21/8/2024.

DSP

Q $x(n) = \{\overset{-2}{2}, \overset{-1}{2}, \overset{0}{2}, \overset{1}{2}, \overset{2}{1}, \overset{3}{1}, \overset{4}{1}, \overset{5}{1}, \overset{6}{1}\}$

Find $(-2n-2)$

Shift $x(n-2) = \{\underset{\uparrow 0}{2}, \underset{1}{2}, \underset{2}{2}, \underset{3}{2}, \underset{4}{1}, \underset{5}{1}, \underset{6}{1}, \underset{7}{1}, \underset{8}{1}\}$

Scale $x(2n-2) = \{2, 2, 1, 1, 1\}$

Reverse $x(-2n-2) = \{\underset{-4}{1}, \underset{-3}{1}, \underset{-2}{1}, \underset{-1}{2}, \underset{0}{2}\}$

Q. For given signal Find 1) $x(n-3)$
2) $x(3-n)$
3) $x(2n)$
4) $x(n)\,u(3-n)$
5) $x[(n-1)^2]$

$x(n) = \left\{\underset{-3}{-\frac{1}{2}}, \underset{-2}{\frac{1}{2}}, \underset{-1}{1}, \underset{\uparrow 0}{1}, \underset{1}{1}, \underset{2}{1}, \underset{3}{\frac{1}{2}}\right\}$

$x(n-3) = \left\{\cancel{\frac{1}{2}}\ \underset{\uparrow 0}{-\frac{1}{2}}, \underset{1}{\frac{1}{2}}, \underset{2}{1}, 1, 1, 1, \frac{1}{2}\right\}$

$x(3-n) = x(n+3)$

$x(-n+3)$

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$x(n+3) = \left\{ -\frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{2} \right\}$

0 1 2 3 4 5 6

$\uparrow$

$x(-n+3) = \left\{ \frac{1}{2}, 1, 1, 1, 1, \frac{1}{2}, -\frac{1}{2} \right\}$

$\uparrow$

0

3) $x(2n) = \left\{ \frac{1}{2}, 1, 1 \right\}$ $u(n) = 1$, $n \geq 0$
$= 0$ else.

$\uparrow$

4) $x(n) \cdot u(3-n)$

$u(-n+3) = \{1, 1, 1, 1, 1\}$

$x(n) \cdot u(3-n) = x(n)$.

5] $x[(n-1)^2]$

n	-3	-2	-1	0^*	1	2	3
$x(n)$	$-\frac{1}{2}$	$1/2$	1	1	1	1	$1/2$
$x[(n-1)^2]$				$x(1)$	$x(0)$	$x(1)$	
	0	0	0	1	1	1	0

$\uparrow$

$\{1, 1, 1\}$

$\uparrow$

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Classification of systems.

Stable	1) Linear	1) Variant	1) causal	Static
Unstable	2) Non-Linear.	2) Invariant	2) non-causal	Dynamic

1) Linear, Non Linear.

$y(n) = \cos[x(n)]$

For given system decide whether it is linear or not.

Step 1: give ip as $x_1(n)$
op $\Rightarrow$ $y_1(n) = \cos[x_1(n)]$

Step 2: give ip as $x_2(n)$
op $\Rightarrow y_2(n)$
$y_2(n) = \cos[x_2(n)]$

Step 3: ~~give ip~~ add response of indiv inps as $y_1(n) + y_2(n) = y'(n)$
$= \cos[x_1(n)] + \cos[x_2(n)]$ - I

Step 4: Give ip as $x_1(n) + x_2(n)$
$y''(n) = \cos[x_1(n) + x_2(n)]$ - II

I $\neq$ II

Hence system is non linear.

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Q 1) $y(n) = x(n-1) + x(n)$. TV

Q.2) $y(n) = \log[x(n)]$ Non Linear TV

Q.3 $y(n) = x^2(n)$ Non Linear TV

Q.4 $y(n) = x[n^2]$ Linear V

$y'(n) = x(n^2-K)$

$y(n-K) = x((n-K)^2)$

Q.1

→ step 1: give ip as $x_1(n)$.

$op \Rightarrow y_1(n) = y_1(n-1) + y_1(n)$

Step 2: ip $x_2(n)$

$op \Rightarrow y_2(n) = y_2(n-1) + y_2(n)$.

step 3 $y'' = y_1(n) + y_2(n)$

$= y_1(n-1) + y_1(n) + y_2(n-1) + y_2(n)$.

step 4: give ip as $x_1(n) + x_2(n)$

Linear

Q.2 Step 1: give ip as — NL

Q.3

Q.4 $y(n) = x[n^2]$.

