CHAPTER 20

Continuity and Differentiability

1. f is defined in [-5, 5] as **[2002]**

$f(x) = x$ if x is rational

$= -x$ if x is irrational. Then

(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

2. If $f(x+y) = f(x).f(y) \forall x.y$ and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is **[2002]**

(a) 0 (b) 1
(c) 6 (d) 2

3. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ **[2002]**
(c) $-y$ (d) $2x^2y$

4. Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x\to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$$

then the value of k is **[2003]**

(a) 0 (b) 4
(c) 2 (d) 1

5. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ then f(x) is

(a) discontinuous every where **[2003]**
(b) continuous as well as differentiable for all x
(c) continuous for all x but not differentiable at $x = 0$
(d) neither differentiable nor continuous at $x = 0$

6. If $f(x) = x^n$, then the value of **[2003]**

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + ... \frac{(-1)^n f^n(1)}{n!}$$

is

(a) 1 (b) 2^n
(c) $2^n - 1$ (d) 0.

7. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P, then $f'(a), f'(b), f'(c)$ are in **[2003]**

(a) Arithmetic -Geometric Progression
(b) A.P
(c) G.P
(d) H.P.

8. Let $f(x) = \dfrac{1-\tan x}{4x - \pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right]$.

If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is **[2004]**

(a) -1 (b) $\dfrac{1}{2}$
(c) $-\dfrac{1}{2}$ (d) 1

9. If $x = e^{y+e^{y+\cdots\text{to}\infty}}, x > 0$, then $\dfrac{dy}{dx}$ is **[2004]**

(a) $\dfrac{1+x}{x}$ (b) $\dfrac{1}{x}$
(c) $\dfrac{1-x}{x}$ (d) $\dfrac{x}{1+x}$

10. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals **[2005]**

(a) 3 (b) 4
(c) 5 (d) 6

11. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then **[2005]**

(a) $f(6) \geq 8$ (b) $f(6) < 8$
(c) $f(6) < 5$ (d) $f(6) = 5$

12. If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals **[2005]**

(a) -1 (b) 0
(c) 2 (d) 1

13. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is **[2005]**

(a) 1 (b) 0
(c) 3 (d) 2

14. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable is **[2006]**

(a) $(-\infty, 0) \cup (0, \infty)$

(b) $(-\infty, -1) \cup (-1, \infty)$

(c) $(-\infty, \infty)$

(d) $(0, \infty)$

15. If $x^m . y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is **[2006]**

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$
(c) xy (d) $\frac{x}{y}$

16. Let $f: R \to R$ be a function defined by $f(x) = \min\{x+1, |x|+1\}$, Then which of the following is true? **[2007]**

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \geq 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x = 1$

17. The function $f: R/\{0\} \to R$ given by **[2007]**

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

(a) 0 (b) 1
(c) 2 (d) -1

18. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$ **[2008]**

Then which one of the following is true?

(a) f is neither differentiable at $x = 0$ nor at $x = 1$
(b) f is differentiable at $x = 0$ and at $x = 1$
(c) f is differentiable at $x = 0$ but not at $x = 1$
(d) f is differentiable at $x = 1$ but not at $x = 0$

19. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals **[2009]**

(a) 1 (b) $\log 2$
(c) $-\log 2$ (d) -1

20. Let $f: (-1, 1) \to$ R be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ **[2010]**

(a) -4 (b) 0
(c) -2 (d) 4

21. The values of p and q for which the function **[2011]**

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$

is continuous for all x in R, are

(a) $p = \frac{5}{2}, q = \frac{1}{2}$

(b) $p = -\frac{3}{2}, q = \frac{1}{2}$

(c) $p = \frac{1}{2}, q = \frac{3}{2}$

(d) $p = \frac{1}{2}, q = -\frac{3}{2}$

22. $\frac{d^2x}{dy^2}$ equals : **[2011]**

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

23. Define $f(x)$ as the product of two real function **[2011RS]**

$$f_1(x) = x, x \in R, \text{and } f_2(x) = \begin{cases} \sin\frac{1}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$$

as follows :

$$f(x) = \begin{cases} f_1(x).f_2(x), & if\ x = 0 \\ 0 & if\ x = 0 \end{cases}$$

Statement - 1 : $f(x)$ is continuous on R.

