Chapter 15

15.1 Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$2 = \frac{15}{(15.5)(H)(\cos^2 30)(\tan 30)} + \frac{\tan 20}{\tan 30}$$

$$2 = \frac{2.235}{H} + 0.63$$

H = **1.63 m**

15.2 Eq. (15.16):

$$H_{\text{cr}} = \frac{c'}{\gamma} \frac{1}{\cos^2 \beta(\tan \beta - \tan \phi')} = \frac{200}{100} \frac{1}{(\cos^2 22)(\tan 22 - \tan 15)} = \mathbf{17.1\,ft}$$

15.3 $\gamma_{\text{sat}} = \dfrac{(1900)(9.81)}{1000} = 18.64 \text{ kN/m}^3$

$\gamma' = 18.64 - 9.81 = 8.83 \text{ kN/m}^3$

Eq. (15.17):

$$F_s = \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan \beta} = \frac{18}{(18.64)(8)(\cos 20)^2 (\tan 20)} + \frac{8.83}{18.64} \frac{\tan 25}{\tan 20}$$

$$= 0.376 + 0.607 = \mathbf{0.98}$$

15.4 $\gamma_{\text{sat}} = \dfrac{(G_s + e)\gamma_w}{1+e} = \dfrac{(2.6+0.5)(62.4)}{1+0.5} = 128.96 \text{ lb/ft}^3$

$\gamma' = 128.96 - 62.4 = 66.56 \text{ lb/ft}^3$

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta}$$

$$= \frac{600}{(128.96)(25)(\cos^2 20)(\tan 20)} + \frac{66.56}{128.96} \frac{\tan 22}{\tan 20}$$

$$= 0.579 + 0.573 \approx \mathbf{1.15}$$

15.5 Eq. (15.31):

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right] = \left[\frac{(4)(400)}{115}\right]\left[\frac{(\sin 50)(\cos 25)}{1-\cos(50-25)}\right] = \mathbf{103.1\,ft}$$

15.6 $F_s = 2;\ c'_d = \frac{c'}{F_s} = \frac{400}{2} = 200\ \text{lb/ft}^2;\ \phi'_d = \tan^{-1}\left(\frac{\tan 25}{2}\right) = 13.12°$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(200)}{115}\right]\left[\frac{(\sin 50)(\cos 13.12)}{1-\cos(50-13.12)}\right] = \mathbf{25.94\,ft}$$

15.7 $H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right];\ \gamma = 115\ \text{lb/ft}^3$

$F_{s(\text{assumed})}$	$\phi'_d = \tan^{-1}\left(\frac{\tan 15}{F_s}\right)$ (deg)	$c'_d = \frac{c'}{F_s}$ (lb/ft^2)	β (deg)	H (m)
1.6	9.5	125	50	13.7
1.8	8.47	111	50	11.63
1.76	8.66	113.6	50	12.0

$F_s \approx \mathbf{1.76}$

15.8 $\rho = 1700\ \text{kg/m}^3;\ \gamma = \frac{(1700)(9.81)}{1000} = 16.68\ \text{kN/m}^3;\ c' = 18\ \text{kN/m}^2;\ \phi' = 20°$

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°;\ F_s = 2;\ c'_d = \frac{c'}{F_s} = \frac{18}{2} = 9\ \text{kN/m}^2$$

$$\phi'_d = \tan^{-1}\left(\frac{\tan\phi'}{F_s}\right) = \tan^{-1}\left(\frac{\tan 20}{2}\right) = 10.31°$$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(9)}{16.68}\right]\left[\frac{(\sin 26.57)(\cos 10.31)}{1-\cos(26.57-10.31)}\right] = \mathbf{23.74\,m}$$

15.9 $m \approx 0.192$ (From Figure 15.13). Eq. (15.36):

$$H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{30}{(17)(0.192)} = \mathbf{9.19\,m\text{ - -Toe failure}}$$

15.10 $m \approx 0.192$ for $\beta = 60°$ (Figure 15.13)

$$c_d = \frac{c_u}{FS_s} = \frac{30}{2} = 15\text{ kN/m}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{15}{(17)(0.192)} = \mathbf{4.6\,m}$$

15.11 $\beta = \tan^{-1}\left[\frac{1}{1}\right] = 45°$. For $\beta = 45°$ and $D = 1.2$, $m = 0.18$ (Figure 15.13).

