CHAPTER 21

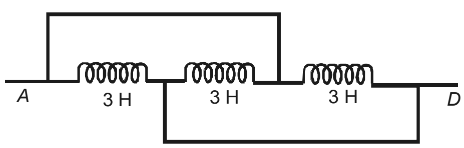
Alternating Current

1. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is **[2002]**

(a) $R/\omega L$ (b) $R/(R^2 + \omega^2 L^2)^{1/2}$

(c) $\omega L/R$ (d) $R/(R^2 - \omega^2 L^2)^{1/2}$

2. The inductance between A and D is **[2002]**



(a) 3.66 H (b) 9 H

(c) 0.66 H (d) 1 H.

3. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is **[2002]**

(a) 4 A (b) 2 A

(c) 6 A (d) 10 A.

4. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is **[2003]**

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

5. The core of any transformer is laminated so as to **[2003]**

(a) reduce the energy loss due to eddy currents

(b) make it light weight

(c) make it robust and strong

(d) increase the secondary voltage

6. Alternating current can not be measured by D.C. ammeter because **[2004]**

(a) Average value of current for complete cycle is zero

(b) A.C. Changes direction

(c) A.C. can not pass through D.C. Ammeter

(d) D.C. Ammeter will get damaged.

7. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50V. The voltage across the LC combination will be **[2004]**

(a) 100 V (b) $50\sqrt{2}$ V

(c) 50 V (d) 0 V (zero)

8. In a LCR circuit capacitance is changed from C to 2 C. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[2004]**

(a) $L/2$ (b) $2L$

(c) $4L$ (d) $L/4$

9. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[2005]**

(a) R, L (b) C alone

(c) L alone (d) L, C

10. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be **[2005]**

(a) 0.4 (b) 0.8

(c) 0.125 (d) 1.25

11. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in **[2005]**

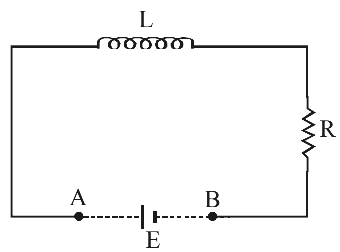
(a) 0.1 s (b) 0.05 s

(c) 0.3 s (d) 0.15 s

12. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1\ k\Omega$ with $C = 2\mu F$. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is **[2006]**

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

13. An inductor ($L = 100$ mH), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[2006]**



(a) 1/eA (b) eA
(c) 0.1 A (d) 1 A

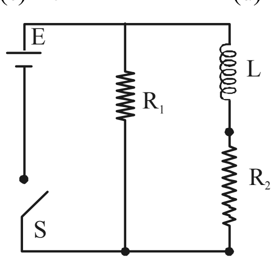
14. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[2007]**

(a) $P = \sqrt{2} E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$
(c) P = zero (d) $P = \frac{E_0 I_0}{2}$

15. An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5V. 2second after the connection is made, the current flowing in ampere in the circuit is **[2007]**

(a) $(1 - e^{-1})$ (b) $(1 - e)$
(c) e (d) e^{-1}

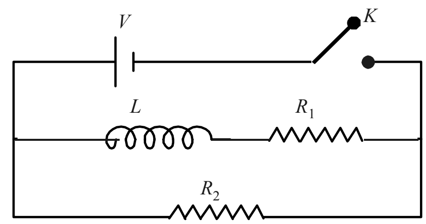
16.



An inductor of inductance L = 400 mH and resistors of resistance $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[2009]**

(a) $\frac{12}{t} e^{-3t}$ V (b) $6\left(1 - e^{-t/0.2}\right)$ V
(c) $12e^{-5t}$ V (d) $6e^{-5t}$ V

17. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[2010]**



(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

18. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is **[2010]**

(a) 305 W (b) 210 W
(c) Zero W (d) 242 W

19. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is: **[2011]**

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$
(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

20. A resistor 'R' and $2\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of *R* to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[2011]**

(a) $1.7 \times 10^5\,\Omega$ (b) $2.7 \times 10^6\,\Omega$

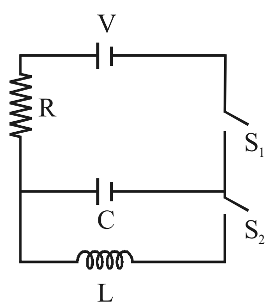
(c) $3.3 \times 10^7\,\Omega$ (d) $1.3 \times 10^4\,\Omega$

21. Combination of two identical capacitors, *a* resistor *R* and *a dc* voltage source of voltage 6V is used in an experiment on a (*C-R*) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time for needed for reducing the voltage of the fully charged series combination by half is **[2011 RS]**

