PAGE No
DATE / /

→ Regular expression.

Reg. exp.	Reg. language.
r	$L(r)$
a	$L(r) = \{a\}$
$a \overset{\text{concatenation}}{\cdot} b$	$L(r) = \{ab\}$
$\underbrace{a+b}_{\text{union}}$ or $\underbrace{a/b}_{\text{or}}$	$L(r) = \{a, b\}$
a^*	$L(r) = \{\epsilon, a, aa, \ldots\}$
$\underbrace{a^+}_{\text{1 or more occurrence}}$	$L(r) = \{a, aa, \ldots\}$
$a^+ = a \cdot a^*$	$L(r) = \{a, aa, \ldots\}$
v.v. Imp → $\boxed{(a+b)^*}$	$L(r) = \{\epsilon, a, b, aa, bb, ab, ba, \ldots\}$

$*$ – 0 or more occurrence of that char.

eg i] write a regular expression for the following:-
(i) set of all strings that starts with 'a' over the input $\Sigma = \{a, b\}$

→

$L = \{a, ab, aba, aa, abb, \ldots\}$

$r.e = a.(a+b)^*$

P.T.O.

Scanned with CamScanner

PAGE No.
DATE / /

ii) For the strings that ends with either "zero" or "11"

$\Sigma = \{0, 1\}$

$\rightarrow$ $r.e = r_1 + r_2$

$= (0+1)^* . 0 + (0+1)^* . 11$

$= (0+1)^* [0 + 11]$

$\therefore L = \{0, 11, 00, 01, 011, 111, \ldots\}$

iii] Strings that starts with "ab" and ends with "ba"

$\Sigma = \{a, b\}$

$\rightarrow$

$r.e = ab(a+b)^* ba + aba$

abba ये term नहीं लेगा r.e se so alag se likhna if b is ... so once written

iv] starts and ends with different letter over $\Sigma = \{a, b\}$

$\rightarrow$

$r.e. = a(a+b)^* b + b(a+b)^* a \cancel{+ ab + ba}$

v] atleast 2a's $\Sigma = \{a, b\}$

$\rightarrow$

$r.e = (a+b)^* a (a+b)^* a (a+b)^*$

vi] exactly 2a's

$\rightarrow$

$r.e = b^* a b^* a b^*$

ज्यादा से ज्यादा

vii] atmost 2a's

$\rightarrow$

0 occurrence of 'a'; 1 occurrence of 'a'; 2 occurrence of 'a'

$r.e = r_1 + r_2 + r_3$

$= b^* + b^* a b^* + b^* a b^* a b^*$

eg:- find R.E consisting of all strings over $\Sigma = \{a,b\}$ starting with any no. of 'a's' followed by one or more 'b's' followed by 1 or more a's. followed by single 'b' followed by any no. of a's followed by b & ending

~~$R.E = a^* b^+$~~

in any string of a's & b's

$$R.E = a^* b^+ a^+ b\, a^* b (a+b)^*$$
$$= a^* b^+ a^+ b\, a^* b\, (a+b)^*$$

$b.b^*$

* How to prove L.H.S = R.H.S part?

$L(M+N) = LM + LN$

$L = a^*$

$M = a a^* b$

$N = b^*$

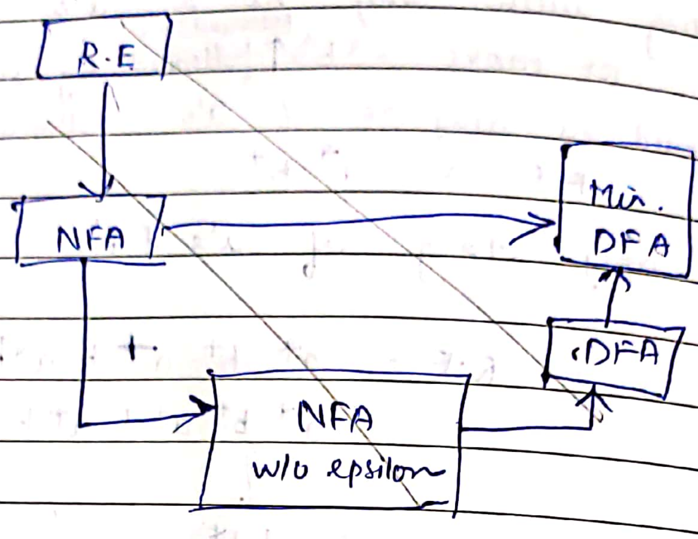
$a^* (a a^* b + b^*) = L.H.S$

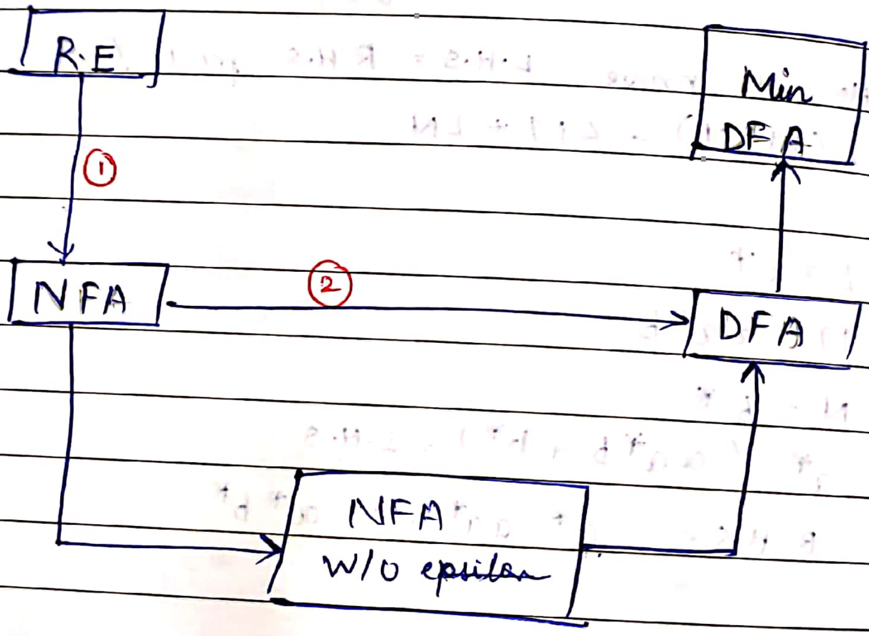
$R.H.S = a^* a a^* b + a^* b^*$

* $a + \epsilon = a, \epsilon$
* $a . \epsilon = a$

DFA → Deterministic finite automata.

NFA → Non-deterministic finite automata.





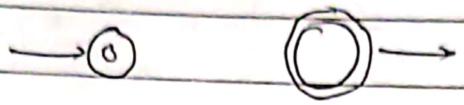
PAGE No

DATE / /

20/2/23

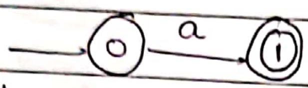
Regular Expression to NFA :-

Rule 1: $r = \phi$

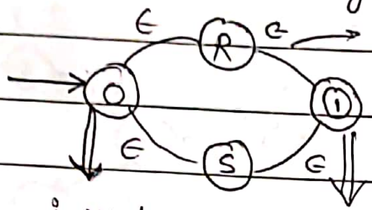


final state

Rule 2: $r = a$



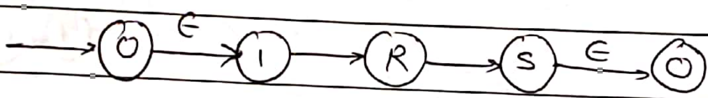
Rule 3: $r = R|S$ (2 RE running parallelly)



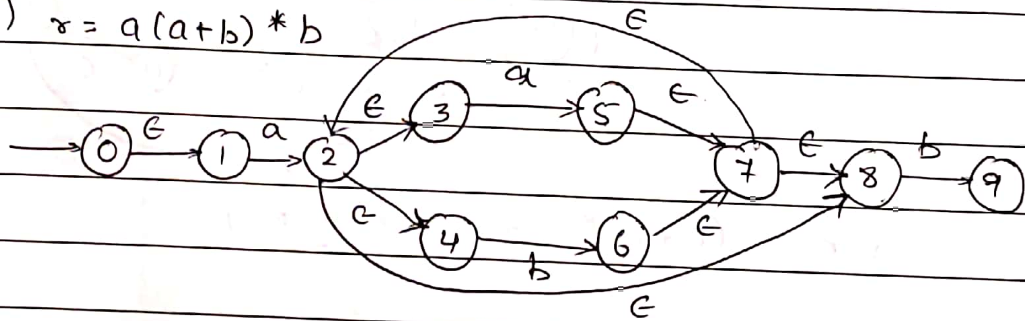
to move away from start state, we don't know what

