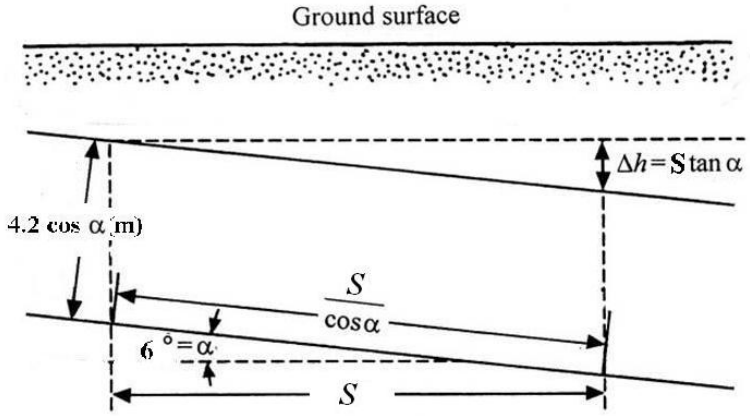
Chapter 7

7.1



From the figure: $i = \dfrac{\text{head loss}}{\text{length}} = \dfrac{S\tan\alpha}{\left(\dfrac{S}{\cos\alpha}\right)} = \sin\alpha$

$q = k\,i\,A = (k)(\sin\alpha)(4.2\cos\alpha)(1)$

$k = 4.8 \times 10^{-3}$ cm/sec $= 4.8 \times 10^{-5}$ m/sec

$$q = (4.8\times10^{-5})(\sin 6°)(4.2\cos 6°)\underbrace{(3600)}_{\substack{\text{to change}\\ \text{to m/hr}}} = 0.0754\ \text{m}^3\text{/hr/m}$$

$\approx$ **7.54×10^{-2} m^3/hr/m**

7.2 $i = \dfrac{h}{\left(\dfrac{S}{\cos\alpha}\right)}$

$$q = kiA = k\left(\frac{h\cos\alpha}{S}\right)(H_1\cos\alpha\times 1) = \left(\frac{0.075}{\underbrace{10^2}_{\text{m/sec}}}\right)\left(\frac{2.75\cos 14}{30}\right)(2\cos 14)$$

= **1.29×10^{-4} m^3/sec/m**

7.3 a. $k = \dfrac{QL}{Aht} = \dfrac{(350\,\text{cm}^2)(30\,\text{cm})}{(176.71\,\text{cm}^2)(50\,\text{cm})(300\,\text{sec})} = \mathbf{3.96 \times 10^{-3}\ cm/sec}$

b. $v_s = v\left(\dfrac{1+e}{e}\right); \quad v = ki$

$$v_s = ki\left(\frac{1+e}{e}\right) = (0.00396)\left(\frac{50\,\text{cm}}{30\,\text{cm}}\right)\left(\frac{1+0.61}{0.61}\right) = \mathbf{0.0174\ cm/sec}$$

7.4 $k = \dfrac{QL}{Aht}; \quad 0.062 = \dfrac{(160)(15)}{(31.67)(h)(60)}$

$h = \mathbf{20.37\ cm}$

7.5 a. $k = 2.303\left(\dfrac{aL}{At}\right)\log_{10}\left(\dfrac{h_1}{h_2}\right) = (2.303)\left[\dfrac{(0.25)(15)}{(19.64)(8\times 60)}\right]\log_{10}\left(\dfrac{40}{20}\right)$

$= \mathbf{2.75 \times 10^{-4}\ cm/sec}$

b. $k = 2.303\left(\dfrac{aL}{At}\right)\log_{10}\left(\dfrac{h_1}{h_2}\right)$

$$0.000275 = (2.303)\left[\frac{(0.25)(15)}{(19.64)(6\times 60)}\right]\log_{10}\left(\frac{40}{h_2}\right) = 0.00122\log\left(\frac{40}{h_2}\right)$$

