Discrete-Time Signal and Discrete-Time System

Signal which has values at certain instant of time is called as discrete time signal.

Classification of Signals

1. Contineous Time / Discrete Time
2. Perodic / Non-Perodic
3. Deterministic / Random
4. Even / ODD
5. Energy / Power

- **Perodic / Non Perodic:** $x(n) = x(n+N)$ {Perodic}
  $x(n) \neq x(n+N)$ {Non-Perodic}

- **Even / Odd:** $x(n) = x(-n)$ {Even} - cosine wave
  $x(n) = -x(n)$ {Odd} - sine wave

- **Deterministic / Random:** signals which can be described mathematically are called discreate and signals which cannot be described mathematically are called random.

- **Energy / Power:** signal is an energy signal if it has a finite value
  $0 < E < \infty$

  signal is power signal if it has a finite value.
  $0 < P < \infty$

  - Power of energy signal is 0, $P[E] = 0$
  - Energy of power signal is ∞, $E[P] = \infty$

Representation Of Signals

1. Equation / Graph
2. Sequence
3. Table

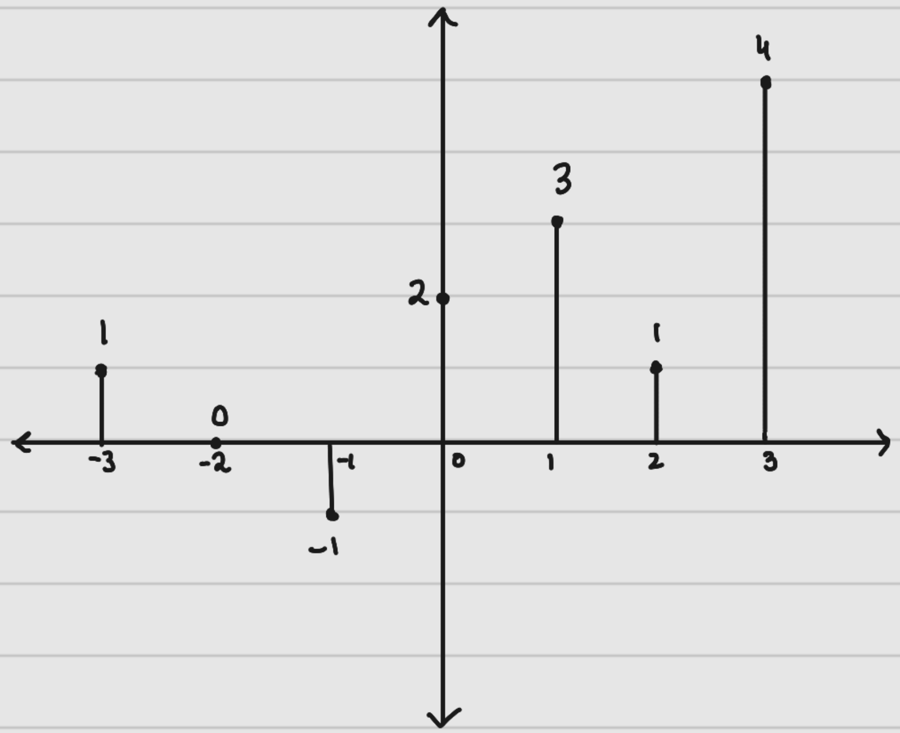
→ **Sequence**: $x(n) = \{1, 2, 3, 4\}$

$n = 0 \quad 1 \quad 2 \quad 3$

If not mentioned, first place is origin.

$x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3, 1, 4\}$

represents origin



→ **Table**:

n	0	1	2	3
$x(n)$	1	2	3	4

Operations On Signals

Operations $\rightarrow$ Dependent
$ \rightarrow$ Independent

- **Dependent Variable:**
  1. Addition
  2. Subtraction
  3. Multiplication
  4. Division
  5. Amplitude Scaling

\# $x_1(n) = \{1, 2, 0, 4\}$, $x_2(n) = \{2, -1, 3, 1\}$

$y(n) = x_1(n) + x_2(n)$, $y(n) = \{3, 1, 3, 5\}$

Q. $x_1(n) = \{3, 2, \underset{\uparrow}{-1}, 0, 1, 2\}$ $\quad x_2(n) = \{1, 2, \underset{\uparrow}{3}, 2\}$

$$y(n) = x_1(n) - x_2(n) = \{2, 0, -4, -2, 1, 2\}$$

\# $x(n) \xrightarrow{\text{'A'}} A \cdot x(n)$

$$x(n) = \{1, 2, 3, 4\}$$
$$3x(n) = \{3, 6, 9, 12\}$$

note: Scaling factor can also be in fraction

- <u>Independent Variable</u>: (Time) → 1. Time shifting
  2. Time scaling
  3. Time reversal

<u>Time shifting</u> : Shifted by K, shifted either left or right by amound K.

$$x(n) \begin{cases} x(n-K) \rightarrow \text{left shift by } K \\ x(n+K) \rightarrow \text{right shift by } K \end{cases}$$

n	-3	-2	-1	0	1	2	3	4	5	6
$x(n)$	2	3	0	-1	2	1	4	0	0	0
$x(n-2)$	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$
value	0	0	2	3 ↑	0	-1	2	1	4	0

The time is shifted by 2

<u>Time scaling</u>:

$$x(n) \longrightarrow x(an) \begin{cases} a>1 \rightarrow \text{compressed} \\ a<1 \rightarrow \text{expanded} \end{cases}$$

$$\begin{array}{r} & -4 & -3 & -2 & -1 & \downarrow 0 & 1 & 2 & 3 & 4 \\ x(n) = \{ & 0, & 0, & 1, & 1, & 1, & 1, & 1, & 1, & 0 \} \\ x(2n) = \{ & 0, & 0, & 0, & 1, & 1, & 1, & 0, & 0, & 0 \} \\ & -8 & -6 & -4 & -2 & \uparrow 0 & 2 & 4 & 6 & 8 \end{array}$$

,

$$\begin{array}{r} & -4 & -3 & -2 & -1 & \downarrow 0 & 1 & 2 & 3 & 4 \\ x(n) = \{ & 0, & 0, & 1, & 1, & 1, & 1, & 1, & 1, & 0 \} \\ x(0.5n) = \{ & 1, & 0, & 1, & 0, & 1, & 0, & 1, & 0, & 1 \} \\ & -2 & -3/2 & -1 & -1/2 & \uparrow 0 & 1/2 & 1 & 3/2 & 2 \end{array}$$

<u>Time reversal</u> (Folding / Flipping): $x(n) \rightarrow x(-n)$

$$x(n) = \{1, 2, \underset{\downarrow}{1}, 2, -1\}$$
$$x(-n) = \{-1, 2, 1, 2, 1\}$$

$$x(n) = \{2, 1, -1, 0, \underset{\uparrow}{3}, 4, 8\} \longrightarrow x(-n) = \{8, 4, \underset{\uparrow}{3}, 0, -1, 1, 2\}$$

* **Operations in Combination**: order to be followed: shift → scale → reverse

$$x(n) = \{2, 0, -3, \underset{\uparrow}{1}, 4, -2, 1, 2\}$$

(i) $x(n-3)$ (ii) $x(n+2)$ (iii) $x(-n-3)$ (iv) $x(-n+2)$

(i) $x(n-3) = \{\underset{\uparrow}{2}, 0, -3, 1, 4, -2, 1, 2\}$

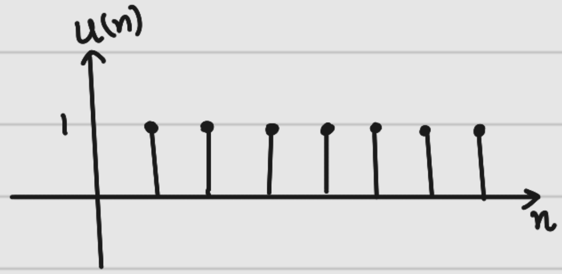
(ii) $x(n+2) = \{2, 0, -3, 1, 4, \underset{\uparrow}{-2}, 1, 2\}$

