

TEST INFORMATION

DATE : 10.05.2015

JEE PREPARATORY TEST (JPT) - 01

Syllabus : Full syllabus

This DPP is to be discussed (12-05-2015)

JPT-1 to be discussed (12-05-2015)

DPP No. # 10

Total Total Marks : 149

Max. Time : 117 min.

Single choice Objective (–1 negative marking) Q. 1 to 13

(3 marks 2½ min.) [39, 32½]

Multiple choice objective (–1 negative marking) Q. 14 to 18

(4 marks, 3 min.) [20, 15]

Single Digit Subjective Questions (no negative marking) Q.19 to Q.24

(4 marks 2½ min.) [24, 15]

Double Digits Subjective Questions (no negative marking) Q.25 to Q.27

(4 marks 2½ min.) [12, 7½]

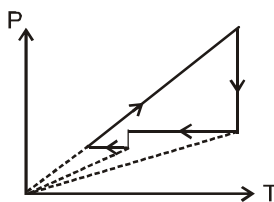
Comprehension (–1 negative marking) Q.28 to 41

(3 marks 2½ min.) [42, 35]

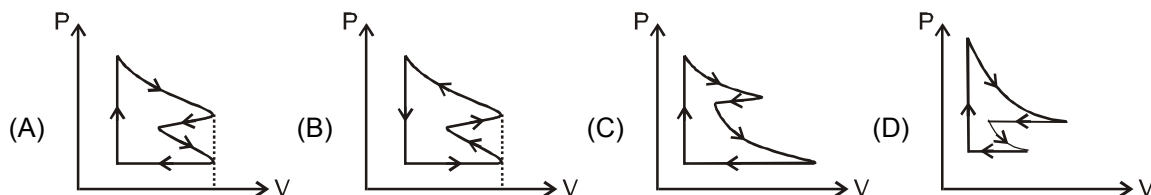
Match Listing (–1 negative marking) Q.42 to Q.45

(3 marks, 3 min.) [12, 12]

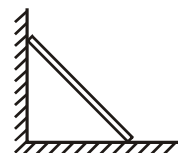
- The ratio of translational and rotational kinetic energies at 100 K temperature is 3 : 2. Then the internal energy of one mole gas at that temperature is [$R = 8.3 \text{ J/mol-K}$]
 (A) 1175J (B) 1037.5 J (C) 2075 J (D) 4150J
- P-T curve for a cyclic process is as shown`



P-V graph for this process will be :

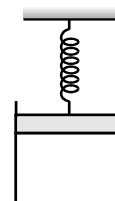


3. A rod of length ℓ is sliding such that one of its ends is always in contact with a vertical wall and its other end is always in contact with horizontal surface. Just after the rod is released from rest, the magnitude of acceleration of end points of the rod is a and b respectively. The angular acceleration of rod at this instant will be



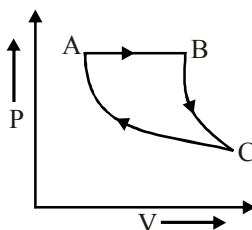
- (A) $\frac{a+b}{\ell}$ (B) $\frac{\sqrt{|a^2 - b^2|}}{\ell}$ (C) $\frac{\sqrt{a^2 + b^2}}{\ell}$ (D) None of these

4. One mole of an ideal gas is kept enclosed under a light piston (area = 10^{-2} m^2) connected by a compressed spring (spring constant 100 N/m). The volume of gas is 0.83 m^3 and its temperature is 100 K . The gas is heated slowly so that it compresses the spring further by 0.1 m . The work done by the gas in the process is: (Take $R = 8.3 \text{ J/K-mole}$ and suppose there is no atmosphere).



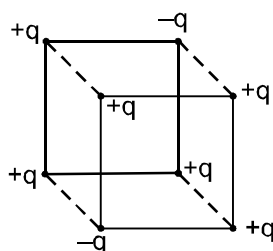
- (A) 3 J (B) 6 J (C) 9 J (D) 1.5 J

5. For a given thermodynamic cyclic process, $P - V$ indicator diagram is as shown in the figure. Process AB, BC & CA are isobaric, adiabatic & isothermal respectively. Then which of the following curve represent correct $V - T$ indicator diagram for the process ABCA?



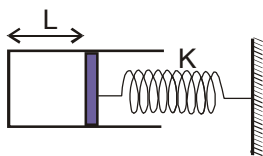
- (A)
- (B)
- (C)
- (D)

6. The side of the cube is ' ℓ ' and point charges are kept at each corner as shown in diagram. Interaction electrostatic potential energy of all the charges is :



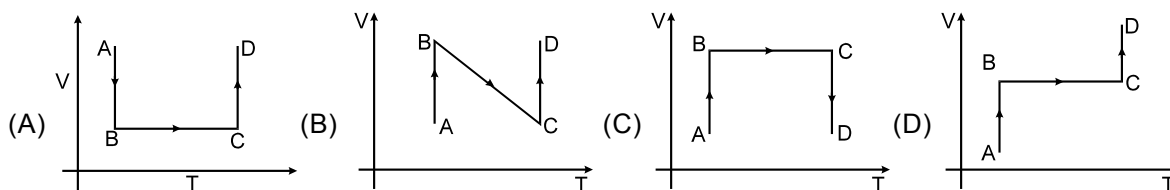
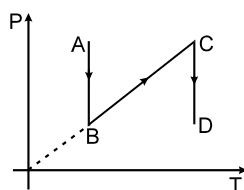
- (A) $\frac{4kq^2}{\sqrt{3}\ell}$ (B) $\frac{\sqrt{3}kq^2}{\ell}$ (C) $\frac{2kq^2}{\sqrt{3}\ell}$ (D) $\frac{kq^2}{\sqrt{3}\ell}$

7. A fixed container is fitted with a piston which is attached to a spring of spring constant k . The other end of the spring is attached to a rigid wall. Initially the spring is in its natural length and the length of container between the piston and its side wall is L . Now an ideal diatomic gas is slowly filled in the container so that the piston moves quasistatically. It pushed the piston by x so that the spring now is compressed by x . The total rotational kinetic energy of the gas molecules in terms of the displacement x of the piston is (there is vacuum outside the container)



- (A) kxL (B) $4kxL$ (C) $kx(x+L)$ (D) $\frac{2kx^2}{L}$

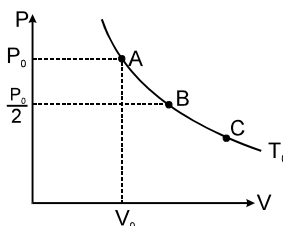
8. P-T diagram is shown below then choose the corresponding V-T diagram



9. Some of the thermodynamic parameters are state variables while some are process variables. Some grouping of the parameters are given. Choose the correct one.

