TARGET : JEE (Advanced) 2015

Course : VIJETA & VIJAY (ADP & ADR)   Date : 24-04-2015

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS
DPP NO. 06

TEST INFORMATION

DATE : 26.04.2015   **CUMULATIVE TEST-02 (CT-02)**

Syllabus : Function & Inverse Trigonometric Function, Limits, Continuity & Derivability, Quadratic Equation, Application of Derivatives, Straight Line, Circle

REVISION DPP OF SOLUTION OF TRIANGLE AND MATRICES & DETERMINANT

Total Marks : 147		**Max. Time : 116 min.**
Single choice Objective (–1 negative marking) Q.1 to 11	**(3 marks 2.5 min.)**	**[33, 27.5]**
Multiple choice objective (–1 negative marking) Q. 12 to 32	**(4 marks, 3 min.)**	**[84, 63]**
Comprehension (–1 negative marking) Q.33 to 38	**(3 marks 2.5 min.)**	**[18, 15]**
Single digit Type (no negative marking) Q. 39	**(4 marks 2.5 min.)**	**[4, 2.5]**
Match the Following (no negative marking) Q.40	**(8 marks, 8 min.)**	**[8, 8]**

1. If A, B and C are the angles of a non-right angled triangle ABC, then the value of

$$\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$$ is equal to

(A) 1   (B) 2   (C) –1   (D) –2

2. The number of 2 × 2 matrices X satisfying the matrix equation $X^2 = I$ (I is 2 × 2 unit matrix) is
(A) 1   (B) 2   (C) 3   (D) infinite

3. If the equation sin x + cos(k + x) + cos(k – x) = 2 has real solutions, then the complete set of values of k is ($n \in I$)

(A) $\left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right]$ (B) $\left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{\pi}{6}\right]$

(C) $\left[2n\pi, 2n\pi + \frac{\pi}{6}\right] \cup \left[2n\pi + \frac{11\pi}{6}, 2n\pi + \pi\right]$ (D) None of these

4. In ΔABC, $\angle ABC = 120°$, AB = 3cm and BC = 4cm. If perpendicular constructed to AB at A and to BC at C meet at D, then CD =

(A) 3   (B) $\frac{8\sqrt{3}}{3}$ (C) 5   (D) $\frac{10\sqrt{3}}{3}$

5. In a triangle ABC, if $2015c^2 = a^2 + b^2$ and cot C = N(cot A + cot B), then the number of distinct prime factor of N is
(A) 0   (B) 1   (C) 2   (D) 4

6. If A is a square matrix and B is singular matrix of same order, then for any positive integer n, $(A^{-1}BA)^n$ equals
(A) $A^{-n} B^n A^n$ (B) $A^n B^n A^{-n}$ (C) $A^{-1} B^n A$ (D) $n(A^{-1} B A)$

7. The number of right angle triangles of integer side lengths whose product of leg lengths is equal to three times the perimeter is
(A) 0   (B) 1   (C) 2   (D) 3

8. The internal bisector of $\angle A$ of triangle ABC meets sides BC at point P and b = 2c. If $9AP^2 + 2a^2 = k.c^2$, then k is equal to
(A) 8   (B) 3   (C) 19   (D) 18

9. If $\begin{vmatrix} {}^{x}C_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} = \lambda \begin{vmatrix} {}^{x}C_r & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_r & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_r & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix}$, then '$\lambda$' is equal to

(A) 1 (B) 2 (C) 3 (D) 4

10. Number of solution(s) of the equation, $\tan 2x = \cot x$ in $0 \le x \le 2\pi$, is
(A) 3 (B) 5 (C) 6 (D) 8

11. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3, 4 and 5 units. Then area of the triangle is equal to:

(A) $\frac{9\sqrt{3}\,(1+\sqrt{3})}{\pi^2}$ (B) $\frac{9\sqrt{3}\,(\sqrt{3}-1)}{\pi^2}$ (C) $\frac{9\sqrt{3}\,(1+\sqrt{3})}{2\pi^2}$ (D) $\frac{9\sqrt{3}\,(\sqrt{3}-1)}{2\pi^2}$

