# LCD

1. $\lim_{x\to 0}\frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is

(A) 2 (B) –2 (C) 1/2 (D) –1/2

2. $f(x) = \begin{cases} ax^2+bx+c, & |x|>1 \\ x+1, & |x|\le 1 \end{cases}$. If f(x) is continuous for all values of x, then;

(A) b = 1, a + c = 0 (B) b = 0, a + c = 2
(C) b = 1, a + c = 1 (D) none of these

3. The equation of the tangent to the curve $f(x) = 1 + e^{-2x}$ where it cuts the line y = 2 is

(A) x + 2y = 2 (B) 2x + y = 2
(C) x – 2y = 1 (D) x – 2y + 2 = 0

4. $\lim_{x\to\infty}\frac{x+\sin x}{x-\sin x}$ = ……..

5. $\lim_{x\to\infty}\left(\frac{x+3}{1+x}\right)^{x+3}$ = ……..

6. $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, then a = ……..b = ……

7. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ is equal to

(A) $\pi$ (B) 1/4
(C) 1/2 (D) 1

8 $\lim_{x\to\infty}\frac{\sqrt{x^2-1}}{2x+1}$ is equal to

(A) 1 (B) 0
(C) -1 (D) 1/2

9. If $f(x) = (1 - x^n)^{1/n}$, $0 < x < 1$, n being an odd positive integer and h(x) = f(f(x)), then $h'\left(\frac{1}{2}\right)$ is equal to

(A) $2^n$ (B) 2
(C) n. $2^{n-1}$ (D) 1

10 Among $\lim_{x\to 0}\sec^{-1}\left(\frac{x}{\sin x}\right)$ …. (1)

and $\lim_{x\to 0}\sec^{-1}\left(\frac{\sin x}{x}\right)$ …. (2)

(A) (1) exists, (2) does not exist (B) (1) does not exist, (2) exists
(C) both (1) and (2) exist (D) neither (1) nor (2) exists

11 A function f (x) is defined as f (x) = $\begin{cases} x^2 - 3x + a, & x < 1 \\ -2, & x = 1 \\ bx + 3, & x > 1 \end{cases}$

What are the values of a and b respectively such that f (x) is continuous at x = 1.

(A) 1, –2 (B) 0, –5 (C) –1, 0 (D) 2, –3

12 Given a function f(x) continuous $\forall$ x $\in$R such that $\lim_{x\to 0}\left[ f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right] = 0$,

then f(0) is

(A) 0 (B) 1 (C) 2 (D) 3

13 The value of $\lim_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\pi$

14 The value of $\lim_{x\to 0}\frac{a^x - b^x}{x}$ is

(A) $\log_e\left(\frac{a}{b}\right)$ (B) $\log_e\left(\frac{b}{a}\right)$ (C) $\log_e$ (ab) (D) none of these

15 If f (x) = $\begin{cases} mx + 1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$, then

(A) m = 1, n = 0 (B) m = $\frac{n\pi}{2}$ + 1 (C) n = $\frac{m\pi}{2}$ (D) m = n = $\frac{\pi}{2}$

16. The value of $\lim_{x\to\infty}\frac{\sqrt{1+x^4}-(1+x^2)}{x^2}$ is equal to

(A) 0 (B) –1
(C) 2 (D) 1

17. $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x - x}$ is equal to

(A) 1 (B) e
(C) –1 (D) 0

18. The function f (x) = $\frac{\tan(\pi - [x-\pi])}{1+[x]^2}$, where [.] denotes greatest integer function, is

(A) discontinuous at some x
(B) continuous at all x, but f′ (x) does not exist for some x
(C) f′ (x) exists for all x
(D) none of these

19 If the function f(x) = $\begin{cases} Ax - B, & x \le 1 \\ 3x, & 1 < x < 2 \\ Bx^2 - A, & x \ge 2 \end{cases}$ be continuous at x = 1 and discontinuous at x = 2, then

(a) A = 3 + B, B $\neq$ 3 (b) A = 3 + B, B = 3 (c) A = 3 + B (d) none of these

41. If $(x) = \begin{cases} ax^2 + b & , \ x \le 1 \\ bx^2 + ax + c & , \ x > 1 \end{cases}$, $b \neq 0$. Then f(x) is continuous and differentiable at x = 1 if

(a) c = 0, a = 2b (b) a = b, $c \in R$

(c) a = b, c = 0 (d) a = b, $c \neq 0$.

