CHAPTER 3

Trigonometric Functions

1. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**

(a) 2 (b) 3

(c) 0 (d) 1

2. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$,

then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**

(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

3. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $2(a^2+b^2)$

4. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**

(a) 4 (b) 6

(c) 1 (d) 2

5. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

6. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**

(a) **A** is false and **B** is true

(b) both **A** and **B** are true

(c) both **A** and **B** are false

(d) **A** is true and **B** is false

7. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[2010]**

(a) $\frac{56}{33}$ (b) $\frac{19}{12}$

(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

8. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

9. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[2011RS]**

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

10. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : 1 **[2012]**

(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

11. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to : **[2013]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

(b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$

(d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

12. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : **[2013]**

(a) sinA cosA + 1
(b) secA cosecA + 1
(c) tanA + cotA
(d) secA + cosecA

13. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals **[2014]**

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

14. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[2015]**

(a) $1:\sqrt{3}$ (b) $2:3$

(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

15. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[2016]**

(a) 20 (b) 5
(c) 6 (d) 10

16. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: **[2016]**

(a) 7 (b) 9
(c) 3 (d) 5

17. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is : **[2017]**

(1) $-\frac{7}{9}$ (2) $-\frac{3}{5}$

(3) $\frac{1}{3}$ (4) $\frac{2}{9}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(a)	(a)	(c)	(b)	(a)	(d)	(d)	(b)	(a)	(b)	(b)	(c)	(b)
16	17													
(a)	(a)													

SOLUTIONS

1. **(b)** The given equation is $\tan x + \sec x = 2\cos x$;

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$;

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1.$;

$\Rightarrow x = 30°, 150°, 270°.$

2. **(d)** $\pi < \alpha - \beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0$$

$$\sin\alpha + \sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65} \qquad ....(1)$$

$$\cos\alpha + \cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65} \qquad ....(2)$$

Square and add (1) and (2)

$$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2+(27)^2}{(65)^2} = \frac{1170}{65\times 65}$$

$$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}}$$

ALTERNATE SOLUTION

Given that $\sin\alpha + \sin\beta = \frac{21}{65}$ ....(1)

$$\cos\alpha + \cos\beta = \frac{-27}{65} \qquad ....(2)$$

Squaring and adding equations (1) and (2) we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha$$

$$+\cos^2\beta + 2\cos\alpha\cos\beta = \left(\frac{-21}{65}\right)^2 + \left(\frac{-27}{65}\right)^2$$

$$\Rightarrow 2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{1170}{4225}$$

$$\Rightarrow 2[1 + \cos(\alpha-\beta)] = \frac{1170}{4425}$$

$$\Rightarrow 4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4425}$$

$$\Rightarrow \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130}$$

$$\Rightarrow \cos\frac{\alpha-\beta}{2} = \frac{-3}{\sqrt{130}}$$

Negative sign is taken because $\frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}$

3. **(a)** $$u^2 = a^2 + b^2 + 2\sqrt{(a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta + \sin^4\theta)} \qquad \ldots(1)$$

Now $(a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta+\sin^4\theta)$

$$= (a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(1 - 2\cos^2\theta\sin^2\theta)$$

$$= (a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta + a^2b^2$$

$$= (a^2-b^2)^2.\frac{\sin^2 2\theta}{4} + a^2b^2 \qquad \ldots(2)$$

$$\because 0 \le \sin^2 2\theta \le 1$$

$$\Rightarrow 0 \le (a^2-b^2)^2\frac{\sin^2 2\theta}{4} \le \frac{(a^2-b^2)^2}{4}$$

$$\Rightarrow a^2b^2 \le (a^2-b^2)^2\,\frac{\sin^2 2\theta}{4}+a^2b^2$$

$$\le (a^2-b^2)^2.\frac{1}{4}+a^2b^2 \quad ....(3)$$

$\therefore$ from (1), (2) and (3)

