Chapter 12

12.1 a. $c' = 0$. From Eq. (12.3): $\tau_f = \sigma' \tan\phi'$. So $175 = 200 \tan\phi'$

$$\phi' = \tan^{-1}\left(\frac{175}{200}\right) = \mathbf{41.2°}$$

b. For $\phi' = 150$ kN/m^2, $\tau_f = 150 \tan 41.2° = 131.3$ kN/m^2

$$\text{Shear force } S = \left(\frac{75 \times 75}{\underbrace{1000 \times 1000}_{\text{area}}}\right)(\underbrace{131.3}_{\tau_f}) = \mathbf{0.739\ kN}$$

12.2 Shear force, $S = (2 \times 2 \text{ in.}^2)(\sigma' \tan\phi') = (4)(20 \tan 38^o) = \mathbf{62.5\ lb}$

12.3 Area of specimen $A = 0.0036$ m^2

Test No.	Normal force N (N)	$\sigma' = \frac{N}{A}$ (kN/m^2)	Shear force S (N)	$\tau_f = \frac{S}{A}$ (kN/m^2)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	200	55.6	155	43.06	37.76
2	300	83.3	230	63.9	37.49
3	400	111.1	310	86.1	37.77
4	500	138.9	385	106.9	37.58

A graph of τ_f vs. σ' will yield $\phi' = \mathbf{37.5^o}$.

12.4 Area of specimen $A = \left(\frac{\pi}{4}\right)(2)^2 = 3.14 \text{ in.}^2$

Test No.	Normal force N (lb)	$\sigma' = \frac{N}{A}$ (lb/in.^{2})	Shear force S (lb)	$\tau_f = \frac{S}{A}$ (lb/in.^{2})	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	60	19.11	37.5	11.94	32.0
2	90	28.66	55	17.52	31.4
3	110	35.03	70	22.29	32.5
4	125	39.81	80	25.48	32.6

A graph of τ_f vs. σ' will yield $\phi' =$ **32.5°**.

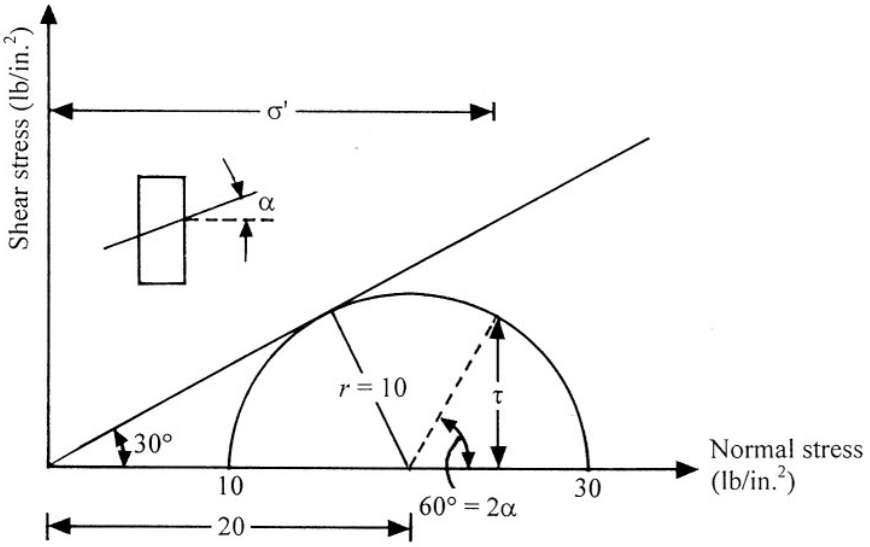
12.5 $c' = 0$. From Eq. (12.8): $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 30°$

$$\sigma_1' = 10 \tan^2\left(45 + \frac{30}{2}\right) = 30 \text{ lb/in.}^2$$

$$\Delta\sigma_{d\,(\text{failure})} = \sigma_1' - \sigma_3' = \mathbf{20\ lb/in.^2}$$

12.6 a. From Eq. (12.4): $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{30}{2} = \mathbf{60}°$

b. Refer to the figure.



$\tau = 10 \sin 60° =$ **8.66 lb/in.^{2};** $\sigma' = 20 + r \cos 60 = 20 + 10 \cos 60 =$ **25 lb/in.^{2}**

For failure, $\tau_f = \sigma' \tan \phi' = 25 \tan 30 = 14.43$ lb/in.^{2}. Since the developed shear stress = 8.66 lb/in.^{2} (which is less than 14.43 lb/in.^{2}), the specimen did not fail along this plane.

12.7 $\phi' = 25 + 0.18D_r = 25 + (0.18)(60) = 35.8°$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = 18\tan^2\left(45 + \frac{35.8}{2}\right) = \mathbf{68.7\ lb/in.^2}$$

12.8 $\sigma_3' = 15\ \text{lb/in.}^2$; $\sigma_1' = \sigma_3' + \Delta\sigma_{d(f)} = 15 + 34 = 49\ \text{lb/in.}^2$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right); \quad 49 = 15\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$\phi' \approx \mathbf{32.1^o}$

12.9 $$\sigma_3' + \Delta\sigma_3' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = \sigma_3' \tan^2(57) = 2.37\sigma_3'$$

$$1 + \frac{\Delta\sigma_3'}{\sigma_3'} = 2.37; \quad \frac{\Delta\sigma_3'}{\sigma_3'} = 1.37$$

$$\sigma_3' = \frac{175}{1.37} = \mathbf{127.7\ kN/m^2}$$

12.10 $$\sigma_1' = \sigma_3' + \Delta\sigma_{d(f)} = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma_3' = \frac{\Delta\sigma_{d(f)}}{\tan^2\left(45 + \frac{\phi'}{2}\right) - 1} = \frac{30}{\tan^2\left(45 + \frac{28}{2}\right) - 1} = \mathbf{16.95\ lb/in.^2}$$

