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ANSWER KEY
DATE: 07-12-2018

COURSE
NUCLEUS

JEE-MAIN MOCK TEST-8
XII

TEST CODE				
1	1	2	9	1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	4	2	2	4	4	2	1	2	3	1	1	2	2	2	4
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans	2	1	1	2	4	1	3	1	4	1	4	3	2	2	1
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans	2	1	4	3	3	3	1	3	4	1	4	2	3	3	2
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans	2	1	4	1	1	1	3	1	2	3	3	3	3	2	2
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans	1	3	4	3	4	2	4	2	3	1	4	3	3	3	4
Q.No.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans	3	2	4	4	2	1	3	3	2	2	3	2	3	2	1

HINTS & SOLUTIONS

PHYSICS

Q.1 $P = \sigma A T^4$

$$\Rightarrow \frac{P_A}{P_B} = \frac{A_A}{A_B}\left(\frac{T_A}{T_B}\right)^4 \Rightarrow T_B = T_A 2^{3/4}$$

as $\lambda_m T$ = constant $\Rightarrow \dfrac{\lambda_A}{\lambda_B} = \dfrac{T_B}{T_A}$

$$\Rightarrow \lambda_B = 5000(2)^{-3/4}\ \text{Å}.$$

Q.2 $\dfrac{1}{\lambda_{k\alpha}} = RZ^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$ & $\dfrac{1}{\lambda_{k_B}} = RZ^2\left(\dfrac{1}{1^2} - \dfrac{1}{3^2}\right)$;

dividing we get,

$\lambda_{k_B} = 0.27$ Å

Q.3 As ; $\dfrac{V}{2\ell} = \dfrac{330 \times 100}{2 \times 33}$

$= 500$ Hz

In second harmonic frequency $= \dfrac{V}{\ell}$

$= 1000$ Hz.

Q.4 $\dfrac{1}{2}LI_0{}^2 = \dfrac{1}{2}(C_1 + C_2)V^2$,

$$V = \left[\frac{LI_0^2}{(C_1 + C_2)}\right]^{1/2},$$

$$Q_1 = C_1V = C_1I_0\sqrt{\frac{L}{C_1 + C_2}}$$

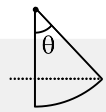
Q.5 $F + f = ma$ .... (1)

Also ; $FR - fR = I\dfrac{a}{R}$

$F - f = ma$ .... (2)

$[I = mR^2]$
From (1) & (2)
$f = 0.$

Q.6 $\sin\theta = \dfrac{\dfrac{mv}{\sqrt{2}\,qB}}{\dfrac{mv}{qB}} = \dfrac{1}{\sqrt{2}}$

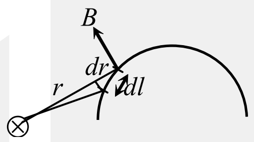


$\Rightarrow \theta = 45°$

$t = \dfrac{T}{8} = \dfrac{\pi m}{4qB}$

Q.7 Magnetic field at a distance r from the wire will be

$B = \dfrac{\mu_0}{2\pi}\dfrac{i_1}{r}$



force on the small element of length dl on semicircular wire is

$dF = i_2\, d\vec{l} \times \vec{B} = i_2(dl_\perp)B = i_2 B\, dr$

$(\because\ dl_\perp = dr)$

$F = \int_R^{3R} i_2\, B\,dr = \dfrac{\mu_0}{2\pi} i_1 i_2 \ln 3$

Q.8 Q = quantity of energy required

$P_1 t_1 = Q,\ \ P_2 t_2 = Q$

$P_{series} = \dfrac{P_1 P_2}{P_1 + P_2}$

$P_{series}\, t_0 = Q, \quad \left(\dfrac{P_1 P_2}{P_1 + P_2}\right) t_0 = Q$

Solving $t_0 = t_1 + t_2$

Q.9 $P_{consumed} = \left(\dfrac{V_A}{V_R}\right)^2 \times P_R$

$= \left(\dfrac{110}{115}\right)^2 \times 500 = 457.46\ \text{W}$

So, percentage drop in power output

$= \dfrac{(500 - 457.46)}{500} \times 100 = 8.6\%$

Q.10 For a black body, wavelength for maximum intensity :