Minimum value of

$$u^2 = a^2+b^2+2\sqrt{a^2b^2}=(a+b)^2$$

Maximum value of u^2

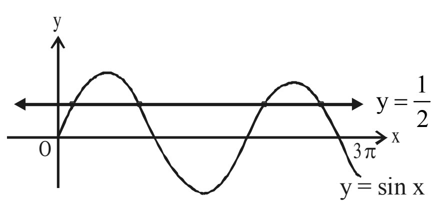
$$= a^2+b^2+2\sqrt{\left(a^2-b^2\right)^2.\frac{1}{4}+a^2b^2}$$

$$= a^2+b^2+\frac{2}{2}\sqrt{(a^2+b^2)^2} \;=2(a^2+b^2)$$

$\therefore$ Max value - Min value

$$= 2(a^2+b^2)-(a+b^2)=(a-b)^2$$

4. **(a)**



$$2\sin^2 x+5\sin x-3=0$$

$$\Rightarrow\ (\sin x+3)(2\sin x-1)=0$$

$$\Rightarrow\ \sin x=\frac{1}{2} \quad \text{and } \sin x\ne -3$$

$\therefore$ In $[0,3\pi]$, x has 4 values.

5. **(c)** $\cos x+\sin x=\frac{1}{2}\ \Rightarrow\ 1+\sin 2x=\frac{1}{4}$

$\Rightarrow\ \sin 2x=-\frac{3}{4}$, so x is obtuse and

$$\frac{2\tan x}{1+\tan^2 x}=-\frac{3}{4}$$

$$\Rightarrow\ 3\tan^2 x+8\tan x+3=0$$

$$\therefore\ \tan x=\frac{-8\pm\sqrt{64-36}}{6}=-\frac{-4\pm\sqrt{7}}{3}$$

as $\tan x<0$ $\quad\therefore\ \tan x=\frac{-4-\sqrt{7}}{3}$

6. **(b)** We have

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)$

$$=-\frac{3}{2}$$

$\Rightarrow$ $2\,[\cos(\beta-\gamma)+\cos(\gamma-\alpha)$
$+\cos(\alpha-\beta)]+3=0$

$\Rightarrow$ $2\,[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]$
$+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta$
$+\sin^2\gamma+\cos^2\alpha=0$

$\Rightarrow$ $[\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\,\sin\beta+2\sin\beta\,\sin\gamma+2\sin\gamma\,\sin\alpha]+[\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\,\cos\beta$
$+2\cos\beta\,\cos\gamma+2\cos\gamma\,\cos\alpha]=0$

$\Rightarrow$ $[\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow$ $\sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

7. **(a)** $\cos(\alpha+\beta)=\frac{4}{5}\ \Rightarrow \tan(\alpha+\beta)=\frac{3}{4}$

$$\sin(\alpha-\beta)=\frac{5}{13}\Rightarrow\tan(\alpha-\beta)=\frac{5}{12}$$

$$\tan 2\alpha=\tan\left[(\alpha+\beta)+(\alpha-\beta)\right]$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\frac{56}{33}$$

8. **(d)** $A=\sin^2 x+\cos^4 x$

$$=\sin^2 x+\cos^2 x(1-\sin^2 x)$$

$$=\sin^2 x+\cos^2 x-\frac{1}{4}(2\sin x.\cos x)^2$$

$$=1-\frac{1}{4}\sin^2(2x)$$

Now $0\le\sin^2(2x)\le 1$

$$\Rightarrow\ 0\ge-\frac{1}{4}\sin^2(2x)\ge-\frac{1}{4}$$

$$\Rightarrow 1 \geq 1 - \frac{1}{4}\sin^2(2x) \geq 1 - \frac{1}{4}$$

$$\Rightarrow 1 \geq A \geq \frac{3}{4}$$

9. **(d)** $\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$

$$\sin 4\theta(1 + 2\cos 3\theta) = 0$$

$\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi ; n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$