initial state final state

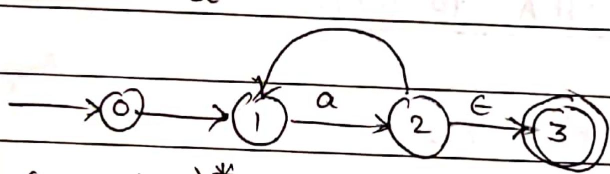
Rule 4: $r = R \cdot S$



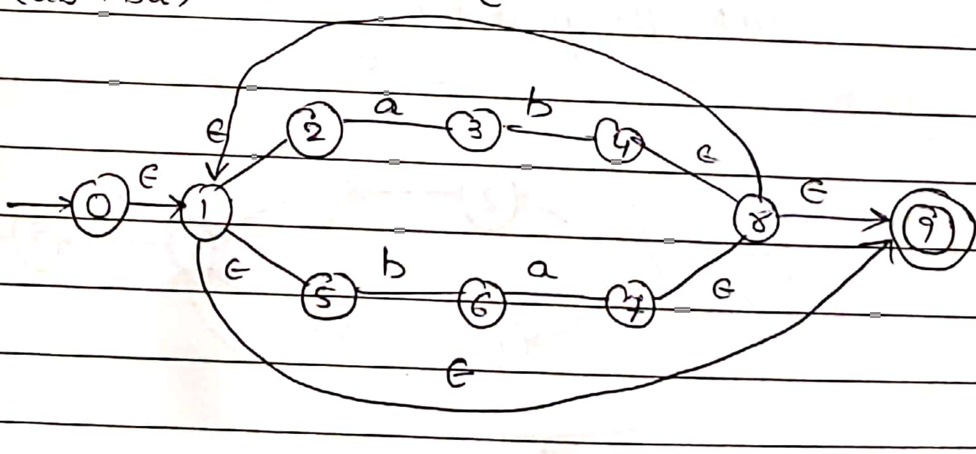
(Q) $r = a(a+b)^*b$



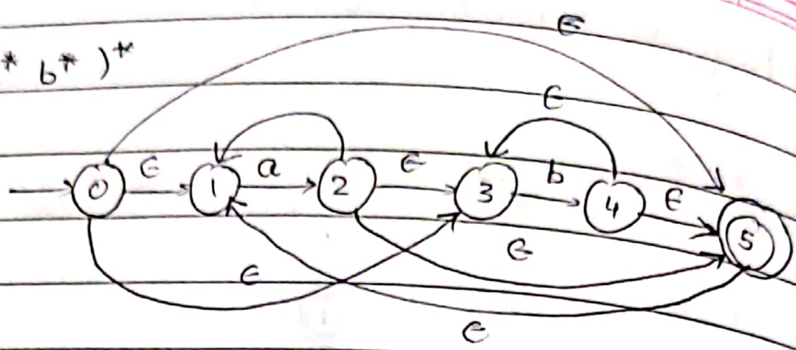
(Q) $r = a.a^* = a^+$



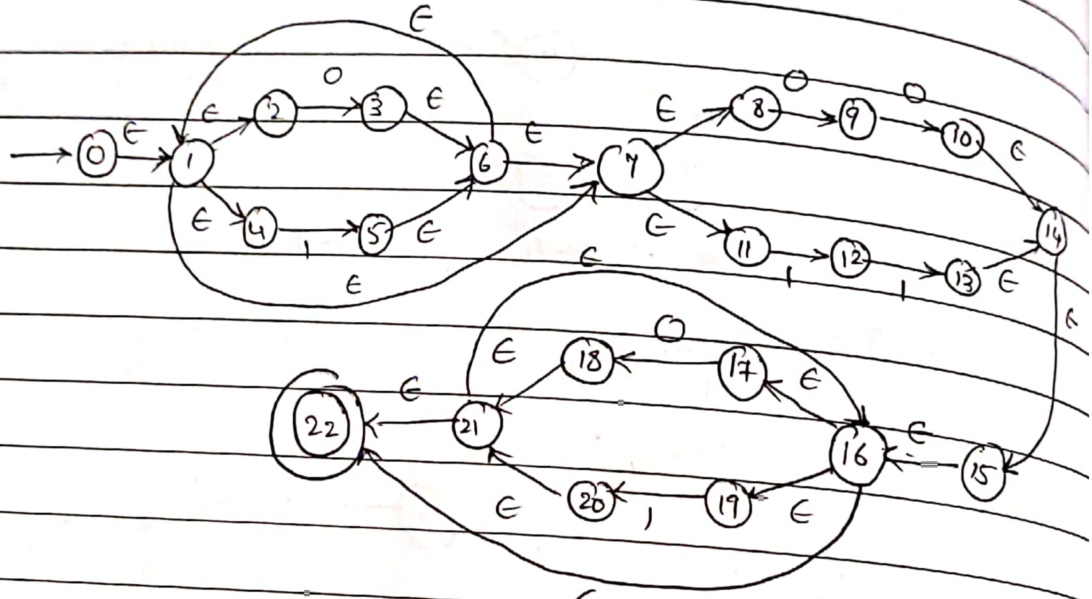
(Q) $r = (ab+ba)^*$



Q) $r = (a^* b^*)^*$



Q) r.e $= (0+1)^* (00+11)(0+1)^*$



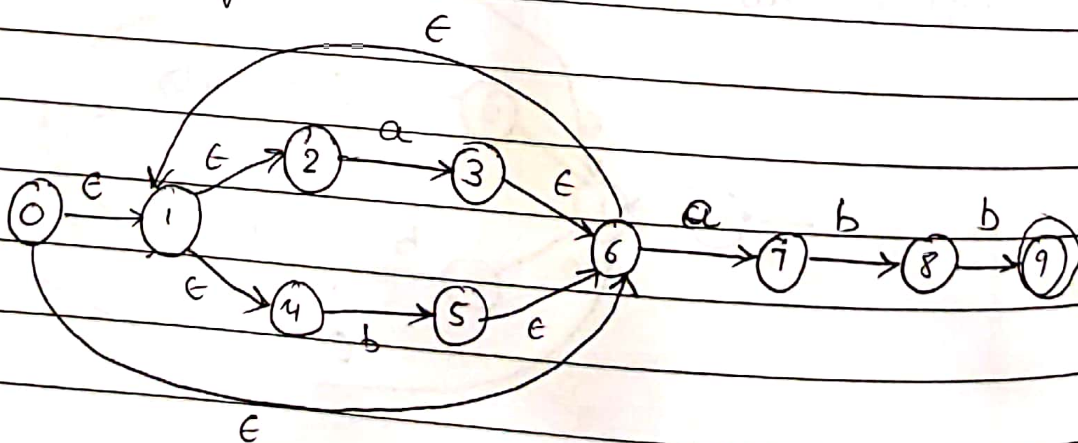
Q) re $= (0+1)^* (0+11)^* 0^* 1$

<u>Convert NFA to DFA</u>

Q1) re $= (a+b)^* abb$

NFA for given R.E

<u>step 1)</u>



step 2]:- Transition table

		E closure (x)	δ(y,a)	δ(y,b)
A	{0}	{0,1,2,4,6}	{3,7} B	{5} C
B	{3,7}	{3,7,6,1,2,4}	{3,7} B	{5,8} D
C	{5}	{5,6,1,2,4}	{3,7} B	{5} C
D	{5,8}	{5,8,6,1,2,4}	{3,7} B	{5,9} E
E	{5,9}*	{5,9,6,1,2,4}	{3,7} B	{5} C

DFA

Minimi

Q \ Σ	a	b
→ A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C

whole sum. Done in next page!

21/2/23

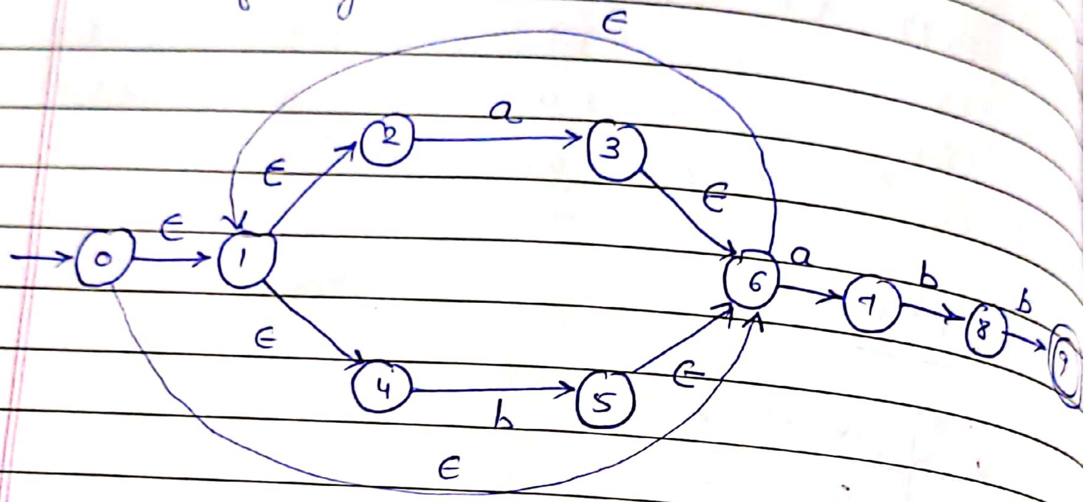
PAGE No.
DATE / /

→ Convert NFA to DFA

eg 1] r.e = $(a+b)^* abb$

→ step 1] :-

NFA for given R.E.



step 2] :- Transition Table.

