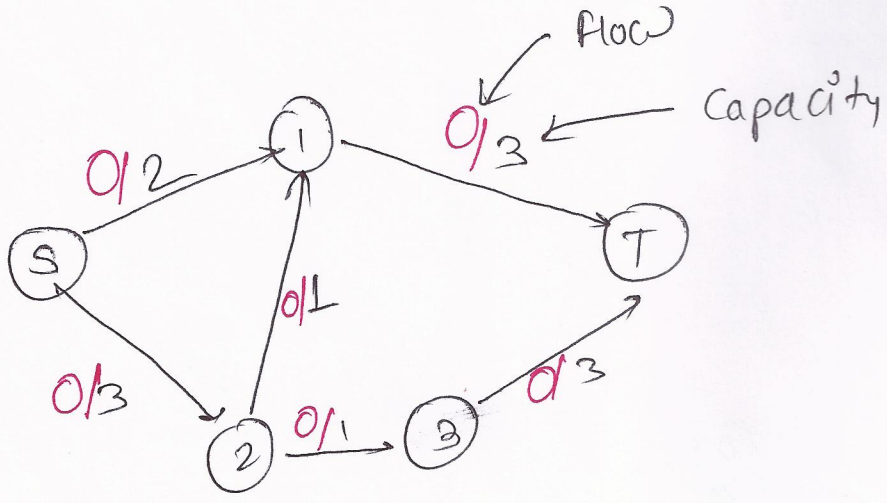
Ford - Fulkerson

Given a graph which represents a flow network where every edge has a capacity.
Also given two vertices source s and sink t in the graph.
find out the maximum possible flow from s to t with following constraints:

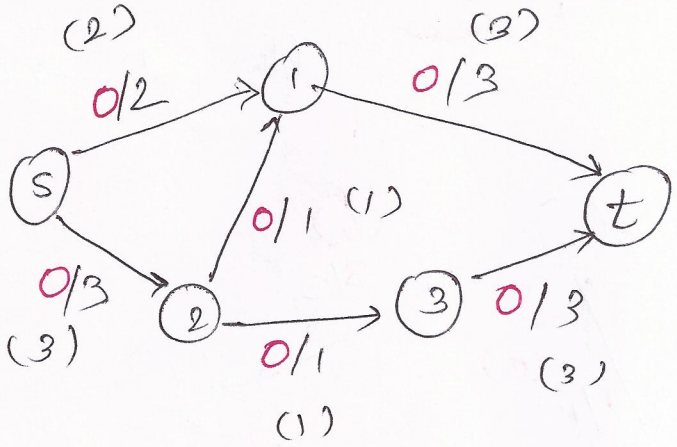
a) Flow on an edge doesn't exceed the given capacity of the edge.
b) In flow is equal to out-flow for every vertex except s and t

algorithm
Ford-Fulkerson Algorithm

1) Start with a initial flow as 0
2) while there is an augmenting path from source to sink
   Add this path flow to flow
3) return flow



residual capacity = total capacity − flow



Augumenting path is from s to t where its residual capacity is greater than 0 (non-negative)

