SECI1013-02

DISCRETE STRUCTURE

ASSIGNMENT 2

GROUP 5

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Question 1

Let D = {1,3,5}. Define R on D where x, y ∈ D, xRy if 3x + y is a multiple of 6.

i) Find the element of R.

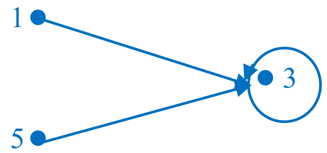
R = {(1,3), (3,3), (5,3)}

ii) Determine the domain and range of R.

Domain = {1,3,5}

Range = {3}

iii) Draw the digraph of the relation



iv) Determine whether the relation R is asymmetric?

Asymmetric = antisymmetric + irreflexive

$$\begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(1,3) , (5,3) ∈ R

(3,1) , (3,5) ∉ R

∴ antisymmetric

$$\begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

∴ Not reflexive

∴ Relation R is not asymmetric because for it to be asymmetric, it has to be both antisymmetric and irreflexive, but the relation is not irreflexive as (3,3) ∈ R

Question 2

Suppose R is an equivalence relation on the set A={x,y,z}. (x,y) ∈ R and (y,z) ∈ R. List all possible member of R and justify your answer.

Equivalence = reflexive + symmetric + transitive

- Reflexive pairs = (x,x), (y,y), (z,z)
- Symmetric pairs = (x,y), (y,x), (y,z), (z,y)
- Transitive pair = (x,y), (y,z) => (x,z) ; symmetric pair of (x,z) = (z,x)

$$\begin{array}{c|ccc|} & x & y & z \\ x & 1 & 1 & 1 \\ y & 1 & 1 & 1 \\ z & 1 & 1 & 1 \end{array} \quad \therefore \text{ reflexive}$$

$$M_R = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} x \quad y \quad z \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \qquad M_R^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{array}{l} M_R = M_R^T \\ \therefore \text{ symmetric} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{array}{l} m_{13} = 1,\ n_{13} = 1 \\ m_{13} = n_{13} \\ \therefore \text{ transitive} \end{array}$$

R = {(x,x), (y,y), (z,z), (x,y), (y,x), (y,z), (z,y), (x,z), (z,x)}

Question 3

Let B = {u, v, w, y} and R={(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)}

i) Construct the matrix of relation, M_R for the relation R on B

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii) List in-degrees and out-degrees of all vertices.

	u	v	w	y
in-degrees	2	2	3	2
out-degrees	2	2	2	3

iii) Determine whether the relation R on the set B is a partial order relation. Check all variances. Justify for answer.

Partial order = reflexive + antisymmetric + transitive

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(u,u), (v,v), (w,w), (y,y) ∈ R

∴ reflexive

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(u,w), (v,w), (w,y), (y,u), (y,v) ∈ R

(w,u), (w,v), (y,w), (u,y), (v,y) ∉ R

∴ antisymmetric

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$m_{14} = 0$, $n_{14} = 1$
$m_{14} \neq n_{14}$

∴ not transitive

R is not partial order relation because for it to be a partial order relation, it must be all reflexive, antisymmetric and transitive but R is not transitive.

Question 4

Let

$$f:[1,\infty) \rightarrow [0,\infty), f(x) = (x-1)^2$$

Determine whether the function f is **one–one, onto**, or **bijective.** Show full working and justify your answer.

One-to-one:

$$f(x_1) = f(x_2)$$
$$(x_1 - 1)^2 = (x_2 - 1)^2$$
$$x_1 - 1 = x_2 - 1$$
$$x_1 = x_2$$
$$\therefore f \text{ is one} - \text{to one because the result is } x_1 = x_1$$
$$\text{that show each values will get different result}$$

Onto:

$$(x-1)^2 = y$$
$$x - 1 = \sqrt{y}$$
$$x = \sqrt{y} + 1$$
$$\therefore \sqrt{y} \geq 0\,, \qquad \left(x = \sqrt{y} + 1\right) \geq 1$$
$$\therefore f \text{ is Onto} - Y$$