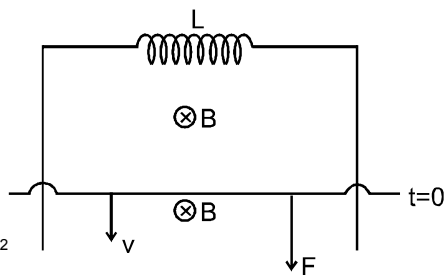
- (A) State variables : Temperature, No. of moles
Process variables : Internal energy, work done by the gas.
- (B) State variables : Volume, Temperature
Process variables : Internal energy, work done by the gas.
- (C) State variables : work done by the gas, heat rejected by the gas
Process variables : Temperature, volume.
- (D) State variables : Internal energy, volume
Process variables : Work done by the gas, heat absorbed by the gas.

10. The state of an ideal gas is changed through an isothermal process at temperature T_0 as shown in figure. The work done by gas in going from state B to C is double the work done by gas in going from state A to B. If the pressure in the state B is $\frac{P_0}{2}$ then the pressure of the gas in state C is :

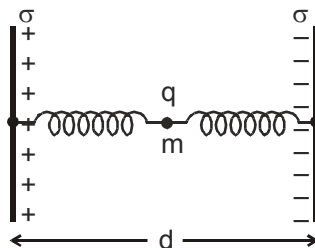


- (A) $\frac{P_0}{2}$ (B) $\frac{P_0}{4}$ (C) $\frac{P_0}{6}$ (D) $\frac{P_0}{8}$

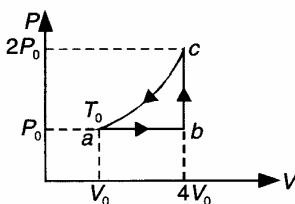
11. As shown in the figure a variable force F is applied on conducting wire of length ℓ such that its velocity remains constant. There is no resistance in any branch in the circuit. Consider the motion of wire from $t = 0$ initially there is no current in inductor. Now when wire has covered a distance x (from initial position) then at that time energy of inductor will be: (Neglect gravity)
- (A) independent of x (B) directly proportional to x
 (C) directly proportional to x^2 (D) directly proportional to $x^{1/2}$



12. Two large non conducting plates having surface charge densities $+\sigma$ and $-\sigma$ respectively, are fixed d distance apart. A small test charge q of mass m is attached to two non conducting springs each of spring constant k as shown in the figure. The sum of lengths of both springs in undeformed state is d . The charge q is released from rest with both the springs nondeformed. Then charge q will (neglect gravity)

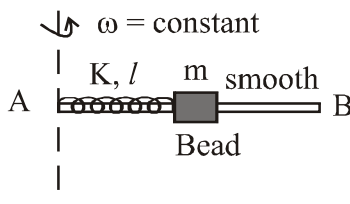


- (A) performs SHM with angular frequency $\sqrt{\frac{2k}{m}}$ and amplitude $\frac{\sigma q}{k \epsilon_0}$.
 (B) performs SHM with angular frequency $\sqrt{\frac{2k}{m}}$ and amplitude $\frac{\sigma q}{2k \epsilon_0}$.
 (C) not perform SHM, but will execute periodic motion.
 (D) remain stationary.
13. Suppose the earth was covered by an ocean of uniform depth h . ($h \ll R$). Let σ be density of ocean and ρ be mean density of earth. Let Δg be the approximate difference of value of net acceleration due to gravity between the bottom of the ocean and top. ($\Delta g = g_{\text{top}} - g_{\text{bottom}}$). Choose the correct option :
- (A) $\Delta g = \frac{4}{3} \pi G h [2\rho - 3\sigma]$ (B) $\Delta g = \frac{4}{3} G \pi h [3\sigma - 2\rho]$
 (C) $\Delta g = \frac{4}{3} G \pi h [2\sigma - 3\rho]$ (D) $\Delta g = \frac{4}{3} G \pi h [3\rho - 3\sigma]$
14. One mole of an ideal monoatomic gas undergoes a cyclic process 'abca' shown in figure. If U denotes the internal energy and W the work done by the gas. Then choose the **incorrect** alternative (s)



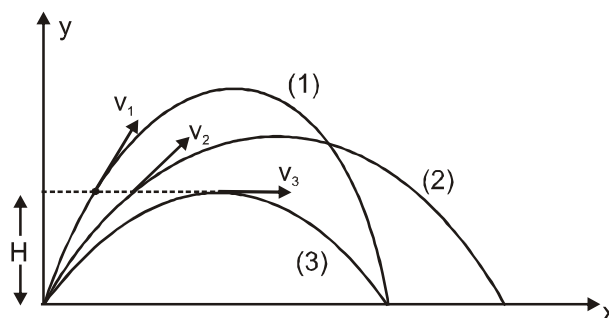
- (A) $U_c - U_a = 10.5RT_0$ (B) $U_b - U_a = +4RT_0$
 (C) $W_{c \rightarrow a} = -3P_0V_0$ (D) $U_c > U_b > U_a$

15. AB is a light rigid rod, which is rotating about a vertical axis passing through A. A spring of force constant K and natural length ℓ is attached at A and its other end is attached to a small bead of mass m . The bead can slide without friction on the rod. At the initial moment the bead is at rest (w.r.t. the rod) and the spring is unstretched. Select correct options :



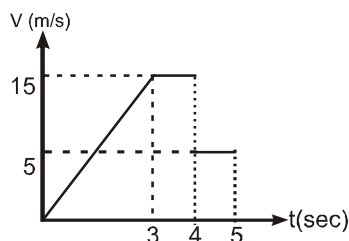
- (A) The maximum velocity attained by the bead w.r.t the rod is given by $V_{\max} = \sqrt{\frac{m\omega^4 \ell^2}{K - m\omega^2}}$
- (B) The maximum velocity attained by the bead w.r.t the rod is given by $V_{\max} = \sqrt{\left(\frac{m\omega^4 + K}{m\omega^2 - K}\right) \omega^2 \ell^2}$
- (C) The maximum extension in the spring is given by $X_{\max} = \frac{2m\omega^2 \ell}{K - m\omega^2}$
- (D) The maximum value of contact force between the bead and the rod is greater than mg

16. Three projectiles are thrown all with same speed u but at different angles of projection ($\theta_1 > \theta_2 > \theta_3$) all taken from horizontal. Maximum height attained by projectile (3) is H . Range of (1) & (3) is same & that of (2) is maximum for the given speed. At height H , speeds are v_1 , v_2 & v_3 as shown. Total time of flights are T_1 , T_2 & T_3 . Choose the correct statement(s).



