60004210075
Medha Shah
A3

TUTORIAL 1 : SETS & LOGIC

24/25

(Q1) Determine the number of integers between 1 & 250 that are divisible by 2 or 3 or 5 or 7

(Q2) Prove the following.

$$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$$

(Q3) Prove that $A \times (B \wedge C) = (A \times B) \cap (A \times C)$

(Q4) Prove using laws of logic

$$[(p \vee q) \wedge (p \vee \sim q)] \vee q \iff p \vee q$$

(Q5) verify that the proposition

$p \vee \sim(p \wedge q)$ is a tautology

(Q6) construct the truth table to determine whether each of the following is tautology or contradiction or contigency.

i) $(q \wedge p) \vee (q \wedge \sim p)$

(ii) $q \rightarrow (q \rightarrow p)$

(iii) $p \rightarrow (q \wedge p)$

(Q7) Write english sentences for the following where

$P(x)$: x is even

$Q(x)$: x is prime

$R(x, y)$: x + y.

i) $\exists x \ \forall y \ R(x, y)$

(ii) $\sim [\exists x \ P(x)]$

(iii) $\sim (\forall x \ Q(x)]$

(iv) $\forall x [\sim Q(x)]$

(A1) Let set A be a set containing all ~~even~~ integers divisible by 2 b/w 1 and 250

$\therefore$ cardinality of A = $\frac{250}{2}$ = 125

Set B be a set containing all integers divisible by 3 b/w 1 & 250

$\therefore$ Cardinality of B = $\frac{250}{3}$ = 83

Similarly, set C → divisible by 5
cardinality = $\frac{250}{5}$ = 50

set D, → divisible by 7
Cardinality = $\frac{250}{7}$ = 35

By addition Principle,

$|A \cup B| = |A| + |B| - |A \cap B|$ $\left\{ |A \cap B| = \frac{250}{6} \right\}$

$|A \cup B| = 125 + 83 - 41$

$|A \cup B| = 167$

$|C \cup D| = |C| + |D| - |C \cap D|$ $\left\{ |C \cap D| = \frac{250}{35} \right.$ 7 $\left. \right\}$

$= 50 + 35 - 7$

$= 85 - 7$

$= 78$

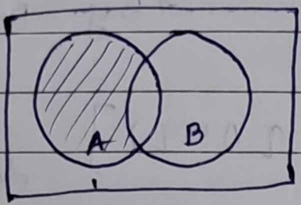
$\big| |A \cup B| \cup |C \cup D| \big| = |A \cup B| + |C \cup D| - |(A \cup B) \cap (C \cup D)|$

~~= |AUB| + |CUD|~~ $167 + 78 - 1$

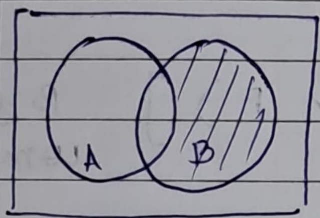
$= \underline{\underline{244}}$

(A2) $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

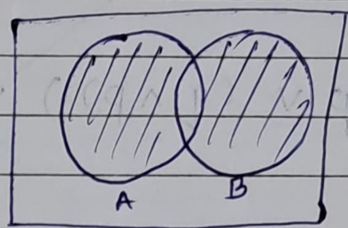
LHS.



$A-B$

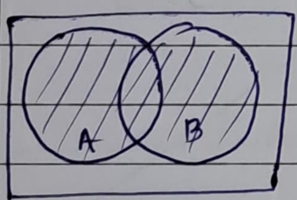


$B-A$

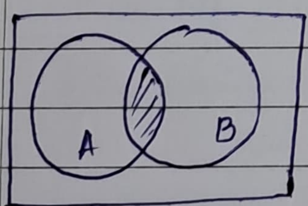


$(A-B) \cup (B-A)$

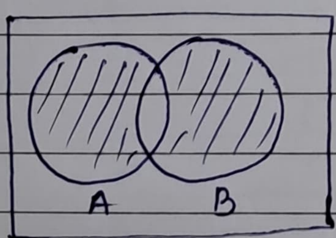
RHS



$A \cup B$



$A \cap B$



$(A \cup B) - (A \cap B)$

LHS = RHS

Hence Proved, $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

(5) $p \vee \sim(p \wedge q)$ is Tautology : TPT.

