CHAPTER 18

Matrices

1. If $A=\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2=\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[2003]**

(a) $\alpha = 2ab, \beta = a^2+b^2$

(b) $\alpha = a^2+b^2,\ \beta = ab$

(c) $\alpha = a^2+b^2,\ \beta = 2ab$

(d) $\alpha = a^2+b^2,\ \beta = a^2-b^2$.

2. If $A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction **[2005]**

(a) $A^n = nA-(n-1)\mathrm{I}$

(b) $A^n = 2^{n-1}A-(n-1)\mathrm{I}$

(c) $A^n = nA+(n-1)\mathrm{I}$

(d) $A^n = 2^{n-1}A+(n-1)\mathrm{I}$

3. If A and B are square matrices of size $n \times n$ such that $A^2-B^2=(A-B)(A+B)$, then which of the following will be always true? **[2006]**

(a) $A = B$

(b) $AB = BA$

(c) either of A or B is a zero matrix

(d) either of A or B is identity matrix

4. Let $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a,b \in N.$

Then **[2006]**

(a) there cannot exist any B such that AB = BA

(b) there exist more than one but finite number of B′s such that AB = BA

(c) there exists exactly one B such that AB = BA

(d) there exist infinitely many B′s such that AB = BA

5. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[2010]**

(a) 5 (b) 6

(c) at least 7 (d) less than 4

6. Let A and B be two symmetric matrices of order 3. **[2011]**

Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

7. If $\omega \neq 1$ is the complex cube root of unity and matrix $H=\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to **[2011RS]**

(a) 0 (b) –H

(c) H^2 (d) H

8. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to: **[2015]**

(a) (2, 1) (b) (–2, – 1)
(c) (2, – 1) (d) (–2, 1)

9. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to : **[2016]**

(a) 4 (b) 13 (c) –1 (d) 5

10. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to : **[2017]**

(a) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$ (b) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$

(c) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$ (d) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$

Answer Key

1	2	3	4	5	6	7	8	9	10					
(c)	(a)	(b)	(d)	(c)	(a)	(d)	(b)	(d)	(c)					

SOLUTIONS

1. **(c)** $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}=\begin{bmatrix}a & b\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ b & a\end{bmatrix}$

$=\begin{bmatrix}a^2+b^2 & 2ab\\ 2ab & a^2+b^2\end{bmatrix}$

$\alpha = a^2+b^2;\ \beta = 2ab$

2. **(a)** We observe that

$A^2=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix},\ A^3=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$ and we can prove by induction that $A^n=\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}$

Now $nA-(n-1)I=\begin{bmatrix}n & 0\\ n & n\end{bmatrix}-\begin{bmatrix}n-1 & 0\\ 0 & n-1\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}=A^n$

$\therefore nA-(n-1)I=A^n$

3. **(b)** $A^2-B^2=(A-B)(A+B)$

$A^2-B^2=A^2+AB-BA-B^2$

$\Rightarrow\ AB=BA$

4. **(d)** $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\quad B=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$

$AB=\begin{bmatrix}a & 2b\\ 3a & 4b\end{bmatrix}$

$BA=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}a & 2a\\ 3b & 4b\end{bmatrix}$

Hence, $AB = BA$ only when $a = b$

$\therefore$ There can be infinitely many B's for which $AB = BA$

5. **(c)** $\begin{bmatrix}1 & \dots & \dots\\ \dots & 1 & \dots\\ \dots & \dots & 1\end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

Similarly, $\begin{bmatrix} ... & ... & 1 \\ ... & 1 & ... \\ 1 & ... & ... \end{bmatrix}$ are 6 non-singular matrices.

So, required cases are more than 7, non-singular 3×3 matrices.

6. (a) $\therefore A' = A$

$B' = B$

Now $(A(BA))' = (BA)'A'$

$= (A'B')A' = (AB)A = A(BA)$

Similarly $((AB)A)' = (AB)A$

So, $A(BA)$ and $(AB)A$ are symmetric matrices.

Again $(AB)' = B'A' = BA$

Now if $BA = AB$, then AB is symmetric matrix.

7. (d) $H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

If $H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega \end{bmatrix}$ then $H^{k+1} = \begin{bmatrix} \omega^{k+1} & 0 \\ 0 & \omega^{k+1} \end{bmatrix}$

So by principle of mathematical induction,

$$H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega^{69}\omega & 0 \\ 0 & \omega^{69}\omega \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$$

8. **(b)** $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & a+4+2b \\ 2+2-4 & 4+1+4 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ ...(i)

$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$

$\Rightarrow a - b = -1$ ...(ii)

On solving (i) and (ii) we get

$-1 + b + 2b = -4$ ...(i)

$-1 + 3b = -4$

$3b = -3$

$b = -1$

and $a = -2$

$(a, b) = (-2, -1)$

9. **(d)** $A(\text{adj } A) = A\,A^T$

$\Rightarrow A^{-1}A\,(\text{adj } A) = A^{-1}A\,A^T$

$\text{adj } A = A^T$

$\Rightarrow \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\Rightarrow a = \frac{2}{5}$ and $b = 3$

$\Rightarrow 5a + b = 5$

10. **(c)** We have $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$\Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$

Also $12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$\therefore 3A^2 + 12A$

$= \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{adj}\,(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$