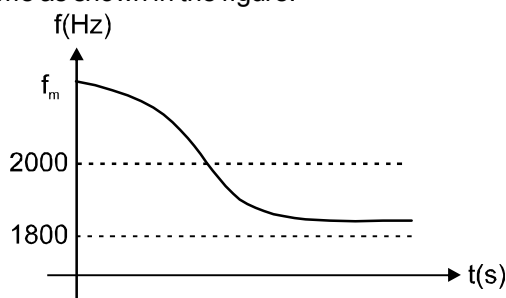
- (A) $v_1 > v_2 > v_3$ (B) $v_1 = v_2 = v_3$
(C) T_1^2, T_2^2 & T_3^2 are in AP (D) θ_1, θ_2 & θ_3 are in AP

17. The figure shows the velocity as a function of the time for an object with mass 10 kg being pushed along a frictionless horizontal surface by external horizontal force. At $t = 3$ s, the force stops pushing and the object moves freely. It then collides head on and sticks to another object of mass 25 kg.



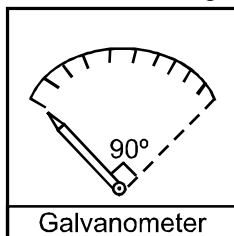
- (A) External force acting on the system is 50 N from $t = 0$ to $t = 3$ sec.
(B) Speed of the 2nd particle just before the collision is 1 m/s.
(C) before collision both bodies are moving in the same direction.
(D) before collision, bodies are moving in opposite direction.

18. In an isobaric process (γ is adiabatic exponent of the gas)
- (A) The heat given to gas is $\frac{\gamma}{\gamma-1}$ times the work done by gas.
 (B) The work done by gas is $(\gamma - 1)$ time the change in internal energy.
 (C) The temperature of gas is increased.
 (D) The temperature of gas is decreased.
19. Solid uniform conducting sphere of mass 'm' and charge Q, rotates about its axis of symmetry with constant angular velocity ' ω ' then the ratio of magnetic moment to the moment of inertia of the sphere is $\frac{xQ\omega}{6m}$ then x is : (Neglect induced charges due to centrifugal force)
20. A stationary observer receives a sound from a source of frequency 2000 Hz moving with constant velocity. The apparent frequency varies with time as shown in the figure.

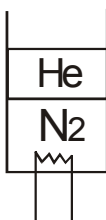


The value of f_m is $(2300 - 10x)$ Hz. Find the value of x. (Take speed of sound = 300 m/s and neglect the time taken by sound to reach the stationary observer).

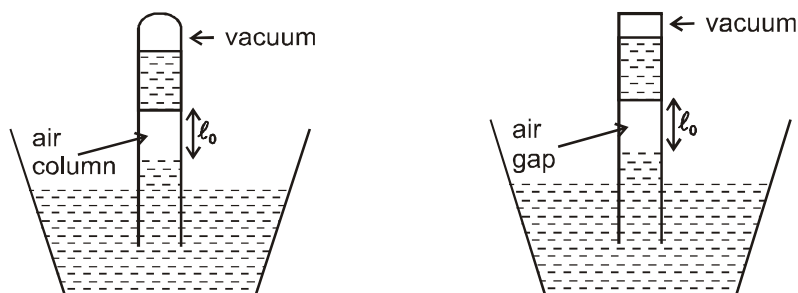
21. In a moving coil galvanometer, a coil of area $\pi \text{ cm}^2$ and 10 windings is used. Magnetic field strength applied on the coil is 1 tesla and torsional stiffness of the torsional spring is $6 \times 10^{-5} \text{ N.m/rad}$. A needle is welded with the coil. Due to limited space, the coil (or needle) can rotate only by 90° . For marking, the 90° space is equally divided into 10 parts as shown. Find the least count of this galvanometer in mA.



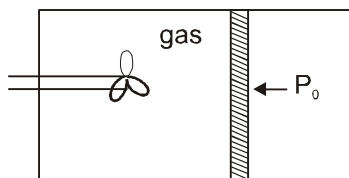
22. A gas consisting of rigid di-atomic molecules (degree of freedom = 5) at pressure $P_0 = 10^5 \text{ N/m}^2$ and temperature 273 K was compressed adiabatically 5 times. The mean kinetic energy of rotating molecules in final state is $n \times 10^{-21} \text{ J}$. Find value of 'n'. ($K = 1.38 \times 10^{-23}$, $(5)^{2/5} = 1.90$).
23. The degree of freedom per molecule for a gas is 6. At constant pressure work done by gas is 25 J. The heat supplied to the gas for this purpose is $25x$ Joule then x is :
24. 5 moles of Nitrogen gas are enclosed in an adiabatic cylindrical vessel. The piston itself is a rigid light cylindrical container containing 3 moles of Helium gas. There is a heater which gives out a power 100 cal/second to the nitrogen gas. A power of 30 cal/sec is transferred to Helium through the bottom surface of the piston. The rate of increment of temperature of the nitrogen gas is (assuming that the piston moves slowly) :



25. At the middle of the mercury barometer tube there is a little column of air with the length ℓ_0 and there is vacuum at the top as shown. Under the normal atmospheric pressure and the temperature of 300 kelvin, $\ell_0 = 10$ cm. What will be the length of the air column if the temperature rises to 330 kelvin ? (Neglect expansion of the tube)



26. One mole of an ideal gas ($\gamma = 1.4$) at 500 K, is filled in an adiabatic cylinder. The piston is free to move against atmospheric pressure. A non-conducting fan is inserted into the gas space and rotated vigorously, so that the gas expands slowly, till its volume is doubled. Find work done by the fan in kJ. (Take $R = 8$ J/mole.K, neglect heat capacity of the cylinder, piston and the fan).



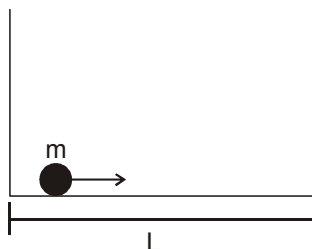
27. The current density \vec{J} inside a long, solid, cylindrical wire of radius $a = 12$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$, where $J_0 = \frac{10^5}{4\pi}$ A/m². Find the magnitude of the magnetic field at $r = \frac{a}{2}$ in μ T.

