Q.P. Code: 24628

(3 hours) **Total Marks: 80**

N.B.
1. Question No 1 is compulsory
2. Solve any three question out of remaining five questions
3. Assumption made should be clearly stated
4. Figure to the right indicates full marks

1 (a) If function f is an isomorphism from semigroup $(S, *)$ to $(T, *')$, then prove that f^{-1} is an isomorphism from $(T, *')$ to $(S, *)$ **5**

(b) Find the generating function for the following sequence **5**
i) 1, 2, 3, 4, 5, ……
ii) 0,0,0,3,3,3,3,3, ……

(c) Find the truth table of $(p \implies q) \wedge ((q \implies r) \implies p)$ **5**

(d) For $x, y \in Z$, xRy if and only if $2x + 5y$ is divisible by 7. Is R an equivalence relation? **5**

2. (a) The college catering service must decide if the mix of food that is supplied for receptions is appropriate. Of 100 people questioned, 37 say they eat fruits, *33* say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings. How many people surveyed eat cheese? How many do not eat any of the offerings? **8**

(b) Prove by laws of logic (do not use truth table) that the following statement is a tautology **6**

$$((p \implies q) \wedge (q \implies r)) \implies (p \implies r)$$

(c) Let $S = \{1, 2, 3, 4, 5\}$and $A = S \times S$. Define the following relation R on $A(a, b)\ R\ (c, d)$ if and only if $ad = bc$. Show that R is an equivalence relation and compute A/R. **6**

3. (a) Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ where the functions are defined by **8**

$$f_1(x) = x \qquad f_2(x) = 1 - x \qquad f_3(x) = \frac{x}{x-1} \qquad f_4(x) = \frac{1}{x}$$
$$f_5(x) = \frac{1}{1-x} \quad \text{and} \quad f_6(x) = 1 - \frac{1}{x}$$

is a group under composition of two functions. Frame the multiplication table.

(b) Solve $a_n - 7a_{n-2} + 6a_{n-3} = 0$, where $a_0 = 8,\ a_1 = 6$ and $a_2 = 22$ **6**

(c) Consider the lattices $L1 = \{1, 2, 4\},\ L2 = \{1, 3, 9\}$ under divisibility. Draw the lattice $L1 \times L2$. **6**

4. (a) **8**

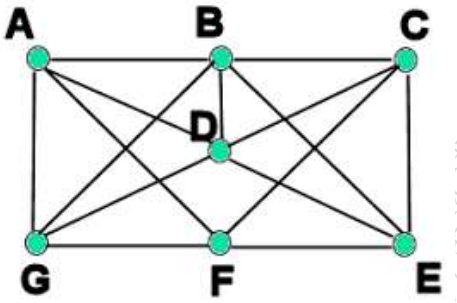
Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be parity check matrix.

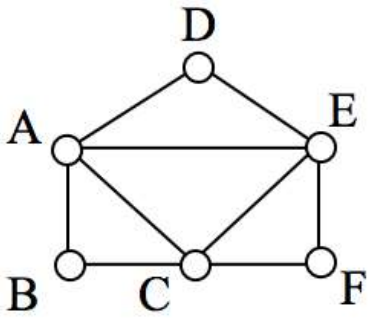
Determine the group code e_H: $B^2 \to B^5$

(b) Prove the following using Mathematical induction $6^{n+2} + 7^{2n+1}$ is divisible by 43 **6**

(c) If 11 numbers are chosen from a set $A = \{1, 2, 3, 4, \ldots\ldots\ldots, 20\}$, then one of them is multiple of the other. **6**

5 (a) Determine the Eulerian and Hamiltonian path/circuit, if any, in the following graphs. **8**

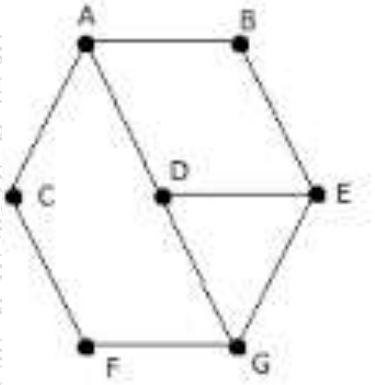


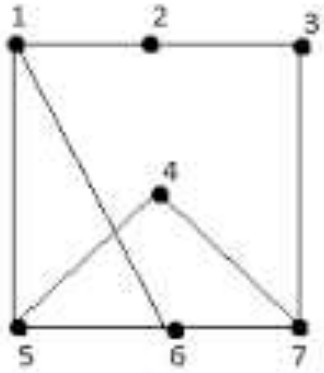


(b) Draw the Hasse diagram of the following poset (i) D_{72} (ii) D_{105} **6**

(c) Let $(G,*)$ is an Abelian group, then for all $a,b \in G$ show that $(a * b)^n = a^n * b^n$ (use mathematical induction) **6**

6. (a) Determine whether following graphs are isomorphic **8**





(b) Solve the recurrence relation : $a_k - 3a_{k-1} = 2$ with initial conditions $a_0 = 1$ using generating function **6**

(c) A connected planar graph has g vertices having degree 2,2,2, 3,3,3, 4,4 & 5. How many edges are there? **6**