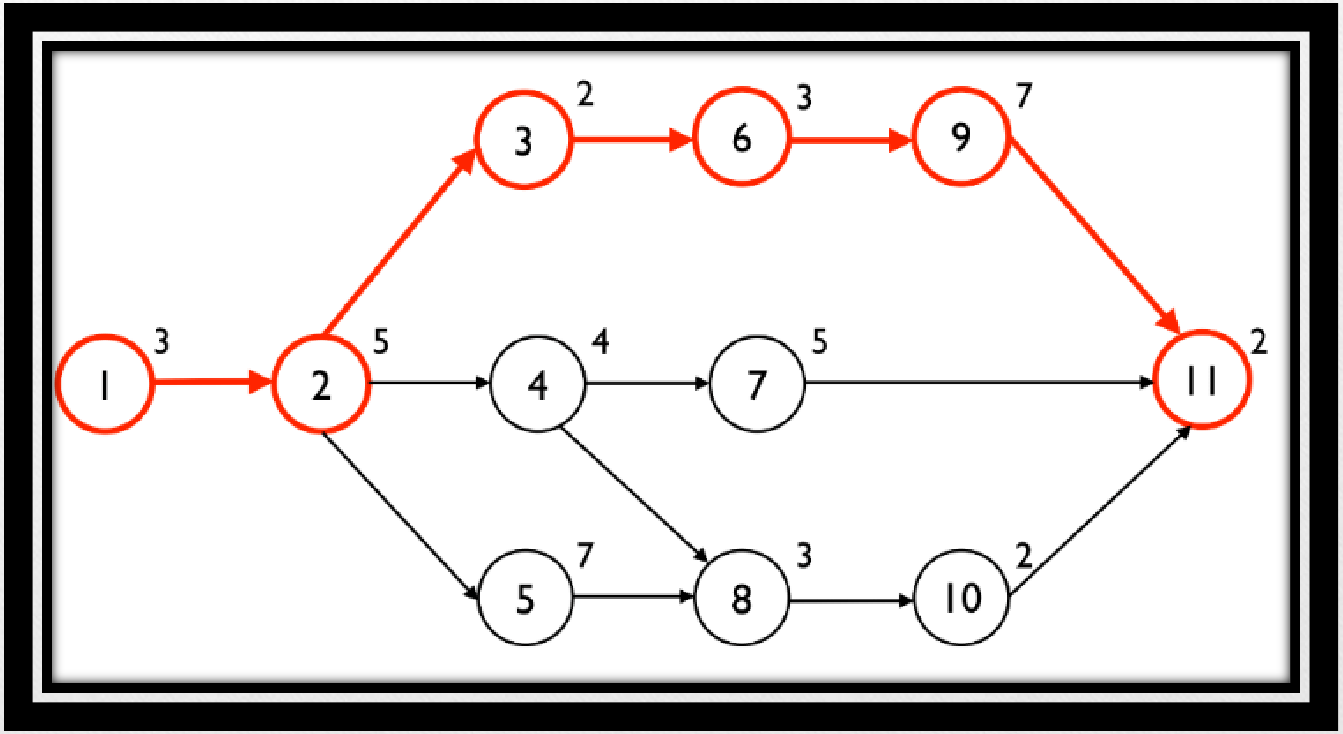
NETWORK ANALYSIS

PERT: Program Evaluation & Review Technique

CPM: Critical Path Method



Introduction

- Network analysis is one of the important tools for project management.
- Whether major or minor, a project has to be completed in a definite time & at a definite cost.
- The necessary information of any particular data can be represented as a project network.
- These techniques are very useful for planning, scheduling and executing large-time bound projects involving careful co-ordination of variety of complex and interrelated activities.

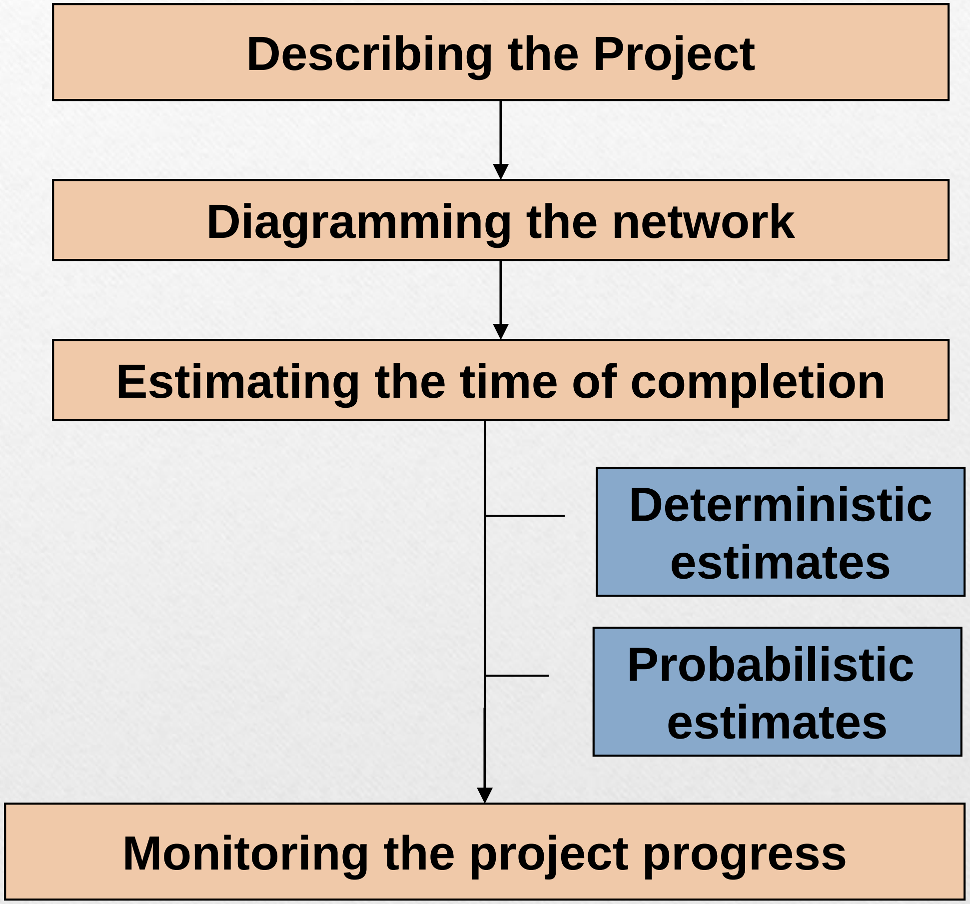
Objectives of network analysis

- Helpful in planning
- Inter-relationship of various activities
- Cost control
- Minimisation of maintenance time
- Reduction of time
- Control on idle resources
- Avoiding delays, interruptions

Real-life Applications of network analysis

- Planning, scheduling, monitoring and control of large and complex projects.
- Construction of factories, highways, building, bridges, cinemas etc.
- Helpful to army for its missile development.
- Assembly line scheduling
- Installation of computers and high tech machineries
- To make marketing strategies

Methodology Involved in Network Analysis



Project Scope, Work Breakdown Structure (WBS), Task Durations, Task Dependencies

CPM

PERT,
Monte-Carlo

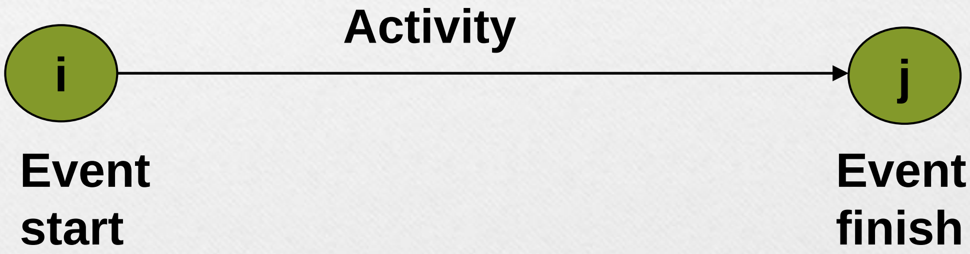
Key Terminologies

- **Activity :** All projects may be viewed as composed of activities. It is the smallest unit of work consuming both time& resources that project manager should schedule & control.

