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BRANCH: COMPUTER ENGINEERING

COURSE ID: DJ19CET303
COURSE: DISCRETE STRUCTURE TUTORIAL
TUTORIAL I

Q1 Determine the number of integers between 1 & 250 that are divisible by 2 or 3 or 5 or 7.

Soln A is a set of all integers from 1 to 250 divisible by 2.
B is a set of all integers from 1 to 250 divisible by 3
C is a set of all integers from 1 to 250 divisible by 5.
D is a set of all integers from 1 to 250 divisible by 7.

Now,

$|A| = \frac{250}{2} = 125$ → cardinality of set A

$|B| = \frac{250}{3} = 83$ → cardinality of set B

$|C| = \frac{250}{5} = 50$ → cardinality of set C

$|D| = \frac{250}{7} = 35$ → cardinality of set D

$$|A \cap B| = \frac{250}{2 \times 3} = \frac{250}{6} = 41$$

$$|\cancel{A} C \cap D| = \frac{250}{5 \times 7} = \frac{250}{35} = 7$$

we know, $|A \cup B| = |A| + |B| - |A \cap B|$

By addition principle

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 125 + 83 - 41$$
$$= 167$$

Similarly,

$$|C \cup B| = |C| + |D| - |C \cap D|$$
$$= 50 + 35 - 7$$
$$= 78$$

$$\big||A \cup B| \cup |C \cup D|\big| = |A \cup B| + |C \cup D| - \big||A \cup B| \cap |C \cup D|\big|$$

$$\big||A \cup B| \cap |C \cup D|\big| = \frac{250}{3 \times 2 \times 5 \times 7} = 1$$

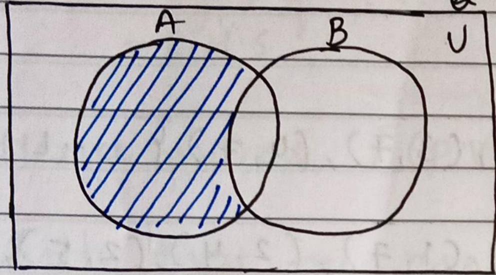
$$\therefore \big||A \cup B| \cup |C \cup D|\big| = 167 + 78 - 1$$
$$= 245 - 1 = 244$$

Q2 Prove the following
$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

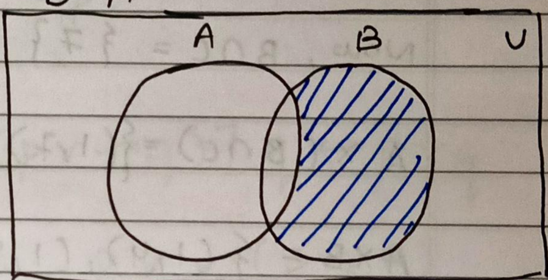
Soln:

L.H.S $\Rightarrow$ $(A-B) \cup (B-A)$

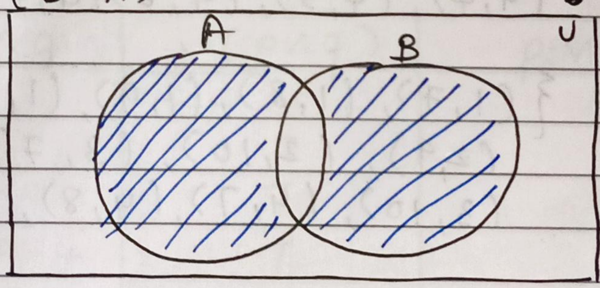
$A-B$



$B-A$

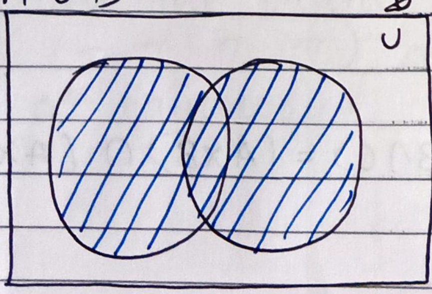


So, $(A-B) \cup (B-A)$

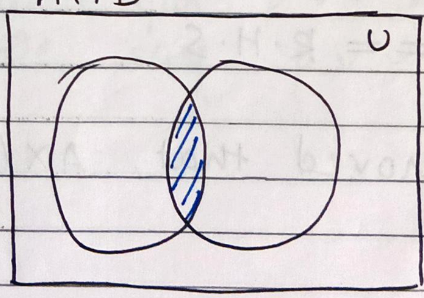


R.H.S $\Rightarrow$ $(A \cup B) - (A \cap B)$

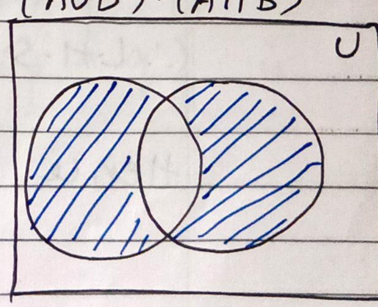
$A \cup B$



$A \cap B$



$(A \cup B)-(A \cap B)$



Hence proved. Since L.H.S == R.H.S
$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

Q3. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Soln: Let the sets be as follows:-

$A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6, 7\}$

$C = \{7, 8, 9, 10\}$

Now, $B \cap C = \{7\}$

$A \times (B \cap C) = \{(1,7), (2,7), (3,7), (4,7)\}$ .... L.H.S

$A \times B = \{(1,4), (1,5), (1,6), (1,7), (2,4), (2,5), (2,6), (2,7), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6) (4,7)\}$

$A \times C = \{(1,7), (1,8), (1,9), (1,10), (2,7), (2,8), (2,9), (2,10), (3,7), (3,8), (3,9), (3,10), (4,7), (4,8), (4,9), (4,10)\}$

then

$(A \times B) \cap (A \times C) = \{(1,7), (2,7), (3,7), (4,7)\}$ .... R.H.S

$\therefore$ L.H.S == R.H.S

Hence proved that, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q4 Prove using Laws of Logic
$[(p \vee q) \wedge (p \vee \sim q)] \vee q \leftrightarrow p \vee q$

Soln $L.H.S = [(p \vee q) \wedge (p \vee \sim q)] \vee q$
$= [p \vee (q \wedge \sim q)] \vee q$ ... By Distribution property
$= [p \vee F] \vee q$ .... ($\because q \wedge \sim q$ is always F)
$= p \vee q$ ... ($\because p \vee F = p$)
$= R.H.S$

Hence proved, $[(p \vee q) \wedge (p \vee \sim q)]\, q \leftrightarrow p \vee q$

Q5 Verify that the preposition $p \vee \sim (p \wedge q)$ is a tautology.

Soln Truth table:

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T
				Tautology

Since the truth table for $p \vee \sim (p \wedge q)$ is (T T T T). We say $p \vee \sim (p \wedge q)$ is a tautology.

Q6 Constract truth table to determine whether each of the following is tautology, contradiction, or contingency.

i) $(q \wedge p) \vee (q \wedge \sim p)$

ii) $q \rightarrow (q \rightarrow p)$

iii) $p \rightarrow (q \wedge p)$

Soln: i) $(q \wedge p) \vee (q \wedge \sim p)$

p	q	$q \wedge p$	$\sim p$	$q \wedge \sim p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

contingency

Since the truth table for $(q \wedge p) \vee (q \wedge \sim p)$ is (T F T F) its contingency.

ii) $q \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$q \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

contingency

Since the truth table for $(q \rightarrow (q \rightarrow p))$ is (T T F T) its contingency.

iii) $p \to (q \wedge p)$

p	q	$q \wedge p$	$p \to (q \wedge p)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Contingency

Since the truth table for $p \to (q \wedge p)$ is (TFTT) it's contingency.

Q7 Write the english sentences for the following
where, $P(x)$: x is even
$Q(x)$: x is prime
$R(x, y)$: $x + y$

i) $\exists x \forall y \ R(x, y)$
ii) $\sim (\exists x \ P(x))$
iii) $\sim (\forall x \ Q(x))$
iv) $\forall x \ (\sim Q(x))$

Soln: i) $\exists x \ \forall y \ R(x, y)$
→ There exist a value of x for all y such that relation $R(x, y)$ where relation $(x+y)$ exists.

ii) $\sim (\exists x \ P(x))$
→ There doesnt exist any value for x for which x is even

iii) $\sim (\forall x \ Q(x))$
→ There doesnt exist any value of x which is prime

iv) $\forall x \ (\sim Q(x))$
→ for all x, x is not a prime

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TUTORIAL II

Q1 Given that the truth values of x, y and z as T, and those of u & v as F, find the truth values of
$(X \wedge (Y \vee Z)) \wedge \sim((X \vee Z) \wedge (U \vee V) \wedge Z)$

Soln:
$X \to T \quad U \to F$
$Y \to T \quad V \to F$
$Z \to T$

$(X \wedge (Y \vee Z)) \wedge \sim((X \vee Z) \wedge (U \vee V) \wedge Z)$
$(T \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$
$(T \wedge T) \wedge \sim(T \wedge F \wedge T)$
$(T) \wedge \sim(F)$
$T \wedge T$
$\boxed{T}$

Q2 Prove using laws of logic
a) $a \to (p \vee c) \Leftrightarrow (a \wedge \sim p) \to c$
b) $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

Soln a) $a \to (p \vee c) \Leftrightarrow (a \wedge \sim p) \to c$

L.H.S : $a \to (p \vee c)$
$\sim a \vee (p \vee c)$ ..... Implicative law.

