Posets

Partially ordered relation: A relation R on a set A is called partial order if R is reflexive, antisymmetric & transitive.

The set A together with the partial order R is called a **partially ordered set** or simply a poset. It is denoted by (A, R).

Example

1. Let A be a set of positive integers and let R be a binary relation such that (a, b) is in R if a divides b.

   Since any integer divides itself. R is reflexive.

Since a divides b means b does not divide a unless $a=b$, R is an antisymmetric relation.

Since a divides b, b divides c, then a divides c, so R is transitive.

Consequently, R is a partial ordered relation.

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2. Let Z^+ be the set of positive integers. The relation '$\leq$' is a partial order on Z^+ because for any element x.

i) $x \leq x$

ii) if $x \leq y$ & $y \leq x$, then $y = x$.

iii) If $x \leq y$ & $y \leq z$, then $x \leq z$

Dual of Poset

Let R be a partial order on a set A, and let R^{-1} be the inverse relation of R.

Then R^{-1} is also a partial order.

The poset (A, R^{-1}) is called the dual of the poset (A, R) and the partial order R^{-1} is called the dual of the partial order R.

Hasse Diagram

A graphical representation of a partial ordering relation in which all arrowheads are understood to be pointing upward is known as the "Hasse Diagram" of the relation.

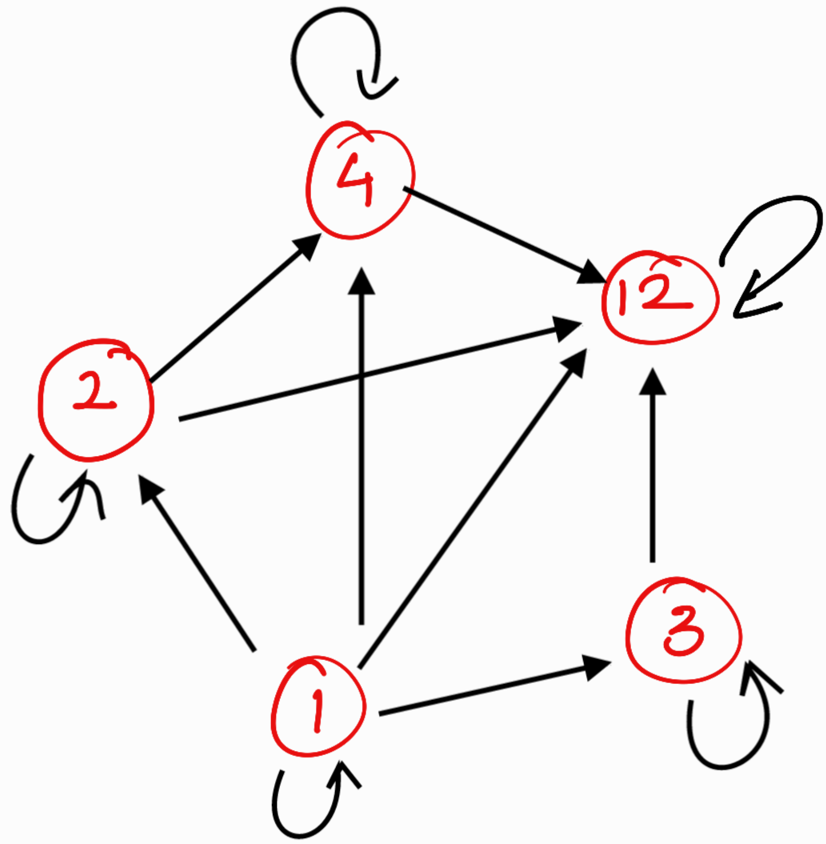
Solved Example

Draw all Hasse Diagrams of posets with three elements.

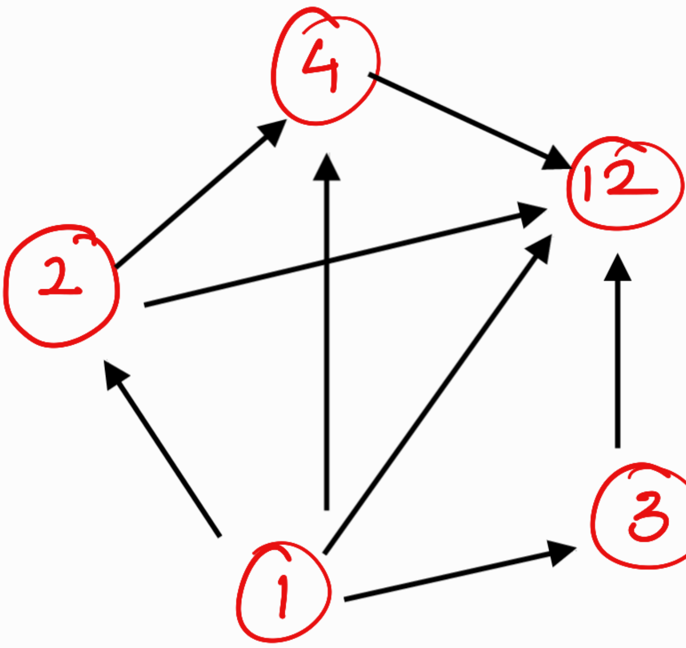
Draw Hasse diagram for the following relations on set $A = \{1, 2, 3, 4, 12\}$

$$R = \{(1,1), (2,2), (3,3), (4,4), (12,12), (1,2), (4,12), (1,3), (1,4), (1,12), (2,4), (2,12), (3,12)\}$$

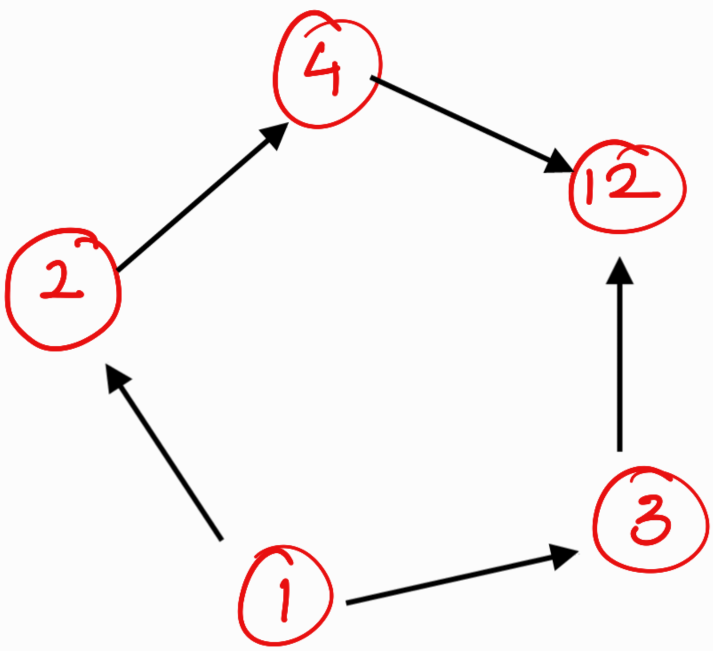
Sol: Digraph



Step 1: Remove Cycle



Step 2: Remove transitive edge



1R2 , 2R4 ∴ 1R4

2R4 , 4R12 ∴ 2R12

1R4 , 4R12 ∴ 1R12

Step 3 Circles are replaced by dots. Arrows are also removed.