$$c_d = \frac{c_u}{FS_s} = \frac{25}{2} = 12.5\text{ kN/m}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{12.5}{(17)(0.18)} = \mathbf{3.86\,m}$$

15.12 $H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{25}{(18)(0.18)} = \mathbf{7.72\,m.}$ It is a **toe circle.**

15.13 a. $D = \frac{12}{8.5} = 1.41$; $\gamma_{sat} = 18.5$ kN/m^3. For $\beta = 40°$ and $D = 1.41$, $m = 0.175$.

$$H_{\text{cr}} = \frac{c_u}{\gamma m};\quad c_u = (8.5)(18.5)(0.175) = \mathbf{27.5\ kN/m^2}$$

b. From Figure 15.13, **midpoint circle**

c. From Figure 15.15, $n \approx 0.7$.

Distance = nH = (0.7)(8.5) = **5.95 m**

15.14 $F_s = \frac{c_u}{\gamma H} M$. $\beta = 30°$; $H = 16$ m; $c_u = 50$ kN/m^2.

From Figure 15.19 for $k_h = 0.4$ and $D = 1$, $M \approx 4$

$$F_s = \frac{50}{(17)(16)}(4) = \mathbf{0.735}$$

15.15 $F_s = \frac{c_u}{\gamma H} M$. $\beta = 60°$; $c_u = 1000$ lb/ft^2; $\gamma = 115$ lb/ft^3; $H = 50$ ft; $k_h = 0.3$.

From Figure 15.20, $M \approx 3$.

$$F_s = \frac{1000}{(115)(50)}(4) = \mathbf{0.57}$$

15.16 a. $\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$

$$\frac{F_s}{\tan\phi'} = \frac{1}{\tan 20} = 2.75$$

From Figure 15.27

$$\frac{c'}{\gamma H_{cr} \tan\phi'} \approx 0.05$$

Or,

$$0.05 = \frac{20}{(16)(H_{cr})(\tan 20)} = \mathbf{68.7\ m}$$

b. $\frac{F_s}{\tan\phi'} = \frac{1}{\tan 25} = 2.14$. $\beta = \tan^{-1}\left(\frac{1}{1.5}\right) = 33.7°$.

Figure 15.27:

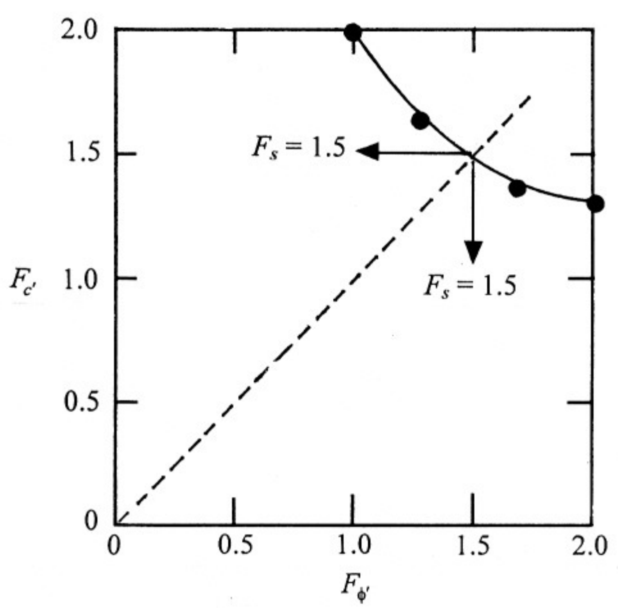
$$\frac{c'}{\gamma H_{cr}\tan\phi'} \approx 0.035; \quad \frac{750}{(110)(H_{cr})(\tan 25)} = 0.035$$

H_{cr} = **417.8 ft**

15.17 $\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$; $\phi' = 10°$; $c' = 700$ lb/ft^2; $\gamma = 110$ lb/ft^3.

