TARGET : JEE (Advanced) 2015

Course: VIJETA & VIJAY (ADP & ADR) **Date : 24-04-2015**

PHYSICS

DPP

DAILY PRACTICE PROBLEMS

NO. 06

TEST INFORMATION

TEST : CUMULATIVE TEST (CT)-2 (6 hours) (**Test Date : 26-04-2015**)

Syllabus : Current electricity, Capacitor, Magnetic field and force, Work, power, energy, Circular motion, Centre of mass.

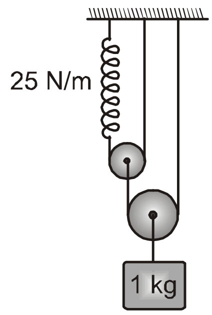
This DPP is to be discussed (28-04-2015)

CT-2 to be discussed (28-04-2015)

DPP No. # 06

Total Total Marks : 151	**Max. Time : 117½ min.**	
Single choice Objective (–1 negative marking) Q. 1 to 16	**(3 marks 2½ min.)**	**[48, 40]**
Multiple choice objective (–1 negative marking) Q. 17 to 22	**(4 marks, 3 min.)**	**[24, 18]**
Single Digit Subjective Questions (no negative marking) Q.23 to Q.31	**(4 marks 2½ min.)**	**[36, 22½]**
Double Digits Subjective Questions (no negative marking) Q. 32	**(4 marks 2½ min.)**	**[4, 2½]**
Comprehension (–1 negative marking) Q.33 to 41	**(3 marks 2½ min.)**	**[27, 22½]**
Match Listing (–1 negative marking) Q.42 to Q.45	**(3 marks, 3 min.)**	**[12, 12]**

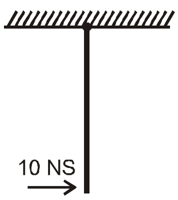
1. Find the natural frequency of oscillation of the system as shown in figure. Pulleys are massless and frictionless. Spring and string are also massless. (Take $\pi^2 = 10$)



(A) $\frac{\pi}{2}$ Hz (B) $\sqrt{\pi}$ Hz

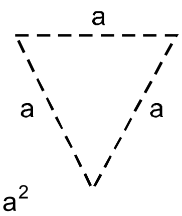
(C) $\frac{10}{\sqrt{\pi}}$ Hz (D) πHz

2. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end. When at rest it receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. The kinetic energy of rod just after impact is



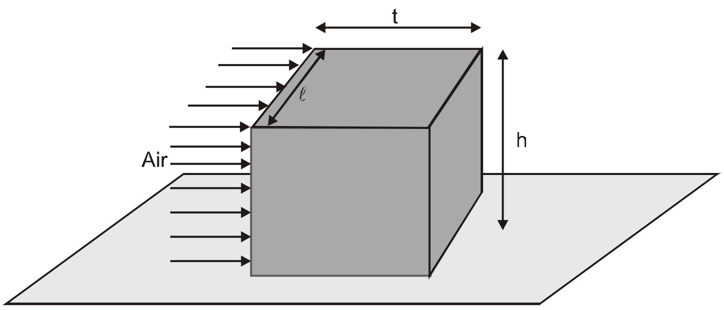
(A) 75 J (B) 100 J (C) 200 J (D) 50 J

3. All sides of an equilateral triangle are diameter of three identical uniform semi-circular rings each of mass m. Plane of each ring is perpendicular to the plane of paper. Then moment of inertia of this system of three semicircular rings about an axis through centroid of triangle and perpendicular to plane of paper is :



(A) $\frac{5ma^2}{24}$ (B) $\frac{5ma^2}{16}$ (C) $\frac{5ma^2}{8}$ (D) $\frac{5ma^2}{6}$

4. A block of dimensions $\ell \times t \times h$ and uniform density ρ_w rests on a rough floor. Wind blowing with speed V and of density ρ_a falls perpendicularly on one face of dimension $\ell \times h$ of the block as shown in figure. Assuming that air is stopped when it strikes the wall and there is sufficient friction on the ground so that the block does not slide, the minimum speed V so that the block topples is :



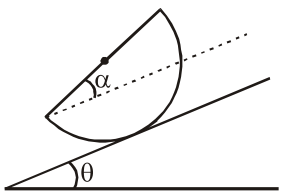
(A) $\left(\frac{\rho_w g}{\rho_a h}\right)^{1/2}.t$ (B) $\left(\frac{\rho_a g}{\rho_w h}\right)^{1/2}.t$ (C) $\left(\frac{g}{h}\right)^{1/2}.t$ (D) None of these

5. An oscillation is superposition of three harmonic oscillations and decribed by the equation $x = A \sin 2\pi \upsilon_1 t$ where A changes with time according to $A = A_0(1 + \cos 2\pi \upsilon_2 t)$ with A_0 to be constant. The frequencies of pure harmonic oscillations forming this oscillation are :

(A) $\upsilon_1, \upsilon_2, |\upsilon_1 - \upsilon_2|$ (B) $\upsilon_1, |\upsilon_1 - \upsilon_2|, \upsilon_1 + \upsilon_2$

(C) $\upsilon_1, \upsilon_2, |\upsilon_2 - \upsilon_1|$ (D) $\upsilon_1, \upsilon_2, \upsilon_1 + \upsilon_2$

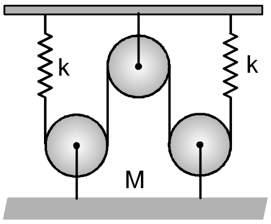
6. A uniform thin hemispherical shell is kept at rest and in equilibrium on an inclined plane of angle of inclination $\theta = 30^\circ$ as shown inf figure. If the surface of the inclined plane is sufficiently rough to prevent sliding then the angle α made by the plane of hemisphere with inclined plane is :



(A) value of μ is needed (B) 30°

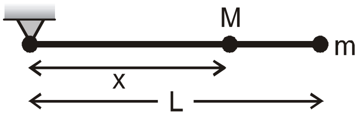
(C) 45° (D) 60°

7. The natural frequency of the system shown in figure is:
{The pulleys are smooth and massless.}



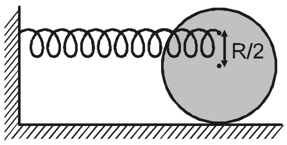
(A) $\frac{1}{\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{2}{\pi}\sqrt{\frac{2k}{M}}$ (C) $\frac{1}{\pi}\sqrt{\frac{k}{M}}$ (D) $\frac{1}{\pi}\sqrt{\frac{4k}{M}}$

8. A massless stick of length L is hinged at one end and a mass m attached to its other end. The stick is free to rotate in vertical plane about a fixed horizontal axis passing through frictionless hinge. The stick is held in a horizontal position. At what distance x from the hinge should a second mass M = m be attached to the stick, so that stick falls as fast as possible when released from rest



(A) $\sqrt{2}L$ (B) $\sqrt{3}L$ (C) $(\sqrt{2}-1)L$ (D) $(\sqrt{3}-1)L$

9. A uniform disc of mass m is attached to a spring of spring constant k as shown in figure and there is sufficient friction to prevent slipping of disc. Time period of small oscillations of disc is:



(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{2m}{3k}}$ (C) $\pi\sqrt{\frac{3m}{2k}}$ (D) $\pi\sqrt{\frac{2m}{3k}}$

10. A particle is executing simple harmonic motion in a conservative force field. The total energy of simple harmonic motion is given by $E = ax^2 + bv^2$ where 'x' is the displacement from mean position x = 0 and v is the velocity of the particle at x then choose the **INCORRECT** statements.{Potential energy at mean position is assumed to be zero}

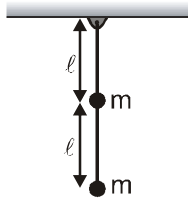
(A) amplitude of S.H.M is $\sqrt{\frac{E}{a}}$

(B) Maximum velocity of the particle during S.H.M is $\sqrt{\frac{E}{b}}$

(C) Time peried of motion is $2\pi\sqrt{\frac{b}{a}}$

(D) displacement of the particle is proportional to the velocity of the particle.

