# Linear Programming Problem

## Type I: Basic solutions of L.P.P.

1. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions? Ans. $x_1 = 1, x_2 = 0, x_3 = 2, z_{max} = 9$

   Maximise $z = x_1 - 2x_2 + 4x_3$

   subject to $x_1 + 2x_2 + 3x_3 = 7$

   $3x_1 + 4x_2 + 6x_3 = 15$

   $x_1, x_2, x_3 \geq 0$

2. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions?

   Maximise $z = x_1 + 3x_2 + 3x_3$

   subject to $x_1 + 2x_2 + 3x_3 = 4$

   $2x_1 + 3x_2 + 5x_3 = 7$

   $x_1, x_2, x_3 \geq 0$

   **[N15/CompIT/5M][M18/Comp/5M][N18/AutoMechCivil/5M] [N19/Comp/5M]**

   **Solution:**

| No | Non-basic var = 0 | Basic var | Equations & solutions | Is the solution feasible? | Is the solution degenerate? | Value of z | Is the solution optimal? |
|---|---|---|---|---|---|---|---|
| 1 | $x_3 = 0$ | $x_1, x_2$ | $x_1 + 2x_2 = 4$<br>$2x_1 + 3x_2 = 7$<br>$x_1 = 2, x_2 = 1$ | Yes | No | 5 | Yes |
| 2 | $x_2 = 0$ | $x_1, x_3$ | $x_1 + 3x_3 = 4$<br>$2x_1 + 5x_3 = 7$<br>$x_1 = 1, x_3 = 1$ | Yes | No | 4 | No |
| 3 | $x_1 = 0$ | $x_2, x_3$ | $2x_2 + 3x_3 = 4$<br>$3x_2 + 5x_3 = 7$<br>$x_2 = -1, x_3 = 2$ | No | No | 3 | No |

3. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions?

Maximise $z = 2x_1 - 2x_2 + 4x_3 - 5x_4$

subject to $x_1 + 4x_2 - 2x_3 + 8x_4 \leq 2$

$-x_1 + 2x_2 + 3x_3 + 4x_4 \leq 1$

$x_1, x_2, x_3, x_4 \geq 0$

**[N19/AutoMechCivil/5M]**

**Solution:**

| No | Non-basic var = 0 | Basic var | Equations & Solutions | Is the solution feasible? | Is the solution degenerate? | Value of z | Is the solution optimal? |
|---|---|---|---|---|---|---|---|
| 1 | $x_3 = 0$, $x_4 = 0$ | $x_1, x_2$ | $x_1 + 4x_2 = 2$<br>$-x_1 + 2x_2 = 1$<br>$x_1 = 0, x_2 = 0.5$ | Yes | Yes | $-1$ | No |
| 2 | $x_2 = 0$, $x_4 = 0$ | $x_1, x_3$ | $x_1 - 2x_3 = 2$<br>$-x_1 + 3x_3 = 1$<br>$x_1 = 8, x_3 = 3$ | Yes | No | 28 | Yes |
| 3 | $x_1 = 0$, $x_4 = 0$ | $x_2, x_3$ | $4x_2 - 2x_3 = 2$<br>$2x_2 + 3x_3 = 1$<br>$x_2 = 0.5, x_3 = 0$ | Yes | Yes | $-1$ | No |
| 4 | $x_2 = 0$, $x_3 = 0$ | $x_1, x_4$ | $x_1 + 8x_4 = 2$<br>$-x_1 + 4x_4 = 1$<br>$x_1 = 0, x_4 = \frac{1}{4}$ | Yes | Yes | $-\frac{5}{4}$ | No |
| 5 | $x_1 = 0$, $x_3 = 0$ | $x_2, x_4$ | $4x_2 + 8x_4 = 2$<br>$2x_2 + 4x_4 = 1$<br>Unbounded soln | - | - | - | - |
| 6 | $x_1 = 0$, $x_2 = 0$ | $x_3, x_4$ | $-2x_3 + 8x_4 = 2$<br>$3x_3 + 4x_4 = 1$<br>$x_3 = 0, x_4 = \frac{1}{4}$ | Yes | No | $-\frac{5}{4}$ | No |

4. Find all basic solutions to the following problem

Maximise $z = x_1 + x_2 + 3x_3$

subject to $x_1 + 2x_2 + 3x_3 = 9$

$3x_1 + 2x_2 + 2x_3 = 15$

$x_1, x_2, x_3 \geq 0$

**[N18/Comp/5M]**

**Solution:**

| No | Non-basic var = 0 | Basic var | Equations & solutions | Is the solution feasible? | Is the solution degenerate? | Value of z |
|---|---|---|---|---|---|---|
| 1 | $x_3 = 0$ | $x_1, x_2$ | $x_1 + 2x_2 = 9$<br>$3x_1 + 2x_2 = 15$<br>$x_1 = 3, x_2 = 3$ | Yes | No | 6 |
| 2 | $x_2 = 0$ | $x_1, x_3$ | $x_1 + 3x_3 = 9$<br>$3x_1 + 2x_3 = 15$<br>$x_1 = \frac{27}{7}, x_3 = \frac{12}{7}$ | Yes | No | 9 |
| 3 | $x_1 = 0$ | $x_2, x_3$ | $2x_2 + 3x_3 = 9$<br>$2x_2 + 2x_3 = 15$<br>$x_2 = \frac{27}{2}, x_3 = -6$ | No | No | -9/2 |

5. Find all basic feasible solutions of the following system of equations

$2x_1 + x_2 - x_3 = 2$

$3x_1 + 2x_2 + x_3 = 3$

6. Find all the basic feasible solutions to the following system of equations.
$2x_1 + 6x_2 + 2x_3 + x_4 = 3$
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$
**[M19/Comp/5M]**
**Solution:**

| No | Non-basic var = 0 | Basic var | Equations & solutions | Is the solution feasible? |
|---|---|---|---|---|
| 1 | $x_3 = 0$ <br> $x_4 = 0$ | $x_1, x_2$ | $2x_1 + 6x_2 = 3$ <br> $6x_1 + 4x_2 = 2$ <br> $x_1 = 0, x_2 = 0.5$ | Yes |
| 2 | $x_2 = 0$ <br> $x_4 = 0$ | $x_1, x_3$ | $2x_1 + 2x_3 = 3$ <br> $6x_1 + 4x_3 = 2$ <br> $x_1 = -2, x_3 = 3.5$ | No |
| 3 | $x_1 = 0$ <br> $x_4 = 0$ | $x_2, x_3$ | $6x_2 + 2x_3 = 3$ <br> $4x_2 + 4x_3 = 2$ <br> $x_2 = 0.5, x_3 = 0$ | Yes |
| 4 | $x_2 = 0$ <br> $x_3 = 0$ | $x_1, x_4$ | $2x_1 + x_4 = 3$ <br> $6x_1 + 6x_4 = 2$ <br> $x_1 = \frac{8}{3}, x_4 = -\frac{7}{3}$ | No |
| 5 | $x_1 = 0$ <br> $x_3 = 0$ | $x_2, x_4$ | $6x_2 + x_4 = 3$ <br> $4x_2 + 6x_4 = 2$ <br> $x_2 = 0.5, x_4 = 0$ | Yes |
| 6 | $x_1 = 0$ <br> $x_2 = 0$ | $x_3, x_4$ | $2x_3 + x_4 = 3$ <br> $4x_3 + 6x_4 = 2$ <br> $x_3 = 2, x_4 = -1$ | No |

7. Find all basic feasible solutions of the following system of equations
$x_1 + 2x_2 + 4x_3 + x_4 = 7$
$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

## Type II: Simplex Method

1. Solve by using Simplex method.

Maximise $z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

**[M17/CompIT/6M]**

**Solution:**

The standard form,

Max $z - 3x_1 - 2x_2 + 0s_1 + 0s_2 = 0$

s.t. $x_1 + x_2 + s_1 + 0s_2 = 4$

$x_1 - x_2 + 0s_1 + s_2 = 2$

$x_1, x_2, s_1, s_2 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | | | |
| 0 | $z$ | **-3** | -2 | 0 | 0 | 0 | 0 | $X + 3Y$ |
| $s_2$ leaves $x_1$ enters | $s_1$ | **1** | 1 | 1 | 0 | 4 | $\frac{4}{1} = 4$ | $X - Y$ |
| | $s_2$ | **1** | **-1** | **0** | **1** | **2** | $\frac{2}{1} = 2$ | - |
| | | | | | | | | |
| 1 | $z$ | 0 | **-5** | 0 | 3 | 6 | - | $X + \frac{5}{2}Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **0** | **2** | **1** | **-1** | **2** | $\frac{2}{2} = 1$ | $\frac{Y}{2}$ |
| | $x_1$ | 1 | **-1** | 0 | 1 | 2 | - | $X + \frac{1}{2}Y$ |
| | | | | | | | | |
| 2 | $z$ | 0 | 0 | $\frac{5}{2}$ | $\frac{1}{2}$ | 11 | | |
| | $x_2$ | 0 | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | | |
| | $x_1$ | 1 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 3 | | |

Thus, the solution is

$x_1 = 3, x_2 = 1, z_{max} = 11$

2. Solve by using Simplex method.

Maximise $z = x_1 + 4x_2$

subject to $2x_1 + x_2 \leq 3$

$3x_1 + 5x_2 \leq 9$

$x_1 + 3x_2 \leq 5$

$x_1, x_2 \geq 0$

**[N14/CompIT/6M]**

**Solution:**

The standard form,

Max $z - x_1 - 4x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$

$3x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 9$

$x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 5$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | -1 | **-4** | 0 | 0 | 0 | 0 | - | $X + \frac{4}{3}Y$ |
| $s_3$ leaves $x_2$ enters | $s_1$ | 2 | **1** | 1 | 0 | 0 | 3 | $\frac{3}{1} = 3$ | $X - \frac{1}{3}Y$ |
| | $s_2$ | 3 | **5** | 0 | 1 | 0 | 9 | $\frac{9}{5} = 1.8$ | $X - \frac{5}{3}Y$ |
| | $s_3$ | **1** | **3** | **0** | **0** | **1** | **5** | $\frac{5}{3} = 1.67$ | $\frac{Y}{3}$ |
| | | | | | | | | | |
| 1 | $z$ | 1/3 | 0 | 0 | 0 | 4/3 | 20/3 | | |
| | $s_1$ | 5/3 | 0 | 1 | 0 | -1/3 | 4/3 | | |
| | $s_2$ | 4/3 | 0 | 0 | 1 | -5/3 | 2/3 | | |
| | $x_2$ | 1/3 | 1 | 0 | 0 | 1/3 | 5/3 | | |

