Chapter 10

10.1 $$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

σ_x = 60 kN/m^2; σ_y = 100 kN/m^2; τ_{xy} = +45 kN/m^2

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{100+60}{2} \pm \sqrt{\left(\frac{100-60}{2}\right)^2 + (45)^2}$$

σ_1= **129.24 kN/m^2;** σ_3 = **30.76 kN/m^2**

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta; \quad \theta = 150°$$

$$\sigma_n = \frac{100+60}{2} + \frac{100-60}{2}\cos[(2)(150)] + 45\sin[(2)(150)] = \mathbf{51.03\ kN/m^2}$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{100-60}{2}\sin[(2)(150)] - 45\cos[(2)(150)] = \mathbf{39.82\ kN/m^2}$$

10.2 σ_x = 750 lb/ft^2; σ_y = 400 lb/ft^2; τ_{xy} = –300 lb/ft^2; θ = 45°

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{400+750}{2} \pm \sqrt{\left(\frac{400-750}{2}\right)^2 + (-300)^2}$$

σ_1= **922.3 lb/ft^2;** σ_3 = **227.7 lb/ft^2**

$$\sigma_n = \frac{400+750}{2} + \frac{400-750}{2}\cos 90 - 300\sin 90 = \mathbf{275\ lb/ft^2}$$

$$\tau_n = \frac{400-750}{2}\sin 90 - (-300)\cos 90 = \mathbf{-175\ lb/ft^2}$$

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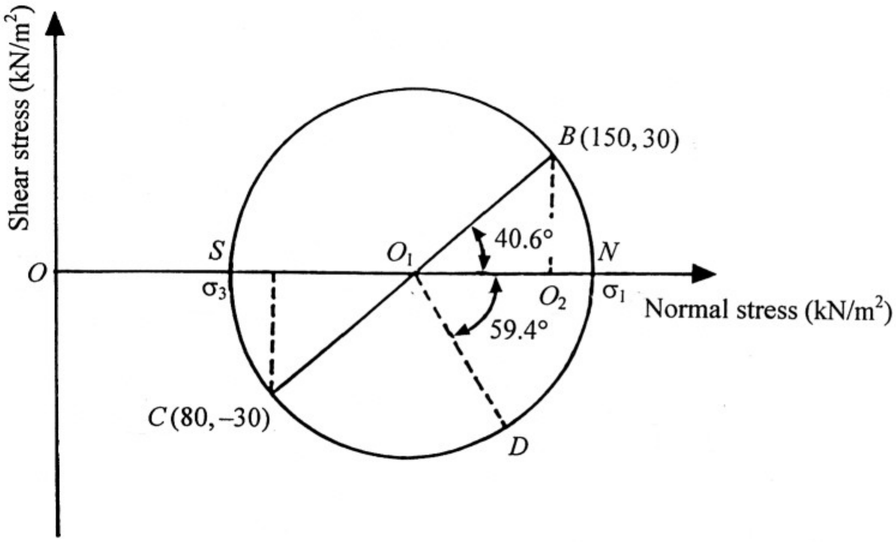
10.3 The Mohr's circle is shown.



$$\overline{OO_1} = \frac{150+80}{2} = 115\,\text{kN/m}^2;\ O_1O_2 = 150-115 = 35\,\text{kN/m}^2$$

$$\overline{O_1B} = \sqrt{\left(\frac{150-80}{2}\right)^2 + (30)^2} = 46.1\,\text{kN/m}^2$$

$$\sigma_3 = \overline{OS} = 115 - 46.1 = \mathbf{68.9\,kN/m^2\ (+)}$$

$$\sigma_1 = \overline{ON} = 115 + 46.1 = \mathbf{161.1\,kN/m^2\ (+)}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{30}{35}\right) = 40.6°$$

$$\sigma_n = \overline{OO_1} + \overline{O_1D}\cos 59.4 = 115 + 46.1\cos 59.4 = \mathbf{138.5\,kN/m^2\ (+)}$$