42. If $f(x) = x^3$ sgn x, then

(a) f is derivable at x = 0 (b) f is continuous, but not derivable at x = 0

(c) LHD at x = 0 is 1 (d) RHD at x = 0 is 0.

43. If f(x) = (x – $x_0$) $\phi$ (x) and $\phi$ (x) is continuous at x = 0, then $f'(x_0)$ is equal to

(a) $\phi'(x_0)$ (b) $\phi(x_0)$

(c) $x_0\ \phi(x_0)$ (d) none of these.

44 If f (x) = $\begin{cases} \dfrac{\sin[x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$ where [x] denotes greatest integer function, then $\lim_{x \to 0} f(x)$ =

(A) 1 (B) 0

(C) -1 (D) doesn't exist

45. If the function $f(x) = \begin{cases} \dfrac{\sin(2x)^2}{x^2} + e^{-x} & , \ x \neq 0 \\ k & , \ x = 0 \end{cases}$ is continuous, then k is

(A) 2 (B) 3

(C) 4 (D) 5.

46. For a function y = f(x), $\dfrac{dy}{dx} = (x-1)(x+2)$. Find the point of local maximum and minimum for the function y = f(x).

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47. Find the function y = f(x) for the above function if it is given that y = 2 at x = 0.

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48. The value of derivative of f (x) = |x –1| + |x –3| at x = 2 is

(A) –2 (B) 0

(C) 2 (D) not defined

49. The function f (x) = |sin x| –1 is

(A) continuous everywhere (B) not differentiable at x = $\dfrac{\pi}{3}$

(C) differentiable at x = 0 (D) differentiable everywhere

50. Let f (x) = $\begin{cases} 3x - 4, & 0 \le x \le 2 \\ 2x + \lambda, & 2 < x \le 3 \end{cases}$, if f (x) is continuous at x = 2, then $\lambda$ is

(A) –1 (B) –2

(C) 2 (D) none of these

51. The number of points at which the function f (x) = $\dfrac{x}{\log|x|}$ is discontinuous is

(A) 1 (B) 2

(C) 3 (D) 4

52. The number of values of x $x \in [0, 2]$ at which the real function $f(x) = |x - 1/2| + |x - 1| + \tan x$ is not differentiable is

(A) 2 (B) 3

(C) 1 (D) 0

**LEVEL−II**

1. The function $(x^2-1)\left|x^2-3x+2\right|+\cos(|x|)$ is not differentiable at
   (A) –1 (B) 0 (C) 1 (D) 2

2. For $x \in R$, $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is
   (A) e (B) $e^{-1}$ (C) $e^{-5}$ (D) $e^5$

3. $\lim\limits_{x\to\frac{\pi}{2}}\left[\frac{6\cos x}{2x-\pi}\right]$, where [.] denotes the greatest integer function, is equal to;
   (A) - 3 (B) - 4 (C) -2 (D) none of these

4. Let $f(x) = (\tan x)^{\frac{1}{x-\frac{\pi}{4}}}$ $\forall$ $x \in (0, \pi/2) \sim \{\pi/4\}$, then the value of $f(\pi/4)$ such that f(x) becomes continuous at $x = \frac{\pi}{4}$ is equal to;
   (A) e (B) $\sqrt{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) $e^2$

5. Let f(x)=[5+3 sinx] $\forall$ $x \in R$. Then total number of points of discontinuity of f(x) in $[0, \pi]$ is equal to;
   (A) 5 (B) 6 (C) 7 (D) 4

6. $f(x) = \sin^{-1}(\sin x)$, $x \in [-2\pi, 2\pi]$. Total number of critical points of f(x) is ;
   (A) 3 (B) 4 (C) 5 (D) 2

7. If the line ax + by + c = 0 is normal to the curve x y + 5 = 0 then
   (A) a > 0 , b > 0 (B) b > 0 , a < 0
   (C) a < 0 , b < 0 (D) b < 0 , a > 0

8. The maximum value of f(x) = |x ln x| in $x \in (0,1)$ is;
   (A) 1/e (B) e
   (C) 1 (D) none of these

9. $f(x) = 3x^3 + 4e^x - k$ is always increasing then value of k =
   (A) 2 (B) –4/9
   (C) 4/9 (D) all of these