$y_1 x_1(n) = x_1(n^2)$

$y_2 x_2(n) = x_2(n^2)$

$y_1 + y_2 = x_1(n^2) + x_2(n^2)$

$x_1(n) + x_2(n) = \cancel{x_1(n^2) + x_2(n^2)}$.

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Variant / Invar

System is invariant if shift in ip signal results in corresponding shift in op.

Step 1: Delay i/p by k
Step 2: Replace n by n-k.
Step 3: Compare 1 and 2.

Q. $y(n) = \cos[x(n)]$

1) Delay i/p by k

$y'(n) = \cos[x(n-k)]$ — i)

2) $n \Rightarrow n-k$

$y(n-k) = \cos[x(n-k)]$ — ii)

3) (i) = (ii) Invariant.

Q. $y(n) = x(n-1) + n\,x(n)$

Delay ip by 'k'

$y(n) = x(n-1-k) + n\,x(n-k)$. — (i)

$n \Rightarrow n-k$

$x(n-k-1) + (n-k)\,x(n-k)$ — ii

(i) $\neq$ (ii)

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Causal / Non-Causal.

If op depends on present and past but does not depend on future then system is called as causal system.

$y(n) = \cos[x(n)]$.

$n = 0$

$y(0) = \cos[x(0)]$

Causal.

$y(n) = x[n^2]$ N C

$n = 2, y(2) = x(4)$

$y(n) = x(n-1) + x(n)$ Causal

$y(n) = \log[x(n)]$ Causal.

Static / Dynamic

(Memoryless) (with Memory).

System in which dp at any instant of time depends on ip sample at same time is called as static

$y(n) = \cos[x(n)]$ Static

$y(n) = x(-n+2)$ Dynamic, Non Causal.

Not dependent only on present

$y(n) = \dfrac{x(n) + x(-n)}{2}$ Linear

☆

Convolution

$$x(n) \longrightarrow \boxed{\text{System}} \longrightarrow y(n)$$
$h(n)$

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

For given two signals, find convolution

$$x(n) = \{1, 1, 1, 1\}$$

$$h(n) = \{1, 2, 0, -3\}$$

	1	1	1	1
1	1	1	1	1
2	2	2	2	2
0	0	0	0	0
-3	-3	-3	-3	-3

$$y(n) = \{1, 3, 3, 0, -1, -3, -3\}$$

$x_1(n) = \{2, 1, \underset{\uparrow}{-1}, -2, -3\}$

$x_2(n) = \{1, \underset{\uparrow}{2}, 0, -3\}$

	2	1	-1	-2	-3
1	2	1	-1	-2	-3
2	4	2	-2	-4	-6
0	0	0	0	0	0
-3	-6	-3	3	6	9

$y(n) = \{2, 5, 1, \underset{\uparrow}{-10}, -10, -3, 6, 9\}$

Samples = $(m + n - 1)$

Correlation

Auto

Cross

Correlation of signal with a delayed copy of itself is auto correlation and correlation of signal with delayed copy of another signal is cross correlation. Co'r

Correlation reaches maximum when two signals considered become most similar to each other.

Find auto correlation for
$x_1(n) = \{1, 1, 2, 2\}$

	2	2	1	↓ 1
→ 1	2	2	1	1
1	2	2	1	1
2	4	4	2	2
2	4	4	2	2

$$y(n) = \{2, 4, 7, \underset{\uparrow}{10}, 7, 4, 2\}$$

Find cross correlation for
$x_1(n) = \{1, 1, 2, 2\}$
$x_2(n) = \{1, 2, 3, 4\}$

$x_2(n)$

	4	3	2	↓ 1
$x(n)$ → 1	4	3	2	1
1	4	3	2	1
2	8	6	4	2
2	8	6	4	2

$$y(n) = \{4, 7, 13, \underset{\uparrow}{17}, 11, 6, 2\}$$

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stable /Unstable

System is said to be stable if we have Bounded Input and bounded output and $\sum_{-\infty}^{\infty} |h(n)| < \infty$

$h(n) \Rightarrow$ Impulse Response

$x(n) = \delta(n)$ $y(n) = h(n)$.