$h_2 = \mathbf{23.82\ cm}$

7.6 a. $k = 2.303\left(\dfrac{aL}{At}\right)\log_{10}\left(\dfrac{h_1}{h_2}\right)$

$$= (2.303)\left(\frac{0.97\times 50}{16\times 10}\right)\log_{10}\left(\frac{41}{18.5}\right) = 0.241\,\text{cm/min} = 0.00402\,\text{cm/sec}$$

$$\overline{K} = \frac{k\eta}{\gamma_w} = \frac{(4.02\times 10^{-5}\ \text{m/sec})(1.005\times 10^{-3}\ \text{N}\cdot\text{s/m}^2)}{9.789\times 10^{3}\ \text{N/m}^3} = \mathbf{4.13 \times 10^{-12}\ m^2}$$

b. $k = 2.303\left(\frac{aL}{At}\right)\log_{10}\left(\frac{h_1}{h_2}\right)$

$$0.241\text{ cm/min} = (2.303)\left(\frac{0.97\times 50}{16\times 7}\right)\log_{10}\left(\frac{41}{h_2}\right)$$

$h_2 = \mathbf{23.5\text{ cm}}$

7.7 From Eq. (7.15) and Table 7.2 for $T = 28^\circ$ C:

$$k_{20^\circ C} = \left(\frac{\eta_{T^\circ C}}{\eta_{20^\circ C}}\right)k_{T^\circ C} = (0.832)(0.009) = \mathbf{7.49\times 10^{-3}\text{ cm/sec}}$$

7.8 Eq. (7.31): $\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}$, or $\frac{0.03}{k_2} = \frac{\left(\frac{0.62^3}{1+0.62}\right)}{\left(\frac{0.48^3}{1+0.48}\right)} = \frac{0.1471}{0.0747}$

$k_2 = \mathbf{0.015\text{ cm/sec}}$

7.9 $e = e_{max} - (e_{max} - e_{min})D_r = 0.68 - (0.68 - 0.42)(0.52) = 0.544$

Eq. (7.32): $k\text{ (cm/sec)} = 2.4622\left[D_{10}^2\frac{e^3}{1+e}\right]^{0.7825}$

$$= (2.4622)\left[(0.4)^2\left(\frac{0.544^3}{1+0.544}\right)\right]^{0.7825} = \mathbf{0.1\text{ cm/sec}}$$

7.10 $e_1 = e_{max} - (e_{max} - e_{min})D_r = 0.72 - (0.72 - 0.46)(0.8) = 0.512$

$e_2 = 0.72 - (0.72 - 0.46)(0.67) = 0.545$

$$\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}},\quad \text{or}\quad \frac{0.006}{k_2} = \frac{\left(\frac{0.512^3}{1+0.512}\right)}{\left(\frac{0.545^3}{1+0.545}\right)} = \frac{0.0887}{0.1047}$$

k_2 = **7.08×10^{-3} cm/sec**

7.11 $n_1 = 0.36; \quad e_1 = \dfrac{n_1}{1-n_1} = \dfrac{0.36}{1-0.36} = 0.562$

$n_2 = 0.48; \quad e_2 = \dfrac{0.48}{1-0.48} = 0.923$

Eq. (7.31): $k_2 = k_1\left(\dfrac{e_2^3}{1+e_2}\right)\left(\dfrac{1+e_1}{e_1^3}\right) = k_1\left(\dfrac{1+e_1}{1+e_2}\right)\left(\dfrac{e_2}{e_1}\right)^3 = (0.072)\left(\dfrac{1.562}{1.923}\right)\left(\dfrac{0.923}{0.562}\right)^3$

k_2 = **0.259 cm/sec**

7.12 $\gamma_{d(\text{field})} = R\gamma_{d(\max)} = (0.9)(16) = 14.4 \text{ kN/m}^3$

$e = \dfrac{G_s\gamma_w}{\gamma_{d(\text{field})}} - 1 = \dfrac{(2.7)(9.81)}{14.4} - 1 = 0.839$

Eq. (7.34): $k \text{ (cm/sec)} = 35\left(\dfrac{e^3}{1+e}\right)(C_u^{0.6})(D_{10} \text{ mm})^{2.32}$

$= 35\left(\dfrac{0.839^3}{1+0.839}\right)(3.1)^{0.6}(0.23)^{2.32} =$ **0.732 cm/sec**

7.13

Sieve no.	Opening (cm)	Percent passing	Fraction between two consecutive sieves (%)
30	0.06	100	
			{– – – – 27
40	0.0425	73	
			{– – – – 14
60	0.02	59	
			{– – – – 36
100	0.015	23	
			{– – – – 23
200	0.0075	0	