- **An activity is represented by an arrow in network diagram**

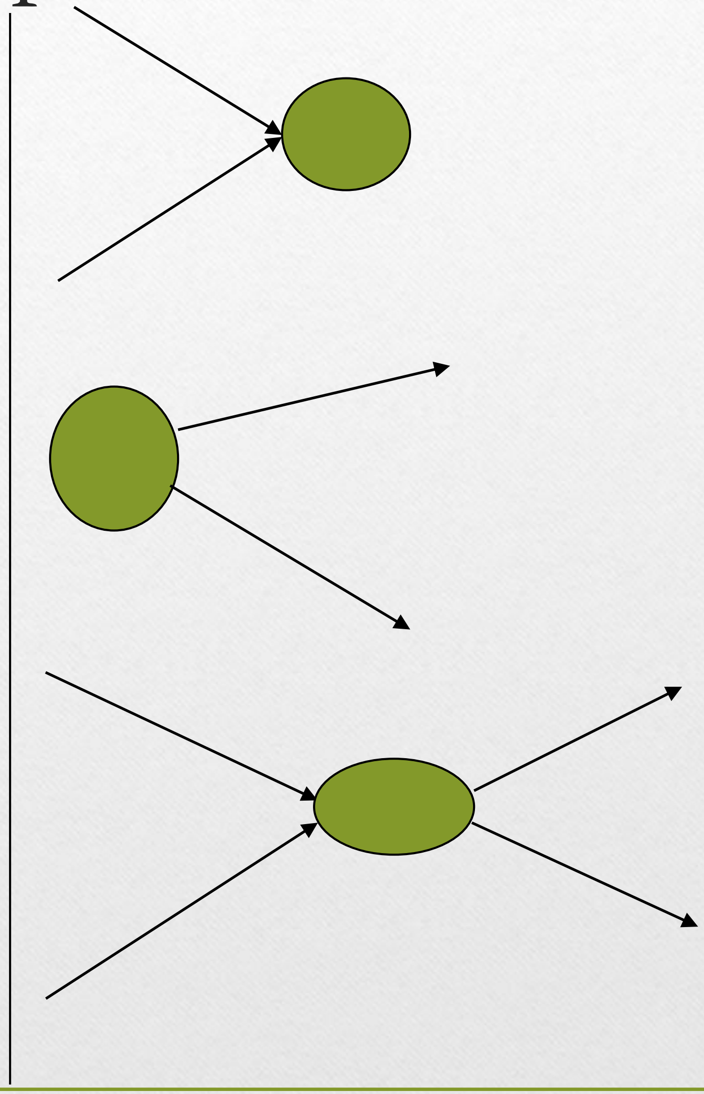
Event

- The beginning & end of an activities are called as events .
- Events are represented by numbered circles called nodes.



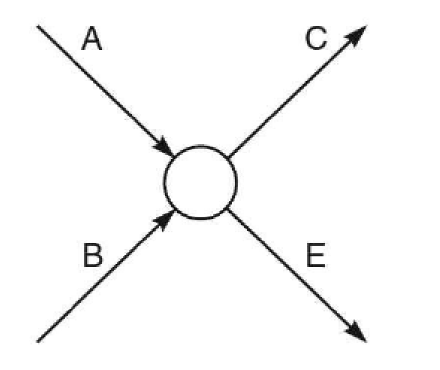
Types of Events

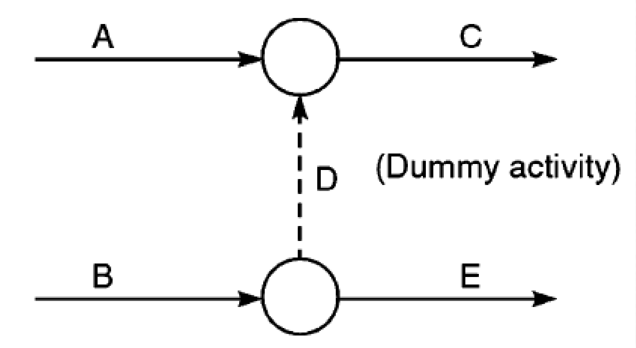
- **Merge event**
- **Burst event**
- **Merge & Burst Event**



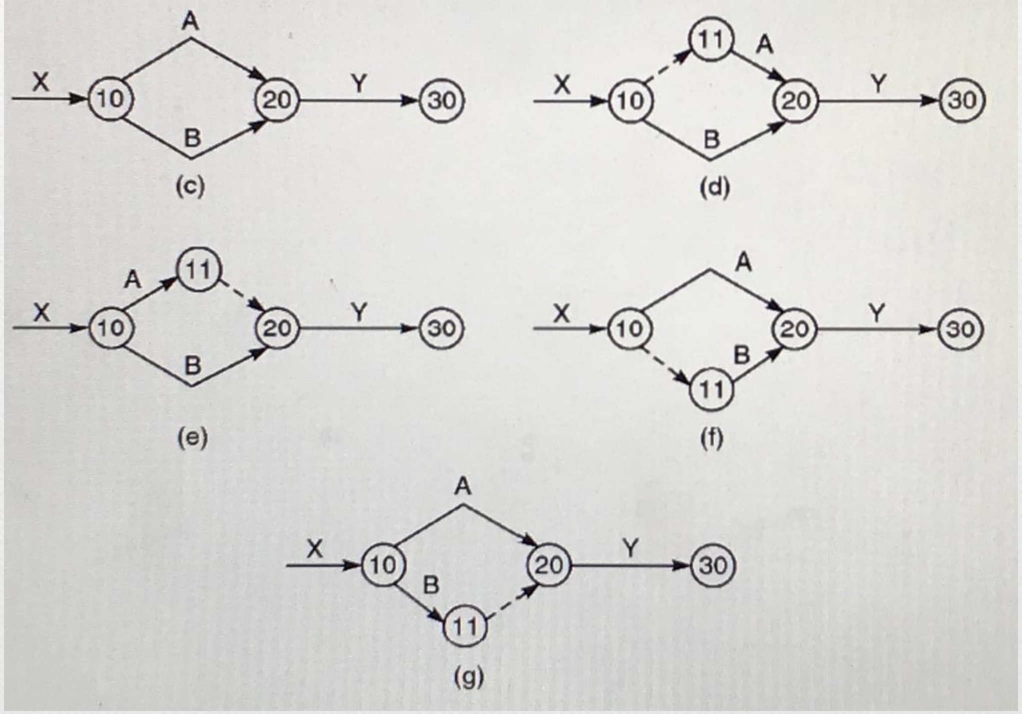
Classification of activities

- **Predecessor activity**: Activities that must be completed immediately prior to the start of another activity are called predecessor activities.
- **Successor activity** : activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.
- **Concurrent activities**: activities that can be accomplished together are known as concurrent activities.
- **Dummy activity**: An activity which does not consume any resource but merely depicts the dependence of one activity on other is called dummy activity. It is introduced in a network when two or more parallel activities have the same start and finish nodes.


A
C
B
E


A
C
D
(Dummy activity)
B
E

(b)


A
X
10
20
Y
30
B
(c)
11
A
X
10
20
Y
30
B
(d)
A
11
X
10
20
Y
30
B
(e)
A
X
10
20
Y
30
B
11
(f)
A
X
10
20
Y
30
B
11
(g)

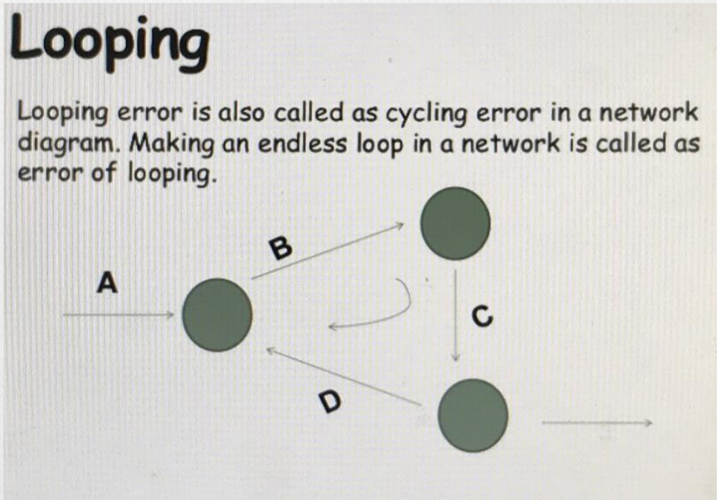
Path & Network

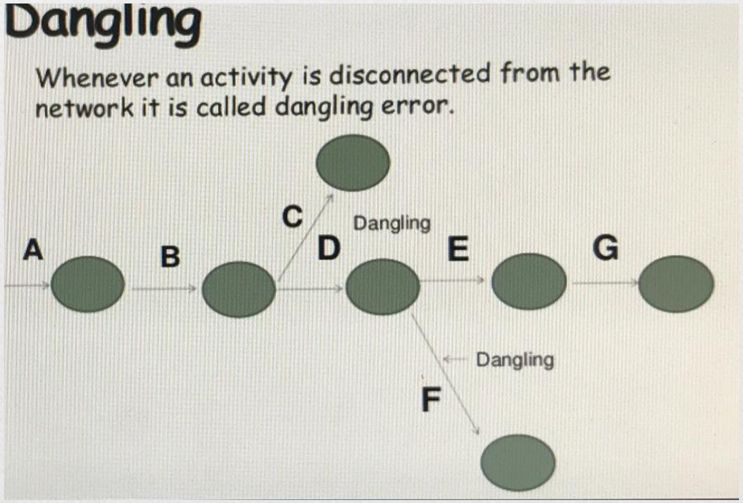
- An unbroken chain of activity arrows connecting the initial event to some other event is called a path.

