CHAPTER 25

Vector Algebra

1. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a}\times\vec{b})^2$ is equal to **[2002]**

(a) 48 (b) 16

(c) $\vec{a}$ (d) none of these

2. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$ **[2002]**

(a) 16 (b) 64

(c) 4 (d) 8

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**

(a) 60° (b) 30°

(c) 45° (d) 90°

4. If $|\vec{a}|=5$, $|\vec{b}|=4$, $|\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**

(a) 25 (b) 50

(c) –25 (d) –50

5. If the vectors $\vec{c}$, $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\hat{b}=\hat{j}$ are such that $\vec{a}, \vec{c}$ and form a right handed system then $\vec{c}$ is: **[2002]**

(a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$

(c) $y\hat{j}$ (d) $-z\hat{i}+x\hat{k}$

6. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$ **[2002]**

(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$

(c) $34:39:45$ (d) $39:35:34$

7. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$ **[2002]**

(a) abc (b) –1

(c) 0 (d) 2

8. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are **[2002]**

(a) 13, 5 (b) 12, 6

(c) 14, 4 (d) 11, 7

9. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are **[2002]**

(a) $T=w\cos\theta$ $R=w\tan\theta$

(b) $T=2w\cos\theta$ $R=w$

(c) $T=w$ $R=w\sin\theta$

(d) $T=w\sin\theta$ $R=w\cot\theta$

10. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**

(a) 3 (b) 0

(c) 1 (d) 2.

11. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is **[2003]**

(a) 50 units (b) 20 units

(c) 30 units (d) 40 units.

12. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**

(a) $\sqrt{288}$ (b) $\sqrt{18}$

(c) $\sqrt{72}$ (d) $\sqrt{33}$

13. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2$, $|\vec{c}|=3$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0

(c) –7 (d) 7

14. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(-1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$

(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

15. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non- coplanar, then the product abc equals **[2003]**

(a) 0 (b) 2

(c) –1 (d) 1

16. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003]**

(a) parallelogram but not a rhombus

(b) square

(c) rhombus (d) rectangle.

17. If $\vec{u},\vec{v}$ *and* $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0

(c) $\vec{u}.(\vec{v}\times\vec{w})$ (d) $\vec{u}.\vec{w}\times\vec{v}$.

18. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead , the force $\vec{P}$ are turned through an angle α , then the moment of couple becomes **[2003]**

(a) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$

(b) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$

(c) $\vec{H}\sin\alpha+\vec{G}\cos\alpha$

(d) $\vec{G}\sin\alpha+\vec{H}\cos\alpha$.

19. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed,then $\vec{R}$ is again doubled. Then $P^2:Q^2:R^2$ is **[2003]**

(a) 2 : 3 : 1 (b) 3 : 1 : 1

(c) 2 : 3 : 2 (d) 1 : 2 : 3.

20. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by **[2003]**

(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$

(c) $\dfrac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

21. Two stones are projected from the top of a cliff h metres high , with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals **[2003]**

(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$

(c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

22. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time **[2003]**

(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$

(c) $\frac{f\cos\alpha}{u}$ (d) $u\sin\alpha$

23. The upper $\frac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**

(a) $80\,m$ (b) $20\,m$

(c) $40\,m$ (d) $60\,m$.

24. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in **[2003]**

(a) H.P (b) A.G.P

(c) A.P (d) G.P.

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (l being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**

(a) 0 (b) $\lambda\vec{b}$

(c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

26. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30

(c) 25 (d) 40

27. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and l is a real number, then the vectors $\overline{a}+2\overline{b}+3\overline{c},\ \lambda\overline{b}+4\overline{c}$ and $(2\lambda-1)\overline{c}$ are non coplanar for **[2004]**

(a) no value of l

(b) all except one value of l

(c) all except two values of l

(d) all values of l

28. Let $\overline{u}, \overline{v}, \overline{w}$ be such that $|\overline{u}|=1, |\overline{v}|=2, |\overline{w}|=3.$ If the projection $\overline{v}$ along $\overline{u}$ is equal to that of $\overline{w}$ along $\overline{u}$ and $\overline{v}$, $\overline{w}$ are perpendicular to each other then $|\overline{u}-\overline{v}+\overline{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$

(c) $\sqrt{14}$ (d) 2

29. Let $\overline{a}, \overline{b}$ and $\overline{c}$ be non-zero vectors such that $(\overline{a}\times\overline{b})\times\overline{c}=\frac{1}{3}|\overline{b}||\overline{c}|\overline{a}$. If q is the acute angle between the vectors $\overline{b}$ and $\overline{c}$, then sinq equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

30. With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are **[2004]**

(a) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$

(b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$

(c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$

(d) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

31. In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is **[2004]**

(a) 9 (b) 4
(c) 5 (d) 3

32. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P}:\vec{Q}:\vec{R}$ is **[2004]**

(a) $\operatorname{cosec}\frac{A}{2}:\operatorname{cosec}\frac{B}{2}:\operatorname{cosec}\frac{C}{2}$

(b) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$

(c) $\sec\frac{A}{2}:\sec\frac{B}{2}:\sec\frac{C}{2}$

(d) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$

33. A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively **[2004]**

(a) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h

(c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h

(d) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

34. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is **[2004]**

(a) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}$ m/s

35. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to **[2004]**

(a) 1 (b) $4u^2/g^2$

(c) $u^2/2g$ (d) u^2/g

36. If C is the mid point of AB and P is any point outside AB, then **[2005]**

(a) $\overline{PA} + \overline{PB} = 2\,\overline{PC}$

(b) $\overline{PA} + \overline{PB} = \overline{PC}$

(c) $\overline{PA} + \overline{PB} + 2\,\overline{PC} = \vec{0}$

(d) $\overline{PA} + \overline{PB} + \overline{PC} = \vec{0}$

37. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is equal to

(a) $3\vec{a}^2$ (b) $\vec{a}^2$ **[2005]**

(c) $2\vec{a}^2$ (d) $4\vec{a}^2$

38. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then **[2005]**

$[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$

for

(a) exactly one value of λ
(b) no value of λ
(c) exactly three values of λ
(d) exactly two values of λ

39. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[2005]**

(a) only y (b) only x
(c) both x and y (d) neither x nor y

40. ABC is a triangle. Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along IA, IB, and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then $P : Q : R$ is **[2005]**

