Set Properties

- **Property 1** (*Properties of Ø and* ∪)
  - $A \cup \varnothing = A$, $A \cap U = A$
  - $A \cup U = U$, $A \cap \varnothing = \varnothing$
- **Property 2** (*The idempotent properties*)
  - $A \cup A = A$, $A \cap A = A$
- **Property 3** (*The commutative properties*)
  - $A \cup B = B \cup A$, $A \cap B = B \cap A$
- **Property 4** (*The associative properties*)
  - $A \cup (B \cup C) = (A \cup B) \cup C$
  - $A \cap (B \cap C) = (A \cap B) \cap C$

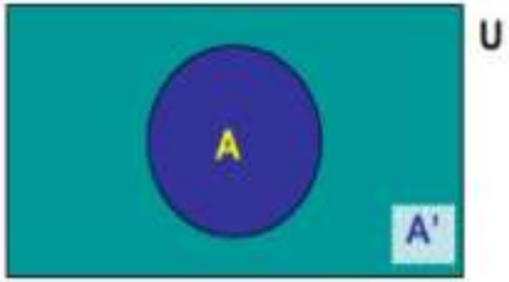
- **Property 5** (*The distributive properties*)
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **Property 6** (*Properties of the complement*)
  - $\varnothing^C = U$, $U^C = \varnothing$
  - $A \cup A^C = U$, $A \cap A^C = \varnothing$
  - $(A^C)^C = A$
- **Property 7** (*De Morgan's laws*)
  - $(A \cup B)^C = A^C \cap B^C$
  - $(A \cap B)^C = A^C \cup B^C$
- **Property 8** (*Absorption laws*)
  - $A \cap (A \cup B) = A$
  - $A \cup (A \cap B) = A$

VENN DIAGRAMS

Often pictures are very helpful in our thinking. In set theory we use closed curves (usually circles) and rectangles to represent sets and a combination of these is named as **Venn-diagrams**.

Below are some examples of Venn-diagrams:

U = universal set
A = subset of U
(i.e. A is contained in U)



A' = complement of A
(i.e. all the elements of U that are not included in A)

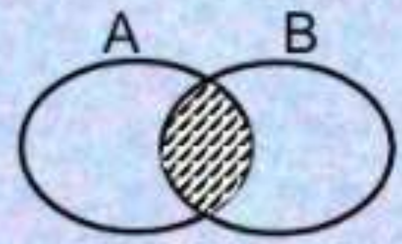
OPERATIONS ON SETS

Union:

- Let A and B be two sets. The union of A and B is the set of all those elements, which belong to either set A or set B or to both A and B. We shall use the notation:
- $A \cup B$ (read as "A union B") to denote the union of A and B.

Intersection:

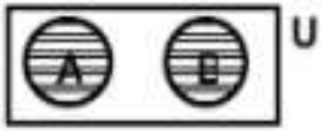
- Let A and B be two sets. The intersection of A and B is the set of only those elements that belong to both A and B. We shall use the notation $A \cap B$ (read as "A intersection B")



