Chapter 15

15.1 Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$2.75 = \frac{31}{(17.8)(H)(\cos^2 25)(\tan 25)} + \frac{\tan 28}{\tan 25}$$

$$2.75 = \frac{4.546}{H} + 1.14$$

H = **2.82 m**

15.2 Eq. (15.16):

$$H_{\text{cr}} = \frac{c'}{\gamma} \frac{1}{\cos^2 \beta(\tan \beta - \tan \phi')} = \frac{300}{115} \frac{1}{(\cos^2 30)(\tan 30 - \tan 21)} = \mathbf{17.97\ ft}$$

15.3 $\gamma' = 19.2 - 9.81 = 9.39$ kN/m^3

Eq. (15.28):

$$F_s = \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan \beta} = \frac{46}{(19.2)(11)(\cos 18)^2 (\tan 18)} + \frac{9.39}{19.2} \frac{\tan 22}{\tan 18}$$

$$= 0.741 + 0.608 \approx \mathbf{1.35}$$

15.4 $$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.73 + 0.69)(62.4)}{1 + 0.69} = 126.27 \text{ lb/ft}^3$$

$\gamma' = 126.27 - 62.4 = 63.87$ lb/ft^3

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

$$F_s = \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan \beta}$$

$$= \frac{1000}{(126.27)(27)(\cos^2 28)(\tan 28)} + \frac{63.87}{126.27} \frac{\tan 18}{\tan 28}$$

$$= 0.707 + 0.309 \approx \mathbf{1.016}$$

15.5 a. Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$F_s = \frac{21}{\left[\frac{(1950)(9.81)}{1000}\right](5)(\cos^2 18)(\tan 18)} + \frac{\tan 26}{\tan 18}$$

$$\approx \mathbf{2.25}$$

b. Eq. (15.15):

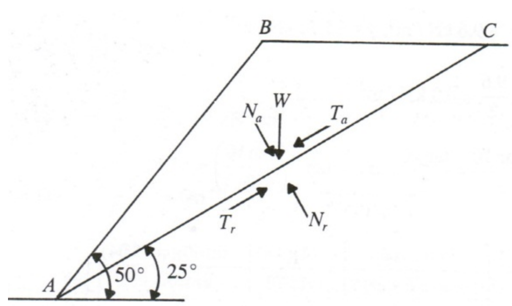
$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$1.75 = \frac{21}{\left[\frac{(1950)(9.81)}{1000}\right](H)(\cos^2 27)(\tan 27)} + \frac{\tan 26}{\tan 27}$$

$$1.75 = \frac{2.713}{H} + 0.957$$

$$H = \mathbf{3.42\ m}$$

15.6 Consider a 1-ft length of the wedge *ABC*



© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Eq. (15.29):

$$W = \frac{1}{2}H^2\gamma\left(\cot\theta - \cot\beta\right) = (0.5)(25^2)(115)\left(\cot 25 - \cot 50\right) = 46{,}913 \text{ lb}$$

$$T_a = 46913 \sin 25 = 19826.3 \text{ lb}$$

$$N_a = 46913 \cos 25 = 42517.6 \text{ lb}$$

$$T_r = \frac{1}{F_s}\left(\overline{AC}\,c' + N_a \tan\phi'\right) = \frac{1}{F_s}\left[\left(\frac{25}{\sin 25}\right)(400) + (42517.6)\tan 25\right] = \frac{43488.3}{F_s}$$

Since $T_r = T_a$,

$$\frac{43488.3}{F_s} = 19826.3$$

Therefore, $F_s = \mathbf{2.19}$

15.7 Eq. (15.42):

$$H_{\text{cr}} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right] = \left[\frac{(4)(28)}{16.5}\right]\left[\frac{(\sin 58)(\cos 14)}{1-\cos(58-14)}\right] = \mathbf{19.9\ m}$$

15.8 $F_s = 2.5$; $c'_d = \dfrac{c'}{F_s} = \dfrac{28}{2.5} = 11.2$ kN/m^2; $\phi'_d = \tan^{-1}\left(\dfrac{\tan 14}{2.5}\right) = 5.69°$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(11.2)}{16.5}\right]\left[\frac{(\sin 58)(\cos 5.69)}{1-\cos(58-5.69)}\right] = \mathbf{5.89\ m}$$

