Chapter 16

16.1 $\phi' = 32°$; $N_c = 44.14$; $N_q = 28.52$; $N_\gamma = 26.87$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{3}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$$

$$= \frac{1}{3}\left[(21)(44.14) + (1)(17.5)(28.52) + \frac{1}{2}(17.5)(1.5)(26.87)\right]$$

$$= \mathbf{593\ kN/m^2}$$

16.2 $\phi' = 24°$; $N_c = 23.36$; $N_q = 11.40$; $N_\gamma = 7.08$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{4}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$$

$$= \frac{1}{4}\left[(1500)(23.36) + (118)(4)(11.40) + \frac{1}{2}(6)(118)(7.08)\right]$$

$$= \mathbf{10{,}732\ lb/ft^2}$$

16.3 $\phi' = 0°$; $N_c = 5.7$; $N_q = 1$; $N_\gamma = 0$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{6}\left(c_u N_c + qN_q\right) = \frac{1}{6}[(37)(5.7) + (19.5)(0.75)(1)] = \mathbf{37.58\ kN/m^2}$$

16.4 For a continuous foundation with vertical loading, all inclination factors and shape factors are equal to one. So,

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{F_s}\left(c'N_c\lambda_{cd} + qN_q\lambda_{qd} + \frac{1}{2}\gamma BN_\gamma\lambda_{\gamma d}\right)$$

$\phi' = 32°$; $N_c = 35.49$; $N_q = 23.18$; $N_\gamma = 30.22$ (Table 16.2)

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{1}{1.5}\right) = 1.266$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1 + 2\tan 32(1-\sin 32)^2\left(\frac{1}{1.5}\right) = 1.184$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{3}\begin{bmatrix}(21)(35.49)(1.266)+(17.5)(1)(23.18)(1.184)\\ +\frac{1}{2}(17.5)(1.5)(30.22)(1)\end{bmatrix} = \mathbf{606.8\ kN/m^2}$$

16.5 $\phi' = 24°$; $N_c = 19.32$; $N_q = 9.60$; $N_\gamma = 9.44$

$$\lambda_{cd} = 1 + 0.4\left(\frac{4}{6}\right) = 1.266$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1 + 2\tan 24(1-\sin 24)^2\left(\frac{4}{6}\right) = 1.209$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{4}\begin{bmatrix}(1500)(19.32)(1.266)+(4)(118)(9.60)(1.209)\\ +\frac{1}{2}(118)(6)(9.44)(1)\end{bmatrix} = \mathbf{11{,}377\ lb/ft^2}$$

16.6 $q_{\text{all}} = \frac{1}{F_s}\left(c'N_c\lambda_{cd} + qN_q\lambda_{qd} + \frac{1}{2}\gamma BN_\gamma\lambda_{\gamma d}\right)$

$\phi' = 0°$; $N_c = 5.14$; $N_q = 1.0$; $N_\gamma = 0$

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{0.75}{2.5}\right) = 1.12$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{6}[(37)(5.14)(1.12)+(0.75)(19.5)+0] = \mathbf{37.94\ kN/m^2}$$

16.7 Eq. (16.12): $q_u = 1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma$

$\phi' = 32^\circ$; $N_c = 44.04$; $N_q = 28.52$; $N_\gamma = 26.87$

$q = \gamma h + \gamma'(D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12$ kN/m^2

$$Q_{\text{all}} = \frac{q_u B^2}{F_s} = \frac{B^2}{F_s}(1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma)$$

$$Q_{\text{all}} = \frac{1.75^2}{3.5}\left[(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(18.9 - 9.81)(1.75)(26.87)\right]$$

$$= \mathbf{1428\ kN}$$

16.8 $q = \gamma D_f = (16 \times 1.2) = 19.2$ kN/m^2

$D = h - D_f = (1.2 + 0.5) - 1.2 = 0.5$ m

Eq. (16.7):

$$\gamma_{av} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right] = \frac{1}{1.75}\left[(16)(0.5) + (18.9 - 9.81)(1.75 - 0.5)\right] = 11.06\ \text{kN/m}^3$$

$$F_s = \frac{q_u B^2}{Q_{\text{all}}} = \frac{B^2}{Q_{\text{all}}}(1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma)$$

$$F_s = \frac{1.75^2}{1428}[(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(11.06)(1.75)(26.87)] = \mathbf{3.58}$$