10. $\lim\limits_{x\to2}\{[2-x]+[x-2]-x\}$ is
   (A) 0 (B) 3
   (C) –3 (D) does not exist

11. Let f (x) be a twice differentiable function and f′′ (0) = 2 then $\lim\limits_{x\to2}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is
   (A) 6 (B) 1
   (C) 12 (D) 3

12 Let $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all real values of x then
   (A) h is ↑ whenever f (x) is ↓ (B) h is ↑ whenever f(x) is ↓ 0

(C) h is ↓ whenever f is ↓ (D) nothing can be said in general

13. Let f′ (x) > 0, g′ (x) < 0 for all x ∈ R, then
(A) f {g (x)} > f {g (x + 1)} (B) f {g (x)} > f {g (x –1)}
(C) g {f (x)} >< g {f (x + 1)} (D) g {f (x)} > g {f (x –1)}

14. $\lim_{x\to\infty} \frac{\ln[x]}{1+x}$ = …….. [.] G. I. F

15. $\lim_{n\to\infty} \left(3^n + 5^n + 7^n\right)^{\frac{1}{n}}$ = ……

16. If $\alpha$, $\beta$ are the roots of $ax^2 + bx + c = 0$ then $\lim_{x\to\alpha} \frac{1-\cos\left(ax^2+bx+c\right)}{(x-\alpha)^2}$ = ……

17. $\lim_{x\to1}\left(1-x+[x-1]+[1-x]\right)$ = ……

18. f (x) = $\sin^{-1}$(cos x) then points of nondifferentiability between [0, 2$\pi$] = ……..

19. Let f (x + y) = f (x) . f (y) for all x & y, if f (5) = 2 and f'(0) = 3, then f' (5) = ……..

20. f(x) = $\begin{cases} \frac{a\,|x^2-x-2|}{2+x-x^2}, & x<2 \\ b, & x=2 \\ \frac{x-[x]}{x-2}, & x>2 \end{cases}$ ( where [.] denotes the greatest integer function ). If f(x) is continuous at x = 2, then
(A) a = 1, b = 2 (B) a = 1, b = 1
(C) a = 0, b = 1 (D) a = 2, b = 1

21. Let f(x) = $\begin{cases} -1, & x\le 0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$ and g(x) sinx + cosx, then points of discontinuity of f{g(x)} in (0, 2$\pi$) is
(A) $\left\{\frac{\pi}{2}, \frac{3\pi}{4}\right\}$ (B) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$
(C) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{5\pi}{4}, \frac{7\pi}{3}\right\}$

22. If $\alpha$ and $\beta$ are the roots at $ax^2 + bx + c = 0$ then $\lim_{x\to\alpha}\left(1+ax^2+bx+c\right)^{1/(x-\alpha)}$ is
(A) a ($\alpha$ – $\beta$) (B) ln|a($\alpha$ – $\beta$)|
(C) $e^{a(\alpha-\beta)}$ (D) $e^{a|\alpha-\beta|}$

23. $\lim_{x\to\pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to

(A) $1/\sqrt{2}$ (B) 1/2

(C) $\frac{1}{2\sqrt{2}}$ (D) 1

24. The function f(x) = $[x]^2 - [x^2]$ where [y] denotes the greatest integer less than or equal to y), is discontinuous at

(A) all integers (B) all integers except 0 and 1

(C) all integers except 0 (D) all integers except 1

25. If the derivative of f(x) w.r. t x is $\frac{\frac{1}{2} - \sin^2 x}{f(x)}$, then f(x) is a periodic function with period

(A) $\pi$ (B) $2\pi$

(C) $\pi/2$ (D) none of these.

