TARGET : JEE (Advanced) 2015

Course : VIJETA & VIJAY (ADP & ADR)   Date : 17-04-2015

TEST INFORMATION

DATE : 19.04.2015   **CUMULATIVE TEST-01 (CT-01)**

Syllabus : Function & Inverse Trigonometric Function, Limits, Continuity & Derivability, Quadratic Equation, Application of Derivatives

REVISION DPP OF SEQUENCE & SERIES AND BINOMIAL THEOREM

Total Marks : 144		**Max. Time : 110.5 min.**
Single choice Objective (–1 negative marking) Q. 1 to Q.13	**(3 marks 2.5 min.)**	**[39, 32.5]**
Multiple choice objective (–1 negative marking) Q. 14 to 34	**(4 marks, 3 min.)**	**[84, 63]**
Comprehension (–1 negative marking) Q.35 to 37	**(3 marks 2.5 min.)**	**[9, 7.5]**
Single digit integer type (no negative marking) Q. 38,39,40	**(4 marks 2.5 min.)**	**[12, 7.5]**

1. The sum $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\ldots+\frac{2008}{2006!+2007!+2008!}$ is equal to

(A) $\frac{1}{2}-\frac{1}{2006!}$ (B) $\frac{1}{2}-\frac{1}{2008!}$ (C) $\frac{1}{2006!-2008!}$ (D) $\frac{1}{2007!}-\frac{1}{2008!}$

2. If $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots\infty=\frac{\pi}{4}$, then the value of $\frac{1}{1.3}+\frac{1}{5.7}+\frac{1}{9.11}+\ldots\infty$ is

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{36}$

3. Let A,G,H are respectively the A.M., G.M. and H.M. between two positive numbers. If xA = yG = zH where x, y, z are non-zero quantities then x, y, z are in

(A) A.P. (B) G.P. (C) H.P. (D) A.G.P.

4. The sum of the coefficients of the polynomial obtained by collection of like terms after the expansion of $(1-2x+2x^2)^{743}(2+3x-4x^2)^{744}$ is

(A) 2974 (B) 1487 (C) 1 (D) 0

5. Let $\alpha_n=(2+\sqrt{3})^n$. If [.] denotes greatest integer function and $n\in N$ then $\lim\limits_{n\to\infty}(\alpha_n-[\alpha_n])$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

6. The number of natural numbers < 300 that are divisible by 6 but not by 18 is

(A) 49 (B) 37 (C) 33 (D) 16

7. If a_i, i = 1, 2, 3, 4 be four real numbers of same sign then the minimum value of $\sum\frac{a_i}{a_j}$ where i, j $\in$ {1,2 3, 4} and $i\neq j$ is

(A) 6 (B) 8 (C) 12 (D) 24

8. If $U_n=U_{n-1}+U_{n-2}$, $n\geq 3$ and $U_1=U_2=1$, then $\sum\limits_{n=2}^{\infty}\frac{U_n}{U_{n-1}\ U_{n+1}}$ is equal to

(A) 1 (B) 3 (C) 2 (D) 4

9. The value of $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)$.........to ∞ is

(A) 3 (B) 6/5 (C) 3/2 (D) 2

10. Let T_r and S_r be the r^{th} term and the sum of first 'r' terms of a series respectively. If for an odd number 'n', $S_n = n$ & $T_n = \frac{T_{n-1}}{n^2}$, then T_m (m being even) is,

(A) $\frac{2}{1+m^2}$ (B) $\frac{2m^2}{1+m^2}$ (C) $\frac{(m+1)^2}{2+(m+1)^2}$ (D) $\frac{2(m+1)^2}{1+(m+1)^2}$

11. The remainder, when $15^{23} + 23^{23}$ is divided by 38, is

(A) 4 (B) 17 (C) 23 (D) 0

12. The value of $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2$ is equal to

(A) $400 \cdot {}^{39}C_{20}$ (B) $400 \cdot {}^{40}C_{19}$ (C) $400 \cdot {}^{39}C_{19}$ (D) $400 \cdot {}^{38}C_{20}$

