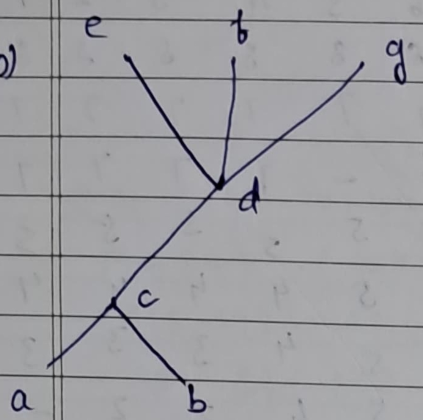
(b)



GLB	a	b	c	d	e	f	g
a	a		a	a	a	a	a
b		b	b	b	b	b	b
c	a	b	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e		
f	a	b	c	d		f	
g	a	b	c	d			g

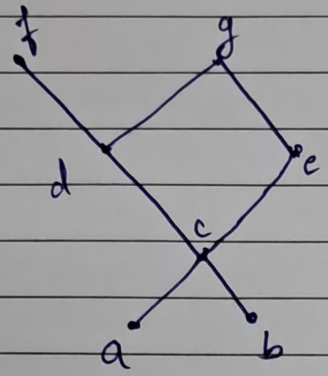
LUB

∨	a	b	c	d	e	f	g
a	a	~~a~~	c	d	e	f	g
b	~~a~~	b	c	d	e	f	g
c	c	c	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	-	-
f	f	f	f	f	-	f	-
g	g	g	g	g	-	-	g

∵ (f, e) (g, e) (e, f) (e, g) (f, g) (g, f)
have no LUB,
it is not a lattice

(4)

(a)



$\{a, b, c\}$

$\Rightarrow$

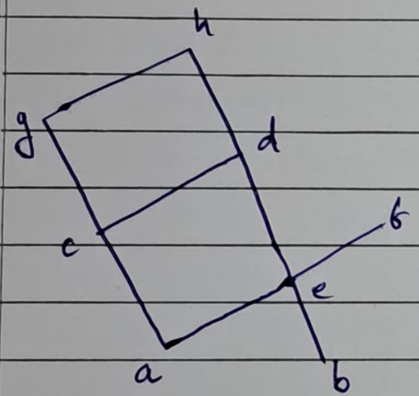
upperbound = $\{d, e, f, c, g\}$

Lowerbound = $\{\phi\}$

LUB = c

GLB = ϕ

(b)



$\{d, e, f\}$

upper bound = $\{\phi\}$

Lower bound = $\{a, c, b\}$

GLB = e

LUB = ϕ

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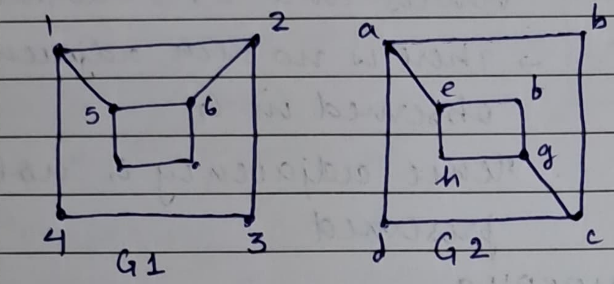
TUTORIAL 5: Graphs, Paths and Colouring

(Q1) Determine whether the following graphs are isomorphic.

(A)



Soln Comparing G_1 & G_2

Feature	G_1	G_2
Vertices	8	8
edges	10	10
degree 2 vertices	4	4
degree 3 vertices	4	4

