Chapter 13

13.1 — 13.4 $K_o = (1 - \sin\phi')(\mathrm{OCR})^{\sin\phi'}$

Problem	ϕ' (deg)	K_o	$P_o = \frac{1}{2} K_o \gamma H^2$	$\bar{z} = \frac{H}{3}$
13.1	38	0.676	**281.55 kN/m**	**2.33 m**
13.2	32	0.679	**8,402.6 lb/ft**	**5 ft**
13.3	35	0.635	**205.7 kN/m**	**2 m**
13.4	30	0.5	**10,500.0 lb/ft**	**6.67 ft**

13.5 — 13.8 $K_a = \tan^2(45 - \phi'/2)$

Prob.	ϕ' (deg)	K_a	$\sigma'_{a(z=H)} = K_a \gamma H$	$P_a = \frac{1}{2} K_a \gamma H^2$	$\bar{z} = \frac{H}{3}$
13.5	30	0.333	(0.333)(105)(15) = **524.5 lb/ft^2**	$\frac{1}{2}(0.333)(105)(15)^2$ = **3933.6 lb/ft**	**5 ft**
13.6	32	0.307	(0.307)(100)(18) = **552.6 lb/ft^2**	$\frac{1}{2}(0.307)(100)(18)^2$ = **4973.4 lb/ft**	**6 ft**
13.7	36	0.26	(0.26)(18)(4) = **18.72 kN/m^2**	$\frac{1}{2}(0.26)(18)(4)^2$ = **37.44 kN/m**	**1.33 m**
13.8	40	0.217	(0.217)(17)(5) = **18.45 kN/m^2**	$\frac{1}{2}(0.217)(17)(5)^2$ = **46.11 kN/m**	**1.67 m**

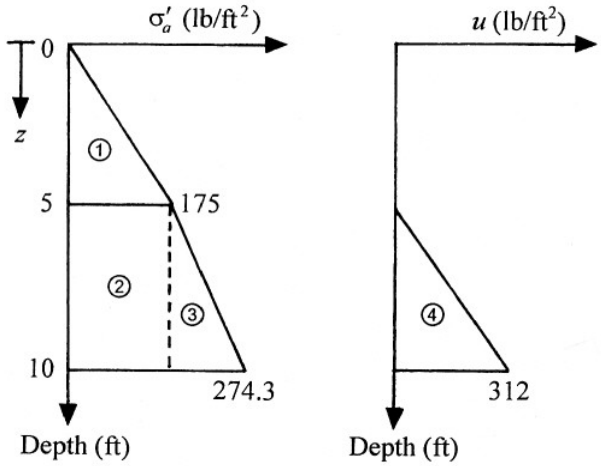
Note: 1. Pressure distribution is similar to that shown in Figure 13.13a., i.e., $\sigma'_a = 0$ at $z = 0$ and $\sigma'_a = K_a \gamma H$ at $z = H$.
2. $\bar{z}$ = distance measured from the bottom of the wall.

13.9 — 13.12 $K_p = \tan^2(45 + \phi'/2)$

Prob.	ϕ' (deg)	K_a	$\sigma'_{a(z=H)} = K_a \gamma H$	$P_a = \frac{1}{2} K_a \gamma H^2$	$\bar{z} = \frac{H}{3}$
13.9	34	3.537	(3.537)(110)(8) = **3112.6 lb/ft^2**	$\frac{1}{2}(3.537)(110)(8)^2$ = **12,450 lb/ft**	**2.67 ft**
13.10	36	3.852	(3.852)(105)(10) = **4044.6 lb/ft^2**	$\frac{1}{2}(3.852)(105)(10)^2$ = **20,223 lb/ft**	**3.33 ft**
13.11	35	3.69	(3.69)(14)(5) = **258.3 kN/m^2**	$\frac{1}{2}(3.69)(14)(5)^2$ = **645.8 kN/m**	**1.67 m**
13.12	30	3	(3)(15)(4) = **1890kN/m^2**	$\frac{1}{2}(3)(15)(4)^2$ = **360 kN/m**	**1.33 m**

Note: 1. $\sigma'_{p(z=0)} = 0$; triangular pressure distribution.
2. $\bar{z}$ = distance measured from the bottom of the wall.