(a) $\sin A : \sin B : \sin C$

(b) $\sin \frac{A}{2} : \sin \frac{B}{2} : \sin \frac{C}{2}$

(c) $\cos \frac{A}{2} : \cos \frac{B}{2} : \cos \frac{C}{2}$

(d) cos A : cos B : cos C

41. A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by **[2005]**

(a) $\frac{u}{3}$ (b) $\frac{u}{2}$

(c) $\frac{2u}{3}$ (d) $\frac{u}{\sqrt{3}}$

42. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance **[2005]**

(a) $\frac{2H}{A-B}$ (b) $\frac{H}{A+B}$

(c) $\frac{H}{2(A+B)}$ (d) $\frac{H}{A-B}$

43. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is: **[2005]**

(a) $2:1$ (b) $3:\sqrt{2}$

(c) $3:2$ (d) $3:2\sqrt{2}$

44. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB}, \overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is **[2006]**

(a) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$ (b) $\frac{(AB)(AC)}{AB + AC}$

(c) $\frac{1}{AB} + \frac{1}{AC}$ (d) $\frac{1}{AD}$

45. If $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$ where $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}.\bar{b} \neq 0$, $\bar{b}.\bar{c} \neq 0$ then $\bar{a}$ and $\bar{c}$ are **[2006]**

(a) inclined at an angle of $\frac{\pi}{3}$ between them

(b) inclined at an angle of $\frac{\pi}{6}$ between them

(c) perpendicular

(d) parallel

46. The values of a, for which points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are **[2006]**

(a) 2 and 1 (b) – 2 and – 1

(c) – 2 and 1 (d) 2 and – 1

47. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is **[2006]**

(a) 90° (b) 120°
(c) 45° (d) 60°

48. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10 m/s^2$, then the height above the point P from where the body began to fall is **[2006]**

(a) 720m (b) 900m
(c) 320m (d) 680m

49. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for **[2007]**

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

50. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**

(a) -4 (b) -2
(c) 0 (d) 1.

51. The resultant of two forces Pn and 3n is a force of 7n. If the direction of 3n force were reversed, the resultant would be $\sqrt{19}\, n$. The value of P is **[2007]**

(a) 3 n (b) 4 n
(c) 5 n (d) 6 n.

52. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is **[2007]**

(a) $\tan^{-1}\dfrac{bc}{a(c-a)}$ (b) $\tan^{-1}\dfrac{bc}{a}$

(c) $\tan^{-1}\dfrac{b}{ac}$ (d) 45°.

53. A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are **[2007]**

(a) 5 kg and 12 kg (b) 5 kg and 13 kg
(c) 12 kg and 13 kg (d) 5 kg and 5 kg

54. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**

(a) 0 (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{2}$ (d) p

55. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\dfrac{6}{5}, \dfrac{-3}{5}, \dfrac{2}{5}$ (b) $\dfrac{6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$

(c) $\dfrac{-6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$ (d) $6, -3, 2$

56. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\; p\vec{v}\; p\vec{\omega}] - [p\vec{v}\; \vec{\omega}\; q\vec{u}] - [2\vec{\omega}\; q\vec{v}\; q\vec{u}] = 0$ holds for : **[2009]**

(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

57. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is **[2010]**

(a) $2\hat{i} - \hat{j} + 2\hat{k}$ (b) $\hat{i} - \hat{j} - 2\hat{k}$

(c) $\hat{i} + \hat{j} - 2\hat{k}$ (d) $-\hat{i} + \hat{j} - 2\hat{k}$

58. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ **[2010]**

(a) (2,–3) (b) (–2, 3)
(c) (3,–2) (d) (–3, 2)

59. If $\vec{a} = \dfrac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \dfrac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is **[2011]**

(a) –3 (b) 5
(c) 3 (d) –5

60. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to **[2011]**

(a) $\vec{c}+\left(\dfrac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b}+\left(\dfrac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c}-\left(\dfrac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b}-\left(\dfrac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

61. If the $p\hat{i}+\hat{j}+\hat{k},\ \hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}\,(p\neq q\neq r\neq 1)$ vector are coplanar, then the value of $pqr-(p+q+r)$ is **[2011RS]**

(a) 2 (b) 0
(c) -1 (d) -2

62. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[2011RS]**

(a) $\vec{a}$ (b) $\vec{c}$

(c) $\vec{0}$ (d) $\vec{a}+\vec{c}$

63. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[2012]**

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

64. Let $ABCD$ be a parallelogram such that $\overline{AB}=\vec{q},\overline{AD}=\vec{p}$ and ÐBAD be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[2012]**

(a) $\vec{r}=3\vec{q}-\dfrac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r}=-\vec{q}+\dfrac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

(c) $\vec{r}=\vec{q}-\dfrac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r}=-3\vec{q}-\dfrac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

65. If the vectors $\overline{AB}=3\hat{i}+4\hat{k}$ and $\overline{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[2013]**

(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{45}$

66. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to **[2014]**

(a) 0 (b) 1
(c) 2 (d) 3

67. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If q is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sin q is : **[2015]**

(a) $\dfrac{2}{3}$ (b) $\dfrac{-2\sqrt{3}}{3}$

(c) $\dfrac{2\sqrt{2}}{3}$ (d) $\dfrac{-\sqrt{2}}{3}$

68. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\dfrac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[2016]**

(a) $\dfrac{2\pi}{3}$ (b) $\dfrac{5\pi}{6}$

(c) $\dfrac{3\pi}{4}$ (d) $\dfrac{\pi}{2}$

69. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3$, $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to : **[2017]**

(a) $\dfrac{1}{8}$ (b) $\dfrac{25}{8}$
(c) 2 (d) 5

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(a)	(a)	(a)	(b)	(c)	(a)	(b)	(a)	(d)	(d)	(c)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(none)	(c)	(c)	(c)	(a)	(a)	(a)	(c)	(a)	(c)	(d)	(c)	(c)	(a)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(c)	(d)	(d)	(a)	(b)	(a)	(c)	(b)	(d)	(c)	(d)	(b)	(d)	(d)	(d)
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
(a)	(b)	(a)	(b)	(b)	(d)	(c)	(a)	(a)	(b)	(d)	(d)	(d)	(d)	(c)
61	62	63	64	65	66	67	68	69						
(d)	(c)	(c)	(b)	(c)	(b)	(c)	(b)	(c)						

SOLUTIONS

1. **(b)** We have, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\frac{\pi}{6}$

$=4\times2\times\frac{\sqrt{3}}{2}=4\sqrt{3}$.

Now, $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=a^2b^2$;

$\Rightarrow\ (\vec{a}\times\vec{b})^2+48=16\times4$

$\Rightarrow\ (\vec{a}\times\vec{b})^2=16$

2. **(a)** We have, $[\vec{a}\times\vec{b}\ \ \vec{b}\,x\,\vec{c}\ \ \vec{c}\times\vec{a}]$

$=(\vec{a}\times\vec{b}).\left\{(\vec{b}\,x\,\vec{c})\times(\vec{c}\times\vec{a})\right\}$

$=(\vec{a}\times\vec{b}).\left\{(\vec{m}\cdot\vec{a})\,\vec{c}-(\vec{m}\cdot\vec{c})\,\vec{a}\right\}$

(where $\vec{m}=\vec{b}\times\vec{c}$)

$=\{(\vec{a}\times\vec{b}).\vec{c}\}.\{(\vec{a}\cdot(\vec{b}\times\vec{c})\}$

$=[\vec{a}\ \vec{b}\ \vec{c}]^2=4^2=16$.

