$T(n) = T(n-1) + 3 \quad \text{if } n>0$

$T(0) = 1$

$T(n) = T(n-1) + 3$

$\quad = T(n-1-1) + 3 + 3$

$f(n)$

{

if $(n>1)$

return $(f(n-1))$.

}

$T(n) = 1 + T(n-1)$ — ①

① Back substitution — It is slow method

$T(n-1) = 1 + T(n-1-1)$

$\quad = 1 + T(n-2)$ — ②

$T(n-2) = 1 + T(n-2-1)$

$\quad = 1 + T(n-3)$ — ③

eq ② in eq ①

$T(n) = 1 + 1 + T(n-2)$

$\quad = 2 + T(n-2)$ ← Put value of $T(n-2)$

$\quad = 2 + 1 + T(n-3)$

$\quad = 3 + T(n-3)$

$T(n) = 1 + T(n-1)$ is valid till $n > 1$

$T(n) = 1$ ~~stop~~ when ~~$n = 1$~~ for

stop when $n = 1$

this is called "Anchor condition"
or "Base condition"

Now in this condition

$$T(n) = k + T(n-k)$$

this has to be '1'

So we need to find for which value

$n - k = 1$ becomes one

$\therefore$ $k = n - 1$

So replace $k$ by $n-1$

$$\therefore \quad T(n) = k + T(n-k)$$

$$= (n-1) + T(n-(n-1))$$

$$= (n-1) + T(1)$$

$T(1) = 1$

Hence

$$= (n-1) + T(1)$$

$$= n - 1 + 1$$

$$= n$$

$$\therefore T(n) = n$$

$$= O(n)$$

Example

$$T(n) = n + T(n-1) \quad ; n > 1 \qquad (1)$$

$$= 1 \quad ; n = 1$$

$\cancel{T(n)}$ $T(n-1) = (n-1) + T((n-1)-1)$

$$T(n-1) = (n-1) + T(n-2) \rightarrow (2)$$

$$T(n-2) = (n-2) + T(n-2-1)$$

$$= (n-2) + T(n-3) \rightarrow (3)$$

eq (2) in eq$^n$ (1)

$$T(n) = n + (n-1) + T(n-2)$$

$$= n + (n-1) + (n-2) + T(n-3)$$

$$= n + (n-1) + (n-2) + \dots (n-k) + \underline{T(n-(k+1))}$$

we can stop by using base condition or Achor condition

in this case base condition is $n = 1$

$\because$

$$n - (k+1) = 1$$

$$(n-k-1) = 1$$

$$\boxed{\therefore k = n-2}.$$

Substitute $k = n-2$

this becomes (1)

$$T(n) = n + (n-1) + (n-2) + \dots \underline{(n-n-2)} + T(n-(n-2+1))$$

$$T(n) = n + (n-1) + (n-2) + \dots + 2 + 1$$ this becomes (2)

$= n + (n-1) + (n-2) + \dots$ since

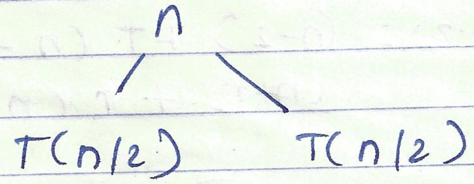
Recursion tree method

$T(n) = 2T(n/2) + n \quad ; n > 1$

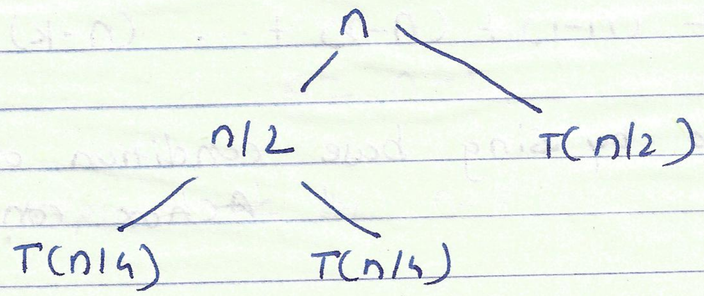
$\quad\;\; = 1 \quad\quad\quad\quad\quad ; n = 1$