13.13 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = \frac{1}{3}$. Refer to the figure.



$z = 0$ ft: $\sigma'_a = \sigma'_a K_a = 0$; $u = 0$

$z = 5$ ft: $\sigma'_a = (105)(5)(\frac{1}{3}) = 175$ lb/ft^2; $u = 0$

$z = 10$ ft: $\sigma'_a = [(105)(5) + (122 - 62.4)(5)](\frac{1}{3}) = 274.3$ lb/ft^2

$u = (62.4)(5) = 312$ lb/ft^2

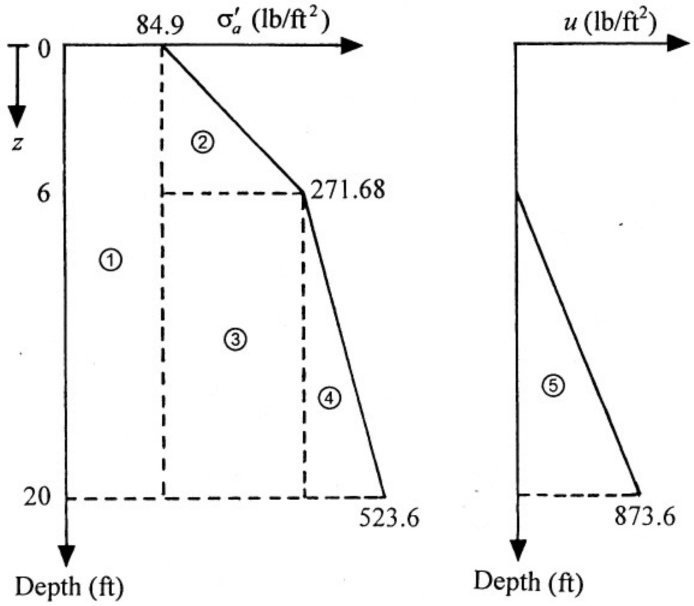
Area No.	Area
1	$(\frac{1}{2})(5)(175) = 437.5$
2	$(175)(5) = 875.0$
3	$(\frac{1}{2})(5)(274.3 - 175) = 248.3$
4	$(\frac{1}{2})(5)(312) = 780.0$
	$P_a = \Sigma$**2,340.8 lb/ft**

Resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(437.5)\left(5 + \frac{5}{3}\right) + (875)\left(\frac{5}{2}\right) + (248.3)\left(\frac{5}{3}\right) + (780)\left(\frac{5}{3}\right)\right]}{2340.8}$$

$$= \frac{2916.7 + 2187.5 + 413.8 + 1300}{2340.8} = \mathbf{2.91\,ft}$$

13.14 $K_a = \tan^2\left(45 - \frac{34}{2}\right) = 0.283$. Refer to the figure.



$z = 0$ ft: $\sigma'_a = \sigma'_a K_a = (300)(0.283) = 84.9$ lb/ft^2; $u = 0$

$z = 6$ ft: $\sigma'_a = [300 + (6)(110)](0.283) = 271.68$ lb/ft^2; $u = 0$

$z = 20$ ft: $\sigma'_a = [300 + (6)(110) + (126 - 62.4)(14)](0.283) = 523.66$ lb/ft^2

