TEST INFORMATION

DATE : 19.04.2015 **CUMULATIVE TEST-01 (CT-01)**

Syllabus : Function & Inverse Trigonometric Function, Limits, Continuity & Derivability, Quadratic Equation, Application of Derivatives

REVISION DPP OF APPLICATION OF DERIVATIVES

Total Marks : 139		**Max. Time : 109 min.**
Single choice Objective (–1 negative marking) Q. 1 to 18	**(3 marks 2.5 min.)**	**[54, 45]**
Multiple choice objective (–1 negative marking) Q. 19 to 36	**(4 marks, 3 min.)**	**[72, 54]**
Comprehension (–1 negative marking) Q.37 to Q.39	**(3 marks 2.5 min.)**	**[9, 7.5]**
Integer Type Questions (no negative marking) Q. 40	**(4 marks 2.5 min.)**	**[4, 2.5]**

1. If $f(x) = \begin{cases} -\cos^2\dfrac{\pi x}{2}, & 0 \le x < 1 \\ (1-x)^2, & 1 \le x \le 2 \end{cases}$ then number of values of 'c' obtained by applying LMVT on f(x) in interval [0, 2] is

(A) 1 (B) 2
(C) 3 (D) LMVT is not applicable

2. Let f(x) be a differentiable function and G be the graph of f(x). Let P = (a, f(a)) be a point on G closest to (0, 0). Then f(a)f'(a) =

(A) a (B) –a (C) 1 (D) –1

3. If $f(x) = x^3 + \log_2 (x + \sqrt{x^2+1})$ and $f(a) + f(b) \ge 0$ is true for any a, b $\in$ R then a & b must satisfy relation

(A) $a + b \ge 0$ (B) $a + b \le 0$ (C) $a \ge 0, b \ge 0$ (D) $a \le 0, b \le 0$

4. If $\theta \in [0, 5\pi]$, $r \in R$ and $2\sin\theta = r^4 - 2r^2 + 3$ then number of possible pairs (r, θ) is

(A) 8 (B) 10 (C) 6 (D) 2

5. The number of real valued continuous functions f(with domain R) such that if x is rational then f(x) is irrational and if x is irrational then f(x) is rational, is/are

(A) 0 (B) 2 (C) 4 (D) Infinite

6. If graphs of $y = \log_a x$ and $y = a^x$ (a > 1) intersect at exactly one point then a =

(A) e (B) $\sqrt{e}$ (C) e^e (D) $e^{1/e}$

7. Tangent lines are drawn at the points of inflexion for the function f(x) = cosx on $[0, 2\pi]$. The lines intersect with the x-axis so as to form a triangle. The area of this triangle is

(A) $\dfrac{\pi^2}{2}$ (B) $\dfrac{\pi^2}{4}$ (C) $\dfrac{\pi^2}{8}$ (D) $\dfrac{\pi^2}{16}$

8. The number of decreasing functions $f : R \to R$ such that $f(f(x)) = x + 1 \ \forall x \in R$ is

(A) 0 (B) 1 (C) 2 (D) infinite

9. Let f(x) be a differentiable real valued function satisfying $f''(x) - 6f'(x) > 6\ \forall\ x \geq 0$. If $f'(0) = -1$ and $g(x) = f(x) + x$ then g(x) is
(A) increasing $\forall\ x \geq 0$ (B) decreasing $\forall\ x \geq 0$
(C) a constant function $\forall\ x \geq 0$ (D) None of these

10. A person is standing at the edge of a slow moving river which is 1 km wide. He wishes to return to the camp ground on the opposite side of the river for which he may swim to any point on the opposite bank and then walk for the rest of the distance. The campground is 1 km away from the point on the opposite bank directly across from where he starts to swim. If he swims at the rate of 2 km/hr and walks at the rate of 3 km/hr, the minimum time taken by him is approximately
(A) 0.6 hr (B) 0.7 hr (C) 0.8 hr (D) 0.9 hr