12. Consider the system of equations in x, y, z as
$x \sin 3\theta - y + z = 0$
$x \cos 2\theta + 4y + 3z = 0$
$2x + 7y + 7z = 0.$
Given system has a non-trivial solution, if $\theta \in$

(A) $\pi\left(n + \frac{(-1)^n}{3}\right)$, $n \in Z$ (B) $\pi\left(n + \frac{(-1)^n}{4}\right)$, $n \in Z$ (C) $\pi\left(n + \frac{(-1)^n}{6}\right)$, $n \in Z$ (D) $n\pi$, $n \in Z$

13. If $a^2 + b^2 + c^2 + ab + bc + ca \le 0 \ \forall \ a, b, c \in R$, then value of the determinant

$\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$ is divisible by

(A) 5 (B) a + b + c (C) $a^2 + b^2 + c^2$ (D) 13

14. If there are three square matrix A, B, C of same order satisfying the equation $A^2 = A^{-1}$ and let $B = A^{2^n}$ & $C = A^{2^{(n-2)}}$ then which of the following statements are true? (where $n \in N$)
(A) $|B - C| = 0$ (B) $(B + C)(B - C) = 0$ (C) $|B - C| = 1$ (D) None of these

15. $\tan |x| = |\tan x|$ if $x \in$

(A) $(\pi k - \pi/2, \pi k]$ where $k \in I - N$ (B) $[\pi k, \pi k + \pi/2)$ where $k \in W$

(C) $(\pi k - \pi/2, \pi k]$ where $k \in I^-$ (D) $[\pi k, \pi k + \pi/2)$ where $k \in I$

16. Let ΔABC be such that $\angle BAC = \frac{2\pi}{3}$ and AB.AC = 1, then the possible length of the angle bisector AD is
(A) 2 (B) 1 (C) 1/2 (D) 1/3

17. If in a triangle whose circumcentre is origin, $a \le \sin A$, then for any point (a, b) lying inside the circumcircle of ΔABC,

(A) $|ab| < 1/8$ (B) $1/8 < |ab| < 1/2$ (C) $|ab| > 1/2$ (D) $|a + b| < \frac{1}{\sqrt{2}}$

18. In a triangle ABC, If D is mid point of side BC and AD is perpendicular to AC, then the value of cosA.cosC is

(A) $\frac{2b^2}{ac}$ (B) $\frac{2\left(a^2-c^2\right)}{3bc}$ (C) $-\frac{2b^2}{ac}$ (D) $\frac{2(c^2-a^2)}{3ac}$

19. Let $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$. If AB is a scalar multiple of B, then

(A) 4a + 7b + 5 = 0 (B) a+ b + 2 = 0 (C) b – a = 4 (D) a + 3b = 0

20. Values of 'α' for which system of equations $x + y + z = 1$, $x + 2y + 4z = \alpha$ and $x + 4y + 10z = \alpha^2$ is consistent, are

(A) 1 (B) 3 (C) 2 (D) 0

21. Consider a matrix $M = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & K \end{bmatrix}$ and the following statements:

Statement **(S_1)** : Inverse of M exists.
Statement **(S_2)** : $K \neq 0$,
Which of the following in respect of the above matrix and statements is/are incorrect?

(A) S_1 implies S_2, but S_2 does not imply S_1. (B) S_2 implies S_1, but S_1 does not imply S_2.
(C) Neither S_1 implies S_2 nor S_2 implies S_1. (D) S_1 implies S_2 as well as S_2 implies S_1.

