Chapter 12

12.1 a. $c' = 0$. From Eq. (12.3): $\tau_f = \sigma' \tan \phi'$

$$\tau = \frac{300}{(1000)(0.063)^2} = 75 \text{ kN/m}^2$$

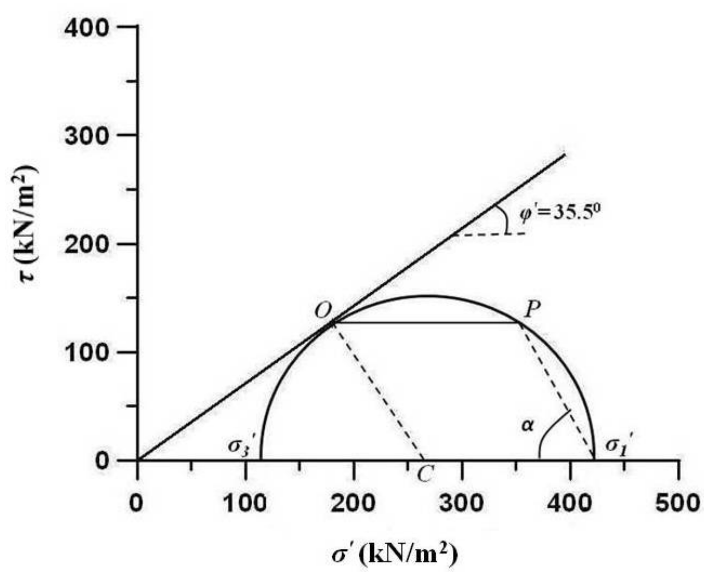
So, $75 = 105 \tan \phi'$

$$\phi' = \tan^{-1}\left(\frac{75}{105}\right) = \mathbf{35.5°}$$

b. For $\sigma' = 180$ kN/m^2, $\tau_f = 180 \tan 35.5° = 128.39$ kN/m^2

Shear force, $S = (128.39)(1000)(0.063)^2 = \mathbf{509.5\ N}$

12.2 The point O (180, 128.4) represents the failure stress conditions on the Mohr-Coulomb failure envelope. The perpendicular line OC to the failure envelope determines the center, C of the Mohr's circle. With the center at C, and the radius as OC, the Mohr's circle is drawn by trial and error such that the circle is tangent to the failure envelope at O. From the graph,



a. $\sigma_3' \approx$ **115 kN/m²**; $\sigma_1' \approx$ **420 kN/m²**

b. The horizontal line *OP* drawn from *O* determines the pole *P*. Therefore, the orientation or the major principal plane with the horizontal is given by the angle $\alpha \approx$ **65°**.

12.3 For $\sigma' = 28$ lb/in², $\tau_f = 28 \tan 33° = 18.18$ lb/in²

Shear force, $S = (18.18)(2.5)^2 =$ **113.65 lb**

12.4 Area of specimen $A = \left(\frac{\pi}{4}\right)(2)^2 = 3.14 \text{ in.}^2$

Test No.	Normal force N (lb)	$\sigma' = \frac{N}{A}$ (lb/in.²)	Shear force S (lb)	$\tau_f = \frac{S}{A}$ (lb/in.²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	15	4.77	5.25	1.67	19.29
2	30	9.55	10.5	3.34	19.27
3	48	15.28	16.8	5.35	19.29
4	83	26.43	29.8	9.5	19.77

A graph of τ_f vs. σ' will yield ϕ' = **19.5°**.

12.5 Area of specimen $A = \left(\frac{\pi}{4}\right)(0.05)^2 = 0.00196 \text{ m}^2$

Test No.	Normal force N (N)	$\sigma' = \frac{N}{A}$ (N/m²)	Shear force S (N)	$\tau_f = \frac{S}{A}$ (N/m²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	250	79.6	139	44.26	29.07
2	375	119.4	209	66.56	29.13
3	450	143.3	250	79.61	29.05
4	540	171.9	300	95.54	29.06

A graph of τ_f vs. σ' will yield $\phi' \approx$ **29°**.