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (lb/ft^2)	$F_{c'} = \frac{c'}{c'_d}$
5	2.01	0.098	539	1.30
6	1.68	0.090	495	1.41
8	1.25	0.078	429	1.63
10	1.00	0.064	352	1.99

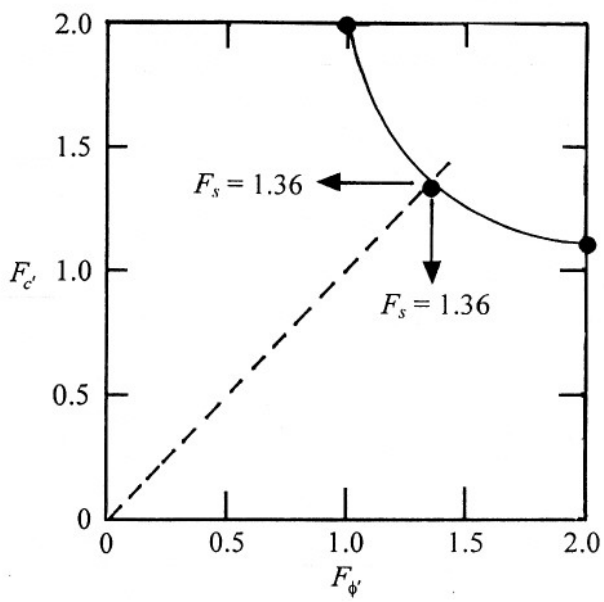
The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown. From the figure, $F_{c'} = F_{\phi'} = F_s =$ **1.5**.



15.18 $n' = 1$; $\phi' = 20°$; $c' = 400$ lb/ft^2; $\gamma = 115$ lb/ft^3; $H = 30$ ft

ϕ'_d (deg)	$F_{\phi'} = \dfrac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (lb/ft^2)	$F_{c'} = \dfrac{c'}{c'_d}$
5	4.16	0.133	458.9	0.87
8	2.06	0.105	362.3	1.1
10	1.36	0.080	276.0	1.45
20	1.00	0.058	200.1	2.0

The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown. From the figure, $F_{\phi'} = F_{c'} = F_s \approx$ **1.36**.

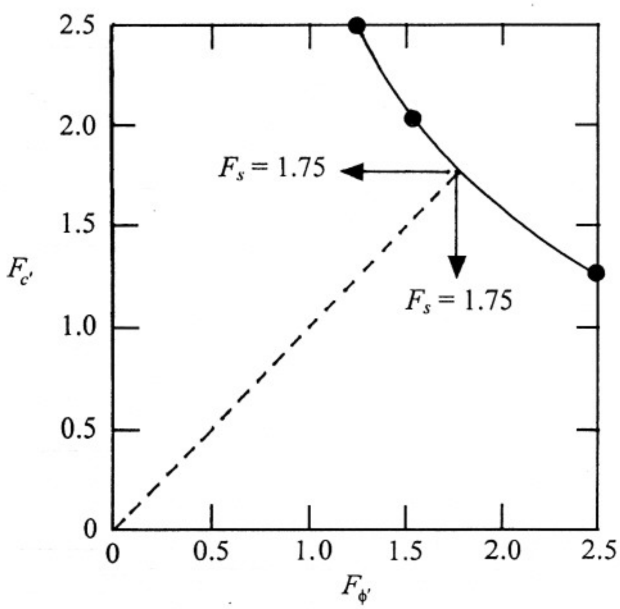


15.19 $n' = 2.5$; $\beta = \tan^{-1}\left(\dfrac{1}{2.5}\right) = 21.8°$; $\phi' = 12°$; $c' = 24$ kN/m^2; $\gamma = 16.5$ kN/m^3; $H = 12$ m

ϕ'_d (deg)	$F_{\phi'} = \dfrac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (kN/m^2)	$F_{c'} = \dfrac{c'}{c'_d}$
5	2.43	0.088	17.42	1.38
8	1.51	0.060	11.88	2.02
10	1.21	0.048	9.50	2.53
12	1.00	0.038	7.52	3.19