(iii) $x(-n-3) = \{2, 1, -2, 4, 1, -3, 0, \underset{\uparrow}{2}\}$

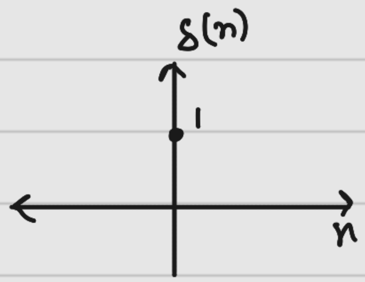
(iv) $x(-n+2) = \{2, 1, \underset{\uparrow}{-2}, 4, 1, -3, 0, 2\}$

• **Basic Signals**

1. **Unit Signal**: $u(n) = 1 \quad ; n \geq 0$
$\qquad = 0 \quad ; \text{else}$

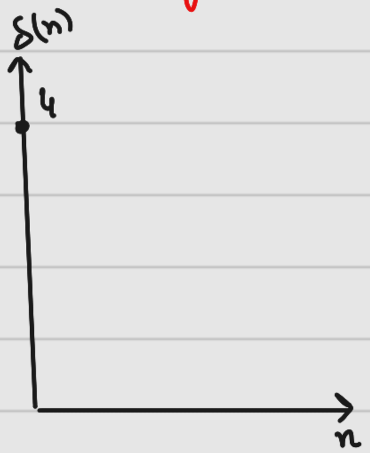


2. **Unit Impulse / DELTA $\delta(n)$**: $\delta(n) = 1 \quad ; n = 0$
$\qquad = 0 \quad ; \text{else}$

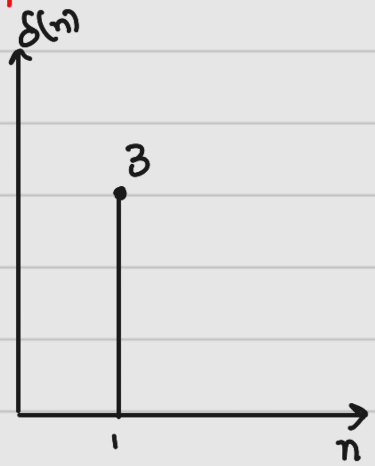


Representation of signal using weighted impulses

$4\delta(n) \rightarrow$



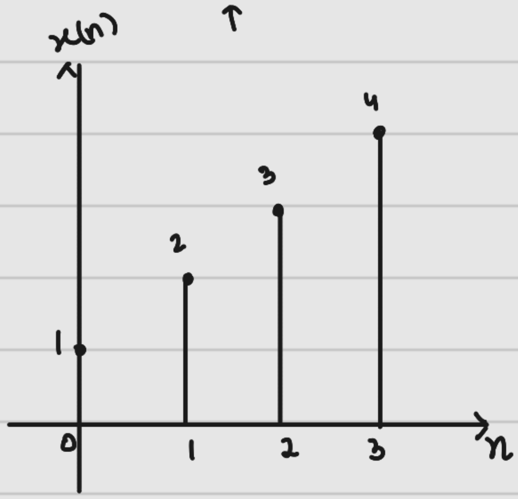
, $3\delta(n-1) \rightarrow$



Consider $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, represent $x(n)$ in $\delta(n)$ form.

$$\Rightarrow x(n) = \underset{\uparrow}{\delta(n)} + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$$

Sketch →



Q sketch given signal $x(n) = 2^{-n}$ for $-2 \leq n \leq 2$, also sketch $y_1(n) = 2x(n) + \delta(n)$ & $y_2(n) = x(n)\,u(2-n)$

$n =$	-2	-1	0	1	2
$x(n) =$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

	-2	-1	0	1	2
(i) $y_1(n) =$	8	4	3	1	$\frac{1}{2}$

(ii) $y_2(n) = x(n)\,u(2-n)$

$u(2-n) = u(-n+2)$

$u(n+2) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$u(-n+2) = \{\ldots 1, 1, \underset{\uparrow}{1}, 1, 1 \ldots\}$

	-2	-1	0	1	2
$\therefore y_2(n) =$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Q $x(n) = \{\overset{-1}{8}, \underset{\uparrow}{\overset{0}{5}}, \overset{1}{2}, \overset{2}{4}, \overset{3}{2}, \overset{4}{1}\}$ (i) $x(n+2)$ (ii) $x(n)u(-n)$ (iii) $x(n-1)u(-n-2)$ (iv) $x(-n-1)u(n)$ (v) $x(2n-1)$

→ (i) $x(n+2) = \{8, 5, 2, \underset{\uparrow}{4}, 2, 1\}$

(ii) $x(n)u(-n) = \{8, \underset{\uparrow}{5}, 2, 4, 2, 1\} \cdot \{\ldots 1, 1, 1, \underset{\uparrow}{1}, 0, 0 \ldots\}$

$= \{8, \underset{\uparrow}{5}, 0, 0, 0, 0\}$

(iii) $x(n-1) = \{\underset{\uparrow}{8}, 5, 2, 4, 2, 1\}$, $u(n-2) = \{\ldots 0, 0, \underset{\uparrow}{0}, 0, 1, 1, 1 \ldots\}$

$u(-n-2) = \{1, 1, 1, 1, 0, \underset{\uparrow}{0}\}$

$x(n-1)u(-n-2) = \{\underset{\uparrow}{0}, 0, 0, 0, 0, 0\}$

(iv) $x(n-1) = \{\underset{\uparrow}{8}, 5, 2, 4, 2, 1\}$

$x(-n-1) = \{1, 2, 4, 2, 5, \underset{\uparrow}{8}\}$

$x(-n-1)u(n) = \{1, 2, 4, 2, 5, \underset{\uparrow}{8}\} \cdot \{0, 0, 0, 0, 0, \underset{\uparrow}{1}\}$

$= \{0, 0, 0, 0, 0, 8\}$

(v) $x(n-1) = \{\underset{\uparrow}{8}, 5, 2, 4, 2, 1\}$

$x(2n-1) = \{\underset{\uparrow}{8}, 2, 2\}$

Condition of Periodicity

Signal is said to be perodic if $x(n) = x(n+N)$,
Condition is given by $f_0 = \frac{k}{N}$, where f_0 is ratio of integers

when signals are added, Then $LCM(N_1, N_2)$ will give time period

Q decide wether $x(n) = \sin\left(\frac{6\pi n}{7} + 1\right)$ is perodic or not.

