Discrete Structures Tutorial 1

24/25

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Q.1] Determine the number of integers between 1 & 250 that are divisible by 2 or 3 or 5 or 7.

Q.2] Prove the following
$(A-B) \cup (B-A) \equiv (A \cup B) - (A \cap B)$

Q3] Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q4] Prove using laws of logic.

$[(p \vee q) \wedge (p \vee \sim q)] \vee q \equiv p \vee q$

Q5] Verify that the preposition $p \vee \sim (p \wedge q)$ is a tautology.

Q.6] Construct truth table to determine whether each of the following is tautology, contradiction or contingency.
i) $(q \wedge p) \vee (q \wedge \sim p)$
ii) $q \rightarrow (q \rightarrow p)$
iii) $p \rightarrow (q \wedge p)$

Q7] Write the english sentences for the following where

$P(x)$: x is even.
$Q(x)$: n is prime
$R(x, y)$: $x + y$.

i) $\exists x \, \forall y \; R(x, y)$
ii) $\sim (\exists x \; P(x))$
iii) $\sim (\forall x \; Q(x))$
iv) $\forall x (\sim Q(x))$

Q.1) Integers between 1 to 250 divisible by 2 or 3 or 5 or 7

→ A is a set of all integers from 1 to 250 divisible by 2
B is a set of all integers from 1 to 250 divisible by 3
C is a set of all integers from 1 to 250 divisible by 5
D is a set of all integers from 1 to 250 divisible by 7

$\therefore |A| = \frac{250}{2} = 125$ → cardinality of set A.

$|B| = \frac{250}{3} = 83$ → cardinality of set B

$|C| = \frac{250}{5} = 50$ → cardinality of set C

$|D| = \frac{250}{7} = 35$ → cardinality of set D

$$|A \cap B| = \frac{250}{2 \times 3} = \frac{125}{3} = \frac{250}{6} = 41$$

$$|C \cap D| = \frac{250}{5 \times 7} = \frac{250}{35} = \frac{50}{7} = 7.$$

∴ We know $|A \cup B| = |A| + |B| - |A \cap B|$
(Addition principle)

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 125 + 83 - 41$$
$$= 125 + 42$$
$$= 167$$

$$|C \cup D| = |C| + |D| - |C \cap D|$$
$$= 50 + 35 - 7$$
$$= 43 + 35$$
$$= 78$$

$$\big||A \cup B| \cup |C \cup D|\big| = |A \cup B| + |C \cup D| - \big||A \cup B| \cap |C \cup D|\big|$$

$$\big||A \cup B| \cap |C \cup D|\big| = \frac{250}{3 \times 2 \times 5 \times 7} = 1$$

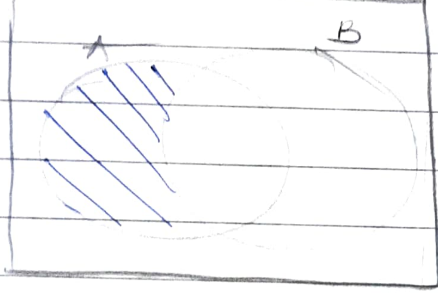
$$\big||A \cup B| \cup |C \cup D|\big| = 167 + 78 - 1$$
$$= 245 - 1$$
$$\boxed{244}$$

Ans :-

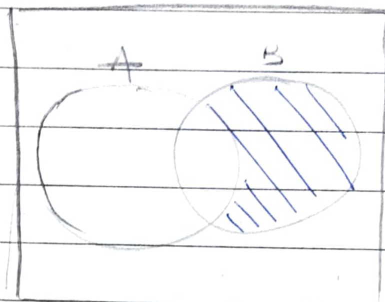
Q.2] $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

LHS :- $(A-B) \cup (B-A)$

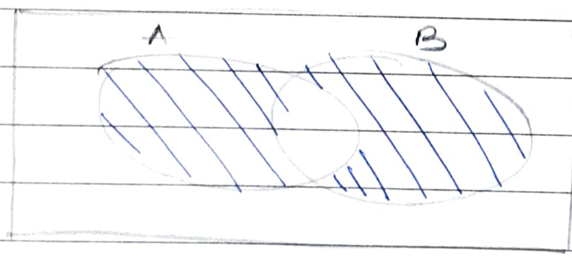
$A-B$



$B-A$

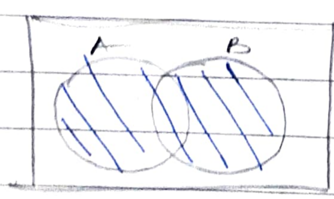


$(A-B) \cup (B-A)$

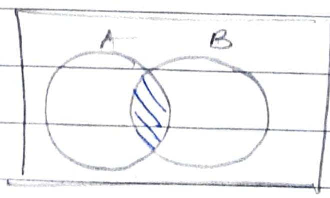


RHS :- $(A \cup B) - (A \cap B)$

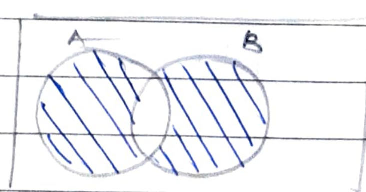
$(A \cup B)$



$(A \cap B)$



$(A \cup B) - (A \cap B)$



Hence proved LHS = RHS

$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$.

Q.5] $p \vee \sim(p \wedge q)$.

Truth table :

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$.
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T
				Tautology

~~Hence~~
Since the truth table for $p \vee \sim(p \wedge q)$ is (T T T T) we say.
$p \vee \sim(p \wedge q)$ is a Tautology.

Q.6] ii) $(q \wedge p) \vee (q \wedge \sim p)$

p	q	$q \wedge p$	$\sim p$	$q \wedge \sim p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F.	F
					Contingency.

Since the truth table for $(q \wedge p) \vee (q \wedge \sim p)$ is (T F T F) ~~its~~ its contingency

ii) $q \to (q \to p)$

p	q	$q \to p$		$q \to (q \to p)$
T	T	T		T
T	F	T		T
F	T	F		F
F	F	T		T

contingency

Since the truth table for $q \to (q \to p)$ is T T F T its contingency.

iii) $p \to (q \wedge p)$

p	q	$q \wedge p$	$p \to (q \wedge p)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

contingency

Since the truth table for $p \to (q \wedge p)$ is T F T T its contengency

~~Q3] Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$~~

~~Q.2] Prove that $(A-B) \cup (B-$~~

Q3] ~~E~~ Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

~~Let set A be {1, 2, 3, 4, 5}~~
~~Let set B be {x, y, z}~~
~~Let set C be {a, b, c, d}~~

~~$\therefore$ LHS = $A \times (B \cap C)$~~
~~$\therefore (B \cap C)$~~

Q.7] $P(x)$: x is even
$P(y)$: x is peeine
$R(x, y)$: $x+y$.

i) ~~\$~~ $\exists x \; \forall y \; R(x, y)$
→ There exist a value of x for all y.
such that relation $R(x, y)$ where relation is $(x+y)$ exists.

ii) $\sim (\exists x \; P(x))$
→ ~~The~~ There ~~exist~~ doesn't exist any value of x for ~~which~~ which x is even.

iii) ~~$\sim(\forall x\, Q(x))$~~

→ ~~x is not prime for all values of x~~

~~For all values of x, x is not prime.~~

iv) $\forall x(\sim Q(x))$

→ For all x, x is not prime

iii) $\sim(\forall x\, Q(x))$

There doesnt exist any value of x which is prime

Discrete Structures Tutorial

25

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Q.1] Given that the truth values of X, Y and Z as T, and those of U & V as F, find the truth values of

$(X \wedge (Y \vee Z)) \wedge \sim((X \vee Z) \wedge (U \vee V) \wedge Z)$

Q.2] Prove using laws of logic.

a) $a \to (p \vee c) \Leftrightarrow (a \wedge \sim p) \to c$

b) $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q)) \Leftrightarrow (\sim p \vee q)$

Q.3] Write english sentences for the following where:

$P(x)$: x is even.

$Q(x)$: x is prime

$R(x,y)$: x · y is even.

i) $\exists x \forall y \, R(x,y)$

ii) $\forall x \exists y \, R(x,y)$

iii) $\sim(\exists x \, P(x))$

iv) $\sim(\forall x \, Q(x))$

v) $\exists y (\sim P(y))$

vi) $\forall x (\sim Q(x))$

~~Q.4~~]

Q.4] Its known that at the university
60% of professors play tennis
50% of professers play bridge
70% jog.
20% Tennis and Bridge
30% Tennis and jog
40% Bridge and jog.

If someone claimed that 20% of the professors jog and play bridge & play tennis
Would you believe the claim? why.