Thus, the solution is

$x_1 = 0, x_2 = \frac{5}{3}, z_{max} = \frac{20}{3}$

3. Solve by using Simplex method.

Maximise $z = 5x_1 + 4x_2$

subject to $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

Ans. $x_1 = 3, x_2 = \frac{3}{2}, z_{max} = 21$

4. Solve the following L.P.P. by simplex method

Maximise $z = 10x_1 + x_2 + x_3$

subject to $x_1 + x_2 - 3x_3 \leq 10$

$4x_1 + x_2 + x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$

**[N17/CompIT/6M]**

**Solution:**

The standard form,

Max $z - 10x_1 - x_2 + x_3 + 0s_1 + 0s_2 = 0$

s.t. $x_1 + x_2 - 3x_3 + s_1 + 0s_2 = 10$

$4x_1 + x_2 + x_3 + 0s_1 + s_2 = 20$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | | | |
| 0 | $z$ | **-10** | -1 | 1 | 0 | 0 | 0 | - | $X + \frac{10}{4}Y$ |
| $s_2$ leaves $x_1$ enters | $s_1$ | **1** | 1 | -3 | 1 | 0 | 10 | $\frac{10}{1} = 10$ | $X - \frac{1}{4}Y$ |
| | $s_2$ | **4** | **1** | **1** | **0** | **1** | **20** | $\frac{20}{4} = 5$ | $\frac{Y}{4}$ |
| | | | | | | | | | |
| 1 | $z$ | 0 | $\frac{3}{2}$ | $\frac{7}{2}$ | 0 | $\frac{5}{2}$ | 50 | | |
| | $s_1$ | 0 | $\frac{3}{4}$ | $-\frac{13}{4}$ | 1 | $-\frac{1}{4}$ | 5 | | |
| | $x_1$ | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 5 | | |

Thus, the solution is

$x_1 = 5, x_2 = 0, x_3 = 0, z_{max} = 50$

5. Solve the following L.P.P. by simplex method

   Minimise $z = x_1 - 3x_2 + 3x_3$

   subject to $3x_1 - x_2 + 2x_3 \le 7$

   $2x_1 + 4x_2 \ge -12$

   $-4x_1 + 3x_2 + 8x_3 \le 10$

   $x_1, x_2, x_3 \ge 0$ Ans. $x_1 = 4, x_2 = 5, x_3 = 0, z_{min} = -11$

6. Solve the following L.P.P. by simplex method

   Maximise $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

   subject to $2x_1 + x_2 + 5x_3 + 6x_4 \le 20$

   $3x_1 + x_2 + 3x_3 + 25x_4 \le 24$

   $7x_1 + x_4 \le 70$

   $x_1, x_2, x_3, x_4 \ge 0$

   **[M14/CompIT/8M]**

   **Solution:**

   Max $z - 15x_1 - 6x_2 - 9x_3 - 2x_4 + 0s_1 + 0s_2 + 0s_3 = 0$

   s.t. $2x_1 + x_2 + 5x_3 + 6x_4 + s_1 + 0s_2 + 0s_3 = 20$

   $3x_1 + x_2 + 3x_3 + 25x_4 + 0s_1 + s_2 + 0s_3 = 24$

   $7x_1 + 0x_2 + 0x_3 + x_4 + 0s_1 + 0s_2 + s_3 = 70$

   $x_1, x_2, x_3, x_4, s_1, s_2, s_3 \ge 0$

   Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | **-15** | -6 | -9 | -2 | 0 | 0 | 0 | 0 | - | $X + 5Y$ |
| $s_2$ leaves $x_1$ enters | $s_1$ | **2** | 1 | 5 | 6 | 1 | 0 | 0 | 20 | $\frac{20}{2} = 10$ | $X - \frac{2}{3}Y$ |
| | $s_2$ | **3** | **1** | **3** | **25** | **0** | **1** | **0** | **24** | $\frac{24}{3} = 8$ | $\frac{Y}{3}$ |
| | $s_3$ | **7** | 0 | 0 | 1 | 0 | 0 | 1 | 70 | $\frac{70}{7} = 10$ | $X - \frac{7}{3}Y$ |
| | | | | | | | | | | | |
| 1 | $z$ | 0 | **-1** | 6 | 123 | 0 | 5 | 0 | 120 | - | $X + 3Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **0** | **1/3** | **3** | **-32/3** | **1** | **-2/3** | **0** | **4** | $\frac{4}{\frac{1}{3}} = 12$ | $3Y$ |
| | $x_1$ | 1 | **1/3** | 1 | 25/3 | 0 | 1/3 | 0 | 8 | $\frac{8}{\frac{1}{3}} = 24$ | $X - Y$ |
| | $s_3$ | 0 | **-7/3** | -7 | -172/3 | 0 | -7/3 | 1 | 14 | - | $X + 7Y$ |
| | | | | | | | | | | | |
| 2 | $z$ | 0 | 0 | 15 | 91 | 3 | 3 | 0 | 132 | | |
| | $x_2$ | 0 | 1 | 9 | -32 | 3 | -2 | 0 | 12 | | |
| | $x_1$ | 1 | 0 | -2 | 19 | -1 | 1 | 0 | 4 | | |
| | $s_3$ | 0 | 0 | 14 | -132 | 7 | -7 | 1 | 42 | | |

Thus, the solution is
$x_1 = 4, x_2 = 12, x_3 = 0, x_4 = 0, z_{max} = 132$

7. Solve by using Simplex method.
Maximise $\quad z = 4x_1 + 10x_2$
subject to $\quad 2x_1 + x_2 \leq 10$
$2x_1 + 5x_2 \leq 20$
$2x_1 + 3x_2 \leq 18$
$x_1, x_2 \geq 0$

**[N19/Comp/8M]**

**Solution:**

Max $\quad z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$
s.t. $\quad 2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$
$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$
$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18$
$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | -4 | **-10** | 0 | 0 | 0 | 0 | - | $X + 2Y$ |
| $s_2$ leaves $x_2$ enters | $s_1$ | 2 | **1** | 1 | 0 | 0 | 10 | $\frac{10}{1} = 10$ | $X - \frac{1}{5}Y$ |
| | $s_2$ | **2** | **5** | **0** | **1** | **0** | **20** | $\frac{20}{5} = 4$ | $\frac{Y}{5}$ |
| | $s_3$ | 2 | **3** | 0 | 0 | 1 | 18 | $\frac{18}{3} = 6$ | $X - \frac{3}{5}Y$ |
| | | | | | | | | | |
| 1 | $z$ | 0 | 0 | 0 | 2 | 0 | 40 | | |
| | $s_1$ | $\frac{8}{5}$ | 0 | 1 | $-\frac{1}{5}$ | 0 | 6 | | |
| | $x_2$ | $\frac{2}{5}$ | 1 | 0 | $\frac{1}{5}$ | 0 | 4 | | |
| | $s_3$ | $\frac{4}{5}$ | 0 | 0 | $-\frac{3}{5}$ | 1 | 6 | | |

Thus, the solution is
$x_1 = 0, x_2 = 4, z_{max} = 40$

8. Solve the following L.P.P. by simplex method

Maximise $z = 3x_1 + 2x_2$

subject to $3x_1 + 2x_2 \leq 18$

$0 \leq x_1 \leq 4$

$0 \leq x_2 \leq 6$

$x_1, x_2 \geq 0$

**[N16/CompIT/6M][N18/AutoMechCivil/8M]**

**Solution:**

Max $z - 3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $3x_1 + 2x_2 + s_1 + 0s_2 + 0s_3 = 18$

$x_1 + 0x_2 + 0s_1 + s_2 + 0s_3 = 4$

$0x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | **-3** | -2 | 0 | 0 | 0 | 0 | - | $X + 3Y$ |
| $s_2$ leaves $x_1$ enters | $s_1$ | **3** | 2 | 1 | 0 | 0 | 18 | $\frac{18}{3} = 6$ | $X - 3Y$ |
| | $s_2$ | **1** | **0** | **0** | **1** | **0** | **4** | $\frac{4}{1} = 4$ | - |
| | $s_3$ | **0** | 1 | 0 | 0 | 1 | 6 | - | - |
| | | | | | | | | | |
| 1 | $z$ | 0 | **-2** | 0 | 3 | 0 | 12 | - | $X + Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **0** | **2** | **1** | **-3** | **0** | **6** | $\frac{6}{2} = 3$ | $\frac{Y}{2}$ |
| | $x_1$ | 1 | **0** | 0 | 1 | 0 | 4 | - | - |
| | $s_3$ | 0 | **1** | 0 | 0 | 1 | 6 | $\frac{6}{1} = 6$ | $X - \frac{1}{2}Y$ |
| | | | | | | | | | |
| 2 | $z$ | 0 | 0 | 1 | 0 | 0 | 18 | | |
| | $x_2$ | 0 | 1 | 1/2 | -3/2 | 0 | 3 | | |
| | $x_1$ | 1 | 0 | 0 | 1 | 0 | 4 | | |
| | $s_3$ | 0 | 0 | -1/2 | 3/2 | 1 | 3 | | |

Thus, the solution is

$x_1 = 4, x_2 = 3, z_{max} = 18$

9. Use Simplex method to

Maximize $z = 3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

**[N18/Comp/8M][M19/Comp/6M]**

**Solution:**

Max $z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $3x_1 + 2x_2 + s_1 + 0s_2 + 0s_3 = 18$

$x_1 + 0x_2 + 0s_1 + s_2 + 0s_3 = 4$

$0x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | -3 | **-5** | 0 | 0 | 0 | 0 | - | $X + 5Y$ |
| $s_3$ leaves $x_2$ enters | $s_1$ | 3 | **2** | 1 | 0 | 0 | 18 | $\frac{18}{2} = 9$ | $X - 2Y$ |
| | $s_2$ | 1 | **0** | 0 | 1 | 0 | 4 | - | - |
| | $s_3$ | **0** | **1** | **0** | **0** | **1** | **6** | $\frac{6}{1} = 6$ | - |
| | | | | | | | | | |
| 1 | $z$ | **-3** | 0 | 0 | 0 | 5 | 30 | - | $X + Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **3** | **2** | **1** | **0** | **-2** | **6** | $\frac{6}{3} = 2$ | $\frac{Y}{3}$ |
| | $s_2$ | **1** | 0 | 0 | 1 | 0 | 4 | $\frac{4}{1} = 4$ | $X - \frac{1}{3}Y$ |
| | $x_2$ | **0** | 1 | 0 | 0 | 1 | 6 | – | – |
| | | | | | | | | | |
| 2 | $z$ | 0 | 2 | 1 | 0 | 3 | 36 | | |
| | $x_1$ | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | $-\frac{2}{3}$ | 2 | | |
| | $s_2$ | 0 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | 1 | $\frac{2}{3}$ | 2 | | |
| | $x_2$ | 0 | 1 | 0 | 0 | 1 | 6 | | |

