Chapter 11

11.1 $$S_{e(\text{flexible, center})} = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_sI_f$$

Given: $\Delta\sigma$ = 4000 lb/ft^2; α = 4; $B' = \frac{3}{2} = 1.5$; μ_s = 0.4;

E_s = 140 ton/ft^2 = 280,000 lb/ft$^{2.}$

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2$$

$$m' = \frac{L}{B} = \frac{6}{3} = 2; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{15}{\left(\frac{3}{2}\right)} = 10$$

From Table 11.1, F_1 = 0.641; from Table 11.2, F_2 = 0.031.

$$I_s = 0.641 + \frac{1-(2)(0.4)}{1-0.4}(0.031) = 0.651$$

Also, with $\frac{D_f}{B} = \frac{3}{3} = 1$ and $\frac{L}{B} = 2$, Table 11.3 gives I_f = 0.75. Hence,

$$S_{e(\text{flexible, center})} = (4000)[(4)(1.5)]\left(\frac{1-0.4^2}{280{,}000}\right)(0.651)(0.75) = 0.0352\text{ ft} = 0.422\text{ in.}$$

$S_{e(\text{rigid})}$ = (0.93)(0.422) ≈ **0.393 in.**

11.2 As in Problem 11.1, $S_{e(\text{rigid})} = 0.93\Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_sI_f$

Given: $\Delta\sigma$ = 100 kN/m^2; $B = L$ = 3 m; E_s = 16,000 kN/m^2; μ_s = 0.3; α = 4;

$$B' = \frac{B}{2} = \frac{3}{2} = 1.5\text{ m.}$$

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2$$

$$m' = \frac{L}{B} = 1; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{20}{\left(\frac{3}{2}\right)} = 13.3$$

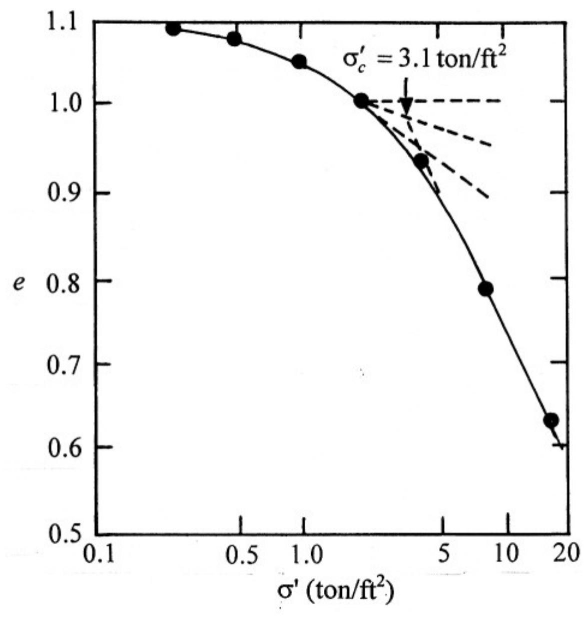
From Table 11.1, $F_1 \approx 0.5$; from Table 11.2, $F_2 \approx 0.01$.

$$I_s = 0.5 + \frac{1-(2)(0.3)}{1-0.3}(0.01) = 0.506$$

Also, $\frac{D_f}{B} = \frac{1.5}{3} = 0.5.$ From Table 11.3, $I_f \approx 0.77$. So,

$$S_{e(\text{rigid})} = (0.93)(100)(4\times 1.5)\left(\frac{1-0.3^2}{280{,}000}\right)(0.506)(0.77) = 0.01237 \text{ m} = \mathbf{12.37\ mm}$$

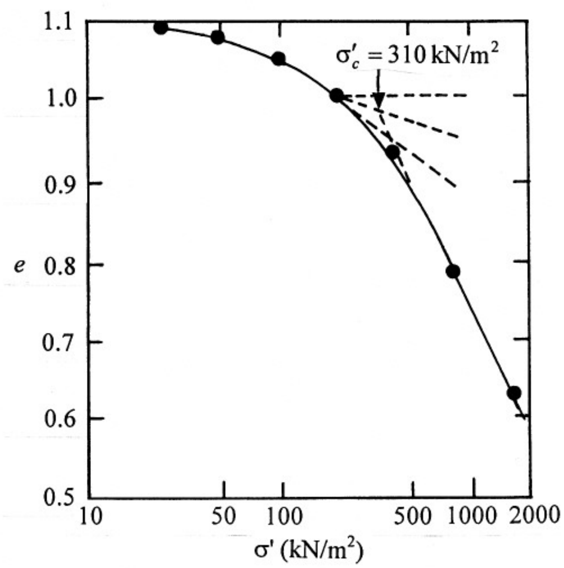
11.3 a. The plot of e vs. σ' is shown.