22. The product of all the values of t, for which the system of equations $(a - t)x + by + cz = 0$, $bx + (c - t)y + az = 0$, $cx + ay + (b - t)z = 0$ has non-trivial solution, is

(A) $\begin{vmatrix} a & -c & -b \\ -c & b & -a \\ -b & -a & c \end{vmatrix}$ (B) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (C) $\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$ (D) $\begin{vmatrix} a & a+b & b+c \\ b & b+c & c+a \\ c & c+a & a+b \end{vmatrix}$

23. Let A and B are square matrices of same order satisfying AB = A and BA = B, then $(A^{2015} + B^{2015})^{2016}$ is equal to

(A) $2^{2015}(A^3 + B^3)$ (B) $2^{2016}(A^2 + B^2)$ (C) $2^{2016}(A^3 + B^3)$ (D) $2^{2015}(A + B)$

24. If p, q, r are in A.P. then value of determinant $\begin{vmatrix} a^2 + 2^{n+1} + 2p & b^2 + 2^{n+2} + 3q & c^2 + p \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix}$ is

(A) 0 (B) Independent from a, b, c
(C) $a^2b^2c^2 - 2^n$ (D) Independent from n

25. If $\begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 - 6x + 9 & 14x - 6 & 21 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then which of the following are correct ?

(A) a = 0 (B) b = 0 (C) c = 0 (D) d = 0

26. If $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, then which of the following are correct ?

(A) $\Delta^2 = \begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ac - b^2 \end{vmatrix}$ (B) $\Delta^2 = \begin{vmatrix} a^2 & c^2 & 2ac - b^2 \\ 2ab - c^2 & b^2 & a^2 \\ b^2 & 2bc - a^2 & c^2 \end{vmatrix}$

(C) $\Delta = 0 \Rightarrow a + b + c = 0$ (D) $a + b + c = 0 \Rightarrow \Delta = 0$

27. If the equations $x + ay - z = 0$, $2x - y + az = 0$, $ax + y + 2z = 0$ have non-trivial solution, then a =

(A) 2 (B) –2 (C) $1 + \sqrt{3}$ (D) $1 - \sqrt{3}$

28. If 'A' is a square matrix of odd order such that $A^2 + A + 2I = 0$, then which of the following is/are true?

(A) A is non-singular (B) A is singular
(C) A cannot be skew symmetric (D) $A^{-1} = -\frac{1}{2}(A + I)$

29. If the elements of a 2 × 2 matrix A are positive and distinct such that $|A + A^T|^T = 0$, then

(A) $|A| \le 0$ (B) $|A| > 0$ (C) $|A - A^T| > 0$ (D) $|AA^T| > 0$

30. If M = {A : A is a 3 × 3 matrix whose entries are –1 and 1}, then

(A) |A| lies from –6 to 6 (B) $|A| \in \{-4, 0, 4\}$
(C) $n(M) = 2^9$ (D) $n(M) = 3^9$

31. Let matrix A = $\begin{bmatrix} 7 & a & b & 1 \\ c & \alpha & \beta & d \\ 3 & e & f & 10 \end{bmatrix}$. All the unknown numbers are distinct integers from the set {2, 4, 5, 6, 8, 9} such that sum of entries of 1st row, 3rd row, 1st column and 4th column are equal to k, then

(A) a + b + c = k + 1 (B) k = 18 (C) ef = d (D) c + d = k – 2

32. A solution of the system of equations x – y = 1/3 and $\cos^2\pi x - \sin^2\pi y = 1/2$ is given by

(A) $\left(\frac{7}{6},\frac{5}{6}\right)$ (B) $\left(\frac{8}{15},\frac{1}{6}\right)$ (C) $\left(\frac{-5}{6},\frac{-7}{6}\right)$ (D) $\left(\frac{1}{6},-\frac{1}{6}\right)$

Comprehension (Q. No. 33 to 35)

The triangle ABC is inscribed in a circle of unit radius. If A : B : C = 1 : 2 : 4, then

33. cos2A + cos2B + cos2C =

(A) 1/2 (B) –1 (C) – 1/2 (D) – 1/3

34. $a^2 + b^2 + c^2$ =

(A) 7/2 (B) 7 (C) 14 (D) 15/2

35. The area of ΔABC is

(A) 7 (B) $\sqrt{7}$ (C) $\frac{\sqrt{7}}{2}$ (D) $\frac{\sqrt{7}}{4}$

Comprehension (Q. No. 36 to 38)

A square matrix A is said to be orthogonal if $A^TA = I = AA^T$.

