Page No.
Date

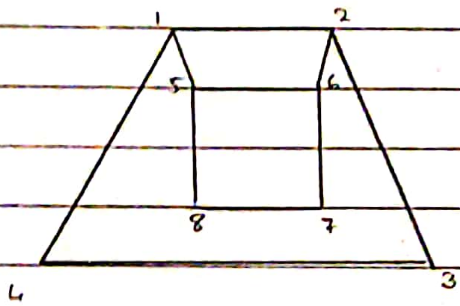
Tutorial 5

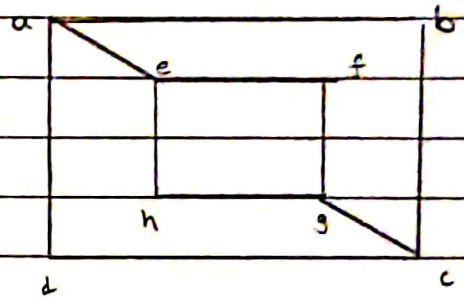
Name: Shasrat Shah
SAP: 60004220126.

Discrete Structure

1 Determine whether the following graph are isomorphic.

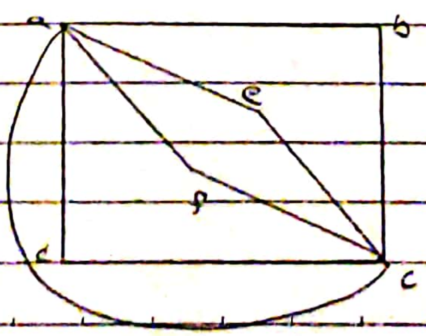
a)

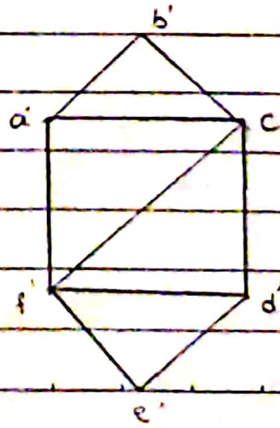




Both the graphs contain 8 vertices and 10 edges.
In graph g1
vertices (1, 2, 5, 6) – 4 vertices have degree 3.
vertices (8, 7, 3, 4) – 4 vertices have degree 2.
In graph g2.
vertices (a, e, g, c) – 4 vertices have degree 3.
vertices (b, d, h, f) – 4 vertices have degree 2.
Also ~~vertex~~ 1 in G1 is adjacent to 2 vertices (2, 5) of degree 3 and 1 vertex of degree 2.
Whereas in graph 2 no vertex follows similar connectivity. Hence G1 & G2 are not isomorphic.

b)





G = 6 vertices 9 edges.
G' = 6 vertices 9 edges.

In G_1
2 vertices (b, d) – degree 4.
2 vertices (a, c) – degree 3.
2 vertices (e, f) – degree 2

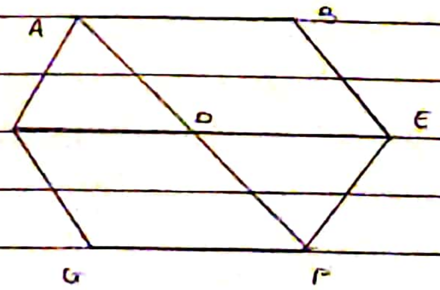
In G_1'
2 vertics (c', f') – degree 4.
2 vertices (d', a') – degree 3.
2 vertices (b', e') – degree 2.

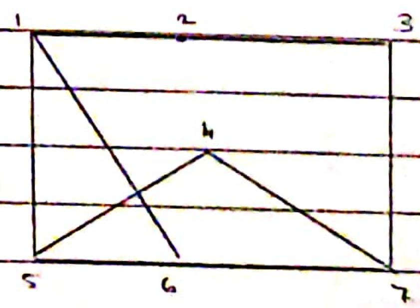
In G_1
4 degree vertex(b) – 2 neighbours (2 degree) (e, f)
– 2 neighbours (3 degree) (a, c)

In G_1'
4 degree vertex(c') – 1 neighbour (2 degree)
2 neighbour (3 degree)
1 neighbour (4 degree)

Similarly is the case for 2nd and 4th degree vertex where the adjacency condition fails.
G & G' are not isomorphic.

c





G — 7 vertices, 9 edges.
G' — 7 vertices, 9 edges.

iii) In G.