$\rightarrow$ $$x(n) = \sin(2\pi f_0 n + \theta)$$

$$\therefore\ 2\pi f_0 n = \frac{6\pi n}{7}$$

$f_0 = \frac{3}{7}$, hence f^n is perodic with time period = 7

Q $x(n) = \cos\frac{2\pi n}{5} + \cos\frac{2\pi n}{7}$

$\rightarrow$ $2\pi f_0 n = \frac{2\pi n}{5}$ & $2\pi f_0' n = \frac{2\pi n}{7}$

$f_0 = \frac{1}{5}$, $N = 5$ $f_0' = \frac{1}{7}$, $N = 7$

$$\frac{N_1}{N_2} = \frac{5}{7}$$

time period of $x(n)$ = $LCM(5,7)$
$= \underline{\underline{35}}$

Q (i) $x(n) = \cos\left(\frac{\pi}{8} n^2\right)$

(ii) $x(n) = e^{i7\pi n}$

(iii) $x(n) = 2\cos \frac{5\pi n}{3} + 3e^{i\frac{3n\pi}{4}}$

(iv) $x(n) = \sin 3n$

(v) $x(n) = \cos(0.01\pi n)$

(vi) $x(n) = e^{i\frac{\pi}{4}n}$

(vii) $x(n) = \sin(0.2n + \pi)$

→ (i) $x(n) = \cos\left(\frac{\pi}{8} n^2\right)$

$2\pi f_0 n = \frac{\pi n^2}{8}$, $f_0 = \frac{n}{16}$ ∴ perodic with period 16

(ii) $x(n) = e^{i7\pi n}$

$2\pi f_0 n = 7\pi n$, $f_0 = \frac{7}{2}$ ∴ perodic with period 2

(iii) $x(n) = 2\cos\frac{5\pi n}{3} + 3e^{i\frac{3n\pi}{4}}$

$2\pi f_0 n = \frac{5\pi n}{3}$, $f_0 = \frac{5}{6}$, $N_1 = 6$

$2\pi f_0 n = \frac{3\pi n}{4}$, $f_0 = \frac{3}{8}$, $N_2 = 8$

∴ perodic with period 24

(iv) $x(n) = \sin 3n$

$2\pi f_0 n = 3n$, $f_0 = \frac{3}{2\pi}$, $N = 2\pi$ ∴ not-perodic

(v) $x(n) = \cos(0.01\pi n)$

$2\pi f_0 n = 0.01\pi n$, $f_0 = \frac{1}{200}$, $N = 200$ ∴ perodic with period 200

(vi) $x(n) = e^{i\frac{\pi}{4}n}$

$2\pi f_0 n = \frac{\pi}{4} n$, $f_0 = \frac{1}{8}$ ∴ perodic with period 8

(vii) $x(n) = \sin(0.2n+\pi)$

$2\pi f_0 n = \frac{n}{5}$, $f_0 = \frac{1}{10\pi}$ ∴ not perodic

Condition for Even / Odd

$$x(n) \begin{cases} x_e(n) = \dfrac{x(n)+x(-n)}{2} \\[2ex] x_o(n) = \dfrac{x(n)-x(-n)}{2} \end{cases}$$

Q. Find even & odd part. $x(n) = \{\underset{\uparrow}{4}, 3, 2, 1, 2\}$

→ $x(-n) = \{2, 1, 2, 3, \underset{\uparrow}{4}\}$

$$x_e(n) = \frac{x(n)+x(-n)}{2} = \{1, \tfrac{1}{2}, 1, \tfrac{3}{2}, \underset{\uparrow}{4}, \tfrac{3}{2}, 1, \tfrac{1}{2}, 1\}$$

$$x_o(n) = \frac{x(n)-x(-n)}{2} = \{-1, -\tfrac{1}{2}, -1, -\tfrac{3}{2}, \underset{\uparrow}{0}, \tfrac{3}{2}, 1, \tfrac{1}{2}, 1\}$$

Q. Find even & odd part: $x(n) = \cos(n) + \sin(n) + \cos(n)\sin(n)$

→ $x(-n) = \cos(n) - \sin(n) - \cos(n)\sin(n)$

$x_e(n) = \cos(n)$

$x_o(n) = \sin(n) + \cos(n)\sin(n)$

Energy / Power

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad \Bigg| \qquad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x(n)|^2$$

Q Calculate energy of $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$\rightarrow$ $$E = \sum_{n=-\infty}^{\infty} \left| \left(\frac{1}{3}\right)^n \right|^2 \quad - \{u(n) = 1 \; ; n \geqslant 0\}$$

$$= \frac{1}{1 - 1/9} = \frac{9}{8} \text{ units}$$

Q Determine energy for $x(n) = \left(\frac{1}{2}\right)^n \; , n \geqslant 0$

$$= 3^n \quad , n < 0$$

$$\therefore E = \sum_{-\infty}^{1} (3^n)^2 + \sum_{0}^{\infty} \left| \left(\frac{1}{2}\right)^n \right|^2$$

$$= \sum_{1}^{\infty} \left(\frac{1}{9}\right)^m + \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n \quad - \{n = -m\}$$

$$= \frac{1/9}{1 - 1/9} + \frac{1}{1 - 1/4}$$

$$= \frac{1}{8} + \frac{4}{3}$$

$$= \frac{35}{24} \text{ units}$$

Q Decide wether it is power or energy signal $x(n) = u(n)$

$\rightarrow$ $$\text{Energy} = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{-\infty}^{\infty} (1)^2 = \infty$$

$$\text{Power} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (1)^2 = \lim_{N\to\infty} \frac{N+1}{2N+1} = \lim_{N\to\infty} \frac{1+1/N}{2+1/N} = \frac{1}{2}$$

$\therefore$ Power signal

Q Find power & energy of $x(n) = 2e^{3jn} u(n)$

$$e^{j\theta} = |\cos\theta + j\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\therefore \text{Energy} = \sum_{0}^{\infty} |2e^{3jn}|^2$$

$$= \sum_{0}^{\infty} 4 = \infty$$

$$\text{Power} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |2e^{3jn}|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} 4$$

$$= 4 \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= 4 \lim_{N\to\infty} \frac{1+1/N}{2+1/N}$$

$$= 4 \times \frac{1}{2}$$

$$= 2 \text{ units}$$

Classification of Discrete System

1. Linear / Non linear
2. Varient / Invarient
3. Casual / Non Casual
4. Static / Dynamic
5. Stable / Unstable

1. Linear / Non-Linear: $y'(n) = f(x_1(n) + x_2(n))$

$\& \; y''(n) = f(x_1(n)) + f(x_2(n))$

if $y'(n) = y''(n)$, Then Linear

Q State if linear or not. $y(n) = |x(n)|$

$\rightarrow$ $y'(n) = |x_1(n) + x_2(n)|$ & $y''(n) = |x_1(n)| + |x_2(n)|$

$y'(n) \neq y''(n)$, hence non linear

2. Varient / Invarient: if shift in input signal results in corresponding shift in output signal, it is invarient

1. delay input by K $(x(n-K))$
2. replace n by n-K $(y(n-K))$
3. If $y(n,K) = y(n-K)$ – (sys is invarient)

Q $y(n) = |x(n)|$

$\rightarrow$

1. $y(n,K) = |x(n-K)|$
2. $y(n-K) = |x(n-K)|$

$y(n,K) = y(n-K)$

hence, invarient

3. Casual / Non-casual: casual if output depends on present and past values and not future values.

Q $y(n) = \cos[x(n+1)]$

→ if $n=5$, $y(5) = \cos[x(6)]$

$x(6)$ can be considered future value, hence it is non casual.

4. Static / Dynamic: if output depends only on current time frame, it is called static, otherwise dynamic.

Q $y(n) = |x(n)|$ & $y(n) = |x(n+1)|$

→ $y(n) = |x(n)|$ depends on current time frame n ∴ Static

$y(n) = |x(n+1)|$ depends on future time frame $n+1$ ∴ dynamic

Q determine L/NL, V/InV, C/NC, S/D

(i) $y(n) = x(2n)$ (ii) $y(n) = \log_{10}(x(n))$

(i) $y(n) = x(2n)$ → $y'(n) = x_1(2n) + x_2(2n)$

$y''(n) = x_1(2n) + x_2(2n)$

→ $y(n,k) = x(2n-k)$

$y(n-k) = x(2n-2k)$

→ $y(5) = x(10)$

→ $n \rightarrow 2n$

∴ Linear, varient, non casual, dynamic

(ii) $y(n) = \log_{10}(x(n))$ → $y'(n) = \log_{10}(x_1(n) + x_2(n))$

$y''(n) = \log_{10}(x_1(n)) + \log_{10}(x_2(n))$

→ $y(n,k) = \log_{10}(x(n-k))$

$y(n-k) = \log_{10}(x(n-k))$

→ $y(5) = \log_{10}(x(5))$

→ $y(n) = \log_{10}(x(n))$

∴ non linear, invarient, casual, static

5. Stable / Unstable: if impulse response is sumable then stable

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

when $x(n) = \delta(n)$, then $y(n) = h(n)$

Q $y(n) = ax(n) . x(n-1)$

→ consider $x(n) = \delta(n)$ $\therefore y(n) = h(n)$

$$h(n) = a\delta(n) . \delta(n-1)$$

$n=0$, $h(0) = a\,\delta(0) . \delta(-1) = 0$

$n=1$, $h(1) = a\delta(1) \cdot \delta(0) = 0$

$n=2$, $h(2) = a\,\delta(2) . \delta(1) = 0$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

∴ The system is stable with summarization value = 0.