VENN DIAGRAMS

Universal set with 2 disjoint sets A & B

U = real numbers

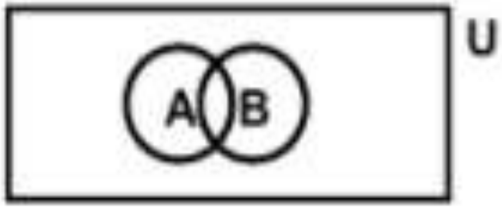


A = odd numbers

B = even numbers

Universal set with two intersecting sets A & B

U = real numbers

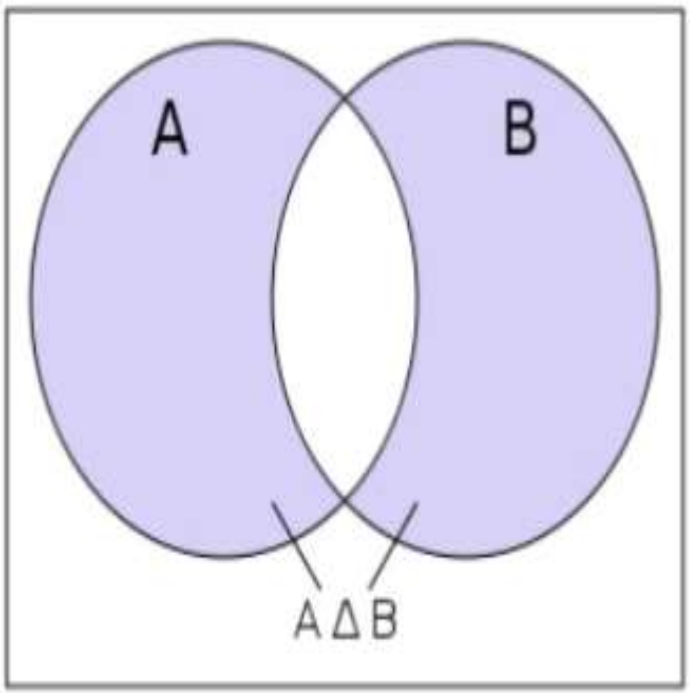


A = even numbers

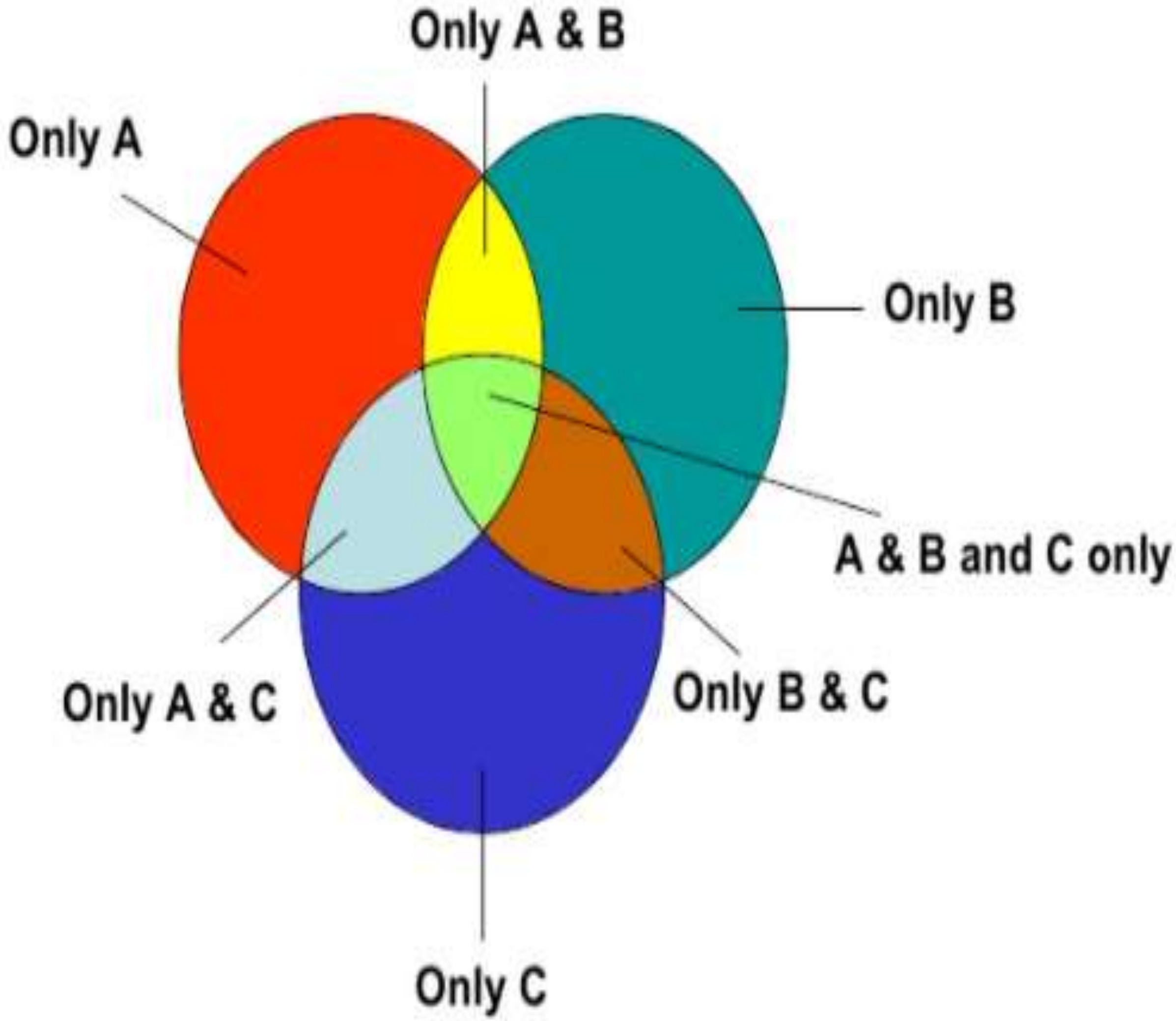
B = number divisible by 5

Symmetric Difference of Two Sets

- It is defined as the union of sets A – B and B – A.
- It is denoted by $A \Delta B$.
- $A \Delta B = (A - B) \cup (B - A)$



CONSIDER THREE SETS A, B AND C



Inclusion-Exclusion Principle

- How many elements are in $A \cup B$?

  $|A \cup B| = |A| + |B| - |A \cap B|$

- Example:

$\{2,3,5\} \cup \{3,5,7\} = \{2,3,5,3,5,7\} = \{2,3,5,7\}$

Find the number of positive integers not exceeding 100 that are not divisible by 5 or 7. Also draw corresponding venn diagram.

