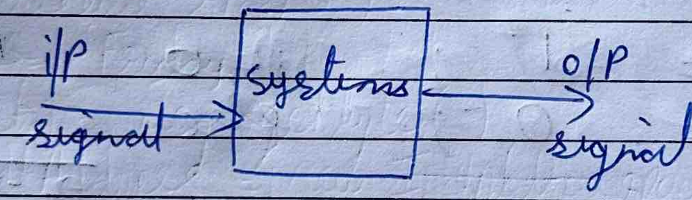
DSP

Signal Systems
↳ 1D

Signal - It is a function of 1/more independent variables. Its a physical quantity which contains some information, if it is dependent on a single variable, it is called as 1D signal. And if it depends on more than 2 variable it is called as multi. Speech is example of 1D signal. image is an example of 2D

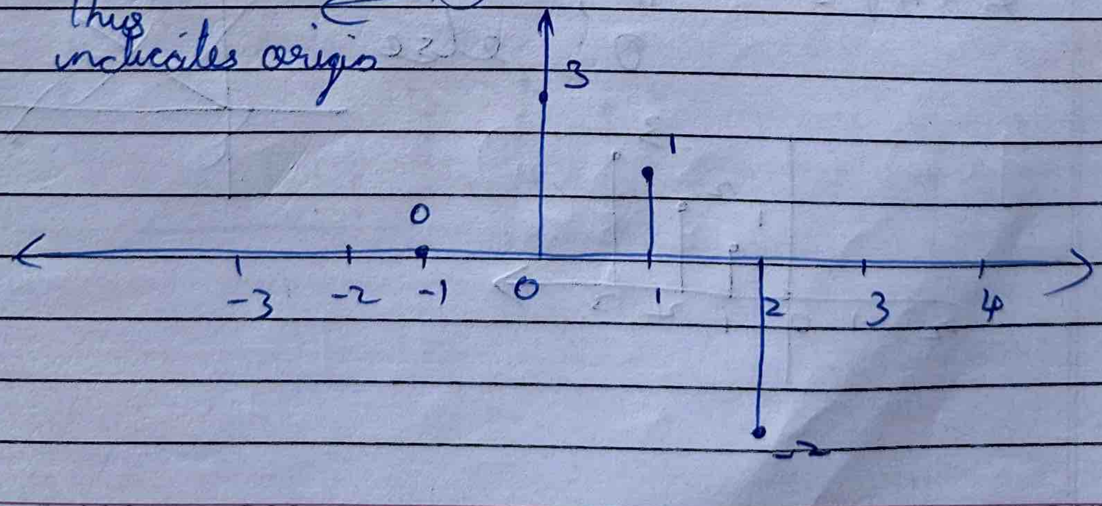
System - It is an entity that operates on 1 or more signal to accomplish a function so as to get modified signal.



Signal representation

$x(n) = \{0, 0, \underset{\uparrow}{3}, 1, -2, 0\}$

This indicates origin

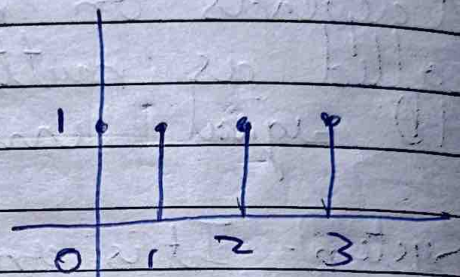


$x_n = \{1, 2, 3, 4\}$
where origin at 1

Basic signals

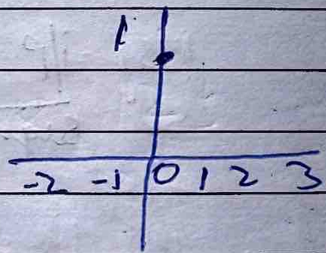
1) Unit step : $U(n)$

$$U(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$



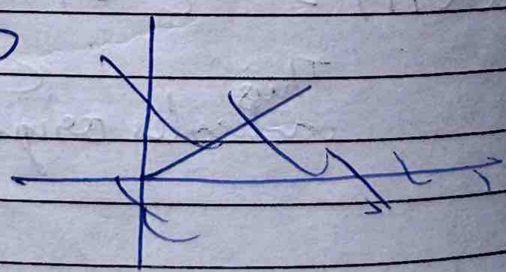
2) Unit Impulse (Delta) $\delta(n)$

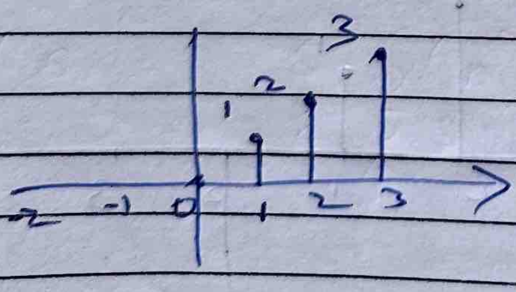
$$\delta(n) = \begin{cases} 1 ; & n = 0 \\ 0 ; & \text{else} \end{cases}$$



3) Ramp $(r(n))$

$$r(n) = \begin{cases} n ; & n \geq 0 \\ 0 ; & \text{else} \end{cases}$$





Operations of Signals

Operations on dependent variable

i) Addⁿ ii) Subtractⁿ iii) Multiplicaⁿ iv) Amp scale

Operation on independent variable

i) Time Shifting
ii) Time scaling
iii) Time reversal (flipping) Inversion

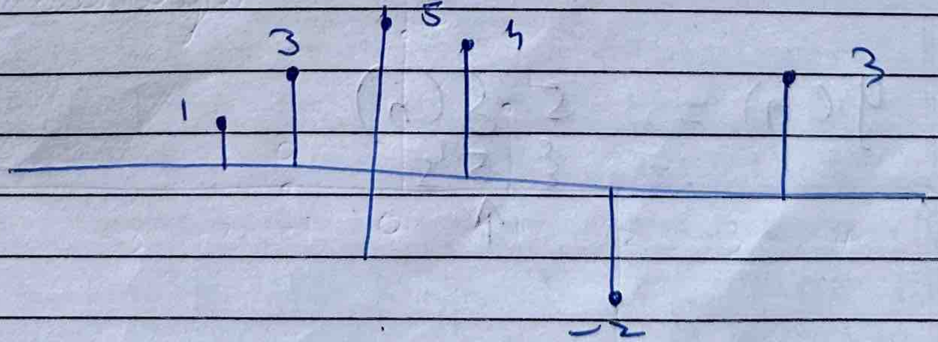
Add given 2 signals

$x_1 = \{1, \underset{\uparrow}{2}, 0, -2, 3\}$

$x_2 = \{1, 2, \underset{\uparrow}{3}, 4, 0\}$

$y(n) = x_1(n) + x_2(n)$
$= \{1, 3, \underset{\uparrow}{5}, 4, -2, 3\}$

Final Signal sketch it

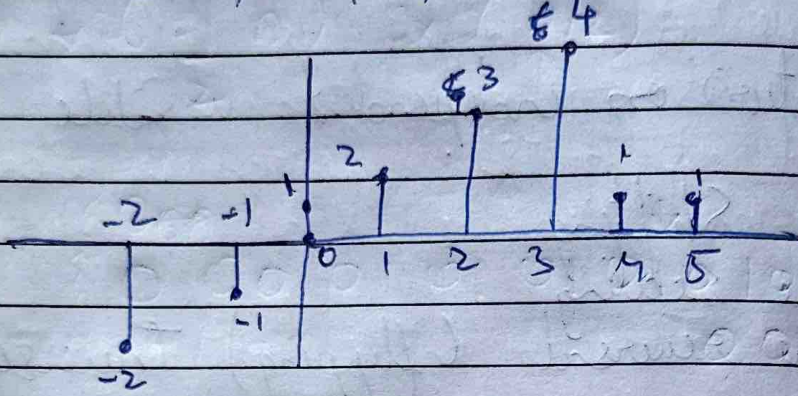


$x_1(n) = u(n)$

$x_2(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2, 3\}$

$y(n) = x_1(n) + x_2(n)$

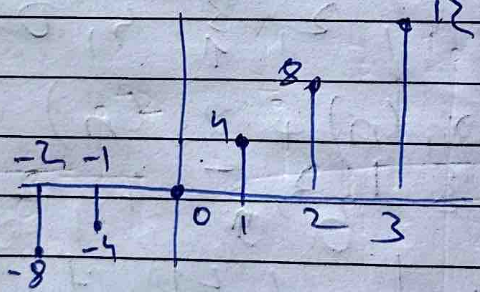
$= \{-2, -1, 1, 2, 3, 4, 1 \ldots\}$



Amplitude scaling

$y_1(n) = 4 * x_2(n)$

$= \{-8, -4, 0, 4, 8, 12\}$



$y(n) = 5\,\delta(n)$

$= \{\underset{\uparrow}{5}\}$

OPERATIONS ON INDEPENDENT VARIABLE

Time shifting operation

Delayed signal

$x(n) \Rightarrow x(n-k)$ Right Shift

Advance signal $\rightarrow x(n+k)$ Left

$x(n) = \{1, \underset{\downarrow}{2}, 3, -2, 4, 1\}$

$x(n-1) = \{\underset{\uparrow}{1}, 2, 3, -2, 4, 1\}$

$x(n-2) = \{\underset{\uparrow}{0}, 1, 2, 3, -2, 4, 1\}$

$x(n+2) = \{1, 2, 3, \overset{\downarrow}{-2}, 4, 1\}$

Time Scalling

$a>1$ Compression

$a<1$ Expansion

$x(n) \Rightarrow x(an)$ scaled

For given signal $x(n)$ find $x(2n)$

$x(n) = \{0, 0, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1, 0, 0\}$

n	-5	-4	-3	-2	-1	0	1	2	3	4	5
x(n)	0	0	1	1	1	1	1	1	1	0	0
x(2n)	0	0	0	0	1	1	1	0	0	0	0
	x(-10)	x(-8)	x(-6)	x(-4)	x(-2)	x(0)	x(2)	x(4)	x(6)	x(8)	x(10)

$$x(n) = \{ \underset{-3}{4}, \underset{-2}{3}, \underset{-1}{2}, \underset{0}{\overset{\downarrow}{1}}, \underset{1}{0}, \underset{2}{1}, \underset{3}{2}, \underset{4}{3}, \underset{5}{4} \}$$

$$x(3n) = \{ \quad 4 \ \underset{\uparrow}{1} \ 2 \}$$

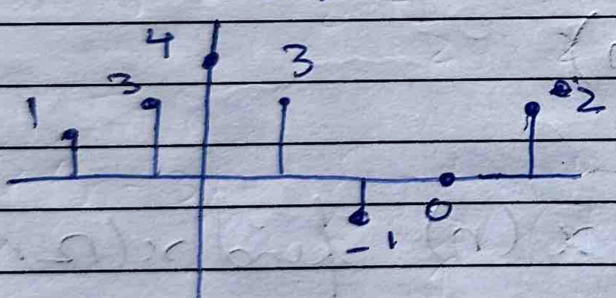
Find $x\left(\frac{1}{2}n\right) = x(n/2)$

$$x(n/2) = \{ \cancel{0 0 0}$$
$$\{ 0 \ 0 0 \ | \ 0 1 0 1 0 1 0 \overset{\downarrow}{1} 0 1 0 1 0 1 0 \ |$$
$$0 0 0 \}$$