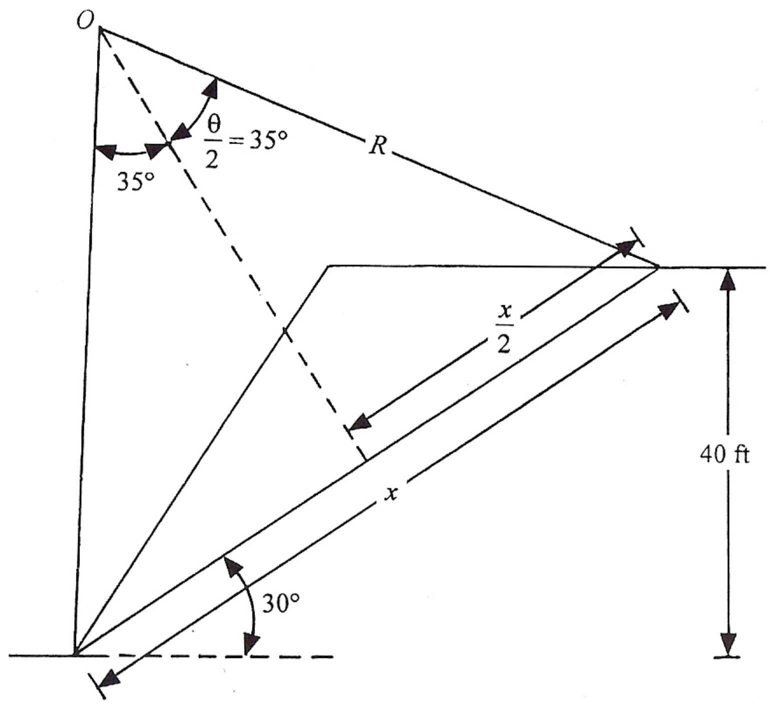
The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown on the next page.

From the figure, $F_{\phi'} = F_{c'} = F_s =$ **1.75**.



Problem 15.19

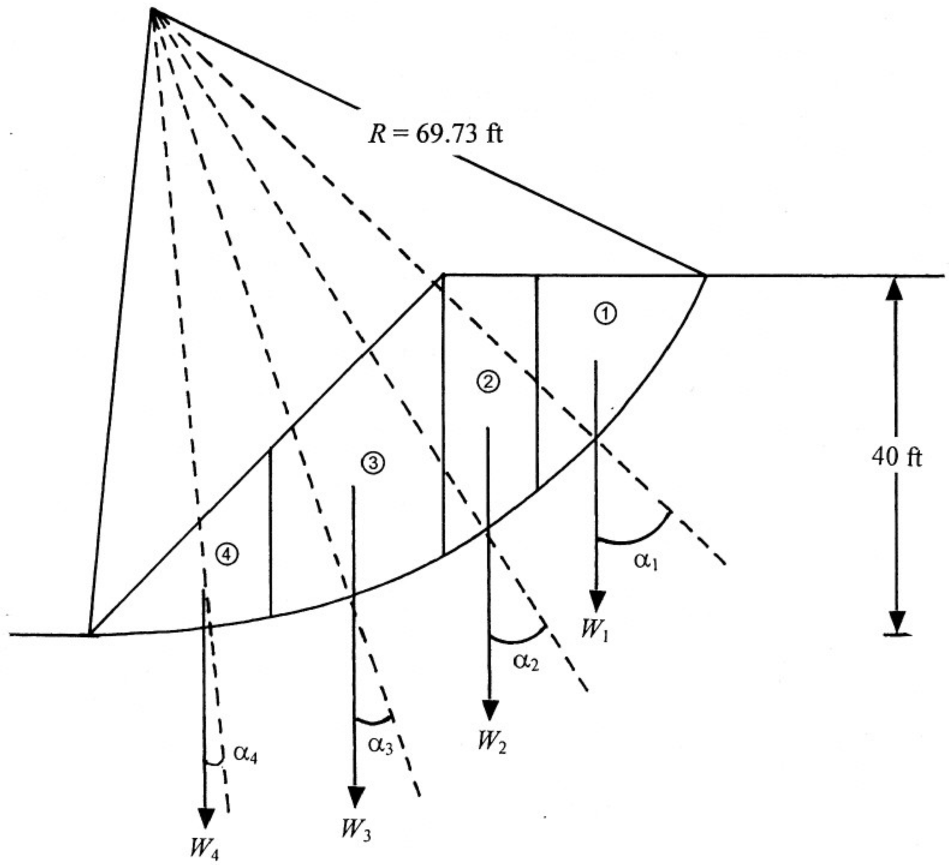
15.20 a. Refer to the figure.



$$\frac{40}{x} = \sin 30°\,;\; x = 80 \text{ ft}$$

$$\frac{40}{\text{Radius}, R} = \sin 35°\,;\;\; R = \frac{40}{\sin 35} = 69.73\,\text{ft}$$

With radius R = 69.73 ft, the trial surface circle has been drawn.



