

Vigenere Cipher

The best known, and one of the simplest, polyalphabetic ciphers is the Vigenère cipher. In this scheme, the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers with shifts of 0 through 25

Method 1

We can express the Vigenère cipher in the following manner. Assume a sequence of plaintext letters $P = p_0, p_1, p_2, \dots, p_{n-1}$ and a key consisting of the sequence of letters $K = k_0, k_1, k_2, \dots, k_{m-1}$, where typically $m < n$. The sequence of ciphertext letters $C = C_0, C_1, C_2, \dots, C_{n-1}$ is calculated as follows:

$$\begin{aligned} C &= C_0, C_1, C_2, \dots, C_{n-1} = E(K, P) = E[(k_0, k_1, k_2, \dots, k_{m-1}), (p_0, p_1, p_2, \dots, p_{n-1})] \\ &= (p_0 + k_0) \bmod 26, (p_1 + k_1) \bmod 26, \dots, (p_{m-1} + k_{m-1}) \bmod 26, \\ &\quad (p_m + k_0) \bmod 26, (p_{m+1} + k_1) \bmod 26, \dots, (p_{2m-1} + k_{m-1}) \bmod 26, \dots \end{aligned}$$

Thus, the first letter of the key is added to the first letter of the plaintext, mod 26, the second letters are added, and so on through the first m letters of the plaintext.

For the next m letters of the plaintext, the key letters are repeated. This process continues until all of the plaintext sequence is encrypted. A general equation of the encryption process is as follows:

$$C_i = (p_i + k_{i \bmod m}) \bmod 26$$

Compare this with the Caesar cipher. In essence, each plaintext character is encrypted with a different Caesar cipher, depending on the corresponding key character. Similarly, decryption can be shown as:

$$p_i = (C_i - k_{i \bmod m}) \bmod 26$$

To encrypt a message, a key is needed that is as long as the message. Usually, the key is a repeating keyword. For example, if the keyword is *deceptive*, the message "we are discovered save yourself" is encrypted as

| | |
|-------------|--|
| key: | <u>d</u> e <u>c</u> e <u>p</u> t <u>i</u> v <u>e</u> d <u>e</u> c <u>e</u> p <u>t</u> i <u>v</u> e <u>e</u> d <u>e</u> c <u>e</u> p <u>t</u> i <u>v</u> e |
| plaintext: | w <u>e</u> a <u>r</u> e <u>d</u> i <u>s</u> c <u>o</u> v <u>e</u> r <u>e</u> d <u>s</u> a <u>v</u> e <u>y</u> <u>o</u> <u>u</u> <u>r</u> <u>s</u> <u>e</u> <u>l</u> <u>f</u> |
| ciphertext: | Z <u>I</u> C <u>V</u> T <u>W</u> Q <u>N</u> G <u>R</u> Z <u>G</u> V <u>T</u> W <u>A</u> V <u>Z</u> H <u>C</u> Q <u>Y</u> G <u>L</u> M <u>G</u> J |

Expressed numerically, we have the following result.

| | | | | | | | | | | | | | | |
|------------|----|---|---|----|----|----|----|----|---|----|----|---|----|----|
| key | 3 | 4 | 2 | 4 | 15 | 19 | 8 | 21 | 4 | 3 | 4 | 2 | 4 | 15 |
| plaintext | 22 | 4 | 0 | 17 | 4 | 3 | 8 | 18 | 2 | 14 | 21 | 4 | 17 | 4 |
| ciphertext | 25 | 8 | 2 | 21 | 19 | 22 | 16 | 13 | 6 | 17 | 25 | 6 | 21 | 19 |

| | | | | | | | | | | | | | |
|------------|----|----|----|----|---|----|----|----|----|----|----|----|---|
| key | 19 | 8 | 21 | 4 | 3 | 4 | 2 | 4 | 15 | 19 | 8 | 21 | 4 |
| plaintext | 3 | 18 | 0 | 21 | 4 | 24 | 14 | 20 | 17 | 18 | 4 | 11 | 5 |
| ciphertext | 22 | 0 | 21 | 25 | 7 | 2 | 16 | 24 | 6 | 11 | 12 | 6 | 9 |

Method 2 (Using Vigenere Table)

→ Plaintext →

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| A | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| B | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A |
| C | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |
| D | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |
| F | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E |
| G | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F |
| H | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G |
| I | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |
| J | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |
| K | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J |
| L | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |
| M | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L |
| N | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M |
| O | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
| P | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| Q | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| R | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q |
| S | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R |
| T | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
| U | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| V | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U |
| W | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |
| X | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |
| Y | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X |
| Z | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y |

↑ KEY ↓

Plaintext : Universal

Key : College

* Here Size of Key < PT,
So repeat it.

Ciphertext Obtained

WBTGIXWCZ 1

Encryption

The first letter of the plaintext is combined with the first letter of the key. The column of plain text "U" and row of key "C" intersects the alphabet of "W" in the vigenere table, so the first letter of ciphertext is "W".

Similarly, the second letter of the plaintext is combined with the second letter of the key. The column of plain text "N" and row of key "O" intersects the alphabet of "B" in the vigenere table, so the second letter of ciphertext is "B".

This process continues continuously until the plaintext is finished.