$u = (62.4)(14) = 873.6$ lb/ft^2

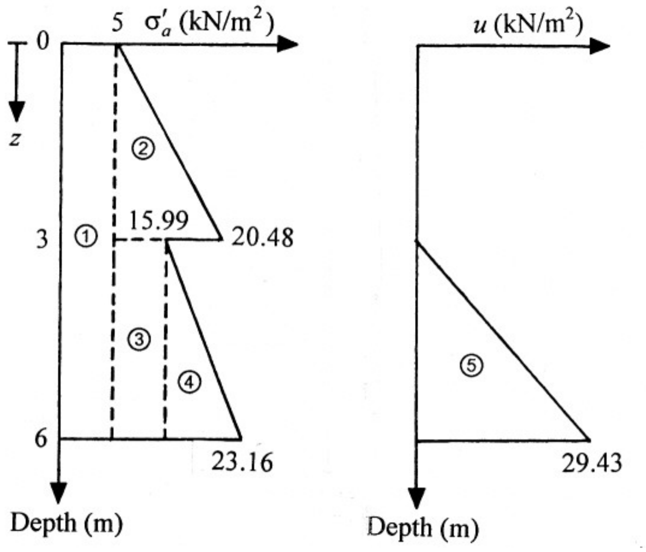
Area No.	Area
1	$(84.9)(20) = 1{,}698.00$
2	$(\frac{1}{2})(6)(271.68 - 84.9) = 560.34$
3	$(14)(271.68 - 84.9) = 2{,}614.92$
4	$(\frac{1}{2})(14)(523.6 - 271.68) = 1{,}763.44$
5	$(\frac{1}{2})(14)(873.6) = 6{,}115.20$
	$P_a = \Sigma$**12,751.90 lb/ft**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[\begin{array}{l}(1{,}698)\left(\frac{20}{2}\right) + (560.34)\left(14 + \frac{6}{3}\right) + (2{,}614.92)\left(\frac{14}{2}\right) \\ + (1{,}763.44)\left(\frac{14}{3}\right) + (6{,}115.2)\left(\frac{14}{3}\right)\end{array}\right]}{12{,}751.9}$$

$$= \frac{16{,}980 + 8{,}965.44 + 18{,}304.44 + 8{,}229.4 + 28{,}573.6}{12{,}751.9} = \mathbf{6.35\ ft}$$

13.15 $K_{a(1)} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333$; $K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$. Refer to the figure.



$z = 0$ m: $\sigma'_a = \sigma'_a K_{a(1)} = (15)(0.333) = 5$ kN/m²; $u = 0$

$z = 3$ m: $\sigma'_a = \sigma'_a K_{a(1)} = [(15.5)(3) + 15](0.333) = 20.48$ kN/m²

$\sigma'_a = \sigma'_a K_{a(2)} = [(15.5)(3) + 15](0.26) = 15.99$ kN/m²

$u = 0$

$z = 6$ m: $\sigma'_a = \sigma'_a K_{a(2)} = [(15.5)(3) + (19 - 9.81)(3)](0.26) = 23.16$ kN/m²

$u = (9.81)(3) = 29.43$ kN/m²

Area No.	Area
1	(6)(5) = 30.00
2	($\frac{1}{2}$)(3)(20.48 – 5) = 23.22
3	(3)(15.99 – 5) = 32.97
4	($\frac{1}{2}$)(3)(23.16 –15.99) = 10.76
5	($\frac{1}{2}$)(3)(29.43) = 44.15
	$P_a = \Sigma$**141.10 kN/m**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(30)\left(\frac{6}{2}\right) + (23.22)\left(3 + \frac{3}{3}\right) + (32.97)\left(\frac{3}{2}\right) + (10.76)\left(\frac{3}{3}\right) + (44.15)\left(\frac{3}{3}\right)\right]}{141.1}$$

$$= \frac{90 + 92.88 + 49.46 + 10.76 + 44.15}{141.1} = \frac{287.25}{141.1} = \mathbf{2.04\ m}$$