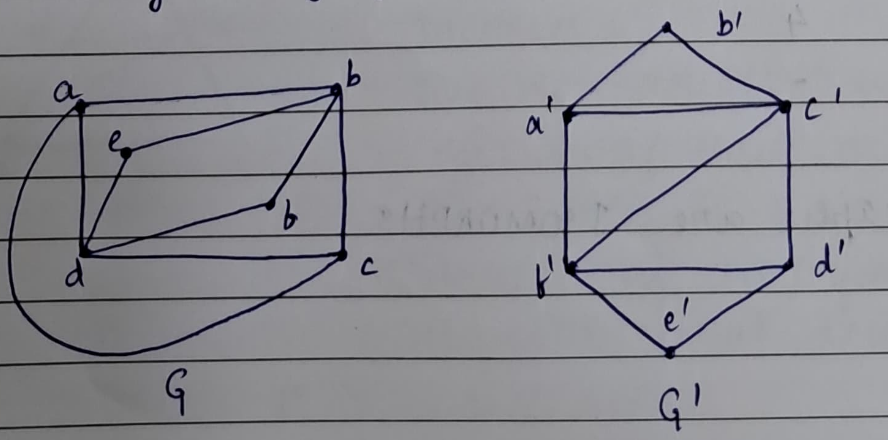
ADJACENCY

→ Vertex 1 of Graph G1 has degree 3 and has 2 vertices of degree 3 & 1 vertex of degree 2 adjacent to it.

→ Whereas, in G2, no similar adjacency can be observed

Hence, adjacency is not preserved,
Hence, given graphs are not ISOMORPHIC.

(B)



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Comparing G & G'

Feature ~~Vertices~~	G	G'
vertices	6	6
edges	9	9
degree 2 vertices	2	2
degree 3 vertices	2	2
degree 4 vertices	2	2

ADJACENCY

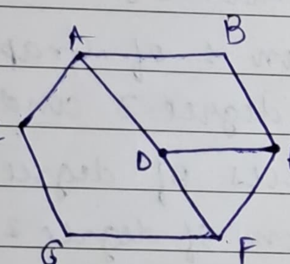
→ In G vertex b which has degree 4 is ~~connected~~ adjacent to 2, 3 degree vertices and 2, 2-degree vertices

→ There is no such adjacency observed in G_1.

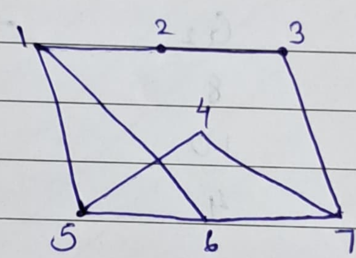
Hence adjacency is not preserved

The graphs are NOT ISOMORPHS

(C)



G



G'

Comparing G & G'

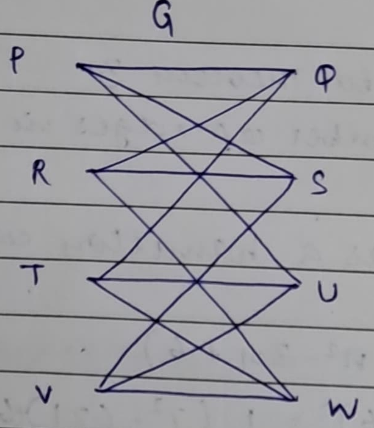
Feature	G	G'
Vertices	7	7
edges	9	9
degree 2 vertices	3	3
degree 3 vertices	4	4
degree 4 vertices	-	-

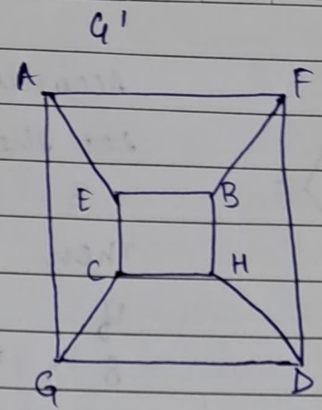
ADJACENCY

- Here adjacency is preserved.

Hence, the graphs are ISOMORPHS

[D]





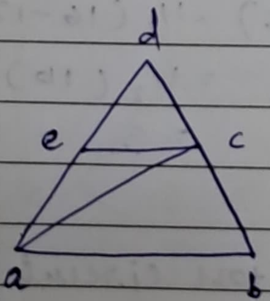
Comparing G & G'

Feature	G	G'
Vertex	8	8
edges	12	12
degree 3 vertices	8	8

Here adjacency is preserved.

