



Shri Vile Parle Kelavani Mandal's  
**DWARKADAS J. SANGHVI COLLEGE OF ENGINEERING**

(Autonomous College Affiliated to the University of Mumbai)  
NAAC Accredited with 'A' Grade (COPA : 3.18)



**RUBRICS INDEX**

Academic Year 2022-23

Name: Shasvat Shah

SAP ID: 60004210126

Department/Branch: Computer

Division: B

Course / Subject Name: Discrete Structure

Course Code: DJ19CEC303

Performance Indicators	T1	T2	T3	T4	T5	T6	T6	T7	T8	Avg.
<b>Course Outcome</b>										
1.Knowledge (Factual/Conceptual)	4	4	4	4	4	4	4	4	4	4
2.Describe (Methods/Procedure) (Procedural)	2	2	2	2	2	2	2	2	2	2
3.Demonstration (Correct solutions) (Procedural/Metacognitive)	2	2	2	2	2	2	2	2	2	2
4. Attitude towards learning (receiving, attending, responding, valuing, organizing, characterization by value)	2	2	2	2	1	1	1	2	2	2
Total (10)	10	10	10	10	9	9	9	10	10	10
Signature	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Exceed Expectations (100%),

Meet Expectations (75%),

Below Expectations (40%)

Signature of the Teacher

Head of the Department

Principal

Name of the Teacher: Mrs. Vishakha Shelke

Date:

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## Tutorial 1

### Discrete Structures

#### Sets & Logics.

- 1 Determine the number of integers between 1 & 250 that are divisible by 2 or 3 or 5 or 7.

→ Let

A be the set of all numbers between 1 to 250 divisible by 2.

B be the set of all numbers between 1 to 250 divisible by 3.

C be the set of all numbers between 1 to 250 divisible by 5.

D be the set of all numbers between 1 to 250 divisible by 7.

$$|A| = \frac{250}{2} = 125$$

$$|B| = \frac{250}{3} = 83$$

$$|C| = \frac{250}{5} = 50$$

$$|D| = \frac{250}{7} = 35$$

$$|A \cup B| = \frac{250}{2 \times 3} = 41$$

$$|C \cup D| = \frac{250}{7 \times 5} = 7$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 125 + 83 - 41$$

$$= 167, \text{ By addition principle}$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 50 + 35 - 7$$

$$= 78, \text{ By addition Principle.}$$

$$|A \cup B \cup C| = |A \cup B| + |C| - (|A \cup B| \cap |C|)$$

$$= 167 + 78 - \frac{250}{2 \times 3 \times 7 \times 5}$$

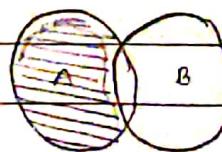
$$= 167 + 78 - 1$$

$$= \underline{\underline{244}}$$

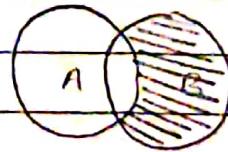
$\therefore$  Out of 250 numbers 244 numbers are divisible by 2, 3, 5 or 7.

2 Prove the following

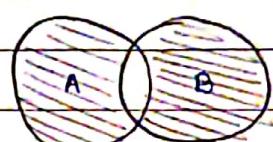
$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$



$A - B$



$B - A$



$$(A - B) \cup (B - A)$$

LHS



$$A \cup B$$



$$A \cap B$$



$$(A \cup B) - (A \cap B)$$

RHS

$$\therefore LHS = RHS$$

3 Prove that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$\text{Let set } A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$C = \{5, 6, 7\}$$

$$B \cap C = \{5\}.$$

$$A \times (B \cap C) = \{(1, 5), (2, 5)\}, \dots \text{ LHS}$$

$$A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}.$$

$$A \times C = \{(1, 5), (1, 6), (1, 7), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)\}$$

$$(A \times B) \cap (A \times C) = \{(1, 5), (2, 5), (3, 5)\}, \dots \text{ RHS}$$

$\therefore$  Hence Proved.

## 4 Proves Using Laws of Logic

$$[(p \vee q) \cap (p \vee \sim q)] \vee q \leftrightarrow p \vee q$$

$$\text{LHS} = [(p \vee q) \cap (p \vee \sim q)] \vee q$$

$$= [p \vee (q \cap \sim q)] \vee q$$

$$= [p \vee F] \vee q$$

$$= p \vee q$$

$$\text{RHS} = p \vee q$$

$\therefore \text{LHS} = \text{RHS}$  Hence Proved.