Col 1	Col 2	Col 3	Col 4	Col 5
p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\therefore$ By col 5, it is a tautology

(6)
(i)

q	p	$\sim p$	$q \wedge p$	$q \wedge \sim p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	T	F	F	F

Contingency : $(q \wedge p) \vee (q \wedge \sim p)$

(ii) $q \rightarrow (q \rightarrow p)$

q	p	$q \rightarrow p$	$q \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Contingency = $q \rightarrow (q \rightarrow p)$

(iii) $p \rightarrow (q \wedge p)$

q	p	$q \wedge p$	$p \rightarrow (q \wedge p)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	T

Contingency = $p \rightarrow (q \wedge p)$.

(Q7) given

$P(x)$: x is even

$Q(x)$: x is prime

$R(x, y)$ = $x+y$

(i) $\exists x \ \forall y \ R(x+y)$

There exist a value of x for all ~~all~~ y for which $R(x,y)$ holds true. Where $R(x,y)$ is a relationship between $(x \& y)$

(ii) $\sim(\exists x \ P(x))$

There doesnot exist a value of x which is even.

(iii) $\sim(\forall x \ Q(x))$

there doesnot exist any value of x which is prime.

(iv) $\forall x \ (\sim Q(x))$

For all x, x is not prime.

25

TUTORIAL 2 :

60004210075
Medha shah
A3

Q1) Given the truth values of x, y and z as T and those of u and v as F, find the truth values of

$(x \wedge (y \vee z)) \wedge \sim((x \vee z) \wedge (u \vee v) \wedge z)$

Q2) Prove using laws of logic

(a) $a \rightarrow (p \vee c) \longleftrightarrow (a \wedge \sim p) \rightarrow c$

(b) $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \longleftrightarrow (\sim p \vee q)$

Q3) Write English sentences for the following where

$p(x)$: x is even $Q(x)$: x is prime

$R(x, y)$: x . y is even.

(i) $\exists x \ \forall y \ R(x, y)$

(ii) $\forall x \ \exists y \ R(x, y)$

(iii) $\sim(\exists x \ P(x))$

(iv) $\sim(\forall x \ Q(x))$

(v) $\exists y \ (\sim P(y))$

(vi) $\forall x \ (\sim Q(x))$

Q4) It is known that at a university, 60% of professors play Tennis, 50% play Bridge, 70% jog, 20% Tennis and Bridge, 30% play Tennis & jog and 40% play bridge & jog. If someone claimed that 80% of professors jog & play bridge & play tennis. Would you believe claim? why?

Q5) Let A = {a, b, c, d, e, f, g, h}. Consider the following subsets of A. A_1 = {a, b, c, d} A_2 = {a, c, e, f, g, h}

A_3 = {a, c, e, g} A_4 = {b, d} A_5 = {f, h}

Determine wether each of the following is partition of A or not?

(a) {A_1, A_2}   (b) {A_1, A_5}   (c) {A_3, A_4, A_5}

(Q6) Let the universal set be $U = \{1, 2, 3 \dots, 10\}$

Let $A = \{2, 4, 7, 9\}$

$B = \{1, 4, 6, 7, 10\}$ $\quad C = \{3, 5, 7, 9\}$

Find

i) $A \cup B$ $\quad$ iii) $B \cap \bar{C}$

ii) $A \cap C$ $\quad$ iv) $(A \cap B) \cup C$

SOLUTIONS

(A1) Given : values of x, y, z as T

u, v as F

expression: $(x \wedge (y \vee z)) \wedge \sim((x \vee z) \wedge (u \vee v) \wedge z)$

Replacing the values,

$(F \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$

$(F \wedge T) \wedge \sim(T \wedge F \wedge T)$

$= T \wedge \sim(F)$

$= T \wedge T$

$= T$

(A2)(a) $a \rightarrow (p \vee c) \longleftrightarrow (a \wedge \sim p) \rightarrow c$

LHS : $a \rightarrow (p \vee c)$

$(\sim a \vee (p \vee c))$ $\quad$ implication Law

$\cancel{((\sim a \vee p) \vee c)}$ $((\sim a \vee p) \vee c)$ $\quad \dots$ associative Law

$= \sim(a \wedge \sim p) \vee c$

$= [(a \wedge \sim p) \rightarrow c]$ $\quad$ implication Law.