Statement - 2 : $f_1(x)$ and $f_2(x)$ are continuous on R.

(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

24. If function $f(x)$ is differentiable at $x = a$,

then $\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$ is : **[2011RS]**

(a) $-a^2 f'(a)$

(b) $a\,f(a) - a^2 f'(a)$

(c) $2af(a) - a^2 f'(a)$

(d) $2a\,f(a) + a^2 f'(a)$

25. If $f : R \to R$ is a function defined by $f(x) = [x]$ $\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is . **[2012]**

(a) continuous for every real x.

(b) discontinuous only at $x = 0$

(c) discontinuous only at non-zero integral values of x.

(d) continuous only at $x = 0$.

26. Consider the function, $f(x) = |x-2|$ `$+ |x-5|$, $x \in R$.

Statement-1 : $f'(4) = 0$

Statement-2 : f is continuous in $[2,5]$, differentiable in $(2,5)$ and $f(2) = f(5)$. **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

27. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to : **[2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) 1 (d) $\sqrt{2}$

28. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0,1[$ **[2014]**

(a) $f'(c) = g'(c)$ (b) $f'(c) = 2g'(c)$

(c) $2f'(c) = g'(c)$ (d) $2f'(c) = 3g'(c)$

29. If the function.

$$g(x)=\begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$$ is differentiable,

then the value of $k+m$ is : **[2015]**

(a) $\frac{10}{3}$ (b) 4

(c) 2 (d) $\frac{16}{5}$

30. For $x \in R$, $f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then : **[2016]**

(a) $g'(0)=-\cos(\log 2)$

(b) g is differentiable at $x=0$ and $g'(0)=-\sin(\log 2)$

(c) g is not differentiable at $x=0$

(d) $g'(0)=\cos(\log 2)$

31. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(a)	(b)	(c)	(d)	(b)	(c)	(c)	(c)	(a)	(b)	(a)	(c)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(b)	(c)	(d)	(a)	(b)	(c)	(c)	(c)	(a)	(c)	(a)	(b)	(c)	(d)
31														
(b)														

SOLUTIONS

1. **(b)** Let a is a rational number other than 0, in [–5, 5], then $f(a)=a$ and $\lim_{x\to a} f(x)=-a$

[As in the immediate neighbourhood of a rational number, we find irrational numbers]

$\therefore f(x)$ is not continuous at any rational number

If a is irrational number, then

$f(a)=-a$ and $\lim_{x\to a} f(x)=a$

$\therefore f(x)$ is not continuous at any irrational number clearly $\lim_{x\to 0} f(x)=f(0)=0$

$\therefore f(x)$ is continuous at $x=0$

2. **(c)** $f(x+y)=f(x)\times f(y)$

Differentiate with respect to x, treating y as constant

$f'(x+y)=f'(x)f(y)$

Putting $x=0$ and $y=x$, we get $f'(x)=f'(0)f(x)$; $\Rightarrow f'(5)=3f(5)=3\times 2=6.$

3. **(a)** $y=(x+\sqrt{1+x^2})^n$

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.\,2x\right)$$

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$$

$$= \frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\frac{dy}{dx} = ny$ or $\sqrt{1+x^2}\,y_1 = ny$

$(y_1 = \frac{dy}{dx})$ Squaring,

$(1+x^2)y_1^2 = n^2y^2$

Differentiating,

$(1+x^2)2y_1y_2 + y_1^2.2x = n^2.2yy_1$

or $(1+x^2)y_2 + xy_1 = n^2y$

4. **(b)** $\lim\limits_{x\to a}\frac{f(a)g'(x)-g(a)f'(x)}{g'(x)-f'(x)} = 4$

(By L' Hospital rule)

$\lim\limits_{x\to a}\frac{k\,g'(x)-k\,f'(x)}{g'(x)-f'(x)} = 4$

$\therefore k = 4.$

5. **(c)** $f(0)=0;\ f(x) = xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}$

R.H.L. $\lim\limits_{h\to 0}(0+h)e^{-2/h} = \lim\limits_{h\to 0}\frac{h}{e^{2/h}} = 0$

L.H.L. $\lim\limits_{h\to 0}(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)} = 0$

therefore, $f(x)$ is continuous.

