CHAPTER 24

Differential Equations

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are **[2002]**

(a) $(1, \frac{2}{3})$ (b) $(3, 1)$

(c) $(3, 3)$ (d) $(1, 2)$

2. The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$ **[2002]**

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4}+cx+d$

(c) $\frac{1}{4}e^{-2x}+cx^2+d$ (d) $\frac{1}{4}e^{-4x}+cx+d$

3. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively.

(a) 2, 3 (b) 2, 1 **[2003]**

(c) 1, 2 (d) 3, 2.

4. The solution of the differential equation

$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$, is **[2003]**

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

(b) $(x-2)=ke^{2\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

5. The differential equation for the family of circle $x^2+y^2-2ay=0$, where a is an arbitrary constant is **[2004]**

(a) $(x^2+y^2)y'=2xy$

(b) $2(x^2+y^2)y'=xy$

(c) $(x^2-y^2)y'=2xy$

(d) $2(x^2-y^2)y'=xy$

6. Solution of the differential equation $ydx+(x+x^2y)dy=0$ is **[2004]**

(a) $\log y=Cx$ (b) $-\frac{1}{xy}+\log y=C$

(c) $\frac{1}{xy}+\log y=C$ (d) $-\frac{1}{xy}=C$

7. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$, is a parameter, is of order and degree as follows : **[2005]**

(a) order 1, degree 2 (b) order 1, degree 1

(c) order 1, degree 3 (d) order 2, degree 2

8. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is **[2005]**

(a) $y\log\left(\frac{x}{y}\right)=cx$ (b) $x\log\left(\frac{y}{x}\right)=cy$

(c) $\log\left(\frac{y}{x}\right)=cx$ (d) $\log\left(\frac{x}{y}\right)=cy$

9. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of **[2006]**

(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**

(a) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\frac{dy}{dx}$

(c) $x^2 = y^2 + xy\frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\frac{dy}{dx}$.

11. The solution of the differential equation $\frac{dy}{dy} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is **[2008]**

(a) $y = \ln x + x$ (b) $y = x\ln x + x^2$
(c) $y = xe^{(x-1)}$ (d) $y = x\ln x + x$

12. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1, and c_2 are arbitrary constants, is **[2009]**

(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$

13. Solution of the differential equation $\cos x\, dy = y(\sin x - y)\, dx,\ 0 < x < \frac{\pi}{2}$ is **[2010]**

(a) $y\sec x = \tan x + c$ (b) $y\tan x = \sec x + c$
(c) $\tan x = (\sec x + c)y$ (d) $\sec x = (\tan x + c)y$

14. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to : **[2011]**

(a) 5 (b) 13
(c) -2 (d) 7

15. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is **[2011]**

(a) $I - \frac{kT^2}{2}$ (b) $I - \frac{k(T-t)^2}{2}$

(c) e^{-kT} (d) $T^2 - \frac{1}{k}$

16. Consider the differential equation **[2011RS]**

$y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then x is given by :

(a) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (b) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

(c) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (d) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

17. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is : **[2012]**

(a) $2\ln 18$ (b) $\ln 9$

(c) $\frac{1}{2}\ln 18$ (d) $\ln 18$

18. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[2013]**

(a) 2500 (b) 3000
(c) 3500 (d) 4500

19. Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$.If $p(0)=100$, then $p(t)$ equals: **[2014]**

(a) $600-500\,e^{t/2}$ (b) $400-300\,e^{-t/2}$

(c) $400-300\,e^{t/2}$ (d) $300-200\,e^{-t/2}$

20. Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,(x\geq 1)$. Then y (e) is equal to: **[2015]**

(a) 2 (b) 2e

(c) e (d) 0

21. If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, y(1 + xy) dx = x dy, then $f\left(-\frac{1}{2}\right)$ is equal to : **[2016]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $-\frac{2}{5}$ (d) $-\frac{4}{5}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(c)	(c)	(c)	(b)	(c)	(c)	(d)	(a)	(d)	(c)	(d)	(d)	(a)
16	17	18	19	20	21									
(c)	(a)	(c)	(c)	(a)	(b)									

SOLUTIONS

1. (c) $\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3$

$$\Rightarrow\left(1+3\frac{dy}{dx}\right)^2=16\left(\frac{d^3y}{dx^3}\right)^3$$

2. (b) $\frac{d^2y}{dx^2}=e^{-2x}$; $\frac{dy}{dx}=\frac{e^{-2x}}{-2}+c$;

$$y=\frac{e^{-2x}}{4}+cx+d$$

3. (c) $y^2=4a(x-h)$, $2yy_1=4a\Rightarrow yy_1=2a$

Differentiating, $\Rightarrow y_1{}^2+yy_2=0$

Degree $=1$, order $=2$.