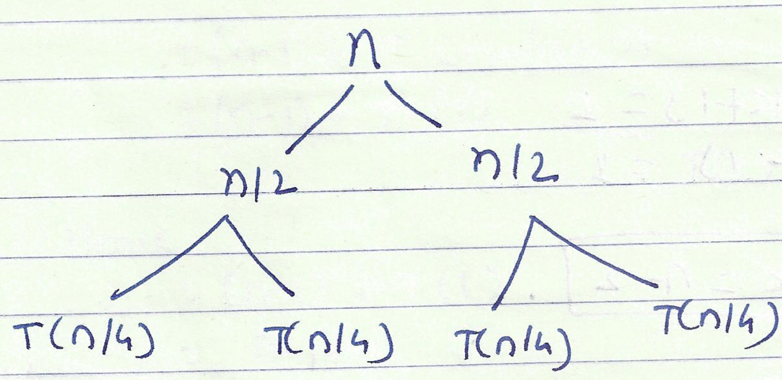
i am doing n amoun[t] of work by dividing the problem in 2 sub problem each problem is of size n/2

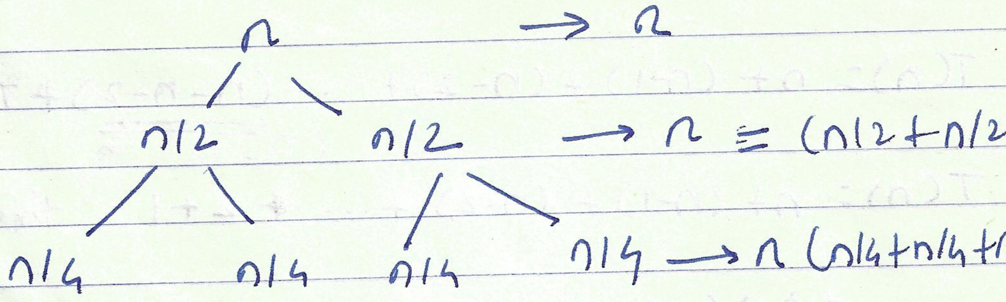
$T(n) \longrightarrow$




$T(n/2) = 2T(n/4) + n/2$



to get total amount of work add all n

Now to count how many levels ?

Now to get how the tree levels are getting formed

initially $n$ or $n/2^0$ — $1^{st}$ level

then $n/2^1$ — $2^{nd}$ level

$n/2^2$ — $3^{rd}$ level

Hence

$$\frac{n}{2^0} \quad \frac{n}{2^1} \quad \frac{n}{2^2} \quad \frac{n}{2^3} \quad \dots \quad \frac{n}{2^k}$$

so levels are 0, 1, 2 3 upto k

means we have $(k+1)$ levels

$k+1$ • $n = 2^k$

to get k value

$n = 2^k$

apply log on both sides

$k = \log n$

Put k value

$(\log n + 1)$ total no of levels

total work done is

# *] Master Theorm

is not applicable to all problems

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$$

?

$(\log n)^2$ $\log^2 n$

$\log n * \log n$ $\log \log n$

$a \geq 1$ , $b > 1$ , $k \geq 0$
& $p$ is reel number

① if $a > b^k$ , then

$$T(n) = \Theta(n^{\log_b a})$$

$(\log n)^2$ $\log^2 n$

$\log n + \log n$ $(\log n)^2$

$\log n$

② if $a = b^k$

a) if $p > -1$ , then $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

b) if $p = -1$ , then

$$T(n) = \Theta(n^{\log_b a} \log \log n)$$

c) if $p < -1$ , then

$$T(n) = \Theta(n^{\log_b a})$$

③ if $a < b^k$

a) if $p \geq 0$ , then

$$T(n) = \Theta(n^k \log^p n)$$

b) if $p < 0$ , then

$$T(n) = O(n^k)$$

Examples

(1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3$ , $b = 2$ , $k = 2$ , $p = 0$

$a = 3 \quad b^k = 2^2$

$3 < 4$

Condition 3 - a

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2 \log^0 n)$

$= \Theta(n^2)$

(2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4 \quad b = 2 \quad k = 2 \quad p = 0$

$4 = 2^2$

con :- 2 - a $\quad T(n) = \Theta(n^{\log_2 4} \log^1 n)$

$= \Theta(n^2 \log n)$

(3) $T(n) = T(n/2) + n^2$

$a = 1 \quad b = 2 \quad k = 2 \quad p = 0$

$1 < 2^2$

④ $T(n) = 2^n T(n/2) + n^n$.