COMPREHENSION-1

The basic idea of Quantum Mechanics is that motion in any system is quantized. The system obeys Classical Mechanics except that not every motion is allowed, only those motions which obey the Bohr - Sommerfeld Quantization,

$$\oint \vec{P} \cdot d\vec{r} = nh, \quad n \in \mathbb{N},$$

where \vec{P} is the momentum, \vec{r} is the position vector and the integral is carried over a closed path. Assuming this is applicable to a particle of mass m moving with a constant speed in a box of length L having elastic collisions with the walls of the box, Answer the following questions. ($h = 6.6 \times 10^{-34}$ J-sec, $L = 3.3$ Å, $m = 10^{-30}$ kg)



28. The allowed momenta are given by :

(A) $\frac{nh}{2L}$ (B) $\frac{nh}{L}$ (C) $\frac{nh}{2\pi L}$ (D) $\frac{nh}{4L}$

29. The allowed kinetic energy of the particle is :

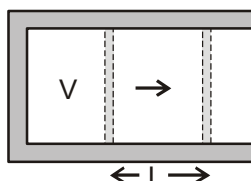
- (A) $\frac{n^2 h^2}{8\pi^2 m L^2}$ (B) $\frac{n^2 h^2}{2m L^2}$ (C) $\frac{n^2 h^2}{8m L^2}$ (D) $\frac{n^2 h^2}{32m L^2}$

30. The difference between 1st and 2nd energy levels is :

- (A) $1.5 \times 10^{-18} \text{ J}$ (B) $3 \times 10^{-18} \text{ J}$ (C) $6 \times 10^{-18} \text{ J}$ (D) $12 \times 10^{-18} \text{ J}$

COMPREHENSION-2

The fixed non-conducting cylinder shown in figure has a nonconducting heavy piston of mass M that can slide without friction. The area of piston is S and the cylinder is filled with an ideal gas ($\gamma = 1.5$), with an initial volume V and an initial pressure P . Assume that the outside pressure on the piston is zero (vacuum). (Neglect acceleration due to gravity).



31. The initial acceleration of piston is :

- (A) $\frac{PA}{\gamma M}$ (B) $\frac{\gamma PA}{M}$ (C) $\frac{PA}{M}$ (D) $\frac{PA}{\gamma - 1}$

32. After the piston has moved by distance S , its velocity is :

- (A) $\left(\frac{4PV}{M} \left[1 - \left(\frac{V}{V+LS} \right)^{1/2} \right] \right)^{1/2}$ (B) $\left(\frac{4PV}{M} \left[1 - \left(\frac{V}{V+LS} \right) \right] \right)^{1/2}$
 (C) $\left(\frac{4PV}{M} \left[1 - \left(\frac{V+LS}{V} \right)^{1/2} \right] \right)^{1/2}$ (D) None of these

33. For the temperature of the gas to drop to one half of its original value, the piston will have to move by a distance :

- (A) $\frac{V}{S}$ (B) $\frac{2V}{S}$ (C) $\frac{3V}{S}$ (D) $\frac{5V}{S}$

COMPREHENSION-3

Sand is dropped vertically downward at the constant rate $\mu \text{ kg/s}$ on a conveyor belt which is moving horizontally with velocity ' v '. Assume that sand particles comes to rest with respect to belt immediately after landing.

34. The magnitude of force must be applied on the belt, so that belt moves with same constant velocity v .

- (A) μv^2 (B) $\frac{\mu v}{2}$ (C) μv (D) $2 \mu v$

35. The rate at which work is done by the external force applied on the belt.

- (A) $\frac{3}{2} \mu v^2$ (B) μv^2 (C) $\frac{\mu v^2}{2}$ (D) $-\frac{\mu v^2}{2}$

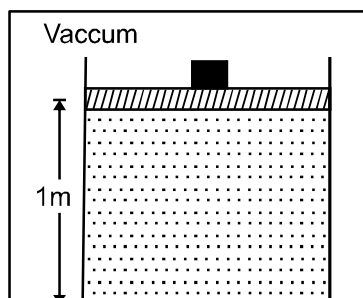
36. The rate at which work is done by friction force on sand is

- (A) $\frac{\mu v^2}{2}$ (B) $-\mu v^2$ (C) $-\frac{\mu v^2}{2}$ (D) $-\frac{3}{2} \mu v^2$



COMPREHENSION-4

One mole of an ideal gas is contained in a perfectly Insulating cylinder. Initially adiabatic piston of unit area and unit mass is in equilibrium. Now a block of same mass is kept gently on piston as shown in figure. (Take $\gamma = 1.5$, $g = 10 \text{ m/s}^2$ and neglect friction and assume change in temperature in whole gas is simultaneous)



37. Depth upto which (piston + block) will move before coming to rest again is :

- (A) $\frac{\sqrt{5}-1}{2} \text{ m}$ (B) $\frac{\sqrt{5}-1}{3} \text{ m}$ (C) $\frac{\sqrt{5}-1}{4} \text{ m}$ (D) None of these

38. Maximum pressure of gas during its motion :

- (A) $\frac{20\sqrt{2}}{(5-\sqrt{3})^{3/2}} \text{ N/m}^2$ (B) $\frac{20\sqrt{2}}{(3+\sqrt{5})^{1/2}} \text{ N/m}^2$ (C) $\frac{20\sqrt{5}}{(3-\sqrt{5})^{3/2}} \text{ N/m}^2$ (D) $\frac{20\sqrt{2}}{(3-\sqrt{5})^{3/2}} \text{ N/m}^2$

COMPREHENSION

One mole of a diatomic gas is heated under a "Kibolinsky Process" in which gas pressure is temperature controlled according to law $P = \frac{4}{5} CT^{3/2}$. Where C is Kibolinsky constant. In this process, gas is heated by 300 K temperature. Answer the following questions.

39. The amount of workdone by gas under above heating is :

- (A) 600 R (B) 750 R (C) - 150 R (D) None of these

40. The amount of heat supplied to the gas under above heating is :

- (A) 600 R (B) - 750 R (C) - 150 R (D) None of these

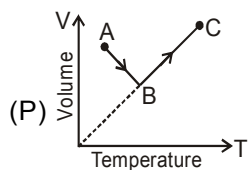
41. Under above process the molar specific heat of gas is :

- (A) $\frac{R}{2}$ (B) R (C) $2R$ (D) None of these

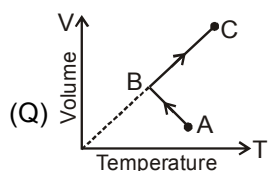
42. In each situation of column-I, a process $A \rightarrow B \rightarrow C$ is given for an ideal gas. Match the proper entries from column-2 to column-1 using the codes given below the columns.

Column-I

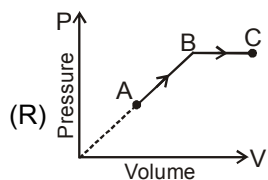
Column-II



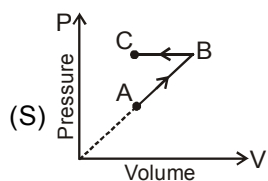
(1) Temperature increases.