4. (c) $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

$$\Rightarrow\frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$$

$$I.F=e^{\int\frac{1}{(1+y^2)}dy}=e^{\tan^{-1}y}$$

$$x(e^{\tan^{-1}y})=\int\frac{e^{\tan^{-1}y}}{1+y^2}e^{\tan^{-1}y}\,dy$$

$$x(e^{\tan^{-1}y})=\frac{e^{2\tan^{-1}y}}{2}+C$$

$$\therefore\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$$

5. (c) $x^2+y^2-2ay=0$ ....(1)

Differentiate,

$$2x+2y\frac{dy}{dx}-2a\frac{dy}{dx}=0\ \Rightarrow a=\frac{x+yy'}{y'}$$

Put in (1), $x^2+y^2-2\left(\frac{x+yy'}{y'}\right)y=0$

$$\Rightarrow(x^2+y^2)y'-2xy-2y^2y'=0$$

$$\Rightarrow(x^2-y^2)y'=2xy$$

6. (b) $ydx + (x + x^2 y)dy = 0$

$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$

It is Bernoulli form. Divide by x^2

$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1.$

put $x^{-1} = t, -x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$

We get,

$-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$

It is linear in t.

Integrating factor

$= e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$

$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})dy + C$

$\Rightarrow \frac{1}{x}.\frac{1}{y} = \log y + C \Rightarrow \log y - \frac{1}{xy} = C$

7. (c) $y^2 = 2c(x + \sqrt{c})$ .... (i)

$2yy' = 2c.1$ or $yy' = c$ .... (ii)

$\Rightarrow \quad y^2 = 2yy'(x + \sqrt{yy'})$

[On putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ .... (iii)

Hence equation (iii) is of order 1 and degree 3.

8. (c) $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put $y = vx$

$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$

$\frac{xdv}{dx} = v \log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$

Put $\log v = z$

$\frac{1}{v}dv = dz \Rightarrow \frac{dz}{z} = \frac{dx}{x}$

$\ln z = \ln x + \ln c$

$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx.$

9. (d) $Ax^2 + By^2 = 1$ ...(1)

$Ax + By\frac{dy}{dx} = 0$ ....(2)

$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ ....(3)

From (2) and (3)

$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$

Dividing both sides by –B, we get

$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

Which is a DE of order 2 and degree 1.

10. (a) General equation of circles passing through origin and having their centres on the x-axis is

$x^2 + y^2 + 2gx = 0$ ...(i)

On differentiating w.r.t x, we get

$2x + 2y.\frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$

$\therefore$ equation (i) be

$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\}.x = 0$

$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx}.y = 0$

$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$

11. (d) $\frac{dy}{dx} = \frac{x + y}{x} = 1 + \frac{y}{x}$

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

we get

$v + x\frac{dv}{dx} = 1 + v \Rightarrow \int \frac{dx}{x} = \int dv$

$\Rightarrow v = \ln x + c \Rightarrow y = x\ln x + cx$

As $y(1) = 1$

$\therefore \; c = 1$ So solution is $y = x\ln x + x$

12. (c) We have $y = c_1 e^{c_2 x}$

$\Rightarrow y' = c_1 c_2 e^{c_2 x} = c_2 y$

$\Rightarrow \frac{y''}{y} = c_2 \Rightarrow \frac{y'' y - (y')^2}{y^2} = 0$

$\Rightarrow y'' y = (y')^2$

13. (d) $\cos x\, dy = y(\sin x - y)\, dx$

$\frac{dy}{dx} = y \tan x - y^2 \sec x$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$ ...(i)

Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

From equation (i)