Thus, the solution is

$x_1 = 2, x_2 = 6, z_{max} = 36$

10. Solve the following L.P.P. by simplex method

Maximise $z = 6x_1 - 2x_2 + 3x_3$

subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

**[N15/CompIT/6M][M18/Comp/8M]**

**Solution:**

The standard form,

Max $z - 6x_1 + 2x_2 - 3x_3 + 0s_1 + 0s_2 = 0$

s.t. $2x_1 - x_2 + 2x_3 + s_1 + 0s_2 = 2$

$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | | | |
| 0 | $z$ | **-6** | 2 | -3 | 0 | 0 | 0 | - | $X + 3Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **2** | **-1** | **2** | **1** | **0** | **2** | $\frac{2}{2} = 1$ | $\frac{Y}{2}$ |
| | $s_2$ | **1** | 0 | 4 | 0 | 1 | 4 | $\frac{4}{1} = 4$ | $X - \frac{1}{2}Y$ |
| | | | | | | | | | |
| 1 | $z$ | 0 | **-1** | 3 | 3 | 0 | 6 | - | $X + 2Y$ |
| $s_2$ leaves $x_2$ enters | $x_1$ | 1 | **-1/2** | 1 | 1/2 | 0 | 1 | - | $X + Y$ |
| | $s_2$ | **0** | **1/2** | **3** | **-1/2** | **1** | **3** | $\frac{3}{\frac{1}{2}} = 6$ | $2Y$ |
| | | | | | | | | | |
| 2 | $z$ | 0 | 0 | 9 | 2 | 2 | 12 | | |
| | $x_1$ | 1 | 0 | 4 | 0 | 1 | 4 | | |
| | $x_2$ | 0 | 1 | 6 | -1 | 2 | 6 | | |

Thus, the solution is

$x_1 = 4, x_2 = 6, x_3 = 0, z_{max} = 12$

11. Solve by using Simplex method.

Maximise $z = 3000x_1 + 2500x_2$

subject to $2x_1 + x_2 \leq 40$

$x_1 + 3x_2 \leq 45$

$x_1 \leq 12$

$x_1, x_2 \geq 0$ Ans. $x_1 = 12, x_2 = 11, z_{max} = 63500$

12. Solve the following LPP by simplex method
    Maximize $z = 1000x_1 + 4000x_2 + 5000x_3$
    Subject to $x_1 + 2x_2 + 3x_3 \leq 14$
    $3x_1 + 2x_2 \leq 14$
    $x_1, x_2, x_3 \geq 0$ Ans.$x_1 = 0, x_2 = 7, x_3 = 0, z_{max} = 28000$
13. Solve the following L.P.P. by simplex method
    Maximise $z = 4x_1 + 3x_2 + 6x_3$
    subject to $2x_1 + 3x_2 + 2x_3 \leq 440$
    $4x_1 + 3x_3 \leq 470$
    $2x_1 + 5x_2 \leq 430$
    $x_1, x_2, x_3 \geq 0$

**[M15/CompIT/6M]**

**Solution:**

Max $z - 4x_1 - 3x_2 - 6x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + 3x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 440$

$4x_1 + 0x_2 + 3x_3 + 0s_1 + s_2 + 0s_3 = 470$

$2x_1 + 5x_2 + 0x_3 + 0s_1 + 0s_2 + s_3 = 430$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | -4 | -3 | **-6** | 0 | 0 | 0 | 0 | - | $X + 2Y$ |
| $s_2$ leaves $x_3$ enters | $s_1$ | 2 | 3 | **2** | 1 | 0 | 0 | 440 | $\frac{440}{2} = 220$ | $X - \frac{2}{3}Y$ |
| | $s_2$ | **4** | **0** | **3** | **0** | **1** | **0** | **470** | $\frac{470}{3} = 156.67$ | $\frac{Y}{3}$ |
| | $s_3$ | 2 | 5 | **0** | 0 | 0 | 1 | 430 | - | - |
| | | | | | | | | | | |
| 1 | $z$ | 4 | **-3** | 0 | 0 | 2 | 0 | 940 | - | $X + Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **-2/3** | **3** | **0** | **1** | **-2/3** | **0** | **380/3** | $\frac{380}{9} = 42.22$ | $\frac{Y}{3}$ |
| | $x_3$ | 4/3 | **0** | 1 | 0 | 1/3 | 0 | 470/3 | - | - |
| | $s_3$ | 2 | **5** | 0 | 0 | 0 | 1 | 430 | $\frac{430}{5} = 86$ | $X - \frac{5}{3}Y$ |
| | | | | | | | | | | |
| 2 | $z$ | 10/3 | 0 | 0 | 1 | 4/3 | 0 | 3200/3 | | |
| | $x_2$ | -2/9 | 1 | 0 | 1/3 | -2/9 | 0 | 380/9 | | |
| | $x_3$ | 4/3 | 0 | 1 | 0 | 1/3 | 0 | 470/3 | | |
| | $s_3$ | 28/9 | 0 | 0 | -5/3 | 10/9 | 1 | 1970/9 | | |

Thus, the solution is

$x_1 = 0, x_2 = \frac{380}{9}, x_3 = \frac{470}{3}, z_{max} = \frac{3200}{3}$

14. Solve the following L.P.P. by simplex method

Maximise $z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Ans. $x_1 = 0, x_2 = 100, x_3 = 230, z_{max} = 1350$

15. Solve the following L.P.P. by simplex method

Maximise $z = 3x_1 + 5x_2 + 4x_3$

subject to $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0$

Ans. $x_1 = \frac{89}{41}, x_2 = \frac{50}{41}, x_3 = \frac{62}{41}, z_{max} = \frac{765}{41}$

16. Solve the following LPP by simplex method

Maximize $z = x_1 - x_2 + 3x_3$

Subject to $x_1 + x_2 + x_3 \leq 10$

$2x_1 - x_3 \leq 3$

$2x_1 - 2x_2 + 3x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Ans. $x_1 = 0, x_2 = 4, x_3 = 6, z_{max} = 14$

17. Solve the following L.P.P. by simplex method

Maximise $z = 4x_1 + 2x_2 + 5x_3$

subject to $12x_1 + 7x_2 + 9x_3 \leq 1260$

$22x_1 + 18x_2 + 16x_3 \leq 19008$

$2x_1 + 4x_2 + 3x_3 \leq 396$

$x_1, x_2, x_3 \geq 0$

Ans. $x_1 = 12, x_2 = 0, x_3 = 124, z_{max} = 668$

18. Solve the following L.P.P. by simplex method

Maximise $z = 4x_1 + x_2 + 3x_3 + 5x_4$

subject to $4x_1 - 6x_2 - 5x_3 - x_4 \leq 2$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

Ans. unbounded solution

19. Solve the following L.P.P. by simplex method

Maximise $z = 4x_1 + x_2 + 3x_3 + 5x_4$

subject to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

**[M16/CompIT/6M]**

**Solution:**

The standard form,

Max $z - 4x_1 - x_2 - 3x_3 - 5x_4 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 + 0s_2 + 0s_3 = 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 + 0s_1 + s_2 + 0s_3 = 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 + 0s_1 + 0s_2 + s_3 = 20$

$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $s_1$ | $s_2$ | $s_3$ | | | |
| 0 | $z$ | -4 | -1 | -3 | **-5** | 0 | 0 | 0 | 0 | - | $X + \frac{5}{4}Y$ |
| $s_1$ leaves $x_4$ enters | $s_1$ | **-4** | **6** | **5** | **4** | **1** | **0** | **0** | **20** | $\frac{20}{4} = 5$ | $\frac{Y}{4}$ |
| | $s_2$ | -3 | -2 | 4 | **1** | 0 | 1 | 0 | 10 | $\frac{10}{1} = 10$ | $X - \frac{1}{4}Y$ |
| | $s_3$ | -8 | -3 | 3 | **2** | 0 | 0 | 1 | 20 | $\frac{20}{2} = 10$ | $X - \frac{1}{2}Y$ |
| | | | | | | | | | | | |
| 1 | $z$ | **-9** | 13/2 | 13/4 | 0 | 5/4 | 0 | 0 | 25 | - | |
| | $x_4$ | **-1** | 3/2 | 5/4 | 1 | 1/4 | 0 | 0 | 5 | - | |
| | $s_2$ | **-2** | -7/2 | 11/4 | 0 | -1/4 | 1 | 0 | 5 | - | |
| | $s_3$ | **-6** | -6 | 1/2 | 0 | -1/2 | 0 | 1 | 10 | - | |

Since, there are no positive ratio obtained and the coefficient is still negative the solution is unbounded.

## Type III: Big M Method

1. Using Penalty (Big-M or Charne’s) method to solve the following L.P.P.

   Minimise $z = 2x_1 + 3x_2$

   subject to $x_1 + x_2 \geq 5$

   $x_1 + 2x_2 \geq 6$

   $x_1, x_2 \geq 0$

   **[N16/CompIT/8M]**

   **Solution:**

   The standard form,

   Max $z' = -z = -2x_1 - 3x_2$

   Max $z' + 2x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ……(1)

   s.t. $x_1 + x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 5$ ……(2)

   $x_1 + 2x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 6$ ……(3)

   Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

   $z' + (2-2M)x_1 + (3-3M)x_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -11M$

   Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | | | |
| 0 | $z'$ | 2-2M | **3-3M** | M | M | 0 | 0 | -11M | - | $X - \frac{(3-3M)}{2}Y$ |
| $A_2$ leaves $x_2$ enters | $A_1$ | 1 | **1** | -1 | 0 | 1 | 0 | 5 | $\frac{5}{1} = 5$ | $X - \frac{1}{2}Y$ |
| | $A_2$ | **1** | **2** | **0** | **-1** | **0** | **1** | **6** | $\frac{6}{2} = 3$ | $\frac{Y}{2}$ |
| | | | | | | | | | | |
| 1 | $z'$ | **1/2-M/2** | 0 | M | 3/2 –M/2 | 0 | | -9-2M | - | $X - (1-M)Y$ |
| $A_1$ leaves $x_1$ enters | $A_1$ | **1/2** | **0** | **-1** | **1/2** | **1** | | **2** | $\frac{2}{\frac{1}{2}} = 4$ | $2Y$ |
| | $x_2$ | **1/2** | 1 | 0 | -1/2 | 0 | | 3 | $\frac{3}{\frac{1}{2}} = 6$ | $X - Y$ |
| | | | | | | | | | | |
| 2 | $z'$ | 0 | 0 | 1 | 1 | | | -11 | | |
| | $x_1$ | 1 | 0 | -2 | 1 | | | 4 | | |
| | $x_2$ | 0 | 1 | 1 | -1 | | | 1 | | |

