Matrix Multiplication by MapReduce.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ 1 & 2 \\ 3 & 4 \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ 5 & 6 \\ 7 & 8 \\ b_{21} & b_{22} \end{bmatrix}$$

$A = i \times j = 2 \times 2$ $\qquad$ $B = j \times k = 2 \times 2$

Resultant (R) $= i, k = i \times k = 2 \times 2$

1 step matrix multiplication has 1 mapper and 1 reducer.

Formula for mapper:-

Matrix A (k, v) = ((i, k), (A, j, A_{ij})) for all k

(key = (i, k), value = (A, j, A_{ij}))

Matrix B (k, v) = ((i, k), (A, j, A_{ij})) for all i

(key = (i, k), value = (A, j, A_{ij}))

matrix A

when a_{11} (i = 1, j = 1) = (i, k) (A, j, A_{ij})

here k is variable and takes all values of k (no. of columns) = 2

i.e k = 1 & k = 2.

k = 1 → (1, 1) (A, 1, A_{11})

= (1, 1) (A, 1, 1)

k = 2 → (1, 2) (A, 1, 1)

a_{12} → i = 1, j = 2, k = 1 & 2.

(i, k)	(A, j, A_{ij})
(1, 1)	(A, 2, 2)
(1, 2)	(A, 2, 2)

a_{21} → $i=2, j=1$ $k=1$ & 2. $a_{21} = 3$.

(2,1) (A, 1, 3)
(2,2) (A, 1, 3)

a_{22} → $i=2, j=2,$ $k=1$ & 2 $a_{22}=4$.

(2,1) (A, 2, 4)
(2,2) (A, 2, 4)

Matrix B ⟹ (i,k) (B, j, B_{ij}) i is variable.

b_{11} → $i=1, k=1, j=1$. $b_{11}=5$

j k
2
$i=2$

(i,k) (B, j, B_{jk})
(1,1) (B, 1, 5)
(2,1) (B, 1, 5)

b_{12} → $j=1, k=2$ $i=1$ & 2. $b_{12}=6$

(1,2) (B, 1, 6)
(2,2) (B, 1, 6)

b_{21} → $j=2, k=1$ $i=1$ & 2. $b_{21}=7$)

(1,1) (B, 2, 7)
(2,1) (B, 2, 7)

b_{22} → $j=2, k=2$ $i=1$ & 2. $b_{22} = 8$.

(1, 2) (B, 2, 8)
(2, 2) (B, 2, 8)

Map common key-pairs.

key	values list A & B.			
(1,1)	(A,1,1)	(A,2,2)	(B,1,5)	(B,2,7)
(1,2)	(A,1,1)	(A,2,2)	(B,1,6)	(B,2,8)
(2,1)	(A,1,3)	(A,2,4)	(B,1,5)	(B,2,7)
(2,2)	(A,1,3)	(A,2,4)	(B,1,6)	(B,2,8)

Reducer :— $(k, v) = (i, k)$

$= (A_{ij} \times B_{jk})$ — Summation.

Reduce (1,1) → (A,1,1) × (B,1,5) + (A,2,2) × (B,2,7)

= 1×5 + 2×7

= 5 + 14 = 19.

Reduce (1,2) → (A,1,1) × (B,1,6) + (A,2,2) × (B,2,8)

= 1×6 + 2×8

= 6 + 16 = 22

Reduce (2,1) → (A,1,3) x (B,1,5) + (A,2,4) x (B,2,7)

= 15 + 28

= 43

Reduce (2,2) → (A,1,3) x (B,1,6) + (A,2,4) x (B,2,8)

= 3x6 + 4x8

= 18 + 32

= 50

Resultant matrix = $\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

Example 2:-

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Resultant = ?

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$i \times j = 3 \times 2$ $\qquad$ $j \times k = 2 \times 2$

Resultant $= i \times k = 3 \times 2$.

Mapper for Mat(A) (K,V) $\Rightarrow$ ((i,k) (A, j, A_{ij}))
for all k.

$a_{11} \rightarrow i=1, j=1 \quad k = 1 \& 2. \quad a_{11} = 1$

(1, 1) (A, 1, 1)
(1, 2) (A, 1, 1)

$a_{12} \rightarrow i=1, j=2 \quad k=1 \& 2 \quad a_{12} = 2$

(1, 1) (A, 2, 2)
(1, 2) (A, 2, 2)

$a_{21} \rightarrow i=2, j=1 \quad k=1 \& 2 \quad a_{21} = 2.$

(2, 1) (A, 1, 2)
(2, 2) (A, 1, 2)

$a_{22} \rightarrow i=2, j=2, \quad k=1 \& 2 \quad a_{22} = 1$

(2, 1) (A, 2, 1)
(2, 2) (A, 2, 1)

a_{31} → $i=3$, $j=1$ $k=1\,2\,2$ $a_{31}=3$.

a_{32} → $i=3, j=2$ $a_{32}=4$

(3,1) (A,1,3)
(3,2) (A,1,3)

(3,1) (A, 2, 4)
(3,2) (A, 2, 4)

Mapper for Mat(B) (K,V) ⟹ ((i,k), (B, j, B_{jk}))
for all i

b_{11} → $j=1$ $k=1$ $i=1\,2\,2\,2\,3$. $b_{11}=1$

(1,1) (B,1,1)
(2,1) (B,1,1)
(3,1) (B,1,1)

b_{12} → $j=1$, $k=2$ $i=1,2,3$. $b_{12}=2$.

(1,2) (B,1,2)
(2,2) (B,1,2)
(3,2) (B,1,2)

b_{21} → $j=2$, $k=1$ $i=1,2,3$ $b_{21}=1$

(1,1) (B,2,1)
(2,1) (B,2,1)
(3,1) (B,2,1)

b_{22} → $j=2$, $k=2$, $i=1,2,3$. $b_{22}=3$.

(1,2) (B,2,3)
(2,2) (B,2,3)
(3,2) (B,2,3)

Map common key-value pairs :-

(1,1) (A,1,1) (A,2,2) (B,1,1) (B,2,1)

(1,2) (A,1,1) (A,2,2) (B,1,2) (B,2,3)

(2,1) (A,1,2) (A,2,1) (B;1,1) (B,2,1)

(2,2) (A,1,2) (A,2,1) (B,1,2) (B,2,3)

(3,1) (A,1,3) (B,1,1) (B,2,1) (A,2,4)

(3,2) (A,1,3) (B,1,2) (B,2,3) (A,2,4)

Reducer → (K,V) = (i,k)
= ($A_{ij} \times B_{jk}$) summation.

(1,1) = (A,1,1) x (B,1,1) + (A,2,2) x (B,2,1)
= 1 + 2 = 3

(1,2) = (A,1,1) x (B,1,2) + (A,2,2) x (B,2,3)
= 2 + 6 = 8

$$(2,1) = (A,1,2) \times (B,1,1) + (A,2,1) \times (B,2,1)$$
$$= 2 + 1 = 3$$

$$(2,2) = (A,1,2) \times (B,1,2) + (A,2,1) \times (B,2,3)$$
$$= 4 + 3 = 7$$

$$(3,1) = (A,1,3) \times (B,1,1) + (A,2,4) \times (B,2,1)$$
$$= 3 + 4 = 7$$

$$(3,2) = (A,1,3) \times (B,1,2) + (A,2,4) \times (B,2,3)$$
$$= 6 + 12 = 18$$

$$\text{Resultant} = \begin{bmatrix} 3 & 8 \\ 3 & 7 \\ 7 & 18 \end{bmatrix}$$