R.H.S: $\sim(a \wedge \sim p) \vee c$
$(\sim a \vee p) \vee c$
$\sim a \vee (p \vee c)$ .....($\because a \vee (b \vee c) = (a \vee b) \vee c$)

L.H.S == R.H.S, Hence proved

b) $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

L.H.S: $(p \wedge q) \vee (\sim p \vee (\sim p \vee q))$

$(p \wedge q) \vee ((\sim p \vee \sim p) \vee (\sim p \vee q))$

$(p \wedge q) \vee (\sim p \vee q)$

$[(p \vee \sim p) \vee (p \vee q)$

$[(p \vee \sim p) \vee (p \vee q)] \wedge [(q \vee \sim p) \vee (q)]$

$T \wedge [(\sim p \vee q) \vee q]$

$T \wedge [\sim p \vee q]$

$\sim p \vee q$

$= R.H.S$

$\therefore$ L.H.S == R.H.S, Hence proved

Q3 Write English sentence for the following where

$P(x)$: x is even

$Q(x)$: x is prime

$R(x, y)$: $x \cdot y$ is even

Soln: i) $\exists x \ \forall y \ R(x, y)$

There exist a value of x for all values of y for which the relationship $x \cdot y$ is even holds true.

ii) $\forall x \ \exists y \ R(x, y)$

For all values of x there exists values of y for which the relationship $x \cdot y$ is even holds true.

iii) $\sim (\exists x \ P(x))$

It is not true that there exist a value of x for which x is even

iv) $\sim (\forall x \ Q(x))$

It is not true for all values x for with which x is prime

v) $\exists y \, (\sim P(y))$

There exists a value of y for which y is not even

vi) $\forall x \, (\sim Q(x))$

For all values of x, x is not prime.

Q4 Its known that at the university

60% of professors play tennis

50% of professors play bridge

70% of professors jog.

20% Tennis and Bridge

30% Tennis and jog

40% Bridge and jog

If someone claimed that 20% of the professors. jog and play bridge & play tennis. Would you belive the claim? why?

soln: Let the professors playing tennis be represented by set T.

Let the professors playing Bridge be represented by set B

Let the professors who jog be represented by set J

Universal set $|U| = 100 = |T \cup B \cup J|$

$\therefore |T| = 60$ & $|T \cup J| = 30$

$|J| = 70$ $|T \cup B| = 20$

$|B| = 50$ $|B \cup J| = 40$

$|T \cup B \cup J| = |T| + |B| + |J| - |T \cup B| - |B \cup J|$
$- |T \cup J| + |T \cap B \cap J|$

$100 = 60 + 50 + 70 - 20 - 30 - 40 + |T \cap B \cap J|$

$\therefore 100 = 180 - 90 + |T \cap B \cap J|$

$\therefore 100 - 180 + 90 = |T \cap B \cap J|$

$\therefore |T \cap B \cap J| = 10$

we can say, professors playing all three Tennis, jog and bridge are 10%.
So, it cannot be claimed that 20%.
IV professors jog, play ~~pri~~ ~~p~~ bridge, play tennis

Q5 Let $A = \{a, b, c, d, e, f, g, h\}$. Consider the following subsets of A.

$A_1 = \{a, b, c, d\}$
$A_2 = \{a, c, e, f, g, h\}$
$A_3 = \{a, c, e, g\}$
$A_4 = \{b, d\}$
$A_5 = \{f, h\}$

Determine whether each of the following is partition of A or not?

Soln: i) $\{A_1, A_2\}$

$\{A_1, A_2\} = \{(a, b, c, d\}, (a, c, e, f, g, h)\}$
It is not a partition because its not mutually disjoint.

ii) $\{A_1, A_3\}$

$\{A_1, A_3\} = \{(a, b, c, d\}, (a, c, e, g)\}$
Since $A_1 \cup A_3 \neq A$,
Its not a partition

iii) $\{A_3, A_4, A_5\}$

$\{A_3, A_4, A_5\} = \{(a, c, eg), (b, d), (f, h)\}$

Since $(A_3 \cup A_4 \cup A_5) = A$

and $A_3 \cap A_4 \cap A_5 = \phi$

Sets A_3, A_4, A_5 are mutually disjoint and their union is A, hence its a partition.

Q6 Let the universal set be

$U = \{1, 2, 3, \ldots 10\}$

Let $A = \{2, 4, 7, 9\}$, $B = \{1, 4, 6, 7, 10\}$

$C = \{3, 5, 9, 7\}$

Find :

Soln : i) $A \cup B$

$A \cup B = \{1, 2, 4, 6, 7, 9, 10\}$

ii) $A \cap C$

$A \cap C = \{7, 9\}$

iii) $B \cap \overline{C}$

$\overline{C} = \{1, 2, 4, 6, 8, 10\}$

$B \cap \overline{C} = \{1, 4, 6, 10\}$

iv) $(A \cap \overline{B}) \cup C$

$\overline{B} = \{2, 3, 5, 8, 9\}$

$(A \cap \overline{B}) = \{2, 9\}$

$(A \cap \overline{B}) \cup C = \{2, 3, 5, 7, 9\}$

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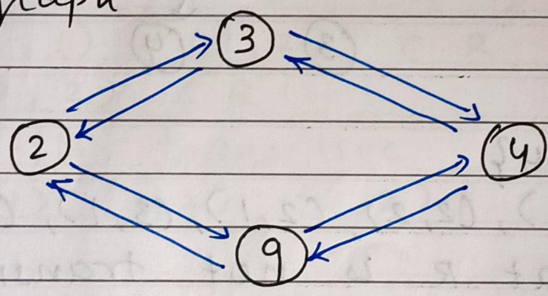
TUTORIAL III

Q1 Given,
$A = \{2, 3, 4, 6, 9\}$
Relation = 'x is relatively prime to y'

Soln: $R = \{(2,3), (2,9), (3,4), (4,9), (9,4), (4,3), (9,2), (3,2)\}$

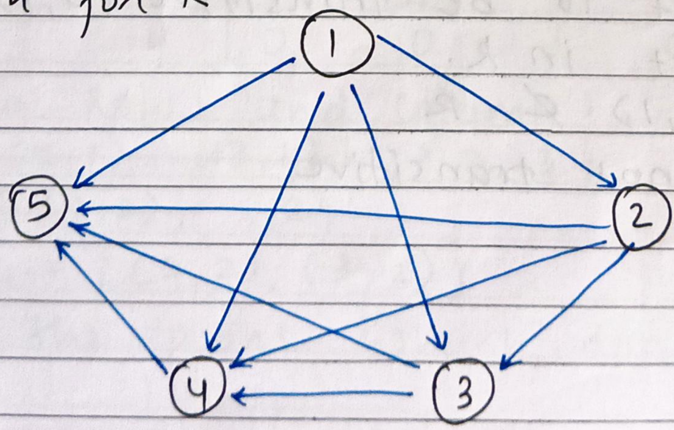
Directed graph



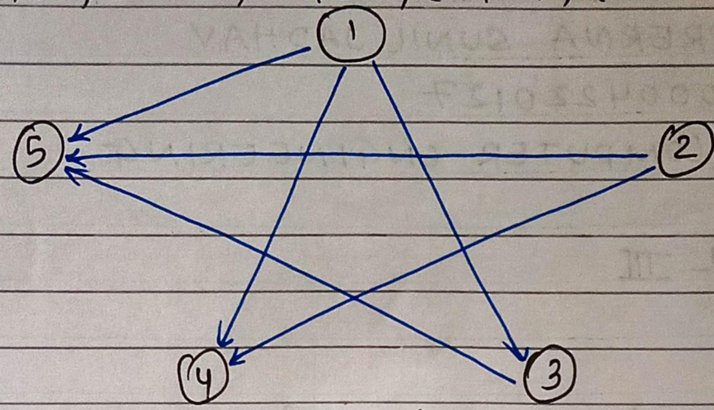
Q2 Given, $A = \{1, 2, 3, 4, 5\}$
To find, R, R^2, R^3

Soln: $R = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

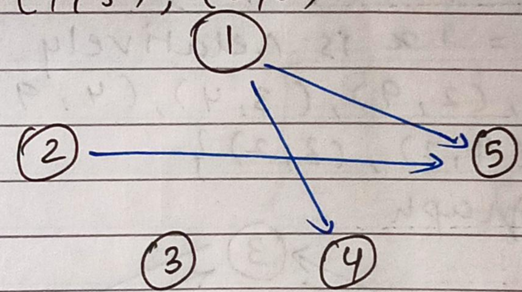
Diagraph for R



$R^2 = \{(1,3),(1,4),(1,5),(2,4),(2,5),(3,5)\}$



$R^3 = \{(1,4),(1,5),(2,5)\}$



Q3 $S = \{1,2,3,4\}$

$R = \{(4,3),(2,2),(2,1),(3,1),(1,2)\}$

(a) Show that R is not transitive.

