# Chapter 10

10.1 a. $$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x$ = 162 kN/m$^2$; $\sigma_y$ = 128 kN/m$^2$; $\tau_{xy}$ = +32 kN/m$^2$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{128+162}{2} \pm \sqrt{\left(\frac{128-162}{2}\right)^2 + (32)^2}$$

$\sigma_1$= **181.23 kN/m$^2$;** $\sigma_3$ = **108.76 kN/m$^2$**

b. $$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta; \quad \theta = 35°$$

$$\sigma_n = \frac{128+162}{2} + \frac{128-162}{2}\cos[(2)(35)] + 32\sin[(2)(35)] = \mathbf{169.25\ kN/m^2}$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{128-162}{2}\sin[(2)(35)] - 32\cos[(2)(35)] = \mathbf{-26.92\ kN/m^2}$$

10.2 a. $\sigma_x$ = 72 kN/m$^2$; $\sigma_y$ = 121 kN/m$^2$; $\tau_{xy}$ = 39 kN/m$^2$; θ = 147°

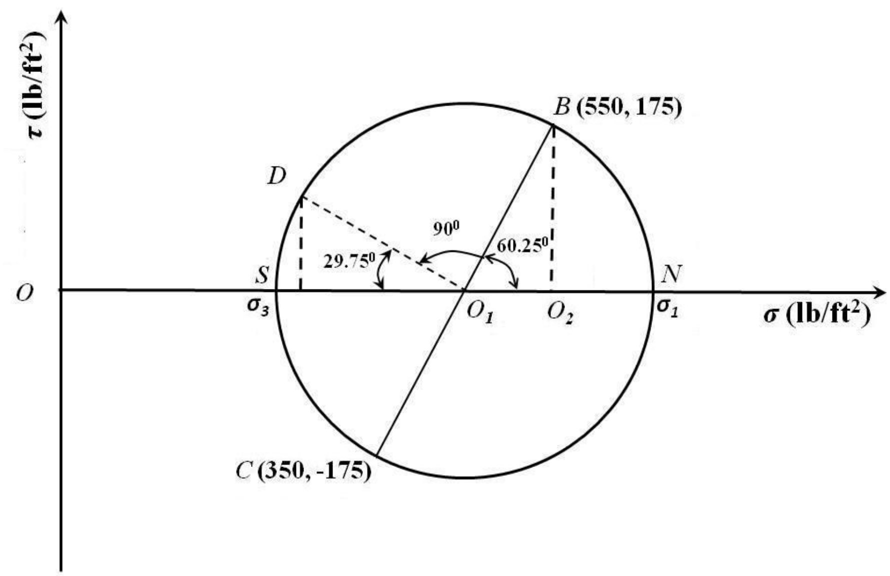
$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{121+72}{2} \pm \sqrt{\left(\frac{121-72}{2}\right)^2 + (39)^2}$$

$\sigma_1$= **142.55 kN/m$^2$;** $\sigma_3$ = **50.45 kN/m$^2$**

b. $$\sigma_n = \frac{121+72}{2} + \frac{121-72}{2}\cos[(2)(147)] + 39\sin[(2)(147)] = \mathbf{131.33\ kN/m^2}$$

$$\tau_n = \frac{121-72}{2}\sin[(2)(147)] - (39)\cos[(2)(147)] = \mathbf{-38.24\ kN/m^2}$$

10.3 a. The Mohr's circle is shown below.




$$\overline{OO_1} = \frac{550+350}{2} = 450\ \text{lb/ft}^2;\ O_1O_2 = 550 - 450 = 100\ \text{lb/ft}^2$$

$$\overline{O_1B} = \sqrt{\left(\frac{350-550}{2}\right)^2 + (-175)^2} = 201.5\ \text{lb/ft}^2$$