11. There are 50 machines in a factory each producing 1000 bolts daily. For each additional machine installed, the output per machine drops by 10 bolts. How many additional machines should be installed to maximize the total output per day?
(A) 20 (B) 30 (C) 50 (D) 25

12. If $a^2 x^4 + b^2y^4 = c^6$ then the maximum value of xy is (a,b,c > 0)
(A) $\frac{c^2}{\sqrt{ab}}$ (B) $\frac{c^3}{ab}$ (C) $\frac{c^3}{\sqrt{2ab}}$ (D) $\frac{c^3}{2ab}$

13. The point on the curve $xy^2 = 1$ nearest to origin is
(A) $\left(2^{-1/3}, \pm 2^{1/6}\right)$ (B) $\left(2^{-1/3}, 2^{-1/6}\right)$ (C) $\left(2^{1/3}, \pm 2^{1/6}\right)$ (D) (1, 1)

14. The fraction exceeding its own n^{th} power ($n \in N$) by the maximum possible value is
(A) $\left(\frac{1}{n}\right)^{\frac{1}{n-1}}$ (B) $\left(\frac{1}{n}\right)^{n-1}$ (C) $\left(\frac{1}{n}\right)^{n}$ (D) $\left(\frac{n}{n}+1\right)^{\frac{1}{n-1}}$

15. Let $f(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial (a, b, c, d $\in$ R). If $f(\alpha) f(\beta) = 0$ where α and β are the distinct real roots of $f'(x) = 0$, then
(A) f(x) = 0 has all three different real roots
(B) f(x) = 0 has three real roots but two of them are equal
(C) f(x) = 0 has only one real root
(D) all three roots of f(x) = 0 are real and equal

16. The equation $\sin x + \sin^{-1}x = \cos x + \cos^{-1}x$, $x \in [-1, 1]$ has
(A) infinitely many solutions (B) at least one solution
(C) no solution (D) exactly one solution

17. Let f(x) be a non-negative continuous function satisfying $f'(x)\cos x \leq f(x)\sin x\ \forall\ x \geq 0$. Then $f\left(\frac{5\pi}{3}\right) =$
(A) $e^{-1/2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 0 (D) $\frac{1}{2}$

18. $f(x) = (4\sin^2x - 1)^n (x^2 + 6x + 11)$ where $n \in N$ has a local minimum at $x = \frac{\pi}{6}$ if
(A) n is even (B) n is odd (C) n is prime number (D) n is any natural number

19. Consider function $f(x) = |x\ell nx|$. Then
(A) maximum value of f(x) in $x \in (0, 1)$ is $\frac{1}{e}$
(B) f'(x) has local minima at x = 1
(C) Rolle's theorem can be applied to f(x) for an interval of maximum length 1 unit
(D) $f'(x + 2) - f'(x) < 2$ for all x > 1

20. If f (x) = |x| – {x} where {.} denotes fractional part function then

(A) f(x) is decreasing in $\left(\frac{-1}{2}, 0\right)$ (B) Rolle's theorem can be applied to f(x) in [0, 1]

(C) Maximum value of f(x) is not defined (D) Minimum value of f(x) is not defined

21. $f(x) = 2e^x + (a^2 - 5a + 6)e^{-x} + (10a - 2a^2 - 11)x - 3$ is increasing for all real values of x if a $\in$

(A) {2} (B) [2, 3] (C) (2, 3) (D) $(3, \infty)$

22. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where [.] denotes greatest integer function then

(A) $f\left(\frac{\pi}{2}\right) = 0$ (B) Maximum value of f(x) is 2

(C) f(x) is even function (D) $f\left(\frac{\pi}{2}\right) = \cos 4$

23. Let $f : R \to R$ be a real function then which of following statements is/are FALSE?

(A) If f is continuous and range of f = R then f is monotonic

(B) If f is monotonic and range of f = R then f is continuous

(C) If f is monotonic and continuous then range of f = R

(D) If f'(c) = 0 then x = c is a point of local extrema

24. Let $f_n(x) = (2 + (-2)^n)x^2 + (n + 3)x + n^2$ where n is a positive integer. A possible value of n for which $f_n(x)$ has a finite maximum value as x varies is