∴ the Graphs are ISOMORPHS

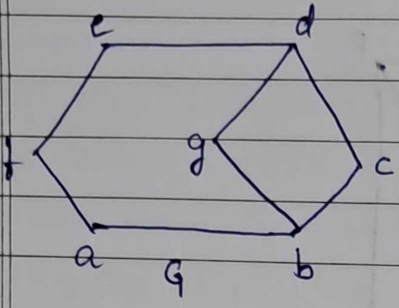
[Q2]
[A]



In this graph, with accordance to THEOREM 1;
Two non-adjacent vertices (say d, a) the sum of their degrees (2+3=5) is equal to number of vertices in the graph. Therefore Hamilton circuit exists.

π : a, b, c, d, e, a

[B]



According to Theorem 3

~~Let~~ The number of edges in G is 8.

Then G has a hamilton circuit if

where,

n = no of verties

$8 \geq 1/2 (n^2 - 3n + 6)$

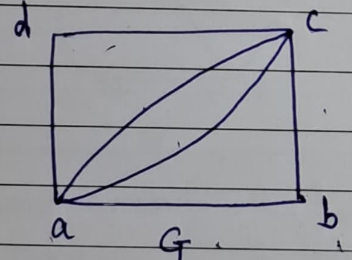
$1/2 (n^2 - 3n + 6) = \frac{1}{2} (7^2 - (21) + 6)$

$= 17$

$8 < 17$; therefore ~~Rule~~ Theorem 3 fails.

It is not a Hamilton circuit.

[C]



According to theorem 3:

The no of edges in G is 6

then G has hamilton circuit

if $m \geq 1/2 (n^2 - 3n + 6)$

where

m = no of edges

n = no of vertices

$1/2 (n^2 - 3n + 6) = 1/2 (16 - 12 + 6)$

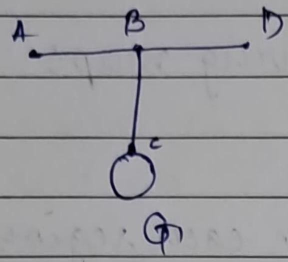
$= 1/2 (10)$

$= 5$

$6 \geq 5$

Hence, theorem 3 holds, true, Hamilton circuit exists.

π : a, b, c, d, a

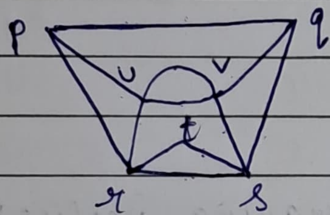
(D) 

Vertice	Degree
A	1
B	3
C	3
D	1

According to theorem 2, G has a hamilton circuit if each vertex has degree greater than or equal to $n/2$ $(= 2)$

$\therefore$ theorem 2 fails here, circuit is not Hamilton.

[Q3]

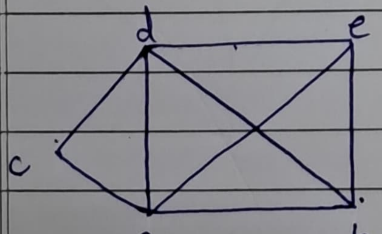
[A] 

This graph fails theorem 2, for Hamilton circuits as t doesnot have degree greater than $n/2$ (here = 3)

But, Hamilton path ~~doesnot~~ exist.

π_H : s, t, r, p, u, v, q, s

Euler path : p, q, s, r, p, u, v, s, t, r, u, v, q

[B] 

According to theorem 3 for hamilton circuits, (Here m = 8 , n = 5)

$\frac{1}{2}(n^2 - 3n + 6) = \frac{1}{2}(25 - 15 + 6)$

$= \frac{1}{2}(10 + 6) = 8$

here $m = \frac{1}{2}(n^2 - 3n + 6)$

$\therefore$ it is a hamilton circuit.

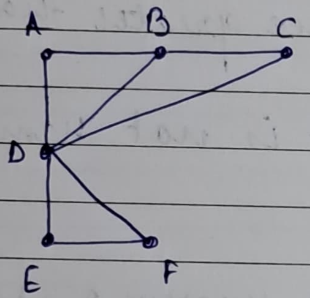
π_1 : a, b, e, d, c, a

Eulerian path : e, d, c, a, b, d, a, e, b

[Q4] Find Chromatic Number for the following maps.