3. **(a)** $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow\vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow(\vec{b}+\vec{c})^2=(\vec{a})^2=5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$

$\Rightarrow2|\vec{b}||\vec{c}|\cos\theta=49-34=15$;

$\Rightarrow2\times5\times3\cos\theta=15$;

$\Rightarrow\ \cos\theta=1/2;\Rightarrow\ \theta=\frac{\pi}{3}=60^\circ$

4. **(a)** We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow(\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2|+|\vec{c}|^2$

$+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25$.

$\therefore|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25$.

5. **(a)** Since $\vec{a},\vec{c},\vec{b}$ form a right handed system,

$\therefore\quad\vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&0\\x&y&z\end{vmatrix}=z\hat{i}-x\hat{k}$

6. **(b)** We have $\vec{a}\times\vec{b}=39\vec{k}=\vec{c}$

Also $|\vec{a}|=\sqrt{34},|\vec{b}|=\sqrt{45},|\vec{c}|=39$;

$\therefore|\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$.

7. (c) Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$. Then

$$\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{r}$$

$$\Rightarrow 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$$

$$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r}\Rightarrow \vec{a}\times\vec{r}=\vec{0}$$

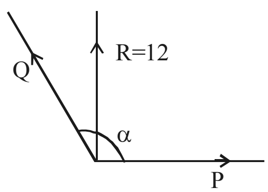
Similarly $\vec{b}\times\vec{r}=\vec{0}$ & $\vec{c}\times\vec{r}=\vec{0}$

Above three conditions will be satisfied for non-zero vectors if and only if $\vec{r}=\vec{0}$

8. (a) Given $P+Q=18$ ....(1)

$P^2+Q^2+2PQ\cos\alpha=144$ ......(2)

$$\tan 90^\circ=\frac{Q\sin\alpha}{P+Q\cos\alpha}$$



$\Rightarrow \quad P+Q\cos\alpha=0$ ......(3)

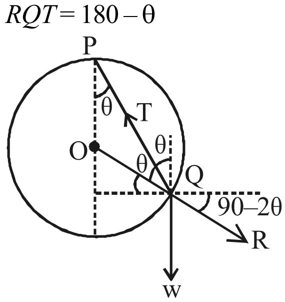
From (2) and (3),

$$Q^2-P^2=144\Rightarrow(Q-P)(Q+P)=144$$

$$\therefore \quad Q-P=\frac{144}{18}=8$$

From (1), On solving, we get $Q=13$, $P=5$

9. (b) $\angle TQW=180-\theta$; $\angle RQW=2\theta$;
$\angle RQT=180-\theta$



Applying Lami's theorem at Q.

$$\frac{T}{\sin 2\theta}=\frac{R}{\sin(180-\theta)}=\frac{W}{\sin(180-\theta)}$$

$\Rightarrow \quad R=W$ and $T=2W\cos\theta$

10. (a) since $\vec{n}$ is perpendicular $\vec{u}$ and $\vec{v}$,

$$\vec{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\hat{n}=\frac{\begin{vmatrix} i & j & k\\ 1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$$

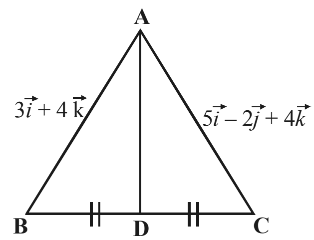
$$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$$

11. (d) $\vec{F}+\vec{F}_1+\vec{F}_2=7i+2j-4k$

$\vec{d}=P.V\ of\ \vec{B}-P.V\ of\ \vec{A}=4i+2j-2k$

$W=\vec{F}.\vec{d}=28+4+8=40\ unit$

12. (d)



$$P.V\ of\ \overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}$$

$$=4i-j+4k\ or\ |\overrightarrow{AD}|=\sqrt{16+16+1}=\sqrt{33}$$

13. (c) $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{-1-4-9}{2}=-7$$

14. (b) Vector perpendicular to the face OAB

$$=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 2 & 1 & 3\end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}$$

Vector perpendicular to the face ABC

$$=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -2 & -1 & 1\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

Angle between the faces = angle between their normals

$$\cos\theta=\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|=\frac{19}{35}\ \text{or}\ \theta=\cos^{-1}\left(\frac{19}{35}\right)$$

15. **(c)** $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$As \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \ (given\ condition)$$

$\therefore \quad abc = -1$

16. (none)

$A = (7,-4,7), B = (1,-6,10), C = (-1,-3,4)$ and $D = (5,-1,5)$

$$AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2}$$
$$= \sqrt{36+4+9} = 7$$

Similarly $BC = 7,\ CD = \sqrt{41},\ \ DA = \sqrt{17}$

$\therefore$ None of the options is satisfied

17. **(c)** $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$= (\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})$

$= \vec{u}.(\vec{u}\times\vec{v})$

$-\vec{u}.(\vec{u}\times\vec{w})+\vec{u}.(\vec{v}\times\vec{w})+\vec{v}.(\vec{u}\times\vec{v})-\vec{v}.(\vec{u}\times\vec{w})$

$+\vec{v}.(\vec{v}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{v}\times\vec{w})$

$= \vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$

$= [\vec{u}\vec{v}\vec{w})]+[\vec{v}\vec{w}\vec{u})]-[\vec{w}\vec{u}\vec{v}] = \vec{u}.(\vec{v}\times\vec{w})$

18. **(c)** $\vec{G} = \vec{r}\times\vec{p}\,; \qquad |\vec{G}| = rp\sin\theta$

$|\vec{H}| = rp\cos\theta \ \left[\because \sin(90^o+\theta) = \cos\theta\right]$

$G = rp\sin\theta$.......(1) $\quad H = rp\cos\theta$........(2)

$x = rp\sin(\theta+\alpha)$......(3)

From (1), (2) & (3),

$x = \vec{G}\cos\alpha + \vec{H}\sin\alpha\cdot$

19. **(c)**

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ....(1)

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ....(2)

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ ....(3)

$On\ (1)+(3),\ 5R^2 = 2P^2 + 2Q^2$ ....(4)

On $(3)\times 2 + (2),\ 12R^2 = 3P^2 + 6Q^2$ .....(5)