13. The term independent from 'x' in the expansion of $\left(1+\sqrt{x}+\frac{1}{\sqrt{x}-1}\right)^{-30}$ is

(A) ${}^{30}C_{20}$ (B) 0 (C) ${}^{30}C_{10}$ (D) ${}^{30}C_5$

14. If $a = \sum_{r=0}^{20} {}^{20}C_r$, $b = \sum_{r=0}^{9} {}^{20}C_r$, $c = \sum_{r=11}^{20} {}^{20}C_r$, then

(A) a = b + c

(B) $b = 2^{19} - \frac{1}{2}\,{}^{20}C_{10}$

(C) $c = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$

(D) $a - 2c = \frac{2^{10}(1.3.5.....19)}{10!}$

15. The age of the father of two children is twice that of the elder one added to 4 times that of the younger one. If the geometric mean of the ages of the two children is $4\sqrt{3}$ and their harmonic mean is 6, then father's age is 8p years. The value of p is contained in the set

(A) $\{4x : |x| \le 5, x \in R\}$

(B) $\{z : Im(z) = 0, z \in C\}$

(C) $\left\{\frac{12x}{x^2+1} : x = \sin\theta, \theta \in R\right\}$

(D) $\{5 + \cos\theta : 2\sin\theta < 1, \tan\theta > 0, \theta \in R\}$

16. The natural numbers are written as a sequence of digits 123456789101112 . . . , then in the sequence

(A) 190^{th} digit is 1

(B) 201^{st} digit is 3

(C) 2014^{th} digit is 8

(D) 2013^{th} digit is same as 2014^{th} digit

17. If $N = 7^{2014}$, then

(A) sum of last four digits of N is 23

(B) Number of divisors of N are 2014

(C) Number of composite divisors of N are 2013

(D) If number of prime divisors of N are p then number of ways to express a non-zero vector coplanar with two given non-collinear vectors as a linear combination of the two vectors is p + 1.

18. Consider the sequence of numbers $\alpha_0, \alpha_1, \ldots, \alpha_n$ where $\alpha_0 = 17.23$, $\alpha_1 = 33.23$ and $\alpha_{r+2} = \frac{\alpha_r + \alpha_{r+1}}{2}$. Then

(A) $|\alpha_{10} - \alpha_9| = \frac{1}{32}$

(B) $\alpha_0 - \alpha_1, \alpha_1 - \alpha_2, \alpha_2 - \alpha_3, \ldots$ are in G.P.

(C) $\alpha_0 - \alpha_2, 2(\alpha_1 - \alpha_2), \alpha_1 - \alpha_3$ are in H.P.

(D) $|\alpha_{10} - \alpha_9| = |\alpha_8 - \alpha_7|$

19. A sequence of numbers A_n where $n \in N$ is defined as :

$A_1 = \frac{1}{2}$ and for each $n \geq 2$, $A_n = \left(\frac{2n-3}{2n}\right) A_{n-1}$, then

(A) $\sum_{K=1}^{5} A_K = 1$ (B) $\sum_{K=1}^{10} A_K < 1$ (C) $A_3 = A_1A_2$ (D) $\sum_{K=1}^{n} A_K > 1 \ \forall\ n \geq 3$

20. Given 'n' arithmetic means are inserted between each of the two sets of numbers a, 2b and 2a, b where $a, b \in R$. If m^{th} mean of the two sets of numbers is same then

(A) $\frac{a}{b} = \frac{m}{n-m+1}$ (B) $\frac{a}{b} = \frac{n}{n-m+1}$ (C) $\frac{a}{b} < n$ (D) $\frac{a}{b} \leq m$

21. If a, b, c are three terms of an A.P. such that $a \neq b$ then $\frac{b-c}{a-b}$ may be equal to

(A) 0 (B) $\sqrt{3}$ (C) 1 (D) 2

22. If $S_n = \frac{1}{3!} + \frac{5}{4!} + \frac{11}{5!} + ... + \frac{n^2+n-1}{(n+2)!}$ is sum of n terms of sequence $<t_n>$ then

(A) $t_{100} = \frac{10099}{102!}$ (B) $S_{2009} = \frac{1}{2} - \frac{1}{2011(2009!)}$

(C) $S_{2009} = \frac{1}{4} - \frac{1}{2011(2009!)}$ (D) $\lim_{n\to\infty} S_n = \frac{1}{2}$

23. Consider the sequence $<a_n>$ given by $a_n = \frac{1000^n}{n!}$, $n \in N$ then correct option is/are

(A) $a_n \to \infty$ as $n \to \infty$ (B) $a_n \to 0$ as $n \to \infty$

(C) $a_n = a_{n+1}$ for exactly one value of n (D) $a_n < a_{n+1} \ \forall\ n \in N$

24. If $a_1, a_2, a_3, \ldots$, are in A.P. with common difference d and $b_K = a_K + a_{K+1} + \ldots + a_{K+n-1}$ for $K \in N$ then

