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CHAPTER 10

Straight Lines & Pair of Straight Lines

1. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **[2002]**

(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

2. Locus of mid point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$ whre p is constant is **[2002]**

(a) $x^2+y^2=\frac{4}{p^2}$ (b) $x^2+y^2=4p^2$

(c) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{2}{p^2}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$

3. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect on the y-axis then **[2002]**

(a) $2fgh=bg^2+ch^2$ (b) $bg^2 \neq ch^2$
(c) $abc=2fgh$ (d) none of these

4. The pair of lines represented by $3ax^2+5xy+(a^2-2)y^2=0$ are perpendicular to each other for **[2002]**

(a) two values of a (b) $\forall\ a$
(c) for one value of a (d) for no values of a

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **[2003]**

(a) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$
(b) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$
(c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$
(d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$.

6. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then **[2003]**

(a) $pq=-1$ (b) $p=q$
(c) $p=-q$ (d) $pq=1$.

7. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is **[2003]**

(a) $(3x+1)^2+(3y)^2=a^2-b^2$
(b) $(3x-1)^2+(3y)^2=a^2-b^2$
(c) $(3x-1)^2+(3y)^2=a^2+b^2$
(d) $(3x+1)^2+(3y)^2=a^2+b^2$.

8. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) **[2003]**

(a) are vertices of a triangle
(b) lie on a straight line
(c) lie on an ellipse
(d) lie on a circle.

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9. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of `$c$` is **[2003]**

(a) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

(b) $\frac{1}{2} a_2^2 + b_2^2 - a_1^2 - b_1^2$

(c) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(d) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$.

10. Let $A(2, -3)$ and $B(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **[2004]**

(a) $3x - 2y = 3$ (b) $2x - 3y = 7$

(c) $3x + 2y = 5$ (d) $2x + 3y = 9$

11. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is **[2004]**

(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

12. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product c has the value **[2004]**

(a) –2 (b) –1

(c) 2 (d) 1

13. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals **[2004]**

(a) –3 (b) 1

(c) 3 (d) 1

14. The line parallel to the x- axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[2005]**

(a) below the x - axis at a distance of $\frac{3}{2}$ from it

(b) below the x - axis at a distance of $\frac{2}{3}$ from it

(c) above the x - axis at a distance of $\frac{3}{2}$ from it

(d) above the x - axis at a distance of $\frac{2}{3}$ from it

15. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is **[2005]**

(a) $\left(-1, \frac{7}{3}\right)$ (b) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

(c) $\left(1, \frac{7}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{7}{3}\right)$

16. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[2006]**

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$

(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

17. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belong to **[2006]**

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(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$

(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$

18. Let A (h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1square unit, then the set of values which 'k' can take is given by **[2007]**

(a) $\{-1, 3\}$ (b) $\{-3, -2\}$

(c) $\{1, 3\}$ (d) $\{0, 2\}$

19. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[2007]**

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$.

20. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is **[2007]**

(a) 1 (b) 2

(c) –1/2 (d) –2.

21. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is **[2008]**

(a) 1 (b) 2

(c) –2 (d) – 4

22. The shortest distance between the line y – x = 1 and the curve $x = y^2$ is : **[2009]**

(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

23. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for : **[2009]**

(a) exactly one values of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

24. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: **[2009]**

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$

(c) $\left(\frac{5}{3}, 0\right)$ (d) $(0, 0)$

25. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[2011]**

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

26. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a in the interval :

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[2011RS]

