CHAPTER 3

Motion in a Plane

1. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? **[2003]**

$[g = 10\text{m/s}^2, \sin 30^o = \frac{1}{2}, \cos 30^o = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m
(c) 2.60m (d) 8.66m

2. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

3. A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) R (b) $\frac{1}{R}$
(c) $\frac{1}{R^2}$ (d) R^2

4. Which of the following statements is **FALSE** for a particle moving in a circle with a constant angular speed ? **[2004]**

(a) The acceleration vector points to the centre of the circle
(b) The acceleration vector is tangent to the circle
(c) The velocity vector is tangent to the circle
(d) The velocity and acceleration vectors are perpendicular to each other.

5. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ ? **[2004]**

(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

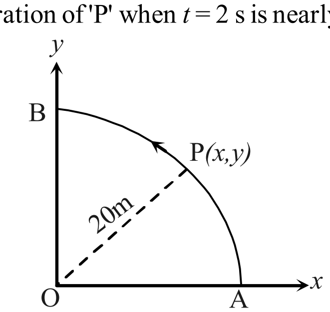
6. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is : **[2009]**

(a) $7\sqrt{2}$ units (b) 7 units
(c) 8.5 units (d) 10 units

7. A particle is moving with velocity $\vec{v} = k(y\hat{i}+x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**

(a) $y = x^2$ + constant
(b) $y^2 = x$ + constant
(c) xy = constant
(d) $y^2 = x^2$ + constant

8. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly. **[2010]**



(a) $13 m/s^2$ (b) $12 m/s^2$
(c) $7.2 ms^2$ (d) $14 m/s^2$

9. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R,θ) on the circle of radius R is (Here θ is measured from the x-axis) **[2010]**

(a) $-\frac{v^2}{R}\cos\theta\ \hat{i}+\frac{v^2}{R}\sin\theta\ \hat{j}$

(b) $-\frac{v^2}{R}\sin\theta\ \hat{i}+\frac{v^2}{R}\cos\theta\ \hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\ \hat{i}-\frac{v^2}{R}\sin\theta\ \hat{j}$

(d) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

10. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$

(c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

11. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(a) $20\sqrt{2}$ m (b) 10m

(c) $10\sqrt{2}$ m (d) 20m

12. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[2013]**

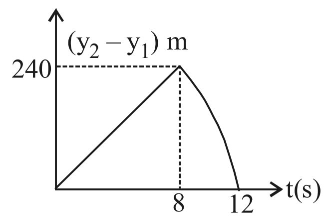
(a) $y=x-5x^2$ (b) $y=2x-5x^2$

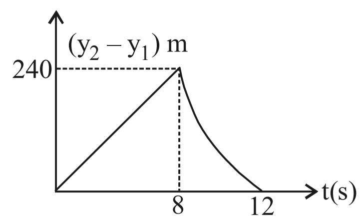
(c) $4y=2x-5x^2$ (d) $4y=2x-25x^2$

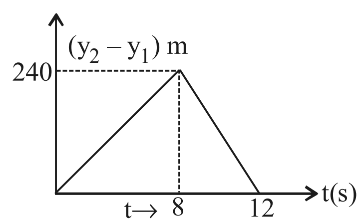
13. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ? **[2015]**

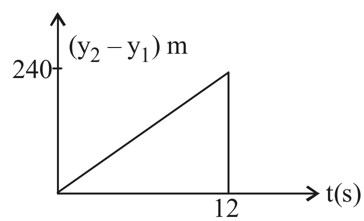
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)

(*The figures are schematic and not drawn to scale*)

(a)


(b)


(c)


(d)


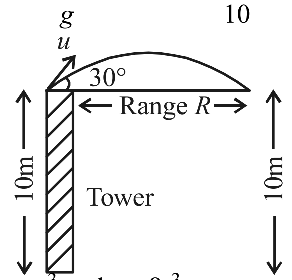
Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(b)	(a)	(b)	(c)	(a)	(a)	(d)	(c)	(a)	(d)	(b)	(b)