5 Verify that the preposition  $p \vee \sim(p \vee q)$  is a tautology

$p$	$q$	$p \vee q$	$\sim(p \vee q)$	$p \vee \sim(p \vee q)$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	F
F	F	F	T	T

$p$	$q$	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

tautology.

Since the truth table for the below equation is  $(T, T, T, T)$ , hence it is a tautology.

G Construct the truth table to determine whether each of the following is a tautology, contradiction or a contingency.

i)  $(q \vee p) \vee (\neg q \wedge \neg p)$

P	$q \vee$	$\neg p$	$q \vee \neg p$	$\neg q \wedge \neg p$	Final
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	F	T	T
F	F	T	F	F	F

contingency

ii)  $q \vee \rightarrow (q \rightarrow p)$

P	$q \vee$	$q \rightarrow p$	$q \vee (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

contingency.

iii)  $p \rightarrow (q \wedge p)$

P	$q \vee$	$q \wedge p$	$p \rightarrow (q \wedge p)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

contingency.

7 Write the english sentences for the following where

$P(x)$  :  $x$  is even.

$Q(x)$  :  $x$  is prime

$R(x, y)$  :  $x + y$ ,

i)  $\exists x \forall y R(x, y)$

There exists a value of  $x$  for all  $y$  such that relation  $R(x, y)$  where relation  $(x+y)$  exists.

ii)  $\sim (\exists x P(x))$

There doesn't exist any values of  $x$  where the value of  $x$  is even.

iii)  $\sim (\forall x Q(x))$

There doesn't exist any values of  $x$  where the value of  $x$  is a prime.

iv)  $\forall x (\sim Q(x))$

For all values of  $x$ , the value of  $x$  is not prime

Tutorial 2

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## Discrete Structures

Given that the truth values of  $x, y$ , and  $z$  as T and those of  $u$  &  $v$  as F. Find the truth values of  $(x \wedge (y \vee z)) \wedge \sim((x \vee z) \wedge (u \vee v) \wedge z)$

∴  $x = T \quad y = T \quad z = T \quad u = F \quad v = F$

$$(T \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$$

$$(T \wedge T) \wedge \sim[(T \wedge F) \wedge T]$$

$$T \wedge \sim[F \wedge T]$$

$$T \wedge \sim[F]$$

$$T \wedge \sim F$$

$$T \wedge T$$

$$= T$$

2 Prove using laws of logic  
i)  $a \rightarrow (p \vee c) \leftrightarrow (a \wedge \sim p) \rightarrow c$

$$\therefore \sim a \vee (p \vee c) — \text{LHS}$$

$$p \rightarrow r = \sim p \vee r$$

$$\sim(a \wedge \sim p) \vee c — \text{RHS}$$

LHS ...

$$\begin{aligned} & \sim q \vee (p \vee c) \\ \therefore & (\sim q \vee p) \vee (\sim q \vee c) \end{aligned}$$

RHS ...

$$\begin{aligned} & \sim (q \wedge \sim p) \vee c \\ & (\sim q \vee p) \vee c \\ & \sim q \vee (p \vee c) \dots \end{aligned}$$

$\therefore LHS = RHS$ , Hence Proved.

(ii)  $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \Leftrightarrow (\sim p \vee q)$

$$LHS = \sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q))$$

$$= \sim(\sim p \wedge \sim q) \vee (\sim p \vee (\sim p \vee q))$$

$$= (p \vee \sim q) \vee (\sim p \vee q)$$

$$= [(p \vee \sim p) \vee (p \vee q)] \wedge [(\sim p \vee \sim p) \vee q]$$

$$= T \wedge [\sim p \vee q]$$

$$= \sim p \vee q$$

$$= RHS,$$

$\therefore LHS = RHS$  Hence Proved.

Q3 Write English sentence for the following where  
 $P(x)$  is even.

$\alpha(x) : x$  is prime,

$R(x, y) : x \cdot y$  is even.

i)  $\exists x \forall y R(x, y)$

There exists a value for  $x$  for all values of  $y$  such that  $R(x, y)$  or  $x \cdot y$  is even.

ii)  $\forall x \exists y R(x, y)$

For all values of  $x$ , there exists a value for  $y$  such that the relation  $R(x, y)$  or  $x \cdot y$  holds true for even.

iii)  $\sim (\exists x P(x))$

There doesn't exist values of  $x$  when  $x$  is even.

iv)  $\sim (\forall x \alpha(x))$

There exists no values of  $x$  when  $x$  is a prime number.

v)  $\exists y [\sim P(y)]$

There exists a value for  $y$  when  $y$  is not even.

vi)  $\forall x (\sim \alpha(x))$

For all values of  $x$ , there is a value when  $x$  is not a prime.