26. $\lim_{x\to 0}\left[(\min(y^2+2y+7))\frac{\sin x}{x}\right]$ = ? (where [.] denotes greatest integer function)

(A) 4 (B) 5

(C) 6 (D) none of these

27. $\lim_{x\to 0}\left[100\frac{\tan x}{x}\right]$ = ? (where [.] denotes greatest integer function)

(A) 100 (B) 99

(C) 101 (D) 0

28. If f (x) = |cos 2x|, then f′ $\left(\frac{\pi}{4}+0\right)$ is equal to

(A) 2 (B) 0

(C) –2 (D) doesn't exist

29. $\lim_{x\to\pi/2}(\sin x)^{1/\cos x}$ =

(A) 0 (B) e

(C) 1 (D) doesn't exist

30. $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$ equals to

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

31. $\lim_{x\to\pi/4}(2-\tan x)^{\frac{1}{\ln(\tan x)}}$ equals to

(A) e (B) 1 (C) 0 (D) $e^{-1}$

32. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ equals to

(A) 0 (B) $\pi$ (C) $-\pi$ (D) not exist

33. f (x) = max {x, $x^3$},then the number of points where f (x) is not differentiable, are

(A) 1 (B) 2 (C) 3 (D) 4

34. $\lim_{x\to 2}\frac{\tan\left(e^{x-2}-1\right)}{\ln(x-1)}$

(A) 2 (B) -2 (C) 1 (D) –1

35. The function defined by f (x) = $\begin{cases}\frac{\sin x^2}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ is

(A) continuos and derivable at x = 0 (B) neither continuous nor derivable at x = 0
(C) continuous but not derivable at x = 0 (D) none of these

36. $\lim_{x\to 0}(1+\tan^2\sqrt{x})^{1/2x}$ is equal to

(A) 1 (B) 0 (C) $e^{1/2}$ (D) $e^{-1/2}$

37. The left hand derivative of f (x) = [x] sin ($\pi$x) at x = k, k is an integer is

(A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1}k\pi$

38. $\lim_{x\to y}\frac{x^y-y^x}{x^x-y^y}$ is

(A) $\frac{\log ey}{1-\log y}$ (B) $\frac{1-\log y}{\log ey}$ (C) $\frac{1-\log y}{1+\log y}$ (D) (1 – log y) log ey

39. $\lim_{x\to 0}(\sin x)^x$ is

(A) 1 (B) 0 (C) $\infty$ (D) does not exist

40 If f(x) is a continuous function $\forall$ x $\in$ R and the range of f(x)=(2, $\sqrt{26}$) and g(x) = $\left[\frac{f(x)}{a}\right]$ is continuous $\forall$ x $\in$ R ([.] denotes the greatest integer function), then the least positive integral value of a is

(A) 2 (B) 3 (C) 6 (D) 5

41 Let $f(x)=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n}+5}$. then the set of values of x for which f(x) = 0, is

(A) |2x| > $\sqrt{3}$ (B) |(2x)| < $\sqrt{3}$ (C)|2x| ≥ $\sqrt{3}$ (D)| 2x | ≤ $\sqrt{3}$

42 If f (x) = $\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim_{x\to 2}f(x)$ is equal to

(A) –2 (B) –1 (C) 0 (D) 1

43 If f (x) = $\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x}, & x\neq 0\\ k, & x=0\end{cases}$ and f (x) is continuous at x = 0, the value of k is

(A) a – b (B) a + b (C) log a + log b (D) none of these

44 The expression of $\frac{dy}{dx}$ of the function y = $a^{x^{a^{x\ldots\ldots\infty}}}$ is

(A) $\frac{y^2}{x(1-y\log x)}$ (B) $\frac{y^2\log y}{x(1-y\log x)}$ (C) $\frac{y^2\log y}{x(1-y\log x\log y)}$ (D) $\frac{y^2\log y}{x(1+y\log x\log y)}$

45 The value of $\lim_{x\to\infty} \frac{x^3 \sin(1/x) - 2x^2}{1+3x^2}$ is

(A) 0 (B) – 1/3 (C) –1 (D) – 2/3

46 $\lim_{x\to 2} \frac{\tan\left(e^{x-2}-1\right)}{\ln(x-1)}$

(A) 2 (B) -2 (C) 1 (D) –1

47 Let f(x) = $\begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$. Then f(x) . sin x is

(A) differentiable at x = 0 (B) continuous at x = 0
(C) not continuous at x =0 (D) none of these

48 The function f(x) = $\frac{\cos x - \sin x}{\sin 4x}$ is not defined at x = $\pi$/4. The value which should be assigned to f at x = $\pi$/4, so that it is continuous there, is

(A) 0 (B) 1 (C) -1 (D) none of these

49 $\lim_{x\to\infty} \frac{\ln x - [x]}{[x]}$, ([.] denotes the greatest integer function)