Soln: For R to be transitive, it must satisfy the condition, That is aRb & bRc then aRc.

Here, $(4,1) \subset R$ should be satisfied.

$\because (4,3),(3,1) \subset R$

For it to be transitive $(4,1)$ should be present in R.

$\therefore (4,1) \not\subset R$

R is not transitive

Q3

(b) Find a Relation $R_1 \subseteq R$ such that R is transitive.

Soln: For a transitive set if aRb & bRc then aRc should be present

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Subset $M_{R_1} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 1 & 1 \end{array}$ is transitive.

since $(2,1), (1,2) \subset R$
& $(2,2) \subset R$
i.e., its transitive

$R_1 = \{(2,1), (1,2), (2,2)\}$

Q3

(c) Transitive closure by Warshalls, $S = \{1,2,3,4\}$
$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$

Soln: Let $W_0 =$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Selecting Row 1 and column 1,

$C_1 = \cancel{\{(2,1), (3,1)\}}\ \{2, 3\}$

$R_1 = \cancel{\{(1,2)\}}\ \{2\}$

$\therefore C_1 \times R_1 = \{(2,2), (3,2)\}$

Adding the pairs which dont exists in W_0.

$$\therefore W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Here, selecting Row 2 and column 2,

$C_2 = \{1, 2, 3\}$

$R_2 = \{1, 2\}$

$C_2 \times R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

Adding these pairs which dont exist in W_1

$$\therefore W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Selecting Row 3 and column 3,

$C_3 = \{4\}$

$R_3 = \{1, 2\}$

$C_3 \times R_3 = \{(4,1), (4,2)\}$

Adding pairs which are not present in W_2

$$\therefore W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

Selecting Row 4 and Column 4

$C_4 = \phi$

$R_4 = \{1, 2, 3\}$

$C_4 \times R_4 = \phi$ $\quad$ $\therefore$ so no new addition.

$\therefore R_T' = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Q4 $A = \{11, 12, 13, 14\}$
$R = \{(11,12), (12,13), (13,14), (12,11)\}$
Find the transitive closure of R using Warshall's algorithm.

Sol'n:

$$W_0 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 0 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

Selecting Row 1 and column 1

$C_1 = \{12\}$
$R_1 = \{12\}$
$\therefore C_1 \times R_1 = \{(12, 12)\}$

$$\therefore W_1 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

Selecting Row 2 and column 2

$C_2 = \{11, 12\}$
$R_2 = \{11, 12, 13\}$
$\therefore C_2 \times R_2 = \{(11,11), (11,12), (11,13), (12,11), (12,12), (12,13)\}$

$$\therefore W_2 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

Selecting Row 3 and ~~P~~ column 3

$C_3 = \{11, 12\}$

$R_3 = \{14\}$

$C_3 \times R_3 = \{(11, 14), (12, 14)\}$

$$\therefore W_3 = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Selecting Row 4 and column 4

$C_{\cancel{3}4} = \{11, 12, 13\}$

$R_4 = \phi$

$\therefore R_4 \times C_4 = \phi$ .... NO new addition.

$\therefore R_T = \{(11,11), (11,12), (11,13), (11,14), (12,11), (12,12), (12,13), (12,14), (13,14)\}$

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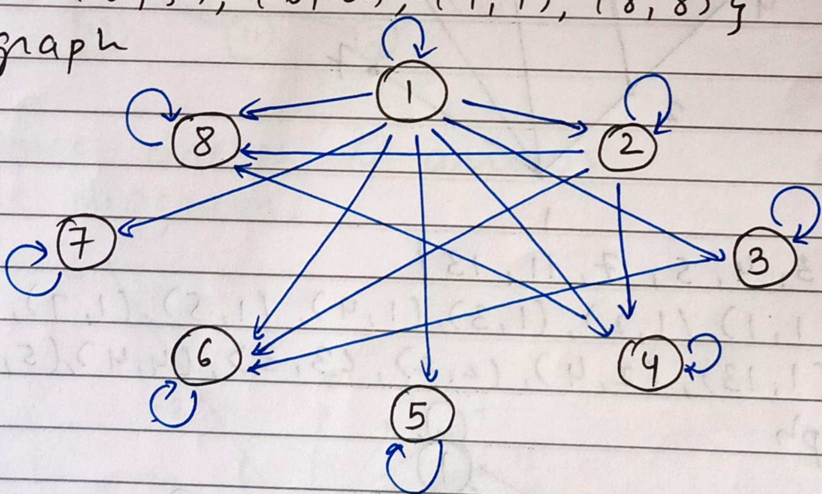
TUTORIAL IV

Q1 Draw the Hasse Diagram for Divisibility set on the set.

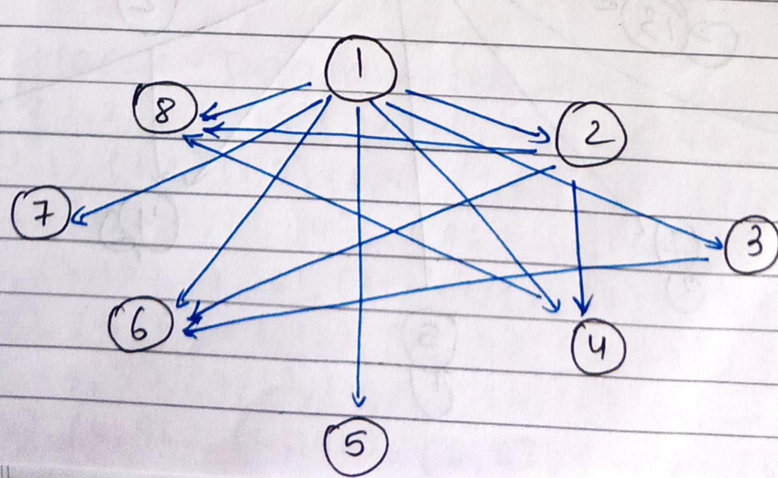
i) {1, 2, 3, 4, 5, 6, 7, 8}

Soln: R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)}

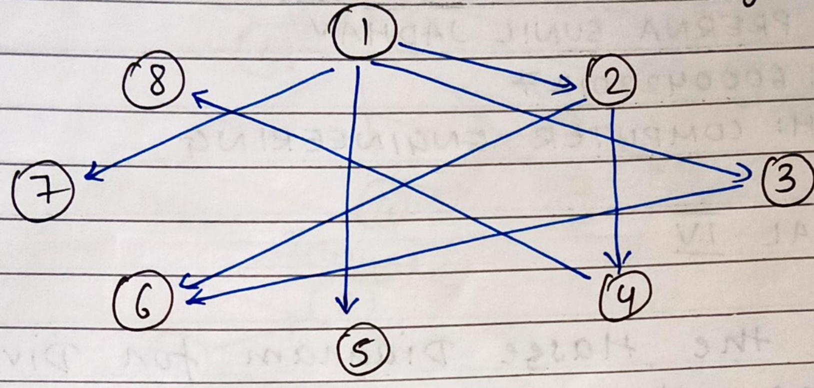
Diagraph



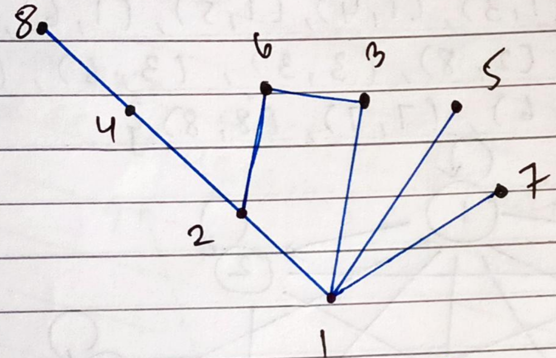
Step 1: Remove the loops



Step 2: Remove the transitive edges.



