Chapter 13

13.1 — 13.4 $K_o = (1 - \sin\phi')(\text{OCR})^{\sin\phi'}$

Problem	ϕ'(deg)	K_o	$P_o = \frac{1}{2}K_o\gamma H^2$	$\bar{z} = H/3$
13.1	35	0.634	**143.44 kN/m**	**1.67 m**
13.2	33	0.627	**8,607.1 lb/ft**	**5.67 ft**
13.3	29	0.515	**176.13 kN/m**	**2 m**
13.4	40	0.463	**8,625.7 lb/ft**	**6 ft**

13.5 — 13.8 $K_a = \tan^2(45 - \phi'/2)$

Problem	ϕ'(deg)	K_a	$\sigma'_{a(z=H)} = K_a\gamma H$	$P_a = \frac{1}{2}K_a\gamma H^2$	$\bar{z} = H/3$
13.5	32	0.307	(0.307)(110)(14) = **472.7 lb/ft²**	($\frac{1}{2}$)(0.307)(110)(14)^2 = **3309.4 lb/ft**	**4.66 ft**
13.6	28	0.361	(0.361)(99)(22) = **786.2 lb/ft²**	($\frac{1}{2}$)(0.361)(99)(22)^2 = **8648.8 lb/ft**	**7.33 ft**
13.7	37	0.248	(0.248)(17.6)(5) = **21.8 kN/m²**	($\frac{1}{2}$)(0.248)(17.6)(5)^2 = **54.56 kN/m**	**1.67 m**
13.8	41	0.207	(0.207)(19.5)(9) = **36.32 kN/m²**	($\frac{1}{2}$)(0.207)(19.5)(9)^2 = **163.47 kN/m**	**3 m**

Note: 1. Pressure distribution is similar to that shown in Figure 13.11a., i.e., $\sigma'_a = 0$ at $z = 0$ and $\sigma'_a = K_a\gamma H$ at $z = H$

2. $\bar{z}$ = distance measured from the bottom of the wall

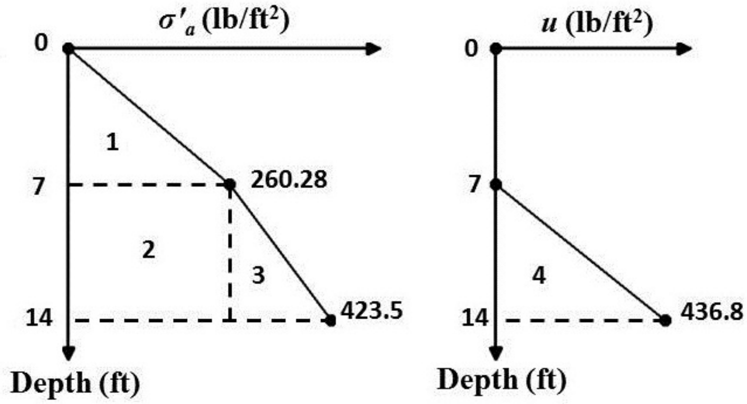
13.9 — 13.12 $K_p = \tan^2(45 + \phi'/2)$

Problem	ϕ'(deg)	K_p	$\sigma'_{p(z=H)} = K_p\gamma H$	$P_p = \frac{1}{2}K_p\gamma H^2$	$\bar{z} = H/3$
13.9	32	3.254	(3.254)(117)(11) = **4187.9 lb/ft²**	($\frac{1}{2}$)(3.254)(117)(11)^2 = **23,033 lb/ft**	**3.67 ft**
13.10	38	4.203	(4.203)(101)(16) = **6792 lb/ft²**	($\frac{1}{2}$)(4.203)(101)(16)^2 = **54,336 lb/ft**	**5.33 ft**
13.11	30	3.0	(3)(16.6)(7) = **348.6 kN/m²**	($\frac{1}{2}$)(3)(16.6)(7)^2 = **1220.1 kN/m**	**2.33 m**
13.12	27	2.662	(2.662)(20.5)(12) = **654.8 kN/m²**	($\frac{1}{2}$)(2.662)(20.5)(12)^2 = **3929.1 kN/m**	**4 m**

Note: 1. $\sigma'_{p(z=0)} = 0$; triangular pressure distribution

2. $\bar{z}$ = distance measured from the bottom of the wall

13.13 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361.$ Refer to the figure.