$\delta(n) = 1$, $n = 0$
$= 0$ else

Q. $y(n) = \cancel{a(n} \; a.x(n).x(n-1)$.

$x(n) = \delta(n)$ $\therefore$ $y(n) = h(n)$
$h(n) = a\,\delta(n).\delta(n-1)$
$n = 0$
$h(0) = a\,\delta(0)\,\delta(-1) = 0$
$n = 1$
$h(1) = 0$

$h(0) + h(1) + \dots = 0$ finite

Q $\cancel{x(n) = \delta}$ $y(n) = e^{-x(n)}$
$x(n) = \delta(n)$
$h(n) = e^{-\delta(n)}$.
$h(n) = e^{-\delta(0)}$
$h(n) = e^{-1}$

unstable.

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1) Image Negation.

$L = max + 1$

$S = (L-1) - r$

Q.

$$\begin{bmatrix} 4 & 3 & 5 & 2 \\ 3 & 6 & 4 & 6 \\ 2 & 2 & 6 & 5 \\ 7 & 6 & 4 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 3 & 4 & 2 & 5 \\ 4 & 1 & 3 & 1 \\ 5 & 5 & 1 & 2 \\ 0 & 1 & 3 & 6 \end{bmatrix}$$

2) Thresholding.

$$S = \begin{cases} 1 & r > \text{threshold} \\ 0 & r <= \text{threshold.} \end{cases}$$

Apply threshold $th = 4$.

$S = L - 1 \quad r \geq T$

$= 0 \quad$ otherwise

$$\begin{bmatrix} 7 & 0 & 7 & 0 \\ 0 & 7 & 7 & 7 \\ 0 & 0 & 7 & 7 \\ 7 & 7 & 7 & 0 \end{bmatrix}$$

~~5)~~

3) Clipping. (Grey Level Slicing without background).
Removes the ~~back~~ foreground from the background

$L = 8$

$L - 1 = 7$

$r_1 = 2, r_2 = 5$

$$s = \begin{cases} L-1 & 2 \le r \le 5 \\ 0 & \text{otherwise} \end{cases}$$

4) Grey Level slicing with background.

$$S = \begin{cases} L-1 & a \le r \le b. \\ r. & \end{cases}$$

5) Bit plane slicing. [Used for compression].

r =

0	7	3	1
3	6	4	6
2	4	2	2
1	2	5	3

000

th = 4.

1)

7	0	4	6
4	1	3	1
5	3	5	5
6	5	2	4

2)

0	7	0	0
0	7	7	7
0	7	0	0
0	0	7	0

3)

0	0	7	0
7	0	7	0
7	7	7	7
0	7	7	7

4)

0	7	7	1
7	6	7	6
7	7	7	7
1	7	7	7

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Bit level

msb — Plane 2

0	1	0	0
0	1	1	1
0	1	0	0
0	0	1	0

centre — Plane1

0	1	1	0
1	1	0	1
1	0	1	1
0	1	0	1

lsb — Plane 0

0	1	1	1
1	0	0	0
0	0	0	0
1	0	1	1

Q. $x(n) = 4\delta(n+4) + 3\delta(n+3) - 2\delta(n+2) + \delta(n) - 3\delta(n-1) + 2\delta(n-2) - 4\delta(n-3)$.

$$x(n) = \begin{cases} 4 & 3 & -2 & 0 & -3 & 2 & -4 \\ n= -4 & -3 & -2 & 1 & -1 & 2 & 3 \end{cases}$$
$\uparrow$

Q. $x(n) = 5\delta(n+2) + 2\delta(n+1) - 3\delta(n-2) + 4\delta(n-3)$

$$x(n) = \begin{cases} 5 & 2\ 0\ 0\ -3 & 4 \\ -2 & -1\ 1\ 0\ 2 & 3 \end{cases}$$
$\uparrow$

Q write given signal in terms of weighted impulses

$x(n) = \{2, 1, \underset{\uparrow}{3}, -2, 1\}$

$2\delta(n+2) + \delta(n+1) + 3\delta(n) - 2\delta(n-1) + \delta(n-2)$

Q. $x(n) = \{3, -1, 0, \underset{\uparrow}{2}, 1\}$

$3\delta(n+3) - \delta(n+2) + \cancel{\delta(n+1)} + 2\delta(n) + \delta(n-1)$

Discrete Fourier Transform (DF

DTFT (Discrete Time F.T.)

