JEE MAIN SOLVED PAPER–2019

9 APRIL 2019 (MORNING SHIFT)

PHYSICS

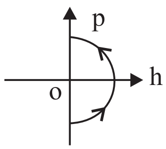
1. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is:

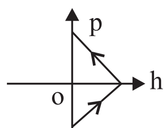
(1) $0.01\ kg/m^3$ (2) $0.10\ kg/m^3$
(3) $0.31\ kg/m^3$ (4) $0.07\ kg/m^3$

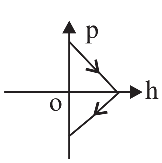
2. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

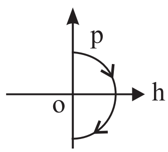
(1) 90° (2) 150° (3) 120° (4) 60°

3. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is:

(1) 

(2) 

(3) 

(4) 

4. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

(1) $\frac{MgL}{2n^2}$ (2) $\frac{MgL}{}$

(3) $\frac{2MgL}{n^2}$ (4) nMgL

5. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.0 kg (2) 1.5 kg
(3) 1.8 kg (4) 1.2 kg

6. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is

(1) $\frac{k}{4I}\theta$ (2) $\frac{k}{I}\theta$ (3) $\frac{k}{2I}\theta$ (4) $\frac{2k}{I}\theta$

7. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 4 : 3 : 2 (2) 10 : 15 : 7
(3) 14 : 15 : 20 (4) 2 : 3 : 4

8. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be:

(1) $\frac{2GM}{9a^2}$ (2) $\frac{GM}{9a^2}$

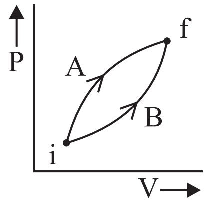
(3) $\frac{GM}{3a^2}$ (4) $\frac{2GM}{3a^2}$

9. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) M (2) $\frac{M}{2}$ (3) 4 M (4) 2 M

10. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and

ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(1) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$

(2) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$

(3) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$

(4) $\Delta Q_A = \Delta Q_B; \Delta U_A = \Delta U_B$

11. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(1) 100 m/s (2) $80\sqrt{5}$ m/s

(3) $100\sqrt{5}$ m/s (4) 80 m/s

12. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be:

(1) $\frac{m\bar{v}^2}{6k_B}$ (2) $\frac{m\bar{v}^2}{3k_B}$

(3) $\frac{m\bar{v}^2}{7k_B}$ (4) $\frac{m\bar{v}^2}{5k_B}$

13. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\frac{1}{10}}$ (2) $2T\sqrt{\frac{1}{14}}$

(3) $4T\sqrt{\frac{1}{15}}$ (4) $4T\sqrt{\frac{1}{14}}$

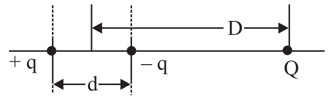
14. The pressure wave, $P = 0.01 \sin[1000t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 4°C (2) 11°C (3) 12°C (4) 15°C

15. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is: (All quantities are in SI units.)

(1) 20 m (2) 80 m (3) 40 m (4) 60 m

16. A system of three charges are placed as shown in the figure:



If D >> d, the potential energy of the system is best given by

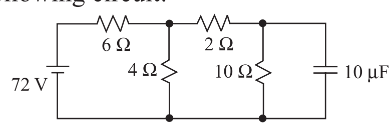
(1) $\frac{1}{4\pi\in_0}\left[\frac{-q^2}{d} - \frac{qQd}{2D^2}\right]$

(2) $\frac{1}{4\pi\in_0}\left[\frac{-q^2}{d} + \frac{2qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\in_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(4) $\frac{1}{4\pi\in_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

17. Determine the charge on the capacitor in the following circuit:



(1) 60 μC (2) 2 μC

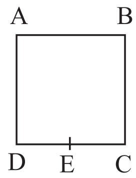
(3) 10 μC (4) 200 μC

18. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(1) 6.25×10^{-6} J (2) 3.75×10^{-6} J

(3) 2.16×10^{-6} J (4) 2.55×10^{-6} J

19. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is: (E is mid-point of arm CD)



(1) R (2) $\frac{7}{64}R$

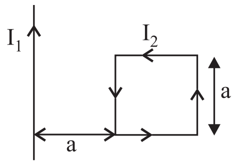
(3) $\frac{3}{4}R$ (4) $\frac{1}{16}R$

20. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is:

(1) 0.38 Nm (2) 0.55 Nm

(3) 0.42 Nm (4) 0.27 Nm

21. A rigid square of loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be:



(1) Repulsive and equal to $\frac{\mu_o I_1 I_2}{2\pi}$

(2) Attractive and equal to $\frac{\mu_o I_1 I_2}{3\pi}$

(3) Repulsive and equal to $\frac{\mu_o I_1 I_2}{4\pi}$

(4) Zero

22. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 k Ω resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

(1) 40 V (2) 15 V (3) 20 V (4) 10 V

23. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to:

(1) L (2) L^2 (3) $1/L^2$ (4) 1/L

24. The magnetic field of a plane electromagnetic wave is given by:

$\vec{B} = B_0 \hat{i}[\cos(kz - \omega t)] + B_1 \hat{j}\cos(kz + \omega t)$

Where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.

The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to:

(1) 0.6 N (2) 0.1 N

(3) 0.9 N (4) 3×10^{-2} N

25. A signal Acosωt is transmitted using $v_0 \sin \omega$ modulated (AM) signal is:

(1) $v_0 \sin\omega_0 \ t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}(\omega_0 + \omega)\ t$

(2) $v_0 \sin[\omega_0(1 + 0.01 \ A\sin\omega t)t]$

(3) $v_0 \sin\omega_0 t + A\cos\omega t$

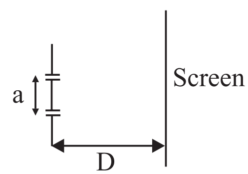
(4) $(v_0 + A)\cos\omega t \sin\omega_0 t$

26. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

(1) 0.24 m (2) 1.60 m

(3) 0.32 m (4) 0.16 m

27. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be:



(1) $\frac{2nD\lambda}{a(\mu-1)}$ (2) $\frac{nD\lambda}{a(\mu-1)}$

(3) $\frac{D\lambda}{a(\mu-1)}$ (4) $\frac{2D\lambda}{a(\mu-1)}$

28. The electric field of light wave is given as

$\vec{E} = 10^3 \cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14} t\right)\hat{x}\frac{N}{C}$

This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is:

Given, E (in eV) = $\dfrac{12375}{\lambda(\text{in Å})}$

(1) 2.0 V (2) 0.72 V
(3) 0.48 V (4) 2.48 V

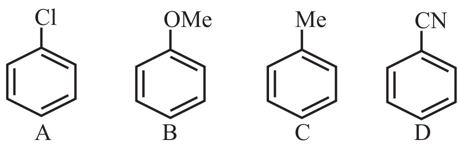
29. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2^{nd} Balmer line (n = 4 to n = 2) will be;
(1) 889.2 nm (2) 488.9 nm
(3) 642.7 nm (4) 388.9 nm

30. An NPN transistor is used in common emitter configuration as an amplifier with 1 k Ω load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are:
(1) 0.33 kΩ, 1.5 (2) 0.67 kΩ, 300
(3) 0.67 kΩ, 200 (4) 0.33 kΩ, 300

CHEMISTRY

31. The element having greatest difference between its first and second ionization energies, is:
(1) Ca (2) Sc
(3) Ba (4) K

32. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is:



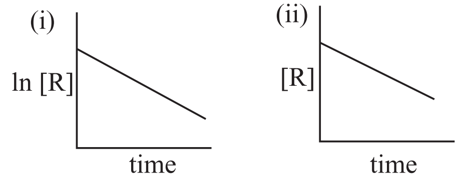
(1) D < A < C < B (2) B < C < A < D
(3) A < B < C < D (4) D < B < A < C

33. Consider the van der Waals constants, a and b, for the following gases,

Gas	Ar	Ne	Kr	Xe
a/ (atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/ (10^{-2} dm^3 mol^{-1})	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?
(1) Kr (2) Ne (3) Xe (4) Ar

34. The given plots represents the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are:



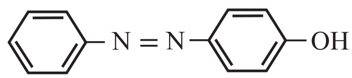
(1) 1, 0 (2) 1, 1
(3) 0, 1 (4) 0, 2

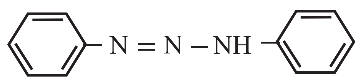
35. Among the following, the set of parameters that represents path functions, is:
(A) q + w (B) q
(C) w (D) H – TS
(1) (B) and (C)
(2) (B), (C) and (D)
(3) (A) and (D)
(4) (A), (B) and (C)

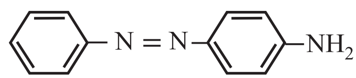
36. The ore that contains the metal in the form of fluoride is:
(1) cryolite (2) malachite
(3) magnetite (4) sphalerite

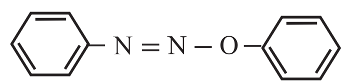
37. Excessive release of CO_2 into the atmosphere results in:
(1) global warming
(2) polar vortex
(3) formation of smog
(4) depletion of ozone

38. Aniline dissolved in dilute HCl is reacted with sodium nitrate at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is:

(1) 

(2) 

(3) 

(4) 

39. Among the following, the molecule expected to be stabilized by anion formation is:
C_2, O_2, NO, F_2
(1) C_2 (2) F_2 (3) NO (4) O_2

40. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is:

(1) $NO_2 < NO < N_2O_3 < N_2O$
(2) $NO_2 < N_2O_3 < NO < N_2O$
(3) $N_2O < N_2O_3 < NO < NO_2$
(4) $N_2O < NO < N_2O_3 < NO_2$

41. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:
(x_M = Mole fraction of 'M' in solution;
x_N = Mole fraction of 'N' in solution;
y_M = Mole fraction of 'M' in vapour phase;
y_N = Mole fraction of 'N' in vapour phase)

(1) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$

(2) $(x_M - y_M) < (x_N - y_N)$

(3) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$

(4) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

42. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is:

(1) 4×10^{-2} (2) 6×10^{-2}
(3) 4×10^{-4} (4) 16×10^{-4}

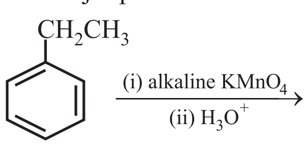
43. The number of water molecules(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is:

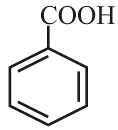
(1) 2 (2) 3 (3) 1 (4) 4

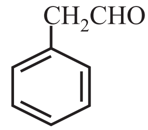
44. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is:
$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$,
$E^\circ = 2$ V at 298 K
(Faraday's constant, F = 96000 C mol^{-1})

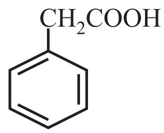
(1) – 384 (2) 384 (3) 192 (4) – 192

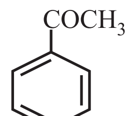
45. The major product of the following reaction is:



(1) 

(2) 

(3) 

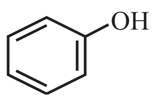
(4) 

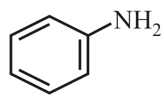
46. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman} / \Delta\bar{v}_{Balmer}$ is :

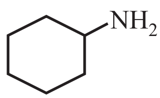
(1) 4 : 1 (2) 9 : 4
(3) 5 : 4 (4) 27 : 5

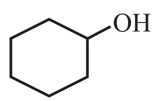
47. The organic compound that gives following qualitative analysis is:

	Test	Inference
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2/water	Decolourization

(1) 

(2) 

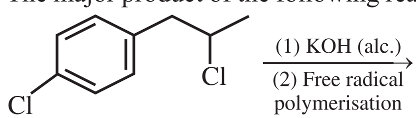
(3) 

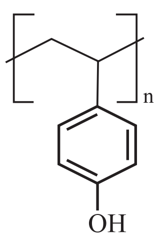
(4) 

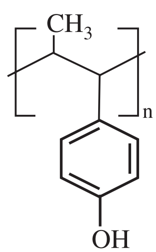
48. C_{60}, an allotrope of carbon cantains:

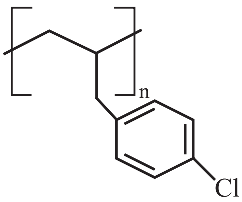
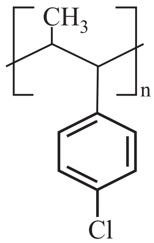
(1) 12 hexagons and 20 pentagons.
(2) 18 hexagons and 14 pentagons.
(3) 16 hexagons and 16 pentagons.
(4) 20 hexagons and 12 pentagons.

49. The major product of the following reaction is:

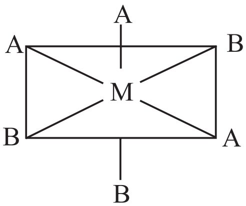


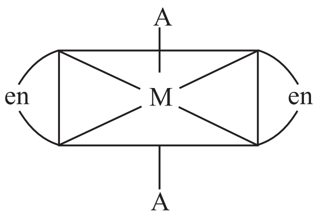
(1) 

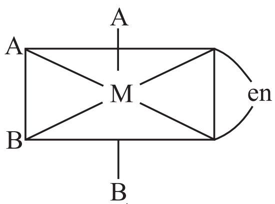
(2) 

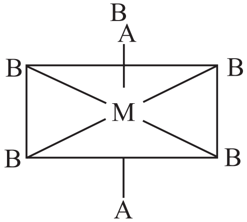
(3)  (4) 

50. The one that will show optical activity is:
(en = ethane 1, 2-diamine)

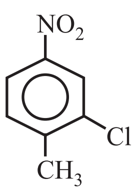
(1) 

(2) 

(3) 

(4) 

51. The correct IUPAC name of the following compound is:



(1) 5-chloro-4-methyl-1-nitrobenzene
(2) 2-chloro-1-methyl-4-nitrobenzene
(3) 3-chloro-4-methyl-1-nitrobenzene
(4) 2-methyl-5-nitro-1-chlorobenzene

52. Match the catalysts **(Column I)** with products **(Column II)**.

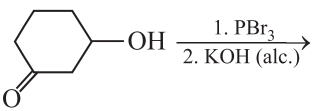
Column I		**Column II**
Catalyst		**Product**
(A) V_2O_5	(i)	Polyethylene
(B) $TiCl_4$/ $Al(Me)_3$	(ii)	ethanol
(C) $PdCl_2$	(iii)	H_2SO_4
(D) Iron Oxide	(iv)	NH_3

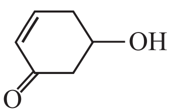
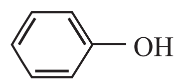
(1) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)
(2) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)
(3) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)
(4) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

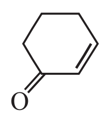
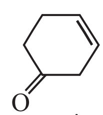
53. Which of the following statements is not true about sucrose?
(1) It is a non reducing sugar.
(2) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose.
(3) It is also named as invert sugar.
(4) On hydrolysis, it produces glucose and fructose.

54. Magnesium powder burns in air to give:
(1) $Mg(NO_3)_2$ and Mg_3N_2
(2) MgO and Mg_3N_2
(3) MgO only
(4) MgO and $Mg(NO_3)_2$

55. The major product of the following reaction is:



(1)  (2) 

(3)  (4) 

56. The major product of the following reaction is:
$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1equiv.)}}$

(1) $CH_3CD(I)CHD(Cl)$
(2) $CH_3CD(Cl)CHD(I)$
(3) $CH_3CD_2CH(Cl)(I)$
(4) $CH_3C(I)(Cl)CHD_2$

57. The major product of the following reaction is:

$CH_3CH = CHCO_2CH_3 \xrightarrow{LiAlH_4}$

(1) $CH_3CH_2CH_2CO_2CH_3$
(2) $CH_3CH = CHCH_2OH$
(3) $CH_3CH_2CH_2CH_2OH$
(4) $CH_3CH_2CH_2CHO$

58. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are:

(1) d_{xz} and d_{yz}
(2) d_{yz} and dz^2
(3) d_{z^2} and d_{xz}
(4) $d_{x^2-y^2}$ and d_{xy}

59. The aerosol is a kind of colloid in which:

(1) solid is dispersed in gas
(2) gas is dispersed in solid
(3) gas is dispersed in liquid
(4) liquid is dispersed in water

60. For a reaction, $N_2(g) + 3H_2(g) \rightarrow 2\ NH_3(g)$; identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures.

(1) 56 g of N_2 + 10 g of H_2
(2) 35 g of N_2 + 8 g of H_2
(3) 28 g of N_2 + 6 g of H_2
(4) 14 g of N_2 + 4 g of H_2

MATHEMATICS

61. Slope of a line passing through P(2, 3) and intersecting the line x + y = 7 at a distance of 4 units from P, is:

(1) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$
(3) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (4) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

62. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to:

(1) $2\sqrt{6}$ (2) $2\sqrt{\frac{10}{3}}$
(3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

63. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set

$S = \{x \in R : f(x) = f(0)\}$

contains exactly:

(1) four irrational numbers.
(2) four rational numbers.
(3) two irrational and two rational numbers.
(4) two irrational and one rational number.

64. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to:

(1) $-3\tan^{-1/3} x + C$ (2) $-\frac{3}{4}\tan^{-4/3} x + C$
(3) $-3\cot^{-1/3} x + C$ (4) $3\tan^{-1/3} x + C$

(Here C is a constant of integration)

65. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is:

(1) $\frac{25}{192}$ (2) $\frac{7}{32}$
(3) $\frac{1}{192}$ (4) $\frac{25}{32}$

66. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is:

(1) $\sqrt{5}/2$ (2) $2\sqrt{5}$
(3) 9/2 (4) 7/2

67. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, –x + y + 4 = 0, then which one of the following points lies on the curve?

(1) (–2, 1) (2) (–2, 2)
(3) (2, –1) (4) (2, –2)

68. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is:

(1) $\frac{\pi-2}{8}$ (2) $\frac{\pi-1}{4}$
(3) $\frac{\pi-2}{4}$ (4) $\frac{\pi-1}{2}$

69. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(1) $\frac{3}{4} + \cos 20°$ (2) 3/4

(3) $\frac{3}{2}(1 + \cos 20°)$ (4) 3/2

70. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{\sqrt{15}}{2}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{3}{\sqrt{5}}$

71. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with $y(1) = 1$, is:

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(3) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (4) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

72. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is:

(1) $\sim p \wedge \sim q$ (2) $p \wedge q$

(3) $p \leftrightarrow q$ (4) $\sim p \vee \sim q$

73. All the points in the set $S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a:

(1) straight line whose slope is 1.

(2) circle whose radius is 1.

(3) circle whose radius is $\sqrt{2}$.

(4) straight line whose slope is –1.

74. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is:

(1) 8^3 (2) 8^2 (3) 8 (4) 8^{-2}

75. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$$

is continuous, then k is equal to:

(1) 2 (2) $\frac{1}{2}$ (3) 1 (4) $\frac{1}{\sqrt{2}}$

76. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1 - x^2}$, is surjective, then A is equal to:

(1) $R - \{-1\}$ (2) $[0, \infty)$

(3) $R - [-1, 0)$ (4) $R - (-1, 0)$

77. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point:

(1) $(-\sqrt{2}, 1, -4)$ (2) $(\sqrt{2}, -1, 4)$

(3) $(-\sqrt{2}, -1, -4)$ (4) $(\sqrt{2}, 1, 4)$

78. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to:

(1) (50, 50 + 46A) (2) (50, 50 + 45A)

(3) (A, 50 + 45A) (4) (A, 50 + 46A)

79. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is:

(1) $\frac{13\pi}{6}$ (2) $\frac{5\pi}{3}$ (3) 2π (4) π

80. Let $p, q \in R$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:

(1) $p^2 - 4q + 12 = 0$ (2) $q^2 - 4p - 16 = 0$

(3) $q^2 + 4p + 14 = 0$ (4) $p^2 - 4q - 12 = 0$

81. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is:

(1) {5, 10, 15} (2) {10, 15}

(3) {5, 10, 15, 20} (4) {10}

82. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (– 1, f(– 1)), then S is equal to:

(1) $\left\{\frac{1}{3}, 1\right\}$ (2) $\left\{-\frac{1}{3}, -1\right\}$

(3) $\left\{\frac{1}{3}, -1\right\}$ (4) $\left\{-\frac{1}{3}, 1\right\}$

83. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is:

(1) $x^2 + y^2 - 4x^2y^2 = 0$ (2) $x^2 + y^2 - 2xy = 0$

(3) $x^2 + y^2 - 16x^2y^2 = 0$ (4) $x^2 + y^2 - 2x^2y^2 = 0$

84. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$.

If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$

(2) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(3) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

(4) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

85. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is:

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$ (3) $\frac{31}{6}$ (4) $\frac{13}{6}$

86. If

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} \cdots\cdots\cdots\cdots \begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix},$$

then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is:

(1) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

87. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies f(x + y) = f(x) f(y) for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is:

(1) 2 (2) 16 (3) 4 (4) 3

88. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:

(1) m + n = 68 (2) m = n = 78

(3) n = m – 8 (4) m = n = 68

89. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for y ≠ 0 in R,

$$\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$$ is equal to:

(1) $y(y^2 - 1)$ (2) $y(y^2 - 3)$

(3) y^3 (4) $y^3 - 1$

90. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is:

(1) 25 (2) 22 (3) 24 (4) 20

Hints and Solutions

PHYSICS

1. **(Bonus)** $d = \frac{M}{V} = \frac{M}{L^3} = ML^{-3}$

$$\frac{\Delta d}{d} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L}$$

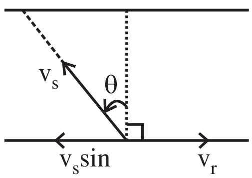
$$= \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31\text{kgm}^{-3}$$

2. **(3)** To cross the river straight

$V_s \sin\theta = V_r \therefore \sin\theta = \frac{v_r}{v_s} = \frac{2}{4} = \frac{1}{2}$

$\therefore \theta = 30°$

Direction of swimmer with respect to flow $= 90° + 30° = 120°$

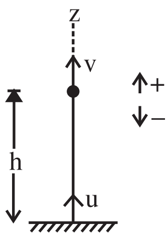


3. **(4)** $v^2 = u^2 - 2gh$

or $v = \sqrt{u^2 - 2gh}$

Momentum, $p = mv$

$\therefore p = m\sqrt{u^2 - 2gh}$

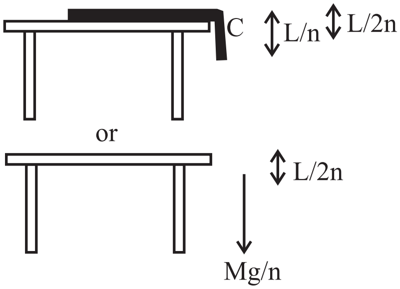


Therefore graph between p and h cannot have straight line.

(2) and (3) are not possible.

During upward journey as h increases, p decreases and in downward journey as h decreases p increases. Therefore 4 is the correct option.

4. **(2)** Length of hanging part = L/n
Mass of hanging part = M/n
Weight of hanging part = Mg/n
Let 'C' be the centre of mass of the hanging part.



The hanging part can be assumed to be a particle of weight Mg/n at a distance L/n below the table top. The work done in lifting it to the table top is equal to increase in its potential energy.

$\therefore W = \left(\frac{Mg}{n}\right)\left(\frac{L}{n}\right)$

$\therefore W = \frac{MgL}{n^2}$

5. **(4)** For head on elastic collision we have

$$V_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

Here $m_1 = 2\text{kg}$, $u_1 = x$, $u_2 = 0$, $v_1 = x/4$

$\therefore \frac{x}{4} = \frac{(2 - m_2)x}{2 + m_2} \Rightarrow m_2 = 1.2\text{kg}$

6. **(4)** Work done by torque is responsible for change in kinetic energy.

$$\therefore \tau = \frac{dE}{d\theta} \quad \therefore I\alpha = 2K\theta \quad \therefore \alpha = \frac{2k\theta}{I}$$

7. **(3)** $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$$= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2$$

$$= \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}^2 = \frac{1m}{2}\left[1 + \frac{K^2}{R^2}\right]$$

$\therefore h \propto 1 + \frac{K^2}{R^2}$

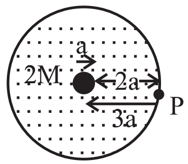
For ring : $h \propto 2$ ($\because K = R$)

For solid cylisnder, $h \propto \frac{3}{2} \quad \left(\because K = \frac{R}{\sqrt{2}}\right)$

For solid sphere, $h \propto \frac{7}{5} \quad \left(\because K = \sqrt{\frac{2}{5}}R\right)$

Ratio of heights $2 : \frac{3}{2} : \frac{7}{5} \Rightarrow 20:15:14$

8. **(3)** $E_P = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$



For a part on the surface of a spherical uniform charge distribution the whole mass acts as a point mass kept at the centre.

9. **(4)** We have, h = $\frac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

m = ρV = ρ × πr²h = $\rho \times \pi r^2 \times \frac{2T\cos\theta}{r\rho g}$

$\Rightarrow$ m α r

$\therefore \frac{m_1}{m_2} = \frac{r}{2r}$

or, $m_2 = 2m_1 = 2m$

10. **(3)** Internal energy depends only on initial and final state

So, $\Delta U_A = \Delta U_B$

Also $\Delta Q = \Delta U + \Delta W$

$\therefore W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$

[Area under P-V graph gives the work done.]

11. **(3)** $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{(273+127)}{(273+227)}}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2} \times 200 = 100\sqrt{5} \text{ m/s.}$$

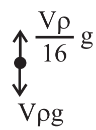
12. **(1)** In this case the total degree of freedom is 6. According to law of equipartition of energy,

$$\frac{1}{2}mv^{-2} = 6\left(\frac{1}{2}k_B T\right)$$

$$\therefore \frac{1}{2}mv^{-2} = 3k_B T$$

or, $T = \frac{mv^{-2}}{6k_B}$

13. **(3)** $T = 2\pi\sqrt{\frac{l}{g}}$



$$V\rho\, g_{eff} = V\rho g - \frac{V\rho}{16}g$$

$$g_{eff} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$$

Now $T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\frac{15g}{16}}} = \frac{4}{\sqrt{15}}T$

14. **(1)** On comparing with $P = P_0 \sin(\omega t - kx)$, we have

$\omega = 1000$ rad/s, $K = 3\ m^{-1}$

$$\therefore v_0 = \frac{\omega}{k} = \frac{1000}{3} = 333.3\text{m/s}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

or, $\frac{333.3}{336} = \sqrt{\frac{273+0}{273+t}}$ $\therefore$ t = 4°C

15. **(2)** Given, y = 0.3 sin (0.157 x) cos (200 πt)

So, k = 0.157 and ω = 200π = 2πf

$\therefore$ f = 100 Hz and $v = \frac{\omega}{k} = \frac{200\pi}{0.157} = 4000$m/s

Now, using $f = \frac{nv}{2l} = \frac{4v}{2l} = \frac{2v}{l}$ [here n = 4]

$$\therefore l = \frac{2v}{f} = \frac{2 \times 4000}{100} = 80m$$

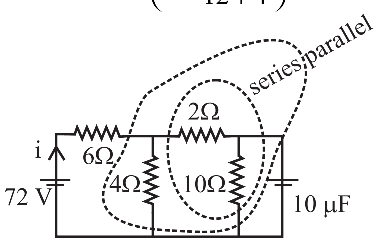
16. (4) $U = \frac{1}{4\pi\in_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D+\frac{d}{2}\right)} + \frac{(-q)Q}{\left(D-\frac{d}{2}\right)}\right]$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d} + \frac{qQ\left(D-\frac{d}{2}\right) - qQ\left(D+\frac{d}{2}\right)}{D^2 - \frac{d^2}{4}}\right]$$

$$= \frac{1}{4\pi\in_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \because \frac{d^2}{4} << D$$

17. (4) At steady state, there is no current in capacitor. 2Ω and 10Ω are in series. There equivalent resistance is 12Ω. This 12Ω is in parallel with 4Ω and there combined resistance is $12 \times 4/(12 + 4)$. This resistance is in series with 6Ω. Therefore, current drawn from battery

$$i = \frac{V}{R} = \left(\frac{72}{6 + \frac{12\times4}{12+4}}\right) = 8A$$



Current in 10Ω resistor

$$i' = \left(\frac{4}{4+12}\right)8 = 2A$$

Pd across capacitor, $V = i'R = 2 \times 10 = 20V$

$\therefore$ Charge on the capacitor, $q = CV$

$= 10 \times 20 = 200\ \mu C$.

18. (2) $W = U_f - U_i = \frac{q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)$ $\left(\because U = \frac{q^2}{2C}\right)$

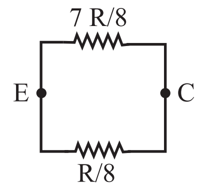
$$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right)\times10^6$$

$= 3.75 \times 10^{-6} J$

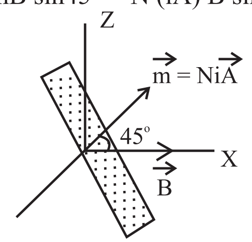
19. (2) Here $R_{DA} = R_{AB} = R_{BC} = R/4$
and $R_{DE} = R_{EC} = R/8$
Now $R_{ED}, R_{DA}, R_{AB}, R_{BC}$ are in series.

$$\therefore R_s = \frac{R}{8} + \frac{R}{4} + \frac{R}{4} + \frac{R}{4} = \frac{R + 2R + 2R + 2R}{8} = \frac{7R}{8}$$

$$\therefore R_{eq} = \frac{\left(\frac{7R}{8}\right)\left(\frac{R}{8}\right)}{R} = \frac{7R}{64}$$



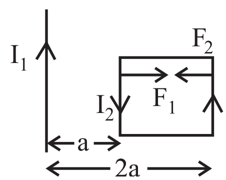
20. (4) $\tau = mB \sin45° = N\,(iA)\,B \sin 45°$



$$= 100\times3(5\times2.5)\times10^{-4}\times1\times\frac{1}{\sqrt{2}} = 0.27\ \text{Nm}$$

21. (3) $F = F_1 - F_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1I_2}{a} - \frac{I_1I_2}{2a}\right)\times a$

$= \frac{\mu_0 i_1 i_2}{4\pi}$ (Repulsive)



22. (3) $V = i_g(G + R) = 4 \times 10^{-3}$
$(50 + 5000) = 20V$

23. (4) Inductance $= \frac{\mu_0 N^2 A}{L}$

24. (1) $B_0 = \sqrt{B_0^2 + B_1^2} = \sqrt{30^2 + 2^2}\times10^{-6}$

$\approx 30 \times 10^{-6} T$

$\therefore E_0 = cB = 3\times10^8\times30\times10^{-6}$

$= 9 \times 10^3$ V/m

$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{9}{\sqrt{2}}\times10^3 V/m$

Force on the charge,

$$F = E_{rms}Q = \frac{9}{\sqrt{2}}\times10^3\times10^{-4} \simeq 0.64N$$

25. (1) The equation of amplitude modulated wave

m = (v_0+A cosωt) sinωt

= $v_0$sinω_0t +A cosωt sinω_0t

$= v_0\sin\omega_0 t + \frac{A}{2}[\sin(\omega_0-\omega)t + \sin(\omega_0+\omega)t]$

26. (3) $+5 = -\frac{v}{u} \Rightarrow v = -5u$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-5u}+\frac{1}{u}=\frac{-1}{0.4}$

$\therefore$ u = – 0.32 m.

27. (Bonus) Shift = nβ (given)

$\therefore\ D\frac{(\mu-1)t}{a}=\frac{n\lambda D}{a}\quad\left[\because \text{Shift}=\frac{D(\mu-1)t}{a}\right]$

or $t=\frac{n\lambda}{(\mu-1)}$

28. (3) Here ω = 2π ×6 × 10^{14}

$\Rightarrow$ f = 6 × 10^{14} Hz

Wavelength

$$\lambda=\frac{c}{f}=\frac{3\times10^8}{6\times10^{14}}=0.5\times10^{-6}m=5000\text{Å}$$

Given $E=\frac{12375}{5000}=2.48\,eV$

Using E = W + eV_s

$\Rightarrow$ 2.48 = 2 + eV_s

or V_s = 0.48 V

29. (2) $\frac{1}{\lambda_1}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}$

$\frac{1}{\lambda_2}=R\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{3R}{16}$

$\therefore\frac{\lambda_2}{\lambda_1}=\frac{80}{108}$

$$\lambda_2=\frac{80}{108}\lambda_1=\frac{80}{108}\times660=488.9\text{nm.}$$

30. (2) Given $\Delta V_i = 10\times10^{-3}V$

$\Delta I_c = 3\times10^{-3}$A

$\Delta I_b = 15\times10^{-6}$A

$$R_i=\frac{\Delta V_i}{\Delta I_b}=\frac{10\times10^{-3}}{15\times10^{-6}}=0.67k\Omega$$

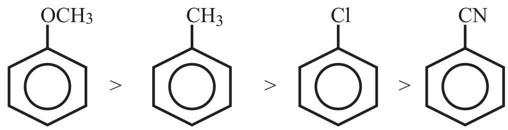
$\therefore$ Voltage gain $=\frac{\Delta I_c}{\Delta I_b}\times\frac{R_0}{R_i}$

$$=\left(\frac{3\times10^{-3}}{15\times10^{-6}}\right)\times\frac{1000}{670}\quad=200\times\frac{1000}{670}\approx300$$

CHEMISTRY

31. (4) Alkali metals have high difference in the first and second ionisation energy as they achieve stable noble gas configuration after first ionisation.