$z = 0$ ft: $\sigma'_a = \sigma'_a K_a = 0;\ u = 0$

$z = 7$ ft: $\sigma'_a = (103)(7)(0.361) = 260.28\ \text{lb/ft}^2;\ u = 0$

$z = 14$ ft: $\sigma'_a = [(103)(7) + (127 - 62.4)(7)](0.361) = 423.5\ \text{lb/ft}^2$

$u = (62.4)(7) = 436.8\ \text{lb/ft}^2$

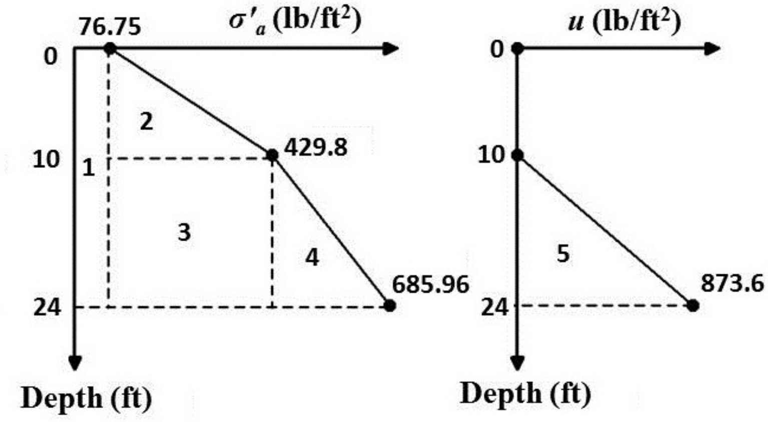
Area No.	Area	
1	$(\frac{1}{2})(7)(260.28) = 910.98 =$	910.98
2	$(260.28)(7) =$	1,821.96
3	$(\frac{1}{2})(7)(423.5 - 260.28) =$	571.27
4	$(\frac{1}{2})(7)(436.8)\ =$	1,528.8
		Σ4,833 lb/ft

Resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(910.98)\left(7 + \frac{7}{3}\right) + (1821.96)\left(\frac{7}{2}\right) + (571.27)\left(\frac{7}{3}\right) + (1528.8)\left(\frac{7}{3}\right)\right]}{4833}$$

$= \mathbf{4.09\ ft}$

13.14 $K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307.$ Refer to the figure.



$z = 0$ ft: $\sigma'_a = qK_a = (250)(0.307) = 76.75$ lb/ft^2; $u = 0$

$z = 10$ ft: $\sigma'_a = [250 + (10)(115)](0.307) = 429.8$ lb/ft^2; $u = 0$

$z = 24$ ft: $\sigma'_a = [250 + (10)(115) + (122 - 62.4)(14)](0.307) = 685.96$ lb/ft^2

$u = (62.4)(14) = 873.6$ lb/ft^2

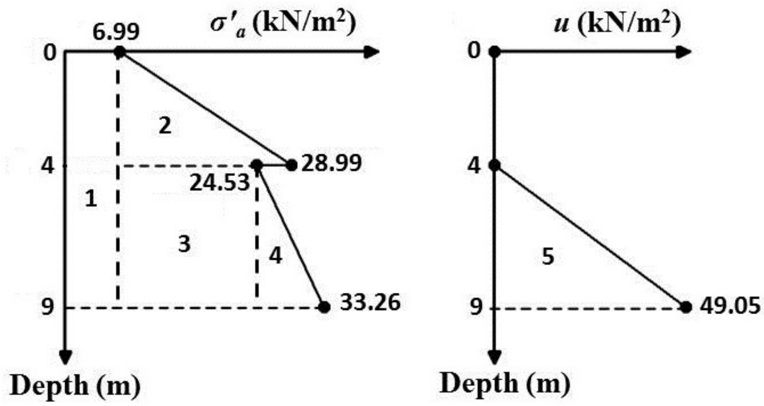
Area No.	Area	
1	$(76.75)(24) =$	1,842
2	$(\frac{1}{2})(10)(429.8 - 76.75) =$	1,765.25
3	$(14)(429.8 - 76.75) =$	4,942.7
4	$(\frac{1}{2})(14)(685.96 - 429.8) =$	1,793.12
5	$(\frac{1}{2})(14)(873.6) =$	6,115.20
	$P_a =$	**Σ16,458 lb/ft**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[\begin{array}{l}(1842)\left(\frac{24}{2}\right) + (1765.25)\left(14 + \frac{10}{3}\right) + (4942.7)\left(\frac{14}{2}\right) \\ + (1{,}793.12)\left(\frac{14}{3}\right) + (6{,}115.2)\left(\frac{14}{3}\right)\end{array}\right]}{16458} = \mathbf{7.55\ ft}$$