11. Two particle of mass m each are fixed to a massless rod of length 2ℓ. The rod is hinged at one end about a smooth hinge and it performs oscillations of small angle in vertical plane. The length of the equivalent simple pendulum is:



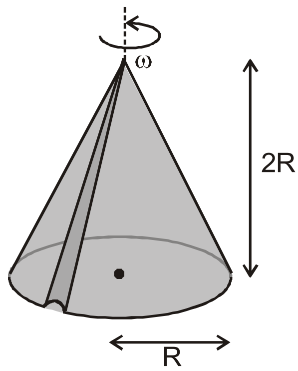
(A) $\frac{3\ell}{2}$ (B) $\frac{10\ell}{3}$

(C) $\frac{5\ell}{3}$ (D) None of these

12. Two copper balls of radius r and 2r are released at rest in a long tube filled with liquid of uniform viscosity. After some time when both the spheres acquire critical velocity (terminal velocity) then ratio of viscous force on the balls is :

(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 18

13. A uniform solid cone of mass m, base radius 'R' and height 2R, has a smooth groove along its slant height as shown in figure. The cone is rotating with angular speed 'ω', about the axis of symmetry. If a particle of mass 'm' is released from apex of cone, to slide along the groove, then angular speed of cone when particle reaches to the base of cone is

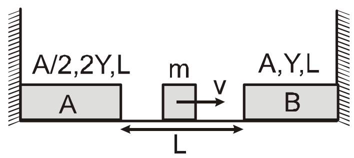


(A) $\frac{3\omega}{13}$ (B) $\frac{4\omega}{13}$ (C) $\frac{5\omega}{13}$ (D) $\frac{9\omega}{13}$

14. A uniform metal rod (fixed at both ends) of 2 mm^2 cross-section is cooled from 40 ºC to 20 ºC. The co-efficient of the linear expansion of the rod is 12×10^{-6} per degree & it's young modulus of elasticity is 10^{11} N/m^2. The energy stored per unit volume of the rod is:

(A) 2880 J/m^3 (B) 1500 J/m^3 (C) 5760 J/m^3 (D) 1440 J/m^3

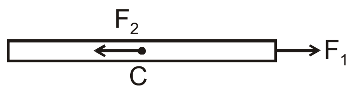
15. In the given figure, two elastic rods A & B are rigidly joined to end supports. A small block of mass ' m ' is moving with velocity v between the rods. All collisions are assumed to be elastic & the surface is given to be smooth. The time period of oscillations of small mass ' m ' will be:
(A = area of cross section, Y = young's modulus, L = length of each rod)



(A) $\frac{2L}{v} + 2\pi\sqrt{\frac{mL}{AY}}$ (B) $\frac{2L}{v} + 2\pi\sqrt{\frac{2mL}{AY}}$

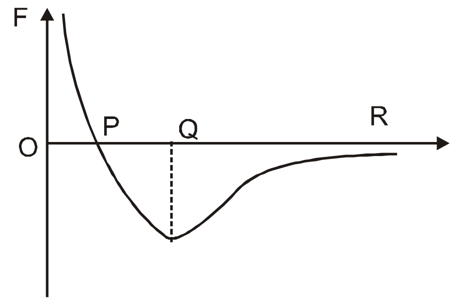
(C) $\frac{2L}{v} + \pi\sqrt{\frac{mL}{AY}}$ (D) $\frac{2L}{v}$

16. Two forces F_1 and F_2 act on a thin uniform elastic rod placed in space. Force F_1 acts at right end of rod and F_2 acts exactly at centre of rod as shown (both forces act parallel to length of the rod).



(i) F_1 causes extension of rod while F_2 causes compression of rod.
(ii) F_1 causes extension of rod and F_2 also causes extension of rod.
(iii) F_1 causes extension of rod while F_2 does not change total length of rod.
The correct order of True / False in above statements is
(A) T F F (B) F T F (C) F F T (D) F F F

17. Figure shows roughly how the force F between two adjacent atoms in a solid varies with inter atomic separation r. Which of the following statements are correct ?



(A) OQ is the equilibrium separation.
(B) Hooke's law is obeyed near P.
(C) The potential energy of the atoms is the gradient of the graph at all points.
(D) The energy to separate the atoms completely is obtained from the magnitude of the area enclosed below the axis of r.

18. A particle constrained to move along x-axis given a velocity u along the positive x-axis. The acceleration 'a' of the particle varies as $a = -bx$, where b is a positive constant and x is the x co-ordinate of the position of the particle . Then select the correct alternative(s): .

(A) The maximum displacement of the particle from the starting point is $\frac{u}{\sqrt{b}}$
(B) The particle will oscillate about the origin
(C) Velocity is maximum at the origin
(D) Given data is insufficient to determine the exact motion of the particle.

19. A uniform ring having mass m, radius R, cross section area of the wire A and young's modulus Y is rotating with an angular speed ω (ω is small) on a smooth horizontal surface. Which of the following options is **correct** :

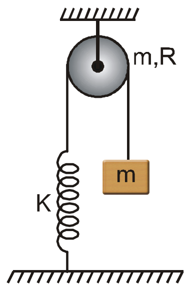
(A) Tension in the wire is $\frac{mR\omega^2}{2\pi}$

(B) Change in length of the wire is $\frac{mR^2\omega^2}{2A.Y}$

(C) Change in radius of the ring is $\frac{mR^2\omega^2}{2\pi A.Y}$

(D) elastic potential energy stored is $\frac{1}{4\pi}\left(\frac{m^2\omega^4R^3}{A.Y}\right)$

20. A uniform disc of mass m and radius R is free to rotate about its fixed horizontal axis without friction. There is sufficient friction between the inextensible light string and disc to prevent slipping of string over disc. At the shown instant extension in light spring is $\frac{3mg}{K}$, where m is mass of block, g is acceleration due to gravity and K is spring constant. Then select the correct alternative(s).



(A) Acceleration of block just after it is released is $\frac{4g}{3}$

(B) Tension in the string continuously increases till extension in the spring reaches maximum value.

(C) Acceleration of the block just after release $\frac{3}{4}g$

(D) Angular acceleration of disc just after release is $\frac{4g}{3R}$

21. A solid glass hemisphere of density d and radius R lies (with curved surface of hemisphere below the flat surface) at the bottom of a tank filled with water of density ρ such that the flat surface of hemisphere is H depth below the liquid surface. Weight of water + tank is W_1 and that of hemisphere is W_2. Then choose the **incorrect** options

(A) Force exerted by the liquid on the flat surface of hemisphere is independent of H and d but depends on R and ρ

(B) Force exerted by the liquid on the curved surface of hemisphere is independent of H and d but depends on R and ρ

(C) Force exerted by the liquid on the hemisphere is independent of H and d but depends on R and ρ

(D) Combined weight of water + tank + hemisphere with hemisphere inside water, taken by a weighing machine is equal to $W_1 + W_2$

22. A 20 gm particle is subjected to two simple harmonic motions

$x_1 = 2 \sin 10t$,

$x_2 = 4 \sin (10t + \frac{\pi}{3})$. where x_1 & x_2 are in metre & t is in sec.