36. Let A = $\begin{bmatrix} 29 & -28 \\ 30 & -29 \end{bmatrix}$ and P is a orthogonal matrix of order 2. if $Q = P^TAP$, then $PQ^{2015}P^T$ =

(A) 2015 A (B) A^{2016} (C) I (D) A

37. P is an orthogonal matrix of order 3 and α, β, γ are direction angles of a straight line.

Let A = $\begin{bmatrix} \sin^2\alpha & \sin\alpha\sin\beta & \sin\alpha\sin\gamma \\ \sin\alpha\sin\beta & \sin^2\beta & \sin\beta\sin\gamma \\ \sin\alpha\sin\gamma & \sin\beta\sin\gamma & \sin^2\gamma \end{bmatrix}$ and $Q = P^TAP$. If $PQ^6P^T = 2^kA$, then k =

(A) 5 (B) 7 (C) 6 (D) 0

38. A = $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal matrix, then 36|abc| =

(A) 4 (B) 6 (C) 9 (D) 1

39. If f(x) = $\begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\beta) & \sin(\alpha-\gamma) \end{vmatrix}$ and f(0) = $\frac{1}{4}$, then $\left[\sum_{r=1}^{15} f(x_r)\right]$ is (where [.] is G.I.F.)

40. Consider a square matrix A of order 2 whose four distinct elements are 0,1,2 and 4. Let N denote the number of such matrices.

	Column–I	Column–II	
(A)	Possible non-negative value of |A| is	(P)	2
(B)	Sum of values of determinants corresponding to all such N matrices is	(Q)	4
(C)	If absolute value of |A| is least, then possible value of | adj(adj(adj A)) | is	(R)	–2
(D)	If |A| is algebraically least, then possible value of $\|4A^{-1}\|$ is	(S)	0

ANSWER KEY

REVISION DPP OF STRAIGHT LINE AND CIRCLE

1.	(C)	2.	(C)	3.	(C)	4.	(B)	5.	(A)	6.	(D)	7.	(A,C)
8.	(B)	9.	(D)	10.	(A)	11.	(D)	12.	(B,D)	13.	(A,C)	14.	(A,C,D)
15.	(A,B)	16.	(B,C)	17.	(B,C,D)	18.	(A,B,C,D)	19.	(A,B,C)	20.	(A,D)	21.	(A,B,C,D)
22.	(A,C)	23.	(A,C,D)	24.	(B,C,D)	25.	(A,D)	26.	(B,C)	27.	(A,D)	28.	(A,B)
29.	(B,C,D)	30.	(A,B)	31.	(B,D)	32.	(B,C,D)	33.	(A,C,D)	34.	(A,C,D)	35.	(C,D)
36.	(B,C,D)	37.	(B,D)	38.	(A,B)	39.	(B)	40.	(C)				

MATHEMATICS

1. Δ = tanA (tanB. tanC – 1) – 1 (tanC – 1) + 1 (1 – tanB)
= tanA. tanB. tanC – tanA – tanB – tanC + 2 = 2 (as Π tanA = $\Sigma \tan A$)

2. $X^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a^2 + bc = 1$ ....(1) $b(a + d) = 0$ ....(2)
$c(a + d) = 0$ ....(3) $bc + d^2 = 1$ ....(4)

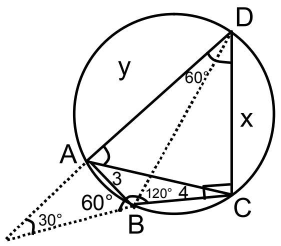
case-I $a + d \neq 0$
$\Rightarrow$ b = 0 and c = 0 $\Rightarrow$ a = ±1 and d = ±1
$\Rightarrow$ (a, d) = (1, 1), (–1, –1) $\Rightarrow$ X = I, –I

case-II a + d = 0
$\Rightarrow$ $a^2 + bc = 1$ $\Rightarrow$ infinite matrices

3. sin x + cos(k + x) + cos(k – x) = 2 $\Rightarrow$ sin x + 2 cos k . cos x = 2

$\therefore$ $2 \le \sqrt{1+4\cos^2 k}$ $\Rightarrow$ $\cos^2 k \ge \frac{3}{4}$ $\Rightarrow$ $k \in \left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right]$

4.