Q.5] Let $A = \{a, b, c, d, e, f, g, h\}$ consider the following subsets of A.
$A_1 = \{a, b, c, d\}$
$A_2 = \{a, c, e, f, g, h\}$
$A_3 = \{a, c, e, g\}$
$A_4 = \{b, d\}$
$A_5 = \{f, h\}$
Determine whether each of the following is partition of A or not?

i) $\{A_1, A_2\}$ ii) $\{A_1, A_5\}$ iii) $\{A_3, A_4, A_5\}$

Q.6] Let the universal set be
$U = \{1, 2, 3 \ldots 10\}$
Let $A = \{2, 4, 7, 9\}$ $B = \{1, 4, 6, 7, 10\}$
$C = \{3, 5, 9, 7\}$
Find 1) $A \cup B$ 2) $A \cap C$ 3) $B \cap \bar{C}$ 4) $(A \cap \bar{B}) \cup C$.

Solution

Q.1] i

$x \to T$

$y \to T$

$z \to T$

$u \to F$

$v \to F$

$(x \wedge (y \vee z)) \wedge \sim((x \vee z) \wedge (u \vee v) \wedge z)$

$(T \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$

$(T \wedge (T)) \wedge \sim(T \wedge F \wedge T)$

$(T) \wedge \sim(F)$

$T \wedge T$

$\boxed{T}$

Q.2] a) $a \to (p \vee c) \Leftrightarrow (a \wedge \sim p) \to c$

LHS : $a \to (p \vee c)$

$\sim a \vee (p \vee c)$ → Implicative law

RHS : $\sim(a \wedge \sim p) \vee c$

$(\sim a \vee p) \vee c$

$\sim a \vee (p \vee c)$ $(\because a \vee (b \vee c) = (a \vee b) \vee c)$

LHS = RHS

Hence proved

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b) $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \equiv (\sim p \vee q)$

LHS: $(p \wedge q) \vee (\sim p \vee (\sim p \vee q))$.

$(p \wedge q) \vee ((\sim p \vee \sim p) \vee (\sim p \vee q))$

$(p \wedge q) \vee (\sim p \vee q)$

$[(p \vee \sim p) \vee (p \vee q)]$

$[(p \vee \sim p) \vee (p \vee q)] \wedge [(q \vee \sim p) \vee (q)]$

$T \wedge [(\sim p \vee q) \vee q]$

$T \wedge [\sim p \vee q]$

$\sim p \vee q$

$=$ RHS

$\therefore$ LHS $=$ RHS Hence Proved.

Q.3] $P(x)$: x is even ; $Q(x)$ x is prime ; $R(x,y)$: $x \cdot y$ is even

i) $\exists x \; \forall y \; R(x,y)$

→ ~~For some value of x there exist~~

→ There exist a value of x for all values of y for which the relationship $x \cdot y$ is even holds true

ii) $\forall x \; \exists y \; R(x,y)$

~~The~~ For all values of x there exists values of y for which the relationship $x \cdot y$ is even holds true

iii) $\sim(\exists x \; P(x))$

~~There doesnot exist a value of x for which x is even.~~

iii) $\sim(\exists x\, P(x))$
It is not true that there exist a value of x for which x is even

iv) $\sim(\forall x\, Q(x))$
It is not true that for all values x for which x is prime

v) $\exists y\, (\sim P(y))$
There exists a value of y for which y is not even.

vi) $\forall x\, (\sim Q(x))$
For all values of x, x is not prime

Q.4] Let the professors playing tennis be represented by set T
Let the professors playing Bridge be represented by set B
Let the professors playing jog be represented by set J
Universal set $|U| = 100 = |T \cup B \cup J|$

$\therefore$ $|T| = 60$ $\quad |J| = 70$ $\quad |T \cup J| = 30$
$|B| = 50$ $\quad |T \cup B| = 20$ $\quad |B \cup J| = 40$.

$|T \cup B \cup J| = |T| + |B| + |J| - |T \cup B| - |B \cup J|$
$- |T \cup J| + |T \cap B \cap J|$
$= 60 + 50 + 70 - 20 - 30 - 40 + |T \cap B \cap J|$

~~$100 = 270 - |T \cap B \cap J|$~~

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$100 = 180 - 90 + |\cancel{T \cup B}\ T \cap B \cap J|$

$100 + 90 - 180 = |T \cap B \cap J|$

$190 - 180 = |T \cap B \cap J|$

$10 = |T \cap B \cap J|$

Therefore. ~~&~~ professors playing all three Tennis, jog and bridge is 10%.

∴ It cannot be claimed that 20% professors jog play bridge and play tennis

Q.5] $A = \{a, b, c, d, e, f, g, h, \}$

~~A1 = {~~ .

i) $\{A_1, A_2\} = \{(a, b, c, d), (a, c, e, f, g, h)\}$

It is not a partition because its not mutualy disjount

~~{A1, A2}~~ $A_1 \cap A_2 \neq \phi$

ii) ~~{A1~~ A hence its not a partition.

iii) $\{A_1, A_3\} = \{(a, b, c, d), (a, c, e, g)\}$

Since $A_1 \cup A_3 \neq A$.

That is the union of A_1 and A_3 set is not the A set hence its not a partition.

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iii) $\{A_3, A_4, A_5\}$
$= \{(a, c, e, g), (b, d), (f, h)\}$

Since $(A_3 \cup A_4 \cup A_5) = A$
and $A_3 \cap A_4 \cap A_5 \; \cancel{\phi} = \phi$

Sets A_3, A_4, A_5 are mutualy disjoint and their unions is ~~a~~ ~~na~~ set A hence its a partition

Q.6] $U = \{1, 2, 3 \dots 10\}$.
$A = \{2, 4, 7, 9\}$, $B \{1, 4, 6, 7, 10\}$, $C \{3, 5, 9, 7\}$

1) $A \cup B = \{\cancel{\ } 1, 2, 4, 6, 7, 9, 10\}$
2) $A \cap C = \{7, 9\}$
3) $B \cap \bar{C}$
$\bar{C} = \{1, 2, 4, 6, 8, 10\}$
$B \cap \bar{C} = \{1, 4, 6, \cancel{\ }, 10\}$

4) $(A \cap \bar{B}) \cup C$
$\bar{B} = \{2, 3, 5, 8, 9\cancel{9}, 10\}$
$(A \cap \bar{B}) = \{2, 9\}$
$(A \cap \bar{B}) \cup C = \{2, 9, 3, 5, 7\}$

Discrete Structures - Tutorial - 3

24

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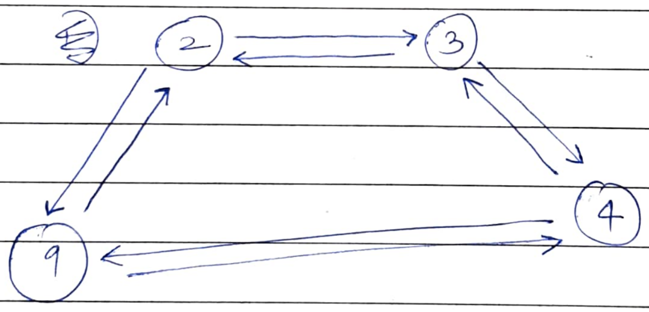
Q.1] $A = \{2, 3, 4, 6, 9\}$

x is relatively prime to y

Ordered pair $= R$

$= \{(2,3), (3,4), (2,9), (3,2), (4,3), (9,2), (4,9), (9,4)\}$



Q.2] $A = \{1, 2, 3, 4, 5\}$

aRb iff. $a < b$

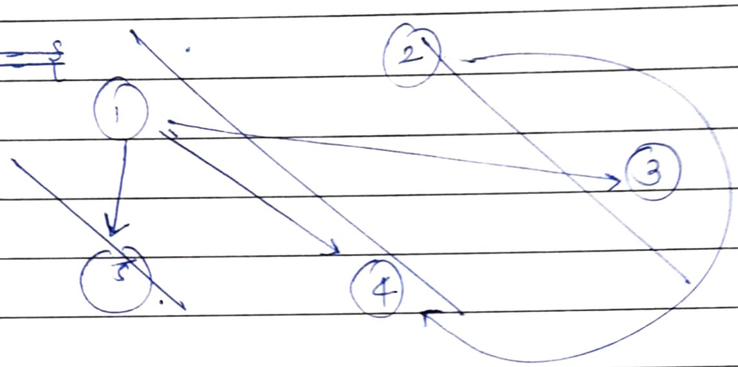
Compute :- R, R^2, R^3.

