$4(k+1)-3$

Discrete Maths

Q.1

a) $1+5+9+\ldots+(4n-3) = n(2n-1)$

So for basis step we must show that $P(1)$ is true

$\therefore$ $P(1)$ is the statement

$\therefore$ $1 = 1(2\times1-1) = 1$, which is clearly true.

$\therefore$ Induction step

we must show that for $k \geq 1$

if $p(k)$ is true then $p(k+1)$ must also be true.

We assume that for some fixed $k \geq 1$

$$1+5+9+\ldots+(4k-3) = k(2k-1) \quad -(1)$$

We have to prove.

$$1+5+9+\ldots+(4k+1) = (k+1)(2k+1) \quad -(2)$$

~~For~~ We now wish to show truth of $P(k+1)$

$\therefore$ From (1)

$$P(k+1) = \left[1+5+9+\ldots(4k-3)\right] + 4k+1$$

$$= k(2k-1)+4k+1$$

$$= \cancel{2k^2-k+k+1}$$

$$= 2k^2-k+4k+1$$

$$= 2k^2+3k+1$$

$$= 2k^2+2k+k+1 = 2k(k+1)+1(k+1)$$

$$= (2k+1)(k+1)$$

b) $2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

→ Let $p(n)$ be the predicate

$\therefore 2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

In this eg, $n_0 = 1$

Basis step.

we must first show $P(1)$ is true.

$\therefore \cancel{P(1)}\ 2 = \frac{1(3\times1+1)}{2} = 2$, which is clearly true.

Induction step:

We must now show that for $k \geq 1$

if $p(k)$ is true then $p(k+1)$ should also be true.

$\therefore$ We assume for some fixed $k \geq 1$

$2+5+8+\dots+(3k-1) = \frac{k(3k+1)}{2}$ — (2)

$\therefore$ we have to prove

$2+5+8+\dots+(3k+2) = \frac{(k+1)(3k+4)}{2}$

$\therefore$ From eqn (2)

$2+5+8\dots+(3k-1)+(3k+2)$ $\cancel{= P(k)+}$ $= P(k+1)$

$\therefore \cancel{2+5+} \frac{k(3k+1)}{2} + (3k+2)$ $= P(k+1)$

$$\therefore \frac{k(3k+1) + 2(3k+2)}{2} = p(k+1)$$

$$\therefore \frac{3k^2+k+6k+4}{2} = p(k+1)$$

$$\therefore \frac{3k^2+7k+4}{2} = p(k+1)$$

$$\therefore \frac{3k^2+3k+4k+4}{2} = p(k+1)$$

$$\therefore \frac{3k(k+1)+4(k+1)}{2} = p(k+1)$$

$$\therefore \frac{(3k+4)(k+1)}{2} = p(k+1).$$

Thus we have shown the L.H.S of $p(k+1)$ equals the R.H.S of $p(k+1)$.

$\therefore$ By principle of mathematical induction, it follows that $p(n)$ is true for $n \geq 1$.

Q.2 $a_n + 2a_{n-1} = n+3$ for $n >= 1$ and with $a_0 = 3$.

The given recurrence relation is

$\rightarrow$ $a_n + 2a_{n-1} = n+3$.

$\therefore$ ~~This~~ The characteristic equation is.

$$r + 2 = 0$$

$$\therefore r = -2$$

$\therefore$ The homogenous solution is

$$a_n^h = b_1(-2)^n$$

$An+B$

$A+B$
$A \cdot B$

Let the particular solution.

$a_n^p = \cancel{an+b}\ An+B$

Putting this value of a_n in the given equation.

$An+B + 2[A(n-1)+B] = n+3$

$\therefore An + B + 2An - 2A + 2B = n+3$

$\therefore 3An + 3B - 2A = n+3$

$\therefore 3A = 1$, $3B - 2A = 3$

$\therefore A = \frac{1}{3}$ $\therefore 3B = 3 + \frac{2}{3}$

$\therefore B = \frac{11}{9}$.

$\therefore a_n^p = \frac{1}{3}n + \frac{11}{9}$

$\therefore a_n = a_n^h + a_n^p$

$= b_1(-2)^n + \frac{1}{3}n + \frac{11}{9}$.