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

DFT

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \quad ; k = 0 \text{ to } (N-1)$$

$$X[k] = X(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$$

Continuous freq spectrum $X(\omega)$ is replaced by discrete fourier spectrum $X[k]$ infinite Σ is in DTFT is replaced by finite Σ in DFT

$$\underset{T.D}{x(n)} \xrightarrow{DFT} \underset{F.D.}{X[k]}$$

$$\xleftarrow[I.DFT]{}$$

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \quad ; k = 0 \text{ to } (N-1).$$

Since Σ is taken for N points this is called N point DFT.

I.D.F.T

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi nk/N} \quad ; n = 0 \text{ to } (N-1).$$

A new term ω is given by $\omega_N = e^{-j2\pi/N}$

$\omega_N = e^{-j2\pi/N} \longrightarrow$ Twiddle factor.

$$X[k] = \sum_{n=0}^{N-1} x(n) \omega_N^{nk} \quad ; k : 0 \text{ to } (N-1).$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \omega_N^{-nk} \quad ; n = 0 \text{ to } (N-1).$$

Q. Find DFT of $x(n) = \{1, 2, 3, 4\}$.
$N = 4$.

$x(n) \xrightarrow{DFT} X[k]$

$$X[k] = \sum_{n=0}^{N-1} x(n) \omega_N^{nk} \quad ; k = 0 \text{ to } (N-1).$$

$$= \sum_{n=0}^{3} x(n) \omega_N^{nk} \quad ; k = 0 \text{ to } 3$$

$$= x(0)\omega_N^{0} + x(1)\omega_N^{k} + x(2)\omega_N^{2k} + x(3)\omega_N^{3k} \quad ; k = 0 \text{ to } 3$$

$2\pi/3$

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$X[k] = 1W_N^0 + 2W_N^k + 3W_N^{2k} + 4W_N^{3k}$

$k=0$

$X[0] = 1+2+3+4 = 10$

$k=1$

$X[1] = 1 + 2W_N^1 + 3W_N^2 + 4W_N^3$

$= 1 - 2j + (-3) + 4j$

$= -2 + 2j$

$W_N^1 = W_4^1$

$W_N^1 = e^{-j2\pi/N}$

$W_4^1 = e^{-j2\pi/4}$

$= e^{-j(\pi/2)}$ $= \underline{\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}}$