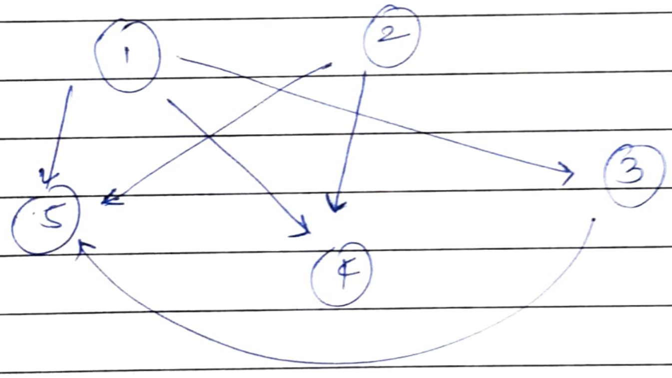
$\Rightarrow R = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

$R = \{(1,2)(1,3)(1,4)(1,5)(2,3)(2,4)(2,5)(3,4)(3,5)(4,5)\}$

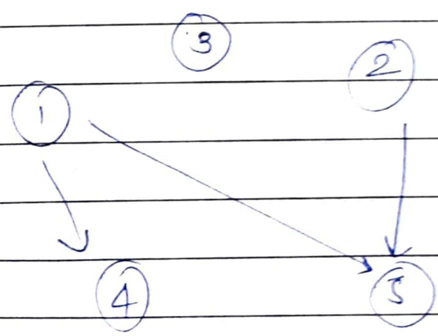
$R^2 = \{(1,3)(1,4)(1,5)(2,4)(2,5)(3,5)\}$

$\cancel{R^3 = \{}$





$R^3 = \{(1,4)(1,5)(2,5)\}$



Q3] $S = \{1, 2, 3, 4\}$

Relation R on S is

$R = \{(4,3), (2,2), (2,1), (3,1) (1,2)\}$

a) → For A Relation R to be transitive it must satisfy the condition That if aRb and bRc then aRc.

Here ~~(,)~~ (~~$~~3,1) and (4,3) exist but for it to be transitive (4,1) should also exist hence its not transitive

b)

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_{R1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Since (2,1) (1,2) C R.

and ~~(x)~~ (2,2) C R

Hence its transitive

$\therefore R1 = \{(1,2) (2,1) (2,2)\}$

c) Transitive closure by warshal.

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore p = (2,1) (3,1)$

$q = (1,2)$

$p, q = (2,2) (3,2)$

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$p = (1,2)\ (\cancel{3,1})\ (2,2)(3,2)$

$q = (2,1)\ (2,2)$

$p,q = (1,1)(2,1)(3,1)(1,2)$
$(2,2)(3,2)$

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$p = (4,3)$

$q = (3,1)(3,2)$

$p,q = (4,1)(4,2)$

$$W_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Since all the elements of column are zero no intersection is possible

$\therefore W_3 = W_4$

Transitive closure

Q.

$$M^T = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

$\therefore R^T = \{(1,1)\ (2,1)(3,1)(4,1)(1,2)(2,2)(3,2)$
$(4,2)(4,3)\}$

Q.4] $A = \{11, 12, 13, 14\}$

$R = \{(11, 12)\ (12, 13)\ (13, 14)\ (12, 11)\}$

Find transitive closure of R using warshal.

→ $R = \{(11, 12)\ (12, 13)\ (13, 14)\ (12, 11)\}$

$$\cancel{W_0 R =} \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 0 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$\therefore \cancel{P = \{(13, 14)\}}$

$q =$

$$W_0 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 0 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$p = (12, 11)$

$q = (11, 12)$

$p, q = (12, 12)$

$$W_1 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 0 & 1 & 0 & 0 \\ 12 & 1 & 1 & 0 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$p = (11, 12)\ (12, 12)$

$q = (12, 11)\ (12, 12)\ (12, 13)$

$p, q = (11, 11)\ (11, 12)\ (11, 13)\ (12, 11)$

$$W_2 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 0 \\ 12 & 1 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$p = (11, 13)\ (12, 13)$

$q = (13, 14)$

$p, q = (11, 14)\ (12, 14$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\because$ all zeros are present hence row intersection is not possible

$\therefore W_3 = M_T$

$$M_T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore R_T = \{ (11,11)(11,12)(11,13)(11,14)(12,11)(12,12)(12,13)$
$(12,14)(13,14) \}$

DATE PAGE NO.

Tutorial - 4

Discrete ~~Ma~~ Structures

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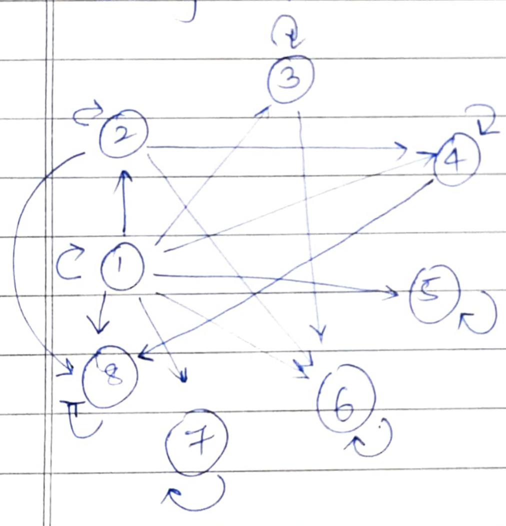
25

Q.1] Hasse Diagram:

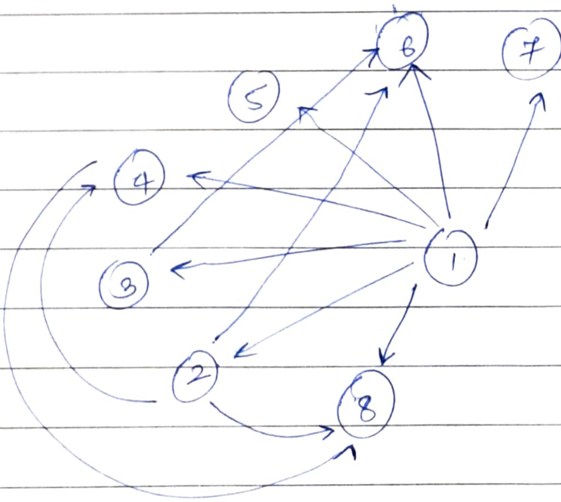
i) {1, 2, 3, 4, 5, 6, 7, 8}

R = {(1,1), (2,2) (1,3), (1,4) (1,5) (1,6), (1,7), (2,4), (2,6), (2,8), (3,6), (4,8), (1,8), (3,3), (4,4) (5,5), (6,6), (7,7), (8,8)}

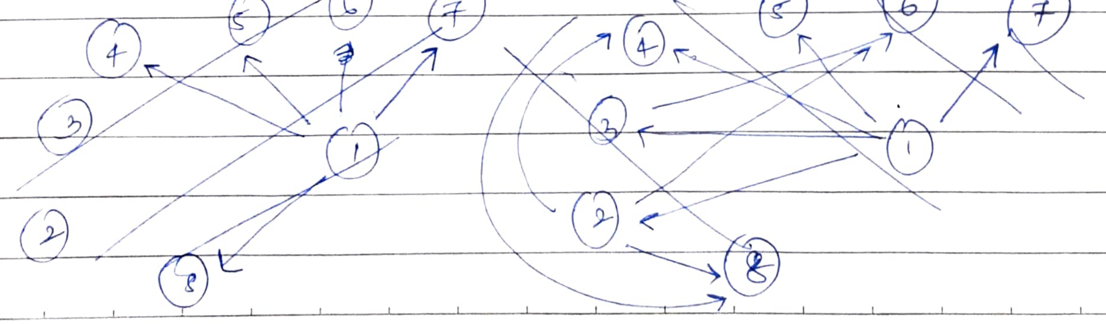
Diagraph.

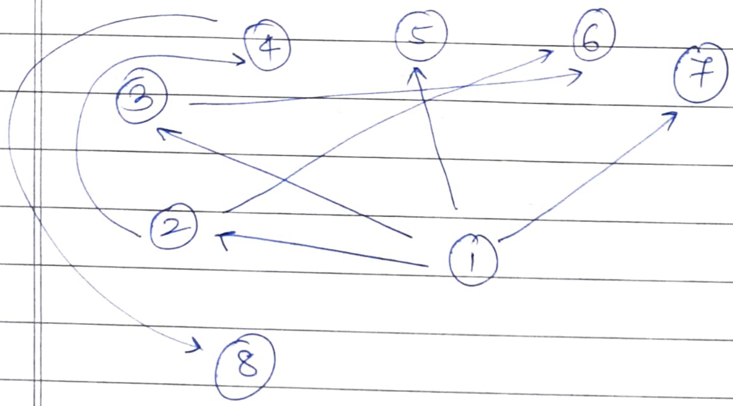


Step 1 :- Remove the loops.

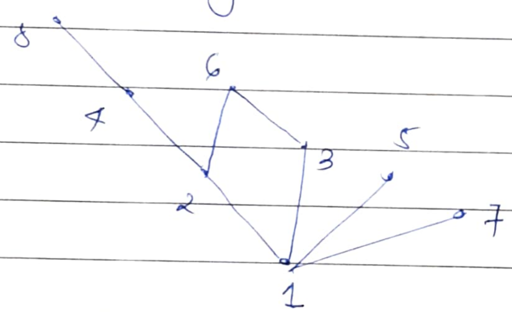


Step 2 :- Remove the transitive edge.





