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Article *in* Sensor Letters · January 2018

DOI: 10.1166/sl.2018.3923

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SENSOR LETTERS
Vol. 16, 1–5, 2018

The Combined Effect of Repulsion Zone and Initial Velocity on Viscek Model

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(Received: 18 December 2017. Accepted: 28 January 2018)

Our aim in this paper is to investigate the kinetic phase transition in the collective motion of self-propelled particles by studying and discussing the impact of the initial velocity v_0 and the repulsive interaction on the dynamical system. This dynamic is affected by the combined effect of radius of repulsion R_1, and density ρ. So in order to go one step further, our focusing attention is to better understand the complex behavior of non-equilibrium multi-agent system by extending the original model proposed by Viscek et al. for one zone. In this study, we include a second zone of interaction named zone of repulsion R_1, where particles repel each other at short distances in order to avoid collision with other members. The analysis is performed over different situations by using numerical simulations. We have found that the radius R_1 of repulsion zone plays an important role in our system. In effect, by varying the parameter R_1 and ρ, a phase transition can be achieved from disordered moving of individuals to aligned collective motion group. Our model results show that, for different sizes of radius of repulsion R_1 and density ρ, kinetic phase diagrams in the plane (η_c; v_0) are found. Implications of these findings are discussed for different situations via technical and numerical simulations, and directions for future research are pointed out.

Keywords: Complex Motion, Noise, Radius of Repulsion, Initial Velocity, Kinetic Phase Transition.

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1. INTRODUCTION

Recently, physicists have made a great effort in order to study widely the motion of complex system of self-propelled particles which remains a natural phenomenon.[1–12] Various biological organisms have been tested to evaluate the main features of the collective motion of flock/swarms[13–19] bacterial populations,[5–7] for ants[8,9] and the same in the motion of pedestrians.[10] One of the reasons why animal groups are such a popular subject for scientific studies is the importance of their cohesion and social interactions. The nature of system describing these phenomena is a non-equilibrium phase transition systems which are of a great interest in the modern statistical mechanics. In this system, the motion depends on lots of different variables and parameters. The agents (particles/cell) are controlled not only by some external fields, but also by the zone of interactions with other agents in their neighborhood. The primary study to modeling this phenomenon was given by Vicsek and his collaborators;[1] they treated the problem with computer simulations. In their elegant model, they considered the same N point particles move with a constant velocity v_0 inside a square simulation box of size $L \times L$ with periodic boundary conditions, and interact locally by trying to align their directions with their neighbors, The Vicsek model was capable to construct the real life of behavior flock phenomena, such as the kinetic phase transition from net transport phase to no transport phase by varying the density of the particles or the amplitude of noise. The transition observed in this new type of non-equilibrium systems, is analogous to the continuous phase transition in equilibrium systems. It is found that the particle density and the noise play a very important role, and their variations can affect significantly the collective behavior of the flock.[15–27]

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 1546-198X/2018/16/001/005 doi:10.1166/sl.2018.3923

In our work we aim to recreate this kinetic phase transition by introducing the second zone named zone of repulsion R_1 and by varying the initial velocity v_0 in the large interval of simulation. More precisely, this contribution is a continuation of our study of complex behavior of interacting particles.[3–7] We aim to study the combined effect between density[8–10] and radius of repulsion[7–12] in order to determine the new diagram of phase (the variation of critical noise η_c as a function of initial velocity v_0. In our results we prove that the transition phase depends not only on the noise of the particles but also by on the initial velocity,[11] on the density and radius of repulsion. These parameters they affect considerably the complex motion of agents.

The remainder of this paper is structured as follows: In the next section, we provide a brief description of the mathematical model used to investigate numerically the effects of the repulsive zone on the kinetic phase transition that occurs in our system. In Section 3, we discuss our numerical results, and finally we conclude the paper by summarizing the main point.

2. THE MODEL

The phenomenon of complex behavior of non-equilibrium multi-agent system is a subject of great interdisciplinary interest and is of central importance in order to be capable of controlling for example development of tumor, the spreading of an epidemic process, in view to produce the necessary medication, which has a devastating socio-economic impact on human's society. The model that we consider here is the Vicsek model with repulsive interaction.[2] By means of a Monte Carlo procedure, we simulate the behavior of agents in the two-dimensional space. Each agent attempts to maintain a minimum distance from others within a zone of repulsion, modeled as a circle, centered on the agent with radius R_1 (Fig. 1).