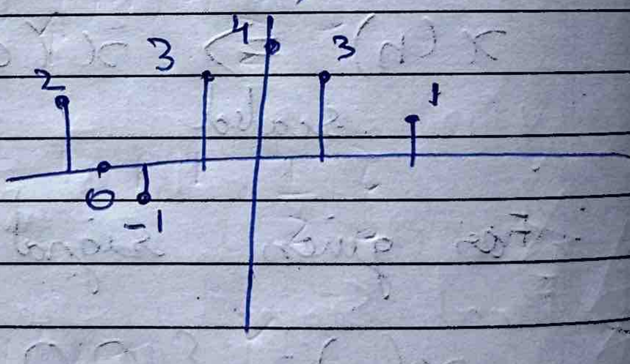
Time reversal

$$x(n) \xrightarrow{\text{reversal}} x(-n)$$

$x(n)$



$x(-n)$



1 Shift 2 Scale 3 Reverse

COMBINATIONS

$x(n) = \{1, 2, 0, \underset{\uparrow}{-1}, 3, 4\}$

find $\{x(n-2), x(n+2), x(-n-2), x(-n+2)$

$x(n-2) = \{1, \overset{\downarrow}{2}, 0, -1, 3, 4\}$

$x(n+2) = \{1, 2, 0, -1, 3, \underset{\uparrow}{4}\}$

$x(-n-2) = \{4, 3, -1, 0, \underset{\uparrow}{2}, 1\}$

$x(-n+2) = \{\underset{\uparrow}{4}, 3, -1, 0, 2, 1\}$

$x(n) = \{\underset{-4}{4}, \underset{-3}{3}, \underset{-2}{2}, \underset{-1}{1}, \underset{\substack{0\\ \uparrow}}{0}, \underset{1}{1}, \underset{2}{2}, \underset{3}{3}, \underset{4}{4}\}$

find $x(3n-1)$

my method

$x(3n) = \{3 \overset{\downarrow}{0} 3\}$

$x(3n-1) = \{\underset{\uparrow}{3} 0 3$

Main method:

$x(n-1) = \{4, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 4\}$

$x(3n-1) = \{4, \underset{\uparrow}{1}, 2\}$

$$x(n) = \{2, 2, \overset{\downarrow}{2}, 2, 1, 1, 1, 1, 1\}$$

$$x(-2n) = \{1, 1, 1, 2, \underset{\Uparrow}{2}\}$$

$$x(\cancel{2}n - 2) = \{1, \cancel{0}, \cancel{0}, \underset{\uparrow}{2}, 2, 2, 2, 1, 1, 1, 1, 1\}$$

$$x(2n-2) = \{\cancel{0}, \underset{\uparrow}{2}, 2, 1, 1, 1\}$$

$$x(-2n-2) = \{1, 1, 1, 2, \underset{\Uparrow}{2}, \cancel{0}\}$$

For a given signal find

$x(n-3)$ $\qquad x(n)\,U(3-n)$

$x(3-n)$ $\qquad x((n-1)^2]$

$x(2n)$

$$x\,n = \{-\tfrac{1}{2}, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}\}$$

$$x(n-3) = \{-\underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}, 1, 1, 1, 1, \tfrac{1}{2}\}$$

$$x(n+3) = \{-\tfrac{1}{2}, \tfrac{1}{2}, 1, 1, 1, 1, \overset{\downarrow}{\tfrac{1}{2}}\}$$

$$x(-n+3) = \{\tfrac{1}{2}, 1, 1, 1, 1, \tfrac{1}{2}, -\tfrac{1}{2}\}$$

$$x(2n) = \{\tfrac{1}{2}, \underset{\uparrow}{1}, 1\}$$

$u(n+3) = \{1, 1, 1 \ldots\}$

$u(n+3) = \{1, 1, 1, \underset{\uparrow}{1}, 1, \ldots\}$

$u(-n+3) = \{\ldots, 1, \underset{\uparrow}{1}, 1, 1, 1\}$

$x(n) * u(3-n) = \{0, 0, -\frac{1}{2}, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}\}$

$x((n-1)^2)$

∵ There is no direct formulae we use the table

n	-3	-2	-1	0	1	2	3
$x(n)$	$-1/2$	$1/2$	1	1	1	1	$1/2$
$x(n-1)^2$	$x(16)$	$x(9)$	$x(4)$	$x(1)$	$x(0)$	$x(1)$	$x(4)$
	0	0	0	1	1	1	0

$x[(n-1)^2] = \{1, 1, 1\}$

Classification of signals ⇒ khud se karo

systems

Classification of signals ⇒ Bank canceled

Linear $y(n) = \cos(x(n))$ $y_1(n) = \cos(x_1(n))$

linear means they are overlapping $y_2(n) = \cos(x_2(n))$

not same at the end Non linear $y' = \cos y_1(n) + y_2(n) = \cos(x_1(n)) + \cos(x_2(n))$ - i

$y'' = \cos(x_1(n) + x_2(n))$ - ii ∵ i ≠ ii ∴ It is non linear

Variant

Invariant - System is invariant if shift in input signal results in corresponding shift in output

Causal - If the output depends on present and past but doesnt depend on future then causal

Non-Causal

Static (Memory less) Output any instant of time depends on input sample at same time is called static

Dynamic (with memory)

Stable

Unstable

Q Prove $y(n) = \sin[x(n)]$ is a non linear system

1 $y(n) = x(n-1) + x(n)$

2 $y(n) = \log(x(n)]$

3 $y(n) = x^2(n)$

4 $y(n) = x[n^2]$

i) $y_1(n) = x_1(n-1) + x_1(n)$

$y_2(n) = x_2(n-1) + x_2(n)$

$y'(n) = y_1(n) + y_2(n) = x_1(n-1) + x_1(n) + x_2(n-1) + x_2(n)$

$y''(n) = x_1(n-1) + x_2(n-1) + x_1(n) + x_2(n)$

linear

ii) $y(n) = \log(x(n))$

$y_1(n) = \log(x_1(n))$

$y_2(n) = \log(x_2(n))$

$y'(n) = \log(x_1(n) * x_2(n))$

$y''(n) = \log(x_1(n) + x_2(n))$

Non linear

iii) $y(n) = x^2(n)$

$y_1(n) = x_1^2(n)$

$y_2(n) = x_2^2(n)$

$y' = x_1^2(n) + x_2^2(n)$

$y'' = (x_1(n) + x_2(n))^2$

Non linear

iv) $y(n) = x[n^2]$

$y_1(n) = x_1(n^2)$

$y_2(n) = x_2(n^2)$

$y'(n) = x_1(n^2) + x_2(n^2)$

$y''(n) = x_1(n^2) + x_2(n^2)$

Linear

Variant Invariant

Steps

1 Delay i/p by k

2 Replace 'n' by n-k

3 Compare

$y(n) = \cos(x(n))$

delay i/p by k

$y'(n) = \cos[x(n-k)]$ - i

$n \Rightarrow n-k$

$y(n-k) = \cos[x(n-k)]$ - ii

i = ii ∴ Invariant

1) $y(n) = x(n-1) + n(x(n))$

i delay i/p by k

$y'(n) = x(n-1-k) + n(x(n-k))$ - i

ii $n \Rightarrow n-k$

$y'(n-k) = x(n-1-k) + (n-k)(x(n-k))$ - ii

i ≠ ii

Variant Signal

2) $y(n) = x(n-1) + x(n)$

i $y'(n) = x(n-1-k) + x(n-k)$

ii $y'(n-k) = x(n-1-k) + x(n-k)$

$y'(n) = y'(n-k)$ NV

3 $y(n) = \log[x(n)]$

i $y'(n) = \log[x[n-k]]$

ii $y''(n-k) = \log(x(n-k)]$

$y'(n) = y''(n-k)$ NV

4 $y(n) = x^2(n)$

i) $y'(n) = x^2(n-k)$

ii) $y''(n) = x^2(n-k)$

$y'(n) = y''(n)$ NV

5 $y(n) = x(n^2)$

$y'(n) = x(n^2-k)$

$y''(n) = x((n-k)^2)$

$y'(n) \neq y''(n)$ V

Causal / Non-Causal

$y(n) = x(-n)$

NC

$y(n) = x(n^2)$

$n = 2$

$y(2) = x(4)$

NC $\because x(4)$ is after $x(2)$

if $x(n) \leq x(2)$ then Causal

Static / Dynamic

$y(n) = \cos(x(n))$

$y(0) = \cos(x(0))$

Static C

$y(n) = x(-n+2)$

$y(0) = x(2)$

Dynamic NC

Number of elements is $(m+n-1)$

$m =$ x_1 length $n = x_2$ length

Convolution

$x(n) \longrightarrow \boxed{\text{system}} \longrightarrow y(n)$

$h(n)$

$$y(n) = x(n) * h(n)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

For given 2 signals find convolution

$x(n) = \{1, 1, 1, 1\}$
$h(n) = \{1, \underset{\uparrow}{2}, 0, -3\}$

	$\downarrow$ 1	1	1	1
1	1	1	1	1
$\rightarrow$ 2	2	2	2	2
0	0	0	0	0
-3	-3	-3	-3	-3

$y(n) = \{1, \underset{\uparrow}{3}, 3, 0, -1, -3, -3\}$

Q. Find the convolution of $x_1(n) = \{2, 1, \overset{\downarrow}{-1}, -2, -3\}$
$x_2(n) = \{1, \underset{\uparrow}{2}, 0, -3\}$

	2	1	$\downarrow$ -1	-2	-3
1	2	1	-1	-2	-3
$\rightarrow$ 2	4	2	-2	-4	-6
0	0	0	0	0	0
-3	-6	-3	3	6	9

$y(n) = \{2, 5, 1, \underset{\uparrow}{-10}, -10, -3, 6, 9\}$

CORRELATION

Correlation of signal with delayed copy of itself is auto correlation

Correlation of signal with delayed copy of another signal is cross correlation

correlation reaches max when 2 signal considered become most similar to each other

1) Auto correlation for $x_1(n) = \{\underset{\uparrow}{1}, 1, 2, 2\}$

	2	2	1	1
→ 1	2	2	1	1
1	2	2	1	1
2	4	4	2	2
2	4	4	2	2

$y(n) = \{2, 4, 7, \underset{\uparrow}{10}, 7, 4, 2\}$

2) $x_1(n) = \{1, 1, 2, 2\}$

$x_2(n) = \{1, 2, 3, 4\}$

	4	3	2	1
→ 1	4	3	2	1
1	4	3	2	1
2	8	6	4	2
2	8	6	4	2

$y(n) = \{4, 7, 13, \underset{\uparrow}{17}, 11, 6, 2\}$

Stable Unstable

System is said to be stable if we have bounded input and bounded output and summation from $-\infty$ to ∞ $< \infty$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$h(n)$ impulse response

$x(n) = \delta(n)$ $\quad y(n) = h(n)$

$$\delta(n) = \begin{cases} 1 & ; n=0 \\ 0 & ; \text{else} \end{cases}$$

$y(n) = a\, x(n) \cdot x(n-1)$

$x(n) = \delta(n)$ $\quad y(n) = h(n)$

$h(n) = a\, \delta(n) \cdot \delta(n-1)$

$n = 0$

$h(0) = a\,\delta(0)\, \delta(-1) = 0$

$n = 1$

$h(1) = a\delta(1)\, \delta(0) = 0$

$h(0) + h(1) + \dots = 0$

$\because$ 0 is finite

$\therefore$ System is Stable

$y(n) = e^{-x(n)}$

$x(n) = \delta(n)$

$y(n) = e^{-\delta(n)}$

$n = 0 \quad yh(0) = e^{-1} = 1/e$

$n = 1 \quad yh(1) = 1$

$n = 2 \quad yh(2) = e^{-0} = 1$

Unstable

Q $x(n) = 4\delta(n+4) + 3\delta(n+3) - 2\delta(n+2) + \delta(n) - 3\delta(n-1) + 2\delta(n-2) - 4\delta(n-3)$