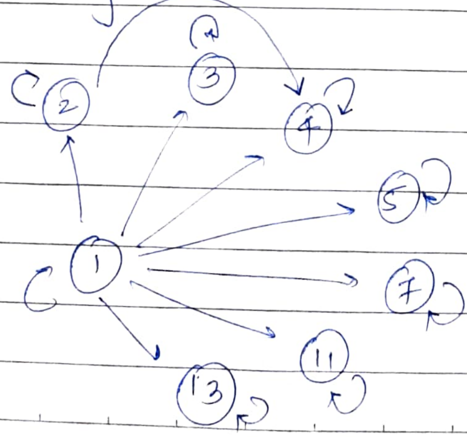
Hasse Diagram



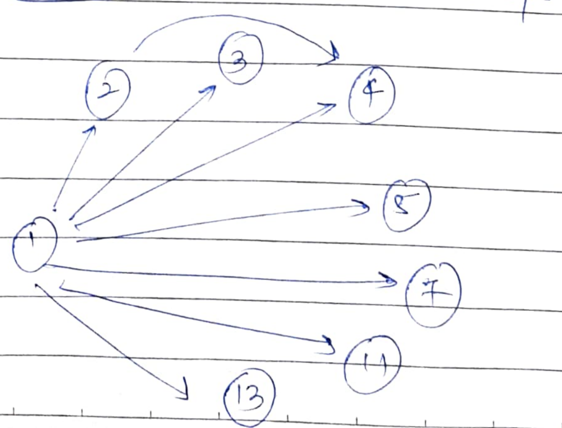
ii) $\{1, 2, 3, 4, 5, 7, 11, 13\}$.

$D = \{ (1,1)(2,2)(3,3)(4,4)(5,5)(7,7)(11,11)(13,13)$
$(1,2), (1,3), (1,4)(1,5)(1,7), (1,11), (1,13),$
$(2,4), \}$

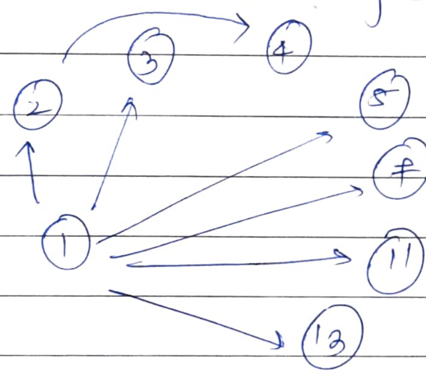
Diagraph.



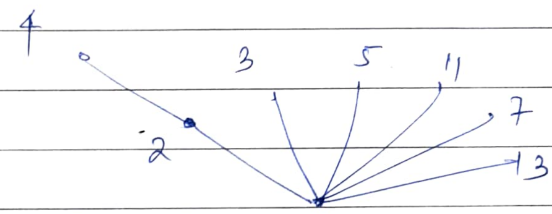
Step 1 : Remove the loops



Step 2 :- Remove Transitive edges



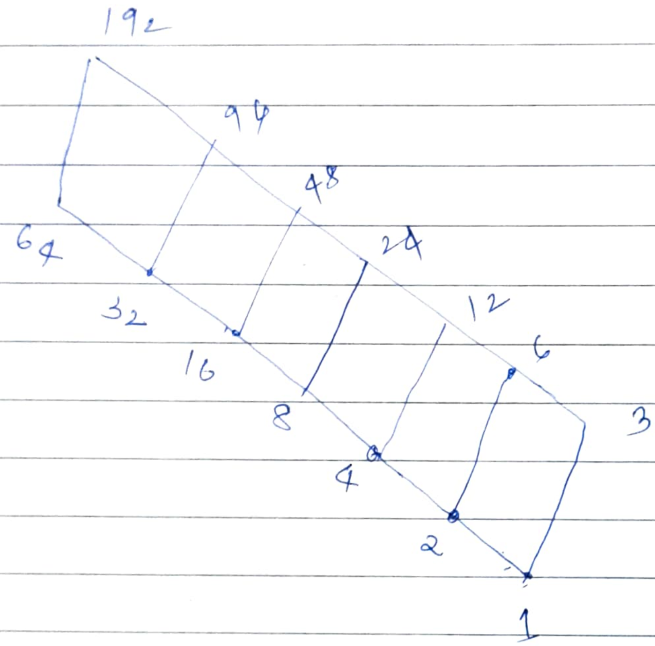
Hasse Diagram



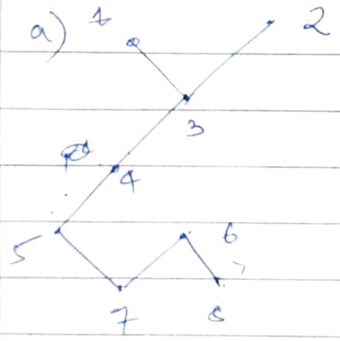
Q-2] D_{192} := { 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192 }.

∴ R = { (1, 2) (1, 3) (1, 4) (1, 6) (1, 8) (1, 12) (1, 16) (1, 24) (1, 32)
(1, 32) (1, 48) (1, 64) (1, 96) (1, 192)
(2, 4) (2, 6) (2, 8) (2, 12) (2, 16) (2, 24) (2, 32) (2, 48)
(2, 64) (2, 96) (2, 192)
(3, 6) (3, 12) (3, 24) (3, 48), (3, 96) (3, 192)
(4, 8) (4, 12) (4, 16) (4, 24) (4, 32) (4, 48) (4, 64)
(4, 96), (4, 192)
(6, 12) (6, 24) (6, 48) (6, 96) (6, 192)
(8, 16) (8, 24) (8, 32) (8, 8) (8, 96) (8, 192).
(12, 24) (12, 48) (12, 96) (12, 192)
(16, 32) (16, 64) (16, 96) (16, 192)
(24, 48) ~~(16, 96)~~ ~~(16, 192)~~ (24, 96) (24, 196)
(32, 64) (32, 96) (32, 192)
(48, 96) (48, 192)
(64, 192) (192, 192)
(2, 2) (3, 3) (4, 4) (6, 6) (8, 8) (12, 12)
(16, 16) (24, 24) (32, 32) (48, 48)
(64, 64) (96, 96) (192, 192) }

Hasse Diagram



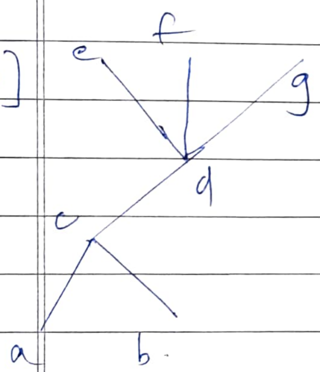
Q.3] a)



V	8	7	6	5	4	3	2	1
8	8	6	6	5	4	3	2	1
7	6	7	6	5	4	3	2	1
6	6	6	6	–	4	3	2	1
5	5	5	–	5	4	3	2	1
4	4	4	4	4	4	3	2	1
3	3	3	3	3	3	3	2	1
2	2	2	2	2	2	2	2	–
1	1	1	1	1	1	1	–	1

∧	8	7	6	5	4	3	2	1
8	8	-	8	8	8	8	8	8
7	-	7	7	7	7	7	7	7
6	8	7	6	~~8~~	7	7	7	7
5	8	7	-	5	5	5	5	5
4	8	7	6	5	4	4	4	4
3	8	7	3	5	4	3	3	3
2	8	7	2	5	4	3	2	
1	8	7	1	5	4	3		1

Q3 b]



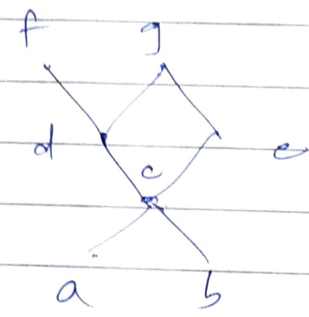
LUB

V	a	b	c	d	e	f	g
a	a	[illegible]	c	d	e	f	g
b	[illegible]	b	c	d	e	f	g
c	c	c	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	-	-
f	f	f	f	f	-	f	-
g	g	g	g	g	-	-	g

GLB

∧	a	b	c	d	e	f	g
a	a		a	a	a	a	a
b	-	b	b	b	b	b	b
c	a	b	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e	-	-
f	a	b	c	d	-	f	-
g	a	b	c	d	-	-	g

Q4 a) a)



→ {a, b, c}

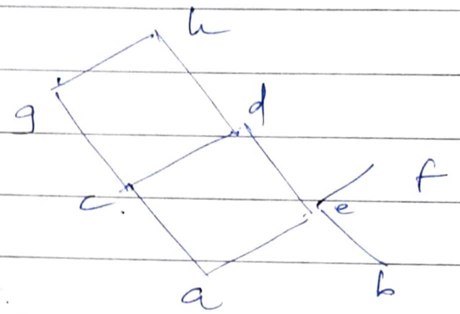
Upper Bound = {d, e, f, g, c}

Lower bound: { }

LUB = {c}

GLB = { }

b)



→ {d, c, f}

Upper bound = { }

Lower bound. {a, e, b}

LUB = ϕ.