   Thus, the solution is

   $x_1 = 4, x_2 = 1, z'_{max} = -11, \therefore\ z_{min} = 11$

2. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 10x_1 + 3x_2$

subject to $x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1, x_2 \geq 0$

**[M18/AutoMechCivil/6M]**

**Solution:**

The standard form,

Max $z' = -z = -10x_1 - 3x_2$

Max $z' + 10x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ……(1)

s.t. $x_1 + 2x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 3$ ……(2)

$x_1 + 4x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 4$ ……(3)

Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

$$z' + (10 - 2M)x_1 + (3 - 6M)x_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -7M$$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | | | |
| 0 | $z'$ | 10-2M | **3-6M** | M | M | 0 | 0 | -7M | - | $X - \frac{(3-6M)}{4}Y$ |
| $A_2$ leaves $x_2$ enters | $A_1$ | 1 | **2** | -1 | 0 | 1 | 0 | 3 | $\frac{3}{2} = 1.5$ | $X - \frac{1}{2}Y$ |
| | $A_2$ | **1** | **4** | **0** | **-1** | **0** | **1** | **4** | $\frac{4}{4} = 1$ | $\frac{Y}{4}$ |
| | | | | | | | | | | |
| 1 | $z'$ | 37/4-M/2 | 0 | M | **3/4 – M/2** | 0 | | -M-3 | - | $X - (3/2 - M)Y$ |
| $A_1$ leaves $s_2$ enters | $A_1$ | **1/2** | **0** | **-1** | **1/2** | **1** | | **1** | $\frac{1}{\frac{1}{2}} = 2$ | $2Y$ |
| | $x_2$ | 1/4 | 1 | 0 | **-1/4** | 0 | | 1 | - | $X + \frac{1}{2}Y$ |
| | | | | | | | | | | |
| 2 | $z'$ | 17/2 | 0 | 3/2 | 0 | | | -9/2 | - | |
| | $s_2$ | 1 | 0 | -2 | 1 | | | 2 | - | |
| | $x_2$ | 1/2 | 1 | -1/2 | 0 | | | 3/2 | - | |

Thus, the solution is

$$x_1 = 0, x_2 = \frac{3}{2}, z'_{max} = -\frac{9}{2}, \therefore\ z_{min} = \frac{9}{2}$$

3. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.
   Maximise $z = 3x_1 - x_2$
   subject to $2x_1 + x_2 \geq 2$
   $x_1 + 3x_2 \leq 3$
   $x_2 \leq 4$
   $x_1, x_2 \geq 0$ Ans.$x_1 = 3, x_2 = 0, z_{max} = 9$
4. Use Penalty method (Big M) to solve
   Minimize $z = 4x + y$
   Subject to $3x + y = 3$
   $4x + 3y \geq 6$
   $x + 2y \leq 4$
   $x, y \geq 0$ Ans. $x = \frac{2}{5}, y = \frac{9}{5}, z_{min} = \frac{17}{5}$
5. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.
   Maximise $z = x_1 + 4x_2$
   subject to $3x_1 + x_2 \leq 3$
   $2x_1 + 3x_2 \leq 6$
   $4x_1 + 5x_2 \geq 20$
   $x_1, x_2 \geq 0$ Ans.No solution

6. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 2x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

**[N17/CompIT/8M][M19/AutoMechCivil/6M]**

**Solution:**

The standard form,

Max $z' = -z = -2x_1 - x_2$

Max $z' + 2x_1 + x_2 + 0s_2 + 0s_3 + MA_1 + MA_2 = 0$ ……(1)

s.t. $3x_1 + x_2 + A_1 = 3$ ……(2)

$4x_1 + 3x_2 - s_2 + A_2 = 6$ ……(3)

$x_1 + 2x_2 + s_3 = 3$

Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

$z' + (2 - 7M)x_1 + (1 - 4M)x_2 + Ms_2 + 0s_3 + 0A_1 + 0A_2 = -9M$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | | | |
| 0 | $z'$ | **2-7M** | 1-4M | M | 0 | 0 | 0 | -9M | - | $X - \frac{(2-7M)}{3}Y$ |
| $A_1$ leaves $x_1$ enters | $A_1$ | **3** | **1** | **0** | **0** | **1** | **0** | **3** | $\frac{3}{3} = 1$ | $\frac{Y}{3}$ |
| | $A_2$ | **4** | 3 | -1 | 0 | 0 | 1 | 6 | $\frac{6}{4} = 1.5$ | $X - \frac{4}{3}Y$ |
| | $s_3$ | **1** | 2 | 0 | 1 | 0 | 0 | 3 | $\frac{3}{1} = 3$ | $X - \frac{1}{3}Y$ |
| | | | | | | | | | | |
| 1 | $z'$ | 0 | $\frac{1-5M}{3}$ | M | 0 | | 0 | -2-2M | - | $X - \frac{1-5M}{5}Y$ |
| $A_2$ leaves $x_2$ enters | $x_1$ | 1 | $\frac{1}{3}$ | 0 | 0 | | 0 | 1 | 3 | $X - \frac{1}{5}Y$ |
| | $A_2$ | **0** | $\frac{5}{3}$ | **-1** | **0** | | **1** | **2** | 1.2 | $\frac{3}{5}Y$ |
| | $s_3$ | 0 | $\frac{5}{3}$ | 0 | 1 | | 0 | 2 | 1.2 | $X - Y$ |
| | | | | | | | | | | |
| 2 | $z'$ | 0 | 0 | $\frac{1}{5}$ | 0 | | | $-\frac{12}{5}$ | | |
| | $x_1$ | 1 | 0 | $\frac{1}{5}$ | 0 | | | $\frac{3}{5}$ | | |
| | $x_2$ | 0 | 1 | $-\frac{3}{5}$ | 0 | | | $\frac{6}{5}$ | | |
| | $s_3$ | 0 | 0 | 1 | 1 | | | 0 | | |

Thus, the solution is

$x_1 = \frac{3}{5}, x_2 = \frac{6}{5}, z'_{max} = -\frac{12}{5}, \therefore\ z_{min} = \frac{12}{5}$

7. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.
Minimise $z = 6x_1 + 4x_2$
subject to $2x_1 + 3x_2 \le 30$
$3x_1 + 2x_2 \le 24$
$x_1 + x_2 \ge 3$
$x_1, x_2 \ge 0$

**[N18/AutoMechCivil/8M]**

**Solution:**

The standard form,

Max $z' = -z = -6x_1 - 4x_2$

Max $z' + 6x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 + MA_3 = 0$ ……(1)

s.t. $2x_1 + 3x_2 + s_1 = 30$
$3x_1 + 2x_2 + s_2 = 24$
$x_1 + x_2 - s_3 + A_3 = 3$ …… (2)

Multiplying eqn (2) by M and subtracting with eqn (1), we get

$z' + (6 - M)x_1 + (4 - M)x_2 + 0s_1 + 0s_2 + Ms_3 + 0A_3 = -3M$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $A_3$ | | | |
| 0 | $z'$ | 6-M | **4-M** | 0 | 0 | M | 0 | -3M | - | $X - (4 - M)Y$ |
| $A_3$ leaves $x_2$ enters | $s_1$ | 2 | **3** | 1 | 0 | 0 | 0 | 30 | $\frac{30}{3} = 10$ | $X - 3Y$ |
| | $s_2$ | 3 | **2** | 0 | 1 | 0 | 0 | 24 | $\frac{24}{2} = 12$ | $X - 2Y$ |
| | $A_3$ | **1** | **1** | **0** | **0** | **-1** | **1** | **3** | $\frac{3}{1} = 3$ | - |
| | | | | | | | | | | |
| 1 | $z'$ | 2 | 0 | 0 | 0 | 4 | | -12 | | |
| | $s_1$ | -1 | 0 | 1 | 0 | 3 | | 21 | | |
| | $s_2$ | 1 | 0 | 0 | 1 | 2 | | 18 | | |
| | $x_2$ | 1 | 1 | 0 | 0 | -1 | | 3 | | |

Thus, the solution is

$x_1 = 0, x_2 = 3, z'_{max} = -12, \therefore\ z_{min} = 12$

8. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.
Minimise $z = x_1 + 2x_2 + x_3$
subject to $x_1 + \frac{x_2}{2} + \frac{x_3}{2} \le 1$
$\frac{3}{2}x_1 + 2x_2 + x_3 \ge 8$
$x_1, x_2, x_3 \ge 0$ Ans. No solution

## Type IV: Dual Simplex Method

1. Use Dual simplex method to solve the following LPP

Minimize $z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

**[M17/CompIT/6M]**

**Solution:**

The standard form,

Min $z = 6x_1 + x_2$

$z - 6x_1 - x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -3$

$-x_1 + x_2 + s_2 = 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | | |
| 0 | $z$ | -6 | **-1** | 0 | 0 | 0 | $X - Y$ |
| $s_1$ leaves | $s_1$ | **-2** | **-1** | **1** | **0** | **-3** | $-Y$ |
| $x_2$ enters | $s_2$ | -1 | **1** | 0 | 1 | 0 | $X + Y$ |
| Ratio | | $\frac{-6}{-2} = 3$ | $\frac{-1}{-1} = 1$ | - | - | - | |
| 1 | $z$ | **-4** | 0 | -1 | 0 | 3 | $X - \frac{4}{3}Y$ |
| $s_2$ leaves | $x_2$ | **2** | 1 | -1 | 0 | 3 | $X + \frac{2}{3}Y$ |
| $x_1$ enters | $s_2$ | **-3** | **0** | **1** | **1** | **-3** | $-\frac{Y}{3}$ |
| Ratio | | $\frac{-4}{-3} = 1.33$ | - | - | - | - | |
| 2 | $z$ | 0 | 0 | $-\frac{7}{3}$ | $-\frac{4}{3}$ | 7 | |
| | $x_2$ | 0 | 1 | $-\frac{1}{3}$ | $\frac{2}{3}$ | 1 | |
| | $x_1$ | 1 | 0 | $-\frac{1}{3}$ | $-\frac{1}{3}$ | 1 | |

The solution is

$x_1 = 1, x_2 = 1, z_{min} = 7$

2. Use Dual simplex method to solve the following LPP

Minimize $z = 6x_1 - x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

**[N19/AutoMechCivil/6M]**

**Solution:**

The standard form,

Min $z = 6x_1 - x_2$

$z - 6x_1 + x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -3$

$-x_1 + x_2 + s_2 = 0$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | | |
| 0 | $z$ | **-6** | 1 | 0 | 0 | 0 | $X - 3Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **-2** | **-1** | **1** | **0** | **-3** | $\frac{Y}{-2}$ |
| | $s_2$ | **-1** | 1 | 0 | 1 | 0 | $X - \frac{1}{2}Y$ |
| Ratio | | $\frac{-6}{-2} = 3$ | – | - | - | - | |
| 1 | $z$ | 0 | 4 | -3 | 0 | 9 | |
| | $x_1$ | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{3}{2}$ | |
| | $s_2$ | 0 | $\frac{3}{2}$ | $-\frac{1}{2}$ | 1 | $\frac{3}{2}$ | |