$\lambda \alpha \dfrac{1}{T}$ & $P \alpha T^4$

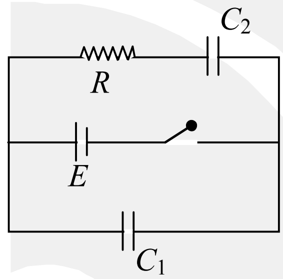
$\Rightarrow \quad P \alpha \dfrac{1}{\lambda^4} \Rightarrow \quad P' = 16\,P.$

$\therefore \quad P'\,T' = 32PT$

Q.11 $\dfrac{q^2}{4\pi\varepsilon_0 a} = k$

Q.12 Rearranging the circuit, we observed that C_1 is joined directly to the cell and acquires its full charge when S is closed. It plays no part in the charging of C_2 through R.

So, $q_2 = Q_0(1 - e^{-t/\tau})$



Q.13 $300 = e\sigma A\left(900^4 - 300^4\right)$ ...(i)

$600 = \dfrac{\sigma A}{2}\left(900^4 - 300^4\right) + \dfrac{e\sigma A}{2}\left(900^4 - 300^4\right)$ ...(ii)

$e = \dfrac{1}{3}$

Q.14 $f \propto \sqrt{g}$

In water $f_w = 0.8\, f_{air}$

$\therefore \quad \dfrac{g'}{g} = (0.8)^2 = 0.64$

or $\quad \dfrac{\rho_w}{\rho_m} = 0.36$ ...(i)

In liquid, $\quad \dfrac{g'}{g} = (0.6)^2 = 0.36$

or $\quad \dfrac{\rho_L}{\rho_m} = 0.64$

From equations (i) and (ii) $\frac{\rho_L}{\rho_w} = \frac{0.64}{0.36}$

$S_L = \rho_L/\rho_w = 1.77$

Q.15 Doppler's effect depends upon velocity of approach and separation of source and observer. hence no change in frequency received by the observer.
$\therefore$ no beat is heard.

Q.16 $f = \frac{(2n+1)v}{4(l+e)}$; $(l_1+e) = \frac{v}{4f}$; $(l_2+e) = \frac{3v}{4f}$

$\Rightarrow \frac{l_2+e}{l_1+e} = 3$

$l_2 = (3.6 - 2.34)$ m and $l_1 = (3.6 - 3.22)$

$\Rightarrow e = 0.06$ m $= 0.6$ r $\Rightarrow r = 0.1$ m

$A = 100\,\pi\ \text{cm}^2$

Q.17 $\Delta Pm = 2\ P_0 \cos kx$
(assuming closed end as origin)

At point Q, $x = L - \frac{7L}{9} = \frac{2L}{9}$

$\Delta Pm = 2\Delta P_0 \cos\left(\frac{2\pi}{\lambda} \times \frac{2L}{9}\right) = \Delta P_0$

$\therefore$ Required ratio = 1 : 2

Q.18 $T = 2\pi\sqrt{\frac{l}{g-a}}$, a is the downward acceleration of box

$T_0 = 2\pi\sqrt{\frac{l}{g}} \Rightarrow a = \frac{3g}{4}$

$Mg - R = Ma \Rightarrow R = \frac{Mg}{4}$, $v = \frac{Mg}{4k}$

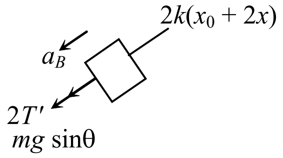
Q.19 Let elongation of spring be x_0 in equilibrium. Then,

$$2T + mg\sin\theta = 2kx_0 \qquad \text{....(i)}$$

and $\quad T = mg \qquad$ ....(ii)

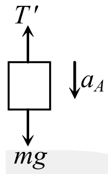
Let Block B is displaced by x down the inclination

F.B.D. of B



$$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta \text{ ...(iii)}$$

F.B.D. of A



$mg - T = ma_A$

Also, $\quad a_A = 2a_B$

$T' = mg - 2ma_B$

$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$

$-ma_B = 4kx + 4ma_B$

$a_B = -\frac{4k}{5m}x$

$\therefore \quad T = 2\pi\sqrt{\frac{5m}{4k}}$

$T = 6.28$ s.