32. (1)



(+M) > (+I and +H) > (–I and +M) > (–M, –I)

33. (1) Critical temperature $=\frac{8a}{27Rb}$

Value of $\frac{a}{b}$ is highest for Kr. Therefore, Kr has greatest value of critical temperature.

34. (1) In graph (i), ln [Reactant vs time is linear with positive intercept and negative slope. Hence it is 1st order In graph (ii), [Reactant] vs time is linear with positive intercept and negative slope. Hence, it is zero order.

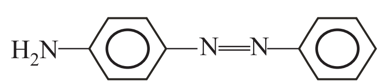
35. (1) We know that heat and work are not state functions but $q + w = \Delta U$ is a state function. $H - TS$ (i.e. G) is also a state function.

36. (1)

Magnetite	Fe_3O_4
Sphalerite	ZnS
Cryolite	Na_3AlF_6
Malachite	$CuCO_3.Cu(OH)_2$

37. (1) Global warming is caused by the emission of green house gases. 72% of the totally emitted green house gases is CO_2. Therefore, excessive release of CO_2 is the main cause of global warming.

38. (3) In acidic medium aniline is more reactive than phenol that's why electrophilic aromatic substitution of $Ph-\overset{+}{N}_2$ takes place with aniline.



39. (1) Configuration of C_2

$= \sigma 1s^2 \sigma^* 1s^2\ \sigma 2s^2 \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2$

Configuration of C_2^-

$= \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \ \sigma 2p_z^1$

Bond order

$$= \frac{\text{No.of bonding e}^- - \text{No.of antibonding e}^-}{2}$$

C_2 has *s-p* mixing and the HOMO is $\pi 2p_x = \pi 2p_y$ and LUMO is $\sigma 2p_z$. So, the extra electron will occupy bonding molecular orbital and this will lead to an increase in bond order.

C_2^- has more bond order than C_2.

40. (4)

(Oxide)	(Oxidation state)
N_2O	+1
NO	+2
N_2O_3	+3
NO_2	+4

So, $N_2O < NO < N_2O_3 < NO_2$

41. (4) $P_M^o = 450$ mmHg, $P_N^o = 700$ mmHg

$P_M = P_M^o x_M = y_M P_T$

$\Rightarrow P_M^\circ = \frac{y_M}{x_M}(P_T)$

Similarly, $P_N^o = \frac{y_N}{x_N}(P_T)$

Given, $P_M^o < P_N^\circ$

$\Rightarrow \frac{y_M}{x_M} < \frac{y_N}{x_N}$

$\Rightarrow \frac{y_M}{y_N} < \frac{x_M}{x_N}$

42. (2) We know, $\pi = iCRT$; $\pi_{xy} = 4\pi_{BaCl_2}$

$\therefore\ 2[XY] = 4 \times (0.01) \times 3$

$[XY] = 0.06$

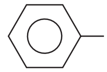
$= 6 \times 10^{-2}$ mol/L

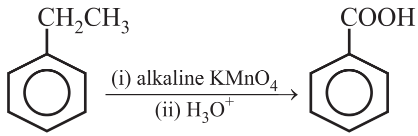
43. (3) In $CuSO_4 \cdot 5H_2O$, four H_2O molecules are directly coordinated to the central metal ion while one H_2O molecule is hydrogen bonded.

44. (1) $\Delta G^\circ = -nFE^\circ_{cell}$

$= -2 \times (96000) \times 2\ V = -384000$ J/mol

$= -384$ kJ/mol

45. (1) Alkaline $KMnO_4$ converts  —R with a benzylic hydrogen into benzoic acid.



46. (2) $\bar{v} \propto \Delta E$

For H-atom

$$\bar{v} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series,

$$\bar{v}\,(\max) = 13.6\left(1 - \frac{1}{\infty}\right)$$

$$\bar{v}\,(\min) = 13.6\left(1 - \frac{1}{4}\right)$$

$$\therefore\ \bar{v}_{\max} - \bar{v}_{\min} = 13.6\left(\frac{1}{4}\right)$$

For Balmer series,

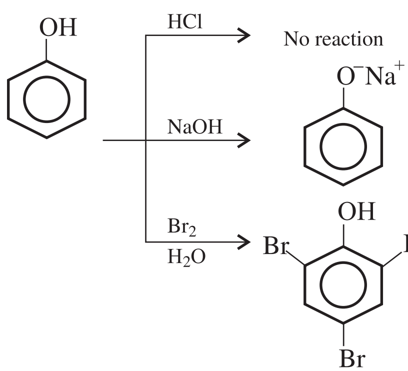
$$\bar{v}\,(\max) = 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right)$$

$$\bar{v}\,(\min) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore\ \bar{v}_{\max} - \bar{v}_{\min} = 13.6\left(\frac{1}{9}\right)$$

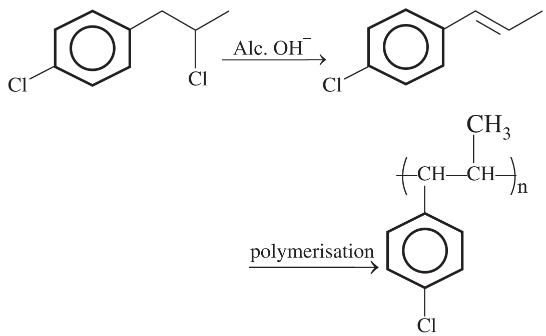
$$\frac{\Delta\bar{v}_{\text{Lyman}}}{\Delta\bar{v}_{\text{Balmer}}} = \frac{9}{4}$$

47. (1)

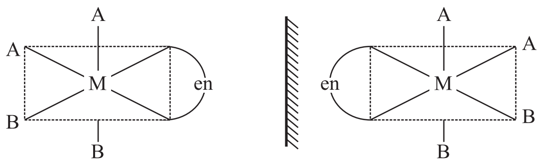


48. (4) Fullerene (C_{60}) contains 20 hexagons (six membered) rings and 12 pentagons (five membered rings):

49. (4)

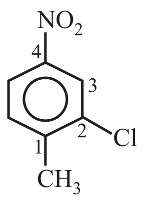


50. (3)



No plane of symmetry or centre of symmetry
Hence it is optically active.

51. (2)



2-Chloro-1-methyl-4-nitrobenzene

52. (3) (A) $V_2O_5 \rightarrow$ Preparation of H_2SO_4 in contact process

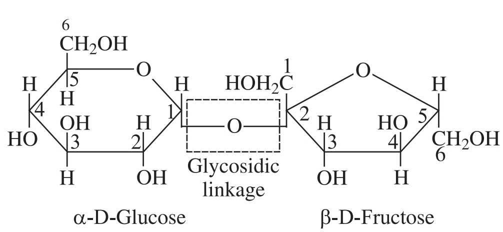
(B) $TiCl_4 + Al(Me)_3 \rightarrow$ Polyethylene (Ziegler-Natta catalyst)

(C) $PdCl_2 \rightarrow$ Ethanal (Wacker's process)

(D) Iron oxide $\rightarrow NH_3$ in (Haber's process)

53. (2) Sucrose contains glycosidic link between C_1 of α-D glucose and C_2 of β-D-fructose.

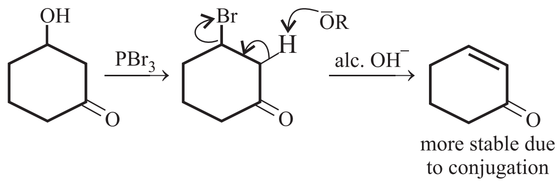
$$C_{12}H_{22}O_{11} + H_2O \longrightarrow \text{Glucose} + \text{Fructose}$$



Sucrose

54. (2) Mg burns in air and produces a mixture of nitride and oxide.

55. (3)



56. (4) $CH_3-C\equiv C-H \xrightarrow[\text{1 eq}]{DCl} CH_3-\underset{Cl}{C}=\underset{D}{CH} \xrightarrow{DI} CH_3-\overset{I}{\underset{Cl}{C}}-\overset{D}{\underset{D}{CH}}$

Both additions follow Markovnikov's rule.

57. (2) $LiAlH_4$ reduces esters to alcohols but does not reduce C = C.

$$CH_3CH=CH-CO_2Me \xrightarrow{LiAlH_4}$$

$$CH_3CH=CHCH_2OH$$

58. (1) Cr^{3+} has d^3 configuration and forms an octahedral inner orbitals complex.

The set of degenerate orbitals are

(d_{xy}, d_{yz} and d_{xz}) and ($d_{x^2-y^2}$ and d_{z^2}).

59. (1) In aerosol, the dispersion medium is gas while the dispersed phase can be both solid or liquid.

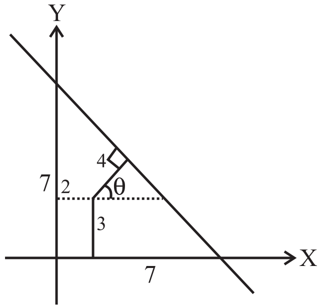
60. (1) According to the stoichiometry of balanced equation 28 g N_2 react with 6 g H_2

$$\underset{\substack{\text{1 mole}\\ \text{28 g}}}{N_2} + \underset{\substack{\text{3 mole}\\ \text{6 g}}}{3H_2} \longrightarrow 2NH_3$$

∴ For 56 g of N_2, 12 g of H_2 is required.

MATHEMATICS

61. (2)



Since point at 4 units from P (2, 3) will be A (4 cosθ + 2, 4 sin (θ + 3) and this point will satisfy the equation of line x + y = 7

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

On squaring

$\Rightarrow \sin 2\theta - \frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6}$ (ignoring –ve sign)

$\Rightarrow \tan\theta = \frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

62. (1) $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\Rightarrow \frac{k^2+2}{4} - \left(\frac{k}{4}\right)^2 = 5$

$\Rightarrow 4k^2 + 8 - k^2 = 80$

$\Rightarrow 3k^2 = 72$

$\Rightarrow k = 2\sqrt{6}$

63. (4) Since, function f(x) have local extreem points at x = –1, 0, 1. Then

f(x) = K (x + 1) x (x – 1)

$= K(x^3 - x)$

$\Rightarrow f(x) = K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$

(using integration)

$\Rightarrow f(0) = C$

$\because f(x) = f(0) \Rightarrow K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) = 0$

$\Rightarrow \frac{x^2}{2}\left(\frac{x^2}{2} - 1\right) = 0 \Rightarrow x = 0, 0, \sqrt{2}, -\sqrt{2}$

$\therefore S = \{0, -\sqrt{2}, \sqrt{2}\}$

64. (1) $I = \int \sec^{\frac{2}{3}} x.\text{cosec}^{\frac{4}{3}} dx$

$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$

Put tan x = z

$\Rightarrow \sec^2 x \, dx = dz$

$\Rightarrow I = \int z^{-\frac{4}{3}} \cdot dz = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C$

$\Rightarrow I = -3(\tan x)^{\frac{-1}{3}} + C$

65. (4) P (at least one hit the target) = 1 – P (none of them hit the target)

$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{7}{32} = \frac{25}{32}$

66. (3) Let point on line be P (2k + 1, 3k – 1, 4k + 2)

Since, point P lies on the plane x + 2y + 3z = 15

$\therefore$ 2k + 1 + 6k – 2 + 12k + 6 = 15

$\Rightarrow k = \frac{1}{2}$ $\therefore P \equiv \left(2, \frac{1}{2}, 4\right)$

Then the distance of the point P from the origin is

$OP = \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$

67. (4) $y = x^3 + ax - b$

Since, the point (1, –5) lies on the curve.

$\Rightarrow 1 + a - b = -5$

$\Rightarrow a - b = -6$ ...(1)

Now, $\frac{dy}{dx} = 3x^2 + a \Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 3 + a$

Since, required line is perpendicular to y = x – 4, then slope of tangent at the point P (1, –5) = –1

$\therefore$ 3 + a = – 1 $\Rightarrow$ a = – 4 $\Rightarrow$ b = 2

$\therefore$ the equation of the curve is $y = x^3 - 4x - 2$

$\Rightarrow$ (2, –2) lies on the curve.

68. (2) Let $I = \int_0^{\pi/2} \frac{\sin^3 x dx}{\sin x + \cos x}$ ...(1)

Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 x\, dx}{\sin x + \cos x}$ ...(2)

Adding equations (1) and (2), we get

$$2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin(2x)\right)dx$$

$$\Rightarrow I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0^{\pi/2} \Rightarrow I = \frac{\pi - 1}{4}$$

69. (2) $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \left(\frac{1+\cos 20°}{2}\right) + \left(\frac{1+\cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

$$= \frac{3}{4}$$

70. (3) Since, lx + my + n = 0 is a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$

then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

but it is given that $mx - y + 7\sqrt{3}$ is normal

to hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$

then $\frac{24}{m^2} - \frac{18}{(-1)^2} = \frac{(24+18)^2}{(7\sqrt{3})^2} \Rightarrow m = \frac{2}{\sqrt{5}}$

71. (3) $\frac{dy}{dx} + \frac{2}{x}y = x$ and y(1) = 1 (given)

Since, the above differential equation is the linear differential equation, then

$$I.F = e^{\int \frac{2}{x}dx} = x^2$$

Now, the solution of the linear differential equation

$$y \times x^2 = \int x^3 dx \Rightarrow yx^2 = \frac{x^4}{4} + C$$

$$\because y(1) = 1 \quad \therefore 1 \times 1 = \frac{1}{4} + C \quad \Rightarrow C = \frac{3}{4}$$

$\therefore$ solution becomes

$$y = \frac{x^2}{4} + \frac{3}{4x^2}$$

72. (4) $\sim (p \vee (\sim p \wedge q))$

$= \sim ((p \vee \sim p) \wedge (p \vee q)) = \sim (t \wedge (p \vee q))$

$= \sim (p \vee q) = \sim p \wedge \sim q$

73. (2) Let z∈ S then $z = \frac{\alpha + i}{\alpha - i}$

Since, z is a complex number and let z = x + iy

Then, $x + iy = \frac{(\alpha + i)^2}{\alpha^2 + 1}$ (by rationalisation)

$$\Rightarrow x + iy = \frac{(\alpha^2 - 1)}{\alpha^2 + 1} + \frac{i(2\alpha)}{\alpha^2 + 1}$$

Then compare both sides

$x = \frac{\alpha^2 - 1}{\alpha^2 + 1}$ ...(1)

$y = \frac{2\alpha}{\alpha^2 + 1}$ ...(2)

Now squaring and adding equations (1) and (2)

$$\Rightarrow x^2 + y^2 = \frac{(\alpha^2 - 1)^2}{(\alpha^2 + 1)^2} + \frac{4\alpha^2}{(\alpha^2 + 1)^2} = 1$$

74. (2) $\because T_4 = 20 \times 8^7$

$$\Rightarrow {}^6C_3 \left(\frac{2}{x}\right)^3 \times (x^{\log_8 x})^3 = 20 \times 8^7$$

$$\Rightarrow 8 \times 20 \times \left(\frac{x^{\log_8 x}}{x}\right)^3 = 20 \times 8^7$$

$$\Rightarrow \frac{x^{\log_8 x}}{x} = 64$$

Now, take $\log_8$ on both sides, we get

$(\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow \log_8 x = -1$ or $\log_8 x = 2$

$$\Rightarrow x = \frac{1}{8} \quad \text{or} \quad x = 8^2$$

75. **(2)** Since, f(x) is continuous, then

$$\lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

Now by L- hospital's rule

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x} = k \Rightarrow \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{(\sqrt{2})^2} = k \quad \Rightarrow k = \frac{1}{2}$$

76. **(3)** $f(x) = \dfrac{x^2}{1-x^2}$

$$\Rightarrow f(-x) = \frac{x^2}{1-x^2} = f(x)$$

$$f'(-x) = \frac{2x}{(1-x^2)^2}$$

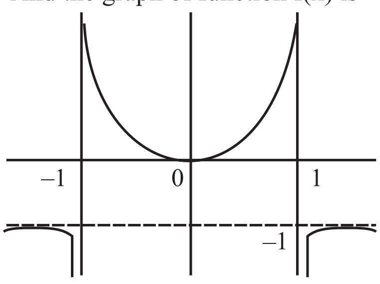
$\therefore$ f(x) increases in $x \in (10, \infty)$

Also f(0) = 0 and

$\lim_{x\to\pm\infty} f(x) = -1$ and f(x) is even function

$\therefore$ Set A = R – [–1, 0)

And the graph of function f(x) is



Alternative

For f to be subjective A = Range of f.