The solution is

$x_1 = \frac{3}{2}, x_2 = 0, z_{min} = 9$

3. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 6x_1 + 3x_2 + 4x_3$

subject to $x_1 + 6x_2 + x_3 = 10$

$2x_1 + 3x_2 + x_3 = 15$

$x_1, x_2, x_3 \geq 0$

**[N18/Comp/8M]**

**Solution:**

Min $z = 6x_1 + 3x_2 + 4x_3$

s.t. $\quad x_1 + 6x_2 + x_3 \leq 10$

$x_1 + 6x_2 + x_3 \geq 10$ i.e. $-x_1 - 6x_2 - x_3 \leq -10$

$2x_1 + 3x_2 + x_3 \leq 15$

$2x_1 + 3x_2 + x_3 \geq 15$ i.e. $-2x_1 - 3x_2 - x_3 \leq -15$

Standard form:

Min $\quad z = 6x_1 + 3x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

$z - 6x_1 - 3x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

s.t. $\quad x_1 + 6x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0s_4 = 10$

$-x_1 - 6x_2 - x_3 + 0s_1 + s_2 + 0s_3 + 0s_4 = -10$

$2x_1 + 3x_2 + x_3 + 0s_1 + 0s_2 + s_3 + 0s_4 = 15$

$-2x_1 - 3x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + s_4 = -15$

Simplex Table,

| Iteration No. | Basic Var | Coefficient of | | | | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | | |
| 0 | $z$ | -6 | **-3** | -4 | 0 | 0 | 0 | 0 | 0 | $X-Y$ |
| $s_4$ leaves $x_2$ enters | $s_1$ | 1 | **6** | 1 | 1 | 0 | 0 | 0 | 10 | $X+2Y$ |
| | $s_2$ | -1 | **-6** | -1 | 0 | 1 | 0 | 0 | -10 | $X-2Y$ |
| | $s_3$ | 2 | **3** | 1 | 0 | 0 | 1 | 0 | 15 | $X+Y$ |
| | $s_4$ | **-2** | **-3** | **-1** | **0** | **0** | **0** | **1** | **-15** | $\frac{Y}{-3}$ |
| Ratio | | $\frac{-6}{-2}=3$ | $\frac{-3}{-3}=1$ | $\frac{-4}{-1}=4$ | - | - | - | - | - | |
| | | | | | | | | | | |
| 1 | $z$ | **-4** | 0 | -3 | 0 | 0 | 0 | -1 | 15 | $X-\left(\frac{4}{3}\right)Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **-3** | **0** | **-1** | **1** | **0** | **0** | **2** | **-20** | $\frac{Y}{-3}$ |
| | $s_2$ | **3** | 0 | 1 | 0 | 1 | 0 | -2 | 20 | $X+Y$ |
| | $s_3$ | **0** | 0 | 0 | 0 | 0 | 1 | 1 | 0 | - |
| | $x_2$ | **2/3** | 1 | 1/3 | 0 | 0 | 0 | -1/3 | 5 | $X+\frac{2}{9}Y$ |
| Ratio | | $\frac{-4}{-3}=1.33$ | - | $\frac{-3}{-1}=3$ | - | - | - | - | - | |
| | | | | | | | | | | |
| 2 | $z$ | 0 | 0 | -5/3 | -4/3 | 0 | 0 | -11/3 | 125/3 | |
| | $x_1$ | 1 | 0 | 1/3 | -1/3 | 0 | 0 | -2/3 | 20/3 | |
| | $s_2$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | |
| | $s_3$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | |
| | $x_2$ | 0 | 1 | 1/9 | 2/9 | 0 | 0 | 1/9 | 5/9 | |

Thus the solution is $x_1 = \frac{20}{3}, x_2 = \frac{5}{9}, x_3 = 0, z_{min} = \frac{125}{3}$

4. Use Dual simplex method to solve the following LPP

Minimize $z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

**[N18/AutoMechCivil/8M][M19/Comp/8M]**

**Solution:**

The standard form,

Min $z = x_1 + x_2$

$z - x_1 - x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -2$

$x_1 + x_2 + s_2 = -1$

Simplex table,

| Iteration No. | Basic Var | Coefficient of $x_1$ | $x_2$ | $s_1$ | $s_2$ | RHS | Formula |
|---|---|---|---|---|---|---|---|
| 0 | $z$ | **-1** | -1 | 0 | 0 | 0 | $X - \frac{1}{2}Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **-2** | **-1** | **1** | **0** | **-2** | $\frac{Y}{-2}$ |
| | $s_2$ | **1** | 1 | 0 | 1 | -1 | $X + \frac{1}{2}Y$ |
| Ratio | | $\frac{-1}{-2} = \frac{1}{2}$ | $\frac{-1}{-1} = 1$ | - | - | - | |
| 1 | $z$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 1 | |
| $s_2$ leaves $x_1$ enters | $x_2$ | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 1 | |
| | $s_2$ | **0** | $\mathbf{\frac{1}{2}}$ | $\mathbf{\frac{1}{2}}$ | **1** | **-2** | |
| Ratio | | - | - | - | - | - | |

Since, there are no positive ratios obtained, the problem has no solution

5. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 2x_1 + x_2$

subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

**[M18/Comp/8M][M18/AutoMechCivil/8M][N19/Comp/8M]**

**Solution:**

The standard form,

Min $\quad z - 2x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $\quad -3x_1 - x_2 + s_1 + 0s_2 + 0s_3 = -3$

$\quad -4x_1 - 3x_2 + 0s_1 + s_2 + 0s_3 = -6$

$\quad x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | | |
| 0 | $z$ | -2 | **-1** | 0 | 0 | 0 | 0 | $X - \frac{1}{3}Y$ |
| $s_2$ leaves $x_2$ enters | $s_1$ | -3 | **-1** | 1 | 0 | 0 | -3 | $X - \frac{1}{3}Y$ |
| | $s_2$ | **-4** | **-3** | **0** | **1** | **0** | **-6** | $\frac{Y}{-3}$ |
| | $s_3$ | 1 | **2** | 0 | 0 | 1 | 3 | $X + \frac{2}{3}Y$ |
| Ratio | | $\frac{-2}{-4} = \frac{1}{2}$ | $\frac{-1}{-3} = \frac{1}{3}$ | - | - | - | - | - |
| 1 | $z$ | **-2/3** | 0 | 0 | -1/3 | 0 | 2 | $X - \frac{2}{5}Y$ |
| $s_1$ leaves $x_1$ enters | $s_1$ | **-5/3** | **0** | **1** | **-1/3** | **0** | **-1** | $-\frac{3}{5}Y$ |
| | $x_2$ | **4/3** | 1 | 0 | -1/3 | 0 | 2 | $X + \frac{4}{5}Y$ |
| | $s_3$ | **-5/3** | 0 | 0 | 2/3 | 1 | -1 | $X - Y$ |
| Ratio | | $\frac{-\frac{2}{3}}{-\frac{5}{3}} = \frac{2}{5}$ | - | - | $\frac{-\frac{1}{3}}{-\frac{1}{3}} = 1$ | - | - | - |
| 2 | $z$ | 0 | 0 | -2/5 | -1/5 | 0 | 12/5 | |
| | $x_2$ | 1 | 0 | -3/5 | 1/5 | 0 | 3/5 | |
| | $x_1$ | 0 | 1 | 4/5 | -3/5 | 0 | 6/5 | |
| | $s_3$ | 0 | 0 | -1 | 1 | 1 | 0 | |

Thus, the solution is

$x_1 = \frac{6}{5}, x_2 = \frac{3}{5}, z_{min} = \frac{12}{5}$

6. Use Dual simplex method to solve the following LPP

Minimize $z = 20x_1 + 16x_2$

Subject to $x_1 + x_2 \geq 12$

$2x_1 + x_2 \geq 17$

$x_1 \geq 2.5$

$x_2 \geq 6$

$x_1, x_2 \geq 0$ Ans. $x_1 = 5, x_2 = 7, z_{min} = 212$

7. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 2x_1 + 2x_2 + 4x_3$

subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

**[N15/CompIT/8M]**

**Solution:**

The standard form,

Min $z - 2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $-2x_1 - 3x_2 - 5x_3 + s_1 + 0s_2 + 0s_3 = -2$

$3x_1 + x_2 + 7x_3 + 0s_1 + s_2 + 0s_3 = 3$

$x_1 + 4x_2 + 6x_3 + 0s_1 + 0s_2 + s_3 = 5$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | | |
| 0 | $z$ | -2 | **-2** | -4 | 0 | 0 | 0 | 0 | $X - \frac{2}{3}Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **-2** | **-3** | **-5** | **1** | **0** | **0** | **-2** | $\frac{Y}{-3}$ |
| | $s_2$ | 3 | **1** | 7 | 0 | 1 | 0 | 3 | $X + \frac{1}{3}Y$ |
| | $s_3$ | 1 | **4** | 6 | 0 | 0 | 1 | 5 | $X + \frac{4}{3}Y$ |
| Ratio | | $\frac{-2}{-2} = 1$ | $\frac{-2}{-3} = 0.67$ | $\frac{-4}{-5} = 0.8$ | - | - | - | - | |
| | | | | | | | | | |
| 1 | $z$ | -2/3 | 0 | -2/3 | -2/3 | 0 | 0 | 4/3 | |
| | $x_2$ | 2/3 | 1 | 5/3 | -1/3 | 0 | 0 | 2/3 | |
| | $s_2$ | 7/3 | 0 | 16/3 | 1/3 | 1 | 0 | 7/3 | |
| | $s_3$ | -5/3 | 0 | -2/3 | 4/3 | 0 | 1 | 7/3 | |

The solution is, $x_1 = 0, x_2 = \frac{2}{3}, x_3 = 0, z_{min} = \frac{4}{3}$

8. Use Dual simplex method to solve the LPP

Minimise $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to $5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

$x_1, x_2, x_3, x_4 \geq 0$

Ans. $x_1 = 0, x_2 = \frac{30}{11}, x_3 = \frac{16}{11}, x_4 = 0, z_{min} = \frac{258}{11}$

9. Use Dual simplex method to solve the LPP

Minimise $z = 3x_1 + 2x_2 + x_3 + 4x_4$

Subject to $2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$

$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$

$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$

$x_1, x_2, x_3, x_4 \geq 0$

Ans. $x_1 = \frac{65}{23}, x_2 = 0, x_3 = \frac{20}{23}, x_4 = 0, z_{min} = \frac{215}{23}$