4 At a university we know,

60% of the professors play tennis

50% of the professors play bridge.

70% of the professors jog.

20% play tennis and bridge.

30% tennis & jog.

40% bridge and jog.

If someone claimed that 20% of the professors jog, play bridge & tennis. Would you believe the claim, why



Let the professors playing tennis be represented by  $\star \cdot T$

Let the professors playing bridge be represented by  $B$

Let the professors playing jogging be represented by  $J$ .

Let  $U$  be the universal set  $|U| = 100$

$$|U| = 100 = |A \cup B \cup C|$$

$$|T| = 60$$

$$|B| = 50$$

$$|J| = 70$$

$$|T \cap J| = 30$$

$$|T \cap B \cap J| = ?$$

$$|T \cap B| = 20$$

$$|B \cap J| = 40$$

$$\text{Ans} |T \cup B \cup J| = |T| + |B| + |J| - |T \cap J| - |T \cap B| - |J \cap B| + |T \cap B \cap J|$$

$$100 = 60 + 50 + 70 - 40 - 20 - 30 + |T \cap B \cap J|$$

$$100 - 90 = |T \cap B \cap J|$$

$$10 = |T \cap B \cap J|$$

Hence the claim wouldnt be believed. As 10% is common

5 Let  $A = \{a, b, c, d, e, f, g, h\}$ . Consider the following subset of  $A$ .

$$A_1 = \{a, b, c, d\}$$

$$A_2 = \{a, c, e, f, g, h\}$$

$$A_3 = \{a, c, e, g\}$$

$$A_4 = \{b, d\}$$

$$A_5 = \{f, h\}$$

Determine whether each of the following is a partition or not.

i)  $A_1 \& A_2$

$$\{A_1, A_2\}$$

Not a partition, because they aren't mutually disjoint.

ii)  $A_2 \& A_3 \& A_4$

$$\{A_2, A_3, A_4\}$$

Yes it is a partition.

iii)  $A_1, A_3$

$$\{A_1, A_3\}$$

Not a partition, because they aren't mutually disjoint  
&  $A_1 \cup A_3 \neq A$ .

iv)  $A_3, A_4, A_5$

$$\{A_3, A_4, A_5\}$$

Yes it is a partition,

because

$$A_3 \cup A_4 \cup A_5 = A$$

$$A_3 \cap A_4 \cap A_5 = \emptyset$$

G Let the Universal set be

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 4, 7, 9\}$$

$$B = \{1, 4, 6, 7, 10\}$$

$$C = \{3, 5, 9, 7\}$$

→ Given

$$|A| = 4$$

$$|B| = 5$$

$$|C| = 4$$

$$|U| = 10$$

i)  $A \cup B \rightarrow \{1, 2, 4, 6, 7, 9, 10\}$

ii)  $A \cap C \rightarrow \{7, 9\}$

iii)  $B \cap \bar{C} \rightarrow \{1, 4, 6, 10\}$

iv)  $A \cup (A \cap \bar{B}) \cup C$        $\bar{B} = \{2, 3, 5, 8, 9\}$

$$(A \cap \bar{B}) \rightarrow \{2, 3, 4, 5, 7, 8, 9\}$$

$$(A \cap \bar{B}) \rightarrow \{2, 9\}$$

$$(A \cap \bar{B}) \cup C \rightarrow \{2, 3, 5, 7, 9\}$$

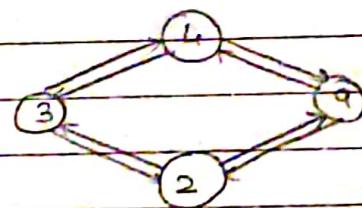
Discrete Structures  
Relations & Functions

1 Given ,

$$A = \{2, 3, 4, 6, 9\}$$

Relation = 'x is negatively prime to y'

$$R = \{(2, 3) (2, 9) (3, 2) (3, 4) (4, 3) (4, 9) (9, 4), (9, 2)\}$$

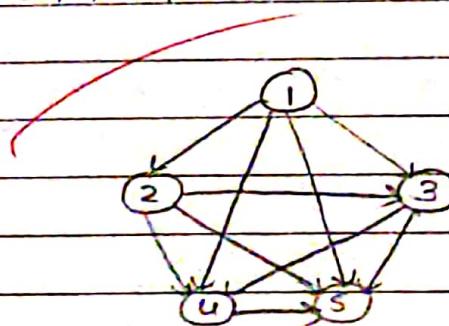


Shows the directed graph of the above relation R.