1 - 3 degree vertex (D)

3 - 3 degree vertex (A, E, F)

3 - 2 degree vertex (B, C, G)

In G'

4 - 3 degree vertices (1, 5, 6, 7)

3 - 2 degree vertices (2, 3, 4)

iv) In G.

D has 3 - 3 degree neighbours

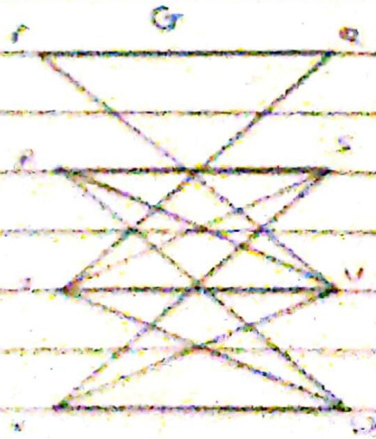
In G,

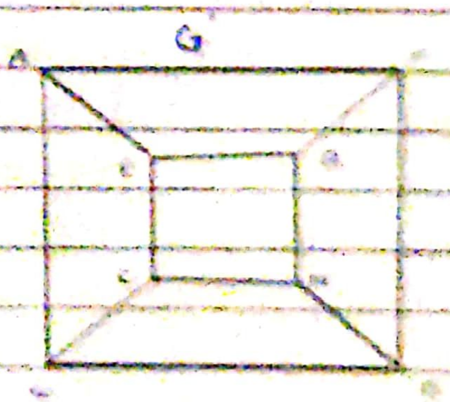
6 has 3 - 3 degree edges.

In G'

6 has 3 - 3 degree edges.

All the vertices are perfectly adjacent hence G & G' are isomorphic.



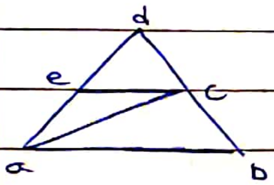


In G — 8 vertices, 12 edges.
In G' — 8 vertices, 12 edges.

In G - All 8 vertices - 3 degree vertices.
In G' - All 8 vertices - 3 degree vertices.
Since all vertices in graph G & G' are 3 degree therefore 1, 3 degree vertex will have 3, 3 edges as neighbours. Hence this is true for all vertices so the graphs are isomorphic.

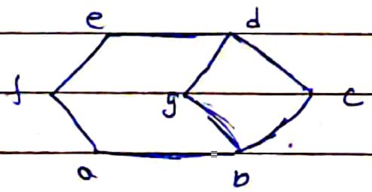
2 Determine the graphs below have a hamiltonian circuit Eulerian circuit. If so find them.

A]



If we consider two non adjacent sides i.e. e & b then the sum of their degrees $(3+2) = 5$ which is equal to the number of vertices. So hamiltonian circuit exists.
Hamiltonian circuit - a b c d e a.
Here the vertices a and e have odd degree of hence the eulerian circuit doesnt exist.

B]

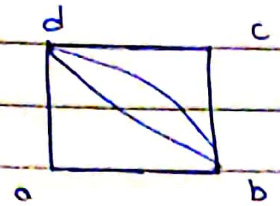


As degree of each vertex is less than $7/2 = 3.5$ Rule 2 fails.
There are no two non-adjacent vertex such that sum of their degree is greater than no. of vertices. Rule 1 fails.
Also the no. of edges $m = 8$
$\frac{1}{2}\left(7^2 - (3 \times 7) + 6\right) = 17$.
As no. of edges $< 17 \rightarrow$ Rule 3 fails
Hamiltonian circuit does not exist.
As B, D have odd degree of ~~ver~~ neighbour, so euler circuit does not exist.