- A network is the graphical representation of logically & sequentially connected arrows & nodes representing activities & events of a project. It is a diagram depicting precedence relationships between different activities.

Errors in network logic

- Looping : looping is known as cycling error and creates an impossible situation and it appears that none of the activities could ever be completed.
- Dangling : sometimes a project network includes an activity which does not fit into the end objective of the project and is carried out without any result related with completion of the project . Such an error in network is called dangling





Guidelines for Network Construction

1. A complete network diagram should have one start point & one finish point.
2. The flow of the diagram should be from left to right.
3. Arrows should not be crossed unless it is completely unavoidable.
4. Arrows should be kept straight & not curved or bent.
5. Angle between arrows should as large as possible.
6. Each activity must have a tail or head event.. No two or more activities may have same tail & head events.
7. Once the diagram is complete the nodes should be numbered from left to right. It should then be possible to address each activity uniquely by its tail & head event.

Advantages

- Planning & controlling projects
- Flexibility
- Designation of responsibilities
- Achievement of objective with least cost
- Better managerial control

Limitations of PERT /CPM

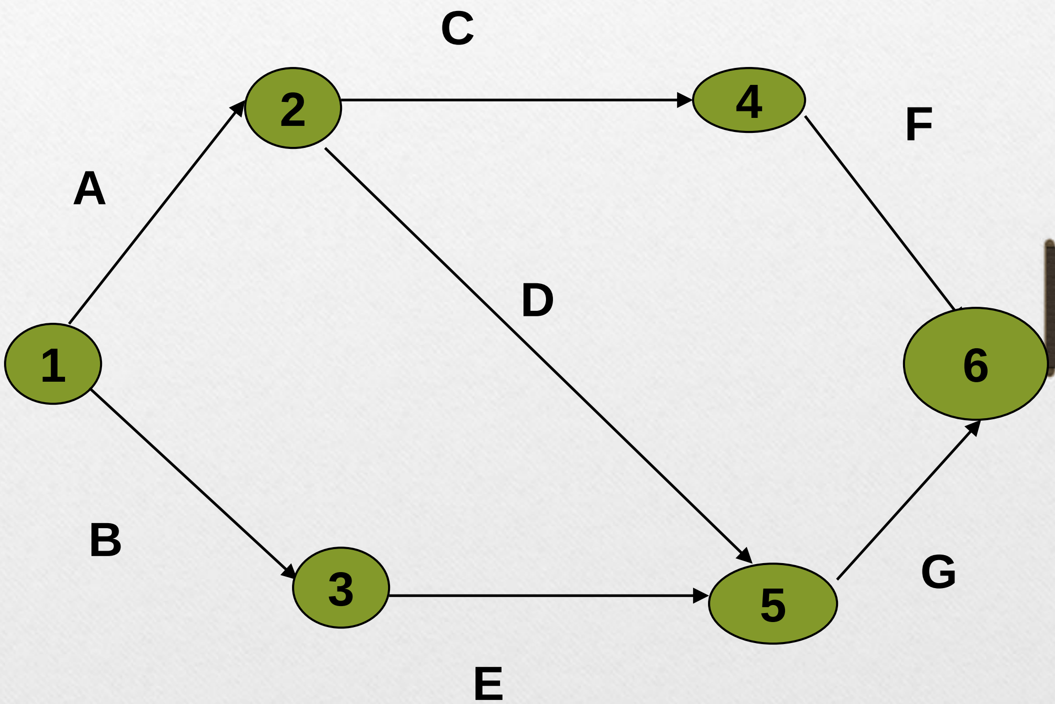
- Network diagrams should have clear starting & ending points , which are independent of each other which may not be possible in real life.
- Another limitation is that it assumes that manager should focus on critical activities.
- Resources will be available when needed for completion for an activity is again unreal.

Difficulties

- Difficulty in securing realistic time estimates.
- The planning & implementation of networks requires trained staff.
- Developing clear logical network is troublesome.

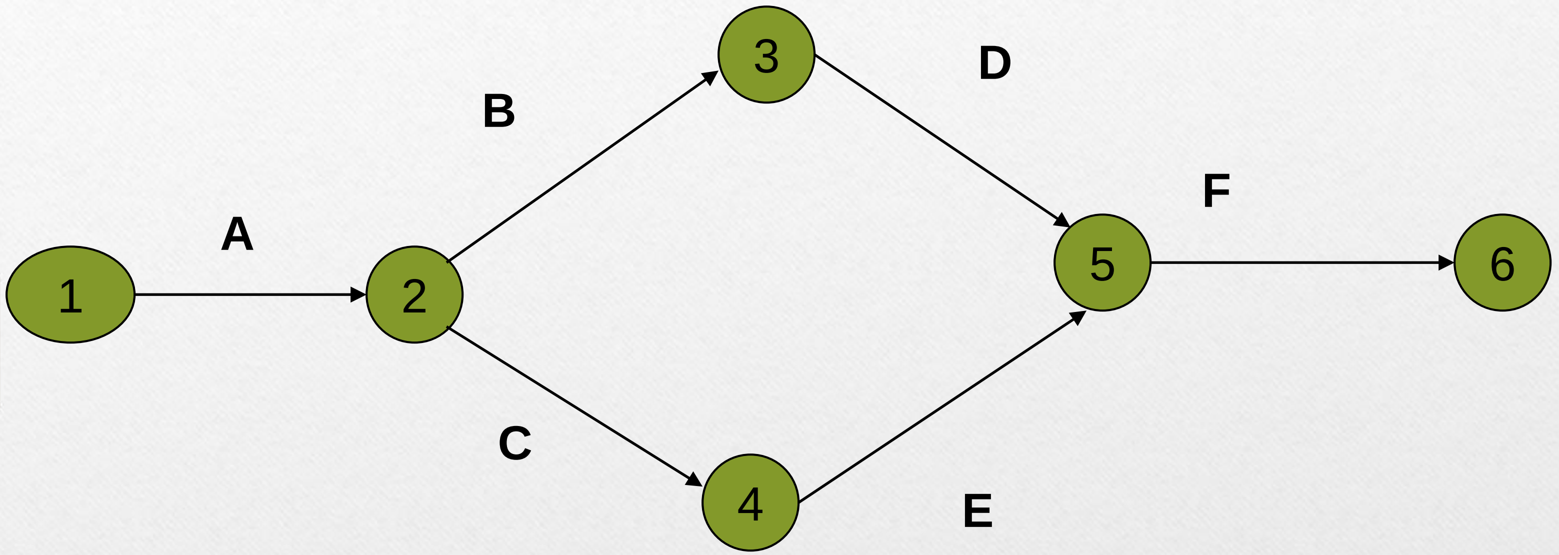
Example

Activity	Predecessor activity
A	none
B	none
C	A
D	A
E	B
F	C
G	D & E



Draw the network diagram for the following

Activity	Predecessor activity
A	none
B	A
C	A
D	B
E	C
F	D ,E



A
B
C
D
E
F
1
2
3
4
5
6

Difference between PERT & CPM

PERT (Programme Evaluation Review Technique)	**CPM** (Critical Path Method)
1. PERT is event oriented.	1. CPM is activity oriented.
2. PERT is probabilistic.	2. CPM is deterministic.
3. PERT is primarily concerned with time only.	3. CPM places dual emphasis on project time as well cost.
4. PERT is generally used for projects where time required to complete the activities is not known a priori. Thus PERT is used for large, R&D type of projects.	4. CPM is used for projects which are repetitive in nature and comparatively small in size.
5. Three time estimates are possible for activities linking up two events.	5.One time estimate is possible for activities (No allowance is made for uncertainty)

Critical Path Method

- Those activities which contribute directly to the overall duration of the project constitute critical activities. The critical activities form a chain running through the network which is called critical path.
- **Critical event** : The slack of an event is the difference between the latest & earliest events time. The events with zero slack time are called as critical events.
- **Critical activities** : The difference between latest start time & earliest start time of an activity will indicate amount of time by which the activity can be delayed without affecting the total project duration. The difference is usually called total float. Activities with 0 total float are called as critical activities.

Critical path

- The critical path is the longest path in the network from the starting event to ending event & defines the minimum time required to complete the project.
- The critical path is denoted by darker or double lines.