(a) $(0, \infty)$ (b) $[1, \infty)$

(c) $(-1, \infty)$ (d) $(-1, 1)$

27. If A (2, – 3) and B (– 2, 1) are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[2011RS]**

(a) $x - y = 1$ (b) $2x + 3y = 1$

(c) $2x + 3y = 3$ (d) $2x - 3y = 1$

28. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : **[2012]**

(a) $\frac{29}{5}$ (b) 5

(c) 6 (d) $\frac{11}{5}$

29. A line is drawn through the point (1,2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[2012]**

(a) $-\frac{1}{4}$ (b) -4

(c) -2 (d) $-\frac{1}{2}$

30. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[2013]**

(a) $y = x + \sqrt{3}$ (b) $\sqrt{3}y = x - \sqrt{3}$

(c) $y = \sqrt{3}x - \sqrt{3}$ (d) $\sqrt{3}y = x - 1$

31. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[2013]**

(a) $2 + \sqrt{2}$ (b) $2 - \sqrt{2}$

(c) $1 + \sqrt{2}$ (d) $1 - \sqrt{2}$

32. Let PS be the median of the triangle vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through (1, –1) and parallel to PS is: **[2014]**

(a) $4x + 7y + 3 = 0$ (b) $2x - 9y - 11 = 0$

(c) $4x - 7y - 11 = 0$ (d) $2x + 9y + 7 = 0$

33. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then **[2014]**

(a) $3bc - 2ad = 0$ (b) $3bc + 2ad = 0$

(c) $2bc - 3ad = 0$ (d) $2bc + 3ad = 0$

34. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? **[2016]**

(a) $\left(\frac{1}{3}, \frac{-8}{3}\right)$ (b) $\left(\frac{-10}{3}, \frac{-7}{3}\right)$

(c) (–3, –9) (d) (–3, – 8)

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(d)	(a)	(a)	(a)	(a)	(c)	(b)	(b)	(d)	(a)	(c)	(a)	(a)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(c)	(a)	(c)	(a)	(d)	(d)	(a)	(a)	(b)	(b)	(b)	(c)	(c)	(b)
31	32	33	34											
(b)	(d)	(a)	(a)											

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SOLUTIONS

1. **(a)** $AB=\sqrt{(4+1)^2+(0+1)^2}=\sqrt{26}$;

$BC=\sqrt{(3+1)^2+(5+1)^2}=\sqrt{52}$

$CA=\sqrt{(4-3)^2+(0-5)^2}=\sqrt{26}$;

In isosceles triangle side $AB = CA$

For right angled triangle, $BC^2 = AB^2 + AC^2$

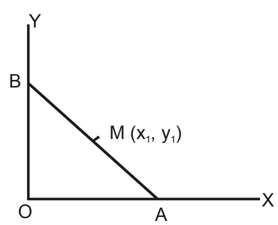
So, here $BC=\sqrt{52}$ or $BC^2 = 52$

or $(\sqrt{26})^2+(\sqrt{26})^2=52$

So, the given triangle is right angled and also isosceles

2. **(d)** Equation of AB is

$x\cos\alpha + y\sin\alpha = p$;



$$\Rightarrow \frac{x\cos\alpha}{p}+\frac{y\sin\alpha}{p}=1;$$

$$\Rightarrow \frac{x}{p/\cos\alpha}+\frac{y}{p/\sin\alpha}=1$$

So co-ordinates of A and B are

$$\left(\frac{p}{\cos\alpha},0\right) \text{ and } \left(0,\frac{p}{\sin\alpha}\right);$$

So coordinates of midpoint of AB are

$$\left(\frac{p}{2\cos\alpha},\frac{p}{2\sin\alpha}\right)=(x_1,\ y_1)(let);$$

$$x_1=\frac{p}{2\cos\alpha}\ \&\ y_1=\frac{p}{2\sin\alpha};$$

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\cos^2\alpha + \sin^2\alpha = 1$

Locus of (x_1, y_1) is $\dfrac{1}{x^2}+\dfrac{1}{y^2}=\dfrac{4}{p^2}$.

