Chapter 3

3.1 $$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \left(\frac{e}{1+e}\right)\gamma_w = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w = \boldsymbol{\gamma_d + n\gamma_w}$$

3.2 $$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \left(\frac{e}{1+e}\right)\gamma_w = \boldsymbol{\gamma_d + \left(\frac{e}{1+e}\right)\gamma_w}$$

3.3 $$\gamma_d = \frac{G_s\gamma_w}{1+e}; \quad G_s = \frac{Se}{w}$$

$$\boldsymbol{\gamma_d = \frac{Se\gamma_w}{(1+e)w}}$$

3.4 a. $$w = \left(\frac{711.2 - 623.9}{623.9}\right)(100) = \mathbf{14\%}$$

b. $$\rho = \frac{M}{V} = \frac{711.2}{0.4} = \mathbf{1778\ kg/m^3}$$

c. $$\rho_d = \frac{623.9}{0.4} = \mathbf{1559.75\ kg/m^3}$$

d. $$\rho_d = \frac{G_s\rho_w}{1+e}$$

$$e = \frac{G_s\rho_w}{\rho_d} - 1 = \frac{(2.68)(1000)}{1559.75} - 1 = \mathbf{0.718}$$

e. $$n = \frac{e}{1+e} = \frac{0.718}{1+0.718} = \mathbf{0.418}$$

3.5 Moisture content: $$w = \frac{W_w}{W_s} = \left(\frac{39.93 - 34.54}{34.54}\right)(100) = \mathbf{15.6\%}$$

Moist unit weight: $\gamma = \frac{W}{V} = \frac{39.93}{0.33} = \mathbf{121\ lb/ft^3}$

Dry unit weight: $\gamma_d = \frac{\gamma}{1+w} = \frac{121}{1+\frac{15.6}{100}} = \mathbf{104.7\ lb/ft^3}$

Void ratio: $e = \frac{G_s\gamma_w}{\gamma_d} - 1 = \frac{(2.67)(62.4)}{104.7} - 1 = \mathbf{0.59}$

Porosity: $n = \frac{e}{1+e} = \frac{0.59}{1+0.59} = \mathbf{0.37}$

Degree of saturation: $S = \frac{wG_s}{e} = \left[\frac{(0.156)(2.67)}{0.59}\right](100) = \mathbf{70.6\%}$

3.6 a. $\gamma = \frac{W}{V} = \frac{23}{0.2} = \mathbf{115\ lb/ft^3}$

b. $\gamma_d = \frac{\gamma}{1+w} = \frac{115}{1+0.11} = \mathbf{103.6\ lb/ft^3}$

c. $e = \frac{G_s\gamma_w}{\gamma_d} - 1 = \frac{(2.7)(62.4)}{103.6} - 1 = \mathbf{0.626}$

d. $n = \frac{e}{1+e} = \frac{0.626}{1+0.626} = \mathbf{0.385}$

e. $S = \frac{wG_s}{e} = \left[\frac{(0.11)(2.7)}{0.626}\right](100) = \mathbf{47.4\%}$

f. $W_s = \frac{W}{1+w} = \frac{23}{1+0.11} = 20.72\ \text{lb}$

$W_w = 23 - 20.72 = 2.28\ \text{lb}$

$V_w = \frac{W_w}{\gamma_w} = \frac{2.28}{62.4} = \mathbf{0.0365\ ft^3}$

3.7 a. $\gamma_d = \frac{\gamma_{sat}}{1+w} = \frac{19.8}{1+0.171} = \mathbf{16.91\ kN/m^3}$

b. $\gamma_d = \dfrac{G_s\gamma_w}{1+e}$; $16.91 = \dfrac{(G_s)(9.81)}{1+0.171G_s}$; $G_s = \mathbf{2.44}$

c. $e = wG_s = (0.171)(2.44) = \mathbf{0.417}$

3.8 a. $\gamma = \dfrac{G_s\gamma_w + wG_s\gamma_w}{1+e}$; $G_s = \dfrac{Se}{w}$; $wG_s = Se$

$$\gamma = \frac{\left(\dfrac{Se}{w}\right)\gamma_w + Se\gamma_w}{1+e}$$

$$95 = \frac{\left[\dfrac{(0.6)(e)}{0.192}\right](62.4) + (0.6)(e)(62.4)}{1+e}; \quad e = \mathbf{0.69}$$

b. $G_s = \dfrac{Se}{w} = \dfrac{(0.6)(0.69)}{0.912} = \mathbf{2.16}$

c. $\gamma_{sat} = \dfrac{\gamma_w(G_s+e)}{1+e} = \dfrac{(62.4)(2.16+0.69)}{1+0.69} = \mathbf{105.2\ lb/ft^3}$