	X	ε closure (x)	δ(y, a)	δ(y, b)
A	{0}	{0, 1, 2, 4, 6}	{3, 7} B	{5} C
B	{3, 7}	{3, 7, 6, 1, 2, 4}	{3, 7} B	{5, 8} D
C	{5}	{5, 6, 1, 2, 4}	{3, 7} B	{5} C
D	{5, 8}	{5, 8, 6, 1, 2, 4}	{3, 7} B	{5, 9} E
E	{5, 9}*	{5, 9, 6, 1, 2, 4}	{3, 7} B	{5} C

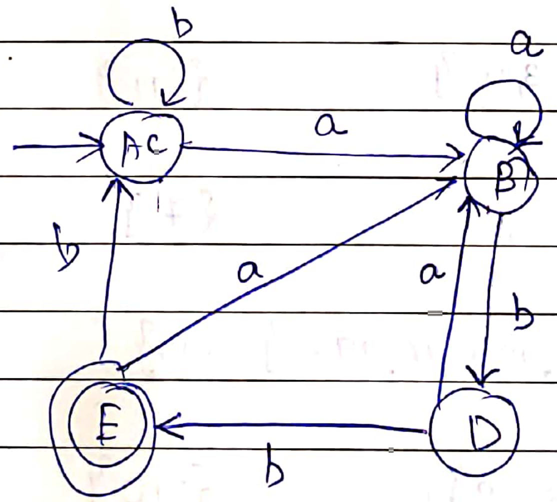
DFA :-

Q \ Σ	a	b
A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C

final & non-final cannot be merged though they have same transition state

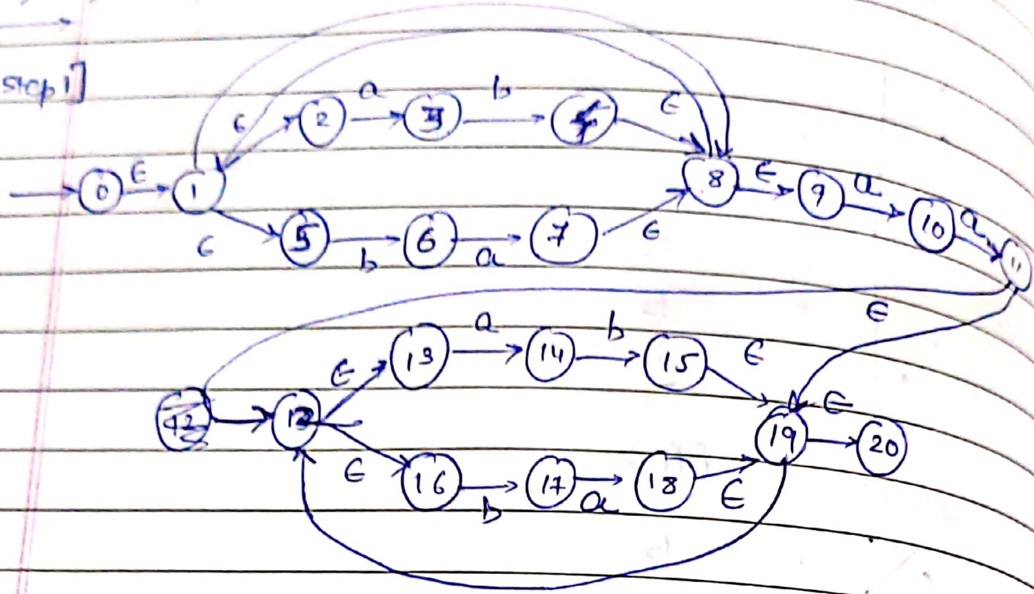
Minimized DFA :

Q \ Σ	a	b
AC	B	AC
B	B	D
D	B	E
E*	B	AC



eg 2] $(ab|ba)^* aa (ab|ba)^*$

step 1]



step 2]

x	ε closure (x)	δ(y,a)	δ(y,b)
{0}	{0,1,2,5,8,9}	{3,10}	{6}
{3,10}	{3,10}	{11}	{4}
{6}	{6}	{7}	{ }
{11}	{11,12,13,16,19,20}	{14}	{17}
{4}	{4,8,9,1,2,5}	{3,10}	{6}
{7}	{7,8,9}	{10}	{6}
{14}	{14}	{ }	{15}
{17}	{17}	{18}	{ }
{10}	{10}	{11}	{ }

note vaparl

PAGE No.
DATE / /

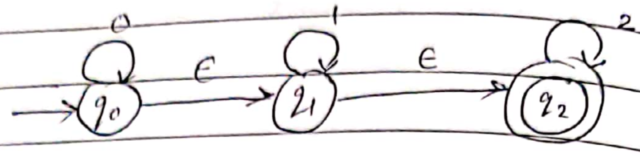
{15}	{15,19}	{ }	{ }
{18}	{18,19}	{ }	{ }
{ }	{ }	{ }	{ }

* • your start state will be same
• input also remains same
• Total no. of state remains same.
• But Transitions changes.

PAGE No.
DATE / /

NFA with epsilon to NFA w/o epsilon

eg 1] :



soln :

NFA 1 = $q_0, \Sigma, Q, \delta, F$

NFA 2 = $q_0, \Sigma, Q, \delta', F'$

step I] :

x	ε closure (x)	δ(y, 0)	δ(y, 1)	δ(y, 2)
$\{q_0\}^*$	$\{q_0, q_1, q_2\}$	$\{q_0\}$	$\{q_1\}$	$\{q_2\}$
$\{q_1\}^*$	$\{q_1, q_2\}$	$\{\ \}$	$\{q_1\}$	$\{q_2\}$
$\{q_2\}^*$	$\{q_2\}$	$\{\ \}$	$\{\ \}$	$\{q_2\}$

$F' = \{q_0, q_1, q_2\}$

step II] :

$\delta(q_0, 0)$

→ Rule :- ε- closure [δ' [ε -closure (x), i/p]]

$\delta'[\ \epsilon\text{-closure}(q_0), 0]$

$= \delta'[\{q_0, q_1, q_2\}, 0]$

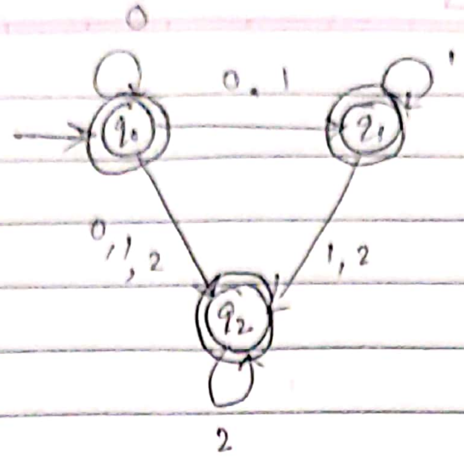
$= \delta'[(q_0, 0) \cup (q_1, 0) \cup (q_2, 0)]$

$= \{q_0\}$

$= \epsilon$ closure $\{q_0\} = \{q_0, q_1, q_2\}$

$(q_0, 0) = \{q_0, q_1, q_2\}$

$(q_0, 1) = \{q_1, q_2\}$



ie. ϵ closure $[\delta'[\epsilon \text{ closure}(q_0), 1]]$

$= \epsilon$ closure $[\delta'\{q_0, q_1, q_2\}, 1]$

$= \epsilon$ closure $[\delta'(q_0, 1) \cup (q_1, 1), (q_2, 1)]$

$= \epsilon$ closure $[\phi \cup q_1 \cup \phi]$

$= \epsilon$ closure $[q_1]$

$= \{q_1, q_2\}$.

can replace q_0 by $\{q_0, q_1, q_2\}$

$Q \backslash \Sigma$	0	1	2
q_0^*	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1^*	$\{\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_2^*	$\{\}$	$\{\}$	$\{q_2\}$
$\{\}$			

NFA

Step 3 :-

			$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
A	$\{q_0\}^*$	$\{q_0, q_1, q_2\}$	$\underbrace{\delta(y, 0)}_{B}$	$\underbrace{\delta(y, 1)}_{C}$	$\underbrace{\delta(y, 2)}_{D}$
B	$\{q_0, q_1, q_2\}$	—			
C	$\{q_1, q_2\}^*$	$\{q_1, q_2\}$	$= \{\}$ E	$\{q_1, q_2\}$ C	$\{q_2\}$ D
D	$\{q_0\}^*$	$\{q_2\}$	$\{\}$ E	$\{\}$ E	$\{q_2\}$ D
E	$\{\}$	$\{\}$	$\{\}$ E	$\{\}$	$\{\}$ E

Step IV] :-

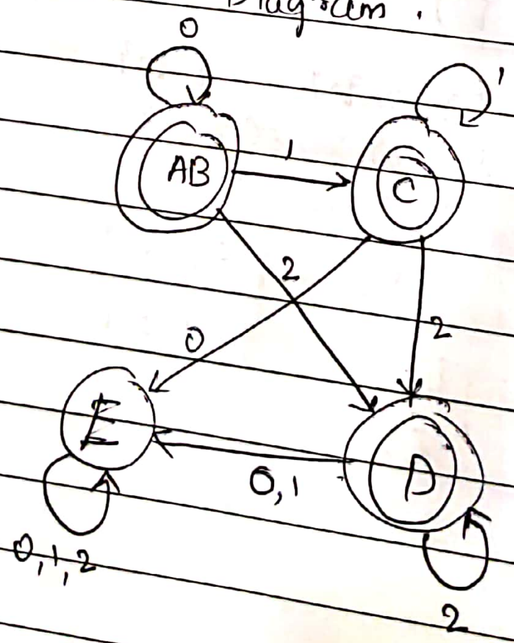
Q \ Σ	0	1	2
→ A*	B	C	D
B*	B	C	D
C*	E	C	D
D*	E	E	D
E	E	E	E

Step V] :-

Min DFA

Q \ Σ	0	1	2
→ AB	AB	C	D
C	E	C	D
D	E	E	D
E	E	E	E

Step VI] :- Diagram.