16.9 $\phi' = 22^\circ$. From Table 16.1, $N_c = 20.27$; $N_q = 9.19$; $N_\gamma = 5.09$

$$\gamma = \frac{(1750)(9.81)}{1000} = 17.16\ \text{kN/m}^3$$

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma_{\text{av}} BN_\gamma$$

$q = \gamma D_f = (1.5)(17.16) = 25.74$ kN/m^2

$$\gamma_{\text{av}} = \frac{1}{B}[\gamma D + \gamma'(B - D)]$$

$D = h - D_f = 2.5 - 1.5 = 1$ m

$$\gamma_{\text{sat}} = \frac{(1950)(9.81)}{1000} = 19.13\ \text{kN/m}^3$$

$$\gamma_{\text{av}} = \frac{1}{2}[(17.16)(1) + (19.13 - 9.81)(2-1)] = 13.24\ \text{kN/m}^3$$

$$q_u = (1.3)(28)(20.27) + (25.74)(9.19) + (0.4)(13.24)(2)(5.09)$$

$$= 1028.3\ \text{kN/m}^2$$

$$Q_{\text{all}} = \frac{(q_u)B^2}{F_s} = \frac{(1028.3)(2)^2}{3.5} = \mathbf{1175\ kN}$$

16.10 From Eq. (16.12): $q_{\text{all}} = \dfrac{1}{F_s}(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma)$

$\phi' = 29^\circ$; $N_c = 34.24$; $N_q = 19.98$; $N_\gamma = 16.18$ (Table 16.1)

$$q_{\text{all}} = \frac{1}{4}[(1.3)(900)(34.24) + (4.5)(116)(19.98) + (0.4)(116)(B)(16.18)]$$

$$= 12622.6 + 187.7B \tag{a}$$

$$q_{\text{all}} = \frac{250000}{B^2} \tag{b}$$

From Eqs. (a) and (b), $\dfrac{250{,}000}{B^2} = 12{,}622.6 + 187.7B$. By trial and error,

$B \approx$ **4.31 ft**

16.11 $\phi' = 25^\circ$. From Table 16.1, $N_q = 12.72$; $N_\gamma = 8.34$.

$$q_{\text{all}} = \frac{1}{2.5}[(2.1\times 19)(12.72) + (0.4)(19)(B)(8.34)] = 203 + 25.35B$$

$$q_{\text{all}} = \frac{550}{B^2} = 203 + 25.35B$$

$B \approx$ **1.5 m**

16.12 $q_{\text{all}} = \dfrac{q_u}{F_s} = \dfrac{1}{F_s}\left(c'N_c\lambda_{cs}\lambda_{cd} + qN_q\lambda_{qs}\lambda_{qd} + \dfrac{1}{2}\gamma BN_\gamma\lambda_{\gamma s}\lambda_{\gamma d}\right)$

$\phi' = 32^\circ$; $N_c = 35.49$; $N_q = 23.18$; $N_\gamma = 30.22$ (Table 16.2)

$$\lambda_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{23.18}{35.49}\right) = 1.653$$

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{1.2}{1.75}\right) = 1.274$$

$$\lambda_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + \tan 32 = 1.624$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\frac{D_f}{B} = 1 + 2\tan 32(1-\sin 32)^2\left(\frac{1}{1.75}\right) = 1.157$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 0.6$$

$$\lambda_{\gamma d} = 1$$

$$q = \gamma h + \gamma'(D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{(1.75)^2}{3.5}\left[\begin{array}{c}(17)(35.49)(1.653)(1.274) + (17.12)(23.18)(1.624)(1.157)\\ + \dfrac{1}{2}(18.9 - 9.81)(1.75)(30.22)(0.6)(1)\end{array}\right]$$

$\approx$ **1890 kN/m^2**

16.13 a. For vertical load, Eq. (16.44): $q_u = qN_q\lambda_{qd}\lambda_{qs} + \dfrac{1}{2}\gamma B'N_\gamma\lambda_{\gamma d}\lambda_{\gamma s}$

$c' = 0$, $\phi' = 31^\circ$. Table 16.2: $N_q = 20.63$; $N_\gamma = 25.99$.