Q.20 Current in the circuit is given by

$i = \frac{\varepsilon}{3+x}$

Power generated in 1Ω

$= \left(\frac{\varepsilon}{3+x}\right)^2 \times 1 = \frac{\varepsilon^2}{3+x}$

Power will be max when $3 + x$ is minimum
i.e., for $x = 0$

Q.21 As the block moves out of the liquid, tension increases.

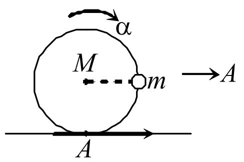
Q.22 $T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$, $T \propto R^{3/2}$

Radius of 2nd satellite is 1% greater

Hence time period is $1 \times \frac{3}{2} = 1.5$ % larger

Q.23 From perpendicular axis theorem
$I_z = I_x + I_y = 2I$

Q.24 $(A_{CM})_x = \dfrac{mA + MA}{m+M} = A$



$(A_{CM})_y = \dfrac{M \times 0 + mR\alpha}{m+M} = \dfrac{mR\alpha}{m+M}$

$f = (M+m)A$ ...(i)

$(M+m)g - N = (M+m)(A_{CM})_y$ ...(ii)

$mgR = I_A \alpha$ ...(iii)

$A = R\alpha$ ...(iv)

$\therefore$ N = 70 N

Q.25 In steady state no current flows through capacitor. The potential difference across capacitor and resistor of resistance R_2 is same.
$\therefore$ charge on capacitor

$= CV = C \times \dfrac{R_2}{r + R_2} \times 3 = 1\mu F \times \dfrac{1}{5+1} \times 3$

$= 2\mu C.$

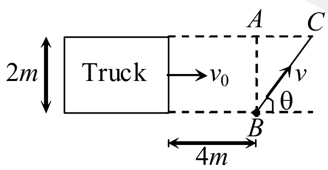
Q.26 $[\beta] = L$

$ML^2T^{-2} = \dfrac{\alpha[L]^{1/2}}{[L]}$

$\alpha = [M][L^{5/2}][T^{-2}]$

Q.27 For safe crossing, the condition is that the man must cross the road by the time the truck covers the distance 4 + AC or 4 + 2cot

$\therefore \quad \dfrac{4 + 2\cot\theta}{8} = \dfrac{2/\sin\theta}{v}$



or $v = \dfrac{8}{2\sin\theta + \cos\theta}$ ...(i)

For minimum v, $\dfrac{dv}{d\theta} = 0 \Rightarrow \tan\theta = 2$

From equation (i), $v_{min} = \dfrac{8}{\sqrt{5}} = 3.57$ m/s

Q.28 Acceleration of block m with respect to inclined plane = 6

Acceleration of inclined plane = $\dfrac{2}{\sqrt{3}}$

Q.29 Work done by friction = $-\mu mgl$
Work done by gravity = $-mgh$
So work done by force = $mgh + \mu mgl$

Q.30 $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = \dfrac{d\vec{r}}{dt} = (2t-4)\hat{i}$

$+ 2t\hat{j}$, $\vec{a} = \dfrac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a}.\vec{v} = 0$

$(2\hat{i} + 2\hat{j}).((2t-4)\hat{i} + 2t\hat{j}) = 0$

$8t - 8 = 0$

t = 1 sec.

Ans. t = 1 sec.