$$\tau_n = \overline{O_1D}\sin 59.4 = \mathbf{39.7\,kN/m^2}\ \mathbf{(-)}$$

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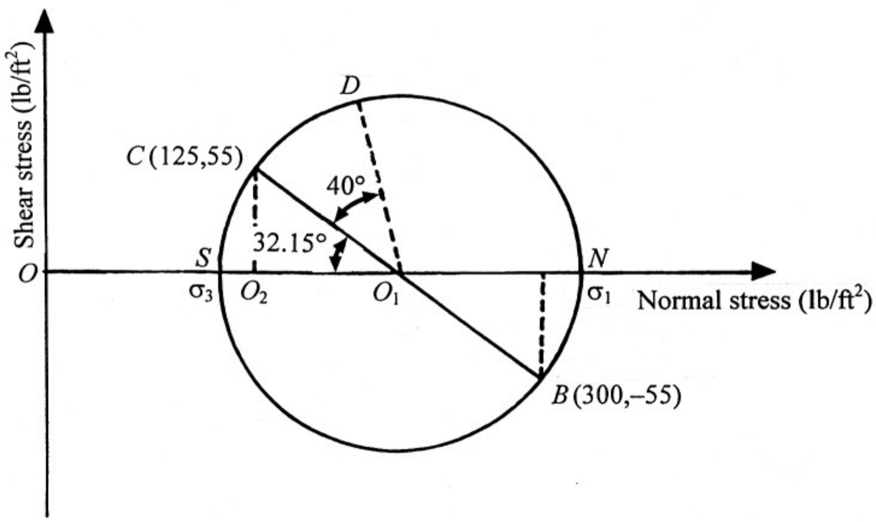
10.4 The Mohr's circle is shown.



$$\overline{OO_1} = \frac{300+125}{2} = 212.5\,\text{lb/ft}^2 \quad O_1O_2 = 212.5 - 125 = 87.5\,\text{lb/ft}^2$$

$$\overline{O_1B} = \sqrt{(87.5)^2 + (55)^2} = 103.35\,\text{lb/ft}^2$$

$$\sigma_1 = \overline{ON} = 212.5 + 103.35 = \mathbf{315.85\ lb/ft^2}$$

$$\sigma_3 = \overline{OS} = 212.5 - 103.34 = \mathbf{109.15\ lb/ft^2}$$

$$\angle CO_1O_2 = \tan^{-1}\left(\frac{55}{87.5}\right) = 32.15°$$

$$\sigma_n = \overline{OO_1} - \overline{O_1D}\cos(32.15 + 40) = 212.5 - 103.35\cos 72.15 = \mathbf{180.8\ lb/ft^2}$$