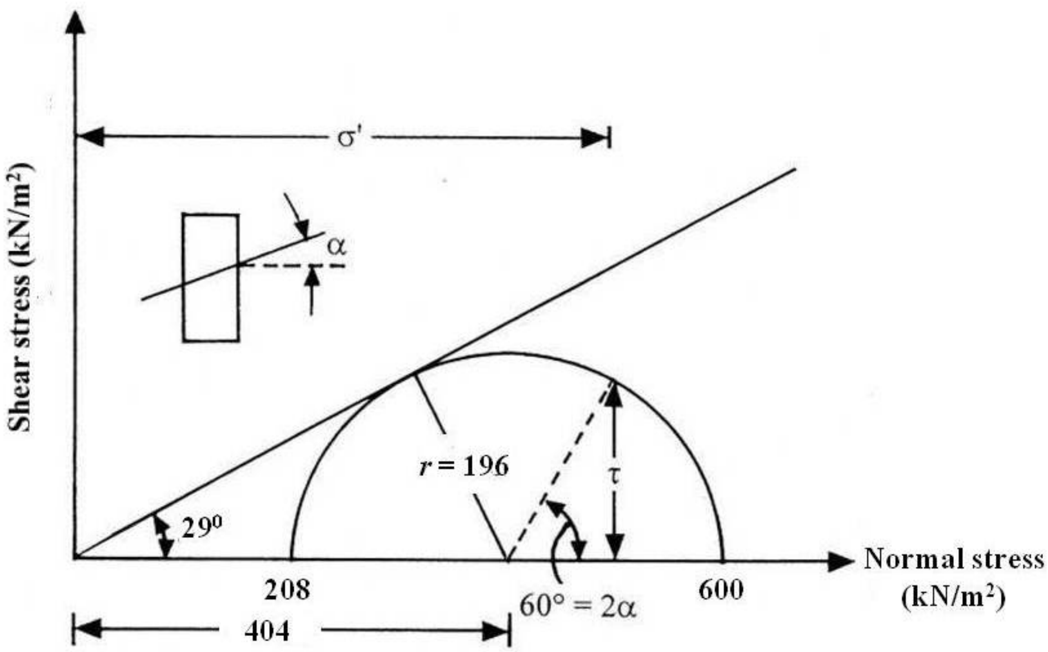
12.6 $c' = 0$. From Eq. (12.8): $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 30°$

$$\sigma_1' = 208\tan^2\left(45+\frac{29}{2}\right) \approx 600\,\text{kN/m}^2$$

$$\Delta\sigma_{d(\text{failure})} = \sigma_1' - \sigma_3' = 600 - 208 = \mathbf{392\ kN/m^2}$$

12.7 a. From Eq. (12.4): $\theta = 45+\dfrac{\phi'}{2} = 45+\dfrac{29}{2} = \mathbf{59.5}°$

b. Refer to the figure.



$\tau = 196\sin 60° = \mathbf{169.7\ kN/m^2}$

$\sigma' = 404 + r\cos 60 = 404 + 196\cos 60 = \mathbf{502\ kN/m^2}$

For failure, $\tau_f = \sigma'\tan\phi' = 502\tan 29 = 278.26\ \text{kN/m}^2$. Since the developed shear stress = 169.5 kN/m^2 (which is less than 278.26 kN/m^2), the specimen did not fail along this plane.

12.8 $\phi' = 28 + 0.18D_r = 28 + (0.18)(68) = 40.24°$

$$\sigma_1' = \sigma_3'\tan^2\left(45+\frac{\phi'}{2}\right) = 150\tan^2\left(45+\frac{40.24}{2}\right) = \mathbf{697.43\,kN/m^2}$$

12.9 $\sigma'_3 = 125\,\text{kN/m}^2;\;\; \sigma'_1 = \sigma'_3 + \Delta\sigma_{d(f)} = 125 + 175 = 300\,\text{kN/m}^2$

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right);\quad 300 = 125\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$\phi' \approx$ **24.3°**