Q $y(n) = e^{-x(n)}$

→ consider $x(n) = \delta(n)$ $\therefore y(n) = h(n)$

$$h(n) = e^{-x\delta(n)}$$

$n=0$, $h(0) = e^{-x\delta(0)} = e^{-1}$

$n=1$, $h(1) = e^{-x\delta(1)} = 1$

$n=2$, $h(2) = e^{-x\delta(2)} = 1$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

Sumarization sum $= e^{-1} + 1 + 1 + 1 + 1 \ldots\ldots\ldots = \infty$

∴ The system is unstable

Correlation

measure of degree to which two signal are similar

Correlation — Auto: signal with itself
Correlation — Cross

Q $x(n) = \{-\underset{\uparrow}{1}, 2, 1\}$. Find auto correlation

$\rightarrow \quad x(-n) = \{\underset{\uparrow}{1}, 2, -1\}$

$\gamma_{xx} =$

	1	2	-1
-1	-1	-2	1
2	2	4	-2
1	1	2	-1

$= \{1, 0, \underline{\underline{6}}, 0, -1\}$

Q $x_1(n) = \{\underset{\uparrow}{1}, 1, 2, 2\}$ $\quad x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, Find cross correlation

$x_2(-n) = \{4, 3, 2, \underset{\uparrow}{1}\}$

	4	3	2	1
→ 1	4	3	2	1*
1	4	3	2	1
2	8	6	4	2
2	8	6	4	2

$\gamma_{x_1 x_2} = \{4, 7, 13, \underset{\uparrow}{17}, 11, 6, 2\}$

Convolution (Linear)

$x(n) = \{2, 1, \underset{\uparrow}{-1}, -2, -3\}$, $\quad h(n) = \{1, \underset{\uparrow}{2}, 0, -3\}$

	2	1	-1	-2	-3
1	2	1	-1	-2	-3
→ 2	4	2	-2*	-4	-6
0	0	0	0	0	0
-3	-6	-3	3	6	9

$\gamma_{xh} = \{2, 5, 1, \underset{\uparrow}{-10}, -10, -3, 6, 9\}$

Discrete Fourier Transform

Time Domain $\xrightarrow{\text{Fourier Transform}}$ Frequency Domain

$x(n)$ $\qquad\qquad$ $X[K]$

$$DTFT = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$DFT = X[K] = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} \quad , K = 0,1,2,\ldots (N-1)$$

$N \Rightarrow$ length of the signal

$$IDFT = x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X[K]\, e^{j\frac{2\pi nk}{N}} \quad , n = 0,1,2,\ldots\ldots (N-1)$$

Twiddle Factor $\quad W_N = e^{-j2\pi/N}$

$$\therefore\ DFT = X[K] = \sum_{n=0}^{N-1} x(n)\, W_N^{nK}$$

$$\&\ IDFT = x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X[K]\, W_N^{-nk}$$

- DFT

Q Find DFT of $x(n) = \{1, 2, 3, 4\}$

$\rightarrow \quad X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \quad , N = 4 \quad , K = 0,1,2,3$

$$X[K] = x(0)\, W_N^{0} + x(1)\, W_N^{K} + x(2)\, W_N^{2k} + x(3)\, W_N^{3k}$$

$$= 1 + 2W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$$

$K=0, \quad X[0] = 1 + 2 + 3 + 4 = 10$

$K=0, \quad X[1] = 1 + 2W_N + 3W_N^{2} + 4W_N^{3}$

$$W_N = e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$W_N^2 = -1$$

$$W_N^3 = j$$

$$\therefore K=1, \quad X[1] = 1 - 2j - 3 + 4j$$
$$= -2 + 2j$$

$$K=2, \quad X[2] = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$$
$$= 1 - 2 + 3 - 4$$
$$= -2$$

$$K=3, \quad X[3] = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$$
$$= 1 + 2j - 3 - 4j$$
$$= -2 - 2j$$

$$\therefore X[K] = \begin{cases} 10 & , \quad K=0 \\ -2+2j & , \quad K=1 \\ -2 & , \quad K=2 \\ -2-2j & , \quad K=3 \end{cases}$$

Q Find DFT of $x(n) = \{1, 2, 2, 1\}$

$$DFT = \sum_{n=0}^{N-1} x(n) \omega_N^{nk} \quad , \quad N = 4 \; , \quad k = 0, 1, 2, 3$$

$$= x(0)\omega_N^{0} + x(1)\omega_4^{k} + x(2)\omega_4^{2k} + x(3)\omega_4^{3k}$$

$$= 1 + 2\omega_4^{k} + 2\omega_4^{2k} + 1\omega_4^{3k}$$

$$k=0 \; , \; X[0] = 1 + 2 + 2 + 1 = 6$$

$$k=1 \; , \; X[1] = 1 + 2\omega_4 + 2\omega_4^{2} + \omega_4^{3}$$

$$\omega_4 = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = -j$$

$$\omega_4^{2} = -1$$

$$\omega_4^{3} = j$$

$$\therefore k=1 \; , \; X[1] = 1 - 2j - 2 + j$$
$$= -1 - j$$

$$k=2 \; , \; X[2] = 1 - 2 + 2 - 1$$
$$= 0$$

$$k=3 \; , \; X[3] = 1 + 2j - 2 - j$$
$$= -1 + j$$

$$X[k] = \begin{bmatrix} 6 \\ -1-j \\ 0 \\ -1+j \end{bmatrix}$$

Q Find DFT of $x(n) = (-1)^n$ for $N=3$

→ $x(n) = \{1, -1, 1\}$

$$X[k] = \sum_{N=0}^{2} x(n) W^{nk}$$

$$= x(0)W_N^0 + x(1)W_N^k + x(2)W_N^{2k}$$
$$= 1 - W_N^k + W_N^{2k}$$

$k=0$; $X[0] = 1$
$k=1$; $X[1] = 1 - W_N + W_N^2$
$k=2$; $X[2] = 1 - W_N^2 + W_N^2$

$$W_N = e^{-j\frac{2\pi}{3}} = \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)$$

$$= -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$W_N^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}j$$

$$W_N^4 = \frac{-1}{2} - \frac{\sqrt{3}}{2}j$$

$$X[1] = 1 + \frac{1}{2} + \frac{\sqrt{3}}{2}j - \frac{1}{2} + \frac{\sqrt{3}}{2}j = 1 + \sqrt{3}j$$

$$X[2] = 1 + \frac{1}{2} - \frac{\sqrt{3}}{2}j - \frac{1}{2} - \frac{\sqrt{3}}{2}j = 1 - \sqrt{3}j$$

$$X[k] = \begin{cases} 1 & , k=0 \\ 1+\sqrt{3}j & , k=1 \\ 1-\sqrt{3}j & , k=2 \end{cases}$$

• IDFT

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X[K]\,W_N^{-nK} \quad , n = 0 \ldots\ldots N-1$$

Q Find IDFT for $X[K] = [4, 1-j, -2, 1+j]$

$\rightarrow$ $N=4$,

$$W_N^{*} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & j & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 4+1-j-2+1+j \\ 4+j-j^2+2-j-j^2 \\ 4-1+j-2-1-j \\ 4-j+j^2+2+j+j^2 \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$x(n) = \{1, 2, 0, 1\}$$