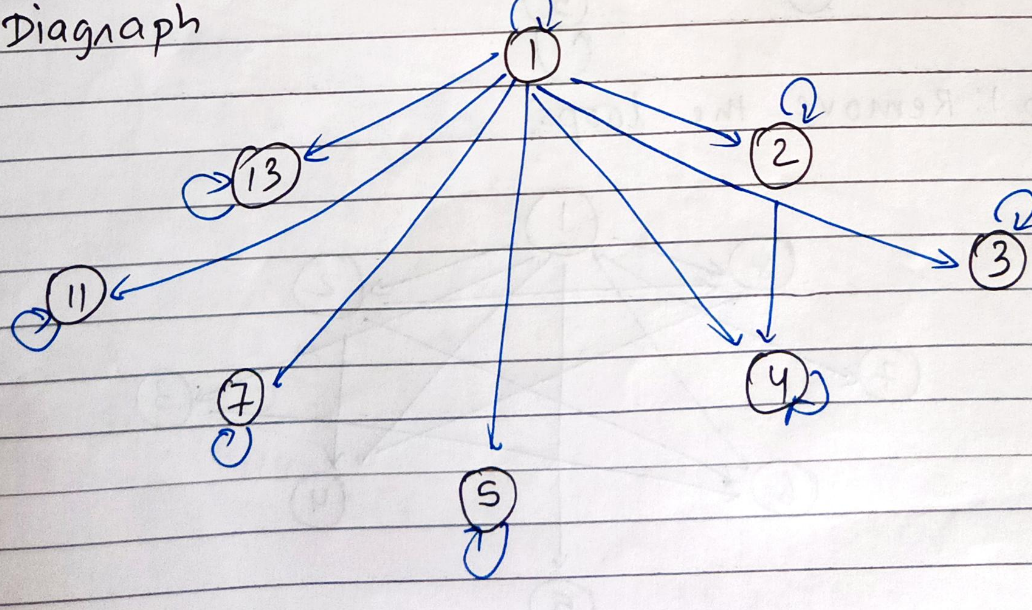
Hasse Diagram



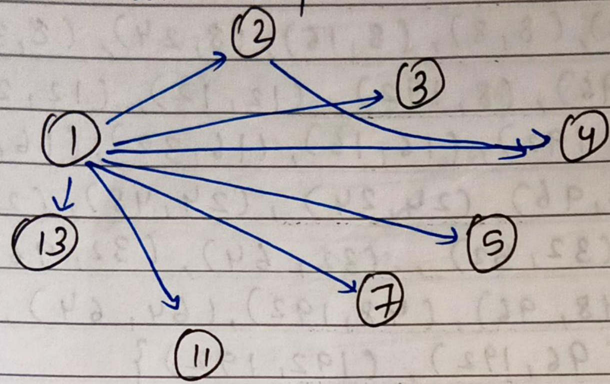
ii) {1, 2, 3, 4, 5, 7, 11, 13}

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (1, 11), (1, 13), (2, 4), (2, 2), (3, 3), (4, 4), (5, 5), (7, 7), (11, 11), (13, 13)}

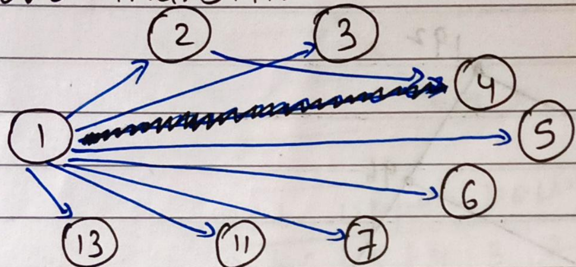
Diagraph



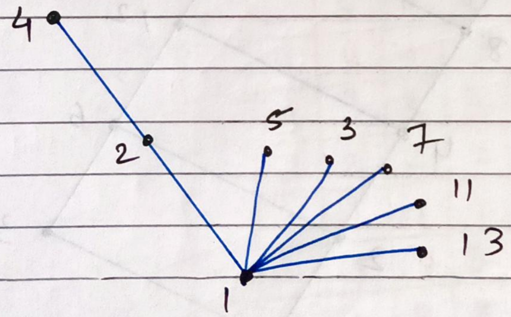
Remove all loops



Remove Transitive



Remove circles & arrows
Hasse diagram.



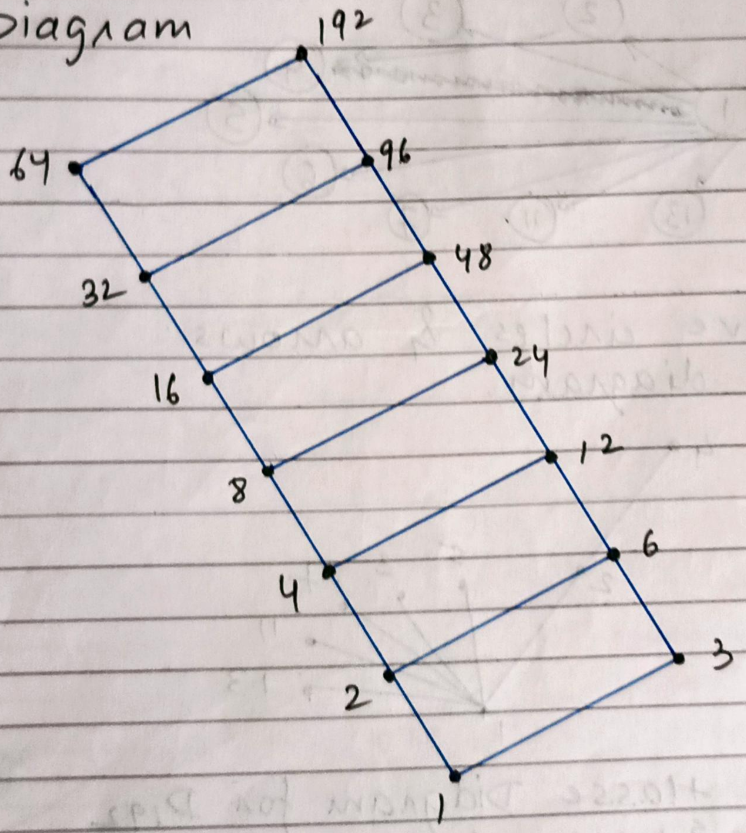
Q2 Draw Hasse Diagram for D_{192}

Soln: $D_{192} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192\}$

$\therefore R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12), (1,16), (1,24),$
$(1,32), (1,48), (1,64), (1,96), (1,192), (2,2), (2,4), (2,6),$
$(2,8), (2,12), (2,16), (2,24), (2,32), (2,48), (2,64), (2,96),$
$(2,192), (3,6), (3,12), (3,24), (3,48), (3,96), (3,192),$
$(4,4), (3,3), (4,8), (4,12), (4,16), (4,24), (4,32), (4,48),$
$(4,64), (4,96), (4,192), (6,6), (6,12), (6,24), (6,48),$

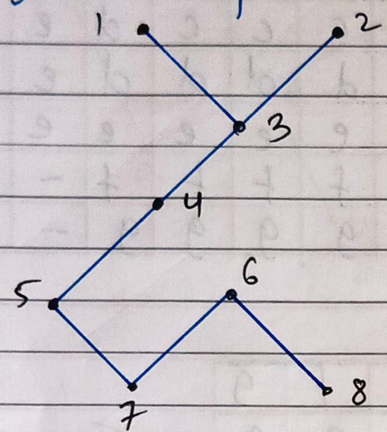
(6,96), (6,192), (8,8), (8,16), (8,24), (8,32), (8,48), (8,64), (8,96), (8,192), (12,12), (12,24), (12,48), (12,96), (12,192), (16,16), (16,32), (16,48), (16,64), (16,96), (24,24), (24,48), (24,96), (24,192), (32,32), (32,64), (32,96), (32,192), (48,48), (48,96), (48,192), (64,64), (64,192), (96,96), (96,192), (192,192)}

Hasse Diagram



Q3 Determine whether the following Hasse diagram represent a lattice or not.

a)



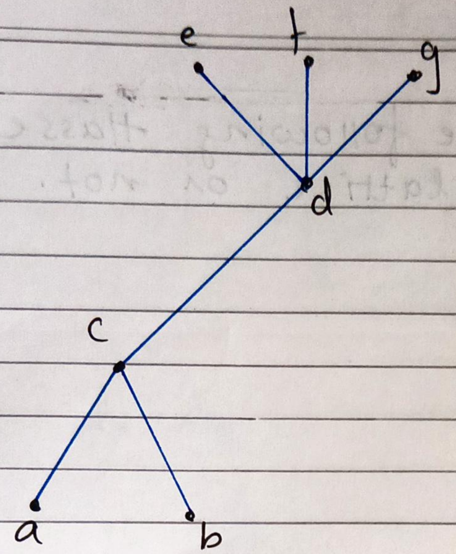
LUB : Least upper bound

∨	8	7	6	5	4	3	2	1
8	8	6	6	5	4	3	2	1
7	6	7	6	5	4	3	2	1
6	6	6	6	-	4	3	2	1
5	5	5	-	5	4	3	2	1
4	4	4	4	4	4	3	2	1
3	3	3	3	3	3	3	2	1
2	2	2	2	2	2	2	2	-
1	1	1	1	1	1	1	-	1

∧GLB	8	7	6	5	4	3	2	1
8	8	-	8	8	8	8	8	8
7	-	7	7	7	7	7	7	7
6	8	7	6	7	7	7	7	7
5	8	7	-	5	5	5	5	5
4	8	7	6	5	4	4	4	4
3	8	7	3	5	4	3	3	3
2	8	7	2	5	4	3	2	-
1	8	7	1	5	4	3	-	1

(2,1), (2,2) have no LUB. So it is not a lattice.