FA with output :-

Moore ~~Moore~~ (state specific)

Mealy
Transition specific

$Q, \Sigma, q_0, \delta, \lambda$

eg 1] Design a Moore machine to output 'A' if it ends in '101'. Output 'B' if it ends in '110' & output 'C' otherwise.

→

step 1]:- Defn :- $M_1 = Q, \Sigma, q_0, \delta, \lambda, \Delta$

λ → mapping fn

Σ → i/p symbol

Δ → o/p symbol

$\Sigma = \{0, 1\}$

$\Delta = \{A, B, C\}$

step 2] :- Logic :-

q_s → initial state

q_0 → ~~initi~~ ending in 0

q_1 → " '1'

q_{10} → ending in 10

q_{101} → " 101

q_{11} → " 11

q_{110} → " 110

P. T. O.

Step 3] Transition Table :-

Q \ Σ	0	1
→ q_s	q_0	q_1
q_0	q_0	q_1
q_1	q_{10}	q_{11}
q_{10}	q_0	q_{101}
q_{101}	q_{10}	q_{11}
q_{11}	q_{110}	q_{11}
q_{110}	q_0	q_{101}

λ :-

Q \ Δ	
→ q_s	C
q_0	C
q_1	C
q_{10}	C
q_{11}	C
q_{101}	A
q_{110}	B

step 5) :- 10101

$\delta(q_s, 10101) \to C$

$\delta(q_1, 0101) \to C$

$\delta(q_{10}, 101) \to C$

$\delta(q_{101}, 01) \to A$

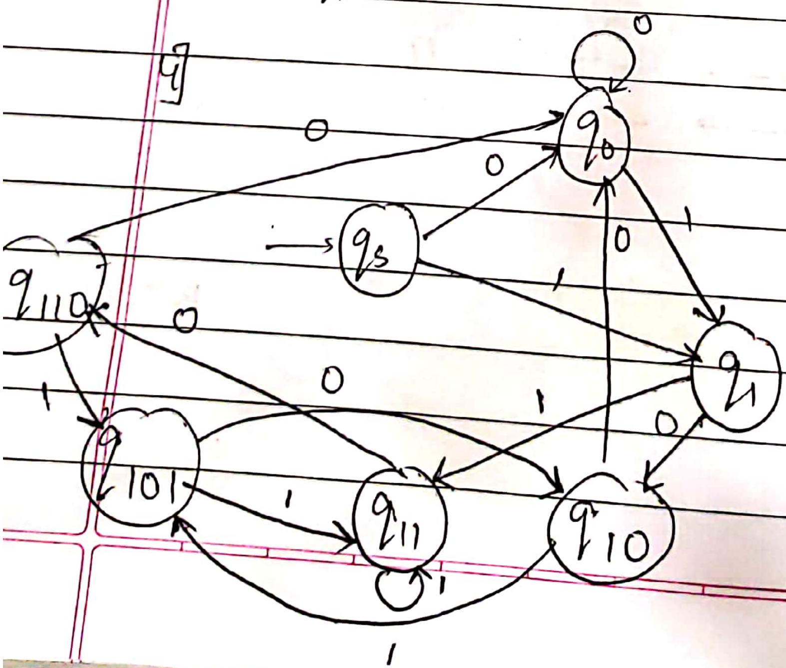
$\delta(q_{10}, 1) \to C$

$\delta(q_{101}) \to A$

CCCACA

String is "n+1"

4]



eg 2] Design a ~~mealy~~ moore machine which determine the residue mod 3 for each binary string over the input $\Sigma = \{0,1\}$ treated as binary integer

eg 3] Design a Moore machine which counts occurence of substring 'aba' in i/p string. $\Sigma = \{a,b\}$.

* Moore M/C

$Q, \Sigma, \delta, q_0, \underbrace{\lambda, \Delta}_{\cancel{F}}$

o/p = state specific

* Mealy m/c

o/p = transition specific

PAGE No.
DATE / /

28/2/23

Q] Design a Mealy machine through o/p 'A' if it ends in 101 else 'B'.

→ step 1] :

$\Delta = \{A, B\}$ $\quad q_0 = \{q_s\}$

$\Sigma = \{0, 1\}$

$Q = \{q_s, q_0, q_1, q_{10}, q_{101}\}$

step 2] : Logic :

q_s → initial state

q_0 → ends with 0

q_1 → — 1

q_{10} → — 10

q_{101} → — 101

Step 3] δ :

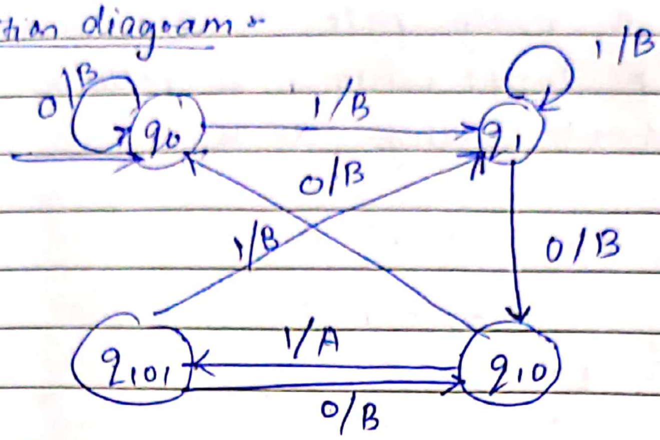
Q \ Σ	0	1
→ q_s	q_0	q_1
q_0	q_0	q_1
q_1	q_{10}	q_1
q_{10}	q_0	q_{101}
q_{101}	q_{10}	q_1

λ :

Q \ Δ	0	1
→ q_s	B	B
q_0	B	B
q_1	B	B
q_{10}	B	A
q_{101}	B	B

→ merge

step 4) Transition diagram :-



Step 5] :- Simulation

$(q_0, 110101)$	~~B~~B	B
$(q_1, 10101)$	B	B
$(q_1, 0101)$	B	B
$(q_{10}, 101)$	B	A
$(q_{101}, 01)$	A	B
$(q_{10}, 1)$	B	A
(q_{101}).	A	

~~BBBBABA~~ ~~BB BB ABA~~

PAGE No.
DATE / /

eg Design a mealy m/c to o/p same characters as i/p except when i is followed by e than e should change to u

$\Sigma = \{a, e, i, o, u\} = \Delta$

~~a e i o u~~

q_s
q_e
q_i
q_o
q_u
q_{ie}

	a	e	i	o	u
q_s	q_a	q_e	q_i	q_o	q_u
q_a	q_a	q_e	q_i	q_o	q_u
q_e	q_a	q_e	q_i	q_o	q_u
q_i	q_a	q_{ie}	q_i	q_o	q_u
q_o	q_a	q_e	q_i	q_o	q_u
q_u	q_a	q_e	q_i	q_o	q_u
q_{ie}	q_a	q_e	q_i	q_o	q_u

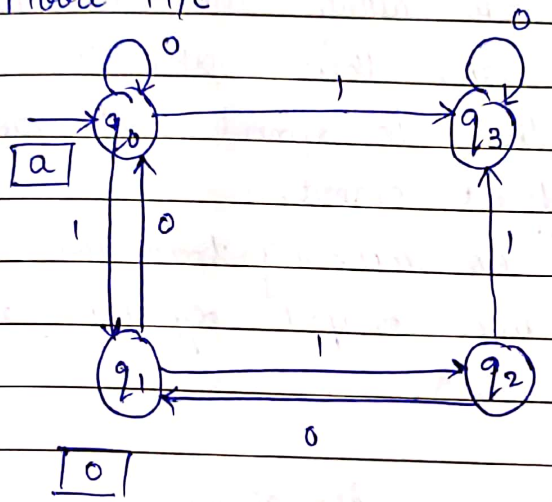
	a	e	i	o	u
q_s	a	e	i	o	u
q_a	a	e	i	o	u
q_e	a	e	i	o	u
q_i	a	u	i	o	u
q_o	a	e	i	o	u
q_u	a	e	i	o	u
q_{ie}	a	e	i	o	u