LHS = RHS

Hence Proved.

(b) $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

LHS

$\sim(p \vee q) \rightarrow (\sim p \vee (\sim p \vee q))$

$\sim(\sim(p \vee q)) \vee (\sim p \vee (\sim p \vee q))$ Implication Law

$(p \vee q) \vee ((\sim p \vee \sim p) \vee (\sim p \vee q))$ Associative

$(p \vee q) \vee (\sim p \vee (\sim p \vee q))$

$p \vee (\sim p \vee (\sim p \vee q)) \wedge q \vee (\sim p \vee (\sim p \vee q))$

$((p \vee \sim p) \vee (p \vee \sim p) \vee (p \vee q)) \wedge ((q \vee \sim p) \vee (q \vee \sim p) \vee (q \vee q))$

$(T) \wedge ((q \vee \sim p) \vee (q \vee \sim p) \vee q)$ $\because p \vee \sim p = T$

$(q \vee p) \vee (q \vee p) = T$

$= (\sim p \vee q)$

LHS = RHS.

Hence, Proved.

(A3) (i) $\exists x \forall y \; R(x, y)$: There exist a value of x, for all values of y for which the relationship $x \cdot y$ is even holds true.

(ii) $\forall x \exists y \; R(x, y)$: ~~There exists a value of~~

For all values of x, there exists a value of y for which the relationship $x \cdot y$ is even, holds true.

(iii) $\sim(\exists x \; P(x))$: There doesnot ~~exist a value of~~ x for which x is even.

~~(iv) $\sim(\forall x$~~

FOR EDUCATIONAL USE

(iii) $\sim(\exists x\ P(x))$: it is not true that, there exist a value of x which is even.

(iv) $\sim(\forall x\ Q(x))$: It is not true that, for all values of x, x is prime.

(v) $\exists y\ (\sim P(y))$: There exists a value of y for which y is even.

(vi) $\forall x\ (\sim Q(x))$: For all values of x, x is not prime.

(A4) Let A be professors playing tennis

$|A| = 60$

Let B be professors playing bridge $|B| = 50$

Let C be professors jogging $|C| = 70$

Also,

Given,
$|A \cap B| = 20$
$|A \cap C| = 30$
$|B \cap C| = 40$

To find : $|A \cap B \cap C|$;

Also, $|A \cup B \cup C| = 100$ (assuming data to be of 100 professors)

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

let $|A \cap B \cap C| = x$

~~100 = (20 + 30 + 40) + t~~

$100 = 60 + 50 + 70 - 20 - 30 - 40 + x$

$100 = 180 - 90 + x$

$100 - 90 = x$

$10 = x$

$\therefore$ the claim doesnot hold true as there are 10% professors play bridge, tennis and jog.

Sundaram

FOR EDUCATIONAL USE

(A5) Given,

$A = \{a, b, c, d, e, f, g, h\}$

$A_1 = \{a, b, c, d\}$

$A_2 = \{a, c, e, f, g, h\}$

$A_3 = \{a, c, e, g\}$

$A_4 = \{b, d\}$

$A_5 = \{f, h\}$

(a) $\{A_1, A_2\}$: $\{(a, b, c, d), (a, c, e, f, g, h)\}$

It is NOT A PARTITION as the cells are not mutually disjoint.

$(a, b, c, d) \cap (a, c, e, f, g, h) = (a, c)$

$\therefore (a, c) \neq (\phi)$.

(b) $\{A_1, A_5\}$: $\{(a, b, c, d), (f, h)\}$

It is NOT A PARTITION because the cells do not consists of all ~~ets~~ elements in set A

$\because (a, b, c, d) \cup (f, h) = (a, b, c, d, f, h)$

It doesnot contain the elements e, g.

(c) $\{A_3, A_4, A_5\} = \{(a, c, e, g), (b, d), (f, h)\}$

$\because \{a, c, e, g\} \cup \{b, d\} \cup \{f, h\} = S$

~~$\{a, c, e, g\} \cup \{b, d\}$~~ $\cup \{f, h\} = \phi$

i.e. mutually disjoint.

Hence, it is a Partition of A.