Bijective:

$$\therefore \text{function } f \text{ is Bijective because it has both one} - \text{to} - \text{one and onto} - y$$

Question 5

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4, \qquad g(x) = \frac{3}{2}x - 1$$

a) Find the inverse of $g(x)$.

$$let\ y = \frac{3}{2}x - 1$$
$$y + 1 = \frac{3}{2}x$$
$$\frac{2y + 2}{3} = x$$
$$g^{-1}(x) = \frac{2x + 2}{3}$$

b) Find the composition $(g \circ f)(x)$.

$$g\big(f(x)\big)$$
$$= \frac{3}{2}\big(f(x)\big) - 1$$
$$= \frac{3}{2}(9x + 4) - 1$$
$$= \frac{27}{2}x + 6 - 1$$
$$= \frac{27}{2}x + 5$$

c) Find the composition $(f \circ g)(x)$.

$$f(g(x))$$
$$= 9\big(f(x)\big) + 4$$
$$= 9\left(\frac{3}{2}x - 1\right) + 4$$
$$= \frac{27}{2}x - 9 + 4$$
$$= \frac{27}{2}x - 5$$

d) Find the composition $(f \circ g \circ g)(x)$.

$$g(g(x))$$
$$= \frac{3}{2}(g(x)) - 1$$
$$= \frac{3}{2}\left(\frac{3}{2}x - 1\right) - 1$$
$$= \frac{9}{4}x - \frac{3}{2} - 1$$
$$= \frac{9}{4}x - \frac{5}{2}$$

$$f\Big(g\big(g(x)\big)\Big)$$
$$= 9(g\big(g(x)\big)) + 4$$
$$= 9\left(\frac{9}{4}x - \frac{5}{2}\right) + 4$$
$$= \frac{81}{4}x - \frac{45}{2} + 4$$
$$= \frac{81}{4}x - \frac{37}{2}$$

Question 6

In a reactor, two intermediates mix to form product P. The initial temperatures are $P_0 = 4.0°F$ and $P_1 = 5.0°F$. Engineers observe that, for $t \geq 2$ minutes, the update rule is: "The new temperature is the previous temperature plus one-quarter of the temperature two minutes ago."

a) Write the recurrence relation that models this.

$P_t = P_{t-1} + \frac{1}{4}P_{t-2}$, $t \geq 2$

b) Using your recurrence, list $P_0, P_1, \ldots, P_5$ (exact values preferred).

$P_2 = P_1 + \frac{1}{4}P_0 = 5 + \frac{1}{4}(4) = 6.0°F$

$P_3 = P_2 + \frac{1}{4}P_1 = 6 + \frac{1}{4}(5) = 7.25°F$

$P_4 = P_3 + \frac{1}{4}P_2 = 7.25 + \frac{1}{4}(6) = 8.75°F$

$P_5 = P_4 + \frac{1}{4}P_3 = 8.75 + \frac{1}{4}(7.25) = 10.5625°F$

$\therefore$ 4.0, 5.0, 6.0, 7.25, 8.75, 10.5625

Question 7

Given the recurrence relation below,

$$s_1 = 2,\ s_n = s_{n-1}^2 - 1, \text{ for } n \geq 2.$$

a) Write a recursive algorithm to calculate the n^{th} term of the sequence.

S(n) {

if (n = 1)

return 2

return $s_n = s_{n-1}^2 - 1$

}

b) Trace the recursive steps to compute s_4. Show your working in a diagram.

S(4) {
n = 4
n $\neq$ 1
return s(3) * s(3) – 1
}

return 8^2 - 1, s(4) = 63

S(3) {
n = 3
n $\neq$ 1
return s(2) * s(2) – 1
}

return 3^2 - 1, s(3) = 8

S(2) {
n = 2
n $\neq$ 1
return s(1) * s(1) – 1
}

return 2^2 - 1, s(2) = 3

S(1) {
n = 1
return 2
}

$\therefore s_4 = 63$