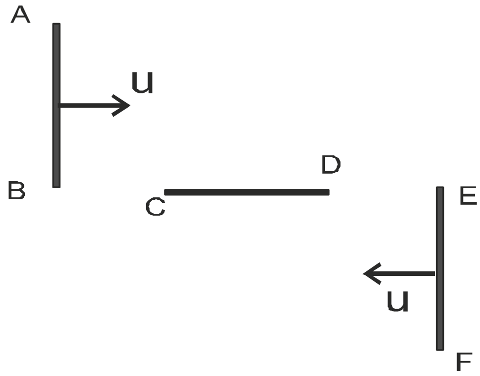
(A) The displacement of the particle at t = 0 will be $2\sqrt{3}$ m.

(B) Maximum speed of the particle will be $20\sqrt{7}$ m/s.

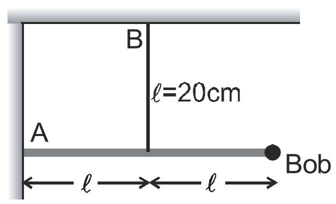
(C) Magnitude of maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2.

(D) Energy of the resultant simple harmonic motion will be 28 J.

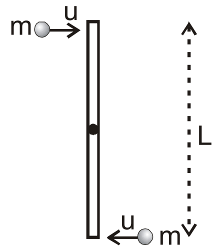
23. Three identical horizontal rods AB, CD and EF each of length 2m are on a smooth horizontal surface. Rod CD is at rest while the rods AB and EF are purely translating with equal and opposite velocities of magnitude 5 m/s.The ends B and E collide simultaneously with the ends C and D respectively, and the rods rigidly join just after the collisions.Find the angular speed of the system in rad/s just after the collision.



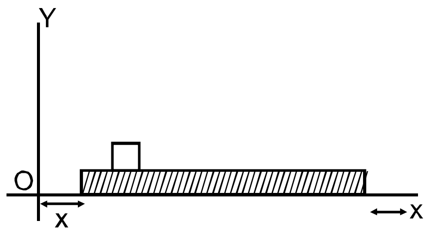
24. A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$sec. and fill value of X. (g = 10 m/s^2)



25. A uniform rod of mass 200 grams and length L = 1m is initially at rest in vertical position. The rod is hinged at centre such that it can rotate freely without friction about a fixed horizontal axis passing through its centre. Two particles of mass m = 100 grams each having horizontal velocity of equal magnitude u = 6 m/s strike the rod at top and bottom simultaneously as shown and stick to the rod. Find the angular speed (in rad/sec.) of rod when it becomes horizontal.

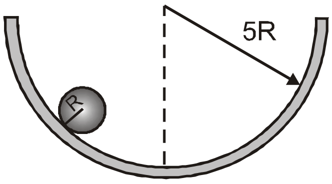


26. A small block is kept on a platform executing SHM in the horizontal plane, described by x = A sinωt. The time period of SHM is T and the coefficient of friction between the block and the platform is μ. The condition that the block does not slip on the platform at any instant is $\mu \geq \frac{x\pi^2 A}{gT^2}$ then write the value of ‘x’.



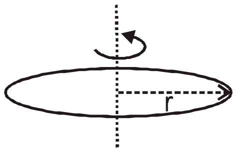
27. Two particles P_1 and P_2 are performing SHM along the same line about the same mean position. Initially they are at their positive extreme position. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time (in s) after which the separation between the particles become 6 cm.

28. A solid sphere (radius = R) rolls without slipping in a cylindrical trough (radius = 5R). The time period of small of oscillations is $2\pi\sqrt{\frac{(k^2+3)R}{kg}}$. Find the value of k (axis of cylinder is fixed and horizontal).



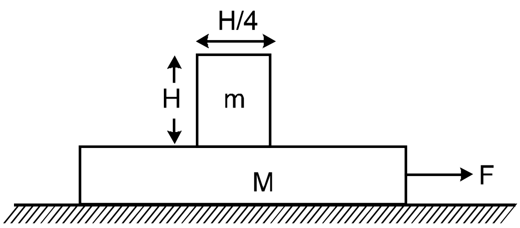
29. Two opposite forces F_1 = 120N and F_2 = 80N act on an heavy elastic plank of modulus of elasticity y = 2×10^{11} N/m^2 and length L = 1m placed over a smooth horizontal surface. The cross-sectional area of plank is A = 0.5m^2. If the change in the length of plank is x × 10^{-9}m, then find x ?

30. A ring of radius r made of wire of density ρ is rotated about a stationary vertical axis passing though its centre and perpendicular to the plane of the ring as shown in figure. Determine the angular velocity (in rad/s) of ring at which the ring breaks. The wire breaks at tensile stress σ. Ignore gravity. (Take $\frac{\sigma}{\rho}$ = 4 m^2/s^2 and r = 1m)



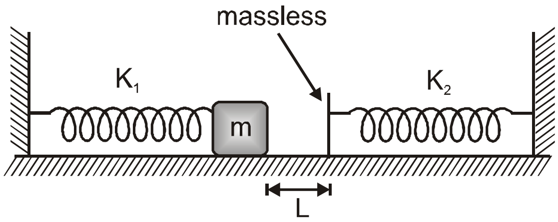
31. The length of an elastic string is 5 metre when the longitudinal tension is 4 N and 6 metre when the tension is 5 N. If the length of the string (in metre) is "2X" when the longitudinal tension is 9 N (assume Hooke's law is valid) then the value of X will be :

32. A block of mass m=2kg of shown dimensions is placed on a plank of mass M = 6Kg which is placed on smooth horizontal plane. The coefficient of friction between the block and the plank is $\mu = \frac{1}{3}$. If a horizontal force F is applied on the plank, then find the maximum value of F (in N) for which the block and the plank move together



COMPREHENSION-1

There are two ideal springs of force constants K_1 and K_2 respectively. When both springs are relaxed the separation between free ends is L. Now the particle of mass m attached to free end of left spring is displaced by distance 2L towards left and then released. Assuming the surface to be frictionless. $\left(\frac{K_1}{K_2} = \frac{4}{3}\right)$. (Neglect size of the block)



33. The time interval after which mass 'm' hits the right spring will be :

(A) $\frac{7\pi}{6}\sqrt{\frac{m}{K_1}}$ (B) $\frac{4\pi}{3}\sqrt{\frac{m}{K_1}}$ (C) $\frac{3\pi}{4}\sqrt{\frac{m}{K_1}}$ (D) $\frac{7\pi}{4}\sqrt{\frac{m}{K_1}}$

34. The maximum compression produced in right spring will be :

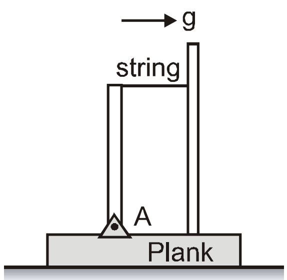
(A) $\frac{6L}{7}$ (B) $\frac{7L}{6}$ (C) $\frac{L}{3}$ (D) $\frac{2L}{3}$

35. Suppose mass m hits the right spring and sticks to it. The extension in left spring when mass 'm' is in equilibrium position during its motion is :

(A) $\frac{4L}{7}$ (B) $\frac{3L}{7}$ (C) L (D) $\frac{L}{2}$

COMPREHENSION-2

A rod of mass 'm and length L is attached to a L shaped plank at 'A'. rod can move freely about A. A string is tied between rod and plank as shown in figure. Whole system is moving with a constant acceleration g in x-direction