b. $\sigma'_c = \mathbf{3.1\ ton/ft^2}$

c. $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.79-0.63}{\log\left(\frac{16}{8}\right)} = \mathbf{0.53}$$

11.4 a. The plot is shown.



b. $\sigma'_c = \mathbf{96\ kN/m^2}$

c. $C_c = \dfrac{e_1 - e_2}{\log\left(\dfrac{\sigma'_2}{\sigma'_1}\right)}$

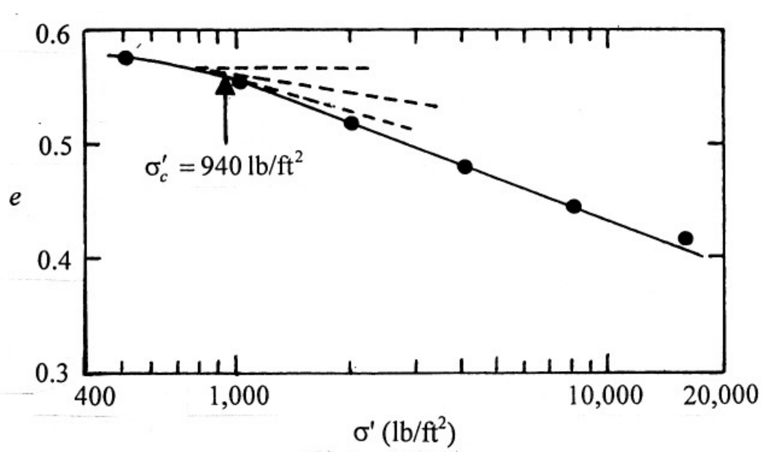
$= \dfrac{1.06 - 0.925}{\log\left(\dfrac{500}{200}\right)}$

$= \mathbf{0.339}$

11.5 a. Height of solids: $H_s = \dfrac{W_s}{AG_s\gamma_w} = \dfrac{95.2\text{ g}}{(4.91)(2.54)^2(2.68)(1)} = 1.12\text{ cm} = 0.441\text{ in.}$

σ' (lb/ft^2)	H (in.)	H_s (in.)	$H_v = H - H_s$ (in.)	$e = \dfrac{H_v}{H_s}$
500	0.6947	0.441	0.2537	0.575
1,000	0.6850	0.441	0.244	0.553
2,000	0.6705	0.441	0.2295	0.52
4,000	0.6520	0.441	0.211	0.478
8,000	0.6358	0.441	0.1948	0.442
16,000	0.6252	0.441	0.1842	0.418

The e-log σ' graph is plotted.



b. From the graph, $\sigma'_c = \mathbf{940\ lb/ft^2}$

c. $C_c = \dfrac{e_1 - e_2}{\log\left(\dfrac{\sigma'_2}{\sigma'_1}\right)} = \dfrac{0.52 - 0.478}{\log\left(\dfrac{4000}{2000}\right)} = \mathbf{0.133}$

11.6 a. $S_c = \dfrac{C_c H}{1+e_o}\log\left(\dfrac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$

$C_c = 0.009(LL - 10) = (0.009)(50 - 10) = 0.36$

$$\sigma'_o = \gamma_{d\,(\text{sand})}H_1 + [\gamma_{\text{sat(sand)}} - 62.4]H_2 + [\gamma_{\text{sat(clay)}} - 62.4]\frac{H_3}{2}$$

$$= (110)(8) + (115 - 62.4)(15) + (120 - 62.4)\left(\frac{17}{2}\right) = 2158.6\ \text{lb/ft}^2$$

$$S_c = \frac{(0.36)(17\times 12)}{1+0.9}\log\left(\frac{2158.6+1000}{2158.6}\right) = \mathbf{6.39\ in.}$$

b. $S_c = \dfrac{C_s H}{1+e_o}\log\left(\dfrac{\sigma'_c}{\sigma'_o}\right) + \dfrac{C_c H}{1+e_o}\log\left(\dfrac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$

$$= \frac{(17)(12)}{1+0.9}\left[\frac{0.36}{6}\log\left(\frac{2600}{2158.6}\right) + 0.36\log\left(\frac{3158.6}{2600}\right)\right] = \mathbf{3.79\ in.}$$