SOLUTIONS

1. **(d)** From the figure it is clear that range is required

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2\times 30°)}{10} = 5\sqrt{3}$$



2. **(b)** $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

3. **(a)** The angle for which the ranges are same is complementary.
Let one angle be θ, then other is $90° - \theta$

$$T_1 = \frac{2u\sin\theta}{g},\ T_2 = \frac{2u\cos\theta}{g}$$

$$T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g} = 2R$$

$$(\because\ R = \frac{u^2\sin^2\theta}{g})$$

Hence it is proportional to R.

4. **(b)** Only option (b) is false since acceleration vector is always radial (i.e., towards the center) for uniform circular motion.

5. **(c)** Yes, the person can catch the ball when horizontal velocity is equal to the horizontal component of ball's velocity, the motion of ball will be only in vertical direction with respect to person for that,

$$\frac{v_o}{2} = v_o\cos\theta \text{ or } \theta = 60°$$

6. **(a)** Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, t=10s

$$\vec{v} = \vec{u} + \vec{a}t = 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j})\times 10$$

$$= 7\hat{i} + 7\hat{j}$$

$$\therefore |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ units}$$

7. **(a)** $\vec{v} = k(y\hat{i} + x\hat{j})$
x-component of $v = ky$

$$\Rightarrow \frac{dx}{dt} = ky \quad ...(1)$$

y-component of $v = kx$

$$\Rightarrow \frac{dy}{dt} = kx \quad ...(2)$$

From (1) and (2), $\frac{dy}{dx} = \frac{x}{y}$

$\Rightarrow ydy = xdx \Rightarrow y^2 = x^2 +$ constant

8. **(d)** $s = t^3 + 5$

$$\Rightarrow \text{velocity, } v = \frac{ds}{dt} = 3t^2$$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At $t = 2s$, $at = 6 \times 2 = 12$ m/s^2

$$a_c = \frac{9\times 16}{20} = 7.2 \text{ m/s}^2$$

$\therefore$ Resultant acceleration

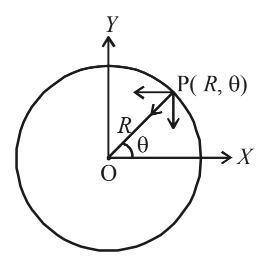
$$= \sqrt{a_t^2 + a_c^2}$$

$$= \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$$

$$= \sqrt{195.84} = 14 \text{ m/s}^2$$

9. **(c)** Clearly $\vec{a} = a_c\cos\theta(-\hat{i}) + a_c\sin\theta(-\hat{j})$

$$= \frac{-v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$



10. **(a)** Total area around fountain

$$A = \pi R_{max}^2$$

Where $R_{max} = \frac{v^2\sin 2\theta}{g} = \frac{v^2\sin 90°}{g} = \frac{v^2}{g}$

$\therefore \; A = \pi \dfrac{v^4}{g^2}$

11. **(d)** $R = \dfrac{u^2 \sin^2 \theta}{g}$, $H = \dfrac{u^2 \sin^2 \theta}{2g}$

H_{max} at $2\theta = 90°$

$H_{max} = \dfrac{u^2}{2g}$

$\dfrac{u^2}{2g} = 10 \Rightarrow u^2 = 10g \times 2$

$R = \dfrac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \dfrac{u^2}{g}$

$R_{max} = \dfrac{10 \times g \times 2}{g} = 20$ metre

12. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

$\Rightarrow x = t$ ...(i)

$y = 2t - \dfrac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

13. **(b)** $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

for $y_1 = -240$m, $t = 8$s

$\therefore$ $y_2 - y_1 = 30t$ for $t \le 8$s.

for $t > 8$s,

$y_2 - y_1 = 240 - 40t - \dfrac{1}{2}gt^2$