13.15 $K_{a(1)} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333;\quad K_{a(2)} = \tan^2\left(45 - \frac{34}{2}\right) = 0.282.$

Refer to the figure.



$z = 0$ m: $\sigma'_a = q\,K_{a(1)} = (21)(0.333) = 6.99\ \text{kN/m}^2;\ u = 0$

$z = 4$ m: $\sigma'_a = [(16.5)(4) + 21](0.333) = 28.99\ \text{kN/m}^2$

$\sigma'_a = [(16.5)(4) + 21](0.282) = 24.53\ \text{kN/m}^2$

$u = 0$

$z = 9$ m: $\sigma'_a = [(16.5)(4) + (20.2 - 9.81)(5)](0.282) = 33.26\ \text{kN/m}^2$

$u = (9.81)(5) = 49.05\ \text{kN/m}^2$

Area No.	Area	
1	$(6.99)(9) =$	62.91
2	$(\frac{1}{2})(4)(28.99 - 6.99) =$	44
3	$(5)(24.53 - 6.99) =$	87.7
4	$(\frac{1}{2})(5)(33.26 - 24.53) =$	21.82
5	$(\frac{1}{2})(5)(49.05) =$	122.62

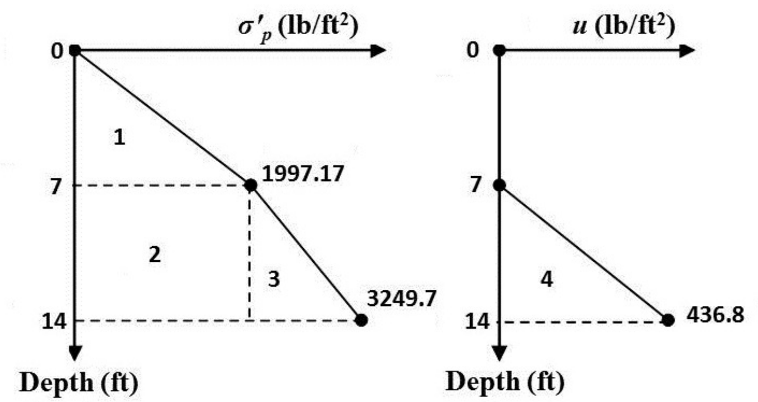
$P_a = \Sigma$**339.05 kN/m**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(62.91)\left(\frac{9}{2}\right) + (44)\left(5 + \frac{4}{3}\right) + (87.7)\left(\frac{5}{2}\right) + (21.82)\left(\frac{5}{3}\right) + (122.62)\left(\frac{5}{3}\right)\right]}{339.05}$$

$= \mathbf{3.01\ m}$

13.16 $K_p = \tan^2\left(45 + \frac{28}{2}\right) = 2.77.$ Refer to the figure.



$z = 0$ ft: $\sigma'_p = 0$; $u = 0$

$z = 7$ ft: $\sigma'_p = \gamma_1 z K_p = (103)(7)(2.77) = 1997.17\ \text{lb/ft}^2$; $u = 0$

$z = 14$ ft: $\sigma'_p = [(103)(7) + (127 - 62.4)(7)](2.77) = 3249.7\ \text{lb/ft}^2$

$u = (62.4)(7) = 436.8\ \text{lb/ft}^2$

Area No.	Area	
1	$(\frac{1}{2})(7)(1997.17) =$	6,990.09
2	$(7)(1997.17) =$	13,980.19
3	$(\frac{1}{2})(7)(3249.7 - 1997.17) =$	4,383.8
4	$(\frac{1}{2})(7)(436.8) =$	1,528.8