R.H.D $= \lim\limits_{h\to 0}\frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h} = 0$

L.H.D. $= \lim\limits_{h\to 0}\frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-0}{-h} = 1$

therefore, L.H.D. $\neq$ R.H.D.

$f(x)$ is not differentiable at $x=0$.

6. **(d)** $f(x) = x^n \Rightarrow f(1) = 1$

$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$

....

.... $f^n(x) = n! \Rightarrow f^n(1) = n!$

$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \ldots + (-1)^n\frac{n!}{n!}$

$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^n\,{}^nC_n = 0$

7. **(b)** $f(x) = ax^2+bx+c$

$f(1) = f(-1)$

$\Rightarrow a+b+c = a-b+c$ or $b=0$

$\therefore f(x) = ax^2+c$ or $f'(x) = 2ax$

Now $f'(a); f'(b);$ and $f'(c)$

are $2a(a); 2a(b); 2a(c)$

i.e. $2a^2, 2ab, 2ac.$

$\Rightarrow$ If a, b, c are in A.P. then $f'(a); f'(b)$ and $f'(c)$ are also in A.P.

8. **(c)** $f(x) = \frac{1-\tan x}{4x-\pi}$ is continuous in $\left[0, \frac{\pi}{2}\right]$

$\therefore f\left(\frac{\pi}{4}\right) = \lim\limits_{x\to\frac{\pi}{4}} f(x) = \lim\limits_{x\to\frac{\pi}{4}^+} f(x)$

$= \lim\limits_{h\to 0} f\left(\frac{\pi}{4}+h\right)$

$= \lim\limits_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}+h\right)}{4\left(\frac{\pi}{4}+h\right)-\pi}, h>0$

$= \lim\limits_{h\to 0}\frac{1-\frac{1+\tan h}{1-\tan h}}{4h}$

$= \lim\limits_{h\to 0}\frac{-2}{1-\tan h}.\frac{\tan h}{4h} = \frac{-2}{4} = -\frac{1}{2}$

ALTERNATE SOLUTION

$\because f(x)$ is continuous at $x = \frac{\pi}{4}$

$\therefore f\left(\frac{\pi}{4}\right) = \lim\limits_{x\to\frac{\pi}{4}} f(x) = \lim\limits_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi}$

$= \lim\limits_{x\to\frac{\pi}{4}}\frac{-\sec^2 x}{4}$ [using L' Hospital's rule]

$= \frac{-\sec^2\frac{\pi}{4}}{4} = \frac{-2}{4} = \frac{-1}{2}$

9. **(c)** $x = e^{y+e^{y+\cdots\infty}} \Rightarrow x = e^{y+x}$.

Taking log.

$\log x = y + x \Rightarrow \frac{1}{x} = \frac{dy}{dx} + 1$

$\therefore \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$

10. **(c)** $f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$;

As function is differentiable so it is continuous as it is given that $\lim_{h \to 0} \frac{f(1+h)}{h}$ $= 5$ and hence $f(1) = 0$

Hence $f'(1) = \lim_{h \to 0} \frac{f(1+h)}{h} = 5$

11. **(a)** As $f(1) = -2$ & $f'(x) \geq 2 \ \forall \ x \in [1, 6]$

Applying Lagrange's mean value theorem

$\frac{f(6) - f(1)}{5} = f'(c) \geq 2$

$\Rightarrow f(6) \geq 10 + f(1)$

$\Rightarrow f(6) \geq 10 - 2 \Rightarrow f(6) \geq 8.$

12. **(b)** $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$|f'(x)| = \lim_{h \to 0} \left| \frac{f(x+h) - f(x)}{h} \right|$