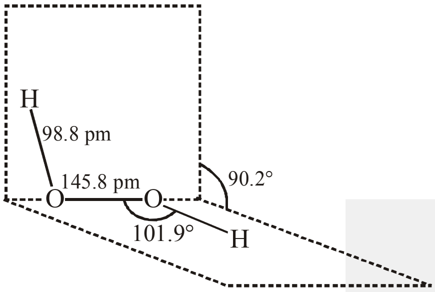
CHEMISTRY

Q.31 $IE_1 \longrightarrow 24.6$ eV
$IE_2 \longrightarrow 54.4$ eV

$He \xrightarrow[(IE_1+IE_2)]{} He^{2+}$

= 24.6 + 54.4 eV
= 79.0 eV
In kJ $\rightarrow 79.0 \times 1.6 \times 10^{-22} \times 6 \times 10^{23}$
= 7584 kJ

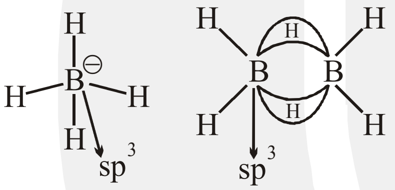
Q.33 Reaction is endothermic $\Rightarrow \Delta H$ = positive
So on increasing temperature reaction will shift forward. On decreasing volume concentration of every species will increases.

Q.34 

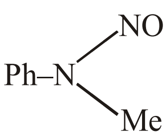
Q.35 due to H-bonding butan-1-ol has highest boiling point out of the three compounds. Butanal has higher boiling point than butane due to its higher polarity.

Q.36 $CN^- + H_2O \rightleftharpoons HCN + OH^-$
$0.1–0.1\ h \qquad\qquad 0.1\ h$
$[OH^-] = 0.1 \times 0.04 = 4 \times 10^{-3}$ M
$K_{sp} = [Al^{3+}]\,[OH^-]^3$
$6.4 \times 10^{-20} = S \times [4 \times 10^{-3}]^3$
$S = 10^{-12}$ mol L^{-1}

Q.37 $2MH + B_2H_6 \longrightarrow 2M^+ [BH_4]^-$ {M = Li or Na}



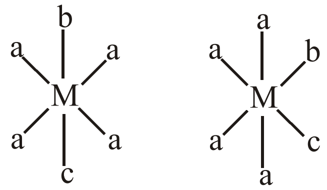
Q.38 (1) $PhNH_2 + CHCl_3 + KOH \rightarrow PhNC + 3KCl + 3H_2O$
unpleasant smell (isocyanide test)
$PhNHMe + CHCl_3 + KOH$ –ve isocyanide test as it not 1° amine.
(2) $PhNH_2 + NaNO_2 + HCl \rightarrow PhN_2^+\ Cl^-$
(Diazonium reaction)
$PhNHMe + NaNO_2 + HCl \rightarrow$



(yellow oil)

Q.39 The formation of micelle only above certain temperature called Kraft temperature suggests positive ΔS of micelle formation which even overcome effect of positive ΔH of micelle formation. Besides kinetic effect also become important at high temperature.

Q.40 Ma_4bc



G.I. = 2, O.I. =0, S.I. = 2
All S.I.are optically inactive

Q.41 Teflon is a addition polymer

Q.42 $Pt \mid H_2(P_1 atm) \mid H^+ (M_1) \mid\mid H^+ (M_2) \mid H_2 (P_2 atm) \mid Pt$

at anode $\quad \underset{P_1}{H_2} \longrightarrow \underset{(M_1)}{2H^+} + 2e^-$

at cathode $\quad \underset{(M_2)}{2H^+} \longrightarrow \underset{P_2}{H_2}$

net cell reaction $\quad 2H^+ (M_2) + H_2(P_1) \longrightarrow 2H^+ (M_1) + H_2(P_2)$

$$E_{Cell} = E^{\circ}_{Cell} - \frac{0.06}{2} \log \frac{[H^+]^2(P_2)}{(H^+)^2(P_1)}$$

$\because \quad P_1 = P_2$

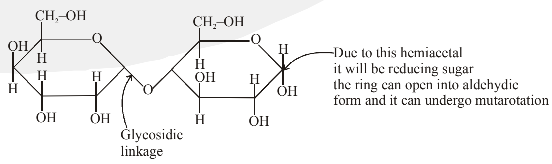
$E^{\circ}_{Cell} = 0$

$$E_{Cell} = -\frac{0.06}{2} \log \frac{[H^+]^2_{M_1}}{[H^+]^2_{M_2}}$$

$$= -\frac{0.06}{2} \log (1.5)^2$$

$= -0.0108$ V **Ans.**

Q.43 $Ni\,(s) + 4CO\,(g) \longrightarrow Ni(CO)_4(g)$
d^{10}, sp^3, tetrahedral complex

