Laplace Transform

Q.1. Find Laplace Transform of $f(t)$ where,

1. $f(t) = t,\ 0 < t < 1/2\ ;\quad f(t) = t - 1,\ 1/2 < t < 1\ ;\ f(t) = 0,\ t > 1$
2. $f(t) = t,\ 0 < t < 3\ ;\quad f(t) = 6,\ t > 3$
3. $f(t) = erf_c\sqrt{t}$

Q.2. Find Laplace Transform of the following:

1. $sin^4 t$
2. $cosh^4 t$
3. $\sqrt{1 + \sin t}$
4. $\frac{\cos\sqrt{t}}{\sqrt{t}}$
5. $\frac{\cos 2t \sin t}{e^t}$
6. $e^{-3t} \cosh 4t \sin 3t$
7. $\sin 2t \cos t \cosh 2t$
8. $t\sqrt{1 + \sin 2t}$
9. $te^{-2t} \sinh 4t$
10. $te^{3t} \sin 2t$
11. $t\, cos^2 t$
12. $\frac{1}{t}[e^{-t} \sin at]$
13. $\frac{1}{t}[sin^2 t]$
14. $\frac{\cosh 2t \sin 2t}{t}$
15. $\frac{\cosh 3t\ sin^2 2t}{t}$
16. $\int_0^t e^{-2u} cos^2 u\ du$
17. $t \int_0^t e^{-2u} cos^2 u\ du$
18. $\int_0^t u\, e^{-3u} \sin 4u\ du$
19. $\int_0^t \frac{e^{-u} \sin u}{u} du$
20. $e^{-3t} \int_0^t u \sin 3u\ du$
21. $\frac{2 \sin t \sin 2t}{t}$

Q. 3. If $\int_0^\infty e^{-2t} \sin(t+\propto) \cos(t-\propto)\, dt = \frac{3}{8}$, then find $\propto$

Q. 4. State and prove first shifting theorem. Hence, find $L[e^{2t} \cos t \cos 2t]$

Q. 5. If $L[f(t)] = \frac{20-4s}{s^2-4s+20}$ find $L[f(3t)]$

Q. 6. If $L[f(t)] = \frac{1}{s(s^2+1)}$ find $L[e^{-t}.f(2t)]$

Q. 7. If $L[\mathrm{erf}\sqrt{t}] = \frac{1}{s\sqrt{s+1}}$ find $L[t\, \mathrm{erf} 3\sqrt{t}]$

Q. 8. Given $f(t) = t,\ 0 \le t < 3\ ;\ f(t) = 6\ , t > 3$ find $L[f(t)]$ and also $L[f'(t)]$

Q. 9. Evaluate the following integral by using Laplace Transform.

1. $\int_0^\infty e^{-2t}\, t^3 \sin t \; dt$

2. $\int_0^\infty \frac{t^2 \sin t}{e^{2t}}$

3. $\int_0^\infty e^{-3t} t \cos t \; dt$

4. $\int_0^\infty e^{-t}(t^2 - 3t + 5 + e^{2t}t^2)dt$

5. $\int_0^\infty e^{-t} \sin\frac{t}{2} \sinh\frac{\sqrt{3}\,t}{2} \; dt$

6. $\int_0^\infty e^{-3t} erf_c \sqrt{t} \; dt$

7. $\int_0^\infty e^{-t}\left(\int_0^t u \cos^2 u \; du\right) dt$

8. $\int_0^\infty e^{-2t}\left[\int_0^t \left(\frac{1-e^{-u}}{u}\right) du\right] dt$