$$\sigma_3 = \overline{OS} = 450 - 201.5 = \mathbf{248.4\ lb/ft^2\ (+)}$$

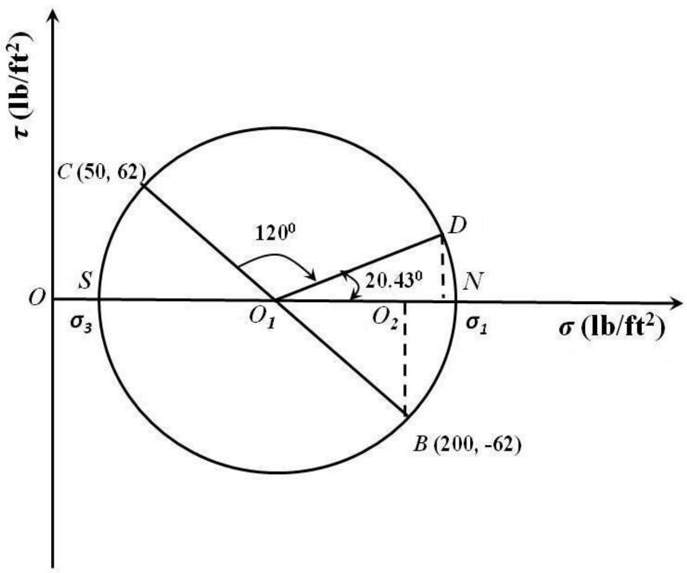
$$\sigma_1 = \overline{ON} = 450 + 201.5 = \mathbf{651.5\ lb/ft^2\ (+)}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{175}{100}\right) = 60.25°$$

b. $\sigma_n = \overline{OO_1} - \overline{O_1D}\cos(29.75) = 450 - 201.5\cos(29.75) = \mathbf{275.1\ lb/ft^2\ (+)}$

$$\tau_n = \overline{O_1D}\sin(29.75) = \mathbf{99.98\ lb/ft^2}\ (+)$$

10.4 a. The Mohr's circle is shown below.




$$\overline{OO_1} = \frac{200+50}{2} = 125 \text{ lb/ft}^2;\ O_1O_2 = 200 - 125 = 75 \text{ lb/ft}^2$$

$$\overline{O_1B} = \sqrt{(75)^2 + (62)^2} = 97.3 \text{ lb/ft}^2$$

$$\sigma_1 = \overline{ON} = 125 + 97.3 = \mathbf{222.3\ lb/ft^2}$$

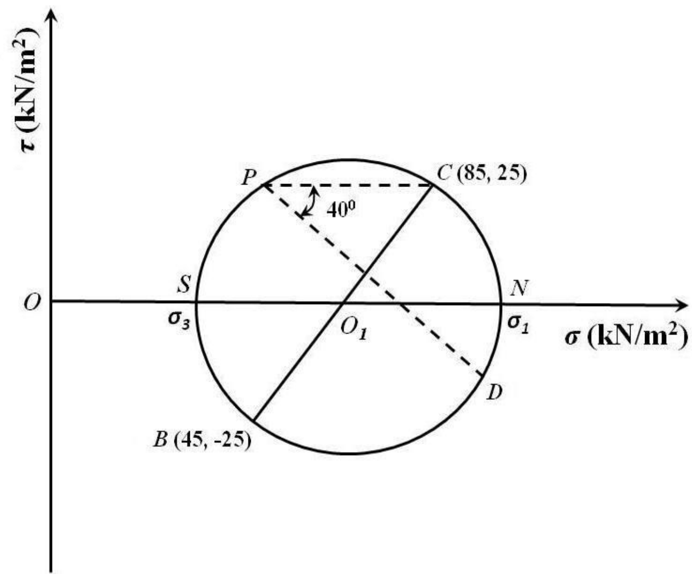
$$\sigma_3 = \overline{OS} = 125 - 97.3 = \mathbf{27.7\ lb/ft^2}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{62}{75}\right) = 39.57°$$

b. $$\sigma_n = \overline{OO_1} + \overline{O_1D}\cos(20.43) = 125 + 97.3\cos(20.43) = \mathbf{216\ lb/ft^2}$$

$$\tau_n = 97.13\sin(20.43) = \mathbf{33.9\ lb/ft^2}$$

10.5 a. The Mohr's circle is shown below.

