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Tutorial 1

Name: Shasvat Shah
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Discrete Structures
Sets & Logics.

1 Determine the number of integers between 1 & 250 that are divisible by 2 or 3 ~~and~~ or 5 or 7.

→ Let

A be the set of all numbers between 1 to 250 divisible by 2.

B be the set of all numbers between 1 to 250 divisible by 3.

C be the set of all numbers between 1 to 250 divisibly by 5.

D be the set of all numbers between 1 to 250 divisible by 7.

$$|A| = \frac{250}{2} = 125$$

$$|B| = \frac{\cancel{20}\ 250}{3} = 83$$

$$|C| = \frac{250}{5} = 50$$

$$|D| = \frac{250}{7} = 35$$

$$|A \cap B| = \frac{250}{2 \times 3} = 41$$

$$|C \cap D| = \frac{250}{7 \times 5} = 7$$

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$|A \cup B| = |A| + |B| - |A \cap B|$

$= 125 + 83 - 41$

$= 167$, By addition principle

CA $|C \cup B| = \cancel{A}\, |C| + |D| - |C \cap D|$

$= 50 + 35 - 7$

$= 78$, By addition Principle.

$|A \cup B| \cup |C \cup B| = |A \cup B| + |C \cup D| - (|A \cup B| \cap |C \cup D|)$

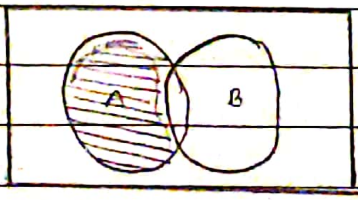
$= 167 + 78 - \dfrac{250}{2 \times 3 \times 7 \times 5}$

$= 167 + 78 - 1$

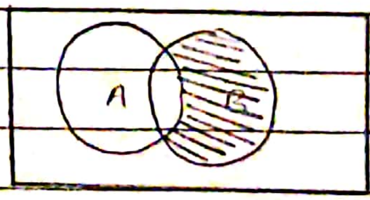
$= \underline{244}$

$\therefore$ Out of 250 numbers 244 numbers are divisible by 2, 3, 5 or 7.

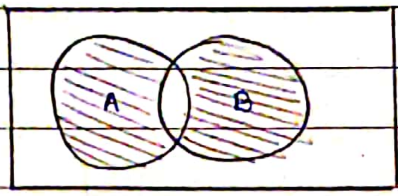
2 Prove the following $(A - B) \cup (B - A) = (A \cup B) - (A$



A − B

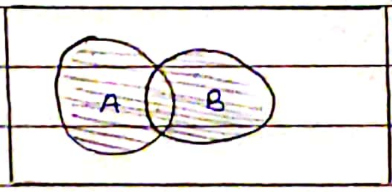


B − A

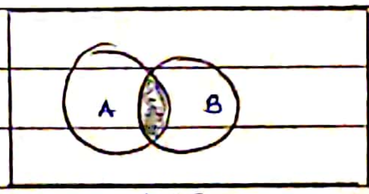


$(A - B) \cup (B - A)$

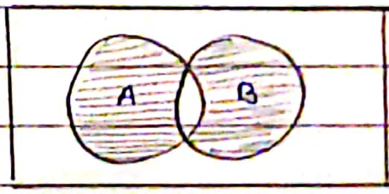
LHS



$A \cup B$



$A \cap B$



$(A \cup B) - (A \cap B)$

RHS

$\therefore$ LHS = RHS

3 Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let set $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$C = \{5, 6, 7\}$

$\therefore$

$B \cap C = \{5\}$.

$A \times (B \cap C) = \{(1,5), (2,5), (3,5)\}$. ... LHS

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5)\}$.

$A \times C = \{(1,5) (1,6) (1,7) (2,5) (2,6) (2,7) (3,5) (3,6) (3,7)$

$(A \times B) \cap (A \times C) = \{(1,5) (2,5) (3,5)\}$. ... RHS

$\therefore$ Hence Proved

4 Proves using Laws of Logic $[(p \vee q) \cap (p \vee \sim q)] \vee q \leftrightarrow p \vee q$

$\therefore$

LHS $= [(p \vee q) \cap (p \vee \sim q)] \vee q$

$= [p \vee (q \wedge \sim q)] \vee q$

$= [p \vee F] \vee q$

$= p \vee q$ RHS $= p \vee q$

$\therefore$ LHS = RHS, Hence Proved.