$$\frac{x^2}{1-x^2} = y \Rightarrow x^2 = y - x^2 y$$

$$\Rightarrow x = \pm\sqrt{\frac{y}{1+y}} \Rightarrow y(1+y) \ge 0 \text{ and } y \ne -1$$

$$\Rightarrow y \in (-\infty, -1) \cup [0, \infty) \Rightarrow y \in R - [-1, 0)$$

$$\Rightarrow A = R - [-1, 0)$$

77. **(4)** Let the required plane passing through the points (0, –1, 0) and (0, 0, 1) be

$\dfrac{x}{\lambda} + \dfrac{y}{-1} + \dfrac{z}{1} = 1$ and the given plane is

$y - z + 5 = 0$

$$\therefore \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2} + 1 + 1\right)}\sqrt{2}}$$

$$\Rightarrow \lambda^2 = \frac{1}{2} \Rightarrow \frac{1}{\lambda} = \pm\sqrt{2}$$

Then, the equation of plane is

$\pm\sqrt{2}x - y + z = 1$

Then the point $(\sqrt{2}, 1, 4)$ satisfies the equation of plane $-\sqrt{2}x - y + z = 1$

78. **(4)** $\because S_n = \left(50 - \dfrac{7A}{2}\right)n + n^2 \times \dfrac{A}{2}$

$\Rightarrow a_1 = 50 - 3A$

$\therefore d = a_2 - a_1 = (S_2 - S_1) - S_1$

$\Rightarrow d = \dfrac{A}{2} \times 2 = A$

Now, $a_{50} = a_1 + 49 \times d$

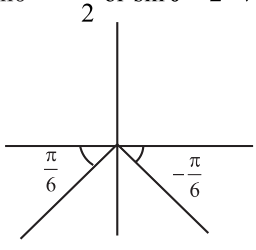
$= (50 - 3A) + 49A = 50 + 46A$

So, $(d, a_{50}) = (A, 50 + 46A)$

79. **(3)** $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\dfrac{1}{2}$ or $\sin\theta = 2 \rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$= \left(\pi+\frac{\pi}{6}\right)+\left(2\pi-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)+\left(-\pi+\frac{\pi}{6}\right)=2\pi$$

80. (4) Since $2-\sqrt{3}$ is a root of the quadratic equation

$x^2 + px + q = 0$

$\therefore 2+\sqrt{3}$ is the other root

$\Rightarrow$ Sum of roots = 4, Product of roots = 1

$\Rightarrow p = -4, q = 1$

$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$

81. (1) Since, $f(x) = 15 - |(10 - x)|$

$\therefore g(x) = f(f(x)) = 15 - |10 - [15 - |10 - x|]|$

$= 15 - ||10 - x| - 5|$

$\therefore$ Then, the points where function g(x) is Non-differentiable are

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow x = 10$ and $x - 10 = \pm 5$

$\Rightarrow x = 10$ and $x = 15, 5$

82. (4) $y = f(x) = x^3 - x^2 - 2x$

$\Rightarrow \dfrac{dy}{dx} = 3x^2 - 2x - 2$

$f(1) = 1 - 1 - 2 = -2, \quad f(-1) = -1 - 1 + 2 = 0$

Since the tangent to the curve is parallel to the line segment joining the points $(1, -2)$ and $(-1, 0)$

And their slopes are equal.

$$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2} \Rightarrow x = 1, \frac{-1}{3}$$

Hence, the required set $S = \left\{\dfrac{-1}{3}, 1\right\}$

83. (1) Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P \equiv \left(\dfrac{1}{\cos\theta}, 0\right)$ & $Q \equiv \left(0, \dfrac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be

$$M \equiv \left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) \equiv (h,k)$$

$\Rightarrow \cos\theta = \dfrac{1}{2h}$ ...(1)

$\sin\theta = \dfrac{1}{2k}$ ...(2)

Now squaring and adding equation (1) and (2)

$$\frac{1}{h^2}+\frac{1}{k^2}=4$$

$\Rightarrow h^2 + k^2 = 4h^2k^2$

$\because$ locus of M is : $x^2 + y^2 - 4x^2y^2 = 0$

84. (3) $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ ...(1)

Since, $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

$\therefore \vec{\beta}_2 . \vec{\alpha} = 0$

Since, $\vec{\beta}_1$ is parallel to $\vec{a}$.

then $\vec{\beta}_1 = \lambda\vec{\alpha}$ (say)

$\vec{a}.\vec{\beta} = \vec{a}.\vec{\beta}_1 - \vec{\alpha}.\vec{\beta}_2$

$\Rightarrow 5 = \lambda\alpha^2 \Rightarrow 5 = \lambda \times 10$ $\quad (\because |\vec{\alpha}| = \sqrt{10})$.

$\Rightarrow \lambda = \dfrac{1}{2}$

$\therefore \vec{\beta}_1 = \dfrac{\vec{\alpha}}{2}$

Cross product with $\vec{\beta}_1$ in equation (1)

$\Rightarrow \vec{\beta} \times \vec{\beta}_1 = -\vec{\beta}_2 \times \vec{\beta}_1$

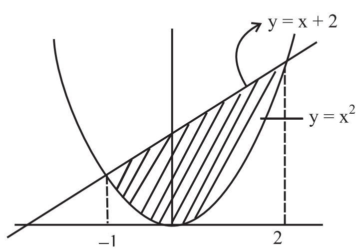
$\Rightarrow \vec{\beta} \times \vec{\beta}_1 = \vec{\beta}_1 \times \vec{\beta}_2 \Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \dfrac{-\vec{\beta} \times \vec{\alpha}}{2}$

$$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \frac{1}{2}\begin{vmatrix} \hat{i} & j & k \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right]$$

$$= \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$$

85. (2)



Required area is equal to the area under the curves $y \ge x^2$ and $y \le x + 2$

$\therefore$ requried area $\displaystyle\int_{-1}^{2} ((x+2) - x^2)\,dx$

$$=\left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{2}$$

$$=\left(2+4-\frac{8}{3}\right)-\left(+\frac{1}{2}-2+\frac{1}{3}\right)=\frac{9}{2}$$

86. **(2)** $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}$

$$=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 1+2+3+...+(n-1)\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

$\Rightarrow \frac{(n-1)n}{2}=78 \quad \Rightarrow n^2-n-15=0$

$\Rightarrow n = 13$

Now, the matrix $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

Then, the required inverse of

$\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

87. **(4)** $\because f(x+y)=f(x).f(y)$

$\Rightarrow$ Let $f(x)=t^x$

$\because f(1)=2 \ \therefore t=2$

$\Rightarrow f(x)=2^x$

Since, $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$

Then, $\sum_{k=1}^{10} 2^{a+k}=16(2^{10}-1)$

$$\Rightarrow 2^a\sum_{k=1}^{10} 2^k=16(2^{10}-1)$$

$$\Rightarrow 2^a\times\frac{((2^{10})-1)\times 2}{(2-1)}=16\times(2^{10}-1)$$

$\Rightarrow 2.2^a = 16 \Rightarrow a = 3$

88. **(2)** Since, m = number of ways the committee is formed with at least 6 males

$= {}^8C_6 . {}^5C_5 + {}^8C_7 . {}^5C_4 + {}^8C_8 . {}^5C_3 = 78$

and n = number of ways the committee is formed with at least 3 females

$= {}^5C_3 . {}^8C_8 + {}^5C_4 . {}^8C_7 + {}^5C_5 . {}^8C_6 = 78$

Hence, m = n = 78

89. **(3)** Let $\alpha=\omega$ and $\beta=\omega^2$ are roots of $x^2+x+1=0$

& *Let* $\Delta=\begin{vmatrix}y+1 & \omega & \omega^2\\ \omega & y+\omega^2 & 1\\ \omega^2 & 1 & y+\omega\end{vmatrix}=\Delta$

Applying $C_1\to C_1+C_2+C_3$, we get

$$\Delta=\begin{vmatrix}y+1+\omega+\omega^2 & \omega & \omega^2\\ y+1+\omega+\omega^2 & y+\omega^2 & 1\\ 1+\omega+\omega^2+y & 1 & y+\omega\end{vmatrix}$$

$$\Delta=\begin{vmatrix}y & \omega & \omega^2\\ y & y+\omega^2 & 1\\ y & 1 & y+\omega\end{vmatrix}\ (\because 1+\omega+\omega^2=0)$$

$$\Delta=y\begin{vmatrix}1 & \omega & \omega^2\\ 1 & y+\omega^2 & 1\\ 1 & 1 & y+\omega\end{vmatrix}$$

Applying $R_2\to R_2-R_1$ & $R_3\to R_3-R_1$, we get

$$\Delta=y\begin{vmatrix}y+\omega^2-\omega & 1-\omega^2\\ 1-\omega & y+\omega-\omega^2\end{vmatrix}$$

$\Rightarrow \Delta = y$

$$\left[y-(\omega-\omega^2)(y+(\omega-\omega^2)-(1-\omega)(1-\omega^2)\right]$$

$$\Rightarrow \Delta=y\left[y^2-(\omega-\omega^2)^2-1+\omega^2+\omega-\omega^3\right]$$

$$\Rightarrow \Delta=y\left[y^2-\omega^2-\omega^4+2\omega^3-1+\omega^2+\omega^4-\omega^3\right]$$

$(\because \omega^4=\omega)$

$\Rightarrow \Delta = y(y^2) = y^3$

90. **(1)** $\because y^2=16x \Rightarrow a=4$

One end of focal of the parabola is at (1, 4)

$\because$ y – coordinate of focal chord is $2at$

$\therefore 2at=4$

$\Rightarrow t=\frac{1}{2}$

Hence, the required length of focal chord

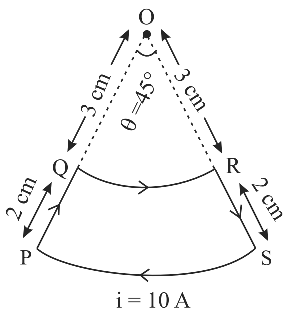
$$=a\left(t+\frac{1}{t}\right)^2=4\times\left(2+\frac{1}{2}\right)^2=25$$

JEE MAIN SOLVED PAPER–2019

9 JANUARY 2019 (MORNING SHIFT)

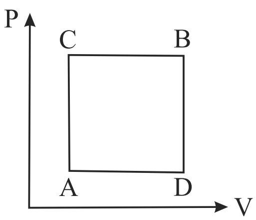
PHYSICS

1. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to:



(1) 1.0×10^{-7} T (2) 1.5×10^{-7} T
(3) 1.5×10^{-5} T (4) 1.0×10^{-5} T

2. A gas can be taken from A to B via two different processes ACB and ADB.



When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

(1) 40 J (2) 80 J (3) 100 J (4) 20 J

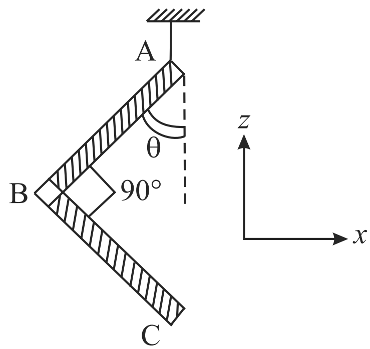
3. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive *x*-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}\ \text{V/m}$. The corresponding magnetic field $\vec{B}$, at that point will be:

(1) $18.9 \times 10^{-8}\ \hat{k}$T (2) $2.1 \times 10^{-8}\ \hat{k}$T
(3) $6.3 \times 10^{-8}\ \hat{k}$T (4) $18.9 \times 10^{8}\ \hat{k}$T

4. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 16 : 9 (2) 25 : 9 (3) 4 : 1 (4) 5 : 3

5. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then:



(1) $\tan\theta = \dfrac{1}{2\sqrt{3}}$ (2) $\tan\theta = \dfrac{1}{2}$

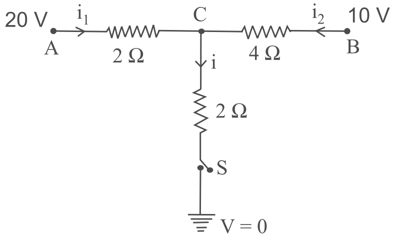
(3) $\tan\theta = \dfrac{2}{\sqrt{3}}$ (4) $\tan\theta = \dfrac{1}{3}$

6. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds

$\left[\dfrac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to :

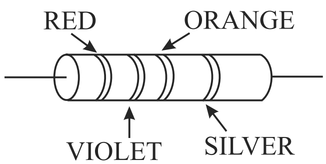
(1) 3.16 (2) 0.32
(3) 0.45 (4) 2.24

7. When the switch S, in the circuit shown, is closed then the valued of current i will be:



(1) 3A (2) 5A (3) 4A (4) 2A

8. A resistance is shown in the figure. Its value and tolerance are given respectively by:



(1) 270 Ω, 10% (2) 27 kΩ, 10%
(3) 27 kΩ, 20% (4) 270 Ω, 5%

9. A bar magnet is demagnetized by inserthing it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:

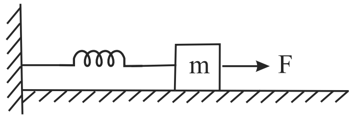
(1) 285 A/m (2) 2600 A/m
(3) 520 A/m (4) 1200 A/m

10. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

(1) $\dfrac{F}{A\alpha\Delta T}$ (2) $\dfrac{F}{A\alpha(\Delta T-273)}$

(3) $\dfrac{F}{2A\alpha\Delta T}$ (4) $\dfrac{2F}{A\alpha\Delta T}$

11. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:



(1) $\dfrac{2F}{\sqrt{mk}}$ (2) $\dfrac{F}{\pi\sqrt{mk}}$

(3) $\dfrac{\pi F}{\sqrt{mk}}$ (4) $\dfrac{F}{\sqrt{mk}}$

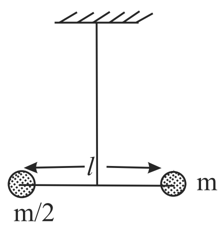
12. Three charges + Q, q, + Q are placed respectively, at distance, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at $x = 0$, is zero, then value of q is:

(1) – Q/4 (2) + Q/2
(3) + Q/4 (4) – Q/2

13. A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3} m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field B (t) = (0.4T) sin (50πt). The the net charge flowing through the loop during t = 0 s and t = 10 ms is close to:

(1) 14 mC (2) 7 mC
(3) 21 mC (4) 6 mC

14. Two masses m and $\dfrac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:

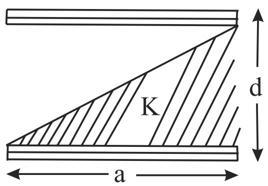


(1) $\dfrac{3k\theta_0^2}{l}$ (2) $\dfrac{2k\theta_0^2}{l}$

(3) $\dfrac{k\theta_0^2}{l}$ (4) $\dfrac{k\theta_0^2}{2l}$

15. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:
(1) 2.0% (2) 2.5% (3) 1.0% (4) 0.5%

16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is:



(1) $\frac{K \in_0 a^2}{2d(K+1)}$ (2) $\frac{K \in_0 a^2}{d(K-1)} \ln K$

(3) $\frac{K \in_0 a^2}{d} \ln K$ (4) $\frac{1}{2} \frac{K \in_0 a^2}{d}$

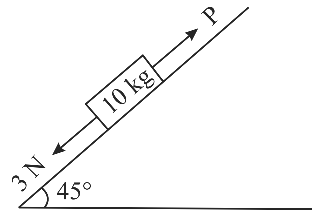
17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is $1.6 m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to:

(1) 2 Ωm (2) 4 Ωm
(3) 0.4 Ωm (4) 0.2 Ωm

18. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is:

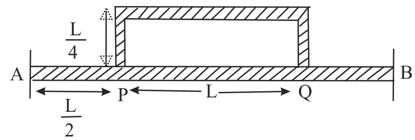
(1) $\frac{L}{m}$ (2) $\frac{4L}{m}$ (3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

19. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward? (take $g = 10 \text{ ms}^{-2}$)



(1) 32 N (2) 18 N (3) 23 N (4) 25 N

20. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to:



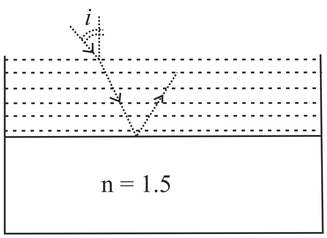
(1) 45°C (2) 75°C (3) 60°C (4) 35°C

21. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speeed of transverse waves in the string is 60 ms^{-1}. when the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to:

(1) $\frac{g}{30}$ (2) $\frac{g}{5}$ (3) $\frac{g}{10}$ (4) $\frac{g}{20}$

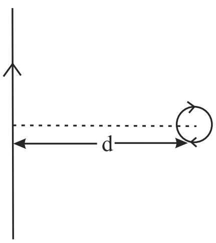
22. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an acitvity of 20 mCi. The correct choices for half-lives of A and B would then be respectively:
(1) 5 days and 10 days
(2) 10 days and 40 days
(3) 20 days and 5 days
(4) 20 days and 10 days

23. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is:



(1) $\sqrt{\frac{5}{3}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{5}{\sqrt{3}}$ (4) $\frac{4}{3}$

24. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The redius of the loop is a and distance of its centre from the wire is d (d>>a). If the loop applies a force F on the wire then:



(1) $F = 0$ (2) $F \propto \left(\frac{a}{d}\right)$

(3) $F \propto \left(\frac{a^2}{d^3}\right)$ (4) $F \propto \left(\frac{a}{d}\right)^2$

25. Surface of certain metal is first illuminated with light of wavelength λ_1 = 350 nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of (2) The work function of the metal (in eV) is close to:

(Energy of photon $= \frac{1240}{\lambda\,(\text{in nm})}$ eV)

(1) 1.8 (2) 2.5 (3) 5.6 (4) 1.4

26. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

(1) $y = x^2 +$ constant (2) $y^2 = x +$ constant
(3) $y^2 = x^2 +$ constant (4) $xy =$ constant

27. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screem is shifted by a distance d. Then d is:

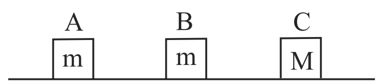
(1) 1.1 cm away from the lens
(2) 0
(3) 0.55 cm towards the lens
(4) 0.55 cm away from the lens

28. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is:

(1) $\frac{R}{\sqrt{5}}$ (2) $\frac{R}{\sqrt{2}}$

(3) R (4) $R\sqrt{2}$

29. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?