Vertex	Degree
a	2
b	3
c	2
d	3
e	2
f	2
g	2

Page No.	
Date	

c]



No. of vertices = 4.

As each vertex has degree $4 = 4/2$

$\therefore$ 2 hence rule two satisfied.

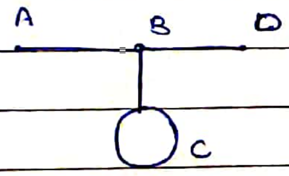
Hamiltonian circuit exists.

Hamiltonian circuit - a b c d a.

Vertex	Degree
a	4
b	2
c	4
d	2

As the above graph we can say that it is connected and each vertex has even degree so eulers circuit exists.

D]

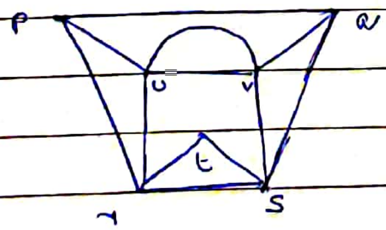


Vertex	Degree
A	1
B	3
C	3
D	1

Visually, we can see that it is impossible to make a circuit without repeating circuits, so here hamiltonian circuit doesnt exist. Also all vertices have degree $4, 2$ (A, D) so rule 2 fails. so no hamiltonian circuit. As all vertices A, B, C, D have all odd degree so eulerian circuit not possible.

3

A]

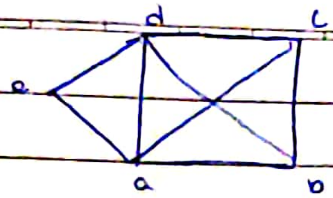


Vertex	Degree
p	3
q	3
r	4
s	4
t	2
u	4
v	4

It has 2 vertices with odd degree so eulerian path is present. Hamiltonian - ~~prsq~~ prtsquv

Eulerian circuit - pqsrpuvstsuvq.

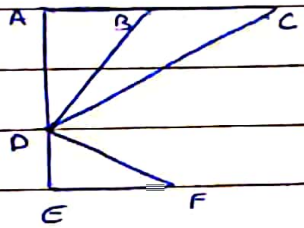
Q]



Vertex	degree
a	4
b	3
c	2
d	4
e	3

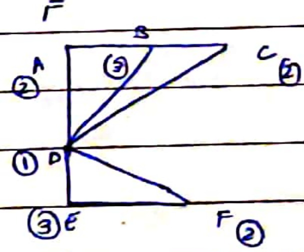
It has 2 vertices of odd degree so euler path
e d c a b d a e f
Hamiltonian : a b c d e

4
A]

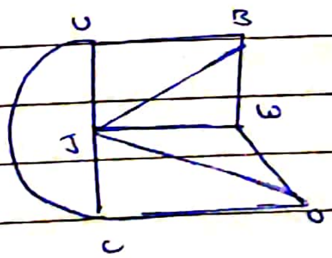


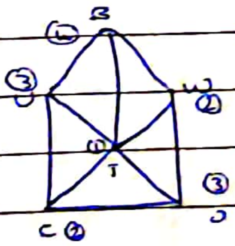
Vertex	Degree
A	2
B	3
C	2
D	5
E	2
F	2

In descending order DBAEFC
Assign 1 to D
Asingn 2 to FAC
Assign 3 to EB.
Graph is 3 colourable.
$(G)\chi = 3$
chromatic number.



B]





vertex	degree
B	3
J	5
U	3
W	3
C	3
O	3

∴ Assign 1 to J
Assign 2 to CW
Assign 3 to OU
Assign 4 to B.

Scanned with CamScanner

Page No.
Date

Tutorial 6

Name : Shasvat Shah
SAP : 60004220126

Discrete Structure.

