

$$A = \{1, 2, 3, 4\}$$

$\begin{array}{c} \swarrow \quad \downarrow \quad \searrow \\ R/\text{IR} \quad S/A/\text{Anti} \quad \text{Transi} \end{array}$

Determine whether relation is reflexive, irreflexive, Symmetric, asymmetric, antisymmetric or transitive

$$1) R = \{ (1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4) \}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$2) R = \{ (1,2), (1,3), (1,4), (2,3), (2,4), (3,4) \}$$

$$3) R = \{ (1,1), (2,2), (3,3) \} \quad (4,4) \notin R$$

$$4) R = \{ (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4) \}$$

$$5) R = \{ (1,3), (4,2), (2,4), (3,1), (2,2) \}$$

$\begin{matrix} a \\ b \\ c \end{matrix} \begin{bmatrix} a & b & c \\ 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$

$\begin{matrix} a \\ b \\ c \end{matrix} \begin{bmatrix} a & b & c \\ 0 & & \\ & 0 & \\ & & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & \\ & 1 & \\ & & 0 \end{bmatrix}$

$a \rightarrow a$  for all  $x \in R$

$\bigcirc$  for some  $x \in R$

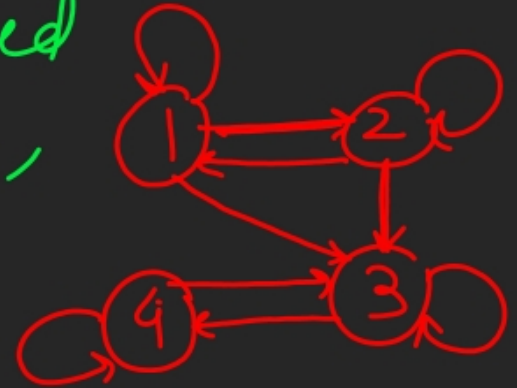


$\begin{bmatrix} 0 & & \\ & 1 & \\ & & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$

$a, b, c$   
 $a \rightarrow b, b \rightarrow c$   
 $a \rightarrow c$

# Equivalence Relation

A relation  $R$  on set  $A$  is called equivalence if it is reflexive, symmetric and transitive



Let  $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4), (1,3), (2,3)\}$

# Equivalence Relations and Partitions

Let  $P$  be a partition of set  $A$ .

relation  $R$  on  $A$  is  $a R b$  if and only if  
 $a$  and  $b$  are members of same block

Then  $R$  is equivalence relation on  $A$

Proof: a) If  $a \in A$ , then  $a R a$

b) If  $a R b$  then  $a$  and  $b$  are same block, so  $b R a$

c) If  $a R b$  and  $b R c$ , then  $a, b$ , and  $c$  lie in same block of  $P$  so  $a R c$



$$A = \{1, 2, 3, 4\}$$

$$P = \{ \{1, 2, 3\}, \{4\} \} \text{ of } A$$

Find equivalence relation  $R$  on  $A$  determined by  $P$

→ The blocks of  $P$  are  $\{1, 2, 3\}$  and  $\{4\}$ . Each element in a block is related to every other element in the same block and only to those elements.

$$R = \{ (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4) \}$$