Q3) b)



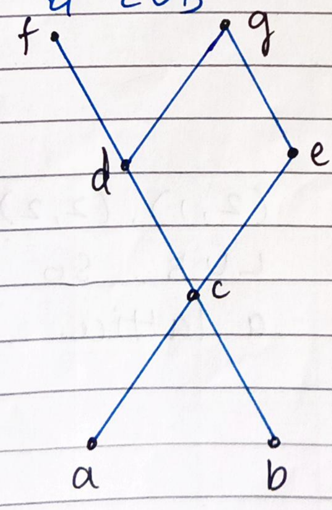
LUB ∨	a	b	c	d	e	f	g
a	a		c	d	e	f	g
b		b	c	d	e	f	g
c	c	c	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	-	-
f	f	f	f	f	-	f	-
g	g	g	g	g	-	-	g

GLB ∧	a	b	c	d	e	f	g
a	a		a	a	a	a	a
b	-	b	b	b	b	b	b
c	a	b	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e	-	-
f	a	b	c	d	-	f	-
g	a	b	c	d	-	-	g

There are NULL values, hence not a lattice.

Q4 Find the upper bounds, lower bound, GLB & LUB

a)



$\Rightarrow \{a, b, c\}$

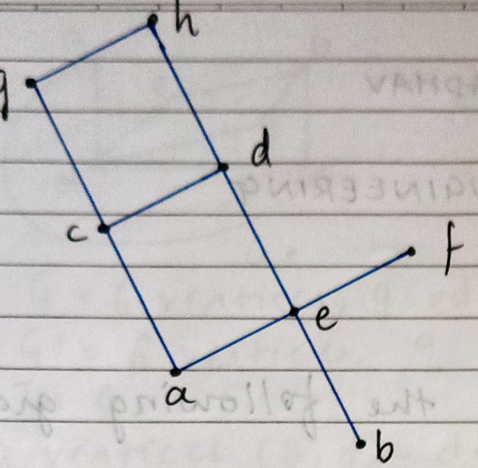
upper bound $= \{d, e, f, g, c\}$

Lower bound $= \{\}$

LUB $= \{C\}$

GLB $= \{\}$

b)



$\Rightarrow \{d, e, f\}$

Upper bound $= \{\ \}$

Lower bound $= \{a, e, b\}$

LUB $= \phi$

GLB $= \{e\}$

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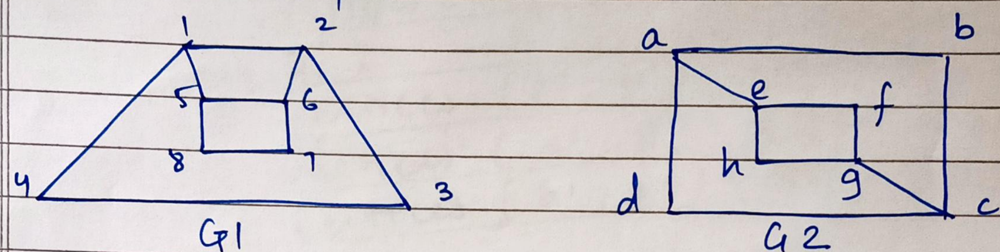
TUTORIAL V

Q1 Determine whether the following graphs are isomorphic

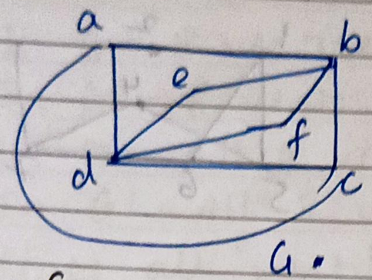
A]

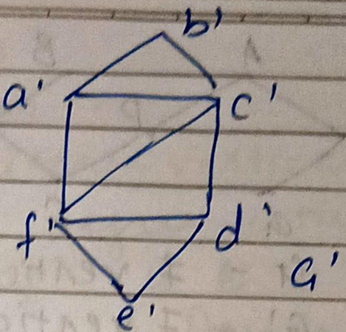


Soln: i) Both the graphs contain 8 vertices & 10 edges
ii) In Graph G1,
Vertices (1, 2, 5, 6) - (4 vertices) have degree 3
& Vertices (8, 7, 3, 4) - (4 vertices) have degree 2
In Graph G2,
Vertices (a, e, g, c) - (4 vertices) have degree 3
& Vertices (b, d, h, f) - (4 vertices) have degree 2
iii) Also vertex 1 in G1 is adjacent to 2 vertices. (2, 5) of degree 3 and 1 vertex of degree 2
Whereas in Graph2 no vertex follows similar connectivity. Hence G1 & G2 are not isomorphic.

Q1

B]





Soln: i> G = 6 vertices, 9 edges
G' = 6 vertices, 9 edges

ii> In G,
2 vertices (b, d) - degree 4
2 vertices (a, c) - degree 3
2 vertices (e, f) - degree 2

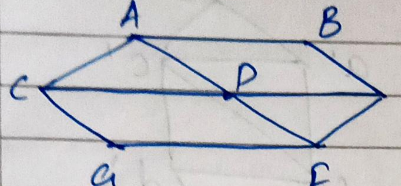
In G',
2 vertices (c', f') - degree 4
2 vertices (a', d') - degree 3
2 vertices (b', e') - degree 2

iii> In G
4 degree vertex (b) - 2 neighbours (2 degree)
(e, b)
- 2 neighbours (3 degree)
(a, c)

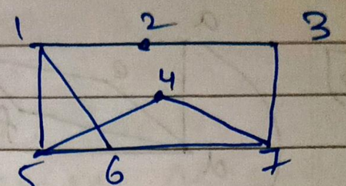
iv> In G'
4 degree vertex (c') - 1 neighour (2 degree)
2 neighbour (3 degree)
1 neighbour (4 degree)

Similar is the case for 2nd 4 degree vertex f'
condition of adjaceny fails.
∴ G & G' are not isomorphic

Q1 c]



G



G'

Soln: i> G - 7 vertices, 9 edges
G' - 7 vertices, 9 edges

ii> In G
1 - 3 degree vertex (D)
3 - 3 degree vertex (A, E, F)
3 - 2 degree vertex (B, C, G)

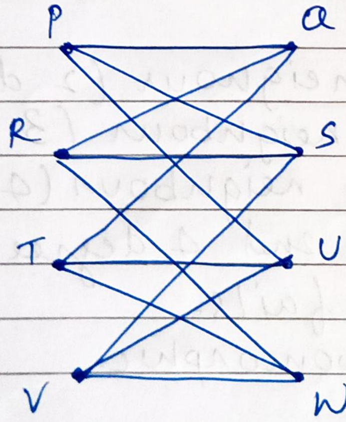
In G'
4 - 3 degree vertices (1, 5, 6, 7)
3 - 2 degree vertices (2, 3, 4)

iii> In G
D has 3 - 3 degree neighbours

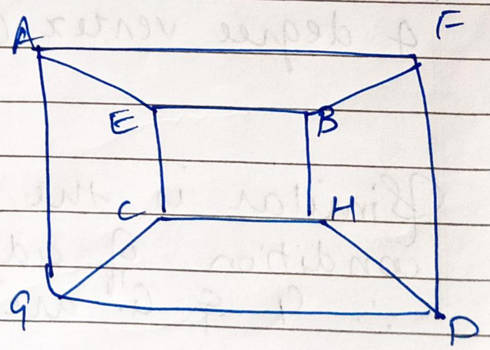
In G'
6 has 3 - 3 degree neighbours

All the vertices are perfectly adjacent hence G & G' are isomorphic

Q1 D]



G



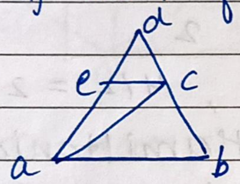
G'

Soln: i) In G - 8 vertices, 12 edges
In G' - 8 vertices, 12 edges

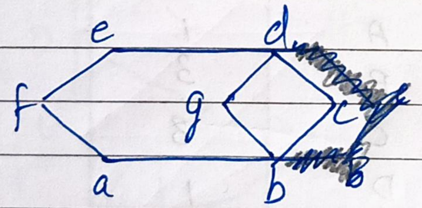
ii) In G - All 8 vertices - 3 degree vertices
In G' - All 8 vertices - 3 degree vertices

Since all vertices in Graph G & G' are 3 degree. Therefore, for 1, 3 Degree vertex we have 3, 3 degree neighbours
As this is true for all vertices so the graphs are isomorphic.

Q2 Determine whether the graphs below have a Hamiltanion circuit, Eulerian circuits. If so, find them.

A] 

If we consider 2 non adjacent vertices e and b and take the sum of ~~theie~~ their degrees $(3 + 2 = 5)$ which is equal to the number of vertices, So Hamiltanion circuit exists. (Acc. to Rule 1)
Hamiltanion circuit :- a, b, c, d, e, a.
Also, Vertex 'e' and 'a' have odd degree so Eulerian circuit doesnt exist.

