Project Presentation

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Overview

1. Recap of Vicek Model
   - Recap
2. Analytical Models
   - Hydrodynamic Descriptions
3. Passive rods among Active Rods
   - Passive rods
   - Mixture
4. Ongoing and Future Work

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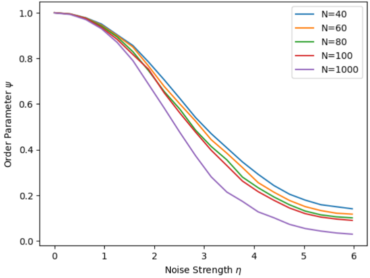
- 

Figure: Original VM

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- $$x_i = v_0 V(\theta_i)$$ $$\theta_i = \gamma \sum_{|x_i - x_j| < R} \sin(2(\theta_j - \theta_i)) + \xi_i(t)$$

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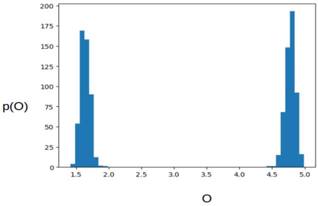
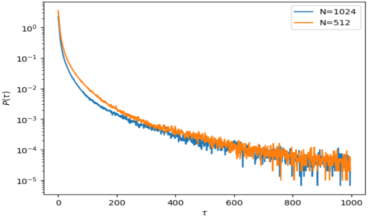
- 

(c) Probability Order Parameter



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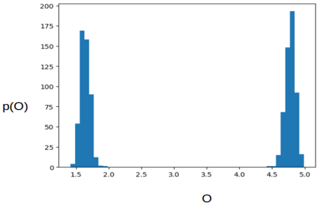

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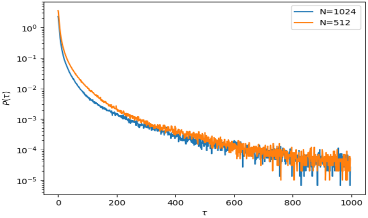

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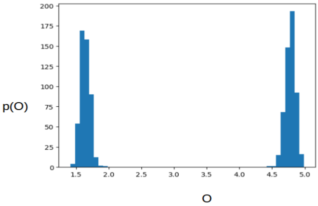
- 

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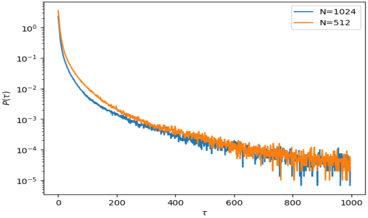

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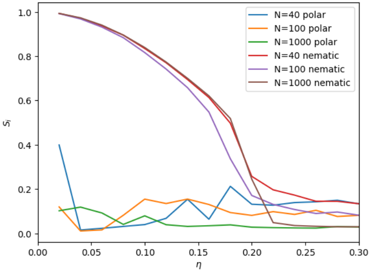
- Polar Order parameter is negligible.



- Figure: Order Parameters against Noise

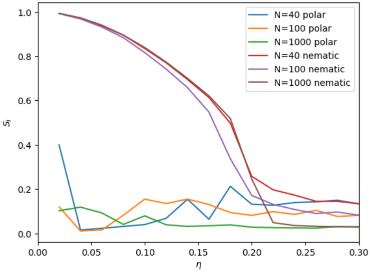


Figure: Order Parameters against Noise

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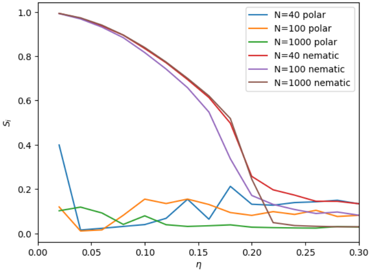
- 

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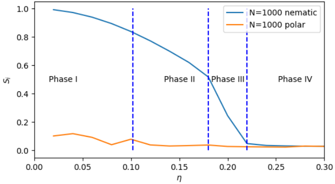
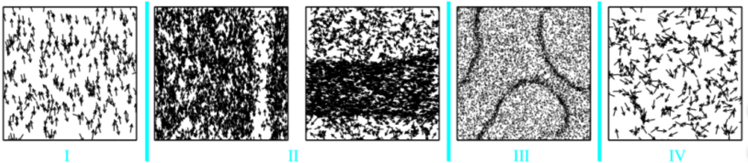
- 

Figure: Phases



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Hydrodynamic Descriptions

- A.Peshkov and group considered a dilute limit where only binary interactions take place.Assuming that the orientations are uncorrelated the Boltzmann equation governing the one particle distribution is given by,

$$\partial_t f(\vec{r}, \theta, t) + v_0 \vec{e}(\theta) \cdot \nabla f(\vec{r}, \theta, t) = I_{dif}[f] + I_{col}[f]$$

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$$\rho(\vec{r},t) = \int_{-\pi}^{\pi} d\theta f(\vec{r},\theta,t)$$

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- Integrating over θ would give us back the continuity equation.