$P_a = \boldsymbol{\Sigma 26{,}883\ \text{lb/ft}}$

Location of the resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(6990.09)\left(7 + \frac{7}{3}\right) + (13980.19)\left(\frac{7}{2}\right) + (4383.8)\left(\frac{7}{3}\right) + (1528.8)\left(\frac{7}{3}\right)\right]}{26883} = \mathbf{4.76\ ft}$$

13.17 a. Use Table 13.2: For $\alpha = 10°$ and $\phi' = 36°$, $K_a = 0.270$

$$\sigma'_a = \gamma z K_a = (19)(4)(0.270) = \mathbf{20.52\ kN/m^2}$$

b. Equation (13.24):

$$P_a = \frac{1}{2}K_a\gamma H^2 = \frac{1}{2}(0.27)(19)(4)^2 = \mathbf{41.04\ kN/m}$$

Location and Direction of Resultant: **At a distance of H/3 = 4/3 = 1.33 m above the bottom of the wall inclined at an angle α = 10° to the horizontal.**

13.18 a. Use Table 13.3: For $\alpha = 10°$ and $\phi' = 36°$, $K_p = 3.598$

$$\sigma'_a = \gamma z K_p = (19)(4)(3.598) = 273.44\ \text{kN/m}^2$$

b. Equation (13.25):

$$P_p = \frac{1}{2}\gamma H^2 K_p = \frac{1}{2}(19)(4)^2(3.598) = \mathbf{546.89\ kN/m}$$

Location and Direction of Resultant: **At a distance of H/3 = 4/3 = 1.33 m above the bottom of the wall inclined at an angle α = 10° to the horizontal.**

13.19 a. $H = 5$ m; $c_u = 17$ kN/m²; $\gamma = 21$ kN/m²; $\phi = 0$

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = 1; \sigma'_a = \gamma z - 2c_u$$

At the top ($z = 0$ m):

$$\sigma'_a = -2c_u = (-2)(17)$$

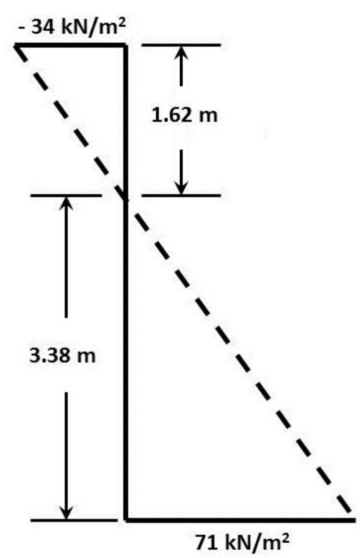
$$= -34\ \text{kN/m}^2$$

At the bottom ($z = 5$ m):

$$\sigma'_a = (21)(5) - (2)(17)$$

$$= 71\ \text{kN/m}^2$$

The pressure diagram is shown.



b. Eq. (13.41): $z_o = \dfrac{2c_u}{\gamma} = \dfrac{(2)(17)}{21} = \mathbf{1.62\ m}$

c. Eq. (13.43): $P_a = \dfrac{1}{2}\gamma H^2 - 2c_u H = \dfrac{1}{2}(21)(5)^2 - (2)(17)(5) = \mathbf{92.5\ kN/m}$

d. Eq. (13.45):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma}$$
$$= \frac{1}{2}(21)(5)^2 - (2)(17)(5) + \frac{(2)(17)^2}{21} = \mathbf{120\ kN/m}$$

Resultant measured from the bottom:

$$\bar{z} = \frac{H - z_o}{3} = \frac{5 - 1.62}{3} = 1.126\ \text{m} \approx \mathbf{1.13\ m}$$

13.20 a. $\sigma_a = \sigma_o K_a - 2c\sqrt{K_a}$

$\sigma_o = \gamma z + q$; $K_a = 1$.