1 Find the generating function for the following sequence
1, 2, 3, 4, 5, 6 ....

→ Using the sequence 1, 2, 3, 4, 5, 6 ... the above expression becomes (using generating function)

$$f(x) = \sum a x^n$$

$$f(x) = 1x^0 + 2x^1 + 3x^2 + 4x^3 + 5x^4 + \dots \quad (1)$$

Multiply eq 1 by x

$$x f(x) = x + 2x^2 + 3x^3 + 4x^3 \dots$$

Subtracting equation (1) from equation (2),

$$f(x) = (x-1)$$

$$= [x + 2x^2 + 3x^3 + 4x^4 + 5x^5 \dots] - [1 + 2x + 3x^2 + 4x^3 + 5x^4 \dots]$$

$$= -[1 + x + x^2 + \dots]$$

$$f(x) = (1-x)$$

$$= 1 + x + x^2 + x^3 \dots$$

$$= \frac{1}{1-x} \quad \text{for} \quad |x| \leq 1$$

$$f(x) = \frac{1}{(1-x)^2} \quad \text{for} \quad |x| \leq 1$$

Page No.	
Date	

2 Solve the recursive relation $a_n = 3a_{n-1} + 2 \quad n \geq 1$

$a_0 = 1$ using generating function.

Consider $g(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_r x^r$

$\therefore 3g(x) = 3a_0 + 3a_1 x + 3a_2 x^2 + \dots 3a_{r-1} x^r$

we know,

$$\frac{2}{1-x} = 2 + 2x + 2x^2 + \dots - 2x^r + \dots$$

Subtract the last 2 series from the 1st two noting that $1 = a_0$, we get.

$$a(x) - 3x\, g(x) - \frac{2}{1-x} = (1-2) + (a_1 - 3a_0 - 2)x + (a_2 - 3a_1 - 2)x^2 + \dots$$

Since $a_0 = 1$.

$$a_r - 3a_{r-1} - 2 = 0$$

each bracket on the right except the first is 0.

Here $(1-3x)\, g(x) = \frac{-1+2}{1-x} = \frac{-1+x+2}{1-x} = \frac{1+x}{1-x}$

$$a(x) = \frac{1+x}{(1-x)(1-3x)} = \frac{2}{1-3x} - \frac{1}{1-x} \quad \therefore \text{By partial fraction}$$

$$= 2\left[1 + (3x) + [3(x)]^2 + \dots\right] - \left[1 + x + x^2 + \dots\right]$$

$$= 2 \sum_{r=0}^{\infty} 3^r a_r - \sum_{r=0}^{\infty} x^r = \sum_{r=0}^{\infty} \left[2 \times (3^r - 1)\right] x^r$$

But $a(x) = \sum a_r x^2 = a_n = 2 \cdot 3^r - 1$

Inorder to make use of $a_n - 3a_n - 1 - 2 = 0$ we multiply $a(x)$ by 1, $a(x)$ by $(3x)$ & 2 by $\frac{1}{1-x}$ and subtract as above.

Scanned with CamScanner

Q3 $a_n - 5a_{n-1} + 6a_{n-2} = 2^n$

4 Solve using recurrence relation,

$a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$, $a_0 = 5$

$a_1 = -9$

$a_2 = 15$

$a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$

Solution is $a_n = r^n$

$r^n = -3(r^{n-1} + r^{n-2}) - r^{n-3}$

$r^n = -3r^{n-1} - 3r^{n-2} - r^{n-3}$

$r^n + 3r^{n-1} + 3r^{n+2} + r^{n-3} = 0$

$r^{n-3}[r^2 + 3r^2 + 3r + 1] = 0$

$r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0$

$r = (-1)$

$a_n = (C_1 + C_2 n + C_3 n^2)(-1)^n$

$a_0 = 5$ $a_1 = -9$ $a_2 = 15$

we get.

$C_1 = 5$ $C_2 = 3$ $C_3 = 1$

$a_n = (5 + 3n + n^2)(-1)^n$

5 $G(x) = \frac{1}{3 - 6x} = \frac{1}{3(1-2x)}$

Now,

$G(x) = \frac{1}{3}(1-2x)^{-1}$

$= \frac{1}{3}(1 + 2x + 4x^2 + 8x^3 + 16x^4 \ldots)$

Scanned with CamScanner

Page No.
Date

$5 \times \frac{1}{3} \times (1, 2, 4, 8, 16)$ time.