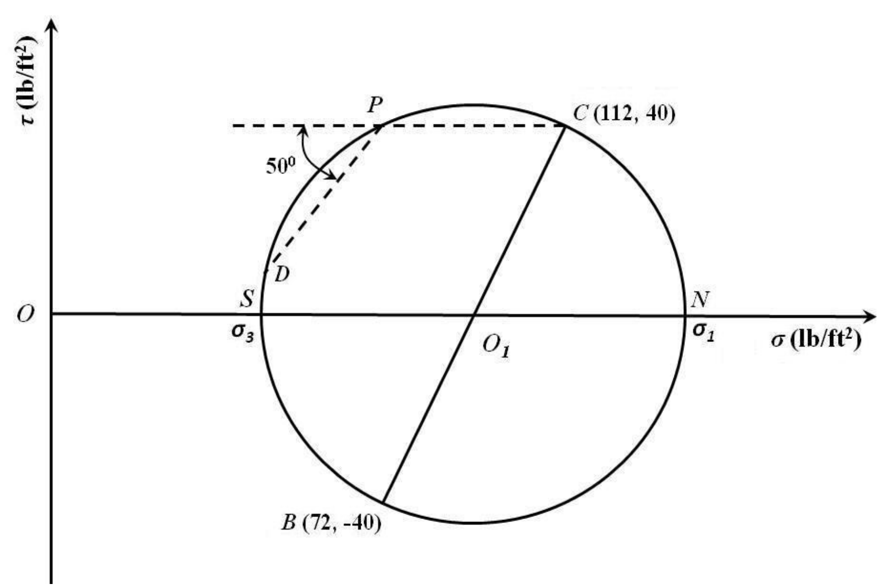



$$\sigma_1 = \overline{ON} = \mathbf{99\,kN/m^2}\,;\ \ \sigma_3 = \overline{OS} = \mathbf{33\,kN/m^2}$$

b. $\sigma_n$ and $\tau_n$ are coordinates of $D$. So

$\sigma_n \approx$ **96 kN/m²**; $\tau_n \approx$ **15.8 kN/m² (–)**

10.6 a. The Mohr's circle is shown below.



$\sigma_1 = \overline{ON} = \mathbf{136.5\ lb/ft^2}$; $\sigma_3 = \overline{OS} = \mathbf{47.3\ lb/ft^2}$

b. $\sigma_n$ and $\tau_n$ are coordinates of $D$. So

$\sigma_n \approx$ **48 lb/ft²**; $\tau_n \approx$ **10 lb/ft²**

10.7

| Load @ | $P$ (kN) | $r$ (m) | $z$ (m) | $\frac{r}{z}$ | $I_1$ (Table 10.1) | $\Delta\sigma_z = \frac{P}{z^2} I_1$ (kN/m²) |
|---|---|---|---|---|---|---|
| $B$ | 100 | 6 | 6 | 1.0 | 0.0844 | 0.234 |
| $C$ | 200 | $(6^2 + 6^2)^{0.5} = 8.48$ | 6 | 1.41 | 0.0311 | 0.173 |
| $D$ | 400 | $(6^2+3^2) = 6.708$ | 6 | 1.118 | 0.0648 | 0.72 |

$\Delta\sigma_z = \sum \mathbf{1.127\ kN/m^2}$

10.8 Eq. (10.15):

$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2} = \frac{(2)(90)(3)^3}{\pi[(6.5)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[2.5^2 + 3^2]^2}$$

$$= \mathbf{24.6\ kN/m^2}$$

10.9 Eq. (10.15): In this case, $x_2 = 0$

$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$= \frac{(2)(90)(3)^3}{\pi[(4+0)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[0^2 + 3^2]^2} = \mathbf{71.44\ kN/m^2}$$

10.10 $$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$48 = \frac{(2)(q_1)(6)^3}{\pi[19^2 + 6^2]^2} + \frac{2(930)(6)^3}{\pi[5^2 + 6^2]^2} = 34.36 + 0.00087 q_1$$

$q_1$ = **15,678 lb/ft**

10.11 $\Delta\sigma_z$ at $A$ due to $q_1 = \dfrac{2q_1 z^3}{\pi[x^2 + z^2]^2}$, or $(\Delta\sigma_z)_1 = \dfrac{(2)(292)(3)^3}{\pi[(3)^2 + (3)^2]^2} = 15.49 \text{ kN/m}^2$

Vertical component of $q_2 = q_2 \sin 45°$

$$(\Delta\sigma_z)_2 = \frac{2q_2(\sin 45)(3)^3}{\pi[(7.5)^2 + (3)^2]^2};\ (\Delta\sigma_z)_2 = 0.00285q_2$$