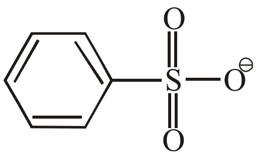
Q.44 

Q.46 Carbonates of alkali metals

(a) Covalent character $\rightarrow Li_2CO_3 > Na_2CO_3 > K_2CO_3 > Rb_2CO_3 > Cs_2CO_3$

(b) Solubility $\rightarrow Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3 < Cs_2CO_3$

(c) Thermal stability $\rightarrow Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3 < Cs_2CO_3$

Q.47 

is most stable conjugate base.

Q.48 $A(g) \rightleftharpoons B(g) + C(g) + D(g)$

$\alpha = 0.2 \quad VD = 60$

$\Rightarrow M_{obs} = 120 \qquad n = 3 - 1 = 2$

$$\alpha = \frac{M_{Th} - M_{Obs}}{M_{Obs}(n-1)}$$

$$0.2 = \frac{M_{Th} - 120}{120(3-1)}$$

$M_{Th} = (0.2 \times 240) + 120$
$= 48 + 120$
$= 168$ Ans.

Q.49 $d^5 \rightarrow WFL\ (\Delta_0 < PE)$

eg $\boxed{1\,|\,1}$

t_2g $\boxed{1\,|\,1\,|\,1}$

] Symmetrical distribution in both eg and t_2g orbitals

Q.51 $$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

For H : Z = 1

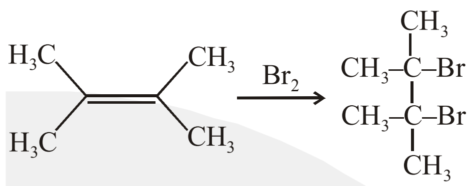
$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \quad \left(K = \frac{1}{4\pi\varepsilon_0}\right)$$

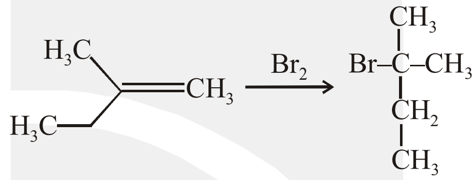
$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$v^2 = \frac{e^2}{4\pi\varepsilon_0 rm}$$

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 rm}}$$

Q.52 $$\underset{\text{(Yellow)}}{2CrO_4^{2-}} \underset{OH^-}{\overset{H^+}{\rightleftharpoons}} \underset{\text{(d}^3\text{s, tetrahedral)}}{Cr_2O_7^{2-}} + H_2O$$

Q.53 



Chiral center present

Q.54 $$\frac{d_{bcc}}{d_{fcc}} = \frac{\left(2M \times 3\sqrt{3}/(N_A \times 64r^3)\right)}{\left(4M \times 2\sqrt{2}/(N_A \times 64r^3)\right)} = 0.918$$

Q.55 Stability of halogen oxide :
$I_2O > Cl_2O > Br_2O$

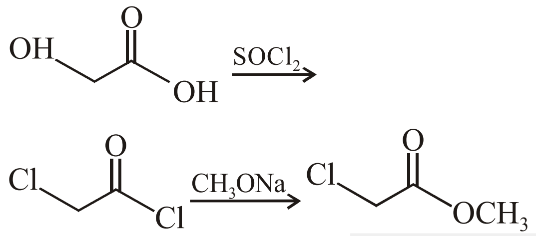
Q.57 Theory based

Q.58 (1) $Pb(NO_3)_2$ (White) $\xrightarrow[NH_4OH]{Excess} Pb(OH)_2\downarrow$ (White)

(2) $Fe(NO_3)_2$ (White) $\xrightarrow[NH_4OH]{Excess} Fe(OH)_2\downarrow$ (Green)

(3) $AgNO_3$ (White) $\xrightarrow[NH_4OH]{Excess} [Ag(NH_3)_2]^+$ (soluble complex)

