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Multistage Graph

→ A multistage graph $G = (V, E)$ is a directed graph in which the vertices are partitioned into $K \geq 2$, disjoint sets V_i.

Here

V → vertices

E → edges

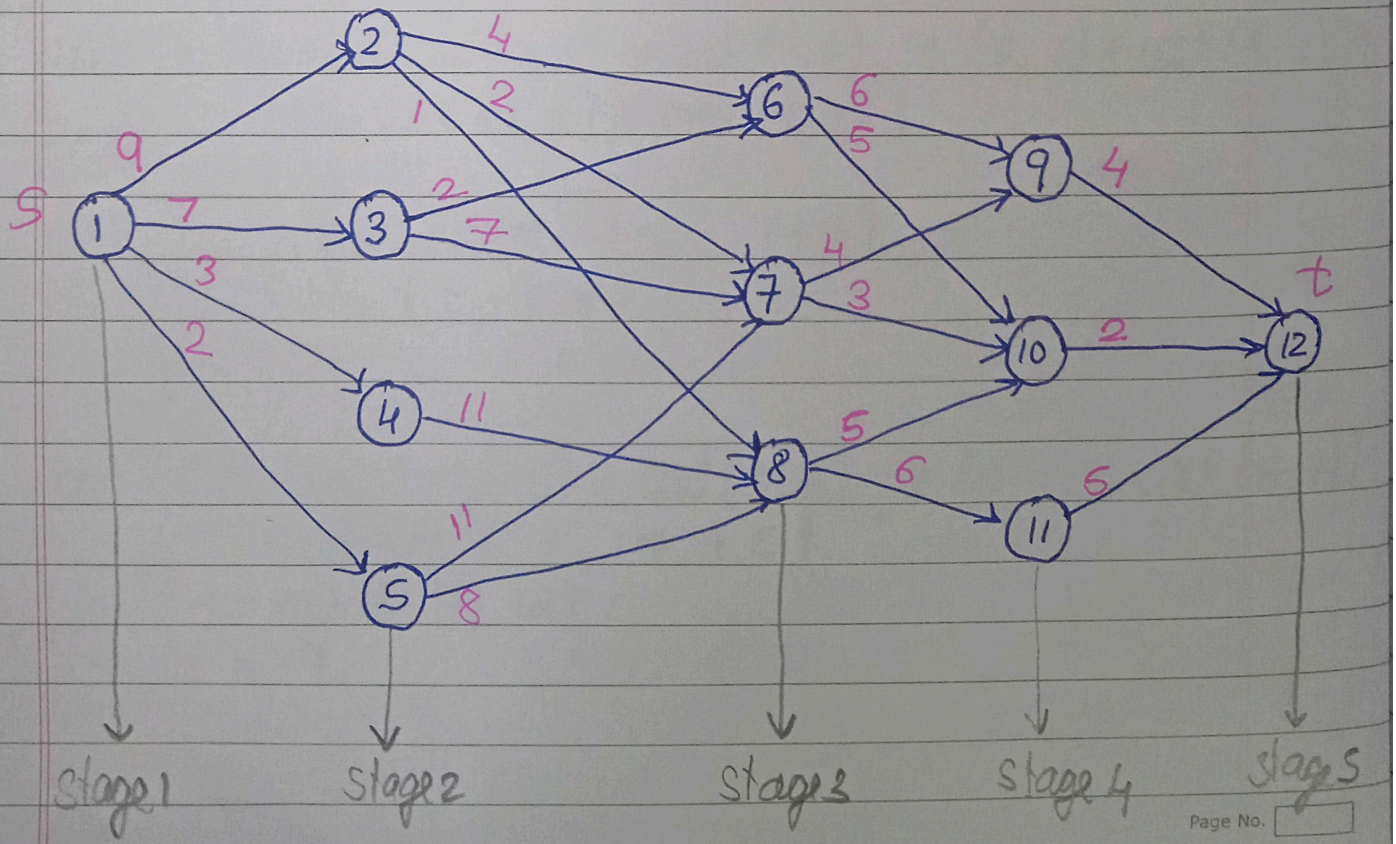
K → no. of sets / stages.

$1 \leq i \leq k$ → i.e no. of vertices must be greater than 1 and less than stage no.