Horizontal component of $q_2 = q_2 \cos 45°$

From Eq. (10.17): $(\Delta\sigma_z)_3 = \dfrac{2q_2 xz^2}{\pi(x^2 + z^2)^2} = \dfrac{2q_2(\cos 45)(7.5)(3)^2}{\pi[7.5^2 + 3^2]^2} = 0.007136q_2$

Total vertical stress,

$\Delta\sigma_z = 42 \text{ kN/m}^2 = (\Delta\sigma_z)_1 + (\Delta\sigma_z)_2 + (\Delta\sigma_z)_3$

$42 = 15.49 + 0.00285q_2 + 0.007136q_2$

$q_2 = \mathbf{2656.3 \text{ kN/m}}$

10.12 $B = 36$ ft; $q = 900$ lb/ft$^2$; $x = 21$ ft; $z = 15$ ft

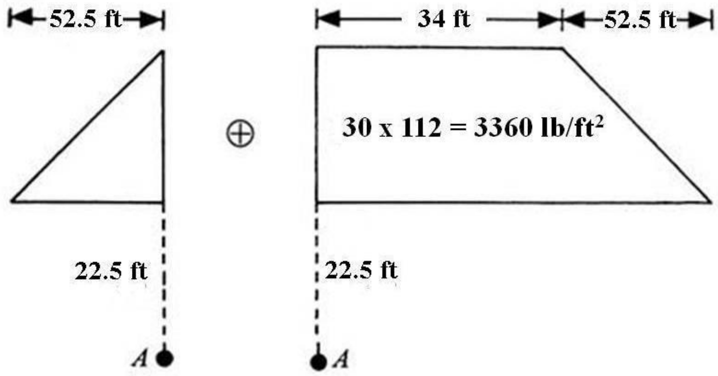
$$\frac{2x}{B} = \frac{(2)(27)}{36} = 1.5;\ \frac{2z}{B} = \frac{(2)(15)}{36} = 0.833.\ \text{From Table 10.4, } \frac{\Delta\sigma_z}{q} = 0.18$$

$\Delta\sigma_z = (0.18)(900) =$ **162 lb/ft$^2$**

10.13 $\dfrac{2x}{B} = \dfrac{(2)(0)}{6} = 0;\ \dfrac{2z}{B} = \dfrac{(2)(5)}{6} = 1.67.\ \text{From Table 10.4, } \dfrac{\Delta\sigma_z}{q} = 0.61$

$\Delta\sigma_z = (120)(0.61) =$ **73.2 kN/m$^2$**

10.14 Refer to the figure.




For the left side (with the notations given in Figure 10.19):

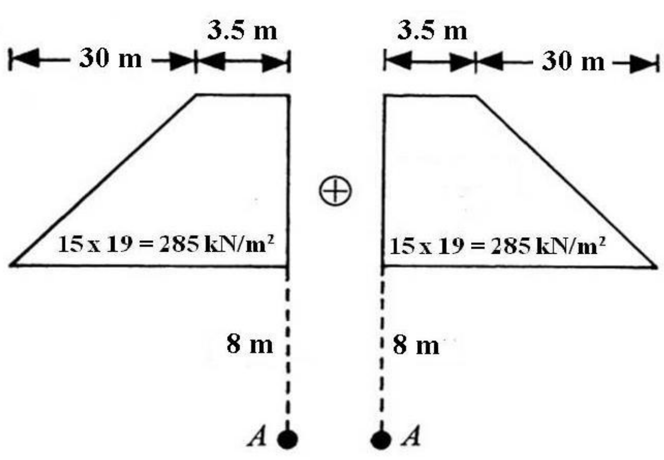
$$\frac{B_1}{z} = \frac{0}{22.5} = 0; \ \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \text{ From Figure 10.20, } I_{2(L)} = 0.375$$

For the right side:

$$\frac{B_1}{z} = \frac{34}{22.5} = 1.51; \ \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \text{ From Figure 10.20, } I_{2(R)} = 0.48$$

$\Delta\sigma_z = q[I_{2(L)} + I_{2(R)}] = (3360)(0.375 + 0.48) =$ **2872.8 lb/ft²**