2/3/23

Conversion Moore to Mealy

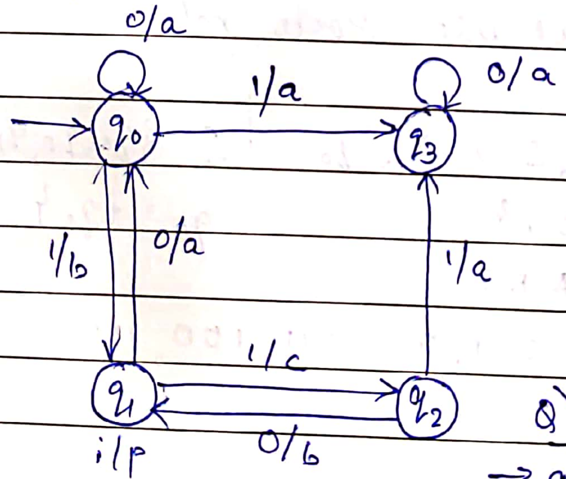
Assign the output symbol of that respective state to all of its incoming edges.

eg:-1) Moore M/c



a	Q \ Σ	0	1	O/P
	q_0			
	q_1			
	q_2			
	q_3			

Mealy M/c



Q \ Σ	0	1	O/P	
	i/p			
$\to q_0$	q_0	q_3	a	a

PAGE No.
DATE / /

Conversion Mealy m/c to Moore m/c

1) If the o/p symbol along with the incoming edges is same then assign that o/p symbol to that respective state.
2) If the o/p symbol along with the incoming edges is not same then split the state as many times as the o/p symbol with each state producing different output.
3) If there are no incoming transitions to a particular state then any output symbol can be assigned.

eg 1]:- Design mealy m/c for given r.e $= (0+1)^* (00+11)$

& convert it into Moore m/c.

step 1] :- $Q, \Sigma, \delta, \lambda, \Delta, q_0$. | $Q = \{q_s, q_0, q_1, q_{11}, q_{00}\}$

$\Sigma = \{0, 1\}$ | $q_0 = \{q_s\}$

A = Accept
R = Reject.

$\Delta = \{A, R\}$

$L = \{00, 11, 000, 011, 100, 111, \ldots\}$

step 2] :- Logic :-

→ q_s : start state

q_0 : ending with '0'

q_1 : ending with '1'

q_{00} : ——— '00'

q_{11} : ——— '11'

! merging is done only with transitions & not output !

PAGE No. DATE / /

Step 3] Transition table :-

δ :-

Q \ Σ	0	1
→ q_s	q_0	q_1
q_0	q_{00}	q_1
q_1	q_0	q_{11}
q_{00}	q_{00}	q_1
q_{11}	q_0	q_{11}

λ :-

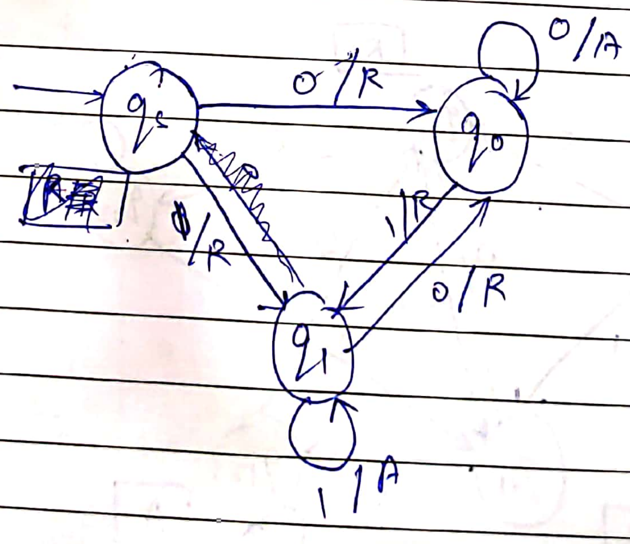
Q \ Δ	0	1
→ q_s	R	R
q_0	A	R
q_1	R	A
q_{00}	A	R
q_{11}	R	A

Merging $\underbrace{(q_0 \ \& \ q_{00})}_{\to q_0}$ AND $\underbrace{(q_1 \ \& \ q_{11})}_{\to q_1}$

Q \ Σ	0	1
→ q_s	q_0	q_1
q_0	q_0	q_1
q_1	q_0	q_1

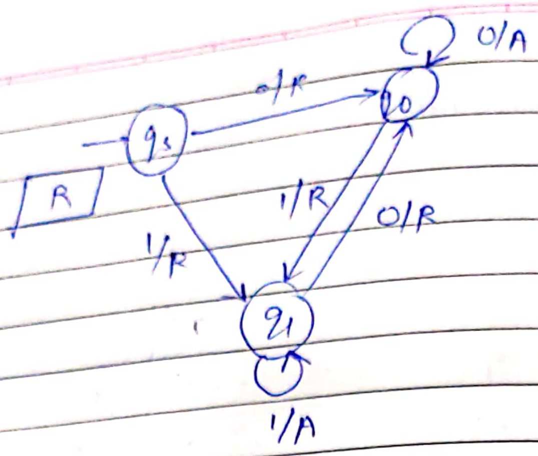
Q \ Δ	0	1
→ q_s	R	R
q_0	A	R
q_1	R	A

Step 4] :-



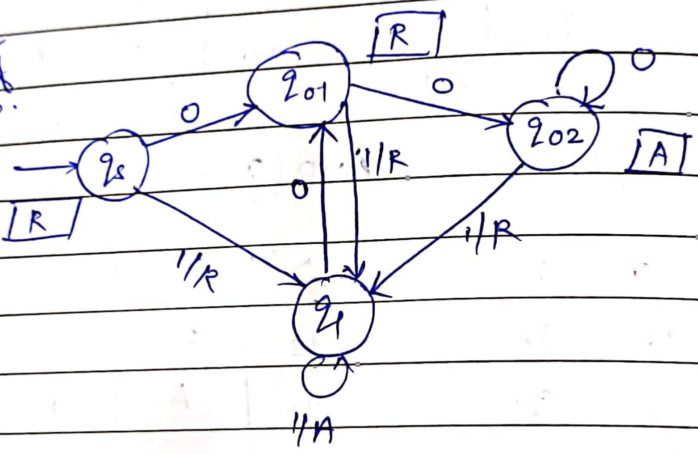
Scanned with CamScanner

ek time पे ek hi state handle kro.

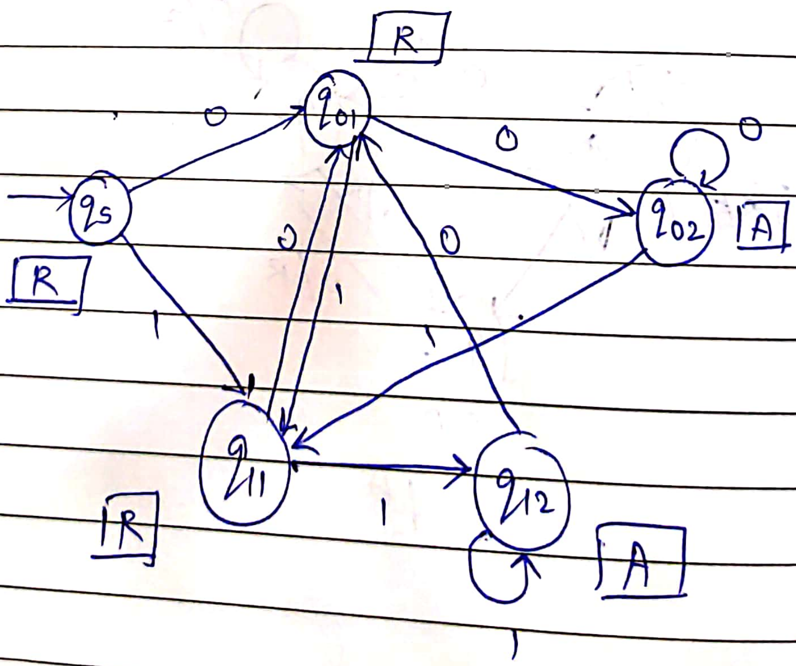


⇓

we cannot q_{01} & q_{02} only bcoz of self loop.



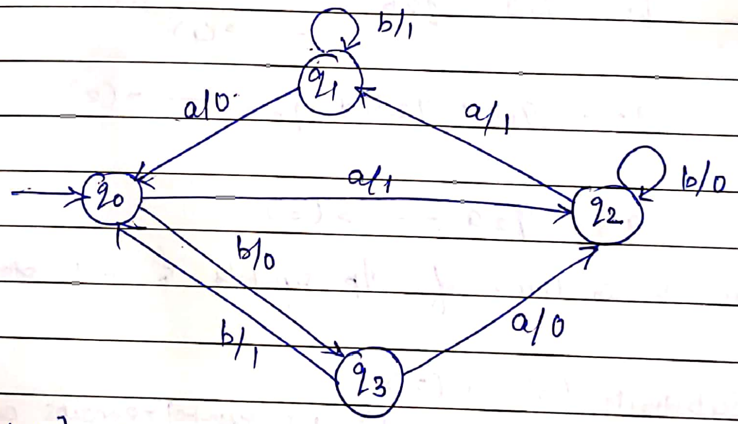
⇓



Moore Machine.