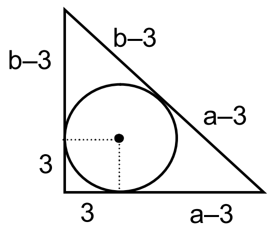
PB = $\frac{3}{\sin 30^\circ}$ = 6 $\therefore$ In ΔPCD $\Rightarrow$ x = 10 tan30° = $\left(\frac{10\sqrt{3}}{3}\right)$

5. cotC = N(cotA + cotB) $\Rightarrow$ $\frac{\cos C}{\sin C} = N\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}\right)$

$\Rightarrow$ $\frac{a^2+b^2-c^2}{4\Delta} = N\left(\frac{b^2+c^2-a^2}{4\Delta} + \frac{a^2+c^2-b^2}{4\Delta}\right)$ $\Rightarrow$ N = 1007 = 19 × 53

6. Consider n = 2
$\therefore$ $(A^{-1}BA) = (A^{-1}BA).(A^{-1}BA) = A^{-1}B^2A$

7.



$\therefore$ ab = 6s $\Rightarrow$ 2Δ = 6s $\Rightarrow$ r = 3
Now, $a^2 + b^2 = (a + b – 6)^2$ $\Rightarrow$ (a – 6) (b – 6) = 18

8. $AP = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{4}{3}\, c.\cos\frac{A}{2}$

Now, $9AP^2 + 2a^2 = 16c^2\cos^2\frac{A}{2} + 2a^2 = 16c^2.\ \frac{S(S-a)}{bc} + 2a^2 = 8.\left(\frac{a+3c}{2}\right)\left(\frac{3c-a}{2}\right) + 2a^2 = 18c^2$

9. R.H.S. = $\begin{vmatrix} {}^{x}C_r & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_r & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_r & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix}$

Apply $C_3 \to C_3 + C_2$

$$\begin{vmatrix} {}^{x}C_r & {}^{x}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^{y}C_r & {}^{y}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^{z}C_r & {}^{z}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

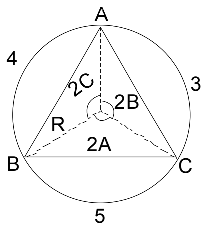
Apply $C_2 \to C_2 + C_1$

$$\begin{vmatrix} {}^{x}C_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^{y}C_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^{z}C_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

Apply $C_3 \to C_3 + C_2$

$$\begin{vmatrix} {}^{x}C_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

10. $\tan^2 x = \frac{1}{3}$ $\Rightarrow$ $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ $\Rightarrow$ 6 solutions

11. 

angle = $\frac{\text{arc}}{\text{radius}}$ ....(1)

$\because$ $4 + 5 + 3 = 2\pi R$ $\Rightarrow$ $R = 6/\pi$ $\because$ $2A = \frac{5}{R} = \frac{5\pi}{6}$,

$2B = \frac{3}{R} = \frac{\pi}{2}$ and $2C = \frac{4}{R} = \frac{2\pi}{3}$

Area of Δ ABC = $\frac{1}{2}R^2\left[\sin\frac{2\pi}{3} + \sin\frac{5\pi}{6} + \sin\frac{\pi}{2}\right]$

$= \frac{R^2}{2}\left[\frac{\sqrt{3}}{2} + \frac{1}{2} + 1\right] = \frac{R^2}{2}\left[\frac{\sqrt{3}+3}{2}\right] = \frac{\sqrt{3}(\sqrt{3}+1)}{4} \times \frac{36}{\pi^2} = \frac{9\sqrt{3}(\sqrt{3}+1)}{\pi^2}$

12. $\Delta = 0$

$\Rightarrow \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0 \Rightarrow \sin\theta = \frac{1}{2}, 0$