Put $n=0$

$\therefore 3 = b_1 + \frac{11}{9}$

$\therefore b_1 = 3 - \frac{11}{9}$

$\therefore b_1 = \frac{16}{9}$

$\therefore a_n = \frac{16}{9}(-2)^n + \frac{1}{3}n + \frac{11}{9}$.

b/a

classmate

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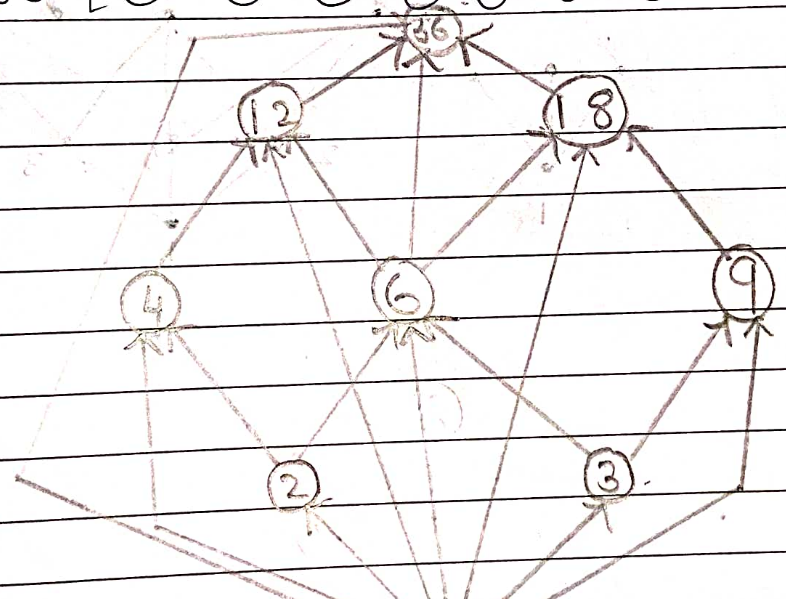
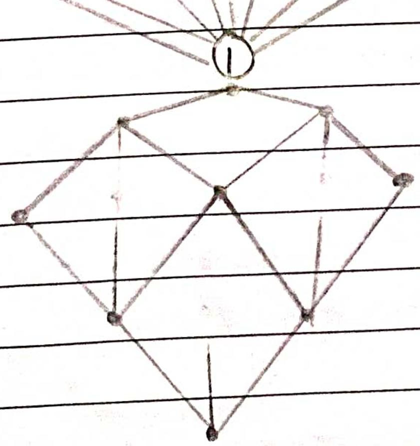
3) a) $A = D_{36}$

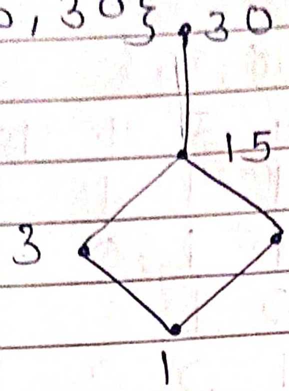
$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

	1	2	3	4	6	9	12	18	36
1	1	1	1	1	1	1	1	1	1
2	0	1	0	1	1	0	1	1	1
3	0	0	1	0	1	1	1	1	1
4	0	0	0	1	0	0	1	0	1
6	0	0	0	0	1	0	1	1	1
9	0	0	0	0	0	1	0	1	1
12	0	0	0	0	0	0	1	0	1
18	0	0	0	0	0	0	0	1	1
36	0	0	0	0	0	0	0	0	1