15.9 $H = \dfrac{4c'_d}{\gamma}\left[\dfrac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right]$; $\gamma = 118$ lb/ft^3

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

$F_{s(\text{assumed})}$	$\phi'_d = \tan^{-1}\left(\dfrac{\tan 22}{F_s}\right)$ (deg)	$c'_d = \dfrac{c'}{F_s}$ (lb/ft^2)	β (deg)	H (ft)
2.0	11.42	350	45	49.27
2.5	9.18	280	45	35.02
3.0	7.67	233.3	45	27
2.75	8.36	254.5	45	30.54

$F_s \approx$ **2.75**

15.10 $\rho = 1800\ \text{kg/m}^3$; $\gamma = \dfrac{(1800)(9.81)}{1000} = 17.65\ \text{kN/m}^3$; $c' = 20\ \text{kN/m}^2$; $\phi' = 17°$

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°;\ F_s = 2.3;\ c'_d = \frac{c'}{F_s} = \frac{20}{2.3} = 8.69\ \text{kN/m}^2$$

$$\phi'_d = \tan^{-1}\left(\frac{\tan\phi'}{F_s}\right) = \tan^{-1}\left(\frac{\tan 17}{2.3}\right) = 7.57°$$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(8.69)}{17.65}\right]\left[\frac{(\sin 26.57)(\cos 7.57)}{1-\cos(26.57-7.57)}\right] \approx \mathbf{16\ m}$$

15.11 $m \approx 0.185$ (From Figure 15.13). Eq. (15.48):

$$H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{26}{(18.5)(0.185)} = \mathbf{7.59\ m\ \text{--}\ Toe\ circle}$$

15.12 $m \approx 0.185$ for $\beta = 55°$ (Figure 15.13)

$$c_d = \frac{c_u}{F_s} = \frac{26}{2.5} = 10.4\ \text{kN/m}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{10.4}{(18.5)(0.185)} = \mathbf{3.04\ m}$$

15.13 $\beta = \tan^{-1}\left[\dfrac{1}{2}\right] = 26.56°$. For $\beta = 26.56°$ and $D = 1.5$, $m = 0.16$ (Figure 15.13).

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

$$c_d = \frac{c_u}{F_s} = \frac{800}{2.5} = 320 \text{ lb/ft}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{320}{(119)(0.16)} = \mathbf{16.8\ ft}$$

15.14 $H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{800}{(119)(0.16)} = \mathbf{42\ ft.}$ It is a **midpoint circle.**

15.15 a. $D = \frac{14}{10} = 1.4$; γ_{sat} = 17 kN/m^3. For β = 48° and D = 1.4, m = 0.18.

$H_{\text{cr}} = \frac{c_u}{\gamma m}$; c_u = (10)(17)(0.18) = **30.6 kN/m^2**

b. From Figure 15.13, **midpoint circle**

c. From Figure 15.15, $n \approx 0.85$.

Distance = nH = (0.85)(10) = **8.5 m**

15.16 a. $\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$

$$\frac{F_s}{\tan\phi'} = \frac{1}{\tan 12} = 4.7$$

From Figure 15.25

$$\frac{c'}{\gamma H_{\text{cr}} \tan\phi'} \approx 0.2$$

Or,

$$H_{\text{cr}} = \frac{750}{(118)(0.2)(\tan 12)} = \mathbf{149.5\ ft}$$

b. $\frac{F_s}{\tan\phi'} = \frac{1}{\tan 18} = 3.07$. $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Figure 15.25:

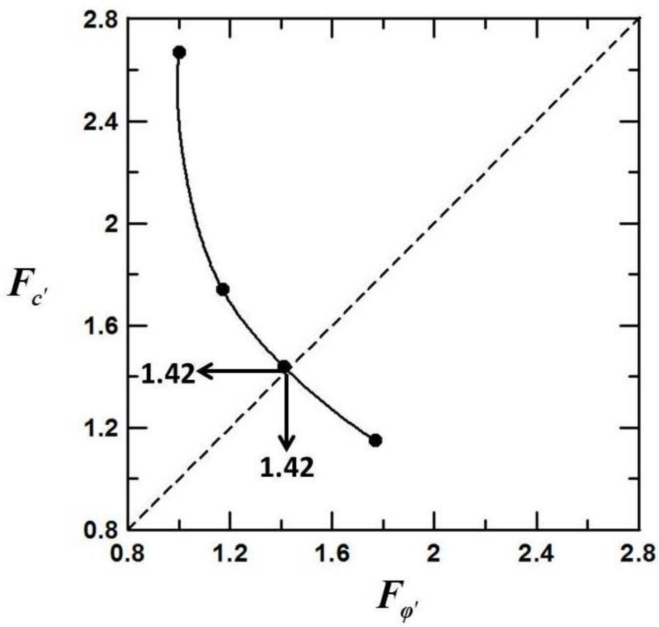
$$\frac{c'}{\gamma H_{\text{cr}} \tan\phi'} \approx 0.22; \quad \frac{30}{(17)(H_{\text{cr}})(\tan 18)} = 0.22$$

H_{cr} = **24.7 m**

15.17 $\beta = \tan^{-1}\left(\frac{1}{2.5}\right) = 21.8°$; $\phi' = 14°$; $c' = 500$ lb/ft^2; $\gamma = 120$ lb/ft^3.

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (lb/ft^2)	$F_{c'} = \frac{c'}{c'_d}$
8	1.77	0.06	432	1.15
10	1.41	0.048	346	1.44
12	1.17	0.04	288	1.74
14	1.00	0.026	187	2.67

The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown. From the figure, $F_{c'} = F_{\phi'} = F_s =$ **1.42**.



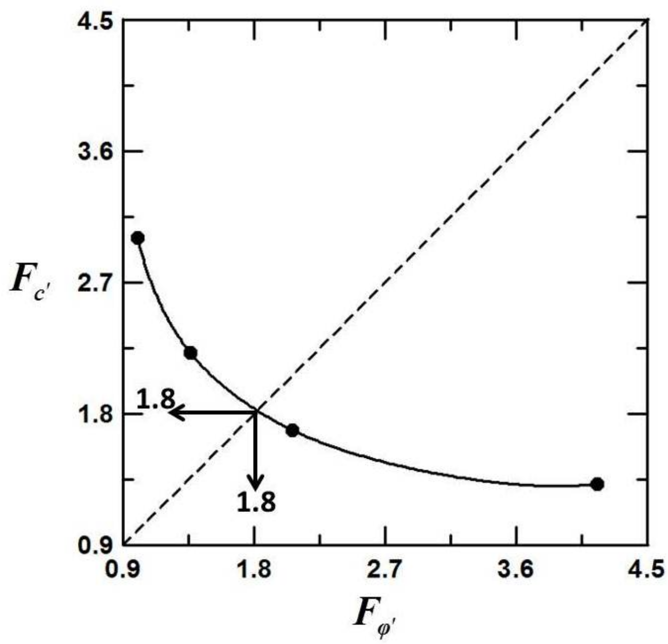
© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

15.18 $n' = 1$; $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$; $\phi' = 20°$; $c' = 32$ kN/m^2; $\gamma = 18$ kN/m^3;

$H = 10$ m

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (kN/m^2)	$F_{c'} = \frac{c'}{c'_d}$
5	4.16	0.134	24.12	1.32
10	2.06	0.105	18.9	1.69
15	1.36	0.08	14.4	2.22
20	1.00	0.059	10.62	3.01

The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown below. From the figure, $F_{\phi'} = F_{c'} = F_s =$ **1.8**.