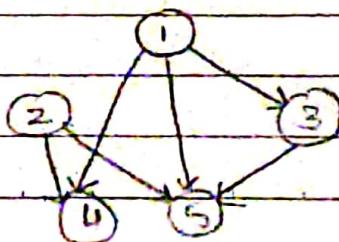
2 Given  $A = \{1, 2, 3, 4, 5\}$ 

Find  $R$ ,  $R^2$ ,  $R^3$  arb 'y  $\Rightarrow b$   $a < b$

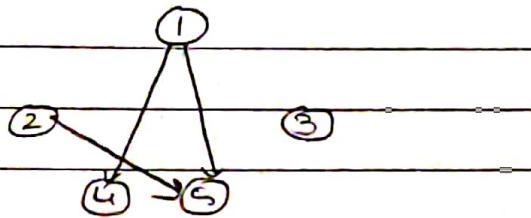
$$R = \{(1, 2) (1, 3) (1, 4) (1, 5) (2, 3) (2, 4) (2, 5) (3, 4) (3, 5) (4, 5)\}$$



$$R^2 = \{(1, 3) (1, 4) (1, 5) (2, 4) (2, 5) (3, 5)\}$$



$$R^3 = \{(1,4), (1,5), (2,5)\}$$



3)  $S = \{1, 2, 3, 4\}$

$$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$$

i) Show that the relation is transitive.

For  $R$  to be transitive it must satisfy the condition  $aRb \& bRc$  then  $aRc$

Here  $(4,1) \in R$  should be satisfied.

Hence for it to be transitive  $(4,1)$  should be present in  $R$ .

$\therefore R$  is not transitive.

ii) Find a relation  $R_1 \subseteq R$  such that  $R$  is transitive.

For a transitive set if  $aRb \& bRc$  then  $aRc$  should be present.

$$MR = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 \\ \hline 1 & [0 & 1] & 0 & 0 \\ 2 & [1 & 1] & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$$\text{Subset } MR_1 = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 1 & 1 \end{array} \text{ is transitive.}$$

since  $(1,2)(2,1)(2,2)$

$\therefore$  its transitive.

$$R_1 = \{(2,1), (1,2), (2,2)\}$$

iii) Transitive closure of  $R$  by warshall's algorithm,

$$R = \{(4,3)(2,2)(2,1)(3,1)(1,2)\}.$$

	1	2	3	4	col row	row col
1	0	1	0	0		
2	1	1	0	0		
3	1	0	0	0		
4	0	0	1	0	(2,2)(2,3)(3,2)	

∴ Add pairs that don't exist in the Relation.

	1	2	3	4	col row	row col
1	0	1	0	0		
2	1	1	0	0		
3	1	0	0	0		
4	0	0	1	0	(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)	

∴ Add pairs that don't exist in  $w_1$ .

	1	2	3	4	col row	row col
1	1	1	0	0		
2	1	1	0	0		
3	1	1	0	0		
4	0	0	1	0	(1,1)(2,1)(4,1)(4,2)	

	1	2	3	4	col row	row col
1	1	1	0	0		
2	1	1	0	0		
3	1	1	0	0		
4	1	1	1	0	= b	

$$RT = \{(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)(4,1)(4,2)(4,3)\}.$$

$$4 \quad A = \{11, 12, 13, 14\}.$$

$$R = \{(11, 12), (12, 13), (13, 14), (12, 11)\}.$$

Find transitive closure of R using warshall's algo.

	11	12	13	14	col row
11	0	1	0	0	11 12
12	1	0	1	0	12 13
13	0	0	0	1	13 14
14	0	0	0	0	(12, 12)

	11	12	13	14	col row
11	0	1	0	0	11 11
12	1	1	1	0	12 12
13	0	0	0	1	13 13
14	0	0	0	0	(11, 11)(11, 12)(11, 13)(12, 11)(12, 12)(12, 13)

	11	12	13	14	col row
11	1	1	1	0	11 14
12	1	1	1	0	12 14
13	0	0	0	1	13 14
14	0	0	0	0	(11, 14) (12, 14)