$(1 + x + x^2 + x^3 \ldots)$

Seauence is $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3} \ldots$

2 $\frac{x}{1-5x+6x^2} = \frac{x}{(1-3x-2x+6x^2)}$

$= \frac{x}{1(1-3x) - 2x(1-3x)}$

$= x/(1-2x)(1-3x)$

$= \frac{3x - 1 - 2x + 1}{\cancel{3x-1}\ (1-3x)(1-2x)}$

$= \frac{1}{2x-1} - \frac{1}{3x-1}$

$\frac{x}{1-5x+6x^2} = \frac{-1}{(1-2x)} + \frac{1}{(1-3x)}$

In $(1-x)^{-1}$ put $x = -3x$
we get.

$\frac{x}{1-5x+6x^2} = 1 + 3x + 9x^2 + 27x^3 \ldots) - (1+2x+4x^2 + 8x^3 + \ldots)$

$= x + 5x^2 + 9x^3 \ldots$

Seauence is : $(1, 5, 9, \ldots)$

Scanned with CamScanner

Page No.
Date

Tutorial 7

Name: Shasvat Shah
SAP: 60004220126

Discrete Structures.

Using pigeonhole principle show that.

i) In any room of people who have been doing some handshaking, there will atleast be two people who have shaken the hands the same number of times.

→ Lets assume there are n members in a room.
Then no. of handshakes possible can be 0, 1, 2, 3 ... n-1
But if there is a person with (n-1) handshakes, there cannot be a person with zero handshakes.
∴
There are (n-1) pigeonholes (possibilities) and n pigeons.
There should be atleast two people with same number of handshake.

ii) A bag contains 10 marbles (red), 10 white marbles and 10 blue marbles. What is the minimum number of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of the same colour.

→ In the current scenario after removing ~~one~~ nine balls, they are 3 red, 3 blue and 3 white balls.
The 10th ball would be the marble with 4 of the same colour. Hence minimum of 10 balls should be removed inorder to get 4 balls of one colour.

2 There are 100 people at a party. Assume that if person A knows person B, then B knows person A. Prove that there are atleast two people at the party who know the same no. of people.

→ Total people in the party are 100.
So there can be 0, 1, 2, 3 ..., n-1 people known a person.
If a person knows 0 people then there can be a person with (n-1) people knowing him.
Therefore there are (n-1) possibilities or pigeon holes, for n pigeons (people).
∴ There must be two people knowing the same number of people.

3 A physician testing a new medication instruct a test patient to take 48 pills over a 30-day period. The patient is at liberty to distribute the pills however he likes over the period as long as he takes atleast one pill a day and finishes all 48 pills by the end of 30 days. Prove that no matter how the patient decides to arrange things, there will be some strech of consecutive days in which the total no. of pills taken is 11.

→ Let P_i be the total number of pills taken by the end of ith days. Now if the patient decides to take one pill each for atleast 11 days, then the condition holds true. But considering the patient takes pills in the following sequence

Page No.
Date

$P_1 < P_2 < P_3 \ldots < P_{30}$ here $P_{30} = 48$

$P_1 < P_2 < P_3 \ldots < P_{30} = 48$ —— ①

Adding 11 to each no. of sequence

$P_{1+11} < P_{2+11} < P_{3+11} \ldots < P_{30} = 59$ — ②

There are 30 no's each in sequence 48 & 59

Hence there are 60 no.s (pigeons) all less than equal to 59 (pigeon holes)

Hence by the pigeon hole principle, atleast two of these no.s must be equal (which violates the condition of strictly increasing sequence). Hence there is one such $P_i = P_j + 11$ $(i > j)$ $\therefore$ $P_i - P_j = 11$ ... which implies that there are exactly 11 pills taken in the consecutive days $j+1, j+2 \ldots i$.