3. **(a)** Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0.$

For unique point of intersection $f^2 - bc = 0$

$\Rightarrow af^2 - abc = 0.$

Since $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

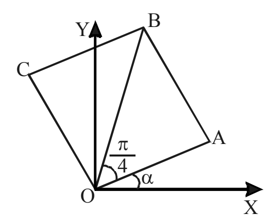
$\Rightarrow 2fgh - bg^2 - ch^2 = 0$

4. **(a)** $3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0.$;

$$\Rightarrow a=\frac{-3\pm\sqrt{9+8}}{2}=\frac{-3\pm\sqrt{17}}{2}$$

5. **(a)** Co-ordinates of $A=(a\cos\alpha, a\sin\alpha)$

Equation of OB,



$$y=\tan\left(\frac{\pi}{4}+\alpha\right)x$$

$CA \perp^r$ to OB

$$\therefore \text{ slope of } CA=-\cot\left(\frac{\pi}{4}+\alpha\right)$$

Equation of CA

$$y-a\sin\alpha=-\cot\left(\frac{\pi}{4}+\alpha\right)(x-a\cos\alpha)$$

$$\Rightarrow (y-a\sin\alpha)\left(\tan\left(\frac{\pi}{4}+\alpha\right)\right)=(a\cos\alpha-x)$$

$$\Rightarrow (y-a\sin\alpha)\left(\frac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\tan\alpha}\right)(a\cos\alpha-x)$$

$\Rightarrow (y-a\sin\alpha)(1+\tan\alpha)$

$=(a\cos\alpha-x)(1-\tan\alpha)$

$\Rightarrow (y-a\sin\alpha)(\cos\alpha+\sin\alpha)=(a\cos\alpha-x)(\cos\alpha-\sin\alpha)$

$\Rightarrow y(\cos+\sin\alpha)-a\sin\alpha\cos\alpha-a\sin^2\alpha$

$=a\cos^2\alpha-a\cos\alpha\sin\alpha-x(\cos\alpha-\sin\alpha)$

$\Rightarrow y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$

$y(\sin\alpha+\cos\alpha)+x(\cos\alpha-\sin\alpha)=a.$

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6. **(a)** Equation of bisectors of second pair of straight lines is,

$$qx^2 + 2xy - qy^2 = 0 \quad ....(1)$$

It must be identical to the first pair

$$x^2 - 2pxy - y^2 = 0 \quad ....(2)$$

from (1) and (2)

$$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1.$$

7. **(c)** $x = \dfrac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow a\cos t + b\sin t = 3x - 1$

$y = \dfrac{a\sin t - b\cos t}{3}$

$\Rightarrow a\sin t - b\cos t = 3y$

Squaring and adding,

$(3x-1)^2 + (3y)^2 = a^2 + b^2$

8. **(b)** Taking co-ordinates as

$\left(\dfrac{x}{r}, \dfrac{y}{r}\right); (x, y) \,\&\, (xr, yr)$.

Then slope of line joining

$$\left(\frac{x}{r}, \frac{y}{r}\right), (x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$$

and slope of line joining (x, y) and (xr, yr)

$= \dfrac{y(r-1)}{x(r-1)} = \dfrac{y}{x}$

$\therefore m_1 = m_2$

$\Rightarrow$ Points lie on the straight line.

9. **(b)** $(x - a_1)^2 + (y - b_1)^2$

$= (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)x + (b_1 - b_2)y$

$$+\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$$

$$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

10. **(d)** Let the vertex C be (h, k), then the centroid of

ΔABC is $\left(\dfrac{2-2+h}{3}, \dfrac{-3+1+k}{3}\right)$

or $\left(\dfrac{h}{3}, \dfrac{-2+k}{3}\right)$. It lies on $2x + 3y = 1$

$\Rightarrow \dfrac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$

$\Rightarrow$ Locus of C is $2x + 3y = 9$

11. **(a)** Let the required line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ....(1)

then $a + b = -1$ ....(2)

(1) passes through (4, 3), $\Rightarrow \dfrac{4}{a} + \dfrac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(3)

Eliminating b from (2) and (3), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\dfrac{x}{2} + \dfrac{y}{-3} = 1$ or $\dfrac{x}{-2} + \dfrac{y}{1} = 1$