(a) 10 second (b) 5 second

(c) 2.5 second (d) 20 second

22. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct ? **[2013]**



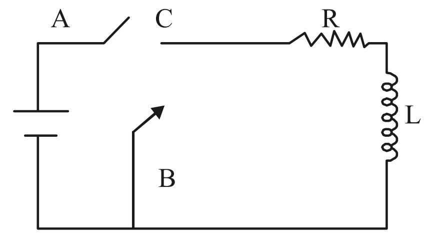
(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At, $t = \tau$, $q = CV/2$

(c) At, $t = 2\tau$, $q = CV(1 - e^{-2})$

(d) At, $t = 2\,\tau$, $q = CV(1 - e^{-1})$

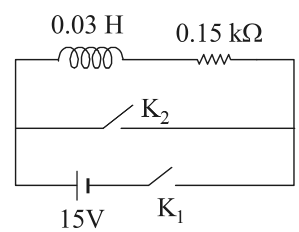
23. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to: **[2014]**



(a) $\dfrac{e}{1-e}$ (b) 1

(c) –1 (d) $\dfrac{1-e}{e}$

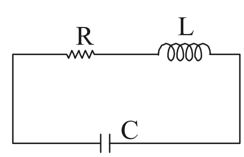
24. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : $\left(e^5 \cong 150\right)$ **[2015]**



(a) 6.7 mA (b) 0.67 mA

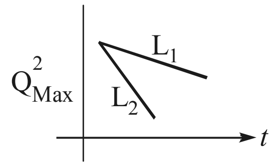
(c) 100 mA (d) 67 mA

25. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below : **[2015]**

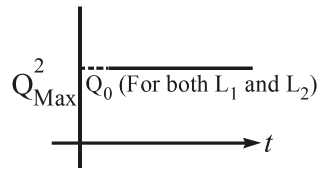


If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time(t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (*plots are schematic and not drawn to scale*)

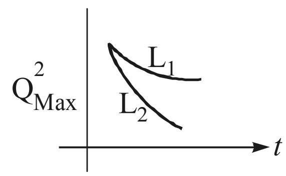
(a)



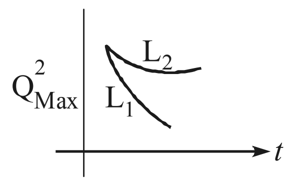
(b)



(c)



(d)

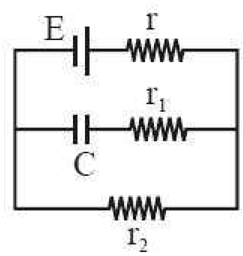


26. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to : **[2016]**

(a) 0.044 H (b) 0.065 H

(c) 80 H (d) 0.08 H

27. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[2017]**



(a) $CE\dfrac{r_2}{(r+r_2)}$ (b) $CE\dfrac{r_1}{(r_1+r)}$

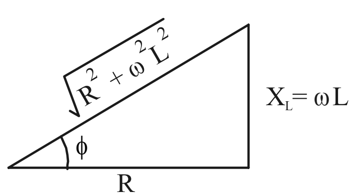
(c) CE (d) $CE\dfrac{r_1}{(r_2+r)}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(b)	(c)	(a)	(a)	(d)	(a)	(a)	(b)	(a)	(c)	(a)	(c)	(a)
16	17	18	19	20	21	22	23	24	25	26	27			
(c)	(c)	(d)	(a)	(b)	(c)	(c)	(c)	(b)	(c)	(b)	(a)			

SOLUTIONS

1. **(b)** The impedance triangle for resistance (*R*) and inductor (*L*) connected in series is shown in the figure.



Power factor $\cos\phi = \dfrac{R}{\sqrt{R^2+\omega^2L^2}}$

2. **(d)** These three inductors are connected in parallel. The equivalent inductance L_p is given by

$$\frac{1}{L_p}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}=1$$

$\therefore L_p = 1$

3. **(b)** $N_p = 140, N_s = 280, I_p = 4A, I_s = ?$

For a transformer $\dfrac{I_s}{I_p}=\dfrac{N_p}{N_s}$

$$\Rightarrow \frac{I_s}{4}=\frac{140}{280} \Rightarrow I_s = 2\,A$$

4. **(c)** When the capacitor is completely charged, the total energy in the LC circuit is with the capacitor and that energy is

$$E = \frac{1}{2}\frac{Q^2}{C}$$

When half energy is with the capacitor in the form of electric field between the plates of the capacitor we get

$\frac{E}{2} = \frac{1}{2}\frac{Q'^2}{C}$ where Q' is the charge on one plate of the capacitor

$$\therefore \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q'^2}{C} \Rightarrow Q' = \frac{Q}{\sqrt{2}}$$

5. **(a)** Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decreases thereby decreasing the eddy current losses.