(2) Pressure first increases and then remains constant



(3) Temperature first decreases and then increases



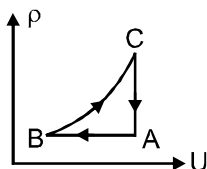
(4) Pressure first decreases and then remains constant

	p	q	r	s
(A)	1	3	1	2
(B)	2	4	3	1
(C)	1	3	4	2
(D)	3	2	1	4

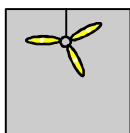
43. In column-II, some situations are given, and in column-I, their results are given. Match the proper entries from column-2 to column-1 using the codes given below the columns.

Column-I

(P) 0°C ice kept at atmospheric pressure, melts and converts into 0°C water ($\rho_{\text{ice}} = 0.9 \rho_{\text{water}}$). For the ice water system during this process.
 (Q) A gas is undergoing a cyclic process. Density (ρ) v/s internal energy (U) graph is as shown. Here process B–C is an adiabatic process. Consider the gas as the system, for the complete cyclic process :



(R) In a closed, non-conducting room, a fan is turned on. Consider the air in the room as a system. During this process.



(S) Gas is filled in a conducting container of negligible heat capacity. The container is kept in ice–water mixture for a long time. Now the piston is slowly brought down, till the gas is very much compressed. Now the cylinder is coated with an insulating material and now the piston is released. When the piston is allowed to reach the initial position slowly then in whole process (Assume the gas in the container to be the system).

	p	q	r	s
(A)	3	1	4	2
(B)	2	1	3	4
(C)	3	2	1	4
(D)	2	4	3	1

Column-II

- (1) Net work is done by the system
- (2) Net work is done on the system
- (3) Internal energy of the system increases
- (4) Internal energy of the system decreases

44. An ideal gas consists of a large number of identical molecules. Absolute temperature of the gas is T (in kelvin), molecular weight of gas is M and R is gas constant. Match the proper entries from column-2 to column-1 using the codes given below the columns.

Column-I

- (P) Root mean square speed of molecules is greater than
 (Q) Most probable speed of molecules is smaller than
 (R) Average velocity of a molecule is smaller than
 (S) Speed of a molecule may be greater than

	p	q	r	s
(A)	2	1	3	4
(B)	3	1	4	2
(C)	1	3	2	4
(D)	4	2	3	1

Column-II

- (1) $\sqrt{\frac{RT}{M}}$
- (2) $1.5\sqrt{\frac{RT}{M}}$
- (3) $2\sqrt{\frac{RT}{M}}$
- (4) $2.5\sqrt{\frac{RT}{M}}$

45. In a hydrogen atom, $a_0 = 0.529 \text{ \AA}$, $E_1 = -13.6 \text{ eV}$. Some modified situations are given compare the energy of levels and radius of allowed orbits.

Column-I

Column-II

(P) An atom consists of positive charge $2e$ and

(1) $\frac{E_1}{2}$ or $2E_1$

mass m and an electron of mass m revolving around the centre of mass of the system

(Q) An atom consists of positron and an electron

(2) $2a_0$ or $\frac{a_0}{2}$

revolving around the centre of mass of system

(R) An atom consists of a particle of mass double that

(3) $\frac{2}{9} E_1$ or $\frac{9}{2} E_1$

of electron and charge same as that of electron revolving around a heavy nucleus of charge $4e$

(S) An atom consists of a nucleus of infinite mass and an electron revolving around the nucleus of charge $2e$

(4) a_0 or $8a_0$

(5) $4E_1$

	P	Q	R	S
(A)	1	3	4	5
(B)	2	2	5	4
(C)	3	1	2	1
(D)	4	2	1	5

ANSWER KEY OF DPP No. # 09

1.	(A)	2.	(A)	3.	(B)	4.	(A)	5.	(A)	6.	(A)	7.	(B)
8.	(A)	9.	(C)	10.	(A)	11.	(C)	12.	(A)	13.	(C)	14.	(B)
15.	(A)	16.	(D)	17.	(A, B, C)	18.	(A, B, D)	19.	(A, C, D)	20.	(B, C, D)		
21.	(A, C, D)	22.	(C, D)	23.	(A, C)	24.	2	25.	6	26.	6 m	27.	5
28.	6	29.	7	30.	90	31.	12	32.	100	33.	(D)	34.	(A)
35.	(B)	36.	(B)	37.	(C)	38.	(B)	39.	(C)	40.	(A)	41.	(A)
42.	(B)	43.	(B)	44.	(D)	45.	(A)						

PHYSICS

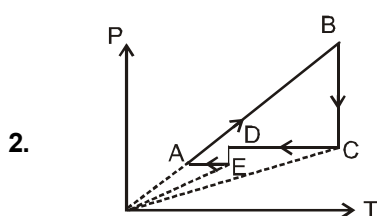
1. According to law of equipartition of energy, energies equally distributed among its degree of freedom, Let translational and rotational degree of freedom be f_1 and f_2 .

$$\therefore \frac{K_T}{K_R} = \frac{3}{2} \quad \text{and} \quad K_T + K_R = U$$

Hence the ratio of translational to rotational degrees of freedom is 3:2. Since translational degrees of freedom is 3, the rotational degrees of freedom must be 2.

$$\therefore \text{Internal energy (U)} = 1 \times (f_1 + f_2) \times \frac{1}{2} RT$$

$$U = \frac{1 \times 5 \times 8.3 \times 100}{2} = U = 2075 \text{ J}$$



$$PV = nRT$$

$$\frac{P}{T} \propto \frac{1}{V}$$

$$\Rightarrow \text{Slope of line joining origin to that point} \propto \frac{1}{V}$$

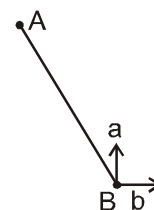
as the slope of line OE is greater than the slope of line OC, So, volume at 'E' is less than that at 'C'.
 So, ans. is (D).