Q \ Σ	0 (i/p)	1	o/p Δ
→ q_s	q_{01}	q_{11}	R
q_{01}	q_{02}	q_{11}	R
q_{02}	q_{02}	q_{11}	A
q_{11}	q_{01}	q_{12}	R
q_{12}	q_{01}	q_{12}	A

eg 2] Convert Mealy to Moore m/c.



step 1] : Transition Table

Q \ Σ	a (i/p)	b	Δ (o/p)	
q_0	q_2	q_3	1	0
q_1	q_0	q_1	0	1
q_2	q_1	q_2	1	0
q_3	q_2	q_0	0	1

$\cancel{P, Q \longrightarrow R.E}$

$\cancel{R = Q + RP}$

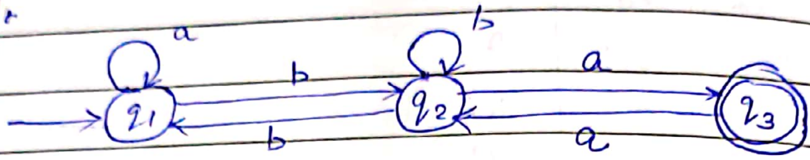
$\cancel{\boxed{Q = RP^*}}$

Arden's theorem

R.E to F.A

1) No ϵ move
2) Only 1 initial state

eg 1)



Construct a reg. exp for above finite automata

$q_1 = q_1 a + q_2 b + \epsilon$ ← (as q_1 is initial state) $\longrightarrow (1)$

$q_2 = q_1 b + q_2 b + q_3 a \longrightarrow (2)$

$q_3 = q_2 a \longrightarrow (3)$

R.E will be in terms of i/p symbols & not states.

Substitute (3) in (2)

*(only state is substituted) i/p symbol remains as it is

$q_2 = q_1 b + q_2 b + q_2 a a$

$q_2 = q_1 b + q_2 (b + aa)$

$R = Q + RP = QP^*$

$\cancel{= RP^*}$

$\cancel{= q_2 (b+aa)^*}$

~~R =~~ $\boxed{R = Q + RP \quad \therefore R = QP^*}$

$q_2 = q_1 b (b + aa)^* \longrightarrow (4)$

Scanned with CamScanner

Substitute (4) in (1)

$$q_1 = q_1 a + q_1 b(b+aa)^* b + \epsilon$$

$$\Rightarrow q_1 = \overset{R}{q_1} \underbrace{(a + b(b+aa)^*) \cdot b}_{P} + \epsilon \rightarrow \varphi$$

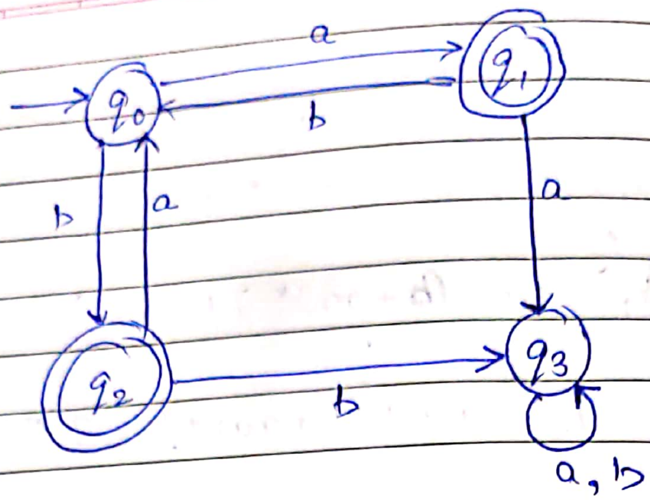
$$q_1 = [(a + b(b+aa)^*) b]^* \longrightarrow (5)$$

$$\cancel{q_1 b(b+aa)^*}$$

Substitute (5) in (4)

$$q_2 = [(a+b(b+aa)^*) b]^* b(b+aa)^* \longrightarrow (6)$$

eg 2]:



$r = r_1 + r_2$

<u>Soln</u>:

$$q_0 = \epsilon + q_1 b + q_2 a \longrightarrow (1)$$

$$q_1 = q_0 a \longrightarrow (2)$$

$$q_2 = q_0 b \longrightarrow (3)$$

$$q_3 = q_1 a + q_2 b + q_3 a + q_3 b \longrightarrow (4)$$

q_3 पे कोई depend नहीं तो उसे आराम करने दो.

$$q_0 = \epsilon + q_0 ab + q_0 \cancel{a} ba$$

$$q_0 = \epsilon + q_0 (ab + ba)$$

$$R = Q + RP$$

$$R = QP^*$$

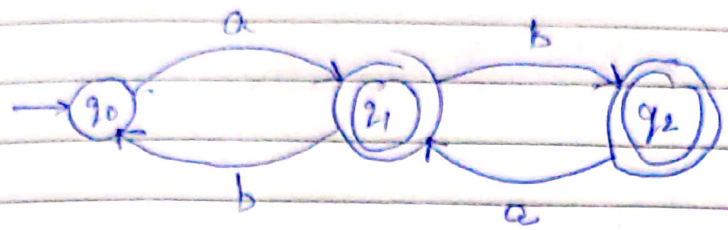
$$\therefore q_0 = \cancel{q_0} (ab+ba)^*$$

$$q_1 = (ab+ba)^* a$$

$$q_2 = (ab+ba)^* b$$

$$r = r_1 + r_2 = q_1 + q_2 = (ab+ba)^* \cancel{ab} (a+b)$$

eg 3]



soln:-

$$q_0 = \epsilon + q_1 b \longrightarrow (1)$$

$$q_1 = q_0 a + q_2 a \longrightarrow (2)$$

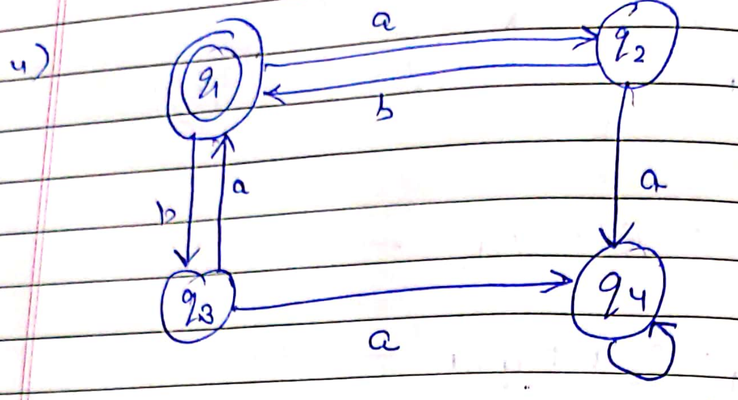
$$q_2 = q_1 b \longrightarrow (3)$$

Substitute (3) in (2),

$$q_1 = q_0 a + q_1 ba$$

PAGE No. DATE / /

4)



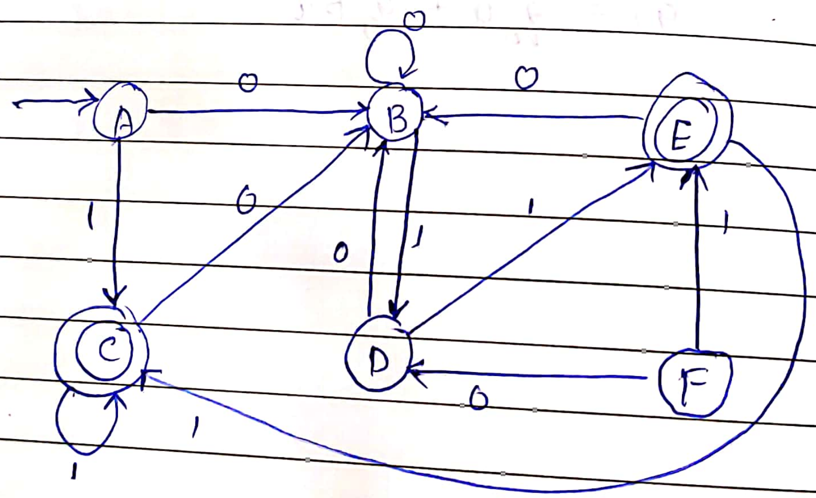
DFA minimization. (No ε) { No epsilon }.

⇓

Box - Method

Myhill - Nerode thm

eg 1]



step 1] Transition table.

Q \ Σ	0	1
→ A	B	C
B	B	D
C*	B	C
D	B	E
E*	B	C
~~F~~	~~D~~	~~E~~

get rid of node which is unreachable to start state.