(A) has value –1 (B) has value 0 (B) has value 1 (D) does not exist

50. The function $\frac{e^{\tan x}-1}{e^{\tan x}+1}$ is discontinuous

(A) at n$\pi$, n $\in$ I (B) at (2n+1) $\frac{\pi}{2}$, n $\in$ I (C) No where (D) Every where

51 If a, b, c, d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$ =

(A) $e^{d/b}$ (B) $e^{c/a}$
(C) $e^{(c+d)/(a+b)}$ (D) e

52 The length of the largest interval in which the function 3 sin x – 4 $\sin^3$ x is increasing, is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$
(C) $\frac{3\pi}{2}$ (D) $\pi$

53. The interval in which f (x) = $e^{|x^2-6x+8|}$ increases, is

(A) $(-\infty, 2) \cup (3, 4)$ (B) R
(C) $(2, 3) \cup (4, \infty)$ (D) (2, 4)

54. If x + |y| = 2y, then y as a function of x is

(A) continuous but not differentiable at x = 0 (B) continuous and differentiable at x = 0
(C) differentiable for all x (D) none of these

55. If y= a log |x| + $bx^2$ + x has its extremum values at x = –1 and x = 2, then

(A) a = 2, b = –1 (B) a = 2, b = –1/2
(C) a = –1/2, b = 1/2 (D) none of these

56. The points of extremum of the function $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$ , is/are

(A) x = 0 (B) x = 1/2
(C) x = –2 (D) $x = \pm 1$

57. Let $f(x) = x^{n+1} + a.\, x^n$, where 'a' is a positive real number. Then x = 0 is a point of
(A) local minimum for any integer n (B) local maximum for any integer n
(C) local minimum if n is an even integer (D) local minimum if n is an odd integer

58. Least natural number 'a' for which $x + ax^{-2} > 2\ \forall\ x \in (0, \infty)$ is
(A) 1 (B) 2 (C) 5 (D) none of these

59. Let $f(x) = \lim_{n\to\infty} (\sin x)^{2n}$ ,then f is
(a) continuous at $x = \pi/2$ , (b) discontinuous at $x = \pi/2$
(c) discontinuous at $x = -\pi/2$
(d) discontinuous at infinite number of points.

60. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ , then f(x) is continuous, but not differentiable at x = 0, if
(a) $n \in (0, 1]$ (b) $n \in [1, \infty)$ (c) $n \in (-\infty, 0)$ (d) n= 0

61. If $f(x) = \{|x| - |x-1|\}^2$ , then $f'(x)$ equals
(a) 0 for all x (b) $2\{|x| - |x-1|\}$
(c) $\begin{cases} 0 \text{ for } x<0 \text{ and for } x>1 \\ 4(2x-1) \text{ for } 0<x<1 \end{cases}$ (d) $\begin{cases} 0 \text{ for } x<0 \\ 4(2x-1) \text{ for } x>0 \end{cases}$

62. If the function $f(x) = \begin{cases} (1+|\sin x|)^{\frac{a}{|\sin x|}} & , \frac{-\pi}{6} < x < 0 \\ b & , x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}} & , 0 < x < \frac{\pi}{6}, \end{cases}$ is continuous at x = 0, then
(a) $a = \log_e b$, $a = \frac{2}{3}$ (b) $\log_e a$, $a = \frac{2}{3}$
(c) $a = \log_e b$, $b = 2$ (d) none of these

63. The function $f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right) & , x<1 \\ |2x-3|[x] & , x \geq 1 \end{cases}$
(a) is continuous at x = 1 (b) is differentiable at x = 1
(c) is continuous but not differentiable at x = 1 (d) none of these

64. The value of p for which the function $f(x) = \begin{cases} \dfrac{\left(4^x-1\right)^3}{\sin\left(\dfrac{x}{p}\right)\log\left\{1+\dfrac{x^2}{3}\right\}} & , x \neq 0 \\ 12(\log 4)^3 & , x = 0 \end{cases}$ is continuous at x = 0 is

(a) 1 (b) 2
(c) 3 (d) 4

65. $\lim\limits_{x\to\infty}\left(\dfrac{1}{1-n^2}+\dfrac{2}{1-n^2}+.....+\dfrac{n}{1-n^2}\right)$ is equal to

(A) 0 (B) –1/2
(C) 1/2 (D) none of these

66 $\lim\limits_{x\to 2}(-1)^{[x]}$, where [x] is the greatest integer function, is equal to