5 Verify that the preposition $p \vee \sim(p \vee q)$ is a tautology

p	q	$p \vee q$	$\sim(p \vee q)$	$p \vee \sim(p \vee q)$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	F
F	F	F	T	T

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

tautology.

Since the truth table for the below eqvation is (T, T, T, T). Hence it is a tautology.

6 Construct the truth table to determine whether each of the following is a tautology, contradiction or a contingency.

i) $(q \wedge p) \vee (q \wedge \sim p)$

p	q	$\sim p$	$q \wedge p$	$q \wedge \sim p$	Final,
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	F	T	T
F	F	T	F	F	F

contingency

ii) $q \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$q \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

contigency.

iii) $p \rightarrow (q \wedge p)$

p	q	$q \wedge p$	$p \rightarrow (q \wedge p)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Contingency.

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7 Write the english sentences for the following where

$P(x)$: x is even.

$Q(x)$: x is prime

$R(x, y)$: $x + y$.

i) $\exists x \forall y \; R(x, y)$

There exists a value of x for all y such that relation $R(x, y)$ where relation $(x+y)$ exists.

ii) $\sim (\exists x \; P(x))$

There doesnt exist any values of x where the value of x is even.

iii) $\sim (\forall x \; Q(x))$

There doesnt exist any values of x where the value of x is a prime.

iv) $\forall x \; (\sim Q(x))$

For all values of x, the value of x is not prime

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Tutorial 2

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Discrete Structures

1 Given that the truth values of x, y, and z as T and those of u & v as F. Find the truth values of $(x \wedge (y \vee z)) \wedge \sim((x \vee z) \wedge (u \vee v) \wedge z)$

$\therefore \quad x = T \quad y = T \quad z = T \quad u = F \quad v = F$

$(T \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$

$(T \wedge T) \wedge \sim[(T \wedge F) \wedge T]$

$T \wedge \sim[F \wedge T]$

$T \wedge \sim[F]$

$T \wedge \sim F$

$T \wedge T$

$= T$

2 Prove using laws of logic

i) $a \to (p \vee c) \leftrightarrow (a \wedge \sim p) \to c$

$p \to q = \sim p \vee q$

$\therefore \quad \sim a \vee (p \vee c)$ — LHS

$\sim(a \wedge \sim p) \vee c$ — RHS

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LHS ...

$$\sim a \vee (p \vee c)$$

$$(\sim a \vee p) \vee (\sim a \vee c)$$

RHS ...

$$\sim (a \wedge \sim p) \vee c$$

$$(\sim a \vee p) \vee c$$

$$\sim a \vee (p \vee c) \ ...$$

$\therefore$ LHS = RHS, Hence Proved.

(iii) $\sim (p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

LHS $= \sim (p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q))$

$= \sim(\sim p \vee \sim q) \vee (\sim p \vee (\sim p \vee q))$

$= (p \vee q) \vee (\sim p \vee q)$

$= [(p \vee \sim p) \vee (p \vee q)] \wedge [(q \vee \sim p) \vee q]$

$= T \wedge [\sim p \vee q]$

$= \sim p \vee q$

$=$ RHS.

$\therefore$ LHS = RHS Hence Proved.

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Q3 Write English sentence for the following where
$P(x)$ is even.
$Q(x)$: x is prime.
$R(x, y)$: $x \cdot y$ is even.

i) $\exists x \ \forall y \ R(x,y)$
There exists a value for x for all values of y such that $R(x,y)$ or $x \cdot y$ is even.

ii) $\forall x \ \exists y \ R(x,y)$
For all values of x, there exists a value for y such that the relation $R(x,y)$ or $x.y$ holds true for even.

iii) $\sim (\exists x \ P(x))$
There doesn't exist values of x when x is even.

iv) $\sim (\forall x \ Q(x))$
There exists no values of x when x is a prime number.

v) $\exists y \ [\sim P(y)]$
There exists a value for y when ~~x~~ y is not even.

vi) $\forall x \ (\sim Q(x))$
For all values of x, there is a value when x is not a prime.