3.9 a. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{112}{1+0.108} = \mathbf{101.1\ lb/ft^3}$

b. $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.67)(62.4)}{101.1} - 1 = \mathbf{0.648}$

c. $n = \dfrac{e}{1+e} = \dfrac{0.648}{1+0.648} = \mathbf{0.39}$

d. $S = \dfrac{wG_s}{e} = \left[\dfrac{(0.108)(2.67)}{0.648}\right](100) = \mathbf{44.5\%}$

3.10 a. $\gamma = \dfrac{(G_s+Se)\gamma_w}{1+e} = \dfrac{[2.67+(0.8)(0.648)](62.4)}{1+0.648} = 120.73\ \text{lb/ft}^3$

Water to be added: $120.73 - 112 = \mathbf{8.73\ lb/ft^3}$

b. $\gamma = \dfrac{(G_s+Se)\gamma_w}{1+e} = \dfrac{(2.67+0.648)(62.4)}{1.648} = 125.6\ \text{lb/ft}^3$

Water to be added = 125.6 – 112 = **13.6 lb/ft^3**

3.11 a. $\rho_d = \dfrac{\rho}{1+w} = \dfrac{1680}{1+0.18} = \mathbf{1423.7\ kg/m^3}$

b. $e = \dfrac{G_s\rho_w}{\rho_d} - 1 = \dfrac{(2.73)(1000)}{1423.7} - 1 = 0.918$

$n = \dfrac{e}{1+e} = \dfrac{0.918}{1+0.918} = \mathbf{0.479}$

c. $S = \dfrac{wG_s}{e} = \left[\dfrac{(0.18)(2.73)}{0.918}\right](100) = \mathbf{53.5\%}$

d. $\rho_{sat} = \dfrac{(G_s+e)\gamma_w}{1+e} = \dfrac{(2.73+0.918)(1000)}{1+0.918} = 1901.98\ \text{kg/m}^3$

Water to be added: $\rho_{sat} - \rho = 1901.98 - 1680 \approx$ **222 kg/m^3**

3.12 $\gamma_d = \dfrac{(1780)(9.81)}{1000} = 17.46\ \text{kN/m}^3$; $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.68)(9.81)}{17.46} - 1 = 0.506$

$w = \dfrac{e}{G_s} = \left(\dfrac{0.506}{2.68}\right)(100) = \mathbf{18.88\%}$

3.13 a. $e = \dfrac{0.35}{1-0.35} = 0.538$

$\gamma_{sat} = \dfrac{(G_s+e)\gamma_w}{1+e} = \dfrac{(2.69+0.538)(9.81)}{1+0.538} = \mathbf{20.59\ kN/m^3}$

b. $\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e}$; $17.5 = \dfrac{(2.69)(9.81)(1+w)}{1+0.538}$; $w = \mathbf{1.99\%}$

3.14 $e = wG_s = (0.23)(2.62) = 0.603$

$\rho_d = \dfrac{G_s\rho_w}{1+e} = \dfrac{(2.62)(1000)}{1+0.603} = \mathbf{1634.4\ kg/m^3}$

$\rho_{sat} = \rho_d(1 + w) = (1634.4)(1 + 0.23) =$ **2010.3 kg/m^3**

3.15 a. $\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e} = \dfrac{(2.71)(62.4)(1+0.215)}{1+0.75} = \mathbf{117.4\ lb/ft^3}$

b. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{117.4}{1+0.215} = \mathbf{96.6\ lb/ft^3}$

c. $S = \dfrac{wG_s}{e} = \left[\dfrac{(0.215)(2.71)}{0.75}\right](100) = \mathbf{77.7\%}$

3.16 $e = \dfrac{wG_s}{S} = \dfrac{(0.182)(2.67)}{0.8} = 0.607$

$\gamma_d = \dfrac{G_s\gamma_w}{1+e} = \dfrac{(2.67)(62.4)}{1.607} = \mathbf{103.7\ lb/ft^3}$