'F' state is not reachable to the 1st edge.

step 2]:

B	X			
C	X	X		
D		X	X	
E	X	X		X
	A	B	C	D

E & C are final state & here EC are cross boxes. X cross in where we have non-final & final state together.

step 3] → crossing in box table

step 4) :- (A, B)

$\delta(A, 0) = B$

$\delta(B, 0) = B$

$\delta(A, 1) = C$

$\delta(B, 1) = D$ > {C, D}

1st stage.

(2) (A, D) :-

$\delta(A, 0) = B$

$\delta(D, 0) = B$

$\delta(A, 1) = C$

$\delta(D, 1) = E$ > {C, E}

(3) (B, D)

$\delta(B, 0) = B$

$\delta(D, 0) = B$

$\delta(B, 1) = D$

$\delta(D, 1) = E$ > $\delta(D, E)$

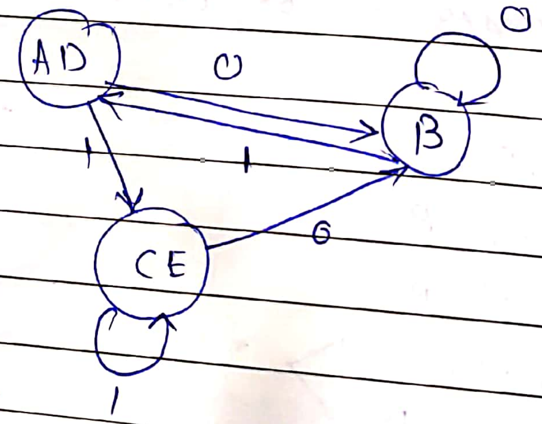
PAGE No.
DATE / /

Blank Boxes gets merged.

∴ AD
B.
CE.

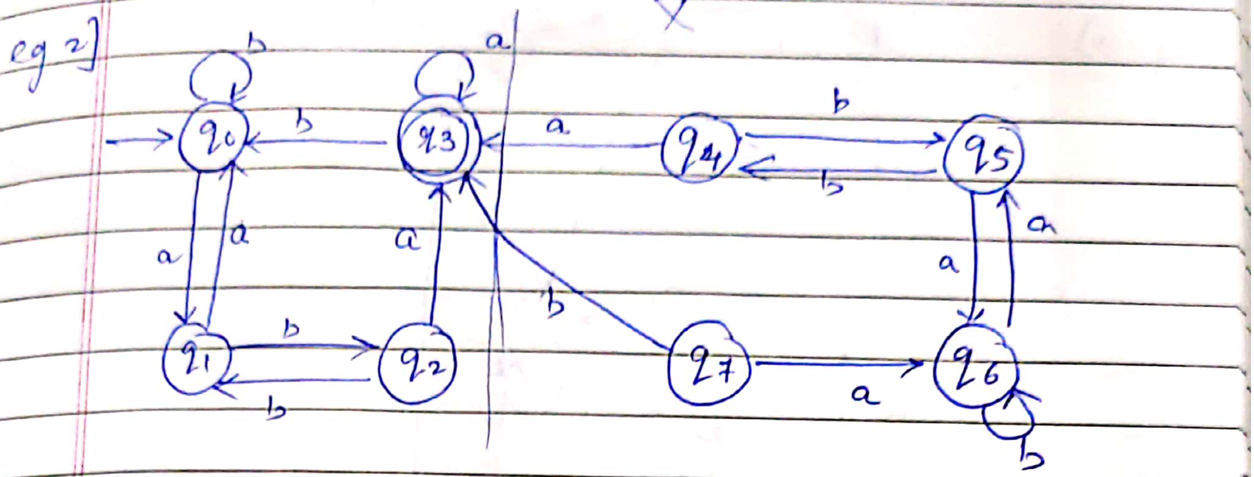
Transition table :

Q \ Σ	0	1
→ AD	B	CE
B	B	AD
CE	B	CE



Scanned with CamScanner

eg 2]



	q_0	q_1	q_2
q_1	X		
q_2	X	X	
q_3	X	X	X

(q_0, q_1)

$(q_0, a) = q_1 \qquad (q_0, b) = q_0$

$(q_1, a) = q_0 \qquad (q_1, b) = q_2$ $\Big\} \; (q_0, q_2)$

$(q_1\, q_2)$

$(q_1, a) = q_0 \qquad (q_1, b) = q_2$

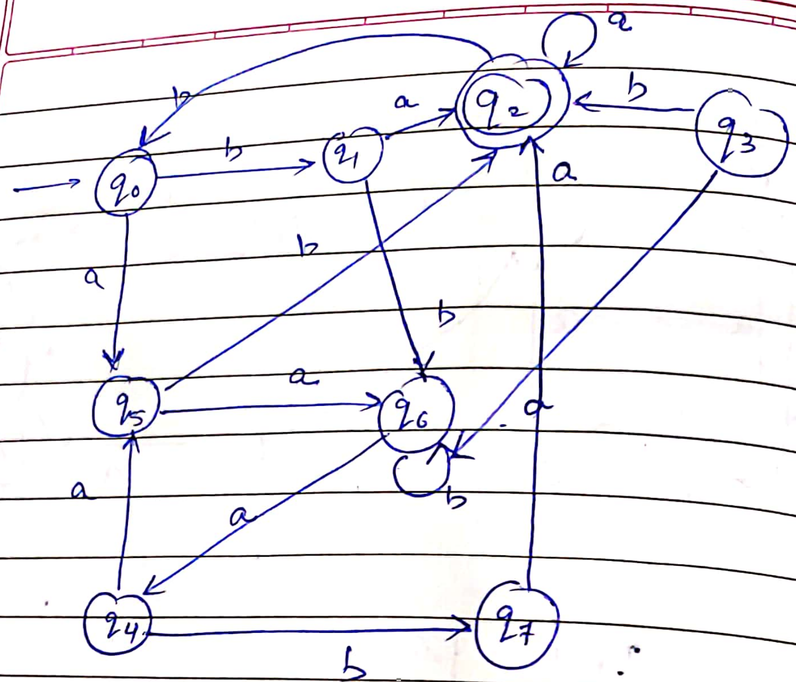
$(q_2, a) = q_3 \qquad (q_2, b) = q_1$ $\Big\} \; (q_2, q_1)$

(q_0, q_2)

$(q_0, a) = q_1 \qquad (q_2, b) = q_0$

$(q_2, a) = q_3 \qquad (q_2, b) = q_1$ $\Big\} \; (q_0, q_1)$

eg :-



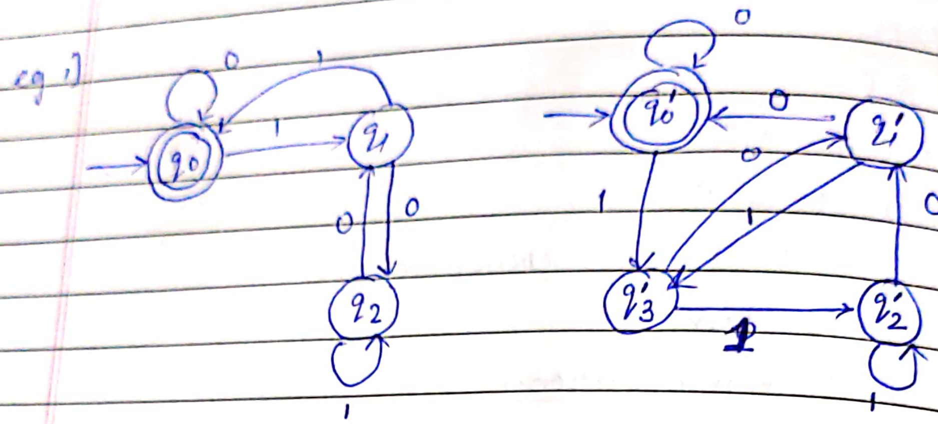
PAGE No.
DATE / /

TT1 Syllabus

1. FSM
2. R.E
3. R.E to NFA
4. NFA to DFA
   (with & w/o epsilon moves)
5. Arden's thm
6. DFA minimization.
7. FSM equivalence
8. CFG : LMD, RMD, parse tree
9. Moore & Mealy m/c's

FSM equivalence

eg 1)



M 1 M2

soln :-

$\Sigma = \{0, 1\}$

$n = 2$

$n+1$ colmn's

Q	0	1
v, v'	v_0, v_0'	v_1, v_1'
(q_0, q_0')	(q_0, q_0')	(q_1, q_3')
(q_1, q_3')	(q_2, q_1')	(q_0, q_2')
(q_2, q_1')	(q_1, q_0')	(q_2, q_3')

As q_0 & q_2' are final & non-final states resp. FSM is not equivalence & we will stop there.

eg 2]

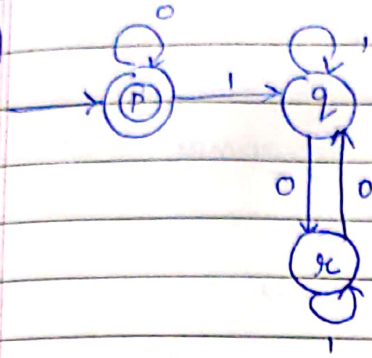


Fig (1)

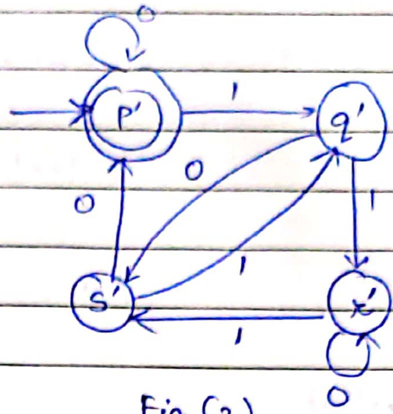


Fig (2)

Soln :-

$\Sigma = \{0, 1\}$

$n = 2$

n+1 columns.

