Single Source Shortest Path

The Bellman - Ford Algorithm

If a graph $G = (V, E)$ contains a negative weight cycle, then some shortest paths may not exist.

Bellman-Ford algorithm finds all shortest-path lengths from a source $s \in V$ to all $v \in V$ or determines that a negative-weight cycle exists.

Thus, Bellman-Ford algorithm solves the single-source shortest-paths problem in the more general case in which edge weights can be negative.

Given a weighted, directed graph $G = (V, E)$ with source s & weight function $w : E \to R$, the Bellman-Ford algorithm returns a boolean value indicating whether or not there is a negative-weight cycle that is reachable from the source.

If there is such a cycle, the algorithm indicates that no solution exists.
If there is no such cycle, the algorithm produces the shortest paths and their weights

Time complexity is $\Theta(V+E)$.

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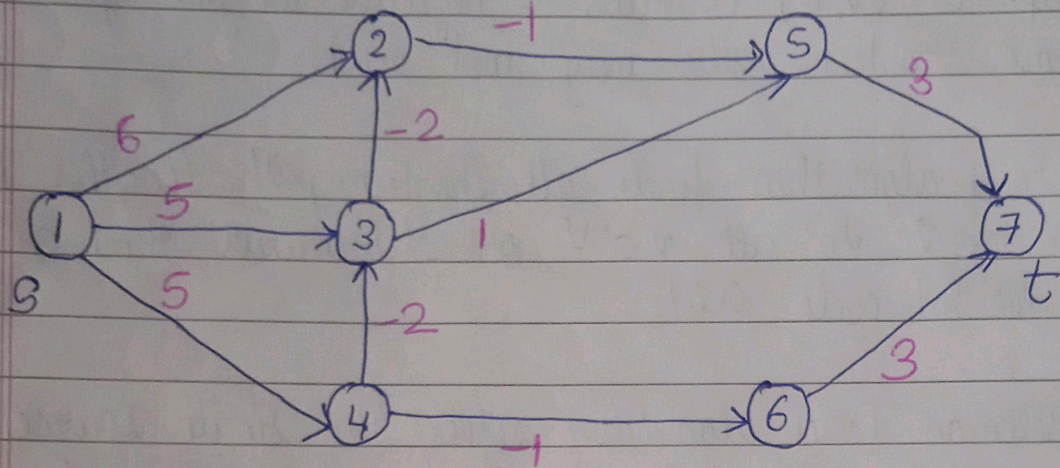
Problems :

Q. Find the minimum cost path from s to t in the given graph.



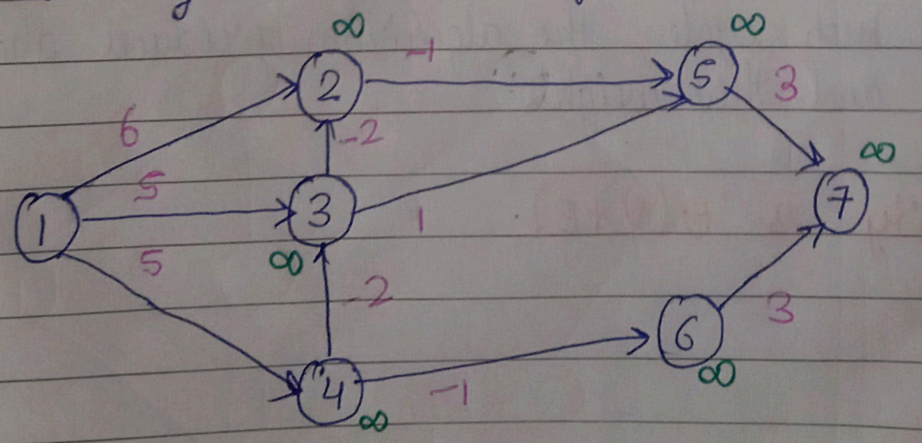
$$\text{if } d[u] + c(u,v) < d[v]$$

$$d[v] = d[u] + c(u,v)$$

⇒ write all the edges of the graph.

(1, 2) (1, 3) (1, 4) (2, 5) (3, 2) (3, 5)
(4, 3) (4, 6) (5, 7) (6, 7).