(A) $\sum_{K=1}^{n} b_K = n^2 a_n$ (B) $\sum_{K=1}^{n} b_K = (n+1)^2 a_n$

(C) $b_K = \frac{n}{2}[a_n + a_1 + 2d(K-1)]$ (D) $\sum_{K=1}^{n} b_K = n(n+1)a_n$

25. If $f(n) = \sum_{i>j\geq 0} {}^{n+1}C_i\, {}^{n}C_j$ then

(A) f(2) = 16 (B) f(5) = 1001

(C) f(6) = 4096 (D) all of these

26. If $(1 + x + x^2)^n = \sum_{k=0}^{2n} a_k x^k$ then $a_r - {}^nC_1 a_{r-1} + {}^nC_2 a_{r-2} - \ldots + (-1)^r\, {}^nC_r a_0$ is equal to

($\lambda \in W$ and $0 \leq \lambda \leq n/3$)

(A) 0 if $r \neq 3\lambda$ (B) 0 if $r = 3\lambda$ (C) non-zero if $r \neq 3\lambda$ (D) non-zero if $r = 3\lambda$

27. Which of the following is true ?

(A) ${}^{26}C_0 + {}^{26}C_1 + \ldots + {}^{26}C_{13} = 2^{25} + \frac{1}{2}\ {}^{26}C_{13}$ (B) ${}^{25}C_0 + {}^{25}C_1 + \ldots + {}^{25}C_{12} = 2^{24}$

(C) ${}^{25}C_1 - {}^{25}C_2 + {}^{25}C_3 - \ldots + {}^{25}C_{25} = -1$ (D) ${}^{25}C_1 . 3^1 - {}^{25}C_2 . 3^2 + \ldots + {}^{25}C_{25} . 3^{25} = 2^{25} + 1$

28. If ${}^{100}C_6 + 4.{}^{100}C_7 + 6.{}^{100}C_8 + 4.{}^{100}C_9 + {}^{100}C_{10}$ has value ${}^xC_y$ then x + y can take value

(A) 112 (B) 114 (C) 196 (D) 198

29. $(2 - 3x + 2x^2 + 3x^3)^{20} = a_0 + a_1x + \ldots + a_{60}x^{60}$, then

(A) $\sum_{r=1}^{30} a_{2r-1} = 0$ (B) $\sum_{r=1}^{30} a_{2r} = 2^{40} - 2^{20}$ (C) $a_0 = 2$ (D) $a_{59} = 40(3^{19})$

30. Let $(1 + x^2)^2 (1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$. If $n \in N$ and a_1, a_2, a_3 are in arithmetic progression then the possible value(s) of n is/are

(A) 2 (B) 3 (C) 4 (D) 5

31. If $f(m) = \sum_{r=0}^{m} {}^{30}C_{30-r}.{}^{20}C_{m-r}$, then (if n < k then take ${}^nC_k = 0$)

(A) Maximum value of f(m) is ${}^{50}C_{25}$

(B) $f(0) + f(1) + f(2) + \ldots + f(25) = 2^{49} + \frac{1}{2}.{}^{50}C_{25}$

(C) f(33) is divisible by 37

(D) $\sum_{m=0}^{50} (f(m))^2 = {}^{100}C_{50}$

32. The value of ${}^{15}C_1 + {}^{16}C_2 + {}^{17}C_3 + \ldots + {}^{39}C_{25}$ is equal to

(A) ${}^{40}C_{15} - 1$

(B) ${}^{40}C_{24}$

(C) ${}^{25}C_1 + {}^{26}C_2 + {}^{27}C_3 + \ldots + {}^{39}C_{15}$

(D) ${}^{40}C_{25} - 1$

33. If $\left(8 + 3\sqrt{7}\right)^n$ = I + f, where 'I' is an integer, $n \in N$ and 0 < f < 1, then

(A) I is an odd integer (B) I is an even integer (C) (I + f) (1 – f) = 1 (D) (I + f) (1 – f) = 2^n

34. For natural numbers m, n, if $(1- y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + \ldots$ & $a_1 = a_2 = 10$, then

(A) m < n (B) m > n (C) m + n = 80 (D) m – n = 20

Comprehension (Q. No. 35 to 37)

Let f(n) denotes the n^{th} term of the sequence 2, 5, 10, 17, 26, ... and g(n) denotes the n^{th} term of the sequence 2, 6, 12, 20,30, ...
Let F(n) and G(n) denote respectively the sum of n terms of the above sequences.