$= -j$

$W_4^2 = W_N^1 W_N^1$

$= (-j)(-j)$

$= j^2 = -1$

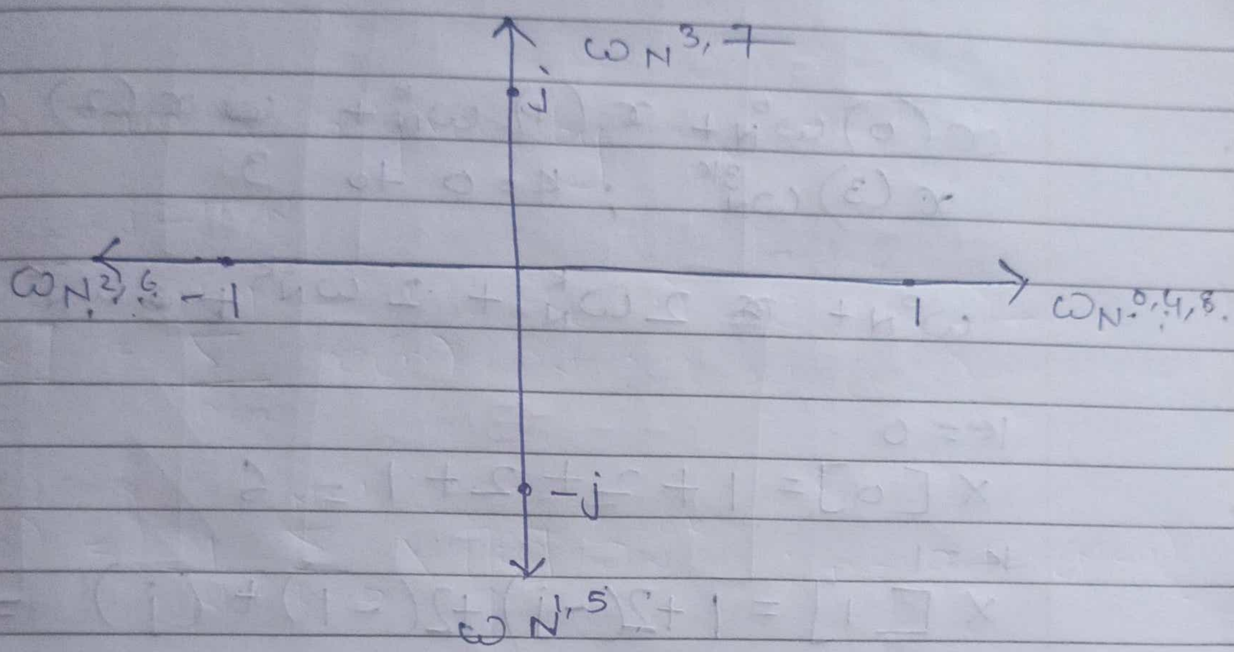
$W_4^3 = (-j)(-j)(-j) = (-1)(-j)$

$= \cancel{-j^2(-1)} = j$

$k = 2$

$$X[2] = 1 + 2\omega_N^{2} + 3\omega_N^{4} + 4\omega_N^{6}$$

$k = 3$



$$X[2] = 1 + 2(-1) + 3(1) + 4(-1)$$
$$= 1 - 2 + 3 - 4$$
$$= -2$$

$$X[3] = 1 + 2j + 3(-1) + 4(-j)$$
$$= 1 + 2j - 3 - 4j$$
$$= -2 - 2j$$

$$X[k] = \{10, -2+2j, -2, -2-2j\}$$

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Q.2 $x(n) = \{1, 2, 2, 1\}$

$\rightarrow X[k] = \sum_{n=0}^{N-1} x(n) \omega^{nk}$; $k = 0$ to $(N-1)$

$x(0)\omega_N^0 + x(1)\omega_N^k + x(2)\omega_N^{2k} + x(3)\omega_N^{3k}$; $k = 0$ to 3

$\omega_N^0 + 2\omega_N^k + 2\omega_N^{2k} + \omega_N^{3k}$; $k = 0$ to

$k = 0$.

$X[0] = 1 + 2 + 2 + 1 = 6$

$k = 1$

$X[1] = 1 + 2(-j) + 2(-1) + (j) = -1 - j$

$k = 2$

$X[2] = 1 + 2(-1) + 2(1) + (-1) = 0$

$k = 3$

$X[3] = 1 + 2(j) + 2(-1) - j = -1 + j$

Q. $x(n) = (-1)^n$ for $N = 3$

$X[k] = \sum_{n=0}^{2} x(n) \, \omega_N^{nk}$ $\omega_{(3)}^1 = e^{-j2\pi/N}$

$= e^{-j2\pi/3}$

$= -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$

$-\frac{1}{2} - \frac{\sqrt{3}j}{2}$

$x(n) = \begin{Bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \end{Bmatrix}$

$X[0] = 1$

$X[1] = 1 - \omega_N^k + \omega_N^{2k}$

$= 1 - \omega_3^1 + \omega_3^2$

$= 1 - \frac{1}{2} - \frac{\sqrt{3}}{2}j + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)^2$

$\frac{1}{4} -$

$X[1] = 1 + 1.732j$

$X[2] = 1 - 1.732j$

$x(n) = \{1, 2, 3, 4\}$

$X[k] = W_{N(4\times4)} \; x(n).$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Q. $x(n) = \{1, 2, 1, 2\}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 1-2j-1+2j \\ 1-2+1-2 \\ 1+2j-1-2j \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

Q. Find IDFT if ~~x(k~~ $X[k] = [4, 1-j, -2, 1+j]$

$$x(n) = \frac{1}{N} \omega_N^{*} X[k]$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4+(1-j)-2+(1+j) \\ 4+j+1+2-j-1 \\ 4-1+j \\ \end{bmatrix} = \cancel{\frac{1}{4}} \begin{bmatrix} \cancel{4}\ 1 \\ \cancel{8}\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Q. Determine if $X[k] = \{3, 2+j, 1, 2-j\}$

$$x(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 3 \\ 2+j \\ 1 \\ 2-j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} \cancel{8}\ \cancel{4}\ \cancel{7} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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Linear