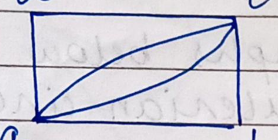
B] 

Vertex	Degree
a	2
b	3
c	2
d	3
e	2
f	2
g	2

As degree of each vertex is less than $7/2 = 3.5$ $\therefore$ Rule 2 fails

There are no two non adjacent vertex such that sum of their degree is greater than no. of vertices. Rule 1 fails
Also the no. of edges $m = 8$,
$1/2\ (7^2 - (3\times7) + 6) = 17$
As no. of edges < 17 ⇒ Rule 3 fails
Hamiltonian circuit doesn't exist.
As b, d, vertices have odd degree, so Euler circuit doesnt exist.

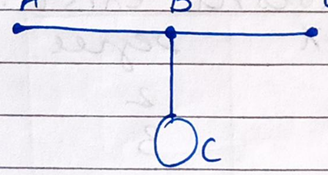
c]



Vertex	Degree
a	4
b	2
c	4
d	2

No. of vertices (n) = 4
As each vertex has degree ≥, 4/2 = 2, so The rule 2 is satisfied. ∴ Hamiltonian circuit exists.
Hamiltanion circuit - a, b, c, d, a
As the above graph is connected & each vertex has an even degree, so euler's circuit exist.

D]

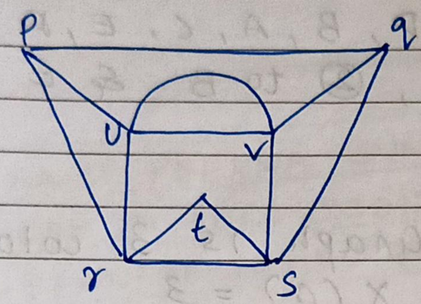


Vertex	Degree
A	1
B	3
C	3
D	1

Visually, we see that is impossible to make a circuit without repeating vertices. So Hamiltanion circuit doesnt exist. Also not all vertices have degree ≥ 2 (A, D) so rule 2 fails & so no ~~hait~~ hamiltanion circuit. As all vertices A, B, C, D have odd degrees so Eulerian circuit not possible.

Q3

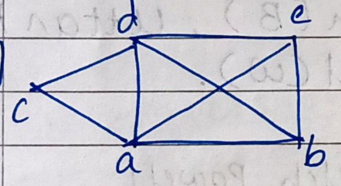
A]



vertex	degree
p	3
q	3
r	4
s	4
t	2
u	4
v	4

It has 2 vertices with odd degree so Eulerian path is present:
p, q, s, r, p, u, v, s, t, s, u, v, q
Hamiltanion : p, q, s, t, r, u, v.

B]

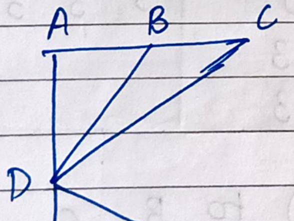


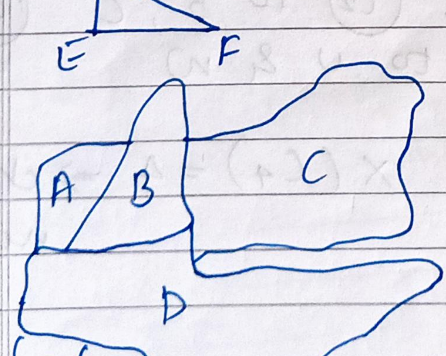
vertex	degree
a	4
b	3
c	2
d	4
e	3

It has 2 vertices of odd degree so
Euler Path: e d c a b d a e f
Hamiltonian: a b c d e

Q4

A]



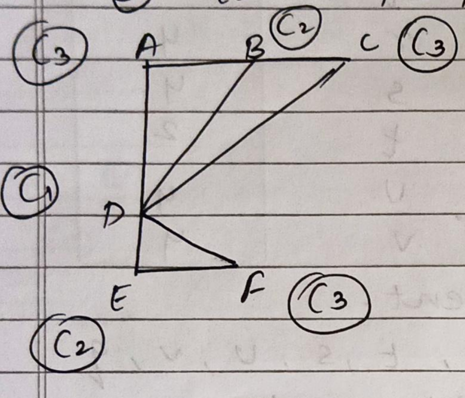


Applying welch Powell

Vertex	Degree
A	2
B	3
C	2
D	5
E	2
F	2

In Descending order D, B, A, C, E, F

Assigning 1 to D, 2 to B & E, 3 to A, C, F

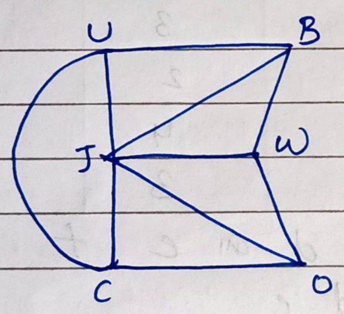


Graph is 3 colorable

$\chi(G) = 3$

chromatic no.

B] Uttar Pradesh (UP), Bihar (B), Uttar khand (J)
Orrissa (O), West Bengal (W).

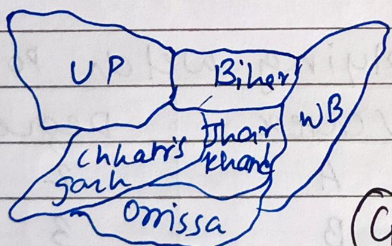


Using Welch Powell

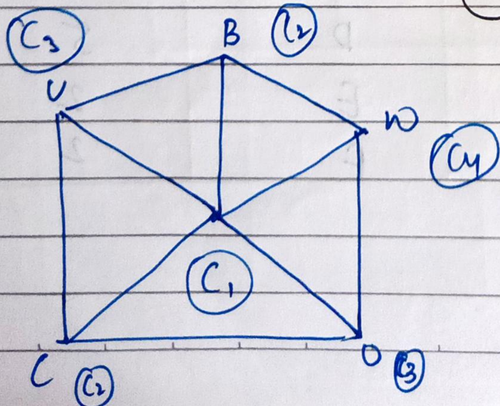
Vertex	Degree
B	3
J	5
U	3
W	3
O	3
C	3

Descending

J	B	C	O	U	W
5	3	3	3	3	3



C1 to J, C2 to B, C C3 to O,
C4 to U & W



$\chi(C_4) = 4 \Rightarrow$ chromatic no.

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TUTORIAL VI

Q1 Find the generating function for the following sequence
1, 2, 3, 4, 5, 6, ...

Soln: Using the sequence 1, 2, 3, 4, 5, 6, ... the above expression becomes (using generating function)

$f(x) = \Sigma a x^n$

$f(x) = 1x^0 + 2x^1 + 3x^2 + 4x^3 + 5x^4 + \dots$ — ①

Multiply eq ① by x

$x f(x) = x + 2x^2 + 3x^3 + 4x^4 \dots$ — ②

Subtracting eq① from eq②

$f(x)[x-1] = [x + 2x^2 + 3x^3 + 4x^4 \dots] - [1 + 2x + 3x^2 + \dots]$

$f(x)[x-1] = -[1 + x + x^2 + \dots]$

$f(x)[1-x] = 1 + x + x^2 + x^3 + \dots$

$= \frac{1}{(1-x)}$ for $|x| \leq 1$

$\therefore f(x) = \frac{1}{(1-x)^2}$ for $|x| \leq 1$

Q2 Solve the recursive relation
$a_r = 3a_{r-1} + 2 \quad r \geq 1$ well $a_0 = 1$ using generating functions.

Soln: consider $g(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_r x^r$

$\therefore 3x\, g(x) = 3a_0 + 3a_1 x + 3a_2 x^2 + \cdots + 3a_{r-1} x^r$

we know, $\frac{2}{1-x} = 2 + 2x + 2x^2 + \cdots 2x^r +$

Subtracting the last 2 series from the 1st 2 noting that $a_0 = 1$, we get

$a(x) - 3x\, g(x) - \frac{2}{1-x} = (1-2) + (a_1 - 3a_0 - 2)x + (a_2 - 3a_1 - 2)x^2 + \cdots$

Since $a_0 = 1$

$\therefore a_r - 3a_{r-1} - 2 = 0$

each bracket on the right except the first is 0

Hence, $(1-3x)\, g(x) = -1 + \frac{2}{1-x} = \frac{-1+x+2}{1-x} = \frac{1+x}{1-x}$

$a(x) = \frac{1+x}{(1-x)(1-3x)} = \frac{2}{1-3x} - \frac{1}{(1-x)}$ By partial fraction

$= 2[1 + (3x) + [3(x)]^2 + \cdots] - [1 + x + x^2 + \cdots]$

$= 2 \sum_{r=0}^{\infty} 3^r a^r - \sum_{r=0}^{\infty} x^r = \sum_{r=0}^{\infty} [2 \times (3^r - 1)] x^r$

But $a(x) = \sum a_r x^2 \Rightarrow a_r = 2 \cdot 3^r - 1$

In order to make use of $a_r - 3a_{r-1} - 2 = 0$ we multiply $a(x)$ by 1, $a(x)$ by $(3x)$ & 2 by $\frac{1}{(1-x)}$ and subtract as above.