At $z = 0$ ft:

$\sigma_o = 11\ \text{kN/m}^2$

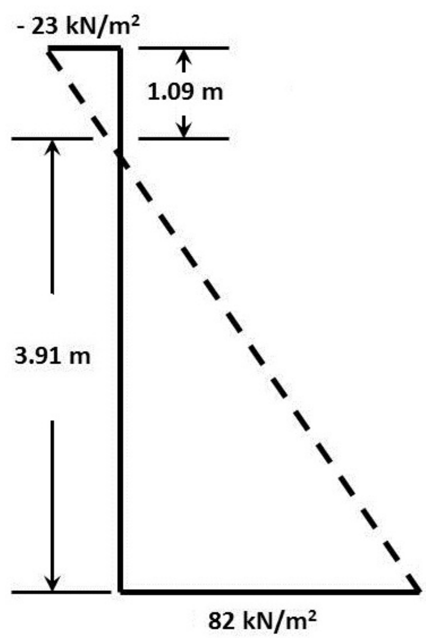
$\sigma_a = 11 - (2)(17) = -23\ \text{kN/m}^2$

At $z = 5$ m:

$\sigma_o = (21)(5) + 11 = 116\ \text{kN/m}^2$

$\sigma_a = 116 - (2)(17) = 82\ \text{kN/m}^2$

The pressure diagram is shown.



b. $\sigma_a = 0$; $(\gamma z_o + q) - 2c = 0$

$$z_o = \frac{2c_u - q}{\gamma} = \frac{34 - 11}{21} = \mathbf{1.09\ m}$$

c. Referring to the diagram in Part a,

$$P_a = \frac{1}{2}(3.91)(82) - \frac{1}{2}(23)(1.09) = \mathbf{147.8\,kN/m}$$

d. $P_a = \frac{1}{2}(3.91)(82) = \mathbf{160.3\,kN/m}$

Location of the resultant from the bottom of the wall: $\frac{3.91}{3} = \mathbf{1.3\,m}$

13.21 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361$; $\sqrt{K_a} = 0.6$. Eq. (13.44):

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'H + \frac{2c'^2}{\gamma}$$

$$= \frac{1}{2}(0.361)(122)(33)^2 - (2)(0.6)(750)(33) + \frac{(2)(750)^2}{122} = \mathbf{3502.18\,lb/ft}$$

13.22 Use Eqs. (13.53) and (13.54). $\alpha = 0$; $\theta = 12^\circ$; $\phi' = 34^\circ$; $\gamma = 119$ lb/ft^3; $H = 32$ ft

Part	δ' (deg)	K_a [Eq. (13.54)]	$P_a = \frac{1}{2}K_a\gamma H^2$ [Eq. (13.53)]
1	14	0.3511	**21,392 lb/ft**
2	21	0.3509	**21,381 lb/ft**

***P*$_a$ is located at a vertical distance of 32/3 = 10.67 ft above the bottom of the wall and is inclined at an angle δ' to the normal drawn to the back face of the wall.**

13.23 a. $\phi' = 38^\circ$; $\psi = 90 - \theta - \delta' = 90 - 5 - 20 = 65^\circ$