(A6) Given: $U = \{1, 2, 3 \dots 10\}$

& $A = \{2, 4, 7, 9\}$ $B = \{1, 4, 6, 7, 10\}$

$C = \{3, 5, 7, 9\}$

(i) $A \cup B = \{1, 2, 4, 6, 7, 9, 10\}$

(ii) $B \cap \bar{C}$

$\bar{C} = \{1, 2, 4, 6, 8, 10\}$

$B \cap \bar{C} = \{1, 4, 6, 10\}$

(iii) $A \cap C$

$A \cap C = \{7, 9\}$

(iv) $(A \cap \bar{B}) \cup C$

$\bar{B} = \{2, 3, 5, 8, 9\}$

$A \cap \bar{B} = \{2, 9\}$

$(A \cap \bar{B}) \cup C = \{2, 3, 5, 7, 9\}$

60004210075
Medha shah
A3

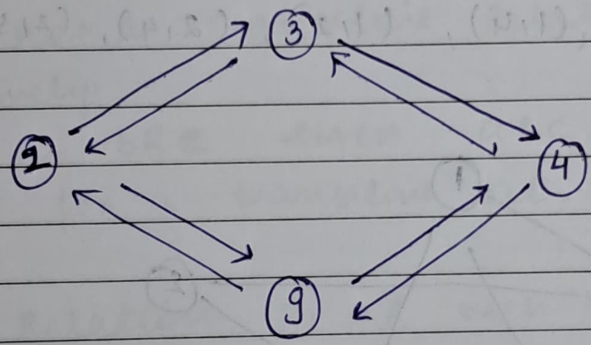
TUTORIAL 3 : RELATIONS & FUNCTIONS

24

(A1) Given,
$A = \{2, 3, 4, 6, 9\}$
Relation: 'x is relatively prime to y'

$R = \{(2,3), (2,9), (3,4), (4,9), (9,4), (4,3), (9,2), (3,2)\}$

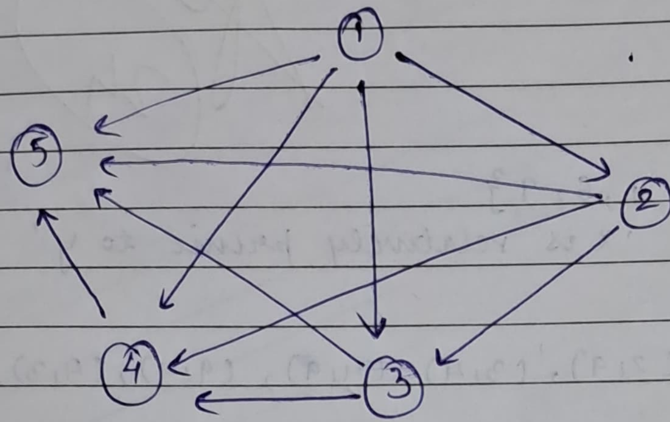
Directed graph



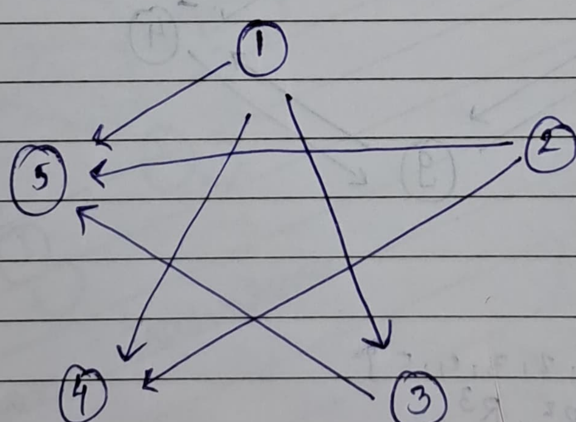
(A2) Given, $A = \{1, 2, 3, 4, 5\}$
To find, R, R^2, R^3
$R = \{(1,2)\ (1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)$
$(3,4)\ (3,5)\ (4,5)\}$