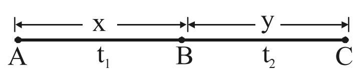
$2P^2 + 2Q^2 - 5R^2 = 0$ .....(6)

$3P^2 + 6Q^2 - 12R^2 = 0$ .....(7)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6}\ or\ P^2 : Q^2 : R^2 = 2:3:2$$

20. **(a)** Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x, BC = y$, time taken from A to $B = t_1$ and time taken from B to $C = t_2$, then $s = x + y$ and $t = t_1 + t_2$

For the motion from A to B

$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx\ (\because u = 0)$

$\Rightarrow \quad x = \dfrac{v^2}{2f}$ ....(1)

and $\quad v = u + ft \quad \Rightarrow v = ft_1$

$\Rightarrow \quad t_1 = \dfrac{v}{f}$ ...(2)

For the motion from B to C

$v^2 = u^2 + 2fs$

$\Rightarrow \quad 0 = v^2 - 2ry \quad \Rightarrow y = \dfrac{v^2}{2r}$ ...(3)

and $v = u + ft \Rightarrow 0 = v - rt_2$

$\Rightarrow \quad t_2 = \dfrac{v}{r}$

Adding equations (1) and (3), we get

$$x + y = \frac{v^2}{2}\left[\frac{1}{f} + \frac{1}{r}\right] = s$$

Adding equations (2) and (4), we get

$$t_1 + t_2 = v\left[\frac{1}{f} + \frac{1}{r}\right] = t$$

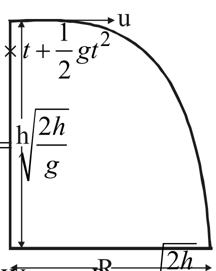
$$\therefore \quad \frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f} + \frac{1}{r}\right]^2}{2 \times \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)} = \frac{1}{f} + \frac{1}{r}$$

$$\Rightarrow \quad t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$

21. (a) For the stone projected horizontally, for horizontal motion, using distance

$= \text{speed} \times \text{time} \Rightarrow R = ut$

and for vertical motion

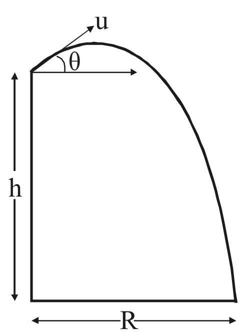


$h = 0 \times t + \frac{1}{2}gt^2$

$\Rightarrow t = \sqrt{\dfrac{2h}{g}}$

$\therefore$ We get $R = u\sqrt{\dfrac{2h}{g}}$ ....(1)

For the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $h = -u\sin\theta \times t + \frac{1}{2}gt^2$ ....(3)

From (1) and (2) we get

$$u\sqrt{\frac{2h}{g}} = u\cos\theta \times t$$

$$\Rightarrow \quad t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$$

Substituting this value of t in eq (3) we get

$$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$$

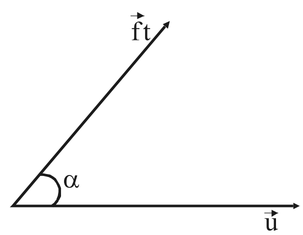
$$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$$

$$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$$

$$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \therefore \quad \tan\theta = u\sqrt{\frac{2}{hg}}$$

22. (a) We can consider the two velocities as

$\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (ft\cos\alpha)\hat{i} + (ft\sin\alpha)\hat{j}$



$\therefore$ Relative velocity of second with respect to first

$$\vec{v} = \vec{v}_2 - \vec{v}_1 = (ft\cos\alpha - u)\hat{i} + ft\sin\alpha\,\hat{j}$$

$$\Rightarrow \quad |\vec{v}|^2 = (ft\cos\alpha - u)^2 + (ft\sin\alpha)^2$$

$$= f^2t^2 + u^2 - 2uft\cos\alpha$$

For $|\vec{v}|$ to be min we should have

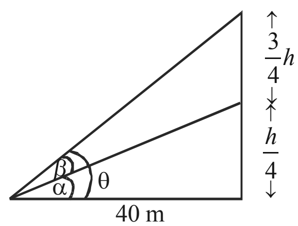
$$\frac{d|v|^2}{dt} = 0 \Rightarrow 2f^2t - 2uf\cos\alpha = 0$$

$$\Rightarrow \quad t = \frac{u\cos\alpha}{f}$$

Also $\dfrac{d^2|v|^2}{dt^2} = 2f^2 = +ve$

$\therefore$ $|v|^2$ and hence $|v|$ is least at the time $\dfrac{u\cos\alpha}{f}$

23. (c)



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\dfrac{3}{5}\right)$

or $\beta = \theta - \alpha$

$\Rightarrow \quad \tan\beta = \dfrac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha}$

or $\quad \dfrac{3}{5} = \dfrac{\dfrac{h}{40} - \dfrac{h}{160}}{1 + \dfrac{h}{40} . \dfrac{h}{160}}$

$h^2 - 200h + 6400 = 0$

$\Rightarrow h = 40$ or 160 metre

$\therefore$ possible height $= 40$ metre

24. (a) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$R_1 = \dfrac{u^2}{g(1+\sin\beta)}$ *and* $R_2 = \dfrac{u^2}{g(1-\sin\beta)}$

$\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2g}{u^2}$ *or* $\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2}{R}\left[\because R = \dfrac{u^2}{g}\right]$

$\therefore R_1, R, R_2$ *are in H.P.*

25. (c) Let $\vec{a} + 2\vec{b} = t\vec{c}$ and $\vec{b} + 3\vec{c} = s\,\vec{a}$, where t and s are scalars. Adding, we get

$\vec{a} + 3\vec{b} + 3\vec{c} = t\vec{c} + s\vec{a}$

$\Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = t\vec{c} + s\vec{a} - \vec{b} + 3\vec{c}$

$= t\vec{c} + (\vec{b} + 3\vec{c}) - \vec{b} + 3\vec{c} = (t+6)\vec{c}$

$\left[\text{using } s\,\vec{a} = \vec{b} + 3\vec{c}\right]$

$= \lambda\vec{c}$, where $\lambda = t + 6$

26. (d) Resultant of forces $\vec{F} = 7\hat{i} + 2\hat{j} - 4\hat{k}$

Displacement $\vec{d} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ Work done $= \vec{F}.\vec{d} = 28 + 4 + 8 = 40$

27. (c) Vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + 4\vec{c}$, and $(2\lambda - 1)\vec{c}$ are coplanar if

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda(2\lambda - 1) = 0 \;\Rightarrow \lambda = 0$ or $\dfrac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda = 0, \dfrac{1}{2}$