3. At the initial moment, angular velocity of rod is zero.
 Acceleration of end B of rod with respect to end A is shown in figure.
 Centripetal acceleration of point B with respect to A is zero ($\because \omega^2 \ell = 0$)
 So at the initial moment, acceleration of end B with respect

to end A is perpendicular to the rod which is equal to $\sqrt{a^2 + b^2}$

$$a_{\text{rel}} = \ell \alpha$$

$$\frac{\sqrt{a^2 + b^2}}{\ell} = \alpha \quad \text{where } \alpha \text{ is angular acceleration}$$



4. Before heating let the pressure of gas be P_1 from the equilibrium piston,
 $PA = kx_1$

$$\therefore x_1 = \frac{PA}{K} = \left(\frac{nRT}{V} \right) \frac{A}{K} = \frac{1 \times 8.3 \times 100 \times 10^{-2}}{0.83 \times 100} = 10^{-1} = 0.1 \text{ m}$$

Since during heating process,

The spring is compressed further by 0.1 m

$$\therefore x_2 = 0.2 \text{ m}$$

$$\text{work done by gas} = \frac{1}{2} \cdot 100(0.2^2 - 0.1^2) = \frac{1}{2} \cdot 100 \cdot (0.1)(0.3) = 1.50 = 1.5 \text{ J}$$

$$= \frac{4kq^2}{\sqrt{3}\ell}$$

7. Rotational K.E. = Rotational degree of freedom $\times \frac{1}{2} nRT$

$$= 2 \times \frac{1}{2} nRT = nRT = PV$$

$$= P_A \cdot \frac{V}{A} = \text{force on piston } (L + x) = kx (L + x)$$

8. BC is isochoric. $V_B > V_A$, $V_B = V_C$, $V_D > V_C$

9. Internal energy and volume depend upon states.

10. Work done by gas in going isothermally from state A to B is

$$\Delta W_{AB} = nRT \ln \frac{P_A}{P_B} = nRT \ln 2 \quad \dots\dots\dots(1)$$

Work done by gas in going isothermally from state B to C is

$$\Delta W_{BC} = nRT \ln \frac{P_B}{P_C} = nRT \ln \frac{P_0}{2P_C} \quad \dots\dots\dots(2)$$

It is given that $\Delta W_{BC} = 2 \Delta W_{AB}$

$$\therefore \ln \frac{P_0}{2P_C} = \ln(2)^2 \quad \therefore P_C = \frac{P_0}{8}$$

11. Current in circuit at any time $t = \left(\frac{B\ell v}{L} t \right)$

$$\text{So, energy of inductor at time } t = \frac{1}{2} L \left(\frac{B\ell v}{L} t \right)^2$$

$$t = \frac{x}{v}$$

$$\text{So, } E \propto x^2$$

12. The electrostatic force on charge is constant, hence it does not effect the time period of spring + particle system

$$\therefore \omega = \sqrt{\frac{2k}{m}}$$

Initially the charge is at rest, i.e., at extreme position.

The equilibrium position, shall be at a distance A towards right, where A (by definition) is amplitude of vibration.

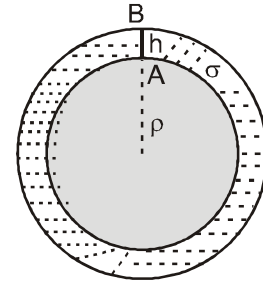
$$\therefore 2kA = \frac{\sigma}{\epsilon_0} q \quad \text{or} \quad A = \frac{\sigma q}{2k \epsilon_0}$$

13. $g_A = \frac{4}{3} G \pi R \rho$

$$g_B = \frac{G \left[\frac{4}{3} \pi \rho R^3 + \left[\frac{4}{3} \pi (R+h)^3 - \frac{4}{3} \pi R^3 \right] \sigma \right]}{(R+h)^2}$$

$$= \frac{4}{3} G \pi \rho R \left[1 - \frac{2h}{R} \right] + \frac{4}{3} G \pi \left[(R+h) - \left(1 - \frac{2h}{R} \right) R \right] \sigma$$

$$\Delta g = g_B - g_A = \frac{4}{3} \pi G h [3\sigma - 2\rho].$$

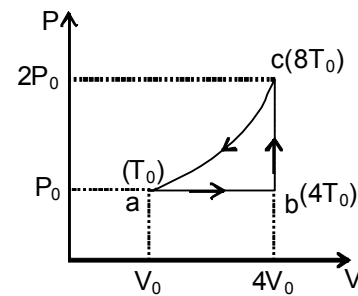


14. (a) $U_c - U_a = nC_v dT = (1) \frac{3R}{2} (8T_0 - T_0) = \frac{21}{2} RT_0$

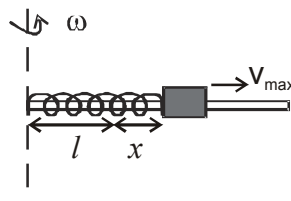
(b) $U_b - U_a = nC_v dT$

$= (1) \frac{3R}{2} (4T_0 - T_0) = 4.5RT_0$ (wrong)

(c) $W_{c \rightarrow a}$ cannot be determined, but
 $|W_{c \rightarrow a}| > |W_{a \rightarrow b}|$
 $|W_{c \rightarrow a}| > 3P_0 V_0$, Hence (C) is wrong



15. Velocity will be maximum at equilibrium position



$$\Rightarrow m\omega^2(\ell + x) = Kx$$

$$\Rightarrow x = \frac{m\omega^2 \ell}{K - m\omega^2}$$

$$\frac{1}{2} m V_{\max}^2 = \int_0^x m\omega^2(\ell + x) dx - \frac{1}{2} Kx^2$$

$$\Rightarrow V_{\max}^2 = \frac{2m\omega^2 \ell x + m\omega^2 x^2 - Kx^2}{m}$$

$$V_{\max}^2 = \frac{(m\omega^2 \ell + m\omega^2(\ell + x) - Kx)x}{m}$$

$$\Rightarrow V_{\max}^2 = \omega^2 \ell = \frac{m\omega^4 \ell^2}{m\omega^2 - K}$$

$$V_{\max} = \sqrt{\frac{m\omega^4 \ell^2}{K - m\omega^2}}$$

For maximum extension

$$\int_0^x m\omega^2(\ell + x) dx - \frac{1}{2} Kx_{\max}^2 = 0 \Rightarrow x_{\max} = \frac{2m\omega^2 \ell}{K - m\omega^2}$$

16. from work energy theorem

$$v_1 = v_2 = v_3$$

$$\theta_1 + \theta_2 = 90^\circ \text{ \& } \theta_2 = 45^\circ$$

$$\Rightarrow \frac{\theta_1 + \theta_3}{2} = \theta_2 \quad \Rightarrow \quad \theta_1, \theta_2 \text{ \& } \theta_3 \text{ are in AP} \quad \text{AP में है।}$$

$$T_1 = \frac{2u \sin \theta_1}{g} \quad \Rightarrow \quad T_2 = \frac{2u \sin 45^\circ}{g} \quad \Rightarrow \quad T_3 = \frac{2u \cos \theta_1}{g}$$

$$\Rightarrow \frac{T_1^2 + T_3^2}{2} = T_2^2 \quad \Rightarrow \quad T_1^2, T_2^2 \text{ \& } T_3^2 \text{ are in AP}$$

17. From graph in time from $t = 0$ to $t = 3$ sec.
acceleration of object of mass $m_1 = 10$ kg is

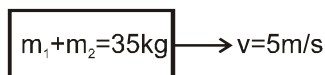
$$a = \frac{15 - 0}{3} = 5 \text{ m/s}^2$$

\therefore Force on object of mass m_1 from $t = 0$ to $t = 3$ sec. (i)
 $= 10 \times 5 = 50$ N

Before and after collision at $t = 4$ sec, the velocities of blocks are as shown.