(A) 1 (B) 2 (C) 3 (D) 5

25. For c > 0, the equation sinx = cx has exactly five solutions and x_0 is the largest of these five solutions, then

(A) $\tan x_0 = x_0$ (B) $\cot x_0 = x_0$ (C) $2\pi < x_0 < \frac{5\pi}{2}$ (D) $x_0 = \frac{5\pi}{2}$

26. Let f : (a, b) $\to$ R is a differentiable function such that $\lim_{x \to a^+} f^2(x) = 0$, $\lim_{x \to b^-} f^2(x) = e - 1$ and $2f(x)\,f'(x) - f^2(x) \geq 1$ for all x $\in$ (a, b) then value of (b – a) can be

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

27. Consider function $f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$ then which of the following hold good for f(x)?

(A) Domain is $[2, \infty)$ (B) Range is $[-10, \infty)$ (C) Domain is $(0, \infty)$ (D) f'(1) = –8

28. If $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are non-zero real numbers in G.P. then

(A) f(x) = 0 has exactly one root in $(-\infty, \infty)$ (B) f''(x) = 0 has one root in $(-\infty, \infty)$

(C) f(x) = 0 has three roots in $(-\infty, \infty)$ (D) $f'(x) > 0\ \forall x \in R$

29. A movie screen on a wall is 20 feet high and 10 feet above the floor. If a man has to position himself at distance x from the screen to have a maximum angle of view θ, then

(A) $x = \frac{10}{\sqrt{3}}$ feet (B) $\theta = 60°$ (C) $x = 10\sqrt{3}$ feet (D) $\theta = 30°$

30. Let $f'(x) = e^{x^2}$ and f(0) = 10, then which of the following is/are true?

(A) $f(1) \in (11, 9 + e)$ (B) $f(1) \in (11, 10 + e)$

(C) Absolute value of integral part of – f(1) is 12 (D) all of these

31. If $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ then equation x – sin x = a has

(A) one solution if $a \in \left[1 - \frac{\pi}{2}, \frac{\pi}{2} - 1\right]$ (B) no solution if $a \in \left(-\infty, 1 - \frac{\pi}{2}\right)$

(C) no solution if $a \in \left[\frac{\pi}{2} - 1, \infty\right)$ (D) All of these

32. Let f(x) be a differentiable function with $f(1)\,f(-1) \neq 0$. Define a function $g(x) = \dfrac{x^2 - 1}{f(x)}$. If g(x) does not follow Rolle's theorem in [–1, 1], then which of the following options is/are FALSE?

(A) f(x) = 0 cannot have any root in [–1, 1]
(B) f(x) = 0 has at least one root in [–1, 1]
(C) $f'(x)$ is zero at at least one point in [–1, 1]
(D) f(x) cannot satisfy Rolle's theorem in [–1,1]

33. Let f(x) be a function satisfying $f'(x) = \ell n\left(x + \sqrt{x^2 + 1}\right)$ and f(0) = 0, then

(A) $f(x) \geq 0 \ \forall \ x \in R$
(B) $f(x) \leq 0 \ \forall \ x \in R$
(C) f'(x) is increasing $\forall x \in R$
(D) f(x) is even function

34. If 'm' is the slope of a tangent to the curve $e^y = 1 + x^2$, then

(A) $|\,m\,| \leq 1$
(B) there exists a value of x for which $m = \cos^{-1}x$
(C) m takes maximum value at x = 1
(D) m is increasing for $x \in [-1, 1]$

35. Which of the following are incorrect given $x \neq y$?

(A) $\dfrac{\cos^{-1}x - \cos^{-1}y}{y - x} \leq 1 \ \forall \ x, y \in [-1, 1]$
(B) $\dfrac{\cot^{-1}x - \cot^{-1}y}{y - x} \geq 1 \ \forall \ x, y \in R$
(C) $\dfrac{\tan^{-1}x - \tan^{-1}y}{x - y} \leq 1 \ \forall \ x, y \in R$
(D) $\dfrac{\sin^{-1}x - \sin^{-1}y}{x - y} \geq 1 \ \forall \ x, y \in [-1, 1]$