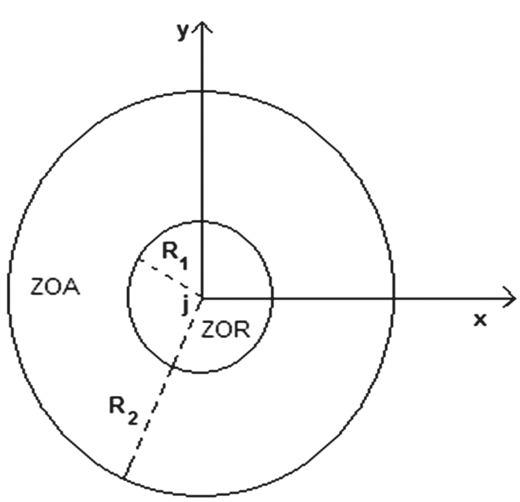

Fig. 1. Schematic representation of the two zones centered at the position of the agent j: ZOR is the zone of repulsion while ZOA is the zone of alignment.

In our implementation, as in the Vicsek model, we consider an $L \times L$ square shaped surface with periodic boundary conditions. We identify N identical pointwise individuals move synchronously in the system ($i = 1, \ldots, N$). Each agent presents a unique position vector $r_i(t)$, and unique direction vector $v_i(t)$ at time t, and partitioned into discrete time steps Δt.

At each time, every agent assesses the other agent positions and velocities within a local neighborhood, to find their preferred travel direction $V_i^d(t)$:

$$V_i^d(t) = \frac{V_i(t)}{v_0} \tag{1}$$

In our model, we consider evolutionary scenarios for models with different nature of zones in which particles are allowed to evolve in the region $[-\eta/2, \eta/2]$.

The zone of repulsion is modeled by a circular surface with radius R_1, the individuals belong this zone have a size N_1. The individual i has N_1 neighbors determined by the condition $0 < r_j(t) - r_i(t) \leq R_1$, where $r_j(t)$ is the position of the j-th neighboring individual ($j = 1, \ldots, N_r$, $j \neq i$). We describe this zone by the following equation:

$$V_i^{\text{repulse}}(t+1) = -\frac{\sum_{j\neq i}^{N_1} r_{i,j}(t)}{|\sum_{j\neq i}^{N_1} r_{i,j}(t)|} \tag{2}$$

Particle i, therefore, turns away from neighbors, j, within the inner-most interaction radius, choosing its desired direction of motion at the next time-step, $r_{i,j}(t)$, given as follows: $\vec{r}_{i,j}(t) = (\vec{r}_j(t) - \vec{r}_i(t))/(|\vec{r}_j(t) - \vec{r}_i(t)|)$. So if neighbors are present in zone of repulsion i.e., $N_1 > 0$ then the preferred direction of travel for the next time step is $V_i^d(t+1) = V_i^{\text{repuls}}(t+1)$.

If there are no neighbors present in the zone of repulsion, then individual i responds to neighbors within the zone of alignment given by the radius R_2. There are N_2 detectable neighbors in the zone of alignment, determined by $R_1 < r_j(t) - r_i(t) \leq R_2$. The preferred travel direction resulting from the zone of alignment is the average of the neighbor's velocities

$$V_i^{\text{aligne}}(t+1) = \frac{\sum_{j\neq i}^{N_2} V_j(t)}{|\sum_{j\neq i}^{N_2} V_j(t)|} \tag{3}$$

Where ($j = 1, \ldots, N_o$, $j \neq i$), if neighbors are found in zone of alignment i.e., $N_2 > 0$ then the preferred way of move for the next time step is given by

$$V_i^d(t+1) = V_i^{\text{align}}(t+1)$$

In the rare case if no neighbors are detected, then $V_i^d(t+1) = V_i^d(t)$.

The noise parameter η should scale with the square root of the time with uniform distribution in interval $[-\eta/2, \eta/2]$, especially when considering ensemble properties of the group, such as the average velocity.

The parameter adequately determined to explain clearly the collective behavior of individuals is the normalized average velocity which presents a good judgment of the ordered/disordered motion for the particles, given by:

$$v_a = \frac{1}{Nv_0}\left|\sum_{i=1}^{N} v_i\right| \tag{4}$$

We have studied in detail the nature of kinetic phase transition by determining absolute value of the local average velocity then we attempt to find a kinetic phase transition analogous to phase transitions found in equilibrium systems.

On the other hand, we will focus next on studying the equilibrium critical behavior which is the susceptibility. This parameter can be obtained by measuring the variance of the average normalized velocity v_a. It is defined as a function of the order parameter v_a and the number of particles N:

$$\aleph = N^2[\langle v_a^2\rangle - \langle v_a\rangle^2] \tag{5}$$

3. SIMULATION RESULTS AND DISCUSSION

Tamas Viscek et al. have used an interval of the initial velocity $(0.003 < v_0 < 0.3)$, where the actual value of v_0 does not affect the results. They used this interval for which the particles always interact with their actual neighbors and move fast enough to change the configuration after a few updates of the directions.