11.7 $\gamma_{d\,(\text{sand})} = \dfrac{(2.65)(9.81)}{1+0.64} = 15.85\ \text{kN/m}^3$

$$\gamma'_{\text{sand}} = \frac{(2.65-1)(9.81)}{1+0.64} = 9.87\ \text{kN/m}^3$$

$$\gamma'_{\text{clay}} = \frac{(2.75-1)(9.81)}{1+0.9} = 9.04\ \text{kN/m}^3$$

$C_c = 0.009(LL - 10) = (0.009)(45) = 0.405$

$$\sigma'_o = (2.5)(15.85) + (2.5)(9.87) + \left(\frac{3}{2}\right)(9.04) = 77.86\ \text{kN/m}^2$$

$$S_c = \frac{(0.405)(3)}{1+0.9}\log\left(\frac{77.86+100}{77.86}\right) = 0.229\text{ m} = \mathbf{229\ mm}$$

11.8 $$\gamma_{s(\text{sand})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.65)(62.4)}{1+0.58} = 104.66\text{ lb/ft}^3$$

$$\gamma'_{\text{sand}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.65-1)(62.4)}{1.58} = 65.16\text{ lb/ft}^3$$

$$\gamma'_{\text{clay}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.72-1)(62.4)}{1+1.1} = 51.11\text{ lb/ft}^3$$

$$\sigma'_o = (5)(104.66) + (7)(65.16) + \left(\frac{6}{2}\right)(51.11) = 1132.75\text{ lb/ft}^2$$

$C_c = 0.009(LL - 10) = (0.009)(45 - 10) = 0.315$

$$S_c = \frac{C_cH}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_cH}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{\left(\frac{0.315}{5}\right)(6)}{2.1}\log\left(\frac{3500}{1132.75}\right) + \frac{(0.315)(6)}{2.1}\log\left(\frac{1132.75+3000}{3500}\right)$$

$$= 0.153\text{ ft} \approx \mathbf{1.84\ in.}$$

11.9 $$C_c = \frac{e_1-e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.82-0.7}{\log\left(\frac{4000}{2500}\right)} = 0.588$$

$$C_c = \frac{e_1-e_3}{\log\left(\frac{\sigma'_3}{\sigma'_1}\right)}$$

$$e_3 = e_1 - C_c\log\left(\frac{\sigma'_3}{\sigma'_1}\right) = 0.82 - 0.588\log\left(\frac{6000}{2500}\right) = \mathbf{0.596}$$

11.10 $C_c = \dfrac{1.1-0.9}{\log\left(\dfrac{3}{1}\right)} = 0.419$

$$e_3 = 1.1 - 0.419\log\left(\frac{3.5}{1}\right) = \mathbf{0.872}$$

11.11 $T_{90} = \dfrac{c_v t}{H_{\text{dr}}^2}$; U = 60%; T_v = 0.286 (Table 11.8)

$$0.286 = \frac{(2.8\times10^{-6}\ \text{m}^2/\text{min})(t)}{\left(\dfrac{3}{2}\text{m}\right)^2};\ \ t = 229{,}821\ \text{min} = \mathbf{159.6\ days}$$

11.12 The plot of e–log σ' is shown.

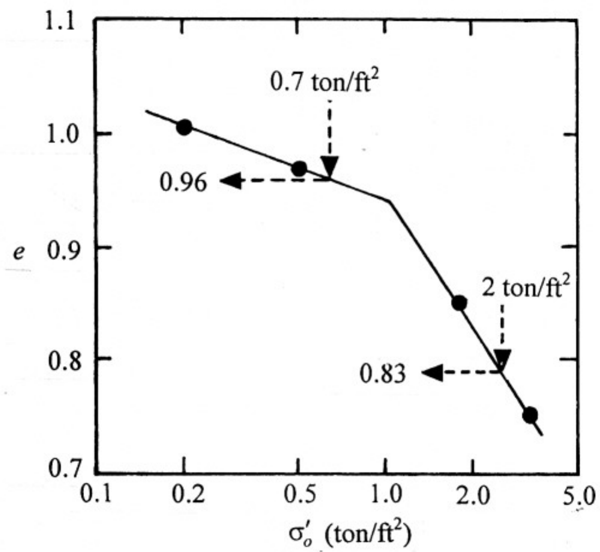
σ'_o = 0.7 ton/ft^2; e_1 = 0.96;