13.16 a. Eq. (13.23): $\sigma'_a = \dfrac{\gamma z \cos\alpha\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta = \sin^{-1}\left(\frac{\sin 15}{\sin 35}\right) - 15 + (2)(10) = 31.82°$$

$$\sigma'_a = \frac{(15)(5)(\cos 15)\sqrt{1+\sin^2(35) - (2)(\sin 35)(\cos 31.82)}}{\cos 15 + \sqrt{\sin^2 35 - \sin^2 15}} = \mathbf{29.18\ kN/m^2}$$

Eq. (13.25):

$$\beta = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1-\sin\phi'\cos\psi_a}\right) = \tan^{-1}\left[\frac{(\sin 35)(\sin 31.28)}{1-(\sin 35)(\cos 31.28)}\right] = \mathbf{30.54}°$$

b. Eq. (13.27):

$$K_{a(R)} = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

$$= \frac{\cos(15-10)\sqrt{1+\sin^2 35 - (2)(\sin 35)(\cos 31.82)}}{\cos^2 10\left(\cos 15 + \sqrt{\sin^2 35 - \sin^2 10}\right)} = 0.4137$$

$$P_a = \frac{1}{2}\gamma H^2 K_{a(R)} = \frac{1}{2}(15)(5)^2(0.4137) = \mathbf{77.57\ kN/m}$$

Location and Direction of Resultant: **At a distance of *H*/3 = 5/3 = 1.67 m above the bottom of the wall inclined at an angle β = 30.54º to the normal drawn to the back face of the wall**

13.17 This is a Rankine case, since δ′ = 0. $P_p = \dfrac{1}{2}\gamma H^2 K_{p(R)}$.

Eq. (13.33): $K_{p(R)} = \dfrac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' + 2\sin\phi'\cos\psi_p}}{\cos^2\theta\left(\cos\alpha - \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$

α = 0; θ = 10º; φ′ = 30º

$$\psi_p = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) + \alpha - 2\theta = \sin^{-1}\left(\frac{\sin 0}{\sin 30}\right) + 0 - (2)(10) = -20°$$

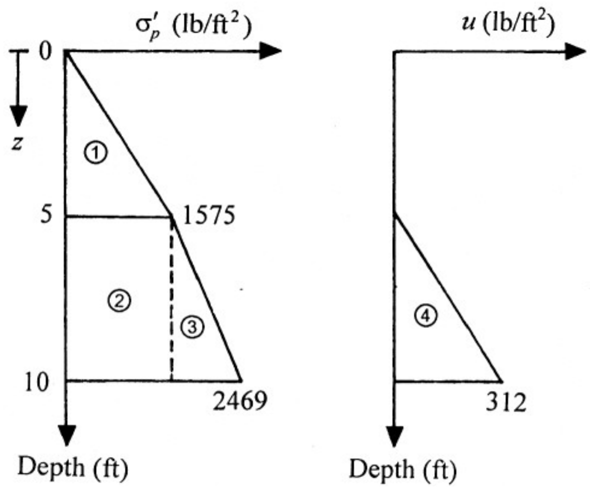
$$P_p = \frac{1}{2}(16.5)(4)^2(3.05) = \mathbf{402.6\ kN/m}$$

Eq. (13.32):

$$\beta = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_p}{1+\sin\phi'\cos\psi_p}\right) = \tan^{-1}\left\{\frac{(\sin 30)[\sin(-20)]}{1+(\sin 30)[\cos(-20)]}\right\} = \mathbf{-6.64°}$$

***P*$_p$ acts at a distance of *H*/3 = 4/3 = 1.33 m from the bottom of the wall inclined at an angle β = –6.64° to the normal drawn to the back face of the wall.**

13.18 $K_p = \tan^2\left(45+\frac{30}{2}\right) = 3$. Refer to the figure.