For fraction between sieve Nos. 30 and 40: $\dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \dfrac{27}{0.06^{0.404} \times 0.0425^{0.595}} = 550.93$

For fraction between sieve Nos. 40 and 60:

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{14}{0.0425^{0.404} \times 0.02^{0.595}} = 514.21$$

For fraction between sieve Nos. 60 and 100:

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{36}{0.02^{0.404} \times 0.015^{0.595}} = 2127.71$$

For fraction between sieve Nos. 100 and 200:

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{23}{0.015^{0.404} \times 0.0075^{0.595}} = 2306.35$$

$$\sum \frac{\sum 100\%}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{100}{550.93 + 514.21 + 2127.71 + 2306.35} = 0.0182$$

$$k = (1.99 \times 10^4)(0.0182)^2 \left(\frac{1}{7.5}\right)^2 \left(\frac{0.68^3}{1 + 0.68}\right) = \mathbf{0.0219\ cm/sec}$$

7.14

$$\frac{k_1}{k_2} = \left(\frac{e_1^n}{1 + e_1}\right)\left(\frac{1 + e_2}{e_2^n}\right) = \left(\frac{1 + e_2}{1 + e_1}\right)\left(\frac{e_1}{e_2}\right)^n$$

$$\frac{0.2 \times 10^{-6}}{0.91 \times 10^{-6}} = \left(\frac{2.6}{1.95}\right)\left(\frac{0.95}{1.6}\right)^n ; \quad 0.1648 = 0.593^n$$

$$n = \frac{\log 0.1648}{\log 0.593} = 3.45$$

$$k_1 = C\left(\frac{e_1^n}{1 + e_1}\right); \quad C = \frac{(0.2 \times 10^{-6})(1 + 0.95)}{0.95^{3.45}} = 4.655 \times 10^{-7}\ \text{cm/sec}$$

$$k_3 = \left(\frac{1.1^{3.45}}{2.1}\right)(4.655 \times 10^{-7}) = \mathbf{3.08 \times 10^{-7}\ cm/sec}$$

7.15 Eq. (7.37): $\log k = A' \log e + B'$

$$A' = \frac{\log k_1 - \log k_2}{\log e_1 - \log e_2} = \frac{\log(0.2 \times 10^{-6}) - \log(0.91 \times 10^{-6})}{\log(0.95) - \log(1.6)} = 2.9$$

$$B' - \log k_1 - A' \log e_1 = \log(0.2 \times 10^{-6}) - 2.9 \log(0.95) = -6.634$$

$$\log (k_3) = (2.9)\log(1.1) - 6.634 = -6.514$$

$k_3 = \mathbf{3.062 \times 10^{-7}\ cm/sec}$

7.16 $k_{H(\text{eq})} = \dfrac{1}{H}(k_1H_1 + k_2H_2 + ...)$

$$k_{H(\text{eq})} = \frac{1}{4}[(10^{-4})(1) + (2.8\times10^{-2})(1) + (3.5\times10^{-5})(2)] = 7.042\times10^{-3}\ \text{cm/sec}$$

$$k_{V(\text{eq})} = \frac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + ...} = \frac{4}{\dfrac{1}{10^{-4}} + \dfrac{1}{2.8\times10^{-2}} + \dfrac{2}{3.5\times10^{-5}}}$$

$$= 5.95\times10^{-5}\ \text{cm/sec}$$

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{7.042\times10^{-3}}{5.95\times10^{-5}} = \mathbf{118.35}$$

7.17 $q = kiA$

$$i = \frac{160-150}{125} = 0.08$$

$q = 250\ \text{m}^3/\text{day}$

$A = 2\times500 = 1000\ \text{m}^2$

$$k = \frac{q}{iA} = \frac{250}{(0.08)(1000)} = \mathbf{3.125\ m/day}$$

CRITICAL THINKING PROBLEM

7.C.1 a. $$k_{v(\text{eq})} = \frac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + \dfrac{H_3}{k_3}} = \frac{60}{\dfrac{20}{5\times10^{-3}} + \dfrac{20}{4.2\times10^{-2}} + \dfrac{20}{3.9\times10^{-4}}} = 1.076\times10^{-3}\ \text{cm/sec}$$

$$q = k_{v(\text{eq})}iA = (0.001076)\left(\frac{47}{60}\right)\left(\frac{\pi}{4}\right)(15^2) = 0.149\ \text{cm}^3/\text{sec} = \mathbf{536.4\ cm^3/hr}$$

b. $x = 0$ mm

$Z = \mathbf{-220\ mm}$;

$$\frac{u}{\gamma_w} = 470 + 220 = \mathbf{690\ mm}$$

$$h = \frac{u}{\gamma_w} + Z = 690 - 220 = \mathbf{470\ mm}$$

$x = 200$ mm

$Z = \mathbf{-220\ mm}$

$$k_{v(\text{eq})} i = k_1 i_1$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.005)\left(\frac{\Delta H}{20}\right)$$