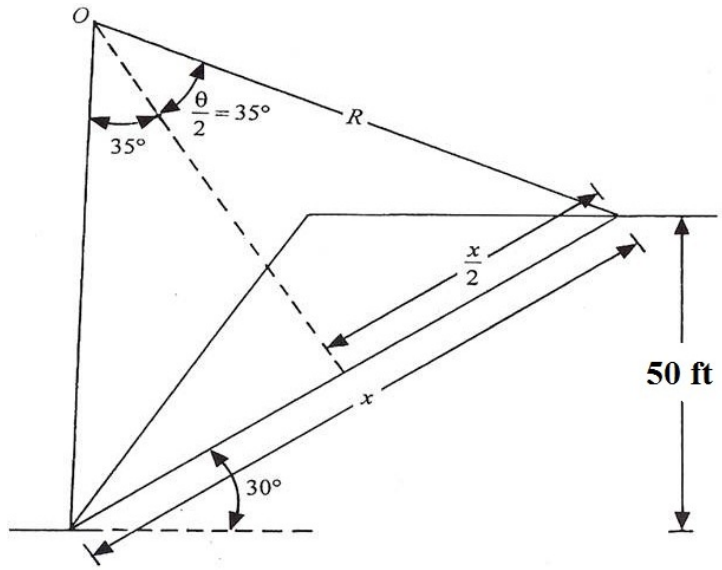
15.19 For $\beta = 21.8°$ and $\frac{c'}{\gamma H \tan\phi'} = \frac{500}{(120)(60)\tan 14} = 0.278$, the value of $\frac{\tan\phi}{F_s}$

obtained from Figure (15.27) is about 0.15. Therefore,

$F_s = \frac{\tan 14}{0.15} = \mathbf{1.66}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

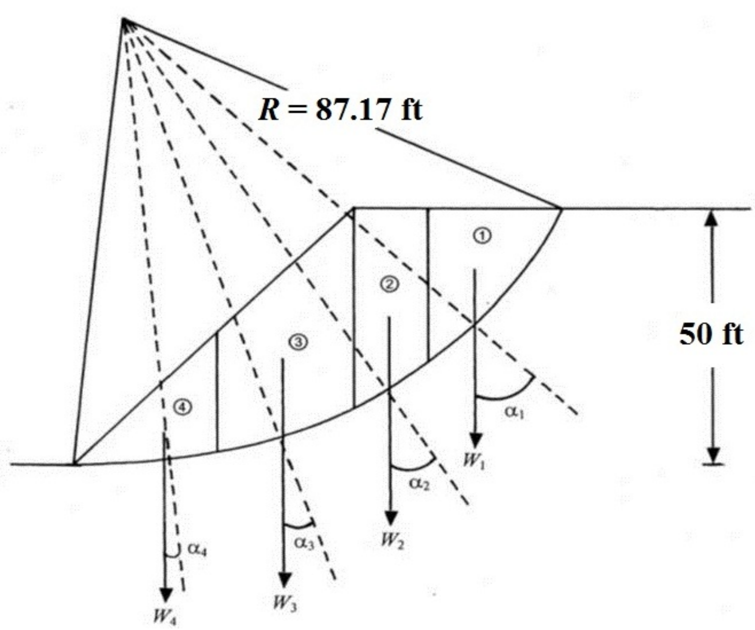
15.20 a. Refer to the figure.



$$\frac{50}{x} = \sin 30°\,;\ x = 100 \text{ ft}$$

$$\frac{50}{\text{Radius}, R} = \sin 35°\,;\ R = \frac{50}{\sin 35} = 87.17 \text{ ft}$$

With radius R = 87.17 ft, the trial surface circle has been drawn.



© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Now the following table can be prepared.

Slice	Area of slices (ft^2)	Weight of slice $W_n = A \times \gamma$ (kip/ft)	α_n (deg)	$W_n \cos \alpha_n$ (kip/ft)	$W_n \sin \alpha_n$ (kip/ft)
1	$\frac{(30)(23)}{2} = 345$	41.75	47	28.47	30.52
2	$\frac{(13)(30+41)}{2} = 462$	55.9	32	47.4	29.62
3	$\frac{(25)(41+25)}{2} = 825$	99.82	20	93.8	34.14
4	$\frac{(25)(25)}{2} = 312$	37.75	5	37.6	3.29
				$\sum$207.27	$\sum$97.57

$$F_s = \frac{R\theta c' + (\sum W_n \cos \alpha_n) \tan \phi'}{\sum W_n \sin \alpha_n}$$

$$= \frac{(87.17)\left[\left(\frac{\pi}{180}\right)(70)\right](0.65) + (207.27)(\tan 18)}{97.57} = \mathbf{1.4}$$