→

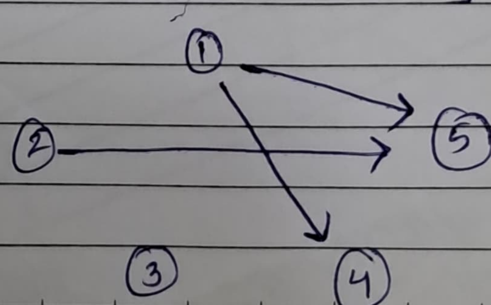
Diagraph for R



$R^2 = \{ (1,3), (1,4), (1,5), (2,4), (2,5), (3,5), \}$



$R^3 = \{ (1,4)\ (1,5)\ (2,5) \}$



(A3) $S = \{1, 2, 3, 4\}$

$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$

(a) Show that R is not transitive

For R to be transitive,

~~(4,1) should~~ $(4,1) \subset R$

$\because (4,3), (3,1) \in R$

For it to be Transitive

(4,1) should be a part as well of R.

$\because (4,1) \notin R$

It is not a Transitive Relation.

Alternatively,

if aRb & bRc then aRc should be present for a transitive set.

(b) Find a Relation $R_1 \subseteq R$ such that R_1 is transitive

Soln For a transitive set if aRb & bRc the aRc should be present.

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

subset $M_{R1} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 1 \\ 2 & 1 & 1 \end{array}$ is transitive

as $(2,1)\ (1,2) \subseteq R$
& and $(2,2) \subseteq R$
i.e Transitive

$R_1 = \{ (2,1)\ (1,2)\ (2,2) \}$

(c) Transitive closure by Warshalls

$$W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$p = (2,1)\ (3,1)$
$q = (1,2)$
$p,q = (2,2)\ (3,2)$

$$
W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}
$$

$p = (1,2)\ (2,2)\ (3,2)$

$q = (2,1)\ (2,2)$

$p.q = (1,1)\ (2,1)\ (3,1)\ (1,2)\ (2,2)\ (3,2)$

$$
W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}
$$

$p = (4,3)$

$q = (3,1)\ (3,2)$

$p.q = (4,1),\ (4,2)$

$$
W_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}
$$

$p = \cancel{(\ldots)}$ $\quad\because$ columns contain all zeroes,

$q =$ $\quad$ ~~not~~ no intersection is possible

$\therefore W_3 = W_4$

4. Transitive closure,

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 1 & 0 & 0 \\
2 & 1 & 1 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 \\
4 & 1 & 1 & 1 & 0
\end{array}
$$

$M_T =$ the above matrix

$R'_T = \{ (1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,1)\ (3,2),\ (4,1)\ (4,2)\ (4,3) \}$

(4) $A = \{11, 12, 13, 14\}$

$R = \{(11,12), (12,13), (13,14), (12,11)\}$

~~MR~~

$$W_0 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 0 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$p = (12, 11)$
$q = (11, 12)$
$p, q = (12, 12)$

$$W_1 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$p = (11, 12), (12, 12)$
$q = (12, 11), (12, 12)\ (12, 13)$
$p, q = (11, 11)\ (11, 12)\ (11, 13)\ (12, 11)$
$(12, 12)\ (12, 13)$

$$W_2 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

p = (11, 13) (12, 13)
q = (13, 14)
p.q = (11, 14) (12, 14)

$$W_3 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ 11 & 1 & 1 & 1 & 1 \\ 12 & 1 & 1 & 1 & 1 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

p =
q =

∵ All zeroes are present in the row, intersection is not possible.

∴ $W_3 = W_4$

$$M_T = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ 11 & 1 & 1 & 1 & 1 \\ 12 & 1 & 1 & 1 & 1 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

R_T = { (11, 11) (11, 12) (11, 13) (11, 14) (12, 11) (12, 12) (12, 13) (12, 14) (13, 14) }

∴ R_T is the Transitive closure on the Relation R.

60004210015
Medha shah
A3

TUTORIAL 4 : DISCRETE STRUCTURES

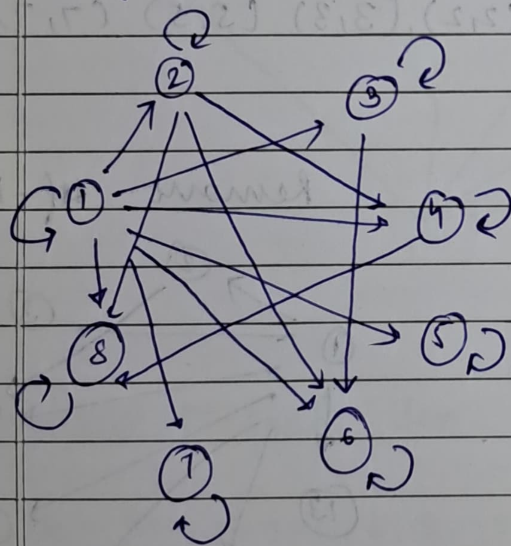
25

Q1)