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4 At a university we know,
60% of the professors play tennis
50% of the professors play bridge.
70% of the professors jog.
20% play tennis and bridge.
30% tennis & jog.
40% bridge and jog.

If someone claimed that 20% of the professors jog, play bridge & tennis, would you believe the claim, why.

→

Let the professors playing tennis be represented by T
Let the professors playing bridge be represented by B
Let the professors ~~playing~~ jogging be represented by J.
Let U be the universal set $|U| = 100$

$|U| = 100 = |A \cup B \cup C|$

$|J| = 70$

$|B| = 50$

$|T| = 60$

$|T \cap J| = 30$ $\qquad |T \cap B \cap J| = ?$

$|T \cap B| = 20$

$|B \cap J| = 40.$

$|T \cup B \cup J| = |T| + |B| + |J| - |T \cap J| - |T \cap B| - |J \cap B| + |T \cap B \cap J|$

$100 = 60 + 70 + 50 - 40 - 30 - 20 + |T \cap B \cap J|$

$100 - 90 = |T \cap B \cap J|$

$10 = |T \cap B \cap J|$

Hence the claim wouldnt be believed. As 10% is common

5 Let $A = \{a, b, c, d, e, f, g, h\}$. Consider the following subset of A.

$$A_1 = \{a, b, c, d\}$$
$$A_2 = \{a, c, e, f, g, h\}$$
$$A_3 = \{a, c, e, g\}$$
$$A_4 = \{b, d\}$$
$$A_5 = \{f, h\}$$

Determine whether each of the following is a partition or not.

i) A_1 & A_2

$$\{A_1, A_2\}.$$

Not a partition, because they arent mutually disjoint.

ii) A_2 & $A_{\cancel{3}4}$

$$\{A_2, A_4\}$$

Yes it is a partition,

iii) A_1, A_3

$$\{A_1, A_3\}$$

Not a partiton, because they arent mutually disjoint & $A_1 \cup A_3 \neq A$.

iv) A_3, A_4, A_5

$$\{A_3, A_4, A_5\}$$

Yes it is a partition,

because

$$A_3 \cup A_4 \cup A_5 = A$$
$$A_3 \cap A_4 \cap A_5 \neq A$$

6 Let the Universal set be

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 7, 9\}$

$B = \{1, 4, 6, 7, 10\}$

$C = \{3, 5, 9, 7\}$

→ Given

$|A| = 4$

$|B| = 5$

$|C| = 4$

$|U| = 10$

i) $A \cup B \rightarrow \{1, 2, 4, 6, 7, 9, 10\}$

ii) $A \cap C \rightarrow \{7, 9\}$

iii) $B \cap \bar{C} \rightarrow \{1, 4, 6, 10\}$

iv) $\cancel{A \cup}\ (A \cap \bar{B}) \cup C$ $\quad \bar{B} = (2, 3, 5, 8, 9)$

$(A \cup \bar{B}) \rightarrow \{2, 3, 4, 5, 7, 8, 9\}$

$(A \cap \bar{B}) \rightarrow \{2, 9\}$

$(A \cap \bar{B}) \cup C \rightarrow \{2, 3, 5, 7, 9\}$

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Tutorial 3

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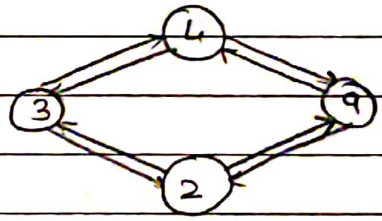
Discrete Structures
Relations & Functions

1 Given,

$A = \{2, 3, 4, 6, 9\}$

Relation = 'x is negatively prime to y'

$R = \{(2,3)(2,9)(3,2)(3,4)(4,3)(4,9)(9,4),(9,2)\}$

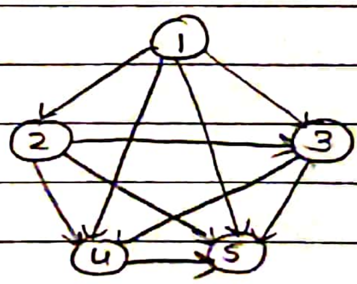


Shows the directed graph of the above relation R.