10. Use the dual simplex method to solve the following L.P.P.

Maximise $z = -3x_1 - 2x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

**[M16/CompIT/8M]**

**Solution:**

Min $z' = -z = 3x_1 + 2x_2$

$z' - 3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

s.t. $-x_1 - x_2 + s_1 + 0s_2 + 0s_3 + 0s_4 = -1$

$x_1 + x_2 + 0s_1 + s_2 + 0s_3 + 0s_4 = 7$

$-x_1 - 2x_2 + 0s_1 + 0s_2 + s_3 + 0s_4 = -10$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 + s_4 = 3$

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | | |
| 0 | $z'$ | -3 | **-2** | 0 | 0 | 0 | 0 | 0 | $X-Y$ |
| $s_3$ leaves $x_2$ enters | $s_1$ | -1 | **-1** | 1 | 0 | 0 | 0 | -1 | $X-\frac{1}{2}Y$ |
| | $s_2$ | 1 | **1** | 0 | 1 | 0 | 0 | 7 | $X+\frac{1}{2}Y$ |
| | $s_3$ | **-1** | **-2** | **0** | **0** | **1** | **0** | -10 | $\frac{Y}{-2}$ |
| | $s_4$ | 0 | **1** | 0 | 0 | 0 | 1 | 3 | $X+\frac{1}{2}Y$ |
| Ratio | | $\frac{-3}{-1}=3$ | $\frac{-2}{-2}=1$ | - | - | - | - | | |
| | | | | | | | | | |
| 1 | $z$ | **-2** | 0 | 0 | 0 | -1 | 0 | 10 | $X-4Y$ |
| $s_4$ leaves $x_1$ enters | $s_1$ | **-1/2** | 0 | 1 | 0 | -1/2 | 0 | 4 | $X-Y$ |
| | $s_2$ | **1/2** | 0 | 0 | 1 | 1/2 | 0 | 2 | $X+Y$ |
| | $x_2$ | **1/2** | 1 | 0 | 0 | -1/2 | 0 | 5 | $X+Y$ |
| | $s_4$ | **-1/2** | **0** | **0** | **0** | **1/2** | **1** | **-2** | $-2Y$ |
| Ratio | | $\frac{-2}{-\frac{1}{2}}=4$ | - | - | - | - | - | - | |
| | | | | | | | | | |
| 2 | $z'$ | 0 | 0 | 0 | 0 | -3 | -4 | 18 | |
| | $s_1$ | 0 | 0 | 1 | 0 | -1 | -1 | 6 | |
| | $s_2$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | |
| | $x_2$ | 0 | 1 | 0 | 0 | 0 | 1 | 3 | |
| | $x_1$ | 1 | 0 | 0 | 0 | -1 | -2 | 4 | |

Thus the solution is $x_1 = 4, x_2 = 3, z'_{min} = 18, \therefore\ z_{max} = -18$

11. Use dual simplex method, solve

Maximise $\quad z = -2x_1 - x_3$

Subject to $\quad x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

**[M14/CompIT/8M]**

**Solution:**

The standard form,

Min $\quad z' = -z = 2x_1 + x_3$

$z' - 2x_1 + 0x_2 - x_3 + 0s_1 + 0s_2 = 0$

s.t. $\quad -x_1 - x_2 + x_3 + s_1 + 0s_2 = -5$

$-x_1 + 2x_2 - 4x_3 + 0s_1 + s_2 = -8$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | RHS | Formula |
|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | | |
| 0 | $z'$ | -2 | 0 | **-1** | 0 | 0 | 0 | $X-\frac{1}{4}Y$ |
| $s_2$ leaves $x_3$ enters | $s_1$ | -1 | -1 | **1** | 1 | 0 | -5 | $X+\frac{1}{4}Y$ |
| | $s_2$ | **-1** | **2** | **-4** | **0** | **1** | **-8** | $\frac{Y}{-4}$ |
| Ratio | | $\frac{-2}{-1}=2$ | - | $\frac{-1}{-4}=0.25$ | - | - | - | |
| | | | | | | | | |
| 1 | $z'$ | -7/4 | **-1/2** | 0 | 0 | -1/4 | 2 | $X-Y$ |
| $s_1$ leaves $x_2$ enters | $s_1$ | **-5/4** | **-1/2** | **0** | **1** | **1/8** | **-7** | $-2Y$ |
| | $x_3$ | 1/4 | **-1/2** | 1 | 0 | -1/4 | 2 | $X-Y$ |
| Ratio | | $\frac{7}{5}$ | 1 | – | - | - | - | |
| | | | | | | | | |
| 2 | $z'$ | -1/2 | 0 | 0 | -1 | -3/8 | 9 | |
| | $x_2$ | 5/2 | 1 | 0 | -2 | -1/4 | 14 | |
| | $x_3$ | 3/2 | 0 | 1 | -1 | -3/8 | 0 | |

The solution is

$x_1=0, x_2=14, x_3=9, z'_{min}=9, \therefore\ z_{max}=-9$

12. Use the dual simplex method to solve the following L.P.P.

Maximise $z=-2x_1-2x_2-4x_3$

subject to $2x_1+3x_2+5x_3\geq 2$

$3x_1+x_2+7x_3\leq 3$

$x_1+4x_2+6x_3\leq 5$

$x_1,x_2,x_3\geq 0$ Ans. $x_1=0, x_2=\frac{2}{3}, z_{min}=\frac{4}{3}$

## Type V: Duality

1. Write the dual of the following L.P.P.

Maximise $z = 2x_1 - x_2 + 4x_3$

subject to $x_1 + 2x_2 - x_3 \le 5$

$2x_1 - x_2 + x_3 \le 6$

$x_1 + x_2 + 3x_3 \le 10$

$4x_1 + x_3 \le 12$

$x_1, x_2, x_3 \ge 0$

**[N17/CompIT/5M]**

**Solution:**

Primal,

Max $z = 2x_1 - x_2 + 4x_3$

s.t. $x_1 + 2x_2 - x_3 \le 5$

$2x_1 - x_2 + x_3 \le 6$

$x_1 + x_2 + 3x_3 \le 10$

$4x_1 + 0x_2 + x_3 \le 12$

$x_1, x_2, x_3 \ge 0$

Its dual,

Min $w = 5y_1 + 6y_2 + 10y_3 + 12y_4$

s.t. $y_1 + 2y_2 + y_3 + 4y_4 \ge 2$

$2y_1 - y_2 + y_3 + 0y_4 \ge -1$

$-y_1 + y_2 + 3y_3 + y_4 \ge 4$

$y_1, y_2, y_3 \ge 0$

2. Write the dual of the following L.P.P.

Maximise $z = 4x_1 + 9x_2 + 2x_3$

subject to $2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 = 5$

$x_1, x_2, x_3 \ge 0$

**[M19/AutoMechCivil/5M]**

**Solution:**

Primal,

Max $z = 4x_1 + 9x_2 + 2x_3$

s.t. $2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 \le 5$

$3x_1 - 2x_2 + 4x_3 \ge 5$ i.e. $-3x_1 + 2x_2 - 4x_3 \le -5$

$x_1, x_2, x_3 \ge 0$

Its dual,

Min $\quad w = 7y_1 + 5y_2' - 5y_2''$

s.t. $\quad 2y_1 + 3y_2' - 3y_2'' \geq 4$

$3y_1 - 2y_2' + 2y_2'' \geq 9$

$2y_1 + 4y_2' - 4y_2'' \geq 2$

$y_1, y_2', y_2'' \geq 0$

Its dual,

Min $\quad w = 7y_1 + 5y_2$

s.t. $\quad 2y_1 + 3y_2 \geq 4$

$3y_1 - 2y_2 \geq 9$

$2y_1 + 4y_2 \geq 2$

$y_1 \geq 0, y_2$ unrestricted

3. Construct dual of the following LPP:

Maximise $\quad z = 8x_1 + 3x_2$

Subject to $\quad x_1 - 6x_2 \geq 2$

$5x_1 + 7x_2 = -4$

$x_1, x_2 \geq 0$

4. Construct the dual of the following LPP:

Maximize $\quad z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject to $\quad 3x_1 - x_2 + x_3 - 4x_4 = 6$

$5x_1 + 3x_2 - x_3 - 2x_4 = 4$

$x_1, x_2 \geq 0, x_3, x_4$ unrestricted

5. Write the dual of the following L.P.P.

Maximise $\quad z = 2x_1 - x_2 + 3x_3$

subject to $\quad x_1 - 2x_2 + x_3 \geq 4$

$2x_1 + x_3 \leq 10$

$x_1 + x_2 + 3x_3 = 20$

$x_1, x_3 \geq 0, x_2$ unrestricted

**[M15/CompIT/5M]**

**Solution:**

Primal,

Max $\quad z = 2x_1 - x_2' + x_2'' + 3x_3$

s.t. $\quad x_1 - 2x_2' + 2x_2'' + x_3 \geq 4$

$2x_1 + 0x_2' + 0x_2'' + x_3 \leq 10$

$x_1 + x_2' - x_2'' + 3x_3 \geq 20$

$x_1 + x_2' - x_2'' + 3x_3 \leq 20$

$x_1, x_2', x_2'', x_3 \geq 0$

Primal,

Max $z = 2x_1 - x_2' + x_2'' + 3x_3$

s.t. $-x_1 + 2x_2' - 2x_2'' - x_3 \leq -4$

$2x_1 + 0x_2' + 0x_2'' + x_3 \leq 10$

$-x_1 - x_2' + x_2'' - 3x_3 \leq -20$

$x_1 + x_2' - x_2'' + 3x_3 \leq 20$

$x_1, x_2', x_2'', x_3 \geq 0$

Its dual,

Min $w = -4y_1 + 10y_2 - 20y_3' + 20y_3''$

s.t. $-y_1 + 2y_2 - y_3' + y_3'' \geq 2$

$2y_1 + 0y_2 - y_3' + y_3'' \geq -1$

$-2y_1 + 0y_2 + y_3' - y_3'' \geq 1$

$-y_1 + y_2 - 3y_3' + 3y_3'' \geq 3$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Min $w = -4y_1 + 10y_2 - 20y_3$