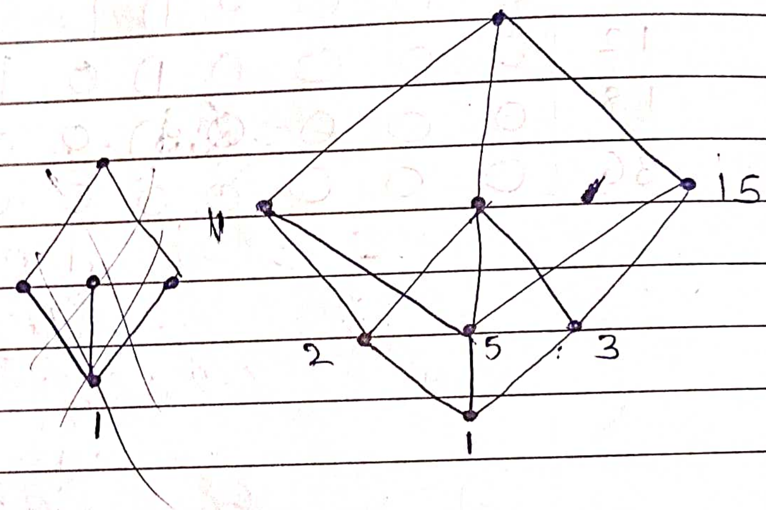




2) $B = \{1, 3, 5, 15, 30\}$



3) $C = \{1, 2, 3, 5, 6, 10, 15, 30\}$

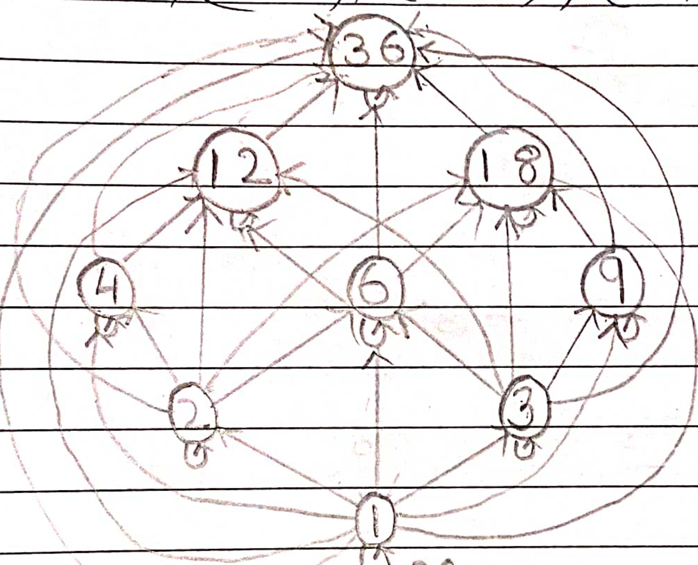


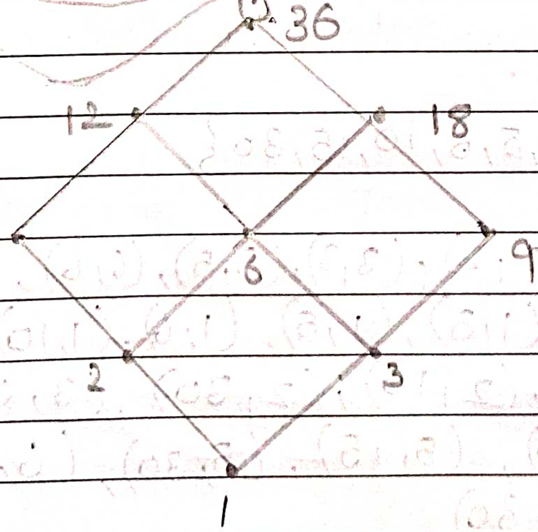
Q.3

a) $A = D36$

$\therefore A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

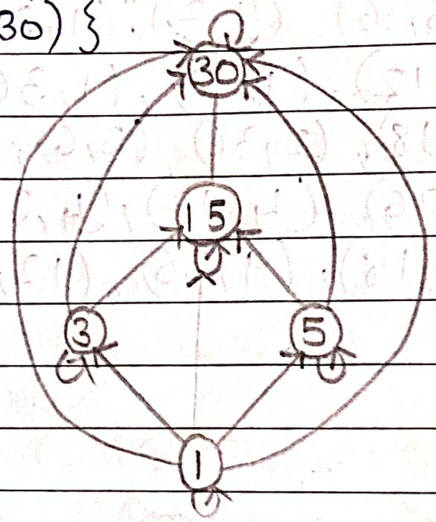
$\therefore R = \{(1,1), (2,2), (3,3), (4,4), (6,6), (9,9), (12,12), (18,18), (36,36), (1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18), (1,36), (2,4), (2,6), (2,12), (2,18), (2,36), (3,6), (3,9), (3,12), (3,18), (3,36), (4,12), (4,36), (6,12), (6,18), (6,36), (9,18), (9,36), (12,36), (18,36)\}$