Decryption

Decryption is done by the row of keys in the vigenere table. First, select the row of the key letter, find the ciphertext letter's position in that row, and then select the column label of the corresponding ciphertext as the plaintext.

For example, in the row of the key "C" and the ciphertext "W", this ciphertext letter appears in the column "U", that means the first plaintext letter is "U".

Next, in the row of the key "O" and the ciphertext "B", this ciphertext letter appears in the column "N", that means the second plaintext letter is "N".

This process continues until the ciphertext is finished.

Plaintext Obtained

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Strength and Weaknesses of Vigenere Cipher

The strength of this cipher is that there are multiple ciphertext letters for each plaintext letter, one for each unique letter of the keyword. Thus, the letter frequency information is obscured. However, not all knowledge of the plaintext structure is lost.

First, suppose that the opponent believes that the ciphertext was encrypted using either monoalphabetic substitution or a Vigenère cipher. A simple test can be made to make a determination.

If a monoalphabetic substitution is used, then the statistical properties of the ciphertext should be the same as that of the language of the plaintext.

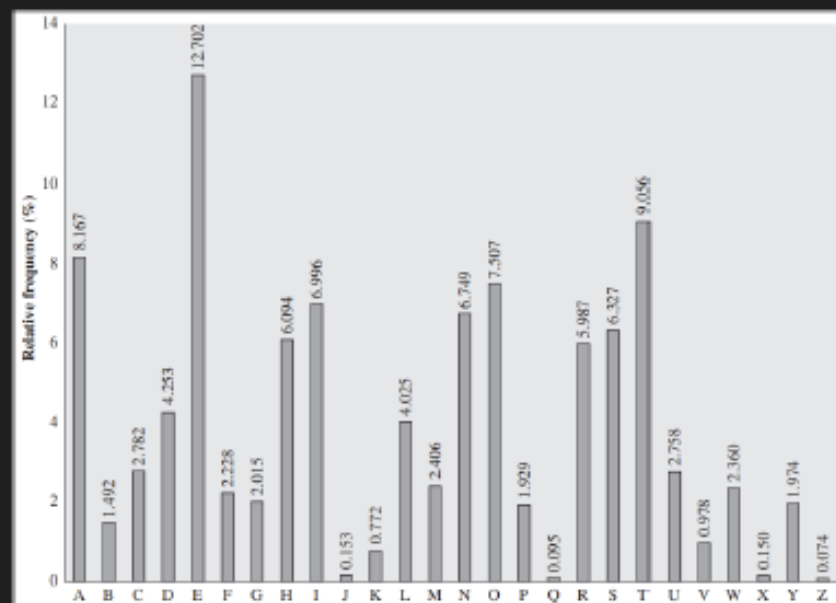


Figure 2.5 Relative Frequency of Letters in English Text

Thus, referring to Figure 2.5, there should be one cipherletter with a relative frequency of occurrence of about 12.7%, one with about 9.06%, and so on. If only a single message is available for analysis, we would not expect an exact match of this small sample with the statistical profile of the plaintext language. Nevertheless, if the correspondence is close, we can assume a monoalphabetic substitution.

| | |
|-------------|-----------------------------|
| key: | deceptive |
| plaintext: | wearediscoveredsaveyourself |
| ciphertext: | ZICVTWQNGRZGVTVAVZHCQYGLMGJ |

What if it is vigenere Cipher??

If, on the other hand, a Vigenère cipher is suspected, then progress depends on determining the length of the keyword, as will be seen in a moment. For now, let us concentrate on how the keyword length can be determined. The important insight that leads to a solution is the following: If two identical sequences of plaintext letters occur at a distance that is an integer multiple of the keyword length, they will generate identical ciphertext sequences. In the foregoing example, two instances of the sequence "red" are separated by nine character positions. Consequently, in both cases, r is encrypted using key letter e, e is encrypted using key letter p, and d is encrypted using key letter t. Thus, in both cases, the ciphertext sequence is VTW. We indicate this above by underlining the relevant ciphertext letters and shading the relevant ciphertext numbers.

An analyst looking at only the ciphertext would detect the repeated sequences VTW at a displacement of 9 and make the assumption that the keyword is either three or nine letters in length. The appearance of VTW twice could be by chance and may not reflect identical plaintext letters encrypted with identical key letters. However, if the message is long enough, there will be a number of such repeated ciphertext sequences. By looking for common factors in the displacements of the various sequences, the analyst should be able to make a good guess of the keyword length.

Solution of the cipher now depends on an important insight. If the keyword length is m , then the cipher, in effect, consists of m monoalphabetic substitution ciphers. For example, with the keyword DECEPTIVE, the letters in positions 1, 10, 19, and so on are all encrypted with the same monoalphabetic cipher. Thus, we can use the known frequency characteristics of the plaintext language to attack each of the monoalphabetic ciphers separately.

The periodic nature of the keyword can be eliminated by using a nonrepeating keyword that is as long as the message itself. Vigenère proposed what is referred to as an **autokey system**, in which a keyword is concatenated with the plaintext itself to provide a running key. For our example,

| | |
|-------------|-----------------------------|
| key: | deceptivewearediscoveredsav |
| plaintext: | wearediscoveredsaveyourself |
| ciphertext: | ZICVTWQNGKZEIIGASXSTSLVWLA |