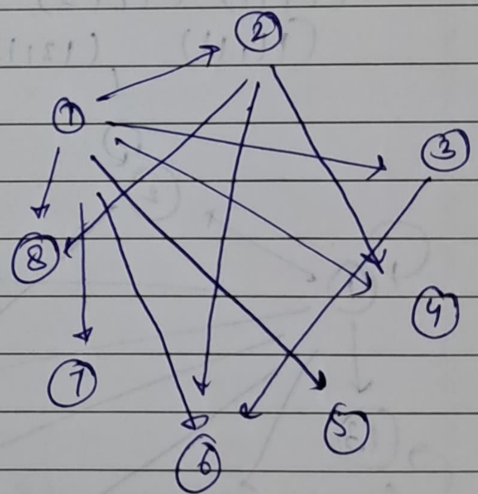
i) {1, 2, 3, 4, 5, 6, 7, 8}

R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2)
(2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5),
(6,6), (7,7), (8,8)}

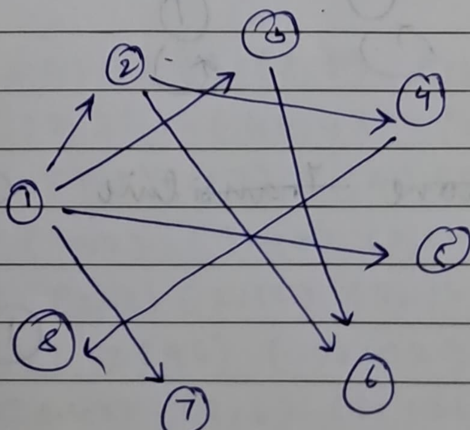
Diagraph



→ Remove the self-loops

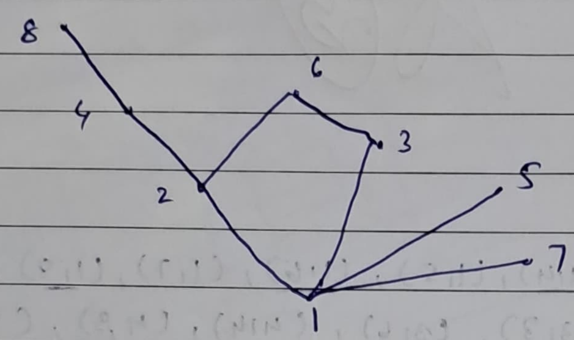


→ Remove Transitive



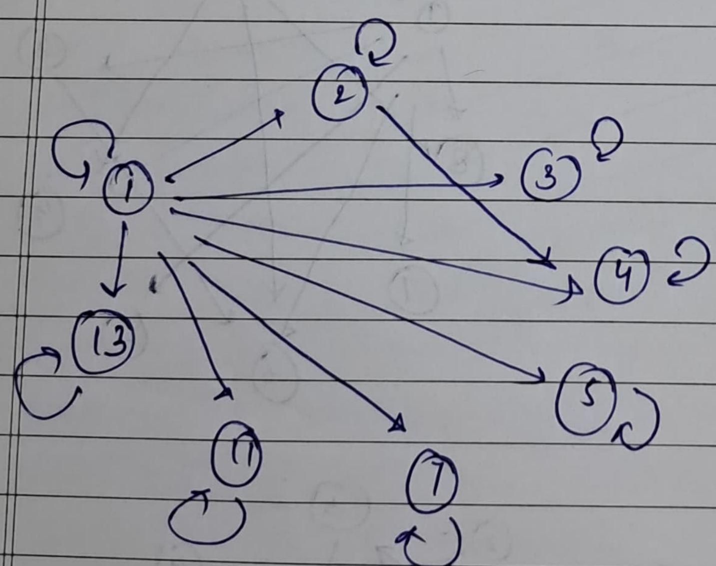
FOR EDUCATIONAL USE