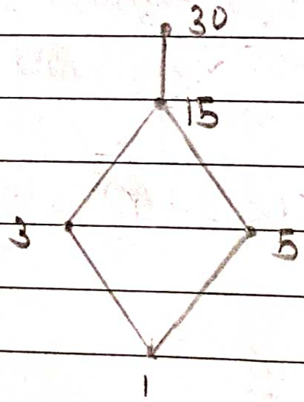




2) $B = \{1, 3, 5, 15, 30\}$

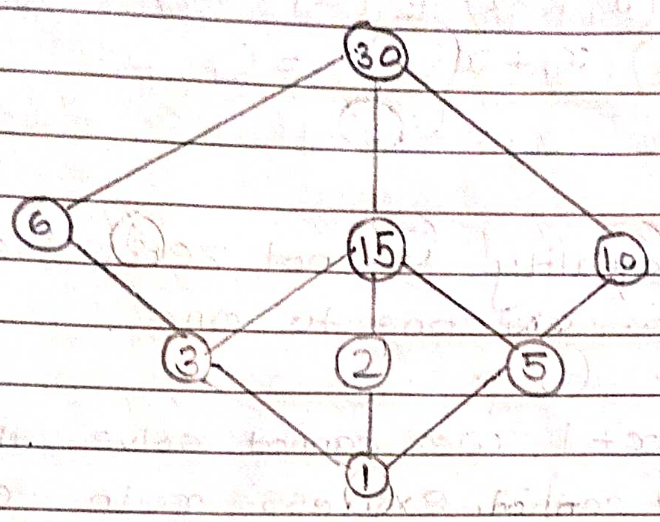
$R = \{(1,1), (3,3), (5,5), (15,15), (30,30),$
$(1,3), (1,5), (1,15), (1,30),$
$(3,15), (3,30), (5,15), (5,30),$
$(15,30)\}.$

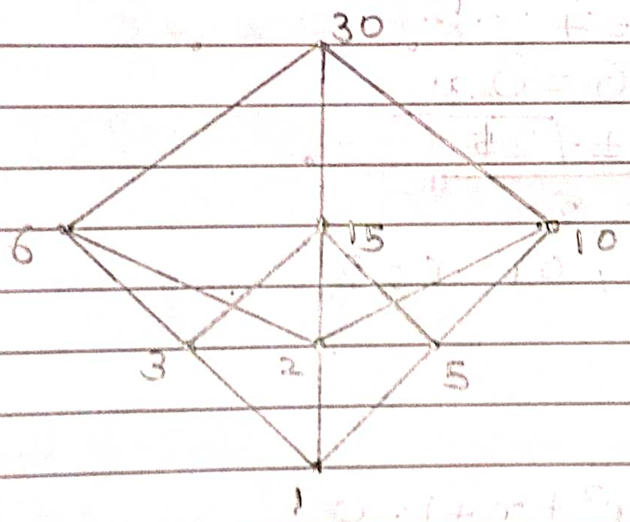




3) $C = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$R = \{(1,1), (2,2), (3,3), (5,5), (6,6), (10,10), (15,15), (30,30),$
$(1,2), (1,3), (1,5), (1,6), (1,10), (1,15), (1,30)$
$(2,6), (2,10), (2,30), (3,6), (3,15), (3,30),$
$(5,10), (5,15), (5,30), (6,30), (10,30),$
$(15,30).$





4) $f: Z \to Z,\ f(x) = x^2 + x + 1.$

Ans The set Z is the set of all integers, positive, negative including zero.

To test whether f is injective we have to test whether $f(x_1) = f(x_2)$ implies $x_1 = x_2$

Now, $f(x_1) = x_1^2 + x_1 + 1$, $f(x_2) = x_2^2 + x_2 + 1$

$\therefore x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\cancel{\therefore x_1^2 + - x_2^2 \; x}$ $\quad x_1^2 - x_2^2 + x_1 - x_2 = 0$

$\cancel{\therefore x_1^2 + x}$

$(x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$

$\therefore (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\therefore x_1 = x_2$ or $x_1 = -x_2 - 1$

As second equality is not zero the function is not injective. or one to one.