$4\delta(n+4) = \begin{cases} 4, & n = 4 \\ 0, & \text{otherwise} \end{cases}$

$x(n) =$

+3	+2	+1	0	-1	-2	3	-4
-4	2	-3	1	0	-2	3	4

Q $x(n) = 5\delta(n+2) + 2\delta(n+1) - 3\delta(n-2) + 4\delta(n-3)$

+3	+2	+1	0	-1	-2
4	-3	0	0	2	5

Represent in terms of weighted impulses

Q $x(n) = \{2, 1, \underset{\uparrow}{3}, -2, 1\}$

$$x(n) = 2\delta(n+2) + \cancel{2}\,\delta(n+1) + 3\delta(n) \overset{-2}{\cancel{+}}\,\delta(n-1) + \delta(n-2)$$

Q $x(n) = \{3, -1, 0, \underset{\uparrow}{2}, 1\}$

$$x(n) = 3\delta(n+3) - \delta(n+2) + 2\delta(n) + \delta(n-1)$$

Discrete Fourier transform (DFT)

Time Domain

DTFT (Discrete time FT)

$$x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

Frequency Domain

DFT

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \; ; \; k = 0 \text{ to } N-1$$

$$X[k] = x(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$$

Continuous frequency spectrum $x(\omega)$ is replaced by discrete fourier spectrum $X[k]$. $\sum^{\infty}$ in DTFT is replaced by finite summation in DFT

DFT

$$\underset{T.D}{x(n)} \underset{IDFT}{\overset{DFT}{\rightleftarrows}} \underset{F.D}{X[k]}$$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad k: 0 \text{ to } N-1$$

~~Since condition is taken for n and point of~~

IDFT ~~DEFT~~ Inverse (IDFT)

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{\frac{j2\pi nk}{N}} \; ; \; n = 0 \text{ to } (N-1)$$

New term ω is given by

DFT

$$W_N = e^{-j2\pi/N}$$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}, \quad k = 0 \text{ to } N-1$$

↳ Twiddle factor

IDFT

$$x(n) = \sum_{k=0}^{N-1} X(n)\, W_N^{-nk}, \quad n : 0 \text{ to } N-1$$

Q. DFT of $x(n) = \{1, 2, 3, 4\}$

$$x(n) \xrightarrow{DFT} X[k]$$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}, \quad k : 0 \text{ to } N-1$$

$$N = 4$$

$$= \sum_{n=0}^{3} x(n)\, W_N^{nk}$$

$$= x(0)\, W_N^0 + x(1)\, W_N^k + x(2)\, W_N^{2k} + x(3)\, W_N^{3k}, \quad k: 0 \text{ to } \cancel{N-1}\ 3$$

$$= W_N^0 + 2W_N^k + 3W_N^{2k} + 4W_N^{3k}$$

$k = 0$

$$X[0] = 1 + 2 + 3 + 4 = 10$$

$k = 1$

$$X[1] = \cancel{W_N}\,1 + 2W_N^1 + 3W_N^2 + 4W_N^3$$

$W_N^1 = W_4^1$

$W_N^1 = e^{-j2\pi/4}$

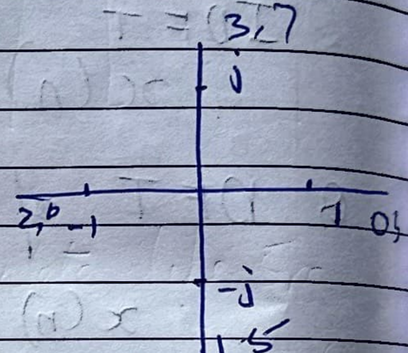
$= e^{-j\pi/2}$

$= -j$

$W_N^2 = -j \times -j = -1$

$W_4^3 = j$

$W_4^4 = -1$



$X[\boxed{1}] = 1 - 2j - 3 + 4j$

$= 2j - 2$

$X[2] = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$

$= 1 - 2 + 3 - 4$

$= -2$

$X[3] = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$

$= 1 + 2j - 3 - 4j$

$= -2 - 2j$

$X[k] = \{10, -2+2j, -2, -2-2j\}$

Q2 $x(n) = \{1, 2, 2, 1\}$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$= \sum_{n=0}^{3} x(n)\, W_4^{nk}$$

$$= x(0)\, W_4^{0} + x(1) \cdot W_4^{k} + x(2)\, W_4^{2k} + x(3) W_4^{3k}$$

$$= W_4^{0} + 2 \cdot W_4^{k} + 2\, W_4^{2k} + W_4^{3k}$$

$X[0] = 6$

$X[1] = 1 - 2j - 2 + j$

$= -j - 1$

$X[2] = 1 - 2 + 2 - 1 = 0$

$X[3] = 1 + 2j - 2 - j$

$= -1 + j$

Q $x(n) = (-1)^n$ for $N = 3$

$$X[k] = \sum_{n=0}^{2} x(n)\, W_N^{nk}$$

$x(n) = \{+1, -1, +1\}$

$X[0] = 1$

$W_3^1 = \frac{-1}{2} + \frac{\sqrt{3}}{2} j$

$X[1] = W_3^{0} - W_3^{1} + W_3^{2}$

$= 1 + 1.732j$

$X[2] = W_3^{0} - W_3^{2} + W_3^{4}$

$= 1 - 1.732j$

Shortcut for DFT (N = 4)

$$X[k] = W_{N\,(4\times4)} \cdot x(n) \qquad x_n = \{1, 2, 3, 4\}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

If only this is asked solve complete like 5m

If there are multiple sub Q's u can use this

$$x(n) = \{1, 2, 1, 2\}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2-2j \\ 6 \\ -2+2j \end{bmatrix}$$

Find IDFT

$X[k] = [4, 1-j, -2, 1+j]$

$$x(n) = \frac{1}{N} W_N^{*} X[k]$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ 4+j+1+2-j+1 \\ 0 \\ 4-j+1+2+j-1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Determine length for sequence $X[k] = [3, 2+j, 1, 2-j]$

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 3 \\ 2+j \\ 1 \\ 2-j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 3+2j-1-1-2j-1 \\ 0 \\ 3-2j+1-1+2j+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Linear

$x(n) = \{1, 2, 3, 4\}$

1) Shift

$x(n-2) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4\}$

$x(n+1) = \{1, \underset{\uparrow}{2}, 3, 4\}$

2) Folding

$x(-n) = \{4, 3, 2, 1\}$

Circular

$x_p(n) \longrightarrow X_p(k)$

$x(n) = \{1, 2, 3, 4\}$

$x(n-2) = \{3, 4, 1, 2\}$

$x(n-1) = \{4, 1, 2, 3\}$

$x(n+1) = \{2, 3, 4, 1\}$

$x(-n) = \{1, 4, 3, 2\}$

TBC ....

Skipped topic from past lects ....

Classification of signals

CT/DT	Periodic / Non-Periodic	Even / odd	Random / Deterministic	Energy / Power

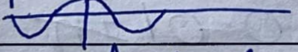
i) Continuous Time signal :- Signal which has every instant of time for eg:- ECG signal

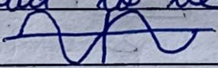
ii) Discrete Time Signal: Signal which has value at certain instant of time eg:- BP measured at every hr.

iii Periodic :- Signal which is periodic if $x(n) = x(n+N)$ where N is the time period

iv) Non Periodic : $x(n) \neq x(n+N)$

Signal is periodic if it is repeated after fixed time interval

v Even :- Signal it is said to be even if $x(n) = x(-n)$
eg:- cosine  symmetric

vi) Odd : Signal is said to be odd if $x(n) = -x(-n)$
eg:- sine  assymmetric

vii) Random :- cannot define using any mathematical equation
eg:- noise signal

viii) Deterministic :- It is one which can be represented using mathematical equation eg :- $x(n) = e^{-3n}$

ix) Signal is said to be energy if energy value lies between $0 < E < \infty$

x) Power signal $0 < P < \infty$

★ Power of energy signal is '0'

Energy of power signal is '∞'

I Periodic / Non Periodic

For discrete time signal condition for periodicity is

① $f_0 = \frac{k}{N}$

Signal is periodic only if its frequency (f_0) is ration of 2 integers . N : Time period

Periodicity of a signal

② $x(n) = x_1(n) + x_2(n)$
$\quad\quad N_1 \quad\quad N_2$

N_1, N_2 } both should integers both numerators and denominators

Time Period : $N = LCM(N_1, N_2)$

Q Decide whether given signal is periodic or not if periodic find time period

$x(n) = \cos 0.01\pi n$

$\cos 2\pi f n = \cos 0.01\pi n$

$2\pi f n = 0.01\pi n$

$f = \frac{0.01}{2} = 0.005 = \frac{1}{200}$

Since this is the ration of 2 integers ∴ It is Periodic

Time period = 200

2 $x(n) = \sin 3\pi n$

$\sin 2\pi f n = \sin 3\pi n$

$2\cancel{\pi} f \cancel{n} = 3\cancel{\pi}\cancel{n}$

$f = \frac{3}{2}$

Time Period = 2

3 $x(n) = \sin 3n$

$\sin 2\pi f n = \sin 3n$

$f = \frac{3}{2\pi}$

Not periodic as it is ~~a~~ not a ratio of 2 integers

4) $x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$

$\frac{2\pi n}{5} = 2\pi f_1 n$ $\qquad$ $\frac{2\pi n}{7} = 2\pi f_2 n$

$f_1 = \frac{1}{5}$ $\quad N_1 = 5$ $\qquad$ $f_2 = \frac{1}{7}$ $\quad N_2 = 7$

$\frac{N_1}{N_2} = \frac{5}{7}$

$N = \text{lcm}(5, 7) = 35$

$x(n) = \sin(0.2n + \pi)$

$\sin(2\pi f n + \phi)$

$2\pi f n = 0.2n$

$f = \frac{1}{10\pi}$ Non periodic

c) $x_n = e^{j(\frac{\pi}{4}n)}$

$= \cos\frac{\pi}{4}n - j\sin\frac{\pi}{4}n$ X not required

$2\pi f n = \frac{\pi}{4}n$

$f = \frac{1}{8}$ $N = 8$ Periodic

II Even and Odd

$x_e(n) = \frac{x(n) + x(-n)}{2}$

$x_o(n) = \frac{x(n) - x(-n)}{2}$

For given signal find even and odd parts

$x(n) = \{1, 0, \underset{\downarrow}{2}, 3, -1\}$

$x(-n) = \{-1, 3, \underset{\uparrow}{2}, 0, 1\}$

$x_e(n) = \{\{0, 3/2, 2, 3/2, 0\}\} = \{\{0, 2, 2, 2, 0\}\}$ X

$x_o(n) = \{\{1, -3/2, 0, 3/2, -1\}\} = \{\{1, -2, 0, 2, -1\}\}$ ✓ not required

III $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

Q Find energy and Power if $x(n) = u(n)$

$$x(n) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2 = \infty$$

$u(n)$ is not energy signal

$$P = \lim_{N = \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

$$= \lim_{n \to \infty} \frac{N+1}{2n+1}$$

$$= \lim_{n \to \infty} \cancel{2E A 2A}$$

$$= \frac{1}{2} \Big\} \text{ finite}$$

$|x+iy| = \sqrt{x^2+y^2}$

Q $x(n) = 2\, e^{3in}\, u(n)$

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \rightarrow |\cos 3n - i \sin 3n|$

$= \sum_{n=0}^{\infty} |2\, e^{3in}\, u(n)|^2$

$= \sum_{n=0}^{\infty} |4|$

$= 4\infty$ Not energy

$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |2\, e^{3in}\, U(n)|^2$

$= \lim_{N \to \infty} \frac{4}{2N+1} \sum_{n=0}^{N} N$

$= \lim_{N \to \infty} \frac{4N}{2N+1}$

$= 2$

Q $x(n) = \left(\frac{1}{3}\right)^n U(n)$

$E = \sum_{n=-\infty} |x(n)|^2$

$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2 = \sum_{n=0}^{\infty} \left(\left(\frac{1}{9}\right)^n\right)$