6. **(a)** D.C. ammeter measure average current in AC current, average current is zero for complete cycle. Hence reading will be zero.

7. **(d)** Since the phase difference between L & C is π,
$\therefore$ net voltage difference across
$LC = 50 - 50 = 0$

8. **(a)** For resonant frequency to remain same LC should be const. LC = const

$$\Rightarrow LC = L' \times 2C \Rightarrow L' = \frac{L}{2}$$

9. **(a)** Phase difference for R–L circuit lies between $\left(0, \frac{\pi}{2}\right)$

10. **(b)** Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

11. **(a)** The charging of inductance given by,

$$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\frac{i_0}{2} = i_0(1 - e^{-\frac{Rt}{L}}) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Taking log on both the sides,

$$-\frac{Rt}{L} = \log 1 - \log 2$$

$$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300 \times 10^{-3}}{2} \times 0.69$$

$\Rightarrow t = 0.1$ sec.

12. **(c)** Across resistor, $I = \frac{V}{R} = \frac{100}{1000} = 0.1\ A$

At resonance,

$$X_L = X_C = \frac{1}{\omega C} = \frac{1}{200 \times 2 \times 10^{-6}} = 2500$$

Voltage across L is

$I X_L = 0.1 \times 2500 = 250\text{ V}$

13. **(a)** Initially, when steady state is achieved,

$$i = \frac{E}{R}$$

Let E is short circuited at $t = 0$. Then

At $t = 0$, $i_0 = \frac{E}{R}$

Let during decay of current at any time the current flowing is $-L\frac{di}{dt} - iR = 0$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt \Rightarrow \int_{i_0}^{i}\frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\Rightarrow \log_e \frac{i}{i_0} = -\frac{R}{L}t \Rightarrow i = i_0\, e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{E}{R}e^{-\frac{R}{L}t} = \frac{100}{100}e^{\frac{-100 \times 10^{-3}}{100 \times 10^{-3}}} = \frac{1}{e}$$

14. **(c)** We know that power consumed in a.c. circuit is given by, $P = E_{rms} . I_{rms} \cos\phi$

Here, $E = E_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

which implies that the phase difference,

$$\phi = \frac{\pi}{2}$$

$$\therefore P = E_{rms} . I_{rms} . \cos\frac{\pi}{2} = 0$$

$$\left(\because \cos\frac{\pi}{2} = 0\right)$$

15. (a) $I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$

(When current is in growth in LR circuit)

$= \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - \mathrm{e}^{-\frac{5}{10}\times 2}\right)$

$= (1 - e^{-1})$

16. (c) Growth in current in LR_2 branch when switch is closed is given by

$$i = \frac{E}{R_2}[1 - e^{-R_2 t/L}]$$

$$\Rightarrow \frac{di}{dt} = \frac{E}{R_2}.\frac{R_2}{L}.e^{-R_2 t/L} = \frac{E}{L}e^{-\frac{R_2 t}{L}}$$

Hence, potential drop across

$$\mathrm{L} = \left(\frac{E}{L}e^{-R_2 t/L}\right)L = Ee^{-R_2 t/L}$$

$$= 12e^{-\frac{2t}{400\times 10^{-3}}} = 12\mathrm{e}^{-5t}\mathrm{V}$$

17. (c) At $t = 0$, no current will flow through L and R_1

$\therefore$ Current through battery $= \frac{V}{R_2}$

At $t = \infty$,

effective resistance, $R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$

$\therefore$ Current through battery $= \frac{V}{R_{eff}}$

$= \frac{V(R_1 + R_2)}{R_1 R_2}$

18. (d) When capacitance is taken out, the circuit is LR.

$\therefore \tan\phi = \frac{\omega L}{R}$

$\Rightarrow \omega L = R\tan\phi = 200\times\frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

Again, when inductor is taken out, the circuit is CR.

$\therefore \tan\phi = \frac{1}{\omega CR}$

$\Rightarrow \frac{1}{\omega c} = R\tan\phi = 200\times\frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

Now, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

$= \sqrt{(200)^2 + \left(\frac{200}{\sqrt{3}} - \frac{200}{\sqrt{3}}\right)^2} = 200\,\Omega$

Power dissipated $= V_{rms} I_{rms}\cos\phi$

$= \mathrm{V_{rms}}\cdot\frac{\mathrm{V_{rms}}}{\mathrm{Z}}.\frac{R}{\mathrm{Z}}\left(\because \cos\phi = \frac{R}{Z}\right)$