12. **(c)** Let the lines be $y = m_1 x$ and $y = m_2 x$ then

$m_1 + m_2 = -\dfrac{2c}{7}$ and $m_1 m_2 = -\dfrac{1}{7}$

Given $m_1 + m_2 = 4\, m_1 m_2$

$\Rightarrow \dfrac{2c}{7} = -\dfrac{4}{7} \Rightarrow c = 2$

13. **(a)** $3x + 4y = 0$ is one of the lines of the pair

$6x^2 - xy + 4cy^2 = 0$, Put $y = -\dfrac{3}{4}x$,

we get $6x^2 + \dfrac{3}{4}x^2 + 4c\left(-\dfrac{3}{4}x\right)^2 = 0$

$\Rightarrow 6 + \dfrac{3}{4} + \dfrac{9c}{4} = 0 \Rightarrow c = -3$

14. **(a)** The line passing through the intersection of lines $ax + 2by = 3b = 0$ and

$bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

As this line is parallel to x-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$\Rightarrow ax + 2by + 3b - \dfrac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow ax + 2by + 3b - ax + \dfrac{2a^2}{b}y + \dfrac{3a^2}{b} = 0$

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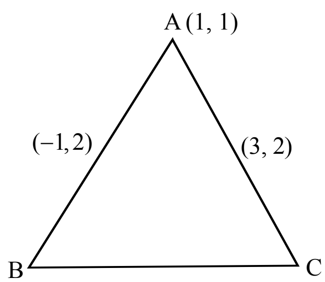
$$y\left(2b+\frac{2a^2}{b}\right)+3b+\frac{3a^2}{b}=0$$

$$y\left(\frac{2b^2+2a^2}{b}\right)=-\left(\frac{3b^2+3a^2}{b}\right)$$

$$y=\frac{-3(a^2+b^2)}{2(b^2+a^2)}=\frac{-3}{2}$$

So it is 3/2 units below x-axis.

15. (c) Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)

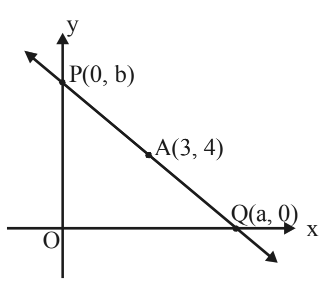


$\Rightarrow$ vertex B and C come out to be (– 3, 3) and (5, 3)

$\therefore$ Centroid is $\dfrac{1-3+5}{3}, \dfrac{1+3+5}{3}$

$$\Rightarrow \left(1, \frac{7}{3}\right)$$

16. (c)



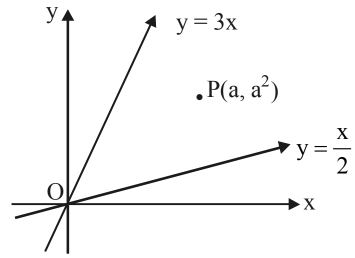
$\because$ A is the mid point of PQ, therefore

$$\frac{a+0}{2}=3, \quad \frac{0+b}{2}=4 \Rightarrow a=6, b=8$$

$\therefore$ Equation of line is $\dfrac{x}{6}+\dfrac{y}{8}=1$

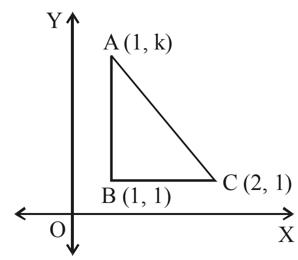
or $4x+3y=24$

17. (c) Clearly for point P,



$$a^2-3a<0 \text{ and } a^2-\frac{a}{2}>0 \Rightarrow \frac{1}{2}<a<3$$

18. (a) **Given :** The vertices of a right angled triangle A(l, k), B(1, 1) and C(2, 1) and Area of ΔABC = 1 square unit