36. Tension in the string is:

(A) Zero (B) 2mg (C) $\frac{mg}{2}$ (D) mg

37. Force exerted by hinge on the rod is :

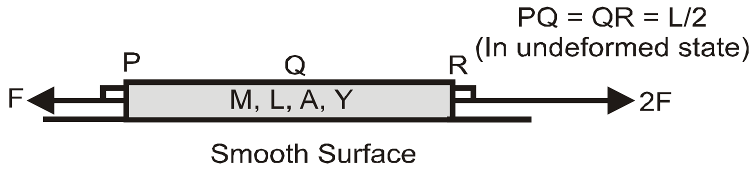
(A) mg (B) $\frac{mg}{2}$ (C) $\frac{mg\sqrt{5}}{2}$ (D) $\frac{5mg}{4}$

38. If string is cut at any instant then the angular acceleration of rod (with respect to the plank) at that instant is

(A) $\frac{2g}{3\ell}\circlearrowleft$ (B) $\frac{6g}{\ell}\circlearrowleft$ (C) $\frac{2g}{3\ell}\circlearrowright$ (D) $\frac{3g}{2\ell}\circlearrowleft$

COMPREHENSION-3

A uniform rod of mass M and length L, area of cross section A is placed on a smooth horizontal surface. Forces acting on the rod are shown in the digram



39. Ratio of elongation in section PQ of rod and section QR of rod is
(A) 1 : 1 (B) 3 : 5 (C) 5 : 7 (D) 1 : 2

40. Ratio of elastic potential energy stored in section PQ and section QR of the rod is
(A) 19 :37 (B) 21 : 39 (C) 23 : 41 (D) 17 : 35

41. Total elastic potential energy stored in the rod is :

(A) $\frac{7F^2L}{6AY}$ (B) $\frac{11F^2L}{6AY}$ (C) $\frac{5F^2L}{6AY}$ (D) $\frac{3F^2L}{2AY}$

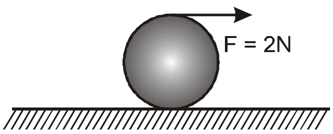
42. In column-I some conditions are mentioned and magnitude of required result ask in column -I is given in column-II, Match the appropriate choice.

	Column–I	Column–II
(P)	A thin layer of water, of surface tension 0.08 N/m present between two massles square plates of area 400 cm^2 then minimum force (in N and perpenducular to be plate) required to pull the plates apart is:	(1) 3.2
(Q)	A water drop at radius 1 cm, suddenly split into 10^3 identical droplets. If surface tension of water is 0.08 N/m. Then the work done is (in Joule)	(2) 0.90
(R)	A soap bubble of radius 1 cm is blown very slowly, so that its radius increase up to 2cm If surface tension of solution is 0.08 N/m then work done (in Joule) in the process is.	(3) less than one
(S)	If water is poured in a container has a circular hole of radius 0.5 mm at its bottom, such that water is just start flow, from the hole. Then the height (in cm.) of water is (surface tension of water is 0.08 N/m.,)	(4) 0.06

	P	Q	R	S
(A)	3	2	3	1
(B)	4	1	4	1
(C)	3	3	3	3
(D)	4	2	2	1

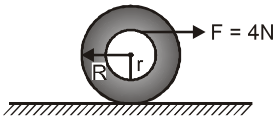
43. In column-I some situations are shown and in column-II information about their resulting motion is given. Select the correct answer using the codes given below the columns. :

	Column–I	Column–II
(P)	A uniform solid sphere of mass 1 kg and radius 1 m, $\mu_s = 0.05$	(1) friction will be in the direction of acceleration of centre of mass of body.



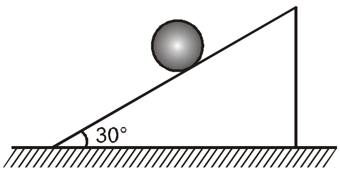
(Q) A uniform body of mass 1 kg, $r = \frac{1}{2}m$, R = 1 m, I (about axis passing through centre and perpendicular to plane of paper) = 2 kg m², $\mu_s = 0.3$

(2) friction wil be opposite to direction of acceleration of centre of mass of body.



(R) A uniform solid cylinder released on a fixed incline plane m = 2 kg, R = 1 m, $\mu_s = \frac{2}{5}$.

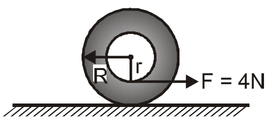
(3) Body rotates clockwise.



(S) A uniform body of mass 1 kg $r = \frac{1}{2}m$, R = 1 m, I(about axis passting through centre and perpendicular to plain of paper) = 2kgm² $\mu_s = 0.5$

(String tightly wound on inner radius is pulled).

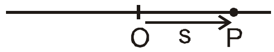
(4) Body rotates anticlockwise.



Codes :

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	4	1	2	3
(D)	4	2	3	1

44. A particle of mass m = 1 kg executes SHM about mean position O with angular frequency ω = 1.0 rad/s and total energy 2J. x is positive if measured towards right from O. At t = 0, particle is at O and moves towards right. Then select the correct answer using the codes given below the columns. :



Column-I	Column-II
(P) speed of particle is $\sqrt{2}$ m/s at	(1) x = + 1m
(Q) Kinetic energy of the particle is 1J at	(2) x = – 1m
(R) At t = π/6 s particle is at	(3) x = + $\sqrt{2}$ m
(S) Kinetic energy is 1.5 J at	(4) x = – $\sqrt{2}$ m

Codes :

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	4	3	1	2
(D)	4	2	3	1

45. **Match the column :**

In a spring block system on a horizontal smooth surface. K = spring constant, A = amplitude, m = mass of the block. In column I some changes are given and column II respective effect is written. Then select the correct answer using the codes given below the columns. :

Column I	Column II
(P) If mass of the block is doubled (keeping K, A unchanged)	(1) time period increases
(Q) If the amplitude of oscillation is doubled (keeping K, m unchanged)	(2) time period decreases
(R) If force constant is doubled (keeping m, A unchanged)	(3) energy of oscillation increases
(S) If another spring of same force constant is attached parallel to the previous one (keeping m, A unchanged)	(4) energy of oscillation remains constant

Codes :

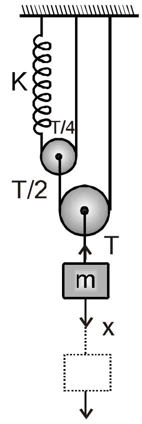
	P	Q	R	S
(A)	1	3	2	3
(B)	2	1	3	4
(C)	3	4	1	2
(D)	4	2	3	1

ANSWER KEY OF DPP NO. # 05

1.	(A)	**2.**	(D)	**3.**	(D)	**4.**	(B)	**5.**	(A)	**6.**	(C)	**7.**	(A)
8.	(C)	**9.**	(C)	**10.**	(C)	**11.**	(C)	**12.**	(B)	**13.**	(D)	**14.**	(A)
15.	(D)	**16.**	(B,C,D)	**17.**	(B,C,D)	**18.**	(A,B,C)	**19.**	(A,B,D)	**20.**	(B,C)	**21.**	(A,B,C)
22.	3	**23.**	1	**24.**	4	**25.**	0	**26.**	2	**27.**	8	**28.**	0
29.	**6**	**30.**	24	**31.**	(D)	**32.**	(C)	**33.**	(C)	**34.**	(C)		
35.	(C)	**36.**	(B)	**37.**	(A)	**38.**	(B)	**39.**	(A)	**40.**	(B)	**41.**	(B)
42.	(C)	**43.**	(A)	**44.**	(D)	**45.**	(D)						

PHYSICS

1. Let mass 'm' falls down by x so spring extends by 4x ;



$\therefore \frac{T}{4} = k(4x)$

T = (16k) x

Where T is the restoring force on mass m

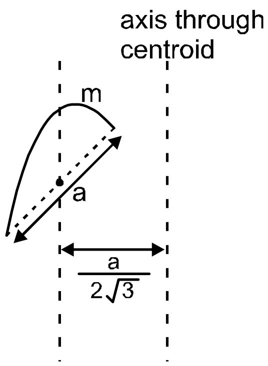
$\therefore\ f = \frac{1}{2\pi}\sqrt{\frac{16k}{m}}$

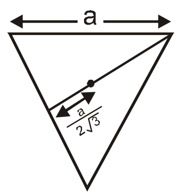
$f = \frac{2}{\pi}\sqrt{\frac{k}{m}} = \frac{2}{\pi} \times \sqrt{\frac{25}{1}} = \pi$ Hz

2. Apply C.O.A.M.,

$10 \times 1 = \frac{ML^2}{3}\omega$; ω = 15 rad. K.E. = $\frac{1}{2}I\omega^2$ = 75 J

3.