Hence its proved that there will always be strech of consecutive days in which the total number of pills taken is 11.

4 Use Mathematical Induction to prove:

i) $1 + 5 + 9 + \ldots + (4n-3) = n(2n-1)$

Let P_n be the predicate.

$n_0 = 1$

Show that $P(1)$ is true.

$P(1) = 1(2(1) - 1) = 1.$

$\therefore$ It is true.

$\therefore$ Induction step.

Now for $\forall k \geq 1$ and if $P(k)$ is true then $P(k+1)$ should be also true.

$P(k) = 1 + 5 + 9 + \ldots + 4k - 3 = k(2k-1)$

$\therefore$ $P(k+1) = (k+1)(2(k+1) - 1)$

Page No.
Date

$= (k+1)(2k+2-1)$
$= (k+1)(2k+1)$ —— (11)

$P(k+1) = \underbrace{1+5+9+\dots+4k-3}_{P(k)} + [4(k+1)-3]$
$= k(2k-1) + 4(k+1) - 3$
$= k(2k-1) + 4k+4-3$
$= (2k-1)k + (4k+1)$
$= 2k^2 + 3k + 1$
$= 2k^2 + k + 2k + 1$
$= k(2k+1) + 1(2k+1)$
$= (k+1)(2k+1)$ — 3

Since RHS of (2) and (3) are equal $P(k+1)$ is also true $\forall k \geq 1$
By principle of Mathematical Induction.

ii) $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$

Let $P(k)$ be the predicate
$n_0 = 1$ here

Base Step
$P(n_0) = 2$
$\therefore P(n_0) = 1(3(1)+1)/2$
$= 4/2$
$= 2$
$\therefore P(n_0) =$ true.

Induction Step.
For all $k > 1$ $P(k+1) \rightarrow P(k)$ should be true.
$P(k) = 2+5+8+\dots+(3k-1) = \dfrac{k(3k+1)}{2}$

Scanned with CamScanner

Page No.
Date

$$P(k+1) = \frac{(k+1)(3(k+1)+1)}{2}$$

$$P(k+1) = \frac{(k+1)(3k+3+1)}{2}$$

$$P(k+1) = \frac{(k+1)(3k+4)}{2} \quad \text{--- ③}$$

Also

$$P(k+1) = \underbrace{2+5+8+\dots+(3k-1)}_{P(k)} + 3(k+1)-1$$

= from 2 we can write,

$$P(k+1) = \frac{k(3k+1)}{2} + 3(k+1) - 1$$

$$= \frac{k(3k+1) + 2(3k+2)}{2}$$

$$= \frac{3k^2 + k + 6k + 4}{2}$$

$$= \frac{(k+1)(3k+4)}{2}$$

$$= P(k+1)$$

By Mathematical Induction this predicate is true.

Page No.
Date

5 Use mathematical induction to prove that $1^3 + 2^3 + 3^3 + \dots + n^3 = n^2(n+1)^2/4$ for all positive integer n.

$n \geq 1$.

Base step.

Let $P(k)$ be the predicate.

$n_0 = 1$.

we must show that $P(1) = 1^3$

& substitute 1.

$$P(1) = \frac{1^2(1+1)^2}{4} = \frac{1(2)^2}{4} = 1$$

Induction step.

Now for $\forall k \geq 1$ & $P(k)$ is true then $P(k+1)$ should also be true

$$P(k) = 1^3 + 2^3 + 3^3 \dots k^3 = k^2(k+1)^2/4 \quad \text{— (H)}$$

$$P(k+1) = (k+1)^2 \left((k+1)+1\right)^2$$

$$= \frac{(k+1)^2 (k+2)}{k} \quad \dots \text{— } 3$$

Also

$$P(k+1) = 1^3 + 2^3 + \dots + k^3 + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= (k+1)^2 \left[\frac{k^2}{4} + (k+1)\right]$$

$$= (k+1)^2 \left[\frac{k^2}{4} + k+1\right]$$

$$= (k+1)^2 \left[k^2 + 4k + 4\right]$$

Scanned with CamScanner

$$\frac{(k+1)^2 \; (k+2)^2}{4} \quad \dots \text{ (6)}$$

The predicate is true $\forall k \geq 1$
By principle of mathematical Induction.