Critical Path Analysis

1. *Calculate the time schedule for each activity*
2. *Calculate the time schedule for the completion of the entire project*
3. *Identify the critical activities and find the critical path*

Activity	:	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration (weeks)	:	15	15	3	5	8	12	1	14	3	14

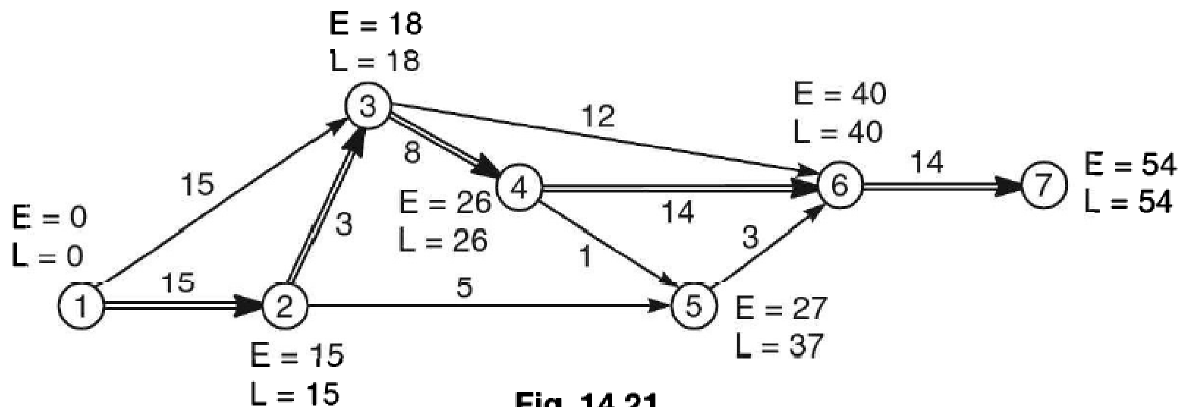

E = 0
L = 0
E = 15
L = 15
E = 18
L = 18
E = 26
L = 26
E = 27
L = 37
E = 40
L = 40
E = 54
L = 54
1
2
3
4
5
6
7
15
15
3
8
12
14
1
5
3
14

Fig. 14.21

•**Early Start Time: The earliest point in the schedule at which a activity can begin.**

EST of activities with no predecessors = First logical starting point.
EST of activities with predecessors = Predecessor EFT (Earliest Finish Time).

•**Early Finish Time: The earliest point in the schedule at which a activity can finish.**

EFT of activity with no predecessors = Estimated activity duration.
EFT of activity with predecessors = (Activity EST + Estimated activity duration).

- **Latest Start Time: The latest point in the schedule at which a activity can start without causing a delay.**

Activity LST = (LFT – Task duration).

- **Latest Finish Time: The latest point in the schedule at which a activity can finish without causing a delay.**

Activity LFT = EST of the next dependent task.

- **Float :** There are many activities where the maximum time available to finish the activity is more than the time required to complete the activity. The difference between the two times is known as float available for the activity.
- **Slack:** The difference between the latest event times and the earliest event time. i.e. Slack = TL - TE.

There are four types of float:

- **Total float** : It is the spare time available when all preceding activities occur at earliest possible times & all succeeding activities occur at latest possible times.

TF = LFT – EFT (or)

TF = LST - EST

- **Free float (FF)** : The maximum number of days the activity can be delayed without delaying any succeeding activity.

Free float = Total float - head event slack

= EST of successor – EFT of current

- **Independent Float (IF)** : It may be defined as the amount of time by which the start of an activity can be delayed without affecting the earliest start time of any successor activity, assuming that preceding activity has finished at its latest finish time.

Independent Float = Free float – tail event slack

Interfering Float: It is that part of the total float which causes a reduction in the floats of the succeeding activities.

It can be defined as the difference between the latest finish time of the activity under consideration and the earliest start time of the following activity, or zero, whichever is larger. i.e = (LFT - EST) = TF-FF = head event slack.

Interfering Float = Total Float – Free Float

Activity (i – j)	Duration (D)	Start time		Finish time		Total Float
		Earliest	Latest	Earliest	Latest	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1-2	15	0	0	15	15	0
1-3	15	0	3	15	18	3
2-3	3	15	15	18	18	0
2-5	5	15	32	20	37	17
3-4	8	18	18	26	26	0
3-6	12	18	28	30	40	10
4-5	1	26	36	27	37	10
4-6	14	26	26	40	40	0
5-6	3	27	37	30	40	10
6-7	14	40	40	54	54	0

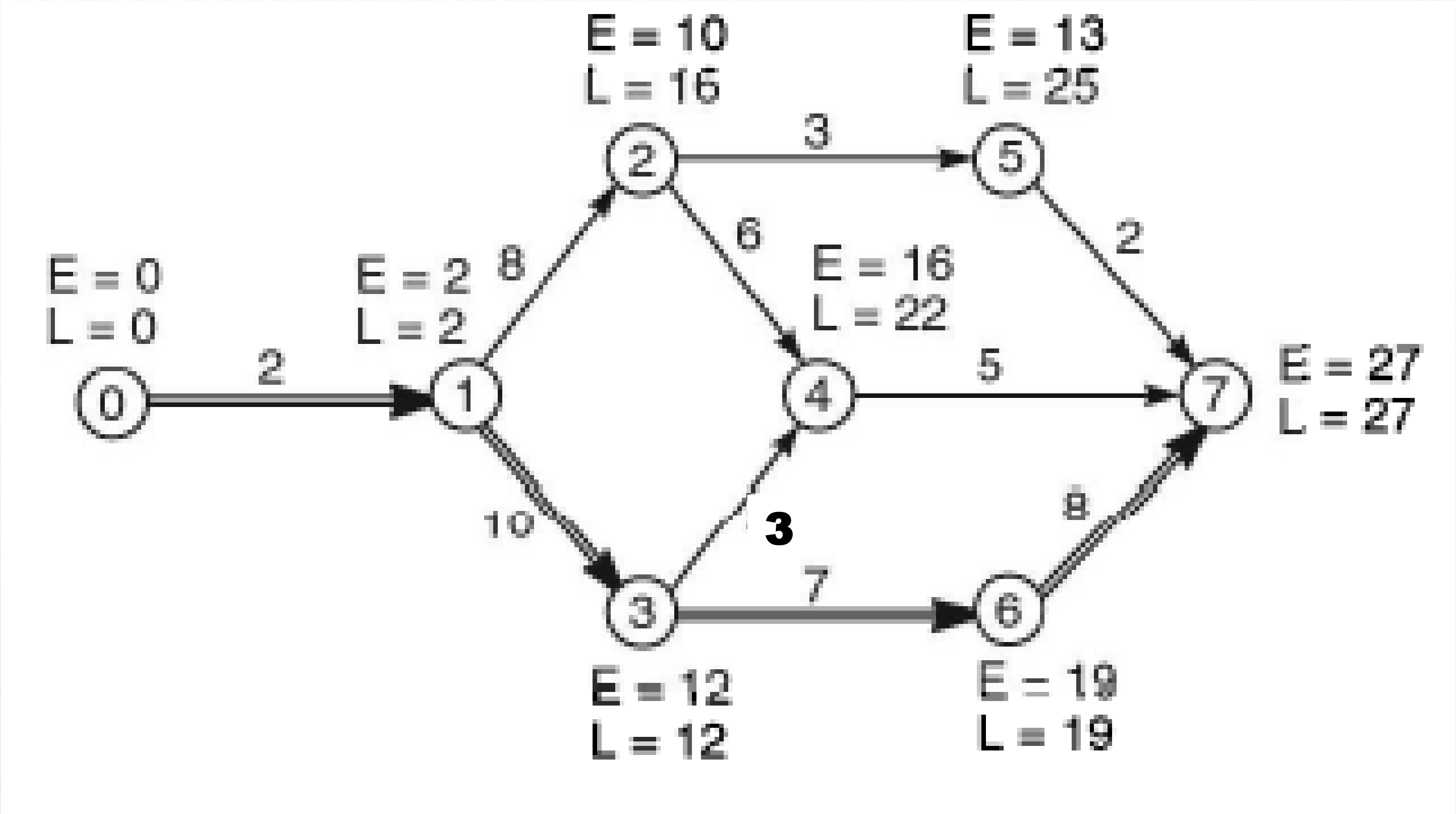
Critical Path = 1-2-3-4-6-7= 54 weeks

Example 2

The utility data for a network are given below. Determine the total, free, independent and interfering floats and identify the critical path.