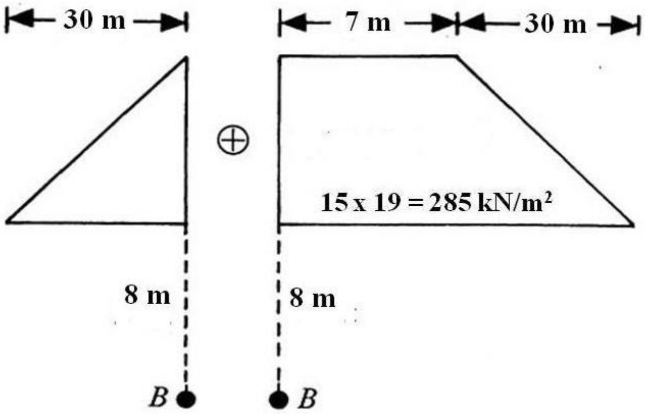
10.15 At *A*:




| For the left side: | For the right side: |
|---|---|
| $\frac{B_1}{z} = \frac{3.5}{8} = 0.437$ | $\frac{B_1}{z} = 0.437$ |
| $\frac{B_2}{z} = \frac{30}{8} = 3.75$ | $\frac{B_2}{z} = 3.75$ |
| $I_2 = 0.46$ | $I_2 = 0.46$ |

$\Delta\sigma_z = (15)(19)(0.46 + 0.46) =$ **262.2 kN/m²**

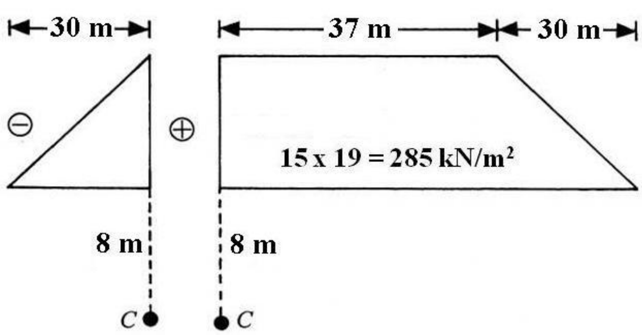
At $B$:




| For the left side: | For the right side: |
|---|---|
| $\frac{B_1}{z} = \frac{0}{8} = 0$ | $\frac{B_1}{z} = \frac{7}{8} = 0.875$ |
| $\frac{B_2}{z} = \frac{30}{8} = 3.75$ | $\frac{B_2}{z} = \frac{30}{8} = 3.75$ |
| $I_2 = 0.41$ | $I_2 = 0.48$ |

$\Delta\sigma_z = (15)(19)(0.41 + 0.48) = \mathbf{253.65\ kN/m^2}$

At $C$:




| For the left side: | For the right side: |
|---|---|
| $\frac{B_1}{z} = 0$ | $\frac{B_1}{z} = \frac{37}{8} = 4.625$ |
| $\frac{B_2}{z} = \frac{30}{8} = 3.75$ | $\frac{B_2}{z} = \frac{30}{8} = 3.75$ |
| $I_2 = 0.41$ | $I_2 = 0.5$ |

$\Delta\sigma_z = (15)(19)(0.5 - 0.41) = \mathbf{25.65\ kN/m^2}$

10.16 Eq. (10.26) and Table 10.6: $q = 2200$ lb/ft$^2$

| $R$ (ft) | $z$ (ft) | $\frac{z}{R}$ | $\frac{\Delta\sigma_z}{q}$ | $\Delta\sigma_z$ (lb/ft$^2$) |
|---|---|---|---|---|
| 12 | 0 | 0 | 1 | **2200** |
| 12 | 4 | 0.333 | 0.9634 | **2119.5** |
| 12 | 8 | 0.666 | 0.8251 | **1815.2** |
| 12 | 16 | 1.333 | 0.4983 | **1096.2** |
| 12 | 32 | 2.667 | 0.1809 | **397.9** |