Q3 $a_n - 5a_{n-1} + 6a_{n-2} = 2^n$

Q4 Solve using reccurence relation
$a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$. $a_0 = 5$
$a_1 = -9 \quad a_2 = 15$

Soln: $a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$

Solution is $a_n = r^n$

$r^n = -3(r^{n-1} + r^{n-2}) - r^{n-3}$

$r^n = -3r^{n-1} - 3r^{n-2} - r^{n-3}$

$r^n + 3r^{n-1} + 3r^{n-2} + r^{n-3} = 0$

$r^{n-3}[r^3 + 3r^2 + 3r + 1] = 0$

$\therefore r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0$

$\therefore r = (-1)^r$

$\therefore a_n = (C_1 + C_2 n = C_3 n^2)(-1)^n$

But $a_0 = 5 \quad a_1 = -9 \quad a_2 = 15$

we get $C_1 = 5$, $C_2 = 3$, $C_3 = 1$

$\therefore a_n = (5 + 3n + 1n^2)(-1)^n$

Q5

1] $G(x) = \dfrac{1}{3-6x} = \dfrac{1}{3(1-2x)}$

Now $G(x) = 1/3\,(1-2x)^{-1} = 1/3\,(1 + 2x + 4x^2 + 8x^3 + 16x^4 + \cdots)$

Now, sequence

$(5 \times \frac{1}{3} \times (1, 2, 4, 8, 16)$ time

$(1 + x + x^2 + x^3 + \cdots)$

Sequence is $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3} \ldots$

2] $\frac{x}{1-5x+6x^2} = \frac{x}{(1-3x-2x+6x^2)}$

$= \frac{x}{1(1-3x)-2x(1+3x)}$

$= \frac{x}{(1-3x)(1-2x)}$

$= \frac{3x-1-2x+1}{(1-3x)(1-2x)}$

$= \frac{(3x-1)-(2x+1)}{(1-3x)(1-2x)}$

$= \frac{1}{2x-1} - \frac{1}{3x-1}$

$\therefore \frac{x}{1-5x+6x^2} = \frac{-1}{(1-2x)} + \frac{1}{(1-3x)}$

$= (1-3x)^{-1} - (1-2x)^{-1}$

In $(1-x)^{-1}$ put $x = -3x$

we get,

$\frac{x}{(1-5x+6x^2)} = (1+3x+9x^2+27x^3 \ldots) - (1+2x+4x^2+8x^3 \ldots)$

$= x + 5x^2 + 9x^3 + \ldots$

Sequence is $(1, 5, 9 \ldots)$

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TUTORIAL VII

Q1 Using pigeonhole principle show that

i) In any room of people who have been doing some handshaking, there will always be atleast two people who have shaken hands the same number of times.

Soln: Lets assume there are n members in a room. Then the no. of handshakes can be

$0, 1, 2, 3, \ldots, n-1$

But if there is a person with $(n-1)$ handshakes then there is cannot be a person with 0 handshakes.

$\therefore$ There are $(n-1)$ possibility (pigeon holes). and n number (pigeons).

There should be atleast 2 people with same, number of handshakes

ii) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color.

Soln: Red $\rightarrow$ 10, White $\rightarrow$ 10, Blue $\rightarrow$ 10

In the current case scenario after removing 9 balls there are 3 red, 3 blue, 3 white marbles. Then the 10^{th} ball would be the marbles with 4 of same colour. Minimum 10 balls should be removed in order to get 4 balls of same colour.

Q2 There are 100 people at a party. Assume that if person A knows person B, then B knows person A. Prove that there are at least two people at the party who know the same no. of people.

Soln: Total people in the party → 100
So there can be 0, 1, 2, 3, ..., n-1 people known a person.
If a person knows 0 people then there could be a person with (n-1) people knowing him. Therefore, there are (n-1) possibility (pigeon hole). for n (pigeon) ~~people~~.
∴ There must be 2 people knowing same number of people

Q3 A physician testing a new medication instruct a test patient to take 48 pills over a 30-day period. The patient is at liberty to distribute the pills however he likes over this period as long as he takes at least one pill a day and finishes all 48 pills by the end of the 30 days. Prove that no matter how the patient decides to arrange things, there will be some stretch of consecutive days in which the total no. of pills taken is 11.

Soln: Let P_i be the total no. of pills taken by the end of i^{th} days. Now if the patient decides to take one pill each for atleast 11 days, then the condition holds true & in many cases. But considering the patient takes pill in the following sequence

$P_1 < P_2 < P_3 \ldots < P_{30}$ here $P_{30} = 48$

$P_1 < P_2 < P_3 \ldots < P_{30} = 48$ — ①

Adding 11 to each no. of sequence

$P_1+11 < P_2+11 < P_3+11 \ldots < P_{30}+11 = 59$ — ②

There are 30 no.s each in sequence ① & ②

Hence there are 60 no.s (pigeons) all less than or equal to 59 (pigeon holes)

Hence, by pigeon hole principle, atleast 2 of these no.s must be equal (which violates the condition of strictly increasing sequence). Hence there is one such

$P_i = P_j + 11$ $(i > j)$. $\therefore P_i - P_j = 11$ ... which implies that there are exactly 11 pills taken in the consecutive days $j+1, j+2, \ldots i$

Hence its proved that there will always be stretch of consecutive day in which the total no. of pills taken is 11.

Q4 Use the mathematical induction to show that:

i) $1 + 5 + 9 + \ldots + (4n-3) = n(2n-1)$

Soln: Let $P(n)$ be the ~~peredicate~~ predicate

$n_0 = 1$

$\therefore$ Show that $P(1)$ is true

$\therefore P(1) = 1(2(1)-1) = 1$ $\therefore$ Its true

Induction step

Now for $\forall k \geq 1$ and if $P(k)$ is true then $P(k+1)$ should also be true.

$P(k) = 1+5+9+\dots+4k-3 = k(2k-1)$

$\therefore P(k+1) = (k+1)(2(k+1)-1)$

$= (k+1)[2k+2-1)$

$= (k+1)[2k+1]$ —— ②

$P(k+1) = \underbrace{1+5+9+\dots\dots+4k-3}_{P(k)} + 4(k+1)-3$

$= k(2k-1) + 4(k+1) - 3$

$= k(2k-1) + 4k+4-3$

$= (2k-1)k + (4k+1)$

$= 2k^2 - k + 4k + 1$

$= 2k^2 + 3k + 1$

$= 2k^2 + k + 2k + 1$

$= k(2k+1) + 1(2k+1)$

$= (2k+1)(k+1)$ —— ③

Since R.H.S of ② and ③ are equal

$\therefore P(k+1)$ is also true

$\forall k \geq 1$

By principle of mathematical induction.

ii) $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$

Soln: Let $P(K)$ be the predicate

$n_0 = 1$ here

Base step

$P(n_0) = 2$

$P(n_0) = 1(3(1)+1)/2 = 4/2 = 2$

$\therefore P(n_0) = \text{True}$

Induction step

For all $K > 1$ $P(K+1) \rightarrow P(K)$ should be true.

$\therefore P(K) = 2 + 5 + 8 + \dots + (3K-1) = \frac{K(3K+1)}{2}$ ... (2)

$P(K+1) = \frac{(K+1)(3(K+1)+1)}{2}$

$P(K+1) = \frac{(K+1)(3K+3+1)}{2}$

$P(K+1) = \frac{(K+1)(3K+4)}{2}$ — (3)

Also,

$P(K+1) = \underbrace{2 + 5 + 8 + \dots + (3K-1)}_{P(K)} + 3(K+1) - 1$

$\therefore$ from 2 we can write

$\therefore P(K+1) = \frac{K(3K+1)}{2} + 3(K+1) - 1$

$= \frac{K(3K+1)}{2} + 3K + 3 - 1$

$= \frac{K(3K+1) + 2(3K+2)}{2}$

$= \frac{3K^2 + K + 6K + 4}{2}$

$$= \frac{(k+1)(3k+4)}{2}$$

$$= P(k+1)$$

By Mathematical induction this Predicate is true

Q5 Use Mathematical induction to prove that $1^3 + 2^3 + 3^3 + \dots + n^3 = n^2(n+1)^2/4$ for all positive Integers n. ①

Soln: $n \geq 1$

Base step

Let P(k) be the predicate

$n_0 = 1$

$\therefore$ we must show that $P(1) = 1^3$ — ②

Substitute n = 1 in ①

$$P(1) = \frac{1^2(1+1)^2}{4} = \frac{1(2)^2}{4} = 1 \quad \text{--- ③}$$

from ③ & ② the statement is true

Induction step

Now for $\forall k \geq 1$ & P(k) is true then p(k+1) should also be true.