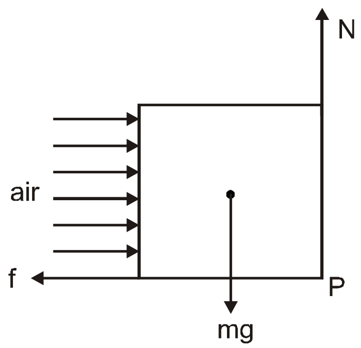




$I' = I_{cm} + m\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{5ma^2}{24}$, $\quad I_{cm} = \frac{m\left(\frac{a}{2}\right)^2}{2} = \frac{ma^2}{8}$

$I = 3I' = \frac{5ma^2}{8}$.

4. $F = V\frac{dm}{dt} = \rho_a \ell hV.V = \rho_a \ell hV^2$ $\qquad \{\because \frac{dm}{dt} = \rho_a \ell hV\}$



Total torque of air about point P is $\rho_a \ell hV^2 \frac{h}{2}$

$\tau_a = \frac{\rho_a \ell h^2 V^2}{2}$; $\tau_w = Mg.\frac{t}{2} = \rho_\omega.\ell.h.t.g.\frac{t}{2}$

for toppling $\quad \tau_a > \tau_\omega \Rightarrow \quad V > \left(\frac{\rho_\omega g}{\rho_A h}\right)^{1/2}.t$

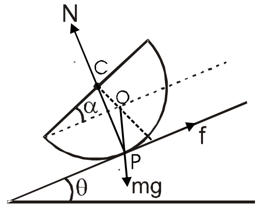
5. $x = A_0(1 + \cos 2\pi\upsilon_2 t).\ \sin 2\pi\upsilon_1 t$

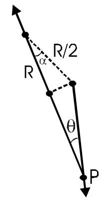
$= A_0 \sin 2\pi\upsilon_1 t + \dfrac{A_0}{2}[(\sin 2\pi(\upsilon_1 + \upsilon_2)t + \sin 2\pi(\upsilon_1 - \upsilon_2)t]$

Hence the frequencies are

$\upsilon_1, |\upsilon_1 - \upsilon_2|, \upsilon_1 + \upsilon_2$.

6. O is the centre of mass of the hollow hemisphere and is $\frac{R}{2}$ from C.





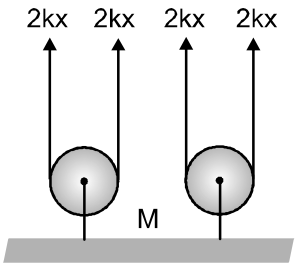
$f = mg \sin\theta$ .... (1)

$N = mg \cos\theta$ .... (2)

$N \times \dfrac{R}{2} \sin\alpha = \left[R - \dfrac{R}{2}\cos\alpha\right] f$ ......(3)

$\therefore \quad \tan\theta = \dfrac{\sin\alpha}{2 - \cos\alpha} \Rightarrow \alpha = 60^\circ$

7. If the mass M is displaced by x from its mean position each spring further stretched by 2x.



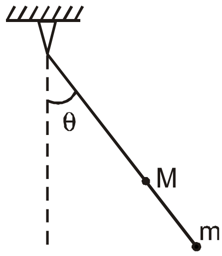
Net restoring force

$F = -8kx$

$M.a = -8\ kx$

$f = \dfrac{1}{2\pi}\sqrt{\left|\dfrac{a}{x}\right|} = \dfrac{1}{2\pi}\sqrt{\dfrac{8k}{M}} = \dfrac{1}{\pi}\sqrt{\dfrac{2k}{M}}$

8. Angular acceleration of rod



$\alpha = \dfrac{m(x+L)g\sin\theta}{m(x^2+L^2)}$

For rod to fall as fast as possible, $\dfrac{d\alpha}{dx} = 0$

or $x = (\sqrt{2} - 1)L$

9. Let centre of disc is displaced by x from its equilibrium position(spring was in its natural length). Now calculate the torque about lowest point of disc.

$k.\dfrac{3}{2}R.\dfrac{3x}{2} = \dfrac{3}{2}mR^2\dfrac{a}{R}$

$\dfrac{3kx}{2m} = a$

So, $T = 2\pi\sqrt{\dfrac{2m}{3k}}$.

10. amplitude is obtained for v = 0

$\therefore \quad A = \sqrt{\frac{E}{a}}$

Maximum velocity is obtained for x = 0

$V_{max} = \sqrt{\frac{E}{b}} \qquad V_{max} = A\,\omega$

$$\omega = \frac{\sqrt{\frac{E}{b}}}{\sqrt{\frac{E}{a}}} = \sqrt{\frac{a}{b}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{b}{a}}$$

Alternative

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$b = \frac{m}{2}, \; a = \frac{k}{2}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{a}{b}}$

$E = \frac{1}{2}mv_{max}^2 \Rightarrow V_{max} = \sqrt{\frac{E}{b}}$

$E = \frac{1}{2}kA^2 \qquad A = \sqrt{\frac{E}{a}}$

11. $T = 2\pi\sqrt{\frac{I}{mg\ell}}, \; I = m\ell^2 + m(2\ell)^2 = 5m\ell^2$

$$= 2\pi\sqrt{\frac{5m\ell^2}{2mg\frac{3\ell}{2}}} = 2\pi\sqrt{\frac{5\ell}{3g}}$$

$\therefore \quad L_{eq} = \frac{5\ell}{3}$

12. $Mg - f_B = F_v$

$\Rightarrow \quad \frac{4}{3}\pi r^3(\rho_m - \rho_\ell)g = F_v$

13. **(a)** Initially

$I_1 = \frac{3}{10}mR^2 \quad \& \quad \omega_1 = \omega$

Finally $I_2 = \frac{13}{10}mR^2 \quad \& \quad \omega_2 = \omega_{new}$

Using conservation of Angular momentum

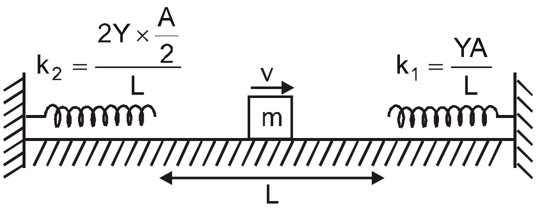
$I_1\omega_1 = I_2\omega_2$

$\omega_2 = \omega_{new} = \frac{3\omega}{13}$

14. Energy Density $= \frac{1}{2}$ stress × strain $= \frac{1}{2}$ Y (strain)^2 = 2880 J/m^3

15. Rod behaves as spring of spring constant $\frac{YA}{\ell}$

Equivalent system is:



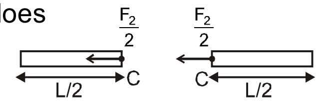
The time period of oscillations of block is

$$T = \frac{2L}{V} + \frac{1}{2}\left(2\pi\sqrt{\frac{mL}{YA}}\right) + \frac{1}{2}\left(2\pi\sqrt{\frac{mL}{2Y.A/2}}\right)$$

$$= \frac{2L}{V} + 2\pi\left(\frac{mL}{AY}\right)$$

16. The force F_1 causes extension in rod.

F_2 causes compression in left half of rod and an equal extension in right half of rod. Hence F_2 does not effectively change length of the rod.