$= \frac{1}{1-1/9} = \frac{9}{8}$ Energy signal

$$x(n) = \left(\frac{1}{2}\right)^n \quad ; \quad n \geq 0$$

$$= 3^n \quad ; \quad n < 0$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{-1} |9^n| + \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \sum_{n=1}^{\infty} \left|\left(\frac{1}{9}\right)^n\right| + \frac{1}{1-1/4}$$

$$= \frac{1/9}{1-1/9} + \frac{4}{3}$$

$$= \frac{1}{8} + \frac{4}{3}$$

$$= \frac{35}{24} \leftarrow \text{Energy Signal}$$

$$1 + \frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3} + \ldots + \frac{1}{9^\infty} = \frac{1}{1-1/9}$$

Properties of DFT

1) Linearity Property

$$a\,x_1(n) + b\,x_2(n) \xrightarrow{DFT} a\,X_1[k] + b\,X_2[k]$$

2) Periodicity property : Both DFT and IDFT produce periodic results

$$x_p(n) \underset{IDFT}{\overset{DFT}{\rightleftharpoons}} X_p[k]$$

3) Time Shift property

Whenever there is shift in time domain there is multiplication factor in frequency domain Sign changes

$$\underset{TD}{x(n)} \xrightarrow{DFT} \underset{FD}{X[k]}$$

$$x(n-m) \xrightarrow{DFT} \underset{\text{Twiddle}}{W}^{mk} X[k]$$

$$x(n+m) \xrightarrow{DFT} W^{-mk} X[k]$$

4 Frequency Shift : There is a shift in frequency domain we have multiplication factor in time domain and sign remains same

$$\underset{TD}{x(n)} \xrightarrow{DFT} \underset{FD}{X[k]}$$

$$x(n)\,W^{-mn} \xrightarrow{DFT} X[k-m]$$

$$x(n)\,W^{mn} \xrightarrow{DFT} X[k+m]$$

5 Time Reversal

$$x(n) \xrightarrow{DFT} X[k]$$
$$x(-n) \xrightarrow{DFT} X[-k]$$

Q1) find $X[k]$, if $x(n) = \{1, 2, 3, 4\}$

Hence find DFT of i) $p(n) = \{4, 1, 2, 3\}$

ii) $q(n) = \{3, 4, 1, 2\}$

iii) $l(n) = \{1, 4, 3, 2\}$

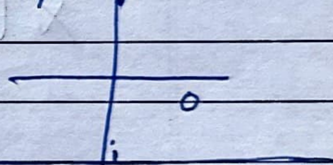
iv) $h(n) = \{1, -2, 3, -4\}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

i) $p(n) = x(n-1)$

$P[k] = W^{1k} X[k]$ by time shift property

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} \begin{matrix} W_4^0 \\ W_4^1 \\ W_4^2 \\ W_4^3 \end{matrix}$$



$$= \begin{bmatrix} 10 \\ 2j+2 \\ 2 \\ -2j+2 \end{bmatrix}$$

$l(n) = x(-n)$

$L[k] = X[-k]$

$= [10\ \ -2-2j,\ -2,\ -2+2j]$

$h(n) = x(n)$; 'n' even

$-x(n)$; 'n' odd

$h(n) = (-1)^n x(n)$

$H[k] = X[k+2]$

$$= \begin{bmatrix} -2 \\ -2-2j \\ 10 \\ -2+2j \end{bmatrix}$$ By frequency shift property

$q(n) = x(n-2)$

$Q[k] = W^{2k} X[k]$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 10 \\ 2-2j \\ -2 \\ 2+2j \end{bmatrix}$$

6 Symmetry

If signal x[n] is real valued then after taking DFT, real part of x[k] is symmetric and imaginary part of x[k] is anti-symmetric

Symmetry at $k = \frac{N}{2}$

$x(n) = \{1, 2, 3, 4\} \xrightarrow{DFT} X[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{matrix} k=0 \\ k=1 \\ k=2 \\ k=3 \end{matrix}$

$$X[k] = \begin{array}{c} \begin{bmatrix} 36 \\ 2+6j \\ 6j \\ -4 \\ 0 \\ -4 \\ -6j \\ 2-6j \end{bmatrix} \end{array} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}$$

$N = 8$

IN NEW NB →

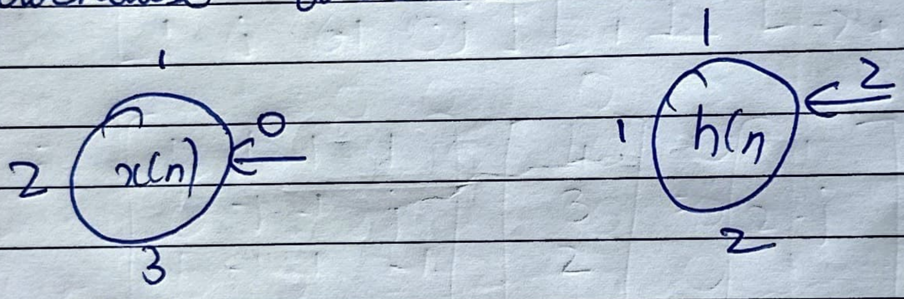
Circular Convolution :
Length of both signals should be same. If not, add zero.

3 methods for Circular Convolution
1) Graphical 2) DFT, IDFT 3) Matrix

$$y(m) = \sum_{n=0}^{N-1} x(n)\, h(m-n)$$

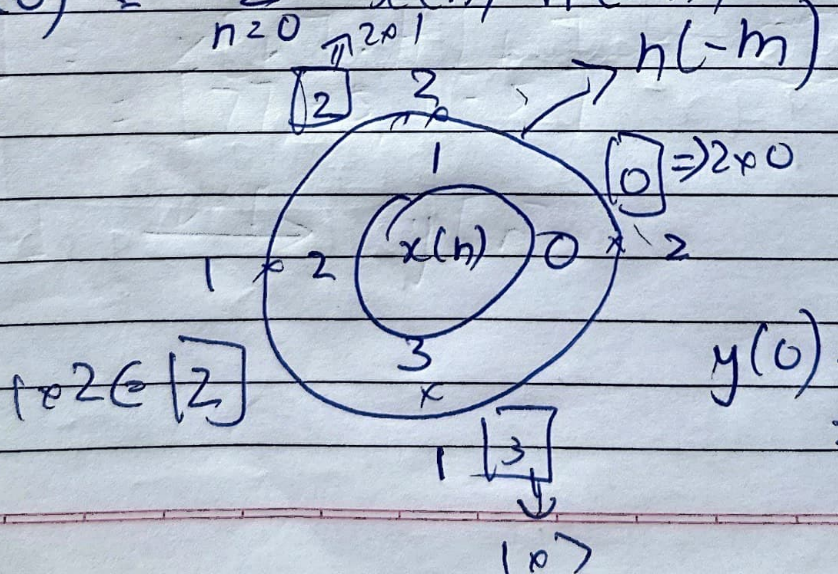
Q. $x(m) = \{0, 1, 2, 3\}$
$h(m) = \{2, 1, 1, 2\}$

1) Graphical
Step 1 : Draw $x(m)$ and $h(m)$ in anticlockwise direction



Step 2 : Put $m = 0$

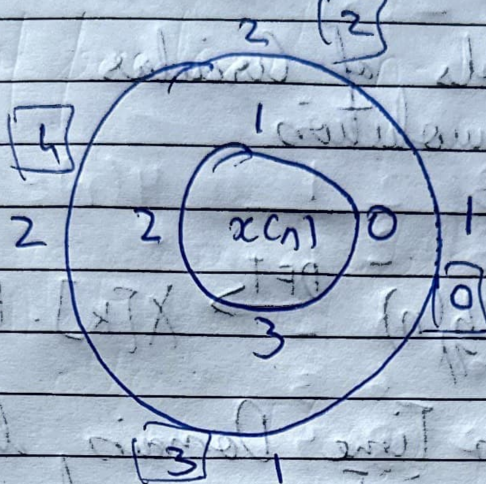
$$y(0) = \sum_{n=0}^{n-1} x(n)\, h(-n)$$



$y(0) = 2 + 2 + 0 + 3$
$= 7$

Step 3:

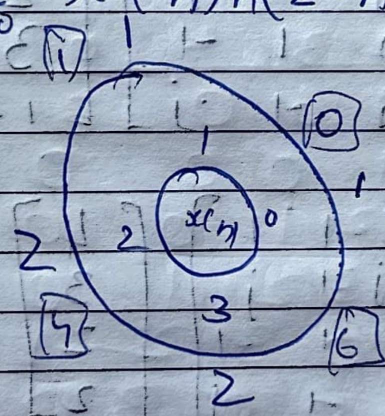
$$y(1) = \sum_{n=0}^{N-1} x(n)\, h(1-n)$$



$$y(1) = 4+3+0+2 = 9$$

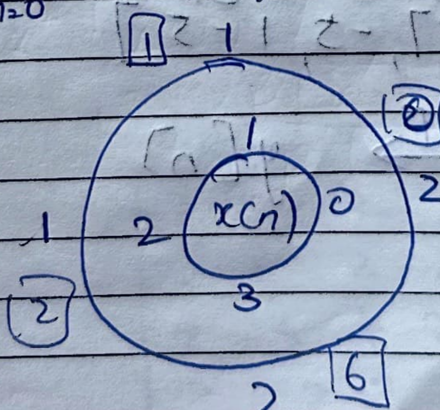
Step 4:

$$y(2) = \sum_{n=0}^{N-1} x(n)\, h(2-n)$$



$$y(2) = 0+1+4+6 = 11$$

$$y(3) = \sum_{n=0}^{N-1} x(n)\, h(3-n)$$



$$y(3) = 0+1+2+6 \Rightarrow 9$$

$$y(n) = \{7, 9, 11, 9\}$$

2) DFT, IDFT

$x(n) = \{1, 2, 3, 1\}$

$h(n) = \{4, 3, 2, 2\}$

$\textcircled{N}$, $\circledast$ } symbols of Circular Convolution

$x(n) \; \textcircled{N} \; h(n) \xrightarrow{DFT} X[k] \cdot H[k]$

Convolution in Time Domain becomes Multiplication in Frequency domain

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$y[k] = X[k] \cdot H[k]$$
$$= [77, -5, 1, -5]$$

$y(k] \xrightarrow{IDFT} y[n]$

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$$y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

3) Matrix

$x(n) = \{6, 1, 2, 3\}$
$h(n) = \{2, 1, 1, 2\}$

$h(-n) = \{2, 2, 1, 1\}$

$$\begin{array}{c} \\ h(-n) \\ h(1-n) \\ h(2-n) \\ h(3-n) \end{array}\begin{array}{c} 0 \quad 1 \quad 2 \quad 3 \\ \left|\begin{matrix} 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{matrix}\right| \end{array}\begin{array}{c} \\ = 7 \\ = 9 \\ = 4 \\ = 9 \end{array}$$

$$\begin{array}{c} 1 \quad 2 \quad 3 \quad 1 \\ \left|\begin{matrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{matrix}\right| \end{array}\begin{matrix} 17 \\ 19 \\ 22 \\ 19 \end{matrix}$$

* Linear Convolution using Circular Convolution

$x(n) = \{1, 2, 3, 4\}$

$h(n) = \{1, 2, 3\}$

	0	3	2	1
1	0	3	2	1
2	0	6	4	2
3	0	9	6	3
4	0	12	8	4

$y(n) = \{1, 4, 10, 16, 17, 12\}$

$x(n) = \{1, 2, 3, 4, 0, 0\}$

$h(n) = \{1, 2, 3, 0, 0, 0\}$

	1	2	3	4	0	0	
$h(-n)$	1	0	0	0	3	2	= 1
$h(1-n)$	2	1	0	0	0	3	= 4
$h(2-n)$	3	2	1	0	0	0	= 10
$h(3-n)$	0	3	2	1	0	0	= 16
$h(4-n)$	0	0	3	2	1	0	= 17
$h(5-n)$	0	0	0	3	2	1	= 12