Now $y = x^2 + x + 1$. we cannot solve the equation for x i.e. we cannot express x in terms of y. So we try positive, zero, negative values of y.

If $y = 7$.

$\therefore x^2 + x + 1 = 7$

$\therefore x^2 + x - 6 = 0$.

$$\therefore x = \frac{-1 \pm \sqrt{25}}{2}$$

$\therefore x = -3$, or $x = 2$

$\therefore x \in Z$

If $y = 0$

$\therefore x^2 + x + 1 = 0$

$$\therefore x = \frac{-1 \pm \sqrt{1-4}}{2} \quad \text{imaginary}$$

Thus $x \notin Z$, if $x = 0$.

If $y = -8$, $x^2 + x + 1 = -8$

$\therefore x^2 + x + 9 = 0$

$$\therefore x = \frac{-1 \pm \sqrt{1-36}}{2}, \text{imaginary}$$

if $y = -8$, $x \notin Z$. Hence, x is not surjectiv

5) Let the functions f, g and h defined as follows.

→ $f: R \to R$ is defined as $f(x) = 2x+3$

$g: R \to R$ is defined as $g(x) = 3x+4$

$h: R \to R$ is defined as $h(x) = 4$

Find i) $hogof$ ii) $go(hof)$

→ i) $hogof = h[g[f(x)]]$

$\therefore g[f(x)] = g[2x+3]$

$\cancel{= (2x+3)(3x+4)}$

$= 3(2x+3)+4$

$= 6x+13$

$\therefore h[g[f(x)]] = h[6x+13]$

$= 6x+13+4$

$= 6x+17$

ii) $go(hof)$

$= g(h(f(x)))$

$\therefore h[f(x)] = h[2x+3]$

$= 2x+3+4$

$= 2x+7$

$\therefore g[h[f(x)]] = g[2x+7]$

$= 3[2x+7]+4$

$= 6x+25$

6) For $x, y \in Z$ xRy if and only if $2x+5$ is divisible by 7. Is R an equivalence relation?

→ i) Putting $y=x$; we see that $2x+5x=7x$ is divisible by 7. $\therefore xRx$

$\therefore$ R is reflexive.

ii) Let xRy i.e $2x+5y$ is divisible by 7

$\therefore$ Let $2x+5y=7m$ where $m \in I$

Now consider $2y+7x$. We write the sum as

$$2y+5x = 7y-5y+7x-2x = 7y+7x-5y-2x$$
$$= 7(y+x)-7m$$
$$= 7(y+x-m)$$

$\therefore 2y+5x$ is divisible by 7. Thus xRy then yRx

iii) Let xRy and yRz

Let $2x+5y=7m$ and $2y+5z=7n$, $m, n \in I$

$$\therefore 2x+5y+2y+5z = 7m+7n$$
$$\therefore 2x+5z = 7m+7n-7y$$
$$\therefore 2x+5z = 7(m+n-y)$$

$\therefore 2x+5z$ is divisible by 7.

If xRy, yRz then xRz

Since R is reflexive, symmetric and transitive, R is a equivalence relation.

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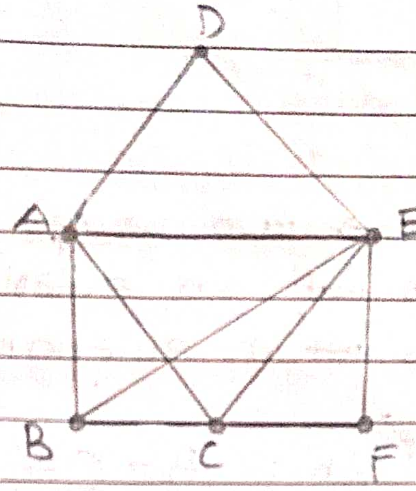
7) Find the euler path, eulerian circuit, Hamiltonian path, Hamiltonian Circuit if any from the following graph.

→ a)



Vertex	Degree
A	4
B	3
C	4
D	2
E	5
F	2

Hamiltonian circuit:
A-B-C-F-E-D-A.