[A]

Soln Constructing graph with one vertex for each region and edge connecting two vertices whose corresponding share a boundary.



Applying Welsh Powell Algorithm

Vertex	Degree
A	2
B	3
C	2
D	5
E	2
F	2

Assigning vertices in descending order

D	B	A	C	E	F
5	3	2	2	2	2

\# Assign Colour 1 to D

\# Assign Colour 2 to B, E

\# Assign Colour 3 to A, C, F

All the vertices have be assigned a color & so G is 3-colorable

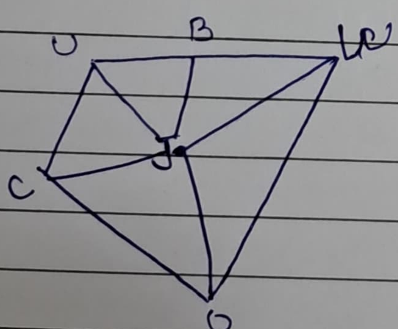
$\therefore \chi(G) = 3$

[B]

Sol^n Constructing graph with one vertex for each region and an edge connecting two vertices whose corresponding regions share a common boundary

Now, applying Welch Powell Algorithm to find Chromaticity

Vertex	Degree	State
U	3	UP
B	3	Bihar
J	5	Jharkhand
W	3	West Bengal
O	3	Orissa
C	3	Chhattisgarh



Arranging vertices in descending order, we get,

J,	B,	C,	O,	U,	W
5	3	3	3	3	3

Assign colour 1 to J
colour 2 to B, C
colour 3 to O, U
Colour 4 to W

∴ All vertices have been assigned a colour so, graph 4 is colourable.

$\chi(G) = 4.$

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TUTORIAL 6 : Generating functions & Recurrence Relations

Que. Find the generating function for the following sequence .. 1, 2, 3, 4, 5, 6 . . .

Sol^n Using the sequence 1, 2, 3, 4, 5, 6 .. the above expression becomes :

$$f(x) = \Sigma a x^n$$
$$f(x) = 1x^0 + 2x^1 + 3x^2 + 4x^3 + 5x^4 \ldots \quad - (1)$$

Multiplying eq^n 1 by x

$$x f(x) = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \ldots \quad - (2)$$

Subtracting (1) from (2)

$$f(x)[x-1] = [x + 2x^2 + 3x^3 + 4x^4] - [1 + 2x + 3x^2 + \ldots]$$
$$= -[1 + x + x^2 + x^3 \ldots]$$

$$= \frac{1}{1-x} \quad \text{for } |x| \leq 1$$

$$\therefore f(x) = \frac{1}{(1-x)^2} \quad \text{for } |x| \leq 1$$

Que 2 Solve the recursive relation
$a_n = 3a_r + 2 \quad r \geq 1$ with $a_0 = 1$ using generating function.

Sol^n 2 Consider $g(x) = a_0 + a_1 x + a_2 x^2 + \ldots$

$\therefore 3x g(x) = 3a_0 + 3a_1 x + 3a_2 x^2 + \ldots$

$\frac{2}{1-x} = 2 + 2x + 2x^2 + \ldots$

Subtracting las from the 1st & substituting $a_0 = 1$ we get,

$g(x) - 3x g(x) = \frac{2}{1-x} = (1-2) + (a_1 - 3a_0 - 2)x + (a_2 - 3a_1 - 2)x^2 \ldots$

$\because a_0 = 1$

$\therefore a_r - 3a_{r-1} - 2 = 0$

each bracket on right except the first is zero,

Hence, $(1-3x) g(x) = -1 + \frac{2}{(1-x)} = \frac{-1+x+2}{1-x} = \frac{1+x}{1-x}$

$g(x) = \frac{1+x}{(1-x)(1-3x)} = \frac{2}{(1-3x)} - \frac{1}{(1-x)}$ (By partial fraction)

$= 2\sum_{r=0}^{\infty} 3^r x^r - \sum_{r=0}^{\infty} x^r = \sum_{r=0}^{\infty} [2(3^r - 1)] x^r$

But $g(x) = \sum a_r x^r - a_n = 2 \cdot 3^r - 1$

In order to make use of $a_n - 3a_n - 2 = 0$
we multiply $g(x)$ by 1, $g(x)$ by $3x$ & 2 by $\frac{1}{(1-x)}$ & subtract as above.