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Convolution for long data sequences

Overlap & Add | Overlap & Save

1) Overlap & Add

→ Divide given sequence into subsequences of length N
→ length of h(n) = 'm'
→ Calculate $L = M + N - 1$
→ Add zeros to $x_i(n)$ and $h(n)$ to make length = 'L'
→ Perform circular conv to find $y_1(n) \ldots, y_n(n)$
→ Overlap and Add to get results

Q Perform convolution using Overlap & Add
$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$
$h(n) = \{1, 1, 1\}$

→ Divide into substring — we decide $N = 4$
$x_1(n) = \{3, -1, 0, 1\}$
$x_2(n) = \{3, 2, 0, 1\}$
$x_3(n) = \{2, 1\}$

→ $M = 3$
→ $L = 4 + 3 - 1 = 6$
$x_1(n) = \{3, -1, 0, 1, 0, 0\}$ $h(n) = \{1, 1, 1, 0, 0, 0\}$
$x_2(n) = \{3, 2, 0, 1, 0, 0\}$
$x_3(n) = \{2, 1, 0, 0, 0, 0\}$

$y_1(n) = x_1(n) \; (N) \; h(n)$

$h(-n) = \{1, 0, 0, 0, 1, 1\}$

$$\begin{array}{cccccc|c} 3 & -1 & 0 & 1 & 0 & 0 & \\ \hline 1 & 0 & 0 & 0 & 1 & 1 & 3 \\ 1 & 1 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 & \cancel{4}\,0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

$y_1 = \{3, 2, 2, 0, 1, 1\}$

$$\begin{array}{cccccc|c} 3 & 2 & 0 & 1 & 0 & 0 & \\ \hline 1 & 0 & 0 & 0 & 1 & 1 & 3 \\ 1 & 1 & 0 & 0 & 0 & 1 & 5 \\ 1 & 1 & 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

$y_2 = \{3, 5, 5, 3, 1, 1\}$

$$\begin{array}{cccccc|c} 2 & 1 & 0 & 0 & 0 & 0 & \\ \hline 1 & 0 & 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}$$

$y_3 = \{2, 3, 3, 1, 0, 0\}$

Number of overlap = $M-1$ = No of 0's

y_1	3	2	2	0	1	1										
y_2					3	5	5	3	1	1						
y_3									2	3	3	1	0	0		

$\{3, 2, 2, 0, 4, 6, 5, 3, 4, 3, 1, 0, 0\}$

2) Overlap and Save

→ Divide the sequence into subsequence of length N

→ $len(h(n)) = M$

→ $L = M + N - 1$

→ To make length of $x_i(n) = N$ use elements from prev sequence

→ Add 0's to $h(n)$ to make length ~~N~~ L

→ Perform circular convolution to find $y_1(n)$, $y_2(n)$, ..., $y_n(n)$

→ Use Overlap and save to find the result

Q $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$

$h(n) = \{1, 1, 1\}$

→ $N = 4$ $M = 3$ $L = 6$

$x_1(n) = \{0, 0, 3, -1, 0, 1\}$

$x_2(n) = \{0, 1, 3, 2, 0, 1\}$

$x_3(n) = \{\cancel{3}, 0, 1, 2, 1, 0, 0\}$ We can take $M-1$ elements only

$$\begin{array}{c} 0\ 0\ 3\ {-1}\ 0\ 1 \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 1 \\ 3 \\ 2 \\ 2 \\ 0 \end{array} \qquad \begin{array}{c} 0\ 1\ 3\ 2\ 0\ 1 \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 4 \\ 6 \\ 5 \\ 3 \end{array}$$

	0	1	2	1	0	0	
	1	0	0	0	1	1	0
	1	1	0	0	0	1	1
	0	1	1	0	0	0	3
	0	1	1	1	0	0	4
	0	0	1	1	1	0	3
	0	0	0	1	1	1	1

$y_1(n) = \boxed{\cancel{1\ 1}}\ 3\ 2\ 2\ 0$

$\boxed{\cancel{1\ 2}}\ 4\ 6\ 5\ 3$

$\boxed{\cancel{0\ 1}}\ 3\ 5\ 3\ 1$

3 2 2 0 4 6 5 3 3 4 3 1

Q. Find convolution of 2 sequences by Overlap & Add and Overlap & Save method

$x(n) = \{1, 2, 3, 4, 1, 2, 3, 4\}$

$N = 4 \quad M = 3 \quad L = 6$

$h(n) = \{1, 2, 3\}$

$x_1(n) = \{1, 2, 3, 4, 0, 0\}$

$x_2 = \{1, 2, 3, 4, 0, 0\}$

	1	2	3	4	0	0	
	1	0	0	0	3	2	1
	2	1	0	0	0	3	4
	2	3	1	0	0	0	10
	0	2	3	1	0	0	16
	0	0	2	3	1	0	17
	0	0	0	2	3	1	12

1 4 10 16 18 8

$\{1, 4, 10, 16, 18, 16, 10, 16, 17, 12\}$ 1 4 10 16 18 12

1 5 7 13 19 13 7 13 18 8

$x_1(n) = \{0, 0, 1, 2, 3, 4\}$
$x_2(n) = \{3, 4, 1, 2, 3, 4\}$
$x_3(n) = \{3, 4, 0, 0, 0, 0\}$

$$\begin{array}{c} 0\ 0\ 1\ 2\ 3\ 4 \\ \left|\begin{array}{cccccc} 1 & 0 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 2 & 1 \end{array}\right| \end{array} \begin{array}{c} \\ 17 \\ 12 \\ 1 \\ 4 \\ 10 \\ 16 \end{array} \qquad \begin{array}{c} 3\ 4\ 1\ 2\ 3\ 4 \\ \left|\begin{array}{cccccc} 1 & 0 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 2 & 1 \end{array}\right| \end{array} \begin{array}{c} \\ 20 \\ 22 \\ 18 \\ 16 \\ 10 \\ 16 \end{array}$$

$$\begin{array}{c} 3\ 4\ 0\ 0\ 0\ 0 \\ \left|\begin{array}{cccccc} 1 & 0 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 2 & 1 \end{array}\right| \end{array} \begin{array}{c} \\ 3 \\ 10 \\ 17 \\ 12 \\ 0 \\ 0 \end{array}$$

$\boxed{17\ \ 12}$ 1 4 10 16
$\boxed{20\ \ 22}$ 18 16 10 16
$\boxed{3\ \ 10}$ 17 12 0 0

1 4 10 16 18 16 10 16 17 12

PAGE NO. :
DATE : / /

	Add^n	Mult^n
DFT	$N(N-1)$	N^2
FFT	$N \log_2 N$	$\frac{N}{2} \log_2 N$

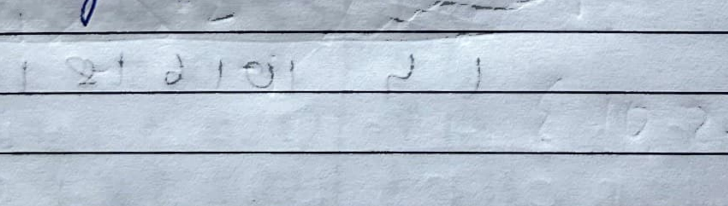
$N=8$	Add	Mul
DFT	56	64
FFT	24	12

Develop flow graph for $N=2$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$X[k] = x(0)\,1 + x(1)\, W_N^{nk}$$

$$X[0] = x(0) + x(1)$$
$$X[1] = x(0) + x(1) W_2^1 = x(0) - x(1)$$



flow graph

Fast fouries transform

Advanced flowgraph

DIT FFT — Decimation in Time

DIF FFT — Decimating in frequency.

Q Develop DIT-FFT flowgraph for N=4

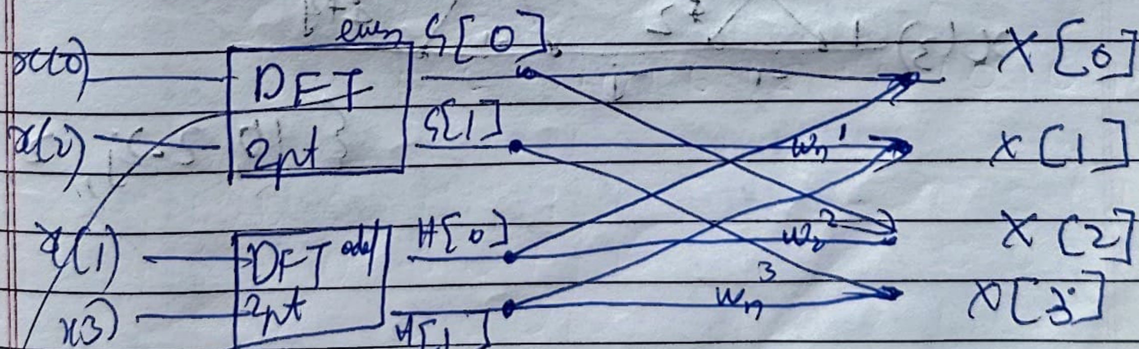
S By DIT - FFT eq

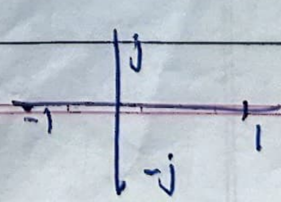
$$X[k] = G[k] + W_N^k H[k]$$

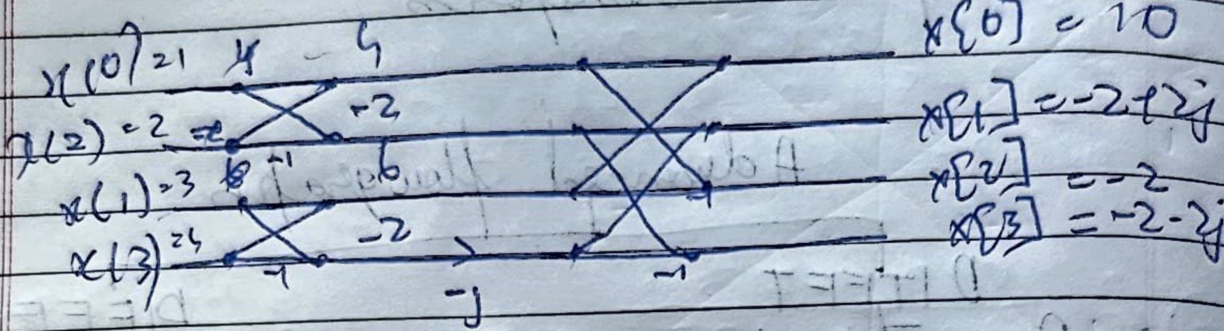
$$X[0] = G[0] + H[0]$$
$$X[1] = G[1] + W_N^1 H[1]$$