(A) 1 (B) –1
(C) $\pm 1$ (D) doesn't exist

67 $\lim\limits_{x\to\infty}\left(\dfrac{x}{2+x}\right)^{2x}$ =

(A) $e^{-4}$ (B) $e^{-6}$
(C) $e^{-2}$ (D) none of these

68 If f (x) = [x sin $\pi$ x] { where [x] denotes greatest integer function}, then f (x) is

(A) continuous at x = 0 (B) continuous in (-1, 0)
(C) differentiable at x = 1 (D) differentiable in (-1, 1)

69 In order that function f (x) = $(x + 1)^{\cot x}$ is continuous at x = 0, f (0) must be defined as

(A) 0 (B) e
(C) 1/e (D) none of these

70 $\lim\limits_{x\to\infty}\left(\dfrac{x^n}{e^x}\right)$ = 0, (n is integer), for

(A) no value of n (B) all value of n
(C) only negative value of n (D) only positive value of n

71 $\lim\limits_{n\to\infty}\left(4^n+5^n\right)^{\frac{1}{n}}$ is equal to

(A) 4 (B) 5
(C) e (D) none of these

72 $\lim\limits_{x\to\frac{\pi}{2}}\left[\sin^{-1}(\sin x)\right]$ equals, where [.] denotes the greatest integer function

(A) $\dfrac{\pi}{2}$ (B) 0
(C) 1 (D) does not exist

73 The value of derivative of f (x) = |x –1| + |x –3| at x = 2 is

(A) –2 (B) 0

(C) 2 (D) not defined

74 The number of points where the function f (x) = $x^2$ –1 + |ln |x|| is not differentiable is

(A) 1 (B) 2

(C) 3 (D) none of these

75 f (x) is a continuous function and takes only rational values. If f (0) = 3, then f (2) equals

(A) 5 (B) 0

(C) 1 (D) none of these

76 $\lim_{x\to 0}\frac{|x+1|+|x-1|-2}{x}$ is equal to

(A) 1 (B) −1 (C) 2 (D) 0

77. $\lim_{x\to 0}\frac{x.2^x-x}{1-\cos x}$ is equal to

(A) log2 (B) $\frac{1}{2}$ log2 (C) 2 log2 (D) none of these

78. If f(x) = $\left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$. Then $\lim_{x\to\infty} f(x)$ is

(A) $e^4$ (B) $e^3$ (C) $e^2$ (D) None of these

79. $\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{x^3}$ is equal to

(A) 1/2 (B) 2 (C) −1/2 (D) None of these

80. $\lim_{x\to 0}\frac{(a+x)^2\sin(a+x)-a^2\sin a}{x}$ is equal to

(A) $a^2$ cosa + a sina (B) $a^2$ cosa + 2a sina (C) $2a^2$ cosa + a cosa (D) None of these

81. Let f : R → R is a differentiable function and f(1) = 4. Then the value of $\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt$ is

(A) 8 f′(1) (B) 4f′(1) (D) 2f′(1) (D) None of these

82. If f(x) = $\begin{cases} x-3 & x<0 \\ x^2-3x+2, & x\geq 0 \end{cases}$ and g(x) = f(|x|) + |f(x)| , then g(x) is continuous at

(A) R − {0} (B) $R^+$

(C) R − {1, 2} (D) R − {0, 1, 2}

83. The value the limit $\lim_{x\to 0}\frac{\left(e^{a/x}-e^{-a/x}\right)}{\left(e^{a/x}+e^{-a/x}\right)}$ , a > 0 is

(A) 0 (B) 1

(C) infinity (D) does not exist

84. The number of points where g(f(x)) is discontinuous given that $g(x) = \frac{1}{x^2 + x - 1}$ and $f(x) = \frac{1}{x - 3}$ is

(A) 1 (B) 2
(C) 3 (D) 4

85. The value of $\lim_{x \to 0} \left( \frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2}$ is

(A) $e^2$ (B) $e^3$
(C) $e^5$ (D) none of these

86. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

87. Let $f(x + y) = f(x)\, f(y)$ $\forall$ x, y $\in$ R. Suppose that f(3) = 3 and f′(0) = 11 then f′(3) is given by

(A) 22 (B) 44 (C) 28 (D) 33

88. The function $f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ then which of the following is not true

(A) continuous at x = 1 (B) continuous at x = 3
(C) differentiable at x = 1 (D) differentiable at x = 3