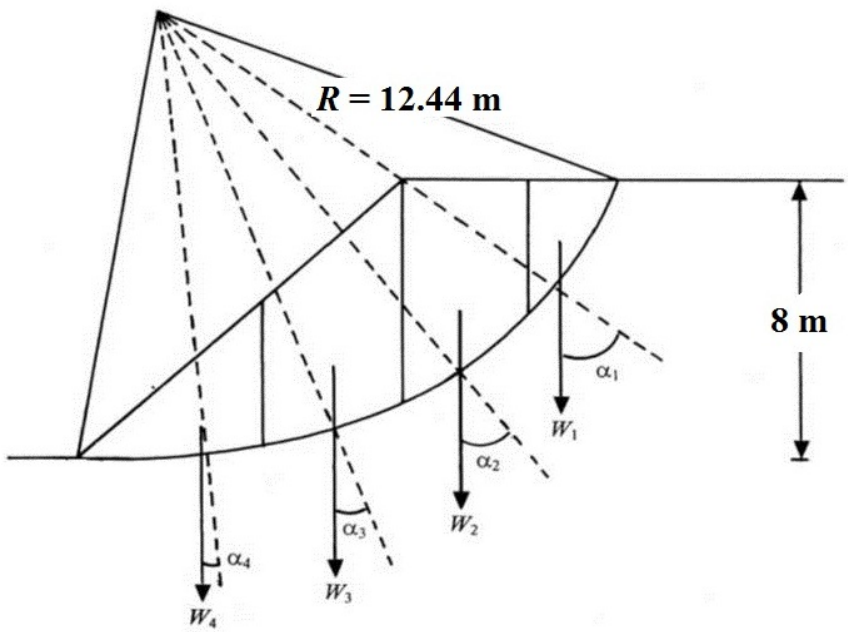
Note: The accuracy can be increased by increasing the number of slices.

b. As in Part a, $\frac{H}{x} = \sin \alpha$; $x = \frac{H}{\sin \alpha} = \frac{8}{\sin 30} = 16 \text{ m}$

$$\frac{\left(\frac{x}{2}\right)}{R} = \sin \frac{\theta}{2}, \text{ or } \frac{8}{\sin 40} = R = 12.44 \text{ m}$$

With a radius R = 12.44 m, the trial surface has been drawn on the next page.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.



The following table can now be prepared.

Slice	Area of slices (m^2)	Weight of slice $W_n = A \times \gamma$ (kN/m)	α_n (deg)	$W_n \cos \alpha_n$ (kN/m)	$W_n \sin \alpha_n$ (kN/m)
1	$\frac{(2.36)(3.85)}{2} = 4.54$	77.18	54	45.36	62.44
2	$\frac{(3.85+6.51)(3.25)}{2} = 16.83$	286.11	38	225.45	176.14
3	$\frac{(6.51+4.14)(3.55)}{2} = 18.9$	321.3	20	301.92	109.89
4	$\frac{(4.14)(4.74)}{2} = 9.81$	166.77	6	165.85	17.43
				$\sum 738.58$	$\sum 365.9$

$$F_s = \frac{R\theta c' + (\sum W_n \cos\alpha_n)\tan\phi'}{\sum W_n \sin\alpha_n}$$

$$= \frac{(12.44)\left[\left(\frac{\pi}{180}\right)(80)\right](27) + (738.58)(\tan 20)}{365.9} = \mathbf{2.01}$$

Note: The accuracy will improve with smaller slices.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

15.21 $\phi' = 25°$; $\beta = 26.56°$; $r_u = 0.5$; $\dfrac{c'}{\gamma H} = \dfrac{20}{(19)(14)} \approx 0.075$.

Using Table 15.3, the following table can be prepared.

D	m'	n'	$F_s = m' - n'r_u$
Toe circle	1.853	1.430	1.138
1.00	1.872	1.386	1.179
1.25	2.004	1.641	1.183
1.50	2.308	1.914	1.351

$F_s \approx$ **1.14**.

15.22 $\phi' = 20°$; $\beta = 18.43°$; $r_u = 0.5$; $\dfrac{c'}{\gamma H} = \dfrac{475}{(118)(40)} = 0.1$

Using Table 15.3, the following table can be prepared.