Q4) Solve using recurrence relations,

$a_n = -3(a_{n-1} + a_{n-2}) + a_{n-3}$ with

$a_0 = 5,\ a_1 = -9$ & $a_2 = 15$

Sol^n $a_n = -3(a_{n-1} + a_{n-2}) + a_{n-3}$

Substituting $a_n = r^n$

$r^n = -3(r^{n-1} + r^{n-2}) + r^{n-3}$

$r^n = -3r^{n-1} - 3r^{n-2} + r^{n-3}$

$r^{n-3}\ \underbrace{[r^3 + 3r^2 + 3r + 1]}_{\text{characteristic eq}^n} = 0$

$\therefore r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0$

$r = (-1)^n$

$a_n = (c_1 + c_2 n + c_3 n^2)(-1)^n$

put $a_0 = 5$; $a_1 = -9$ & $a_2 = 15$

we get,

$c_1 = 5$

$c_2 = 3$ & $c_3 = 1$

$\therefore a_n = (5 + 3n + n^2)(-1)^n$

Que 5 Given a generating function find out

(i) $\frac{1}{3-6x}$

(ii) $\frac{x}{1-5x+6x^2}$

(i) $\frac{1}{3-6x} = \frac{1}{2(1-2x)}$

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Now $G(x) = \frac{1}{3}(1-2x)^{-1}$

$= \frac{1}{3}(1+2x+4x^2+8x^3+\dots)$

Now Sequence, $(\frac{1}{3} \times (1, 2, 4, 8, 16 \dots)$

$(1+x+x^2+x^3+\dots)$

Sequence is $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3} \dots$

(ii) $\frac{x}{1-5x+6x^2} = \frac{x}{1-3x-2x+6x^2} = \frac{x}{(1-3x)(1-2x)}$

$= \frac{x}{(3x-1)(2x-1)}$

$= \frac{3x-1-(2x-1)}{(3x-1)(2x-1)}$

$= \frac{1}{2x-1} - \frac{1}{3x-1}$

$\frac{x}{1-5x+6x^2} = \frac{-1}{1-2x} + \frac{1}{1-3x}$

$= (1-3x)^{-1} + (1-2x)^{-1}$

in $(1-\cancel{x})^{-1}$ we put $\cancel{x} = (3x)$

we get.

$\frac{x}{1-5x+6x^2} = (1+3x+9x^2 \dots) - (1+2x+4x^2 \dots)$

$= x+5x^2+9x^3 \dots$

$\therefore$ Sequence $= (1, 5, 9 \dots)$

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TUTORIAL 7 : COMBINATORICS

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Q1.
Soln
(i) Lets assume n number of members in a room
Then the number of handshakes can be 0, 1, 2, 3 .. n-1
But if there is a person with (n-1) handshakes
then there must be a person with 0 handshakes.
∴ There are (n-1) possibility (pigeon holes) & n members (pigeons)
∴ There should be atleast 2 people with same no. of handshakes.

(ii) 10 red marbles
10 white marbles
10 blue marbles
In the worst case scenario after removing 9 balls there are 3 red, 3 white, 3 blue marbles.
Then the 10th ball would be marble with 4 of same color.
∴ Minimum 10 balls should be removed in order to get 4 balls of same color.