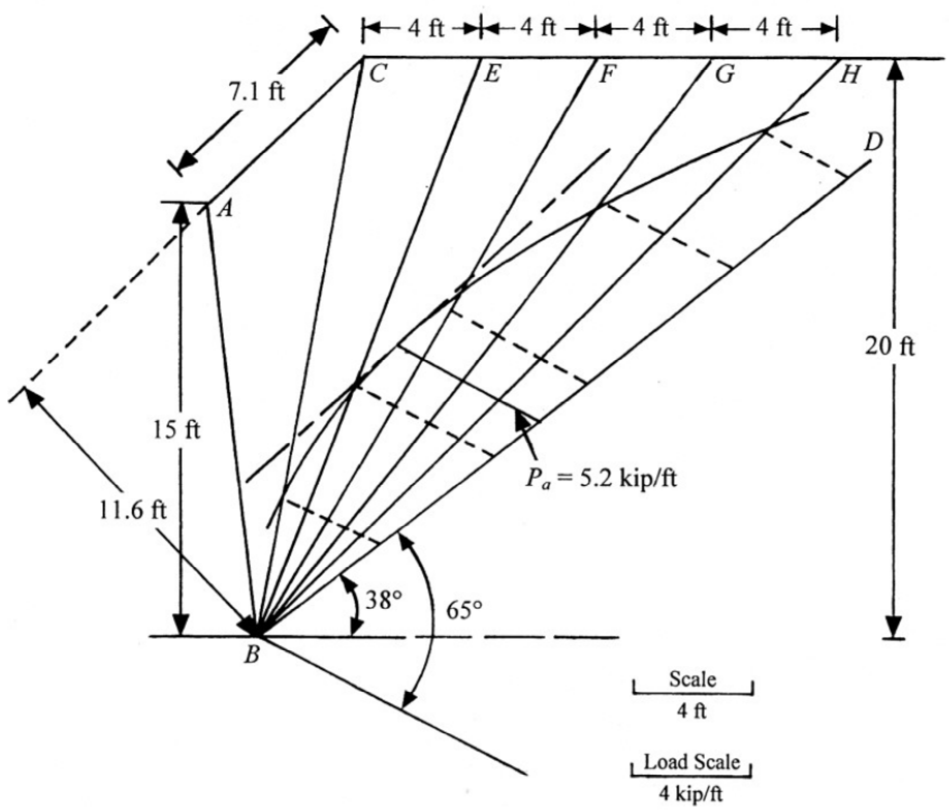
$$\text{Weight of wedge } ABC = \frac{1}{2}(11.6)(7.1)\underbrace{(128)}_{\gamma} = 5271\,\text{lb/ft} = 5.271\,\text{kip/ft}$$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(20)(4)(128) = 5120 \text{ lb/ft} = 5.12 \text{ kip/ft}$$

Wedge	Weight (kip/ft)
ABC	5.271
ABE	5.271 + 5.12 = 10.391
ABF	10.391 + 5.12 = 15.511
ABG	15.511 + 5.12 = 20.631
ABH	20.631 + 5.12 = 25.751

The graphical construction is shown. P_a = **5.2 kip/ft**



b. $\gamma = \frac{(1680)(9.81)}{1000} = 16.48 \text{ kN/m}^3; \phi' = 30°; \ \psi = 90 - 10 - 30 = 50°$

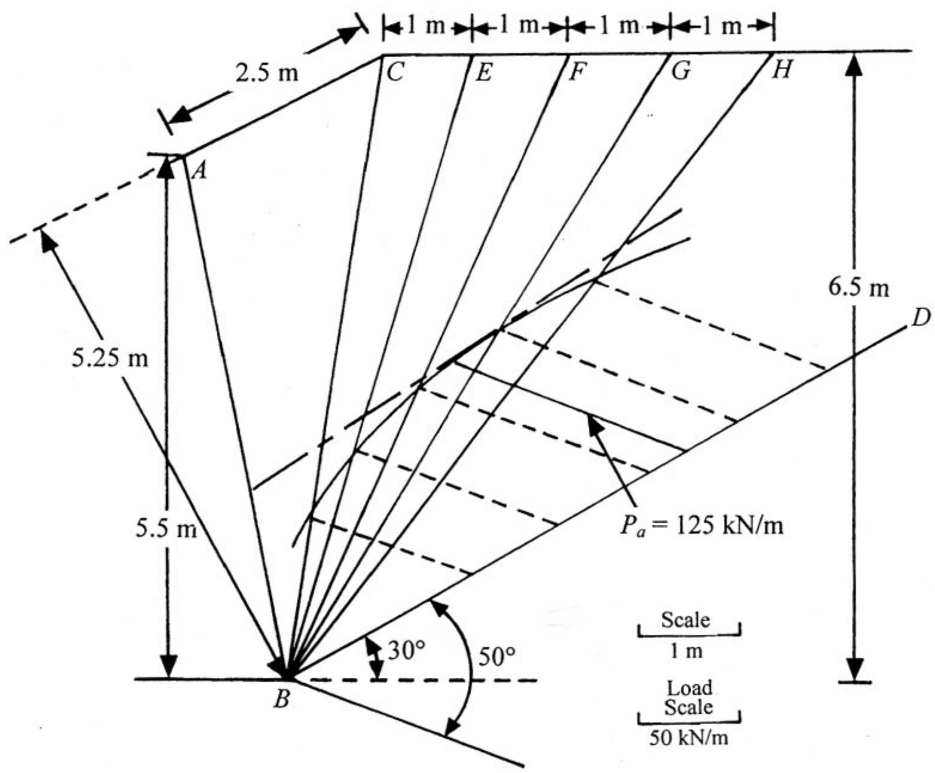
$$\text{Weight of wedge } ABC = \frac{1}{2}(5.25)(2.5)(16.48) = 108.15 \text{ kN/m}$$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(1)(6.5)(16.48) = 53.56 \text{ kN/m}$$