$[\because \theta, \in (0, \pi)]$

10. **(b)** Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$t^2 - 4t - 1 = 0$

$$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5} \quad \left(\because t = e^{\sin x}\right)$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} \text{ and } e^{\sin x} = 2 + \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$$

and $\sin x = \ln\left(2 + \sqrt{5}\right) > 1$

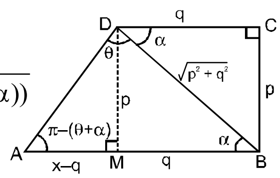
So, rejected.

Hence given equation has no solution.

$\therefore$ The equation has no real roots.

11. **(a)** From Sine Rule

$$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin(\pi - (\theta+\alpha))}$$



$$AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha} = \frac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$$

$$\left(\because \cos\alpha = \frac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha = \frac{p}{\sqrt{p^2+q^2}}\right)$$

12. **(b)** Given expression can be written as

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$\left(\because \tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{\cos A}{\sin A}\right)$$

$$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A \sin A}\right\}$$

$$= \frac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\cos A}$$

$= 1 + \sec A \operatorname{cosec} A$

13. **(b)** Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

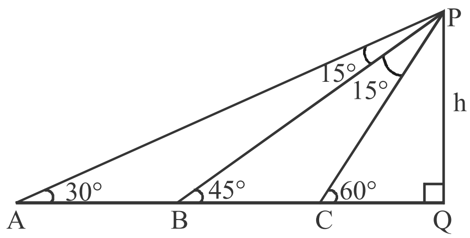
Consider

$$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4}[1 - 2\sin^2 x\cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x\cos^2 x]$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

14. (c)



$\because$ PB bisects $\angle APC$, therefore

$AB : BC = PA : PC$

Also in ΔAPQ, $\sin 30° = \dfrac{h}{PA} \Rightarrow PA = 2h$

and in ΔCPQ, $\sin 60° = \dfrac{h}{PC} \Rightarrow PC = \dfrac{2h}{\sqrt{3}}$

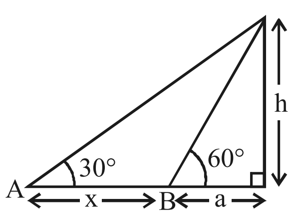
$\therefore$ $AB : BC = 2h : \dfrac{2h}{\sqrt{3}} = \sqrt{3} : 1$

15. (b) $\tan 30° = \dfrac{h}{x+a}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+a} \Rightarrow \sqrt{3}h = x + a$ ...(1)

$\tan 60° = \dfrac{h}{a} \Rightarrow \sqrt{3} = \dfrac{h}{a}$

$\Rightarrow h = \sqrt{3}a$ ...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \dfrac{10}{2}$ minutes = 5 minutes

16. (a) $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow 2 \cos 2x \cos x + 2 \cos 3x \cos x = 0$

$\Rightarrow 2\cos x\left(2\cos\dfrac{5x}{2}\cos\dfrac{x}{2}\right) = 0$

$\cos x = 0,\ \cos\dfrac{5x}{2} = 0,\ \cos\dfrac{x}{2} = 0$

$x = \pi, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}$

17. (a) We have

$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\tan^2 x - 5\cos^2 x = 4\cos^2 x - 2 + 9$

$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$

$\Rightarrow 5(\sec^2 x - 1) = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$\Rightarrow \dfrac{5}{t} - 9t - 12 = 0$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$

$\Rightarrow (3t - 1)(3t + 5) = 0$

$\Rightarrow t = \dfrac{1}{3}$ as $t \neq -\dfrac{5}{3}$.

$\cos 2x = 2\cos^2 x - 1 = 2\left(\dfrac{1}{3}\right) - 1 = -\dfrac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\dfrac{1}{3}\right)^2 - 1 = -\dfrac{7}{9}$