(1) 5 (2) 2 (3) 4 (4) 3

30. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s close to (Take charge of electron to be $= 1.6 \times 10^{-19}$C)

(1) 0.02 (2) 3 (3) 2 (4) 0.2

CHEMISTRY

31. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is:

(1) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

(2) both are paramagnetic with three unpaired electrons.

(3) both absorb energies corresponding to their complementary colors.

(4) Δ_0 value for (A) is less than that of (B).

32. The correct decreasing order for acid strength is:

(1) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(2) $FCH_2COOH > CNCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(3) $CNCH_2COOH > NO_2CH_2COOH > FCH_2COOH > ClCH_2COOH$

(4) $NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$

33. The major product of following reaction is:

$$R-C\equiv N \xrightarrow[\text{(ii) } H_2O]{\text{(i) } AlH(i-Bu)_2}$$

(1) RCOOH (2) $RCONH_2$

(3) RCHO (4) RCH_2NH_2

34. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is :

(1) 5.92 (2) 6.93

(3) 3.87 (4) 4.90

35. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in $JK^{-1}\ mol^{-1}$, x is:

(1) $\frac{2R}{4+R}$ (2) $\frac{2R}{4-R}$

(3) $\frac{4+R}{2R}$ (4) $\frac{4-R}{2R}$

36. The one that is extensively used as a piezoelectric material is:

(1) tridymite

(2) amorphous silica

(3) quartz

(4) mica

37. Correct statements among 'A' to 'D' regarding silicones are:

(A) They are polymers with hydrophobic character.

(B) They are biocompatible.

(C) In general, they have high thermal stability and low dielectric strength.

(D) Usually, they are resistant to oxidation and used as greases.

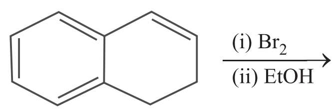
(1) (A), (B), (C) and (D)

(2) (A), (B) and (C) only

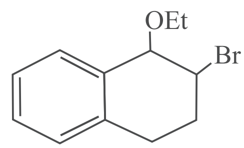
(3) (A) and (B) only

(4) (A), (B) and (D) only

38. The major product of the following reaction is:



(1)



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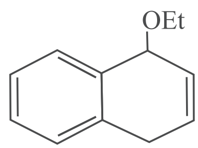

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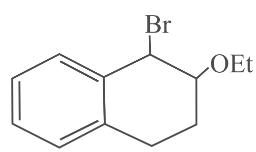
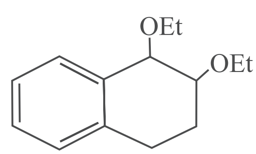


(4)



39. In general, the properties that decrease and increase down a group in the periodic table, respectively, are:
(1) atomic radius and electronegativity.
(2) electron gain enthalpy and electronegativity.
(3) electronegativity and atomic radius.
(4) electronegativity and electron gain enthalpy.

40. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:
(1) 12 (2) 4 (3) 8 (4) 16

41. The correct match between Item-I and Item-II is:

	Item-I (Drug)		**Item-II (Test)**
A	Chloroxylenol	P	Carbylamine test
B	Norethindrone	Q	Sodium hydrogen carbonate test
C	Sulphapyridine	R	Ferric chloride test
D	Penicillin	S	Bayer's test

(1) A → R ; B → P ; C → S ; D → Q
(2) A → Q ; B → S ; C → P ; D → R
(3) A → R ; B → S ; C → P ; D → Q
(4) A → Q ; B → P ; C → S ; D → R

42. A water sample has ppm level concentration of the following metals: Fe = 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is:
(1) Cu (2) Mn (3) Fe (4) Zn

43. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4$ = 303 g mol^{-1})
(1) 22.8 (2) 15.2 (3) 7.6 (4) 11.4

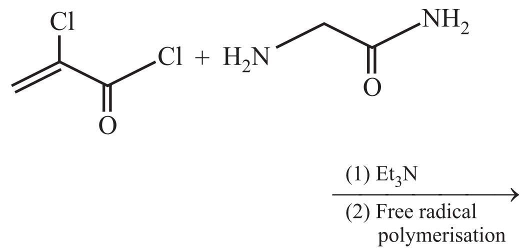
44. Which one of the following statements regarding Henry's law is not correct?
(1) Higher the value of K_H at a given pressure, higher is the solubility of the gas in liquids.
(2) Different gases have different K_H (Henry's law constant) values at the same temperature.
(3) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.
(4) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.

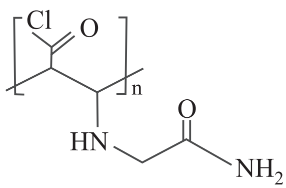
45. The following results were obtained during kinetic studies of the reaction;
$2A + B \rightarrow$ Products

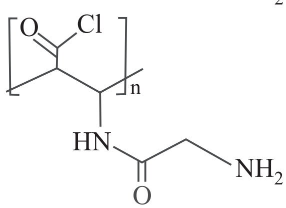
Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

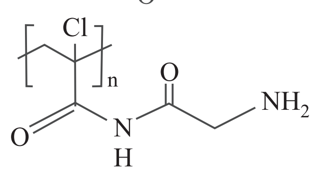
The time (in minutes) required to consume half of A is:
(1) 5 (2) 10 (3) 1 (4) 100

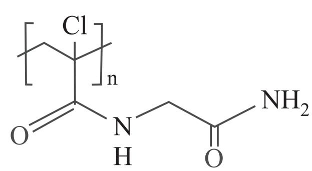
46. Major product of the following reaction is:



(1) 

(2) 

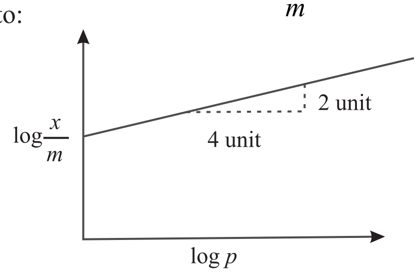
(3) 

(4) 

47. The alkaline earth metal nitrate that does not crystallise with water molecules, is:
(1) $Mg(NO_3)_2$ (2) $Sr(NO_3)_2$
(3) $Ca(NO_3)_2$ (4) $Ba(NO_3)_2$

48. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is:
[pK_b of NH_4OH = 4.7].
(1) 5.2 (2) 9.0
(3) 5.0 (4) 9.4

49. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\frac{x}{m}$ is proportional to:



(1) p^2 (2) $p^{1/4}$
(3) $p^{1/2}$ (4) p

50. Which amongst the following is the strongest acid?
(1) $CHBr_3$ (2) CHI_3
(3) $CH(CN)_3$ (4) $CHCl_3$

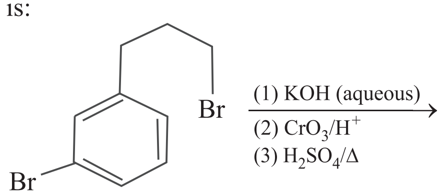
51. The ore that contains both iron and copper is:
(1) copper pyrites (2) malachite
(3) dolomite (4) azurite

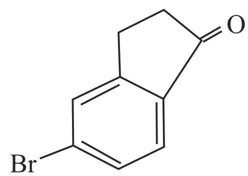
52. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number ($\overline{v}$) against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)
(1) Linear with intercept $-R_H$
(2) Non linear
(3) Linear with slope R_H
(4) Linear with slope $-R_H$

53. The isotopes of hydrogen are:
(1) Tritium and protium only
(2) Protium and deuterium only
(3) Protium, deuterium and tritium
(4) Deuterium and tritium only

54. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-?
(1) Li_2^+ is unstable and Li_2^- is stable
(2) Li_2^+ is stable and Li_2^- is unstable
(3) Both are stable
(4) Both are unstable

55. The major product of the following reaction is:



(1)


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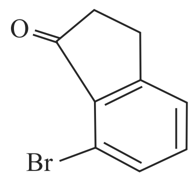

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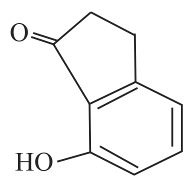

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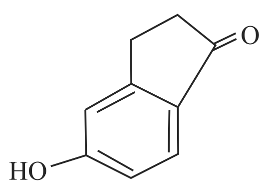


56. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and + 3 oxidation states. This is due to:

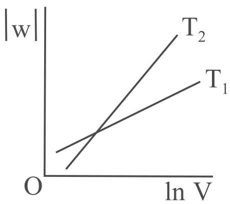
(1) inert pair effect
(2) diagonal relationship
(3) lattice effect
(4) lanthanoid contraction

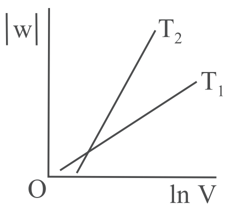
57. The increasing order of pKa of the following amino acids in aqueous solution is:

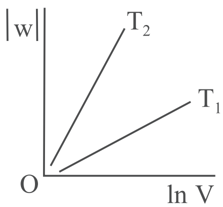
Gly Asp Lys Arg

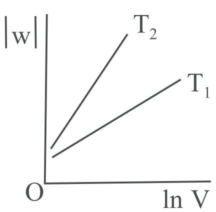
(1) Asp < Gly < Arg < Lys
(2) Gly < Asp < Arg < Lys
(3) Asp < Gly < Lys < Arg
(4) Arg < Lys < Gly < Asp

58. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

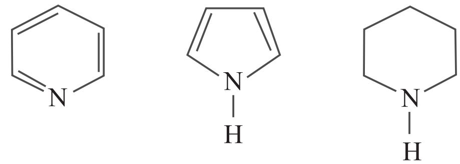
(1) 

(2) 

(3) 

(4) 

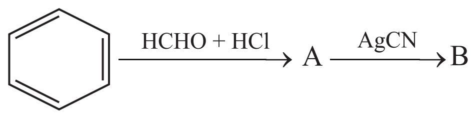
59. Arrange the following amines in the decreasing order of basicity :



I II III

(1) I > II > III
(2) III > I > II
(3) III > II > I
(4) I > III > II

60. The compounds A and B in the following reaction are, respectively :



(1) A = Benzyl alcohol, B = Benzyl cyanide
(2) A = Benzyl chloride, B = Benzyl cyanide
(3) A = Benzyl alcohol, B = Benzyl isocyanide
(4) A = Benzyl chloride, B = Benzyl isocyanide

MATHAMETICS

61. The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is:

(1) 0 (2) $\frac{4}{3}$
(3) $\frac{2}{3}$ (4) $-\frac{4}{3}$

62. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is:

(1) 6π (2) $3\sqrt{3}\pi$
(3) $\frac{4}{3}\pi$ (4) $2\sqrt{3}\pi$

63. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2\left(x^2-1\right)}{2\sin(x^2-1)+\sin 2\left(x^2-1\right)}}\,dx \text{ is equal}$$

to:

(1) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

(2) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

64. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1)=1$, then $y\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{7}{64}$ (2) $\frac{1}{4}$

(3) $\frac{49}{16}$ (4) $\frac{13}{16}$

65. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(1) $(5, 2\sqrt{6})$ (2) $(8, 6)$

(3) $(6, 4\sqrt{2})$ (4) $(4, -4)$

66. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola

$\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval:

(1) $(3, \infty)$ (2) $(3/2, 2]$

(3) $(2, 3]$ (4) $(1, 3/2]$

67. For $x\in$ **R** – {0, 1}, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to:

(1) $f_3(x)$ (2) $\frac{1}{x}f_3(x)$

(3) $f_2(x)$ (4) $f_1(x)$

68. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to:

(1) $\frac{19}{2}$ (2) 9

(3) 8 (4) $\frac{17}{2}$

69. If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be:

(1) – 2 (2) – 3

(3) 4 (4) 2

70. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$, then x is equal to:

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

71. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :

(1) $2\sqrt{3}y = 12x + 1$ (2) $\sqrt{3}y = x + 3$

(3) $2\sqrt{3}y = -x - 12$ (4) $\sqrt{3}y = 3x + 1$

72. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) is inconsistent when a = 4

(2) has a unique solution for |a| = $\sqrt{3}$

(3) has infinitely many solutions for a = 4

(4) is inconsistent when |a| = $\sqrt{3}$

73. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

(1) 6 (2) 8

(3) 4 (4) 14

74. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(2) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(3) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(4) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

75. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true?

(1) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(2) Each line passes through the origin.

(3) The lines are all parallel .

(4) The lines are not concurrent.

76. $\lim\limits_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) does not exist

77. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point:

(1) (– 3, 0, – 1) (2) (– 3, 1, 1)

(3) (3, 3, – 1) (4) (3, 2, 1)

78. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to:

(1) $\frac{4}{9}$ (2) $\frac{8}{15}$

(3) $\frac{7}{17}$ (4) $\frac{8}{17}$

79. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to:

(1) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

80. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$ then the ordered pair $(\oplus, \odot)$ is:

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$

(3) $(\wedge, \vee)$ (4) $(\wedge, \wedge)$

81. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 16 (2) 22

(3) 20 (4) 18

82. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

(1) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(2) $13 - 4\cos^6\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$

(4) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

83. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is:

(1) $\frac{8}{3}$ (2) $\frac{32}{3}$

(3) $\frac{56}{3}$ (4) $\frac{14}{3}$

84. Let $a_1, a_2, \ldots, a_{30}$ be an A.P., $S \sum^{30} a$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.

If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to:

(1) 52 (2) 57

(3) 47 (4) 42

85. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a + bx, & \text{if} \quad 1 < x < 3 \\ b + 5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5

(2) continuous if a = – 5 and b = 10

(3) continous if a = 0 and b = 5

(4) not continuous for any values of a and b

86. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} \text{ is purely imaginary}\right\}$

Then the sum of the elements in A is:

(1) $\frac{5\pi}{6}$ (2) π

(3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

87. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 500 (2) 200

(3) 300 (4) 350

88. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

(1) – 256 (2) 512

(3) – 512 (4) 256

89. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then:

(1) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(2) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

(3) a, b, c are in A.P

(4) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

90. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals:

(1) 49/169 (2) 52/169

(3) 24/169 (4) 25/169

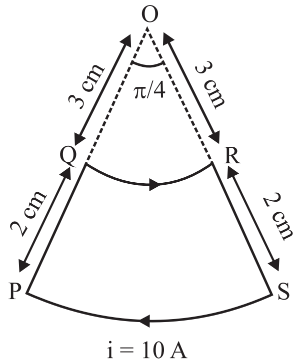
Hints and Solutions

PHYSICS

1. **(4)** There will be no magnetic field at O due to wire PQ and RS

Magnetic field at 'O' due to arc QR $= \dfrac{\mu_0}{4\pi}\dfrac{(10)}{\left(3\times10^{-2}\right)}\times\dfrac{\pi}{4}$

(Perpendicular outwards)



Magnetic field at 'O' due to arc PS

$= \dfrac{\mu_0}{4\pi}\times\dfrac{(10)}{\left(5\times10^{-2}\right)}\times\dfrac{\pi}{4}$ (Perpendicular inwards)

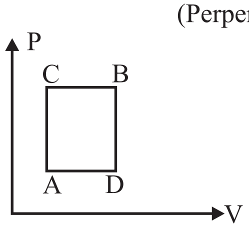
$\therefore$ Net magnetic field at 'O'

$$B = \frac{\mu_0}{4\pi}\times10\left[\frac{1}{\left(3\times10^{-2}\right)}-\frac{1}{\left(5\times10^{-2}\right)}\right]\times\frac{\pi}{4}$$

$$\Rightarrow B = \frac{\pi}{3}\times10^{-5}\,T \approx 1\times10^{-5}\,T$$

(Perpendicular outwards)

2. **(1)**



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow 60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

$\Rightarrow U_{ACB} = 30\text{ J}$

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$

$[\because\ \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}]$

3. **(2)** As we know,

$$|B| = \frac{|E|}{C} = \frac{6.3}{3\times10^8} = 2.1\times10^{-8}\,T$$

As $\vec{V}\perp\vec{E}\perp\vec{B}$ therefore direction of $\vec{B}$ is in z direction

$$\vec{B} = 2.1\times10^{-8}\,\hat{k}\,T$$

4. **(2)** $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{\dfrac{I_1}{I_2}}+1\right)^2}{\left(\sqrt{\dfrac{I_1}{I_2}}-1\right)^2} = 16\,(\text{given})$

$$\therefore\ \sqrt{\frac{I_1}{I_2}}+1 = 4\left(\sqrt{\frac{I_1}{I_2}}-1\right)$$

$$\therefore\ \sqrt{\frac{I_1}{I_2}} = \frac{5}{3}\quad\therefore\ \frac{I_1}{I_2} = \frac{25}{9}$$

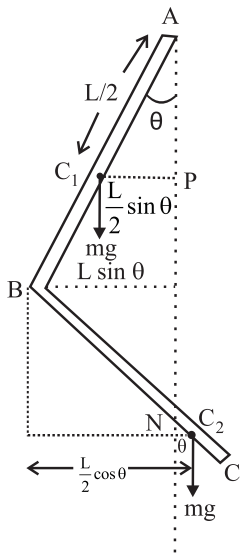
5. **(4)** Given that, the rod is of uniform mass density and AB = BC

Let mass of one rod is m.