(4) $Hg(NO_3)_2$ (White) $\xrightarrow[NH_4OH]{Excess} HgO.HgNH_2NO_3\downarrow$

Q.59 

Q.60 Reaction is exothermic $\therefore \Delta H = -ve$
$\Delta G = \Delta H - T\Delta S$
Since process is spontaneous
$\Delta G = -$ ve
This is possible only if magnitude of
$\Delta H > T\Delta S$

MATHEMATICS

Q.61 $\because$ $gof(x) = g(x^3 + 3) = 2x^3 + 7$
$gof(2) = 2 \cdot 8 + 7 = 23$
$\therefore$ $gof(2) = 23 \Rightarrow f^{-1}o\, g^{-1}(23) = 2$

Q.62 Required probability = 1 – all students are evaluating their own answer sheet

$$= 1 - \frac{1}{120} = \frac{119}{120}$$

Q.63 number of elements in $A \times B = 6$
$\therefore$ number of required subsets =
${}^6C_2 + {}^6C_3 + {}^4C_4$
$= 15 + 20 + 15 = 50$

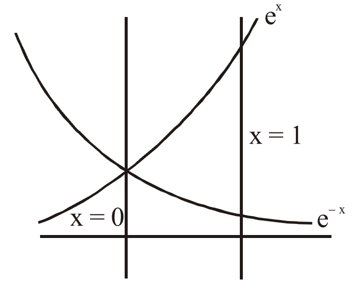
Q.64 $\because$ ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Its eccentricity $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2} = 0.5$

$\because$ eccentricity of auxiliary circle = 0
$\therefore$ ellipse will coincide with auxiliary circle in 5 seconds.

Q.65 Area $= \int_0^1 \left(e^x - e^{-x}\right)dx$

$= \left(e^x + e^{-x}\right)_0^1$



$= e + \frac{1}{e} - 2$

Q.66 given sum
$= {}^5C_0 \cdot {}^{50}C_5 - {}^5C_1 \cdot {}^{40}C_5$ ....
= coefficient of x^5 in
$({}^5C_0 \cdot (1 + x)^{50} - {}^5C_1 \cdot (1 + x)^{40}$ ....)
= coefficient of x^5 in $((1 + x)^{10} - 1)^5$
= coefficient of x^5 in
$({}^{10}C_1 \cdot x + {}^{10}C_2 \cdot x^2 + ...+ {}^{10}C_{10} \cdot x^{10})^5$
$= ({}^{10}C_1)^5 = 10^5$

Q.67 ellipse is

$$\frac{\left(\frac{3x + 4y - 1}{5}\right)^2}{10} + \frac{\left(\frac{4x - 3y + 2}{5}\right)^2}{2} = 1$$

$\therefore$ radius of auxiliary circle $= \sqrt{10}$
$\therefore$ radius of its director circle =
$\sqrt{2} \cdot \sqrt{10} = \sqrt{20}$

Q.68 The number will be divisible by 9 if sum of the digits is divisible by 9.
$\therefore$ digits should be 1, 2, 3, 5, 7 or 1, 2, 4, 5, 6

$\therefore$ Probability $= \frac{2 \times 5!}{{}^7C_5 \times 5!} = \frac{2}{21}$

Q.69 $\because$ for non trivial solution

$$D = 0 \Rightarrow \begin{vmatrix} 2 & k & 0 \\ 0 & -2 & k \\ k & 0 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(-4 - 0) - k(0 - k^2) = 0$
$\Rightarrow k^3 = 8 \Rightarrow k = 2$

Q.70 z-axis is $\vec{r} = \vec{0} + \lambda(\hat{k})$

line is $\vec{r} = (2\hat{i} + 5\hat{j} - \hat{k}) + \mu\,(3\hat{i} + 2\hat{j} - 5\hat{k})$

$\therefore$ shortest distance

$$= \left| \frac{(2\hat{i}+5\hat{j}-\hat{k})\cdot\left((3\hat{i}+2\hat{j}-5\hat{k})\times\hat{k}\right)}{\left|(3\hat{i}+2\hat{j}-5\hat{k})\times\hat{k}\right|} \right| = \frac{11}{\sqrt{13}}$$