$x(n) = \{1, 2, 3, 4\}$

$x(n-2)$
$= \{0, 0, 1, 2, 3, 4\}$
$\uparrow$

$x(n+1) = \{1, 2, 3, 4\}$
$\uparrow$

Folding

$x(-n) = \{4, 3, 2, 1\}$

Circular

$x(n-2)$
$= \{3, 4, 1, 2\}$

$x(n+1)$
$= \{2, 3, 4, 1\}$

$x(n-1) = \{4, \cancel{2}, 1, 2, 3\}$

Folding

$x(-n) = \{1, 4, 3, 2\}$

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Contrast stretching (Piecewise Linear Transformation)

$$s = \begin{cases} \alpha \cdot r & 0 \le r \le r_1 \\ \beta(r - r_1) + s_1 & r_1 \le r \le r_2 \\ \gamma(r - r_2) + s_2 & s \le \end{cases}$$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$r_1 = 3$ and $r_2 = 5$
$s_1 = 2$ and $s_2 = 6$

$\alpha = 0.66$
$\beta = 2$
$\gamma = 0.5$

r	s
0	0
1	0.66
2	1.32
3	2
4	4
5	
6	
7	

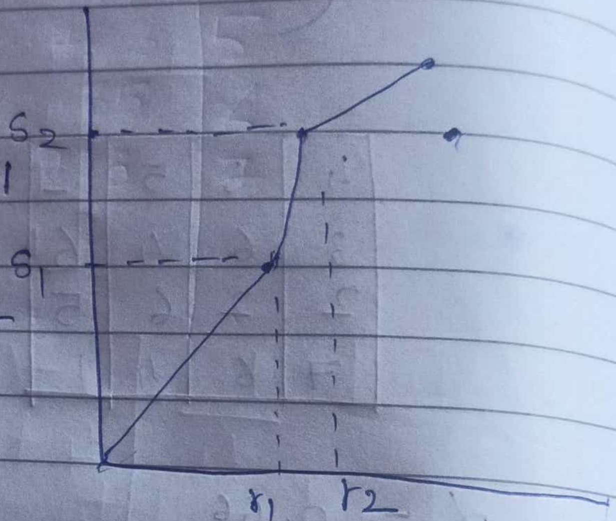
7			
10			

$r1 = 8$ | $s_1 = 4$

$r_2 = 12$ | $s2 = 8$

$\alpha =$

r	s
~~0~~	0
1	0.5 / 1
2	1
3	1.5 / 2
4	2
~~5~~	
6	3
7	3.5 / 4
8	4
~~9~~	
10	6
~~11~~	
12	8
~~13~~	
~~14~~	
15	15



$\alpha = 0.5$

$\beta = 1$

$\gamma = 2.3$

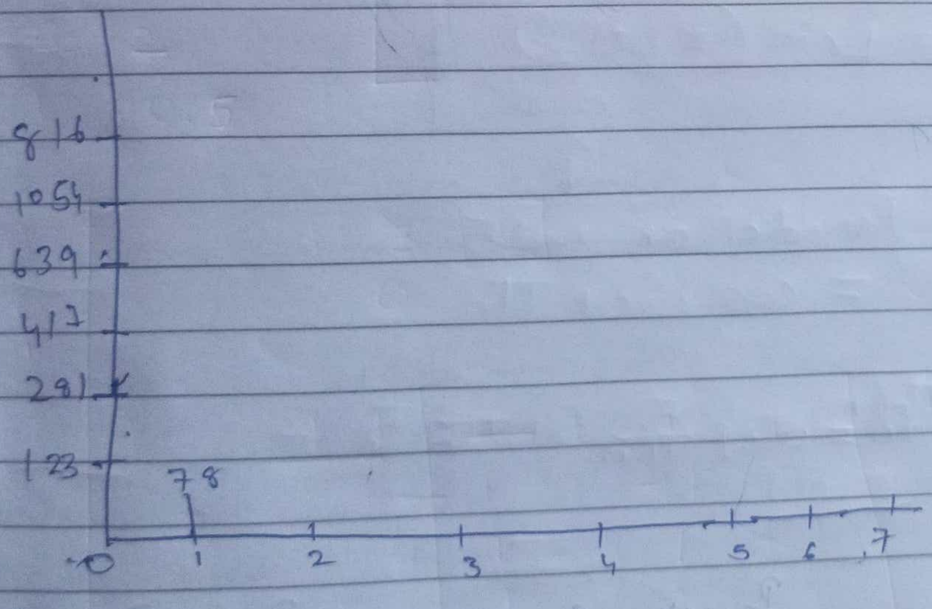
4	8	1	2
6	15	1	3
8	2	3	

Log Transformation (Dynamic Range Compression).
$s = c \log(1+r)$

Histogram Equalization

Gray Level	Freqn	PDF	CDF	CDF × (L-1)	Round off
0	123	0.03	0.03	0.21	0
1	78	0.019	0.049	0.343	0
2	281	0.068	0.117	0.819	1
3	417	0.101	0.218	1.526	2
4	639	0.156	0.314	2.618	3
5	1054	0.257	0.631	4.417	4
6	816	0.199	0.83	5.81	6
7	688	0.168	1	7	7