Activity	:	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration	:	2	8	10	6	3	3	7	5	2	8

[*P.T.U. B. Tech. (Mech.) 2010; H.P.U.B. Tech. (Mech.) Nov., 2006; P.U.B. Com. Sept., 2004*]



E = 0
L = 0
E = 2
L = 2
E = 10
L = 16
E = 13
L = 25
E = 16
L = 22
E = 12
L = 12
E = 19
L = 19
E = 27
L = 27
0
1
2
3
4
5
6
7
2
8
10
3
6
3
7
5
2
8

Activity	Duration	Start time		Finish time		Float			
		Earliest	Latest	Earliest	Latest	Total	Free	Independent	Interfering
1	2	3	4	5	6	7	8	9	10
0 - 1	2	0	0	2	2	0	0	0	0
1 - 2	8	2	8	10	16	6	0	0	6
1 - 3	10	2	2	12	12	0	0	0	0
2 - 4	6	10	16	16	22	6	0	$-6 \approx 0$	6
2 - 5	3	10	22	13	25	12	0	$-6 \approx 0$	12
3 - 4	3	12	19	15	22	7	1	1	6
3 - 6	7	12	12	19	19	0	0	0	0
4 - 7	5	16	22	21	27	6	6	0	0
5 - 7	2	13	25	15	27	12	12	0	0
6 - 7	8	19	19	27	27	0	0	0	0

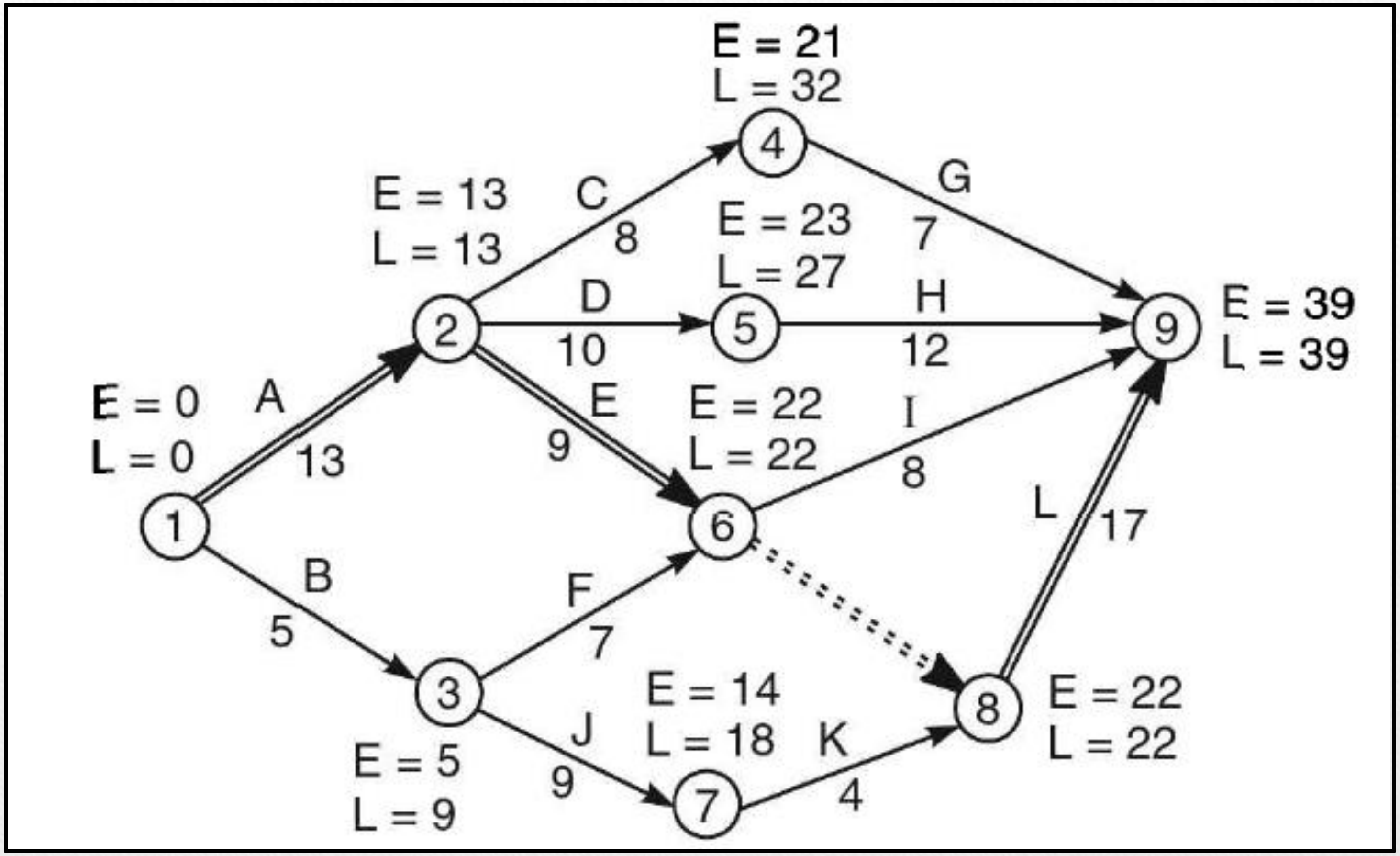
Example 3

Estimated times for the jobs of a project are given below:

Job	:	A	B	C	D	E	F	G	H	I	J	K	L
Time (weeks)	:	13	5	8	10	9	7	7	12	8	9	4	17

The constraints governing the jobs are as follows:

A and B are start jobs; A controls C, D and E; B controls F and J; G depends upon C; H depends on D; E and F control I and L; K follows J ; L is also controlled by K; G, H, I and L are the last jobs. Draw the network, determine project duration and the critical path.



Critical path is 1-2-6-8-9 project duration is 39 weeks.

Method 2. For identifying the critical path, the following conditions are checked. If an activity satisfies all the three conditions, it is critical.

(*i*) E = L for the tail event.

(*ii*) E = L for the head event.

(*iii*) $E_j - E_i = L_j - L_i = t_{ij}$.

Method 3: The various paths and their durations are calculated

PERT

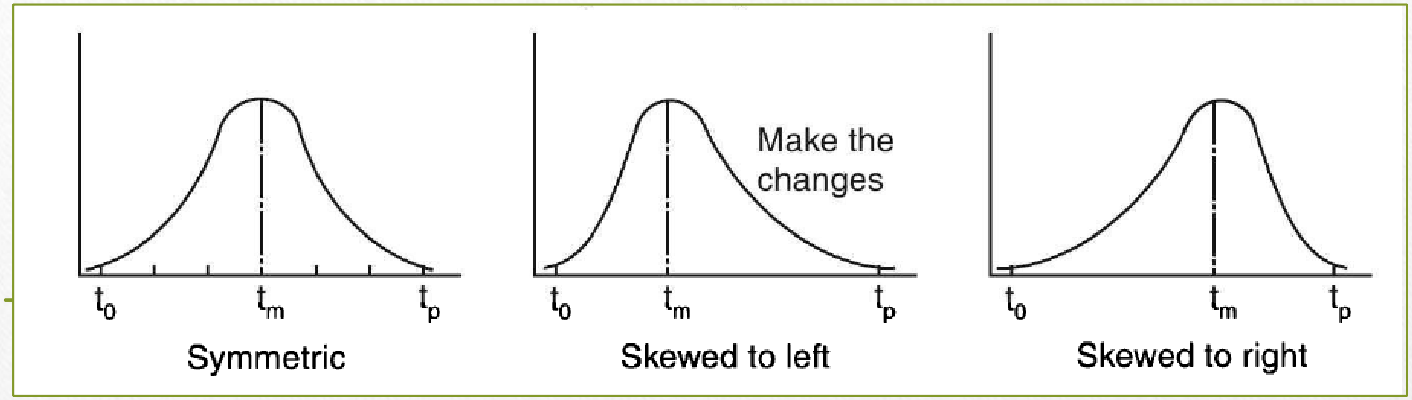
PERT is designed for scheduling complex projects that involve many inter-related tasks. it improves planning process because:

1. **It forms planner to define the projects various components activities.**
2. **It provides a basis for normal time estimates & yet allows for some measure of optimism or pessimism in estimating the completion dates.**
3. **It shows the effects of changes to overall plans they contemplated.**
4. **It provides built in means for ongoing evaluation of the plan.**

ESTIMATING ACTIVITY TIMES

- **Optimistic time (t_0)** : is that time estimate of an activity when everything is assumed to go as per plan. In other words it is the estimate of minimum possible time which an activity takes in completion under ideal conditions.
- **Most likely time (t_m)** : the time which the activity will take most frequently if repeated number of times.
- **Pessimistic time (tp)** : the unlikely but possible performance time if whatever could go wrong , goes wrong in series. In other words it is the longest time the activity can take.