10.17 Eq. (10.27) and Tables 10.7 and 10.8: $q = 380$ kN/m$^2$

| $z$ (m) | $r$ (m) | $R$ (m) | $\frac{z}{R}$ | $\frac{r}{R}$ | $A'$ | $B'$ | $\Delta\sigma_z$ (kN/m$^2$) |
|---|---|---|---|---|---|---|---|
| 3 | 0 | 5 | 0.6 | 0 | 0.48550 | 0.37831 | **328.24** |
| 3 | 1 | 5 | 0.6 | 0.2 | 0.47691 | 0.37531 | **323.84** |
| 3 | 3 | 5 | 0.6 | 0.6 | 0.40427 | 0.32822 | **278.34** |
| 3 | 5 | 5 | 0.6 | 1 | 0.25588 | 0.14440 | **152.1** |
| 3 | 7 | 5 | 0.6 | 1.4 | 0.12657 | 0.00085 | **48.42** |

10.18 Refer to the Newmark's chart.

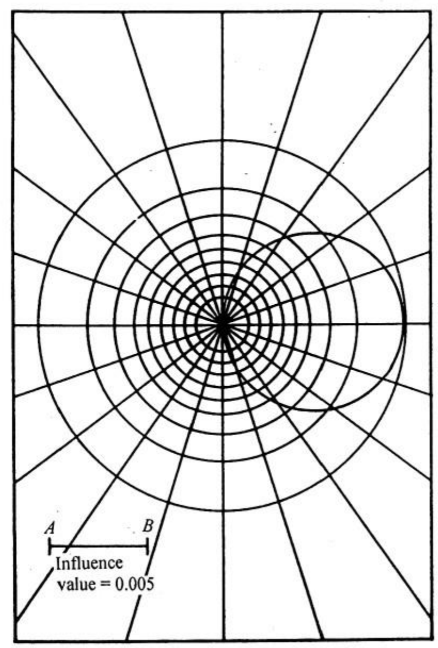
The plan is drawn to scale.

$\overline{AB} = 6$ m. $M \approx 65$.

$\Delta\sigma_z = (IV)\, q\, M$

$= (0.005)(450)(65)$

$=$ **146.25 kN/m²**




10.19 Point $A$:

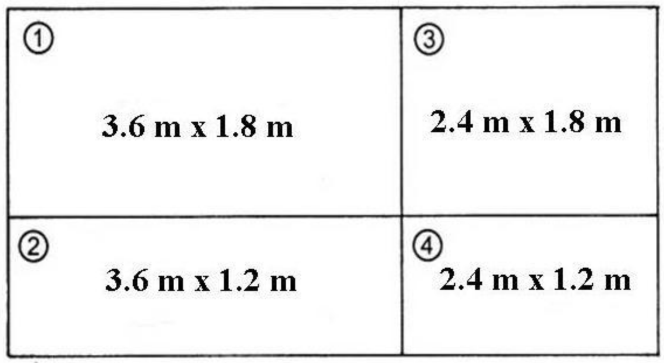
Eqs. (10.32) and (10.33): $n = \dfrac{L}{z} = \dfrac{6}{3} = 2;\; m = \dfrac{B}{z} = \dfrac{3}{3} = 1$

Eq. (10.30): $\Delta\sigma_z = q I_3$; Table 10.9: $I_3 = 0.1999$

$\Delta\sigma_z = (225)(0.1999) = 44.97$ kN/m$^2$ $\approx$ **45 kN/m²**

Point $B$:

Refer to the figure on the next page.




For rectangle 1: $m = \dfrac{3.6}{3} = 1.2;\quad n = \dfrac{1.8}{3} = 0.6;\quad I_3 = 0.1431$

For rectangle 2: $m = \dfrac{3.6}{3} = 1.2;\quad n = \dfrac{1.2}{3} = 0.4;\quad I_3 = 0.1063$

For rectangle 3: $m = \dfrac{2.4}{3} = 0.8;\quad n = \dfrac{1.8}{3} = 0.6;\quad I_3 = 0.1247$