$B' = B - 2x = 2.5 - (2)(0.2) = 2.1$ m; $L' = 2.5$ m

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{2.1}{2.5}\right)\tan 31 = 1.504$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{2.1}{2.5}\right) = 0.664$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B'} = 1 + 2\tan 31(1-\sin 31)^2\left(\frac{1}{2.1}\right) = 1.134$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (19)(1)(1.504)(1.134)(20.63) + \frac{1}{2}(19)(2.1)(25.99)(1)(0.664)$$

$$= 1012.8 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B'L'}{F_s} = \frac{(1012.8)(2.1)(2.5)}{(5)} = \mathbf{1063.4\,kN}$$

b. $B' = 6 - (2)(0.9) = 4.2$ ft; $L' = 6$ ft. $\phi' = 26°$

$N_c = 22.25$; $N_q = 11.85$; $N_\gamma = 12.54$

$$q_u = c'N_c\lambda_{cs}\lambda_{cd} + qN_q\lambda_{qs}\lambda_{qd} + \frac{1}{2}\gamma B'N_\gamma\lambda_{\gamma s}\lambda_{\gamma d}$$

$$\lambda_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{4.2}{6}\right)\left(\frac{11.85}{22.25}\right) = 1.373$$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{4.2}{6}\right)\tan 26 = 1.341$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{4.2}{6}\right) = 0.72$$

$$\lambda_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B'}\right) = 1 + 0.4\tan^{-1}\left(\frac{4}{4.2}\right) = 1.006$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B'} = 1 + 2\tan 26(1-\sin 26)^2\left(\frac{4}{4.2}\right) = 1.293$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (900)(22.25)(1.373)(1.006) + (115)(4)(11.85)(1.341)(1.293)$$
$$+(0.5)(115)(4.2)(12.54)(0.72)(1)$$

$$= 39{,}291 \text{ lb/ft}^2 = 39.29 \text{ kip/ft}^2$$

$$Q_{\text{all}} = \frac{(39.29)(4.2)(6)}{5} = \mathbf{198\ kip}$$

c. $\phi' = 38°$; $c' = 0$. $N_q = 48.93$; $N_\gamma = 78.03$.

$B' = 1.5 - (2)(0.1) = 1.3$ m; $L' = 1.5$ m

$$\gamma = \frac{(1800)(9.81)}{1000} = 17.66\ \text{kN/m}^3$$

For vertical load, Eq. (16.44): $q_u = qN_q\lambda_{qd}\lambda_{qs} + \frac{1}{2}\gamma B' N_\gamma \lambda_{\gamma d}\lambda_{\gamma s}$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.3}{1.5}\right)\tan 38 = 1.677$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.3}{1.5}\right) = 0.653$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\frac{D_f}{B'} = 1 + 2\tan 38(1-\sin 38)^2\left(\frac{1.5}{1.3}\right) = 1.266$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (17.66)(1.5)(1.266)(1.677)(48.93) + \frac{1}{2}(17.66)(1.3)(78.03)(1)(0.653)$$

$$= 3336.7\ \text{kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B'L'}{F_s} = \frac{(3336.7)(1.3)(1.5)}{(5)} = \mathbf{1301.3\ kN}$$

16.14 Eq. (16.57): $q_{u(F)} = q_{u(P)}\left(\frac{B_F}{B_P}\right) = (6800)\left(\frac{5}{2}\right) = 170{,}000\ \text{lb/ft}^2$

$$Q_{\text{all}} = \frac{Aq_{u(F)}}{5} = \left(\frac{170{,}000}{5}\right)(5)^2 = 85{,}000\ \text{lb} = \mathbf{85\ kip}$$

16.15 $q_{u(P)} = 320$ kN/m^2. Eq. (16.56): $q_{u(F)} = q_{u(P)}$; $q_{u(F)} = 320\ \text{kN/m}^2$

$$Q_{\text{all}} = \frac{Aq_{u(F)}}{F_s} = \frac{\left(\frac{\pi}{4}\right)(2.5)^2(320)}{4} = \mathbf{392.7\ kN}$$

CRITICAL THINKING PROBLEM

C.16.1 The footing is placed at a depth of 1.5 m.

Part (a)

<u>B = 1 m</u>

Eq. (16.49): $q_{\text{net}} = \frac{N_{60}}{0.05} F_d \left[\frac{S_e(\text{mm})}{25}\right]$

2 × B = 2 × 1 = 2 m

N_{60} should be averaged up to a distance of 2 m below the foundation or up to a depth = 1.5 + 2 = 3.5 m

Therefore, $N_{60\text{-avg}}$ = (12 + 7)/2 ≈ 10; S_e = 20 mm

$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{1}\right) = 1.495 \le 1.33$ So, F_d = 1.33

$$q_{\text{net}} = \frac{10}{0.05}(1.33)\left[\frac{20}{25}\right] = 212.8\ \text{kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(212.8)(1)^2}{3} \approx 71\ \text{kN}$$

<u>B = 1.5 m</u>

Eq. (16.50): $q_{\text{net}} = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d \left[\frac{S_e(\text{mm})}{25}\right]$