17. Since F-r curve is continuous, so

$\left.\frac{dF}{dr}\right|_{p^+} = \left.\frac{dF}{dr}\right|_{p^-} = \left.\frac{dF}{dr}\right|_{p} = -\alpha$ and F(at P) = 0 so Hooke's law valid near point P.

Energy required to separate the atoms $= |\Delta U| = \left|-\int \vec{F}.d\vec{r}\right| = |\text{Area enclosed between curve and r - axis}|$

18. (A) $\therefore \quad \frac{dv}{dt} = -bx = v\frac{dv}{dx}$

$$\int_u^0 v\,dv = \int_0^x -bx\,dx$$

$$\Rightarrow \quad \left.\frac{v^2}{2}\right|_u^0 = -b\left.\frac{x^2}{2}\right|_0^x$$

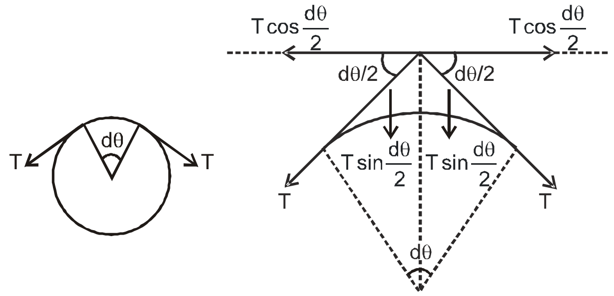
$$\Rightarrow \quad -\frac{u^2}{2} = -\frac{bx^2}{2} \quad \Rightarrow \quad x = \frac{u}{\sqrt{b}}$$

(B) F = m (– bx)

$a = -bx = -\omega^2 x$

(C) acceleration is always towards origin and acceleration is zero at origin which is the mean position of SHM.

19. Let T be the tension in the string.



$$2T\sin\frac{d\theta}{2} = \frac{m}{2\pi R}\times R.\omega^2 \cdot Rd\theta.$$

$$T = \frac{mR\omega^2}{2\pi}$$

$$Y = \frac{T/A}{\Delta\ell/\ell}$$

$$\frac{\Delta\ell}{\ell} = \frac{T}{Y.A} \Rightarrow \Delta\ell = \frac{T}{Y.A}\times\ell = \frac{m.R\omega^2}{2\pi}\times\frac{1}{Y.A}\times 2\pi R = \frac{mR^2\omega^2}{Y.A}$$

$$\frac{\Delta\ell}{\ell} = \frac{\Delta R}{R} = \frac{T}{Y.A} = \frac{m.R\omega^2}{2\pi A.Y} \Rightarrow \quad \Delta R = \frac{mR^2\omega^2}{2\pi A.Y}$$

$$V = \frac{1}{2}K.X^2 = \frac{1}{2}\left(\frac{Y.A.}{\ell}\right)\times(\Delta\ell)^2 = \frac{1}{2}\frac{Y.A}{2\pi R}\times\left(\frac{m.R^2\omega^2}{Y.A}\right)^2 = \frac{1}{4\pi}\left(\frac{m^2.R^3\omega^4}{Y.A}\right)$$

20. For disc, from torque equation

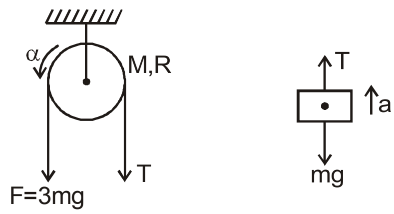
$$3\text{ mg R} - \text{TR} = \frac{mR^2}{2}\alpha \quad .... (1)$$

By application of Newton's second law on block we get,

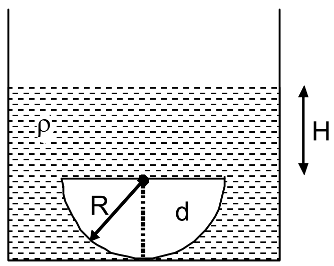
$T - mg = ma$ .... (2)

where $a = R\alpha$ ..... (3)

solving $a = \frac{4g}{3}$



21.



(a) force on flat surface depends on H
(b) Pressure at the location of curved surface depends on H

(c) Net force on hemisphere by liquid = $\left(\frac{2}{3}\pi R^3\right)(\rho)g$

22. At t = 0

Displacement $x = x_1 + x_2 \quad = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$ m.

Resulting Amplitude A = $\sqrt{2^2 + 4^2 + 2(2)(4)\cos\pi/3} = \sqrt{4+16+8} = \sqrt{28} = 2\sqrt{7}$ m

Maximum speed = $A\omega = 20\sqrt{7}$ m/s

Maximum acceleration = $A\omega^2 = 200\sqrt{7}$ m/s²

Energy of the motion = $\frac{1}{2} m\omega^2 A^2$ = **28 J Ans.**

23. Applying conservation of the angular momentum of the system of three rods about midpoint of the rod CD .

$$\Rightarrow m \times 5 \times 1 + m \times 5 \times 1 = \left[2\left(\frac{m2^2}{12} + m(\sqrt{2})^2\right) + \frac{m2^2}{12}\right] \Rightarrow \omega = \frac{30}{15} = 2 \text{ rad/sec.}$$

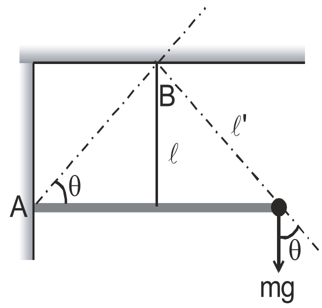
24. The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ' then

$$T = 2\pi\sqrt{\frac{\ell'}{g'}}$$

$\ell' = \ell/\sin\theta = \sqrt{2}\,\ell$ (because θ = 45)

$g' = g\cos\theta = g/\sqrt{2}$

$$T = 2\pi\sqrt{\frac{2\ell}{g}} \quad = 2\pi\sqrt{\frac{2\times 0.2}{10}} = \frac{2\pi}{5}\text{s.}$$



25. From conservation of angular momentum.

$$mu\,\frac{L}{2} + mu\,\frac{L}{2} = \left[2m\frac{L^2}{12} + m\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2\right]\omega$$

$$muL = \left[\frac{mL^2}{6} + \frac{mL^2}{4} + \frac{mL^2}{4}\right]\omega = \frac{2mL^2}{3}\omega \quad \text{or} \quad \omega = \frac{3u}{2L} = \frac{3\times 6}{2\times 1} = 9 \text{ rad/s}$$

26. N = mg
f = ma
As f must be static friction (No slip condition)
$f \le \mu N \quad \Rightarrow \quad ma \le \mu\, mg$
or $ma_o \le \mu mg$
$\therefore \quad mA\omega^2 \le \mu mg$

$\therefore \quad \omega \le \sqrt{\frac{\mu g}{A}}$

$\omega = \frac{2\pi}{T} \le \sqrt{\frac{\mu g}{A}}$

$\therefore \quad T \geq 2\pi\sqrt{\dfrac{A}{\mu g}} \quad \Rightarrow \quad \mu \geq \dfrac{4\pi^2 A}{gT^2}$

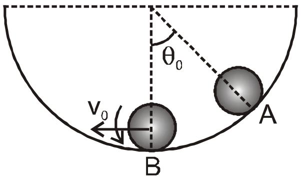
27. The x coordinates of the particles are

$x_1 = A_1 \cos \omega t$, $x_2 = A_2 \cos \omega t$

separation $= x_1 - x_2 = (A_1 - A_2) \cos \omega t = 12 \cos \omega t$

Now $x_1 - x_2 = 6 = 12 \cos \omega t$

$\Rightarrow \quad \omega t = \dfrac{\pi}{3} \Rightarrow \dfrac{2\pi}{12} . t = \dfrac{\pi}{3} \quad \Rightarrow \quad t = 2s$ **Ans.**

28. 