35. $\lim_{n\to\infty} \frac{f(n)}{g(n)} =$

(A) 1 (B) 2 (C) 3 (D) does not exist

36. $\lim_{n\to\infty} \frac{F(n)}{G(n)} =$

(A) 0 (B) 1 C) 2 (D) does not exist

37. $\lim_{n\to\infty} \left(\frac{F(n)}{G(n)}\right)^n - \lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right)^n =$

(A) $\frac{\sqrt{e}-1}{e\sqrt{2}}$ (B) $\frac{\sqrt{e}+1}{e\sqrt{e}}$ (C) $\frac{1-\sqrt{e}}{e\sqrt{e}}$ (D) $\frac{e\sqrt{e}}{1+\sqrt{e}}$

38. Let S denote the sum of the series $\frac{3}{2^3} + \frac{4}{2^4.3} + \frac{5}{2^6.3} + \frac{6}{2^7.5} + \frac{7}{2^7.15} + \ldots\ldots \infty$, then the value of S^{-1} is

39. If S = 1 + $\frac{4}{3}$ + 1 + $\frac{16}{27}$ + ... ∞, then find the value of [S] (where [.] is G.I.F.)

40. The value of $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\sum_{t=0}^{r-1} \frac{1}{5^n} {}^nC_r \, {}^rC_t \, 3^t\right)$ is equal to

DPP # 3

REVISION DPP OF APPLICATION OF DERIVATIVES

1.	(C)	**2.**	(B)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(D)	**7.**	(B)
8.	(A)	**9.**	(A)	**10.**	(B)	**11.**	(D)	**12.**	(C)	**13.**	(A)	**14.**	(A)
15.	(B)	**16.**	(D)	**17.**	(C)	**18.**	(A)	**19.**	(A,D)	**20.**	(A,C,D)		
21.	(A,B,C)	**22.**	(B,D)	**23.**	(A,C,D)	**24.**	(C,D)	**25.**	(A,C)	**26.**	(B,C)		
27.	(B,C)	**28.**	(A,B)	**29.**	(C,D)	**30.**	(A,B,C,D)			**31.**	(A,B)		
32.	(A,C,D)	**33.**	(A,C,D)	**34.**	(A,B,C,D)		**35.**	(A,B)		**36.**	(A,B,C,D)		
37.	(B)	**38.**	(A)	**39.**	(D)	**40.**	5						

MATHEMATICS

1_. $S = \sum_{K=1}^{2006} \frac{K+2}{K!+(K+1)!+(K+2)!} = \sum_{K=1}^{2006} \frac{K+2}{K!(K+2)^2} = \sum_{K=1}^{2006} \frac{1}{K!(K+2)}$

$= \sum_{K=1}^{2006} \frac{K+1}{(K+2)!} = \sum_{K=1}^{2006} \frac{K+2-1}{(K+2)!} = \sum_{K=1}^{2006}\left[\frac{1}{(K+1)!} - \frac{1}{(K+2)!}\right] = \frac{1}{2} - \frac{1}{2008!}$

2_. $\frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \ldots \infty = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots\right) = \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$

3_. $xA = yG \Rightarrow \frac{x}{y} = \frac{G}{A} = \frac{2\sqrt{ab}}{a+b}$

$yG = zH \Rightarrow \frac{y}{z} = \frac{2\sqrt{ab}}{a+b} \quad \therefore \quad \frac{x}{y} = \frac{y}{z}$

4_. $(1 - 2x + 2x^2)^{743} (2 + 3x - 4x^2)^{744} = a_0 + a_1x + \ldots + a_{2974}x^{2974}$

Put $x = 1 \Rightarrow 1 = a_0 + a_1 + \ldots + a_{2974}$

5_. Let $\alpha_n = (2 + \sqrt{3})^n = I + f$ where $0 < f < 1$

Let $G = (2 - \sqrt{3})^n \Rightarrow I + f + G = 2[{}^nC_0\, 2^n + {}^nC_2 \,.\, 2^{n-2} \,.\, 3 + \ldots] \Rightarrow f + G$ is integer

But $0 < f + G < 2 \Rightarrow f + G = 1$

$\therefore \alpha_n - [\alpha_n] = f = 1 - G = 1 - (2 - \sqrt{3})^n \Rightarrow \lim_{n\to\infty} (\alpha_n - [\alpha_n]) = 1 - 0 = 1$