D	m'	n'	$F_s = m' - n'r_u$
Toe circle	2.286	1.588	1.49
1.00	2.421	1.472	1.68
1.25	2.283	1.558	1.50
1.50	2.387	1.742	1.51

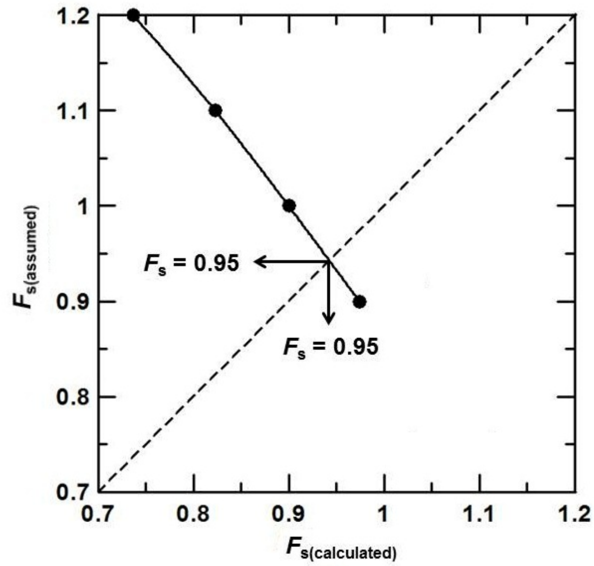
$F_s \approx$ **1.49**.

15.23 $\beta = 26°$; $\phi' = 21°$; $r_u = 0.5$; $\gamma = 19$ kN/m^3; $c' = 21$ kN/m^2; $H = 17$ m

$F_{s(\text{assumed})}$	$\dfrac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \dfrac{\tan\phi'}{\tan\phi'_d}$
1.2	0.0541	27.5	0.737
1.1	0.0591	25	0.823
1.0	0.065	23	0.9
0.9	0.0722	21.5	0.974

From the plot on the next page, $F_s \approx$ **0.95**.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.



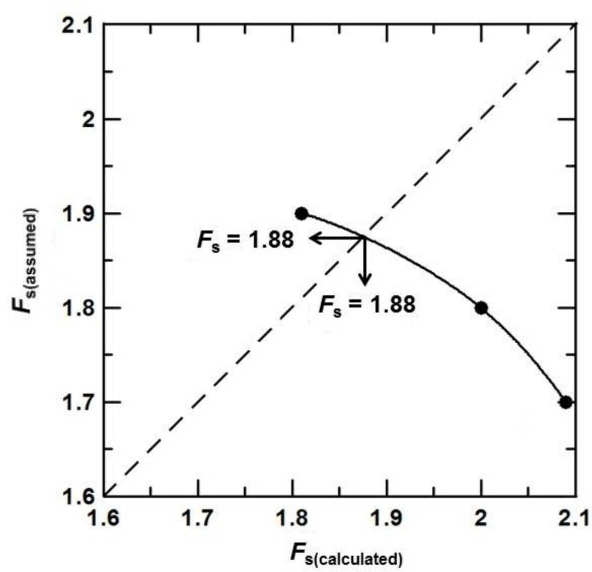
15.24 $\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$; $\phi' = 24°$; $c' = 27$ kN/m^2; $\gamma = 17.5$ kN/m^3; $r_u = 0.25$; $H = 18$ m

a. Spencer's solution (Figure 15.35):

$F_{s(\text{assumed})}$	$\dfrac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \dfrac{\tan\phi'}{\tan\phi'_d}$
1.7	0.0504	12	2.09
1.8	0.0476	12.5	2.0
1.9	0.045	13.8	1.81

From the graph,

$F_s \approx$ **1.88**.



© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

b. Michalowski's solution (Figure 15.36)

$\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$; $\phi' = 24°$; $c' = 27$ kN/m^2; $\gamma = 17.5$ kN/m^3; $r_u = 0.25$;

$H = 18$ m

$$\frac{c'}{\gamma H \tan\phi'} = \frac{27}{(17.5)(18)\tan 24} 0.192$$

Figure (15.36): $\frac{F_s}{\tan 24} \approx 4.5$; Therefore, $F_s \approx \mathbf{2.0}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.