$\gamma = \gamma_d(1 + w) = (103.7)(1.182) = \mathbf{122.6\ lb/ft^3}$

3.17 a. $\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e}$; $112.32 = \dfrac{(2.7)(62.4)(1.1)}{1+e}$; $e = \mathbf{0.65}$

b. $\gamma_{sat} = \dfrac{\gamma_w(G_s+e)}{1+e} = \dfrac{(62.4)(2.7+0.65)}{1.65} = \mathbf{126.7\ lb/ft^3}$

3.18 a. $\gamma = \dfrac{\gamma_w(G_s+Se)}{1+e}$; $105.73 = \dfrac{(62.4)(G_s+0.5e)}{1+\mathrm{e}}$

$G_S = 1.694 + 1.194e$ (a)

$112.67 = \dfrac{(62.4)(G_s+0.75e)}{1+\mathrm{e}}$ (b)

From Eqs. (a) and (b),

$112.67 = \dfrac{(62.4)(1.694+1.194e+0.75e)}{1+\mathrm{e}}$; $e = \mathbf{0.81}$

b. From Eq. (a):

$G_S = 1.694 + (1.194)(0.81) = \mathbf{2.66}$

3.19 $\gamma_d = \dfrac{G_s\gamma_w}{1+e} = \dfrac{(2.66)(62.4)}{1+0.81} = 91.7 \text{ lb/ft}^3$

$\gamma_{sat} = \dfrac{\gamma_w(G_s+e)}{1+e} = \dfrac{(62.4)(2.66+0.81)}{1+0.81} = 119.6 \text{ lb/ft}^3$

Water = $(2.5)(\gamma_{sat} - \gamma_d) = (2.5)(119.6 - 91.7) =$ **69.75 lb**

3.20 $e = e_{max} - D_r(e_{max} - e_{min}) = 0.78 - (0.65)(0.78 - 0.43) = 0.553$

$\gamma = \dfrac{G_s\gamma_w}{1+e} = \dfrac{(2.67)(9.81)}{1+0.553} =$ **16.87 kN/m^3**

3.21 $e = e_{max} - D_r(e_{max} - e_{min}) = 0.75 - (0.78)(0.75 - 0.46) = 0.524$

$\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e} = \dfrac{(2.68)(9.81)(1.09)}{1.524} =$ **18.8 kN/m^3**

3.22 $\gamma_{d(min)} = \dfrac{G_s\gamma_w}{1+e_{max}}; \quad 92 = \dfrac{(2.65)(62.4)}{1+e_{max}}; \quad e_{max} = 0.8$

$\gamma_{d(max)} = \dfrac{G_s\gamma_w}{1+e_{min}}; \quad 108 = \dfrac{(2.65)(62.4)}{1+e_{min}}; \quad e_{min} = 0.532$

$e = e_{max} - D_r(e_{max} - e_{min}) = 0.85 - (0.6)(0.8 - 0.532) = 0.639$

$\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e} = \dfrac{(2.65)(62.4)(1.08)}{1.639} =$ **109 lb/ft^3**

3.23 a. $\gamma_d = \dfrac{G_s\gamma_w}{1+e}; \quad 100 = \dfrac{(2.65)(62.4)}{1+e}; \quad e = 0.654$

$S = \dfrac{wG_s}{e} = \dfrac{(0.184)(2.65)}{0.654} =$ **74.6%**

b. This is when $S = 100\%$, so $e = wG_s = (0.184)(2.65) = 0.488$

$\gamma_d = \dfrac{G_s\gamma_w}{1+e} = \dfrac{(2.65)(62.4)}{1+0.488} =$ **111.1 lb/ft^3**

3.24 a. $D_r = \dfrac{e_{max} - e_1}{e_{max} - e_{min}}$

$e_1 = e_{max} - D_r(e_{max} - e_{min}) = 0.9 - (0.4)(0.9 - 0.46) = 0.724$

$\gamma_d = \dfrac{G_s\gamma_w}{1+e_1} = \dfrac{(2.65)(62.4)}{1+0.724} = \mathbf{95.9\ lb/ft^3}$

b. $e_2 = e_{max} - D_r(e_{max} - e_{min}) = 0.9 - (0.75)(0.9 - 0.46) = 0.57$

$\dfrac{\Delta H}{H} = \dfrac{\Delta e}{1+e_1} = \dfrac{e_1 - e_2}{1+e_1};\ \dfrac{\Delta H}{6} = \dfrac{0.724 - 0.57}{1+0.724}$

$\Delta H = 0.536$ ft = **6.43 in.**