6_. 6, 12, 18, ... , 294 $\Rightarrow$ 49 numbers

18, 36, 54, ... , 288 $\Rightarrow$ 16 numbers $\therefore$ 49 – 16 = 33

7_. Let $E = \frac{a_1}{a_2} + \frac{a_1}{a_3} + \frac{a_1}{a_4} + \frac{a_2}{a_1} + \frac{a_2}{a_3} + \frac{a_2}{a_4} + \frac{a_3}{a_1} + \frac{a_3}{a_2} + \frac{a_3}{a_4} + \frac{a_4}{a_1} + \frac{a_4}{a_2} + \frac{a_4}{a_3}$

A.M. $\geq$ G.M. $\Rightarrow \frac{E}{12} \geq \left(\frac{a_1}{a_2}.\frac{a_1}{a_3}. \ldots\ldots\frac{a_4}{a_3}\right)^{1/12} \Rightarrow E \geq 12$

8. $\sum_{n=2}^{\infty} \frac{U_n}{U_{n-1}U_{n+1}} = \sum_{n=2}^{\infty} \frac{U_{n+1} - U_{n-1}}{U_{n-1}U_{n+1}} = \sum_{n=2}^{\infty}\left(\frac{1}{U_{n-1}} - \frac{1}{U_{n+1}}\right) = \frac{1}{U_1} + \frac{1}{U_2} = 2$

9. $\frac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\ldots\ldots \text{ n terms}}{\left(1-\frac{1}{3}\right)} = \frac{3}{2}\left[1 - \frac{1}{3^{2^n}}\right] = \frac{3}{2} \text{ as } n \to \infty$

10. $S_n - S_{n-2} = 2$ (for odd $n \geq 3$)

$\Rightarrow T_n + T_{n-1} = 2 \Rightarrow \left(\frac{1}{n^2} + 1\right) T_{n-1} = 2 \Rightarrow T_{n-1} = \frac{2n^2}{1+n^2} \Rightarrow T_m = \frac{2(m+1)^2}{1+(m+1)^2}$

11. $(19-4)^{23} + (19+4)^{23} = 2[{}^{23}C_0\ 19^{23}4^{\circ} + \ldots + {}^{23}C_{22}\ 19^{1}4^{22}]$

12. $\sum_{r=0}^{20} r(20-r).{}^{20}C_r.{}^{20}C_r = \sum_{r=0}^{19} r(20-r)^{20}C_r{}^{20}C_{20-r} = 400\sum_{r=0}^{19} {}^{19}C_{r-1}.{}^{19}C_{19-r} = 400\ .\ {}^{38}C_{18} = 400\ .\ {}^{38}C_{20}$

13. $\left(1+\sqrt{x}+\frac{1}{\sqrt{x}-1}\right)^{-30} = \left(\frac{\sqrt{x}-1}{x}\right)^{30} = \frac{\left(\sqrt{x}-1\right)^{30}}{x^{30}}$ $\Rightarrow$ these is no constant term

14_*. $b = {}^{20}C_0 + {}^{20}C_1 + \ldots. + {}^{20}C_9 = {}^{20}C_{20} + \ldots. + {}^{20}C_{11} = c$

$\Rightarrow$ $a = b + c + {}^{20}C_{10}$ $\Rightarrow$ $a = 2b + {}^{20}C_{10}$

$\Rightarrow$ $a - 2b = \frac{20!}{10!10!} = \frac{(2.4......20)(1.3.5....19)}{10!10!} = \frac{2^{10}(1.3.5.....19)}{10!}$

15_*. $z = 2x + 4y$, $xy = 48$, $\frac{2xy}{x+y} = 6$ $\Rightarrow$ $x + y = 16$ $\Rightarrow$ $x = 12, y = 4$

$\Rightarrow$ $z = 24 + 16 = 40$ $\Rightarrow$ $p = 5$

When $-1 < x < 1$ then $\frac{12x}{x^2+1} \in [-6, 6]$

$\theta \in \left(0,\frac{\pi}{6}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$ $\Rightarrow$ $\cos\theta \in (-1, 0) \cup \left(\frac{\sqrt{3}}{1},1\right)$ $\Rightarrow$ $5 + \cos\theta \in (4, 5) \cup \left(5+\frac{\sqrt{3}}{2},6\right)$

16_*. 1 2 . . . . 9 $\Rightarrow$ 9

10 11 . . . . 99 $\Rightarrow$ 180

190th digit is1 ($\therefore$ 100)

201st digit is 3 (100 101 102 103)

100 101 102 . . . . 707 $\Rightarrow$ $608 \times 3 = 1824$ $\Rightarrow$ $9 + 180 + 1824 = 2013$

so 2014th digit is 7. ($\because$ 708)

17_*. $7^{2014} = 49(1 + 2400)^{503} = 49(1207201 + 10^4\lambda) = 59152849 + 10^4K$