28. (c) Projection of $\vec{v}$ along $\vec{u}$ $= \dfrac{\vec{v}.\vec{u}}{|\vec{u}|} = \dfrac{\vec{v}.\vec{u}}{2}$

projection of $\vec{w}$ along $\vec{u}$ $= \dfrac{\vec{w}.\vec{u}}{|\vec{u}|} = \dfrac{\vec{w}.\vec{u}}{2}$

Given $\dfrac{\vec{v}.\vec{u}}{2} = \dfrac{\vec{w}.\vec{u}}{2}$ ....(1)

Also , $\vec{v}.\vec{w} = 0$ ....(2)

Now $|\vec{u} - \vec{v} + \vec{w}|^2$

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u}.\vec{v} - 2\vec{v}.\vec{w} + 2\vec{u}.\vec{w}$

$= 1 + 4 + 9 + 0$ [From (1) and (2)] $= 14$

$\therefore |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$

29. (a) Given $(\vec{a} \times \vec{b}) \times \vec{c} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are noncollinear

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\therefore\ \vec{a}.\vec{c}=0$ and $-\vec{b}.\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta=\frac{-1}{3}$

$\therefore\ \sin\ \theta=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

30. (c) Let forces be P and Q. then $P+Q=4$ ....(1)

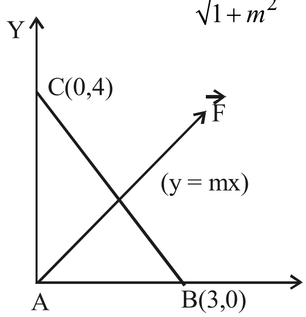
and $P^2+Q^2=3^2$ ....(2)

Solving we get the forces

$$\left(2+\frac{\sqrt{2}}{2}\right)N \text{ and } \left(2-\frac{\sqrt{2}}{2}\right)N$$

31. (c) Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y=mx$. (see figure)

Moment about $B=\frac{3m}{\sqrt{1+m^2}}|\vec{F}|=9$ ....(1)



Moment about $C=\frac{4}{\sqrt{1+m^2}}|\vec{F}|=16$....(2)

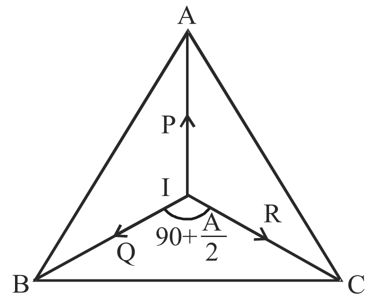
Dividing (1) by (2), we get

$m=\frac{3}{4}\Rightarrow|\vec{F}|=5N.$

32. (d) IA, IB, IC are bisectors of the angles A, B and C as I is incentre of ΔABC.

Now $\angle BIC=180-\frac{B}{2}-\frac{C}{2}=90°+\frac{A}{2}$ *etc.*

Applying Lami's theorem at I

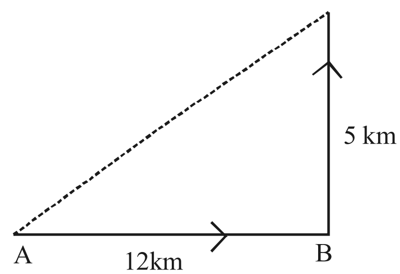


$$\frac{P}{\sin(90°+\frac{A}{2})}=\frac{Q}{\sin\left(90°+\frac{B}{2}\right)}=\frac{R}{\sin\left(90°+\frac{C}{2}\right)}$$

$$\Rightarrow\quad P:Q:R=\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$$

33. (d) Time taken by the particle in complete journey

$$T=\frac{12}{4}+\frac{5}{5}=4\ hr.$$



$\therefore$ Average speed $=\frac{12+5}{4}=\frac{17}{4}$

Average velocity $=\sqrt{\frac{12^2+5^2}{4}}=\frac{13}{4}$

[using vector addition]

34. (a) If $v=\frac{1}{4}$, component along OB

$$=\frac{v\sin 30°}{\sin(45°+30°)}=\frac{\frac{1}{4}\times\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}=\frac{\sqrt{6}-\sqrt{2}}{8}$$

35. (b) For same horizontal range the angles of projection must be α and $\frac{\pi}{2}-\alpha$

$\therefore t_1=\frac{2u\sin\alpha}{g}$ and

$$t_2 = \frac{2u\sin\left(\frac{\pi}{2}-\alpha\right)}{g} = \frac{2u\cos\alpha}{g}$$

$$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$$

36. **(a)** $\overrightarrow{PA}+\overrightarrow{AP}=0$ and $\overrightarrow{PC}+\overrightarrow{CP}=0$

$\Rightarrow \quad \overrightarrow{PA}+\overrightarrow{AC}+\overrightarrow{CP}=0$

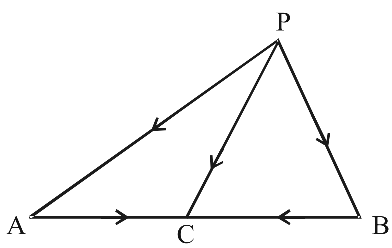
and $\overrightarrow{PB}+\overrightarrow{BC}+\overrightarrow{CP}=0$

Adding, we get

$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{AC}+\overrightarrow{BC}+2\overrightarrow{CP}=0.$

Since $\overrightarrow{AC}=-\overrightarrow{BC}$ & $\overrightarrow{CP}=-\overrightarrow{PC}$

$\Rightarrow \quad \overrightarrow{PA}+\overrightarrow{PB}-2\overrightarrow{PC}=0.$



37. **(c)** Let $\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}$

$\vec{a}\times\vec{i}=z\vec{j}-y\vec{k} \Rightarrow (\vec{a}\times\vec{i})^2=y^2+z^2$

Similarly,

$(\vec{a}\times\vec{j})^2=x^2+z^2$ and $(\vec{a}\times\vec{k})^2=x^2+y^2$

$\Rightarrow \quad (\vec{a}\times\vec{i})^2+(\vec{a}\times\vec{j})^2+(\vec{a}\times\vec{k})^2$

$=2(x^2+y^2+z^2)=2\vec{a}^2$

38. **(b)** Let us consider

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

then as per question

$\left[\lambda\left(\vec{a}+\vec{b}\right)\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$

$$\Rightarrow \begin{vmatrix} \lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3) \\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda b_3 \\ \lambda c_1 & \lambda c_2 & \lambda c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1+b_1 & a_2b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$R_1 - R_2 \qquad R_2 - R_3$

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\Rightarrow \lambda^4 = -1$

Hence λ has no real values.