$P(k) = 1^3 + 2^3 + \dots + k^3 = k^2(k+1)^2/4$ --- ④

$$\therefore P(k+1) = \frac{(k+1)^2[(k+1)+1]}{4}$$

$$= \frac{(k+1)^2[(k+1)+1]}{4}$$

$$= \frac{(k+1)^2(k+2)}{k} \quad \text{--- ⑤}$$

Also,

$$P(K+1) = \underbrace{1^3 + 2^3 + \cdots + k^3}_{P(K)} + (k+1)^3$$

$$= \frac{k^2 (k+1)^2}{4} + (k+1)^3$$

$$= (k+1)^2 \left[\frac{k^2}{4} + (k+1)\right]$$

$$= (k+1)^2 \left[k^2/4 + (k+1)\right]$$

$$= (k+1)^2 \left[k^2 + 4k + 4\right] / 4$$

$$= \frac{(k+1)^2 (k+2)^2}{4} \quad \text{--- (6)}$$

The predicate is true $\forall\, k \geq 1$

By the principle of mathematical induction

Q6 Prove that for any positive integer number n, $n^3 + 2n$ is divisible by 3.

Soln: $n^3 + 2n$ is divisible by 3 $\forall\, n \geq 1$

Base steps

show that it is true for $n = 1$

for $n = 1$ $n^3 + 2n = 1^3 + 2(1) = 1 + 2 = 3$

$\therefore$ at $n = 1$ it is divisible by 3.

for $n = k$

we assume that $k^3 + 2k$ is divisible by 3

$\therefore k^3 + 2k = 3m$

Now for $(k+1)$

$n = k+1$

$$(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + 2(k+1)$$

$$= k^3 + 3k^2 + 3k + 1 + 2k + 2$$

$$= 3m + 3(k^2 + k + 1)$$

$$= 3(m + k^2 + k + 1)$$

By mathematical induction $n^3 + 2n$ is divisible by 3.

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TUTORIAL VIII

Q1 A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	Less than 4000	4000-9000	9000-14000	More than 14000
Frequency	20	210	325	445

If a tyre is bought from this company what is the probability that:

i) It has to be substituted before 4000 km is covered?

Soln: The total no. of trials covered = 100
Frequency of vehicles whose tyres have to be replaced before 4000 km $(E_1) = 20$
Probability that tyre will be substituted before 4000 km $= \frac{E_1}{N} = \frac{20}{1000} = 0.02$

ii) It will last more than 9000 km?

Soln: Frequency of tyre that last more than ~~from~~ 9000km
= Frequency of tyre that last from 9001 to 14000 km + Frequency of tyre that last more than 14000 km
= 325 + 445 (E_2)
= 770

Probability that a tyre will last more than 9000 km $= \frac{E_2}{N} = \frac{770}{1000} = 0.77$

iii) It has to be replaced after 4000 km & 14000 km is covered by it?

Soln: P (tyre has to be replaced after it has covered distance ranging between 4000 and 14000)

$$= \frac{\Sigma \text{ frequency of whose tyre are replaced between 4000 km \& 9000 km} + \text{Frequency of vehicle whose tyre are replaced b/n 9001 to 14000 km}}{N}$$

$$= \frac{210 + 325}{1000} = \frac{535}{1000} = \underline{\underline{0.535}}$$

Q2 The percentage of marks obtained by a student in the monthly test are given below:

Test	1	2	3	4	5
percentage of marks obtained	69	71	73	68	74

Based on the above table, find the probability of students getting more than 70% marks in a test.

Soln: The test in which students get more than 70% are Test 2, Test 3, Test 5

Total no. of Tests conducted (N) = 5

No. of tests in which students score more than 70% marks = Test 2, Test 3, Test 5
= 3 tests

∴ Probability that the student gets more than 70% marks $= \frac{3}{\text{Total no. of tests}} = \frac{3}{5} = 0.6$

Q3 One card is drawn from a deck of 52 cards, well-shuffled. Calculate the probability that the card will be (i) an ace (ii) not an ace

Sol: Total no. of cards in a deck (N) = 52

Total no. of aces in a deck (E) = 4

i) Probability that the card drawn is an ace
$= \frac{\text{no. of aces}}{\text{total cards}} = \frac{4}{52} = \frac{1}{13}$

ii) Probability that the card drawn is not an ace
$= 1 - \left(\text{Probability that the card drawn is an ace}\right) = 1 - \frac{1}{13} = \frac{12}{13}$

Q4 A bag contains 4 balls. Two balls are drawn without replacement and are found to be blue. What is the probability that all the balls in the bag are blue?

Soln: Total balls in the bag = (N) = 4
Of the remaining 2 balls we can have 3 possibilities,
E_1 = Both balls are blue
E_2 = One ball is blue
E_3 = None of the balls are blue
All the events are equally likely and mutually exclusive so there are equal chances that any of the three events will occur

$\therefore P(E_1) + P(E_2) + P(E_3) = 1$

Since equal chances to occur

$\therefore P(E_1) = P(E_2) = P(E_3)$

$\therefore P(E_1) + P(E_2) + P(E_3) = 1$

$\therefore P(E_1) = P(E_2) = P(E_3) = 1/3$

Let A = Event that 2 balls already drawn are blue.

We need to find (E_1/A)

Now $P\left(\frac{A}{E_1}\right) = \frac{\text{Total no. of ways to pick 2 blue balls}}{\text{Total no. of ways to pick 2 balls}}$

$= \frac{{}^4C_2}{{}^4C_2} = 1$ ($\because$ all balls are blue)

$P\left(\frac{A}{E_2}\right) = \frac{\text{Total no of ways to pick 2 blue balls when one blue ball is already picked}}{\text{Total no. of way to pick 2 balls}}$

$$P\left(\frac{A}{E_2}\right) = \frac{3}{6} = \frac{1}{2}$$

$P\left(\frac{A}{E_3}\right)$ = Here we have 2 blue balls

$$= \frac{\text{No. of ways to pick 2 blue balls out of 4}}{\text{No. of ways to pick 2 balls}}$$

$$= \frac{2C2}{4C2} = \frac{1}{6}$$

using Bayes Theorem

$$P\left(\frac{E_1}{A}\right) = \frac{P(A/E_1) \times P(E_1)}{P(A/E_1) \times (P(E_1)) + P(A/E_2)P(E_2) + P(A/E_3)P(E_3)}$$

$$= \frac{1 \times 1/3}{1 \times 1/3 + 1/2 \times 1/3 + 1/6 \times \frac{1}{3}}$$

$$= \frac{1}{1 + 1/2 + 1/6} = \frac{6}{10} = \frac{3}{5} = 0.6$$

Ans: Probability that the balls are blue.
$= 0.6$

Q5 In a neighbourhood, 90% children were falling sick due to flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child developes rashes, find the child's probability of having flu.

Soln: Let A be the doctor finding rash (R)
Let B_1 be the event that child has measles
Let B_2 be the event that child has flu.

S → Sick children

∴ Probability of being sick with flu
$= P(B_2) = 0.9$

∴ Probability of being sick with measles
$= P(B_1) = 0.1$

∴ Probability of finding a rash given that the child has measles $= P\left(\frac{R}{B_1}\right) = 0.95$

∴ Probability of finding a rash given that the child has flu $= P\left(\frac{R}{B_2}\right) = 0.08$

Probability of child having flu given he has rashes $= P\left(\frac{B_2}{A}\right) = \frac{P(A/B_2) \times P(B_2)}{P(A/B_2) \times P(B_2) + P(A/B_1) \times P(B_1)}$

$$= \frac{0.9 \times 0.08}{0.9 \times 0.08 + 0.1 \times 0.95} = \frac{0.072}{0.167} = \frac{72}{167}$$

Q6 It is observed that 50% of mails are spam. There is a software that filters spam mail before reaching inbox. Its accuracy for detecting a spam mail is 99% and chances of tagging a non-spam mail as spam mail is 5%. If a certain mail is tagged as spam find the probability that it is not a spam mail.

Soln: Let E_1 be the event of spam mail
Let E_2 be the event of non-spam mail
A = Event of detecting spam mail
$P(E_1) = 0.5$
$P(E_2) = 0.5$] → given

$\therefore P(A/E_1)$ = Probability of detecting a spam given it is a spam.
$= 0.99$

$P(A/E_2)$ = Probability of detecting a spam given it is not a spam
$= 0.05$

$\therefore$ Probability of event being non spam given a spam mail is detected

$$= P\left(\frac{E_2}{A}\right) = \frac{P(A/E_2) \times P(E_2)}{P(A/E_2) \times P(E_2) + P(A/E_1)\, P(E_1)}$$

$$= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.99 \times 0.5}$$

$$= \frac{0.05}{1.04}$$

$$= \frac{5}{104}$$

$$\boxed{\simeq 0.048}$$

—— x —— x ——