Divisors are $7^0, 7^1, 7^2, \ldots, 7^{2014}$

$\Rightarrow$ No. of divisors are 2015, composite divisors 2013 and prime divisors 1 $\Rightarrow$ $p = 1$

Also no of ways to express a non-zero vector coplanar with two given non-collinear vectors as a linear combination of the two vectors = 1

18_*. $2\alpha_{r+2} = \alpha_r + \alpha_{r+1}$

$\Rightarrow$ $2(\alpha_{r+2} - \alpha_{r+1}) = \alpha_r - \alpha_{r+1}$ $\Rightarrow$ $\alpha_{r+2} - \alpha_{r+1} = -\frac{1}{2}(\alpha_{r+1} - \alpha_r)$

$\Rightarrow$ $\alpha_{10} - \alpha_9 = -\frac{1}{2}(\alpha_9 - \alpha_8) = \frac{1}{4}(\alpha_8 - \alpha_7) = \ldots. = -\frac{1}{2^9}(\alpha_1 - \alpha_0) = \frac{-16}{2^9} = \frac{-1}{32}$

As $\alpha_0 - \alpha_1, \alpha_1 - \alpha_2, \alpha_2 - \alpha_3,$ are in G.P.

$\Rightarrow$ $\alpha_0 - \alpha_2, 2(\alpha_1 - \alpha_2), \alpha_1 - \alpha_3$ are in H.P.(Adding middle term to all terms)

19_*. $2K\,A_K = (2K-3)A_{K-1}$ $\Rightarrow$ $2K\,A_K - 2(K-1)A_{K-1} = -A_{K-1}$

put $K = 2, 3, 4, 5, \ldots$

$\Rightarrow$ $4A_2 - 2A_1 = -A_1$

$6A_3 - 4A_2 = -A_2$

....

$2KA_K - 2(K-1)A_{K-1} = -A_{K-1}$

$\Rightarrow$ $2KA_K - 2A_1 = -(A_1 + \ldots + A_{K-1})$ $\Rightarrow$ $A_1 + A_2 + \ldots + A_k = 1 - (2k-1)A_k$

As $(2K-1)A_K > 0$ $\Rightarrow$ $A_1 + A_2 + \ldots + A_k < 1$ where $k \geq 2$

20_*. $A_m = a + m\left(\frac{2b-a}{n+1}\right)$

$A_m' = 2a + m\left(\frac{b-2a}{n+1}\right)$

$\Rightarrow \quad a(n + 1) + m(2b - a) = 2a(n + 1) + m(b - 2a)$

$\Rightarrow \quad bm = a(n - m + 1) \quad \Rightarrow \quad \frac{a}{b} < n \quad \Rightarrow \quad m < n^2 - mn + n$

$\Rightarrow \quad m - n < n(n - m)$ which is false for n = m

$\frac{a}{b} \le m \quad \Rightarrow \quad \frac{m}{n-m+1} \le m \quad \Rightarrow \quad 0 \le m(n - n)$ which is true.

21_. $\frac{b-c}{a-b} = \frac{\left[A+(q-1)D\right]-\left[A+(r-1)D\right]}{\left[A+(p-1)D\right]-\left[A+(q-1)D\right]} = \frac{q-r}{p-q}$ Rational Number

22_*. $t_n = \frac{n^2+n-1}{(n+2)!} = \frac{(n^2+2n)-(n+1)}{(n+2)!} = \frac{n}{(n+1)!} - \frac{n+1}{(n+2)!} = \left(\frac{1}{n!} - \frac{1}{(n+1)!}\right) - \left(\frac{1}{(n+1)!} - \frac{1}{(n+2)!}\right)$

$S_n = \left(1 - \frac{1}{(n+1)!}\right) - \left(\frac{1}{2} - \frac{1}{(n+2)!}\right) = \frac{1}{2} - \frac{1}{(n+1)!} + \frac{1}{(n+2)!}$

23_*. $a_n = \frac{1000}{1}.\frac{1000}{2}......\frac{1000}{1000}.\frac{1000}{1001}.\frac{1000}{1002}......\frac{1000}{n}$, $n > 1000$

$\Rightarrow \quad a_n \to 0$ as $n \to \infty$

$a_n = a_{n+1} \quad \Rightarrow \quad \frac{1000^n}{n!} = \frac{1000^{n+1}}{(n+1)!} \quad \Rightarrow \quad n + 1 = 1000 \quad \Rightarrow \quad n = 999.$