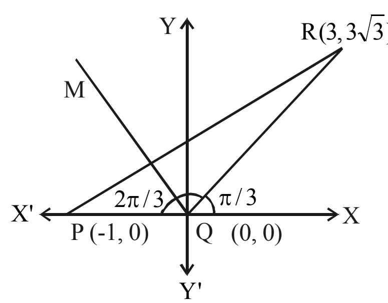


We know that, area of right angled triangle

$$=\frac{1}{2}\times BC\times AB \;=\; 1=\frac{1}{2}(1)\,|\,(k-1)|$$

$$\Rightarrow \pm(k-1)=2 \;\Rightarrow\; k=-1, 3$$

19. (c) **Given :** The coordinates of points P, Q, R are (–1, 0), (0, 0), $(3, 3\sqrt{3})$ respectively.



$$\text{Slope of QR}=\frac{y_2-y_1}{x_2-x_1}=\frac{3\sqrt{3}}{3}$$

$$\Rightarrow \quad \tan\theta=\sqrt{3} \Rightarrow \theta=\frac{\pi}{3}$$

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$\Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

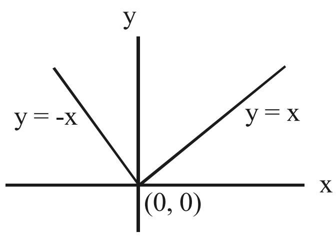
Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line QM $= \tan \frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}(x - 0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

20. (a) Equation of bisectors of lines, $xy = 0$ are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation $my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore mx^2 + (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow 1 - m^2 = 0 \Rightarrow m = \pm 1$

21. (d) Slope of $PQ = \frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ $= (k - 1)$

Also mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow 2y - 7 = 2(k - 1)x - (k^2 - 1)$

$\Rightarrow 2(k - 1)x - 2y + (8 - k^2) = 0$

$\therefore$ y-intercept $= -\frac{8-k^2}{-2} = -4$

$\Rightarrow 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

22. (d) Let (a^2, a) be the point of shortest distance on $x = y^2$

Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \frac{a^2 - a + 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[(a - \frac{1}{2})^2 + \frac{3}{4}\right]$$

It is min when $a = \frac{1}{2}$ and D_{min}

$= \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

23. (a) If the lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore m_1 = m_2 \Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2 + 1)(p + 1) = 0$

$\Rightarrow p = -1$

$\therefore p$ can have exactly one value.

24. (a) Given that $P(1, 0), Q(-1, 0)$

and $\frac{AP}{AQ} = \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{1}{3}$

$\Rightarrow 3AP = AQ$

Let $A = (x, y)$ then

$3AP = AQ \Rightarrow 9AP^2 = AQ^2$

$\Rightarrow 9(x - 1)^2 + 9y^2 = (x + 1)^2 + y^2$

$\Rightarrow 9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow 8x^2 - 20x + 8y^2 + 8 = 0$

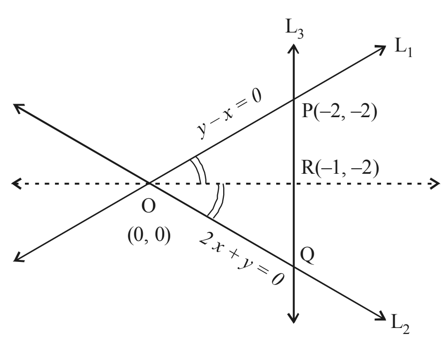
$\Rightarrow x^2 + y^2 - \frac{5}{2}x + 1 = 0$ ....(1)

$\therefore$ A lies on the circle given by eq (1). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) $= \left(\frac{5}{4}, 0\right)$

25. (b)



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$L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$
On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.
On solving the equation of line L_1 and L_3, we get $P = (-2, -2)$.
Similarly, we get $Q = (-1, -2)$
We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

26. **(b)** $x + y = |a|$

and $ax - y = 1$

Case I : If $a > 0$

$x + y = a$ .... (1)

$ax - y = 1$ .... (2)