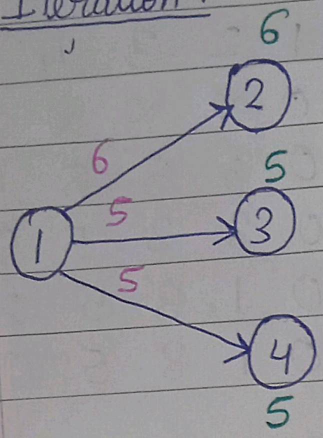
Initially all distance from source will be infinite



iteration ↓ → distance

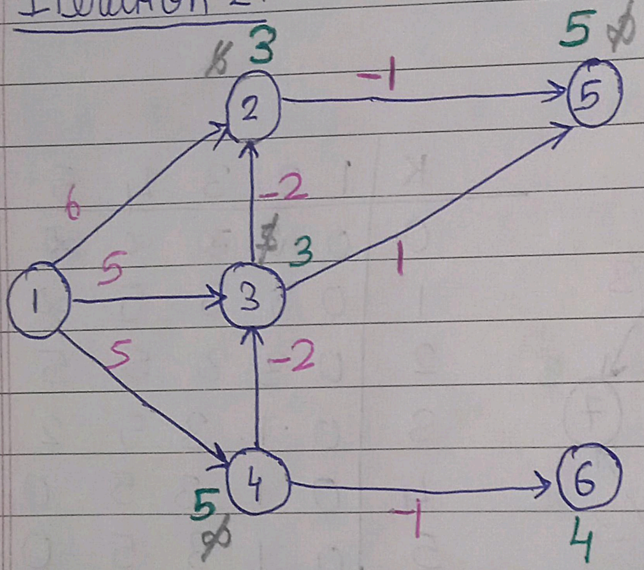
k	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞

Iteration 1:



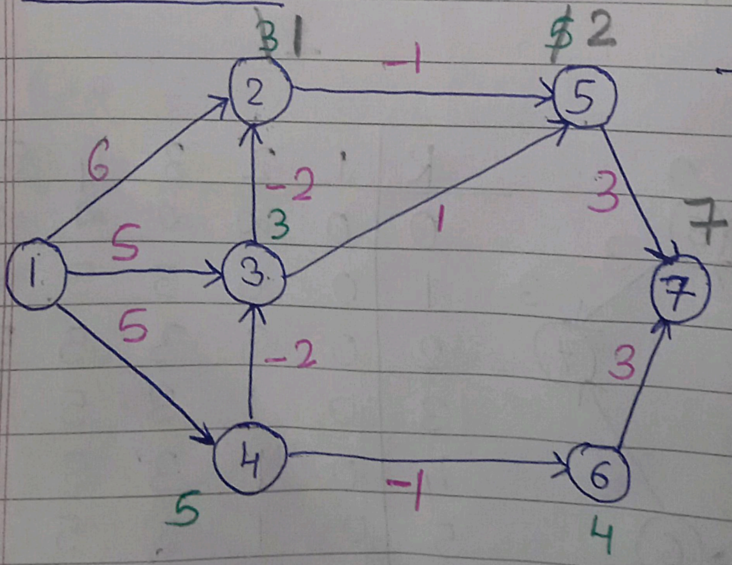
K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞

Iteration 2:



K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	~~∞~~ 4	∞

Iteration 3:



K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7

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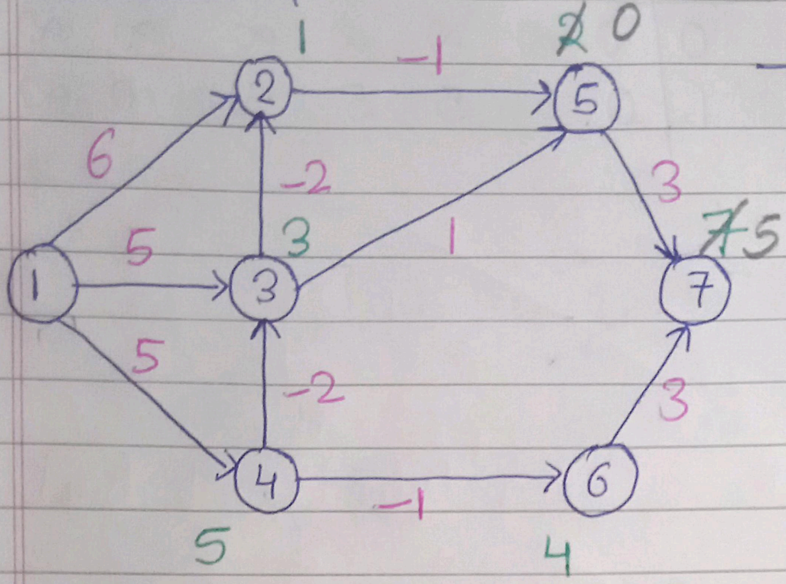
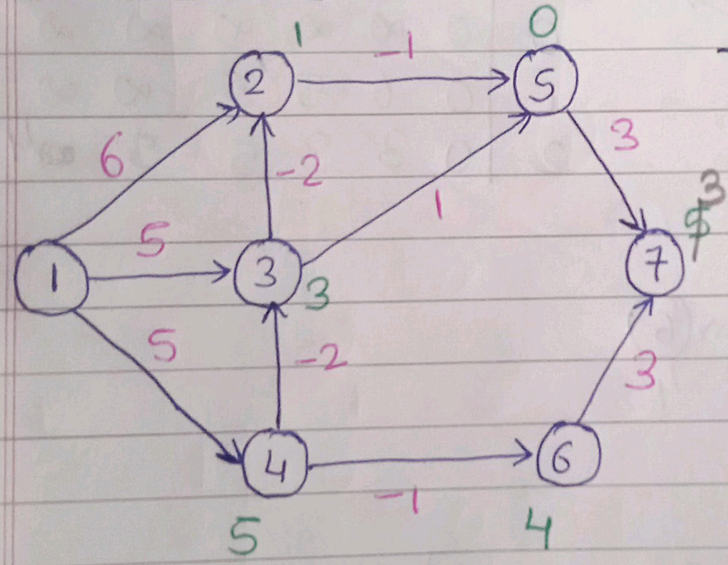
Iteration 4:



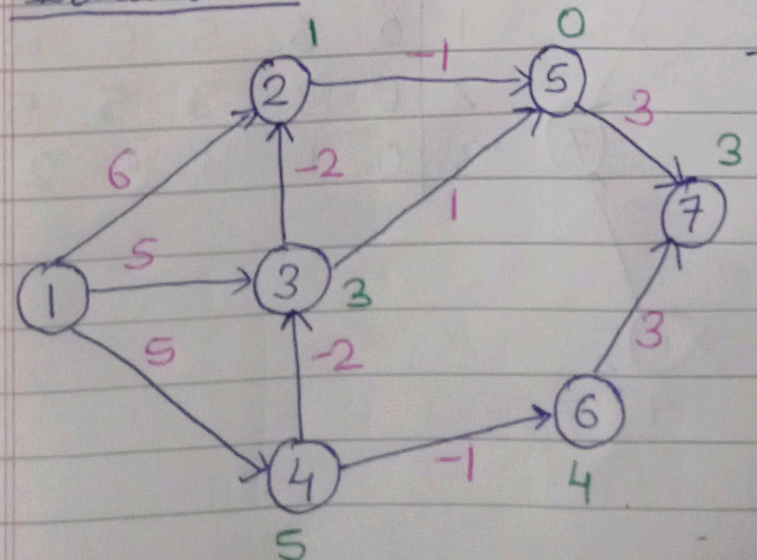
K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7
4	0	1	3	5	0	4	5

Iteration 5:



K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7
4	0	1	3	5	0	4	5
5	0	1	3	5	0	4	3

Iteration 6 :



K	1	2	3	4	5	6	7
0	0	∞	∞	∞	∞	∞	∞
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7
4	0	1	3	5	0	4	5
5	0	1	3	5	0	4	3
6	0	1	3	5	0	4	3

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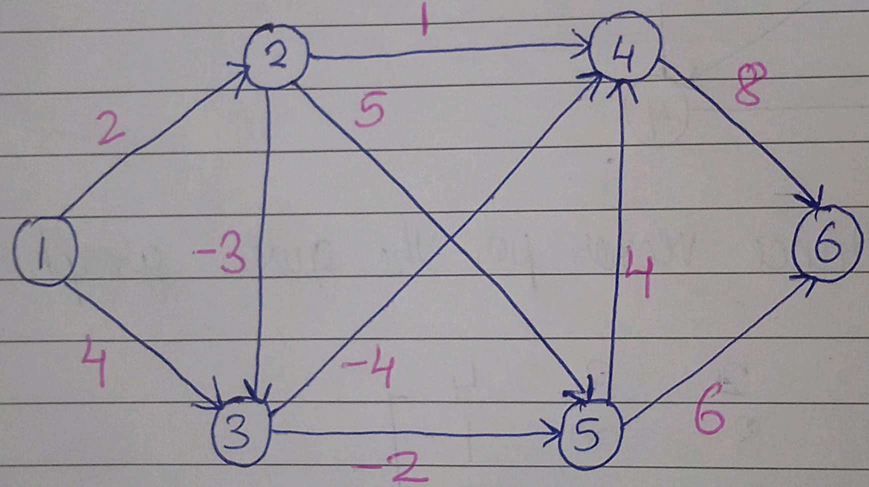
Stop here as the no. of iterations = n−1 where n is the no. of vertices.

Shortest Path:

$1 \to 4 \to 3 \to 2 \to 5 \to 7$

$$\text{Cost} = 5 + (-2) + (-2) + (-1) + 3$$
$$= 5 - 2 - 2 - 1 + 3$$
$$= 5 - 5 + 3$$
$$= 3$$

Q. Find the shortest path from 1 to 6 in the given graph.



Q. Find the minimum shortest path from A to E in the given graph.