$n \Rightarrow 4096.$



Gray level	Freqn	P	C.D.F	
0	200	0.18	0.18	1
1	270	0.245	0.425	3
2	130	0.118	0.543	4
3	60	0.054	0.597	4
4	60	0.054	0.651	5
5	80	0.072	0.723	5
6	140	0.127	0.85	6
7	160	0.145	0.995	7

$n = 1100$

Power signal: $0 < P < \infty$

Energy signal: $0 < P < \infty$.

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Signal is Continuous Time ⟹ Every instant

District Time: Signal which has value at certain instancce of time
Eg. BP after e

Signal is periodic if $x(n) = x(n+N)$ where N is time period.

Signal is non-periodic if $x(n) \neq x(n+N)$.

Signal is periodic if it is repeated after fix time interval.

Signal is even if $x(n) = x(-n)$. [~~Si~~ Cosine]

Signal is odd if $x(n) = -x(-n)$. [sine].

Even signal are symmetric and odd signals are assymmetric

Random: Cannot be defined Eg noise

Deterministic is one which can be represented using mathematical equation.

Eg $x(n) = e^{-3n}$.

Signal is said to be energy if energy values lies betweeen 0 to ∞.

Power of an energy s/g is '0'.
Energy of a power s/g is '∞'.

Periodic / Non Periodic

For discrete time signal condition for periodicity

$f_0 = \frac{k}{N}$

Signal is periodic only if its frequency if f_0 is ratio of 2 integers

N: Time period

Periodicity of signal: $x(n) = x_1(n) + x_2(n)$

N_1 N_2

Check Periodic $= \frac{N_1}{N_2}$

TP: $N = LCM(N_1, N_2)$.

Decide Periodic or not.

1) $x(n) = \cos 0.01 \pi n$

$\cos 2\pi f n = \cos 0.01 \pi n$

$2\pi f n = 0.01 \pi n$

$f = \frac{0.01}{2} = \frac{1}{200}$

∴ ratio of 2 integers.

$N = 200$.

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2) $x(n) = \sin 3\pi n$

$\sin 2\cancel{\pi f n} = \sin 3\cancel{\pi n}$

$2f = 3$

$f_0 = \frac{3}{2}$ $N = 2$.

3) $x(n) = \sin 3n$ Not periodic $\because f_0 = \frac{3}{2\pi}$

4) $x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$

~~cos = 2~~

$x(n) = x_1(n) + x_2(n)$

$N_1 = 5, \ N_2 = 7$

$\frac{N_1}{N_2} = \frac{5}{7}$ which is ratio of integers

$N = LCM(5, 7)$
$= 35$

5) $x(n) = \sin(0.2n + \pi)$

$\sin(2\pi f n + \phi)$

$2\pi f n = 0.2n$

$f_0 = \frac{0.2}{2\pi}$

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6) $x(n) = e^{j\left(\frac{8\pi}{4}\right)n}$ $\cos \frac{n\pi}{4} + i\sin \frac{n\pi}{4}$

Even and odd :

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

For given signal find even and odd part

$x(n) = \{1, 0, \underset{\uparrow}{2}, 3, -1\}$

$x(-n) = \{-1, 3, \underset{\uparrow}{2}, 0, 1\}$

Even = $\{0, 1.5, \underset{\uparrow}{2}, 1.5, 0\}$

Odd = $\{1, -1.5, 0, 1.5, -1\}$

1, 0, 2, 3, -1

Power and Energy.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Find Energy and Power of $x(n) = u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left|\overset{u}{\cancel{x}}(n)\right|^2 = \sum_{n=0}^{\infty} (1)^2$$

$$= \infty$$

$u(n)$ is not an energy signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

Mihir

2) $x(n) = 2e^{3jn} u(n)$.