89. The function f(x) = max{1 − x, 1 + x, 2}, x $\in$ (−∞, ∞) is

(A) differentiable at all points
(B) differentiable at all points except at x = 1 and x = −1
(C) continuous at all points except at x = 1 and x = −1, where it is discontinuous
(D) None of these

90. Let $f(x) = [\tan^2 x]$ where [.] is greatest integer function then

(A) $\lim_{x \to 0} f(x)$ does not exist (B) f(x) is continuous at x = 0
(C) f(x) is not differentiable x = 0 (D) f′(0) = 1

**LEVEL−III**

1. The number of critical points of f (x) = max (sin x , cos x) for $x \in (0 , 2\pi)$

(A) 2 (B) 5
(C) 3 (D) non

2. If $f(x) = \int_0^x (t+1)\ (e^t - 1)\ (t-2)\ (t+4)\ dt$ then f (x) would assume the local minima at;

(A) x = - 4 (B) x = 0
(C) x = -1 (D) x = 2.

3. Let f (x) = [cos x + sin x], $0 < x < 2\pi$ where [x] denotes the greatest integer less than or equal to x. The number of points of discontinuity of f (x) is

(A) 6 (B) 5
(C) 4 (D) 3

4. $\lim_{x \to -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$ ……

5. f (x) = $\frac{1}{\log |x|}$ is discontinuous at x = ……

6. The value of the limit

$\lim_{x \to 0} \left\{1^{1/\sin^2 x} + 2^{1/\sin^2 x} + ..... + n^{1/\sin^2 x}\right\}^{\sin^2 x}$

(A) $\infty$ (B) 0
(C) $\frac{n(n+1)}{2}$ (D) n

7. $\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(A) 1/5 (B) 1/6
(C) 1/4 (D) ½

8. If $\tan^{-1}(x + h) = \tan^{-1}(x) + (h \sin y)(\sin y) - (h \sin y)^2 \cdot \frac{\sin 2y}{2} + (h \sin y)^3 \cdot \frac{\sin 3y}{3} + ....$, where $x \in (0, 1)$, $y \in (\pi/4, \pi/2)$, then

(A) $y = \tan^{-1} x$ (B) $y = \sin^{-1} x$
(C) $y = \cot^{-1} x$ (D) $y = \cos^{-1} x$

9. The value of $\lim_{|x| \to \infty} \cos[\tan^{-1}(\sin(\tan^{-1} x))]$ is equal to

(A) -1 (B) $\sqrt{2}$ (C) $-\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}}$

10. If $\lim_{x \to 0} \int_0^x \frac{t^2 dt}{(x - \sin x)\sqrt{a+t}} = 1$, then the value of a is

(A) 4 (B) 2 (C) 1 (D) none of these

12 For some g, let $f(x) = x(x+3)\, e^{g(x)}$ be a continuous function. If there exists only one point x = d such that f′(d) = 0, then

(A) d < -3 (B) d > 0 (C) $-3 \le d \le 0$ (D) $-3 < d < 0$

13 $\lim_{n\to\infty}\left[1-\ln\left(1+\frac{1}{n}\right)^{n-1}\right]$ is equal to

(A) 0 (B) 1 (C) e (D) none of these

14 The value of $\lim_{x\to\infty}\frac{x^n+nx^{n-1}+1}{e^{[x]}}$, $n \in I$ is

(A) 1 (B) 0 (C) n (D) n(n –1)

15 Given a function f(x) continuous $\forall\ x \in R$ such that $\lim_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right] = 0$,

then f(0) is

(A) 0 (B) 1 (C) 2 (D) 3

16 Let R be the set of real numbers and $f : R \to R$ be such that for all x and y in R $|f(x) - f(y)| \le |x - y|^7$. Then f (x) is.

(A) linear (B) constant

(C)quadratic (D) none of these.

17. Find the value of $\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$

(A) 2/5 (B) 2/3

(C) 1/4 (D) 1/5.