$\sigma'_o + \Delta\sigma'$ = 2.0 ton/ft^2; e = 0.83

Δe = 0.96 – 0.83 = 0.13

$$S_c = \frac{H\Delta e}{1+e_o} = \frac{(4.5\times12)(0.13)}{1+0.96}$$

= **3.58 in.**



11.13 a. $m_v = \dfrac{a_v}{1+e_{\text{av}}} = \dfrac{\left(\dfrac{\Delta e}{\Delta\sigma'}\right)}{1+e_{\text{av}}}$

$\Delta e = e_1 - e_2$ = 1.7 – 1.48 = 0.22

$$\Delta\sigma' = \sigma'_2 - \sigma'_1 = 400 - 150 = 250\ \text{kN/m}^2$$

$$e_{\text{av}} = \frac{1.7+1.48}{2} = 1.59$$

$$m_v = \frac{a_v}{1+e_{\text{av}}} = \frac{\left(\frac{0.22}{250}\right)}{1+1.59} = \mathbf{0.00034\ m^2/kN}$$

b. $$c_v = \frac{k}{m_v \gamma_w} = 0.002\ \text{cm}^2/\text{sec}$$

$$0.002\ \text{cm}^2/\text{sec} = \frac{k}{(0.00034 \times 100^2\ \text{cm}^2/\text{kN}\left(\frac{9.81}{100^3}\right)\text{kN/cm}^3}$$

k = **6.67 × 10^{-8} cm/sec**

11.14 In the laboratory:

$$T_{50} = \frac{c_v t_{50}}{H_{\text{dr}}^2}$$

$$0.197 = \frac{(c_v)(140\ \text{sec})}{\left(\frac{1}{2}\text{in.}\right)^2};\ c_v = 35.17 \times 10^{-5}\ \text{in.}^2/\text{sec}$$

U = 30%; T_{30} = 0.0707 (Table 11.8)

In the field:

$$T_{30} = \frac{c_v t_{30}}{H_{\text{dr}}^2}$$

$$0.0707 = \frac{(35.17 \times 10^{-5}\ \text{in.}^2/\text{sec})(t_{30})}{(8 \times 12\ \text{in.})^2}$$

T_{30} = 1,852,634 sec = **21.44 days**

11.15 a. $$m_v = \frac{a_v}{1+e_{\text{av}}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'}\right)}{1+e_{\text{av}}}$$

Δe = 1.21 – 0.96 = 0.25

$\Delta\sigma'$ = 4 – 2 = 2 ton/ft^2

$$e_{av} = \frac{1.21+0.96}{2} = 1.085$$

$$m_v = \frac{\left(\frac{0.25}{2}\right)}{1+1.085} = 0.06\ \text{ft}^2/\text{ton}$$

$$c_v = \frac{k}{m_v \gamma_w} = \frac{1.8\times10^{-4}}{(0.06\ \text{ft}^2/\text{ton})\left(\frac{62.4}{2000}\ \text{ton/ft}^3\right)} = 0.0962\ \text{ft}^2/\text{day}$$

$$t_{50} = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.286)(9)^2}{0.0962} = \mathbf{240.8\ days}$$

b. $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma_2'}{\sigma_1'}\right)} = \frac{1.21-0.96}{\log\left(\frac{4}{2}\right)} = 0.83$$

$$S_c = \frac{C_c H}{1+e_o}\log\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'}\right) = \frac{(0.83)(9)}{1+1.21}\log\left(\frac{4}{2}\right) = 1.018\ \text{ft}$$