$\Delta H = 3.371$ cm $= 33.71$ mm

Therefore, $h = 470 - 33.71 = \mathbf{436.29\ mm}$

$$\frac{u}{\gamma_w} = 436.29 - (-220) = \mathbf{656.29\ mm}$$

$x = 400$ mm

$Z = \mathbf{-220\ mm}$

$$k_{v(\text{eq})} i = k_2 i_2$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.042)\left(\frac{\Delta H}{20}\right)$$

$\Delta H = 0.4$ cm $= 4$ mm

Therefore, $h = 436.29 - 4 = \mathbf{432.29\ mm}$

$$\frac{u}{\gamma_w} = 432.29 - (-220) = \mathbf{652.29\ mm}$$

$\underline{x = 600 \text{ mm}}$

$Z = \mathbf{-220\ mm}$

$k_{v(\text{eq})} i = k_3 i_3$

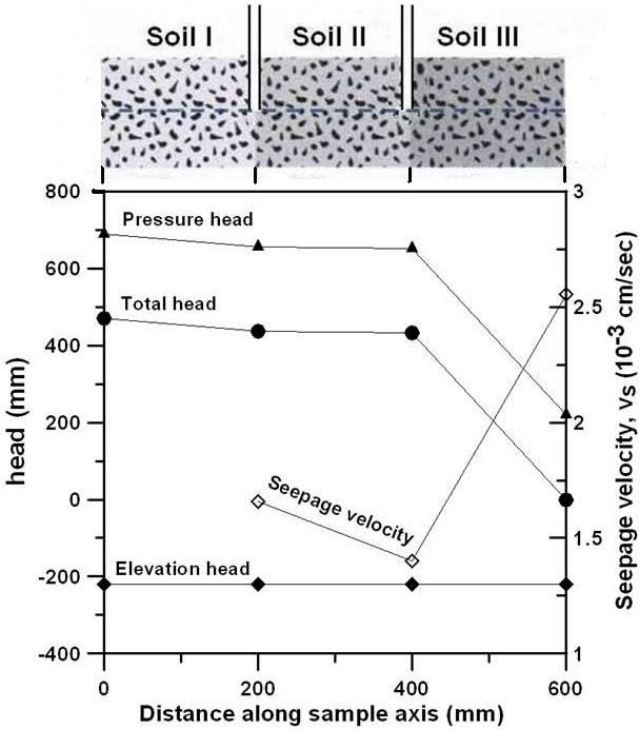
$$(0.001076)\left(\frac{47}{60}\right) = (0.00039)\left(\frac{\Delta H}{20}\right)$$

$\Delta H = 43.224 \text{ cm} = 432.24 \text{ mm}$

Therefore, $h = 432.29 - 432.24 \approx \mathbf{0\ mm}$

$$\frac{u}{\gamma_w} = 0 - (-220) = \mathbf{220\ mm}$$

c. The variation of heads with distance is shown in the following figure.



d. $v = k_{v(\text{eq})} i = (0.001076)\left(\frac{47}{60}\right) = 0.000843 \text{ cm/sec}$

$$v_s = \frac{v}{n}$$

Soil I: $v_s = \frac{0.000843}{0.5} = \mathbf{0.00168\ cm/sec}$

Soil II: $v_s = \frac{0.000843}{0.6} = \mathbf{0.0014\ cm/sec}$

Soil III: $v_s = \frac{0.000843}{0.33} = \mathbf{0.00255\ cm/sec}$

These values are plotted in the figure on the previous page.

e. Height of water column is equal to the piezometric or pressure head at a point.
Therefore: height of water in A = pressure head at x = 200 mm = **656.29 mm**
height of water in B = pressure head at x = 400 mm = **652.29 mm**