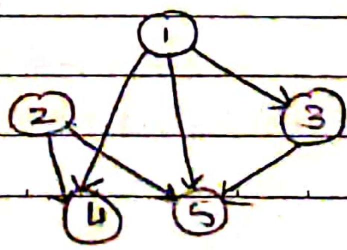
2 Given $A = \{1, 2, 3, 4, 5\}$

Find R, R^2, R^3 aRb if $\cancel{a \neq b}$ $a < b$

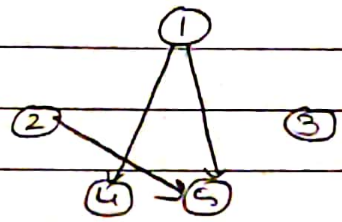
$R = \{(1,2)(1,3)(1,4)(1,5)(2,3)(2,4)(2,5)(3,4)(3,5)(4,5)\}$.



$R^2 = \{(1,3)(1,4)(1,5)(2,4)(2,5)(3,5)\}$



$R^3 = \{((1,4)\ (1,5)\ (2,5)\}$



3 $S = \{1,2,3,4\}$

$R = \{(4,3),\ (2,2),\ (2,1)\ (3,1)\ (1,2)\}$

i) Show that the relation is transitive.

For R to be transitive it must satisfy the condition aRb & bRc then aRc

Here $(4,1) \in R$ should be satisfied.

Hence for it to be transitive (4,1) should be present in R.

$\therefore$ R is not transitive.

ii) Find a relation $R_1 \subseteq R$ such that R is transitive.

For a transitive set if aRb & bRc then aRc should be present.

$$M_R = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Subset $M_{R_1} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 1 & 1 \end{array}$ is transitive.

since (1,2) (2,1) (2,2)

$\therefore$ its transitive.

$R_1 = \{(2,1)\ (1,2)\ (2,2)\}$

iii) Transitive closure of R by warshalls algorithm,

$R = \{(4,3)(2,2)(2,1)(3,1)(1,2)\}$.

$$W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

col ~~row~~ — row ~~col~~

2, 3 — 2, ~~3~~

(2,2) ~~(2,3)~~ (3,2)

∴ Add pairs that dont exist in the Relation.

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

col ~~row~~ — row ~~col~~

1, 2, 3 — 1, 2

(1,1)(1,2) (2,1)(2,2) (3,1) (3,2)

∴ Add pairs that dont exist in W_1

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

col — row

4 — 1, 2

~~(1,4)~~ ~~(2,4)~~ (4,1) (4,2)

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

col — row

0 — 1, 2, 3

$= \phi$

$RT = \{(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)(4,1)(4,2)(4,3)\}$

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4 $A = \{11, 12, 13, 14\}$.

$R = \{(11, 12)\ (12, 13)\ (13, 14)\ (12, 11)\}$.

Find transitive closure of R using warshalls algo.

$$W_0 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 0 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

col row
12 12

(12, 12)

$$W_1 = \begin{array}{c|cccc} 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

col row
11 11
12 12
13

(11, 11) (11, 12) (11, 13) (12, 11) (12, 12) (12, 13).

$$W_2 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

col row
11 14
12

(11, 14) (12, 14)

$$W_3 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 1 \\ 12 & 1 & 1 & 1 & 1 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

col row.
11 0
12
13

$\therefore R_T = \{(11, 11)(11, 12)(11, 13)(11, 14)(12, 11)(12, 12)(12, 13)(12, 14)(13, 14)\}$

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Tutorial 4

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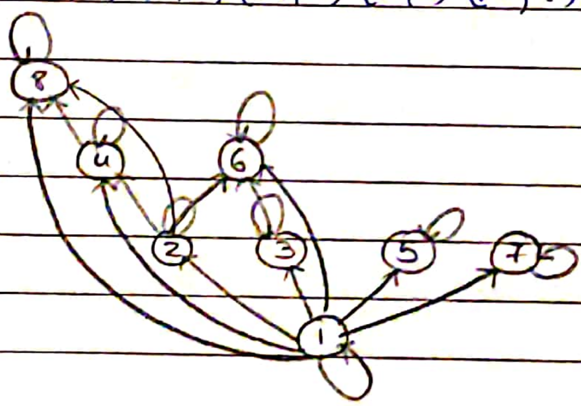
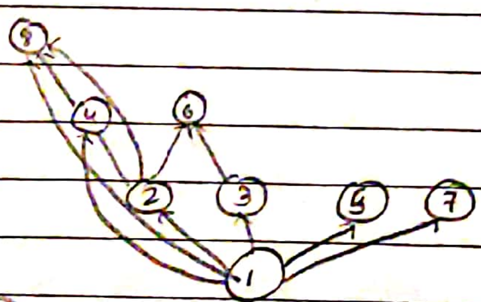
Discrete Structures.