12.10 $\sigma'_3 + \Delta\sigma'_3 = \sigma'_3 \tan^2\left(45 + \dfrac{\phi'}{2}\right) = \sigma'_3 \tan^2(60.5°) = 3.12\sigma'_3$

$$1 + \frac{\Delta\sigma'_3}{\sigma'_3} = 3.12;\quad \frac{\Delta\sigma'_3}{\sigma'_3} = 2.12$$

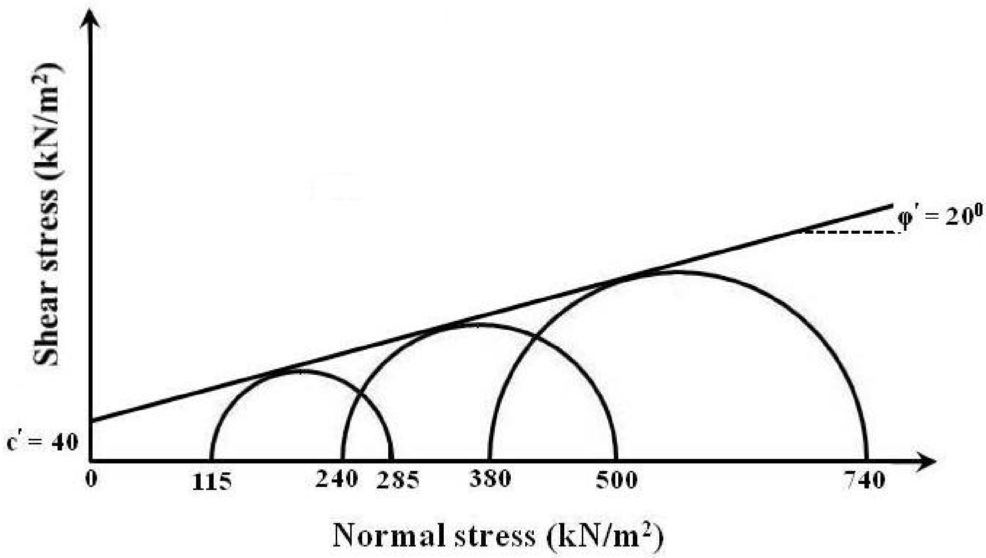
$$\sigma'_3 = \frac{18}{2.12} = \mathbf{8.49\ lb/in.^2}$$

12.11 $\sigma'_1 = \sigma'_3 \tan^2\left(45 + \dfrac{\phi'}{2}\right) = \left(\sigma_3 - \Delta u_{d(f)}\right)\tan^2\left(45 + \dfrac{31}{2}\right)^2$

$$\sigma'_1 = (15 - 4.8)\tan^2(60.5)^2 = \mathbf{31.86\,lb/in.^2}$$

12.12 a. The effective principal stresses at failure are calculated as follows and the Mohr-Coulomb failure envelope is drawn from the Mohr's circles on the following page.

Test no.	σ_3 (kN/m^2)	$(\Delta\sigma_d)_f$ (kN/m^2)	$(\Delta u_d)_f$ (kN/m^2)	$\sigma'_3 = \sigma_3 - (\Delta u_d)_f$ (kN/m^2)	$\sigma'_1 = \sigma'_3 + (\Delta\sigma_d)_f$ (kN/m^2)
1	100	170	–15	115	285
2	200	260	–40	240	500
3	300	360	–80	380	740



From the graph: $c' \approx$ **40 kN/m²** and $\phi' \approx$ **20°**

b. Effective stress in the middle of the clay layer:

$$\sigma'_0 = (2)(19 - 9.8) = 18.4 \text{ kN/m}^2$$

$$\tau = c' + \sigma' \tan\phi' = 40 + 18.4\tan(20) = \mathbf{46.7\ kN/m^2}$$

12.13 a. $\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$; $(25 + 33) = 25\tan^2\left(45 + \frac{\phi'}{2}\right)$