Using conservation of mechanical energy

. $E_A = E_B$

$mg\,4R\,(1 - \cos\theta) = \dfrac{7}{10}\, mv_0^{\,2} \Rightarrow 8\,mg\,R\sin^2\dfrac{\theta}{2} = \dfrac{7}{10}\, mv_0^{\,2}$

since θ is very small

$\dfrac{7}{10}\, v_0^{\,2} = 2\, gR\,\theta_0^2 \qquad v_0^{\,2} = \dfrac{20gR}{7}\,\theta_0^2$

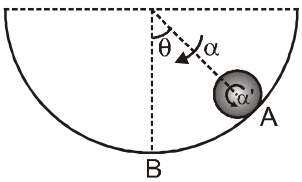
Linear amplitude of SHM $a = 4R\theta_0 \Rightarrow \theta_0 = \dfrac{a}{4R}$

$v_0^{\,2} = \dfrac{20gR}{7}\,\dfrac{a^2}{16R^2} = \dfrac{5}{28}\dfrac{g}{R}a^2$

comparing $v_0^{\,2} = \omega^2 a^2$

$\omega = \sqrt{\dfrac{5g}{28R}}, \qquad T = 2\pi\sqrt{\dfrac{28R}{5g}}$

Alternate solution :



$mg\sin\theta - f = m\,\alpha' R.$

$f\,R = \dfrac{2}{5}\, mR^2(\alpha')$

acceleration of center of mass of solid sphere $= \alpha' R = \alpha 4R$

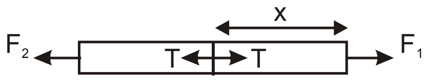
solving above 3 equations $\Rightarrow mg\sin\theta = \dfrac{28}{5}\, m\alpha R$

For small θ

$\alpha = \dfrac{5g\theta}{28R}$

$\omega = \sqrt{\dfrac{5g}{28R}}$, $T = 2\pi\sqrt{\dfrac{28R}{5g}}$

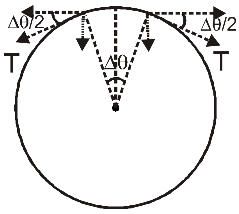
29. $dL = \dfrac{T}{A}\dfrac{dx}{y}$



$T = F_1 - (F_1 - F_2)\dfrac{x}{L}$

$\int_0^{\Delta L} dl = \dfrac{(F_1 + F_2)L}{2Ay} = 1\times10^{-9}$ m

30. $2T \sin\dfrac{\Delta\theta}{2} = dm \times \omega^2 r$



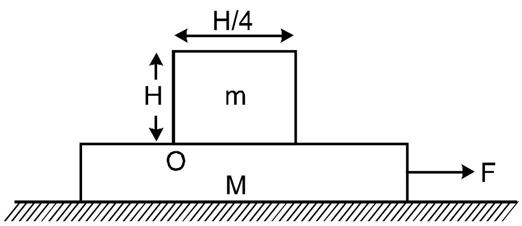
$2T\left(\dfrac{\Delta\theta}{2}\right) = \rho \times A \times r\Delta\theta \times \omega^2 \times r$

$\sigma = \dfrac{T}{A} = \rho r^2 \omega^2$

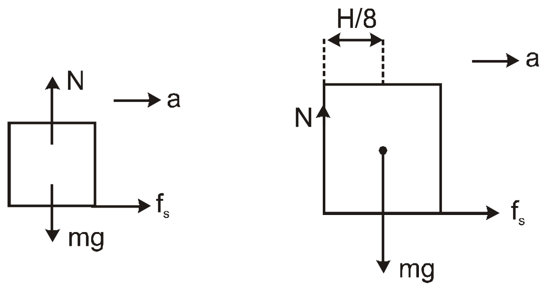
$\therefore \quad \omega = \dfrac{1}{r}\sqrt{\dfrac{\sigma}{\rho}} = 2$ rad/s

31. Let the original length of the string be L.
Applying F = kx, we have 4 = k (5 – L)
5 = k (6 – L)
9 = k (2X – L). From these equations x= 5

32. The block has two tendencies,



(i) to slide w.r.t. plank
(ii) to topple over the point O maximum acceleration for sliding



$f_s = ma \le f_L$
$ma \le \mu mg$

$a \le \dfrac{g}{3}$ $a_{max} = \dfrac{g}{3}$

Maximum acceleration for toppling,
$N = mg$, $f_s = ma$

$$N.\left(\frac{H}{8}\right) = \frac{H}{2} f_{s,} \qquad a_{max} = \frac{g}{4}$$

So, the block will topple before sliding. Hence, $f_{max} = (M + m)\frac{g}{4}$.

33 to 35 Time taken by particle to go from

$x = 0$ to $x = A/2$ is $\frac{T}{12}$

$\therefore \qquad$ time interval $= \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$

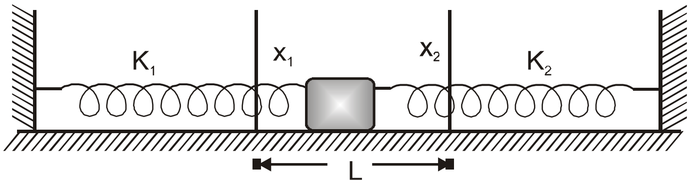
$$= \frac{7}{12}.2\pi\sqrt{\frac{m}{K_1}} = \frac{7\pi}{6}\sqrt{\frac{m}{K_1}}.$$

Assume, maximum compression in right spring is x. Hence,

$$\frac{1}{2}K_1(2L)^2 = \frac{1}{2}K_1(L + x)^2 + \frac{1}{2}K_2x^2$$

put $\quad K_2 = \frac{3}{4}K_1$, we get $\quad x = \frac{6L}{7}$.

When mass m is in equilibrium both spring will be in extended state.

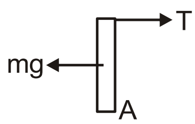


$K_1x_1 = K_2x_2 \qquad$ and $\qquad x_1 + x_2 = L$

$x_1 = \frac{3L}{7}$.

36. Torque about 'A'

$$\frac{mg\ell}{2} - T\ell = 0$$

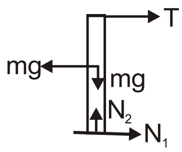


$T = \frac{mg}{2}$ newton

37. $N_2 = mg$

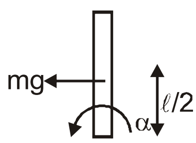
$N_1 + T = mg$

$N_1 = \frac{mg}{2}, \qquad N = \sqrt{N_1^2 + N_2^2} = mg\frac{\sqrt{5}}{2}$

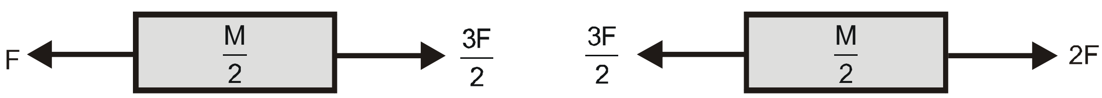


38. $mg\frac{\ell}{2} = \left(\frac{m\ell^2}{3}\right)\alpha$

$\Rightarrow \qquad \alpha = \frac{3g}{2\ell}$



39.