Periodic property:- $G_p(n) = \begin{bmatrix} G[0] \\ G[1] \\ G[0] \\ G[1] \end{bmatrix}$

$$X[2] = G[0] + W_N^2 H[0]$$
$$X[3] = G[1] + W_N^3 H[1]$$



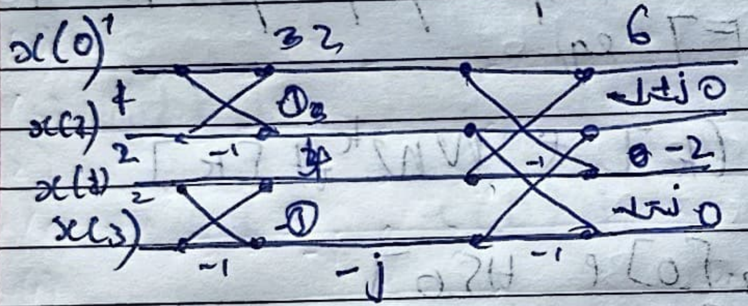




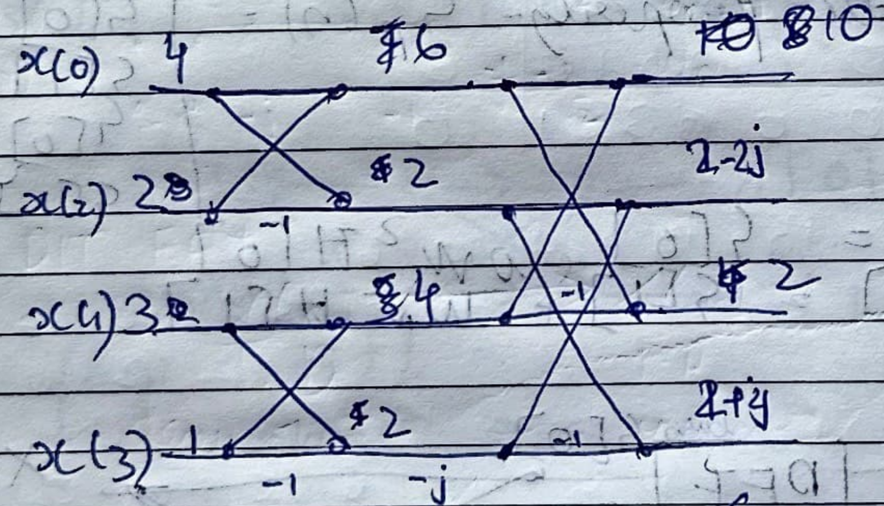
Q. Find DFT using flowgraph

if 1) $x(n) = \{1, 2, 1, 2\}$

2) $x(n) = \{4, 3, 2, 1\}$



$\{6, 0, -2, 0\}$



$\{10, 2-2j, 2, 2+2j\}$

PAGE NO.:
DATE: / /

Develop $n=8$ flowgraph usind DITFFT

$2^{3} \rightarrow$ total 3 levels

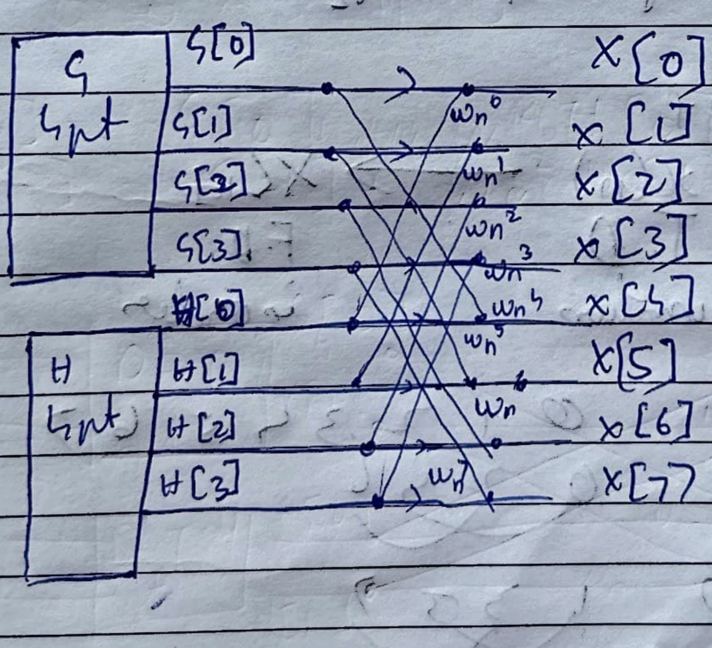
$N=2, \quad N=4, \quad N=8$

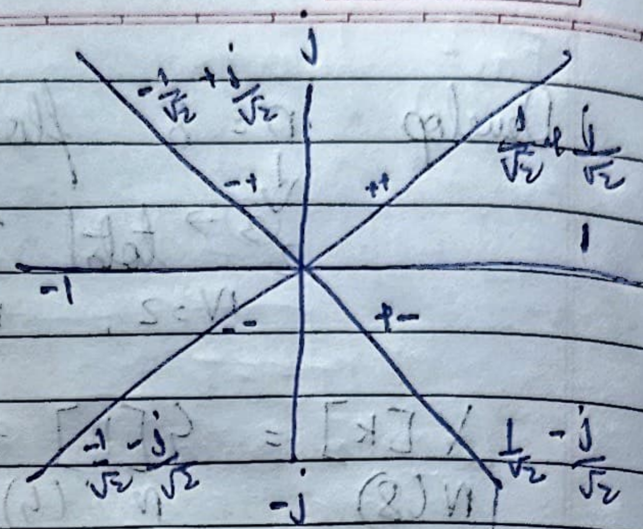
$X[k] = G[k] + W_N^k H[k]$

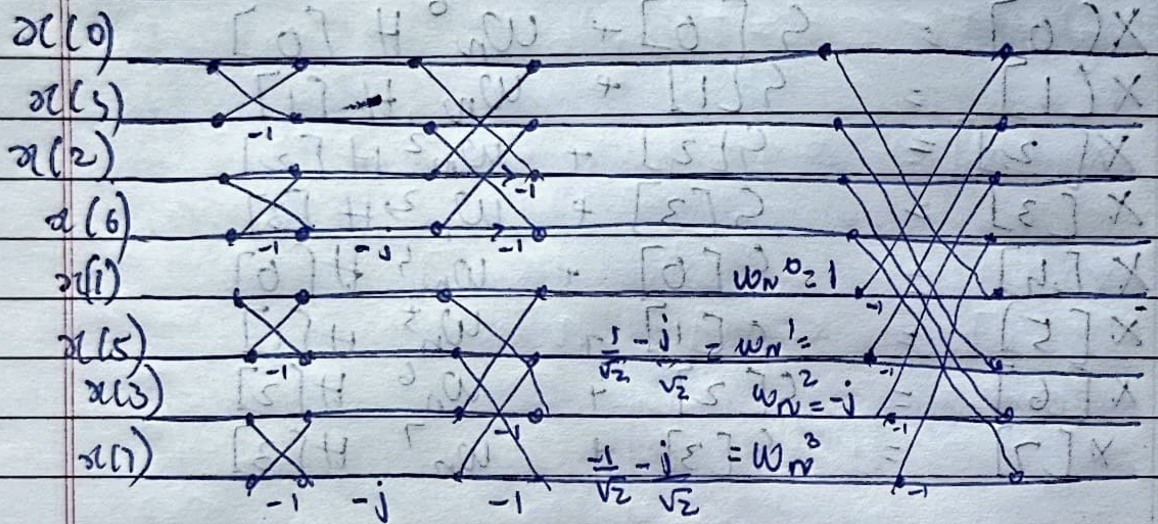
$N(8) \quad \frac{N}{4}(4) \quad \frac{N}{2}(4)$

$k(0-7) \quad (0\to3) \quad (0\to3)$

$X[0] = G[0] + W_N^0 H[0]$
$X[1] = G[1] + W_N^1 H[1]$
$X[2] = G[2] + W_N^2 H[2]$
$X[3] = G[3] + W_N^3 H[3]$
$X[4] = G[0] + W_N^4 H[0]$
$X[5] = G[1] + W_n^5 H[1]$
$X[6] = G[2] + W_n^6 H[2]$
$X[7] = G[3] + W_n^7 H[3]$







DIT FFT

x(n) ⟶ X[k]

T.D F.D

decimation in order

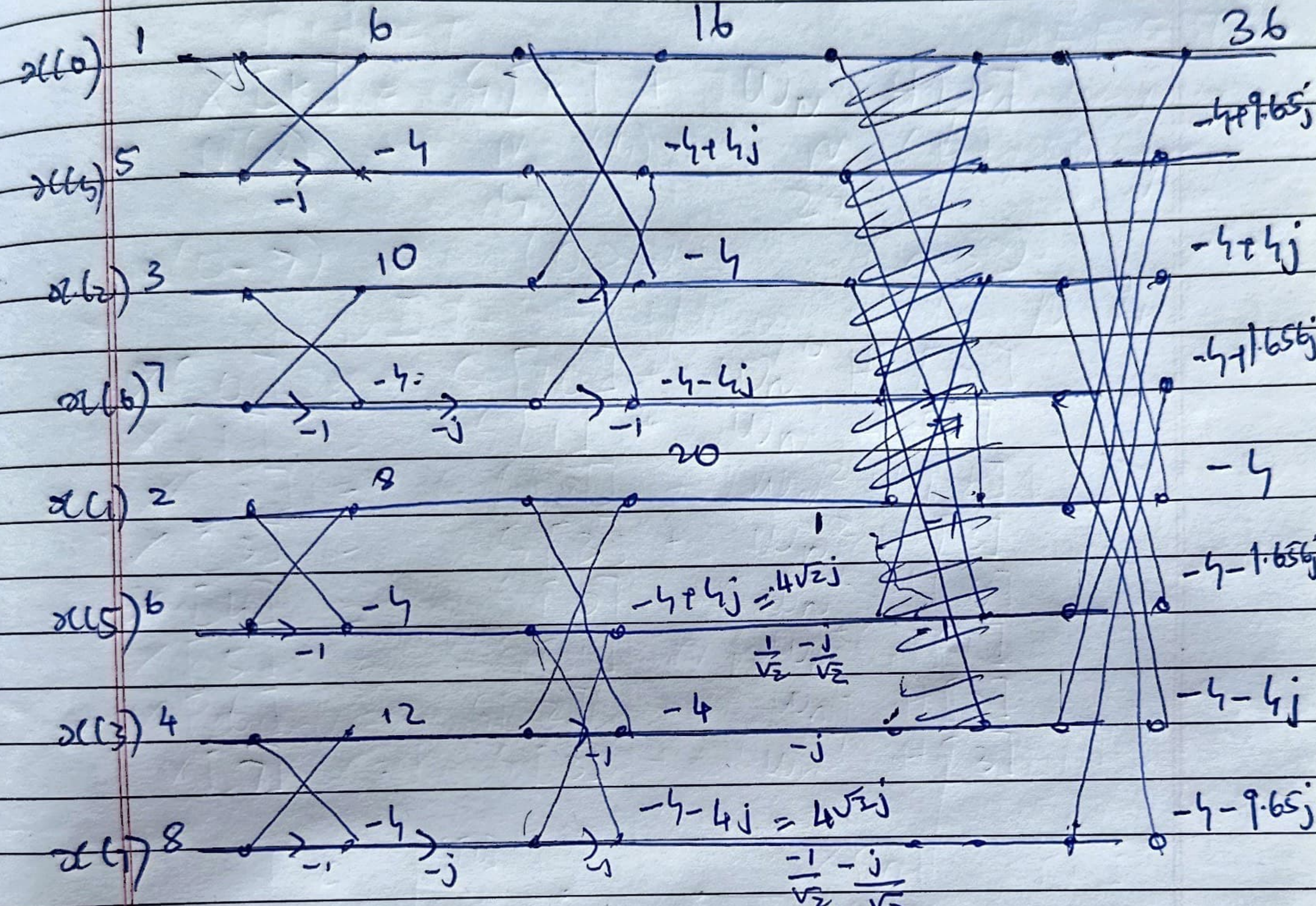
0 1 2 3 4 5 6 7

E 0 2 4 6

O 1 3 5 7

Q Find DFT of signal $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$ using DIT FFT

$x(n) \rightarrow X[k]$

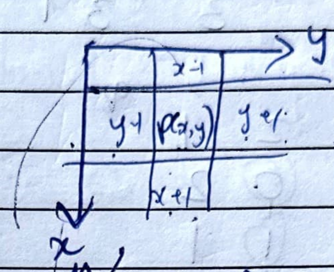


Answer is symetric only if signal is real.