24_*. $b_K = \frac{n}{2}[a_K + a_{K+n-1}] = \frac{n}{2}[a_1 + (K - 1)d + a_1 + (n + K - 2)d]$

$= \frac{n}{2}[2a_1 + (K - 1)d + (n - 1)d + (K - 1)d] = \frac{n}{2}[a_n + a_1 + 2(K - 1)d]$

$\sum_{K=1}^{n} b_K = \frac{n}{2}\left[na_n + na_1 + 2d\frac{n(n-1)}{2}\right] = \frac{n^2}{2}\left[a_n + a_1 + d(n-1)\right] = n^2 a_n$

25_*. $f(n) = {}^nC_0({}^{n+1}C_1 + {}^{n+1}C_2 + \ldots + {}^{n+1}C_{n+1})$

$+ {}^nC_1({}^{n+1}C_2 + {}^{n+1}C_3 + \ldots\ldots + {}^{n+1}C_{n+1})$

+ ---

$+ {}^nC_n\ {}^{n+1}C_{n+1}$

$= ({}^nC_0\ {}^{n+1}C_1 + {}^nC_1\ {}^{n+1}C_2 + \ldots\ldots + {}^nC_n\ {}^{n+1}C_{n+1})$

$+ ({}^nC_0\ {}^{n+1}C_2 + {}^nC_1\ {}^{n+1}C_3 + \ldots\ldots + {}^nC_{n-1}\ {}^{n+1}C_{n+1})$

+ --

$= {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \ldots\ldots + {}^{2n+1}C_0$

$= 2^{2n}$

26_*. $(1 - x)^n (1 + x + x^2)^n = ({}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \ldots\ldots)(a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n})$

$\Rightarrow \quad (1 - x^3)^n = ({}^nC_0\ a_r - {}^nC_1\ a_{r-1} + \ldots\ldots)\ x^r$ + other terms

Required expression = coefficient of x^r in ${}^nC_0 - {}^nC_1\ x^3 + {}^nC_2x^6 - \ldots\ldots = 0$ if $r \ne 3\lambda$ and $(-1)^{r/3}\ {}^nC_{r/3}$ if $r = 3\lambda$

27_. $2({}^{26}C_0 + {}^{26}C_1 + \ldots\ldots + {}^{26}C_{13}) = ({}^{26}C_0 + \ldots\ldots + {}^{26}C_{26}) + {}^{26}C_{13} = 2^{26} + {}^{26}C_{13}$

$(1 - 3)^{25} = {}^{25}C_0 - {}^{25}C_1\ 3^1 + \ldots - {}^{25}C_{25}\ 3^{25}$

28_*. ${}^{100}C_6 + {}^{100}C_7 + 3({}^{100}C_7 + {}^{100}C_8) + 3({}^{100}C_8 + {}^{100}C_9) + {}^{100}C_9 + {}^{100}C_{10} = {}^{101}C_7 + 3({}^{101}C_8) + 3({}^{101}C_9) + {}^{101}C_{10}$

$= {}^{101}C_7 + {}^{101}C_8 + 2({}^{101}C_8 + {}^{101}C_9) + {}^{101}C_9 + {}^{101}C_{10} = {}^{102}C_8 + 2.{}^{102}C_9 + {}^{102}C_{10}$

$= {}^{103}C_9 + {}^{103}C_{10} = {}^{104}C_{10} \quad \Rightarrow \quad {}^xC_y = {}^{104}C_{10}$ or ${}^{104}C_{94}$

29^. Put x = 1 & –1 and add $4^{20} + 4^{20} = 2(a_0 + a_2 + \ldots + a_{60})$

Now subtract $\Rightarrow \quad 0 = 2(a_1 + a_3 + \ldots + a_{59})$

$a_0 = 2^{20}$ and a_{59} = coeff of x^{59} in $(2 - 3x + 2x^2 + 3x^3)^{20} = {}^{20}C_1.2.3^{19}$

30. $a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots\ldots\ldots = (1 + 2x^2 + x^4)(1 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots\ldots.)$

$= 1 + {}^nC_1 x + (2 + {}^nC_2)x^2 + (2\,{}^nC_1 + {}^nC_3)x^3 + \ldots\ldots\ldots..$

Now $2a_2 = a_1 + a_3$

for n = 2 we have $a_1 = 2, a_2 = 3, a_3 = 4$ which are in A.P.

for $n \geq 3$ we have $2({}^nC_2 + 2) = {}^nC_1 + ({}^nC_3 + 2\,{}^nC_1) \Rightarrow n^3 - 9n^2 + 26n - 24 = 0 \Rightarrow n = 2, 3, 4 \Rightarrow n = 3, 4$

