35/25 (b)

Term Test 1 (DSGT)

Shasvat Shah
60004220126
Comps

Assignment

1 $A = \{1, 4, 7, 13\}$

$R = \{\{1,4\}, \{4,7\}, \{7,4\}, \{1,13\}\}$

Find transitive closure using warshells algorithm.

$M_R =$

	1	4	7	13
1	0	1	0	1
4	0	0	1	0
7	0	1	0	0
13	0	0	0	0

Let $W_0 =$

	1	4	7	13
1	0	1	0	1
4	0	0	1	0
7	0	1	0	0
13	0	0	0	0

Select column 1, row 1

$C_1 = \{\}$ $R_1 = \{4, 13\}$

$C_1 \times R_1 = \phi$ (no new addition)

$W_1 =$

	1	4	7	13
1	0	1	0	1
4	0	0	1	0
7	0	1	0	0
13	0	0	0	0

$C_2 = \{1, 7\}$ $R_2 = \{7\}$

$W_2 =$

	1	4	7	13
1	0	1	1	1
4	0	0	1	0
7	0	1	1	0
13	0	0	0	0

$C_3 = \{1, 4, 7\}$ $R_3 = \{4, 7$

$C_3 \times R_3 = \{(1,4), (1,7), (4,4), (4,7), (7,4), (7,7)\}$

Pair (4,4) does not exist therefore needs to be added. in W_3

$W_3 =$

	1	4	7	13
1	0	1	1	1
4	0	1	1	0
7	0	1	1	0
13	0	0	0	0

$C_4 = \{$ $\}$ $R_4 = \{1$

$\therefore C_4 \times R_4 = \phi$ (no new pair added

$W_4 =$

	1	4	7	13
1	0	1	1	1
4	0	1	1	0
7	0	1	1	0
13	0	0	0	0

Thus the ~~reflexi~~ transitive closure of R =

$R = \{(1,4)(1,7)(1,13)(4,4)(4,7)(7,4)(7,7)\}$

2 Define reflexive and symmetric closure of a relation.

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(1,4)(2,4)(3,1)(3,2)(4,2)(4,3)(4,4)\}$

Reflexive closure - If R is a relation defined on A which is not reflexive, and if Δ denotes equality relation Δ on A then $R_1 = R \cup \Delta$ is the reflexive closure of a relati

R on A is obtained by adding (a,a) to R for each (a,b) ∈ A

Symmetric Closure - If R is a relation defined on A which is not symmetric then $R_1 = R \cup R^{-1}$ is the symmetric closure of R. The symmetric closure of R is obtained by adding (b,a) to R for each (a,b) ∈ R.