$\leq \lim_{h \to 0} \left| \frac{(h)^2}{h} \right|$

$\Rightarrow |f'(x)| \leq 0 \Rightarrow f'(x) = 0$

$\Rightarrow f(x) = \text{constant}$

As $f(0) = 0$

$\Rightarrow f(1) = 0.$

13. **(a)** $x^2 - (a-2)x - a - 1 = 0$

$\Rightarrow \alpha + \beta = a - 2$; $\alpha\beta = -(a+1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6 = (a-1)^2 + 5$

For min. value of $\alpha^2 + \beta^2$ where α is an integer

$\Rightarrow a = 1.$

14. **(c)** $f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} \frac{x}{(1-x)^2}, & x < 0 \\ \frac{x}{(1+x)^2}, & x \geq 0 \end{cases}$

$\therefore$ $f'(x)$ exist at everywhere.

15. **(a)** $x^m . y^n = (x+y)^{m+n}$

$\Rightarrow mlnx + nlny = (m+n)ln(x+y)$

Differentiating both sides.

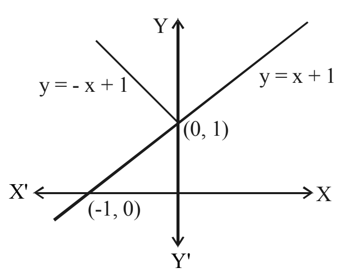
$\therefore \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \left(\frac{m}{x} - \frac{m+n}{x+y}\right) = \left(\frac{m+n}{x+y} - \frac{n}{y}\right)\frac{dy}{dx}$

$\Rightarrow \frac{my - nx}{x(x+y)} = \left(\frac{my - nx}{y(x+y)}\right)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

16. **(a)** $f(x) = \min\{x+1, |x|+1\} \Rightarrow f(x) = x + 1 \ \forall \ x \in R$



Hence, $f(x)$ is differentiable everywhere for all $x \in R$.

17. **(b)** Given, $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$

$\Rightarrow f(0) = \lim_{x \to 0} \frac{1}{x} - \frac{2}{e^{2x} - 1}$

$$= \lim_{x\to 0}\frac{(e^{2x}-1)-2x}{x(e^{2x}-1)};\left[\frac{0}{0}\text{ form}\right]$$

$\therefore$ using, L'Hospital rule

$$f(0) = \lim_{x\to 0}\frac{4e^{2x}}{2(xe^{2x}2+e^{2x}.1)+e^{2x}.2}$$

$$= \lim_{x\to 0}\frac{4e^{2x}}{4xe^{2x}+2e^{2x}+2e^{2x}}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to 0}\frac{4e^{2x}}{4(xe^{2x}+e^{2x})} = \frac{4.e^{0}}{4(0+e^{0})} = 1$$

18. **(c)** We have

$$f(x) = \begin{cases}(x-1)\sin\left(\frac{1}{x-1}\right), & \text{if } x \neq 1\\ 0 & \text{, if } x = 1\end{cases}$$

$$Rf'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{h\sin\frac{1}{h}-0}{h} = \lim_{h\to 0}\sin\frac{1}{h}$$

= a finite number

Let this finite number be l

$$L\ f'(1) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$= \lim_{h\to 0}\frac{-h\sin\left(\frac{1}{-h}\right)}{-h}$$

$$= \lim_{h\to 0}\sin\left(\frac{1}{-h}\right) = -\lim_{h\to 0}\sin\left(\frac{1}{h}\right)$$

$= -$ (a finite number) $= -l$

Thus $Rf'(1) \neq Lf'(1)$

$\therefore$ f is not differentiable at x = 1

Also,

$$f'(0) = \sin\frac{1}{(x-1)} - \frac{x-1}{(x-1)^2}\cos\left(\frac{1}{x-1}\right)\Bigg]_{x=0}$$