On adding equation (1) and (2), we get

$x(1 + a) = 1 + a \Rightarrow x = 1$

$y = a - 1$

It is in first quadrant

so $a - 1 \geq 0$

$\Rightarrow a \geq 1$

$\Rightarrow a \in [1, \infty)$

Case II : If $a < 0$

$x + y = -a$ .... (3)

$ax - y = 1$ .... (4)

On adding equation (3) and (4), we get

$x(1 + a) = 1 - a$

$$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$$

Since $a - 1 < 0$

$\therefore a + 1 > 0$

$\Rightarrow a > -1$ .... (5)

$$y = -a - \frac{1-a}{1+a} > 0 = \frac{-a - a^2 - 1 + a}{1+a} > 0$$

$$\Rightarrow -\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$$

Since $a^2 + 1 > 0$

$\therefore a + 1 < 0$

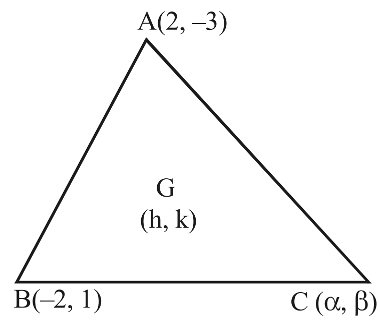
$\Rightarrow a < -1$ .... (6)

From (5) and (6), a $\in \phi$

Hence Case-II is not possible.

So, correct answer is $a \in [1, \infty)$

27. **(b)**



$\alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

Third vertex (α, β) lies on the line

$2x + 3y = 9$

$2\alpha + 3\beta = 9$

$2(3h) + 3(3k + 2) = 9$

$2h + 3k = 1$

$2x + 3y = 1$

28. **(c)** Let the joining points be $A(1,1)$ and $B(2,4)$. Let point C divides line AB in the ratio 3 : 2.
So, by section formula we have

$$C = \left(\frac{3\times2 + 2\times1}{3+2}, \frac{3\times4 + 2\times1}{3+2}\right)$$

$$= \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

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$\therefore$ C satisfies the equation $2x + y = k$.

$\Rightarrow \dfrac{2+8}{5} + \dfrac{14}{5} = k \Rightarrow k = 6$

29. **(c)** Equation of a line passing through (x_1, y_1) having slope m is given by $y - y_1 = m(x - x_1)$

Since the line PQ is passing through (1,2) therefore its equation is $(y - 2) = m(x - 1)$ where m is the slope of the line PQ.

Now, point P $(x,0)$ will also satisfy the equation of PQ

$\therefore$ $y - 2 = m(x - 1) \Rightarrow 0 - 2 = m(x - 1)$

$\Rightarrow -2 = m(x - 1) \Rightarrow x - 1 = \dfrac{-2}{m}$

$\Rightarrow x = \dfrac{-2}{m} + 1$

Also, $OP = \sqrt{(x-0)^2 + (0-0)^2} = x$

$= \dfrac{-2}{m} + 1$

Similarly, point $Q(0,y)$ will satisfy equation of PQ

$\therefore$ $y - 2 = m(x - 1)$

$\Rightarrow y - 2 = m(-1)$

$\Rightarrow y = 2 - m$ and $OQ = y = 2 - m$

Area of $\Delta POQ = \dfrac{1}{2}(OP)(OQ)$

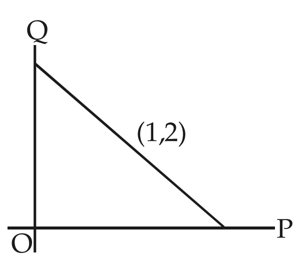
$= \dfrac{1}{2}\left(1 - \dfrac{2}{m}\right)(2 - m)$

($\because$ Area of $\Delta = \dfrac{1}{2} \times \text{base} \times \text{height}$)