Solution:

Let

A: All positive integers not exceeding 100

A1: Divisible by 5

A2:Divisible by 7

There are 100 integers not exceeding 100

|A|=100

There are 100 integers not exceeding 100, while a number divisible by5 is every 5^{th} element in the lost of positive integers. Use the division rule:

$|A1|=\frac{|A|}{d}$ =100/5=20

Similarly we obtain for numbers divisible by 7 (round down)

$|A2|=\frac{|A|}{d}=100/7=14$

Numbers divisible by 5 and 7 are divisible by 35(round down)

$|A1 \cup A2|=\frac{|A|}{d}=100/35=2$

By principal of inclusion-exclusion

$$|A1 \cup A2|= |A1|+ |A2|- |A1 \cap A2|$$

$$= 20+14-2$$

$$= 32$$

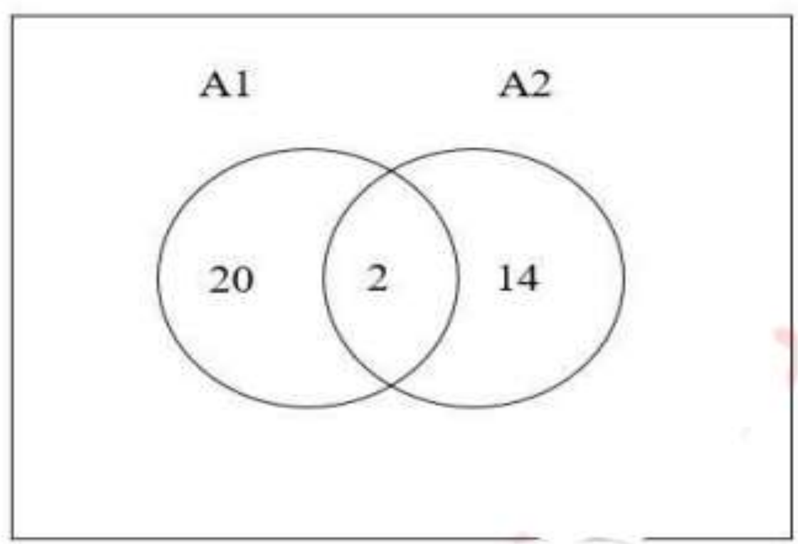
Thus 32 of the 100 integers are divisible by 5 or 7 , then the number of integers not divisible by 5 or 7 are

$|(A1 \cup A2)^C| = |A| - |A1 \cup A2|$

$= 100 - 32$

$= 68$

Venn Diagram:



Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2,3 or 5.

Solution:-

n(S) = no of integers in the set = 100

n(T) = no of integers divisible by 2 = 10

n(T') = no of integers divisible by 3 = 1

n(F) = no of integers divisible by 5 = 10

$$n(T \cap T') = \frac{100}{2 \times 3} = 16; \quad n(T' \cap F) = \frac{100}{5 \times 3}$$

$|U| = 100$

$A \to$ Div by 2 $\to |A| = \frac{100}{2}$

$B \to$ Div by 3 $\to |B| = 33$

$C \to$ Div by 5 $\to |C| = 20$

$|A \cup B \cup C|$

$= |A| + |B| + |C|$

$- |A \cap B| - |A \cap C| - |B \cap C|$

$+ (|A \cap B \cap C|)$

$|A \cap B| = \frac{100}{2 \cdot 3} = 16$

$|A \cap C| = \frac{100}{10} = 10$

$|B \cap C|$

$= \frac{100}{15}$

$= 6$

$$n(T \cap F) = \frac{100}{2 \times 5} = 10; \quad n(T \cap T' \cap F) = \frac{100}{2 \times 3 \times 5} = 3$$

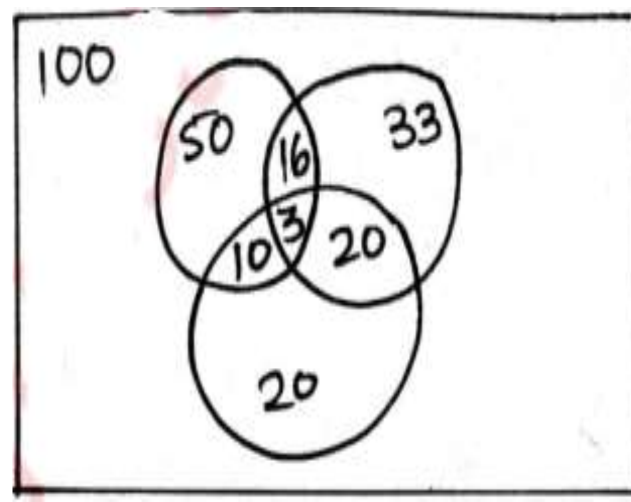
No of integers divisible by 2 ,3 & 5

n(T U T' u F) =

$$= n(T) + n(T') + n(F) - n(T \cap T') - n(T \cap F) - n(T' \cap F) + n(T \cap T' \cap F)$$