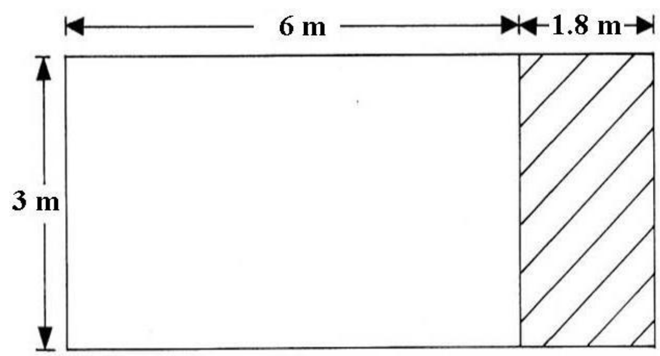
For rectangle 4: $m = \dfrac{2.4}{3} = 0.8;\quad n = \dfrac{1.2}{3} = 0.4;\quad I_3 = 0.0931$

$\Delta\sigma_z = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}] = (225)(0.1431 + 0.1063 + 0.1247 + 0.0931)$

$= \mathbf{105.12\ kN/m^2}$

Point $C$:

Refer to the figure.




$$\Delta\sigma_z = \begin{pmatrix}\text{stress at } C \text{ due to} \\ \text{area } 7.8\,\text{m} \times 3\,\text{m}\end{pmatrix} - \begin{pmatrix}\text{stress at } C \text{ due to} \\ \text{area } 1.8\,\text{m} \times 3\,\text{m}\end{pmatrix}$$

For rectangular area 7.8 m × 3 m: $m = \frac{7.8}{3} = 2.6; \quad n = \frac{3}{3} = 1; \quad I_3 = 0.2026$

For rectangular area 1.8 m × 3 m: $m = \frac{1.8}{3} = 0.6; \quad n = \frac{3}{3} = 1; \quad I_3 = 0.1361$

$\Delta\sigma_z = q(0.2026 - 0.1361) = (225)(0.2026 - 0.1361) =$ **14.96 kN/m²**

10.20 Eqs. (10.35), (10.37), (10.38), and (10.39):

$$b = \frac{B}{2} = \frac{3}{2} = 1.5\text{ m}$$

$$m_1 = \frac{L}{B} = \frac{6}{3} = 2$$

| | *z* (m) | | | | |
|---|---|---|---|---|---|
| | 2 | 4 | 6 | 8 | 10 |
| $n_1 = \frac{z}{b}$ | 1.33 | 2.66 | 4 | 5.33 | 6.66 |
| $I_4$ (Table 10.10) | 0.682 | 0.356 | 0.190 | 0.119 | 0.079 |
| $\Delta\sigma_z = qI_4$ (kN/m²) | **153.4** | **80.1** | **42.7** | **26.8** | **17.8** |

## CRITICAL THINKING PROBLEM

10.C.1

1. Vertical stress increase due to wheel load:

   $y = 0.305$ m; $R = 0.15$ m; $q = 565$ kN/m²

| Element | *r* (m) | $\frac{y}{R}$ | $\frac{r}{R}$ | $A'$ | $B'$ | $\Delta\sigma_y$ (kN/m²) |
|---|---|---|---|---|---|---|
| *A* | 0.457 | 2.03 | 3.05 | 0.02221 | 0.00028 | **12.7** |
| *B* | 0.267 | 2.03 | 1.78 | 0.05278 | 0.04391 | **54.63** |
| *C* | 0 | 2.03 | 0 | 0.10557 | 0.17889 | **160.71** |

Overburden pressure at the middle of the layer = 0.305 × 19.4 = 5.92 kN/m$^2$

Total vertical pressure, $\Delta\sigma_y$:

At $A$: $\sigma_{y\text{-}A}$ = 12.7 + 5.92 = **18.62 kN/m$^2$**

At $B$: $\sigma_{y\text{-}B}$ = 54.63 + 5.92 = **60.55 kN/m$^2$**

At $C$: $\sigma_{y\text{-}C}$ = 160.71 + 5.92 = **166.63 kN/m$^2$**

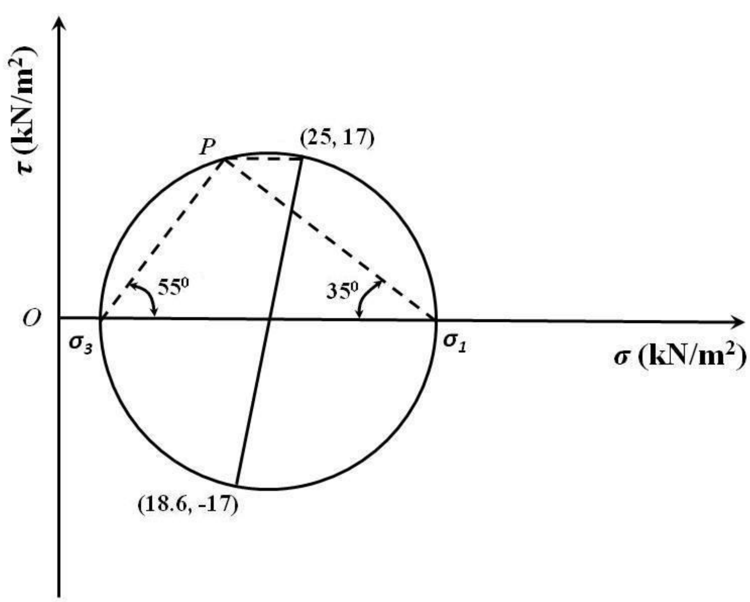
These values are entered into the following table.

| Element at | Horizontal stress, $\sigma_x$ (kN/m$^2$) | Shear stress, $\tau$ (kN/m$^2$) | Vertical stress, $\sigma_y$ (kN/m$^2$) | $\sigma_1$ (kN/m$^2$) | $\sigma_3$ (kN/m$^2$) | $\alpha_i$ (deg) |
|---|---|---|---|---|---|---|
| $A$ | 25 | 17 | **18.62** | **39** | **4.5** | **55** |
| $B$ | 32 | 45 | **60.55** | **93** | **1** | **48** |
| $C$ | 7 | 0 | **166.63** | **167** | **7** | **0** |

2. Element at $A$:

The Mohr's circle is shown. $\sigma_1 \approx$ **39 kN/m$^2$**; $\sigma_3 \approx$ **4.5 kN/m$^2$**

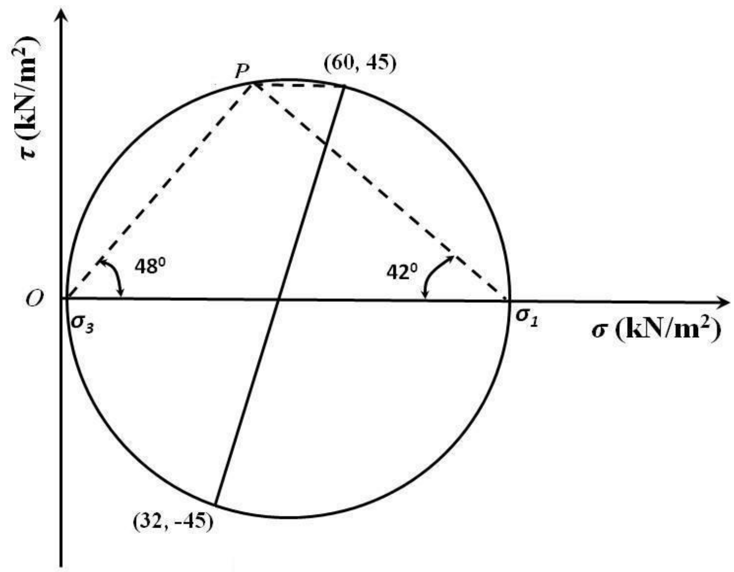
These values are entered in the above table. The pole is located at point $P$. The maximum principal stress acts on a plane which is inclined at 35° with the horizontal. Therefore, $\alpha_A$ = 90 – 35 = 55**°**.



Element at *B*:

The Mohr's circle is shown. $\sigma_1 \approx$ **93 kN/m²**; $\sigma_3 \approx$ **1 kN/m²**

These values are entered in the table on the previous page. The pole is located at point *P*. The maximum principal stress acts on a plane which is inclined at 42° with the horizontal. Therefore, $\alpha_B = 90 - 42 =$ **48°**.




Element at *C*:

Since there is no shear stress, the horizontal and vertical stresses are principal stresses. Therefore, $\sigma_1 \approx$ **167 kN/m²**; $\sigma_3 \approx$ **7 kN/m²;** $\alpha_C =$ **0°.** These values are entered in the table on the previous page.

3. The plot is shown in the figure.