Now the following table can now be prepared.

Slice	Area of slices (ft^2)	Weight of slice $W_n = A \times \gamma$ (kip/ft)	α_n (deg)	$W_n \cos \alpha_n$ (kip/ft)	$W_n \sin \alpha_n$ (kip/ft)
1	$\frac{(26)(20)}{2} = 260$	29.9	47	20.39	21.86
2	$\frac{(10)(26+32)}{2} = 290$	33.35	32	28.28	17.67
3	$\frac{(20)(32+20)}{2} = 520$	59.8	20	59.19	20.45
4	$\frac{(20)(20)}{2} = 200$	23	5	22.91	2.00
				$\sum$127.77	$\sum$61.98

$$F_s = \frac{R\theta c' + (\sum W_n \cos\alpha_n)\tan\phi'}{\sum W_n \sin\alpha_n}$$

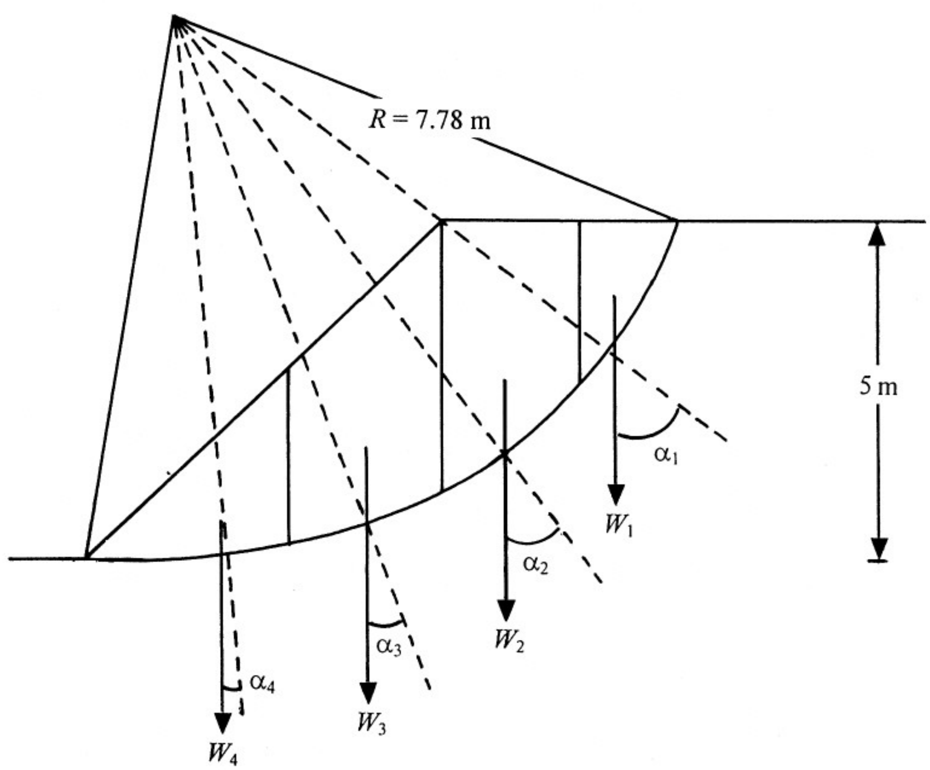
$$= \frac{(69.73)\left[\left(\frac{\pi}{180}\right)(70)\right](0.4) + (127.77)(\tan 20)}{61.98} = \mathbf{1.3}$$

Note: The accuracy can be increased by increasing the number of slices.

b. As in Part a, $\frac{H}{x} = \sin\alpha;\ x = \frac{H}{\sin\alpha} = \frac{5}{\sin 30} = 10\ \text{m}$

$$\frac{\left(\frac{x}{2}\right)}{R} = \sin\frac{\theta}{2},\ \text{or}\ \frac{5}{\sin 40} = R = 7.78\ \text{m}$$

With a radius R = 7.78 m, the trial surface has been drawn.



The following table can now be prepared.