	11	12	13	14	col row
11	1	1	1	1	11 0
12	1	1	1	1	12 0
13	0	0	0	1	13 0
14	0	0	0	0	

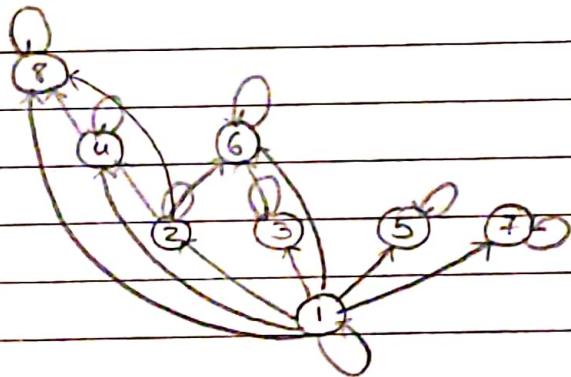
$$\therefore R_T = \{(11, 11)(11, 12)(11, 13)(11, 14)(12, 11)(12, 12)(12, 13)(12, 14)(13, 14)\}$$

Discrete Structures.

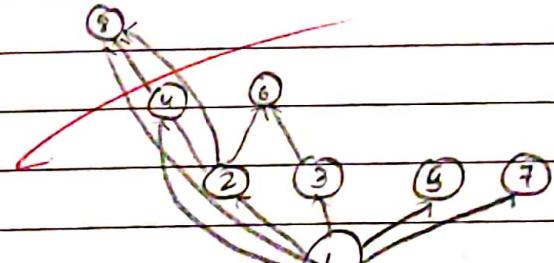
1 Draw hasse diagram for the divisibility set

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$

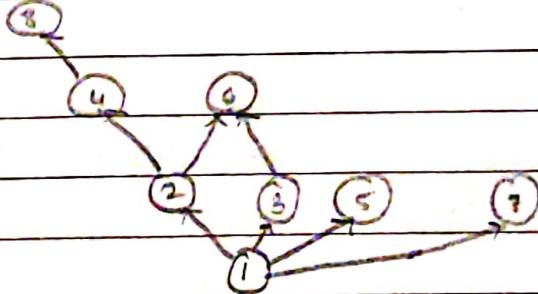
$$R = \{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (1,7) (1,8) (2,2) (2,4) (2,6) (2,8) (3,3) (3,6) (4,4) (4,8) (5,5) (6,6) (7,7) (8,8)\}.$$



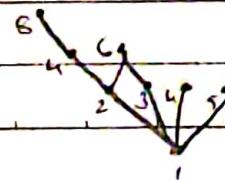
Shows the digraph of the above relation.



Step one removes the loops.



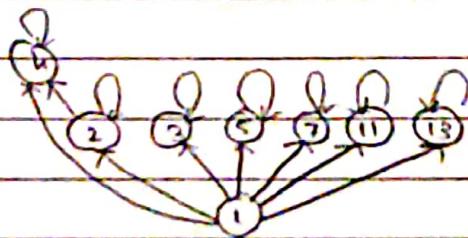
Step 2 removes the transitive edges



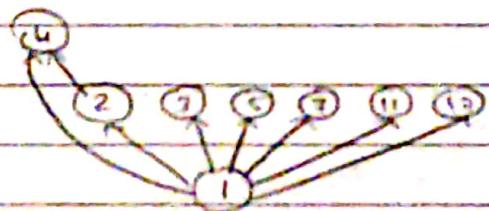
Shows the hasse diagram.

ii)  $\{1, 2, 3, 4, 5, 7, 11, 13\}$ .

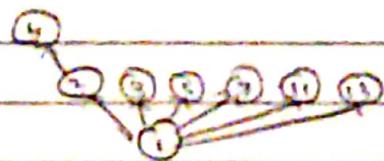
$$R = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,7)(1,11)(1,13)(2,2)(2,4)(3,3)(4,4)(5,5)(7,7)(11,11)(13,13)\}$$



shows the digraph  
of the following relation



shows the step 1  
without loops



shows the step 2  
without transitive edges.