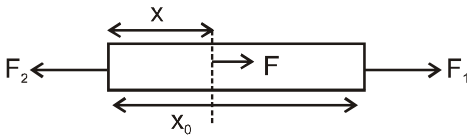


$$\Delta\ell_1 = \left(\frac{3F}{2}\right)\frac{\frac{L}{2}}{2AY} + \frac{F\frac{L}{2}}{2AY} = \frac{5FL}{8AY}$$

$$\Delta\ell_2 = \frac{2F\frac{L}{2}}{2AY} + \frac{\frac{3F}{2}\frac{L}{2}}{2AY} = \frac{7FL}{8AY}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{5}{7}$$

40. $F = \left(\frac{F_1 - F_2}{x_0}\right)x + F_2 = \alpha x + \beta$



Energy density at any x

$$\frac{dU}{dV} = \frac{1}{2}\left(\frac{\alpha x+\beta}{A}\right)\left(\frac{\alpha x+\beta}{AY}\right) = \frac{1}{2A^2Y}\ (\alpha x + \beta)^2$$

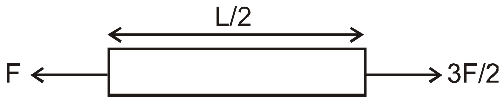
Energy stored in small segment dx

$$dU = \frac{1}{2A^2Y}\ (\alpha^2x^2 + \beta^2 + 2\alpha\beta x)\ Adx$$

$$U = \int dU = \frac{1}{2AY}\int_0^{x_0}(\alpha^2x^2+\beta^2+2\alpha\beta x)dx = \frac{1}{2AY}\left(\frac{\alpha^2x_0^3}{3}+\beta^2x_0+\beta x_0^2\right)$$

Consider section PQ

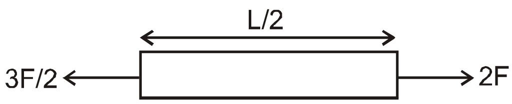
$\alpha = F/L$, $\beta = F$, $x_0 = L/2$



$$U_1 = \frac{19F^2L}{48AY}$$

Consider secton QR

$\alpha = F/L$, $\beta = 3F/2$, $x_0 = L/2$



$$U_2 = \frac{37F^2L}{48AY}$$

41. $U = \frac{19F^2L}{48AY} + \frac{37F^2L}{48AY} = \frac{7F^2L}{6AY}$

42. $F = T.4\ell$ $\quad$ $A = 400\ cm^2$, $\ell = 20$ cm = 0.2 m

$= \frac{8}{100} \times 4 \times \frac{2}{10}$

$= \frac{64}{100} = 0.064\ N \cong 0.06$ N.

(B) $\quad W = T.4\pi r^2\ (n^{1/3} - 1)$

$= \frac{8}{100} \times 4\pi \times \frac{1}{100\times100} \times (9)$

$= 32 \times 9\pi \times 10^{-6}$

$= 90432 \times 10^{-6}$

= 0.09432 Joule

(C) $\quad W = 2 \times T4\pi\ [(2R)^2 - R^2]$

$= 8\pi TR^2 \times 3$

$= 24\pi \frac{8}{100} \times \frac{1}{100\times100}$

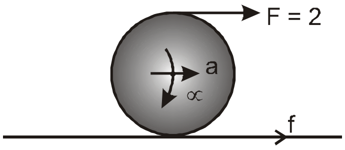
$= \frac{59088}{1000000} = 0.059088$ 0.06 Joule

(D) $h = \dfrac{2T.\cos\theta}{r\rho g}$

$= 2 \times \dfrac{8}{100} \times \dfrac{1\times10^4}{5\times10^3\times10}$

$= \dfrac{32}{10}\times\dfrac{10^4}{10^6}$ = 32× 10^{4-7} = 0.032 m = 3.2 cm

43. (A)



Let friction be static

$F + f = ma$

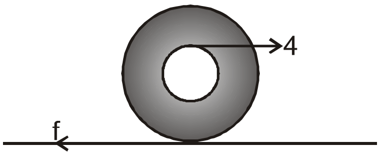
$FR - fR = \dfrac{2}{5}mR^2\alpha$

$a = R\alpha$

$f = \dfrac{6}{7}N$

$f_L = 0.5N \Rightarrow$ friction is kinetic

(B)



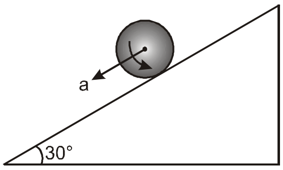
$4 - f = ma$

$4\times\dfrac{1}{2}+f\times1 = 2a$

$f = 2N$

$f_L = 3N$

(C)

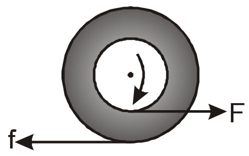


$a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \dfrac{10\times\dfrac{1}{2}}{1+\dfrac{1}{2}} = \dfrac{10}{3}$ m/s²

$mg\sin\theta - f = ma$

$f = \dfrac{mg}{2} - ma = 10 - \dfrac{20}{3} = \dfrac{10}{3}N$

$f_L = u_sN = \dfrac{2}{5}\times2\times10\times\dfrac{\sqrt{3}}{2} = 4\sqrt{3}$ N Static

(D) 

$f_L = 5N$
$f = 10/3\,N$ static
$F - f = ma \Rightarrow 4 - f = 1a$

$$f \times 1 - 4\times\frac{1}{2} = 2\times\frac{a}{R} \Rightarrow f - 2 = 2a; \quad a = 2/3\ m/s^2$$

44. $KE_{max} = \frac{1}{2}mv_{max}^2 = TE$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2\times 2}{1}} = 2\ m/s$$

amplitude $A = \frac{v_{max}}{\omega} = 2m.$

$x = A\sin\omega t = 2\sin t$

$v = 2\cos t = \sqrt{4 - x^2}$

(P) $v = \sqrt{2}$ m/s $\Rightarrow x = \pm\sqrt{2}$ m.

(Q) $KE = \frac{1}{2}mv^2 \Rightarrow 1 = \frac{1}{2}\times 1\times v^2 \Rightarrow v = \sqrt{2}$ m/s.

$\therefore x = \pm\sqrt{2}$ m.

(R) at $t = \pi/6$ s, $x = 2\sin\pi/6 = 1m.$

(S) $KE = \frac{3}{2} \Rightarrow 1.5 = \frac{1}{2}\times mv^2$

$\Rightarrow \quad v = \sqrt{3} \Rightarrow x = \pm 1m.$

45. (P) $T = 2\pi\sqrt{\frac{m}{k}}$ $\quad m\uparrow \quad T\uparrow$

$E = \frac{1}{2}kA^2$

(Q) $E = \frac{1}{2}kA^2 \quad A\uparrow \quad E\uparrow$

(R) $T = 2\pi\sqrt{\frac{m}{k}} \quad k\uparrow \quad T\downarrow$

$E = \frac{1}{2}kA^2 \quad k\uparrow \quad E\uparrow$

(S) $T = 2\pi\sqrt{\frac{m}{k_{eq}}} \quad k_{eq}\uparrow \quad T\downarrow$

$k_{eq} = 2k$

$E = \frac{1}{2}k_{eq}A^2 \quad k_{eq}\uparrow \quad E\uparrow$