31. $f(m) = \sum_{r=0}^{m} {}^{30}C_{30-r}\ {}^{20}C_{m-r} = \sum_{r=0}^{m} {}^{30}C_r\ {}^{20}C_{m-r} \Rightarrow f(m) = {}^{50}C_m$

$f(33) = {}^{50}C_{33} = {}^{50}C_{17} = \dfrac{34.35.36\ldots\ldots..50}{17!}$ which is multiple of 37

32. ${}^{15}C_1 + {}^{16}C_2 + {}^{17}C_3 + \ldots\ldots + {}^{39}C_{25} = {}^{15}C_0 + {}^{15}C_1 + {}^{16}C_2 + {}^{17}C_3 + \ldots\ldots + {}^{39}C_{25} - {}^{15}C_0 = {}^{40}C_{25} - 1$

33. $\left(8 + 3\sqrt{7}\right)^n = I + f$; $\left(8 - 3\sqrt{7}\right)^n = f'$

Adding $I + f + f' = 2$ (integer) $\Rightarrow$ $f + f' =$ integer $\Rightarrow$ $f + f' = 1$

34. $(1 - y)^m (1 + y)^n = 1 + (n - m)y + \left\{\dfrac{m(m-1)}{2} + n\left(\dfrac{n-1}{2}\right) - mn\right\}y^2\ldots\ldots.$

Also $a_1 = a_2 = 10 \Rightarrow n - m = 10$ & $a_2 = \dfrac{m^2 + n^2 - m - n - 2mn}{2} = 10$

$\Rightarrow (m - n)^2 - (m + n) = 20 \Rightarrow m + n = 80 \Rightarrow m = 35 \Rightarrow n = 45$

Sol. (35 to 37)

$f(n) = n^2 + 1,\ g(n) = n^2 + n \Rightarrow \lim\limits_{n\to\infty} \dfrac{f(n)}{g(n)} = 1$

$F(n) = \sum(n^2 + 1) = \dfrac{n}{6}(2n^2 + 3n + 7)$

$G(n) = \sum(n^2 + n) = \dfrac{n(n+1)(n+2)}{3} \Rightarrow \lim\limits_{n\to\infty} \dfrac{F(n)}{G(n)} = 1$

$$\lim_{n\to\infty}\left(\frac{F(n)}{G(n)}\right)^n - \lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right)^n = \lim_{n\to\infty}\left(\frac{2n^3 + 3n^2 + 7n}{6} \times \frac{3}{n^3 + 3n^2 + 2n}\right)^n - \lim_{n\to\infty}\left(\frac{n^2 + 1}{n^2 + n}\right)^n$$

$$= e^{\lim\limits_{n\to\infty}\frac{(-3n^2+3n)n}{n(2n^2+6n+4)}} - e^{\lim\limits_{n\to\infty}\left(\frac{n^2+1-n^2-n}{n^2+n}\right)n} = e^{-3/2} - e^{-1}$$

38. $S = \sum_{r=1}^{\infty} \dfrac{r+2}{2^{r+1}\ .\ r\ (r+1)} = \sum\left(\dfrac{2}{r} - \dfrac{1}{r+1}\right)\dfrac{1}{2^{r+1}} = \sum\left(\dfrac{1}{r.2^r} - \dfrac{1}{(r+1)2^{r+1}}\right) = 1/2$

39. $S = 1 + \dfrac{4}{3} + \dfrac{9}{9} + \dfrac{16}{27} + \ldots\ldots\infty$

$\dfrac{1}{3}S = \dfrac{1}{3} + \dfrac{4}{9} + \dfrac{9}{27} + \ldots\ldots\infty \Rightarrow \dfrac{2}{3}S = 1 + \dfrac{3}{3} + \dfrac{5}{9} + \dfrac{7}{27} + \ldots\ldots\ldots\infty \Rightarrow \dfrac{2}{3}S = 3$

40. $E = \lim\limits_{n\to\infty} \sum_{r=1}^{n}\left(\sum_{t=0}^{r-1} \dfrac{1}{5^n}{}^nC_r\ {}^rC_t 3^t\right) = \lim\limits_{n\to\infty} \sum_{r=1}^{n} \dfrac{{}^nC_r}{5^n}(4^r - 3^r) = \lim\limits_{n\to\infty}\left(\dfrac{(5^n - 1)}{5^n} - \dfrac{(4^n - 1)}{5^n}\right) = 1 - 0 = 1.$