Q IDFT for $X[k] = \{3, 2+j, 1, 2-j\}$

$\rightarrow \quad N=4 \quad , \quad W^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

$$x(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 3 \\ 2+j \\ 1 \\ 2-j \end{bmatrix}$$

$$x(n) = \frac{1}{4} \begin{bmatrix} 3+2+j+1+2-j \\ 3+2j+j^2-1-2j+j^2 \\ 3-2-j+1-2+j \\ 3-2j-j^2-1+2j-j^2 \end{bmatrix}$$

$$x(n) = \frac{1}{4} \begin{bmatrix} 8 \\ 0 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x(n) = \{\underset{\uparrow}{2}, 0, 0, 1\}$$

• Properties of DFT

(1) <u>Linearity</u>: $a x_1(n) + b x_2(n) \xrightarrow{DFT} a X_1[k] + b X_2[k]$

(2) <u>Perodicity</u>: $x_p(n) \xrightarrow{DFT} X_p[K]$

<u>Linear Shifting</u>: $x(n) = \{1, 2, 3, 4\}$ $\quad x(n+2) = \{0, 0, 1, 2, 3, 4\}$

$x(n-1) = \{1, 2, 3, 4\}$ $\quad x(-n) = \{4, 3, 2, 1\}$

<u>Circular Shifting</u>: $x(n) = \{1, 2, 3, 4\}$

$x(n-1) = \{4, 1, 2, 3\}$

$x(n-2) = \{3, 4, 1, 2\}$

$x(n+1) = \{2, 3, 4, 1\}$

$x(n+2) = \{3, 4, 1, 2\}$

$x(-n) = \{1, 4, 3, 2\}$

(3) <u>Symmetric</u>: If $x(n)$ is real The Real $(X[K])$ is symmetric at $K = \frac{N}{2}$

& $Img(X[K])$ is anti-symmetric

(4) <u>Time Shifting</u>: when There is shift in time, multiplicative comes in freq. domain

$x(n-m) \xrightarrow{DFT} W_N^{mK} X[K]$ $\quad$ (-m to m)

$x(n+m) \xrightarrow{DFT} W_N^{-mK} X[K]$ $\quad$ (m to -m)

(5) <u>Frequency Shifting</u>: when There is shift in freq. domain, multiplicative comes in time domain

$W_N^{-mn} x(n) \xrightarrow{DFT} X[k-m]$

$W_N^{mn} x(n) \xrightarrow{DFT} X[k+m]$

(6) <u>Time Reversal</u>: $x(n) \xrightarrow{DFT} X[K]$

$x(-n) \xrightarrow{DFT} X[-K]$

Q Find $X[K]$ if $x(n) = \{1,2,3,4\}$, ∴ find (i) $p(n)=\{4,1,2,3\}$ (ii) $g(n) = \{3,4,1,2\}$
(iii) $l(n) = \{1,4,3,2\}$ (iv) $h(n) = \{1,-2,3,-4\}$

→ $$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

(i) $p(n) = x(n-1)$
$$P[K] = W_N^{1K} X[K]$$
$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} W_N^0 \\ W_N^1 \\ W_N^2 \\ W_N^3 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} = \begin{bmatrix} 10 \\ 2+2j \\ 2 \\ 2-2j \end{bmatrix}$$

(ii) $g(n) = x(n-2)$
$$G[K] = W_N^{2K} X[K]$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} W_N^0 \\ W_N^2 \\ W_N^4 \\ W_N^6 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 2-2j \\ -2 \\ 2+2j \end{bmatrix}$$

(iii) $l(n) = x(-n)$

$$\therefore L[k] = X[-k]$$

$$X[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \qquad \therefore X[-k] = \begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2+2j \end{bmatrix}$$

(iv) $h(n) = \{1, -2, 3, -4\}$

$$h(n) = \begin{cases} x(n) & \text{even} \\ -x(n) & \text{odd} \end{cases}$$

$$\therefore h(n) = (-1)^n x(n)$$

$$h(n) = (j^2)^n x(n)$$

$$h(n) = (\omega_N^2)^n x(n)$$

$$H[k] = X[k+2]$$

$$\therefore H[k] = \begin{bmatrix} -2 \\ -2-2j \\ 10 \\ -2+2j \end{bmatrix}$$

Q Find DFT of $x(n) = \{1+j,\ 2+2j,\ 3+2j,\ 4+j\}$, hence find DFT of

(i) $p(n) = \{1,2,3,4\}$ (ii) $g(n) = \{1,2,2,1\}$

$$\rightarrow \quad X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+4j \end{bmatrix} = \begin{bmatrix} 10+6j \\ -1+j \\ -2 \\ -3-3j \end{bmatrix}$$

(i)
$$x(n) = p(n) + j\,g(n)$$
$$x^*(n) = p(n) - j\,g(n)$$

$$\therefore\ x(n) + x^*(n) = 2p(n)$$
$$\therefore\ 2P[k] = X[k] + X^*[-k] \qquad \{x^*(n) = X^*[-k]\}$$
$$P[k] = \frac{X[k] + X^*[-k]}{2}$$

$$X^*[k] = \begin{bmatrix} 10-6j \\ -1-j \\ -2 \\ -3+3j \end{bmatrix}, \quad X^*[-k] = \begin{bmatrix} 10-6j \\ -3+3j \\ -2 \\ -1-j \end{bmatrix} \qquad \therefore\ P[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

(ii) $x(n) - x^*(n) = 2g(n)$

$$\therefore\ G[k] = \frac{(X[k] - X^*[-k])}{2}j$$

$$G[k] = \frac{j}{2}\left[\begin{bmatrix} 10+6j \\ -1+j \\ -2 \\ -3-3j \end{bmatrix} - \begin{bmatrix} 10-6j \\ -3+3j \\ -2 \\ -1-j \end{bmatrix}\right]$$

$$\therefore\ G[k] = \begin{bmatrix} 6 \\ -1-j \\ 0 \\ -1+j \end{bmatrix}$$

- **Circular Convolution** (length should be same, else padd)

3 methods : 1. Graphical Method
2. Matrix Method
3. DFT, IDFT

(1) DFT, IDFT : $x_1(n) \circledast x_2(n) = y(n)$

$$\begin{array}{ccc} x_1(n) \circledast x_2(n) & = & y(n) \\ \downarrow \text{DFT} & & \uparrow \text{IDFT} \\ X_1[K]X_2[K] & = & Y[K] \end{array}$$

Q Using DFT, IDFT, find circular convolution of $x_1(n) = \{1, 2, 3, 1\}$
$x_2(n) = \{4, 3, 2, 2\}$

→ $$X_1[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$X_2[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+3+2+2 \\ 4-3j-2+2j \\ 4-3+2-2 \\ 4+3j-2-2j \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$Y[k] = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix} \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix} = \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$y(n)=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\begin{bmatrix}77\\-5\\1\\-5\end{bmatrix}, \quad y(n)=\begin{bmatrix}17\\19\\22\\19\end{bmatrix}$$

Q Using DFT, IDFT, find circular convolution of $x_1(n)=\{0,1,2,3\}$
$h(n)=\{2,1,1,2\}$

$$\rightarrow \quad X_1[K]=\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}\begin{bmatrix}0\\1\\2\\3\end{bmatrix}=\begin{bmatrix}6\\-2+2j\\-2\\-2-2j\end{bmatrix}$$

$$X_2[K]=\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}\begin{bmatrix}2\\1\\1\\2\end{bmatrix}=\begin{bmatrix}6\\1+j\\0\\1-j\end{bmatrix}$$

$$Y[K]=\begin{bmatrix}6\\-2+2j\\-2\\-2-2j\end{bmatrix}\begin{bmatrix}6\\1+j\\0\\1-j\end{bmatrix}=\begin{bmatrix}36\\-4\\0\\-4\end{bmatrix}$$