(Q2) 100 people are there in a party.
So there can be 0, 1, 2, 3 ... n-1 people knowing a person.
If a person knows 0 people then there cannot be a person with (n-1) people knowing him. ∴ There are (n-1) possibility (pigeon hole) for n people (pigeon)
∴ There must be 2 people knowing same no. of people

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Q9) Let p_i be the total no of pills taken by the end of the ith day, Now, if the patient decides to take one pill each for atleast 11 days, then the condition holds true in many cases. But considering patient takes them in order,

$p_1 < p_2 < p_3 \dots < p_{30}$ here $p_{30} = 48$

$p_1 < p_2 < \dots < p_{30} = 48$ — ①

Adding 11 to sequence.

$p_1+11 < p_2+11 < \dots < p_{30}+11 = 59$ — ②

There are 30 nos in each seq ① & ②.
Hence, there are 60 numbers and all less than or equal to 59.
Hence, by pigeon hole principle, atleast two of the numbers must be equal.
Hence, there is one such $p_i = p_j + 11$ $(i > j)$
$\therefore p_i - p_j = 11$ which implies that there are exactly 11 pills taken in the consecutive days.
Hence it is proved that there will be always a stretch of consecutive days in which the total no of pills taken is 11.

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Q4)
(i) $1+5+9 \dots + 4n-3 = n(2n-1)$ — ①

let $P(n)$ be the predicate.
$n_0 = 1$ here,
Basis step, we must show $P(1)$ is true.
$P(1) = 1$ — ②
Substituting $n=1$ in ①.
$P(1) = 1(2(1)-1) = 1$ — ③
$\because$ 2 & 3 are equal statement is true.
Induction step,
Now, for $\forall k \geq 1$ & $P(k)$ is true then $P(k+1)$ should also be true.
$P(n+1) = (n+1)(2(n+1)-1) = (n+1)(2n+1)$

$P(n+1) = 1+5+9 \dots + 4n-3 + 4n+1$
$= 2n^2 - n + 4n + 1$
$= 2n^2 + 3n + 1 = 2n(n+1) + 1(n+1)$
$= P(n+1)$
The predicate is true for all $n \geq 1$ by principal of mathematical induction.

(ii) $2+5+8 \dots (3n-1) = \frac{n(3n+1)}{2}$
let $P(n)$ be predicate.
$n_0 = 1$ here
Basis steps:-
$P(n_0) = 2$
$P(n_0) = \frac{1(3\times1+1)}{2} = 2$

$P(n_0)$ is True

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Induction step,

$\forall$ $k \geq 1$ $P(k+1) \longrightarrow P(k)$ should hold true

$P(k+1) = (k+1)\left(\frac{3(k+1)+1}{2}\right)$

$P(k+1) = 2+5+8 \cdots (3k-1) + (3(k+1)-1)$

$= P(k) + 3k+2$

$= \frac{(3k+1)k}{2} + 3k+2$

$= \frac{(k+1)(3k+4)}{2}$

$= P(k+1)$

$\therefore$ By mathematical induction this predicate is true.

(Q5) $1^3 + 2^3 + 3^3 + \cdots n^3 = \frac{n^2(n+1)^2}{4}$ — (1)

$n \geq 1$

Basis step.

Let $P(n)$ be a predicate, $n_0 = 1$

$P(1) = 1^3 = 1$ — (2)

$P(1) = \frac{1^2(1+1)^2}{4} = 1$ — (3)

(2) & (3) are equal hence, true.

Induction step,

Now $\forall$ $k \geq 1$ if $P(k)$ is true then $P(k+1)$ should also be true

$P(n+1) = 1^3 + 2^3 + 3^3 \cdots (n+1)^3$

$= \frac{(n+1)^2(n+2)^2}{4}$

$\therefore$ the predicate is true $\forall$ $n \geq 1$. By mathematical induction

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(2b) n^3+2n is divisible by 3 ∀ $n \geq 1$

Basis step

Show that it is true for $n=1$

for $n=1$

$n^3+2n = 1^3+2(1) = 3$

at $n=1$ it is divisible by 3

for $n=k$

we assume k^3+2k is divisible by 3

$$k^3+2k = 3m$$

now for $k+1$

$n=k+1$

$$(k+1)^3+2(k+1) = (k^3+3k^2+3k+1)+(2k+2)$$
$$= 3m+3(k^2+k+1)$$
$$= 3(m+k^2+k+1)$$

at $n=k+1$ it is divisible by 3.