Problem

I) Forward Approach

$$cost(i,j) = \min \{c(j,l) + cost(i+1, l)\}$$



Stage 1:

1. cost (4, 9) = 4
   cost (4, 10) = 2
   cost (4, 11) = 5

2. cost (3, 6) = min {c(6, 9) + cost (4, 9), c(6, 10) + cost (4, 10)}
   = min {6 + 4, 5 + 2}
   = min {10, 7}
   = 7

   cost (3, 7) = min {c(7, 9) + cost (4, 9), c(7, 10) + cost (4, 10)}
   = min {4 + 4, 3 + 2}
   = min {8, 5}
   = 5

   cost (3, 8) = min {c(8, 10) + cost (4, 10), c(8, 11) + cost (4, 11)}
   = min {5 + 2, 6 + 5}
   = min {7, 11}
   = 7

Stage 2:

cost (2, 2) = min {c (2, 6) + cost (3, 6), c (2, 7) + cost (3, 7),
c (2, 8) + cost (3, 8)}

= min {4 + 7, 2 + 5, 1 + 7}
= min {11, 7, 8}
= 7

cost (2, 3) = min {c(2, 6) + cost(3, 6), c (3, 7) + cost (3, 7)}
= min {2 + 7, 7 + 5}
= min {9, 12}
= 9

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$cost(2,4) = \min\{c(4,8) + cost(3,8)\}$
$= \min\{11 + 7\}$
$= 18$

$cost(2,5) = \min\{c(5,7) + cost(3,7), c(5,8) + cost(3,8)\}$
$= \min\{11+5, 8+7\}$
$= \min\{16, 15\}$
$= 15$

Stage 1:

$cost(1,1) = \min\{c(1,2) + cost(2,2), c(1,3) + cost(2,3),$
$c(1,4) + cost(2,4), c(1,5) + cost(2,5)\}$
$= \min\{9+7, 7+9, 3+18, 2+15\}$
$= \min\{16, 16, 21, 17\}$
$= 16$

Trace the solution:

$d(4,10) = 2$
$d(3,6) = 10$, $d(3,7) = 10$, $d(3,8) = 10$
$d(2,2) = 7$, $d(2,3) = 6$, $d(2,4) = 8$, $d(2,5) = 8$;
$d(1,1) = 2$

$1 \rightarrow 2 \rightarrow 7 \rightarrow 10 \rightarrow 12$ → Minimum cost path

Cost of the path is: $9+2+3+2 = 16$

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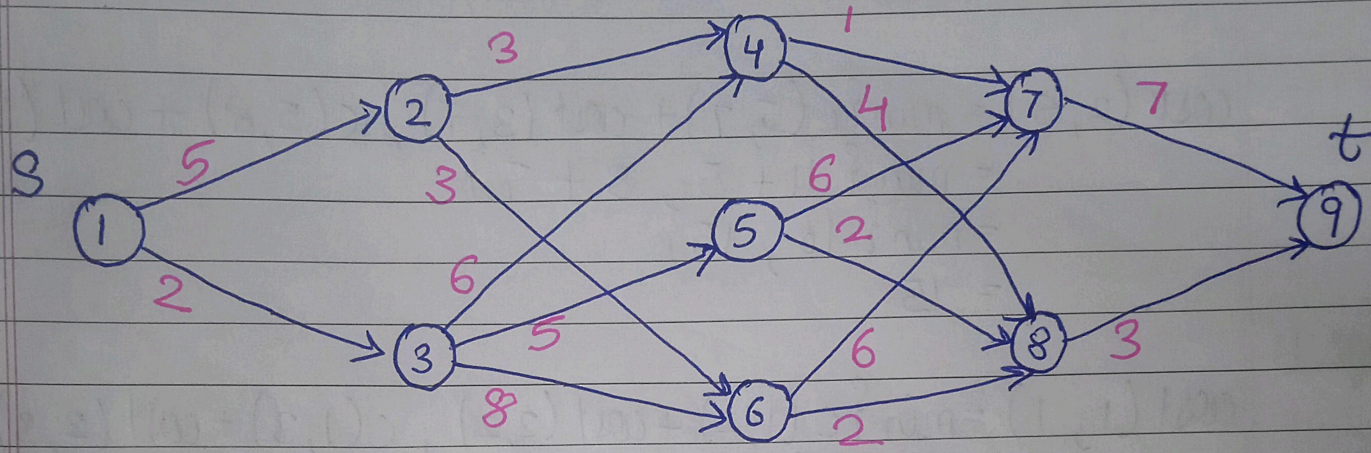
Q. Find a minimum cost path from 1 to 9 in the given graph using Multistage graph. (Forward Approach)