18 $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ is

(A) 1 (B) –1

(C) 0 (D) doesn't exist

19 Given that f (x) is a non-zero differentiable function such that f (x + y) = f (x). f (y), $\forall$ x, y $\in$ R, and f′ (0) = 1 then ln f (1) is equal to

(A) 0 (B) 1

(C) e (D) none of these

20 The largest interval where the function $f(x) = \frac{x}{1+|x|}$ is differentiable

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(-\infty, 0) \cup (0, \infty)$ (D) none of these

21 $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{x^2}$ is equal to

(A) $\frac{11e}{24}$ (B) $-\frac{11e}{24}$ (C) $\frac{e}{24}$ (D) None of these

22 The value of the limit $\lim_{x\to 0}\left(\frac{1-3^x-4^x+12^x}{\sqrt{2\cos x+7}-3}\right)$ is

(A) 0 (B) − 6(log3) (log4)
(C) 1 (D) none of these

23 Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$, for all x, y ∈ R and if f(x) is differentiable, and f′(0) = −1, f(0) = 1 then the function f(x) is

(A) −x + 1 (B) x + 1
(C) $x^2$ − 1 (D) x −1

24 The points of discontinuity of the function fog where g(x) = $\frac{1}{x-1}$ and f(x) = $\frac{1}{x^2+x-2}$ are

(A) $\frac{1}{2}$, 2, 1 (B) 2, 1 (B) 2, $\frac{1}{2}$ (D) none of these

**ANSWERS**

**LEVEL −I**

1. C 2. B 3. A 4. C

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5. | B | 6. | C | 7. | 1 | 8. | $e^2$ |
| 9. | −2, −1 | 10. | C | 11. | D | 12. | B |
| 13. | D | 14. | D | 15. | C | 16. | A |
| 17. | A | 18. | D | 19. | A | 20. | B |
| 21. | B | 22. | B | 23. | D | 24. | A |
| 25. | A | 26. | B | 27. | B | 28. | C |
| 29. | A | 30. | B | 31. | A | 32. | C |
| 33. | C | 34. | A | 35. | C | 36. | B |
| 37. | A | 38. | D | 39. | A | 40. | A |
| 41. | A | 42. | B | 43. | B | 44. | D |
| 45. | D | 46. | −2, 1 | 47. | $f(x) = x^3/3 + x^2/2 – 2x + 2$ | | |
| 48. | B | | | | | | |
| 49. | A | 50. | B | 51. | C | 52. | A |

**LEVEL −II**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | D | 2. | C | 3. | A | 4. | D |
| 5. | A | 6. | B | 7. | A, C | 8. | A |
| 9. | D | 10. | C | 11. | A | 12. | C |
| 13. | B, D | 14. | 0 | 15. | 7 | 16. | $(2a\alpha + b)^2 / 2$ |
| 17. | −1 | 18. | $0, \pi, 2\pi$ | 19. | 6 | 20. | B |
| 21. | B | 22. | C | 23. | B | 24. | B |
| 25. | A | 26. | B | 27. | B | 28. | A |
| 29. | C | 30. | B | 31. | D | 32. | C |
| 33. | C | 34. | C | 35. | A | 36. | C |
| 37. | A | 38. | C | 39. | D | 40. | C |
| 41. | A | 42. | D | 43. | B | 44. | A |
| 45. | B | 46. | C | 47. | B | 48. | D |
| 49. | A | 50. | B | 51. | A | 52. | B |
| 53. | C | 54. | A | 55. | B | 56. | D |
| 57. | C | 58. | B | 59. | D | 60. | B |
| 61. | C | 62. | A | 63. | C | 64. | D |
| 65. | B | 66. | D | 67. | A | 68. | A |
| 69. | B | 70. | B | 71. | B | 72. | C |
| 73. | B | 74. | B | 75. | D | 76. | D |
| 77. | C | 78. | A | 79. | A | 80. | B |
| 81. | A | 82. | A | 83. | D | 84. | C |
| 85. | A | 86. | C | 87. | D | 88. | D |
| 89. | B | 90. | B | | | | |

**LEVEL −III**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | C | 2. | C | 3. | B | 4. | $1 / \sqrt{2\pi}$ |
| 5. | $0, \pm 1$ | 6. | D | 7. | D | 8. | C |
| 9. | D | 10. | A | | | 12. | D |
| 13. | A | 14. | B | 15. | A | 16. | B |
| 17. | B | 18. | D | 19. | B | 20. | A |
| 21. | A | 22. | B | 23. | A | 24. | A |