Fourier Expansion

- Fourier series expansion of f with respect to θ,

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- $(\hat{f}_1, \hat{f}_2) = (0, \sqrt{\mu/\xi})$ which confirms that the polar order is always 0 while the nematic order udergoes a phase transition when $\mu > 0$.

Passive Rods

- $$x_i = \sigma_i(t)$$
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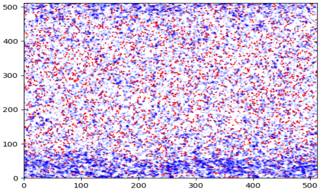
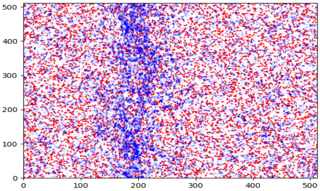
- System of passive rods evolving according to the above equations exhibits QLRO (decays with system size).

- We introduce passive rods in dilute amounts in a system of active rods.



(a) Passive Rods (10%)



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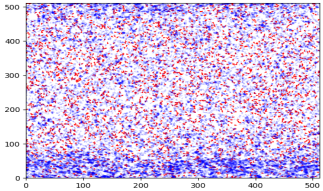

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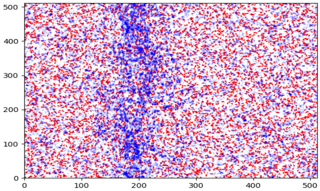

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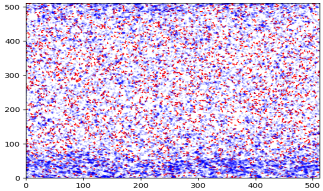

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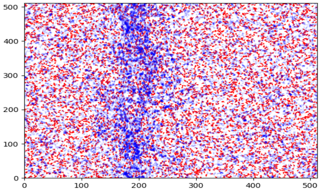

(f) [Passive Rods (20%)

- The band structure exists even though the active and passive rods interact among each other. However the band structure completely disappears when we consider a smaller system size.

Mixture

- Possible reason could be the low density of the bands in smaller system size.

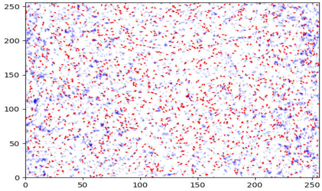

Figure: Smaller System Mixture

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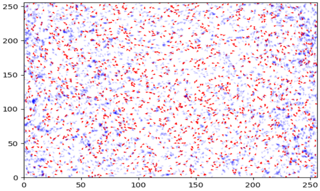

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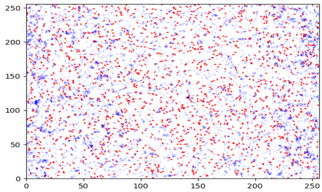

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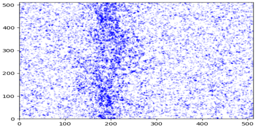
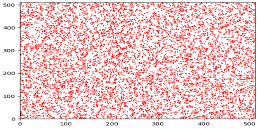


(a) Active Rods



(b) Passive Rods

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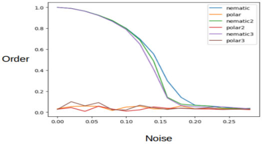
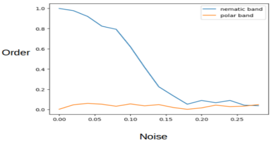


(c) Mixture Phase Transition



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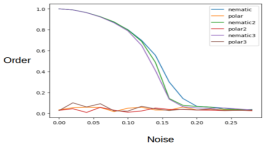

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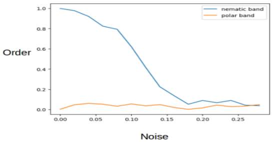

(f) Passive particles phase transition

- There is a small decrease in the critical noise of phase transition. Furthermore there seems to be a phase transition in the passive particles as well which is probably induced by the interactions with the active rods.

Ongoing and Future Works

Volume Exclusion Effects

Steric Effects have only been considered in the angular orientation till now. By considering a potential penalizing the overlap in the translation term as well we can get a much more exact model.

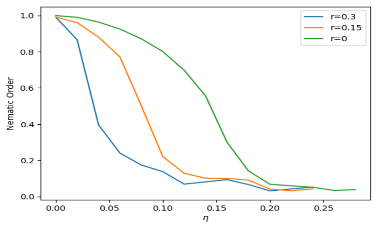

Figure: Effects of Size

Kihara Potential

$$\Phi(r) = \Phi_0 \left(\frac{m}{n-m} (\frac{\rho_0}{\rho})^n - \frac{n}{n-m} (\frac{\rho_0}{\rho})^m \right)$$

Ongoing and Future Works

Formation of Band like structures

High density nematic bands are formed only when the size of the system is greater than λ_C. They are also formed in at specific noise strengths.The occurrence of these bands is poorly understood.

References

- F. Peruani,Active Brownian Rods
- E.Bertin,Michel Droz, Guillaume Gregoire,Boltzmann and hydrodynamic description for self propelled rods
- T.Vicsek,A. Czirok,E. Ben-Jacob,I.Cohen Novel Phase Transitions in Self Driven Systems,P