$$\tau_n = 103.35\sin 72.15 = \mathbf{98.4\ lb/ft^2}$$

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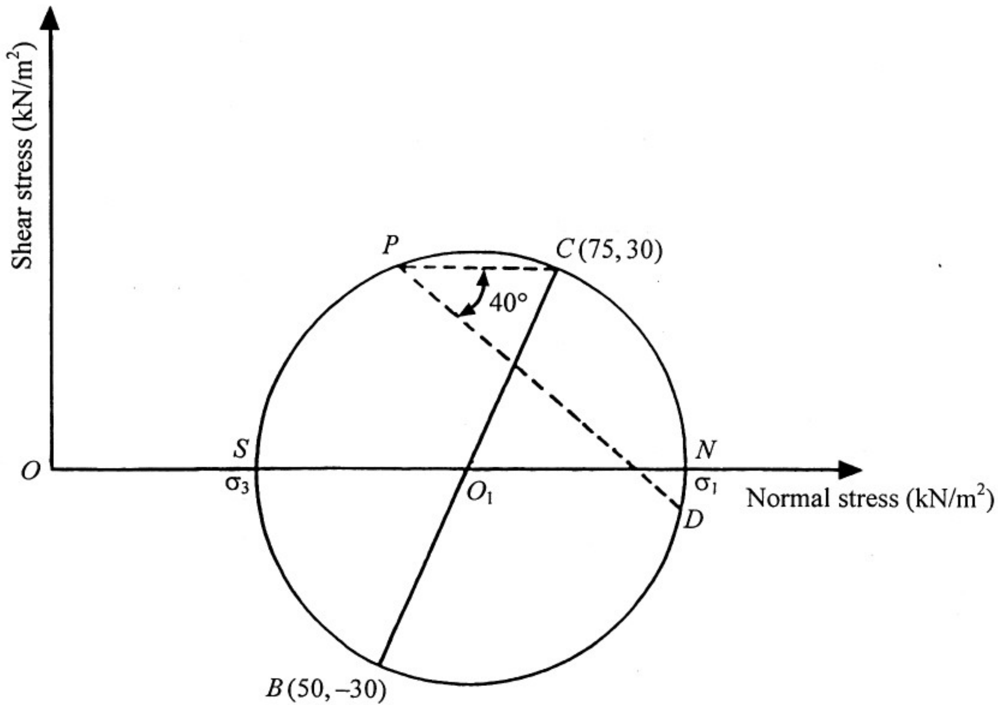
10.5 a. The Mohr's circle is shown.



$\sigma_1 = \overline{ON} = \mathbf{95\ kN/m^2}$; $\sigma_3 = \overline{OS} = \mathbf{30\ kN/m^2}$

b. σ_n and τ_n are coordinates of D. So

$\sigma_n \approx$ **94.2 kN/m²**; $\tau_n \approx$ **7.1 kN/m² (–)**

10.6 a. The Mohr's circle is shown on the next page.

$\sigma_1 = \overline{ON} = \mathbf{109.1\ lb/ft^2}$; $\sigma_3 = \overline{OS} = \mathbf{25.9\ lb/ft^2}$

b. σ_n and τ_n are coordinates of D. So

$\sigma_n \approx$ **29.1 lb/ft²**; $\tau_n \approx$ **16.08 lb/ft²**

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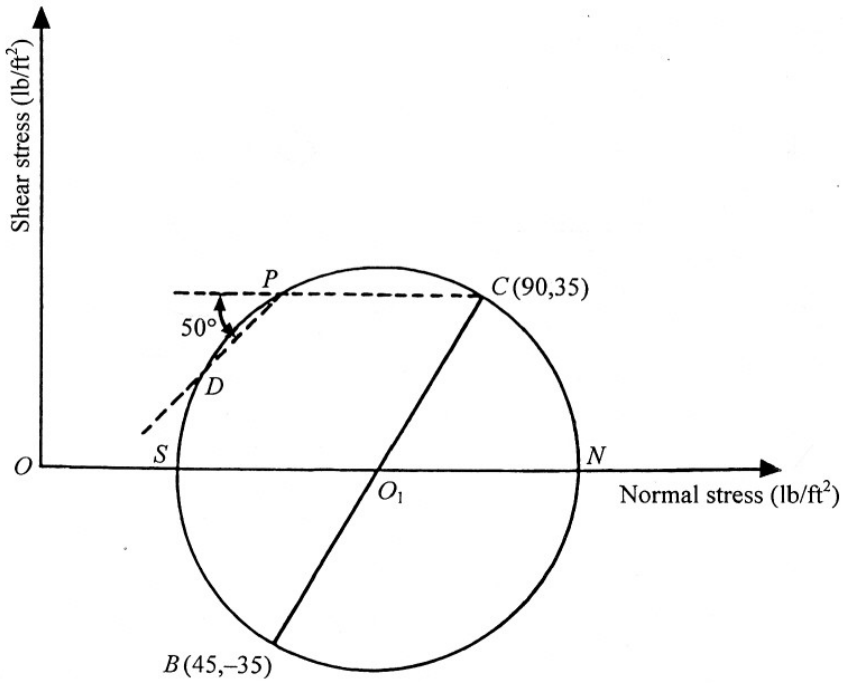