12.11 a. $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right); \ (250 + 275) = 250\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$\phi' \approx \mathbf{20.8^o}$

b. $$\theta = 45 + \frac{\phi'}{2} = 45 + \frac{20.8}{2} = \mathbf{55.4}^\circ$$

c. From Eqs. (10.8) and (10.9):

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\theta = \frac{525+250}{2} + \frac{525-250}{2}\cos(2\times 55.4)$$

$$= \mathbf{338.7\ kN/m^2}$$

$$\tau = \sigma' \tan\phi' = 338.7 \tan 20.8 = \mathbf{128.7\ kN/m^2}$$

12.12 $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

Specimen I: $$(15 + 31.4) = 46.4 = 15\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right) \qquad \text{(a)}$$

Specimen II: $$(25 + 47) = 72 = 25\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right) \qquad \text{(b)}$$

Subtracting Eq. (a) from Eq. (b): $72 - 46.4 = 10\tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = \mathbf{26}°$

From Eq. (b):

$$c' = \frac{72 - 25\tan^2\left(45 + \frac{26}{2}\right)}{2\tan\left(45 + \frac{26}{2}\right)} = \mathbf{2.49\ lb/in.^2}$$

12.13 $$\sigma_1' = 25\tan^2\left(45 + \frac{26}{2}\right) + (2)(2.49)\tan^2\left(45 + \frac{26}{2}\right) = \mathbf{72\ lb/in.^2}$$

12.14 $c' = 0$. $$\sigma_1' = \sigma_3' + \Delta\sigma_{d(f)} = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma_3' = \frac{\Delta\sigma_{d(f)}}{\tan^2\left(45 + \frac{\phi'}{2}\right) - 1} = \frac{175}{\tan^2\left(45 + \frac{38}{2}\right) - 1} = \mathbf{54.6\ kN/m^2}$$

12.15 $$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right); \quad \phi = 2\left[\tan^{-1}\left(\frac{15+11}{15}\right)^{0.5} - 45\right] = \mathbf{15.6}°$$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right); \quad \phi' = 2\left[\tan^{-1}\left(\frac{15+11-7.2}{15-7.2}\right)^{0.5} - 45\right] = \mathbf{24.4}°$$

12.16 $$\phi = 2\left[\tan^{-1}\left(\frac{140+125}{140}\right)^{0.5} - 45\right] = \mathbf{18}°$$

$$\phi' = 2\left[\tan^{-1}\left(\frac{140+125-75}{140-75}\right)^{0.5} - 45\right] = \mathbf{29.4}°$$

12.17 a. $\sigma_3 = 112$ kN/m^2; $\sigma_1 = 112 + 100 = 212$ kN/m^2

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right); \quad \frac{212}{112} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$\phi = \mathbf{18°}$

b. $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{\sigma_1 - \Delta u_{d(f)}}{\sigma_3 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{31}{2}\right) = 3.124$$

$$\frac{212 - \Delta u_{d(f)}}{112 - \Delta u_{d(f)}} = 3.124$$

Or, $212 - \Delta u_{d(f)} = 349.9 - 3.124\Delta u_{d(f)}$; $2.124\Delta u_{d(f)} = 137.9$

$\Delta u_{d(f)} = \mathbf{64.9\ kN/m^2}$

12.18 $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = 112 \tan^2(45 + 15.5) = 349.9 \text{ kN/m}^2$$

$$\Delta\sigma_{d(f)} = \sigma_1' - \sigma_3' = 349.9 - 112 = \mathbf{237.9\ kN/m^2}$$

12.19 $$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) = 15 \tan^2\left(45 + \frac{22}{2}\right) = 32.97 \text{ lb/in.}^2$$

$$\Delta\sigma_{d(f)} = 32.97 - 15 = \mathbf{17.97\ lb/in.^2}$$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{32.97 - \Delta u_{d(f)}}{15 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{32}{2}\right)$$

$\Delta u_{d(f)}$ = **7.01 lb/in.^{2}**

12.20 $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{100 - \Delta u_{d(f)}}{0 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{25}{2}\right)$$

$\Delta u_{d(f)}$ = **–68.5 kN/m^2**

12.21 $$\frac{120 - \Delta u_{d(f)}}{0 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{23}{2}\right) = 2.283$$

$\Delta u_{d(f)}$ = **–93.5 kN/m^2**

12.22 a.

Test no.	$\frac{\sigma_1' + \sigma_3'}{2} = p'$ (lb/in.^{2})	$\frac{\sigma_1' - \sigma_3'}{2} = q'$ (lb/in.^{2})
1	50	23
2	30	18

$q' = m + p' \tan\alpha$

$23 = m + 50 \tan\alpha$ (a)

$18 = m + 30 \tan\alpha$ (b)

m = **10.5 lb/in.^{2},** α = **14°**

b. $\phi' = \sin^{-1}(\tan\alpha) = \sin^{-1}(\tan 14)$ = **14.44°**

$$c' = \frac{m}{\cos\alpha} = \frac{10.5}{\cos(14)} = \mathbf{10.82\ lb/in.^2}$$

12.23 $$\frac{c_{u(\mathrm{VST})}}{\sigma'_o} = 0.11 + 0.0037PI$$

$\sigma' = (3)(16) + (5)(18.6 - 9.81) = 91.95\ \mathrm{kN/m^2}$

$c_{u(\mathrm{VST})} = [0.11 + (0.0037)(18)](91.95) = \mathbf{16.24\ kN/m^2}$