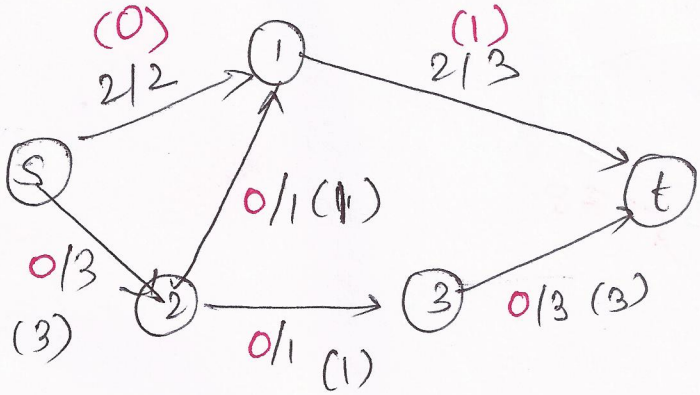
(1) ∴ s → 1 → t
2 3

minimum is = 2

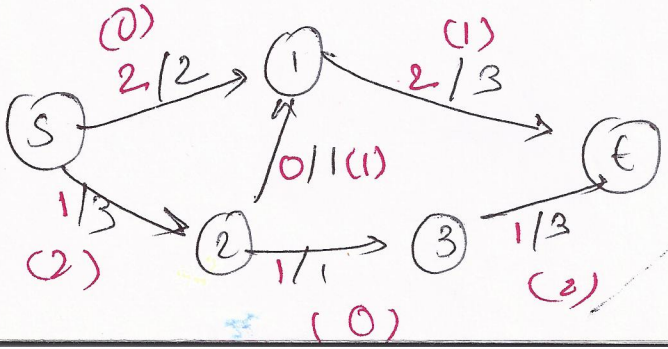
∴ at max we can move 2 unit

flow = 2 ⟶ x

∴



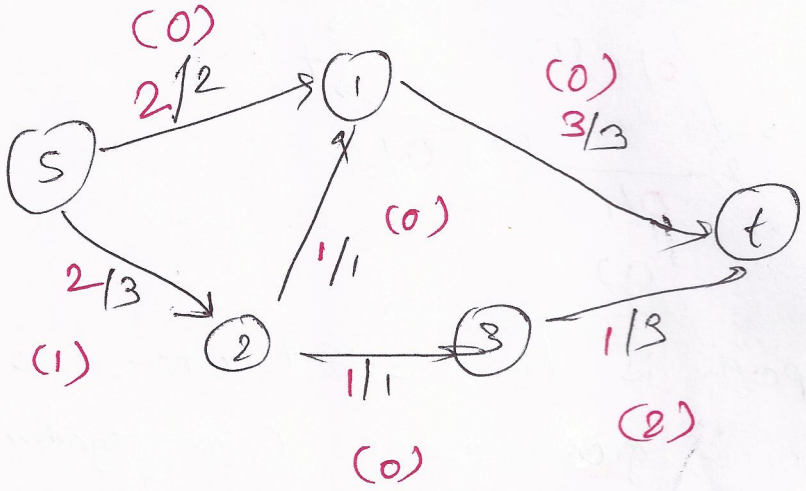
(2) s → 2 → 3 → t
3 | 3 ∴ flow = 1 ⟶ y



(3) $s \to 2 \to 1 \to t$

2 1 1

$\therefore$ flow = $1 \to 2$

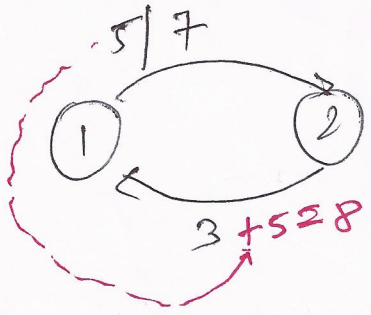


$\therefore$ Max flow = $x + y + z$

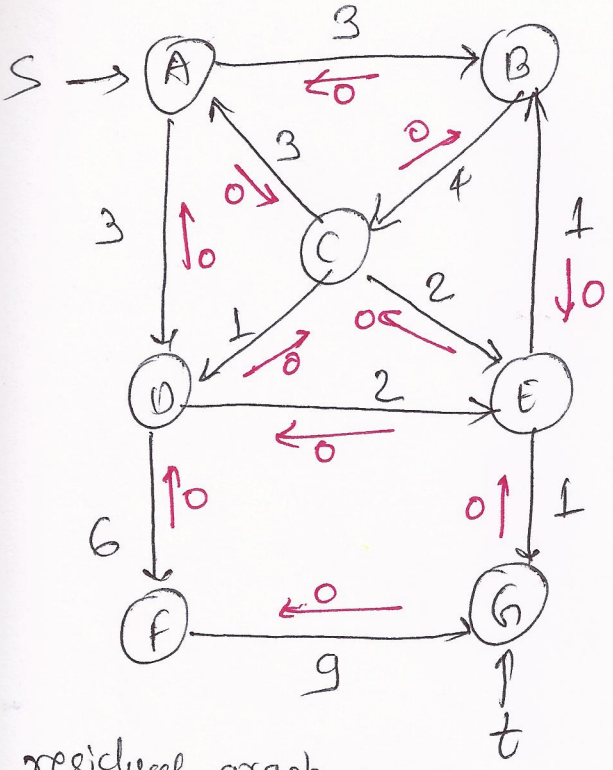
$= 2 + 1 + 1$

$= 4$

Note:-



Example:-



residual graph

Queue
~~A~~
~~B~~
~~D~~
~~C~~
~~E~~
F
G

Parent map
B → A
D → A
C → B
E → D
F → D
G → E

visited set
A, B, D
C, E, F
G

Current = A, B, D, C, E

Flow = 1

max Flow = 1+

(1)

Trace path

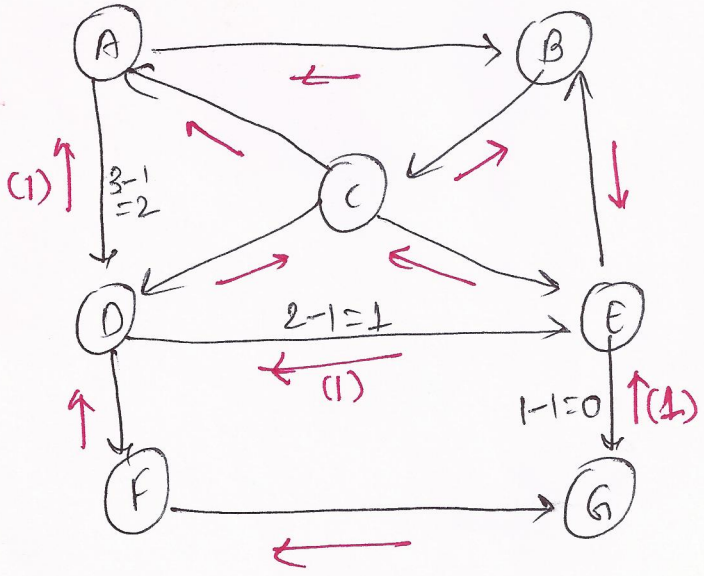
A → D → E → G

3 2 1

minimum = 01

(2)

Subtract this from the path.



(final ans is 5)

(1+2+1+1=5)