13. We have $a^2 + b^2 + c^2 + ab + bc + ca \le 0 \Rightarrow (a+b)^2 + (b+c)^2 + (c+a)^2 \le 0$

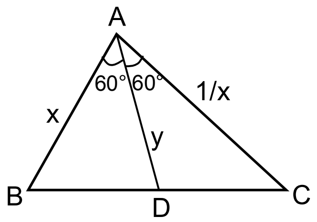
$\therefore$ $a + b = 0, b + c = 0, c + a = 0 \Rightarrow a = b = c = 0 \Rightarrow \Delta = \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$

14. $B = A^{2^n} = A^{2 \cdot 2^{n-1}} = (A^2)^{2^{n-1}} = (A^{-1})^{2^{n-1}} = \left(A^{2^{n-1}}\right)^{-1} = \left(A^{2 \cdot 2^{n-2}}\right)^{-1} = \left((A^2)^{2^{n-2}}\right)^{-1}$

$= \left((A^{-1})^{-1}\right)^{2^{n-2}} = A^{2^{(n-2)}} = C \Rightarrow B - C = 0$

15. R.H.S. ≥ 0 for all x, the given condition is true for those values of |x| which lie in the I or III quadrant and the values of x given by B and D satisfy these conditions.

16.



$AD = y = \dfrac{2bc}{b+c}\cos\dfrac{A}{2} \Rightarrow y = \dfrac{1}{x + \dfrac{1}{x}} \Rightarrow y_{max.} = \dfrac{1}{2}$

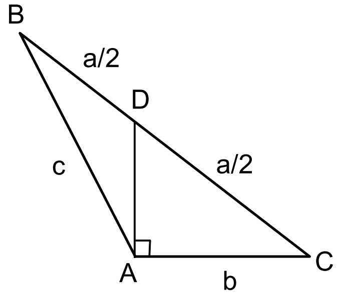
17. $\dfrac{a}{2\sin A} = R \le \dfrac{1}{2} \Rightarrow a^2 + b^2 < \dfrac{1}{4}$ $\therefore$ By A.M. $\ge$ G.M.

$\Rightarrow \dfrac{a^2+b^2}{2} \ge |ab| \Rightarrow |ab| < \dfrac{1}{8}$

now, $\dfrac{a^2+b^2}{2} \ge \left(\dfrac{a+b}{2}\right)^2$

$(a + b)^2 \le 2(a^2 + b^2) < \dfrac{1}{2}$

18.



From ΔACD

$\cos C = \dfrac{2b}{a} \Rightarrow \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{2b}{a} \Rightarrow 3b^2 = a^2 - c^2$

Now $\cos A \cdot \cos C = \dfrac{b^2+c^2-a^2}{2bc}\cdot\dfrac{2b}{a} = \dfrac{b^2+c^2-a^2}{ac} = \dfrac{2(c^2-a^2)}{3ac}$

19. $AB = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} -3a-7b-5 \\ 2a+4b+3 \\ a+2b+2 \end{bmatrix} = \lambda\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$\Rightarrow \begin{cases} (3+\lambda)a + 7b + 5 = 0 \\ 2a + (4-\lambda)b + 3 = 0 \\ a + 2b + 2 - \lambda = 0 \end{cases} \therefore \begin{vmatrix} 3+\lambda & 7 & 5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = 1 \Rightarrow a = -3$ & $b = 1$

20. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0 \quad \Rightarrow \quad \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2 & 4 \\ \alpha^2 & 4 & 10 \end{vmatrix} = 2(\alpha^2 - 3\alpha + 2) = 0$

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 4 \\ 1 & \alpha^2 & 10 \end{vmatrix} = 3(\alpha^2 - 3\alpha + 2) = 0 \Rightarrow \quad \Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 4 & \alpha^2 \end{vmatrix} = \alpha^2 - 3\alpha + 2 = 0 \quad \therefore$

$\Rightarrow \quad \alpha = 1, 2$

21. $|M| = \begin{vmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & K \end{vmatrix} = -5K$

22. $\begin{vmatrix} a-t & b & c \\ b & c-t & a \\ c & a & b-t \end{vmatrix} = -t^3 + \alpha t^2 + \beta t + \gamma = 0$

product of roots $= \gamma = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