Before collision



after collision

\therefore initial momentum of system

$$= m_1 u_1 + m_2 u_2 = 150 + 25 u_2$$

final momentum of system

$$= (m_1 + m_2) v = 35 \times 5 = 175$$

From conservation of momentum

$$\therefore 150 + 25 u_2 = 175$$

$$\text{or } u_2 = +1 \text{ m/s}$$

\therefore speed of second particle just before collision is 1 m/s and before collision both blocks move in same direction.

18. At constant pressure.

If volume increases, temperature also increases

volume decreases, temperature decreases.

In Isobaric process.

$$\Delta Q = \gamma \Delta U$$

$$\therefore \Delta Q = \Delta U + \Delta W$$

$$\therefore \Delta W = \gamma \Delta U - \Delta U = (\gamma - 1) \Delta U$$

$$\Delta W = (\gamma - 1) \Delta U$$

$$\therefore \Delta Q = \frac{\gamma}{(\gamma - 1)} \Delta W$$

$$19. \quad M = \frac{Q}{2m} \times \frac{2}{3} m R^2 \cdot \omega$$

$$I = \frac{2mR^2}{5}$$

$$\frac{M}{I} = \frac{5Q\omega}{6m}$$

$$20. \quad f_m = \frac{v}{(v - v_s \cos \theta)_{\min}} \times f$$

$$1800 = \frac{v}{(v + v_s \cos \theta)_{\max}} \times f$$

On solving,

$$f_m = 2250 \text{ Hz} \\ = 2300 - 50$$

$$21. \quad (NBA)_i = c\theta$$

$$i = \frac{C\theta}{NBA} = \frac{(6 \times 10^{-5})}{10 \times 1 \times \pi \times 10^{-4}} \left(\frac{\pi}{2} \right) \text{ sss} \\ i = 30 \text{ mA}$$

$$\text{So current corresponding 1 part} = \frac{30}{10} = 3 \text{ mA.}$$

22. In an adiabatic expansion,

$$TV^{\gamma-1} = \text{constant}$$

$$T_0 V^{\gamma-1} = T \left(\frac{V}{5} \right)^{\gamma-1} \quad \gamma = 1 + \frac{2}{5}$$

$$\therefore T = (273) \cdot (5)^{2/5} \\ < (KE)_{\text{rotational}} > = kT \\ = 1.38 \times 10^{-23} \times 273 \times (5)^{2/5} \\ = 7 \times 10^{-21} \text{ J (approx).}$$

$$23. \quad \gamma = 1 + \frac{2}{f} = 1 + \frac{1}{3}$$

$$\gamma = \frac{4}{3}$$

$$\text{As we know} \quad \Delta W = \Delta Q - \Delta U$$

$$\therefore \frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q} = 1 - \frac{C_v}{C_p}$$

$$\therefore \frac{\Delta W}{\Delta Q} = 1 - \frac{1}{\gamma} = \frac{1}{4}$$

$$\therefore \Delta Q = 4 \cdot \Delta W \\ \Delta Q = 100 \text{ J.}$$

24. Net power given to N_2 gas = $100 - 30 = 70$ cal/s
The nitrogen gas expands isobarically.

$$\therefore Q = n C_p \frac{dT}{dt} \quad \text{or } 70 = 5 \times \frac{7}{2} R \frac{dT}{dt}$$

$$\therefore \frac{dT}{dt} = 2 \text{ K/sec}$$

25. Pressure in the air inside the column of mercury is equal to the weight of mercury over the air divided by the internal cross sectional area of the tube. When the temperature increases, the weight of the upper part of the mercury column does not change. That is why the pressure in the air is also constant. For the isobaric process, the change in volume is proportional to the change in temperature. The same is true for the lengths of the air column.

$$\frac{\ell}{\ell_0} = \frac{T}{T_0} \Rightarrow \ell = \frac{\ell_0 T}{T_0} = 11$$

26. Using 1st law of TD

$$Q = W + U \uparrow$$

$$0 = ((-W_{\text{fan}}) + P\Delta v) + n \frac{f}{2} R \Delta T$$

$$W_{\text{fan}} = n R \Delta T + n \frac{f}{2} R \Delta T$$

$$W_{\text{fan}} = n C_p \Delta T$$

$$W_{\text{fan}} = (1) \left(R + \frac{f}{2} R \right) (500 \text{ K}) \quad (\text{as the gas is expanding slowly so } p = \text{constant, so } T \propto v)$$

$$W_{\text{fan}} = 14 \text{ kJ}$$

27. Current in the element = $J(2\pi r \cdot dr)$

$$\text{Current enclosed by Amperian loop of radius } \frac{a}{2}$$

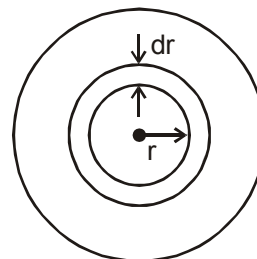
$$I = \int_0^{a/2} \frac{J_0 r}{a} \cdot 2\pi r \cdot dr = \frac{2\pi J_0}{3a} \left(\frac{a}{2} \right)^3 = \frac{\pi J_0 a^2}{12}$$

Applying Ampere's law

$$B \cdot 2\pi \cdot \frac{a}{2} = \mu_0 \cdot \frac{\pi J_0 a^2}{12} \Rightarrow B = \frac{\mu_0 J_0 a}{12}$$

On putting values

$$B = 10 \mu\text{T}$$



28. From the condition given in the paragraph,

$$P \times 2L = nh, \text{ where}$$

P is the momentum,

$$P = \frac{nh}{2L}$$

29. Kinetic energy $E = \frac{P^2}{2m}$

$$E = \frac{n^2 h^2}{8mL^2}$$

31. The initial force on the piston is PA

$$\therefore a = \frac{PA}{M}$$

32. Workdone by gas in the adiabatic process

$$= 2PV \left[1 - \left(\frac{V}{V+LS} \right)^{1/2} \right]$$

$$\therefore \frac{1}{2} Mv^2 = \Delta W$$

$$\therefore v = \left(\frac{4PV}{M} \left[1 - \left(\frac{V}{V+LS} \right)^{1/2} \right] \right)^{1/2}$$

33. $\frac{T_f}{T} = \left(\frac{V}{V_f} \right)^{\gamma-1} = \sqrt{\frac{V}{V+LS}} = \frac{1}{2}$

$$\therefore L = \frac{3V}{S}$$

34 to 36.