Q.71

p	q	~ p	~ p ∧ q	q → p	~ (q → p)
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	F	T	F

$\therefore \sim p \wedge q = \sim(q \to p)$

Q.72 Given limit =

$$\lim_{x\to\infty} \frac{(x+1)^{2010} + (x+2)^{2010} + \ldots\ldots + (x+10)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} = \frac{10}{2} = 5$$

Q.73 $\because\ y'(x) = f'(e^x)\, e^{f(x)} + f(e^x)\cdot e^{f(x)}\cdot f'(x)$

$\therefore\ y'(0) = f'(1)\, e^{f(0)} + f(1)\cdot e^{f(0)}\cdot f'(0)$

$= 2\cdot 1 + 0 = 2$

Q.74 Clearly $x^2 - 7x + a$ should have $x - 4$ as a factor

$\therefore\ 16 - 28 + a = 0 \Rightarrow a = 12$ and $x^2 + 5x + b$ should have $x + 1$ as a factor

$\therefore\ +1 - 5 + b = 0 \Rightarrow b = 4$

Q.75 $\because\ C{\cdot}V{\cdot} = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow \sigma = \frac{C{\cdot}V{\cdot}\times\bar{x}}{100}$

$\therefore\ \sigma_1 = \frac{50\times 30}{100} = 15$

and $\sigma_2 = \frac{60\times 25}{100} = 15$

Q.76

given sum $= i + i + i^2 + i^6 + i^{4!} + \ldots\ldots\ldots\ldots\ldots + i^{100!}$

$i + i - 1 - 1 + \underbrace{1 + 1 + \ldots\ldots\ldots 1}_{97\text{ times}} = 95 + 2i$

Q.77 $|A|^{2^4} = (2.5)^{16} \Rightarrow |A| = \pm 10$

$\because\ |A| = x + y + z$, where $x, y, z \in N$

$\therefore\ x + y + z = 10$

$\therefore$ number of solutions $= {}^{10-1}C_{3-1}$

$= {}^9C_2 = \frac{9\times 8}{2} = 36$

Q.78 $\because\ \left|\vec{c} - \vec{a}\right| = 2\sqrt{2} \Rightarrow \left|\vec{c} - \vec{a}\right|^2 = 8$

$\Rightarrow \left|\vec{c}\right|^2 + 9 - 2\,\vec{c}\cdot\vec{a} = 8 \Rightarrow \left|\vec{c}\right|^2 - 2\left|\vec{c}\right| + 1$

$= 0 \Rightarrow \left|\vec{c}\right| = 1$

$$\Rightarrow \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$\therefore\ 10\left|(\vec{a}\times\vec{b})\times\vec{c}\right| = 10\left|\vec{a}\times\vec{b}\right|\,\left|\vec{c}\right| \sin 30°$

$= 10 \times 3 \times 1 \times \frac{1}{2} = 15$

Q.79 Differentiating, $f'(x) = \frac{1}{(f(x))^2}$

$\Rightarrow \int (f(x))^2 f'(x)\,dx = \int 1\,dx$

$\Rightarrow \frac{(f(x))^3}{3} = x + C$

putting $x = 2$,

$\frac{6}{3} = 2 + C \Rightarrow C = 0$

$\therefore\ f(x) = (3x)^{\frac{1}{3}}$

$\therefore\ f(9) = 3$

Q.80 $\because$ Using LMVT for $f(x)$ in $[1, 6]$

$f'(c) = \frac{f(6) - f(1)}{6 - 1} \geq 2$

$\Rightarrow f(6) + 2 \geq 10 \Rightarrow f(6) \geq 8$

Q.81 number of ways =

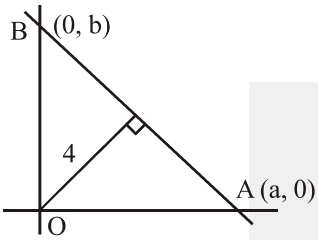
Total selections – (number of ways when exactly two consecutive) – (number of ways when all three consecutive

$= {}^{10}C_3 - 10\cdot{}^6C_1 - 10$

$= 120 - 60 - 10 = 50$

Q.82 Let equation of line be $\frac{x}{a}+\frac{y}{b}=1$

$\because$ Perpendicular distance from $(0, 0) = 4$



$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = 4 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{16}$$

$$\because \text{AM} \geq \text{HM} \Rightarrow \frac{a^2+b^2}{2} \geq \frac{2}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$\Rightarrow a^2 + b^2 \geq 64$

$\therefore$ minimum value of $OA^2 + OB^2$ is equal to 64.