S_c at 60% = (0.6)(1.018) = 0.611 ft ≈ **7.33 in.**

11.16 $$m_v = \frac{a_v}{1+e_{av}} = \frac{\left(\frac{\Delta e}{\Delta\sigma'}\right)}{1+e_{av}} = \frac{\left(\frac{0.73-0.61}{400-200}\right)}{1+\left(\frac{1.73+0.61}{2}\right)} = 0.3592\times10^{-3}\ \text{m}^2/\text{kN}$$

$$c_v = \frac{T_v H_{dr}^2}{t_{50}} = \frac{(0.197)\left(\frac{0.025}{2}\right)^2}{2.8} = 1.09\times10^{-5}\ \text{m}^2/\text{min}$$

$k = c_v m_v \gamma_w = (0.3592 \times 10^{-3}\ \text{m}^2/\text{kN})(1.09 \times 10^{-5}\ \text{m}^2/\text{min})(9.81\ \text{kN/m}^3)$

= $\mathbf{3.84 \times 10^{-8}}$ **m/min**

11.17 $$T_{50} = \frac{c_v t_L}{H_{(dr)L}^2} = \frac{c_v t_F}{H_{dr(F)}^2}$$

$$\frac{t_L}{H_{(dr)L}^2} = \frac{t_F}{H_{dr(F)}^2};\ \frac{225\ \text{sec}}{\left(\frac{0.025}{2}\ \text{m}\right)^2} = \frac{t_F}{(2\ \text{m})^2}$$

T_F = 5,760,000 sec = **66.7 days**

11.18 a. $T_{90} = \dfrac{c_v t_{90}}{H_{\text{dr}}^2}$; $0.848 = \dfrac{c_v(100\times 24\times 60\times 60)}{\left(\dfrac{3}{2}\times 100\text{ cm}\right)^2}$

c_v = **2.21 × 10^{-3} cm^2/sec**

b. $T_{80} = \dfrac{c_v t_{80}}{H_{\text{dr}}^2}$

From Table 11.8, U = 80%; T_{80} = 0.567

$$t_{80} = \frac{T_{80} H_{\text{dr}}^2}{c_v} = \frac{(0.567)\left(\dfrac{2.5}{2}\text{ cm}\right)^2}{2.21\times 10^{-3}} = \mathbf{400.9\ sec}$$

11.19 a. $U(\%) = \left(\dfrac{25}{80}\right)(100) = \mathbf{31.25\%}$

b. $T_v = \dfrac{c_v t}{H_{\text{dr}}^2}$; U = 50%; T_v = 0.197

$$0.197 = \frac{(0.002)(t)}{(300\text{ cm})^2}$$

t_{50} = 8,865,000 sec = **102.6 days**

c. $T_v = 0.197 = \dfrac{(0.002)(t)}{\left(\dfrac{300}{2}\right)^2}$

t_{50} = 2,216,250 sec = **25.65 days**

11.20 Eq. (11.70): $\Delta\sigma'_{\text{av}} = \dfrac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$

Eq. (10.34): $\Delta\sigma' = qI_4$

$$m_1 = \frac{L}{B} = \frac{3}{1} = 3;\ b = \frac{B}{2} = 0.5\text{ m};\ n_1 = \frac{z}{b}$$

$$q = \frac{110\ \text{kN}}{(3)(1)} = 36.7\ \text{kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m^2)	I_4 (Table 10.8)	$\Delta\sigma' = qI_4$ (kN/m^2)
3	1.5	0.5	3	36.7	0.348	12.77
3	2.75	0.5	5.5	36.7	0.15	5.51
3	4.0	0.5	8	36.7	0.079	2.9

$$\Delta\sigma'_{av} = \frac{12.77 + (4 \times 5.51) + 2.9}{6} = 6.29\ \text{kN/m}^2$$

$$\gamma_{sat(clay)} = \frac{G_s\gamma_w + wG_s\gamma_w}{1 + wG_s} = \frac{(2.7)(9.81)(1 + 0.35)}{1 + (0.35)(2.7)} = 18.38\ \text{kN/m}^3$$

$$\sigma'_o = (1.5 \times 15) + (1.5)(18 - 9.81) + (1.25)(18.38 - 9.81) = 45.5\ \text{kN/m}^2$$

$$C_c = 0.009(LL - 10) = (0.009)(38 - 10) = 0.252$$

$$S_c = \frac{(0.252)(2.5)}{1 + 0.945}\log\left(\frac{45.5 + 6.29}{45.5}\right) = 0.0812\ \text{m} = \mathbf{18.2\ mm}$$