39. **(d)** $\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.\vec{b}\times\vec{c}=\begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$=1\left[1+x-y-x+x^2\right]-\left[-x^2-y\right]$

$=1-y+x^2-x^2+y$

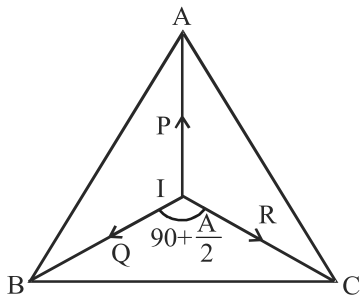
$=1$

Hence $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is independent of x and y both.

40. **(c)** IA, IB, IC are bisectors of the angles A, B and C as I is incentre of ΔABC.

Now $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90° + \frac{A}{2}$ etc.

Applying Lami's theorem at I



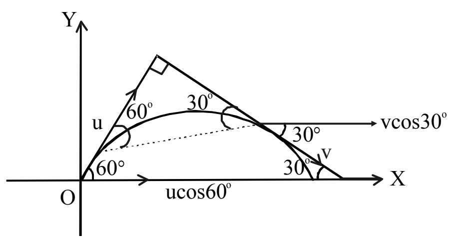
$$\frac{P}{Sin(90°+\frac{A}{2})}=\frac{Q}{\sin\left(90°+\frac{B}{2}\right)}=\frac{R}{\sin\left(90°+\frac{C}{2}\right)}$$

$$\Rightarrow P:Q:R=\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$$

41. **(d)** $u\cos 60°=v\cos 30°$

(as horizontal component of velocity remains the same)

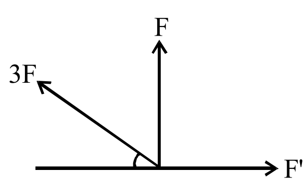
$$\Rightarrow u\cdot\frac{1}{2}=v\cdot\frac{\sqrt{3}}{2} \text{ or } v=\frac{1}{\sqrt{3}}u$$



42. **(b)** Let A and B be displaced by a distance x then Change in moment of $(A+B)$ =applied moments

$$\Rightarrow (A+B)\times x=H \Rightarrow x=\frac{H}{A+B}$$

43. **(d)** $F'=3F\cos\theta$ and $F=3F\sin\theta$



$$\Rightarrow F'=2\sqrt{2}F \Rightarrow F:F'::3:2\sqrt{2}.$$

44. **(d)** If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively, then as per quesiton, forces along AB and AC respectively are

$\left(\frac{1}{AB}\right)\hat{i}$ and $\left(\frac{1}{AC}\right)\hat{j}$

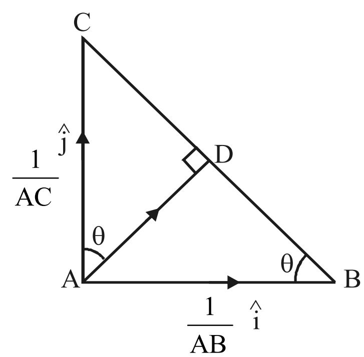
$\therefore$ Their resultant along AD

$$=\left(\frac{1}{AB}\right)i+\left(\frac{1}{AC}\right)j$$

$\therefore$ Magnitude of resultant is

$$=\sqrt{\left(\frac{1}{AB}\right)^2+\left(\frac{1}{AC}\right)^2}=\sqrt{\frac{AC^2+AB^2}{AB^2+AC^2}}$$

$$=\frac{BC}{AB.AC}$$



But from figure $\Delta ABC\sim\Delta DBA$

$$\Rightarrow \frac{BC}{AB}=\frac{AC}{AD}\Rightarrow\frac{BC}{AB\times AC}=\frac{1}{AD}$$

$\therefore$ The required magnitude of resultant becomes $\frac{1}{AD}$.

45. **(d)** $(\bar{a}\times\bar{b})\times\bar{c}=\bar{a}\times(\bar{b}\times\bar{c}),\ \bar{a}.\bar{b}\neq 0,$

$\bar{b}.\bar{c}\neq 0$

$\Rightarrow (\bar{a}.\bar{c}).\bar{b}-(\bar{b}.\bar{c})\bar{a}=(\bar{a}.\bar{c}).\bar{b}-(\bar{a}.\bar{b}).\bar{c}$

$\Rightarrow (\bar{a}.\bar{b}).\bar{c}=(\bar{b}.\bar{c})\bar{a} \Rightarrow \bar{a}\parallel\bar{c}$.

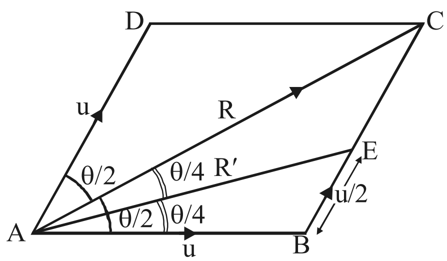
46. **(a)** $\overline{CA}=(2-a)\hat{i}+2\hat{j}$;

$\overline{CB}=(1-a)\hat{i}-6\hat{k}$

$\overline{CA}.\overline{CB}=0 \Rightarrow (2-a)(1-a)=0$

$\Rightarrow a=2,1$

47. **(b)** For two velocities u and u at an angle θ to each other the resultant is given by



$R^2 = u^2 + u^2 + 2u^2\cos\theta = 2u^2(1+\cos\theta)$

$\Rightarrow R^2 = 4u^2\cos^2\theta/2$ or $R = 2u\cos\frac{\theta}{2}$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u$

$\Rightarrow 2u\cos\frac{\theta}{2} = u$

$\Rightarrow \quad \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60°$

or $\quad \theta = 120°$

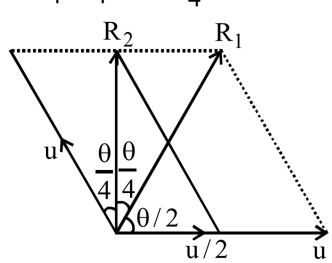
ALTERNATE SOLUTION

$$\tan\frac{\theta}{4} = \frac{\frac{u}{2}\sin\theta}{u + \frac{u}{2}\cos\theta}$$

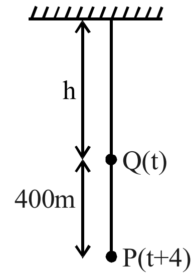
$\Rightarrow \quad \sin\frac{\theta}{4} + \frac{1}{2}\sin\frac{\theta}{4}\cos\theta = \frac{1}{2}\sin\theta\cos\frac{\theta}{4}$

$\therefore \quad 2\sin\frac{\theta}{4} = \sin\frac{3\theta}{4} = 3\sin\frac{\theta}{4} - 4\sin^3\frac{\theta}{4}$

$\therefore \quad \sin^2\frac{\theta}{4} = \frac{1}{4} \Rightarrow \frac{\theta}{4} = 30°$ or $\theta = 120°$



48. **(a)** Using $h = \frac{1}{2}gt^2$ and $h + 400 = \frac{1}{2}g(t+4)^2$



Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow \quad t = 8$ sec

$\therefore \quad h = \frac{1}{2} \times 10 \times 64 = 320m$

$\therefore$ Desired height $= 320 + 400 = 720\ m$

49. **(b)** Given $|2\hat{u} \times 3\hat{v}| = 1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}| = 1, |\hat{v}| = 1$

$\Rightarrow 6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\Rightarrow \quad 6|\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u} \times 3\hat{v}$ is a unit vector.