$= \dfrac{1}{2}\left[2 - m - \dfrac{4}{m} + 2\right]$

$= \dfrac{1}{2}\left[4 - \left(m + \dfrac{4}{m}\right)\right]$

$= 2 - \dfrac{m}{2} - \dfrac{2}{m}$



Let Area $= f(m) = 2 - \dfrac{m}{2} - \dfrac{2}{m}$

Now, $f'(m) = \dfrac{-1}{2} + \dfrac{2}{m^2}$

Put $f'(m) = 0$

$\Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

Now, $f''(m) = \dfrac{-4}{m^3}$

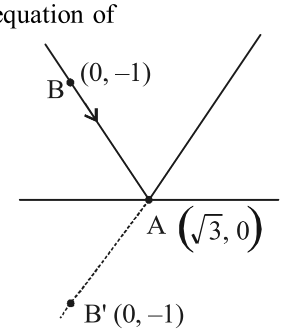
$f''(m)\big|_{m=2} = -\dfrac{1}{2} < 0$

$f''(m)\big|_{m=-2} = \dfrac{1}{2} > 0$

Area will be least at $m = -2$

Hence, slope of PQ is -2.

30. **(b)** Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of



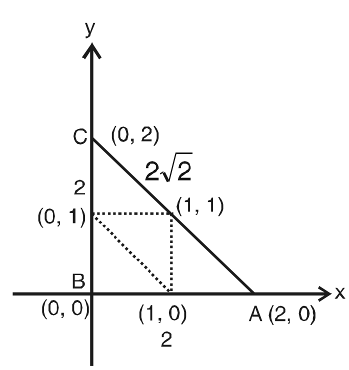
Reflected Ray is $\dfrac{-1-0}{0-\sqrt{3}} = \dfrac{y-0}{x-\sqrt{3}}$

$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

31. **(b)** From the figure, we have

$a = 2, b = 2\sqrt{2}, c = 2$

$x_1 = 0, x_2 = 0, x_3 = 2$



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w w w . c r a c k j

Now, x-co-ordinate of incentre is given as

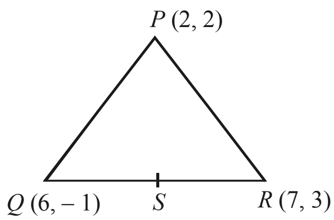
$$\frac{ax_1 + bx_2 + cx_3}{a + b + c}$$

$\Rightarrow$ x-coordinate of incentre

$$= \frac{2 \times 0 + 2\sqrt{2}.0 + 2.2}{2 + 2 + 2\sqrt{2}}$$

$$= \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$$

32. **(d)** Let P, Q, R, be the vertices of ΔPQR



Since PS is the median

S is mid-point of QR

So, $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of $PS = \dfrac{2-1}{2 - \frac{13}{2}} = -\dfrac{2}{9}$

Since, required line is parallel to PS therefore

slope of required line = slope of PS

Now, eqn of line passing through (1, –1) and having slope $-\frac{2}{9}$ is

$$y - (-1) = -\frac{2}{9}(x - 1)$$

$$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$$

33. **(a)** Given lines are

$$4ax + 2ay + c = 0$$
$$5bx + 2by + d = 0$$

The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

$$\Rightarrow \quad x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$$

$$\Rightarrow \quad y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$$

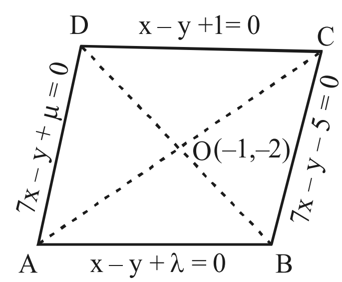
$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

Also distance from axes is same

So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)

$$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$$

34. **(a)**



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore$ $|-1 + 2 + 1| = |-1 + 2 + \lambda| \Rightarrow \lambda = -3$

and $|-7 + 2 - 5| = |-7 + 2 + \mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

On solving the eq^ns of sides pairwise, we get the vertices as

$$\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$$