9. $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} \; dt$

10. $\int_0^\infty \frac{\sin 2t}{t} \; dt$

11. $\int_0^\infty \frac{e^{-2t} \cos 2t \sin 3t}{t} \; dt$

12. $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} \; dt$

13. $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} \; dt$

14. $\int_0^\infty \left(\frac{\sin 2t + \sin 3t}{te^t}\right) dt$

Inverse Laplace Transform

Q. 1. Find the inverse Laplace Transform of the following:

1. $\frac{2s}{s^2+4}$
2. $\frac{4s+15}{16s^2-25}$
3. $\frac{(s^2-1)^2}{s^5}$
4. $\frac{(s-3)}{(s-3)^2+2^2}$
5. $\frac{6s-4}{s^2-4s+20}$
6. $\frac{s+2}{s^2+4s+7}$
7. $\log\left(1+\frac{a^2}{s^2}\right)$
8. $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$
9. $\log\left(\frac{s^2+1}{s(s+1)}\right)$
10. $\tan^{-1}\left(\frac{2}{s^2}\right)$
11. $\tan^{-1}\left(\frac{s+a}{b}\right)$
12. $\cot^{-1}\frac{1}{s}$
13. $\log\sqrt{\frac{s^2+a^2}{s^2}}$
14. $\tan^{-1}(s+1)$
15. $\frac{54}{s^3(s-3)}$
16. $e^{-3t}\,H(t-2)$
17. $e^{-t}\sin t\;H(t-\pi)$
18. $\frac{se^{-\pi s}}{s^2+2s+2}$
19. $e^{-s}\frac{(1+\sqrt{s})}{s^3}$
20. $\frac{e^{4-3s}}{(s+4)^{5/2}}$
21. $\frac{(s+1)e^{-s}}{s^2+s+1}$
22. $t^2\,H(t-2)-\cosh t\;\delta(t-4)$

Q. 2. Find the inverse Laplace Transform of the following by using partial fraction.

1. $\frac{s+29}{(s+4)(s^2+9)}$
2. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$
3. $\frac{2s}{s^4+4}$
4. $\frac{s}{s^4+4a^4}$
5. $\frac{s^2}{(s+1)^3}$
6. $\frac{s^2+1}{s^3+3s^2+2s}$
7. $\frac{3s+7}{s^2-2s-3}$
8. $\frac{2s^2-1}{(s^2+1)(s^2+4)}$
9. $\frac{1}{(s-2)^4(s+3)}$

Q. 3. Find the inverse Laplace Transform of the following by using convolution theorem.

1. $\frac{1}{s(s+4)^2}$
2. $\frac{1}{(s-2)(s+2)^2}$
3. $\frac{s^2}{(s^2+2^2)^2}$
4. $\frac{s}{(s^2+a^2)(s^2+b^2)}$, $(a \neq b)$
5. $\frac{16}{(s-2)(s+2)}$
6. $\frac{1}{(s^2+1)^2}$
7. $\frac{s^2}{(s^2+1)(s^2+4)}$
8. $\frac{1}{(s+3)(s^2+2s+2)}$
9. $\frac{1}{(s-2)^4(s+3)}$
10. $\frac{s+2}{(s^2+4s+5)^2}$
11. $\frac{(s-1)^2}{(s^2-2s+5)^2}$

Q. 4. Find the Laplace Transform of

$$f(t) = \frac{t}{a}\ ,\ 0 < t \le a\ \ ;\ \ f = \frac{1}{a}(2a - t)\ ,\ a < t < 2a \qquad and\ f(t) = f(t + 2a)$$

Q. 5. Find the Laplace Transform of $f(t) = \sin 2t\ \ ,\ 0 < t < \pi/2\ ,$

$f(t) = 0\ ,\ \pi/2 < t < \pi \qquad and\ \ f(t) = f(t + \pi)$

Q. 6. Express the function in terms of Heaviside unit step function and hence find the Laplace transform.

1. $f(t) = \begin{cases} 0 & ,0 < t < 4 \\ (t-4)^3 & ,\quad t > 4 \end{cases}$

2. $f(t) = \begin{cases} \sin t & ,\quad 0 < t < \pi \\ \cos t & ,\quad t > \pi \end{cases}$

3. $f(t) = \begin{cases} t & ,\quad 0 < t < 2 \\ t^2 & ,\quad t > 2 \end{cases}$

Q. 7. Evaluate the following integral by using Laplace Transform

$$\int_0^\infty e^{-2t}(1 + t + t^2)H(t-3)\,dt$$

Q. 8. If $f(t)$ is a periodic function of period a, prove that

$$L[f(t)] = \frac{1}{1 - e^{-as}}\int_0^a e^{-st}\,f(t)\,dt$$

Q. 9. Using Laplace Transform solve the following differential equations with the given conditions.

1. $(D^2 - D - 2)y = 20\sin 2t \quad with\ \ y(0) = 1\ \ and\ \ y'(0) = 2$
2. $\dfrac{dy}{dt} + 2y + \displaystyle\int_0^t y\,dt = \sin t \qquad given\ that\ \ y(0) = 1$
3. $(D^2 + 4D + 3)y = e^{-t}\ \ ;\ \ y(0) = y'(0) = 1$
4. $\dfrac{d^2y}{dt^2} + y = t\ \ ;\ \ y(0) = 1\ , \qquad y'(0) = 0$
5. $2y'' + 5y' + 2y = e^{-2t}\ ;\ \ y(0) = y'(0) = 1$