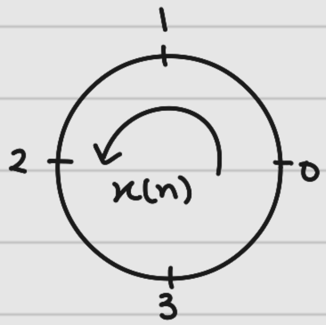
$$y(n)=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\begin{bmatrix}36\\-4\\0\\-4\end{bmatrix}, \quad \therefore y(n)=\begin{bmatrix}7\\9\\11\\9\end{bmatrix}$$

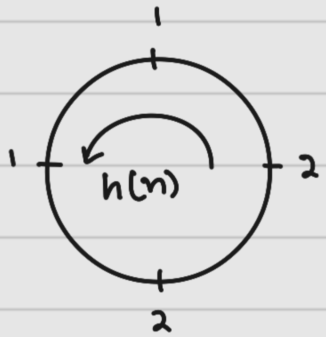
(2) Graphical Method: 1. Draw $x(n)$ & $h(n)$ anticlockwise

2. Calculate using $y(m) = \sum_{n=0}^{N-1} x(n)h(m-n)$

Q $x(n) = \{0, 1, 2, 3\}$, $h(n) = \{2, 1, 1, 2\}$

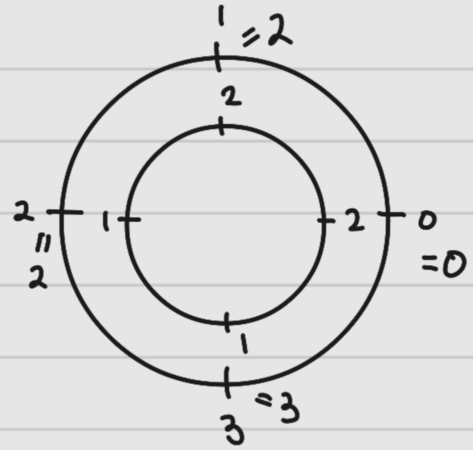
→ Step 1:





Step 2: Calculate , → m = 0

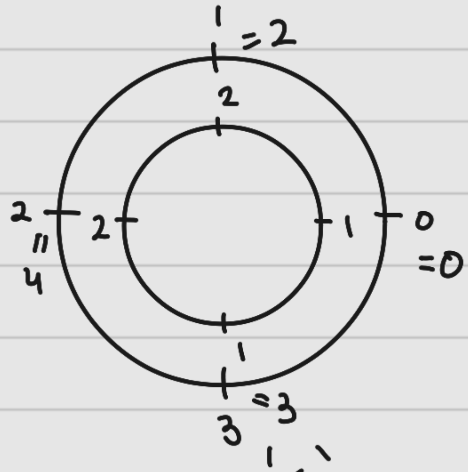
$$y(0) = \sum_{n=0}^{N-1} x(n)h(-n)$$



$$y(0) = 0 + 2 + 2 + 3 = 7$$

→ m = 1

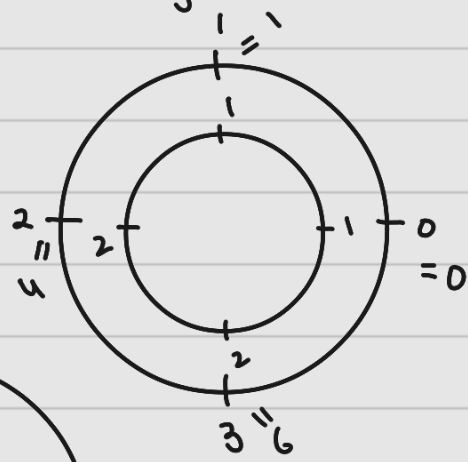
$$y(1) = \sum_{n=0}^{N-1} x(n)h(1-n)$$



$$y(1) = 0 + 2 + 4 + 3 = 9$$

→ m = 2

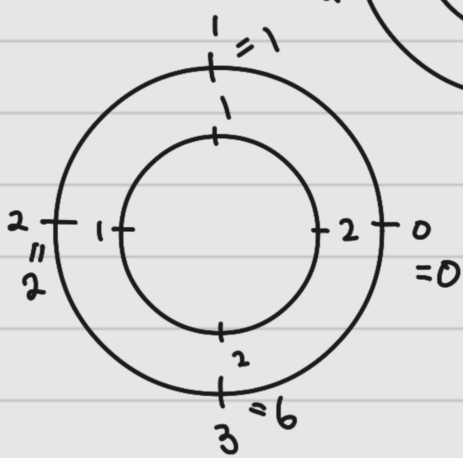
$$y(2) = \sum_{n=0}^{N-1} x(n)h(2-n)$$



$$y(2) = 0 + 1 + 4 + 6 = 11$$

→ m = 3

$$y(3) = \sum_{n=0}^{N} x(n)h(3-n)$$



$$y(3) = 0 + 1 + 2 + 6 = 9$$

$\therefore$ $y(n) = \{7, 9, 11, 9\}$

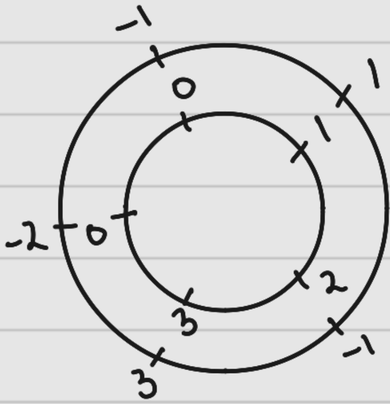
Q use graphical method $x(n)=\{1,-1,-2,3,-1\}$, $h(n)=\{1,2,3\}$

