2\sqrt{a}x.\sqrt{b}t]}$

$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

$= e^{-\left(x + \sqrt{\frac{b}{a}}t\right)^2}$

It is a function of type $y = f(x + vt)$

$\Rightarrow$ Speed of wave $= \sqrt{\frac{b}{a}}$

21. (d) $y = A\sin(\omega t - kx) + A\sin(\omega t + kx)$

$y = 2A\sin\omega t\cos kx$

For standing wave nodes

$\cos kx = 0$

$$\frac{2\pi}{\lambda}.x = (2n+1)\frac{\pi}{2}$$

$$\therefore x = \frac{(2n+1)\lambda}{4},\ n = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$$

22. (a) Since, $I \propto A^2\omega^2$

$I_1 \propto (2a)^2\omega^2$

$I_2 \propto a^2(2\omega)^2$

$I_1 = I_2$

Intensity depends on frequency also.

23. (a) The fundamental frequency of open tube

$+\ \nu_0 = \frac{v}{2l_0}$ ...(i)

where l is the length of the tube

v = speed of sound

That of closed tube

$\nu_c = \frac{v}{4l_c}$ ...(ii)

According to the problem $l_c = \frac{l_0}{2}$

Thus $\nu_c = \frac{v}{l_0/2} \Rightarrow \nu_c = \frac{v}{2l}$ ...(iii)

From equations (i) and (iii)

$$\nu_0 = \nu_c$$

Thus, $\nu_c = f$ $(\because \nu_0 = f$ is given)

24. **(b)** Fundamental frequency,

$$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$$

$$\left[\because v = \sqrt{\frac{T}{\mu}} \text{ and } \mu = \frac{m}{\ell}\right]$$

Also, $Y = \dfrac{T\ell}{A\Delta\ell} \Rightarrow \dfrac{T}{A} = \dfrac{Y\Delta\ell}{\ell}$

$$\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}} \qquad \text{....(i)}$$

$\ell = 1.5$ m, $\dfrac{\Delta\ell}{\ell} = 0.01$,

$\rho = 7.7 \times 10^3$ kg/m^3 (given)
$\gamma = 2.2 \times 10^{11}$ N/m^2 (given)

Putting the value of ℓ, $\dfrac{\Delta\ell}{\ell}$, ρ and γ in eq^n.

(i) we get,

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3} \quad \text{or} \quad f \approx 178.2 \text{ Hz}$$

25. **(c)** Length of pipe $= 85$ cm $= 0.85$m
Frequency of oscillations of air column in closed organ pipe is given by,

$$f = \frac{(2n-1)\upsilon}{4L}$$

$$f = \frac{(2n-1)\upsilon}{4L} \le 1250$$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85\times 4} \le 1250$$

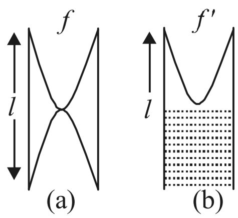
$$\Rightarrow 2n - 1 \le 12.5 \approx 6$$

26. **(d)** $f_1 = f\left[\dfrac{v}{v - v_s}\right] = f \times \dfrac{320}{300} Hz$

$$f_2 = f\left[\frac{v}{v + v_s}\right] = f \times \frac{320}{340} Hz$$

$$\left(\frac{f_2}{f_1} - 1\right) \times 100 = \left(\frac{300}{340} - 1\right) \times 100 \simeq 12\%$$

27. **(b)**



The fundamental frequency in case (a) is

$$f = \frac{v}{2\ell}$$

The fundamental frequency in case (b) is

$$f' = \frac{v}{4(\ell/2)} = \frac{u}{2\ell} = f$$

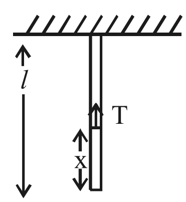
28. **(a)** We know that velocity in string is given by

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{...(i)}$$

where $\mu = \dfrac{m}{l} = \dfrac{\text{mass of string}}{\text{length of string}}$

The tension $T = \dfrac{m}{\ell} \times x \times g$ ..(ii)

From (1) and (2)



$$\frac{dx}{dt} = \sqrt{gx}$$

$$x^{-1/2} dx = \sqrt{g}\, dt$$

$$\therefore \int_0^\ell x^{-1/2} dx - \sqrt{g}\int_0^\ell dt\, 2\sqrt{1}$$

$$= \sqrt{g} \times t \quad \therefore t = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}$$