EXPECTED TIME



- The times are combined statically to develop the expected time t_e

$$t_e = \mu = \frac{t_0 + 4t_m + t_p}{6}$$

- Standard deviation

$$\sigma = \frac{t_p - t_o}{6}$$

- Variance

$$V = \sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

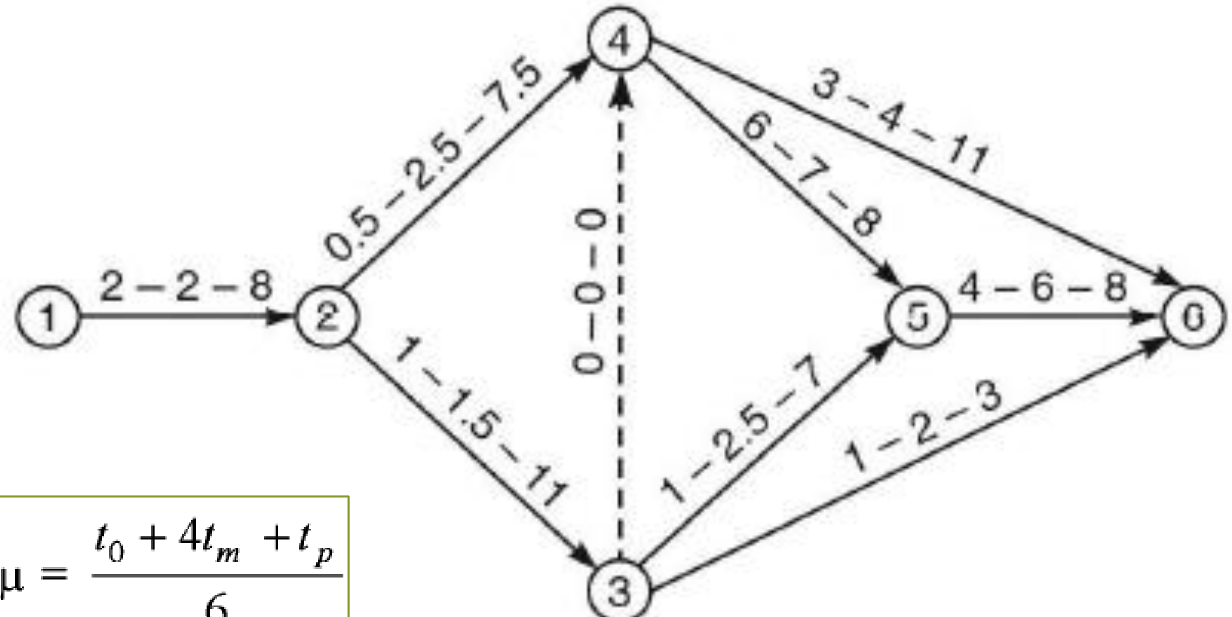
The expected time is then used as the activity duration and the critical path is obtained by the analytical method explained earlier.

STEPS INVOLVED IN PERT

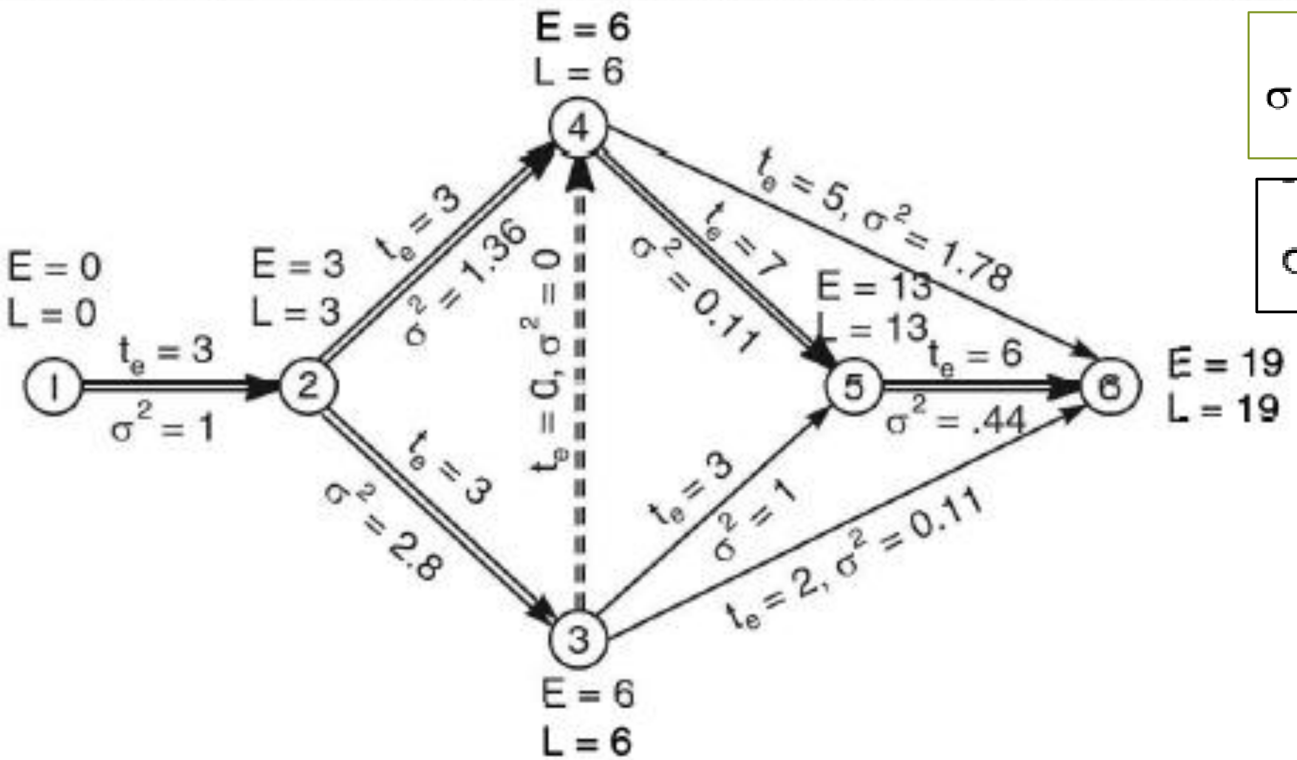
- Develop list of activities.
- A rough network for PERT is drawn.
- Events are numbered from left to right.
- Time estimates for each activity are obtained.
- Expected time for each activity is calculated : to+4tm+tp / 6
- Using these expected times calculate earliest & latest finish & start times of activities.
- Estimate the critical path.
- Using this estimate compute the probability of meeting a specified completion date by using the standard normal equation.

$$Z = \frac{\text{Due date – expected date of completion}}{\text{standard deviation of critical path}}$$

Consider the network shown in Fig 2. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 20 days?



$$t_e = \mu = \frac{t_0 + 4t_m + t_p}{6}$$



$$\sigma = \frac{t_p - t_o}{6}$$

$$\sigma = \sqrt{\Sigma\sigma_{ij}^2}$$

$\therefore$ σ for path 1-2-4-5-6 $= \sqrt{1+1.36+0.11+0.44} = 1.70,$

σ for path 1-2-3-4-5-6 $= \sqrt{1+2.8+0+0.11+0.44} = 2.08.$

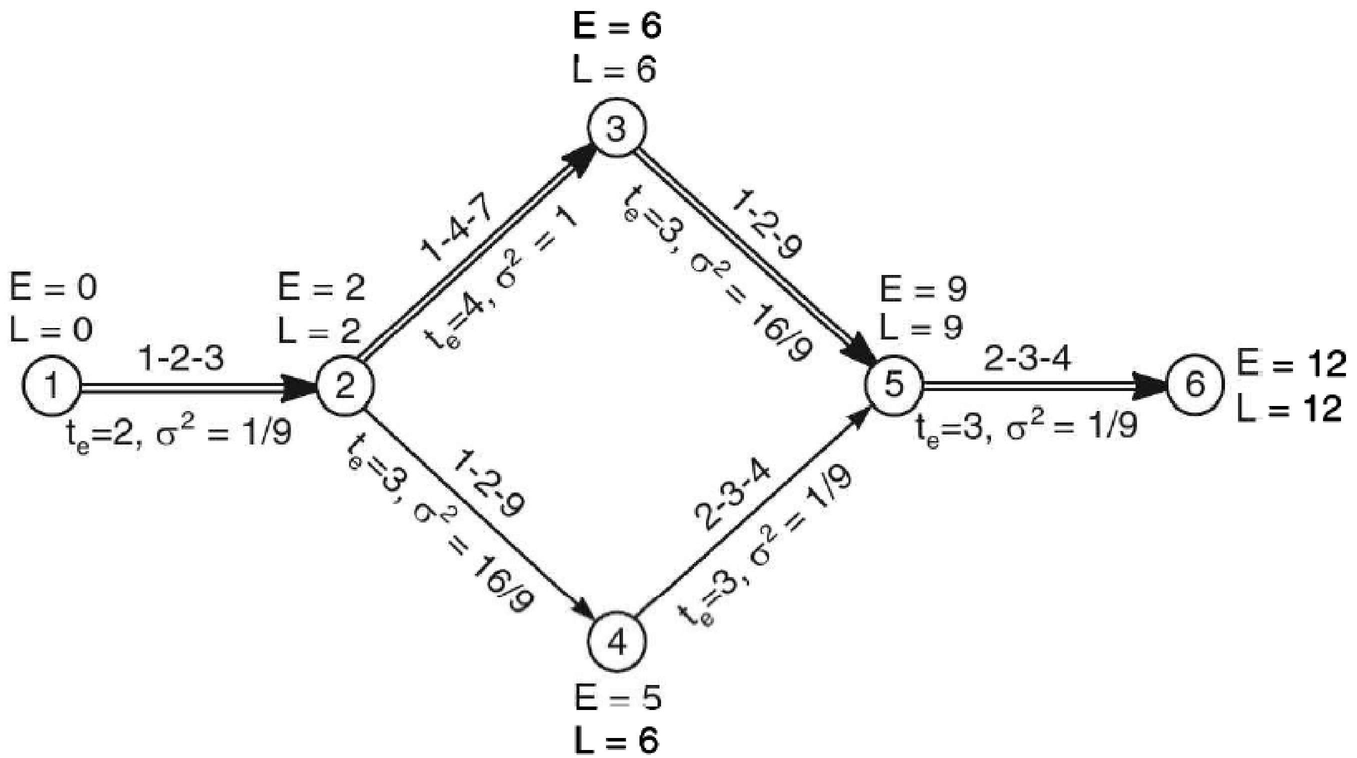
$\therefore$ $\sigma = 2.08$ is chosen as it is higher of the two values.