$a = 2^n$ a has to be a constant which has to be greater than or equal to 1

↓

for this master theorm is not applicable

⑤ $T(n) = 16T(n/4) + n$

$a = 16$ $b = 4$ $k = 1$ $p = 0$

$b^k = 4^1$ $16 > 4$

Con :- I then

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_4 16})$

$= \Theta(n^2)$

⑥ $T(n) = 2T(n/2) + n \log n$.

$a = 2$ $b = 2$ $k = 1$ $p = 1$

$b^k = 2$ $\therefore a = b$

$2 - a$

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$= \Theta(n^1 \log^2 n)$

$= \Theta(n \log^2 n)$

$= \Theta(n \log \log n)$

(7) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$= 2T(n/2) + n \log^{-1} n$

$a = 2 \quad b = 2 \quad k = 1 \quad p = -1$

$b^k = 2 \quad a = b$

Con : 2 - b

$T(n) = \Theta(n^{\log_b a} \log(\log n))$

$= \Theta(n^{\log_2 2} \log \log n)$

$= \Theta(n \log \log n)$

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad k = 0.51 \quad p = 0$

$2 \quad b^k = 4^{0.51}$

$\therefore 2 < 4^{0.51}$

case 3 - a

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^{0.51} \log^0 n)$

$= \Theta(n^{0.51})$

(9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5$

condition not valid $a \geq 1$

not solvable

(10) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6$ $b = 3$ $k = 2$ $p = 1$

$a$ $b^k = 3^2$

$6 < 9$

Cond: 3 — a

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2 \log^1 n)$

$= \Theta(n^2 \log n)$

(11) $T(n) = 64T(n/8) - n^2 \log n$

$\downarrow$

Condition is not valid

master theorm can not be applicable

(12) $T(n) = 7T(n/3) + n^2$

$a = 7$ $b = 3$ $k = 2$ $p = 0$

3 — a

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2 \log^0 n)$

$= \Theta(n^2)$

(13) $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2, \quad k = 0 \quad p = 1$

$a = 4 \quad b^k = 2^0$

$4 > 2^0$

Case (1)

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 4})$

$= \Theta(n^2)$

(13) $T(n) = \sqrt{2}\, T(n/2) + \log n$

$a = \sqrt{2} \quad b = 2 \quad k = 0 \quad p = 1$

$\sqrt{2} \quad b^k = 2^0$

$\sqrt{2} > 1$

Case (1)

$T(n) = \Theta(n^{\log_2 \sqrt{2}})$

(15) $T(n) = 2T(n/2) + \sqrt{n}$

$a=2 \quad b=2 \quad k=1/2 \quad p=0$

$a \quad\quad b^k$

$2 > 2^{1/2}$

Case (1)

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 2})$

$= \Theta(n)$

(16) $T(n) = 3T(n/2) + n$

$a=3, \quad b=2, \quad k=1, \quad p=0$

$a > 2$

Case (1) $\quad T(n) = \Theta(n^{\log_2 3})$

(17) $T(n) = 3T(n/3) + \sqrt{n}$

$a=3 \quad b=3 \quad k=1/2 \quad p=0$

$b^k = 3^{1/2}$

$3 > \sqrt{3}$

Case (1) $\quad T(n) = \Theta(n^{\log_3 3})$

$= \Theta(n)$

(18) $T(n) = 4T(n/2) + cn$

$a = 4$, $b = 2$, $k = 1$ $p = 0$

$b^k = 2^1$

$4 > 2$

Case (1)

$T(n) = \Theta(n^{\log_2 4})$

$= \Theta(n^2)$

(19) $T(n) = 3T(n/4) + (n \log n)$

$a = 3$, $b = 4$, $k = 1$, $p = 1$

$b^k = 64$

$a < b^k$

Case 3 - a

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^1 \log^1 n)$

$= \Theta(n \log n)$