In contrast to the Vicsek model, we aim to prove the effects of initial velocity v_0 on Viscek model with repulsive interaction R_1, we report in Figure 2, the variation of order parameter v_a versus the noise, for various sizes of v_0, from $v_0 = 0.05$ to $v_0 = 0.45$. We find that, for some range of values of v_0, the average velocity v_a decreases with noise η from its maximal value until its minimum value. So, the system undergoes a kinetic phase transition, which occurs at some critical noise η_c, from net transport phase to no transport phase. Furthermore, we note that for weak values of v_0 the model displays long-range ordered motion that disappears by increasing the initial velocity v_0. So, phase transition no longer occurs if one increases the v_0. By comparing our results with the ones of (see Ref. [6]), we find the important role of R_1. In this logic, we understand that, the parameter gives a disorder for our system for high value of v_0 is the radius if repulsion R_1.

In this context, our findings are not in good agreement with the elegant results obtained in the framework of the similar model developed by Gabriel Baglietto et al.

In our study the calculation of the susceptibility permits us to compute the critical values of noise. In this situation, we show in Figure 3, that for some values of v_0, the susceptibility increases with the noise, reaches a maximum, and decreases to zero in the limit $\eta \to \infty$. In this figure, the critical noise η_c which is given by the absolute maximum in the curve depends strongly on the initial velocity v_0. In effect, by increasing the value of v_0, the position of the peak is shifted towards lower noises. The variation of peak occurs by the effect of radius of repulsion R_1 that we added on the Viscek model. Therefore, these calculations prove that the critical values $\eta_c(v_0, R_1)$, which allowed us to describe at what value of the critical noise the flock/swarm no long move together and will give a description of the order/disorder motion. This depends not only on the initial velocity v_0 as found in Ref. [6] but also on the value of R_1.

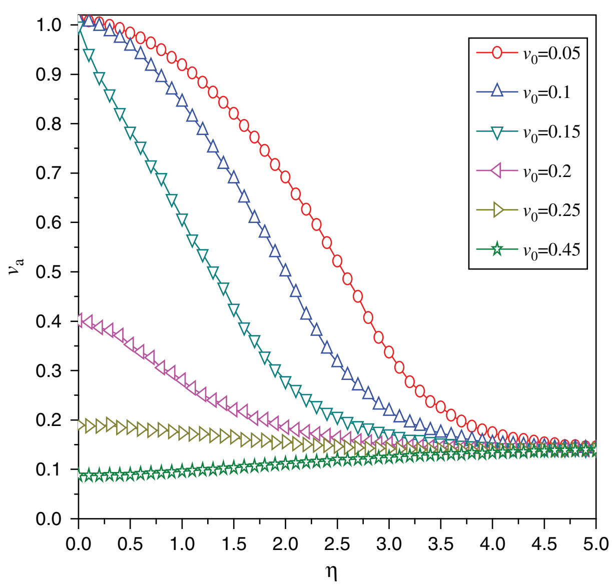

Fig. 2. Variation of the average velocity v_a as a function of noise η for different values of initial velocity v_0 and for fixed values of radius of repulsion and radius of alignment respectively $R_1 = 0.5$, $R_2 = 1.0$.

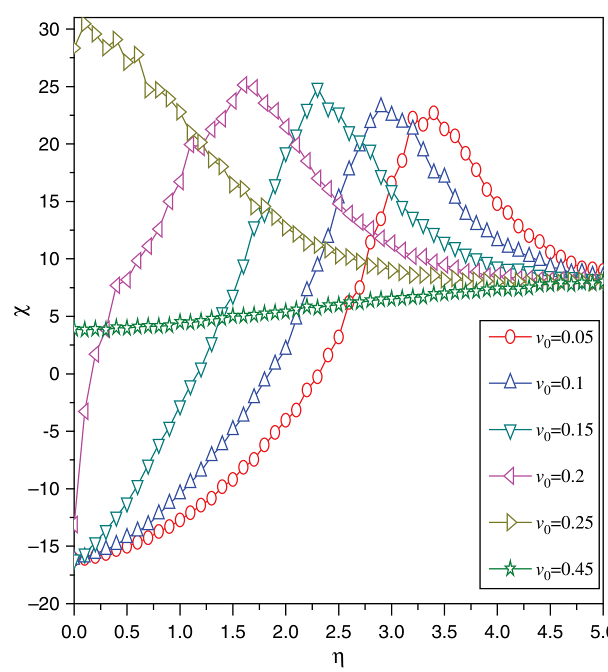

Fig. 3. Variation of the susceptibility χ as a function of noise η for different values of initial velocity v_0 and for fixed values of radius of repulsion and radius of alignment respectively $R_1 = 0.5$, $R_2 = 1.0$.