Let 'F' be the force to be applied on belt to move with same 'v'

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} + \frac{dm}{dt} v = \mu v$$

$$\frac{dw}{dt} = \frac{d}{dt} (F v dt) = F \cdot v = \mu v^2$$

By work energy theorem

$$\frac{dw_F}{dt} + \frac{dw_{fr}}{dt} = \frac{dK}{dt}$$

$$K = \frac{mv^2}{2}$$

$$\frac{dK}{dt} = \frac{dm}{dt} \cdot \frac{1}{2} \cdot v^2 = \frac{\mu v^2}{2}$$

$$\Rightarrow \mu v^2 + \frac{dw_{fr}}{dt} = \frac{\mu v^2}{2}$$

$$\frac{dw_{fr}}{dt} = -\frac{\mu v^2}{2}$$

37 to 38. By energy conservation,

$$2mgh = \frac{4}{2} R \Delta T$$

$$\Delta T = \frac{10h}{R} \quad \dots\dots\dots(i)$$

$$\text{Initially, } \frac{mg}{A} (A \cdot \ell) = RT_1 \quad \dots\dots\dots(ii)$$

$$\text{and finally, } P_{\max} A(\ell - h) = RT_2 \quad \dots\dots\dots(iii)$$

(iii) – (ii)

$$P_m(1 - h) - 10 = 10h \quad \dots\dots\dots(iv)$$

By equation of adiabatic process $PV^\gamma = \text{constant}$

$$\frac{mg}{A} (A\ell)^\gamma = P_{\max} \{A(\ell - h)\}^\gamma$$

$$10 = P_m(1 - h)^\gamma$$

Put this P_m in equation (iv)

$$10(1 - h)^{1-\gamma} - 10 = 10h$$

$$(1 - h)^{1-\gamma} - 1 = h$$

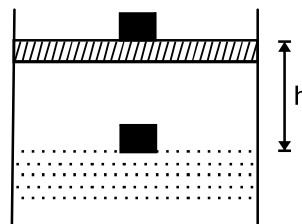
$$(1 - h)^{1-\gamma} = (h + 1)$$

$$\frac{1}{\sqrt{1-h}} = (1 + h) \quad \Rightarrow \quad \frac{1}{(1-h)} = (1 + h)^2$$

$$h(h^2 + h - 1) = 0$$

$$h = \frac{\sqrt{5}-1}{2} \text{ m}$$

$$\text{and } P_n = \frac{10}{\left(1 - \frac{\sqrt{5}-1}{2}\right)^{3/2}} = \frac{20\sqrt{2}}{(3-\sqrt{5})^{3/2}} \text{ N/m}^2.$$



39 TO 41

$$\text{For } P = \frac{4}{5} CT^{3/2}$$

We have $PV^3 = \text{Constant}$

Thus molar specific heat of gas is

$$C = C_V + \frac{R}{1-3} = C_V - \frac{R}{2} = 2R \quad [\text{as } C_V = \frac{5R}{2}]$$

Heat supplied to gas in temperature increment by $\Delta T = 300 \text{ K}$ in this process is

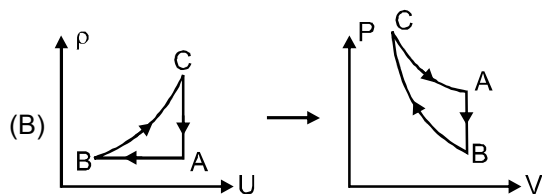
$$Q = nC\Delta T = n(2R)(300) = 600R$$

Change in internal energy of gas in this process is

$$\Delta U = nC_V\Delta T = \frac{5R}{2} \times 300 = 750R.$$

$$\text{Thus work done by the gas is } \Delta W = Q - \Delta U = -150R.$$

43. (A) When 0°C ice converts into 0°C water volume decreases slightly, so $W_{\text{system}} = -ve$. To melt the ice, some heat has to be given ($Q = mL_f$) which is almost equal to increase in internal energy.



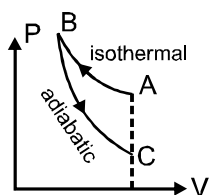
Since, P-v cycle is clockwise, so $W_{\text{net}} = +ve$
and $(\Delta u)_{\text{cycle}} = 0$

(C) By the fan, some work is done on the room air. Done to this, temperature of the gas increases slightly, so internal energy will increase slightly. Mathematically,

$$Q = W + \Delta u$$

$$Q = -ve + \Delta u \Rightarrow \Delta u = +ve.$$

(D) P-V diagram for the process is



From the diagram

$$W_{A \rightarrow B \rightarrow C} = -ve$$

$$(PV)_C < (PV)_A \Rightarrow T_C < T_A$$

So, internal energy decrease.

$$(E) dQ = -2dU$$

$$dQ = 2h \frac{5}{2} R dT$$

$$C = \frac{dQ/dT}{n} = 5R \quad \dots\dots\dots(i)$$

$$C = C_v + \frac{R}{1-x} = \frac{5}{2}R + \frac{R}{1-x} \quad \dots\dots\dots(ii)$$

From equ. (i) & (ii),

$$\text{get } x = \frac{3}{5}$$

So, process equ. is

$$PV^{3/5} = \text{const.}$$

$$\text{If } P \downarrow \Rightarrow V \uparrow \Rightarrow W = +ve$$

To find relation between T and V, put $P = \frac{nRT}{V}$

$$\left(\frac{nRT}{V} \right) (V^{3/5}) = \text{constant}$$

$$T \propto V^{2/5}$$

$$V \uparrow \Rightarrow T \uparrow \Rightarrow \text{internal energy will increase.}$$

44. Root mean square speed of molecules = $\sqrt{\frac{3RT}{M}} = 1.732 \sqrt{\frac{RT}{M}}$

Most probable speed of molecules = $\sqrt{\frac{2RT}{M}} = 1.44 \sqrt{\frac{RT}{M}}$

Average velocity of a molecule is zero

Speed of any individual molecule may be anything.

45. $E_n = E_1 \frac{z^2}{n^2} \left(\frac{\mu}{m} \right)$

$(\mu = \frac{mM}{m+M})$

$r_n = a_0 \left(\frac{n^2}{z} \right) \left(\frac{m}{\mu} \right)$