Remove circles & Arrows

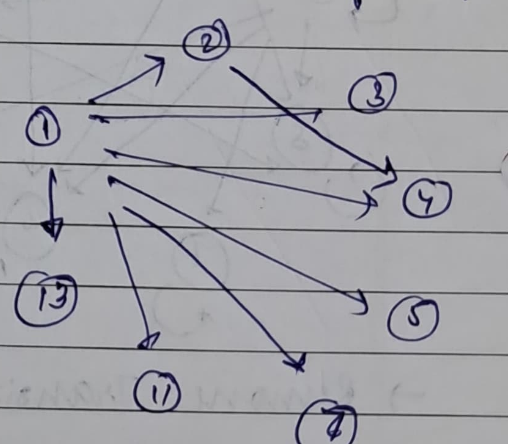


(ii) {1, 2, 3, 4, 5, 7, 11, 13}

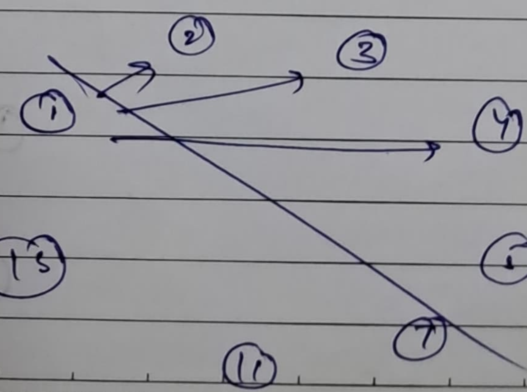
R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7)
(1,11) (1,13) (2,4), (2,2), (3,3) (5,5) (7,7)
(11,11) (13,13) }



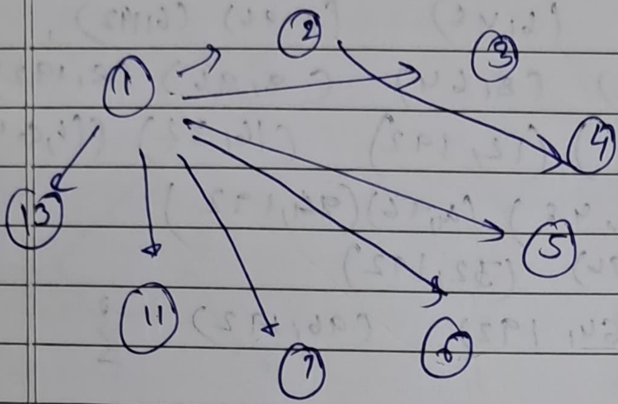
Remove self-Loops



Remove Transitive

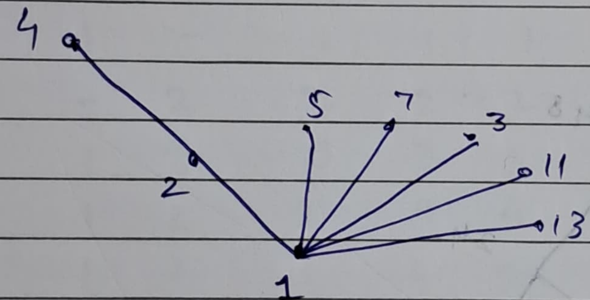


Remove Transitive



Remove circle & arrows.