23. $AB = A$ & $BA = B$

$\Rightarrow \quad AB.A = A^2$ & $BA.B = B^2$

$\Rightarrow \quad A.BA = A^2$ & $B.AB = B^2$

$\Rightarrow \quad AB = A^2$ & $BA = B^2$

$\Rightarrow \quad A = A^2$ & $B = B^2$

$\therefore \quad A^n = A$ & $B^n = B$

Now, $(A^{2015} + B^{2015})^2 = (A + B)^2 = A^2 + B^2 + AB + BA = 2(A + B)$

$(A + B)^3 = 2(A + B)^2 = 4(A + B)$

$(A + B)^4 = 4(A + B)^2 = 8(A + B) \qquad \therefore \qquad (A + B)^n = 2^{n-1}(A + B)$

24. $\Delta = \begin{vmatrix} a^2 + 2^{n+1} + 2p & b^2 + 2^{n+2} + 3q & c^2 + p \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3)$

$\Delta = \begin{vmatrix} 0 & 0 & p + r - 2q \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix} = 0$

25. $f'(x) = \begin{vmatrix} 2x-5 & 2x-5 & 3 \\ 6x+1 & 6x+1 & 9 \\ 14x-6 & 14x-6 & 21 \end{vmatrix} + \begin{vmatrix} x^2 - 5x + 3 & 2 & 3 \\ 3x^2 + x + 4 & 6 & 9 \\ 7x^2 - 6x + 9 & 14 & 21 \end{vmatrix} = 0$

$\therefore \quad f(x)$ is a constant polynomial & $f(0) \neq 0 \quad \Rightarrow \quad d \neq 0$

26. (A) Replace each element by its cofactor.

(B) $\Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} . \begin{vmatrix} a & b & c \\ -c & -a & -b \\ b & c & a \end{vmatrix} = \begin{vmatrix} a^2 & c^2 & 2ac - b^2 \\ 2ab - c^2 & b^2 & a^2 \\ b^2 & 2bc - a^2 & c^2 \end{vmatrix}$

27. $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0 \quad \Rightarrow \quad (a + 2)(a^2 - 2a - 2) = 0$

28. $A^2 + A + 2I = 0 \Rightarrow \quad A(A + I) = -2I \quad \Rightarrow \quad |A|\ |A + I| = (-2)^n \neq 0 \quad \Rightarrow \quad |A| \neq 0$

29. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A + A^T| = \begin{vmatrix} 2a & b+c \\ b+c & 2d \end{vmatrix} = 4ad - (b + c)^2 = 0$

$\Rightarrow \quad \dfrac{b+c}{2} = \sqrt{ad}$

$\because \quad \dfrac{b+c}{2} > \sqrt{bc}$

$\Rightarrow \quad \sqrt{ad} > \sqrt{bc} \quad \Rightarrow \quad ad > bc$

$\Rightarrow \quad ad - bc > 0 \quad \Rightarrow \quad |A| > 0$

$|A - A^T| = \begin{vmatrix} 0 & b-c \\ c-b & 0 \end{vmatrix} = (b - c)^2 > 0$

30. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$|A| = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_1 b_3 c_2 - a_2 b_1 c_3 - a_3 b_2 c_1$

$\Rightarrow \quad \det(A) = P_1 + P_2 + P_3 - P_4 - P_5 - P_6 \quad$ where $|P_i| = 1$

$\therefore \quad |\det(A)| \leq |P_1| + |P_2| + |P_3| + |P_4| + |P_5| + |P_6|$

$\Rightarrow \quad |\det(A)| \leq 6$

Hence option (A) is correct.

Now, applying $C_1 \to C_1 + C_2$ & $C_2 \to C_2 + C_3$, we get elements of 1st and 2nd column as even number

$\therefore \quad |A| =$ multiple of 4

Hence option (B) is correct.