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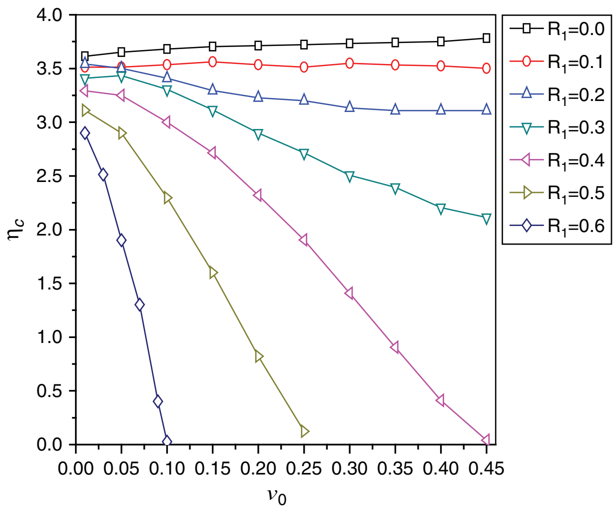

Fig. 4. Variation of the critical noise η_c as a function of initial velocity v_0 for different sizes of zone of repulsion R_1.

In Figure 4, we determine the kinetic phase diagram of the flocking model in the plane of noise versus the initial velocity v_0. From this figure when the zone of repulsion does not exist ($R_1 = 0.0$) or with small value of $R_1 = 0.1$ the critical noise remains practically constant as a function of v_0. As a result, we find almost the same behavior found by (see Ref. [6]). But when we increase the radius of repulsion R_1 the critical noise η_c decreases. There exist some critical value of R_1 which the net transport phase cannot exist. This phase diagram show clearly the non-universality of curve behavior found by Viscek et al. and by Baglietto et al.

The last Figure 5, we represent the variation of the critical noise η_c as a function of initial velocity v_0 for different densities ρ. In the light of these results $\eta_c(v_0, \rho)$ it is evident to find η_c decreases with v_0 and ρ. This phase diagram that we find in this figure looks to kinetic phase diagram of the Vicsek model with repulsive interaction [see Ref. [3]].

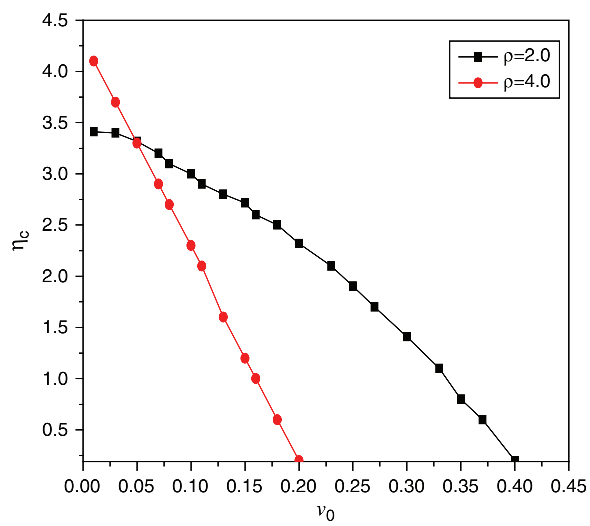

Fig. 5. Variation of the critical noise η_c as a function of initial velocity v_0 for different densities ρ.

Finally, the results found by Aldana et al.[27] look like exactly to our research, but when no repulsion between particle and in the interval of initial velocity v_0 already determined by Viscek et al.

4. CONCLUSION

According to the above discussed findings, we can conclude that the initial velocity and the repulsive interaction play an important role in the kinetic phase transition from an ordered state where all the particles move in the same direction, to a disordered state where the particles move in random uncorrelated directions. These parameters influence the behavior of flocks which become much more coherent in the limit of small noise and low values of v_0. In the Viscek model and in the results found by Baglietto et al., the critical noise appears constant as function of initial velocity. But in light of the obtained results the critical noise depends clearly on the initial velocity, under the effect of the zone of repulsion and the particles density. In this work, we have not considered the movement in three dimensions, and the open boundaries which has allowed us to have much more information about the complex collective behavior motion of the flock. Studies along these lines are currently in progress.

Acknowledgment: We acknowledge the financial support from: Laboratoire de Physique de la Matière Condensée Faculté des Sciences Ben M'Sik, Casablanca, Université Hassan II Casablanca, Maroc; Laboratoire de Bio-Géosciences et ingénierie des matériaux, Ecole Normale Supérieure, Casablanca, Université Hassan II Casablanca, Maroc.

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