GLB = {e}

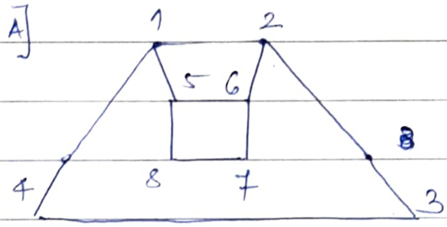
DIS - TUTORIAL - 5

24

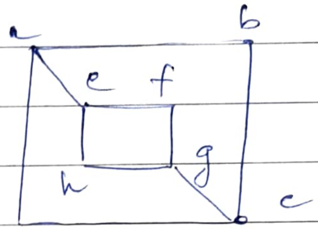
Vaishnavi Padiya
60004210065
DW4 comps.

Q.1] Determine whether the following graphs are isomorphic.

A]



G1



G2

i) Both the graphs contain 8 vertices & 10 edges.

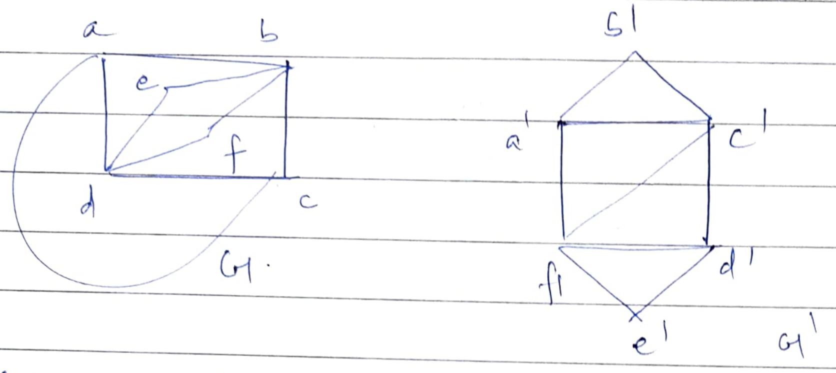
ii) In Graph G1,
Vertices (1, 2, 5, 6) - (4 vertices) have degree 3
& Vertices (8, 7, 3, 4) - (4 vertices) have degree 2.

In Graph G2.
Vertices (a, ~~b~~, e, g, c) - (4 vertices) have degree 3
& Vertices (b, d, h, f) - (4 vertices) have degree 2.

iii) Also Vertex 1 in G1 is adjacent to 2 vertices (2, 5) of degree 3 and and 1 vertex of degree 2 (4)

Whereas in Graph 2 no vertex ~~fo~~ follows similar connectivity
Hence G1 & G2 are ~~not~~ not isomorphic

B]



i) G1 – 6 vertices, 9 edges
G1' – 6 vertices, 9 edges

ii) In G1.

2 vertices (b, d) – degree 4
2 vertices (a, c) – degree 3
2 vertices (e, f) – degree 2

In G1'

2 vertices (c' f') – degree 4
2 vertices (a', d') – degree 3
2 vertices (b', e') – degree 2

iii) ~~4 degree vertex~~ b

In G1.

4 degree vertex (b) – 2 neighbours (2 degree) (e, b)
~~2d~~ – 2 neighbours (3 degree) (a, c)

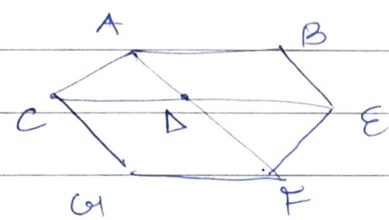
iv) In G1'

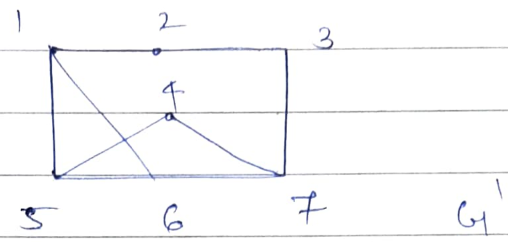
4 degree vertex (c') – 1 neighbour (2 degree)
2 neighbours (3 degree)
1 neighbours (4 degree)

Similar is the case for 2nd 4 degree vertex f'

condition of ~~adjauceny~~ ~~fails~~ adjacency fails.
∴ G1 & G1' are not isomorphous

c]





G1

i) G1 – 7 vertices, 9 edges
G1' 7 vertices, 9 edges.

ii) ~~G~~ In G1.
~~1~~ – ~~1 vertex~~ 1 – 3 degree vertex (D)
3 – 3 degree vertices (A, E, F)
3 – 2 degree vertices (B, C, G)

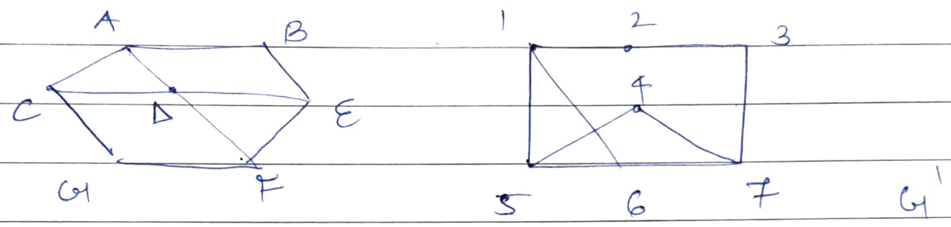
In G1'
4 – 3 degree vertices (1, 5, 6, 7)
3 – 2 degree vertices (2, 3, 4)

iii) In G1 –
D has 3 – 3 degree neighbours
In G1'
6 has 3 – 3 degree neighbours

All the ~~graphs~~ vertices are perfectly adjacent
hence G1 & G1' are isomorphs.

condition of ~~adjancency~~ ~~fails~~ adjacency fails.
∴ G & G' are not isomorphous

c]



G

i) G – 7 vertices, 9 edges
G' 7 vertices, 9 edges.

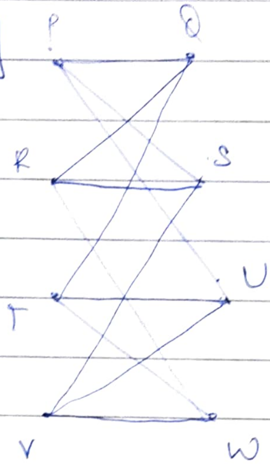
ii) ~~G~~ In G.
~~1~~ – ~~1 vertex~~ 1 – 3 degree vertex (D)
3 – 3 degree vertices (A, E, F)
3 – 2 degree vertices (B, C, G)

In G'
4 – 3 degree vertices (1, 5, 6, 7)
3 – 2 degree vertices (2, 3, 4)

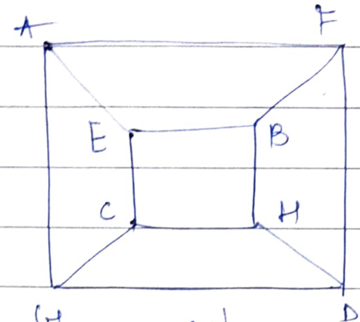
iii) In G –
D has 3 – 3 degree neighbours
In G'
~~5~~ 6 has 3 – 3 degree neighbours

All the ~~graphs~~ vertices are perfectly adjacent
hence G & G' are isomorphs.

D]



G.



G'

i) ~~G~~. In G – 8 vertices, 12 edges
In G' – 8 vertices 12 edges.

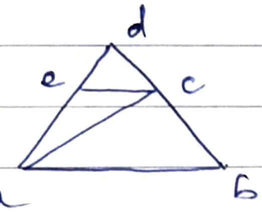
ii) In G – All 8 vertices – 3 degree vertices
In G' All 8 vertices – 3 degree vertics

Since All vertices in graph G & G' are 3 degree. Therefore ~~one~~ for 1, 3 Degree vertex, we have 3, 3 degree neighbours.

As this is true for all vertices so the graphs are isomorphic

Q.2] Determine whether the graphs below have a Hamiltonian circuit, Eulerian circuit. If so. find them

A]



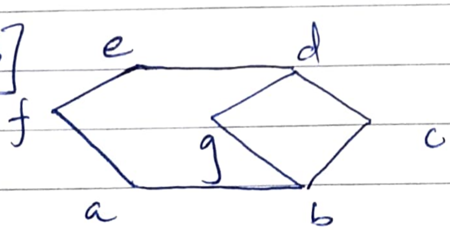
If we consider 2 non adjacent vertices e and b, and take the sum of their degrees $(3+2=5)$. the sum of their degrees $= 5$ which is equal to the number of vertices in the above graph. So Hamiltonian circuit exist (Acc to Rule 1)

Hamiltonian circuit :- a b c d e a.