36. Let f(x) = 3sin x – 4cos x + ax + b, then

(A) f(x) = 0 has only one real root which is positive if a > 5 and b < 0
(B) f(x) = 0 has only one real root which is negative if a > 5 and b > 0
(C) f(x) = 0 has only one real root which is negative if a < –5 and b < 0
(D) f(x) = 0 has only one real root which is positive if a < –5 and b > 0

Comprehension (Q. No. 37 to 39)

f(x) is a polynomial function $f : R \rightarrow R$ such that $f(2x) = f'(x)\,f''(x)$.

37. Value of f(3) is

(A) 4
(B) 12
(C) 15
(D) 18

38. f(x) is

(A) one-one and onto
(B) one-one but not onto
(C) many-one onto
(D) many one into

39. The equation f(x) = x has

(A) no real roots
(B) one real root
(C) four real and distinct roots
(D) three real and distinct roots

40. Water is leaking at the rate of $2m^3$/sec from a cone of semi-vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 meters is d, then | 5d| is equal to

DPP # 2

REVISION DPP OF LIMITS, CONTINUITY & DERIVABILITY AND QUADRATIC EQUATION

1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(B)	**7.**	(D)
8.	(A)	**9.**	(C)	**10.**	(D)	**11.**	(B)	**12.**	(C)	**13.**	(A)	**14.**	(B)
15.	(B)	**16.**	(A,B,C)	**17.**	(C,D)	**18.**	(B,D)	**19.**	(A,C,D)	**20.**	(A,D)	**21.**	(C,D)
22.	(A,B,D)	**23.**	(B,C,D)	**24.**	(A,C)	**25.**	(A,B,C)	**26.**	(A,B)	**27.**	(A,B,C,D)	**28.**	(A,C)
29.	(A,B)	**30.**	(C,D)	**31.**	(A,B,C)	**32.**	(B,D)	**33.**	(C)	**34.**	(A)	**35.**	(B)
36.	(B)	**37.**	(C)	**38.**	(A)	**39.**	(B)	**40.**	(A)				

MATHEMATICS

1_. $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$ $\Rightarrow$ $f'(c) = 1$ $\Rightarrow$ $\dfrac{\pi}{2}\sin(\pi c) = 1$, $-2(1 - c) = 1$

$\Rightarrow$ $\sin \pi c = \dfrac{2}{\pi}$, $c = \dfrac{3}{2}$ $\Rightarrow$ three values of c

2_. $D(x) = x^2 + f^2(x)$ $\Rightarrow$ $D'(x) = 2x + 2f(x)\, f'(x)$
D(x) is minimum $\Rightarrow$ $D'(a) = 0$ $\Rightarrow$ $2a + 2f(a)\, f'(a) = 0$

3_. $f(a) \ge -f(b)$ $\Rightarrow$ $f(a) \ge f(-b)$ $\Rightarrow$ $a \geqq -b$ $\Rightarrow$ $a + b \ge 0$

4_. $-2 \le r^4 - 2r^2 + 3 \le 2$ $\Rightarrow$ $(r^2 - 1)^2 + 2 \le 2$ $\Rightarrow$ $r^2 - 1 = 0$ $\Rightarrow$ $r = \pm 1$

and $\sin\theta = 1$ $\Rightarrow$ $\theta = \dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}$

5_. Let $h(x) = f(x) + x$ $\Rightarrow$ h(x) is irrational for all $x \in R$
But h(x) is continuous
So h(x) can take single irrational value say k
$\Rightarrow$ $h(x) = k$ $\Rightarrow$ $f(x) = k - x$ $\Rightarrow$ $f(-k) = 2k$
But this is contradiction to given condition in question

6_. $\dfrac{d}{dx}(\log_a x) = \dfrac{d}{dx}(a^x) = 1$ $\Rightarrow$ $a^x \log a = \dfrac{1}{x \log a} = 1$