s.t. $-y_1 + 2y_2 - y_3 \geq 2$

$2y_1 + 0y_2 - y_3 \geq -1$

$-2y_1 + 0y_2 + y_3 \geq 1$

$-y_1 + y_2 - 3y_3 \geq 3$

$y_1, y_2 \geq 0$, $y_3$ unretsricted

Its dual,

Min $w = -4y_1 + 10y_2 - 20y_3$

s.t. $-y_1 + 2y_2 - y_3 \geq 2$

$2y_1 + 0y_2 - y_3 = -1$

$-y_1 + y_2 - 3y_3 \geq 3$

$y_1, y_2 \geq 0$, $y_3$ unretsricted

6. Find dual of the following LP model

Maximise $z = 2x_1 + 3x_2 + 5x_3$

Subject to $x_1 + x_2 - x_3 \geq -5$

$x_1 + x_2 + 4x_3 = 10$

$-6x_1 + 7x_2 - 9x_3 \leq 4$

$x_1, x_2 \geq 0, x_3$ unrestricted

**[M14/CompIT/5M]**

**Solution:**

Primal,

Max $\quad z = 2x_1 + 3x_2 + 5x_3' - 5x_3''$

s.t. $\quad x_1 + x_2 - x_3' + x_3'' \geq -5$

$x_1 + x_2 + 4x_3' - 4x_3'' \leq 10$

$x_1 + x_2 + 4x_3' - 4x_3'' \geq 10$

$-6x_1 + 7x_2 - 9x_3' + 9x_3'' \leq 4$

$x_1, x_2, x_3', x_3'' \geq 0$

Primal,

Max $\quad z = 2x_1 + 3x_2 + 5x_3' - 5x_3''$

s.t. $\quad -x_1 - x_2 + x_3' - x_3'' \leq 5$

$x_1 + x_2 + 4x_3' - 4x_3'' \leq 10$

$-x_1 - x_2 - 4x_3' + 4x_3'' \leq -10$

$-6x_1 + 7x_2 - 9x_3' + 9x_3'' \leq 4$

Its dual,

Min $\quad w = 5y_1 + 10y_2' - 10y_2'' + 4y_3$

s.t. $\quad -y_1 + y_2' - y_2'' - 6y_3 \geq 2$

$-y_1 + y_2' - y_2'' + 7y_3 \geq 3$

$y_1 + 4y_2' - 4y_2'' - 9y_3 \geq 5$

$-y_1 - 4y_2' + 4y_2'' + 9y_3 \geq -5$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Min $\quad w = 5y_1 + 10y_2 + 4y_3$

s.t. $\quad -y_1 + y_2 - 6y_3 \geq 2$

$-y_1 + y_2 + 7y_3 \geq 3$

$y_1 + 4y_2 - 9y_3 \geq 5$

$-y_1 - 4y_2 + 9y_3 \geq -5$

$y_1, y_3 \geq 0, y_2$ is unrestricted

Its dual,

Min $\quad w = 5y_1 + 10y_2 + 4y_3$

s.t. $\quad -y_1 + y_2 - 6y_3 \geq 2$

$-y_1 + y_2 + 7y_3 \geq 3$

$y_1 + 4y_2 - 9y_3 = 5$

$y_1, y_3 \geq 0, y_2$ is unrestricted

7. Construct the dual of the following LPP

Maximise $z = 3x_1 + 17x_2 + 9x_3$

Subject to $x_1 - x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 = 1$

$x_1, x_2 x_3 \geq 0$

**[M17/CompIT/5M]**

**Solution:**

Primal,

Max $z = 3x_1 + 17x_2 + 9x_3$

s.t. $-x_1 + x_2 - x_3 \leq -3$

$-3x_1 + 0x_2 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 \leq 1$

$-2x_1 - x_2 + 5x_3 \leq -1$

$x_1, x_2, x_3 \geq 0$

Its dual,

Min $w = -3y_1 + y_2 + y_3' - y_3''$

s.t. $-y_1 - 3y_2 + 2y_3' - 2y_3'' \geq 3$

$y_1 + 0y_2 + y_3' - y_3'' \geq 17$

$-y_1 + 2y_2 - 5y_3' + 5y_3'' \geq 9$

$y_1, y_2, y_3', y_3'' \geq 0$

Its dual,

Min $w = -3y_1 + y_2 + y_3$

s.t. $-y_1 - 3y_2 + 2y_3 \geq 3$

$y_1 + 0y_2 + y_3 \geq 17$

$-y_1 + 2y_2 - 5y_3 \geq 9$

$y_1, y_2 \geq 0$ and $y_3$ unrestricted

8. Write dual of the given LPP

Minimize $z = 2x_1 + 3x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_3 \geq 0$ and $x_2$ is unrestricted

**[M18/AutoMechCivil/5M]**

**Solution:**

Primal,

Minimize $z = 2x_1 + 3(x_2' - x_2'') + 4x_3$

Subject to $2x_1 + 3(x_2' - x_2'') + 5x_3 \geq 2$

$3x_1 + (x_2' - x_2'') + 7x_3 \geq 3$

$3x_1 + (x_2' - x_2'') + 7x_3 \leq 3$

$x_1 + 4(x_2' - x_2'') + 6x_3 \leq 5$

$x_1, x_3, x_2', x_2'' \geq 0$

Primal,

Minimize $z = 2x_1 + 3x_2' - 3x_2'' + 4x_3$

Subject to $2x_1 + 3x_2' - 3x_2'' + 5x_3 \geq 2$

$3x_1 + x_2' - x_2'' + 7x_3 \geq 3$

$-3x_1 - x_2' + x_2'' - 7x_3 \geq -3$

$-x_1 - 4x_2' + 4x_2'' - 6x_3 \geq -5$

$x_1, x_3, x_2', x_2'' \geq 0$

Its dual,

Maximise $w = 2y_1 + 3y_2' - 3y_2'' - 5y_3$

Subject to $2y_1 + 3y_2' - 3y_2'' - y_3 \leq 2$

$3y_1 + y_2' - y_2'' - 4y_3 \leq 3$

$-3y_1 - y_2' + y_2'' + 4y_3 \leq -3$

$5y_1 + 7y_2' - 7y_2'' - 6y_3 \leq 4$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Maximise $w = 2y_1 + 3y_2 - 5y_3$

Subject to $2y_1 + 3y_2 - y_3 \leq 2$

$3y_1 + y_2 - 4y_3 = 3$

$5y_1 + 7y_2 - 6y_3 \leq 4$

$y_1, y_3 \geq 0$ and $y_2$ is unrestricted

9. Construct dual of the following LPP and solve its dual

Minimise $z = 0.7x_1 + 0.5x_2$

Subject to $x_1 \geq 4$

$x_2 \geq 6$

$x_1 + 2x_2 \geq 20$

$2x_1 + x_2 \geq 18$

$x_1, x_2 \geq 0$

**[M19/AutoMechCivil/8M]**

**Solution:**

Its dual,

Max $z = 4y_1 + 6y_2 + 20y_3 + 18y_4$

s.t. $y_1 + 0y_2 + y_3 + 2y_4 \le 0.7$

$0y_1 + y_2 + 2y_3 + y_4 \le 0.5$

$y_1, y_2 \ge 0$

Standard form,

Max $z = 4y_1 + 6y_2 + 20y_3 + 18y_4 + 0s_1 + 0s_2$

$z - 4y_1 - 6y_2 - 20y_3 - 18y_4 + 0s_1 + 0s_2 = 0$

s.t. $y_1 + 0y_2 + y_3 + 2y_4 + s_1 + 0s_2 = 0.7$

$0y_1 + y_2 + 2y_3 + y_4 + 0s_1 + s_2 = 0.5$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $s_1$ | $s_2$ | | | |
| 0 | $z$ | -4 | -6 | **-20** | -18 | 0 | 0 | 0 | | $X + 10Y$ |
| $s_2$ leaves $y_3$ enters | $s_1$ | 1 | 0 | **1** | 2 | 1 | 0 | 0.7 | $\frac{0.7}{1} = 0.7$ | $X - \frac{1}{2}Y$ |
| | $s_2$ | **0** | **1** | **2** | **1** | **0** | **1** | **0.5** | $\frac{0.5}{2} = 0.25$ | $\frac{Y}{2}$ |
| | | | | | | | | | | |
| 1 | $z$ | -4 | 4 | 0 | **-8** | 0 | 10 | 5 | | $X + \frac{8}{1.5}Y$ |
| $s_1$ leaves $y_4$ enters | $s_1$ | **1** | $\mathbf{-0.5}$ | **0** | **1.5** | **1** | $\mathbf{-0.5}$ | **0.45** | $\frac{0.45}{1.5} = 0.3$ | $\frac{Y}{1.5}$ |
| | $y_3$ | 0 | 0.5 | 1 | **0.5** | 0 | 0.5 | 0.25 | $\frac{0.25}{0.5} = 0.5$ | $X - \frac{0.5}{1.5}Y$ |
| | | | | | | | | | | |
| 2 | $z$ | $\frac{4}{3}$ | $\frac{4}{3}$ | 0 | 0 | $\frac{16}{3}$ | $\frac{22}{3}$ | 7.4 | | |
| | $y_4$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | 1 | $\frac{2}{3}$ | $-\frac{1}{3}$ | 0.3 | | |
| | $y_3$ | $-\frac{1}{3}$ | $\frac{2}{3}$ | 1 | 0 | $-\frac{1}{3}$ | $\frac{2}{3}$ | 0.1 | | |

Thus, the solution is

$x_1 = \frac{16}{3}, x_2 = \frac{22}{3}, z_{min} = 7.4$

10. Using Duality solve the following L.P.P.

Minimize $z = 4x_1 + 3x_2 + 6x_3$

subject to $x_1 + x_3 \ge 2$

$x_2 + x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$ Ans. $x_1 = 0, x_2 = 3, x_3 = 2, z_{min} = 21$

11. Using Duality to solve
Minimise $z = 4x_1 + 14x_2 + 3x_3$
Subject to $x_1 - 3x_2 - x_3 \le -3$
$2x_1 + 2x_2 - x_3 \ge 2$
$x_1, x_2, x_3 \ge 0$ Ans. $x_1 = 0, x_2 = 1, x_3 = 0, z_{min} = 14$

12. Using Duality solve the following L.P.P.
Maximise $z = 5x_1 + 8x_2$
subject to $x_1 + x_2 \le 2$
$x_1 + 2x_2 \ge 0$
$-x_1 + 4x_2 \le 1$
$x_1, x_2 \ge 0$ Ans.$x_1 = \frac{7}{5}, x_2 = \frac{3}{5}, z_{max} = \frac{59}{5}$

13. Using Duality to solve
Maximise $z = 3x_1 + 4x_2$
Subject to $x_1 - x_2 \le 1$
$x_1 + x_2 \ge 4$
$x_1 - 3x_2 \le 3$
$x_1, x_2 \ge 0$ Ans. no solution

14. Using Duality to solve
Maximise $z = 3x_1 + 4x_2$
Subject to $2x_1 + x_2 \le 5$
$x_1 + x_2 \le 3$
$x_1, x_2 \ge 0$

**[N19/AutoMechCivil/8M]**

**Solution:**

The standard form,
Max $z = 3x_1 + 4x_2$
s.t. $2x_1 + x_2 \le 5$
$x_1 + x_2 \le 3$

Its dual,
Min $w = 5y_1 + 3y_2$
s.t. $2y_1 + y_2 \ge 3$
$y_1 + y_2 \ge 4$

Max $w' = -w = -5y_1 - 3y_2$
$w' + 5y_1 + 3y_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ .........(1)
s.t. $2y_1 + y_2 - s_1 + 0s_2 + A_1 + 0A_2 = 3$ ............(2)
$y_1 + y_2 + 0s_1 - s_2 + 0A_1 + A_2 = 4$ ..........(3)