$\therefore$ Normal deviate, $Z = \dfrac{T - T_{cp}}{\sigma} = \dfrac{20-19}{2.08} = 0.48.$

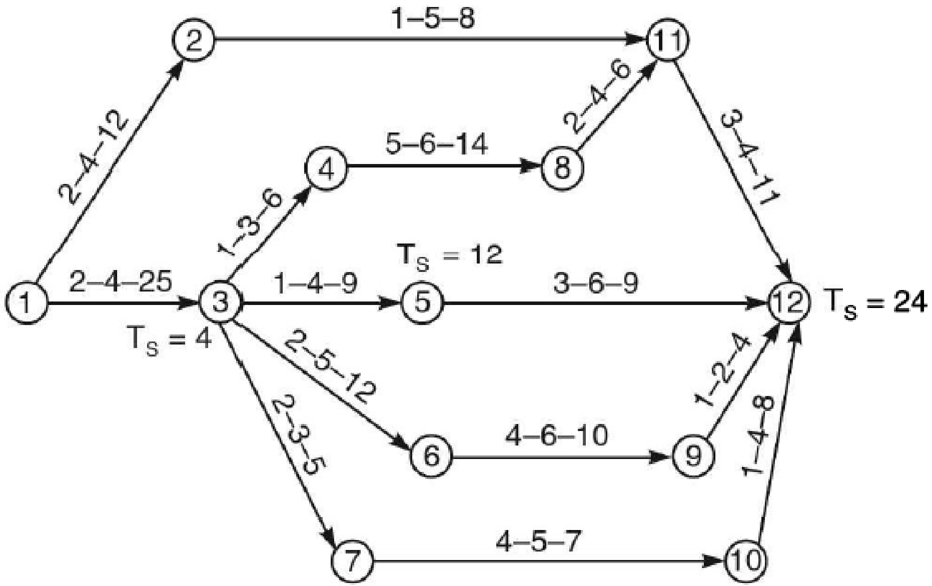
From table C-2, probability = 68.44%.

1-2-3-4-5-6 and 1-2-4-5-6 are the two critical paths.

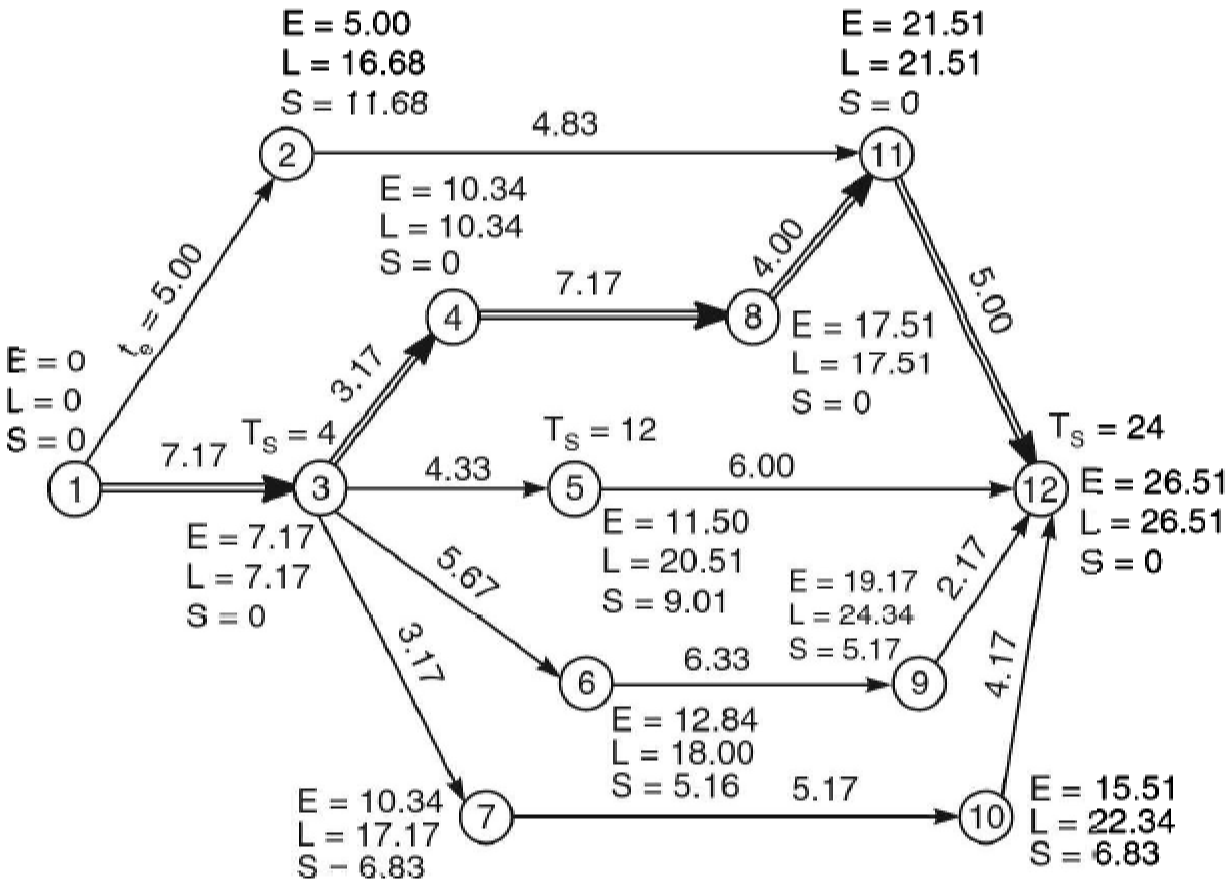
Consider the network shown in Fig. 14.34. The three time estimates, the expected activity durations and the variances are shown along the arrows. The earliest expected times and the latest allowable occurrence times are computed and put along the nodes. What is the probability of completing the project in (i) 12 days (ii) 14 days (iii) 10 days ?



A PERT network is shown in Fig. 14.37. The activity times in days are given along the arrows. The scheduled times for some important events are given along the nodes. Determine the critical path and probabilities of meeting the scheduled dates for the specified events. Tabulate the results and determine slack for each event.



The arrow diagram for the given data is shown in Fig. 14.38. The expected activity times are shown along the arrows. The earliest and latest occurrence times as well as the slacks of the events are also written along the nodes.



Probability of completing the project in the scheduled completion time of 24 days (since T_s (12) = 24):

$$Z = \frac{24 - 26.51}{\sqrt{14.69 + 0.69 + 2.25 + 0.44 + 1.78}} = \frac{-2.51}{\sqrt{19.85}} = -0.5634.$$

$\therefore$ $p\ (T_s \leq 24) = 1$ – value of probability for Z = 0.5634 = 1 – 0.7146 = 29.54%.

Probability that event 3 will occur on the scheduled date:

$$T_s\ (3) = 4,\ E = L = 7.17.$$

$$\therefore \quad Z = \frac{4 - 7.17}{\sqrt{14.69}} = -0.8271.$$

$\therefore$ p = 1 – value of probability for Z = 0. 8271 = 1 – 0.7956 = 20.44%.

Probability of meeting the scheduled date for event 5:

The earliest occurrence time of event 5 is 11.50, while the scheduled time is 12. Event 5 is not on critical path and hence its occurrence can be delayed by 9 days.