$\Rightarrow$ $x = \log_a e$ and $a^x = \dfrac{1}{\log a}$ $\Rightarrow$ $e = \dfrac{1}{\log a}$ $\Rightarrow$ $a = e^{1/e}$

7_. Infection points are $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$; Tangents are $y = -x + \dfrac{\pi}{2}$, $y = x - \dfrac{3\pi}{2}$

Area of triangle $= \dfrac{1}{2} \times \dfrac{\pi}{2} \times \pi = \dfrac{\pi^2}{4}$

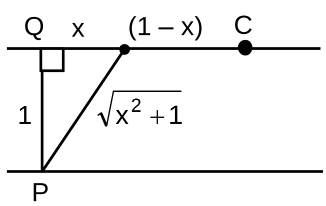
8. $f(f(x)) = x + 1$ $\Rightarrow$ $f(f(f(x))) = f(x) + 1$
$\Rightarrow$ $f(x + 1) = f(x) + 1$ $\Rightarrow$ $f(x + 1) > f(x)$ $\Rightarrow$ f can not be decreasing

9. $f''(x) - 6f'(x) > 6$ $\Rightarrow$ $e^{-6x}(f''(x) - 6f'(x)) > 6e^{-6x}$

$\Rightarrow$ $\dfrac{d}{dx}(e^{-6x} f'(x) + e^{-6x}) > 0$ $\Rightarrow$ $e^{-6x}(f'(x) + 1)$ is increasing

also $h(x) = e^{-6x}[f'(x) + 1] = 0$ at $x = 0$ $\Rightarrow$ $h(x) \ge 0$ $\Rightarrow$ $f'(x) + 1 \ge 0$
$\Rightarrow$ $[f(x) + x]' \ge 0$ $\Rightarrow$ $g'(x) \ge 0$ so g(x) is increasing

10. $T = D/S$ $\Rightarrow$ $T = \dfrac{\sqrt{x^2 + 1}}{2} + \dfrac{1 - x}{3} = f(x)$ (say)



$f'(x) = 0 \Rightarrow$ $x = \dfrac{2}{\sqrt{5}}$ for minima $\Rightarrow f(x)$ = minimum time = 0.7 hr approximately

11. Let number of additional machines installed = x

$P(x) = (50 + x)(1000 - 10x) \Rightarrow P'(x) = 0 \Rightarrow x = 25$

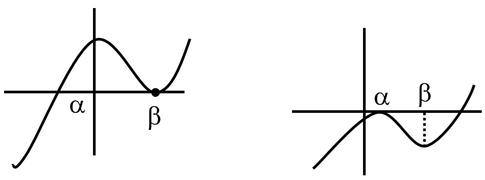
12. Let $\dfrac{ax^2}{c^3} = \sin\theta$ and $\dfrac{by^2}{c^3} = \cos\theta \Rightarrow xy = \sqrt{\dfrac{c^6\sin\theta\cos\theta}{ab}} \Rightarrow (xy)_{max.} = \sqrt{\dfrac{c^6}{2ab}}$

13. $S = \sqrt{x^2+y^2} = \sqrt{x^2+\dfrac{1}{x}} \Rightarrow \dfrac{ds}{dx} = 0 \Rightarrow x = \left(\dfrac{1}{2}\right)^{1/3}$

14. Let 'x' be the fraction $\Rightarrow y = x - x^n$ has to be maximum

$\Rightarrow \dfrac{dy}{dx} = 0 \Rightarrow x = \left(\dfrac{1}{n}\right)^{\frac{1}{n-1}}$ & $\dfrac{d^2y}{dx^2} < 0$

15.