$e^{3jn} = |\cos\theta \pm j\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta}$

$$x(n) = \sum_{n=-\infty}^{\infty} |2e^{3jn} u(n)|^2$$

$$= 4 \, |(1)^2|$$
$$= 4$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= 6 \; \frac{1}{2N+1} \sum_{n=0}^{N} |(1)^2|$$

$$= \frac{\cancel{N}^{2}}{2\cancel{N}+1}$$

$=$

$\frac{1}{1-\frac{1}{2}}$ $\frac{1}{1-\frac{1}{9}}$

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$$x(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$\sum_{n=0}^{\infty} A^n = \frac{1}{1-A}$$

$A < 1.$

$$E = \sum_{n=-\infty}^{\infty} \left| x(n)^2 \right|$$

$$= \sum_{n=0}^{\infty} \left| \left(\frac{1}{3}\right)^n \right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n.$$

$$= \cancel{\frac{1}{1-9}} \; \frac{9}{8}$$

$$x(n) = \left(\frac{1}{2}\right)^n \quad n \geq 0.$$

$$= 3^n \quad , \quad n < 0.$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left| \left(\frac{1}{4}\right)^n \right|^2 + \sum_{n=-\infty}^{-1} \left| 3^n \right|^2$$

$$= \frac{4}{3} + \frac{1}{8}$$

$$= \frac{32+3}{24} = \frac{35}{24}.$$

Properties of DFT

1) Linearity Property.

$$a x_1(n) + b x_2(n) \xrightarrow{DFT} a X_1[k] + b X_2[k].$$

2) Periodic

DFT IDFT

$$x_p(n) \underset{IDFT}{\overset{DFT}{\rightleftarrows}} X_p[k]$$

3) Time shift

Whenever there is shift in time domain there is multiplication factor there is shift in time domain and sign changes

$$\underset{TD}{x(n)} \xrightarrow{DFT} \underset{FD}{X[k]}$$

$$x(n-m) \longrightarrow \omega^{mk} X[k].$$

$$x(n+m) \xrightarrow{DFT} \omega^{-mk} X[k].$$

4) Frequency shift.

There is shift in frequency domain we have multiplication factor in time domain and ~~time~~ sign remains same

$$x(n) \xrightarrow{DFT} X[k]$$
TD FD

$$x(n)\, \omega^{-mn} \xrightarrow{DFT} X[k-m]$$

$$x(n)\, \omega^{mn} \xrightarrow{DFT} X[k+m]$$

5) Time Reversal

If $x(n) \xrightarrow{DFT} X[K]$

$$x(-n) \xrightarrow{DFT} X[-K]$$

Q. Find $X[k]$, if $x(n) = \{1, 2, 3, 4\}$.
Hence find DFT of

1) $p(n) = \{4, 1, 2, 3\}$ | $x(n-1)$

2) $q(n) = \{3, 4, 1, 2\}$ | $x(n-2)$

3) $r(n) = \{1, 4, 3, 2\}$ | $x(-n)$

4) $h(n) = \{1, -2, 3, -4\}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

-1 $\quad j$ $\quad b_1 2\ (-1)^2$

$-j$

$p(n) = x(n-1)$

$P[k] = \omega^{1k} X[k]$ By Time Shift Property.

$$p[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} \omega_N^0 (1) \\ \omega_N^1 (-j) \\ \omega_N^2 (-1) \\ \omega_N^3 (j) \end{bmatrix} \begin{array}{l} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}$$

$$= \begin{bmatrix} 10 \\ 2j+2 \\ 2 \\ -2j+2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2+2j \\ 2 \\ 2-2j \end{bmatrix}$$

$q(n) = x(n-2)$

$Q[k] = \omega^{2k} X[k]$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} \omega_N^0 & 1 \\ \omega_N^2 & -1 \\ \omega_N^4 & 1 \\ \omega_N^6 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 2-2j \\ -2 \\ 2+2j \end{bmatrix}$$

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$l(n) = x(-n)$

$$\cancel{\begin{bmatrix} -10 \\ 2-2j \\ 2 \\ 2+2j \end{bmatrix}}$$

By Time Reversal property.

$$[\,10,\ -2-2j,\ -2,\ -2+2j\,]$$

$h(n) = x(n)$; 'n' even

$= -x(n)$; 'n' odd

$h(n) = (-1)^n x(n)$

$= (\omega_N^{2})^n x(n)$

$\downarrow$ DFT

$H[k] = X[k+2]$

$$= \begin{bmatrix} -2 \\ -2-2j \\ 10 \\ -2+2j \end{bmatrix}$$