31. $8 + a + b = 13 + e + f = 10 + c = 11 + d = k$

$\Rightarrow \quad c = 9, d = 8, (a, b) = (5, 6)$ or $(6, 5)$, $(e, f) = (2, 4)$ or $(4, 2)$

32. $\cos^2\pi x - \sin^2(\pi x - \pi/3) = \dfrac{1}{2}$

$\Rightarrow \quad \cos^2\pi x - \left(\sin \pi x.\dfrac{1}{2} - \cos \pi x.\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{1}{2}$

$\Rightarrow \quad \cos^2\pi x - \left(\sin^2 \pi x.\dfrac{1}{4} + \cos^2 \pi x.\dfrac{3}{4} - \dfrac{\sqrt{3}}{4}\sin 2\pi x\right) = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{4}(\cos^2\pi x - \sin^2\pi x) + \dfrac{\sqrt{3}}{4}\sin 2\pi x = \dfrac{1}{2}$

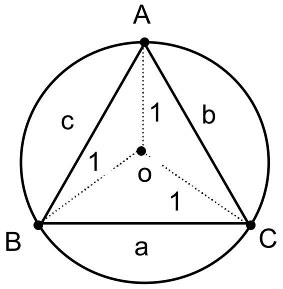
$\Rightarrow \quad \dfrac{1}{2}\cos 2\pi x + \dfrac{\sqrt{3}}{2}\sin 2\pi x = 1$

$\Rightarrow \quad \cos\left(2\pi x - \dfrac{\pi}{3}\right) = 1 \quad \Rightarrow \quad 2\pi x - \dfrac{\pi}{3} = 2n\pi$

$\Rightarrow \quad x = n + \dfrac{1}{6}\ ; N \in I$

Sol. **(33 to 35)**

$A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$



(33) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$

$= -1 - 4\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -1 - 4\frac{\sin\left(\frac{8\pi}{7}\right)}{8\sin\frac{\pi}{7}} = -\frac{1}{2}$

(34) $\cos 2A + \cos 2B + \cos 2C = -\frac{1}{2}$

$\Rightarrow \quad \frac{1+1-a^2}{2.1.1} + \frac{1+1-b^2}{2.1.1} + \frac{1+1-c^2}{2.1.1} = -\frac{1}{2} \quad \Rightarrow \quad a^2 + b^2 + c^2 = 7$

(35) $\Delta = \frac{1}{2}(\sin 2A + \sin 2B + \sin 2C) = 2\sin A \sin B \sin C$

$= 2.\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{4\pi}{7} = 2\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$

$= 2.\sqrt{\sin^2\frac{\pi}{7}.\sin^2\frac{2\pi}{7}\sin^2\frac{3\pi}{7}} = 2.\sqrt{\frac{7}{2^{7-1}}} = \frac{\sqrt{7}}{4}$ square units

Sol. **(36)** $Q^2 = P'AP.P'AP = P'A^2P$

$\Rightarrow \quad Q^{2015} = P'A^{2015}P$

$\therefore \quad PQ^{2015}P' = PP'A^{2015}PP' = A^{2015} = A^{2014}.A = (A^2)^{1007}.A = (I)^{1007}.A = A$

(37) $PQ^6P' = A^6$

Now, $A^2 = 2A$

$\Rightarrow \quad A^3 = 2A.A = 4A$

$\Rightarrow \quad A^6 = 16A^2 = 32A = 2^5A$

(38) $AA^T = I$

$\Rightarrow \quad 2a^2 = 1, 6b^2 = 1, 3c^2 = 1$

$\Rightarrow \quad 36a^2b^2c^2 = 1$

$\Rightarrow \quad 6|abc| = 1$

39. $f'(x) = 0 \quad \Rightarrow \quad$ f(x) is a constant function $\quad \therefore \quad f(x) = \frac{1}{4}$

40. Here 24 matrices are possible.

Values of determinants can be –8, – 4, – 2, 2, 4, 8

(A) Possible non-negative values of |A| are 2, 4, 8

(B) Sum of these 24 determinants is 0

(C) Mod. (det(A)) is least $\quad \therefore |A| = \pm 2 \quad \Rightarrow |\text{adj}(\text{adj}(\text{adj}(A)))| = |A|^{(n-1)^3} = \pm 2$

(D) Least value of det.(A) is –8 $\quad$ Now $|4A^{-1}| = 16\frac{1}{|A|} = \frac{16}{-8} = -2$