Also,

B] vertex 'e' and 'a' have odd degree so Eulerian circuit doesnt exist.

B]



Vertex	Degree
a	2
b	3
c	2
d	3
e	2
f	2
g	2

As, degree of each vertex is less than $7/2 = 3.5$

$\therefore$ Rule 2 fails.

There are no two non adjacent vertes such that sum of their degrees is greater than no of vertices. Rule 1 fails

Also, number of edges $m = 8$

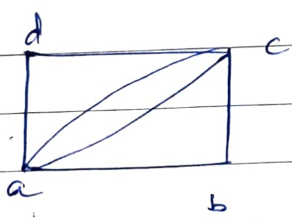
$\frac{1}{2}(7^2 - (3 \times 7) + 6) = 17$

As no of edges $< 17 \Rightarrow$ Rule 3 fails

Hamiltonian circuit doesnt exist.

As b, d, vertices have odd degree, so euler circuit doesnt exist

e]



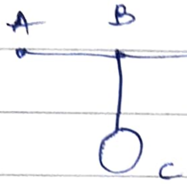
Vertex	Degree
a	4
b	2
c	4
d	2

No of vertices $(n) = 4$.

As each vertex has degree $\geq 4/2 = 2$ So
The Rule 2 is satisfied
Hamiltonian Circuit exists
Hamiltonian Circuit
a, b, c, d, a

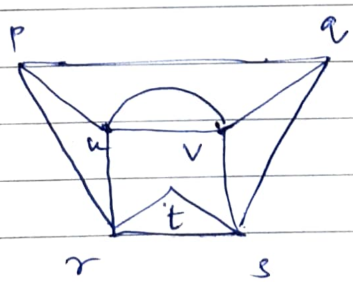
As the above graph is connected & each vertex has an even degree so
Euler Circuit exist
eulerian Circuit

FOR EDUCATIONAL USE

D] 

Vertice	Degree
A	1
B	3
C	3
D	1

Visually, we see that it is impossible to make a circuit without repeating vertices.
So, Hamiltonian circuit not possible
Also looking at vertices ~~so~~ & their degrees. as not all vertices have degree ≥ 2 (A, D) so rule 2 fails & so Hamiltonian circuit is not possible
As all vertices A, B, C, D have odd degrees so Eulerian circuit not possible

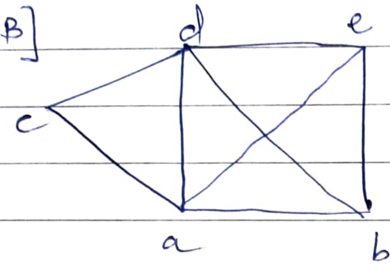
Q.3] A] 

vertex	degree
p	3
q	~~4~~ 3
r	4
s	4
t	2
u	4
v	4

It has 2 vertices of odd degree so Eulerian path is present: ~~p, q, r, s~~, t
p, q, ~~r~~, s, r, p, u, v, s, t, s, u, v, q.

Hamiltonian p, q s t r u v.

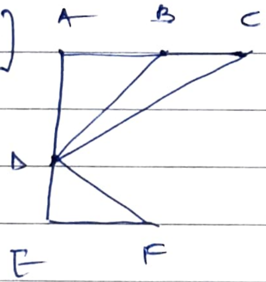
B]



Vertex	degree
a	4
b	3
c	2
d	4
e	3

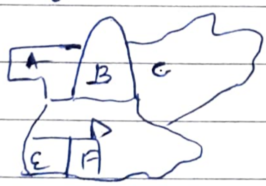
It has 2 vertices of odd degree so
Euler path : e d c a b d a e f
Hamiltonian a b c d e.

Q 4] A]



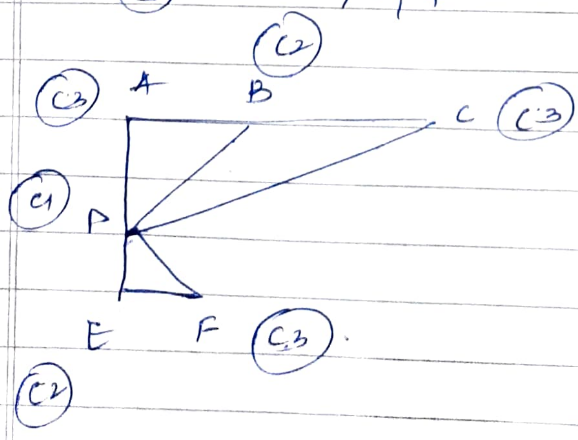
Applying Welch Powell.

Vertex	Degree
A	2
B	3
C	2
D	5
E	2
F.	2.



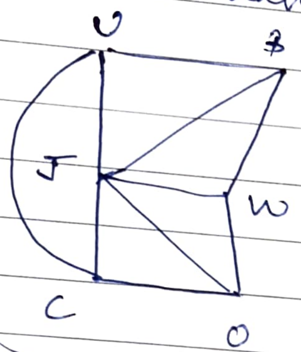
In Descending order D, B, A C E F.
5 3 2 2 2 2.

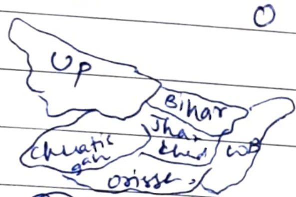
Assigning 1 to D, 2 to B & E
3 to A, C, F



Graph is 3 colorable
$\chi(G) = 3$
chromatic no.

B] Uttar Pradesh (UP), Bihar (B)
Uttarakhand (J), Orrissa (O), West Bengal (W)





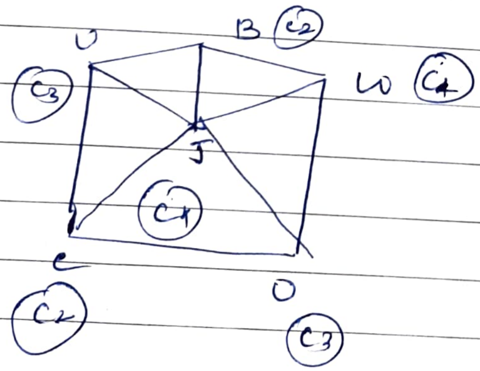
Using welch Powell.

Vertex	Degree
B	3
J	5
U	3
W	3
O	3
C	3

Descending

J	B	C	O	U	W
5	3	3	3	3	3

C1 to J, C2 to B, C, C3 to O, ~~U & C~~
C4 to U & W.



$\chi(G) = 4 \Rightarrow$ chromatic no.

Name: Vaishnavi Pandya
Sap Id: 60004200065

25

DATE

DIS TUTORIAL - 6

Q.1] Find the generating function for the following sequence
$1, 2, 3, 4, 5, 6, \ldots$

→ Using the sequence $1, 2, 3, 4, 5 \ldots$ the above expression becomes ~~f(x)~~ = (using generating function)

$f(x) = \sum a_n x^n$.

$f(x) = \cancel{1 + 2x}\ 1x^0 + 2x^1 + 3x^2 + 4x^3 + 5x^4 + \ldots$ — (1)

Multiply eq (1) by x.

$x f(x) = x + 2x^2 + 3x^3 + 4x^4 \ldots$ — (2).

Subtracting eq (1) from (2)

$f(x)[x-1] = [x + 2x^2 + 3x^3 + 4x^4 + \ldots] - [1 + 2x + 3x^2 + \ldots]$

$f(x)[x-1] = -[1 + x + x^2 + \ldots]$

$f(x)[1-x] = 1 + x + x^2 + x^3 + \ldots$

$= \frac{1}{(1-x)}$ for $|x| \leq 1$

$$\boxed{\therefore f(x) = \frac{1}{(1-x)^2}}$$ for $|x| \leq 1$

Q.2] Solve the recursive relation
$a_n = 3a_{n-1} + 2$ $n \geq 1$ ~~with~~ with $a_0 = 1$ using generating functions

→ Consider $g(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ a_r x^r$

$3x g(x) = 3a_0 + 3a_1 x + 3a_2 x^2 + \ldots + 3a_{r-1} x^r + \ldots$

We know $\frac{2}{1-x} = 2 + 2x + 2x^2 + \ldots 2x^r + \ldots$

Subtracting the last 2 series from the 1st 2, noting that $a_0 = 1$ we get

$g(x) - 3xg(x) - \frac{2}{1-x} = (1-2) + (a_1 - 3a_0 - 2)x + (a_2 - 3a_1 - 2)x^2 + \ldots$

Since $a_0 = 1$

$\therefore a_r - 3a_{r-1} - 2 = 0$

each bracket on the right except the first is zero

Hence $(1-3x)g(x) = -1 + \frac{2}{1-x} = \frac{-1+x+2}{1-x} = \frac{1+x}{1-x}$

$g(x) = \frac{1+x}{(1-x)(1-3x)} = \frac{2}{(1-3x)} - \frac{1}{(1-x)}$ By partial fraction

$= 2[1 + (3x) + [3(x)]^2 + \ldots] - [1 + x + x^2 + \ldots]$

$= 2\sum_{r=0}^{\infty} 3^r x^r - \sum_{r=0}^{\infty} x^r = \sum_{r=0} [2 \times (3^r - 1)] x^r$

But $g(x) = \sum a_r x^r \therefore a_n = 2 \cdot 3^r - 1$.