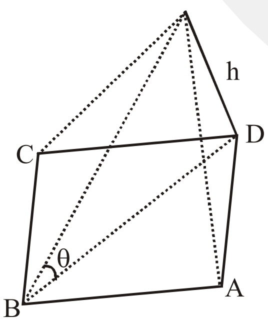
Q.83 $$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)dx + \underbrace{\int_a^{2a} f(x)dx}_{\substack{\text{Let } x = 2a-t \\ dx = -dt}}$$

$$= \int_0^{a} f(x)dx - \int_a^{0} f(2a-t)dt$$

$$= \int_0^{a} f(x)dx + \int_0^{a} f(2a-x)\,dx = 2 + 4 = 6$$

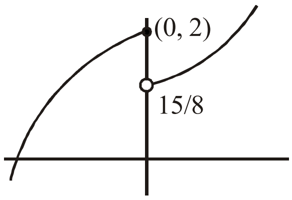
Q.84 Clearly, $AD = CD = h\cot 30° = h\sqrt{3}$

$\therefore BD = AD\sqrt{2} = h\sqrt{6}$



$\therefore \tan\theta = \frac{1}{\sqrt{6}}$

Q.85 Clearly, $x = 0$ is point of local max.



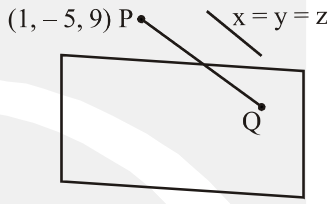
Q.86 equation of line parallel to given line through P is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$$

Its point of intersection with plane is

$Q(\lambda + 1, \lambda - 5, \lambda + 9)$

$\therefore (\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$



$\Rightarrow \lambda = -10$

$\therefore Q = (-9, -15, -1)$

$\therefore PQ = 10\sqrt{3}$

Q.87 $\because x^2 + 4y^2 = 4 \Rightarrow \frac{x^2}{4}+\frac{y^2}{1} = 1$

$\therefore x = 2\cos\theta$ and $y = \sin\theta$

$\therefore x^2 - xy = 4\cos^2\theta - 2\cos\theta \cdot \sin\theta$

$= 2(1 + \cos 2\theta) - \sin 2\theta$

$= 2 + 2\cos 2\theta - \sin 2\theta$

$\therefore$ maximum value $= 2 + \sqrt{5}$

Q.88 $\because \frac{2b^2}{a} = a \Rightarrow 2a^2(e^2 - 1) = a^2$

$\Rightarrow 2e^2 = 3 \Rightarrow e = \sqrt{\frac{3}{2}}$

Q.89 Let $w = 5(z - i) - 6$

$\Rightarrow w + 1 = 5(z - i - 1)$

$\Rightarrow |w + 1| = 5|z - i - 1| = 5$

$\therefore$ locus of w is a circle with centre $(-1, 0)$ of radius 5.

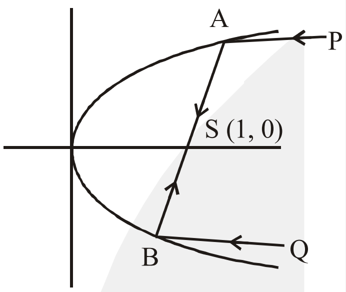
Q.90 After reflection both passes through locus

$\therefore$ AB is a focal chord

Let A be $(t^2, 2t)$

$\therefore 2t = 4 \Rightarrow t = 2$

$\therefore B = \left(\frac{1}{t^2}, \frac{-2}{t}\right)$



$\therefore$ Distance of QB from axis $= \left|\frac{-2}{t}\right| = 1$