$\phi' \approx$ **23.4°**

b. $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{23.4}{2} = \mathbf{56.7}°$

c. From Eqs. (10.8) and (10.9):

$$\sigma'_f = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2}\cos 2\theta = \frac{58+25}{2} + \frac{58-25}{2}\cos(2 \times 56.7)$$

$$= \mathbf{34.94\ lb/in.^2}$$

$$\tau_f = \sigma' \tan\phi' = 34.94 \tan 23.4 = \mathbf{15.12\ lb/in.^2}$$

12.14 $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$

Specimen I: $(105 + 220) = 325 = 105 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$ (a)

Specimen II: $(210 + 400) = 610 = 210 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$ (b)

Subtracting Eq. (a) from Eq. (b): $610 - 325 = 105 \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = \mathbf{27.48}°$

From Eq. (b):

$$c' = \frac{610 - 210 \tan^2\left(45 + \frac{27.48}{2}\right)}{2 \tan\left(45 + \frac{27.48}{2}\right)} = \mathbf{12.48\ kN/m^2}$$

12.15 a. From Eqs. (10.8) and (10.9): $\theta = 40°$

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\theta = \frac{105 + 325}{2} + \frac{325 - 105}{2}\cos(2 \times 40)$$

$$= \mathbf{234.1\ kN/m^2}$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\theta = \frac{325 - 105}{2}\sin(2 \times 40) = \mathbf{108.32\ kN/m^2}$$

b. The angle of inclination of the failure plane:

$$\theta = 45 + \frac{\phi'}{2} = 45 + \frac{27.48}{2} = \mathbf{58.73}°$$

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\theta = \frac{610 + 210}{2} + \frac{610 - 210}{2}\cos(2 \times 58.73)$$

$$= \mathbf{317.74\ kN/m^2}$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\theta = \frac{610 - 210}{2}\sin(2 \times 58.73) = \mathbf{177.46\ kN/m^2}$$

12.16 $\sigma_1 = \sigma_3 \tan^2\left(45 + \dfrac{\phi}{2}\right)$; $\phi = 2\left[\tan^{-1}\left(\dfrac{22+28}{22}\right)^{0.5} - 45\right] = \mathbf{22.9°}$

$\sigma_1' = \sigma_3' \tan^2\left(45 + \dfrac{\phi'}{2}\right)$; $\phi' = 2\left[\tan^{-1}\left(\dfrac{22+28+4}{22+4}\right)^{0.5} - 45\right] = \mathbf{20.48°}$

12.17 a. $\sigma_3 = 150$ kN/m^2; $\sigma_1 = 150 + 120 = 270$ kN/m^2

$\sigma_1 = \sigma_3 \tan^2\left(45 + \dfrac{\phi}{2}\right)$; $\dfrac{270}{150} = \tan^2\left(45 + \dfrac{\phi}{2}\right)$

$\phi = \mathbf{16.6°}$

b. $\sigma_1' = \sigma_3' \tan^2\left(45 + \dfrac{\phi'}{2}\right)$

$$\frac{\sigma_1 - \Delta u_{d(f)}}{\sigma_3 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{27}{2}\right) = 2.662$$

$$\frac{270 - \Delta u_{d(f)}}{150 - \Delta u_{d(f)}} = 2.662$$

Or, $270 - \Delta u_{d(f)} = 399.3 - 2.662\Delta u_{d(f)}$

$\Delta u_{d(f)} = \mathbf{77.8\ kN/m^2}$

12.18 $\sigma_1' = \sigma_3' \tan^2\left(45 + \dfrac{\phi'}{2}\right) = 150 \tan^2 (45 + 27/2) = 399.4$ kN/m^2

$\Delta\sigma_{d(f)} = \sigma_1' - \sigma_3' = 399.4 - 150 = \mathbf{249.4\ kN/m^2}$

12.19 $\sigma_1 = \sigma_3 \tan^2\left(45 + \dfrac{\phi}{2}\right) = 20 \tan^2\left(45 + \dfrac{31}{2}\right) = 62.48$ lb/in.^{2}

$\Delta\sigma_{d(f)} = 62.48 - 20 = \mathbf{42.48\ lb/in.^2}$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{62.48 - \Delta u_{d(f)}}{20 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{24}{2}\right)$$

$\Delta u_{d(f)} = \mathbf{-10.98\ lb/in.^2}$

A dense sand tends to expand during shear. Due to undrained condition, it creates a negative pore water pressure. A loose sand tends to contract during shear, and a positive pore water pressure is developed in undrained conditions. Therefore, for loose sand, $\sigma' < \sigma$, and so, $\phi' > \phi$.