z = 0 ft: $\sigma'_p = 0$; $u = 0$

z = 5 ft: $\sigma'_p = \gamma_1 z K_p = (105)(5)(3) = 1575\ \text{lb/ft}^2$; $u = 0$

z = 10 ft: σ'_p = [(105)(5) + (122 – 62.4)(5)](3) = 2469 lb/ft^2

u = (62.4)(5) = 312 lb/ft^2

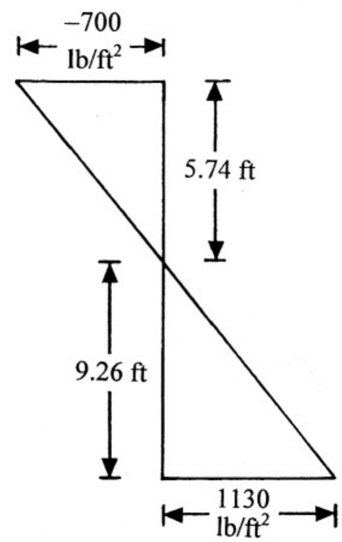
Area No.	Area
1	(½)(5)(1575) = 3,937.5
2	(5)(1575) = 7,875.0
3	(½)(5)(2469 – 1575) = 2,235.0
4	(½)(5)(312) = 780.0
	P_a = **Σ14,828.0 lb/ft**

Location of the resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(3937.5)\left(5+\frac{5}{3}\right)+(7875)\left(\frac{5}{2}\right)+(2235)\left(\frac{5}{3}\right)+(780)\left(\frac{5}{3}\right)\right]}{14{,}828} = \mathbf{3.44\ ft}$$

13.19 a. H = 15 ft; c_u = 350 lb/ft^2; γ = 122 lb/ft^2; ϕ = 0

$$K_a = \tan^2\left(45-\frac{\phi}{2}\right) = 1;\ \sigma'_a = \gamma z - 2c_u$$



At the top (z = 0 ft):

$$\sigma'_a = -2c_u = (-2)(350)$$

$$= -700\ \text{lb/ft}^2$$

At the bottom (z = 15 ft):

$$\sigma'_a = (122)(15) - (2)(350)$$

$$= 1830 - 700 = 1130\ \text{lb/ft}^2$$

The pressure diagram is shown.

b. Eq. (13.49): $z_o = \frac{2c_u}{\gamma} = \frac{(2)(350)}{122} = \mathbf{5.74\ ft}$

c. Eq. (13.51): $P_a = \frac{1}{2}\gamma H^2 - 2c_u H = \frac{1}{2}(122)(15)^2 - (2)(350)(15) = \mathbf{3225\ lb/ft}$

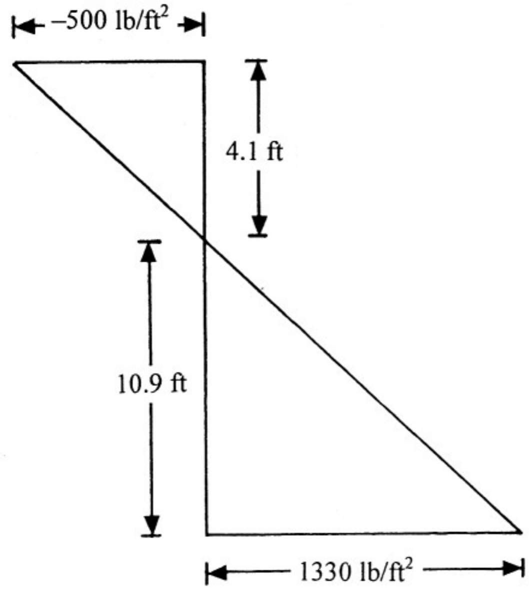
d. Eq. (13.53):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma}$$

$$= \frac{1}{2}(122)(15)^2 - (2)(350)(15) + \frac{(2)(350)^2}{122} = \mathbf{5233\ lb/ft}$$

Resultant measured from the bottom: $\bar{z} = \frac{H - z_o}{3} = \frac{15 - 5.74}{3} = \mathbf{3.09\ ft}$

13.20 a. $\sigma_a = \sigma_o K_a - 2c\sqrt{K_a}$



$\sigma_o = \gamma z + q$; $K_a = 1$.