50. **(b)** Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$

$\Rightarrow 1[1-2(x-2)] - 1[-1-2x] + 1[x-2+x] = 0$

$\Rightarrow 1 - 2x + 4 + 1 + 2x + 2x - 2 = 0$

$\Rightarrow 2x = -4 \Rightarrow x = -2$

51. **(d)** Clearly $\vec{a} = -\frac{8}{7}\vec{c}$

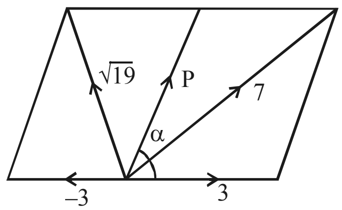
$\Rightarrow \vec{a} \parallel \vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

ALTERNATE SOLUTION

$$\cos\theta = \frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|} = \frac{8\vec{b}\cdot(-7\vec{b})}{8|\vec{b}|7|\vec{b}|} = -1 \Rightarrow \theta = \pi$$

52. (c) Given : Force $P = Pn$, $Q = 3n$, resultant $R = 7n$ & $P' = Pn$, $Q' = (-3)n$, $R' = \sqrt{19}\,n$



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow \quad (7)^2 = P^2 + (3)^2 + 2 \times P \times 3\cos\alpha$

$\Rightarrow \quad 49 = P^2 + 9 + 6P\cos\alpha$

$\Rightarrow \quad 40 = P^2 + 6P\cos\alpha \qquad$ .....(i)

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P \times -3\cos\alpha$

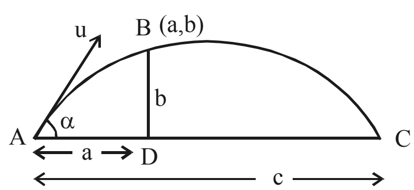
$\Rightarrow \quad 19 = P^2 + 9 - 6P\cos\alpha$

$\Rightarrow \quad 10 = P^2 - 6P\cos\alpha \qquad$ .....(ii)

Adding (i) and (ii) $50 = 2P^2$

$\Rightarrow \quad P^2 = 25 \Rightarrow P = 5n.$

53. (a) Let B be the top of the wall whose coordinates will be (a, b). Range $(R) = c$



B lies on the trajectory

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$

$$\Rightarrow \quad b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$$

$$\Rightarrow \quad b = a\tan\alpha\left[1 - \frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{\frac{2u^2}{g}\cos^2\alpha.\frac{\sin\alpha}{\cos\alpha}}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{R}\right]$$

$$\left(\because R = \frac{u^2\sin^2\alpha}{g}\right)$$

$$\Rightarrow \quad b = a\tan\alpha\left[1 - \frac{a}{c}\right]$$

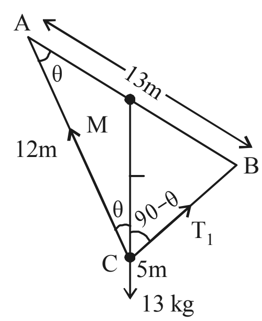
$$\Rightarrow b = a\tan\alpha . \left(\frac{c-a}{c}\right)$$

$$\Rightarrow \quad \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection,

$$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$$

54. (a)



$\because\ 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

Q m is mid point of the hypotenuse AB, therefore $MA = MB = MC$

$\Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin 90°}$$

$\Rightarrow \quad T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow \quad T_1 = 13 \times \frac{5}{13}$ and $T_2 = 13 \times \frac{12}{13}$

$\Rightarrow \quad T_1 = 5$ kg and $T_2 = 12$ kg

55. (b) Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6, – 3, 2 then

$x_2 - x_1 = 6$; $y_2 - y_1 = -3$; $z_2 - z_1 = 2$

So that direction ratios of $\overrightarrow{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}}, \frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

56. (d) $\because$ $\vec{u}, \vec{v}, \vec{w}$ are non coplanar vectors

$$\therefore [\vec{u}, \vec{v}, \vec{w}] \neq 0$$

Now,

$$[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, p\vec{w}, q\vec{u}] [2\vec{w}, q\vec{v}, q\vec{u}] = 0$$

$\Rightarrow$ $3p^2[\vec{u}, \vec{v}, \vec{w}] - pq[\vec{v}, \vec{w}, \vec{u}] - 2q^2 [\vec{w}, \vec{v}, \vec{u}] = 0$

$\Rightarrow$ $3p^2[\vec{u}, \vec{v}, \vec{w}] - pq[\vec{u}, \vec{v}, \vec{w}] + 2q^2[\vec{u}, \vec{v}, \vec{w}]$

$\Rightarrow$ $(3p^2 - pq + 2q^2)[\vec{u}, \vec{v}, \vec{w}] = 0$

$\Rightarrow$ $3p^2 - pq + 2q^2 = 0$

$\Rightarrow$ $2p^2 + p^2 - pq + \frac{q^2}{4} + \frac{7q^2}{4} = 0$

$\Rightarrow$ $2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0$

$\Rightarrow$ $p = 0, q = 0, p = q/2$

This is possible only when $p = 0, q = 0$

$\therefore$ There is exactly one value of (p, q).

57. (d) $\vec{c} = \vec{b} \times \vec{a} \Rightarrow \vec{b}.\vec{c} = \vec{b}.(\vec{b} \times \vec{a}) \Rightarrow \vec{b}.\vec{c} = 0$

$\Rightarrow (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}).(\hat{i} - \hat{j} - \hat{k}) = 0,$

where $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$b_1 - b_2 - b_3 = 0$ ...(i)

and $\vec{a}.\vec{b} = 3 \Rightarrow (\hat{j} - \hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 3$

$\Rightarrow b_2 - b_3 = 3$

From equation (i)

$b_1 = b_2 + b_3 = (3 + b_3) + b_3 = 3 + 2b_3$

$\vec{b} = (3 + 2b_3)\hat{i} + (3 + b_3)\hat{j} + b_3\hat{k}$

From the option given, it is clear that b_3 equal to either 2 or –2.