12.20 $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{148 - \Delta u_{d(f)}}{0 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{28}{2}\right)$$

$\Delta u_{d(f)} = \mathbf{-83.6\ kN/m^2}$

12.21 a.

Test no.	$\frac{\sigma_1' + \sigma_3'}{2} = p'$ (kN/m^2)	$\frac{\sigma_1' - \sigma_3'}{2} = q'$ (lb/in.^{2})
1	212	108
2	362	155

$q' = m + p' \tan \alpha$

$108 = m + 212 \tan \alpha$ (a)

$155 = m + 362 \tan \alpha$ (b)

$m = \mathbf{41.56\ kN/m^2}$, $\alpha = \mathbf{17.4^\circ}$

b. $\phi' = \sin^{-1}(\tan \alpha) = \sin^{-1}(\tan 17.4) = \mathbf{18.26^\circ}$

$$c' = \frac{m}{\cos \alpha} = \frac{41.56}{\cos(17.4)} = \mathbf{43.55\ kN/m^2}$$

12.22 $\dfrac{c_{u(\mathrm{VST})}}{\sigma'_o} = 0.11 + 0.0037PI$

$\sigma' = (2)(18) + (7)(19.5 - 9.81) = 103.83 \text{ kN/m}^2$

$c_{u(\mathrm{VST})} = [0.11 + (0.0037)(23)](103.83) = \mathbf{20.25 \text{ kN/m}^2}$

CRITICAL THINKING PROBLEM

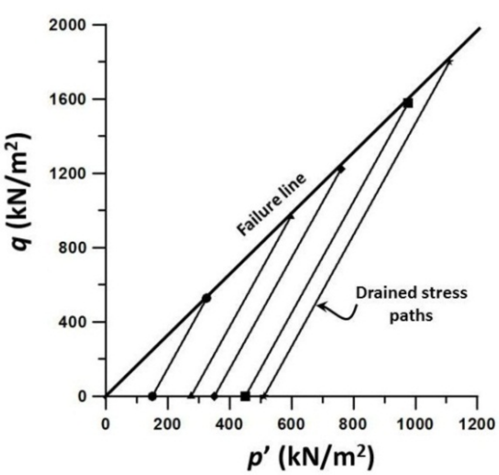
12.C.1 <u>Task 1</u>

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) = \sigma'_3 + \frac{1}{3}\Delta\sigma_d \; ; q = \Delta\sigma_d$$

At the end of consolidation, $p'_0 = \sigma'_3; \; q_0 = 0$

σ_3 (kN/m^2)	p'_0 (kN/m^2)	$(\Delta\sigma_d)_f = q_f$ (kN/m^2)	p'_f (kN/m^2)
150	150	527	325.67
275	275	965	596.67
350	350	1225	758.33
450	450	1580	976.67
510	510	1800	1110

The drained stress paths and the failure line are shown below.



Task 2

At point O: $p' = 0$; $q = 0$

At point P: $p' = \sigma'_3 = 250\,\text{kN/m}^2$; $q = 0$

At point A: $q = 1000 = \Delta\sigma_d$

$$p' = 675 = \sigma'_3 + \frac{1}{3}\Delta\sigma_d = \sigma'_3 + \frac{1}{3}(1000)$$

Therefore, $\sigma'_3 = 341.67\,\text{kN/m}^2$

Stress path O to P: Increase confining pressure from 0 to 250 kN/m^2 under drained condition (effective stress).

Stress path P to A: Increase confining pressure from 250 kN/m^2 to 341.67 kN/m^2 under drained condition (effective stress). Simultaneously increase axial stress from 0 to 1000 kN/m^2.