As there are two vertices with odd degree there is no eulerian circuit. However there exist a eulerian path

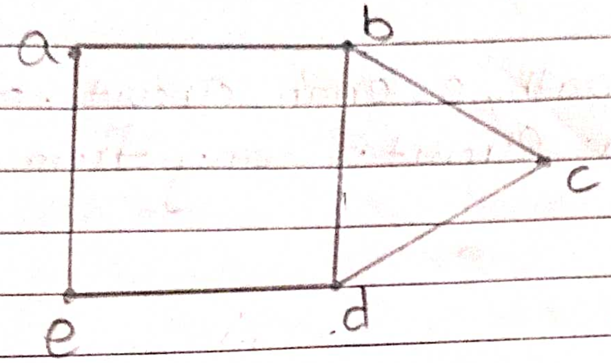
Eulerian path - B-C-F-E-C-A-D-E-A-B-E

I.F Any two vertex which are not adjacent has sum of their degrees greater than or equal to vertices (n) then there is an hamiltonian circuit.

$D(A+F) = 4+2=6$

$\therefore$ Condition is satisfied $(\because 6 \geq 6)$

b)



Vertex	Degree
A	2
B	3
C	2
D	3
E	2

As there are two nodes of two degree 3 there is not an eulerian ~~path~~ circuit.
But there is ~~not~~ a eulerian path.

Eulerian Path : B - C - D - E - A - B - D

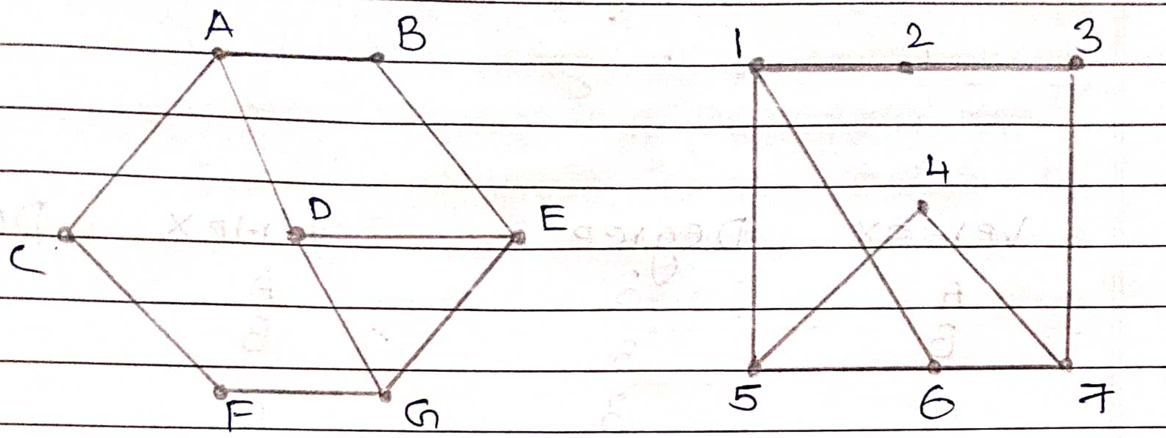
~~If any two vertex that are not~~
Degree of ~~any~~ each vertex must be greater than equal to $\frac{n}{2}$, [n is no. of vertices]

∴ for eg A has degree 2
There exist a hamiltonian circuit

Hamiltonian circuit : A - B - C - D - E - A.

Check following graphs are isomorphic or not. Justify your answer.

a)



Vertex	Degree	Vertex	Degree
A	3	1	3
B	2	2	2
C	2	3	2
D	3	4	2
E	3	5	3
F	2	6	3
G	3	7	3

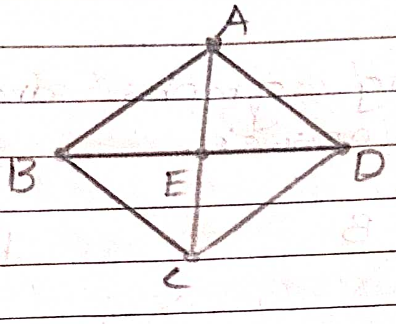
So there are 4 vertices of deg 3 and 3 vertice of deg 2.