6 Prove for any positive integer number n, $n^3 + 2n$ is divisible by 3.

$n^3 + 2n$ is divisible by 3 $\forall n \geq 1$

Base step :

Show that it is true for $n = 1$

for $n = 1$ $\quad n^3 + 2n = 1^3 + 2(1) = 1 + 2 = 3$

at $n = 1$ it is divisible by 3,

for $n = k$,

We assume that $k^3 + 2k$ is divisible by 3.

$\therefore k^3 + 2k = 3m.$

Now for $(k+1)$

$n = k + 1$

$$(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + 2(k+1))$$
$$= k^3 + 3k^2 + 3k + 1 + 2k + 2$$
$$= 3m + 3(k^2 + k + 1)$$
$$= 3(m + k^2 + k + 1)$$

$\therefore$ By mathematical Induction, $n^3 + 2n$ is divisibly by 3.

Page No.
Date

Tutorial 8

Name : Shasvat Shah

SAP : 60004220126

Discrete Structure

1 A tyre manufacturing company kept a record of the distance covered by a tyre needed to be replaced. The table shows the result of 1000 cases,

Distance in km	< 4000	4000 - 9000	9001 - 14000	>14000
Frequency	20	210	325	445

If a tyre is bought from the company what is the probability

i) It has to be substituted before 4000 km is covered.

The total no. of trials covered = 1000.

Frequency of vehicles whose tyres have to be replaced be for 4000 km $(E_1) = 20$. Probability that tyre will be substituted before 4000km $= \frac{E_1}{n}$

$= \frac{20}{1000} = 0.02$.

ii) It will last more than 9000 km

Frequency of tyre that last more than 9000km

= Frequency of tyre that last from 9000 to 14000km

+ frequency of tyre that last more than 14000 km

= 325 + 445

= 770

Probability that a tyre will last more than 9000 km $\frac{E_2}{n} = \frac{770}{1000} = 0.77$.

iii) It hast to be replaced after 4000km & 14000 km

Page No	
Date	

P (tyre has to be replaced after it has covered distance ranging between 4000 to 14000

= frequency of whose tyres are replaced between 4000 to 9000 km + Frequency of vehicle whose tyres are replaced between 9001 to 14000 km.

$= \frac{210 + 325}{1000} = \frac{535}{1000} = 0.535$

2 Thes percentage of marks obtained by a student in the monthly test are given below.

test	1	2	3	4	5
percentage	69	71	73	68	74

Based on the above table, find the probability of students getting more than 70% in a test

→

The test in which students get more than 70% are test 2, test 3 and test 5, Total no. of tests conducted (N) = 5.

No. of test in which students score more than 70% marks = Test 2, Test 3, Test 5 = 3 tests

Probability that the student gets more than 70% marks $= \frac{3}{\text{total no. of tests}} = \frac{3}{5} = 0.6$.

3 One card is drawn from a deck of 52 cards, well shuffled, calculate the probability that the card will be ~~ance~~. an ace, not an ace.

i) $\frac{4}{52} = \frac{1}{3} = \frac{\text{no. of aces}}{\text{no. of cards}}$

Page No.
Date

ii) Not an ace.

$$\frac{48}{52} = \frac{12}{13} = \frac{\text{not an ace}}{\text{no. of cards}}.$$

4 A bag contains 4 balls. Two balls are drawn without replacement and are found to be blue. What is the probability that all balls drawn are blue.

Total Balls in the bag = N = 4.

Of the remaining two balls we have 3 possibilities

E_1 — Both balls are blue.

E_2 — One ball is blue.

E_3 — None of the balls are blue.