In order to make use of

$a_n - 3a_{n-1} - 2 = 0$ we multiply

$g(x)$ by 1, $g(x)$ by $(3x)$ &

2 by $\frac{1}{(1-x)}$ and subtract as above.

Q.4] Solve using recurrence relation

$a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$. $a_0 = 5$ $a_1 = -9$ $a_2 = 15$

$\rightarrow a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$

Solution ~~of the~~ ~~ass~~ is $a_n = r^n$

$\therefore r^n = -3(r^{n-1} + r^{n-2}) - r^{n-3}$

$r^n = -3r^{n-1} - 3r^{n-2} - r^{n-3}$

$r^n + 3r^{n-1} + 3r^{n-2} + r^{n-3} = 0$

$r^{n-3}[\underbrace{r^3 + 3r^2 + 3r + 1}] = 0$

$\therefore$ characteristic eqn

$\therefore r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0$

$\therefore r = (-1)^n$

$a_n = (c_1 + c_2 n + c_3 n^2)(-1)^n$.

~~$a_n = c_1 + c_2 n$~~

Put $a_0 = 5$ $a_1 = -9$ $a_2 = 15$

we get $c_1 = 5$ $c_2 = 3$ $c_3 = 1$

$\therefore \boxed{a_n = (5 + 3n + 1n^2)(-1)^n}$

Q.5] 1] $G(x) = \frac{1}{3-6x} = \frac{1}{3(1-2x)}$.

Now $G(x) = \frac{1}{3}(1-2x)^{-1} = \frac{1}{3}(1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots)$

Now, sequence $(3 \times \frac{1}{3} \times (1, 2, 4, 8, 16 \dots))$ times $(1 + x + x^2 + x^3 + \dots)$

Sequence is $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}$

2] $\dfrac{x}{1-5x+6x^2} = \dfrac{x}{(1-3x-2x+6x^2)}$

$= \dfrac{x}{1(1-3x)-2x(1-3x)}$

$= \dfrac{x}{(3x-1)(2x-1)}$

$= \dfrac{3x-1-2x+1}{(3x-1)(2x-1)}$

$= \dfrac{(3x-1)-(2x-1)}{(3x-1)(2x-1)}$

$= \dfrac{1}{2x-1} - \dfrac{1}{(3x-1)}$

$\therefore \dfrac{x}{1-5x+6x^2} = \dfrac{-1}{(1-2x)} + \dfrac{1}{(1-3x)}$

$= (1-3x)^{-1} - (1-2x)^{-1}$

In $(1-x)^{-1}$ put $x = 3x$.

We get

$\dfrac{x}{(1-5x+6x^2)} = (1+3x+9x^2+27x^3 \dots) - (1+2x+4x^2+8x^3 \dots)$

$= x + 5x^2 + 9x^3 + \dots$

Sequence is $(1, 5, 9, \dots)$.

Camlin Page

DIS TUTORIAL - 7.

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A. Div

Q.1] i) ~~Le~~

25

Lets assume there are n members in a room.
Then the number of handshakes can be:
0, 1, 2, 3 ...., n-1

But if there is a person with (n-1) handshakes then there cannot be a person with 0 handshakes.
∴ There are (n-1) possibility (piegon holes).
and n members (pigeons).
There should be atleast 2 people with same number of handshakes.

ii) Red marbles → 10. blue marbles → 10.
White marbles → 10
In the worst case senario after removing 9 balls there are 3 red, 3 blue, 3 white ~~to~~ marbles. Then the 10th ball would the marble with 4 of same colour
Minimum 10 balls should be removed inorder to get 4 balls of same colour.

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Q.2] Total people in the party → 100.
So there can be 0, 1, 2, 3 . . . , n-1 people knowing a person.
If a person knows 0 people then there could be a person with (n-1) people knowing him.
∴ There are (n-1) possibility (piegon hole). ~~of~~ ~~of~~ for n (pigeons) people.
∴ There must be 2 people knowing same number of people.

Q.3] Let p_i be the total no of pills taken by the end of i^{th} days. Now if the patient decides to take one pill each for atleast 11 days, then the condition holds true & in many cases But considering the patient takes pill in the foll sequence

$p_1 < p_2 < p_3 \ldots < p_{30}$ here $p_{30} = 48$.

$p_1 < p_2 < p_3 \ldots < p_{30} = 48$. —(1)

Adding 11 to each no of sequence

$p_1+11 < p_2+11 < p_3+11 \ldots < p_{30}+11 = 59$ —(2)

There are 30 nos each in sequences (1) & (2)
Hence there are 60 nos (pigeons) all less than or equal to 59 (pigeon holes)
Hence by pigeon hole principle, atleast 2 of these nos must be equal (which violates the condition of strictly increasing sequence)
Hence there is one such $p_i = p_j + 11$ $(i > j)$.
∴ $p_i - p_j = 11$ ... which implies that there are exactly 11 pills taken in the consecutive days $j+1, j+2, \ldots i$ Hence it is proved that there will always be stretched of consecutive days in which the total no of pills taken is 11.

Camlin Page

Q.4] i) $1 + 5 + 9 + \dots + 4n - 3 = n(2n-1)$ — (1)

Let P(n) be the predicate

$n_0 = 1$.

∴ Show that P(1) is true.

∴ $P(1) = 1(2(1)-1) = 1$ ∴ Its TRUE.

Induction step:

Now for $\forall\ k \geq 1$ and if P(k) is true then.

P(k+1) should also be true.

$P(k) = 1 + 5 + 9 + \dots 4k - 3 = k(2k-1)$.

$P(k+1) = \cancel{1+5+9+\dots}\ (k+1)(2(k+1)-1)$

$= (k+1)[2k+2-1]$

$= (k+1)[2k+1]$ — (2)

Also

$P(k+1) = \underbrace{1 + 5 + 9 + \dots}\ + 4(k+1) - 3.$

$= 1 + 5 + 9 + \dots \quad 4k + 4 - 3$.

$= \underbrace{1 + 5 + 9 + \quad 4k - 3}_{\downarrow} + 4$

P(k)

$= k(2k-1) + 4$

$P(k+1) = \underbrace{1 + 5 + 9 + \dots \quad 4k - 3}_{P(k)} + 4(k+1) - 3.$

$= k(2k-1) + 4(k+1) - 3$

$= k(2k-1) + 4k + 4 - 3$

$= (2k-1)k + (4k+1)$

Camlin Page

$= 2k^2 - k + 4k + 1$

$= 2k^2 + 3k + 1$

$= 2k^2 + k + 2k + 1$

$= k(2k+1) + 1(2k+1)$

$\Rightarrow (k+1)(2k+1)$ — (3)

Since RHS of (2) and (3) are equal.
∴ $P(k+1)$ is also true.
$\forall k \geq 1$.

By principal of mathematical induction

ii) $2 + 5 + 8 + \ldots + (3n-1) = \frac{n(3n+1)}{2}$.

Let $P(k)$ be the predicate
$n_0 = 1$ here.
Base step.
$P(n_0) = 2$.
$P(n_0) = \frac{1(3(1)+1)}{2} = \frac{4}{2} = 2$

∴ $P(n_0)$ = True.

Induction step
For all $k \geq 1$ $P(k+1) \to P(k)$ should be true.
∴ $P(k) = 2 + 5 + 8 + \ldots (3k-1) = \frac{k(3k+1)}{2}$ — (2)

~~$P(k+1) = 2 + 5 + 8 + \ldots (3k-1) + (3(k+1) - 1)$~~
~~Also.~~
$P(k+1) = \frac{(k+1)(3(k+1)+1)}{2}$

Camlin Page

$$P(k+1) = (k+1)\frac{(3k+3+1)}{2}$$

$$= \frac{(k+1)(3k+4)}{2} \quad -(3)$$

Also

$$P(k+1) = \underbrace{2+5+8+\cdots(3k-1)}_{P(k)} + 3(k+1)-1$$

$\therefore$ from (2) we can write

$$P(k+1) = \frac{k(3k+1)}{2} + 3(k+1)-1$$

$$= \frac{k(3k+1)}{2} + 3k+3-1$$

$$= \frac{k(3k+1) + 2(3k+2)}{2}$$

$$= \frac{3k^2+k+6k+4}{2}$$

$$= \frac{(k+1)(3k+4)}{2}$$

$$= P(k+1)$$

By mathematical induction this Predicate is true.

Camlin Page

Q.5] $1^3 + 2^3 + 3^3 + \dots n^3 = \frac{n^2 (n+1)^2}{4}$ — (1)

$n \geq 1$ ..