16. $f(x) = \sin x - \cos x + \sin^{-1}x - \cos^{-1}x$

$\Rightarrow f'(x) = \cos x + \sin x + \dfrac{2}{\sqrt{1-x^2}} \Rightarrow f'(x) = \sqrt{2}\,\sin\left(x+\dfrac{\pi}{4}\right) + \dfrac{2}{\sqrt{1-x^2}}$

As $\dfrac{2}{\sqrt{1-x^2}} > 2$ so $f'(x) > 0$ for $x \in [-1, 1]$

Also $f(-1) < 0$ and $f(1) > 0$ so one solution

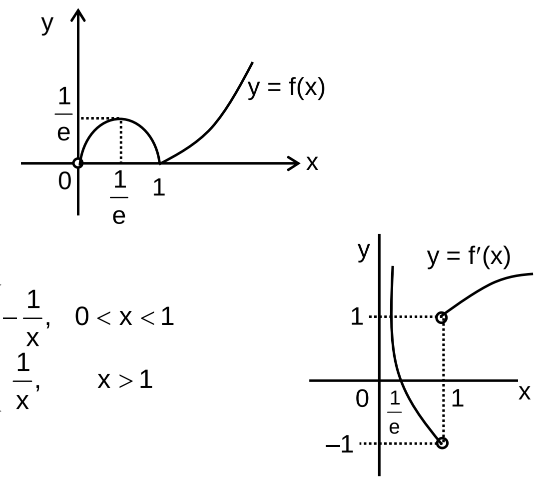
17. Let $g(x) = f(x)\cos x \Rightarrow g'(x) \le 0 \Rightarrow g\left(\dfrac{5\pi}{3}\right) \le g\left(\dfrac{\pi}{2}\right)$

$\Rightarrow g\left(\dfrac{5\pi}{3}\right) \le 0 \Rightarrow \dfrac{1}{2}f\left(\dfrac{5\pi}{3}\right) \le 0 \Rightarrow f\left(\dfrac{5\pi}{3}\right) = 0$

18. $x^2 + 6x + 11 > 0 \quad \forall x \in R$

$f\left(\dfrac{\pi}{6}\right) = 0,\ f\left(\dfrac{\pi^+}{6}\right) = (0^+)^n > 0,\ f\left(\dfrac{\pi^-}{6}\right) = (0^-)^n > 0$

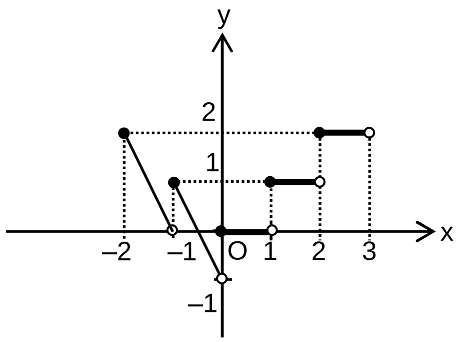
19_*. $f(x) = \begin{cases} -x\ell nx, & 0 < x \le 1 \\ x\ell nx, & x \ge 1 \end{cases}$



$\Rightarrow f'(x) = \begin{cases} -1-\ell nx, & 0 < x < 1 \\ 1+\ell nx, & x > 1 \end{cases} \Rightarrow f''(x) = \begin{cases} -\dfrac{1}{x}, & 0 < x < 1 \\ \dfrac{1}{x}, & x > 1 \end{cases}$

By LMVT, $\dfrac{f'(x+2) - f'(x)}{2} = f''(c)$. As $f''(c) < 1 \Rightarrow f'(x+2) - f'(x) < 2$

20_*. $f(x) = \begin{cases} -2x-2, & -2 < x < -1 \\ -2x-1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$



21_*. $f'(x) \ge 0 \Rightarrow 2e^x - (a^2 - 5a + 6)\, e^{-x} + 10a - 2a^2 - 11 \ge 0$

$\Rightarrow e^{-x}(2e^{2x} - (a^2 - 5a + 6) + (10a - 2a^2 - 11)e^x) \ge 0$

$\Rightarrow e^{2x} + (5a - a^2 - \frac{11}{2})e^x + (5a - a^2 - 6)\left(\frac{1}{2}\right) \ge 0$

$\Rightarrow (e^x + 5a - a^2 - 6)\left(e^x + \frac{1}{2}\right) \ge 0$

$\Rightarrow a^2 - 5a + 6 \le e^x \Rightarrow a^2 - 5a + 6 \le 0 \Rightarrow 2 \le a \le 3$