Problem 10.6

10.7

Load @	P (lb)	r (ft)	z (ft)	$\frac{r}{z}$	I_1 (Table 10.1)	$\Delta\sigma_z = \frac{P}{z^2} I_1$ (lb/ft^2)
A	2000	$(10^2 + 5^2)^{0.5} = 11.18$	10	1.12	0.0626	1.25
B	4000	$(10^2 + 5^2)^{0.5} = 11.18$	10	1.12	0.0626	2.5
C	6000	5	10	0.5	0.2733	16.4

$\Delta\sigma_z = \sum$ **20.15 lb/ft^2**

10.8 Eq. (10.15):

$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2} = \frac{(2)(75)(2)^3}{\pi[(5)^2 + (2)^2]^2} + \frac{(2)(300)(2)^3}{\pi[3^2 + 2^2]^2}$$

$$= \mathbf{9.49\ kN/m^2}$$

10.9 $$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$= \frac{(2)(300)(3)^3}{\pi[(4+3)^2 + (3)^2]^2} + \frac{(2)(260)(3)^3}{\pi[4^2 + 3^2]^2} = \mathbf{15.32\ kN/m^2}$$

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10.10 $\Delta\sigma_z = \dfrac{2q_1 z^3}{\pi[(x_1+x_2)^2+z^2]^2} + \dfrac{2q_2 z^3}{\pi[x_2^2+z^2]^2}$

$$35 = \frac{(2)(750)(3)^3}{\pi[12^2+3^2]^2} + \frac{2q_2(3)^3}{\pi[4^2+3^2]^2} = 0.55 + 0.0275q_2$$

q_2 = **1252.7 lb/ft**

10.11 $\Delta\sigma_z$ at A due to $q_1 = \dfrac{2q_1 z^3}{\pi[x^2+z^2]^2}$, or $(\Delta\sigma_z)_1 = \dfrac{(2)(250)(2)^3}{\pi[(2)^2+(2)^2]^2} = 19.89\ \text{kN/m}^2$

Vertical component of q_2 = $q_2 \sin 45$

$$(\Delta\sigma_z)_2 = \frac{2q_2(\sin 45)z^3}{\pi[(5)^2+(2)^2]^2};\ (\Delta\sigma_z)_2 = 0.0043q_2$$

Horizontal component of q_2 = $q_2 \cos 45$

From Eq. (10.17): $(\Delta\sigma_z)_3 = \dfrac{2q_2 xz^2}{\pi(x_1^2+x_2^2)^2} + \dfrac{2q_2(\cos 45)(5)(2)^2}{\pi[5^2+2^2]^2} = 0.0107q_2$

Total vertical stress,

$\Delta\sigma_z$ = 30 kN/m^2 = $(\Delta\sigma_z)_1 + (\Delta\sigma_z)_2 + (\Delta\sigma_z)_3$

30 = 19.89 + 0.0043q_2 + 0.0107q_2

$$q_2 = \frac{30-19.89}{0.015} = \mathbf{674\ kN/m}$$

10.12 B = 12 ft; q = 350 lb/ft^2; x = 9 ft; z = 5 ft

$$\frac{2x}{B} = \frac{(2)(9)}{12} = 1.5;\ \frac{2z}{B} = \frac{(2)(5)}{12} = 0.833.\ \text{From Table 10.4, } \frac{\Delta\sigma_z}{q} = 0.2$$

$\Delta\sigma_z$ = (0.12)(350) = **70 lb/ft^2**

10.13 $\dfrac{2x}{B} = \dfrac{(2)(1.5)}{3} = 1;\ \dfrac{2z}{B} = \dfrac{(2)(3)}{3} = 2.$ From Table 10.4, $\dfrac{\Delta\sigma_z}{q} = 0.409$