$-\frac{dt}{dx} - t\tan x = -\sec x$

$\Rightarrow \frac{dt}{dx} + (\tan x)t = \sec x$

I.F. $= e^{\int \tan x\, dx} = (e)^{\log|\sec x|} \sec x$

Solution : $t(\text{I.F}) = \int (\text{I.F}) \sec x\, dx$

$\Rightarrow \frac{1}{y}\sec x = \tan x + c$

14. (d) $\frac{dy}{dx} = y + 3 \Rightarrow \int \frac{dy}{y+3} = \int dx$

$\Rightarrow \ell n|y+3| = x + c$

Since $y(0) = 2$, $\therefore \ell n 5 = c$

$\Rightarrow \ell n|y+3| = x + \ell n 5$

When $x = \ell n 2$, then $\ell n|y+3| = \ell n 2 + \ell n 5$

$\Rightarrow \ell n|y+3| = \ell n 10$

$\therefore y + 3 = \pm 10 \Rightarrow y = 7, -13$

15. (a) $\frac{dV(t)}{dt} = -k(T - t)$

$\Rightarrow \int dVt = -k\int (T-t)dt$

$V(t) = \frac{k(T-t)^2}{2} + c$

at $t = 0$, $V(t) = I \Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

$V(T) = I + \frac{k}{2}(T^2 - 2T^2) = I - \frac{k}{2}T^2$

16. (c) $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$

So $x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

Let $\frac{-1}{y} = t$

$\Rightarrow \frac{1}{y^2} dy = dt$

$\Rightarrow I = -\int te^t dt = e^t - te^t$

$= e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$

$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$

$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$

Since $y(1) = 1$

$\therefore c = -\frac{1}{e}$

$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$

17. (a) Given differential equation is

$\frac{dp(t)}{dt} = 0.5p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t) - 900}{2}$

$\Rightarrow 2\frac{dp(t)}{dt} = -[900 - p(t)]$

$\Rightarrow 2\frac{dp(t)}{900 - p(t)} = -dt$

Integrate both the side, we get

$-2\int \frac{dp(t)}{900 - p(t)} = \int dt$

Let $900 - p(t) = u$

$\Rightarrow \quad -dp(t) = du$

$\therefore \quad$ We have,

$$2\int \frac{du}{u} = \int dt \Rightarrow 2\,ln\,u = t + c$$

$\Rightarrow \quad 2ln[900 - p(t)] = t + c$

when $t = 0, p(0) = 850$

$2\,ln(50) = c$

$$\Rightarrow \quad 2\left[ln\left(\frac{900 - p(t)}{50}\right)\right] = t$$

$$\Rightarrow \quad 900 - p(t) = 50e^{\frac{t}{2}}$$

$$\Rightarrow \quad p(t) = 900 - 50e^{\frac{t}{2}}$$

let $p(t_1) = 0$

$$0 = 900 - 50e^{\frac{t_1}{2}} \therefore \; t_1 = 2ln\,18$$

18. **(c)** Given, Rate of change is $\frac{dP}{dx} = 100 - 12\sqrt{x}$

$\Rightarrow \quad dP = (100 - 12\sqrt{x})\,dx$

By integrating

$$\int dP = \int (100 - 12\sqrt{x})\,dx$$

$P = 100x - 8x^{3/2} + C$

Given when $x = 0$ then $P = 2000$

$\Rightarrow \quad C = 2000$

Now when $x = 25$ then

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000$

$= 4500 - 1000$

$\Rightarrow \quad P = 3500$

19. **(c)** Given differential equation is

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$$

By separating the variable, we get

$$dp(t) = \left[\frac{1}{2}p(t) - 200\right]dt$$

$$\Rightarrow \quad \frac{dp(t)}{\frac{1}{2}p(t) - 200} = dt$$

Integrate on both the sides,

$$\int \frac{d(p(t))}{\frac{1}{2}p(t) - 200} = \int dt$$

Let $\frac{1}{2}p(t) - 200 = s \Rightarrow \frac{dp(t)}{2} = ds$

So, $$\int \frac{d\,p(t)}{\left(\frac{1}{2}p(t) - 200\right)} = \int dt$$

$$\Rightarrow \quad \int \frac{2ds}{s} = \int dt$$

$\Rightarrow \quad 2\log s = t + c$

$$\Rightarrow 2\log\left(\frac{p(t)}{2} - 200\right) = t + c$$

$$\Rightarrow \quad \frac{p(t)}{2} - 200 = e^{\frac{1}{2}}k$$

Using given condition $p(t) = 400 - 300\,e^{t/2}$

20. **(a)** Given, $\frac{dy}{dx} + \left(\frac{1}{x\log x}\right)y = 2$

$$\text{I.F.} = e^{\int \frac{1}{x\log x}dx}$$

$$= e^{\log(\log x)} = \log x$$

$y.\log x = \int 2\log x dx + c$

$y\log x = 2[x\log x - x] + c$

Put $x = 1, y.0 = -2 + c$

$c = 2$

Put $x = e$

$y\log e = 2e(\log e - 1) + c$

$y(e) = c = 2$

21. **(b)** $y(1 + xy)dx = xdy$

$$\frac{xdy - ydx}{y^2} = xdx$$

$$\Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx$$

$$-\frac{x}{y} = \frac{x^2}{2} + C \quad \text{as } y(1) = -1 \Rightarrow C = \frac{1}{2}$$

Hence, $y = \frac{-2x}{x^2 + 1} \Rightarrow f\left(\frac{-1}{2}\right) = \frac{4}{5}$