Reflexive closure of R

$\Delta = \{(1,1), (2,2), (3,3), (4,4)\}$

$R_1 = R \cup \Delta$

$R_1 = \{(1,1), (1,2), (1,4), (2,2), (2,4), (3,1), (3,2), (3,3), (4,2), (4,3), (4,4)\}$

metric closure of R

$R = \{(1,1)(1,2)(1,4)(2,4)(3,1)(3,2)(4,2)(4,3)(4,4)\}$

$R^{-1} = \{(1,1)(2,1)(4,1)(4,2)(1,3)(2,3)(2,4)(3,4)(4,4)\}$

$R \cup R^{-1} = R_1$

$R_1 = \{(1,1)(1,2)(2,1)(1,4)(4,1)(2,4)(4,2)(3,1)(1,3)(3,2)(2,3)(4,3)(3,4)(4,4)\}$

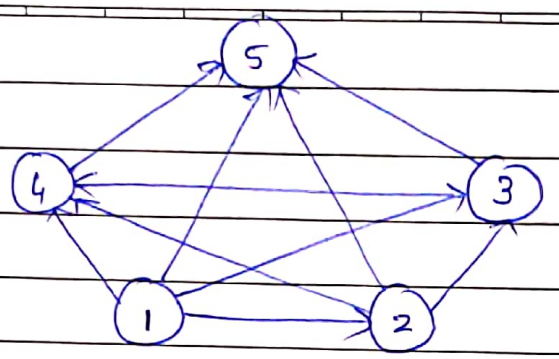
3 $A = \{1,2,3,4,5\}$

A relation R is defined on A if a R b if a < b

Compute R^2 and R^∞

$R = \{(1,2)(1,3)(1,4)(1,5)(2,3)(2,4)(2,5)(3,4)(3,5)(4,5)\}$

R_1



From the given digraph we can conclude

$1 R^2 3$ since $1 R 2$ and $2 R 3$

$1 R^2 4$ since $1 R 3$ and $3 R 4$

$1 R^2 5$ since $1 R 4$ and $4 R 5$

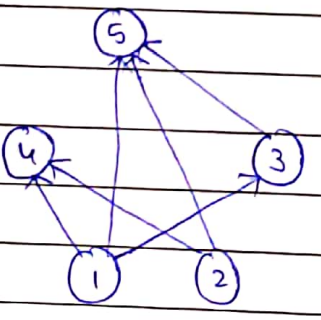
$2 R^2 5$ since $2 R 4$ and $4 R 5$

$2 R^2 4$ since $2 R 3$ and $3 R 4$

$3 R^2 5$ since $3 R 4$ and $4 R 5$

$R^2 = \{(1,3)(1,4)(1,5)(2,5)(2,4)(3,5)\}$.

R_2



$1 R^\infty 2$ since $1 R 2$

$1 R^\infty 3$ since $1 R 3$

$1 R^\infty 4$ since $1 R 4$

$1 R^\infty 5$ since $1 R 5$

$2 R^\infty 3$ since $2 R 3$

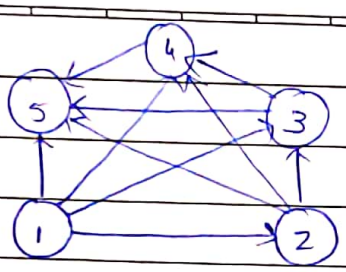
$2 R^\infty 4$ since $2 R 4$

$2 R^\infty 5$ since $2 R 5$

$3 R^\infty 4$ since $3 R 4$

$4 R^\infty 5$ since $4 R 5$

$3 R^\infty 5$ since $3 R 5$



R^{∞}

4 Construct truth table and check: $(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$	$\sim Q \rightarrow \sim P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	T

$(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$

T
T
T
T

So from the truth table we can say that it is a tautlogy

5 Find the number of integers between 1 to 60 which are not divisible by 2, nor by 3, nor 5.

$n(A)$ = numbers divisible by 2 = $\frac{60}{2}$ = 30

$n(B)$ = numbers divisible by 3 = $\frac{60}{3}$ = 20

$n(C)$ = numbers divisbe by 5 = $\frac{60}{5}$ = 12

$n(A \cap B)$ = no. divisible by 2 & 3 = $\frac{60}{2 \times 3}$ = 10

$|A \cup B \cup C| = |A| + |B| + |C| + |A \cap B \cap C| - |A \cap B| - |A \cap C| - |B \cap C|$

$n(A \cap C)$ = no. divisible by 2 & 5 = $\frac{60}{2 \times 5}$ = 6

$n(B \cap C)$ = no. divisible by 3 & 5 = $\frac{60}{3 \times 5}$ = 4

$n(A \cap B \cap C)$ = no. divisible by 2, 3, 5 = $\frac{60}{2 \times 3 \times 5}$ = 2

$\therefore |A \cup B \cup C| = 30 + 20 + 12 - 6 - 4 - 10 + |A \cap B \cap C|$

$60 - 42 = |A \cap B \cap C|$

$18 - 2 = |A \cup B \cup C|$

16 = No.s not divisible by 2, 3 & 5.