Multiplying eqn (2) & (3) by M and subtracting from eqn (1),
$w' + (5 - 3M)y_1 + (3 - 2M)y_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -7M$

Simplex table,

| Iteration No. | Basic Var | Coefficient of | | | | | | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $y_1$ | $y_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | | | |
| 0 | $w'$ | **5–3M** | 3 – 2M | M | M | 0 | 0 | -7M | - | $X-\frac{5-3M}{2}Y$ |
| $A_1$ leaves $y_1$ enters | $A_1$ | **2** | **1** | **-1** | **0** | **1** | **0** | **3** | 1.5 | $\frac{Y}{2}$ |
| | $A_2$ | **1** | 1 | 0 | -1 | 0 | 1 | 4 | 4 | $X-\frac{Y}{2}$ |
| | | | | | | | | | | |
| 1 | $w'$ | 0 | $\frac{1}{2}-\frac{M}{2}$ | $\frac{5}{2}-\frac{M}{2}$ | M | | 0 | $-\frac{15}{2}-\frac{5M}{2}$ | - | $X-(1-M)Y$ |
| $y_1$ leaves $y_2$ enters | $y_1$ | **1** | $\frac{1}{2}$ | $-\frac{1}{2}$ | **0** | | **0** | $\frac{3}{2}$ | 3 | $2Y$ |
| | $A_2$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | -1 | | 1 | $\frac{5}{2}$ | 5 | $X-Y$ |
| | | | | | | | | | | |
| 2 | $w'$ | -1+M | 0 | **3-M** | M | | 0 | -9-M | - | $X-(3-M)Y$ |
| $A_2$ leaves $s_1$ enters | $y_2$ | 2 | 1 | **-1** | 0 | | 0 | 3 | - | $X+Y$ |
| | $A_2$ | **-1** | **0** | **1** | **-1** | | **1** | **1** | 1 | - |
| | | | | | | | | | | |
| 3 | $w'$ | 2 | 0 | 0 | 3 | | | -12 | | |
| | $y_2$ | 1 | 1 | 0 | -1 | | | 4 | | |
| | $s_1$ | -1 | 0 | 1 | -1 | | | 1 | | |

The solution,

$s_1=0, s_2=3, w'_{max}=-12, w_{min}=12$

$\therefore x_1=0, x_2=3, z_{max}=12$

15. Using Duality solve the following L.P.P

Minimise $z=430x_1+460x_2+420x_3$

Subject to $x_1+3x_2+4x_3 \geq 3$

$2x_1+4x_3 \geq 2$

$x_1+2x_2 \geq 5$

$x_1, x_2, x_3 \geq 0$  Ans. $x_1=1, x_2=2, x_3=0, z_{min}=1350$

16. Using Duality solve the following L.P.P.

Maximise $z=2x_1+x_2$

subject to $2x_1-x_2 \leq 2$

$x_1+x_2 \leq 4$

$x_1 \leq 3$

$x_1, x_2 \geq 0$

**[N14/CompIT/6M]**

**Solution:**

The standard form,

Max $z = 2x_1 + x_2$

s.t. $2x_1 - x_2 \le 2$

$x_1 + x_2 \le 4$

$x_1 + 0x_2 \le 3$

Its dual,

Min $w = 2y_1 + 4y_2 + 3y_3$

s.t. $2y_1 + y_2 + y_3 \ge 2$

$-y_1 + y_2 + 0y_3 \ge 1$

Max $w' = -w = -2y_1 - 4y_2 - 3y_3$

$w' + 2y_1 + 4y_2 + 3y_3 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ………(1)

s.t. $2y_1 + y_2 + y_3 - s_1 + 0s_2 + A_1 + 0A_2 = 2$ …………(2)

$-y_1 + y_2 + 0y_3 + 0s_1 - s_2 + 0A_1 + A_2 = 1$ ……….(3)

Multiplying eqn (2) & (3) by M and subtracting from eqn (1),

$w' + (2-M)y_1 + (4-2M)y_2 + (3-M)y_3 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -3M$

Simplex table,

| Iteration No. | Basic Var | Coefficient of $y_1$ | $y_2$ | $y_3$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $w'$ | 2-M | **4-2M** | 3-M | M | M | 0 | 0 | -3M | - | $X-(4-2M)Y$ |
| $A_2$ leaves | $A_1$ | 2 | **1** | 1 | -1 | 0 | 1 | 0 | 2 | 2 | $X-Y$ |
| $y_2$ enters | $A_2$ | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | 1 | 1 | - |
| | | | | | | | | | | | |
| 1 | $w'$ | **6-3M** | 0 | 3-M | M | 4-M | 0 | | -4-M | - | $X-\frac{(6-3M)}{3}Y$ |
| $A_1$ leaves | $A_1$ | **3** | **0** | **1** | **-1** | **1** | **1** | | 1 | 0.33 | $\frac{Y}{3}$ |
| $y_1$ enters | $y_2$ | **-1** | 1 | 0 | 0 | -1 | 0 | | 1 | - | $X+\frac{1}{3}Y$ |
| | | | | | | | | | | | |
| 2 | $w'$ | 0 | 0 | 1 | 2 | 2 | | | -6 | | |
| | $y_1$ | 1/3 | 0 | 1/3 | -1/3 | 1/3 | | | 1/3 | | |
| | $y_2$ | 0 | 1 | 1/3 | -1/3 | -2/3 | | | 4/3 | | |

the solution is

$s_1 = 2, s_2 = 2, w'_{max} = -6, w_{min} = 6$

$\therefore x_1 = 2, x_2 = 2, z_{max} = 6$

17. Using Duality solve the following L.P.P

Maximise $z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \ge 2$

$3x_1 - 4x_2 \le 3$

$x_1 + 3x_3 \le 5$

$x_1, x_2, x_3 \ge 0$

**[M15/CompIT/6M]**

**Solution:**

The standard form,

Max $\quad z = 5x_1 - 2x_2 + 3x_3$

s.t. $\quad -2x_1 - 2x_2 + x_3 \leq -2$

$3x_1 - 4x_2 + 0x_3 \leq 3$

$x_1 + 0x_2 + 3x_3 \leq 5$

Its dual,

Min $\quad w = -2y_1 + 3y_2 + 5y_3$

s.t. $\quad -2y_1 + 3y_2 + y_3 \geq 5$

$-2y_1 - 4y_2 + 0y_3 \geq -2$ i.e. $2y_1 + 4y_2 + 0y_3 \leq 2$

$y_1 + 0y_2 + 3y_3 \geq 3$

Max $\quad w' = -w = 2y_1 - 3y_2 - 5y_3$

$w' - 2y_1 + 3y_2 + 5y_3 + 0s_1 + 0s_2 + 0s_3 + MA_1 + MA_3 = 0$….(1)

s.t. $\quad -2y_1 + 3y_2 + y_3 - s_1 + 0s_2 + 0s_3 + A_1 + 0A_3 = 5$ …………(2)

$2y_1 + 4y_2 + 0y_3 + 0s_1 + s_2 + 0s_3 + 0A_1 + 0A_3 = 2$ ……….(3)

$y_1 + 0y_2 + 3y_3 + 0s_1 + 0s_2 - s_3 + 0A_1 + A_3 = 3$……..(4)

Multiplying eqn (2) & (4) by M and subtracting from eqn (1),

$w' + (-2 + M)y_1 + (3 - 3M)y_2 + (5 - 4M)y_3 + Ms_1 + 0s_2 + Ms_3 + 0A_1 + 0A_3 = -8M$

Simplex table,

| Iteration No. | Basic Var | Coefficient of $y_1$ | $y_2$ | $y_3$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_3$ | RHS | Ratio | Formula |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $w'$ | -2+M | 3-3M | **5-4M** | M | 0 | M | 0 | 0 | -8M | - | $X - \frac{5-4M}{3}Y$ |
| $A_3$ leaves $y_3$ enters | $A_1$ | -2 | 3 | **1** | -1 | 0 | 0 | 1 | 0 | 5 | 5 | $X - \frac{1}{3}Y$ |
| | $s_2$ | 2 | 4 | **0** | 0 | 1 | 0 | 0 | 0 | 2 | - | - |
| | $A_3$ | **1** | **0** | **3** | **0** | **0** | **-1** | **0** | **1** | **3** | 1 | $\frac{Y}{3}$ |
| | | | | | | | | | | | | |
| 1 | $w'$ | $\frac{-11+7M}{3}$ | **3-3M** | 0 | M | 0 | $\frac{5-M}{3}$ | 0 | | -5-4M | - | $X - \frac{3-3M}{4}Y$ |
| $s_2$ leaves $y_2$ enters | $A_1$ | -7/3 | **3** | 0 | -1 | 0 | 1/3 | 1 | | 4 | 1.33 | $X - \frac{3}{4}Y$ |
| | $s_2$ | **2** | **4** | **0** | **0** | **1** | **0** | **0** | | **2** | 0.5 | $\frac{Y}{4}$ |
| | $y_3$ | 1/3 | **0** | 1 | 0 | 0 | -1/3 | 0 | | 1 | - | - |
| | | | | | | | | | | | | |
| 2 | $w'$ | $\frac{-31+23M}{6}$ | 0 | 0 | M | $\frac{-3+3M}{4}$ | $\frac{5-M}{3}$ | 0 | | $\frac{-13-5M}{2}$ | - | $X - (5 - M)Y$ |
| $A_1$ leaves $s_3$ enters | $A_1$ | **-23/6** | **0** | **0** | **-1** | **-3/4** | **1/3** | **1** | | **5/2** | 7.5 | $3Y$ |
| | $s_2$ | 1/2 | 1 | 0 | 0 | 1/4 | **0** | 0 | | 1/2 | - | - |
| | $y_3$ | 1/3 | 0 | 1 | 0 | 0 | **-1/3** | 0 | | 1 | - | $X + Y$ |
| | | | | | | | | | | | | |
| 3 | $w'$ | 14 | 0 | 0 | 5 | 3 | 0 | | | -19 | - | |
| | $s_3$ | -23/2 | 0 | 0 | -3 | -9/4 | 1 | | | 15/2 | - | |
| | $s_2$ | 1/2 | 1 | 0 | 0 | 1/4 | 0 | | | 1/2 | 1 | |
| | $y_3$ | -7/2 | 0 | 1 | -1 | -3/4 | 0 | | | 7/2 | - | |

The solution is

$s_1 = 5, s_2 = 3, s_3 = 0, w'_{max} = -19, w_{min} = 19$

$\therefore x_1 = 5, x_2 = 3, x_3 = 0, z_{max} = 19$