Minimum cost path is:

$1 \rightarrow 3 \rightarrow 5 \rightarrow 8 \rightarrow 9$

Cost of the path is:

$2 + 5 + 2 + 3 = 12$

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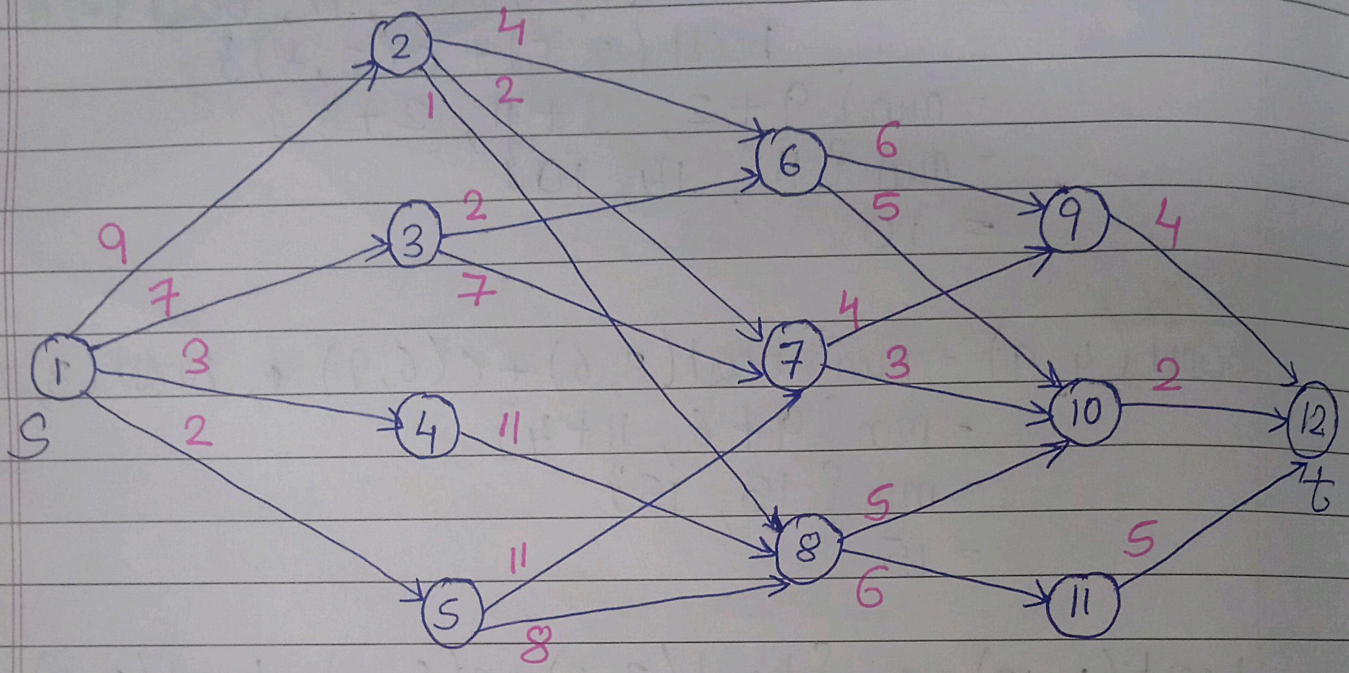
II) Backward Approach

$$bcost(i,j) = \min\{bcost(i-1, l) + c(l, j)\}$$



Stage 2:

bcost (2, 2) = 9
bcost (2, 3) = 7
bcost (2, 4) = 3
bcost (2, 5) = 2

Stage 3:

$$bcost(3,6) = \min\{bcost(2,2) + c(2,6),\ bcost(2,3) + c(3,6)\}$$
$$= \min\{9+4,\ 7+2\}$$
$$= \min\{13, 9\}$$
$$= 9$$

$$bcost(3,7) = \min\{bcost(2,2) + c(2,7),\ bcost(2,3) + c(3,7),\ bcost(2,5) + c(5,7)\}$$
$$= \min\{9+2,\ 7+7,\ 2+11\}$$

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$= \min\{11, 14, 13\}$
$= 11$

$bcost(3,8) = \min\{bcost(2,2)+c(2,7),\ bcost(2,4)+c(4,8),\ bcost(2,5)+c(5,8)\}$
$= \min\{9+2,\ 3+11,\ 2+8\}$
$= \min\{11, 14, 10\}$
$= 10$

Stage 4:

$bcost(4,9) = \min\{bcost(3,6)+c(6,9),\ bcost(3,7)+c(7,9)\}$
$= \min\{9+6,\ 11+4\}$
$= \min\{15, 15\}$
$= 15$

$bcost(4,10) = \min\{bcost(3,6)+c(6,10),\ bcost(3,7)+c(7,10),\ bcost(3,8)+c(8,10)\}$
$= \min\{9+5,\ 11+3,\ 10+5\}$
$= \min\{14, 14, 15\}$
$= 14$

$bcost(4,11) = \min\{bcost(3,8)+c(8,11)\}$
$= \min\{10+6\}$
$= 16$

Stage 5:

$bcost(5,12) = \min\{bcost(4,9)+c(9,12),\ bcost(4,10)+c(10,12),\ bcost(4,11)+c(11,12)\}$
$= \min\{15+4,\ 14+2,\ 16+5\}$
$= \min\{19, 16, 21\}$
$= 16$

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Trace the Solution :

~~d(3,6)~~ = Solution 1

d(5, 12) = 10

d(4, 10) = 7

d(3, 7) = 2

d(2, 2) = 1

So 1 → 2 → 7 → 10 → 12 → cost Path

∴ 9 + 2 + 3 + 2 = 16 → cost of the Path

Solution 2.

d(5, 12) = 10

d(4, 10) = 6

d(3, 6) = 3

d(2, 3) = 1

So 1 → 3 → 6 → 10 → 12 → cost Path

∴ 7 + 2 + 5 + 2 = 16 → cost of the Path.

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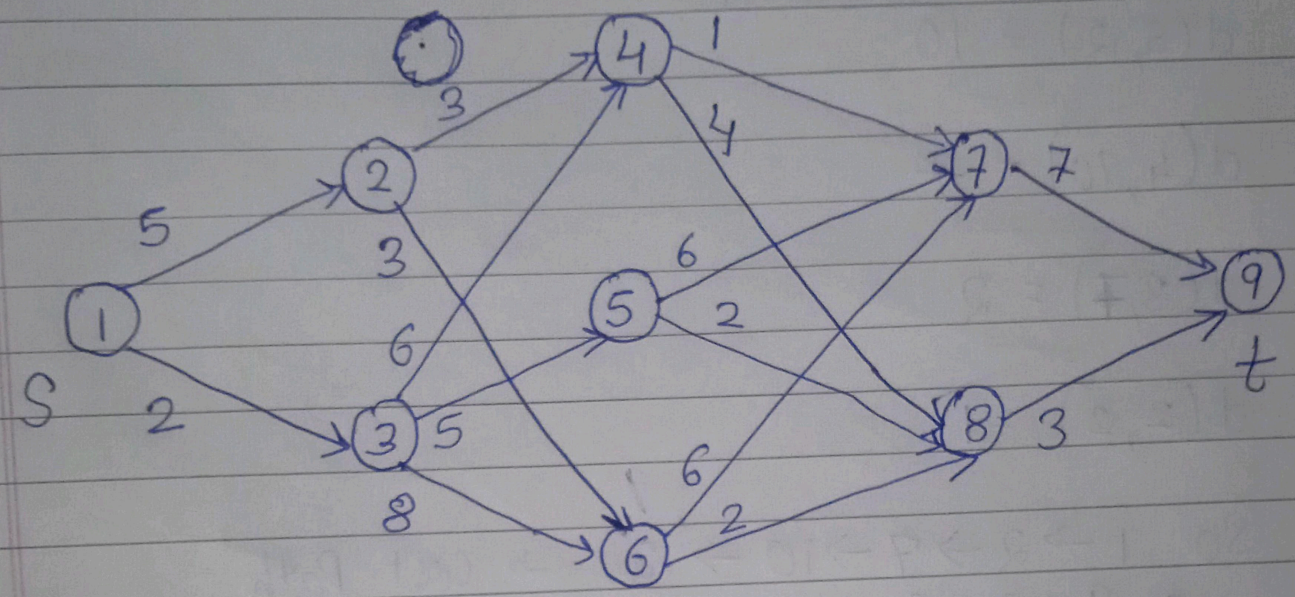
Q. Find the minimum cost path from s to t in the given graph using Multistage graph (backward Approach)



Path :→ 9 → 8 → 5 → 3 → 1

Cost = 3 + 2 + 5 + 2

= 12

Time Complexity for Multistage graph is $O(|V|+|E|)$

V - vertex

E - edges.