Balancing torque about hinge point.

$mg\,(C_1P) = mg\,(C_2N)$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$



$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3}$$

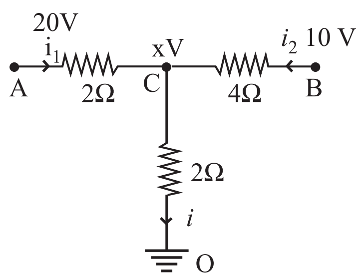
or, $\tan\theta = \frac{1}{3}$

6. **(1)** Using $V_{rms} = \sqrt{\frac{\gamma RT}{M}}$ we have

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}}$$

$= 3.16$

7. **(2)**



Let voltage at C = x volt

From kirchhoff's current law,

KCL : $i_1 + i_2 = i$

$$\frac{20-x}{2} + \frac{10-x}{4} = \frac{x-0}{2} \Rightarrow x = 10$$

$$\therefore\ i = \frac{V}{R} = \frac{x}{R} = \frac{10}{2} = 5A$$

8. **(2)** Using colour code we have

$R = 27 \times 10^3\ \Omega \pm 10\%$

$= 27\ k\Omega \pm 10\%$

9. **(2)** Corecivity, $H = \frac{B}{\mu_0}$ and $B = \mu_0 ni\left(n = \frac{N}{\ell}\right)$

or, $H = \frac{N}{\ell}i = \frac{100}{0.2} \times 5.2 = 2600$ A/m

10. **(1)** Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{(\Delta\ell/\ell)}$

Using, coefficient of linear expansion,

$$\frac{\Delta\ell}{\ell} = \alpha\Delta T$$

$$\therefore Y = \frac{F}{A(\alpha\Delta T)}$$

11. **(4)** Maximum speed is at equilibrium position where

$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,

$$W_F + W_{sp} = \Delta KE$$

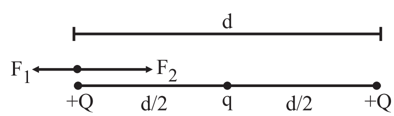
$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2$$

or, $v_{max} = \frac{F}{\sqrt{mk}}$

12. **(1)**



Force due to charge + Q,

$$F_1 = \frac{KQQ}{d^2}$$

Force due to charge q,

$$F_2 = \frac{KQq}{\left(\frac{d}{2}\right)^2}$$

For equilibrium,

$$\frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \qquad \therefore q = -\frac{Q}{4}$$

13. [Bonus]

Net charge $Q = \frac{\Delta\phi}{R} = \frac{1}{10}A(B_f - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3}$ $\left(0.4\sin\frac{\pi}{2} - 0\right)$

$$= \frac{1}{10}(3.5\times10^{-3})(0.4-0)$$

$= 1.4 \times 10^{-4}$

No option matches, so it should be a bonus.

14. (3) Distance of c.m from m/2

$$= \frac{\frac{m}{2}\times 0 + m\times\ell}{\frac{m}{2}+m} = \frac{2\ell}{3}$$

$$I_{cm} = \frac{m}{2}\left(\frac{2\ell}{3}\right)^2 + m\left(\frac{\ell}{3}\right)^2 = \frac{1}{3}m\ell^2$$

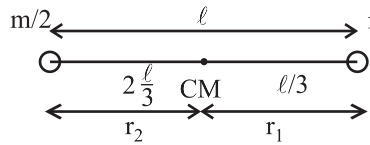
At the mean position

$$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}k\theta_0^2$$

$$\therefore\ \omega^2 = \frac{k}{I_{cm}}\theta_0^2$$

$$\omega^2 = \frac{3k}{m\ell^2}\theta_0^2$$

As we know, $\omega = \sqrt{\frac{k}{I_{cm}}}$



Tension in the rod when it passes through the mean position,

$$= m\omega^2\frac{\ell}{3} = m\left[\frac{3k}{m\ell^2}\theta_0^2\right]\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

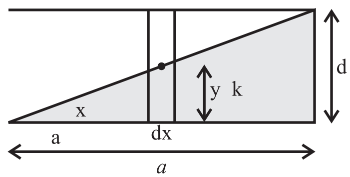
15. (3) Resistance, $R = \frac{\rho\ell}{A}$

$$R = \rho\frac{\ell}{A}\times\frac{\ell}{\ell} = \frac{\rho\ell^2}{V} \quad [\because \text{Volume } (V) = A\ \ell\,]$$

Since resistivity and volume remains constant therefore % change in resistance

$$\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 2\times(0.5) = 1\%$$

16. (2)



From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dC} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$$

$$\frac{1}{dC} = \frac{y}{\varepsilon_0 adx}\left(\frac{y}{k} + d - y\right)$$

$$\int dC = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$$

$$\text{or, } C = \varepsilon\ a.\frac{a}{}\int\frac{dy}{d + y\left(--1\right)}$$

$$\left[\because dy = \frac{d}{a}dx\right]$$

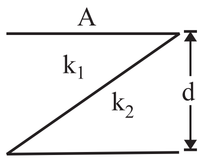
$$=\frac{\varepsilon_0 a^2}{\left(\frac{1}{k}-1\right)d}\left[\ell n\left(d+y\left(\frac{1}{k}-1\right)\right)\right]_0^d$$

$$=\frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{d+d\left(\frac{1}{k}-1\right)}{d}\right)$$

$$=\frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right)=\frac{k\in_0 a^2\ell nk}{(k-1)d}$$

Alternatively remember

$$C=\frac{k_1k_3A\,\varepsilon_0}{d(k_1-k_2)}\log_e\frac{k_1}{k_2}$$



Here $k_1 = 1$, $k_2 = k$, $A = a^2$

$$\therefore\ C=\frac{k\,a^2\varepsilon_0}{d(1-k)}\log_e\frac{1}{k}=\frac{ka^2\varepsilon_0}{d(k-1)}\log_e k$$

17. **(3)** $\rho=\frac{1}{\sigma}=\frac{1}{n_e e\mu_e}$

$$\left[\begin{array}{l}\because\ \sigma=e(n_e\mu_e+n_h\mu_h)\\ \text{Here } n_h\mu_h \text{is neglected}\end{array}\right]$$

$$=\frac{1}{10^{19}\times1.6\times10^{-19}\times1.6}$$

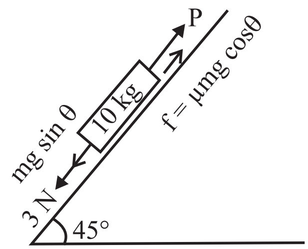
or $\rho = 0.4\ \Omega m$

18. **(3)** Areal velocity $=\frac{\pi R^2}{T}=\frac{\pi R^2}{(2\pi R/v)}=\frac{vR}{2}$

$$\therefore\frac{dA}{dt}=\frac{R}{2}\times\frac{L}{mR}\qquad[\because\ L=mvR]$$

$$\therefore\ \frac{dA}{dt}=\frac{1}{2}\frac{L}{m}$$

19. **(1)**



For equilibrium

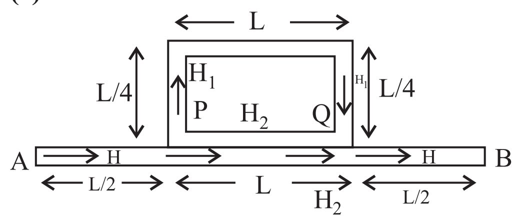
$3 + mg\sin\theta = P + \mu mg\cos\theta$

$$\Rightarrow 3+10\times10\times\frac{1}{\sqrt{2}}$$

$= P + 0.6 \times 10 \times 10 \times \cos45°$

$\therefore\ P = 31.28 \approx 32$ N

20. **(1)**



At P,

$H = H_1 + H_{2s}$

$$\frac{kA(T_A-T_P)}{L/2}$$

$$=\frac{kA(T_P-T_Q)}{3L/2}+\frac{kA(T_P-T_Q)}{L}$$

$\therefore\ 2\,(T_A-T_P)$

$$=\frac{2}{3}(T_P-T_Q)+(T_P-T_Q)$$

$$\therefore 2\,(T_A-T_P)=\frac{5}{3}(T_P-T_Q)\qquad\ldots(i)$$

At, Q

$H_1 + H_2 = H$

$$\therefore\ \frac{kA(T_P-T_Q)}{3L/2}+\frac{kA(T_P-T_Q)}{L}$$

$$= \frac{kA\left(T_Q - T_B\right)}{L/2}$$

$$\therefore\ 2\left(T_Q - T_P\right) = \frac{5}{3}\left(T_P - T_Q\right) \qquad \ldots\text{(ii)}$$

From (i) & (ii)

$$2\,(T_A - T_P) + 2\,(T_Q - T_B) = \frac{10}{3}\left(T_P - T_Q\right)$$

$$T_A - T_B = \frac{8}{3}\left(T_P - T_Q\right)$$

$$\therefore\ T_P - T_Q = \frac{3}{8}\times 120 = 45°\text{C}$$

21. **(2)** Wave speed $V = \sqrt{\dfrac{T}{\mu}}$

when car is at rest a = 0

$$\therefore 60 = \sqrt{\frac{Mg}{\mu}}$$

Similarly when the car is moving with acceleration a,

$$60.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}}$$

on solving we get

$$a = \frac{g}{\sqrt{30}} \qquad \text{[which is closest to g/5]}$$

22. **(3)** Activity A = λ N

For material, A

$10 = (2\,N_0)\lambda_A$

For material, B $\quad 20 = N_0\lambda_B$

$$\Rightarrow \lambda_B = 4\lambda_A$$

$$\therefore\ T_{\frac{1}{2}A} = 4\,T_{\frac{1}{2}B}\left[\because T_{\frac{1}{2}} = \frac{0.693}{\lambda}\right]$$

i.e. 20 days half-lives for A and 5 days $\left(T_{\frac{1}{2}}\right)_B$ for material B.

23. **(2)** For i ≈ 90° at air liquid interface we have by Snell's law

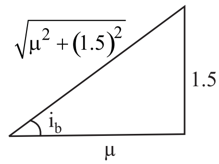
$$\mu = \frac{\sin 90°}{\sin r} \quad \therefore\ \sin r = \frac{1}{\mu}$$

According to Brewster's law, refractive index of liquid (μ) is equal to tangent of polarising angle

$$\because \tan i_b = \frac{1.5}{\mu}$$

$$\therefore\ \sin i_b = \frac{1.5}{\sqrt{\mu^2 + 1.5^2}}$$

Here $\sin r < \sin i_b$



$$\therefore\ \frac{1}{\mu} \le \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$$

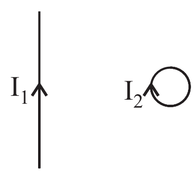
$$\text{or, } \sqrt{\mu^2 + (1.5)^2} \le 1.5 \times \mu$$

$$\Rightarrow \mu^2 + (1.5)^2 \le (\mu \times 1.5)^2$$

$$\Rightarrow \mu \ge \frac{3}{\sqrt{5}} \quad \text{i.e. minimum}$$

value of μ should be $\dfrac{3}{\sqrt{5}}$

24. **(4)** We know that $F = -\dfrac{dv}{dr}$ where r = distance of the loop from straight current carrying wire



Here

$$U = -\vec{m}.\vec{B} = -I_2\pi a^2 \times \frac{\mu_0}{4\pi}\frac{I_1}{r} \times 2 \times \cos 0$$

$$= -\frac{\mu_0\ I_1\ I_2\ a^2}{2r}$$

$$\therefore\ F = -\frac{d}{d(r)}\left[-\frac{\mu_0\ I_1 I_2 a^2}{2r}\right] = -\frac{\mu_0\ I_1 I_2 a^2}{r^2}$$

Here r = d

$$\therefore\ F \propto \frac{a^2}{d^2} \quad \text{(attractive)}$$

25. **(1)** From Einstein's photoelectric equation,

$$\frac{hc}{\lambda_1}-\phi=\frac{1}{2}m(2v)^2 \qquad ...(i)$$

and $\frac{hc}{\lambda_2}-\phi=\frac{1}{2}mv^2 \qquad ....(ii)$

From eqn. (i) & (ii)

$$\Rightarrow \frac{\frac{hc}{\lambda_1}-\phi}{\frac{hc}{\lambda_2}-\phi}=4 \Rightarrow \frac{hc}{\lambda_1}-\phi=\frac{4hc}{\lambda_2}-4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2}-\frac{hc}{\lambda_1}=3\phi \Rightarrow \phi=\frac{1}{3}hc\left(\frac{4}{\lambda_2}-\frac{1}{\lambda_1}\right)$$

$$=\frac{1}{3}\times 1240\left(\frac{4\times 350-540}{350\times 540}\right)$$

$= 1.8$ eV

26. **(3)** From given equation,

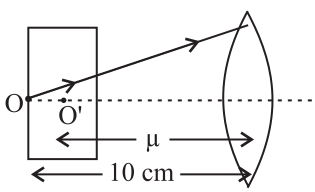
$$\vec{v}=k\left(y\hat{i}+x\hat{j}\right)=ky\hat{i}+kx\hat{j}=V_x\hat{i}+V_y\hat{j}$$

$$\frac{dx}{dt}=ky \text{ and } \frac{dy}{dt}=kx$$

$$\text{Now, } \frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{x}{y}=\frac{dy}{dx}\Rightarrow ydy=xdx$$

Integrating both sides we get $y^2 = x^2 + c$

27. **(4)**



As the object and image distance is same, object is placed at 2f. Therefore 2f = 10 or f = 5 cm.

Shift due to slab, $d=t\left(1-\frac{1}{\mu}\right)$

in the direction of incident ray

$$\Rightarrow d=1.5\left(1-\frac{2}{3}\right)=0.5 \text{ cm}$$

Now, $u = -9.5$ cm

Again using lens formulas $\frac{1}{v}-\frac{1}{-9.5}=\frac{1}{5}$

$\Rightarrow v = 10.55$ cm

Thus, screen is shifted by a distance d = 10.55 – 10 = 0.55 cm away from the lens.