$\rightarrow x(n)=\{1,-1,-2,3,-1\}$, $h(n)=\{1,2,3,0,0\}$

$\rightarrow m=0$

$$y(0)=\sum_{n=0}^{N-1} x(n)h(-n)$$

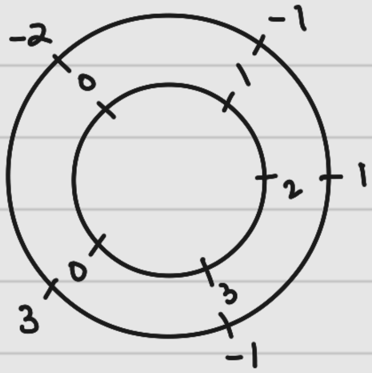
$$y(0)=1+0+0+9-2$$
$$=8$$



$\rightarrow m=1$

$$y(1)=\sum_{n=0}^{N-1} x(n)h(1-n)$$

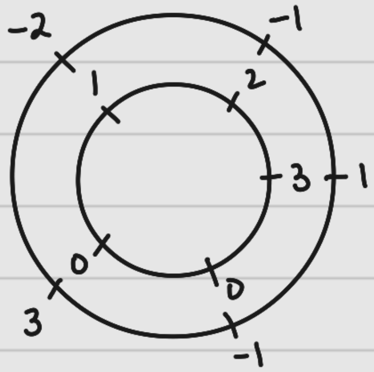
$$y(1)=2-1+0+0-3$$
$$=-2$$



$\rightarrow m=2$

$$y(2)=\sum_{n=0}^{N-1} x(n)h(2-n)$$

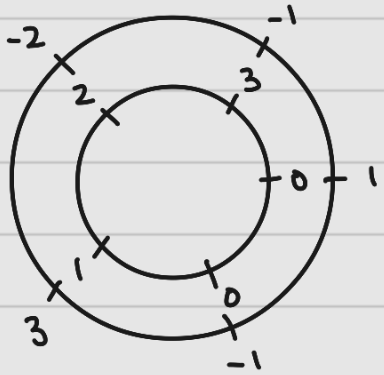
$$y(2)=3-2-2+0+0$$
$$=-1$$



$\rightarrow m=3$

$$y(3)=\sum_{n=0}^{N-1} x(n)h(3-n)$$

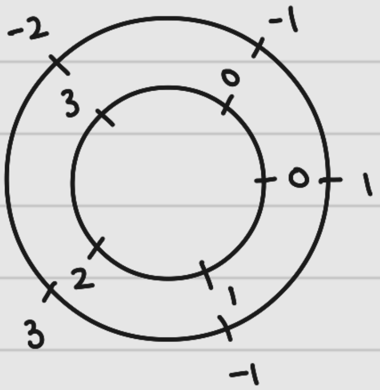
$$y(3)=0-3-4+3+0$$
$$=-4$$



$\rightarrow m=4$

$$y(4)=\sum_{n=0}^{N-1} x(n)h(4-n)$$

$$y(4)=0+0-6+6-1$$
$$=-1$$



$\therefore$ $y(m)=\{8,-2,-1,-4,-1\}$

(3) Matrix Method: $x(n) = \{1, 2, 3, 4\}$

$h(n) = \{1, 2, 3\}$, $h(n) = \{1, 2, 3, 0\}$

	1	2	3	4	
$h(-n)$	1	0	3	2	$\rightarrow 1+0+9+8=18$
$h(1-n)$	2	1	0	3	$\rightarrow 2+2+0+12=16$
$h(2-n)$	3	2	1	0	$\rightarrow 3+4+3+0=10$
$h(3-n)$	0	3	2	1	$\rightarrow 0+6+6+4=16$

$\therefore y(m) = \{18, 16, 10, 16\}$

Q Find circular convolution using matrix method

$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$

$h(n) = \{0, 1, 2, 3, 4, 3, 2, 1\}$

	1	1	1	1	-1	-1	-1	-1	
$h(-n)$	0	1	2	3	4	3	2	1	$\rightarrow -4$
$h(1-n)$	1	0	1	2	3	4	3	2	$\rightarrow -8$
$h(2-n)$	2	1	0	1	2	3	4	3	$\rightarrow -8$
$h(3-n)$	3	2	1	0	1	2	3	4	$\rightarrow -4$
$h(4-n)$	4	3	2	1	0	1	2	3	$\rightarrow 4$
$h(5-n)$	3	4	3	2	1	0	1	2	$\rightarrow 8$
$h(6-n)$	2	3	4	3	2	1	0	1	$\rightarrow 8$
$h(7-n)$	1	2	3	4	3	2	1	0	$\rightarrow 4$

$\therefore y(m) = \{-4, -8, -8, -4, 4, 8, 8, 4\}$

Linear Convolution using Circular Convolution

Q $x(n) = \{1,2,3,4\}$

$h(n) = \{1,2,3\}$

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12

$\therefore x(n) * h(n) = \{\underset{\uparrow}{1},4,10,16,17,12\}$

$x(n) = \{1,2,3,4,0,0\}$ padd till $L = M+N-1$

$h(n) = \{1,2,3,0,0,0\}$

	1	2	3	4	0	0		
$h(-n)$	1	0	0	0	3	2	=	1
$h(1-n)$	2	1	0	0	0	3	=	4
$h(2-n)$	3	2	1	0	0	0	=	10
$h(3-n)$	0	3	2	1	0	0	=	16
$h(4-n)$	0	0	3	2	1	0	=	17
$h(5-n)$	0	0	0	3	2	1	=	12

$y(n) = \{1,4,10,16,17,12\}$

Convolution For Long DATA Sequences

Two methods can be used: 1. Overlap and Add
2. Overlap and Save

⇒ Overlap and Add : a. Divide $x(n)$ into subsequences of length N. Let $len(h(n)) = M$
b. Calculate $L = M + N - 1$
∴ Add 0's to $x(n)$ & $h(n)$ to reach L elements
c. Perform Circular Convolution to get answers y_1, y_2, etc.
d. Overlap and add values to get final answer.

Q Using overlap and add find convolution of $x(n) = \{3,-1,0,1,3,2,0,1,2,1\}$
$h(n) = \{1,1,1\}$

→ let $N = 4$
$M = 3$
$\therefore L = M+N-1 = 6$

$x_1(n) = \{3,-1,0,1\} \rightarrow \{3,-1,0,1,0,0\}$
$x_2(n) = \{3,2,0,1\} \rightarrow \{3,2,0,1,0,0\}$
$x_3(n) = \{2,1\} \rightarrow \{2,1,0,0,0,0\}$
$h(n) = \{1,1,1\} \rightarrow \{1,1,1,0,0,0\}$

$y_1(n)$	3	-1	0	1	0	0	
$h(-n)$	1	0	0	0	1	1	= 3
$h(1-n)$	1	1	0	0	0	1	= 2
$h(2-n)$	1	1	1	0	0	0	= 2
$h(3-n)$	0	1	1	1	0	0	= 0
$h(4-n)$	0	0	1	1	1	0	= 1
$h(5-n)$	0	0	0	1	1	1	= 1

$y_2(n)$

	3	2	0	1	0	0	
$h(-n)$	1	0	0	0	1	1	= 3
$h(1-n)$	1	1	0	0	0	1	= 5
$h(2-n)$	1	1	1	0	0	0	= 5
$h(3-n)$	0	1	1	1	0	0	= 3
$h(4-n)$	0	0	1	1	1	0	= 1
$h(5-n)$	0	0	0	1	1	1	= 1

$y_3(n)$

	2	1	0	0	0	0	
$h(-n)$	1	0	0	0	1	1	= 2
$h(1-n)$	1	1	0	0	0	1	= 3
$h(2-n)$	1	1	1	0	0	0	= 3
$h(3-n)$	0	1	1	1	0	0	= 1
$h(4-n)$	0	0	1	1	1	0	= 0
$h(5-n)$	0	0	0	1	1	1	= 0

★ number of elements to overlap $= M-1 = 2$

$y(0) =$ 3 2 2 0 1 1
$y(1) =$ 3 5 5 3 1 1
$y(2) =$ 2 3 3 1 0 0

$\therefore y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1, 0, 0\}$

⇒ Overlap and Load: a. Divide into subsequences of N. let length of $h(n) = M$

b. Calculate $L = M + N - 1$.

∴ to make length L, take elements from previous sequence

c. Make length of $h(n)$ equal to L by adding 0.

d. Calculate $y_1(n), y_2(n), \ldots - y_i(n)$

Q Using overlap and save, find circular convolution of $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ & $h(n) = \{1, 1, 1\}$

→ $N = 4$

$M = 3$

$\therefore L = M + N - 1 = 6$

$h(x) = \{1, 1, 1, 0, 0, 0\}$

$x_1(n) = \{3, -1, 0, 1\}$

$x_2(n) = \{3, 2, 0, 1\}$

$x_3(n) = \{2, 1\}$

$x_1(n) = \{0, 0, 3, -1, 0, 1\}$

now $x_2(n) = \{0, 1, 3, 2, 0, 1\}$

$x_3(n) = \{0, 1, 2, 1, 0, 0\}$

$y_1(n)$

	0	0	3	-1	0	1	
h(-n)	1	0	0	0	1	1	= 1
h(1-n)	1	1	0	0	0	1	= 1
h(2-n)	1	1	1	0	0	0	= 3
h(3-n)	0	1	1	1	0	0	= 2
h(4-n)	0	0	1	1	1	0	= 2
h(5-n)	0	0	0	1	1	1	= 0