At z = 0 ft:

$\sigma_o = 200$ lb/ft^2

$\sigma_a = 200 - (2)(350)$

$= -500 \text{ lb/ft}^2$

At z = 15 ft:

$\sigma_o = (122)(15) + 200$

$= 2030$ lb/ft^2

$\sigma_a = 2030 - (2)(350) = 1330 \text{ lb/ft}^2$

The pressure diagram is shown.

b. $\sigma_a = 0$; $(\gamma z_o + q) - 2c = 0$

$$z_o = \frac{2c_u - q}{\gamma} = \frac{700 - 200}{122} = \mathbf{4.1\,ft}$$

c. Referring to the diagram in Part a,

$$P_a = \frac{1}{2}(10.9)(1330) - \frac{1}{2}(500)(4.1) = \mathbf{6223.5\ lb/ft}$$

d. $P_a = \frac{1}{2}(10.9)(1330) = \mathbf{7248.5\ lb/ft}$

Location of the resultant from the bottom of the wall: $\frac{10.9}{3} = \mathbf{3.63\,ft}$

13.21 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{16}{2}\right) = 0.568$; $\sqrt{K_a} = 0.754$. Eq. (13.52):

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'H + \frac{2c'^2}{\gamma}$$

$$= \frac{1}{2}(0.568)(19)(5)^2 - (2)(0.754)(26)(5) + \frac{(2)(26)^2}{19} = \mathbf{10.02\ kN/m}$$

13.22 Eq. (13.63): $z_o = \frac{2c'}{\gamma}\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}} = \frac{(2)(5)}{18}\sqrt{\frac{1+\sin 25}{1-\sin 25}} = 0.872\text{ m}$

At z = 0 m: $\sigma'_a = 0$

At z = 5 m: $\sigma'_a = \gamma z K'_{a(R)} \cos\alpha$

$$\frac{c'}{\gamma z} = \frac{5}{(18)(5)} = 0.055$$

For α = 10º, ϕ' = 25º and $\frac{c'}{\gamma z} = 0.055$, the value of $K'_{a(R)} \approx 0.366$

$$\sigma'_a = (18)(5)(0.366)(\cos 10) = 32.44\text{ kN/m}^2$$

$$P_a = \frac{1}{2}(5 - 0.872)(32.44) = \mathbf{66.96\ kN/m}$$

13.23 Use Eqs. (13.68) and (13.69). α = 0; θ = 10º; ϕ' = 30º; γ = 18 kN/m^3; H = 5 m

Part	δ' (deg)	K_a [Eq. (13.69)]	$P_a = \frac{1}{2}K_a\gamma H^2$ [Eq. (13.68)]
1	15	0.3784	85.14 kN/m
2	20	0.3769	84.80 kN/m

P

P_a is located at a vertical distance of 5/3 = 1.67 m above the bottom of the wall and is inclined at an angle δ' to the normal drawn to the back face of the wall.

13.24 a. ϕ' = 38º; $\psi = 90 - \theta - \delta' = 90 - 5 - 20 = 65°$

Weight of wedge $ABC = \frac{1}{2}(11.6)(7.1)\underbrace{(128)}_{\gamma} = 5271\,\text{lb/ft} = 5.271\,\text{kip/ft}$