All the events are equally likely and mutually exclusive so there are equal chances that any of the three events well occur.

$$P(E_1) + P(E_2) + P(E_3) = 1$$

Since equal chances to occur.

$$P(E_1) = P(E_2) = P(E_3)$$

$$\therefore P(E_1) + P(E_2) + P(E_3) = 1$$

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A = Event that 2 balls already drawn are blue.

we need to find (E_1/A)

Now $P\left(\frac{A}{E_1}\right) = \frac{\text{Total no. of ways to pick 2 blue balls}}{\text{Total no. of ways to pick 2 balls}}$

$$= \frac{4/2}{4/2} = 1 \quad \therefore \text{all bals are blue.}$$

$P\left(\frac{A}{E_2}\right) = \frac{\text{Total no. of ways to pick 2 blue balls when one blue ball is already picked}}{\text{Total no. of way to pick 2 balls.}}$

Scanned with CamScanner

Page No.
Date

$$P\left(\frac{A}{E_2}\right) = \frac{3}{6} = \frac{1}{2}$$

$P\left(\frac{A}{E_3}\right)$ = Here we have 2 blue balls.

$= \frac{\text{No. of ways to pick 2 blue balls out of 4}}{\text{No. of ways to pick 2 balls.}}$

$$= \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}.$$

using Bayes Theorum.

$$P\left(\frac{E_1}{A}\right) = \frac{P(A/E_1) \times P(E_1)}{P(A/E_1) \times P(E_1)) + P(A/E_2) P(A/E_3) P(E_3)}$$

$$= \frac{1 \times 1/3}{1 \times 1/3 + 1/2 \times 1/3 + 1/6 \times 1/3}$$

$$= \frac{1}{1 + 1/2 + 1/6} = \frac{6}{10} = 0.6$$

Probability that the balls are blue.
$= 0.6$

5 In a neighbourhood, 90% children were falling sick due to flue and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flue is 0.08! If a child develops rashes, find the child's probability of having flue.

Page No.			
Date			

→ Let A be the doctor finding rash (R)

Let B_1 be the event that child has measles

Let B_2 be the event that child has flue.

S – Sick children.

Probability of being sick with flue.

$= P(B_2) = 0.9$,

Probability of being sick with measles

$- P(B_1) = 0.1$

Probability of finding a rash given that the child has flue $= P\left(\frac{R}{B_2}\right) = 0.08$.

Probability of finding a rash given that the child has measles $= P\left(\frac{R}{B_1}\right) = 0.95$.

Probability of child having flue given he has rashes.

$$= P\left(\frac{B_2}{A}\right) = \frac{P(A/B_2) \times P(B_2)}{P(A/B_2) \times P(B_2) \times P(A/B_1) \times P(B_1)}$$

$$= \frac{0.9 \times 0.08}{0.9 \times 0.08 + 0.1 \times 0.95}$$

$$= \frac{0.072}{0.167} = \frac{72}{167}.$$

6 It is observed that 50% of the mails are spam. There is a software that filters spam mails before reaching the inbox. Its accuracy for detecting a spam mail is 99% and chances of tagging a non-spam mail as spam mail is 5%. If a certain mail is tagged as spam find the probability that it is not a spam mail.

Page No.
Date

→ let E_1 be the event of spam mail.
let E_2 be the event of non-spam mail.
A = Event of detecting spam mail.

$$\left.\begin{array}{l} P(E_1) = 0.5 \\ P(E_2) = 0.5 \end{array}\right] \text{given.}$$

$P(A/E_1)$ = Probability of detecting a spam given it is a spam = 0.99.
$P(A/E_2)$ = Probability of detecting a spam given it is not a spam = 0.05.

Probability of event being non-spam given a spam mail is detected.

$$= P\left(\frac{E_2}{A}\right) = \frac{P(A/E_2) \times P(E_2)}{P(A/E_2) \times P(E_2) + P(A/E_1)\, P(E_1)}$$

$$= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.99 \times 0.5}$$

$$= \frac{0.05}{1.04}$$

$$= \frac{5}{104}$$

$$= 0.048.$$