$= -\sin 1 + \cos 1$

$\therefore$ f is differentiable at x = 0

19. **(d)** $x^{2x} - 2x^x \cot y - 1 = 0$

$\Rightarrow \quad 2\cot y = x^x - x^{-x}$

$\Rightarrow \quad 2\cot y = u - \frac{1}{u}$ where $u = x^x$

Differentiating both sides with respect to x, we get

$$-2\operatorname{cosec}^2 y\frac{dy}{dx} = \left(1+\frac{1}{u^2}\right)\frac{du}{dx}$$

where $u = x^x \Rightarrow \log u = x\log x$

$$\Rightarrow \quad \frac{1}{u}\frac{du}{dx} = 1+\log x$$

$$\Rightarrow \quad \frac{du}{dx} = x^x(1+\log x)$$

$\therefore$ We get

$-2\operatorname{cosec}^2 y$

$$\frac{dy}{dx} = (1+x^{-2x}).x^x(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^x+x^{-x})(1+\log x)}{-2(1+\cot^2 y)} \quad \ldots(i)$$

Now when $x = 1$, $x^{2x} - 2x^x \cot y - 1 = 0$, gives

$1 - 2\cot y - 1 = 0$

$\Rightarrow \quad \cot y = 0$

$\therefore$ From equation (i), at $x = 1$ and $\cot y = 0$, we get

$$y'(1) = \frac{(1+1)(1+0)}{-2(1+0)} = -1$$

20. **(a)**

$$g'(x) = 2\big(f(2f(x)+2)\big)\left(\frac{d}{dx}\big(f(2f(x)+2)\big)\right)$$

$$= 2f(2f(x)+2)f'(2f(x))+2).(2f'(x))$$

$$\Rightarrow g'(0) = 2f(2f(0)+2).f'(2f(0)+2)$$

$$.2f'(0) = 4f(0)(f'(0))^2 = 4(-1)(1)^2 = -4$$

21. **(b)** $\underset{(at\,x=0)}{L.H.L} = \lim_{x\to 0^-} f(x)$

$$= \lim_{h\to 0}\frac{\sin\{(p+1)(-h)\}-\sin h}{-h}$$

$= p+1+1 = p+2$

$$\underset{(at\,x=0)}{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} = \frac{1}{1+1} = \frac{1}{2}$$

$$f(0) = q \Rightarrow p = -\frac{3}{2}, q = \frac{1}{2}$$

22. (c) $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dx}\left(\frac{dx}{dy}\right)\frac{dx}{dy}$

$=\frac{d}{dx}\left(\frac{1}{dy/dx}\right)\frac{dx}{dy}$

$=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d^2y}{dx^2}\cdot\frac{1}{\frac{dy}{dx}}$ $\quad=-\frac{1}{\left(\frac{dy}{dx}\right)^3}\frac{d^2y}{dx^2}$

23. (c) $f(x)=\begin{cases}x\sin(1/x), x\neq 0\\ 0 \quad, x=0\end{cases}$ at $x=0$

LHL $=\lim_{h\to 0^-}\left\{-h\sin\left(-\frac{1}{h}\right)\right\}$

$=0\times$ a finite quantity betwen -1 and $1=0$

RHL $=\lim_{h\to 0^+} h\sin\frac{1}{h}=0$

Also, $f(0)=0$

Thus LHL $=$ RHL $=f(0)$

$\therefore$ $f(x)$ is continuous on R.

$f_2(x)$ is not continuous at $x=0$

24. (c) $\lim_{x\to a}\frac{x^2f(a)-a^2f(x)}{x-a}$

$=\lim_{x\to a}\frac{2xf(a)-a^2f'(x)}{1}$

$=2af(a)-a^2f'(a)$

25. (a) Let $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$

Doubtful points are $x=n,\ n\in I$

L.H.L $=\lim_{x\to n^-}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

($\because$ $[x]$ is the greatest integer function)

R.H.L $=\lim_{x\to n^+}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=n\cos\left(\frac{2n-1}{2}\right)\pi=0$

Now, value of the function at $x=n$ is $f(n)=0$

Since, L.H.L = R.H.L. $=f(n)$

$\therefore$ $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$ is continuous for every real x.