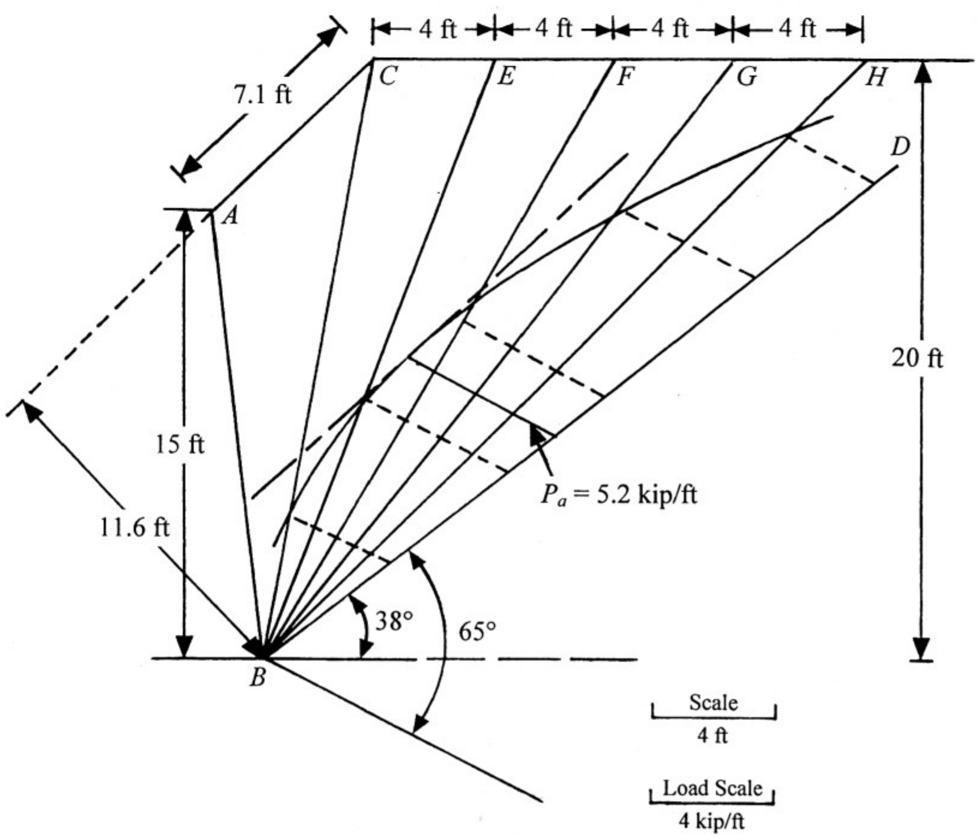
The weight of each of the wedges

CBE, EBF, FBG, $GBH = \frac{1}{2}(20)(4)(128) = 5120\,\text{lb/ft} = 5.12\,\text{kip/ft}$

Wedge	Weight (kip/ft)
ABC	5.271
ABE	5.271 + 5.12 = 10.391
ABF	10.391 + 5.12 = 15.511
ABG	15.511 + 5.12 = 20.631
ABH	20.631 + 5.12 = 25.751

The graphical construction is shown.

P_a = **5.2 kip/ft**



b. $\gamma = \dfrac{(1680)(9.81)}{1000} = 16.48\ \text{kN/m}^3;\ \phi' = 30°;\ \psi = 90 - 10 - 30 = 50°$