∴ By mathematical Induction n^3+2n is divisible by 3 for all $n \geq 1$

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TUTORIAL 8

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(Q1)

Distance (d)	<4000	4000<d<9000	9000<d<14	d>14000
Frequency	20	210	325	445

Total = 1000

① Frequency of vehicles whose tyres have to be replaced before 4000 km (f1) = 20

$P(E_1) = \frac{20}{1000}$

$= 0.02$

② Frequency of tyre that will last more than 9000km
$= 325 + 445$
$= 770$

$P(E_2) = \frac{770}{1000} = 0.77$

Que 2 The test in which students get more than 70% marks are Test 2, test 3 and Test 5

Total no. of Tests conducted = 5 (N)
No of Tests in which students scored more than 70% (E) = 3

Probability that students get more than 70% = $\frac{E}{N}$

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$= \frac{3}{5}$

$= \underline{0.6}$

Q3) Total no of cards in deck (N) = 52

Total no of aces (E) = 4

(1) Probability that card drawn is an ace $= \frac{E}{N} = \frac{4}{52}$

$= \frac{1}{13}$

(2) Probability card is an ace P(E) + Probability card isn't an ace (P(E')) = 1

$P(E) + P(E') = 1$

$\therefore P(E') = 1 - P(E)$

$= 1 - \frac{1}{13}$

$= \frac{12}{13}$

Q4)

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Q4) Total no of balls in bag $(N) = 4$
of remaining 2 balls, we have 3 possible events

$E_1 =$ Both are blue
$E_2 =$ One ball is Blue
$E_3 =$ None of them are Blue

All of them are equally likely & are mutually exclusive so there is one in three chance any one of these events occur.

$P(E_1) = P(E_2) = P(E_3) = 1/3$

Let A be event that 2 blue already drawn are blue
we have to find $P(E_1|A)$

$P(A|E_1) = \dfrac{\text{Total no of ways in which 2 blue ball}}{\text{ways of Picking up}}$

$= \dfrac{^4C_2}{^4C_2} = 1$

$P(A|E_2) = \dfrac{^3C_2}{^4C_2} = \dfrac{3}{6} = \dfrac{1}{2}$

$P(A|E_3) = \dfrac{^2C_2}{^4C_2} = \dfrac{1}{6}$

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Using Baye's theorem,

$$P(E_1|A) = \frac{P(A|E_1) \times P(E_1)}{P(E_1)\,P(A|E_1) + P(A|E_2)\,P(E_2) + P(A)(P(E_3|A))}$$

$$= \frac{1 \times 1/3}{1/3 + 1/3 \times 1/2 + 1/6 \times 1/3}$$

$$= \frac{6}{10} = \frac{3}{5}$$

Q5 Let A be an event finding rash.
B_1 is event that child has measles B_2 is event child has flu.

S → Sick children

P(S) flute = 0.9

$P(B_2) = 0.9$

$P(S|m) = 0.1 = P(B_1)$

$P(A|B_1) = 0.95$

$P(A|B_2) = 0.08$

$$P(B_2|A) = \frac{P(A|B_2) \times P(B_2)}{P(A|B_1)\,P(B_1) + P(A|B_2)\,P(B_2)}$$

$$= \frac{0.9 \times 0.08}{0.9 \times 0.08 + 0.1 \times 0.95}$$

$$= \frac{72}{167}$$

Teacher's Sign.:

Q6) E_1 is event of spam email.

E_2 is event of non spam email.

A = event of detecting spam email

$P(E_1) = 0.5$

$P(E_2) = 0.5$

$P(A|E_1) = 0.99$

$P(A|E_2) = 0.05$

$$P(E_2|A) = \frac{P(A|E_2) \times P(E_2)}{P(A|E_1)\,P(E_1) + P(A|E_2)\,P(E_2)}$$

$$= \frac{0.05}{1.04} = \frac{5}{104}$$