22_*. $f(0) = 2$ is maximum value

23_*. Take counter examples (A) $f(x) = x^3 - x$ (C) $f(x) = e^x$ (D) $f(x) = x^3$ at $x = 0$

24_*. $f_n(x)$ must be quadratic $\Rightarrow n \ne 1$
$f_n(x)$ has maxima $\Rightarrow 2 + (-2)^n < 0 \Rightarrow$ n can be odd

25_*. $\sin x_0 = cx_0$ and $\frac{d}{dx}(\sin x)_{x_0} = \frac{d}{dx}(cx)_{x_0} \Rightarrow \cos x_0 = c \Rightarrow \tan x_0 = x_0$

26_*. Let $g(x) = \log(1 + f^2(x)) - x$; $(a, b) \to R \Rightarrow g'(x) = \frac{2f(x)f'(x) - 1 - f^2(x)}{1 + f^2(x)} \ge 0$

$\Rightarrow \lim\limits_{x\to b^-} g(x) \ge \lim\limits_{x\to a^+} g(x) \Rightarrow 1 - b \ge -a \Rightarrow b - a \le 1$

27^*. $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 - 3.\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) - 4.\left[\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 - 2\right]$

$\sqrt{x} + \frac{1}{\sqrt{x}} = t \Rightarrow f(x) = t^3 - 4t^2 - 3t + 8,\ t \in [2, \infty)$

Let $f(x) = g(t) \Rightarrow g'(t) = (t - 3)(3t + 1) \Rightarrow g(t) \in [g(3)\ g(\infty)) \Rightarrow f(x) \in [-10, \infty)$

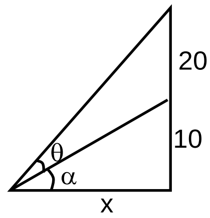
Also $f'(x) = (3t^2 - 8t - 3)\left(\frac{x-1}{2x\sqrt{x}}\right) \Rightarrow f'(1) = 0$

28^*. Since a, b, c, d are in G.P. so $a \ne 0$
Further $f'(x) = 3ax^2 + 2bx + c$
$D = 4b^2 - 12ac = 4b^2 - 12b^2 = -8b^2 < 0$
$\Rightarrow f'(x) > 0$ if $a > 0$ and $f'(x) < 0$ if $a < 0 \Rightarrow f(x)$ is monotonic $\Rightarrow f(x) = 0$ has only one real root.

Now, $f''(x) = 0 \Rightarrow 6ax + 2b = 0 \Rightarrow x = \frac{-b}{3a}$ so one root.

29^*. $\tan(\theta + \alpha) = 30/x$ ...(i)
$\tan\alpha = 10/x$ ...(ii)

$$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta \ \tan\alpha} = \frac{30}{x} \quad \Rightarrow \quad \tan\theta = \frac{20x}{x^2 + 300} = f(x)$$



Now $\theta \to$ max $\Rightarrow$ $\tan\theta \to$ max $\{\because \theta < 90°\}$

$f'(x) = 0$ $\Rightarrow$ $x = 10\sqrt{3}$ $\Rightarrow$ $\theta = 30°$

30^*. Let $f(x) = 10 + \int_0^x e^{x^2} dx \Rightarrow f(1) = 10 + \int_0^1 e^{x^2} dx$. As $1 < e^{x^2} < e^x$ for $x \in (0, 1) \Rightarrow 11 < f(1) < e + 9$.

31^*. Let $f(x) = x - \sin x - a$ $\Rightarrow$ $f'(x) = 1 - \cos x$ $\Rightarrow$ $f'(x) \geq 0$ $\Rightarrow$ f(x) is increasing

$\Rightarrow f(x) \in \left[f\left(\frac{-\pi}{2}\right), f\left(\frac{\pi}{2}\right)\right]$ $\Rightarrow$ $f(x) \in \left[1 - \frac{\pi}{2}, \frac{\pi}{2} - 1\right]$.