$y_2(n)$

	0	1	3	2	0	1	
$h(-n)$	1	0	0	0	1	1	= 1
$h(1-n)$	1	1	0	0	0	1	= 2
$h(2-n)$	1	1	1	0	0	0	= 4
$h(3-n)$	0	1	1	1	0	0	= 6
$h(4-n)$	0	0	1	1	1	0	= 5
$h(5-n)$	0	0	0	1	1	1	= 3

$y_3(n)$

	0	1	2	1	0	0	
$h(-n)$	1	0	0	0	1	1	= 0
$h(1-n)$	1	1	0	0	0	1	= 1
$h(2-n)$	1	1	1	0	0	0	= 3
$h(3-n)$	0	1	1	1	0	0	= 4
$h(4-n)$	0	0	1	1	1	0	= 3
$h(5-n)$	0	0	0	1	1	1	= 1

number of elements to overlap $= M-1 = 2$
number of elements to discard $= M-1 = 2$

$y_1(n) =$ 1 1 3 2 2 0
$y_2(n) =$ 1 2 4 6 5 3
$y_3(n) =$ 0 1 3 4 3 1

$\therefore\ y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

• Fast Fourier Transform

Q Develop a flow graph for N=4, we can use formula

$$X[k] = G[k] + W_N^k H[k]$$

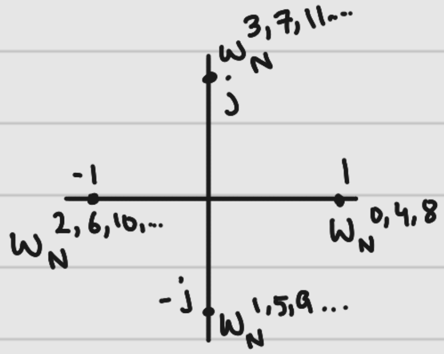
$X[k]$ for N, $G[k]$ for $\frac{N}{2}$, W_N^k for N, $H[k]$ for $\frac{N}{2}$

$\therefore N=4 \quad (0-3) \quad (0-1) \quad (0-3) \quad (0-1)$

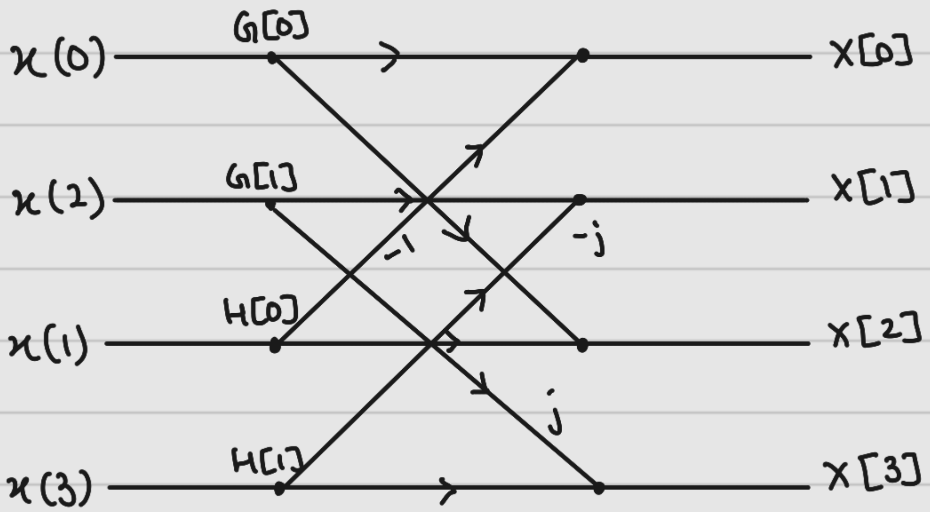
here $G[k]$ = even part of $x(n) = x_e(n)$

& $H[k]$ = odd part of $x(n) = x_o(n)$

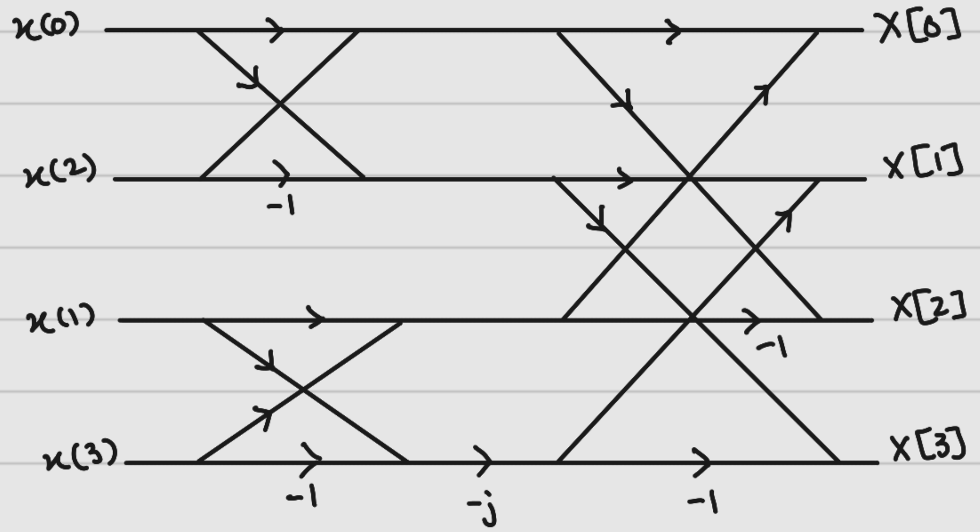
$$\therefore X[0] = G[0] + W_N^0 H[0]$$
$$X[1] = G[1] + W_N^1 H[1]$$
$$X[2] = G[0] + W_N^2 H[0]$$
$$X[3] = G[1] + W_N^3 H[1]$$



For N=4,

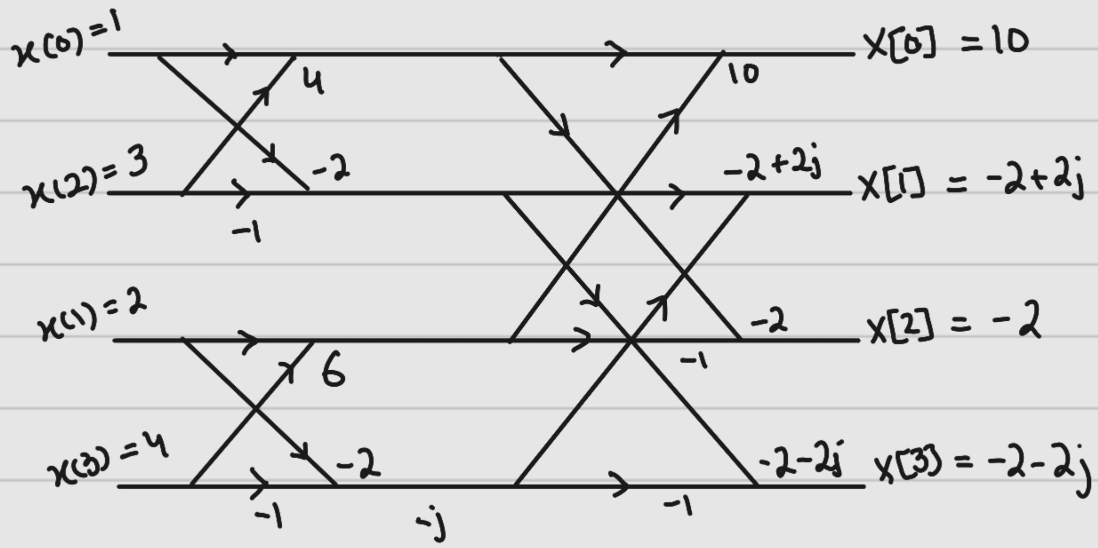


• For N=4



Q Using N=4 flowgraph, find DFT of $x(n) = \{1,2,3,4\}$

→ DFT FFT for $N=4$ (2^2)



$$\therefore \text{DFT of } x(n) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

- <u>DIF-FFT</u> (N=8)