If $b_3 = 2$ then $\vec{b} = 7\hat{i} + 5\hat{j} + 2\hat{k}$ which is not possible

If $b_3 = -2$, then $\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

58. (d) Since, $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually orthogonal

$\therefore$ $\vec{a}.\vec{b} = 0$, $\vec{b}.\vec{c} = 0$, $\vec{c}.\vec{a} = 0$

$\Rightarrow$ $2\lambda + 4 + \mu = 0$ ...(i)

$\lambda - 1 + 2\mu = 0$ ...(ii)

On solving (i) and (ii), we get $\lambda = -3, \mu = 2$

59. (d) $(2\vec{a} - \vec{b}).((\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}))$

$= (2\vec{a} - \vec{b}).((\vec{a} \times \vec{b}) \times \vec{a} + 2(\vec{a} + \vec{b}) \times \vec{b})$

$= (2\vec{a} - \vec{b})\left((\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a} + 2(\vec{a}.\vec{b})\vec{b} - 2(\vec{b}.\vec{b})\vec{a}\right)$

$= (2\vec{a} - \vec{b})(\vec{b} - 0 + 0 - 2\vec{a})$

$[\because \vec{a}.\vec{b} = 0$ and $\vec{b}.\vec{b} = 1]$

$= -4\vec{a}.\vec{a} - \vec{b}.\vec{b} = -5$

60. (c) $\vec{a}.\vec{b} \neq 0$, $\vec{a}.\vec{d} = 0$

Now, $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$

$\Rightarrow$ $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{d})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{d})\vec{b} - (\vec{a}.\vec{b})\vec{d}$

$\Rightarrow$ $(\vec{a}.\vec{b})\vec{d} = -(\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$$\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$$

61. (d) The given vectors are collinear if

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$\Rightarrow$ $p(qr - 1) + 1(1 - r) + 1(1 - q) = 0$

$\Rightarrow$ $pqr - p + 1 - r + 1 - q = 0$

$\Rightarrow$ $pqr - (p + q + r) = -2$

62. (c) $\vec{a} + 3\vec{b} = \lambda\vec{c}$ ....(i)

$\vec{b} + 2\vec{c} = \mu\vec{a}$ ....(ii)

On solving equation (i) and (ii)

$$(1 + 3\mu)\vec{a} - (\lambda + 6)\vec{c} = 0$$

As $\vec{a}$ and $\vec{c}$ are non collinear,

$\therefore$ $1 + 3\mu = 0$ and $\lambda + 6 = 0$

From (i), $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

63. **(c)** Let $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$

Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other

$\therefore \quad \vec{c}.\vec{d} = 0$

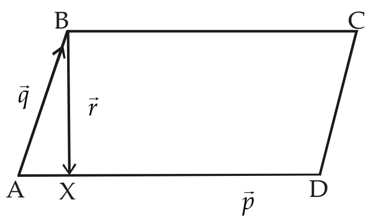
$\Rightarrow \quad \left(\hat{a} + 2\hat{b}\right).\left(5\hat{a} - 4\hat{b}\right) = 0$

$\Rightarrow \; 5 + 6\hat{a}.\hat{b} - 8 = 0 \qquad (\because \vec{a}.\vec{a} = 1)$

$\Rightarrow \quad \hat{a}.\hat{b} = \dfrac{1}{2} \;\Rightarrow\; \theta = \dfrac{\pi}{3}$

64. **(b)** Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.

We have



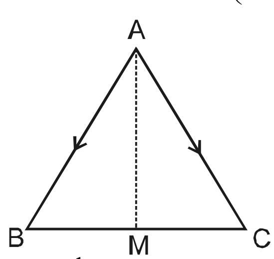
$\overrightarrow{AX} = \left(\dfrac{\vec{p}\,.\,\vec{q}}{|\vec{p}|}\right)\left(\dfrac{\vec{p}}{|\vec{p}|}\right) = \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

Let $\vec{r} = \overrightarrow{BX} = \overrightarrow{BA} + \overrightarrow{AX} \;= -\vec{q} + \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

65. **(c)** We have,

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0 \Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

Now, $\overrightarrow{BM} = \dfrac{\overrightarrow{AC} - \overrightarrow{AB}}{2} \quad \left(\because \quad \overrightarrow{BM} = \dfrac{\overrightarrow{BC}}{2}\right)$



Also, we have

$\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\Rightarrow \quad \overrightarrow{AB} + \dfrac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$

$\Rightarrow \quad 1\,\overrightarrow{AM} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \quad \left|\overrightarrow{AM}\right| = \sqrt{33}$

66. **(b)** L.H.S $= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$

$= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}.\vec{a})\vec{c} - (\vec{b} \times \vec{c}.\vec{c})\vec{a}]$

$= (\vec{a} \times \vec{b}).[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \qquad [\because \vec{b} \times \vec{c}.\vec{c} = 0]$

$= [\vec{a}\;\vec{b}\,\vec{c}].(\vec{a} \times \vec{b}.\vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]^2$

$= [\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

So $\lambda = 1$

67. **(c)** $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \dfrac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow \quad -\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \dfrac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow \quad -\left(\vec{c}.\vec{b}\right)\vec{a} + \left(\vec{c}.\vec{a}\right)\vec{b} = \dfrac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow \quad -\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta\vec{a} + \left(\vec{c}.\vec{a}\right)\vec{b} = \dfrac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\because \quad \vec{a}, \vec{b}, \vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \dfrac{1}{3}$ and $\vec{c}.\vec{a} = 0$

$\Rightarrow \quad \cos\theta = -\dfrac{1}{3} \quad \Rightarrow \sin\theta = \dfrac{2\sqrt{2}}{3}$; $\theta \in$ II quad

68. **(b)** $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \dfrac{\sqrt{3}}{2}\vec{b} + \dfrac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

$\vec{a} \cdot \vec{b} = -\dfrac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\dfrac{\sqrt{3}}{2}$

$[\because \vec{a}$ and $\vec{b}$ are unit vectors$]$

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \dfrac{5\pi}{6}$

69. **(c)** Given :

$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \quad \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

$\therefore \; \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

We have $(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}||\vec{c}|\,\sin 30\,n$

$\Rightarrow |(\vec{a} \times \vec{b}) \times \vec{c}| = 3|\vec{c}|.\dfrac{1}{2} \Rightarrow 3 = 3|\vec{c}|.\dfrac{1}{2}$

$\therefore \; |\vec{c}| = 2$

Now $|\vec{c} - \vec{a}| = 3$

On squaring, we get

$\Rightarrow c^2 + a^2 - 2 - \vec{c}.\vec{a} = 9 \Rightarrow \; 4 + 9 - 2 - \vec{a}.\vec{c} = 9$

$\Rightarrow \vec{a}.\vec{c} = 2 \qquad [\because \vec{c}.\vec{a} = \vec{a}.\vec{c}\,]$