$\Delta\sigma_z$ = (60)(0.409) = **24.54 kN/m^2**

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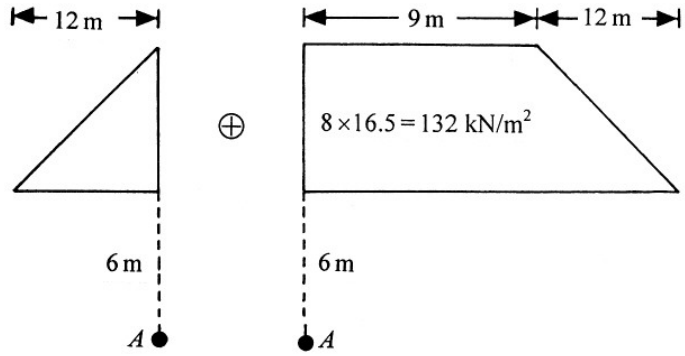
10.14 Refer to the figure.



For the left side (with the notations given in Figure 10.14):

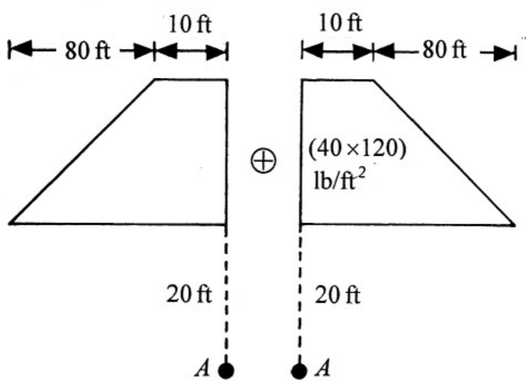
$$\frac{B_1}{z} = \frac{0}{6} = 0;\ \frac{B_2}{z} = \frac{12}{6} = 2.\ \text{From Figure 10.15, } I_{2(L)} = 0.37$$

For the right side:

$$\frac{B_1}{z} = \frac{9}{6} = 1.5;\ \frac{B_2}{z} = \frac{12}{6} = 2.\ \text{From Figure 10.15, } I_{2(R)} = 0.485$$

$\Delta\sigma_z = q[I_{2(L)} + I_{2(R)}] = (132)(0.37 + 0.485) =$ **112.86 kN/m²**

10.15 At *A*:



For the left side:	For the right side:
$\frac{B_1}{z} = \frac{10}{20} = 0.5$	$\frac{B_1}{z} = 0.5$
$\frac{B_2}{z} = \frac{80}{20} = 4$	$\frac{B_2}{z} = 4$
$I_2 = 0.468$	$I_2 = 0.468$

$\Delta\sigma_z = (40)(120)(0.468 + 0.468) \approx$ **4492.8 lb/ft²**

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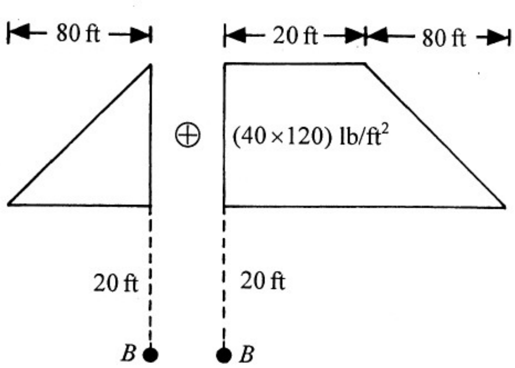
At B:



For the left side:

$$\frac{B_1}{z} = \frac{0}{20} = 0$$

$$\frac{B_2}{z} = \frac{80}{20} = 4$$

$I_2 = 0.42$

For the right side:

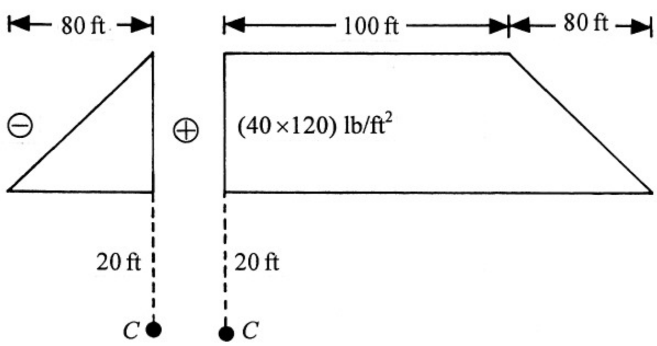
$$\frac{B_1}{z} = \frac{20}{20} = 1$$

$$\frac{B_2}{z} = \frac{80}{20} = 4$$

$I_2 = 0.48$

$\Delta\sigma_z = (40)(120)(0.42 + 0.48) \approx$ **4320 lb/ft²**

At C:



For the left side:

$$\frac{B_1}{z} = 0$$

$$\frac{B_2}{z} = \frac{80}{20} = 4$$

$I_2 = 0.42$

For the right side:

$$\frac{B_1}{z} = \frac{100}{20} = 5$$

$$\frac{B_2}{z} = \frac{80}{20} = 4$$

$I_2 = 0.5$

$\Delta\sigma_z = (40)(120)(0.5 - 0.42) \approx$ **384 lb/ft²**

10.16 Eq. (10.25) and Table 10.5: $q = 200$ kN/m²

R (m)	z (m)	$\frac{z}{R}$	$\frac{\Delta\sigma_z}{q}$	$\Delta\sigma_z$ (kN/m²)
4	1.5	0.375	0.9567	**191.34**
4	3	0.75	0.784	**156.8**
4	6	1.5	0.4240	**84.8**
4	9	2.25	0.2369	**47.38**
4	12	3.0	0.1436	**28.72**

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10.17 Eq. (10.26) and Tables 10.6 and 10.7: $q = 2000$ lb/ft^2

z (ft)	r (ft)	R (ft)	$\frac{z}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_z$ (lb/ft^2)
5	0	10	0.5	0	0.55279	0.35777	**1821**
5	2	10	0.5	0.2	0.54403	0.35752	**1803**
5	4	10	0.5	0.4	0.51622	0.35323	**1739**
5	8	10	0.5	0.8	0.38390	0.26236	**1293**
5	12	10	0.5	1.2	0.18556	0.02165	**414**

10.18 Refer to the Newmark's chart.

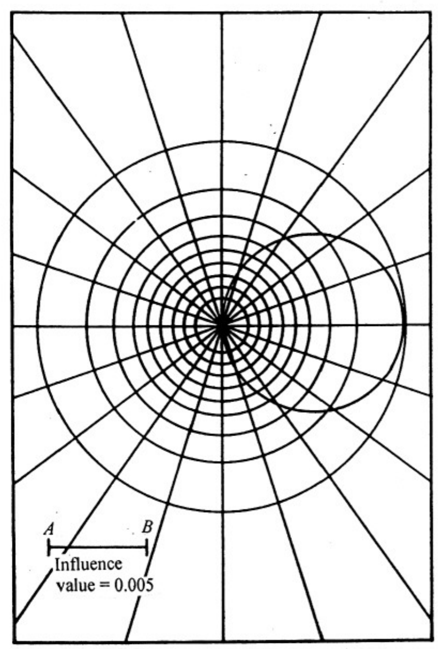
The plan is drawn to scale.

$\overline{AB} = 4$ m. $M \approx 65$.

$\Delta\sigma_z = (IV)\,q\,M$

$= (0.005)(300)(65)$

$=$ **97.5 kN/m^2**



10.19 a. Eqs. (10.31) and (10.32): $n = \frac{L}{z} = \frac{4}{2} = 2$; $m = \frac{B}{z} = \frac{2}{2} = 1$

Eq. (10.29): $\Delta\sigma_z = q\,I_3$; $I_3 = 0.1999$

$\Delta\sigma_z = (100)(0.1999) = 19.99$ kN/m^2 $\approx$ **20 kN/m^2**

b. Refer to the figure.