$= \frac{V^2{}_{\mathrm{rms}}R}{Z^2} = \frac{(220)^2\times 200}{(200)^2}$

$= \frac{220\times 220}{200} = 242\ \mathrm{W}$

19. (a) Energy stored in magnetic field $= \frac{1}{2}\mathrm{Li}^2$

Energy stored in electric field $= \frac{1}{2}\frac{q^2}{C}$

$\therefore \frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$

Also $q = q_0\cos\omega t$ and $\omega = \frac{1}{\sqrt{LC}}$

On solving $t = \frac{\pi}{4}\sqrt{LC}$

20. (b) We have, $\mathrm{V} = \mathrm{V_0}(1 - \mathrm{e}^{-t/RC})$

$\Rightarrow 120 = 200(1 - e^{-t/RC})$

$\Rightarrow t = RC\,\mathrm{in}\,(2.5)$

$\Rightarrow R = 2.71\times 10^6\,\Omega$

21. (c) Time constant for parallel combination $= 2RC$

Time constant for series combination

$= \frac{RC}{2}$

In first case :

$$V = V_0 e^{-\frac{t_1}{2RC}} = \frac{V_0}{2} \quad ...(1)$$

In second case :

$$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2} \quad ....(2)$$

From (1) and (2)

$$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)}$$

$$\Rightarrow t_2 = \frac{t_1}{4} = \frac{10}{4} = 2.5 \text{ sec.}$$

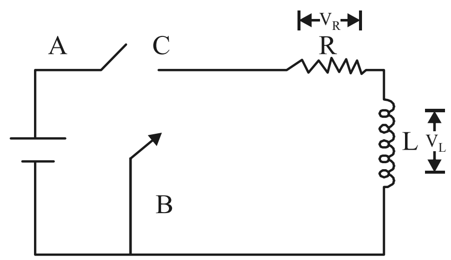
22. **(c)** Charge on he capacitor at any time t is given by $q = CV\,(1 - e^{t/\tau})$

at $t = 2\tau$

$q = CV\,(1 - e^{-2})$

23. **(c)** Applying Kirchhoff's law of voltage in closed loop

$$-V_R - V_C = 0 \;\Rightarrow\; \frac{V_R}{V_C} = -1$$



24. **(b)** $I(0) = \dfrac{15 \times 100}{0.15 \times 10^3} = 0.1\text{A}$

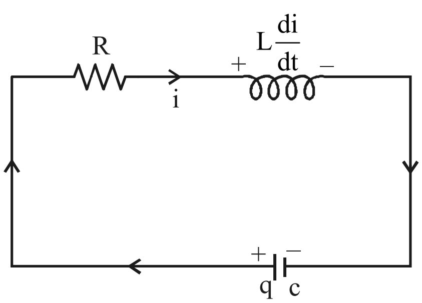
$I(\infty) = 0$

$I(t) = [I(0) - I(\infty)]\; e^{\frac{-t}{L/R}} + i(\infty)$

$I(t) = 0.1\; e^{\frac{-t}{L/R}} = 0.1\, e^{\frac{R}{L}}$

$I(t) = 0.1\; e^{\frac{0.15 \times 1000}{0.03}} = 0.67\text{mA}$

25. **(c)** From KVL at any time t



$$\frac{q}{c} - iR - L\frac{di}{dt} = 0$$

$$i = -\frac{dq}{dt} \Rightarrow \frac{q}{c} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{Lc} = 0$$

From damped harmonic oscillator, the amplitude is given by $A = A_o e - \dfrac{dt}{2m}$

Double differential equation

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$Q_{max} = Q_o e^{-\frac{Rt}{2L}} \Rightarrow Q^2_{max} = Q^2_o e^{-\frac{Rt}{L}}$$

Hence damping will be faster for lesser self inductance.

26. **(b)** Here

$$i = \frac{e}{\sqrt{R^2 + X_L^2}} = \frac{e}{\sqrt{R^2 + \omega^2 L^2}} = \frac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$$

$$10 = \frac{220}{\sqrt{64 + 4\pi^2 (50)^2 L}}$$

$$[\because R = \frac{V}{I} = \frac{80}{10} = 8]$$

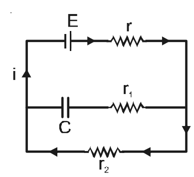
On solving we get

$L = 0.065\,\text{H}$

27. **(a)** In steady state, flow of current through capacitor will be zero.

Current through the circuit,

$$i = \frac{E}{r + r_2}$$



Potential difference through capacitor

$$V_c = \frac{Q}{C} = E - ir = E - \left(\frac{E}{r + r_2}\right) r$$

$$\therefore \quad Q = CE\frac{r_2}{r + r_2}$$