Q	0	1
V, V'	V_0, V_0'	V_1, V_1'
(p, p')	(p, p')	(q, q')
(q, q')	(r, s')	(q, r')
(r, s')	(q, p')	(r, q')
(q, r')	(r, r')	(q, s')

1/3/23

Context-free Language &

Context-free Grammar (CFG) → P...

↓ NLP

~~tools~~

3/3/23

1. Lexical
2. Syntax
3. Semantical

1. Lexical

R.E ↓ R.L } $\xrightarrow{i/p}$ F.A → o/p → tokens.

4 Tuples

(sigma) terminal (Σ)

$G = (V, T, P, S)$

V → variables

P → Production (Δ)

S → Start variable (~q0)

o/p is always in terms of Terminals.

1) Write CFG to generate the following :-

i) set of all strings that start with 'a' over the input a, b.

$L = \{a, aa, ab, aba, \ldots\} = L(G)$

R.E = $a(a+b)^*$

$$\left.\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA \mid bA \mid \epsilon \end{aligned}\right\} \text{Production Rules.}$$

$S = \{\ \}$

$T = \{a, b\}$

$V = \{S, A\}$

ii) set of strings that ends with either '0' or '1'

$T = \{0, 1\}$

$$\left.\begin{aligned} S &\rightarrow A0 \mid A1 \\ A &\rightarrow 0A \mid 1A \mid \epsilon \end{aligned}\right\} P$$

$S = \{S\}$

$V = \{S, A\}$

$L(G) = \{0, 1, 00, 01, 10, 11, \ldots\}$

R.E = $(0+1)^* 0 + (0+1)^* 1$

PAGE No.
DATE / /

iii) start & end with same symbol over $\Sigma = \{0,1,2\}$

$T = \{0,1,2\}$

$S = 0A0 \mid 1A1 \mid 2A2$
$A = 0A \mid 1A \mid 2A \mid \epsilon$ } P

$S = \{S\}$

$V = \{S, A\}$

$L(G) = \{00, 11, 22, 010, 020, \ldots\}$

$R.E = 0(0+1+2)^*0 + 1(0+1+2)^*1 + 2(0+1+2)^*2$

iv) exactly 2a's

$T = \{a, b\}$

$S = AaAaA$
$A = bA \mid \epsilon$ } P
$S = \{S\}$
$V = \{S, A\}$

v) atleast 2a's

$S = AaAaA$
$A = aA \mid bA \mid \epsilon$

vi) atmost 2a's

$S = AaAaA \mid AaA \mid A$
$A = bA \mid \epsilon$

eg 5] *

$L = \{a^n b^n \mid n \geq 1\}$

write down sample strings first to write rules easily

~~$L = \{ab, a^2b^2, a^3b^3, \ldots\}$~~

$L = \{ab, aabb, aaabbb, \ldots\}$

$S \to aSb \mid ab$

eg 6]

$L = \{0^n 1^{2n} \mid n \geq 0\}$

$L = \{\epsilon, 011, 001111, \ldots\}$

$S \to 0S11 \mid \epsilon$

eg 7]

$L = \{a^n b^{n+1} \mid n \geq 1\}$

$L = \{abb, aabbb, aaabbbb, \ldots$

$S =$

Scanned with CamScanner

PAGE No.
DATE / /

CFG
↓
Sentential form :-

Parsers → LMD Left most dirxn
Parsers → RMD Right most dirxn

eg i) $S \rightarrow aAS \mid a$
$A \rightarrow SbA \mid SS \mid ba$
derive using LMD & RMD "aabbaa"

LMD

case ①

$S \rightarrow aAS$
$\xrightarrow{Lmd}$ $aSbAS$ ...... using $A \rightarrow SbA$
$\xrightarrow{Lmd}$ $aabAS$ .... using $S \rightarrow a$
$\xrightarrow{Lmd}$ $aabbaS$ ... using $A \rightarrow ba$
$\xrightarrow{Lmd}$ $aabbaa$ ..... using $S \rightarrow a$

RMD

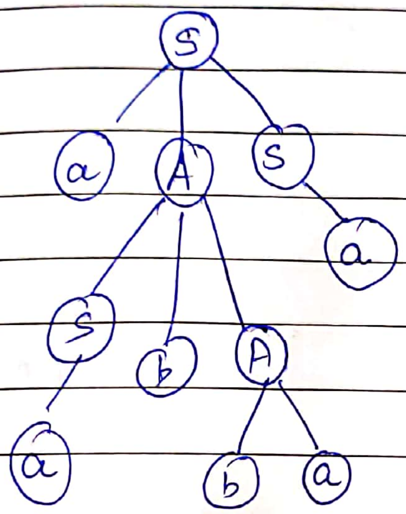
$S \rightarrow aAS$
$\xrightarrow{rmd}$ aAa ... using $S \rightarrow a$
$\xrightarrow{rmd}$ $aSbAa$ $A \rightarrow$
$\xrightarrow{rmd}$ $aSbbaa$ $A \rightarrow ba$
$\xrightarrow{rmd}$ $aabbaa$ $S \rightarrow a$

case ②

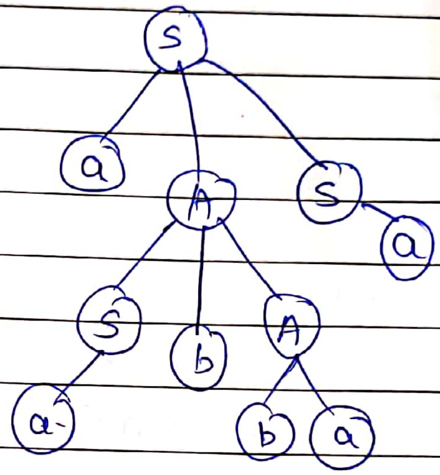
$S \rightarrow aAS$
$\xrightarrow{Lmd}$ $aASS$ ... using $A \rightarrow SS$
$aASa$ using $S \rightarrow a$
X

Derivation / Parse Tree

LMD



RMD



eg 2]
$S \rightarrow aB \mid bA$
$A \rightarrow a \mid aS \mid bAA$
$B \rightarrow b \mid bS \mid aBB$

Derive using LMD & RMD

i) aabb
ii) baab
iii) abb

i) aabb

LMD

RMD

Lmd $S \rightarrow aB$ .. using $B \rightarrow aBB$

Lmd $S \rightarrow aaBB$

Lmd $S \rightarrow aabb$ using $B \rightarrow b$

Rmd, $S \rightarrow aB$

ii) baab

LMD

RMD

$S \rightarrow aB$

$S \rightarrow aB$

iii) abb

LMD

RMD

$S \rightarrow aB$

$S \rightarrow a$

eg 3)

$E \rightarrow E * E$

$E \rightarrow E + E \Rightarrow E \rightarrow E * E \mid E + E \mid id$

$E \rightarrow id$

Derive "id + id * id *

$T = \{id, +, *\}$

LMD

$E \rightarrow E * E$ $\qquad$ $E \rightarrow E + E$

$\xrightarrow{Lmd} E + E * E$ $\qquad$ $\xrightarrow{Lmd} E + E * E$

$\xrightarrow{Lmd} id + id * id$ $\qquad$ $\xrightarrow{Lmd} id + id * id$

Ambiguity → Derived using 2 rules hence no need to do RMD

Scanned with CamScanner

Normal forms 10M
→ CNF
→ GNF

PAGE No.
DATE / /

Reduction of CFG :-

i) elimination of useless variables.
ii) elimination of unit variables
iii) elimination of null variables.

① → elimination of useless variables.

not-reachable — not-generating

C, B — D, A

eg: $S \to AB$
$A \to AA \mid D$ — reachable है पर उसका rule ही नहीं है hence not-generating
$B \to b \mid \epsilon$
$C \to \epsilon$

B is reachable वैसे but A is in loop hence not-reachable

eg] $S \to aSb \mid a \mid \cancel{bAB}$ ⟹ $S \to a$
~~$A \to aA \mid c$~~ — $A \to \ldots$
$B \to \cancel{b} \mid B \mid dB$ — $B \to B \mid dB$

⇓

∴ soln : B is not generating, eliminate rule of B & also $S \to bAB$.
Now, A is not-reachable.
So, eliminate Rule of A
∴ final P: $S \to aSb \mid a$

Scanned with CamScanner