$$= 50 + 33 + 20 - 16 - 10 - 6 + 3$$

$$= 74$$



No of integers no divisible by 2 or 3 or 5

$$\Rightarrow \quad N(T \cup T' \cup F) = n(S) - n(T \cup T' \cup F)$$

$$= 100 - 74 = 26$$

- How many integers between 1 and 300 (inclusive) are divisible by at least one of 3,5,7?

  Answer: $|A \cup B \cup C|$

- By the principle of inclusion-exclusion

$$|A \cup B \cup C| = |A|+|B|+|C|-[|A \cap B|+|A \cap C|+|B \cap C|]+|A \cap B \cap C|$$

- How big are these sets? We use the floor function

$$|A| = \lfloor 300/3 \rfloor = 100 \qquad |A \cap B| = \lfloor 300/15 \rfloor = 20$$

$$|B| = \lfloor 300/5 \rfloor = 60 \qquad |A \cap C| = \lfloor 300/21 \rfloor = 100$$

$$|C| = \lfloor 300/7 \rfloor = 42 \qquad |B \cap C| = \lfloor 300/35 \rfloor = 8$$

$$|A \cap B \cap C| = \lfloor 300/105 \rfloor = 2$$

- Therefore:

$$|A \cup B \cup C| = 100 + 60 + 42 - (20+14+8) + 2 = 162$$

Basic Counting Principles

- THE SUM RULE: If task can be done either in o ne of *n1* ways or in one of *n2* ways, where none of the set of *n1* ways is the same as any of the s et of *n2* ways, then there are *n1+n2* ways to do t he task.

THE PRODUCT RULE: Suppose that a procedur e can be broken down into a sequence of two ta sks. If there are *n1* way to do the first task and fo r each of these ways of doing the first task, ther e are *n2* ways to do the second task, then there are *n1n2* ways to do the procedure.

H.W

Q Among the integers 1 to 1000
How many are not divisible by 5 & 7 but divisible by 3

Ans : 229

Permutation – Arrangement of elements in a set

$$^nP_r = \frac{n!}{(n-r)!}$$

Combination – Selection of elements in a set

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

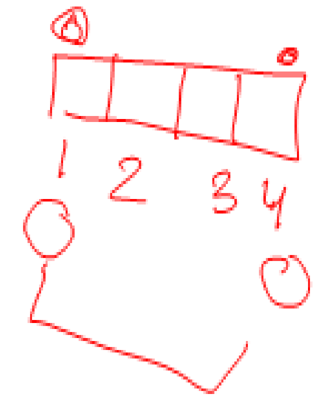
40 → students

Select 5 students → $^{40}C_r$

40 → students

20 20

B G



In how many ways a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students.

Solution:

A committee of 3 faculty and 2 students need to be formed.

Available faculty and students are 7 and 8 respectively.

Out of 7 faculty members 3 faculty members can be chosen in 7C3 ways.

Out of 8 students 2 students can be chosen in 8C2 ways.

Total number of ways of forming a committee =(7C3) * (8C2)

=(7 * 6 * 5 / 1 * 2 * 3) x (8 * 7 / 1 * 2)

=980 ways.

We can form a committee of three faculty members and 2 students from 7 faculty and 8 students in 980 ways.

11

3001, 3002

7	6	5	4
5	2	7	
Th	H	T	U

$7 \times 6 \times 5 \times 4 = 840$

7	7	7	7
5	5		
Th	H	T	U

$(7 \times 7 \times 7 \times 7$
$= 2401$

5	6	5	4
Th	H	T	U

$\rightarrow 5 \times 6 \times 5 \times 4$
$= 600$

How many four digits can be formed out of digits 1,2,3,5,7,8,9 if no digits repeated twice? How many of these will be greater than 3000?

Solution:

We have to make 4 digit number without repetition using 1,2,3,5,7,8,9

For this we have to fill 4 spaces (_ _ _ _) with required numbers.

1st space can be filled in 7 ways. (7 _ _ _)

2nd space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (7 6 _ _)

Similarly 3rd and 4th space can be filled in 5 and 4 respectively. (7 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 =7*6*5*4

=840 digits

The four digit number greater than 3000 are:

The first place can have number 3,5,7,8,9 i.e 5 digits. 1st space can be filled in 5 ways.

(5 _ _ _)

2nd space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (5 6 _ _)

Similarly 3rd and 4th space can be filled in 5 and 4 respectively. (5 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 which are greater than

3000 are =5*6*5*4

=600 digits