Variance of path 1-3-5 = 14.69 + 1.78 = 16.47.

$T_s = 12$,
E = 11.50.

$$\therefore \quad Z = \frac{12 - 11.50}{\sqrt{16.47}} = 0.123.$$

$\therefore$ Probability = 54.89%.

$T_s = 12$,
L = 20.51.

$$\therefore \quad Z = \frac{12 - 20.51}{\sqrt{16.47}} = -2.1.$$

$\therefore$ Probability = 1–0.982 = 0.018 = 1.8%.

Thus the probability of meeting the scheduled date in case of event 5 is less than or equal 54.89% with minimum of 1.8% *i.e.*, it lies between 1.8% and 54.89%.

QUESTION : The table shows activities of project.
Duration (days)

JOB	OPTIMISTIC	MOST LIKELY	PESSIMISTIC
1-2	1	4	7
1-3	5	10	15
2-4	3	3	3
2-6	1	4	7
3-4	10	15	26
3-5	2	4	6
4-5	5	5	5
5-6	2	5	8

1. Draw the network &find expected project completion time.
2. What is the probability that it would be completed in 41 days.

Variance of critical path

Activity	To	Tp	variance
1-3	5	15	2.77
3-4	10	26	7.11
4-5	5	5	0
5-6	2	8	1
			Total= 10.88

So, standard deviation = 3.30

Probability for completing the job in 41 days.

$$Z = \frac{\text{DUE DATE} - \text{EXPECTED DATE OF COMPLETION}}{\text{S.D. OF CRITICAL PATH}}$$

$$= \frac{41 - 36}{3.30}$$

$$= 1.51$$

The tabulated value of corresponding to calculated value i.e. 1.51 is .9345 i.e. 93.45% that project will be completed on 41 day.

QUESTION:

Activity predecessor time estimates (weeks)

activity	Preceding activity	to	tm	tp
A	-	2	3	10
B	-	2	3	4
C	A	1	2	3
D	A	4	6	14
E	B	4	5	12
F	C	3	4	5
G	D,E	1	1	7

- Find the expected duration and variance of each activity.
- What is the expected project length?
- Calculate the variance &standard deviation of the project length.

activity	to	Tm	tp	te	variance
A	2	3	10	4	16/9
B	2	3	4	3	1/9
C	1	2	3	2	1/9
D	4	6	14	7	25/9
E	4	5	12	6	16/9
F	3	4	5	4	1/9
G	1	2	7	2	1

Various paths &expected project length

A-C-F = 4+2+4 = 10

A-D-G = 4+7+2 = 13 CRITICAL PATH

B-E-G = 3+6+2 = 11

Thus critical path is A-D-G with an expected length of 13 days. Hence the expected project length is 13 days.

Project variance = 16/9 + 25/9 + 1 = 50/9

TABLE C-2

Proportion of total area under the normal curve from $-\infty$ to z.

where $z = \frac{x - \mu}{\sigma}$.

z	$\psi(z)$	z	$\psi(z)$	z	$\psi(z)$	z	$\psi(z)$
0.00	0.5000	0.65	0.7422	1.30	0.9032	1.95	0.9744
0.01	0.5040	0.66	0.7454	1.31	0.9049	1.96	0.9750
0.02	0.5080	0.67	0.7486	1.32	0.9066	1.97	0.9756
0.03	0.5120	0.68	0.7517	1.33	0.9082	1.98	0.9761
0.04	0.5160	0.69	0.7549	1.34	0.9099	1.99	0.9767
0.05	0.5199	0.70	0.7580	1.35	0.9115	2.00	0.9772
0.06	0.5239	0.71	0.7611	1.36	0.9131	2.02	0.9783
0.07	0.5279	0.72	0.7642	1.37	0.9147	2.04	0.9793
0.08	0.5319	0.73	0.7673	1.38	0.9162	2.06	0.9803
0.09	0.5359	0.74	0.7703	1.39	0.9177	2.08	0.9812
0.10	0.5398	0.75	0.7734	1.40	0.9192	2.10	0.9821
0.11	0.5438	0.76	0.7764	1.41	0.9207	2.12	0.9830
0.12	0.5478	0.77	0.7794	1.42	0.9222	2.14	0.9838
0.13	0.5517	0.78	0.7823	1.43	0.9236	2.16	0.9846
0.14	0.5557	0.79	0.7852	1.44	0.9251	2.18	0.9854
0.15	0.5596	0.80	0.7881	1.45	0.9265	2.20	0.9861
0.16	0.5636	0.81	0.7910	1.46	0.9279	2.22	0.9868
0.17	0.5675	0.82	0.7939	1.47	0.9292	2.24	0.9875
0.18	0.5714	0.83	0.7967	1.48	0.9306	2.26	0.9881
0.19	0.5753	0.84	0.7995	1.49	0.9319	2.28	0.9887
0.20	0.5793	0.85	0.8023	1.50	0.9332	2.30	0.9893
0.21	0.5832	0.86	0.8051	1.51	0.9345	2.32	0.9898
0.22	0.5871	0.87	0.8078	1.52	0.9357	2.34	0.9904
0.23	0.5910	0.88	0.8106	1.53	0.9370	2.36	0.9909
0.24	0.5948	0.89	0.8133	1.54	0.9382	2.38	0.9913
0.25	0.5987	0.90	0.8159	1.55	0.9394	2.40	0.9918
0.26	0.6026	0.91	0.8186	1.56	0.9406	2.42	0.9922
0.27	0.6064	0.92	0.8212	1.57	0.9418	2.44	0.9927
0.28	0.6103	0.93	0.8238	1.58	0.9429	2.46	0.9931
0.29	0.6141	0.94	0.8264	1.59	0.9441	2.48	0.9934
0.30	0.6179	0.95	0.8289	1.60	0.9452	2.50	0.9938
0.31	0.6217	0.96	0.8315	1.61	0.9463	2.52	0.9941
0.32	0.6255	0.97	0.8340	1.62	0.9474	2.54	0.9945
0.33	0.6293	0.98	0.8365	1.63	0.9484	2.56	0.9948
0.34	0.6331	1.99	0.8389	1.64	0.9495	2.58	0.9951
0.35	0.6368	1.00	0.8413	1.65	0.9505	2.60	0.9953
0.36	0.6406	1.01	0.8438	1.66	0.9515	2.62	0.9956
0.37	0.6443	1.02	0.8461	1.67	0.9525	2.64	0.9959
0.38	0.6480	1.03	0.8485	1.68	0.9535	2.66	0.9961
0.39	0.6517	1.04	0.8508	1.69	0.9545	2.68	0.9963
0.40	0.6554	1.05	0.8531	1.70	0.9554	2.70	0.9965

0.41	0.6591	1.06	0.8554	1.71	0.9564	2.72	0.9967
0.42	0.6628	1.07	0.8577	1.72	0.9573	2.74	0.9969
0.43	0.6664	1.08	0.8599	1.73	0.9582	2.76	0.9971
0.44	0.6700	1.09	0.8621	1.74	0.9591	2.78	0.9973
0.45	0.6736	1.10	0.8643	1.75	0.9599	2.80	0.9974
0.46	0.6772	1.11	0.8665	1.76	0.9608	2.82	0.9976
0.47	0.6808	1.12	0.8686	1.77	0.9616	2.84	0.9977
0.48	0.6844	1.13	0.8708	1.78	0.9625	2.86	0.9979
0.49	0.6879	1.14	0.8729	1.79	0.9633	2.88	0.9980
0.50	0.6915	1.15	0.8749	1.80	0.9641	2.90	0.9981
0.51	0.6950	1.16	0.8770	1.81	0.9649	2.92	0.9982
0.52	0.6985	1.17	0.8790	1.82	0.9656	2.94	0.9984
0.53	0.7019	1.18	0.8810	1.83	0.9664	2.96	0.9985
0.54	0.7054	1.19	0.8830	1.84	0.9671	2.98	0.9986
0.55	0.7088	1.20	0.8849	1.85	0.9678	3.00	0.99865
0.56	0.7123	1.21	0.8869	1.86	0.9686	3.20	0.99931
0.57	0.7157	1.22	0.8888	1.87	0.9693	3.40	0.99966
0.58	0.7190	1.23	0.8907	1.88	0.9699	3.60	0.999841
0.59	0.7224	1.24	0.8925	1.89	0.9706	3.80	0.999928
0.60	0.7257	1.25	0.8944	1.90	0.9713	4.00	0.999968
0.61	0.7291	1.26	0.8962	1.91	0.9719	4.50	0.999997
0.62	0.7324	1.27	0.8980	1.92	0.9726	5.00	0.999997
0.63	0.7357	1.28	0.8997	1.93	0.9732		
0.64	0.7389	1.29	0.9015	1.94	0.9738		