Weight of wedge *ABC* = $\dfrac{1}{2}(5.25)(2.5)(16.48) = 108.15\ \text{kN/m}$

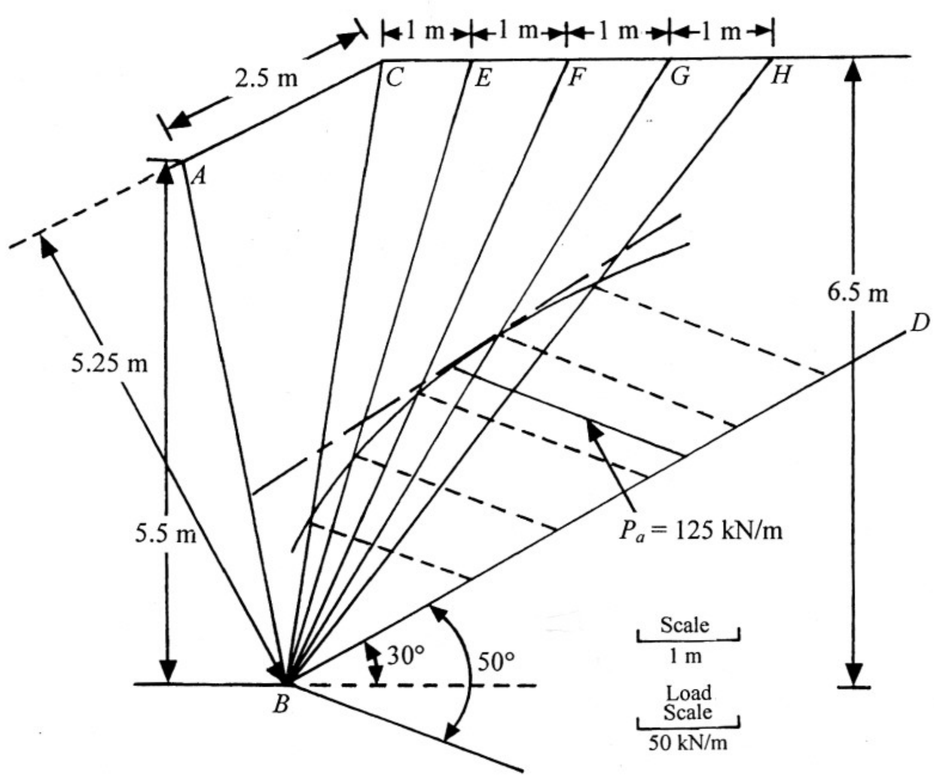
The weight of each of the wedges

CBE, *EBF*, *FBG*, *GBH* = $\dfrac{1}{2}(1)(6.5)(16.48) = 53.56\ \text{kN/m}$

Wedge	Weight (kip/ft)
ABC	108.15
ABE	108.15 + 53.56 = 161.71
ABF	161.71 + 53.56 = 215.27
ABG	215.27 + 53.56 = 268.83
ABH	268.83 + 53.56 = 322.39

The graphical construction is shown.

P_a = **125 kN/m**



13.25 From Eqs. (13.81) and (13.82), $\theta^* = \theta + \bar{\beta}$ and $\alpha^* = \alpha + \bar{\beta}$.

$$\bar{\beta} = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) = \tan^{-1}\left(\frac{0.15}{1-0}\right) = 8.53°$$

θ^* = 10° + 8.53° = 18.53°

α^* = 10° + 8.53° = 18.53°

$$P_a(\theta^*, \alpha^*) = \frac{1}{2}\gamma H^2 K_a$$

$$\frac{\delta'}{\phi'} = \frac{2}{3}$$

From Table 13.7, for θ^* = 18.53° and α^* = 18.53°, the value of $K_a \approx 0.61$.

From Eq. (13.85):

$$P_{ae} = P_a(\theta^*, \alpha^*)(1-k_v)\left[\frac{\cos^2(\theta+\bar{\beta})}{\cos\theta\cos^2\bar{\beta}}\right]$$

$$= \left[\left(\frac{1}{2}\right)(0.61)(15)(4)^2\right](1-0)\left[\frac{\cos^2(10+8.53)}{\cos(10)\cos^2(8.53)}\right] = \mathbf{68.3\ kN/m}$$

13.26 Eq. (13.93): $P_{ae} = \gamma(H-z_o)^2 N'_{a\gamma} - c'(H-z_o)^2 N'_{ac}$

Given $z_o = 0$; θ = 10°; ϕ' = 15°; $k_h = 0.15$

$N'_{ac} = N_{ac} = 1.75$ (Figure 13.31); $N_{a\gamma} = 0.3$ (Figure 13.33); λ = 1.3 (Figure 13.34);

$N'_{a\gamma} = \lambda N_{a\gamma}$. So,

P_{ae} = (19)(6 – 0)^2(1.3 × 0.3) – (20)(6 – 0)(1.75) = **56.76 kN/m**