32^*. g(x) is not necessarily be defined in the complete interval [–1, 1] and g(x) will satisfy Rolle's theorem unless f(x) = 0 for some $x \in (-1, 1)$.

So far as $f'(x)$ is concerned, without further information nothing definite can be said.

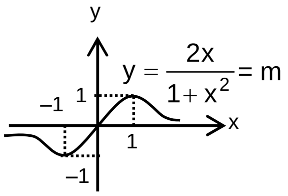
33^*. $f'(x) > 0 \ \forall \ x > 0$ & $f'(x) < 0 \ \forall \ x < 0$ $\Rightarrow$ $f(x) \geq 0 \ \forall \ x \in R$ $\Rightarrow$ option (A) is true.

$f''(x) = \frac{1}{\sqrt{x^2 + 1}} > 0 \Rightarrow f'(x)$ is increasing.

$f'(x)$ is odd $\Rightarrow f'(-x) = -f'(x)$ $\Rightarrow -f(-x) = -f(x) + c$

$f(0) = 0$ $\Rightarrow c = 0$ $\Rightarrow f(-x) = f(x)$ $\Rightarrow$ f(x) is even.

34^*.



$e^y \frac{dy}{dx} = 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2x}{1+x^2}$ $\Rightarrow$ $m = \frac{2x}{1+x^2}$

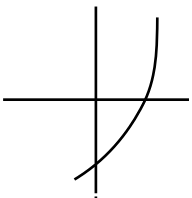
35*. $\frac{\cos^{-1}x - \cos^{-1}y}{x - y}$ = slope of chord = slope of tangent at c (LMVT) = $\frac{-1}{\sqrt{1-c^2}} \in (-\infty, -1]$

$\Rightarrow \frac{\cos^{-1}x - \cos^{-1}y}{y - x} \geq 1$.

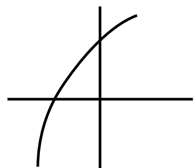
36. $f'(x) = 3\cos x + 4\sin x + a > 0$ if $a > 5$

$f'(x) < 0$ if $a < -5$

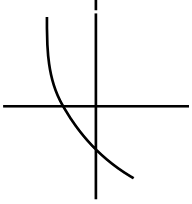
(A)



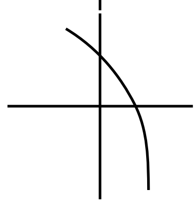
(B)



(C)



(D)



39. Let f(x) has degree 'n' then $f'(x)$ has degree $(n - 1)$ and $f''(x)$ has degree $(n - 2)$

Now, since $f(2x) = f'(x)\, f''(x)$ ...(i)

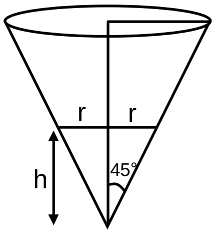
$\therefore \quad n = (n - 1) + (n - 2)$

$\therefore \quad \Rightarrow \quad n = 3$

$\therefore \quad \Rightarrow \quad f(x) = ax^3 + bx^2 + cx + d$

Put in equation (i) to get $4a = 9a^2$, $4b = 18ab$, $2c = 6ac + 4b^2$, $d = 2bc$

$\Rightarrow a = 4/9,\ b = c = d = 0 \Rightarrow f(x) = \dfrac{4x^3}{9}$.

40. 

$\dfrac{dv}{dt} = -2 \quad \Rightarrow \quad \dfrac{d}{dt}\left(\dfrac{1}{3}\pi r^2 h\right) = -2 \Rightarrow \dfrac{d}{dt}\left(\dfrac{1}{3}\pi r^3\right) = -2 \quad \Rightarrow \quad \pi r^2 \dfrac{dr}{dt} = -2$ ...(i)

Also $\dfrac{d}{dt}(2\pi r) = 2\pi \dfrac{dr}{dt} = 2\pi\left(\dfrac{-2}{\pi r^2}\right) = \dfrac{-4}{r^2}$. At r = 2, $\dfrac{d}{dt}(2\pi r) = \dfrac{-4}{4} = -1 \Rightarrow d = -1$