1 Draw hasse diagram for the divisibity set $\{1,2,3,4,5,6,7,8\}$.

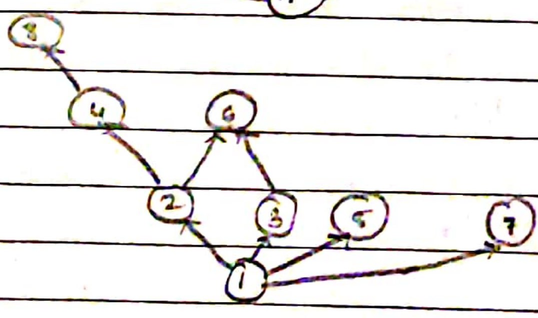
$R = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)(2,2)(2,4)(2,6)(2,8)$
$(3,3)(3,6)(4,4)(4,8)(5,5)(6,6)(7,7)(8,8)\}$.



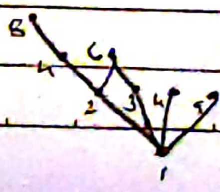
Shows the digraph of the above relation



Step one removes the loops.



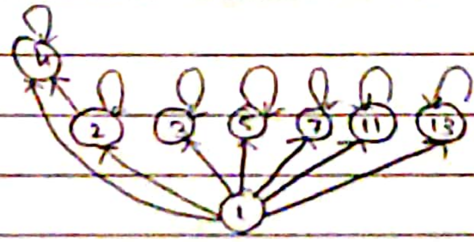
Step 2 removes the transitive edges



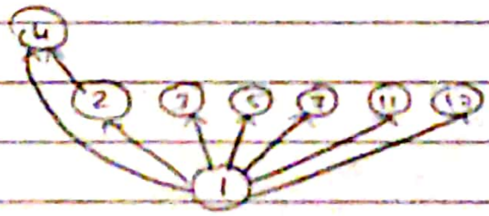
Shows the hasse diagram

iii) $\{1, 2, 3, 4, 5, 7, 11, 13\}$.

R = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,7) (1,11) (1,13) (2,2) (2,4) (3,3)
(4,4) (5,5) (7,7) (11,11) (13,13)

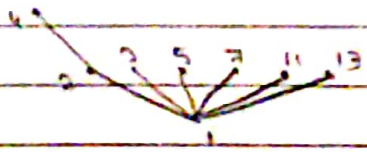


shows the digraph of the following relation



shows the step 1 without loops

Shows the step 2 without transitive edges.