I P   Book Rafel Gonzalez - IP

Binary image 1 bit
Grayscale image 8 bits
Color image 24 bits

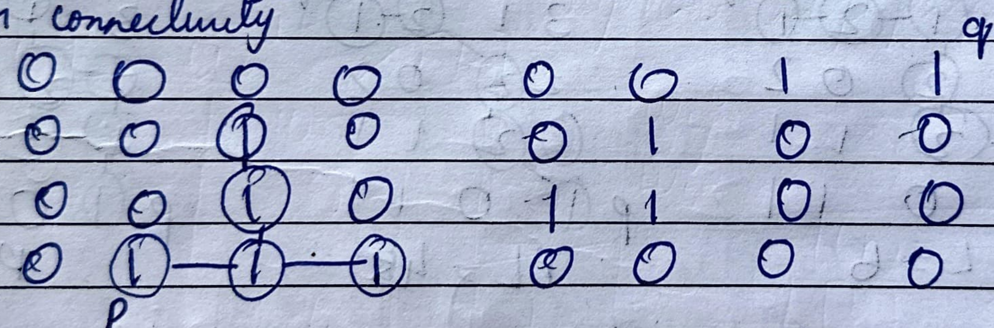
Vertical - x
Horizontal - y



4 Neighbourhood $N_4(P)$
Diagonal Neighbourhood $N_D(P)$
8   "   " $N_8(P)$

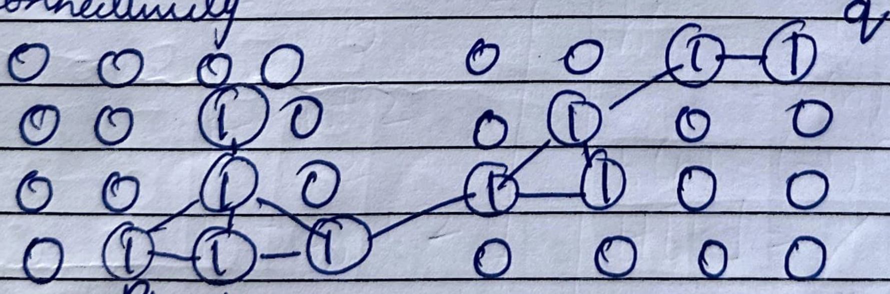
$N_8(P) = N_4(P) \cup N_D(P)$

4 connectivity

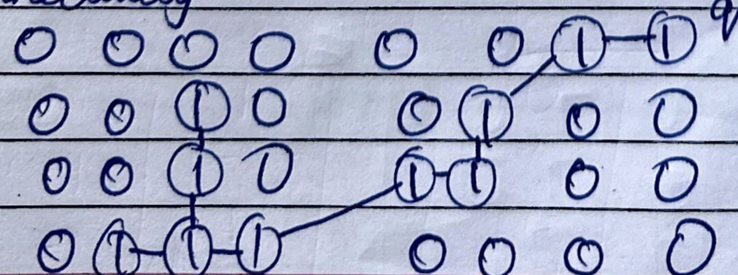


using 4 connectivity no path exist

8 connectivity



m- connectivity



4, 8, m Give proper explainations for

PL = Path length

Exam Question

p, q — i $V = \{0, 1\}$

ii $V = \{1, 2\}$

4 2 3 2
3 3 1 3

4

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	1

X

8

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	1

PL = 4

m

3	1	2	1
2	2	0	2
1	2	1	1
1	0	1	1

PL = 5

$V \in \{1, 2\}$

4

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	1

PL = 6

8

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	1

PL = 4

m

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	1

PL = 5

Research on PNG
next class

Good question :-
$V = \{1\}$
determine whether these 2 subset are 4/8/m

	S_1					S_2			
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

Theory

Image File format

BMP (Bit mapped Graphic Image)

TIFF (Tagged Image File Format)
Huffman coding for compression
JPEG (Joint Photographic Expert Grp)
DCT for compression
DISTANCE MEASURE
Eucledian
$$D_e(p,q) = \sqrt{(x-s)^2+(y-t)^2}$$
City block distance / Manhattan
$$D_4(p,q) = |x-s| + |y-t|$$

Chess Board Distance

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

CHP 1

H11

Image Enhancement

Fourier transform → spatial to frequency
Inverse fourier transform → frequency to spatial

Spatial Domain Methods

i Point Processing - Only a single bit is changed

→ Image negative :- Complete inversion

$s = (L-1) - r$

$s = (256-1) - r = 255 - r$

→ Thresholding → ↑ treshold
→ clipping → Gray level without background
→ Gray level with background
→ Bit plane slicing

$x =$

0	7	3	1
3	6	4	6
2	4	2	2
1	2	5	3

i) Image negative

7	0	4	6
4	1	3	1
5	3	5	5
6	5	2	4

ii) Thresholding threshold = 4

0	7	0	0
0	7	7	7
0	7	0	0
0	0	7	0

iii Clipping between $2 \leq x \leq 3$

iv

0	0	7	0
7	0	7	0
7	7	7	7
0	7	7	7

8 bpp ⇒ take 256 as L

Intensity level slicing between $2 \le x \le 5$

$7, \; 2 < x \le 5$

r else

0	7	7	1
7	6	7	6
7	7	7	7
1	7	7	7

Bit plane slicing

000111011001

~~010001011001~~ 011011000110

010100010010

001010101011

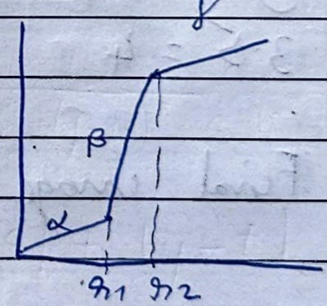
0	1	0	0
0	1	1	1
0	1	0	0
0	0	1	0

0	1	1	0
1	1	0	1
1	0	1	1
0	1	0	1

0	1	1	1
1	0	0	0
0	0	0	0
1	0	1	1

Contrast streching

$$s = \begin{cases} \alpha r & 0 \le r \le r_1 \\ \beta (r - r_1) + s_1 & r_1 \le r \le r_2 \\ \gamma (r - r_2) + s_2 & r_2 \le r \end{cases}$$



Q

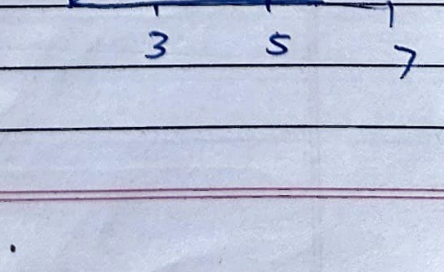
4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$r_1 = 3 \quad r_2 = 5$

$s_1 = 2 \quad s_2 = 6$

$\alpha = \frac{2-0}{3-0} = \frac{2}{3} \quad \beta = \frac{6-2}{5-3} = 2$

$\gamma = \frac{7-6}{7-5} = \frac{1}{2} = 0.5$



Q

7	12	2	3
10	15	1	6
12	4	6	15
8	2	7	15

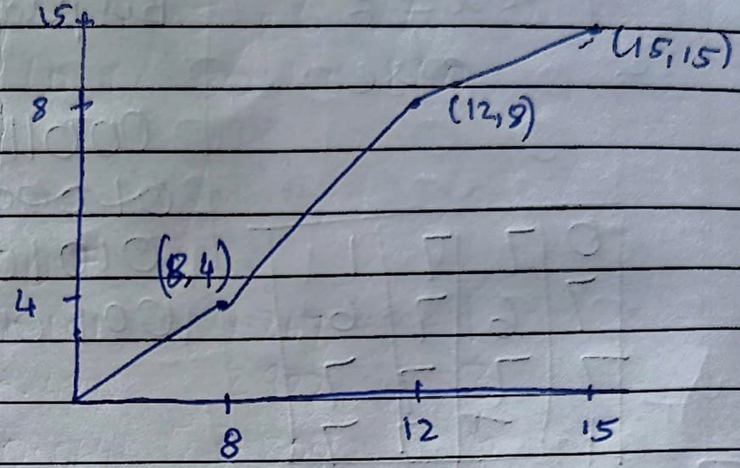
$r_1 = 8$ $\quad s_1 = 4$

$r_2 = 12$ $\quad s_2 = 8$

$\alpha = \frac{4}{8} = 0.5$

$\beta = \frac{8-4}{12-8} = 1$

$\gamma = \frac{15-8}{15-12} = \frac{7}{3}$



0		8	4
1	$0.5\times1 = 0.5 \approx 1$	9	
2	$0.5\times2 = 1$	10	6
3	$1.5 = 2$	11	
4	2	12	$\frac{7}{3}(0) + 8 = 8$
5		13	
6	3	14	
7	$3.5 = 4$	15	$\frac{7}{3}(3) + 8 = 15$

Final image =

4	8	1	2
6	15	1	3
8	2	3	15
4	1	4	15

Log transformation Dynamic Range Compression
It works for frequency domain not for spatial domain

HISTOGRAM EQUALIZATION

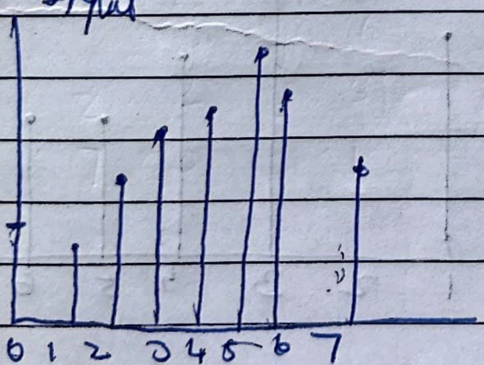
Q

Gray	0	1	2	3	4	5	6	7
Frequency	123	78	281	417	639	1054	816	688

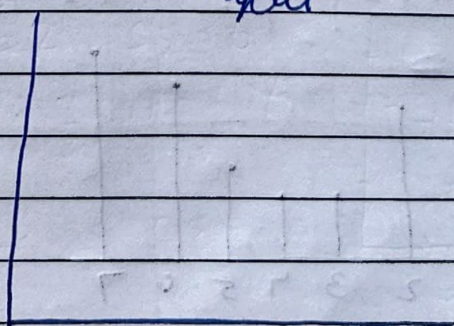
Apply histogram equalization

Gray lvl	Freq^n	PDF n_k/n	CDF	CDF×(N-1)	Round off
0	123	0.03	0.03	0.21	0
1	78	0.019	0.049	0.343	0
2	281	0.068	0.17	0.819	1
3	417	0.101	0.218	1.526	2
4	639	0.156	0.374	2.618	3
5	1054	0.257	0.631	4.417	4
6	816	0.199	0.83	5.81	6
7	688	0.167	1	7	7
	4096				