2 × B = 2 × 1.5 = 3 m

N_{60} should be averaged up to a distance of 3 m below the foundation or up to a depth = 1.5 + 3 = 4.5 m

Therefore, $N_{60\text{-avg}} = (12 + 7)/2 \approx 10$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{1.5}\right) \approx 1.33$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{1.5 + 0.3}{1.5}\right)^2 (1.33)\left[\frac{20}{25}\right] = 153.2 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(153.2)(1.5)^2}{3} \approx 115 \text{ kN}$$

<u>$B = 2$ m</u>

Eq. (16.50): $q_{\text{net}} = \dfrac{N_{60}}{0.08}\left(\dfrac{B + 0.3}{B}\right)^2 F_d\left[\dfrac{S_e(\text{mm})}{25}\right]$

$2 \times B = 2 \times 2 = 4$ m

N_{60} should be averaged up to a distance of 4 m below the foundation or up to a depth = 1.5 + 4 = 5.5 m

Therefore, $N_{60\text{-avg}} = (12 + 7 + 8)/3 = 9$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{2}\right) = 1.247$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{2 + 0.3}{2}\right)^2 (1.247)\left[\frac{20}{25}\right] = 114.7 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(114.7)(2)^2}{3} \approx 229 \text{ kN}$$

<u>$B = 3$ m</u>

Eq. (16.50): $q_{\text{net}} = \dfrac{N_{60}}{0.08}\left(\dfrac{B + 0.3}{B}\right)^2 F_d\left[\dfrac{S_e(\text{mm})}{25}\right]$

$2 \times B = 2 \times 3 = 6$ m

N_{60} should be averaged up to a distance of 6 m below the foundation or up to a depth = 1.5 + 6 = 7.5 m

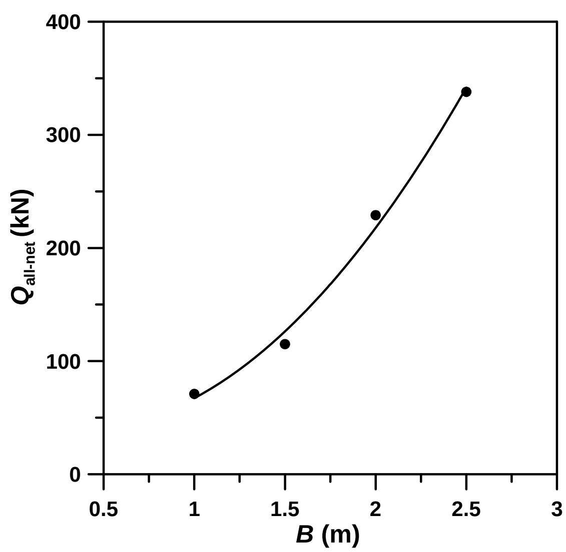
Therefore, $N_{60\text{-avg}} = (12 + 7 + 8 + 19)/4 \approx 12$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{3}\right) = 1.165$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{3+0.3}{3}\right)^2 (1.165)\left[\frac{20}{25}\right] = 112.77\ \text{kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(112.77)(3)^2}{3} \approx 338\ \text{kN}$$

The design chart is shown below.



Part (b)

For any design footing size B,

$Q_{\text{all-net}} \geq Q_{\text{applied}}$ ………………………………………………………….. (1)

$Q_{\text{applied}} = 250$ kN

From the chart it is found that condition (1) is satisfied when $B \approx 2.25$ m

Part (c)

$\phi' = 33^{\circ}$; $c' = 0$; $N_q = 32.33$; $N_\gamma = 31.94$ (Table 16.1); $B = 2.25$ m

$$q_{\text{all-net}} = \frac{q_{\text{u-net}}}{F_s} = \frac{q_u - q}{3} = \frac{1}{3}\left(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma - q\right)$$

$$= \frac{1}{3}\left[(1.5)(17)(32.33) + 0.4(17)(2.25)(31.94) - (1.5)(17)\right]$$

$$= 429.2 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = q_{\text{all-net}}(B)^2 = (429.2)(2.25)^2 \approx \mathbf{2173\ kN}$$

Part (d)

Net allowable column load calculated by the Terzaghi's bearing capacity equation (Part c) is significantly higher than that calculated by the method based on N_{60} and limting settlement value (Part b). In actual design of foundations, both bearing capacity (based on shear strength) and settlement criteria need to be satisfied. It is possible that the allowable column load calculated in Part (c) will change (decrease) if a settlement limit is imposed. Considering the uncertainty in sampling and testing of granular materials for the determination of shear strength, the method based on N_{60} and settlement criteria may be preferable for these conditions.