shows the given hasse diagram

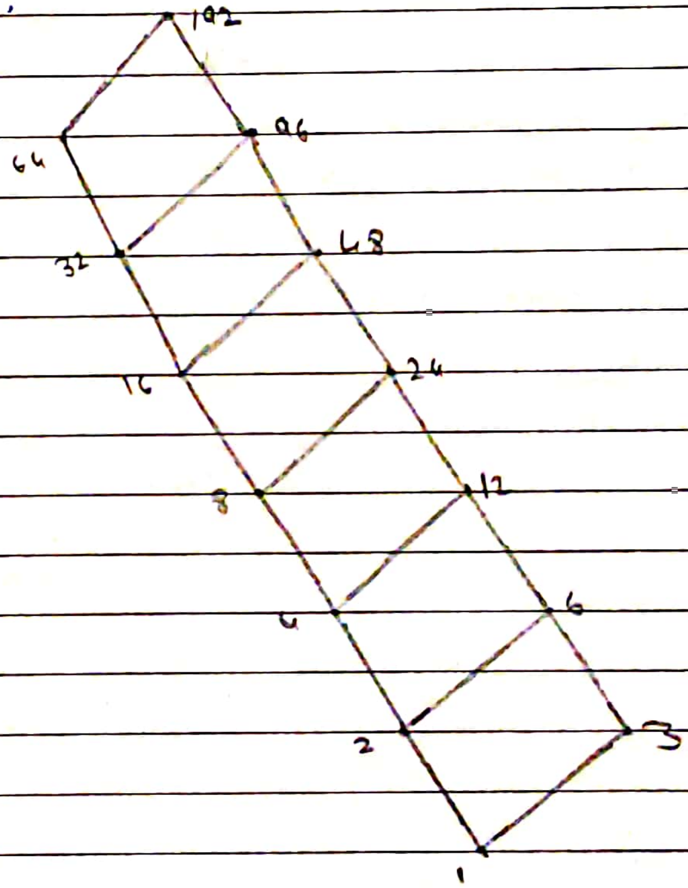
2 Draw hasse diagram for D_{192}

D = {1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192}

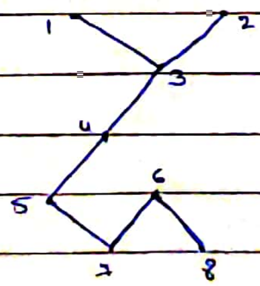
R = {(1,1) (1,2) (1,3) (1,4) (1,6) (1,8) (1,12) (1,16) (1,24) (1,32) (1,48) (1,64
(1,96) (1,192) (2,2) (2,4) (2,6) (2,8) (2,12) (2,16) (2,24) (2,48) (2,32
(2,64) (2,96) (2,192) (3,3) (3,12) (3,24) (3,48) (3,96) (3,192)
(4,4) (4,8) (4,12) (4,16) (4,32) (4,24) (4,48) (4,64) (4,96) (4,192
(6,6) (6,12) (6,24) (6,48) (6,96) (6,192) (8,8) (8,16) (8,32)
(8,24) (8,64) (8,96) (8,192) (12,12) (12,24) (12,48) (12,96) (12,192)
(16,16) (16,32) (16,48) (16,64) (16,96) (16,192), (24,24) (24,48)

(24, 96) (24, 192) (32, 32) (32, 64) (32, 96) (32, 192) (~~64~~ 48, 48) (48, 96)
(48, 192) (64, 64) (64, 192) (96, 96) (96, 192) (192, 192) }.

Hasse Diagram,



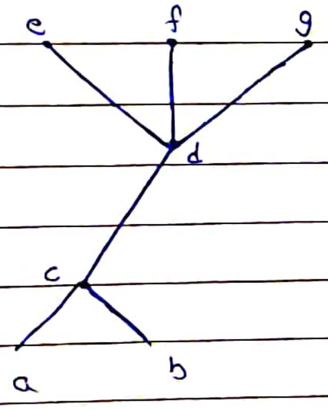
Q3 Determine whether the following hasse diagram represents a lattice or not.



LUB	1	2	3	4	5	6	7	8
1	1	-	1	1	1	1	1	
2	-	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3
4	1	2	3	4	4	4	4	4
5	1	2	3	4	5	5	5	5
6	1	2	3	4	5	6	6	6
7	1	2	3	4	5	6	~~6~~ 7	6
8	1	2	3	4	5	6	~~7~~ 6	8

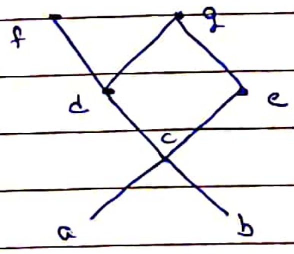
Since we can find null values available, hence it is not a lattice.

b



Here again since we can see that for the case of e, f, g, we cannot find the LUB. Hence it is not a lattice.

4



$\Rightarrow \{a, b, c\}$

upper bound

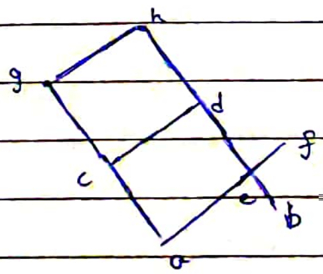
$= \{d, e, f, g, c\}$

lower bound

$\{\}$

LUB $= \{c\}$

GLB $= \{\ \}$



$\Rightarrow \{d, e, f\}$

upperbound =

~~lower~~

$= \{\ \}$

lower bound

$= \{a, e, b\}$

LUB $= \phi$

GLB $= \{e\}$