Input



Output



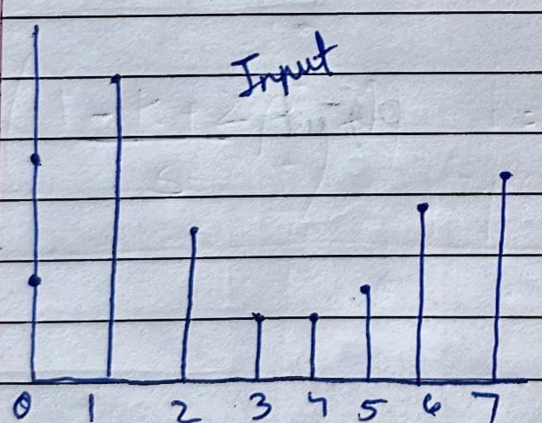
Q Equalise the following image

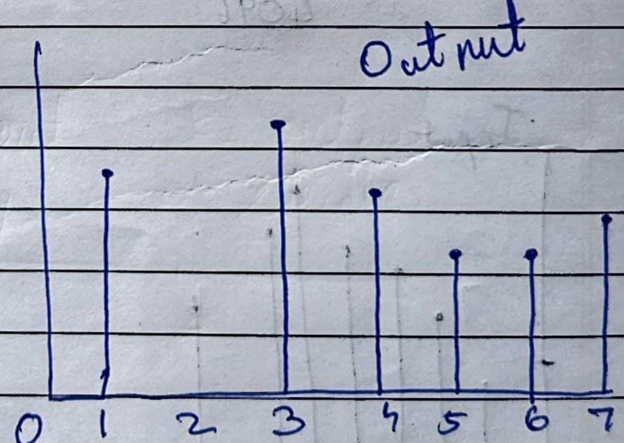
0	1	2	3	4	5	6	7
200	270	130	60	60	80	140	160

		P_K	CDF		
0	200	0.181	0.181	1.267	1
1	270	0.245	0.426	2.982	3
2	130	0.118	0.544	3.808	4
3	60	0.055	0.599	4.193	4
4	60	0.055	0.654	4.578	5
5	80	0.073	0.727	5.089	5
6	140	0.127	0.854	5.978	6
7	160	0.145	1	7	7
	~~900~~1100				

~~0 1 2 3 4~~

0	1	2	3	4	5	6	7
0	200	0	270	190	140	140	160





TT1

LPF ka $\Sigma = 1$

HPF ka $\Sigma = 0$

ii Neighbourhood Processing

• Spatial filtering

→ Smoothening → reducing the noise   Low Pass Filter (LPF)

→ Sharpenning → Improving the quality   High " " (HPF)

Q What happens to boundary if no lagging is involved

LPF (Averaging filter

$$3 \times 3 \text{ LPF Mask} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

→ NUMERICAL (8/10m)

Median Filter

arrange in ascending order and choose the median

High Pass filter

Noise will affect the compilation as it will also enhanced

— ChP5 —

* ⇒ Convolution

CHP 6 :- Edge detection or Segmentation

Highlighting a certain set of segmentation

Process of dividing an image into its constituent part

VIVA Derive the Mask for Pt detection of size 5

Edge based segmentation

Robert

-1	0
0	1

0	-1
1	0

Prewitt

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Sobel

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Most commonly used

Output size $\left(\left(\frac{i - k + 2p}{s}\right) + 1\right)$ i = input k = output p = padding s = stride

Q

10	9	9	4	0
0	6	6	2	2
5	9	8	4	3
7	5	5	4	3
8	10	8	5	0

Sobel

$$\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$$

X

$$\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

Y

X-direction gradient

$$\begin{matrix} 14 & -18 & -22 \\ 10 & -15 & -16 \\ -1 & -12 & -17 \end{matrix}$$

Y-direction gradient

$$\begin{matrix} +6 & +2 & -2 \\ -4 & 1 & -4 \\ -5 & -2 & 1 \end{matrix}$$

$$\begin{matrix} \sqrt{13^2+6^2} & \sqrt{18^2+2^2} & \sqrt{22^2+2^2} \\ \sqrt{10^2+4^2} & \sqrt{15^2+1^2} & \sqrt{16^2+4^2} \\ \sqrt{1^2+5^2} & \sqrt{12^2+2^2} & \sqrt{17^2+1^2} \end{matrix}$$

Magnitude =
$$\begin{matrix} 15 & 18 & 22 \\ 11 & 15 & 16 \\ 5 & 12 & 17 \end{matrix}$$

Threshold = 15

avg of this

if magnitude > threshol then 1 else 0

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$$

Laplacian Edge Detection (2nd derivative)

edges as well as direction

VIVA → not used very sensitive to noise

we use it in LOG laplacian of gaussian

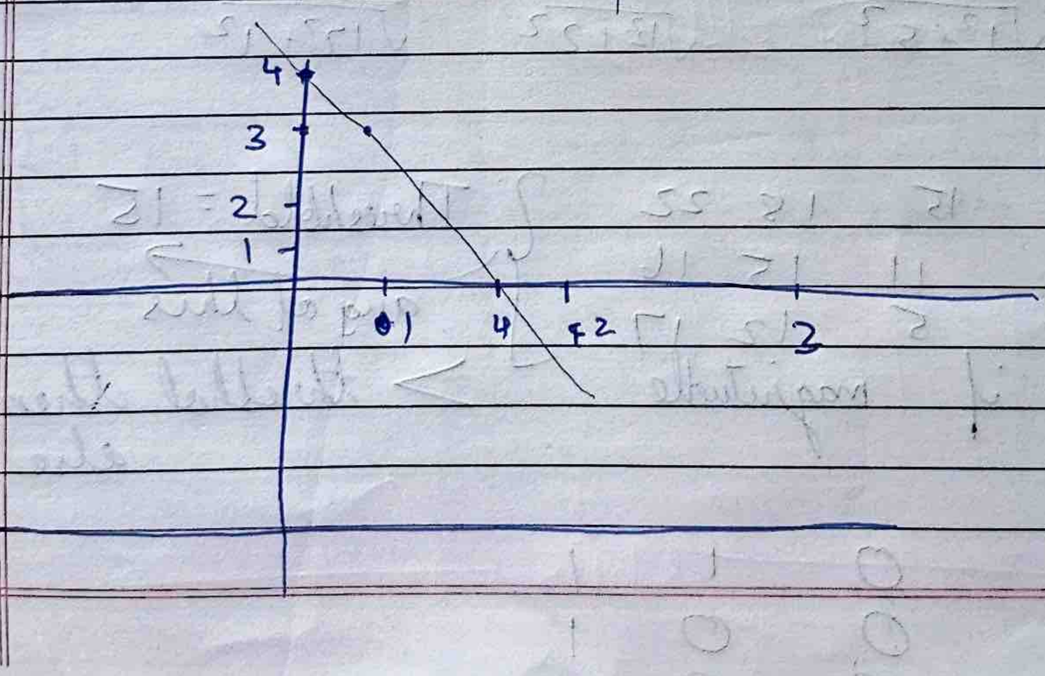
Short Note → **Hough transform**

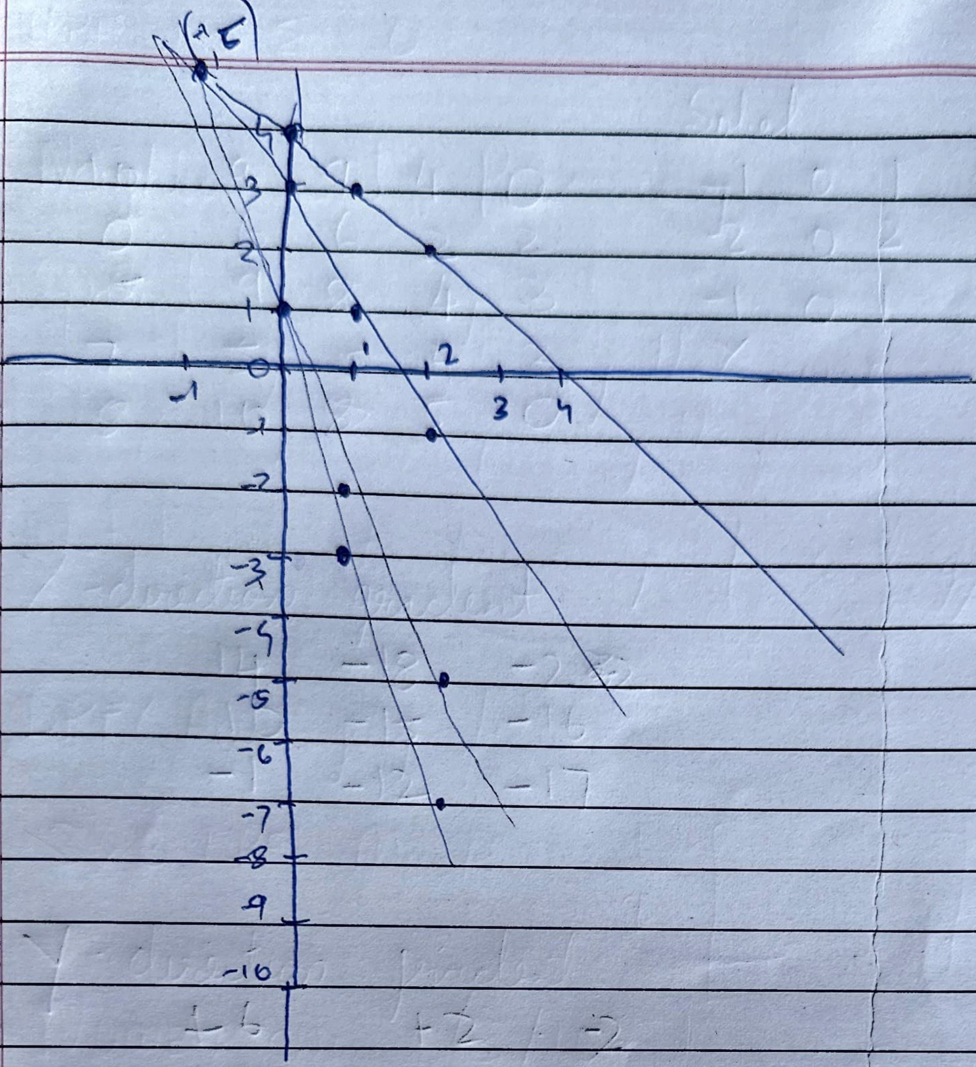
Broken edges → finds ~~images~~ lines from broken lines

Image space is transformed in parameter space

Numerical :-

A (1,4)		B (2,3)		C (3,1)		D (4,1)		E (5,0)	
$c = -mx + y$									
$c = -m + 4$		$c = -2m + 3$		$c = -3m + 1$		$c = -4m + 1$		$c = -5m$	
m	c	m	c	m	c	m	c	m	c
0	4	0	3	0	1	0	1	0	0
1	3	1	1	1	-2	1	-3	1	-5
2	2	2	-1	2	-5	2	-7	2	-10





(-1,5)
4
3
2
1
0
-1
1
2
3
4
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10

For given Image find (1) digital Negative (2) Image after Thresholding Th=3

(3) Grey level Slicing with and without background a=3 b=5

Sketch function for each operation and write equation

1	0	2	5	6	7	0
2	3	4	5	6	7	0
4	5	0	0	1	2	3
6	7	4	2	0	2	3
6	5	3	1	1	1	6
7	6	5	4	3	2	1
0	1	2	3	4	5	2

0	0	0	1	1	1	0
0	1	1	1	1	1	0
1	1	0	0	0	0	1
1	1	1	0	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0
0	0	0	1	1	1	0

6	7	5	2	1	0	7
5	4	3	2	1	0	7
3	2	7	7	6	5	4
1	0	3	5	7	5	4
1	2	4	6	6	6	1
0	1	2	3	4	5	6
7	6	5	4	3	2	5

132

eps=10 128

160

10

120

10 x 10

1.2

$\left(\frac{P}{E}\right)/4$

120

10 12/100

1.2

140/10x15

14/5

0.5 ↑

0	0	0	7	0	0	0
0	7	7	7	0	0	0
7	7	0	0	0	0	7
0	0	7	0	0	0	7
0	7	7	0	0	0	0
0	0	7	7	7	0	0
0	0	0	7	7	7	0

w/0

1	0	~~0~~2	7	6	7	0
2	7	7	7	6	7	0
7	7	0	0	1	2	7
6	7	7	2	0	2	7
6	7	7	1	1	1	6
7	6	7	7	7	2	1
0	1	2	7	7	7	2

with