28. **(2)** Electric field on the axis of a ring of radius R at a distance h from the centre,

$$\frac{kQh}{\left(h^2 \quad R^2\right)^{3/2}}$$

Condition for maximum electric field

we have $\frac{dE}{dh}=0$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2+h^2\right)^{3/2}}\right]=0$$

On solving we get, $h=\frac{R}{\sqrt{2}}$

29. **(3)** Kinetic energy of block A

$$k_1=\frac{1}{2}mv_0^2$$

∴ From principle of linear momentum conservation

$$mv_0=(2m+M)v_f \Rightarrow v_f=\frac{mv_0}{2m+M}$$

$$K.E_f=\frac{1}{6}K.E_i \qquad \text{(given)}$$

$$\frac{1}{2}(2m+M)v_f^2=\frac{1}{6}\times\frac{1}{2}mv_0^2$$

$$6(2m+M)\frac{m^2v_0^2}{(2m+M)^2}=mv_0^2$$

$\Rightarrow 6m = 2m + M$

$\Rightarrow 4m = M$

$$\therefore \frac{M}{m}=\frac{4}{1}$$

30. **(1)** Using, $I = neAv_d$

$\therefore$ Drift speed $v_d = \dfrac{I}{neA}$

$$\frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}}$$

$= 0.02 \times 10^{-3}\ ms^{-1}$
$= 0.02\ mms^{-1}$

CHEMISTRY

31. **(1)** E.C. of Cr^{3+} $(3d^3)$:

↿	↿	↿		

$3d$

For complex A $[Cr(H_2O)_2]^{3+}$:

↿	↿	↿	xx	xx		xx		xx	xx	xx

$3d$ $\quad$ $\underbrace{4s \quad 4p}_{d^2sp^3 \text{ hybridisation}}$

For complex B $[Cr(NH_3)_6]^{3+}$:

↿	↿	↿	xx	xx		xx		xx	xx	xx

$3d$ $\quad$ $\underbrace{4s \quad 4p}_{d^2sp^3 \text{ hybridisation}}$

Here, both the complexes (A) and (B) are paramagnetic with 3 unpaired electrons each. Also H_2O is a weak field ligand which causes lesser splitting than NH_3 which is comparatively stronger field ligand. Hence, the (Δ_0) value of (A) and (B) are calculated from the wavelengths of light absorbed and not from the wavelengths of light emitted.

32. **(4)** The acidic strength of a compound or an acid depends on the inductive effect (–I). Higher the (–I) effect of a substituent higher will be acidic strength. Now, the decreasing order of (–I) effect of the given substituents is $NO_2 > CN > F > Cl$.

$\therefore$ The correct decreasing order of acidic strength amongst the given carboxylic acids is:

$NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$

33. **(3)** $R{-}C{\equiv}N \xrightarrow[\text{(ii) } H_2O]{\text{(i) } AlH(i\text{-}Bu)_2} R{-}CHO$

The reduction of nitriles to aldehydes can be done using DIBAL-H$[AlH(i\text{-}Bu)_2]$.

34. **(1)** Magnetic moment, $\mu = \sqrt{n(n+2)}$ BM
(where, n = no. of unpaired electrons)
As transition metal atom/ion in a complex may have unpaired electrons ranging from zero to 5. So, maximum number of unpaired electrons that may be present in a complex is 5.
$\therefore$ Maximum value of magnetic moment among all the transition metal complexes is

$= \sqrt{5(5+2)} = \sqrt{35} = 5.92$ BM

35. **(4)** Ideal gas equation: $PV = nRT$
After putting the values we get,
$200 \times 10 = (0.5 + x) \times R \times 1000$
(total no. of moles are $0.5 + x$)

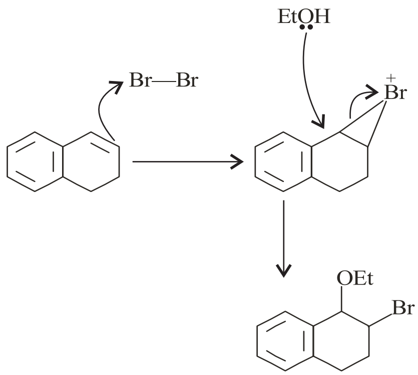
$$x = \frac{4-R}{2R}$$

36. **(3)** Quartz exhibits piezoelectricity and thus can be used as a piezoelectric material.

37. **(4)** Silicones are polymers containing Si—O—Si linkages with strong hydrophobic character.

Generally, they exhibit high thermal stability with high dielectric strength. Silicon greases are resistant to oxidation which are commonly used for greasing purposes.

38. **(1)** Mechanism involved for the given reaction is:



(major product)

39. **(3)** Generally, electronegativity decreases down the group as the size increases. This can also be formulated as:

Electronegativity $\propto \dfrac{1}{\text{size}}$

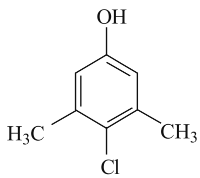
40. **(2)** Number of moles in 92 g of $Na^+ = \dfrac{92}{23}$

= 4 moles

$$\text{Molality } (m) = \frac{\text{Number of moles}}{\text{Mass of solvent (in kg)}}$$

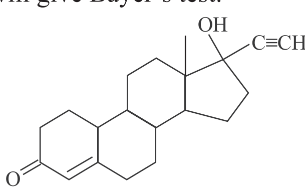
$$\therefore \quad m = \frac{4}{1} = 4 \text{ mol kg}^{-1}$$

41. **(3)** As chloroxylenol contains phenolic group so it gives positive ferric chloride test.



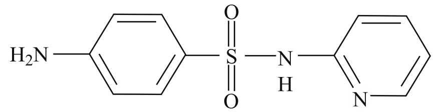
Chloroxylenol

Norethindrone has double bond, thus it will give Bayer's test.



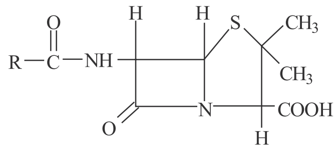
Norethindrone

Sulphapyridine contains $–NH_2$ group so it gives carbylamine test.



Sulphapyridine

Penicillin contains –COOH group so it will give sodium hydrogen carbonate ($NaHCO_3$) test.



Pencillin

42. **(2)** The water sample containing Mn = 5 ppm is unsuitable for drinking as the prescribed level for Mn in drinking water is 0.5 ppm.

43. **(3)** Half cell reaction: $PbSO_4 \rightarrow Pb^{4+} + 2e^-$

According to the reaction:

$PbSO_4 \rightarrow Pb^{4+} + 2e^-$

We require 2F for the electrolysis of 1 mol or 303 g of $PbSO_4$

$\therefore$ Amount of $PbSO_4$ electrolysed by 0.05F

$$= \frac{303}{2} \times .05 = 7.575 \text{ g} \approx 7.6 \text{ g}$$

44. **(1)** The solubility of the gas in liquids decreases with the increase in value of K_H at a given pressure.

45. **(1)** From experiment 1 and II, it is observed that order of reaction w.r.t. (c) is zero.

From experiment II and III, α can be calculated as:

$$\frac{1.386 \times 10^{-2}}{6.93 \times 10^{-3}} = \left(\frac{0.2}{0.1}\right)^{\alpha}$$

$\therefore \alpha = 1$

Now, $\quad$ Rate = $K[A]^1$

or, $\quad 6.93 \times 10^{-3} = K(0.1)$

$K = 6.93 \times 10^{-2}$

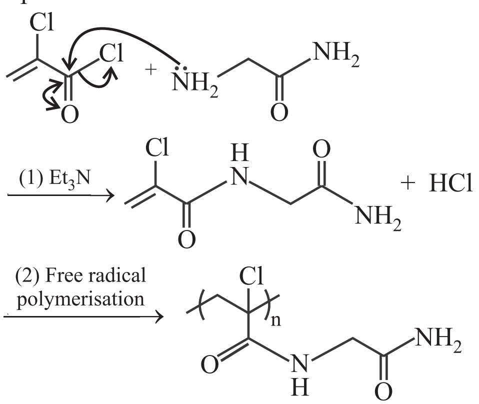
For the reaction, $2A + B \rightarrow$ Products

$$2Kt = \ln\frac{[A]_0}{[A]}$$

$$\therefore \quad t_{1/2} = \frac{0.693}{2K} = \frac{0.693}{06.93\times10^{-2}\times2}$$

$$t_{1/2} = 5$$

46. **(4)** Mechanism for the formation of major product is as follows:



47. **(4)** The chances of formation of hydrate decreases with the decrease in the charge density down the group. This is why, $Ba(NO_3)_2$ does not crystallise with water molecules.

48. **(2)** m. mol of $H_2SO_4 = 20 \times 0.1 = 2$

m. mol of $NH_4OH = 30 \times 0.2 = 6$

	H_2SO_4 +	$2NH_4OH \rightarrow$	$(NH_4)_2SO_4 + 2H_2O$
Initial	2 m mol	6 m mol	0
Final	(2-2) = 0 m mol	$(6 - 2 \times 2)$ = 2 m mol	2 m mol

$[NH_4OH]_{left} = 2$ m mol

$[(NH_4)_2SO_4] = 2$ m mol

$[\overset{+}{N}H_4] = 2 \times 2 = 4$ m mol

Total Volume = 30 + 20 = 50 mL

$$pOH = pK_b + \log\left[\frac{\text{Salt}}{\text{Base}}\right]$$

$$= 4.7 + \log\frac{4/50}{2/50}$$

$$= 4.7 + \log 2 = 5$$

pH = 14 – pOH

pH = 14 – 5 = 9

49. **(3)** In Freundlich adsorption isotherm the extent of adsorption (x/m) of a gas on the surface of a solid is related to the pressure of the gas (P) which can be formulated as:

$$\frac{x}{m} = k(p)^{1/n}$$

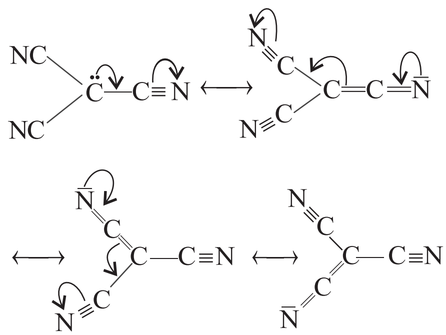
$$\Rightarrow \quad \log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

In the given plot, the slope between $\log\frac{x}{m}$ versus $\log P = \frac{2}{4} = \frac{1}{2}$

$$\therefore \quad \frac{x}{m} \propto p^{1/2}$$

50. **(3)** Due to the resonance stabilisation of the conjugate base, $CH(CN)_3$ is the strongest acid amongst the given compounds.

$$CH(CN)_3 \rightleftharpoons H^+ + \overset{-}{C}(CN)_3$$



The conjugate bases of $CHBr_3$ and CHI_3 are stabilised by inductive effect of halogens. This is why, they are less stable. Also, the conjugate base of $CHCl_3$ involves back-bonding between $2p$ and $3p$ orbitals.

51. **(1)** Amongst the given ores, copper pyrite ($CuFeS_2$), dolomite ($MgCO_3.CaCO_3$), malachite [$CuCO_3.Cu(OH)_2$], azurite [$2CuCO_3.Cu(OH)_2$], copper pyrite contains both copper and iron.

52. (4) As we know,

$$\bar{v} = -R_H\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)Z^2 \text{ (where, } Z = 1)$$

After putting the values, we get

$$\bar{v} = -R_H\left(\frac{1}{n^2}-\frac{1}{8^2}\right)$$

$$\Rightarrow \quad \bar{v} = \frac{R_H}{64}-\frac{R_H}{n^2}$$

Comparing to $y = mx + c$, we get

$$x = \frac{1}{n^2} \text{ and } m = -R_H \text{ (slope)}$$

53. (3) Hydrogen has three isotopes:

Protium (${}_1H^1$), deuterium (${}_1H^2$) and tritium (${}_1H^3$).

54. (3) Electronic configuratios of Li_2^+ and Li_2^-:

Li_2^+: $\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^1$

Li_2^-: $\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^1$

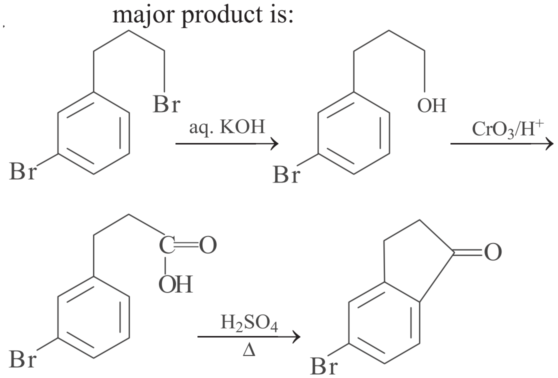
Now,

Bond order of $Li_2^+ = \frac{1}{2}(3-2) = \frac{1}{2}$

Bond order of $Li_2^- = \frac{1}{2}(4-3) = \frac{1}{2}$

Here, both Li_2^+ and Li_2^- have positive bond order, thus both are stable.

55. (1) For the given reaction condition, the major product is:



56. (1) Due to the inert pair effect, thallium exists in more than one oxidation state. Also, for thallium + 1 oxidation state is more stable than +3 oxidation state.

57. (3) Structure of the given α-amino acids are:

$H\overset{+}{N}_3—CH_2—COO^-$
Glycine (Gly)

$HOOC—CH_2—\underset{\overset{+}{N}H_3}{CH}—COO^-$
Aspartic acid (Asp)

$H_2N—(CH_2)_4—\underset{\overset{+}{N}H_3}{CH}—COO^-$
Lysine (Lys)

$H_2N—\overset{\overset{NH}{\|}}{C}—NH—(CH_2)_3—\underset{\overset{+}{N}H_3}{CH}—COO^-$
Arginine (Arg)

Here, aspartic acid is an acidic and glycine is a neutral amino acid while lysine and arginine are basic amino acids. Also, arginine is more basic due to the stronger basic functional groups.

∴ The order of *pKa* value is directly proportional to the basic strength of amino acids, i.e. Arg > Lys > Gly > Asp.

58. (2) For reversible isothermal expansion,

$$w = -nRT \ln \frac{V_2}{V_1}$$

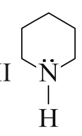
$$\Rightarrow \quad |w| = nRT \ln \frac{V_2}{V_1}$$

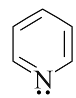
$|w| = nRT\ (\ln V_2 - \ln V_1)$

$|w| = nRT \ln V_2 - nRT\ V_1$

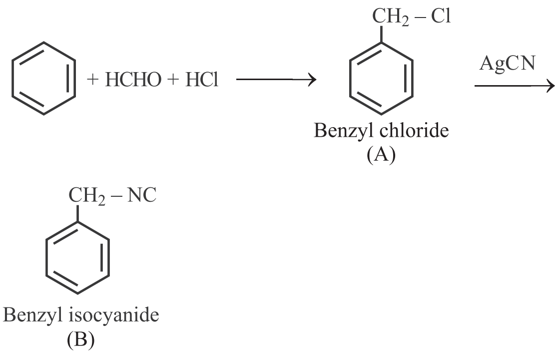
$y = mx + c$

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative than curve 1.

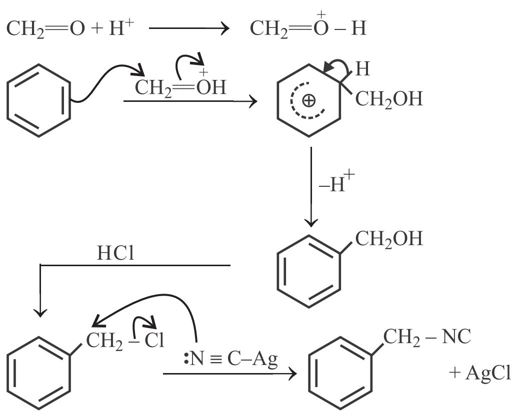
59. (2) Compound, III  is most basic as the lone pair of nitrogen is easily available for the donation.

In case of compound (I)  lone pair is not involved in resonance but nitrogen atom is sp^2 hybridised whereas in compound II the lone pair of nitrogen is involved in aromaticity which makes it least basic.