C)



shows the given hasse  
diagram

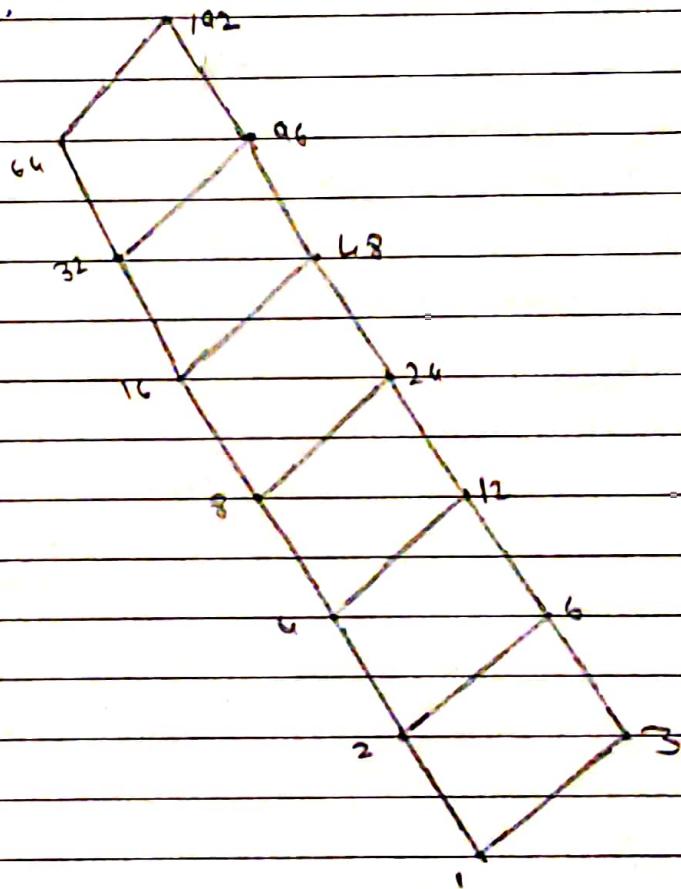
2 Draw hasse diagram for  $D_{192}$

$$D = \{1, 2, 3, 4, 6, 8, 12, 16, 32, 48, 64, 96, 192\}$$

$$R = \{(1,1)(1,2)(1,3)(1,4)(1,6)(1,8)(1,12)(1,16)(1,24)(1,32)(1,48)(1,64)(1,96)(1,192)(2,2)(2,4)(2,6)(2,8)(2,12)(2,16)(2,24)(2,48)(2,96)(2,192)(3,3)(3,6)(3,12)(3,24)(3,48)(3,96)(3,192)(4,4)(4,8)(4,12)(4,16)(4,32)(4,24)(4,48)(4,64)(4,96)(4,192)(6,6)(6,12)(6,24)(6,48)(6,96)(6,192)(8,8)(8,16)(8,32)(8,24)(8,64)(8,96)(8,192)(12,12)(12,24)(12,48)(12,96)(12,192)(16,16)(16,32)(16,48)(16,64)(16,96)(16,192)(24,24)(24,48)(24,96)(24,192)\}$$

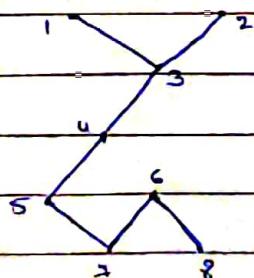
$(24, 96) (24, 192) (32, 32) (32, 64) (32, 96) (32, 192) (64, 48, 48) (48, 96)$   
 $(48, 192) (64, 64) (64, 192) (96, 96) (96, 192) (192, 192) \}$

Hasse Diagram,



Q3

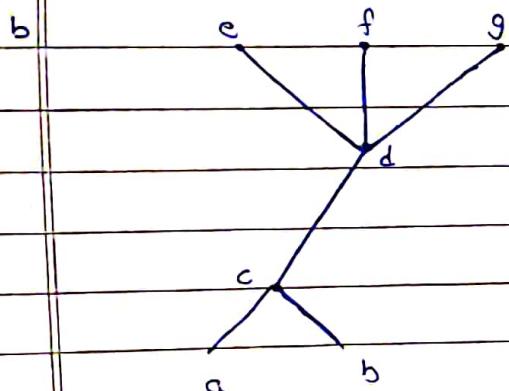
Determine whether the following hasse diagram represents a lattice or not.



LUB

	1	2	3	4	5	6	7	8
1	1	-	1	1	1	1	1	1
2	-	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3
4	1	2	3	4	4	4	4	4
5	1	2	3	4	5	5	5	5
6	1	2	3	4	5	6	6	6
7	1	2	3	4	5	6	6	6
8	1	2	3	4	5	6	6	6

Since we can find null values available, hence it is not a lattice.



Here again since we can see that for the case of e, f, g, we cannot find the LUB. Hence it is not a lattice.