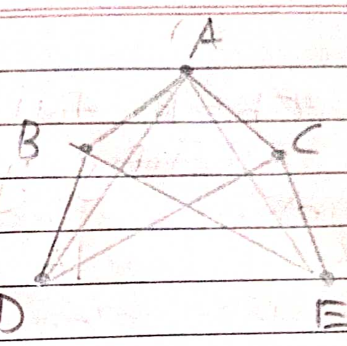
Now for one to one correspondence.

A - 7, B - 4, C - 3, D - 6, E - 1, F - 2, G - 5

Hence adjacency is also preserved. Graphs are isomorphic.

b).





Vertex	Degree
A	3
B	3
C	3
D	3
E	4

Vertex	Degree
A	4
B	3
C	3
D	3
E	3

There is one vertex of degree 4 and 4 vertex of degree 3. in both graphs.

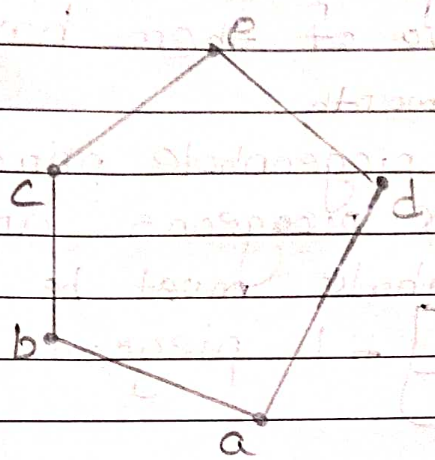
Now for one to one correspondence

E – A

~~A – E~~, B – B, C – C, D – D, A – E

∴ The graphs are isomorphic.

Define Lattice. Check if the following diagram is Lattice or not.



A lattice is a poset in which every subset $\{a, b\}$ consisting of two elements, has a LUB and GLB.

LUB

$\vee$	a	b	c	d	e
a	a	b	c	d	e
b	b	b	c	e	e
c	c	c	c	e	e
d	d	e	e	d	e
e	e	e	e	e	e

GLB

$\wedge$	a	b	c	d	e
a	a	a	a	a	a
b	a	b	b	a	b
c	a	b	c	a	c
d	a	a	a	d	d
e	a	b	c	d	e

Here, every subset has a least upper bound and greatest lower bound, hence it is a lattice.

10) Define Extended Piegeonhole Principle. How many friends you must have to guarantee that at least five of them have birthday in the same month.

→ The extended piegeonhole principle states that if there are x pigeons and y pigeonholes then one pigeonhole must be occupied by atleast $\left\lfloor \frac{x-1}{y} \right\rfloor + 1$ piegeons.

Since there are 12 months considering even distribution, if there are 48 friends, four will have birthday in the same month:
Hence, if there are 49 friends then 5 of them will have birthday in the same month.

Or by extended pigeonhole principle.

$$\left\lfloor \frac{n-1}{12} \right\rfloor + 1 = 5 \qquad \therefore \frac{n-1}{12} = 4$$

$$\therefore n-1 = 48$$

$$\therefore n = 49.$$

11) If any 14 integers from 1 to 26 are chosen then show that at least one of them is a multiple of another.

→ We can write each number between 1 and 26 in the form $2^k \cdot m$ where $k \geq 0$ and m is a positive integer.

Thus $1 = 2^0$, $2 = 2^1 \cdot 1$, $3 = 2^0 \cdot 3$, $4 = 2^2 \cdot 1$,
$5 = 2^0 \cdot 5$, $6 = 2^1 \cdot 3$, $7 = 2^0 \cdot 7$,
$8 = 2^3 \cdot 1$, $9 = 2^0 \cdot 9$, $10 = 2^1 \cdot 5$ ....
$25 = 2^0 \cdot 25$, $26 = 2^1 \cdot 13$

Thus, if we write all numbers between 1 and 26 in this way then it will have a form of one of the numbers. In the set we have.

$$\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$$

There will be thus 13 integers between 1 to 26 in this set.
Every number chosen from 1 to 26 will be one of the above forms.

If 14 are chosen then two will belong to the same piegeonhole, hence they will be multiple of each other.