→ Basic.

Let P(k) be the predicate

$n_0 = 1$

∴ we must show that $P(1) = 1^3$ — (2)

Substitute $n=1$ in (1)

$P(1) = \frac{1^2 (1+1)^2}{4} = \frac{1 (2)^2}{4} = 1$ — (3)

from (2) & (3) the statement is true.

Induction ~~is to~~ step.

Now for ∀ $k \geq 1$ & P(k) is true then,

P(k+1) should also be true.

~~P(k+1) = (~~

$P(k) = 1^3 + 2^3 + \dots k^3 = \frac{k^2 (k+1)^2}{4}$ — (4)

∴ $P(k+1) = \cancel{1^3 + 2^3} + \dots \frac{(k+1)^2 ((k+1)+1)}{4}$

$= \frac{(k+1)^2 (k+2)}{4}$ — (5)

Also

$P(k+1) = \underbrace{1^3 + 2^3 + \dots k^3}_{P(k)} + (k+1)^3$.

$= \frac{k^2 (k+1)^2}{4} + (k+1)^3$

$= (k+1)^2 \left[\frac{k^2}{4} + (k+1) \right]$

Camlin Page

$$P(k+1) = (k+1)^2 \left[\frac{k^2 + 4(k^2+1+2k)}{4} \right]$$

$$= (k+1)(k^2 + 4k^2 + 4 + 8k)/4$$

$$= (k+1)(\cancel{k^2+}\ 5k^2 + 8k + 4)/4$$

$$=$$

$$P(k+1) = (k+1)^2 [k^2/4 + (k+1)]$$

$$= (k+1)^2 [k^2 + 4k + 4]/4$$

$$= \frac{(k+1)^2 (k+2)^2}{4} \quad (6)$$

The predicate is true $\forall\ k \geq 1$

By principle of mathematical induction.

Q.6] $n^3 + 2n$ is divisible by $3\ \forall\ n \geq 1$.

→ Basic step

Show that it is true for $n = 1$

For $n = 1$ $n^3 + 2n = \cancel{1^3}\ 1^3 + 2(1) = 1 + 2 = 3$.

$\therefore$ at $n = 1$ it is divisible by 3.

for $n = k$.

we assume that $k^3 + 2k$ is divisible by 3.

$\therefore k^3 + 2k = 3m$.

Now for $(k+1)$

$n = k+1$.

$$(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + 2(k+1)$$

$$= k^3 + 3k^2 + 3k + 1 + 2k + 1$$

$$= \cancel{k^3}\ \cancel{k^3 + 2k} +$$

$$= 3m + 3(k^2 + k + 1).$$

$$= 3(m + k^2 + k + 1).$$

By mathematical induction $n^3 + 2n$ is divisible by 3.

Camlin Page

DIS TUTORIAL - 8

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Div : A

Q.1)

Distance	d < 4000	4000 < d < 9000	9001 < d < 14,000	d > 14,000
Frequency	20	~~20~~ 210	325	445

The total number of trials conducted = 100.

i) Frequency of vehicles whose tyres have to be replaced before 4000 km (E1) = 20.

Probability that tyre will be substituted before 4000 km

$= \frac{E1}{N} = \frac{20}{1000} = 0.02$

ii) Frequency of tyre that will last more than 9000 km

= Freq of ~~typ~~ tyre that last from 9001 to 14000 km + Freq of tyres that last more than 14000 km

$= 325 + 445$ (E2)

$= 770.$

Probability that a tyre will last more than 9000 km $= \frac{E2}{N} = \frac{770}{1000} = 0.77$.

iii) P(tyre has to be replaced after it has covered distance ranging between 4000 and 14000)

$= \dfrac{\text{frequency of whose tyre are replaced b/n 4000 km \& 9000 km} + \text{Frequency of vehicles whose tyres have been replaced between 9000 \& 14000 km}}{N}$

$= 210 + 325/1000 = 535/1000 = \boxed{0.535}$

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Q.2] The tests in which students get more than 70% marks are Test 2, Test 5, Test 3.

Total no. of Tests conducted (N) = 5

No. of tests in which students score more than 70% marks = Test 2, Test 5, Test 3
= 3 tests.

∴ Probability that the student gets more than 70% marks = $\frac{3}{\text{Total no of tests conducted}}$

$= \frac{3}{5}$

$= \boxed{0.6}$

Q.3] Total number of cards in a deck (N) = 52.
Total number of aces in deck (E) = 4.

i) Probability that the card drawn is an ace

$\Rightarrow \frac{\text{no of aces}}{\text{total cards}} = \frac{E}{N} = \frac{4}{52} = \boxed{\frac{1}{13}}$

ii) Probability that the card is not an ace.

$= 1 -$ (Probability that the card drawn is an ace)

$= 1 - \frac{1}{13}$

$= \boxed{\frac{12}{13}}$

Camlin Page

Q4) Total balls in bag (N) = 4.
Of the remaining 2 balls we can have 3.
possible events
E_1 = Both balls are blue.
E_2 = One ball is blue
E_3 = None of the balls are blue.

All the events are equally likely and mutually exclusive so there are ~~three~~ equal chances that any of the three events will occur

$\therefore P(E_1) + P(E_2) + P(E_3) = 1$

Since equal chances to occur.

$P(E_1) = P(E_2) = P(E_3)$

$\therefore P(E_1) + P(E_1) + P(E_1) = 1$

$\therefore P(E_1) = P(E_2) = P(E_3) = 1/3$.

Let A = event that ~~blue~~ 2 balls already drawn are blue.

$\therefore$ We need to find (E_1/A)

Now $P\left(\frac{A}{E_1}\right) = \frac{\text{Total no of ways to pick 2 blue balls}}{\text{Total no of ways to pick 2 balls}}$

$= \frac{{}^4C_2}{{}^4C_2}$

$= 1$ ($\because$ all balls are blue)

$P\left(\frac{A}{E_2}\right) = \frac{\text{Total no of ways to pick 2 blue balls when one blue ball is already picked}}{\text{Total no of ways to pick 2 balls.}}$

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$$P\left(\frac{A}{E_2}\right) = \frac{3}{6} = \boxed{\frac{1}{2}}$$

$P\left(\frac{A}{E_3}\right)$ = Here we have 2 blue balls ~~left~~.

$$= \frac{\text{No of ways to pick 2 blue balls out of 4}}{\text{No of ways to pick 2 balls}}$$

$$= \frac{^2C_2}{^4C_2}$$

$$= \boxed{\frac{1}{6}}$$

Using Baye's Theorem.

$$P\left(\frac{E_1}{A}\right) = \frac{P(A/E_1) \times P(E_1)}{P(A/E_1)\times P(E_1) + P(A/E_2)P(E_2) + P(A/E_3)P(E_3)}$$

$$= \frac{1 \times 1/3}{1\times 1/3 + 1/2 \times 1/3 + 1/6 \times 1/3}$$

$$= \frac{1}{1 + 1/2 + 1/6}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$= 0.6$$

$\therefore$ Ans: probability ~~of~~ that the balls are blue.

$\therefore = 0.6$

Camlin Page

Q.5] Let A be the doctor finding rash (A).
Let B_1 be the event that child has measles.
Let B_2 be the event that child has flue.
S → Sick children

∴ ~~Pr~~ Probability of being sick with flue.
$= P(B_2) = 0.9$.
Probability of being sick with measles
$= P(B_1) = 0.1$.
Probability of finding a rash given that the child has measles $= P\left(\frac{R}{B_1}\right) = 0.95$

Probability of finding rash given that the child has flue $= P\left(\frac{R}{B_2}\right) = 0.08$.

Probability of child having flue given he has rash.

$$= P\left(\frac{B_2}{A}\right) = \frac{P(A/B_2) \times P(B_2)}{P(A/B_2) \times P(B_2) + P(A/B_1) \times P(B_1)}$$

$$= \frac{0.9 \times 0.08}{0.9 \times 0.08 + 0.1 \times 0.95}$$

$$= \frac{0.072}{0.167}$$

$$= \boxed{\frac{72}{167}}$$

Camlin Page

Q6] Let E_1 be the event of spam mail
Let E_2 be the event of non spam mail

A = Event of detecting spam mail.
$P(E_1) = 0.5$
$P(E_2) = 0.5$ } → given.

∴ $P(A/E_1)$ = Probability of detecting a spam. given it is a spam
$= 0.99$.

$P(A/E_2)$ = Probability of detecting a spam. given its not a spam
$= 0.05$

∴ Probability of event being non spam given a spam mail is detected

$$= P\left(\frac{E_2}{A}\right) = \frac{P(A/E_2) \times P(E_2)}{P(A/E_2) \times P(E_2) + P(A/E_1)\,P(A\,E_1)}$$

$$= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.99 \times 0.5}$$

$$= \frac{0.05}{1.04}$$

$$= \frac{5}{104}$$

$$= \boxed{0.048}$$