Wedge	Weight (kip/ft)
ABC	108.15
ABE	108.15 + 53.56 = 161.71
ABF	161.71 + 53.56 = 215.27
ABG	215.27 + 53.56 = 268.83
ABH	268.83 + 53.56 = 322.39

The graphical construction is shown. P_a = **125 kN/m**



13.24 From Eqs. (13.66) and (13.67), $\theta^* = \theta + \bar{\beta}$ and $\alpha^* = \alpha + \bar{\beta}$.

$$\bar{\beta} = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) = \tan^{-1}\left(\frac{0.1}{1-0}\right) = 5.71°$$

$\theta^* = 9° + 5.71° = 14.71°$

$\alpha^* = 12^{\circ} + 5.71^{\circ} = 17.71^{\circ}$

$$P_a(\theta^*, \alpha^*) = \frac{1}{2}\gamma H^2 K_a$$

$$\frac{\delta'}{\phi'} = \frac{2}{3}$$

From Table 13.5, for $\theta^* = 14.71^{\circ}$ and $\alpha^* = 17.71^{\circ}$, the value of $K_a \approx 0.582$.

From Eq. (13.70):

$$P_{ae} = P_a(\theta^*, \alpha^*)(1-k_v)\left[\frac{\cos^2(\theta+\overline{\beta})}{\cos\theta\cos^2\overline{\beta}}\right]$$

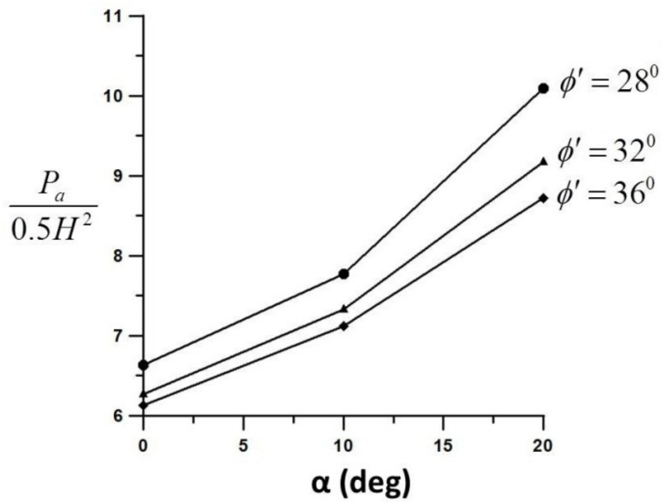
$$= \left[\left(\frac{1}{2}\right)(0.582)(19)(6)^2\right](1-0)\left[\frac{\cos^2(9+5.71)}{\cos(9)\cos^2(5.71)}\right] = \mathbf{190.41\,kN/m}$$

CRITICAL THINKING PROBLEM

13.C.1 Refer to Table A.1 to prepare the following table:

$\theta = 10°$	γ (kN/m^3)	ϕ'	$K_{a(R)}$	$K_{a(R)}\gamma$
$\alpha = 0°$	16.5	28°	0.402	6.633
	17.7	32°	0.354	6.265
	19.5	36°	0.314	6.123
$\alpha = 10°$	16.5	28°	0.471	7.771
	17.7	32°	0.414	7.327
	19.5	36°	0.365	7.117
$\alpha = 20°$	16.5	28°	0.612	10.09
	17.7	32°	0.518	9.168
	19.5	36°	0.447	8.716

The graph of $\frac{P_a}{0.5H^2}$ versus backfill inclination angle, α, is shown on the next page.



The above chart shows that for any backfill inclination α and any wall height H, the active force P_a per unit length of the wall decreases as the soil friction angle (or the compacted unit weight) increases. For a desired level of P_a (at a given α), a compaction unit weight could be estimated from the chart for field specification.