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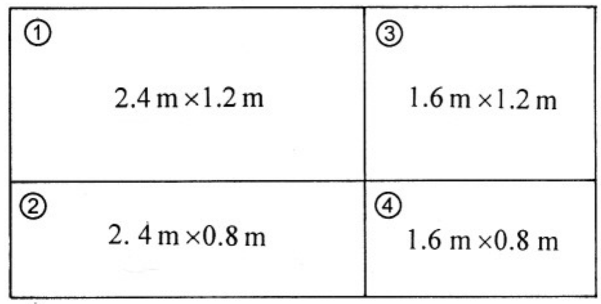


For rectangle 1: $m = \dfrac{1.2}{2} = 0.6;\quad n = \dfrac{2.4}{2} = 1.2;\quad I_3 = 0.1431$

For rectangle 2: $m = \dfrac{0.8}{2} = 0.4;\quad n = \dfrac{2.4}{2} = 1.2;\quad I_3 = 0.1063$

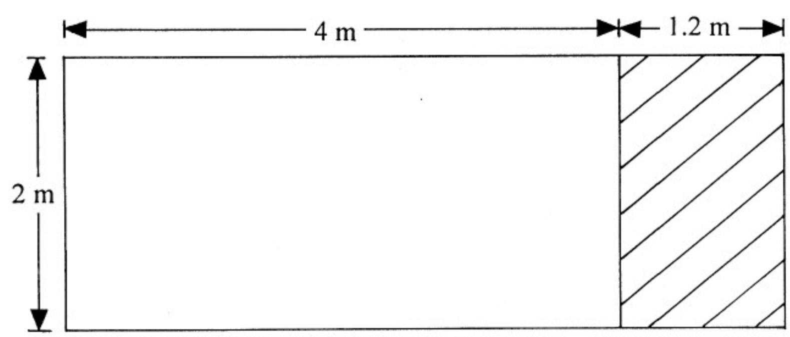
For rectangle 3: $m = \dfrac{1.2}{2} = 0.6;\quad n = \dfrac{1.6}{2} = 0.8;\quad I_3 = 0.1247$

For rectangle 4: $m = \dfrac{0.8}{2} = 0.4;\quad n = \dfrac{1.6}{2} = 0.8;\quad I_3 = 0.0931$

$\Delta\sigma_z = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}] = (100)(0.1431 + 0.1063 + 0.1247 + 0.0931)$

$= \mathbf{46.72\ kN/m^2}$

c. Refer to the figure.



$$\Delta\sigma_z = \begin{pmatrix}\text{stress at } C \text{ due} \\ \text{to rectangular} \\ \text{area } 5.2\,\text{m} \times 2\,\text{m}\end{pmatrix} - \begin{pmatrix}\text{stress at } C \text{ due} \\ \text{to rectangular} \\ \text{area } 2\,\text{m} \times 1.2\,\text{m}\end{pmatrix}$$

For rectangular area $5.2\,\text{m} \times 2\,\text{m}$: $m = \dfrac{2}{2} = 1;\quad n = \dfrac{5.2}{2} = 2.6;\quad I_3 = 0.202$

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For rectangular area $1.2\,\text{m} \times 2\,\text{m}$: $m = \frac{1.2}{2} = 0.6$; $n = \frac{2}{2} = 1$; $I_3 = 0.1361$

$\Delta\sigma_z = q(0.202 - 0.1361) = (100)(0.202 - 0.1361) = \mathbf{6.59\ kN/m^2}$

10.20 Eqs. (10.36), (10.37), and (10.38):

$$b = \frac{B}{2} = \frac{2}{2} = 1\,\text{m}$$

$$m_1 = \frac{L}{B} = \frac{4}{2} = 2$$

$$n_1 = \frac{z}{b} = \frac{3.5}{1} = 3.5$$

From Table 10.9, $I_4 \approx 0.242$

$\Delta\sigma_z = q\,I_4 = (100)(0.242) = \mathbf{24.2\ kN/m^2}$

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