Slice	Area of slices (m^2)	Weight of slice $W_n = A \times \gamma$ (kN/m)	α_n (deg)	$W_n \cos \alpha_n$ (kN/m)	$W_n \sin \alpha_n$ (kN/m)
1	$\frac{(2.6)(1.5)}{2} = 1.95$	33.35	54	19.60	29.98
2	$\frac{(2.6+4.2)(2)}{2} = 6.80$	116.28	38	91.63	71.59
3	$\frac{(4.2+2.8)(2)}{2} = 7.00$	119.70	20	112.48	40.94
4	$\frac{(3)(2.8)}{2} = 4.20$	71.82	6	71.43	7.52
				$\sum 295.14$	$\sum 150.02$

$$F_s = \frac{R\theta c' + (\sum W_n \cos\alpha_n)\tan\phi'}{\sum W_n \sin\alpha_n}$$

$$= \frac{(7.78)\left[\left(\frac{\pi}{180}\right)(80)\right](18) + (295.14)(\tan 15)}{150.02} = \mathbf{1.83}$$

Note: The accuracy will improve with smaller slices.

15.21 $\phi' = 20°$; $\beta = 26.57°$; $r_u = 0.5$; $\frac{c'}{\gamma H} = \frac{300}{(120)(25)} = 0.1$.

Using Table 15.3, the following table can be prepared.

D	m'	n'	$F_s = m' - n' r_u$
Toe circle	1.804	2.101	0.75
1.00	1.841	1.143	1.27
1.25	1.874	1.301	1.22
1.50	2.079	1.528	1.32

$F_s \approx$ **0.75**.

15.22 $\phi' = 20°$; $\beta = 18.43°$; $r_u = 0.5$; $\frac{c'}{\gamma H} = \frac{6}{(20)(6)} = 0.05$.

Using Table 15.3, the following table can be prepared.

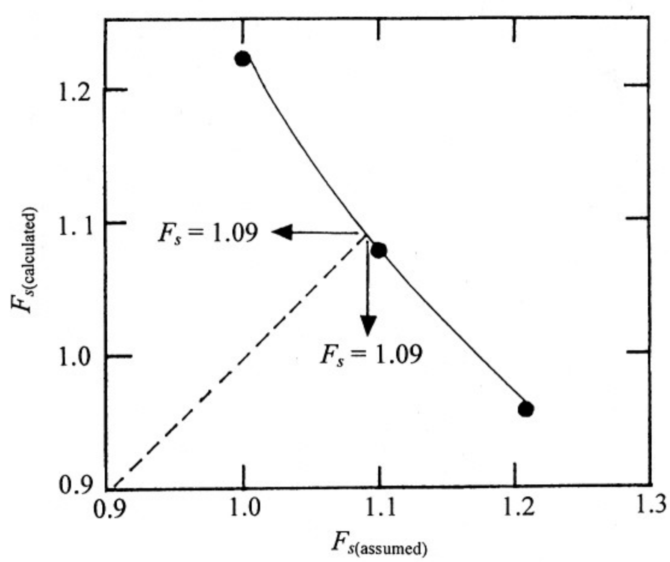
D	m'	n'	$F_s = m' - n' r_u$
1.00	1.840	1.387	1.15
1.25	1.834	1.493	1.09
1.50	2.011	1.705	1.16

$F_s \approx$ **1.09**.

15.23 $\beta = 20^\circ$; $\phi' = 15^\circ$; $r_u = 0.5$; $\gamma = 17.5$ kN/m^3; $c' = 20$ kN/m^2; $H = 15$ m

$F_{s(\text{assumed})}$	$\dfrac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \dfrac{\tan\phi'}{\tan\phi'_d}$
1.2	0.0635	15.5	0.966
1.1	0.0693	14.0	1.075
1.0	0.0762	12.5	1.209
0.9	0.0847	11.5	1.317

From the plot, F_s = **1.09**.



15.24 $\beta = \tan^{-1}\left(\dfrac{1}{3}\right) = 18.43^\circ$; $\phi' = 25^\circ$; $c' = 12$ kN/m^2; $\gamma = 19$ kN/m^3; $r_u = 0.25$;

$H = 12.63$ m

$F_{s(\text{assumed})}$	$\dfrac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \dfrac{\tan\phi'}{\tan\phi'_d}$
1.7	0.0294	15	1.74
1.8	0.0278	16	1.63
1.9	0.0263	17	1.53

From the graph, $F_s \approx \mathbf{1.72}$.