60. (4)



Mechanism:



MATHEMATICS

61. (2) $I = \int_0^{\pi} |\cos x|^3\, dx$

$$= 2\int_0^{\pi/2} \cos^3 x\, dx$$

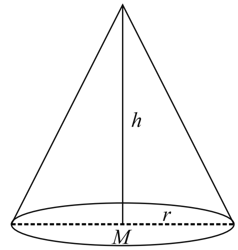
$$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)dx$$

$[\because\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$$= \frac{1}{2}\left[3\sin x + \frac{\sin 3x}{3}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(3 - \frac{1}{3}\right) = \frac{4}{3}$$

62. (4)



$$h^2 + r^2 = \ell^2 = 9 \quad \text{...(1)}$$

Volume of cone

$$V = \frac{1}{3}\pi r^2 h \quad \text{...(2)}$$

From (1) and (2),

$$\Rightarrow\ V = \frac{1}{3}\pi(9 - h^2)h$$

$$\Rightarrow\ V = \frac{1}{3}\pi(9h - h^3)$$

$$\Rightarrow\ \frac{dv}{dh} = \frac{1}{3}\pi\left(9 - 3h^2\right)$$

For maxima/minima,

$$\frac{dV}{dh} = 0 \Rightarrow \frac{1}{3}\pi(9 - 3h^2) = 0$$

$$\Rightarrow\ h = \pm\sqrt{3} \Rightarrow h = \sqrt{3} \qquad (\because\ h > 0)$$

Now; $\dfrac{d^2V}{dh^2} = \dfrac{1}{3}\pi(-6h)$

Here, $\left(\dfrac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$

Then, $h = \sqrt{3}$ is point of maxima

Hence, the required maximum volume is,

$$V = \frac{1}{3}\pi(9-3)\sqrt{3} = 2\sqrt{3}\pi$$

63. (3, 4) Consider the given integral

$$I = \int x\sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}}dx$$

$(\therefore\ \sin 2\theta = 2\sin\theta\cos\theta)$

$$\Rightarrow\ I = \int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}dx$$

$\Rightarrow \quad I=\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx,$

Now let $\frac{x^2-1}{2}=t \quad \Rightarrow \quad \frac{2x}{2}dx=dt$

$\therefore \quad I=\int|\tan(t)|\,dt=\ln|\sec t|+C$

or $\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

64. (3) Since, $x\frac{dy}{dx}+2y=x^2$

$\Rightarrow \quad \frac{dy}{dx}+\frac{2}{x}y=x$

I.F. $=e^{\int\frac{2}{x}dx}=e^{2\ln x}=e^{\ln x^2}=x^2.$

Solution of differential equation is:

$y\cdot x^2=\int x\cdot x^2dx$

$y\cdot x^2=\frac{x^4}{4}+C \qquad \text{...(1)}$

$\because \quad y(1)=1$

$\therefore \quad C=\frac{3}{4}$

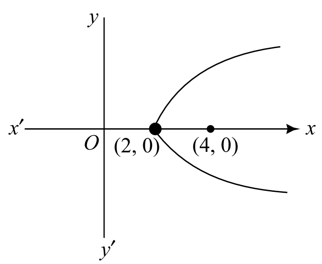
Then, from equation (1)

$y\cdot x^2=\frac{x^4}{4}+\frac{3}{4}$

$\therefore \quad y=\frac{x^2}{4}+\frac{3}{4x^2}$

$\therefore \quad \left(-\right)=\frac{1}{16}+3=\frac{49}{16}$

65. (2) Since, vertex and focus of given parabola is (2, 0) and (4, 0) respectively



Then, equation of parabola is

$(y-0)^2=4\times 2(x-2)$

$\Rightarrow \quad y^2=8x-16$

Hence, the point (8, 6) does not lie on given parabola.

66. (1) $\because\ a^2=\cos^2\theta,\ b^2=\sin^2\theta$

and $e>2 \quad \Rightarrow \quad e^2>4 \quad \Rightarrow \quad 1+b^2/a^2>4$

$\Rightarrow \quad 1+\tan^2\theta>4$

$\Rightarrow \quad \sec^2\theta>4 \quad \Rightarrow \quad \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$

Latus rectum,

$LR=\frac{2b^2}{a}=\frac{2\sin^2\theta}{\cos\theta}=2(\sec\theta-\cos\theta)$

$\Rightarrow \quad \frac{d(LR)}{d\theta}=2(\sec\theta\tan\theta+\sin\theta)>0\ \forall\ \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$

$\therefore \quad \min(LR)=2\left(\sec\frac{\pi}{3}-\cos\frac{\pi}{3}\right)=2\left(2-\frac{1}{2}\right)=3$

max (LR) tends to infinity as $\theta\to\frac{\pi}{2}$

Hence, length of latus rectum lies in the interval $(3,\infty)$

67. (1) The given relation is

$(f_2 o\ J o\ f_1)(x)=f_3(x)=\frac{1}{1-x}$

$\Rightarrow (f_2 o J)(f_1(x))=\frac{1}{1-x}$

$\Rightarrow (f_2 o\ J)\left(\frac{1}{x}\right)=\frac{1}{1-x} \qquad \left[\because f_1(x)=\frac{1}{x}\right]$

$\Rightarrow f_2\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$

$\Rightarrow (f_2\ J(x))=\frac{1}{1-\frac{1}{x}}=\frac{x}{x-1} \qquad \left[\frac{1}{x}\text{ is replaced by }x\right]$

$\Rightarrow 1-J(x)=\frac{x}{x-1} \qquad [\because f_2(x)=1-x]$

$\therefore J(x)=1-\frac{x}{x-1}=\frac{1}{1-x}=f_3(x)$

68. (1) $\because |\vec{a}\times\vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2 - (\vec{a}.\vec{c})^2$

$\Rightarrow |-\vec{b}|^2 = 2|\vec{c}|^2 - 16$

$\Rightarrow 3 = 2|\vec{c}|^2 - 16$

$\Rightarrow |\vec{c}|^2 = \dfrac{19}{2}$

69. (4) $\because a, b, c,$ are in G.P.

$\Rightarrow b^2 = ac$

Since, $a + b + c = xb$

$\Rightarrow a + c = (x - 1)b$

Take square on both sides, we get

$a^2 + c^2 + 2ac = (x - 1)^2 b^2$

$\Rightarrow a^2 + c^2 = (x - 1)^2 ac - 2ac\ [\because b^2 = ac]$

$\Rightarrow a^2 + c^2 = ac[(x - 1)^2 - 2]$

$\Rightarrow a^2 + c^2 = ac[x^2 - 2x - 1]$

$\because a^2 + c^2$ are positive and $b^2 = ac$ which is also positive. Then, $x^2 - 2x - 1$ would be positive but for $x = 2$, $x^2 - 2x - 1$ is negative.

Hence, x cannot be taken as 2.

70. (1) $\cos^{-1}\left(\dfrac{2}{3x}\right) + \cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2}; \left(x > \dfrac{3}{4}\right)$

$\Rightarrow \cos^{-1}\left(\dfrac{2}{3x}\right) = \dfrac{\pi}{2} - \cos^{-1}\left(\dfrac{3}{4x}\right)$

$\Rightarrow \cos^{-1}\left(\dfrac{2}{3x}\right) = \sin^{-1}\left(\dfrac{3}{4x}\right)$

$\left[\because \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}\right]$

Put $\sin^{-1}\left(\dfrac{3}{4x}\right) = \theta \quad \Rightarrow \sin\theta = \dfrac{3}{4x}$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \dfrac{9}{16x^2}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{\sqrt{16x^2 - 9}}{4x}\right)$

$\therefore \cos^{-1}\left(\dfrac{2}{3x}\right) = \cos^{-1}\left(\dfrac{\sqrt{16x^2 - 9}}{4x}\right)$

$\Rightarrow \dfrac{}{} = \dfrac{\sqrt{16x^2 - 9}}{4x} \Rightarrow x^2 = \dfrac{64 + 81}{9 \times 16}$

$\Rightarrow x = \pm\sqrt{\dfrac{145}{144}}$

$\Rightarrow x = \dfrac{\sqrt{145}}{12} \qquad \left(\because x > \dfrac{3}{4}\right)$

71. (2) Since, the equation of tangent to parabola $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$ ...(1)

The line (1) is also the tangent to circle

$x^2 + y^2 - 6x = 0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

$\therefore \dfrac{\left|3m + \dfrac{1}{m}\right|}{\sqrt{1 + m^2}} = 3 \quad \Rightarrow \left(3m + \dfrac{1}{m}\right)^2 = 9(1 + m^2)$

$\Rightarrow m = \pm\dfrac{1}{\sqrt{3}}$

Then, from equation (1): $y = \pm\dfrac{1}{\sqrt{3}}x \pm \sqrt{3}$

Hence, $\sqrt{3}y = x + 3$ isoneoftherequiredcommon tangent.

72. (4) Since the system of linear equations are

$x + y + z = 2$ ...(1)

$2x + 3y + 2z = 5$ ...(2)

$2x + 3y + (a^2 - 1)z = a + 1$ ...(3)

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are $2x + 3y + 2z = 5$ and

$2x + 3y + 2z = \pm\sqrt{3} + 1$ which are parallel, i.e., have no solution.

Hence, the given system of equations is inconsistent, for $|a| = \sqrt{3}$

73. (2) $2^{403} = 2^{400} \cdot 2^3$

$= 2^{4 \times 100} \cdot 2^3$

$= (2^4)^{100} \cdot 8$

$= 8(2^4)^{100} = 8(16)^{100}$

$= 8\,(1 + 15)^{100}$

$= 8 + 15\,\mu$

When 2^{403} is divided by 15, then remainder is 8.

Hence, fractional part of the number is $\frac{8}{15}$

Therefore value of k is 8

74. (3) Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda \text{ (say)}$$

is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$

Since, the above point lies on a line which passes through the point $(-4, 3, 1)$

Then, direction ratio of the required line

$= \langle -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 \rangle$

or $\langle -3\lambda + 3, 2\lambda, -\lambda + 1 \rangle$

Since, line is parallel to the plane

$x + 2y - z - 5 = 0$

Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$

$\Rightarrow \lambda = -1$

Now direction ratio of the required line

$= \langle 6, -2, 2 \rangle$ or $\langle 3, -1, 1 \rangle$

Hence required equation of the line is

$$\frac{(x+4)}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

75. (1) The given equations of the set of all lines

$px + qy + r = 0$ ...(1)

and given condition is :

$3p + 2q + 4r = 0$

$\Rightarrow \frac{3}{4}p + \frac{2}{4}q + r = 0$ ...(2)

From (1) & (2) we get :

$\therefore\ x = \frac{3}{4}, y = \frac{1}{2}$

Hence the set of lines are concurrent and passing through the fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$

76. (1) $L = \lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

$$= \lim_{y \to 0} \frac{\left(\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y \to 0} \frac{1+\sqrt{1+y^4} - 2}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{1+y^4} - 1\right)\left(\sqrt{1+y^4} + 1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \lim_{y \to 0} \frac{1+y^4 - 1}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \frac{1}{2\sqrt{2} \times 2} = \frac{1}{4\sqrt{2}}$$

77. (4) Since, equation of plane through intersection of planes

$x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

But, the above plane is parallel to y-axis then

$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$

$\Rightarrow \lambda = -3$

Hence, the equation of required plane is

$-x - 4z + 7 = 0$

$\Rightarrow x + 4z - 7 = 0$

Therefore, (3, 2, 1) the passes through the point.

78. (2) Since, the equation of curves are

$y = 10 - x^2$ ...(1)

$y = 2 + x^2$ ...(2)

Adding eqn (1) and (2), we get

$2y = 12 \Rightarrow y = 6$

Then, from eqn (1)

$x = \pm 2$

Differentiate equation (1) with respect to x

$$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = -4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$$

Differentiate equation (2) with respect to x

$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = 4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

$$\text{At } (2, 6) \tan\theta = \left(\frac{(-4)-(4)}{1+(-4)\times(4)}\right) = \frac{-8}{15}$$

$$\text{At } (-2, 6), \tan\theta = \frac{(4)-(-4)}{1+(4)(-4)} = \frac{8}{-15}$$

$$\therefore |\tan\theta| = \frac{8}{15}$$

79. (3) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$(A^{-50})_{\theta=\frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\left[\because \cos\left(\frac{50\pi}{12}\right) = \cos\left(4\pi + \frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$$

80. (3) Check each option

(1) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(2) $(p \vee q) \wedge (\sim p \vee q) = (p \wedge \sim p) \vee q = F \vee q = q$

(3) $(p \wedge q) \wedge (\sim p \vee q) = (p \wedge q \wedge \sim p) \vee (p \wedge q) \wedge q$
$= F \vee (p \wedge q) = p \wedge q$

(4) $(p \wedge q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge q = F \sim q = F$

81. (3) $\because$ Variance $= \sigma^2 = \frac{\Sigma x_i^2}{N} - (\bar{x})^2$

$$\Rightarrow 18 = \frac{\Sigma x_i^2}{5} - (150)^2$$

$$\Rightarrow \Sigma x_i^2 = 90 + 112590 = 112590$$

Then, variance of the height of six students

$$V' = \frac{112590 + (156)^2}{6} - \left(\frac{750+156}{6}\right)^2$$

$= 22821 - 22801$

$= 20$

82. (2) $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - 2\sin\theta\cos\theta)^2 + 6$
$(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6$
$+ 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

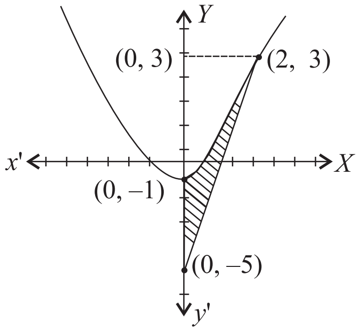
$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta$
$+ 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

83. (1)



$\because$ Curve is given as :
$y = x^2 - 1$

$$\Rightarrow \frac{dy}{dx} = 2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = 4$$

$\therefore$ equation of tangent at (2, 3)

$(y - 3) = 4\,(x - 2)$

$\Rightarrow y = 4x - 5$

but $x = 0$

$\Rightarrow y = -5$

Here the curve cuts Y–axis
$\therefore$ required area

$$= \frac{1}{4}\int_{-5}^{3}(y+5)dy - \int_{-1}^{3}\sqrt{y+1}\,dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2}+5y\right]_{-5}^{3} \frac{-2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$$

$$= \frac{1}{4}\left[\frac{9}{2}+15-\frac{25}{2}+25\right] - \frac{2}{3}\left[4^{3/2}-0\right]$$

$$= \frac{32}{4} - \frac{16}{3} = \frac{8}{3} \text{ sq-units.}$$

84. (1) $S = \sum_{i=1}^{30} a_i = \frac{30}{2}[2a_1 + 29d]$

$$T = \sum_{i=1}^{15} a_{(2i-1)} = \frac{15}{2}[2a_1 + 28d]$$

Since, $S - 2T = 75$

$\Rightarrow 30\,a_1 + 435d - 30a_1 - 420\text{d} = 75$
$\Rightarrow d = 5$
Also, $a_5 = 27 \quad \Rightarrow \quad a_1 + 4d = 27$
$\Rightarrow a_1 = 7,$
Hence, $\quad a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

85. (4) Let $f(x)$ is continuous at $x = 1$, then

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow \quad 5 = a + b \qquad . \qquad$ ..(1)

Let $f(x)$ is continuous at $x = 3$, then

$f(3^-) = f(3) = f(3^+)$

$\Rightarrow \quad a + 3b = b + 15 \quad \Rightarrow$ a + 2b = 15 $\qquad$ ...(2)

Solving (i) & (2) we get b = 10, a = –5

Now $f(x)$ is continuous at $x = 5$, then

$f(5^-) = f(5) = f(5^+)$

$\Rightarrow \quad b + 25 = 30$

Which is not satisfied by a = –5 and b = 10.

Hence, $f(x)$ is not continuous for any values of a and b

86. (4) Suppose $z = \dfrac{3+2i\sin\theta}{1-2i\sin\theta}$

Since, z is purely imaginary, then $z + \overline{z} = 0$

$$\Rightarrow \frac{3+2i\sin\theta}{1-2i\sin\theta} + \frac{3-2i\sin\theta}{1+2i\sin\theta} = 0$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta} = 0$$

$$\Rightarrow \sin^2\theta = \frac{3}{4} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

Now, the sum of elements in A

$$= -\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$$

87. (3) Since, the number of ways to select 2 girls is ${}^5C_2$.

Now, 3 boys can be selected in 3 ways.

(1) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways

(2) Selection of B and selection of any 2 two other boys (except A) in $^5C_2$ ways

(3) Selection of 3 boys (except A and B) in $^5C_3$ ways

Hence, required number of different teams

$= {}^5C_2\,({}^5C_2 + {}^5C_2 + {}^5C_3) = 300$

88. (1) Consider the equation

$$x^2 + 2x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$$

Let $\alpha = -1 + i$, $\beta = -1 - i$

$$\alpha^{15} + \beta^{15} = (-1+i)^{15} + (-1-i)^{15}$$

$$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

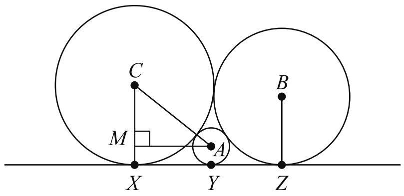
$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15}.2\cos\frac{45\pi}{4} = \left(\sqrt{2}\right)^{15}.2\cos\frac{3\pi}{4}$$

$$= = \frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$$

$$= -2\left(\sqrt{2}\right)^{14} = -256$$

89. (1)



$$AM^2 = AC^2 - MC^2$$

$= (a+c)^2 - (a-c)^2 = 4ac$

$\Rightarrow\ AM^2 = XY^2 = 4ac$

$\Rightarrow\ XY = 2\sqrt{ac}$

Similarly, $YZ = 2\sqrt{ba}$ and $XZ = 2\sqrt{bc}$

Then, $\quad XZ = XY + YZ$

$\Rightarrow\ 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$

$$\Rightarrow\ \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$$

90. (4) X = number of aces drawn

$\therefore\ P(X=1) + P(X=2)$

$$= \left\{\frac{4}{52}\times\frac{48}{52} + \frac{48}{52}\times\frac{4}{52}\right\} + \left\{\frac{4}{52}\times\frac{4}{52}\right\}$$

$$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$$