26. (c) $f(x)=|x-2|=\begin{cases}x-2 &, x-2\geq 0\\ 2-x &, x-2\leq 0\end{cases}$

$=\begin{cases}x-2 &, x\geq 2\\ 2-x &, x\leq 2\end{cases}$

Similarly,

$f(x)=|x-5|=\begin{cases}x-5 &, x\geq 5\\ 5-x &, x\leq 5\end{cases}$

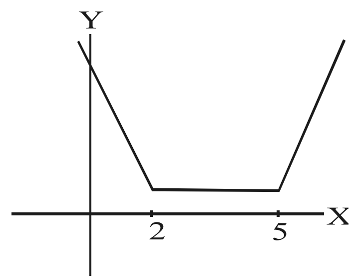
$\therefore$ $f(x)=|x-2|+|x-5|$

$=\{x-2+5-x=3,\ 2\leq x\leq 5\}$

Thus $f(x)=3,\ 2\leq x\leq 5$

$f'(x)=0,\ 2<x<5$

$f'(4)=0$



Clearly, statement-2 is also true.

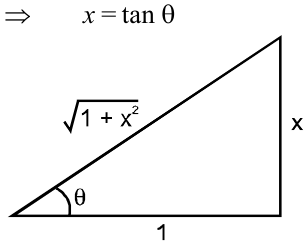
$\because$ $f(2)=0+|2-5|=3$

and $f(5)=|5-2|+0=3$

$a=0,\ b=0$ and c is any real number.

27. (a) Let $y=\sec(\tan^{-1}x)$ and $\tan^{-1}x=\theta$.

$\Rightarrow$ $x=\tan\theta$



Thus, we have $y=\sec\theta$

$\Rightarrow$ $y=\sqrt{1+x^2}$ $(\because \sec^2\theta=1+\tan^2\theta)$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{2\sqrt{1+x^2}}\cdot 2x$

At $x=1$,

$\frac{dy}{dx}=\frac{1}{\sqrt{2}}$ .D

28. **(b)** Since, f and g both are continuous function on $[0, 1]$ and differentiable on $(0, 1)$ then $\exists c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1} = \frac{6-2}{1} = 4$$

and $$g'(c) = \frac{g(1) - g(0)}{1} = \frac{2-0}{1} = 2$$

Thus, we get $f'(c) = 2g'(c)$

29. **(c)** Since g (x) is differentiable, it will be continuous at x = 3

$$\therefore \lim_{x \to 3^-} g(x) = \lim_{x \to 3^+} g(x)$$

$2k = 3m + 2$ ...(1)

Also g(x) is differentiable at x = 0

$$\therefore \lim_{x \to 3^-} g'(x) = \lim_{x \to 3^+} g'(x)$$

$$\frac{k}{2\sqrt{3+1}} = m$$

$k = 4m$ ...(2)

Solving (1) and (2), we get

$$m = \frac{2}{5},\ k = \frac{8}{5}$$

$k + m = 2$

30. **(d)** $g(x) = f(f(x))$

In the neighbourhood of $x = 0$,

$f(x) = |\log 2 - \sin x| = (\log 2 - \sin x)$

$\therefore\ g(x) = |\log 2 - \sin| \log 2 - \sin x||$

$= (\log 2 - \sin(\log 2 - \sin x))$

$\therefore\ g(x)$ is differentiable

and $g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$\Rightarrow g'(0) = \cos(\log 2)$

31. **(b)** We have $(2 + \sin x)\frac{dy}{dx} + (y+1)\cos x = 0$

$$\Rightarrow \frac{d}{dx}(2+\sin x)(y+1) = 0$$

On integrating, we get

$(2 + \sin x)(y + 1) = C$

At $x = 0$, $y = 1$ we have

$(2 + \sin 0)(1 + 1) = C$

$\Rightarrow C = 4$

$$\Rightarrow y + 1 = \frac{4}{2+\sin x}$$

$$y = \frac{4}{2+\sin x} - 1$$

Now $$y\left(\frac{\pi}{2}\right) = \frac{4}{2+\sin\frac{\pi}{2}} - 1$$

$$= \frac{4}{3} - 1 = \frac{1}{3}$$