Hasse,



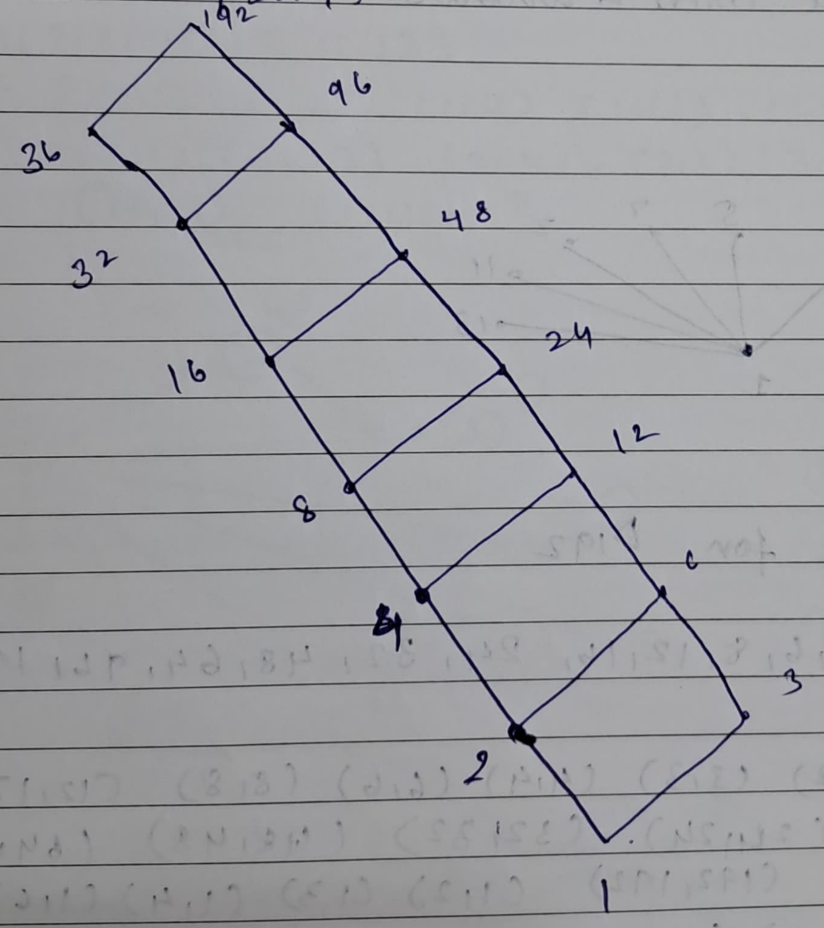
② Hasse Diagram for D_{192}

A = { 1,2,3,4,6,8,12,16, 24, 32, 48,64,96,192}

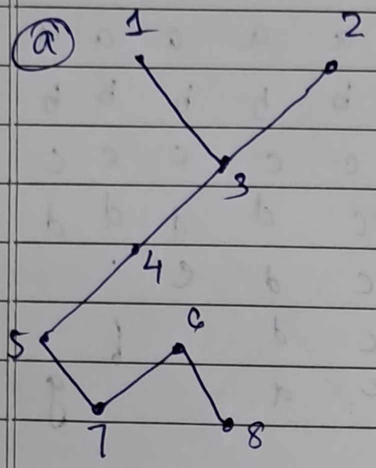
R = { (1,1) (2,2) (3,3) (4,4) (6,6) (8,8) (12,12)
(16,16) (24,24) (32,32) (48,48) (64,64)
(96,96) (192,192) (1,2) (1,3) (1,4) (1,6) (1,8)
(1,12) (1,16) (1,24) (1,32) (1,48) (1,64) (1,96)
(1,192) (2,4) (2,6) (2,8) (2,12) (2,16) (2,24)
(2,32) (2,48) (2,64) (2,96) (2,192) (3,6)
(3,12) (3,24) (3,48) (9,6) (3,192) (4,8)

(4,12) (4,16) (4,24) (4,32) (4,48) (4,64) (4,96)
(4,192) (6,12), (6,24) (6,48) (6,96) (6,192),
(8,24) (8,32) (8,48) (8,64) (8,96) (8,192)
(12,24) (12,48) (12,96) (12,192) (16,32) (16,64)
(16,192) (16,96) (24,48) (24,96) (24,192)
(32,64) (32,96) (32,192)
(48,96) (48,192) (64,192) (96,192) }

Hasse Diagram:-



(3) (a)



GLB	8	7	6	5	4	3	2	1
8	8	-	8	8	8	8	8	8
7	-	7	7	7	7	7	7	7
6	8	7	6	-	7	7	7	7
5	8	7	7	5	5	5	5	5
4	8	7	6	5	4	4	4	4
3	8	7	6	5	4	3	3	3
2	8	7	7	5	4	3	2	-
1	8	7	7	5	4	3	-	1

LUB

$\vee$	1	2	3	4	5	6	7	8
1	1	-	1	1	1	1	1	1
2	-	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3
4	1	2	3	4	4	4	4	4
5	1	2	3	4	5	-	5	5
6	1	2	3	4	-	6	6	6
7	1	2	3	4	5	6	7	6
8	1	2	3	4	5	6	6	8

(2,1) (2,2) have no LUB

: it is not a lattice